A Water Resources Technical Publication

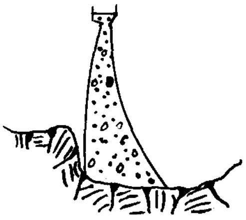
# DESIGN OF ARCH DAMS

UNITED STATES DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF RECLAMATION

# DESIGN OF ARCH DAMS



## DESIGN MANUAL FOR CONCRETE ARCH DAMS

A Water Resources Technical Publication

Denver, Colorado
1977

As the Nation's principal conservation agency, the Department of the Interior has responsibility for most of our nationally owned public lands and natural resources.

This includes fostering the wisest use of our land and water resources, protecting our fish and wildlife, preserving the environmental and cultural values of our national parks and historical places, and providing for the enjoyment of life through outdoor recreation.

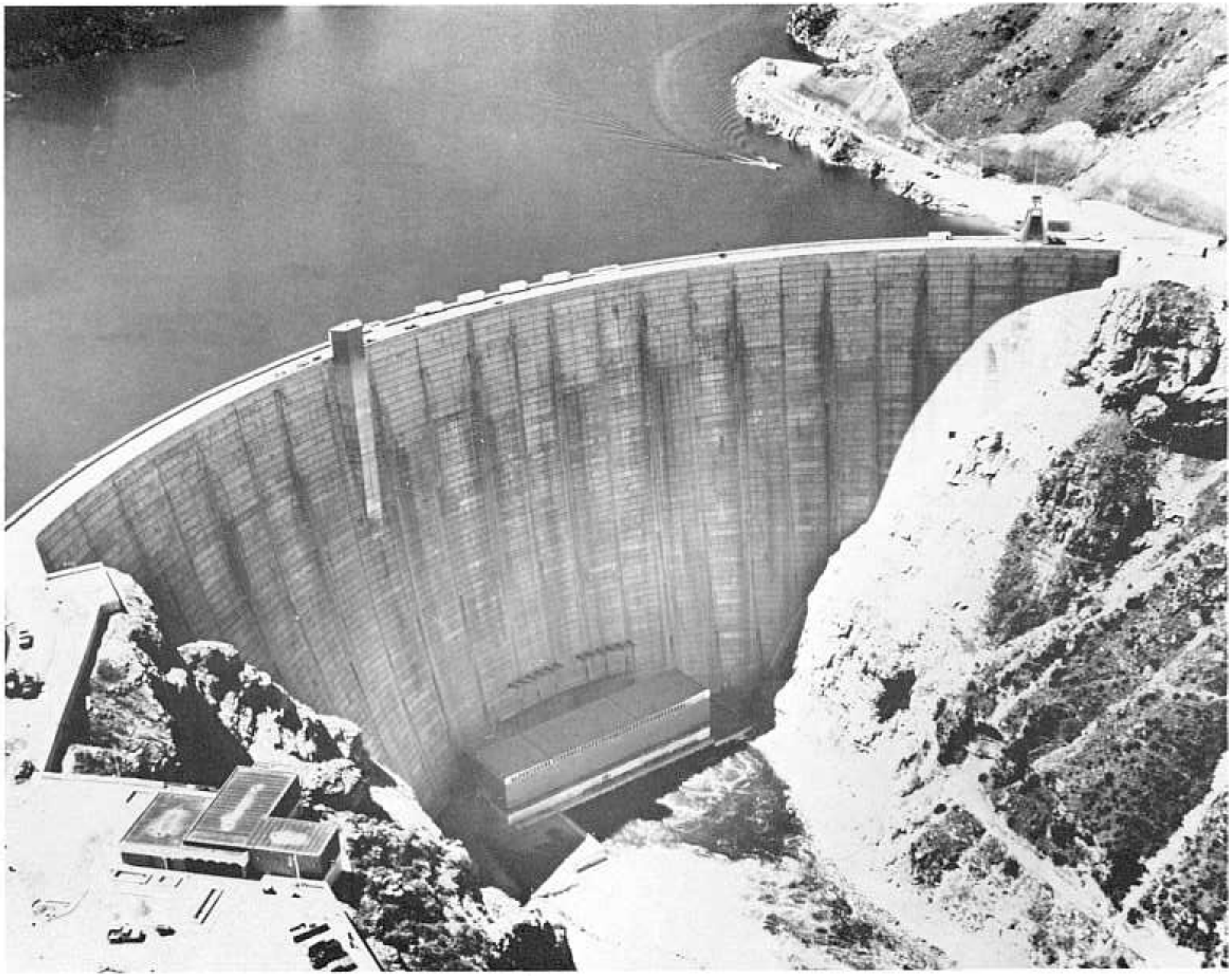
The Department assesses our energy and mineral resources and works to assure that their development is in the best interests of all our people.

The Department also has a major responsibility for American Indian reservation communities and for people who live in Island Territories under U.S. administration.

UNITED STATES GOVERNMENT PRINTING OFFICE

DENVER: 1977

**For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402,**
**and the Bureau of Reclamation, Engineering and Research Center, Attention: 922, P.O. Box 25007,**
**Denver Federal Center, Denver, Colorado 80225.**
**Stock Number 024-003-00118-0**



Frontispiece.—**Yellowtail Dam and Powerplant.** A459-600-137

# Preface

This manual presents instructions, examples, procedures, and standards for use in the design of concrete arch dams. It serves as a guide to sound engineering practices in the design of concrete arch dams and provides the technically trained, qualified design engineer with specialized and technical information that can be readily used in the design of such a dam.

The manual came into being because of the numerous requests made to the Bureau for its latest concepts on the design of concrete dams. A companion Bureau manual "Design of Gravity Dams" has been published.

Certain material in this book has been adapted from "Design of Small Dams." Although most of this text is related exclusively to the design of dams and appurtenant structures, it is important that the designer be familiar with *the purpose of the project of which the dam is a part, the considerations influencing its justification, and the manner of arriving at the size and type of structure to be built.* Factors which affect the selection of the type of dam and its location are discussed in chapter II, "Design Considerations." Chapter XV discusses the ecological and environmental considerations required in constructing a dam. The integrity of the structural design requires strict adherence to specifications for the concrete and to the practice of good workmanship in concrete production. Therefore, a summary of Bureau of Reclamation concrete construction practices or methods is included in chapter XIV, "Concrete Construction."

The manual should be of service to all concerned with the planning and designing of water storage projects, but it cannot relieve the agency or person using it of the responsibility for a safe and adequate design. The limitations stated in the design procedures should be heeded.

This book was prepared by engineers of the Bureau of Reclamation, U.S. Department of the Interior, at the Engineering and Research Center, Denver, Colorado, under the direction of H. G. Arthur,* Director of Design and Construction, and Dr. J. W. Hilf,* Chief, Division of Design. The text was written by members of the Concrete Dams Section, Hydraulic Structures Branch, Division of Design, except for Appendix L "Inflow Design Flood Studies," which was written by D. L. Miller,* of the Flood and Sedimentation Section, Water and Management Planning Branch, Division of Planning Coordination. Members of the Concrete Dams Section who made substantial contributions to the text include: M. D. Copen,* James Legas, E. A. Lindholm, G. S. Tarbox, L. H. Roehm, H. L. Boggs, C. W. Cozart, R. O. Atkinson, G. F. Bowles, M. A. Kramer, F. D. Reed,* C. L. Townsend,* J. S. Conrad,* R. R. Jones, C. W. Jones,* J. L. Von Thun, and J. T. Richardson.* The major editing and coordinating of the text was done by E. H. Larson,* and the final preparation of the text for printing was done by R. E. Haefele and J. M. Tilsley, all of the Publications Section, Technical Services and Publications Branch, Division of Engineering

---

*Retired

Support. The authors and editors wish to express their appreciation to the personnel in the General Services Branch for their contributions and to the technicians of Concrete Dams Section and Drafting Branch who prepared charts, tables, and drawings for use in the text.

The methods of design and analysis were developed through the efforts of dedicated Bureau engineers during the many years the Bureau of Reclamation has been designing and constructing concrete arch dams. Their efforts are gratefully acknowledged.

There are occasional references to proprietary materials or products in this publication. These must not be construed in any way as an endorsement of the Bureau of Reclamation since such endorsement cannot be made for proprietary products or processes of manufacturers or the services of commercial firms for advertising, publicity, sales, or other purposes.

# Contents

## Chapter I—Introduction

## Chapter II—Design Considerations

## Chapter II—Design Considerations—Continued

## Chapter II—Design Considerations—Continued

### K. MISCELLANEOUS CONSIDERATIONS

### L. BIBLIOGRAPHY

## Chapter III—Design Data and Criteria

### A. INTRODUCTION

### B. CONCRETE

### C. FOUNDATION

### D. LOADS

## Chapter III—Design Data and Criteria—Continued

### E. LOADING COMBINATIONS

### F. FACTORS OF SAFETY

### G. BIBLIOGRAPHY

## Chapter IV—Layout and Analysis

### A. LAYOUT OF DAM

## Chapter IV—Layout and Analysis—Continued

### A. LAYOUT OF DAM—Continued

### B. TRIAL-LOAD METHOD OF ANALYSIS

### *1. Theory of the Trial-Load Method*

### *2. Discussion of Method*

## Chapter IV—Layout and Analysis—Continued

### B. TRIAL-LOAD METHOD OF ANALYSIS—Continued

### *3. Preparation for Trial-Load Adjustments*

## Chapter IV—Layout and Analysis—Continued

### B. TRIAL-LOAD METHOD OF ANALYSIS—Continued

## Chapter IV—Layout and Analysis—Continued

### B. TRIAL-LOAD METHOD OF ANALYSIS—Continued

### *4. Procedure for Trial-Load Adjustments*

### *5. Calculation of Stresses*

## Chapter IV—Layout and Analysis—Continued

### B. TRIAL-LOAD METHOD OF ANALYSIS–Continued

### C. ARCH DAM STRESS ANALYSIS SYSTEM

## Chapter IV—Layout and Analysis—Continued

### C. ARCH DAM STRESS ANALYSIS SYSTEM—Continued

### D. DYNAMIC ANALYSIS

### E. THE FINITE ELEMENT METHOD OF ANALYSIS

#### *1. Two-Dimensional Finite Element Program*

## Chapter IV—Layout and Analysis—Continued

### E. THE FINITE ELEMENT METHOD OF ANALYSIS—Continued

## Chapter IV—Layout and Analysis—Continued

### F. FOUNDATION ANALYSIS—Continued

## Chapter V—River Diversion

### A. DIVERSION REQUIREMENTS

## Chapter V—River Diversion—Continued

### B. METHODS OF DIVERSION

### C. SPECIFICATIONS REQUIREMENTS

## Chapter VI—Foundation Treatment

### A. EXCAVATION

### B. GROUTING

### C. DRAINAGE

### D. BIBLIOGRAPHY

## Chapter VII—Temperature Control of Concrete

### A. INTRODUCTION

## Chapter VII—Temperature Control of Concrete—Continued

### A. INTRODUCTION—Continued

### B. METHODS OF TEMPERATURE CONTROL

### C. TEMPERATURE STUDIES

### D. DESIGN CONSIDERATIONS

## Chapter VII—Temperature Control of Concrete—Continued

### D. DESIGN CONSIDERATIONS–Continued

### E. CONSTRUCTION OPERATIONS

### F. BIBLIOGRAPHY

## Chapter VIII—Joints in Structures

## Chapter VIII—Joints in Structures—Continued

## Chapter IX—Spillways

### A. GENERAL DESIGN CONSIDERATIONS

### B. DESCRIPTION OF SPILLWAYS

## Chapter IX—Spillways—Continued

### B. DESCRIPTION OF SPILLWAYS—Continued

### C. CONTROL STRUCTURES

### D. HYDRAULICS OF DISCHARGE CHANNELS

## Chapter IX—Spillways—Continued

### D. HYDRAULICS OF DISCHARGE CHANNELS—Continued

### E. HYDRAULICS OF TERMINAL STRUCTURES

### F. HYDRAULICS OF MORNING GLORY (DROP INLET) SPILLWAYS

### G. STRUCTURAL DESIGN

## Chapter IX—Spillways—Continued

### H. BIBLIOGRAPHY

## Chapter X—Outlet Works and Power Outlets

### A. INTRODUCTION

### B. OUTLET WORKS OTHER THAN POWER OUTLETS

### *1. Hydraulic Design of Outlet Works*

## Chapter X—Outlet Works and Power Outlets—Continued

### B. OUTLET WORKS OTHER THAN POWER OUTLETS—Continued

### *2. Structural Design of Outlet Works*

### C. POWER OUTLETS

### D. BIBLIOGRAPHY

## Chapter XI—Galleries and Adits

## Chapter XII—Miscellaneous Appurtenances

## Chapter XIII—Structural Behavior Measurements

## Chapter XIII—Structural Behavior Measurements—Continued

## Chapter XIV—Concrete Construction

## Chapter XV—Ecological and Environmental Considerations

### A. INTRODUCTION

## Chapter XV—Ecological and Environmental Considerations—Continued

## Appendix A—Preliminary Layout and Volume Nomographs

## Appendix B—Cantilever Computations for Radial and Unit Loads

## Appendix B—Cantilever Computations for Radial and Unit Loads—Continued

## Appendix C—Arch Computations for Initial and Unit Loads

## Appendix C—Arch Computations for Initial and Unit Loads—Continued

### C. VARIABLE-THICKNESS ARCH WITH TRIANGULAR WEDGE ABUTMENTS

## Appendix D—Radial, Tangential, and Twist Adjustments

## Appendix D—Radial, Tangential, and Twist Adjustments—Continued

## Appendix E—Effects of Poisson's Ratio and Vertical Displacement

## Appendix F—Stress Computations

## Appendix F—Stress Computations—Continued

## Appendix G—Arch Dam Stress Analysis System (ADSAS)

## Appendix H—Tables for Arch Analysis

## Appendix I—Special Studies

## Appendix I—Special Studies—Continued

## Appendix J—Finite Element Method—Two-Dimensional Analysis

## Appendix K—Hydraulic Data and Tables

## Appendix K—Hydraulic Data and Tables—Continued

## Appendix L—Inflow Design Flood Studies

### A. COLLECTION OF HYDROLOGIC DATA FOR USE IN ESTIMATING FLOODFLOWS

### B. ANALYSES OF BASIC HYDROLOGIC DATA

## Appendix L—Inflow Design Flood Studies—Continued

### B. ANALYSES OF BASIC HYDROLOGIC DATA—Continued

## Appendix L—Inflow Design Flood Studies—Continued

### B. ANALYSES OF BASIC HYDROLOGIC DATA—Continued

### C. SYNTHETIC UNIT HYDROGRAPH

### D. STREAMFLOW ROUTING

### E. DESIGN STORM STUDIES

## Appendix L—Inflow Design Flood Studies—Continued

### E. DESIGN STORM STUDIES—Continued

### F. PRELIMINARY INFLOW DESIGN FLOOD, RAINFALL ONLY

## Appendix L—Inflow Design Flood Studies—Continued

### F. PRELIMINARY INFLOW DESIGN FLOOD, RAINFALL ONLY—Continued

### G. SNOWMELT RUNOFF CONTRIBUTIONS TO INFLOW DESIGN FLOODS

## Appendix L—Inflow Design Flood Studies—Continued

### G. SNOWMELT RUNOFF CONTRIBUTIONS TO INFLOW DESIGN FLOODS—Continued

### H. ENVELOPE CURVES

### I. STATISTICAL ANALYSES—ESTIMATES OF FREQUENCY OF OCCURRENCE OF FLOODS

### J. FINAL-TYPE INFLOW DESIGN FLOOD STUDIES

## Appendix L—Inflow Design Flood Studies—Continued

### K. BIBLIOGRAPHY

## Appendix M—Sample Specifications for Concrete

## Appendix M—Sample Specifications for Concrete—Continued

## Appendix M—Sample Specifications for Concrete—Continued

## Appendix N—Sample Specifications for Controlling Water and Air Pollution

# TABLES

# TABLES IN APPENDICES

# FIGURES

# FIGURES IN APPENDICES

# Introduction

1-1. *Scope.*—An arch dam is a solid concrete dam, curved upstream in plan. In addition to resisting part of the pressure of the reservoir by its own weight, it obtains a large measure of stability by transmitting the remainder of the water pressure and other loads by arch action into the canyon walls. Successful arch action is dependent on a unified monolithic structure, and special care must be taken in the construction of an arch dam to ensure that no structural discontinuities, such as open joints or cracks, exist at the time the structure assumes its waterload.

The complete design of a concrete arch dam includes not only the determination of the most efficient and economical proportions for the water impounding structure, but also the determination of the most suitable appurtenant structures for the control and release of the impounded water consistent with the purpose or function of the project. This manual presents the basic assumptions, design considerations, methods of analysis, and procedures used by designers within the Engineering and Research Center, Bureau of Reclamation, for the design of an arch dam and its appurtenant works. Discussions in this manual are limited to those dams on rock foundations.

1-2. *Classifications.*—Arch dams are generally classified as thin, medium-thick, or thick arch dams. A thin arch dam is defined as an arch dam with a $b/h$ ratio of 0.2 or less, where $b$ is the base thickness of the crown cantilever and $h$ is the structural height of the dam. A medium-thick arch dam is defined as an arch dam with a $b/h$ ratio between 0.2 and 0.3. A thick arch dam is an arch dam with a $b/h$ ratio of 0.3 or greater.

For statistical purposes, arch dams are classified with reference to their structural height. Dams up to 100 feet high are generally classified as low dams; dams from 100 to 300 feet high as medium-height dams; and dams over 300 feet high as high dams.

1-3. *General Dimensions.*—For uniformity within the Bureau of Reclamation, certain general dimensions have been established and are defined as follows:

The *structural height* of a concrete arch dam is defined as the difference in elevation between the top of the dam and the lowest point in the excavated foundation area, exclusive of such features as narrow fault zones. The top of the dam is the crown of the roadway if a roadway crosses the dam, or the level of the walkway if there is no roadway. Although curb and sidewalk may extend higher than the roadway, the level of the crown of the roadway is considered to be the top of the dam.

The *hydraulic height,* or height to which the water rises behind the structure, is the difference in elevation between the lowest point of the original streambed at the axis of the dam and the maximum controllable water surface.

The *length* of the dam is defined as the distance, measured along the axis of the dam at the level of the top of the main body of the dam or of the roadway surface on the crest, from abutment contact to abutment contact, exclusive of abutment spillway; provided that, if the spillway lies wholly within the dam and not in any area especially excavated for the spillway, the length is measured along the axis extended through the spillway to the abutment contacts.

The *volume* of a concrete arch dam should include the main body of the dam and all mass concrete appurtenances not separated from the dam by construction or contraction joints. Where a powerplant is constructed on the downstream toe of the dam, the limit of concrete in the dam should be taken as the downstream face projected to the general excavated foundation surface.

1-4. ***Arch Dam Definitions.***–Terminology used in the design and analysis of arch dams and definitions of the various types and parts of arch dams as used herein are as follows:

A *plan* is an orthographic projection on a horizontal plane, showing the main features of a dam and its appurtenant works with respect to the topography and available geological data. A plan should be oriented so that the direction of streamflow is toward the top or toward the right of the drawing.

The *reference plane* is a vertical plane which passes through the crown cantilever and the axis radius center.

A *profile* is a developed elevation of the intersection of a dam with the original ground surface, rock surface, or excavation surface along the axis of the dam, the upstream face, the downstream face, or other designated location. Profiles are commonly classified as U-shape or V-shape, with variations between these two classifications.

A *section* is a representation of a dam as it would appear if cut by a plane. An *arch section* is taken horizontally through the dam. A *cantilever section* is a vertical section taken normal to the extrados, usually oriented with the reservoir on the left.

An *arch element, or arch,* is a portion of a dam bounded by two horizontal planes 1 foot apart. For purposes of analysis the edges of the elements are assumed to be vertical.

A *section of an arch* is that part of the arch which is selected for ease of computation. The section must have a constant extrados radius but may be variable in thickness.

A *voussoir* is that smaller segment of a section of an arch which, for ease of computation, is assumed to have constant thickness.

A *cantilever element, or cantilever,* is that portion of a dam which is contained within two vertical planes radial to the extrados and spaced 1 foot apart at the axis. Cantilevers of arch dams other than the constant-center type are bounded by warped surfaces owing to the fact that the locations of arch extrados centers vary with the elevations of the arches. The crown cantilever is located at the point of maximum depth.

The *extrados* is the curved upstream surface of horizontal arch elements.

The *intrados* is the curved downstream surface of horizontal arch elements.

The *thickness* of a dam at any point on the arch centerline is the length of a line along an extrados radius from the upstream to the downstream face which passes through the point.

An *arch centerline* is the locus of all midpoints of the thickness of an arch section.

The *axis* of a dam is a vertical reference surface curved horizontally and usually defined by the upstream edge of the top of the dam.

The *length of an arch* is the length along a curve which is concentric with the extrados and passes through the midpoint of the arch thickness at the crown.

The *height of a cantilever* is the vertical distance between the base elevation of the cantilever section and the top of the dam, which may or may not be the top arch.

The *central angle of an arch* is the angle bounded by lines radiating from the arch extrados center to points of intersection of the arch centerline with the arch abutments.

The *abutment of an arch element* is the surface, at either end of the arch, which contacts the rock of the canyon wall. Arch loads are transferred through the arch abutments into the canyon walls.

A *fillet* is an increase in thickness of a dam at and near the abutments of the arches. Fillets are usually placed at the downstream face, but they may also be used at the upstream face.

*Single curvature* relates to a dam which is curved in plan only.

*Double curvature* relates to a dam which is curved in plan and in elevation, with undercutting of the heel, and in most instances, downstream overhang near the crest.

The *line of centers* is a line in space which is the loci of centers for circular arcs used to describe a face of the dam or a portion thereof. The number of lines of centers necessary to describe an arch dam varies from one for a circular dam with uniform-thickness arches to six for a three-centered dam with variable-thickness arches. Polycentered dams may require more lines of centers if more than three arch segments are used to describe each face of the arch elements. Lines of centers are described with respect to a reference plane.

A *constant-center dam* is one whose arch centers for the upstream face and the downstream face are coincident with the axis center at all elevations. A profile of these centers is a vertical straight line. The arches are of uniform thickness, except as modified by the use of fillets. All cantilevers are of identical shape, varying only in base elevation, except in the case of a constant-center dam with fillets. In the latter instance, increases in arch thickness within the region of the fillets are also evident as increases in cantilever thickness.

*Variable-center dams* include all classes of arch dams whose arch centers for either or both the upstream face and the downstream face vary in location with respect to the axis center at different elevations. The arches may be of uniform thickness or variable thickness, with or without fillets. Cantilevers vary in shape and thickness at different locations within the dam according to the difference in curvature between the arches, and according to the kind of arches used. Single-centered, two-centered, polycentered, three-centered, and constant-angle dams are variable-center dams.

*Single-centered dams* have one set of lines of centers on the reference plane. Both sides of each face are described by the same circular arc. This is the usual type of arch dam for U-shaped or V-shaped canyons with a symmetrical or near symmetrical profile having a crest length to height ratio of about 2 or less.

*Two-centered dams* have two sets of lines of centers, one set for each side of the dam. The two sets are coplaner on the reference plane. Each face of an arch element is described by two circular arcs compounded at the reference plane. Two-centered dams are usually designed for sites with pronounced nonsymmetry.

*Polycentered or three-centered dams* are variable-center dams in which the location of centers and radii associated with them also vary horizontally to produce dams with elliptically shaped or multicentered arch elements. For three-centered or elliptically shaped structures, three centers are used on the extrados and three on the intrados curves for each arch element. Usually, the shorter radius curves are used in the central parts of the arch and the longer radius curves near the abutments. Polycentered dams may use more centers to vary the horizontal curvature. Arch elements may be uniform or variable in thickness.

A *constant-angle dam* is a special type of variable-center dam in which the central angles of the arches are of approximately the same magnitude at all elevations.

*Abutment pads* are structures between arch dams and their foundations. They are used to reduce load intensity and distribute it more uniformly on the foundation rock, reduce effects of foundation irregularities, and produce symmetry for the arch dam. The pads, usually trapezoidal in cross section, may be monolithic with the dam or separated from it by peripheral joints.

$R_{axis}$ is the radius to the axis of the dam.

The *upstream projection* is the horizontal distance from the extrados to the axis on a line normal to the extrados.

The *downstream projection* is the horizontal distance from the intrados to the axis on a line normal to the extrados.

The *crest* is the top of the dam.

## «Chapter II

# Design Considerations

## A. LOCAL CONDITIONS

2-1. *General.*—Although not of immediate concern to the designer of a dam and its appurtenances, the early collection of data on local conditions which will eventually relate to the design, specifications, and construction stages is advisable. Local conditions are not only needed to estimate construction costs, but may be of benefit when considering alternative designs and methods of construction. Some of these local conditions will also be used to determine the extent of the project designs, including such items as access roads, bridges, and construction camps.

2-2. *Data to be Submitted.*—Local conditions should be described and submitted as part of the design data as follows:

(1) The approximate distance from the nearest railroad shipping terminal to the structure site; load restrictions and physical inadequacies of existing roads and structures and an estimate of improvements to accommodate construction hauling; an estimate of length and major structures for access roads; and possible alternative means for delivering construction materials and equipment to the site.

(2) Local freight or trucking facilities and rates.

(3) Availability of housing and other facilities in the nearest towns; requirements for a construction camp; and need for permanent buildings for operating personnel.

(4) Availability or accessibility of public facilities or utilities such as water supply, sewage disposal, electric power for construction purposes, and telephone service.

(5) Local labor pool and general occupational fields existing in the area.

## B. MAPS AND PHOTOGRAPHS

2-3. *General.*—Maps and photographs are of prime importance in the planning and design of a concrete dam and its appurtenant works. From these data an evaluation of alternative layouts can be made preparatory to determining the final location of the dam, the type and location of its appurtenant works, and the need for restoration and/or development of the area.

2-4. *Survey Control.*—Permanent horizontal and vertical survey control should be established at the earliest possible time. A grid coordinate system for horizontal control should be established with the origin located so that all of the features (including borrow areas) at a major structure will be in one quadrant. The coordinate system should be related to a State or National coordinate system, if practicable. All previous survey work, including topography and location and ground surface elevation of subsurface exploration holes, should be corrected to agree with the permanent control system; and all subsequent survey work, including location and ground surface elevations, should be based on the permanent control.

2-5. ***Data to be Submitted.***—A general area map should be obtained locating the general area within the State, together with county and township lines. This location map should show existing towns, highways, roads, railroads, and shipping points. A vicinity map should also be obtained using such a scale as to show details on the following:

(1) The structure site and alternative sites.

(2) Public utilities.

(3) Stream gaging stations.

(4) Existing manmade works affected by the proposed development.

(5) Locations of potential construction access roads, sites for a Government camp and permanent housing area, and sites for the contractor's camp and construction facilities.

(6) Sources of natural construction materials.

(7) Existing or potential areas or features having a bearing on the design, construction, operation, or management of project features such as recreational areas, fish and wildlife areas, building areas, and areas of ecological interest.

The topography of the areas where the dam and any of its appurtenant works are to be located is of prime concern to the designer. Topography should be submitted covering an area sufficient to accommodate all possible arrangements of dam, spillway, outlet works, diversion works, construction access, and other facilities; and should be based on the permanently established horizontal and vertical survey control. A scale of 1 inch equals 50 feet and a contour interval of 5 feet will normally be adequate. The topography should extend a minimum of 500 feet upstream and downstream from the estimated positions of the heel and toe of the dam and a sufficient distance beyond each end of the dam crest to include road approaches. The topography should also cover the areas for approach and exit channels for the spillway. The topography should extend to an elevation sufficiently high to permit layouts of access roads, spillway structures, and visitor facilities.

Ground and aerial photographs are beneficial and can be used in a number of ways. Their principal value is to present the latest data relating to the site in such detail as to show conditions affecting the designs. Close-up ground photographs, for example, will often give an excellent presentation of local geology to supplement that obtained from a topographic map. Where modifications are to be made to a partially completed structure, such photographs will show as-constructed details which may not show on any drawings.

## C. HYDROLOGIC DATA

2-6. ***Data to be Submitted.***—In order to determine the potential of a site for storing water, generating power, or other beneficial use, a thorough study of hydrologic conditions must be made. Necessary hydrologic data will include the following:

(1) Streamflow records, including daily discharges, monthly volumes, and momentary peaks.

(2) Streamflow and reservoir yield.

(3) Project water requirements, including allowances for irrigation and power, conveyance losses, reuse of return flows, and stream releases for fish; and dead storage requirements for power, recreation, fish and wildlife, etc.

(4) Flood studies, including inflow design floods and floods to be expected during periods of construction.

(5) Sedimentation and water quality studies, including sediment measurements, analysis of dissolved solids, etc.

(6) Data on ground-water tables in the vicinity of the reservoir and damsite.

(7) Water rights, including interstate compacts and international treaty effects,

and contractual agreements with local districts, power companies, and individuals for subordination of rights, etc.

Past records should be used as a basis for predicting conditions which will develop in the future. Data relating to streamflow may be obtained from the following sources:

(1) Water supply papers—U.S. Department of the Interior, Geological Survey, Water Resources Division.
(2) Reports of state engineers.
(3) Annual reports—International Boundary and Water Commission, United States and Mexico.
(4) Annual reports—various interstate compact commissions.
(5) Water right filings, permits—state engineers, county recorders.
(6) Water right decrees—district courts.

Data on sedimentation may be obtained from:

(1) Water supply papers—U.S. Department of the Interior, Geological Survey, Quality of Water Branch.
(2) Reports—U.S. Department of the Interior, Bureau of Reclamation; and U.S. Department of Agriculture, Soil Conservation Service.

Data for determining the quality of water may be obtained from:

(1) Water supply papers—U.S. Department of the Interior, Geological Survey, Quality of Water Branch.
(2) Reports—U.S. Department of Health, Education, and Welfare, Public Health Service, and Environmental Protection Agency, Federal Water Control Administration.
(3) Reports—state public health departments.

2-7. ***Hydrologic Investigations.***—Hydrologic investigations which may be required for project studies include the determination of the following: yield of streamflow, reservoir yield, water requirements for project purposes, sediment which will be deposited in the reservoir, floodflows, and ground-water conditions.

The most accurate estimate possible must be prepared of the portion of the streamflow yield that is surplus to senior water rights, as the basis of the justifiable storage. Reservoir storage will supplement natural yield of streamflow during low-water periods. Safe reservoir yield will be the quantity of water which can be delivered on a firm basis through a critical low-water period with a given reservoir capacity. Reservoir capacities and safe reservoir yields may be prepared from mass curves of natural streamflow yield as related to fixed water demands or from detailed reservoir operation studies, depending upon the study detail which is justified. Reservoir evaporation and other incidental losses should be accounted for before computation of net reservoir yields.

The critical low-water period may be one drought year or a series of dry years during the period of recorded water history. Water shortages should not be contemplated when considering municipal and industrial water use. For other uses, such as irrigation, it is usually permissible to assume tolerable water shortages during infrequent drought periods and thereby increase water use during normal periods with consequent greater project development. What would constitute a tolerable irrigation water shortage will depend upon local conditions and the crops to be irrigated. If the problem is complex, the consulting advice of an experienced hydrologist should be secured.

The annual rate at which sediment will be deposited in the reservoir should be ascertained to ensure that sufficient sediment storage is provided in the reservoir so that the useful functions of the reservoir will not be impaired by sediment deposition within the useful life of the project or the period of economic analysis, say 50 to 100 years. The expected elevation of the sediment deposition may also influence the design of the outlet works, necessitating a type of design which will permit raising the intake of the outlet works as the sediment is deposited.

Water requirements should be determined for all purposes contemplated in the project. For irrigation, consideration should be given to climatic conditions, soil types, type of crops, crop distribution, irrigation efficiency and conveyance losses, and reuse of return flows. For municipal and industrial water supplies, the anticipated growth of demand over the life of the project must be considered. For power generation, the factors to be considered are load requirements and anticipated load growth.

Knowledge of consumptive uses is important in the design and operation of a large irrigation project, and especially for river systems as a whole. However, of equal and perhaps more importance to an individual farm or project is the efficiency with which the water is conveyed, distributed, and applied. The losses incidental to application on the farm and the conveyance system losses and operational waste may, in many instances, exceed the water required by the growing crops. In actual operation, the amount of loss is largely a matter of economics. In areas where water is not plentiful and high-value crops are grown, the use of pipe or lined conveyance systems and costly land preparation or sprinkler systems can be afforded to reduce losses to a minimum. A part of the lost water may be consumed nonbeneficially by nonproductive areas adjacent to the irrigated land or in drainage channels. Usually most of this water eventually returns to a surface stream or drain and is referred to as return flow.

In planning irrigation projects, two consumptive use values are developed. One, composed of monthly or seasonal values, is used with an adjustment for effective precipitation and anticipated losses mentioned above to determine the total water requirement for appraising the adequacy of the total water supply and determining reservoir storage requirements. The other, a peak use rate, is used for sizing the canal and lateral system.

Evapotranspiration, commonly called consumptive use, is defined as the sum of evaporation from plant and soil surfaces and transpiration from plants and is usually expressed in terms of depth (volume per unit area). Crop consumptive use is equal to evapotranspiration plus water required for plant tissue, but the two are usually considered the same. Predictions or estimates of evapotranspiration are basic parameters for the engineer or agronomist involved in planning and developing water resources. Estimates of evapotranspiration are also used in assessing the disposition of water in an irrigation project, evaluating the irrigation water-management efficiency, and projecting drainage requirements.

Reliable rational equations are available for estimating evapotranspiration when basic meteorological parameters such as net radiation, vapor pressure and temperature gradients, wind speed at a prescribed elevation above the crops or over a standard surface, and soil heat flux are available. When information on these parameters is not available, which is the usual case, recourse is made to empirical methods. Numerous equations, both empirical and partially based on theory, have been developed for estimating potential evapotranspiration. Estimates from these methods are generally accepted as being of suitable accuracy for planning and developing water resources. Probably the methods most widely used at this time are the Blaney-Criddle method shown in reference [1][1] and the Soil Conservation Service adaptation of the Blaney-Criddle method, shown in reference [2].

A more recent method, nearly developed sufficiently for general usage, is the Jensen-Haise solar radiation method shown in reference [3]. In general terms, these methods utilize climatic data to estimate a climatic index. Then coefficients, reflecting the stage of growth of individual crops and their actual water requirement in relationship to the climatic index, are used to estimate the consumptive use requirements for selected crops.

Project studies must include estimates of floodflows, as these are essential to the determination of the spillway capacity. Consideration should also be given to annual minimum and mean discharges and to the magnitudes of relatively common floods having 20-, 10-, and 4-percent chances of occurrence, as this knowledge is essential for construction

---

[1] Numbers in brackets refer to items in the bibliography, sec. 2-30.

purposes such as diverting the stream, providing cofferdam protection, and scheduling operations. Methods of arriving at estimates of floodflows are discussed in appendix L. If the feasibility studies are relatively complete, the flood determination may be sufficient for design purposes. If, however, floodflows have been computed for purposes of the feasibility study without making full use of all available data, these studies should be carefully reviewed and extended in detail before the actual design of the structure is undertaken. Frequently, new data on storms, floods, and droughts become available between the time the feasibility studies are made and construction starts. Where such changes are significant, the flood studies should be revised and brought up to date.

Project studies should also include a ground-water study, which may be limited largely to determining the effect of ground water on construction methods. However, some ground-water situations may have an important bearing on the choice of the type of dam to be constructed and on the estimates of the cost of foundations. Important ground-water information sometimes can be obtained in connection with subsurface investigations of foundation conditions.

As soon as a project appears to be feasible, steps should be taken in accordance with State water laws to initiate a project water right.

## D. RESERVOIR CAPACITY AND OPERATION

2-8. ***General.***—Dam designs and reservoir operating criteria are related to the reservoir capacity and anticipated reservoir operations. The loads and loading combinations to be applied to the dam are derived from the several standard reservoir water surface elevations. Reservoir operations are an important consideration in the safety of the structure and should not be overlooked in the design. Similarly, the reservoir capacity and reservoir operations are used to properly size the spillway and outlet works. The reservoir capacity is a major factor in flood routings and may determine the size and crest elevation of the spillway. The reservoir operation and reservoir capacity allocations will determine the location and size of outlet works for the controlled release of water for downstream requirements and flood control.

Reservoir area-capacity tables should be prepared before the final designs and specifications are completed. These area-capacity tables should be based upon the best available topographic data and should be the official document for final design and administrative purposes until superseded by a reservoir resurvey. Electronic computer programs are an aid in preparation of reservoir area and capacity data. These computers enable the designer to quickly have the best results obtainable from the original field data.

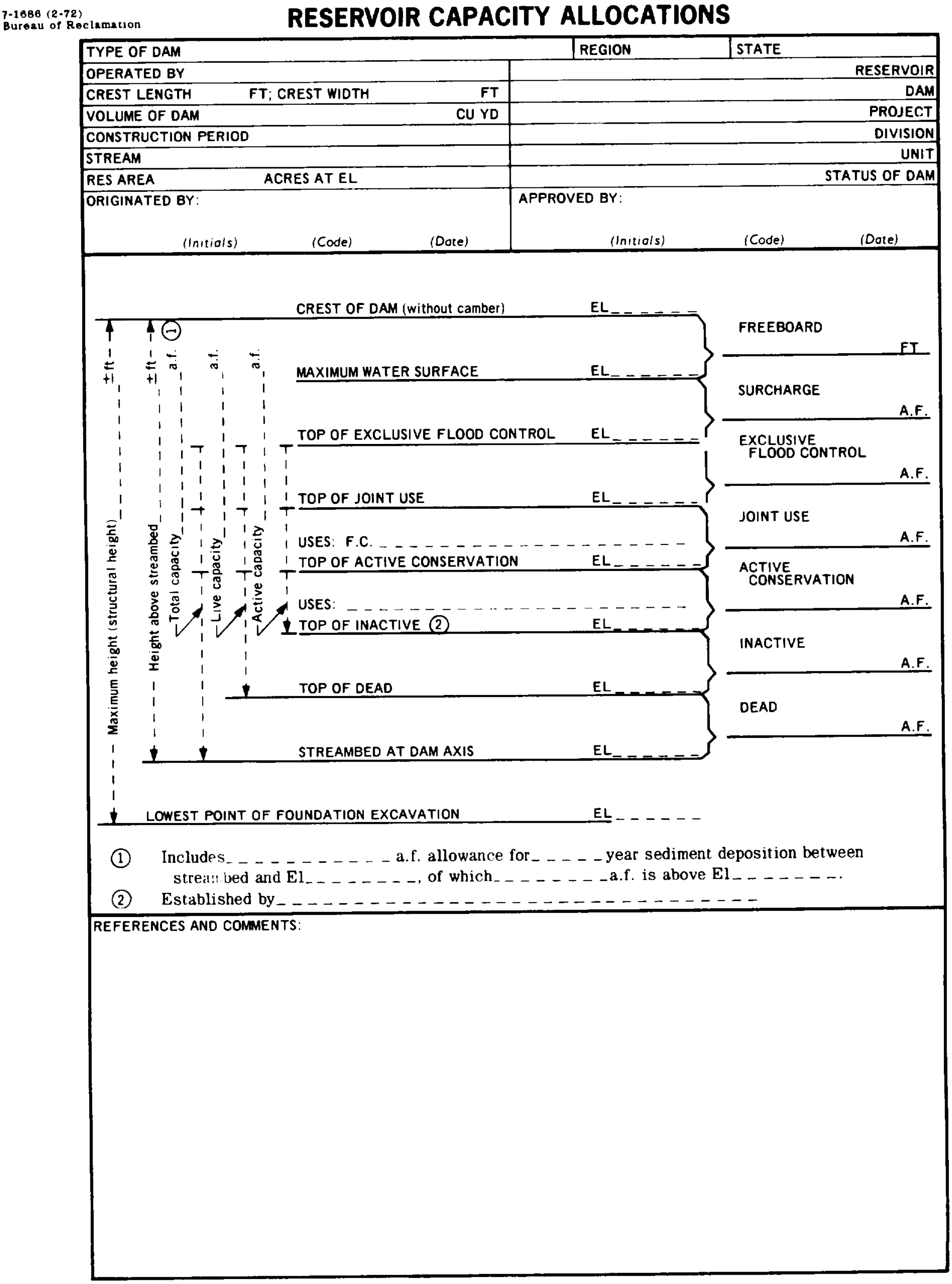
2-9. ***Reservoir Allocation Definitions.***—To ensure uniform reporting of data for design and construction, the following standard designations of water surface elevations and reservoir capacity allocations are used by the Bureau of Reclamation:

(a) *General.* Dam design and reservoir operation utilize reservoir capacity and water surface elevation data. To ensure uniformly in the establishment, use, and publication of these data, the following standard definitions of water surface elevations and reservoir capacities shall be used. Reservoir capacity as used here is exclusive of bank storage capacity.

(b) *Water Surface Elevation Definitions.* (Refer to fig. 2-1.)

(1) *Maximum Water Surface* is the highest acceptable water surface elevation with all factors affecting the safety of the structure considered. Normally, it is the highest water surface elevation resulting from a computed routing of the inflow design flood through the reservoir on the basis of established operating criteria. It is the top of surcharge capacity.

(2) *Top of Exclusive Flood Control Capacity* is the reservoir water surface



7-1686 (2-72)
Bureau of Reclamation

**RESERVOIR CAPACITY ALLOCATIONS**

TYPE OF DAM
REGION
STATE
OPERATED BY
RESERVOIR
CREST LENGTH FT; CREST WIDTH FT
DAM
VOLUME OF DAM CU YD
PROJECT
CONSTRUCTION PERIOD
DIVISION
STREAM
UNIT
RES AREA ACRES AT EL
STATUS OF DAM
ORIGINATED BY: *(Initials)* *(Code)* *(Date)*
APPROVED BY: *(Initials)* *(Code)* *(Date)*

CREST OF DAM (without camber) EL_ _ _ _ _ _
FREEBOARD ___FT
MAXIMUM WATER SURFACE EL_ _ _ _ _ _
SURCHARGE ___A.F.
TOP OF EXCLUSIVE FLOOD CONTROL EL_ _ _ _ _ _
EXCLUSIVE FLOOD CONTROL ___A.F.
TOP OF JOINT USE EL_ _ _ _ _ _
JOINT USE ___A.F.
USES: F.C. _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
TOP OF ACTIVE CONSERVATION EL_ _ _ _ _ _
ACTIVE CONSERVATION ___A.F.
USES: _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
TOP OF INACTIVE ② EL_ _ _ _ _ _
INACTIVE ___A.F.
TOP OF DEAD EL_ _ _ _ _ _
DEAD ___A.F.
STREAMBED AT DAM AXIS EL_ _ _ _ _ _
LOWEST POINT OF FOUNDATION EXCAVATION EL_ _ _ _ _ _

① Includes_ _ _ _ _ _ _ _ _ _ _ a.f. allowance for_ _ _ _ _year sediment deposition between streambed and El_ _ _ _ _ _ _ _, of which_ _ _ _ _ _ _ _a.f. is above El_ _ _ _ _ _ _ _.
② Established by_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

REFERENCES AND COMMENTS:

*Figure 2-1.* **Reservoir capacity allocation sheet used by Bureau of Reclamation.**

elevation at the top of the reservoir capacity allocated to exclusive use for regulation of flood inflows to reduce damage downstream.

(3) *Maximum Controllable Water Surface Elevation* is the highest reservoir water surface elevation at which gravity flows from the reservoir can be completely shut off.

(4) *Top of Joint Use Capacity* is the reservoir water surface elevation at the top of the reservoir capacity allocated to joint use, i.e., flood control and conservation purposes.

(5) *Top of Active Conservation Capacity* is the reservoir water surface elevation at the top of the capacity allocated to the storage of water for conservation purposes only.

(6) *Top of Inactive Capacity* is the reservoir water surface elevation below which the reservoir will not be evacuated under normal conditions.

(7) *Top of Dead Capacity* is the lowest elevation in the reservoir from which water can be drawn by gravity.

(8) *Streambed at the Dam Axis* is the elevation of the lowest point in the streambed at the axis of the dam prior to construction. This elevation normally defines the zero for the area-capacity tables.

(c) *Capacity Definitions.*

(1) *Surcharge Capacity* is reservoir capacity provided for use in passing the inflow design flood through the reservoir. It is the reservoir capacity between the maximum water surface elevation and the highest of the following elevations:

a. Top of exclusive flood control capacity.
b. Top of joint use capacity.
c. Top of active conservation capacity.

(2) *Total Capacity* is the reservoir capacity below the highest of the elevations representing the top of exclusive flood control capacity, the top of joint use capacity, or the top of active conservation capacity. In the case of a natural lake which has been enlarged, the total capacity includes the dead capacity of the lake. If the dead capacity of the natural lake has not been measured, specific mention of this fact should be made. Total capacity is used to express the total quantity of water which can be impounded and is exclusive of surcharge capacity.

(3) *Live Capacity* is that part of the total capacity from which water can be withdrawn by gravity. It is equal to the total capacity less the dead capacity.

(4) *Active Capacity* is the reservoir capacity normally usable for storage and regulation of reservoir inflows to meet established reservoir operating requirements. Active capacity extends from the highest of the top of exclusive flood control capacity, the top of joint use capacity, or the top of active conservation capacity, to the top of inactive capacity. It is the total capacity less the sum of the inactive and dead capacities.

(5) *Exclusive Flood Control Capacity* is the reservoir capacity assigned to the sole purpose of regulating flood inflows to reduce flood damage downstream. In some instances the top of exclusive flood control capacity is above the maximum controllable water surface elevation.

(6) *Joint Use Capacity* is the reservoir capacity assigned to flood control purposes during certain periods of the year and to conservation purposes during other periods of the year.

(7) *Active Conservation Capacity* is the reservoir capacity assigned to regulate reservoir inflow for irrigation, power, municipal and industrial use, fish and wildlife, navigation, recreation, water quality, and other purposes. It does not include exclusive flood control or joint use capacity. The active conservation capacity extends from the top of the active conservation capacity to the top of the inactive capacity.

(8) *Inactive Capacity* is the reservoir capacity exclusive of and above the dead capacity from which the stored water is normally not available because of operating agreements or physical restrictions. Under abnormal conditions, such as a shortage of

water or a requirement for structural repairs, water may be evacuated from this space after obtaining proper authorization. The highest applicable water surface elevation described below usually determines the top of inactive capacity.

a. The lowest water surface elevation at which the planned minimum rate of release for water supply purposes can be made to canals, conduits, the river, or other downstream conveyance. This elevation is normally established during the planning and design phases and is the elevation at the end of extreme drawdown periods.

b. The established minimum water surface elevation for fish and wildlife purposes.

c. The established minimum water surface elevation for recreation purposes.

d. The minimum water surface elevation as set forth in compacts and/or agreements with political subdivisions.

e. The minimum water surface elevation at which the powerplant is designed to operate.

f. The minimum water surface elevation to which the reservoir can be drawn using established operating procedures without endangering the dam, appurtenant structures, or reservoir shoreline.

g. The minimum water surface elevation or the top of inactive capacity established by legislative action.

(9) *Dead Capacity* is the reservoir capacity from which stored water cannot be evacuated by gravity.

2-10. ***Data to be Submitted.***—To complete the designs of the dam and its appurtenant works, the following reservoir design data should be submitted:

(1) Area-capacity curves and/or tables computed to an elevation high enough to allow for storage of the spillway design flood.

(2) A topographic map of the reservoir site prepared to an appropriate scale.

(3) Geological information pertinent to reservoir tightness, locations of mines or mining claims, locations of oil and natural gas wells.

(4) Completed reservoir storage allocations and corresponding elevations.

(5) Required outlet capacities for respective reservoir water surfaces and any required sill elevations. Give type and purpose of reservoir releases and the time of year these must be made. Include minimum releases required.

(6) Annual periodic fluctuations of reservoir levels shown by tables or charts summarizing reservoir operation studies.

(7) Method of reservoir operation for flood control and maximum permissible releases consistent with safe channel capacity.

(8) Physical, economic, or legal limitations to maximum reservoir water surface.

(9) Anticipated occurrence and amounts of ice (thickness) and floating debris, and possible effect on reservoir outlets, spillway, and other appurtenances.

(10) Extent of anticipated wave action, including a discussion of wind fetch.

(11) Where maintenance of flow into existing canals is required, determine maximum and probable carrying capacity of such canal, and time of year when canals are used.

## E. CLIMATIC EFFECTS

2-11. ***General.***—The climatic conditions which are to be encountered at the site affect the design and construction of the dam. Temperature loads may be one of the major loads imposed on a dam, depending upon its height and configuration. Measures which should be employed during the construction period to prevent cracking of concrete must be related to the ambient temperatures encountered at the site. Construction methods and procedures may also be dependent upon the weather conditions, since weather affects the rate of construction and the overall construction schedule. Accessibility of the site during periods of inclement weather affects the construction schedule and should be investigated.

2-12. ***Data to be Submitted.***—The following data on climatic conditions should be submitted as part of the design data:

(1) Weather Service records of mean monthly maximum, mean monthly minimum, and mean monthly air temperatures for the nearest station to the site. Data on river water temperatures at various times of the year should also be obtained.

(2) Daily readings of maximum and minimum air temperatures should be submitted as soon as a station can be established at the site.

(3) Daily readings of maximum and minimum river water temperatures should be submitted as soon as a station can be established at the site.

(4) Amount and annual variance in rainfall and snowfall.

(5) Wind velocities and prevailing direction.

## F. CONSTRUCTION MATERIALS

2-13. ***Concrete Aggregates.***—The construction of a concrete dam requires the availability of suitable aggregates in sufficient quantity to construct the dam and its appurtenant structures. Aggregates are usually processed from natural deposits of sand, gravel, and cobbles. However, if it is more practical, they may be crushed from suitable rock. For small dams, the aggregates may be obtained from existing commercial sources. If the aggregates are obtained from borrow pits or rock quarries, provisions should be made to landscape and otherwise restore the areas to minimize adverse environmental effects. If aggregates are available from the reservoir area, particularly below minimum water surface, their adverse effects would be minimized. However, any early storage in the reservoir, prior to completion of the dam, may rule out the use of aggregate sources in the reservoir.

2-14. ***Water for Construction Purposes.***—For large rivers, this item is relatively unimportant except for quality of the water. For small streams and offstream reservoirs, water for construction purposes may be difficult to obtain. An adequate supply of water for construction purposes such as washing aggregates and cooling and batching concrete should be assured to the contractor, and the water rights should be obtained for him. If necessary to use ground water, information on probable sources and yields should be obtained. Information on locations and yields of existing wells in the vicinity, restrictions if any on use of ground water, and necessary permits should also be obtained.

2-15. ***Data to be Submitted.***—In addition to the data on concrete aggregates and water for construction purposes, the following data on construction materials should be obtained:

(1) An earth materials report containing information on those potential sources of soils, sand, and gravel which could be used for backfill and bedding materials.

(2) Information on riprap for protection of slopes.

(3) Information on sources and character of acceptable road surfacing materials, if required.

(4) References to results of sampling, testing, and analysis of construction materials.

(5) Photographs of sources of construction materials.

(6) Statement of availability of lumber for structural work.

## G. SITE SELECTION

2-16. ***General.***—A water resources development project is designed to perform a certain function and to serve a particular area. Once the purpose and the service area are defined, a preliminary site selection can be made.

Following the determination of the adequacy of the water supply, as discussed in subchapter C, the two most important considerations in selecting a damsite are: (1) the site must be adequate to support the dam and the appurtenance structures, and (2) the area upstream from the site must be suitable for a reservoir. There are often several suitable sites along a river where the dam can be located.

The site finally selected should be that where the dam and reservoir can be most economically constructed with a minimum of interference with local conditions and still serve their intended purpose. An experienced engineer can usually eliminate some of the sites from further consideration. Cost estimates may be required to determine which of the remaining sites will provide the most economical structure.

2-17. ***Factors in Site Selection.***—In selecting a damsite, the following should be considered:

| | |
|---|---|
| Topography | A narrow site will be favorable to an arch dam; however, the abutments must be massive enough to accept the arch loads. In addition the topography must be such that the lines of thrust are directed into the abutments at favorable angles. |
| Geology | The foundation of the dam should be relatively free of major faults and shears. If these are present, they may require expensive foundation treatment to assure an adequate foundation. |
| Appurtenant Structures | While the cost of these structures is usually less than the cost of the dam, economy in design may be obtained by considering their effect at the time of site selection. For example, if a river has a large flow, a large spillway and diversion works will be required. Selecting a site which will better accommodate these appurtenances will reduce the overall cost. |
| Local Conditions | Some sites may have roads, railroads, powerlines, canals, etc., which have to be relocated, thus increasing the overall costs. |
| Access | Accessibility of the site has a very definite effect on the total cost. Difficult access may require the construction of expensive roads. An area suitable for the contractor's plant and equipment near the site will reduce the contractor's construction costs. |

## H. CONFIGURATION OF DAM

2-18. *General.*—The shape and curvature of a dam and its contact with the foundation are extremely important in providing stability and favorable stress conditions. Although stability may be improved, and stresses decreased by adding to the thickness of the dam, this method is not generally economical. The desired results can be achieved by proper shaping and the use of both horizontal and vertical curvature. An arch, for example, transfers its load to the abutment by thrust and shear, thus reducing the bending stress and adding load carrying capacity as compared to a flat beam.

For an arch dam in a relatively narrow canyon, it is advantageous to use a high degree of horizontal curvature, consistent with requirements for site topography and vertical shaping. More equitable stress distributions may be obtained in dams designed for the wider sites by using polycentered or elliptically shaped arches. Vertical curvature and shaping may be used to improve the distribution of vertical stresses by changing the dead load moments. Where necessary to reduce stresses in the rock, the thickness of the dam near the foundation can be increased by using a fillet dam, a variable-thickness dam, or abutment pads.

In general, abrupt changes in the contact between the dam and the canyon profile should be avoided. Such irregularities will induce stress concentrations.

For a full discussion of the design of an arch dam, see chapters III and IV.

## I. FOUNDATION INVESTIGATIONS

2-19. ***Purpose.***—The purpose of a foundation investigation is to provide the data necessary to properly evaluate a foundation. A properly sequenced and organized foundation investigation will provide all the data necessary to evaluate and analyze the foundation at any stage of investigation.

2-20. ***Field Investigations.***—The collection, study, and evaluation of foundation data is a continuing program from the time of the appraisal investigation to the completion of construction. The data collection begins with an appraisal and continues on a more detailed basis through the design phase. Data are also collected continuously during construction to correlate with previously obtained information and to evaluate the need for possible design changes.

(a) *Appraisal Investigation.*—The appraisal investigation includes a preliminary selection of the site and type of dam. All available geologic and topographic maps, photographs of the site area, and data from field examinations of natural outcrops, road cuts, and other surface conditions should be utilized in the selection of the site and preliminary evaluation of the foundation.

The amount of investigation necessary for appraisal will vary with the anticipated difficulty of the foundation. In general, the investigation should be sufficient to define the major geologic conditions with emphasis on those which will affect design. A typical geologic map and profile are shown on figures 2-2 and 2-3.

The geologic history of a site should be thoroughly studied, particularly where the geology is complex. Study of the history may assist in recognizing and adequately investigating hidden but potentially dangerous foundation conditions.

Diamond core drilling during appraisal investigations may be necessary in more complex foundations and for the foundations for larger dams. The number of drill holes required will depend upon the areal extent and complexity of the foundation. Some foundations may require as few as three or four drill holes to define an uncertain feature. Others may require substantially more drilling to determine foundation treatment for a potentially dangerous foundation condition.

Basic data that should be obtained during the appraisal investigation, with refinement continuing until the construction is complete, are:

(1) Dip, strike, thickness, composition, and extent of faults and shears.

(2) Depth of overburden.

(3) Depth of weathering.

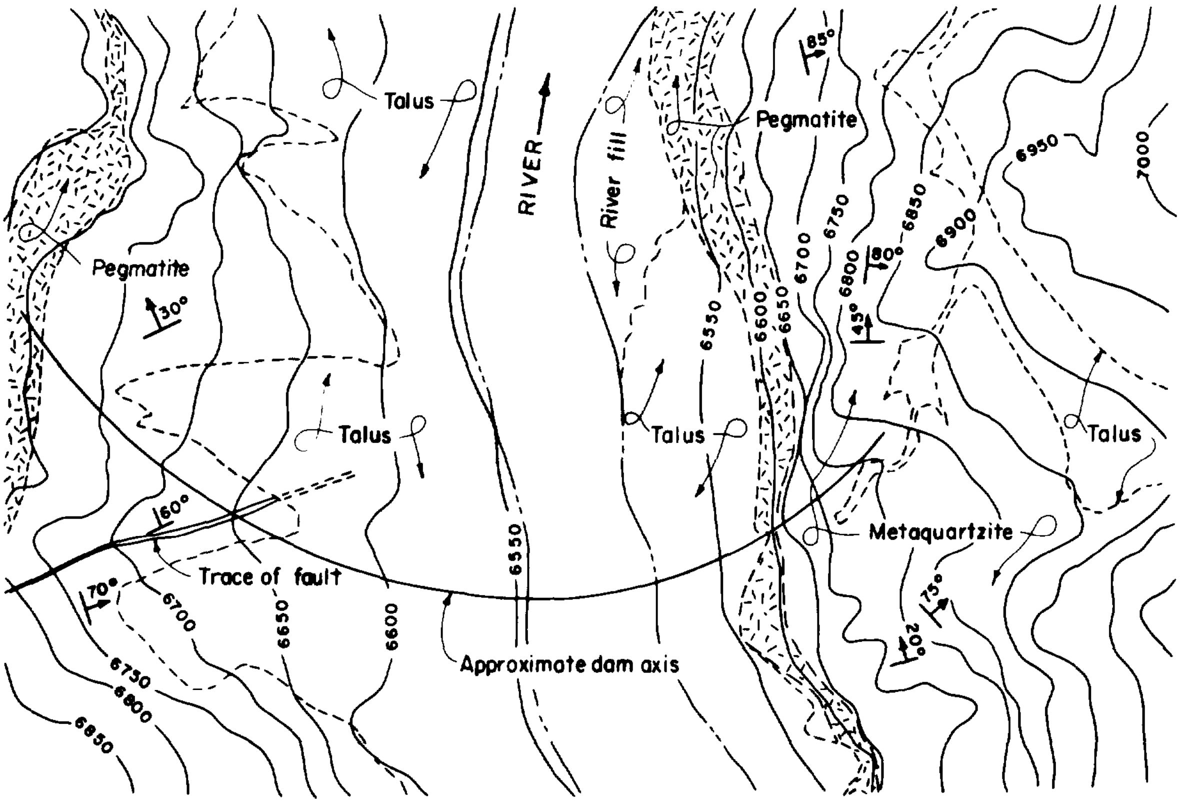
(4) Joint orientation and continuity.

(5) Lithology throughout the foundation.

(6) Physical properties tests of the foundation rock. Tests performed on similar foundation materials may be used for estimating the properties in the appraisal phase.

(b) *Feasibility Investigation.*—During the feasibility phase, the location of the dam is usually finalized and the basic design data are firmed up. The geologic mapping and sections are reviewed and supplemented by additional data such as new surveys and additional drill holes. The best possible topography should be used. In most cases, the topography is easily obtained by aerial photogrammetry to almost any scale desired.

The drilling program is generally the means



T60° Indicates strike and dip of fault T10° Indicates strike and dip of joints

*Figure 2-2.* A typical geologic map of an arch damsite.–288-D-2952

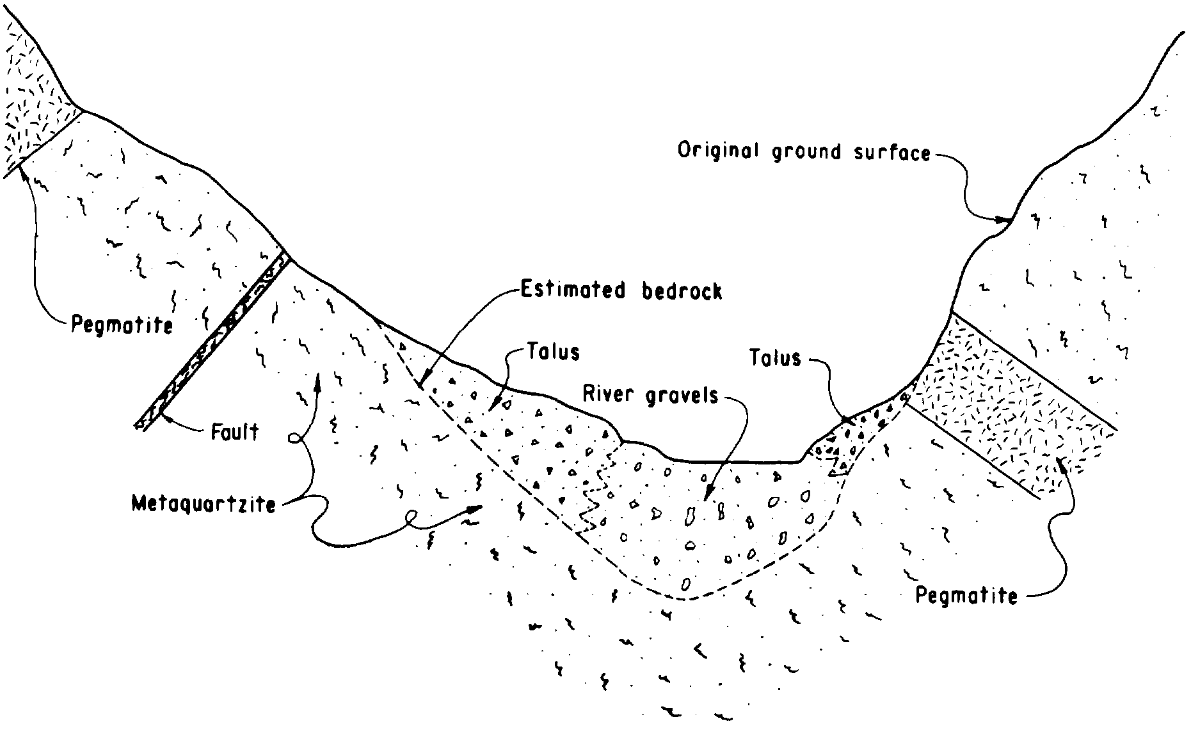


*Figure 2-3.* A typical geologic profile of a damsite.–288-D-2954

of obtaining the additional data required for the feasibility stage. The program takes advantage of any knowledge of special conditions revealed during the appraisal investigation. The drill holes become more specifically oriented and increased in number to better define the foundation conditions and to determine the amount of foundation treatment required.

The rock specimens for laboratory testing during the feasibility investigations are usually nominal, as the actual decision for construction of the dam has not yet been made. Test specimens should be obtained to determine more accurately physical properties of the foundation rock and for petrographic examination. Physical properties of joint or fault samples may be estimated by using conservative values from past testing of similar materials. The similarity of materials can be judged from the cores retrieved from the drilling.

(c) *Final Design Data.*—Final design data are required prior to the preparation of the specifications. A detailed foundation investigation is conducted to obtain the final design data. This investigation involves as many drill holes as are necessary to accurately define the following items:

(1) Strike, dip, thickness, continuity, and composition of all faults and shears in the foundation.
(2) Depth of overburden.
(3) Depth of weathering throughout the foundation.
(4) Joint orientation and continuity.
(5) Lithologic variability.
(6) Physical properties of the foundation rock, including material in the faults and shears.

The foundation investigation may involve, besides diamond core drilling, detailed mapping of surface geology and exploration of dozer trenches and exploratory openings such as tunnels, drifts, and shafts. The exploratory openings can be excavated by contact prior to issuing final specifications. These openings provide the best possible means of examining the foundation.

In addition to test specimens for determining the physical properties, specimens may be required for final design for use in determining the shear strength of the rock types, healed joints, and open joints. This information may be necessary to determine the stability of the foundation and is discussed as the shear-friction factor in subchapter F of chapter III.

Permeability tests should be performed as a routine matter during the drilling program. The information obtained can be utilized in establishing flow nets which will aid in studying uplift conditions and establishing drainage systems. The permeability testing methods presently used by the Bureau of Reclamation are described in designation E-18 of the Earth Manual [4] and the report entitled "Drill Hole Water Tests—Technical Instructions," published by the Bureau of Reclamation in July 1972.

2-21. ***Construction Geology.***—The geology as encountered in the excavation should be defined and compared with the preexcavation geology. Geologists and engineers should consider carefully any geologic change and check its relationship to the design of the structure.

As-built geology drawings should be developed even though revisions in design may not be required by changed geologic conditions, since operation and maintenance problems may develop requiring detailed foundation information.

2-22. ***Foundation Analysis Methods.***—Arch dams are keyed into the foundation so that the foundation will normally be adequate if it has enough bearing capacity to resist the loads from the dam. However, a foundation may have faults, shears, seams, joints, or zones of inferior rock that could develop unstable rock masses when acted on by the loads of the dam and reservoir. The safety of the dam against sliding along a joint, fault, or seam in the foundation can be determined by computing the shear-friction factor of safety. This method of analysis is explained in subchapter F of chapter III. If there are several joints, faults, or seams along which failure can occur, the potentially unstable rock mass can be analyzed by a method called rigid block analysis. This method is explained in detail in subchapter F of chapter IV. These methods of analysis may also be applied to slope stability problems.

The data required for these two methods of analysis are:

(1) Physical properties.
(2) Shearing and sliding strengths of the discontinuities and the rock.
(3) Dip and strike of the faults, shears, seams, and joints.
(4) Limits of the potentially unstable rock mass.
(5) Uplift pressures on the failure surfaces.
(6) Loads to be applied to the rock mass.

When a foundation is interspersed by many faults, shears, joints, seams, and zones of inferior rock, the finite element method of analysis can be used to determine the bearing capacity and the amount of foundation treatment required to reduce or eliminate areas of tension in the foundation. This method of analysis can be utilized to evaluate the effective foundation moduli for use in analyzing the dam, and provides a way to combine markedly different physical properties. The description of this method can be found in subchapter E of chapter IV. In addition to the data required for the rigid block analysis, the finite element analysis requires the deformation moduli of the various parts of the foundation.

2-23. ***In Situ Testing.***—In situ shear tests [5] are more expensive than similar laboratory tests; and consequently, comparatively few can be run. The advantage of a larger test surface may require that a few in situ tests be supplemented by a greater number of laboratory tests. The shearing strength relative to both horizontal and vertical movement should be obtained by either one or a combination of both methods.

Foundation permeability tests may be run in conjunction with the drilling program or as a special program. The tests should be performed according to designation E-18 of the Earth Manual [4] and the report entitled "Drill Hole Water Tests—Technical Instructions," published by the Bureau of Reclamation in July 1972.

Foundation deformation tests can be performed within the exploratory shafts and tunnel. The deformation tests applicable to rock masses are the radial jacking test [6] and the uniaxial jacking test [7]. For softer and less thick or massive materials, the plate gouge test [8] may be used.

2-24. ***Laboratory Testing.***—The following laboratory tests are standard and the methods and test interpretations should not vary substantially from one laboratory to another. A major problem involved with laboratory tests is obtaining representative samples. Sample size is often dictated by the laboratory equipment and is a primary consideration. Following is a list of laboratory tests:

*Physical Properties Tests*

(1) Compressive strength
(2) Elastic modulus
(3) Poisson's ratio
(4) Bulk specific gravity
(5) Porosity
(6) Absorption

*Shear Tests*

| | |
|---|---|
| (1) Direct shear<br>(2) Triaxial shear | Perform on intact specimens and those with healed joints |
| (3) Sliding friction | Perform on open joints |

*Other Tests*

(1) Solubility
(2) Petrographic analysis

2-25. ***Consistency of Presentation of Data.***—It is important that the design engineers, laboratory personnel, and geologists be able to draw the same conclusions from the information presented in the investigations. The standardization of the geologic information and laboratory test results is therefore essential and is becoming increasingly so with the newer methods of analysis.

## J. CONSTRUCTION ASPECTS

2-26. ***General.***—The construction problems that may be encountered by the contractor in constructing the dam and related features should be considered early in the design stage. One of the major problems, particularly in narrow canyons, is adequate area for the contractor's construction plant and equipment and for storage of materials in the proximity of the dam. Locating the concrete plant to minimize handling of the concrete and the aggregates and cement can materially reduce the cost of the concrete.

Permanent access roads should be located to facilitate the contractor's activities as much as practicable. This could minimize or eliminate unsightly abandoned construction roads. Structures should be planned to accommodate an orderly progression of the work. The length of the construction season should be considered. In colder climates and at higher elevations it may be advantageous to suspend all or part of the work during the winter months. Adequate time should be allowed for construction so that additional costs for expedited work are not encountered.

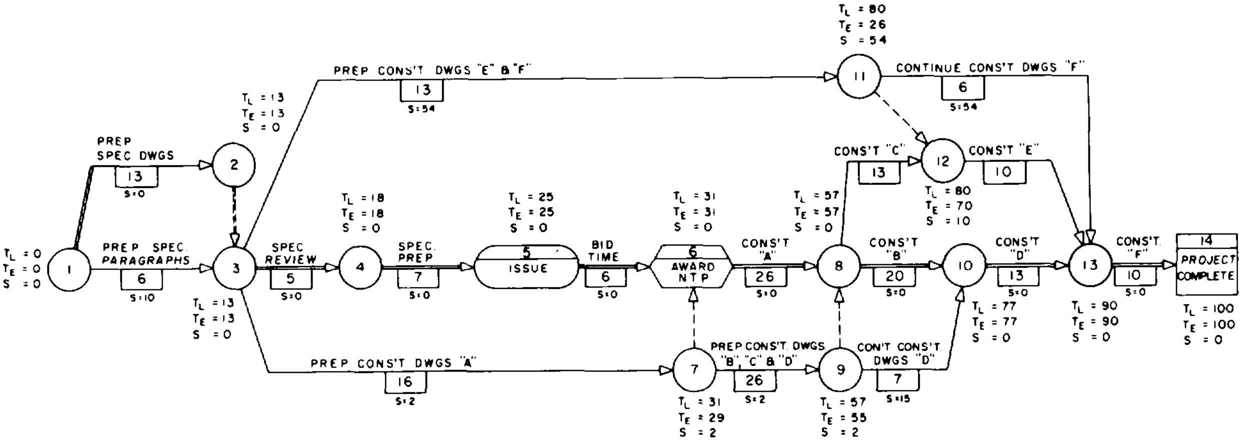
2-27. ***Construction Schedule.***—The contractor's possible methods and timing of construction should be considered at all times during the design of the dam and its appurtenant structures. Consideration of the problems which may be encountered by the contractor can result in significant savings in the cost of construction. By developing an anticipated construction schedule, potential problems in the timing of construction of the various parts can be identified. If practicable, revisions in the design can be made to eliminate or minimize the effect of the potential problems. The schedule can be used to program supply contracts and other construction contracts on related features on the project. It is also useful as a management tool to the designer in planning his work so that specifications and construction drawings can be provided when needed.

The construction schedule can be made by several methods such as Critical Path Method (CPM), Program Evaluation and Review Technique (PERT), and Bar Diagram. Figure 2-4 shows a network for a portion of a hypothetical project for a CPM schedule. Data concerning the time required for various parts of the work and the interdependencies of parts of the work can be programmed into a computer which will calculate the critical path. It will also show slack time or areas which are not critical. In this example, there are two paths of activities. The path which is critical is the preparation of specifications, awarding of contract, and the construction of "A," "B," and "D." The second path, through construction of "C" and "E" is not critical. As the work progresses, the current data on the status of all the phases of work completed and in progress can be fed back into the computer. The computer will then recompute the critical path, thus establishing a new path if another phase of the work has become critical, and will point out any portion of the work that is falling behind the required schedule.

Figure 2-5 shows the construction schedule for the hypothetical project on a bar diagram. This diagram is made by plotting bars to the length of time required for each portion of the work and fitting them into a time schedule, checking visually to make sure interrelated activities are properly sequenced.

## K. MISCELLANEOUS CONSIDERATIONS

2-28. ***Data to be Submitted.***—Many items not covered above affect the design and construction of a dam. Some of these are noted below. In securing and preparing design data, the adequacy and accuracy of the data should contemplate their possible subsequent utility for expansion into specifications design data.



NOTES

$T_L$ – Latest allowable time for activity start or completion.

$T_E$ – Earliest expected time for activity start or completion.

S – Slack or Float Time.

12 – Estimated activity time

═══ CRITICAL PATH (S=0)

*Figure 2-4.* **Typical construction schedule using Critical Path Method (CPM).–288-D-2955**

(1) Details of roadway on crest of dam (and approaches) if required.

(2) Present or future requirement for highway crossing on dam.

(3) Details of fishways and screens, with recommendations of appropriate fish authorities.

(4) Existing works to be replaced or incorporated into dam.

(5) Future powerplant or power development.

(6) Navigation facilities.

(7) Possibility of raising crest of dam in future.

(8) Anticipated future river channel improvement or other construction which might change downstream river regimen.

(9) Recreational facilities anticipated to be authorized, and required provisions for public safety.

(10) Recommended period of construction.

(11) Commitments for delivery of water or power.

(12) Designation of areas within right-of-way boundaries for disposal of waste materials.

2-29. *Other Considerations.*–Design considerations must take into account construction procedures and costs. An early evaluation and understanding of these is necessary if a rapid and economical construction of the dam is to be attained.

Designs for mass concrete structures and their appurtenances should be such that sophisticated and special construction equipment will not be required. Thin, curved walls with close spacing of reinforcement may be desirable for several reasons, and may represent the minimum cost for materials such as cement, flyash, admixtures, aggregates, and reinforcing steel. However, the cost of forming and labor for construction of this type and the decreased rate of concrete placement may result in a much higher total cost than would result from a simpler structure of greater dimensions.

Design and construction requirements should permit and encourage the utilization of machine power in place of manpower wherever practicable. Any reduction in the requirement for high cost labor will result in a significant cost savings in the completed structure. Work

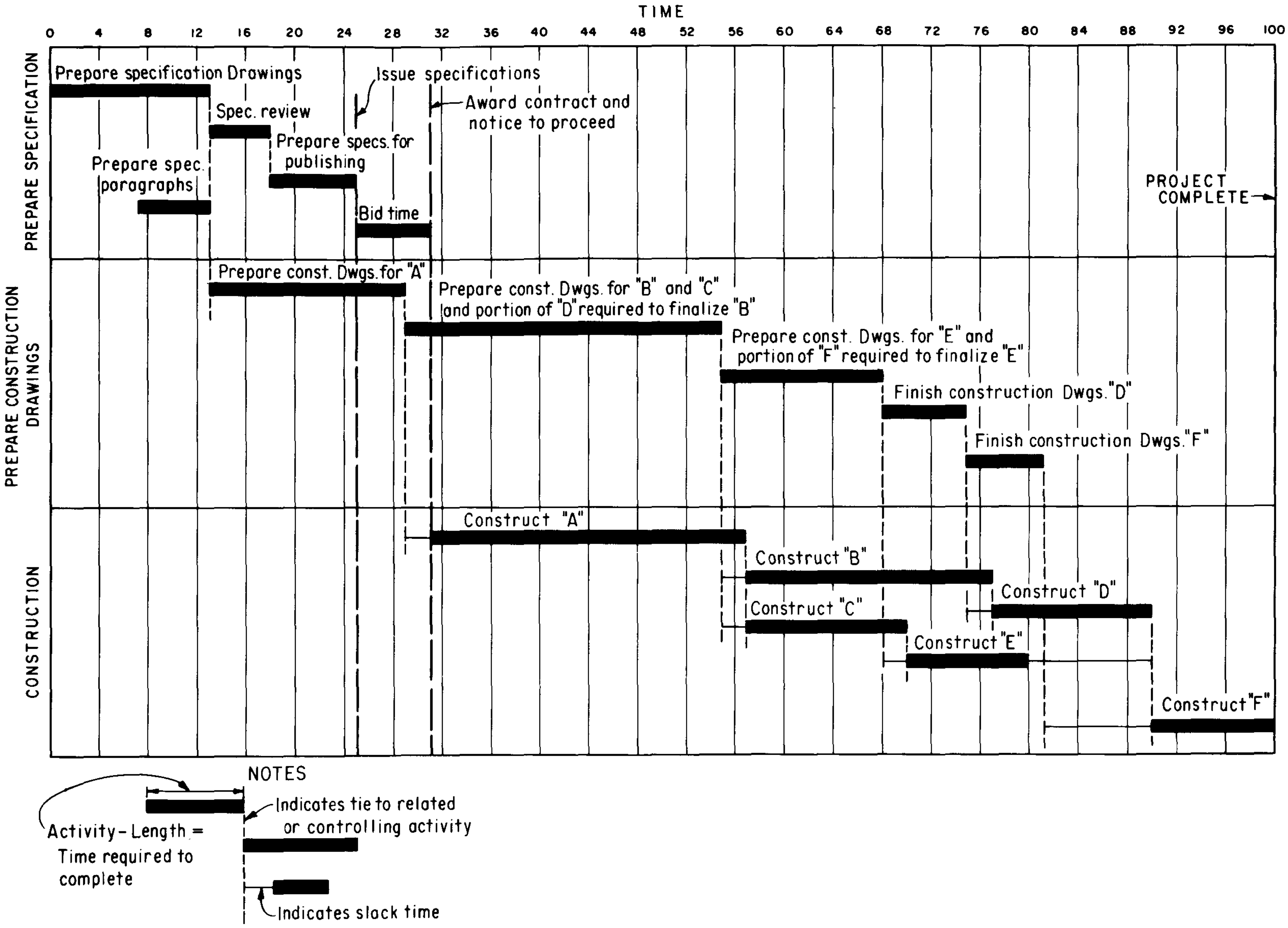


*Figure 2-5.* **Typical construction schedule using a bar diagram.—288-D-2956**

areas involved in a high labor use include the placing, compaction, and curing of the concrete, the treatment and cleanup of construction joints, and the repair and finishing of the concrete surfaces.

Forming is a significant cost in concrete structures. Designs should permit the simpler forms to be used, thus facilitating fabrication, installation, and removal of the forms. Repetitive use of forms will materially reduce forming costs. Although wooden forms are cheaper in initial cost, they can only be used a limited number of times before they warp and fail to perform satisfactorily. The reuse of steel forms is limited only by the designs and the demands of the construction schedule.

## L. BIBLIOGRAPHY

### 2-30. *Bibliography.*

[1] U.S. Department of Agriculture, Agricultural Research Service, "Determining Consumptive Use and Irrigation Water Requirements," Technical Bulletin No. 1275, December 1962.

[2] U.S. Department of Agriculture, Soil Conservation Service, "Irrigation Water Requirements," Technical Release No. 21, April 1967.

[3] Jensen, M. E., "Water Consumption by Agricultural Plants," Water Deficits and Plant Growth, vol. II, Academic Press, New York, N.Y., 1968, pp. 1-22.

[4] "Field Permeability Tests in Boreholes," Earth Manual, Designation E-18, Bureau of Reclamation, 1974.

[5] "Morrow Point Dam Shear and Sliding Friction Tests," Concrete Laboratory Report No. C-1161, Bureau of Reclamation, 1965.

[6] Wallace, G. B., Slebir, E. J., and Anderson, F. A., "Radial Jacking Test for Arch Dams," Tenth Rock Mechanics Symposium, University of Texas, Austin, Tex., 1968.

[7] Wallace, G. B., Slebir, E. J., and Anderson, F. A., "In Situ Methods for Determining Deformation Modulus Used by the Bureau of Reclamation," American Society for Testing and Materials, Denver, Colo., 1969.

[8] Wallace, G. B., Slebir, E. J., and Anderson, F. A., "Foundation Testing for Auburn Dam," Eleventh Symposium on Rock Mechanics, University of California, Berkeley, Calif., 1969.

# Design Data and Criteria

## A. INTRODUCTION

3-1. *Basic Assumptions.*—Computational methods require some basic assumptions for the analysis of an arch dam. The assumptions which cover the continuity of the dam and its foundation, competency of the concrete in the dam, adequacy of the foundation, and variation of stresses across the sections of the dam are as follows:

(1) Rock formations at the damsite are, or will be after treatment, capable of carrying the loads transmitted by the dam with acceptable stresses.

(2) The dam is thoroughly bonded to the foundation rock throughout its contact with the canyon, so that the arch and cantilever elements may be considered to move with the foundation.

(3) The concrete in the dam is homogeneous, uniformly elastic in all directions, and strong enough to carry the applied loads with stresses below the elastic limit.

(4) The dam is a monolithic structure, and arch action occurs if the forces on the dam are sufficient and act to produce positive horizontal thrusts so that vertical closure surfaces (contraction joints, closure slots, etc.) are closed. If the forces acting cause negative horizontal thrusts and tensile stresses exist over 50 percent of the arch thickness, arch action will not occur.

(5) Vertical stresses on horizontal planes vary linearly from the upstream face to the downstream face.

(6) Horizontal stresses normal to vertical radial planes vary linearly from the upstream face to the downstream face.

(7) Horizontal shearing stresses acting in tangential directions vary linearly from the upstream face to the downstream face.

(8) Horizontal shearing stresses acting along horizontal planes in radial directions and along vertical radial planes have parabolic distributions from the upstream face to the downstream face.

## B. CONCRETE

3-2. *Concrete Properties.*—An arch dam must be constructed of concrete which will meet the design criteria for strength, durability, permeability, and other properties. Although mix proportions are usually controlled by strength and/or durability requirements, the cement content should be held to an acceptable minimum in order to minimize the heat of hydration. Properties of concrete vary with age and with proportions and types of ingredients.

Tests must be made on specimens using the full mass mix and the specimens must be of sufficient age to adequately evaluate the strength and elastic properties which will exist for the concrete in the dam [1][1].

(a) *Strength.*—The strength of concrete should satisfy early load and construction requirements, and at some specific age should have the specified compressive strength as determined by the designer. This specific age is often 365 days but may vary from one structure to another.

Tensile strength of the concrete mix should be determined as a companion test series using the direct tensile test method.

Shear strength is a combination of internal friction, which varies with the normal compressive stress, and cohesive strength. Companion series of shear strength tests should be conducted at several different normal stress values covering the range of normal stresses to be expected in the dam. These values should be used to obtain a curve of shear strength versus normal stress.

(b) *Elastic Properties.*—Concrete is not a truly elastic material. When concrete is subjected to a sustained load such as may be expected in a dam, the deformation produced by that load may be divided into two parts—the elastic deformation, which occurs immediately due to the instantaneous modulus of elasticity; and the inelastic deformation, or creep, which develops gradually and continues for an indefinite time. To account for the effects of creep, the sustained modulus of elasticity is used in the design and analysis of a concrete dam.

The stress-strain curve is, for all practical purposes, a straight line within the range of usual working stresses. Although the modulus of elasticity is not directly proportional to the strength, the high strength concretes usually have higher moduli. The usual range of the instantaneous modulus of elasticity for concrete at 28-day age is between $2.0 \times 10^6$ and $6.0 \times 10^6$ pounds per square inch.

(c) *Thermal Properties.*—The effects of temperature change on an arch dam are dependent on the thermal properties of the concrete. Thermal properties necessary for the evaluation of temperature effects are the coefficient of thermal expansion, thermal conductivity, and specific heat [10]. The coefficient of thermal expansion is the length change per unit length per degree temperature change. Thermal conductivity is the rate of heat conduction through a unit thickness over a unit area of the material subjected to a unit temperature difference between faces. The specific heat is defined as the amount of heat required to raise the temperature of a unit mass of the material 1 degree. Diffusivity of concrete is an index of the facility with which concrete will undergo temperature change. Diffusivity is a function of the values of specific heat, thermal conductivity, and density.

(d) *Dynamic Properties.*—Concrete, when subjected to dynamic loadings, may exhibit characteristics unlike those occurring during static loadings. Testing is presently underway in the Bureau's laboratory to determine the properties of concrete when subjected to dynamic loading. Until sufficient test data are available, static strengths and the instantaneous modulus of elasticity should be used.

(e) *Other Properties.*—In addition to the strength, elastic modulus, and thermal properties, several other properties of concrete should be evaluated during the laboratory testing program. These properties, which must be determined for computations of deformations and stresses in the concrete structures, are Poisson's ratio, unit weight, and any autogenous growth or drying shrinkage.

(f) *Average Concrete Properties.*—For preliminary studies until laboratory test data are available, the necessary values may be estimated from published data [2] for similar tests. Until long-term load tests are made to determine the effects of creep, the sustained modulus of elasticity should be taken as 60 to 70 percent of the laboratory value of the instantaneous modulus of elasticity.

If no tests or published data are available, the following may be assumed for preliminary studies:

Specified compressive strength = 3,000 to 5,000 p.s.i.

---

[1] Numbers in brackets refer to items in the bibliography, sec. 3-21.

Tensile strength = 4 to 6 percent of the compressive strength
Shear strength:
  Cohesion = 10 percent of the compressive strength
  Coefficient of internal friction = 1.0
Sustained modulus of elasticity = 3.0 x $10^6$ p.s.i. (static load including effects of creep)
Instantaneous modulus of elasticity = 5.0 x $10^6$ p.s.i. (dynamic or short time load)
Coefficient of thermal expansion = 5.0 x $10^{-6}$ per degree F.
Poisson's ratio = 0.20
Unit weight of concrete = 150 pounds per cubic foot.

## C. FOUNDATION

3-3. *Foundation Deformation.*—The reaction of the foundation to the loads from the dam controls to some extent the stresses within the dam. Conversely, the response of the dam to the external loads and the foundation determines the stresses on the foundation. The proper determination of the interaction between the dam and the foundation requires an accurate knowledge of the deformation characteristics of the foundation.

Whereas the dam is considered to be homogeneous, elastic, and isotropic, its foundation is in general heterogeneous, inelastic, and anisotropic. This variation in the foundation can affect the distribution of load in the dam and as a result on the foundation. Thus, the foundation deformation characteristics should be evaluated over the entire extent of the dam contact. The design of the dam and any treatment to the foundation (see sec. 6-3) to improve its properties are considered separate problems. If treatments are applied to the foundation, the data used for the design of the dam should be based on the properties of the foundation after treatment.

The discussion of foundation investigation in chapter II (secs. 2-19 through 2-25) lists the physical properties normally required and the samples desired for various foundation materials.

The foundation deformation properties used in the analysis of the dam and foundation should represent the composite action of all the materials present in the foundation. The variation in the materials and their composite deformation properties along the foundation contact are required for analysis of the dam.

The foundation investigation should provide information related to or giving deformation moduli and elastic moduli. (Deformation modulus is the ratio of stress to elastic plus inelastic strain. Elastic modulus is the ratio of stress to elastic strain.) The information includes elastic modulus of drill core specimens, elastic modulus and deformation modulus from in situ jacking tests [3], deformation modulus of fault or shear zone material, and logs of the jointing occurring in recovered drill cores. Knowledge of the variation in materials and their relative prevalence at various locations along the foundation is provided by the logs of drill holes and by any tunnels in the foundation.

The amount of analysis required or extrapolation allowed in the establishment of deformation properties over the extent of the foundation is dependent on the uniformity of the foundation as indicated by the foundation investigation. An example of a foundation requiring little analysis for deformation properties might be one which exhibits the following characteristics:

(1) Presence of only one or two rock types.

(2) A closely spaced and regular pattern of discontinuities in these rock types such that their effect could be obtained in an in situ jacking test.

(3) No major low-modulus zones.

(4) In situ jacking tests that show consistent results in the same rock type at

various locations or a distinct relation to some parameter such as elevation or weathering.

The next level of complexity might replace characteristic (3) above with the following:

(3) Widely spaced parallel clay seams and several low-modulus fault zones with regular geometry.

The deformation of a foundation with such a composite could be computed by using stress-strain relationships along with assumptions of stress distributions.

Foundations which do not exhibit these or similar characteristics that simplify the problem of determining the deformation modulus should be considered complex. For these foundations, procedures should be established to aid in the selection of deformation moduli for design purposes. One such procedure, the "Finite Element Method," is described below. The procedure allows for a wide range of complexities of foundations. This is because the detail used in either defining the geometry of the materials or in determining their properties is independent of the basic framework of the method. In the description of the procedure some examples of detail which may or may not be required are discussed.

(a) *Determination of Material Deformability.*—Laboratory and in situ shear tests provide the deformation modulus for fault and shear zones. Good compositional description of the zones tested for deformation modulus permits extrapolation of results to untested zones of similar description.

(b) *Determination of Foundation Deformability.*—For a complex foundation the two-dimensional finite element method should be used to determine the deformation modulus in selected directions and locations along the foundation [4, 5]. A finite element model is prepared which presents each material in its appropriate location and quantity (see secs. 4-58 through 4-64). The deformation properties used for the materials are determined from information gained from the foundation investigation program.

Variations in the characteristics of the zones (for example, pinching and swelling) which are too detailed for inclusion in the total finite element model may modify the results of localized testing of the material. If geological investigations indicate any significant variations, these can be evaluated by additional analysis and appropriate corrections can be made to the deformation moduli determined from tests.

The deformation modulus of the rock masses presented in the finite element model may be taken directly from large-scale jacking tests if the jointing in the foundation is uniform. Rock masses are considered to be the material between major shears and faults. The rock masses in general contain joints, very narrow shears, discontinuous shears, and other minor discontinuities.

Another procedure is to reduce the elastic modulus of test specimens of a rock type by a percentage based on the quality of the rock [6], as estimated on the percentage of recovery of drill core. When the discontinuities at a site are highly variable, a technique known as the "Joint-Shear Index" [7] may be used to determine the modulus of the rock masses. The basic procedure in using this method follows.

For each large-scale jacking test the deformation modulus is obtained. The location, condition, and type of discontinuities in the NX core extracted for instrumentation at each site are logged. The elastic moduli of the best specimens of the core are determined.

After the test has been conducted, the ratio of the deformation modulus obtained to the elastic modulus of the drill core ($E_d/E_c$) can be computed. This ratio is plotted versus the sum of the discontinuities weighted for location, condition, and type. A correlation curve is established when the results of several jacking tests have been plotted. This curve may then be used to determine deformation moduli for rock at any location along the foundation by simply logging the drill core according to a set procedure and determining its elastic modulus. Reference [7] discusses the application of the methods described above to a specific foundation.

(c) *Effective Deformation Modulus Determination.*—The need to reduce the collective deformation moduli to a single value, called the effective deformation modulus, for each arch or cantilever stems from the manner in which the stress analysis method includes the effects of foundation modulus (see sec. 4-30).

An effective deformation modulus is one which, when substituted for the various moduli in a composite foundation, produces an equivalent behavior. The two-dimensional finite element method of analysis can be used to analyze a difficult and complex foundation composed of several different materials and determine the effective deformation modulus for a particular section or location. The technique is well suited to the purpose, since each zone of varying material can be modeled according to its geometric limits and its particular deformation modulus. By analyzing an adequate number of sections or locations, a plot of the variation in effective deformation modulus for the entire foundation can be made and the effective deformation modulus for each arch and cantilever abutment determined. Thus, the variation in deformation modulus along the contact of the dam and foundation can be included in the stress analysis for the dam.

(d) *Modification of Deformation Modulus by Treatment.*—The presence of a zone of low-modulus material in close proximity to the dam may produce an equivalent deformation modulus which is too low to be acceptable or which causes too abrupt a change with respect to adjacent sections. Undesirable stresses in the dam could be produced due to these zones. Treatment may be modeled directly in the finite element studies by simply replacing the material properties of the elements in the zone to be treated. After the correct amount of treatment has been determined, the effective deformation modulus of the foundation should be determined as described in (c) above, with the treatment included.

3-4. ***Foundation Strength.***—Compressive strength of the foundation rock can be an important factor in determining thickness requirements for a dam at its contact with the foundation. Where the foundation rock is nonhomogeneous, a sufficient number of tests, as determined by the designer, should be made to obtain compressive strength values for each type of rock in the loaded part of the foundation.

A determination of tensile strength of the rock is seldom required because discontinuities such as unhealed joints and shear seams cannot transmit tensile stress within the foundation.

Resistance to shear within the foundation and between the dam and its foundation results from the cohesion and internal friction inherent in the foundation materials and at the concrete-rock contact. These properties are found from laboratory and in situ testing as discussed in sections 2-23 and 2-24. However, when test data are not available, values of the properties may be estimated (subject to the limitations discussed below) from published data [2, 8, 9] and from tests on similar materials.

The results of laboratory triaxial and direct shear tests, as well as in situ shear tests, will typically be reported in the form of the Coulomb equation,

$$R = CA + N \tan \phi \qquad (1)$$

where:

$R$ = shear resistance,
$C$ = unit cohesion,
$A$ = area of section,
$N$ = effective normal force, and
$\tan \phi$ = tangent of angle of friction.

which defines a linear relationship between shear resistance and normal load. Experience has shown that such a representation of shear resistance is usually realistic for most intact rock. For other materials, the relationship may not be linear and a curve of shear strength versus normal load should be used for the condition of an existing joint, as discussed later. Also, it may be very difficult to differentiate between cohesive and friction resistance for materials other than intact rock.

In the case of an existing joint in rock, the shear strength is derived basically from sliding friction and usually does not vary linearly with the normal load. Therefore, the shear resistance should be represented by a curve of shear resistance versus normal load, as shown by the curve *OA* in figure 3-1. If a straight line, *BC*, had been used, it would have given values of shear resistance too high where it is above the curve *OA*, and values too low where it is below. A linear variation may be used to represent a portion of the curve. Thus, the line *DE* can be used to determine the shear resistance for actual normal loads between $N_1$ and $N_2$ without significant error. However, for normal loads below $N_1$ or above $N_2$, its use would give a shear resistance which is too high and the design would therefore be unsafe.

Other potential sliding planes, such as shear zones and faults, should be checked to determine if the shear resistance should be linear or curvilinear. As with the jointed rock, a linear variation can be assumed for a limited range of normal loads if tests on specimens verify this type of variation for that range of normal loads.

The specimens tested in the laboratory or in situ are usually small with respect to the planes analyzed in design. Therefore, the scale effect should be carefully considered in determining the shear resistance to be used in design. Among the factors to be considered in determining the scale effect at each site are the following:

(1) Comparisons of tests of various sizes.

(2) Geological variations along the potential failure planes.

(3) Current research on scale effect.

When a foundation is nonhomogeneous, the potential sliding surface may be made up of different materials. The total resistance can be determined by adding the shear resistances offered by the various materials, as shown in the following equation:

$$R_t = R_1 + R_2 + R_3 + \ldots . R_n \qquad (2)$$

where:

$R_t$ = total resistance, and
$R_1$, $R_2$, $R_3$, etc. = resistance offered by the various materials.

When determining the shear resistance offered by the various materials, the effect of deformation should be considered. The shear resistance given by the Coulomb equation or the curves of shear resistance versus normal load are usually the maximum for the test specimen without regard to deformation. Some materials obtain their maximum resistance with less deformation than others. For example, intact rock will not deform as much as a joint in rock or a sheared zone when maximum shear resistance of the material is reached.

The following example illustrates the importance of including the effect of deflection in determining the resistance offered by each material in nonhomogeneous foundations. This example has only 5 percent intact rock to emphasize that a small quantity of high-strength intact rock can make a significant contribution to the total resistance. Such a situation is not normally encountered but can and has occurred.

*Example*: Determine the shear resistance on a potential sliding plane which is 1,000 square feet in area for the following conditions:

(1) Normal load, $N$ = 10,000 kips.

(2) The plane is 5 percent intact rock ($A_r$ = 50 square feet), 20 percent sheared material ($A_s$ = 200 square feet), and 75 percent joint ($A_j$ = 750 square feet).

(3) The values of cohesion and tan $\phi$ for each material are as follows:

| Material | Cohesion (p.s.f.) | Tan $\phi$ |
|---|---|---|
| Intact rock | 200,000 | 1.80 |
| Sheared material | 3,000 | 0.30 |
| Joint | 0 | 0.75 |

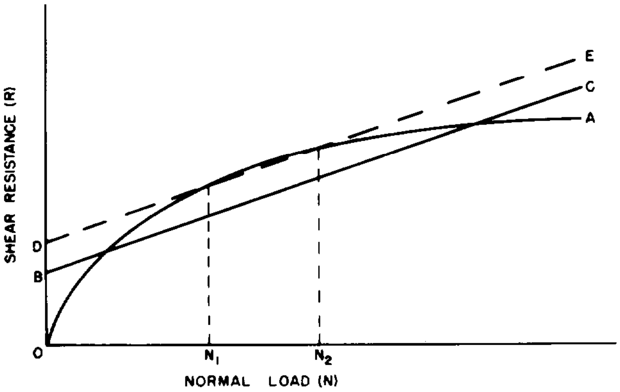


*Figure 3-1.* Shear resistance on an existing joint in rock.–288-D-2957

(4) The normal load on each material is:

| | |
|---|---|
| Intact rock | $N_r$ = 2,000 kips |
| Sheared material | $N_s$ = 1,000 kips |
| Joint | $N_j$ = 7,000 kips |

The shear resistance is determined as follows:

$$R_r = \frac{200{,}000(50)}{1{,}000} + 1.8(2{,}000) = 13{,}600 \text{ kips}$$

$$R_s = \frac{3{,}000(200)}{1{,}000} + 0.3(1{,}000) = 900 \text{ kips}$$

$$R_j = \frac{0(750)}{1{,}000} + 0.75(7{,}000) = 5{,}250 \text{ kips}$$

$$R_t = 13{,}600 + 900 + 5{,}250 = 19{,}750 \text{ kips}$$

For this example, an analysis of the shear strength versus deflection shows that the movement of the intact rock at failure is 0.02 inch. At this deflection the sheared material will have developed only 50 percent of its strength and the joint only 5 percent. Therefore, the actual developed strength at the time the rock would fail is:

$$13{,}600 + 900 \times 0.50 + 5{,}250 \times 0.05 = 14{,}312 \text{ kips}$$

This is about 70 percent of the maximum shear strength computed above without considering deformation.

In some situations, the potential sliding surface comprised of several different materials may exhibit greater total shear resistance after any intact materials are sheared. For example, if the cohesive strength of intact rock is low but the normal load acting on the total surface is large, the sliding friction strength of the combined materials can exceed the shear resistance determined before the rock sheared. For this reason, a second analysis should be performed which considers only the sliding friction strength of the surfaces.

3-5. ***Foundation Permeability.***—The analysis of an arch dam foundation requires a knowledge of the hydrostatic pressure distribution throughout the foundation. The exit gradient for shear zone materials that surface near the downstream toe of the dam should also be determined to check against the possibility of piping (see sec. 6-4).

The laboratory values for permeability of sample specimens are applicable only to that portion or portions of the foundation which they represent. The permeability is controlled by a network of geological features such as joints, faults, and shear zones. The permeability of the geologic features can be determined best by in situ testing. The pressure distribution for design should include the appropriate influence of the permeability and extent of all the foundation materials and geologic features. Such a determination may be made by several methods including two- and three-dimensional physical models, two- and three-dimensional finite element models, and electric analogs.

## D. LOADS

3-6. ***Reservoir and Tailwater.***—Reservoir and tailwater loads to be applied to the structure are obtained from reservoir operation studies and tailwater curves. These studies are based on operating and hydrologic data such as reservoir capacity, storage allocations, streamflow records, flood hydrographs, and reservoir releases for all purposes. A design reservoir can be derived from these operation studies which will reflect a normal high water surface, seasonal drawdowns, and the usual low water surface.

The hydrostatic pressure at any point on the dam is equal to the hydraulic head at that point times the unit weight of water (62.4 lb. per cu. ft.).

The normal design reservoir elevation is the highest elevation that water is normally stored. It is the *Top of Joint Use Capacity*, if joint use capacity is included. If not, it is the *Top of*

*Active Conservation Capacity*. For definitions of reservoir capacities, see section 2-9.

The minimum design reservoir elevation is defined as the usual low water surface as reflected in seasonal drawdowns. Unless the reservoir is drawn down to *Top of Inactive Capacity* at frequent intervals, the minimum design reservoir elevation will be higher than that level.

Maximum design reservoir elevation is the highest anticipated water surface elevation and usually occurs in conjunction with the routing of the inflow design flood through the reservoir.

The tailwater elevation used with a particular reservoir elevation should be the minimum that can be expected to occur with that reservoir elevation.

3-7. ***Temperature.***—Temperature loads are imposed on a concrete dam when the concrete undergoes a temperature change and volumetric change is restrained. The magnitude of the temperature load is related to the closure temperature, to the thermal coefficient of expansion of the concrete, and to the temperature difference between the closure temperature and the operating temperatures [10].

The closure temperature of an arch dam is defined as the mean concrete temperature at the time that the structure is assumed to be monolithic and arch action begins. Two examples would be temperature of the concrete at the time of grouting of contraction joints or at the time of backfilling of a closure slot. If the concrete temperature is not the same throughout the dam at the time of contraction joint grouting, the individual arches will have different closure temperatures.

The closure temperature or temperatures incorporated in the design of a dam should be determined from results of stress analyses and modified as necessary by practical considerations such as costs of temperature control measures, site conditions, and construction program. By artificially cooling the concrete with embedded temperature control systems, the closure temperature may be uniform throughout the dam or it may be varied as desired over the height of the dam to achieve the desired stress distribution. Natural cooling of the concrete will result in varying closure temperatures, depending upon the height and thickness of the dam and upon the climatic conditions and construction schedule.

Operating temperatures are obtained from temperature studies using anticipated ambient air temperatures, reservoir water temperatures, and solar radiation. For reconnaissance and feasibility designs, temperature studies which define the range of mean concrete temperatures will be sufficient. Final design studies should use such methods as Schmidt's method or the finite element method to determine concrete temperatures and temperature gradients as they vary throughout the year in the different parts of the dam.

Secondary stresses can occur around openings and at the faces of the dam due to temperature differentials. These temperature differentials are caused by the difference in the temperature of the concrete surfaces due to ambient air and water temperature variations, solar radiation, temperature of air or water in openings, and temperature of the concrete mass. These secondary stresses are usually localized near the faces of the dam and may produce cracks which give an unsightly appearance. If stress concentrations occur around openings because of these temperature differentials, cracking could lead to progressive deterioration. Openings filled with water such as outlets are of particular concern since cracks, once formed, would fill with water which could increase the pore pressure within the dam and propagate the crack.

When making studies to determine concrete temperature loads and temperature gradients, varying weather conditions can be applied. Similarly, a widely fluctuating reservoir water surface will affect the concrete temperatures. In determining temperature loads, the following conditions and temperatures are used:

(1) *Mean air temperatures.*—The average air temperature which is expected to occur at the site. These are normally obtained from Weather Bureau records of the mean monthly air temperatures and the mean daily maximum and minimum air temperatures.

(2) *Usual weather conditions.*–The combination of the daily air temperatures, a 1-week cycle representative of the cold (hot) periods associated with barometric pressure changes, and the mean monthly air temperatures. This condition will account for temperatures which are halfway between the mean monthly air temperatures and the minimum (maximum) recorded air temperatures at the site.

(3) *Extreme weather conditions.*–The combination of the daily air temperatures, a 2-week cycle representative of the cold (hot) periods associated with barometric pressure changes, and the mean monthly air temperatures. This condition will account for the minimum (maximum) recorded air temperatures at the site. This is an extreme condition and is seldom used.

(4) *Mean concrete temperatures.*–The average concrete temperatures between the upstream and downstream faces which will result from mean air temperatures, reservoir water temperatures associated with the design reservoir operation, and solar radiation.

(5) *Usual concrete temperatures.*–Same as above, except that *usual* weather conditions are applied.

(6) *Extreme concrete temperatures.*–Same as above, except that *extreme* weather conditions are applied.

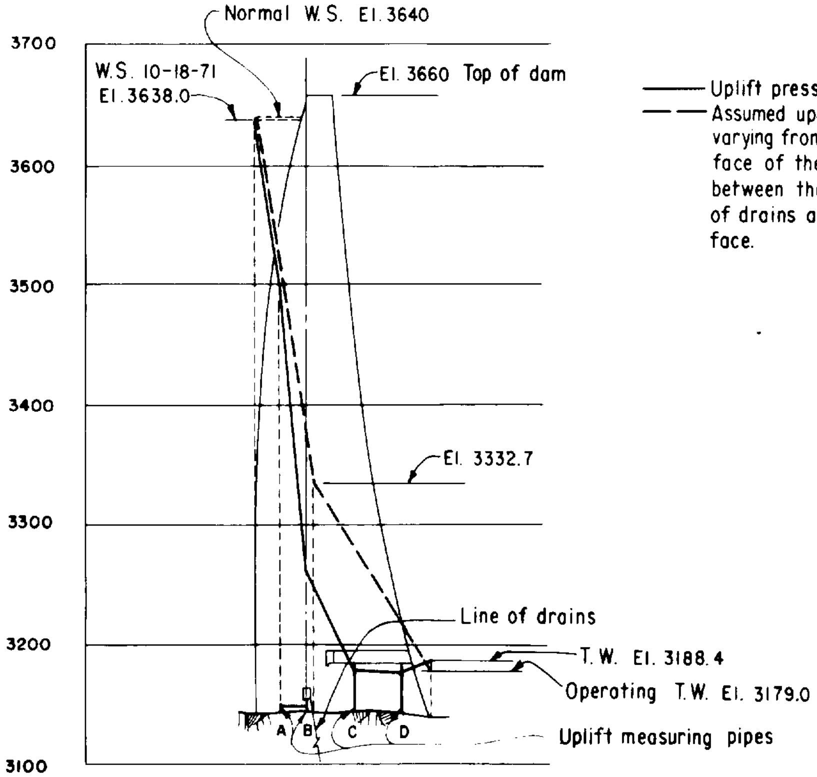
3-8. ***Internal Hydrostatic Pressures.***–Hydrostatic pressures from reservoir water and tailwater act on the dam and occur within the dam and foundation as internal pressures in the pores, cracks, joints, and seams. The distribution of pressure through a horizontal section of the dam is assumed to vary linearly from full hydrostatic head at the upstream face to zero or tailwater pressure at the downstream face, provided the dam has no drains or unlined water passages. When formed drains are constructed, the internal pressure should be modified in accordance with the size, location, and spacing of the drains. Large unlined penstock transitions or other large openings in dams will require special modification of internal pressure patterns. Pressure distribution in the foundation may be modified by the ground water in the general area.

The internal pressure distribution through the foundation is dependent on drain size, depth, location and spacing, and on rock porosity, jointing, faulting, and to some extent on the grout curtain. Determination of such pressure distribution can be made from flow nets computed by several methods including two- and three-dimensional physical models, two- and three-dimensional finite element models, and electric analogs. Such a flow net, modified by effects of drainage and grouting curtains, should be used to determine internal pressure distribution. However, the jointing, faulting, variable permeability, and other geologic features which may further modify the flow net should be given full consideration.

The component of internal hydrostatic pressure acting to reduce the vertical compressive stresses in the concrete on a horizontal section through the dam or at its base is referred to as uplift or pore pressure. Records are kept of the pore pressure measurments in most Bureau of Reclamation dams. Figure 3-2 illustrates actual measured uplift pressures at the concrete-rock contact as compared with design assumptions for Yellowtail Dam.

Laboratory tests indicate that for practical purposes pore pressures act over 100 percent of the area of any section through the concrete. Because of possible penetration of water along construction joints, cracks, and the foundation contact, internal pressures should be considered to act throughout the dam. It is assumed that the pressures are not affected by earthquake acceleration because of the transitory nature of such accelerations.

Internal hydrostatic pressures should be used for analyses of the foundation and for studies of overall stability of an arch dam at its contact with the foundation. These pressures reduce the compressive stresses acting within the concrete, thereby lowering the frictional shear resistances. Unlike gravity dams, which depend on shear resistance for stability, arch dams resist much of the applied load by transferring it horizontally to the abutments by arch actions. Therefore, uplift pressures within arch dams are usually of little importance and are not considered unless horizontal cracking is



NOTES

——— Uplift pressure measured on 10-18-71.

— — — Assumed uplift pressure based on a gradient varying from full reservoir pressure at the upstream face of the dam to one-third the differential pressure between the faces plus operating tailwater at the line of drains and to operating tailwater at the downstream face.

*Figure 3-2.* Comparison of assumed and actual uplift pressure data on an arch dam (Yellowtail Dam in Montana).–288-D-2958

indicated in the analysis (see sec. 4-22). When horizontal cracking is being considered, uplift pressures are assumed to be equal to the reservoir head in the crack and to vary linearly from reservoir head at the end of the crack to tailwater pressure or zero at the downstream face.

If uplift pressures are to be considered for an arch dam, the following criteria apply. For preliminary design purposes uplift pressure distribution in concrete dams is assumed to have an intensity at the line of drains that exceeds the tailwater pressure by one-third the differential between reservoir and tailwater levels. The pressure gradient is then extended to reservoir and tailwater levels, respectively, in straight lines. If there is no tailwater, the pressure diagram is zero at the downstream face. The pressure is assumed to act over 100 percent of the area. In the final design for a dam, the internal pressures within the foundation rock and at the contact with the dam will depend on the location, depth, and spacing of drains as well as on the joints, shears, and other geologic structures in the rock. Internal pressures within the dam depend on the location and spacing of the drains. These internal hydrostatic pressures should be determined from flow nets computed by electric analogy analysis, three-dimensional finite element analysis, or other comparable means.

3-9. *Dead Load.*–The magnitude of dead load is considered equal to the weight of concrete plus appurtenances such as gates and bridges. For preliminary design the unit weight of concrete is assumed to be 150 pounds per cubic foot. For final design the unit weight of concrete should be determined by laboratory tests.

It is assumed that shear stresses are not transmitted across ungrouted contraction joints. Thus, when these joints are left ungrouted until all concrete has been placed, dead load stresses are transferred vertically to the foundation by cantilever action alone.

However, contraction joints in the lower part of the dam may be grouted before concrete placements are completed. In this case, the stresses due to concrete placed after a part of the dam is grouted are transmitted partly by vertical cantilever action and partly by horizontal arch action through the grouted joints.

**3-10.** ***Ice.***—Existing design information on ice pressure is inadequate and somewhat approximate. Good analytical procedures exist for computing ice pressures, but the accuracy of results is dependent upon certain physical data which must come from field and laboratory tests [11].

Ice pressure is created by thermal expansion of the ice and by wind drag. Pressures caused by thermal expansion are dependent on the temperature rise of the ice, the thickness of the ice sheet, the coefficient of expansion, the elastic modulus, and the strength of the ice. Wind drag is dependent on the size and shape of the exposed area, the roughness of the surface, and the direction and velocity of the wind. Ice loads are usually transitory. Not all dams will be subjected to ice pressure, and the designer should decide after consideration of the above factors whether an allowance for ice pressure is appropriate. The method of Monfore and Taylor [12] may be used to determine the anticipated ice pressures. An acceptable estimate of ice load to be expected on the face of a structure may be taken as 10,000 pounds per linear foot of contact between the ice and the dam for an assumed ice depth of 2 feet or more when basic data are not available to compute pressures.

**3-11.** ***Silt.***—Not all dams will be subjected to silt pressure, and the designer should consider all available hydrologic data before deciding whether an allowance for silt pressure is necessary. Horizontal silt pressure is assumed to be equivalent to that of a fluid weighing 85 pounds per cubic foot. Vertical silt pressure is determined as if silt were a soil having a wet density of 120 pounds per cubic foot, the magnitude of pressure varying directly with depth. These values include the effects of water within the silt.

**3-12.** ***Earthquake.***—Concrete dams are elastic structures which may be excited to resonance when subjected to seismic disturbances. Two steps are necessary to obtain loading on a concrete dam due to such a disturbance. First, an estimate of magnitude and location must be made of the earthquake to which the dam will be subjected and the resulting rock motions at the site determined. The second step is the analysis of the response of the dam to the earthquake by either the response spectrum or time-history method.

Most earthquakes are caused by crustal movements of the earth along faults. Geologic examinations of the area should be made to locate any faults, determine how recently they have been active, and estimate the probable length of fault. Seismological records should also be studied to determine the magnitude and location of any earthquakes recorded in the area. Based on these geological and historical data, hypothetical earthquakes usually of magnitudes greater than the historical events are estimated for any active faults in the area. These earthquakes are considered to be the most severe earthquakes associated with the faults and are assumed to occur at the point on the fault closest to the site. This defines the *Maximum Credible Earthquake* and its location in terms of Richter Magnitude $M$ and distance $d$ to the causative fault.

Methods of determining a design earthquake that represents an operating-basis event are under development. These methods should consider historical records to obtain frequency of occurrence versus magnitude, useful life of the structure, and a statistical approach to determine probable occurrence of various magnitude earthquakes during the life of the structure. When future developments produce such methods, suitable safety factors will be included in the criteria.

The necessary parameters to be determined at the site using attenuation methods [13] are acceleration, predominant period, duration of shaking, and frequency content.

Attenuation from the fault to the site is generally included directly in the formulas used to compute the basic data for response spectra. A response spectrum graphically represents the maximum response of a structure with one degree of freedom having a specific damping and subjected to a particular excitation. A response spectrum should be determined for each magnitude-distance relationship by each of three methods as described in appendix D of reference [13]. The design response spectrum of a structure at a site is the composite of the above spectra.

Time-history analyses of a dam are sometimes desirable. The required accelerograms may be produced by appropriate adjustment of existing or artificially generated accelerograms. The previously mentioned parameters are necessary considerations in the development of synthetic accelerograms or in the adjustment of actual recorded accelerograms.

The analytical methods used to compute material frequencies, mode shapes, and structural response are discussed in chapter IV.

## E. LOADING COMBINATIONS

3-13. ***General.***—Arch dam designs should be based on the most severe usual load combination in the following list, unless special considerations dictate otherwise. Combinations of transitory load, each of which has only a remote probability of occurrence at any given time, have a negligible probability of simultaneous occurrence and should not be considered as a reasonable basis for design. When large fluctuations of the water level may be expected, the design should give an acceptable balance of stresses for the various applicable usual load combinations. The dam should be designed for the appropriate following loading combinations using the safety factors prescribed in sections 3-18 through 3-20.

3-14. ***Usual Loading Combinations.***—

(1) Effects of minimum usual concrete temperature and the most probable reservoir elevation occurring at that time, with appropriate dead loads, tailwater, ice, and silt.

(2) Effects of maximum usual concrete temperature and the most probable reservoir elevation occurring at that time, with appropriate dead loads, tailwater, and silt.

(3) Normal design reservoir elevation and the effects of usual concrete temperature occurring at the time, with appropriate dead loads, tailwater, ice, and silt.

(4) Minimum design reservoir elevation and the effects of usual concrete temperature occurring at that time, with appropriate dead loads, tailwater, ice, and silt.

3-15. ***Unusual and Extreme Loading Combinations.***—

(1) *Unusual Loading Combination.*—Maximum design reservoir elevation and the effects of mean concrete temperature occurring at that time, with appropriate dead loads, tailwater, and silt.

(2) *Extreme Loading Combination.*—Any of the above Usual Loading Combinations plus the effects of the *Maximum Credible Earthquake.*

3-16. ***Other Studies and Investigations.***—

(1) Any of the above loading combinations plus hydrostatic pressures within the foundation for foundation stability.

(2) Dead load.

(3) Effects of construction and grouting sequences. Grouting of the contraction joints in an arch dam provides continuity so the structure acts monolithically. Grouting of contraction joints may be performed in stages while concrete placement is in progress. This produces arch action in the grouted part of the dam while none exists in the part where construction is continuing. Water and temperature loadings may also be present on the grouted portion of the dam. This load application sequence may force the lower arches to carry more of the total load than would otherwise be the case.

If the contraction joints in an arch dam are grouted in stages, water level changes, temperature changes, and effects of concrete placement between grouting stages should be studied by separate stress analyses to determine the effects of the construction and grouting program on the stress distribution in the dam. Stresses for all of the construction stages should be combined by superposition.

(4) Any other loading combination which, in the designer's opinion, should be analyzed for a particular dam.

## F. FACTORS OF SAFETY

3-17. *General.*—All design loads should be chosen to represent as nearly as can be determined the actual loads which will act on the structure during operation. Methods of determining load-resisting capacity of the dam should be the most accurate available. All uncertainties regarding loads or load-carrying capacity must be resolved as far as practicable by field or laboratory tests, thorough exploration and inspection of the foundation, good concrete control, and good construction practices. On this basis, the factor of safety will be as accurate an evaluation as possible of the capacity of the structure to resist applied loads. All safety factors listed are minimum values.

Dams, like other important structures, should be frequently inspected. In particular, where uncertainties exist regarding such factors as loads, resisting capacity, or characteristics of the foundation, it is expected that adequate observations and measurements will be made of the structural behavior of the dam and its foundation to assure that the structure is at all times behaving as designed.

The factors of safety for the dam are based on analyses using the "Trial-Load Method of Analysis" (secs. 4-11 through 4-47) or its computerized version, "Arch Dam Stress Analysis System" (secs. 4-48 through 4-54). Although lower safety factors may be permitted for limited local areas, overall safety factors for the dam and the foundation (after beneficiation) should meet the requirements for the loading combination being analyzed. Somewhat higher safety factors should be used for foundation studies because of the greater amount of uncertainty involved in assessing foundation load resisting capacity. For other loading combinations where the safety factors are not specified, the designer is responsible for the selection of safety factors consistent with those for loading combination categories discussed in sections 3-13 through 3-16.

3-18. ***Allowable Stresses.***—The maximum allowable compressive stress for concrete for the Usual Loading Combinations should be determined by dividing the specified compressive strength by a safety factor of 3.0. However, in no case should the allowable compressive stress for the Usual Loading Combinations exceed 1,500 pounds per square inch. In the case of Unusual Loading Combinations the maximum allowable compressive stress should be determined by dividing the specified compressive strength by a safety factor of 2.0 and in no case should this value exceed 2,250 pounds per square inch. The allowable compressive stress for the Extreme Loading Combination should be less than the specified compressive strength.

Although concrete possesses some tensile strength, quantitative evaluations have been uncertain. The importance of tensile stresses must be determined for individual cases by considering location, magnitude, and direction of stress; probable duration of loading which would produce tensile stress; and the effects of cracking on the behavior of the structure.

Whenever practical, tensile stresses should be avoided by redesign of the structure. However, limited amounts of tensile stress may be permitted in localized areas at the upstream face for the Usual Loading Combinations at the discretion of the designer. Under no circumstances should this tensile stress exceed 150 pounds per square inch for the Usual and 225 pounds per square inch for the Unusual Loading Combinations. Tensile stress equal to the tensile strength of concrete at the lift surfaces may be permitted for localized areas on the downstream face during construction or for the combination of low water level and high temperature loading. The point of application of the resultant dead load force must also remain within the vertical section to maintain stability during construction. For the Extreme Loading Combination which includes the *Maximum Credible Earthquake*, the concrete should be assumed to crack whenever the tensile strength is exceeded and the cracks assumed to propagate to the point of zero stress. The structure may be considered safe for the Extreme Loading Combination if, after cracking effects have been included, the stresses are less than the specified compressive strength of the concrete and stability of the structure is maintained.

The maximum allowable compressive stress in the foundation should be less than the compressive strength of the foundation material divided by safety factors of 4.0, 2.7, and 1.3 for the Usual, Unusual, and Extreme Loading Combinations, respectively.

3-19. ***Shear Stress and Sliding Stability.***—The maximum allowable average shearing stress on any plane within the dam shall be less than the shear strength divided by the appropriate safety factor. Safety factors shall be greater than 3.0 for Usual, 2.0 for Unusual, and 1.0 for Extreme Loading Combinations.

The shear-friction factor of safety, $Q$, as computed using equation (3), is a measure of the safety against sliding or shearing at the contact of the dam and the foundation. The shear-friction factor of safety should also be used to check the stability of the remainder of the partially cracked section after cracking has been included for the Extreme Loading Combination.

The shear-friction factor of safety, $Q$ is the ratio of resisting to driving forces as computed by the expression:

$$Q = \frac{CA + (\Sigma N + \Sigma U) \tan \phi}{\Sigma V} \qquad (3)$$

where:

$C$ = unit cohesion,
$A$ = area of the section considered,
$\Sigma N$ = summation of normal forces,
$\Sigma U$ = summation uplift forces,
$\tan \phi$ = coefficient of internal friction, and
$\Sigma V$ = summation of shear forces.

All parameters must be specified using consistent units and with proper signs according to the convention shown on figure 4-26.

Values of cohesion and internal friction should be determined by actual tests of the foundation materials and the concrete proposed for use in the dam.

3-20. ***Foundation Stability.***—Joints, shears, and faults which form identifiable blocks of rock are often present in the foundation. Effects of such planes of weakness on the stability of the foundation should be carefully evaluated. Methods of analysis for foundation stability under these circumstances are discussed in section 4-75. Determination of shear resistance for such foundation conditions is discussed in section 3-4.

The factor of safety against sliding failure of these foundation blocks, as determined by the shear-friction factor, $Q$ using equation (3), should be greater than 4.0 for Usual Loading Combinations, 2.7 for Unusual Loading Combinations, and 1.3 for the Extreme Loading Combination. If the computed safety factor is less than required, foundation treatment can be included to increase the safety factor to the required value.

Treatment to accomplish specific stability objectives such as prevention of differential displacements (sec. 4-78) or stress concentrations due to bridging (sec. 4-79) should be designed to produce the safety factor required for the loading combination being analyzed.

## G. BIBLIOGRAPHY

3-21. ***Bibliography.***—

[1] "Concrete Manual," Bureau of Relamation, eighth eaition, 1975.

[2] "Properties of Mass Concrete in Bureau of Reclamation Dams," Concrete Laboratory Report No. C-1009, Bureau of Reclamation, 1961.

[3] Deere, D. U., Merritt, A. H., and Coon, R. F., "Engineering Classification of In Situ Rock," Technical Report No. AFWL-TR-67-144, Air Force Weapons Laboratory, Kirtland Air Force Base, N. Mex., January 1969.

[4] Christiansen, L.M., Von Thun, J. L., and Tarbox, G. S., "A New Method for Evaluating Foundations," Water Power, March 1971, London, England.

[5] Christiansen, L. M., Misterek, D. L., and Bowles, G. F., "Foundation Analysis of Auburn Damsite," Proceedings, International Symposium on Rock Mechanics, Nancy, France, 1971.

[6] Stagg, K. G., and Zienkiewicz, O. C., "Rock Mechanics in Engineering Practice," John Wiley & Sons, London, England, 1968.

[7] Von Thun, J. L., and Tarbox, G. S., "Deformation Moduli Determined by Joint Shear Index and Shear Catalog," Proceedings, International Symposium on Rock Mechanics, Nancy, France, 1971.

[8] "Physical Properties of Some Typical Foundation Rocks," Concrete Laboratory Report No. SP-39, Bureau of Reclamation, 1953.

[9] Link, Harald, "The Sliding Stability of Dams," Water Power–Part I, March 1969–Part II, April 1969–Part III, May 1969, London, England.

[10] Townsend, C. L., "Control of Cracking in Mass Concrete Structures," Engineering Monograph No. 34, Bureau of Reclamation, 1965.

[11] Monfore, G. E., "Experimental Investigations by the Bureau of Reclamation," Trans, ASCE, vol. 119, 1954, p. 26.

[12] Monfore, G. E., and Taylor, F. W., "The Problem of an Expanding Ice Sheet," Bureau of Reclamation Memorandum, March 18, 1954 (unpublished).

[13] Boggs, H. L., Campbell, R. B., Klein, I. E., Kramer, R. W., McCafferty, R. M., and Roehm, L. H., "Method for Estimating Design Earthquake Rock Motions," Bureau of Reclamation, April 1972.

## <<Chapter IV

# Layout and Analysis

4-1. *General.*—The order in which the topics are presented in this chapter does not necessarily represent the sequence that would be followed for design and analysis, but it does represent a logical chronology of four steps beginning with an initial design layout.

The design of a complex, highly indeterminate structure like an arch dam must include analysis as an integral part of the design procedure. The designer conceives a design and uses analytical methods to determine the stress distributions throughout the structure. The design is continually improved by alternately modifying the layout and checking the results of the analysis until the design objectives are achieved within the allowable design criteria. The four steps in designing a dam can be summarized as:

(1) Layout.
(2) Analysis.
(3) Evaluation.
(4) Modification.

Subchapter A is a discussion of layout procedures currently used by the Bureau of Reclamation and some general discussion concerning design philosophy. Methods used for comprehensive stress and stability analysis of concrete arch dams are discussed in subchapters B through F. The trial-load method of analysis, which was developed prior to 1940 [1, 2],[1] has been expanded and programed for an electronic computer. The computerized version, referred to as the Arch Dam Stress Analysis System (ADSAS), is now used for most stress studies. The reliability of the trial-load method has been confirmed by extensive research measurements [3, 4, 5]. The efficacy of the ADSAS has been demonstrated by comparisons with structural behavior measurements of prototypes [6, 7]. The chief limitation of the trial-load method is its complexity and the amount of time and labor required to make a complete analysis. With the advent of high-speed electronic computers and development of ADSAS, the average time required for a static analysis has been reduced to 15 minutes or less depending on the number of arch and cantilever elements used.

The development and application of static loads are included with the development of the trial-load method of analysis. Determination of the equivalent static loads from dynamic loadings, such as earthquake, are discussed in subchapter D.

A two-dimensional finite element analysis program is now being used to determine stress distributions around openings and near rapid changes in the geometry of the dam. It has also been useful in the determination of the deformation modulus for the foundation and the need for treatment of weak zones in the foundation and stress patterns in the fill concrete or abutment pads. Three-dimensional finite element computer programs are presently under development for the analysis of arch dams and their foundations.

Special studies made for Hoover Dam to show the effects of abutment spreading, foundation closing loads, strains in the canyon floor, canyon wall tilting, stresses in the canyon floor, and strain measurements are included in appendix I. Structural model tests and photoelastic analysis examples are also shown in appendix I.

---

[1] Numbers in brackets refer to items in the bibliography, section 4-80.

4-2. *Notations.*—The following is a list of notations and their meanings as used in this chapter. Other special symbols are included with the appropriate discussion.

*Dimensions and Properties*

$\Phi$ = central angle.
$P$ = unit load intensity.
$R_E$ = extrados (upstream face) radius.
$R_I$ = intrados (downstream face) radius.
$R_{axis}$ = radius of dam axis for arch computations, and distance from extrados center to axis for cantilever computations.
$R_D$ = distance from extrados center to downstream face.
$r$ = radius or distance from extrados center to centerline.
$L_L, L_R$ = arc length of arch centerline.
$s$ = arc length.
$F$ = function for determining properties and functions of uncracked portion of a section.
$t$ = temperature change in degrees Fahrenheit, the plus sign indicating a rise in temperature.
$c$ = coefficient of thermal expansion for concrete per degree Fahrenheit.
$\mu_r$ = Poisson's ratio for foundation material.
$\mu_c$ = Poisson's ratio for concrete.
$E_r$ = deformation modulus of foundation material.
$E_c$ = modulus of elasticity of concrete.
$T$ = thickness of dam element, in horizontal radial direction.
$a$ = short dimension of loaded foundation area, *ab*.
$b$ = long dimension of loaded foundation area, *ab*.
$G$ = modulus of elasticity of concrete in shear stress $= \frac{E_c}{2(1 + \mu_c)}$.
$\phi$ = angle which dam face makes with vertical.
$lg$ = distance from upstream face to center of gravity.
$A$ = area of cross section.
$I$ = moment of inertia of the cross section about a circumferential line through the center of gravity.

*Forces and Deflections*

$W$ = vertical force.
$M$ = bending moment.
~~$M$~~ = twisting moment.
$H$ = tangential force (thrust) in a horizontal plane.
$V$ = shear force.
$w$ = unit weight (without a subscript, assumed to be unit weight of water).
$h$ = vertical distance from water surface to horizontal cross section.
$e$ = eccentricity of a resultant force.
$p$ = water pressure.
$U$ = uplift force on horizontal section.
$\Delta r$ = radial deflection of centerline.
$\Delta s$ = tangential deflection of centerline.

$\theta$ = angular movement, in horizontal planes, about centerline.
$\Delta\nu$ = vertical deflection of centerline.
$A_1$ = angular movement at a point due to a unit moment at the point.
$B_1$ = angular movement at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit negative moment at the point.
$B_2$ = radial deflection at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit negative shear at the point.
$B_3$ = tangential deflection at a point due to a unit negative thrust at the point.
$C_1$ = angular movement at a point due to a unit shear at the point; or, it is the radial deflection at a point due to a unit moment at the point.
$C_2$ = radial deflection at a point due to a unit shear at the point.
$D_1$ = angular movement at a point due to loads between the point and the abutment.
$D_2$ = radial deflection at a point due to loads between the point and the abutment.
$D_3$ = tangential deflection at a point due to loads between the point and the abutment.
$\Psi$ = angle from point where loading begins to any differential element of the arch under load.
$\epsilon$ = unit strain.
$\rho$ = curvature at a cantilever point in a vertical direction; or, the curvature at an arch point in a horizontal direction.

*Abutment Constants*

$\psi$ = angle between vertical plane and plane of foundation surface.
$\alpha'$ = average rotation of foundation in a plane normal to foundation surface, due to bending-moment load.
$\beta'$ = average deformation of foundation normal to foundation surface, due to normal load.
$\gamma'$ = average deformation of foundation in plane of foundation surface, due to tractive or shear load.
$\delta'$ = average rotation of foundation in plane of foundation surface, due to twisting-moment load.
$\alpha''$ = average rotation of foundation in a plane normal to foundation surface, due to tractive load.
$\gamma''$ = average deformation of foundation in plane of foundation, due to bending-moment load.
$\enclose{circle}{\alpha'}$, $\enclose{circle}{\gamma'}$, $\enclose{circle}{\alpha''}$, and $\enclose{circle}{\gamma''}$ are at right angles to the directions of the corresponding foundation deformations not designated by the circles.
$\alpha$ = for cantilevers, angular movement of foundation in vertical radial plane due to one unit of bending moment, $M_a$, per unit length normal to plane of cantilever; for arches, angular movement of abutment due to unit moment at abutment.

$\beta$ = tangential movement of arch abutment due to unit thrust at abutment.

$\textcircled{\gamma}$ = for cantilevers, radial movement of foundation due to one unit of radial shear force, $V_a$, per unit length normal to plane of cantilever; for arches, radial movement of arch abutment due to unit shear at abutment.

$\gamma$ = tangential movement of foundation due to one unit of tangential shear force, $H_a$, per unit length normal to plane of cantilever.

$\delta$ = angular movement of foundation in horizontal plane due to one unit of twisting moment, $\cancel{M_a}$, per unit length normal to plane of cantilever.

$\alpha_2$ = for cantilevers, angular movement of foundation in radial plane due to one unit of radial shear force, $V_a$, per unit length normal to plane of cantilever, or radial movement of foundation due to one unit of bending moment, $M_a$, per unit length normal to plane of cantilever; for arches, angular movement of arch abutment due to unit shear at abutment; or radial movement of arch abutment due to unit moment at abutment.

*Stresses*

$\sigma_x$ = horizontal arch stress normal to a vertical radial plane.

$\sigma_y$ = horizontal radial stress normal to a vertical tangential plane.

$\sigma_z$ = vertical cantilever stress normal to a horizontal plane.

$\eta$ = angle a horizontal line tangent to a face makes with normal to radial arch section.

$\xi$ = angle first principal stress makes with the vertical axis.

*Subscripts*

$L$ = left side
$R$ = right side
$O$ = crown
$a$ = abutment
$E$ = extrados
$I$ = intrados
$D$ = downstream face
$r$ = rock
$c$ = concrete
$w$ = water
$h$ = horizontal }
$v$ = vertical } used as prefix subscript
$A$ = arch
$CA$ = cantilever
$u$ = uplift
$i$ = initial
$m$ = maximum
$f$ = fillet
$W$ = wedge
$g$ = crack
$v$ = voussoir
$p$ = arch point
$TA$ = tangential

$N$ = normal

*Constants*

$$\left.\begin{array}{l} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \end{array}\right\} = \text{foundation-abutment constants}$$

$K_6$ = shear detrusion ratio

$K_o$ = used in determining arch forces at crowns

$$\left.\begin{array}{c} J \\ K \\ L \\ Q \\ R \\ S \end{array}\right\} = \text{constants for transferring } D\text{-terms}$$

## A. LAYOUT OF DAM

4-3. ***Current Practice.***–A layout drawing for an arch dam includes a plan, profile, and section along the reference plane. The drawing is made to describe the dam geometrically and to locate the dam in the site. The necessary data for analyzing the structure are also taken from the layout drawing.

The primary objective in making a layout for an arch dam at a particular site is to determine the arches which will fit the topographic and geologic conditions most advantageously, provide for the installation of adequate facilities for reservoir operation, and distribute the load with the most economical use of materials within allowable stress limitations. The load distribution, and the stresses resulting from such distribution, depend largely on the shape of the canyon, length and height of dam, shape and thickness of dam, and loading conditions.

In producing a satisfactory design, the engineer conceives and constructs a design layout, makes a stress analysis for the design, reviews the results to determine appropriate changes in the design shape which will improve the stress distributions, and draws a new design layout incorporating the changes. The process is repeated until a design is evolved which meets the following criteria as nearly as practicable:

(1) A uniformly varying distribution of stress.

(2) A compressive stress level throughout as nearly equal as practicable to the defined allowable limits.

(3) A minimum volume of concrete.

It is very difficult to design an arch dam which has compressive stresses throughout that are near the maximum allowable and is still economical. Therefore, a good design is usually a compromise solution which yields a dam that has some very low compressive stresses and may even have limited zones of tensile stresses, provided they are within the allowable defined limits. Nevertheless, the three objectives of a good design, as stated above, still represent the goal toward which to strive.

Computer-assisted layout capabilities are planned for development in the near future which should expedite the layout procedure and produce optimum designs.

4-4. ***Level of Design.***–There are three levels of design for which layouts are made–appraisal, feasibility, and final. The level of a design determines the degree of refinement to which the design is taken. An appraisal design is made in conjunction with field investigations during project planning to estimate the concrete volume required for use in determining the feasibility of the project. Appraisal designs may be based on previous designs which are similar in height and shape of profile and for which the stresses are satisfactory. Another means of determining the information is by using the nomographs given in appendix A. The nomographs were constructed from empirical formulas which were derived by statistical analyses of several analytical studies already completed for previously designed dams. A formal development is given for the nomographs and equations in reference [38].

Feasibility designs are used in the selection of the final location and design as a basis for a request for construction funds. Feasibility designs are made in greater detail than appraisal designs since a closer approximation to final design is required. Analyses should be made of the adopted structure for the usual and the most severe loading conditions expected in actual service. The best of the alternative designs will have stresses distributed as uniformly as possible within allowable limits and will have a minimum cost.

Final designs are used to develop specification drawings and construction drawings. Complete analyses using ADSAS are made for normal operating loading conditions and dynamic loading conditions due to earthquake ground accelerations. The temperature distribution applied during final design is determined using detailed analysis and is based on finalized temperature and operating data. The final design will be one which best satisfies the requirements for acceptable stresses most economically.

4-5. ***Required Data.***—The principal data which should be on hand before preparing a layout are: (1) a topographic map of the proposed location, (2) geological data from which rock types and characteristics, depth of overburden, and the locations of faults, jointing, etc., can be obtained, (3) reservoir water surface and tailwater elevations, (4) probable sediment accrual in the reservoir, and (5) the size and location of openings in the dam required for spillways, outlets, etc. These data should be supplemented by (1) climatological records for studies of temperature variations within the dam, and (2) laboratory and possibly in situ tests for determining the strength and elastic properties of the rock and concrete.

4-6. ***Procedure.***—A single-centered, variable-thickness arch is assumed for the purpose of discussion. The procedure for laying out other types of arch dams differs only in the way the arches are defined. The following dimensions are parameters used in empirical formulas to determine initial values of the axis radius and thicknesses of the crown cantilever: $H$, structural height in feet (vertical distance from the crest of the dam to the lowest assumed point of the foundation); $L_1$, straight line distance in feet at crest elevation between abutments excavated to assumed sound rock; $L_2$, straight line distance in feet between abutments excavated to assumed sound rock and measured at an elevation 0.15 $H$ feet above the base.

(a) *Determination of $R_{axis}$ and Central Angle.*—The first step in making a layout is to draw a tentative axis for the dam in plan on transparent paper. The paper is overlaid on the site topography and shifted about until an optimum orientation for the axis is found; this will be one for which the angle of incidence to the surface contour at the crest elevation is approximately equal on each side. The following formula can be used as a guide to selecting a tentative radius for the axis:

$$R_{axis} = 0.6\,L_1$$

Because empirical formulas are only a guide to choosing $R_{axis}$, the designer should make appropriate adjustments in $R_{axis}$, so that the central angle of the top arch and intersection of the axis with the topography are satisfactory. The top arch and the axis are discussed interchangeably because the extrados and the axis have the same radius by definition. The magnitude of the central angle of the top arch is a controlling value which influences the curvature of the entire dam. Objectionable tensile stresses will develop in arches of insufficient curvature, such a condition being apt to occur in the lower elevations of a dam having a V-shape profile. The largest central angle practicable should be used, and consideration given to the fact that the bedrock topography may be inaccurately mapped and the arch abutments may need to be extended to points of somewhat deeper excavation than originally planned. Owing to limitations imposed by topographic conditions and foundation requirements, it will be found that, for most layouts, the largest practicable central angle for the top arch varies between $90^\circ$ and $110^\circ$.

(b) *Defining the Reference Plane and Crown Cantilever.*—The next step after the axis has been located is to define the reference plane and crown cantilever section. The crown cantilever is usually located at the point of maximum depth. The reference plane for a single-centered dam is a vertical plane which passes through the crown cantilever and the $R_{axis}$ center. Ideally, the reference plane should be at the midpoint of the axis. This seldom occurs, however, because most canyons are not symmetrical about their low point.

After the crown cantilever and reference plane have been located, the thickness and shape of the crown cantilever should be determined. Proportioning the crown cantilever is facilitated by considering separately the top thickness, intermediate thickness, and base thickness (see fig. 4-1). Estimates for these crown cantilever thicknesses may be computed using the following empirical equations. The equations are to be used as guides and only for initial layouts.

(1) Crest thickness, in feet,

$$T_C = 0.01(H + 1.2L_1)$$

(2) Base thickness, in feet,

$$T_B = \sqrt[3]{0.0012\ H L_1 L_2 \left(\frac{H}{400}\right)^{\frac{H}{400}}}$$

(3) Thickness at 0.45 $H$, in feet,

$$T_{0.45H} = 0.95\ T_B$$

Nomographs from which $T_C$ and $T_B$ can be obtained are given in appendix A, figures A-2 and A-3, respectively. After values for thickness have been determined, they can be divided into upstream and downstream projections (see sec. 1-4) according to the following formulas:

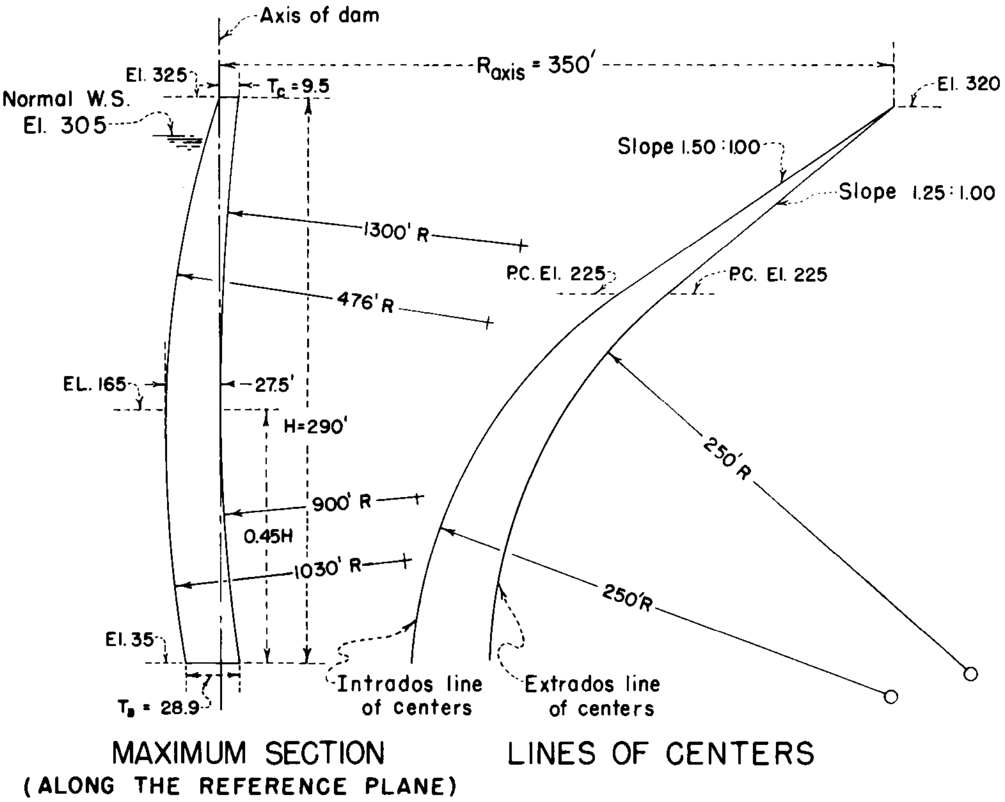


*Figure 4-1.* **Crown cantilever and lines of centers for a preliminary design.—288-D-2961**

| Thickness | Upstream projection | Downstream projection |
|---|---|---|
| At crest | 0.0 | $T_C$ |
| At 0.45 $H$ | 0.95 $T_B$ | 0.0 |
| At base | 0.67 $T_B$ | 0.33 $T_B$ |

The crown cantilever can be constructed after the controlling thicknesses have been determined, as shown on figure 4-1.

(c) *Laying Out the Arches.*—The next step is to draw the arches in plan at convenient elevations for the stress analysis. Usually 5 to 10 arch elevations are selected such that the entire dam is represented by a system of horizontal elements evenly spaced over the height of the structure. Usually the intervals are not greater than 100 feet nor less than 20 feet. The radius centers defining the intrados and extrados for each arch elevation are plotted along the reference plane of the dam. The locations of the radius centers are a function of the canyon width at the particular arch elevation, the required rise for the arch, and the abutment thickness. The radius centers are determined by selecting trial positions until a location is found which defines the desired arch shape. To ensure that the dam is smooth in both the horizontal and vertical directions, the arch radii centers must lie along the reference plane in plan and be connected by smooth, continuous curves in elevation, called lines of centers. Slight adjustments in the trial locations of the arch radii centers are usually necessary as a result. Figures 4-1 and 4-2 show examples of how the arch radii centers are defined. The plan is completed by drawing in the arch

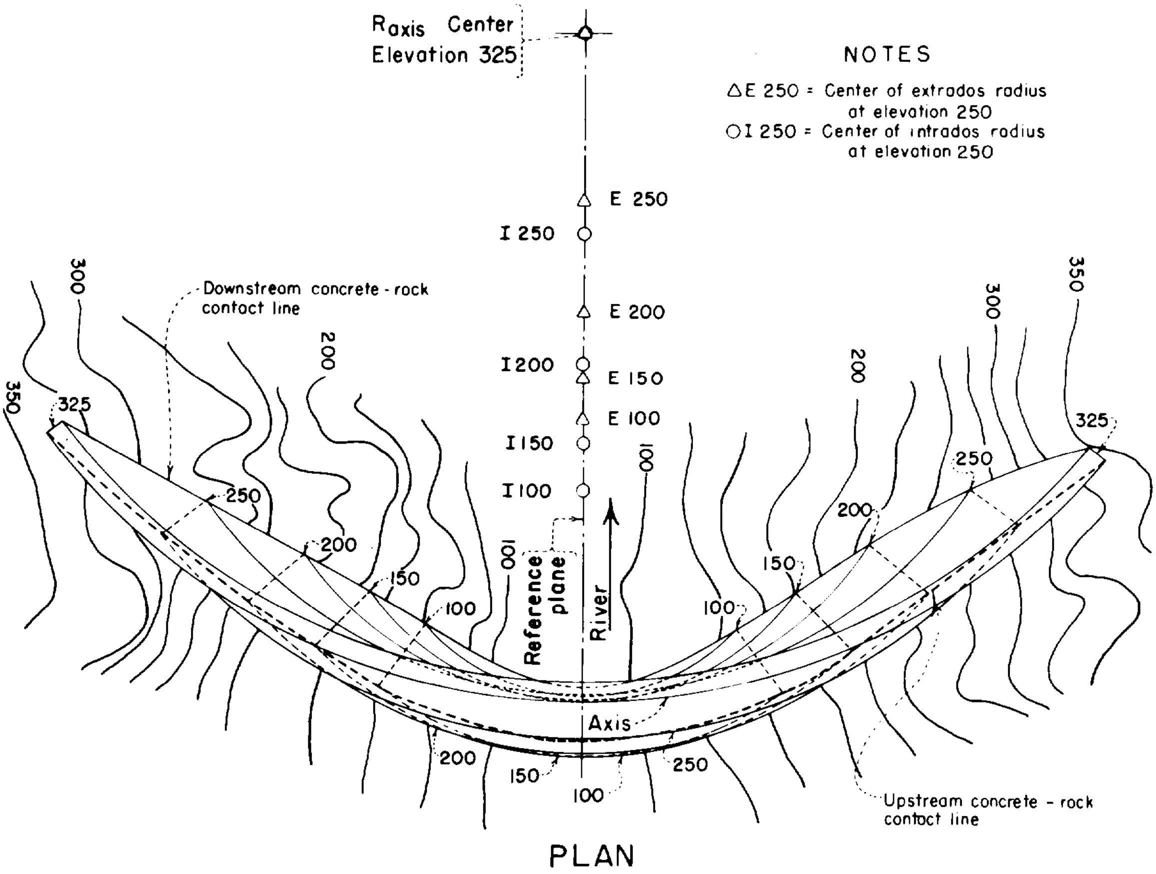


*Figure 4-2.* **Plan for a preliminary design of a single-centered arch in a nearly symmetrical canyon.—288-D-2962**

abutment contacts and the perimetrical contact of the dam and foundation as shown on figure 4-2. The perimetrical contact should be smooth and continuous. This may also require adjustments in the radius centers.

The final step in constructing a layout is to draw a profile of the dam developed along the axis. Surface irregularities such as ridges, depressions, or undulation in the profile will be revealed. Pronounced anomalies should be removed by reshaping the affected arches until a smooth profile is obtained. Smoothness may be thought of as a continuous uniformly varying change in slope along the excavated surface.

**4-7. *Factors to be Considered in the Layout*.**—All factors affecting the layout should be considered in designing a dam. These will include:

(a) *Length-Height Ratio.*—The length-height ratios of dams may be used as a basis for comparison of proposed designs with existing designs. Such comparisons should be made in conjunction with the relative effects of other controlling factors such as central angle, shape of profile, and type of layout. The length-height ratio also gives a rough indication of the economic limit of an arch dam as compared with a dam of gravity design. Generally, the economic limit of an arch dam occurs for a maximum length-height ratio between 4 to 1 and 6 to 1, depending somewhat on the height of dam and local conditions. Even if the length-height ratio for an arch dam falls within the economic range, the combined cost of dam and spillway may be such that another type of dam would be more economical.

(b) *Symmetry.*—Although not an absolute necessity, a symmetrical or nearly symmetrical profile is desirable from the standpoint of stress distribution. A region of stress concentration is likely to exist in an arch dam having a nonsymmetrical profile, a condition tending toward an uneconomical section compared with that of a symmetrical dam. In some cases improvements of a nonsymmetrical layout by one or a combination of the following methods may be warranted: by excavating deeper in appropriate places, by constructing an artificial abutment, or by reorienting and/or relocating the dam.

When a nonsymmetrical canyon is encountered such that a single-centered arch cannot be satisfactorily fitted to the site, a two-centered scheme can usually be used to define the dam. This type of layout is constructed by using two separate pairs of lines of centers, one for each side of the dam. In order to maintain continuity, however, each pair of lines must lie along the reference plane. In some cases the axis radius ($R_{axis}$) may be different on each side, and the arches may be uniform or variable in thickness. Figure 4-3 is an example of a two-centered arch.

(c) *Canyon Shape.*—In dams constructed in U-shaped canyons, the lower arches have chord lengths nearly as long as those near the top. In such cases, use of a variable-thickness arch layout will normally give a relatively uniform stress distribution. Undercutting on the upstream face may be desirable to eliminate areas of tensile stress at the bases of cantilevers.

In dams having narrow V-shape profiles, the lower arches are relatively short, and the greater portion of the load is carried by arch action. From the standpoint of avoiding excessive tensile stresses in the arch, a type of layout should be used which will provide as much curvature as possible in the arches. This may be accomplished by using variable-thickness arches with a variation in location of centers to produce greater curvature in the lower arches. Figure 4-4 shows an example of a two-centered variable-thickness arch dam for a nonsymmetrical site.

Assuming for comparison that factors such as central angle, height of dam, and shape of profile are equal, the arches of dams designed for wider canyons would be more flexible in relation to cantilever stiffness than those of dams in narrow canyons, and a proportionately larger part of the load would be carried by cantilever action. In dams for wide canyons in which there is a tendency for cantilever stresses to be greater than arch stresses, it is desirable to obtain the maximum possible advantage from dead weight by using a crown section having both faces curved, with undercutting at

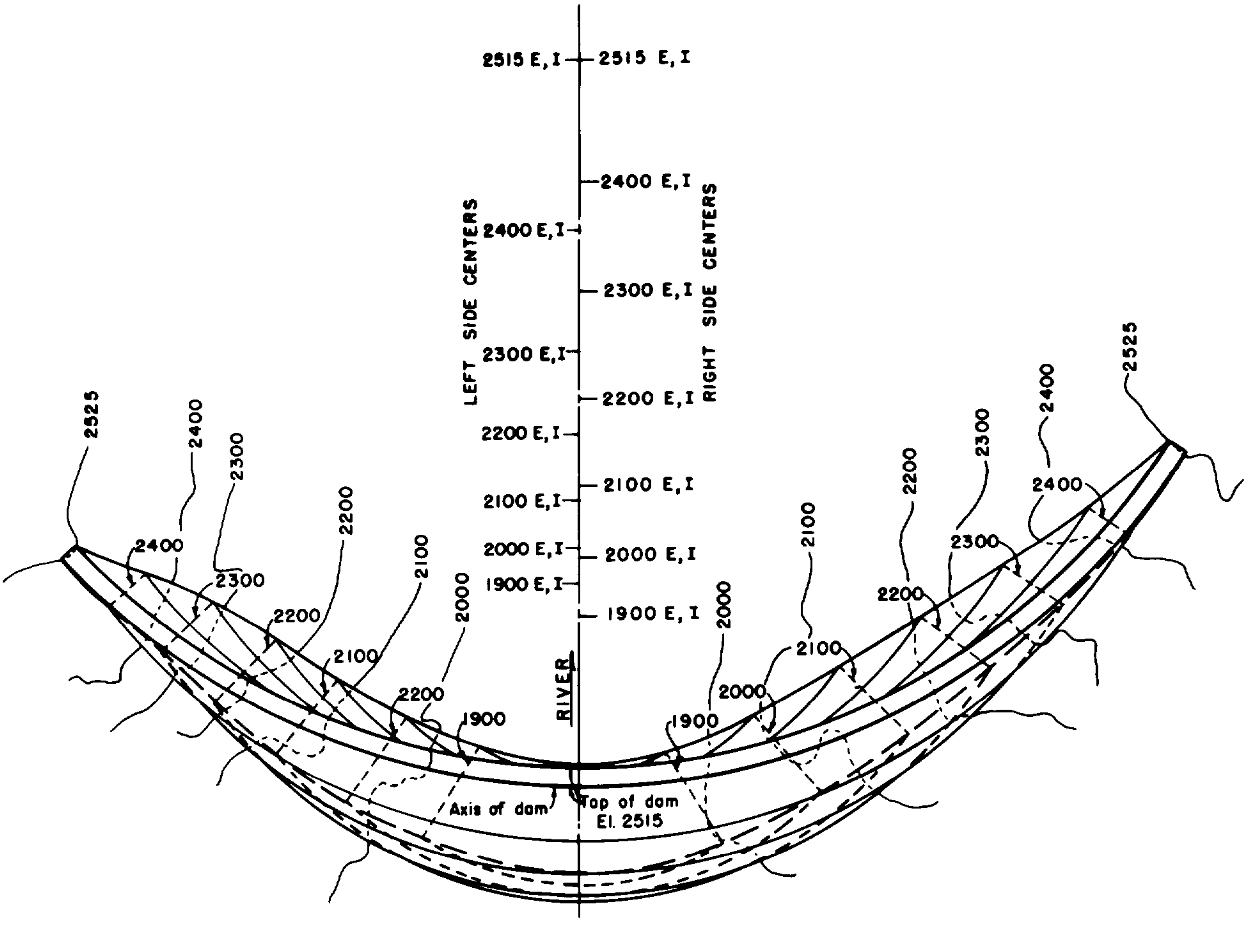


*Figure 4-3.* **Two-centered arch dam with uniform-thickness arches.–288-D-2963**

the base of the upstream face and an overhang at the top of the downstream face. The layout would normally be a variable-thickness or polycentered-type arch.

(d) *Arch Shapes.*—In most cases, uniform-thickness arches may be used in the upper part of the dam since the thinner, longer arches are more flexible and do not carry as much of the load as those in the lower portion of the dam. The need for additional thickness at the abutments will vary with each layout, but extra thickness is normally not required in the upper few arches.

In the most efficient and economical design, the stresses approach uniform values close to the established allowable limits. Variable-thickness arches will have a more uniform stress distribution than arches with fillets, since the thickness varies gradually without any change in curvature. Varying the thickness of the arches also results in adequate thickness for those cantilevers with bases near the midheight of the dam. The angle of intersection of the intrados and a line generally parallel with the corresponding surface contour should be not less than 30° to ensure stability of the arch abutments. If the angle of intersection is less than 30°, a special study should be made to evaluate the abutment stability.

When uniform-thickness arches are used and abutment thickening is desirable, short-radius fillets may be added to the downstream face. The length of radii used at one side need not be equal to that used at the other side. The fillet centers at each side of the dam should fall on smooth curves in plan, to avoid irregularly warped surfaces. The locus of points of

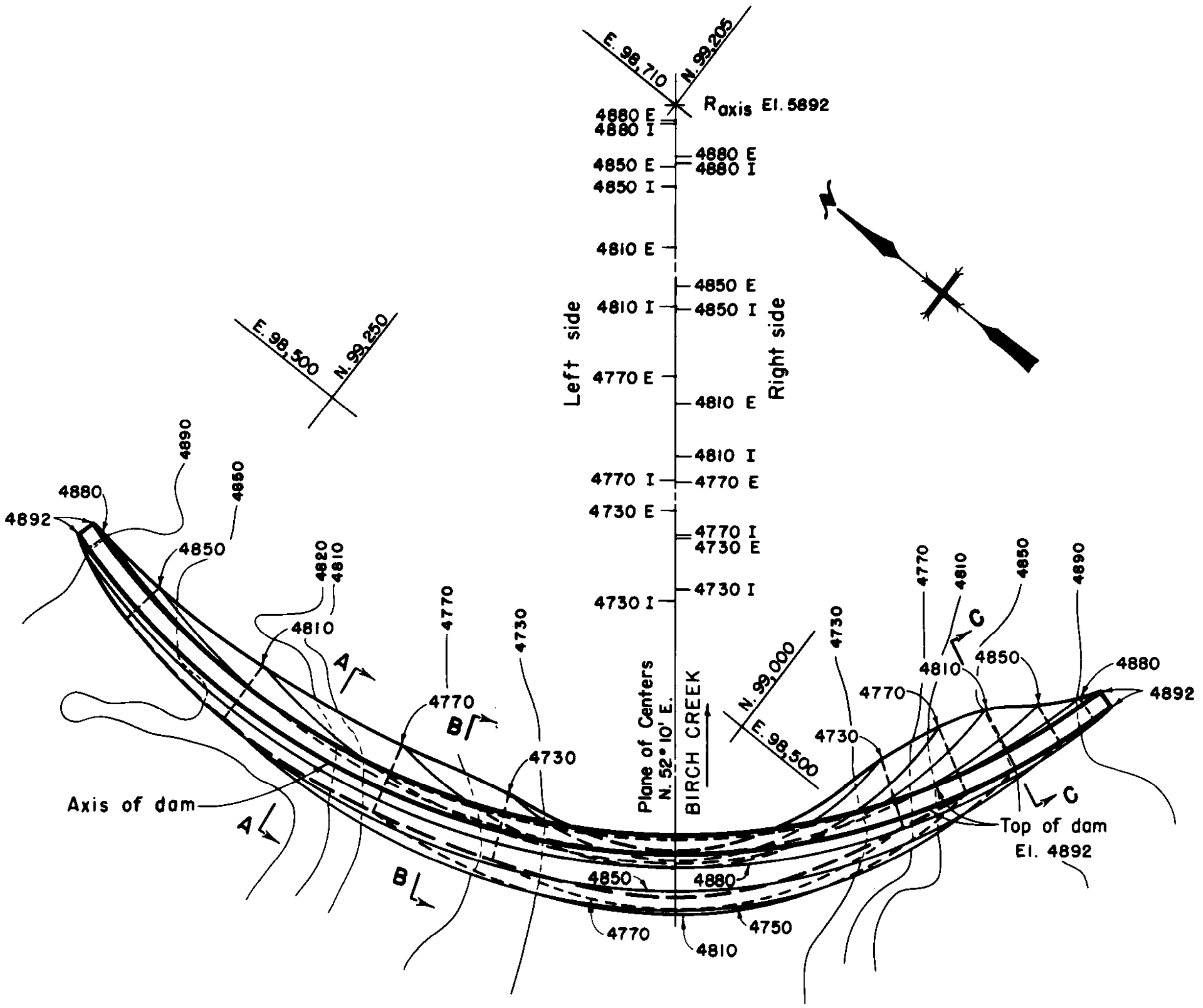


*Figure 4-4.* **Two-centered arch dam with variable-thickness arches at a nonsymmetrical site.–288-D-2964**

tangency between the intrados of the arches and the fillets at each side of the dam, called the trace of beginning of fillets, also requires a smooth curve.

Fillet radii should have enough length to ensure that the resultants of arch forces are directed safely into the abutment rock, and that curvatures at the downstream face of both the arch and cantilever elements are not so great as to produce excessive stresses parallel with the face of the dam. For this reason, the angle of intersection between fillet and arch abutment should be greater than 45°. Fillets should be laid out, as a general rule, so that the traces of beginning of fillets will intersect the top arch at its abutment and intersect approximately the three-fourths points of the arches in the region of greatest arch abutment stresses, that is, at about one-half to three-fourths of the height of the dam above the base. Figure 4-5 shows an example of short-radius fillets used with arches of uniform thickness.

Three-centered or elliptical arches can be used advantageously in wide U- or V-shaped canyons. Elliptical arches have the inherent characteristic of conforming more nearly to the line of thrust for wide sites than do circular arches. Consequently, the concrete is stressed more uniformly throughout its thickness. Because of the smaller influences from moments, elliptical arches require little if any

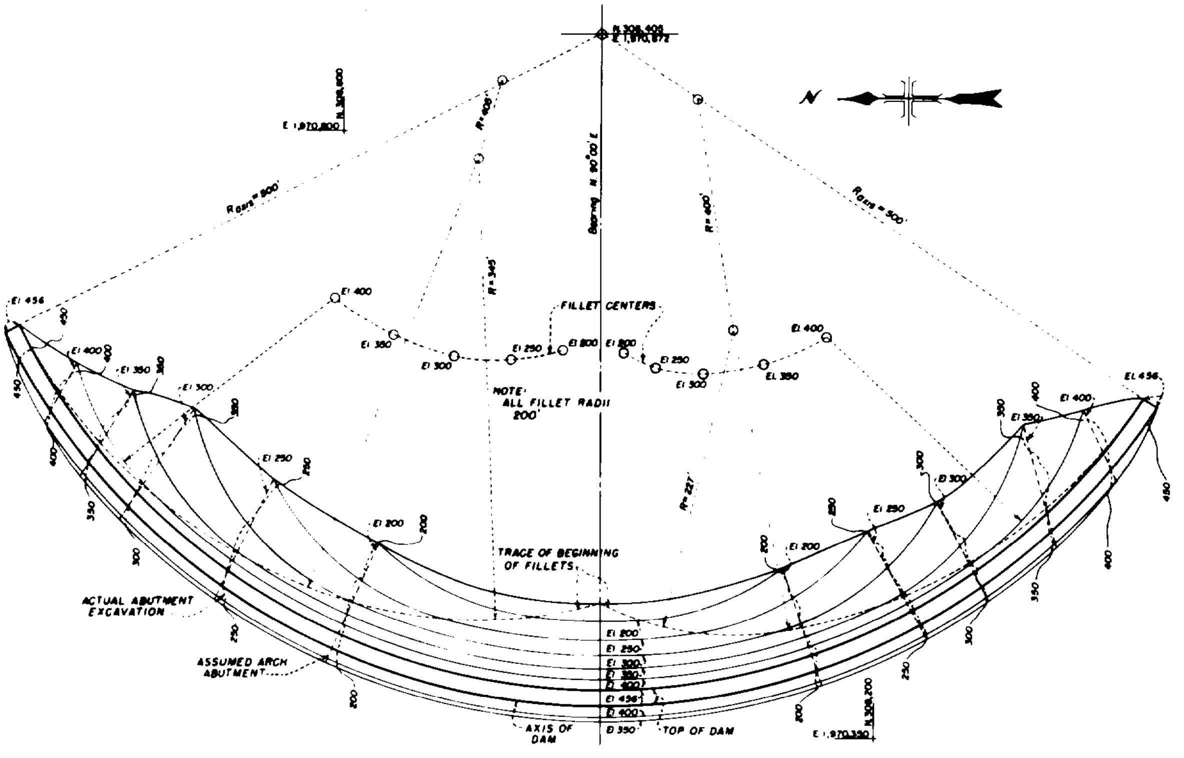


*Figure 4-5.* **Uniform-thickness arches with short-radius fillets.–288-D-2965**

variable thickness. The direct benefit is less required volume and consequently increased economy. Figure 4-6 shows a typical three-centered arch.

(e) *Arch Abutments.*—Care must be taken to make sure that the arch abutments are well keyed into sound rock, and that there is sufficient rock mass to withstand the applied loads. The directions of joint systems in the rock should be given careful consideration in making the layout, to ensure stable abutments under all conditions of loading.

Full-radial arch abutments (normal to the axis) are advantageous for good bearing against the rock, but where excessive excavation at the extrados would result from the use of full-radial abutments, and the rock has the required strength and stability, the abutments may be reduced to half-radial as shown on figure 4-7(a). Where excessive excavation at the intrados would result from the use of full-radial abutments, greater than radial abutments may be used as shown on figure 4-7(b). In such cases, shearing resistance should be carefully investigated. Where full-radial arch abutments cannot be used and excessive excavation would result from the use of either of the two shapes mentioned, special studies may be made for determining the possible use of other shapes having a minimum of excavation. These special studies would determine to what extent the arch abutment could vary from the full-radial and still fulfill all requirements for stability and stress distribution.

Artificial abutments such as pads or thrust blocks may be desirable in many layouts to provide symmetry of profile, as previously mentioned, or to reduce the span of the arches toward the top of the dam. Where topographical conditions are favorable, a thrust block may sometimes be utilized advantageously to provide an overflow section for a spillway. Thrust blocks must be designed to transfer all applied loads into the rock adequately. Resultant forces should be considered in the analysis, because their magnitude and direction represent the combined effects of all the applied forces.

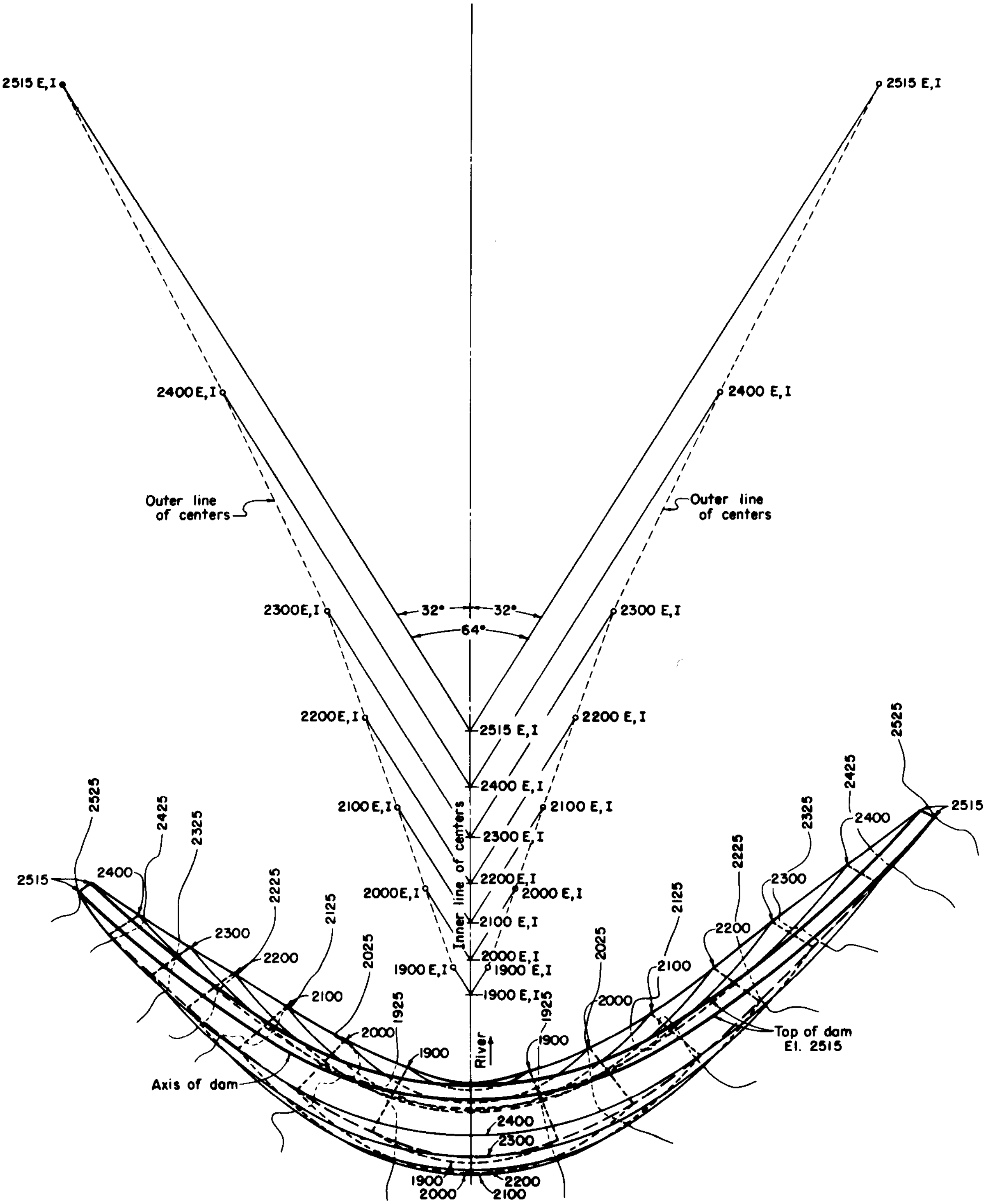
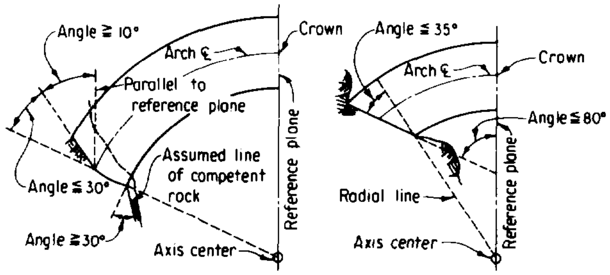


*Figure 4-6.* Three-centered arch dam with uniform-thickness arches.–288-D-2966



(a) HALF-RADIAL ARCH ABUTMENT (b) GREATER THAN RADIAL ABUTMENT

*Figure 4-7.* **Arch abutment types.–288-D-2967**

(f) *Spillway Through Arch.*–Only those aspects of spillway design which influence the design of the dam will be considered here. In the event an overflow type of spillway is to be used, no arch action is considered to take place above the crest elevation of the spillway. The upper portion of the dam must be designed, therefore, to withstand the gravity effects of the loading imposed above the crest by water pressure, concrete mass, piers, bridge, gates, and gate operating mechanisms.

(g) *Freeboard.*–Current practice in the Bureau of Reclamation is to allow the maximum water surface to equal the elevation of the top of the nonoverflow dam and depend on the standard height (3.5-foot) solid parapet to act as freeboard. Exceptional cases may point to the need for more freeboard depending on the anticipated wave heights.

(h) *Corbeling.*–Specified widths for roadways, walks, curbs, and parapets often require a greater thickness at the top of a dam than is necessary for satisfactory arch and cantilever stresses. In lieu of thickening the dam, corbeling of the walkway and parapet along the extrados or intrados of the top arch may be used to provide the required additional width. Wide corbeling along the upstream face is not recommended in climates where forces caused by ice might be excessive.

4-8. ***Analyses.***–The dam, as defined by the layout, is analyzed for stresses and deflections due to the applied loads. Stress analyses are made using methods discussed in subchapters B, C, D, and E.

4-9. ***Evaluation of Analyses.***–The results of an analysis serve two purposes. The designer can evaluate the adequacy of the design, and if improvement is required, the designer can utilize the analysis to ascertain the appropriate modifications to be made.

Evaluation requires a thorough examination of all the analytical output. The following represents the type of information to be reviewed: crown cantilever description; intrados and extrados lines of centers; geometrical statistics; dead load stresses and stability of blocks during construction; radial, tangential, and angular deflections; loading distributions; arch and cantilever stresses; and principal stresses. If any aspect of the design is either incorrect or does not comply with established criteria, modifications must be made to improve the design.

4-10. ***Modifications to the Layout.***–The primary means of effecting changes in the behavior of the dam is by adjusting the shape of the structure. Whenever the overall stress level in the structure is below the allowable limits,concrete volume can be reduced, thereby utilizing the remaining concrete more efficiently and improving the economy. Following are some examples of how a design can be improved by shaping:

(1) When cantilevers are too severely undercut, they are unstable and tend to overturn upstream during construction. The cantilevers must then be shaped to redistribute the dead weight such that the sections are stable.

(2) If an arch exhibits tensile stress on the downstream face at the crown, one alternative would be to reduce the arch thickness by cutting concrete from the downstream face at the crown while maintaining the same intrados contact at the abutment. Another possibility would be to stiffen the crown area of the arch by increasing the horizontal curvature which increases the rise of the arch.

(3) Load distribution and deflection patterns should vary smoothly from point to point. Often when an irregular pattern occurs, it is necessary to cause load to be shifted from the vertical cantilever elements to the horizontal arches. Such a transfer can be effected by changing the

stiffness of the cantilever relative to the arch.

Shaping is the key to producing a complete and balanced arch dam design. The task of the designer is to determine where and to what degree the shape should be adjusted. Figure 4-8 has been included to help the designer determine appropriate changes in the structural shape. If an unsatisfactory stress condition exists, the forces causing those stresses and the direction in which they act can be determined from figure 4-8. For example, the equations of stress indicate which forces combine to produce a particular stress. Knowing the force involved and its algebraic sign, it is possible to determine its direction from the sign convention shown on the figure. With that information the proper adjustment in shape can be made so that the forces act to produce the desired stresses.

There are two fundamental principles to follow in the design of an arch dam. The first is to keep the design simple and ensure that all surfaces vary smoothly without abrupt changes in direction. The second is to remember the structure is a continuum, and therefore the behavior of the entire dam must be considered whenever any change in shape is contemplated.

## B. TRIAL-LOAD METHOD OF ANALYSIS

4-11. ***Introduction.***—The trial-load method is based on the assumption that the waterload is divided between arch and cantilever elements; that the division may or may not be constant from abutment to abutment for each horizontal element; and that the true division of load is the one which causes equal arch and cantilever deflections at all points in all arches and cantilevers instead of at the crown cantilever only. Furthermore, the method assumes that the distribution of load must be such as to cause equal arch and cantilever deflections in all directions; that is, in tangential and rotational directions as well as in radial directions. To accomplish the preceding agreement, it is necessary to introduce internal, self-balancing trial-load patterns on the arches and cantilevers.

4-12. ***Types of Trial-Load Analyses.***—Trial-load analyses may be classified according to their relative accuracy and corresponding complexity. Progressing from the simplest to the most comprehensive, these analyses are called crown-cantilever analysis, radial deflection analysis, and complete trial-load analysis.

A *crown-cantilever analysis* consists of an adjustment of radial deflections at the crown cantilever with the corresponding deflections at the crowns of the arches. This type of analysis assumes a uniform distribution of radial load from the crowns of the arches to their abutments, and neglects the effect of tangential shear and twist. While the results obtained from this analysis are rather crude, it has the advantage of requiring a very short time to complete. When used with judgment it is a very effective tool for appraisal studies.

A *radial deflection analysis* is one in which radial deflection agreement is obtained at arch quarter points with several representative cantilevers by an adjustment of radial loads between these structural elements. With the use of this type of analysis, loads may be varied between the crowns and abutments of arches, thus producing a more realistic distribution of load in the dam. The effects of nonsymmetry may also be included in this analysis. The time required to complete a radial deflection analysis is only slightly greater than that necessary for a crown-cantilever analysis. Since the effects of tangential shear and twist are neglected in this analysis, the results are not complete, but do furnish a much better estimate of the stresses than is possible from a crown adjustment. A radial deflection analysis may be used for a feasibility study.

With the *complete trial-load analysis*, agreement of three linear and three angular displacements is obtained by properly dividing the radial, tangential, and twist loads between the arch and cantilever elements. The accuracy

$$\sigma_z = \frac{W_C}{A_C} \pm \frac{M_C c}{I_C}\ ; \quad \sigma_z'' = \sigma_z \sec^2\phi - p\tan^2\phi - 2\tau_{xz}\tan\eta\tan\phi$$

$$\sigma_x = \frac{H_a}{A_a} \pm \frac{M_a c}{I_a}; \quad \sigma_x'' = \sigma_x \sec^2\eta - p\tan^2\eta - 2\tau_{xz}\tan\phi\tan\eta$$

$$\tau_{xz} = \frac{-V_{TA}}{A_C} \pm \frac{-M_{TW} c}{I_C}\ ; \quad \tau_{xz}' = (\tau_{xz}\cos\eta \pm \tau_{zy}\sin\eta)\sec\phi'$$

$$\sigma_z' = \sigma_z \sec^2\phi' - p\tan^2\phi'\ ; \quad \tan\phi' = \tan\phi\cos\eta$$

$$\sigma_p = \frac{\sigma_z' + \sigma_x''}{2} \pm \sqrt{\left(\frac{\sigma_z' - \sigma_x''}{2}\right)^2 + \left(\tau_{xz}'\right)^2}$$

$$\tan 2a = \frac{-2\tau_{xz}'}{\sigma_z' - \sigma_x''}$$

Note: All forces and moments shown are positive.

The plus sign in the stress equations corresponds to those values computed for the upstream face and the minus for downstream face.

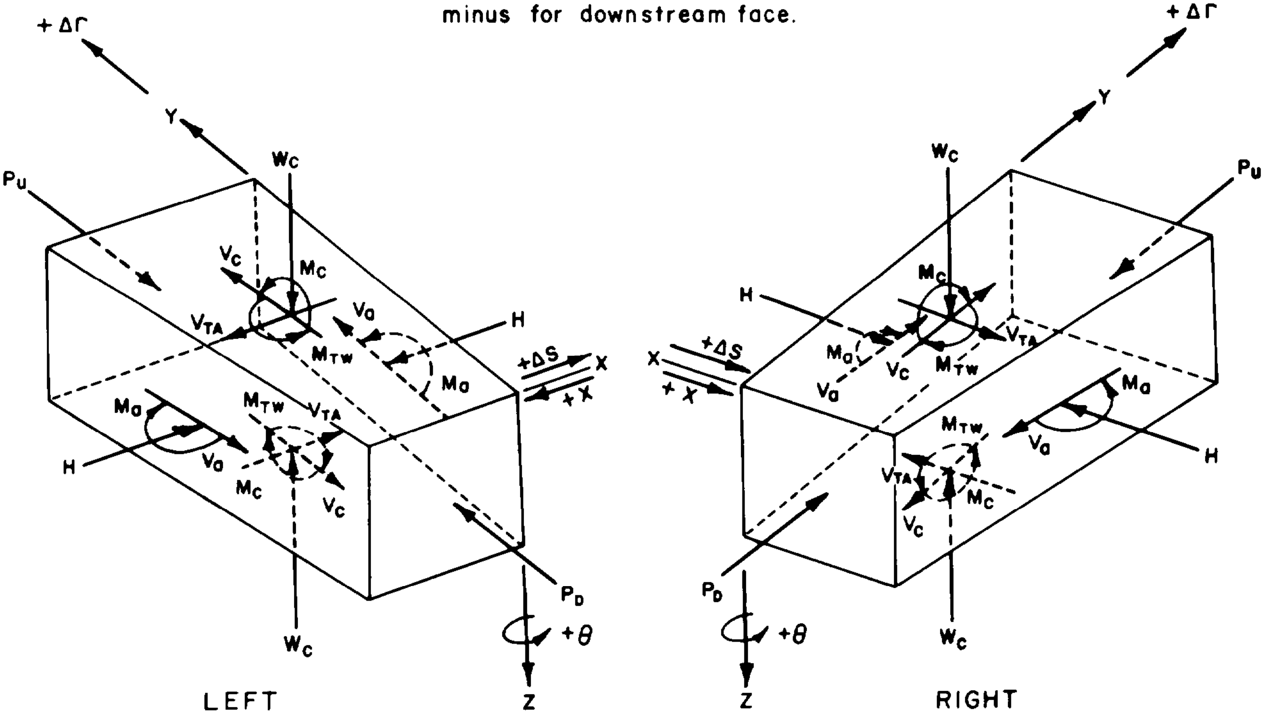


*Figure 4-8.* **Sign convention for arch computations (looking upstream).–288-D-2968**

of this analysis is limited only by the exactness of the basic assumptions, the number of horizontal and vertical elements chosen, and the magnitude of error permitted in the slope and deflection adjustments. A complete trial-load analysis should be made for a specifications design.

To illustrate and compare the three types of analyses, results of each are shown for the specifications design of Yellowtail Dam.

Table 4-1.–*Crown-cantilever analysis.*

Arch and Cantilever Stresses Normal to Extrados Radius

| Elevation | Arch |  |  |  | Crown cantilever |  |
|---|---|---|---|---|---|---|
|  | Abutment |  | Crown |  |  |  |
|  | Ext. | Int. | Ext. | Int. | US | DS |
| 3660 | +342 | + 703 | +619 | +424 | 0 | 0 |
| 3600 | +319 | + 901 | +769 | +443 | +127 | – 29 |
| 3550 | +263 | + 977 | +828 | +415 | +209 | – 37 |
| 3500 | +153 | +1,059 | +933 | +369 | +247 | 0 |
| 3450 | + 33 | +1,037 | +983 | +278 | +262 | + 61 |
| 3400 | – 55 | + 968 | +973 | +171 | +256 | +143 |
| 3350 | –104 | + 867 | +907 | + 74 | +226 | +245 |
| 3300 | –112 | + 718 | +771 | + 3 | +164 | +375 |
| 3250 | – 75 | + 555 | +606 | – 10 | + 66 | +535 |
| 3200 | – 26 | + 457 | +467 | + 45 | – 58 | +718 |
| 3140 | – | – | – | – | –217 | +947 |

Table 4-2.–*Radial deflection analysis.*

(a) Arch Stresses Parallel to Faces

| Elevation | Abutment |  | 3/4 |  | 1/2 |  | 1/4 |  | Crown |  |
|---|---|---|---|---|---|---|---|---|---|---|
|  | Ext. | Int. | Ext. | Int. | Ext. | Int. | Ext. | Int. | Ext. | Int. |
| 3660 | +440 | +469 | +339 | +571 | +431 | +478 | +534 | +375 | + 638 | +270 |
| 3600 | +491 | +566 | +337 | +723 | +468 | +589 | +691 | +363 | + 808 | +243 |
| 3550 | +542 | +488 | +245 | +799 | +403 | +642 | +745 | +294 | + 915 | +121 |
| 3500 | +358 | +748 | +278 | +874 | +570 | +607 | +854 | +335 | + 972 | +221 |
| 3450 | +174 | +845 | +226 | +870 | +560 | +592 | +897 | +290 | +1,026 | +173 |
| 3400 | – 5 | +894 | +213 | +750 | +527 | +502 | +836 | +242 | + 967 | +129 |
| 3350 | – 87 | +871 | +192 | +645 | +485 | +412 | +754 | +192 | + 871 | + 94 |
| 3300 | – 97 | +757 | +124 | +566 | +405 | +323 | + 61 | +102 | + 768 | + 10 |
| 3250 | – 65 | +614 | – | – | +337 | +239 | – | – | + 611 | 0 |
| 3200 | – 25 | +540 | – | – | +297 | +224 | – | – | + 491 | + 50 |

(b) Cantilever Stresses Parallel to Faces

| Elevation | A |  | B |  | C |  | D |  | E |  | F |  | Crown |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | US | DS | US | DS | US | DS | US | DS | US | DS | US | DS | US | DS |
| 3600 | +177 | –67 | +163 | – 52 | +151 | – 42 | +142 | – 34 | +131 | – 26 | +128 | – 24 | +132 | – 29 |
| 3550 | +222 | –57 | +207 | – 36 | +195 | – 22 | +189 | – 14 | +183 | – 7 | +187 | – 10 | +197 | – 20 |
| 3500 | – | – | +196 | + 34 | +191 | + 45 | +191 | + 50 | +197 | + 49 | +209 | + 40 | +227 | + 24 |
| 3450 | – | – | +154 | +139 | +162 | +139 | +176 | +130 | +197 | +119 | +213 | +111 | +237 | + 89 |
| 3400 | – | – | – | – | +128 | +245 | +152 | +222 | +187 | +198 | +204 | +194 | +228 | + 175 |
| 3350 | – | – | – | – | +111 | +329 | +131 | +311 | +165 | +289 | +175 | +298 | +199 | + 280 |
| 3300 | – | – | – | – | – | – | +118 | +394 | +125 | +402 | +117 | +431 | +138 | + 418 |
| 3250 | – | – | – | – | – | – | +101 | +480 | + 70 | +532 | + 32 | +593 | + 43 | + 590 |
| 3200 | – | – | – | – | – | – | – | – | + 8 | +678 | – 70 | +777 | – 77 | + 791 |
| 3180 | – | – | – | – | – | – | – | – | – 32 | +753 | – | – | – | – |
| 3150 | – | – | – | – | – | – | – | – | – | – | –175 | +961 | – | – |
| 3140 | – | – | – | – | – | – | – | – | – | – | – | – | –226 | +1,045 |

Table 4-3.–*Complete trial-load analysis.*

(a) Arch Stresses Parallel to Faces

| Elevation | Abutment | | 3/4 | | 1/2 | | 1/4 | | Crown | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Ext. | Int. | Ext. | Int. | Ext. | Int. | Ext. | Int. | Ext. | Int. |
| 3620 | + 45 | +412 | +288 | +363 | +369 | +391 | +476 | +329 | +541 | +278 |
| 3560 | +151 | +466 | +264 | +524 | +456 | +463 | +608 | +393 | +658 | +368 |
| 3500 | +163 | +646 | +300 | +640 | +563 | +520 | +760 | +441 | +840 | +405 |
| 3440 | +201 | +667 | +258 | +683 | +503 | +532 | +760 | +367 | +879 | +287 |
| 3380 | + 90 | +678 | +259 | +571 | +485 | +441 | +693 | +324 | +789 | +261 |
| 3320 | + 68 | +565 | +214 | +464 | +395 | +346 | +567 | +232 | +643 | +175 |
| 3260 | + 15 | +461 | +221 | +341 | +369 | +264 | +476 | +211 | +524 | +178 |
| 3200 | +162 | +369 | +242 | +295 | +324 | +223 | +390 | +167 | +416 | +145 |

(b) Cantilever Stresses Parallel to Faces

| Elevation | A | | B | | C | | D | | E | | F | | Crown | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | US | DS | US | DS | US | DS | US | DS | US | DS | US | DS | US | DS |
| 3620 | + 33 | + 38 | + 30 | + 41 | + 27 | + 44 | + 23 | + 48 | + 19 | + 52 | + 16 | + 54 | + 15 | + 55 |
| 3560 | + 91 | + 66 | + 85 | + 73 | + 80 | + 79 | + 73 | + 89 | + 59 | +104 | + 52 | +113 | + 49 | +115 |
| 3500 | +119 | +117 | +116 | +126 | +112 | +131 | +107 | +139 | + 88 | +165 | + 68 | +189 | + 60 | +198 |
| 3440 | – | – | +117 | +214 | +120 | +214 | +126 | +208 | +118 | +223 | + 83 | +268 | + 65 | +291 |
| 3380 | – | – | – | – | +125 | +320 | +143 | +291 | +164 | +268 | +127 | +317 | + 98 | +354 |
| 3320 | – | – | – | – | – | – | +158 | +380 | +192 | +330 | +158 | +373 | +123 | +418 |
| 3260 | – | – | – | – | – | – | +165 | +483 | +182 | +437 | +143 | +475 | +109 | +515 |
| 3200 | – | – | – | – | – | – | – | – | +137 | +594 | + 80 | +626 | + 52 | +657 |
| 3175 | – | – | – | – | – | – | – | – | +112 | +667 | – | – | – | – |
| 3145 | – | – | – | – | – | – | – | – | – | – | – 2 | +801 | – | – |
| 3140 | – | – | – | – | – | – | – | – | – | – | – | – | – 38 | +834 |

All stresses are in p.s.i.
Ext. = extrados
Int. = intrados
US = upstream face
DS = downstream face
+ indicates compression
– indicates tension

## 1. *Theory of the Trial-Load Method*

4-13. *General.*–A comparatively elaborate analysis is required if a dependable estimate of stress distribution in an arch dam is to be obtained, because of the redundant nature of this type of structure. The requirements for a correct solution of the stress problem may be inferred from the Kirchhoff uniqueness theorem [8] in the theory of elasticity. The implied requirements are the following:

(1) The elastic properties of the body must be completely expressible in terms of two constants: Young's modulus and Poisson's ratio.

(2) If the volume of the body in the unstressed state is divided into small elements by passing through it a series of intersecting planes or surfaces, each of the elements so formed must be in equilibrium under the forces and stresses which act upon it.

(3) Each of the elements described above must deform in such a way as the body passes into the stressed state that it will continue to fit with its neighbors on all sides.

(4) The stresses or displacements at the boundaries of the body must conform to the stresses or displacements imposed.

Under these conditions Kirchhoff proved that it is impossible for more than one stress system to exist. It follows, therefore, that if the actual structure conforms to requirement (1), and a stress system is obtained which meets requirements (2), (3), and (4), this stress system is the one which must exist in the structure under the assumed conditions. A stress system meeting the above requirements may be obtained by the trial-load method worked out by Bureau of Reclamation engineers. Before proceeding with a description of this method, however, it will be profitable to give some additional consideration to requirements (1) to (4).

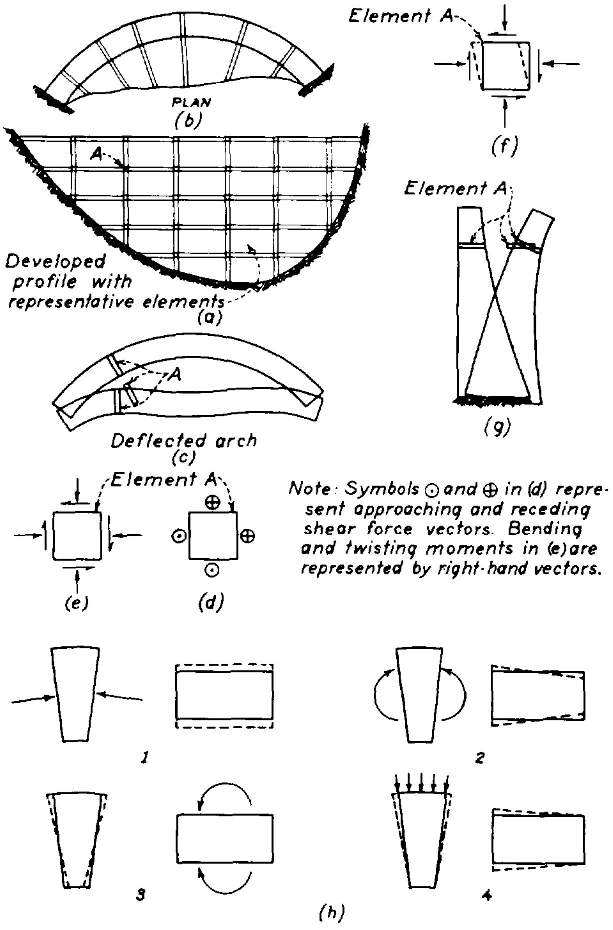
It is well known that concrete subjected to a sustained loading will flow, and it is therefore a fair question whether concrete meets the first requirement. The results of flow tests are erratic, but on the average seem to indicate that the flow rate is proportional to the stress. If this conclusion may be accepted, the linear relation between stress and strain remains and the theorem retains its validity. This is true whether Poisson's ratio is the same for the elastic and flow strains or not, so long as it is proportional to the direct strain in each case.

The conditions expressed in requirements (2) and (3) are usually called the equilibrium and continuity conditions, respectively, while those expressed in requirement (4) are customarily spoken of as the boundary conditions since these are conditions which must be met at the boundaries of the body. In an arch dam the stress distribution to meet this requirement must conform to the water pressure at the upstream face, and show the stress to be zero on the downstream face. At the surface of contact between the dam and the abutment, the displacements of the dam and the abutment must agree. The necessity of meeting the equilibrium and boundary conditions should be self-evident, but these requirements alone are not enough to determine a unique solution since there exist a multitude of stress systems which will meet them. Several such systems for a special case will, in fact, usually be obtained in the course of a trial-load analysis. The result of imposing the continuity condition is, therefore, to select from the multitude of statically possible cases that *one* which must exist. For a dam built of material fulfilling the first requirement, the equilibrium, continuity, and boundary conditions are both necessary and sufficient. If the stress distribution does not satisfy all three, it is incorrect; if it satisfies all three, it is correct. If an attempt is made to impose additional conditions, no solution can be found.

It should also be mentioned that Kirchhoff's proof is valid only if the displacements are so small that the changes of stress due to the changes in the shape of the structure are insignificant. This is almost invariably true in the case of arch dams. If the dam were made thin enough to violate this requirement it would be in danger of failure due to elastic buckling, but the allowable stress-carrying capacity of concrete will generally dictate thicknesses which will be amply safe against this type of failure.

**4-14.** ***Trial-Load Procedure.***—The dam to be analyzed may be represented by contours drawn on a contour map of the site. The dam may be assumed to be divided into a series of horizontal arch and vertical cantilever elements. An *arch element* is a portion of a dam bounded by two horizontal planes 1 foot apart. For purposes of analysis the edges of the elements are assumed to be vertical. A *cantilever element* is that portion of a dam which is contained within two vertical planes radial to the extrados and spaced 1 foot apart at the axis. Cantilevers of arch dams other than the constant-center type are bounded by warped surfaces because the locations of arch extrados centers vary with the elevations of the arches. The totality of the arch elements so defined contains the entire volume of the dam and this is the case also with the cantilever elements. Only a limited number of representative elements are used in the computation. An arch element is shown in (c) of figure 4-9, and cantilever elements in (b) and (g). A representative set of arch and cantilever elements is shown in (a). The dimensions of the representative elements may be readily obtained from the contour representation previously described.

The analysis will be facilitated by introducing the assumption of linear stress distribution as a temporary expedient. This will permit calculation of stresses and deflections by means of the usual arch and beam formulas. Suppose, as a first approximation, that the



Note: Symbols ⊙ and ⊕ in (d) represent approaching and receding shear force vectors. Bending and twisting moments in (e) are represented by right-hand vectors.

***Figure 4-9.*** **Structural elements used in a trial-load analysis.–288-D-1380**

entire waterload has been placed upon the arch elements and the deflections of the representative arches calculated. These computations will, in general, show that the element of arch volume *A* shown in (a) of figure 4-9 and also in (c), (d), (e), (f), (g), and (h) of the same figure, has been deflected downstream and toward the abutment and has, in addition, been turned about a vertical axis. A review of this result in the light of Kirchhoff's requirements will show that the equilibrium and boundary conditions have been met, but that the continuity condition has been violated since no load has been assigned to the cantilever to bend and twist it to a shape which will conform to that of the deflected dam, as arrived at from the arch computations. It will be found necessary to transfer a part of the load to the cantilevers before the position of the element *A*, as obtained from the arch and cantilever computations, can be made the same. Although these loading conditions are discussed separately, in actual practice they are applied simultaneously.

The movement which can occur when force is applied to a dam is described by three linear directions and three rotational directions. These are radial, tangential, and vertical linear directions, and rotation in horizontal, vertical radial, and vertical tangential planes. Four of these movements are considered in the trial-load method of analysis. Because their effects on stresses are minor, vertical deflections and rotations in vertical tangential planes are not usually considered.

**4-15.** ***Radial Adjustment.***–It will be found advisable to begin the adjustments by making a transfer of load which will bring the deflections of the arch and cantilever elements into agreement in the upstream-downstream direction. Horizontal deflections parallel to the extrados radii and normal thereto will be called the radial and tangential deflections, respectively. In this terminology it is the radial deflections which are now to be brought into agreement, and it is in this sense that the adjustment is the radial adjustment. The type of loads required may be explained by reference to (d) and (e) of figure 4-9, which show the element *A* as viewed from the downstream side of the dam. Loads may be applied to the cantilever by introducing shearing forces on the cantilever cross sections, such as indicated by the receding and approaching shear-force vectors at the top and bottom of the element shown in (d). If these differ, a net force will exist, tending to move the element in the upstream or downstream direction, which will require the introduction of shear forces on the arch sections having an equal difference. These shear forces are shown at the sides of the element in (d). It is true that shear forces of the latter type already were present in the arch because of the assumed application of the waterload thereto. These shear forces are assumed to remain, and the shear forces under consideration are to be added algebraically. By this arrangement equilibrium in the radial direction has been maintained, but the shear forces applied exert couples on the element *A*. These are to be balanced by differences between bending moments applied to the sides of the element, as shown by the right-hand vectors parallel to the sides of the element in (e). For those who may be unfamiliar with the vector representation of moments, it is essential to explain that a right-hand vector represents a rotation in the sense that a right-handed screw would turn if it advanced in the direction indicated by the vector. In (e), the vectors represent moments applied to the faces of the element *A*. Thus, equilibrium against rotation is secured. The net result of the addition of these forces is to transfer load from the arch elements to the cantilever elements without altering the total load applied to the dam. Such loads are termed self-balancing. The amounts to be applied must be found by trial, which accounts for the designation of the process as the trial-load method.

When a set of self-balancing radial loads has been chosen, bending moments in the arch and cantilever elements and the deflections due to them are computed. A comparison of radial deflections can now be made. If the self-balancing loads have been skillfully chosen, the radial deflection of the arch elements will have been reduced and the cantilever elements will have acquired radial deflections approximately equal to those of the arch elements at corresponding points. If the agreement is unsatisfactory the self-balancing loads must be modified and the process repeated. It is of interest to note that the bending of the cantilever element has produced a rotation of the element *A* about a horizontal axis, as shown in (g) of figure 4-9. The arch computation indicates no rotation, and the position of the element as arrived at from the arch computation is shown in that sketch.

4-16. ***Tangential Adjustment.***—The defect of the tangential displacements shown in (c) of figure 4-9 may now be eliminated. This is accomplished in a manner very similar to that previously described for the radial displacements, by introducing a set of self-balancing tangential loads of the type shown in (f) of figure 4-9. The vectors shown in (f) represent forces, and equilibrium is obtained by balancing the difference between the tangential shear forces at the top and bottom of the element *A* against a corresponding difference between the arch thrusts applied to the sides. In a similar fashion, a difference between the vertically directed shear forces on the sides of the element is balanced against a difference in the thrusts applied to the top and bottom faces. Equilibrium of rotation about a radial line will require approximate equality between the shear forces applied to the vertical and horizontal faces of the element. A small inequality may exist, due to the action of twist on the converging sides of the element. This effect will be considered in connection with the twist adjustment. The shear forces are assumed to be equal for the purpose of making the tangential adjustment, and a correction is introduced later, if necessary.

The self-balancing tangential-shear loads produce bending in the arch and cantilever elements and a tangential deflection of the cantilever elements due to shearing deformation. Radial, tangential, and rotational displacements about a vertical axis are produced in the arch elements, due to the bending moments introduced. It is customary to compute the tangential displacements of the cantilever upon the assumption that top and bottom faces of the element *A* remain parallel to their original position. A certain unknown amount of error is introduced by using this assumption, the effect of which does not appear to be large.

4-17. ***Twist Adjustment.***—After the arch and cantilever deflections have been brought into radial and tangential agreement, there will still be present an angular defect, the nature of which is shown in (c) and (g) of figure 4-9. In the twist adjustment, twist loads are applied to the arches and cantilever to rotate them into angular agreement. The twist loads must be applied in such a way as to eliminate simultaneously the defects of rotations about the vertical and horizontal axes. The self-balancing twist loads may be described by reference to (e) of figure 4-9. The difference in the twisting moments applied to the top and bottom of the element is held in equilibrium by an equal difference introduced between the bending moments applied to the sides, and similarly, the difference between the twist moments applied to the sides of the element is balanced by a corresponding difference introduced between the bending moments applied to the top and bottom. The twisting moments cause shear stresses to be set up in the vertical and horizontal faces of the block, and since the shear stresses on planes at right angles must be equal, it follows that the twisting moments applied per unit of height and per unit of length, measured along the centerline of the arch element, must be equal.

The simultaneous elimination of the rotation defects about the vertical and horizontal axes with one set of loads would at first glance appear to be impossible, but Westergaard has pointed out that the desired result may be obtained if the twists are adjusted in one direction only, provided radial adjustment is maintained during the process. This conclusion may be arrived at from the following considerations. The cross derivatives obtained by differentiating the radial component of displacement, first with respect to length measured vertically, and second with respect to length measured horizontally along the arch centerline, represents the twist of the arch element, whereas the differentiations performed in the reverse order represent the twist of the cantilever element. Now, the cross derivatives of a continuous surface must be equal if they exist at all, and thus must ensure equality of the twists and compatibility of the rotations if the deflected surfaces, as obtained from the arch and cantilever computations, are the same; that is, if radial adjustment is obtained. It is customary to adjust the twists of the cantilevers to the arch rotations, or, in other words, to adjust the rotations about the vertical axes. As in the previous work, agreement is obtained by trial.

As has been previously mentioned, the twisting moments exert an influence on the magnitudes of the tangential shears. If the vectors in sketch (1) of figure 4-9(h) are assumed to represent twisting moments applied to the side of the element *A* according to the right-hand convention, it will be apparent that their sum taken vectorially will represent a small vector tending to turn the element about an axis lying in the radial direction. This tendency to turn is resisted by shearing forces of the type shown in (f) of figure 4-9, but having one pair reversed so that the resisting couples add. In this way a small difference in the tangential shear forces shown in (f) is introduced. It has not been found necessary to take this difference into account when making the trial-load analysis.

4-18. ***Radial, Tangential, and Twist Readjustments.***—In each of the adjustments described above, the work was carried out without regard to the effect of the added loads on the previous adjustments. Such a procedure depends for its success on making the adjustments in the order of their structural importance. The order described above has been successful in actual use. Even with the most favorable possible order, however, the loads applied for each succeeding adjustment will impair, to some extent, the adjustments previously obtained. The readjustments are made for the purpose of rectifying the errors thus introduced. They are made in the same way as described for the adjustments and in the same order, but in the readjustments the deflections due to all the loads previously applied are included. The process described is rapidly convergent and it is seldom that more than a second set of readjustments need be made.

Up to this time no mention has been made of the effect of Poisson's ratio. The changes of shape produced in the element *A* through lateral expansion are shown on figure 4-9(h). The change of shape produced by thrust in the arch is shown on sketch (1) of that figure. The effects of bending moments in the arch and cantilever are shown in sketches (2) and (3), and the effect of water pressure on the upstream face is shown in sketch (4). The influence of Poisson's ratio is most easily taken care of by computing the deflections due to the lateral strains and introducing them into the readjustments. These computations are based on the results of the adjustments previously made.

The movement of the element *A* can be completely described in terms of deflections in the radial, tangential, and vertical directions, and rotations about axes in these three directions. The following tabulation shows the adjustments which take care of the several components of displacement and rotation, both for an analysis as complete as can be made without discarding the assumption of linear stress distribution and for an analysis as above described and customarily performed.

| Analysis | Deflection | | |
|---|---|---|---|
| | Radial | Tangential | Vertical |
| Complete | Radial | Tangential | Tangential |
| As described | Radial | Tangential | *0 |

*This effect may be included as described in section 4-43.

| Analysis | Rotation about | | |
|---|---|---|---|
| | Radius | Tangent | Vertical |
| Complete | Tangential | Twist | Twist |
| As described | 0 | Twist | Twist |

It will be noted that one of the deflections and one of the rotations are customarily assumed to be zero because the effects are minor.

Assuming that a complete analysis, as shown in the above table, has been made, the requirements implied in the Kirchhoff uniqueness theorem have been met as well as is possible so long as the assumption of linear stress distribution is retained; and tests of models have demonstrated the essential adequacy of such an analysis for a thin dam. What has actually been done is to restore the continuity of the element *A* at the midsurface of the dam. Though continuity has thus been restored at the middle of the element, continuity may not be obtained at the upstream and downstream faces due to the tendency of the element *A* to grow S-shaped as a result of the elastic deformations. This tendency gives rise to nonlinear stress distributions and becomes stronger as the dams become relatively thicker.

4-19. ***Abutment Deformations.***—Formulas for the deformation of abutments can be obtained by making use of the solution of Bousinnesq and Cerruti in the theory of elasticity. Such formulas may be found in a treatise entitled, "Uber die Berechnung der Fundament Deformationen,"by Dr. Frederick Vogt of Oslo, Norway. The abutment deformations should always be taken into account and this is especially true if the dam is thick.

4-20. ***Effect of Temperature Changes.***—The analysis for the effect of temperature changes follows closely the procedure described for the analysis of stresses due to waterloads. In the analysis of temperature stresses the deflections of the arch elements due to the temperature change are used as a first approximation in place of the deflections due to the external loads in the case previously considered. The remainder of the analysis is identical with that of the procedure described.

The temperature range is obtained from data based on actual exposure conditions at the particular dam. Although air temperatures largely govern the final concrete temperatures, solar radiation upon exposed surfaces of the dam and the presence of the water against the upstream face of the dam will also influence the mean temperature of the concrete under conditions of use. Temperature stresses are of sufficient importance to warrant an investigation of the temperature conditions to be met at each specific site [9]. Refer also to chapter VII of this manual.

4-21. ***Stresses Due to Weight of Dam.***—It is customary to build dams in sections separated by contraction joints which are usually filled with grout after the dam has been raised to its full height. With radial joints only, each vertical element becomes statically determinate and it is a simple matter to calculate the dead-load stresses. These stresses are not ordinarily changed by the grouting process and may be assumed to exist without change after the dam is grouted. If circumferential joints are used, the dead-load stresses will be affected by details of construction such as height differential between adjacent blocks and the time of grouting the joints, as well as the grouting pressures used.

4-22. ***Effect of Cracking.***—If tension develops in an arch dam, the limited ability of unreinforced concrete or masonry to carry tension may cause the formation of a crack. The development of a crack will modify the structural action to a considerable extent, and in such cases the analysis must be correspondingly modified. Tension areas may develop in any dam, and cracks may occur on the upstream face at the junction of the dam and the foundation and on the downstream face in the center of the dam near the top. The effect of cracking may be approximately accounted for by ignoring the structural action of all material which develops excessive tension. A preliminary analysis based upon the assumption that the material is capable of carrying tension is usually necessary in order to discover the approximate location of the tension areas. It is important to note that since the development of a crack changes the structure, the law of superposition does not hold if cracking occurs, and it is therefore no longer possible to estimate the effect of several loads by adding together the effects of the individual loads applied separately. This makes it necessary to introduce the effects of vertical loads, waterloads, and temperature changes into the analyses simultaneously. The difficulty of the analysis is greatly increased but this cannot be helped.

4-23. *Nonlinear Stress Distribution.*—After a trial-load analysis is made, based on the assumption of linear stress distribution, a knowledge of the effect of nonlinear stress distribution may be obtained from a two-dimensional finite element analysis, by computations using the methods of the theory of elasticity, or by photoelastic means. In any case, the loads applied are obtained from the trial-load analysis. The greatest deviations generally occur at the junction of the dam and the abutment, where stress concentrations occur. A local thickening of the dam may be required to keep the stresses in this region within bounds.

4-24. *Abutment Stability.*—In gravity dams, the safety against sliding depends upon the angle the resultant of the loads makes with the normal to the plane of the base. The safety of an arch dam against sliding at any given point in the abutment may be estimated in a similar fashion. This is most easily done by computing the *direction cosines* of the normal to the abutment surfaces at the location chosen and the *direction cosines* of the resultant thrust at the same point. For a given strip crossing the abutment surface, the resultant thrust applied to the strip can be estimated from the arch thrusts, tangential and radial shears, and vertical cantilever loads applied to the area of the strip. The angle between the resultant and the normal can then be found by the well-known formula of analytic geometry for obtaining the angle between two lines in space whose *direction cosines* are given. The data for calculating the *direction cosines* of the resultant thrust are obtained from the trial-load analysis.

## 2. Discussion of Method

4-25. *Assumptions and Their Explanations.*—Assumptions which are peculiar to the trial-load analysis or require explanation are given in the following list:

(1) The concrete in the dam, as well as the rock in the abutment and foundation, is a homogeneous, isotropic, and uniformly elastic material.

(2) It is usually assumed that no differential movements occur at the damsite due to waterload on the reservoir walls or floor.

(3) Vertical displacements due to dead load, shrinkage, and temperature changes prior to joint closure take place in the cantilevers before the beginning of the arch action, so that no lateral transfer of these loads takes place.

(4) Average temperature changes in arches vary with the average horizontal thickness for each arch element. Other temperature changes in the arches depend also on location and orientation, varying from upstream to downstream faces and from abutment to abutment.

(5) Excessive tensile stresses are relieved by cracking and all loads are carried by compressive, tensile, and shearing stresses in the uncracked portion of the dam.

(6) Stresses calculated from the final load distribution on a simplified structure represent the stresses in the dam for the assumed condition of loading.

The first assumption is a fundamental requirement which makes possible the use of the theory of elasticity for a mathematical solution of stresses and strains in a body such as a dam. Actually, the foundation rock is never homogeneous nor isotropic. It may contain joints, cracks, and fissures, and be composed of different materials with different properties. Extensive treatment of the rock by grouting seams with cement and excavating weak zones and backfilling them with concrete is necessary to obtain a monolithic foundation. The rock must be sufficiently stable so that it will withstand disintegration, and must be of adequate strength, for without these conditions the dam may fail.

With modern methods of control of concrete, a fairly uniform concrete is readily obtained. The grouting of joints and control of temperature make possible a monolithic structure. Normally, concrete produced by modern methods is of stable composition, although special investigations are made to determine the effect of chemical reaction between alkali aggregate and cement. The effect of water-soaking of concrete is usually negligible.

In accord with the second assumption, movements of the canyon walls due to waterload on the reservoir floor are neglected in the usual analysis. Effects of such movements were investigated for Hoover Dam and are described in appendix I. These effects were found to be negligible.

The third assumption, that the vertical displacements due to dead load, shrinkage, and temperature changes prior to joint closure take place in the cantilevers before the beginning of arch action, is valid because the construction program requires subcooling before grouting. Thus the contraction joints are opened and no arch action will take place until after grouting.

In applying the fourth assumption, the average temperature changes within the arch elements are considered to vary with the average thickness of the arch and the temperature differentials at the faces of the dam. This temperature change is assumed to be constant from abutment to abutment and from upstream face to downstream face of an element. The change from the grouting temperature to the selected operating temperatures is the temperature change usually used in the analysis. The nonuniform temperature changes are the variations from upstream to downstream faces and from abutment to abutment. These variations depend on the orientation of the dam with respect to the sun, the water level, and the location and thickness of the arch.

In applying the fifth assumption, cracking is generally assumed to occur where the allowable tensile stress is exceeded by the greatest amount in each element. Tensile stresses and resultant cracking can be largely eliminated by subcooling the concrete and grouting the contraction joints while concrete temperatures are below ultimate mean annual values. In dams for which studies indicate tensile stresses in excess of the tensile strength of the concrete, cantilevers and arches are analyzed on the assumption that they crack to the point of zero stress at the location of maximum tension in each element. If the redistribution of load caused by this assumption does not relieve the tensions at other points, it may be found necessary to assume cracking at more than one point in an element.

The sixth assumption is based on the premise that all elements in the dam are in deflection agreement after completion of the analysis for a simplified structure.

The effects of assumed loading conditions, uplift pressure, temperature change, ice load, and earthquake shock can be included in the analysis. However, load conditions chosen for design should include only those loads having reasonable probability of simultaneous occurrence. Combinations of transient loads, each of which has only a remote probability of occurrence at any given time, have negligible probability of simultaneous occurrence, and cannot be considered as reasonable bases for design. For example, maximum earthquake will not likely be combined with maximum design flood, nor will maximum ice pressure normally be combined with maximum design flood or maximum earthquake. In special cases, however, some fraction of the maximum ice pressure may best be considered as a long-continuing rather than a transient load. The design of an arch dam should therefore be based on the loading combinations listed in section 3-14, unless special considerations dictate otherwise.

4-26. *Arches and Cantilevers.*—(a) *Coordinate Systems.*—A dam is essentially a three-dimensional elastic body. By means of the elastic theory, conditions of deformation of a three-dimensional structure may be expressed in terms of three mutually perpendicular linear displacements and three angular displacements. In trial-load analyses a system of cylindrical coordinates is ordinarily used. The axes are: first, radial along the radius of the arch; second, tangential along the tangents to the arc through the midpoint of the crown; and third, vertical. Linear displacements are referred to these axes and angular displacements are measured as rotations about the axes. Other systems of coordinates are used in some phases of the analyses. Descriptions of those systems are given in the applicable sections.

(b) *Selection of Arches and Cantilevers.*—For a trial-load analysis, the dam as proposed by preliminary design is assumed to be replaced by two systems of elements. The first is a system of vertical cantilevers each bounded by radial, vertical surfaces 1 foot apart at the axis of the dam, which converge from the upstream to the downstream face. (In special studies cantilevers of a greater width may be assumed.) The cantilevers resist vertical and radial forces applied at the upstream or downstream faces as external loads; and tangential forces, twisting moments, and bending moments applied at a distance of one-half the crown thickness from the upstream face as internal loads. Weight, usually being purely a cantilever loading, is considered only in the final computation of stresses. The second system of elements consists of horizontal arches 1 foot high with parallel, horizontal top and bottom surfaces and with vertical upstream and downstream faces. These are statically indeterminate elements terminating at elastic abutments. They resist radial forces applied at the faces as external loads, tangential forces and horizontal moments applied along the arc through the midpoint of the crown, and twisting moments in vertical radial planes.

Both of these structural systems are assumed to occupy the entire volume of the dam, as shown on figure 4-10. Note that the right and left sides of the dam in the figure are reversed from those determined by the normal convention. Arches and cantilevers may move independently of each other, but elements at corresponding points must have identical linear and angular displacements so that the continuous structures of arches and cantilevers occupy the position of the loaded dam. These elements are assumed to be held in place by a pattern of external loads and internal

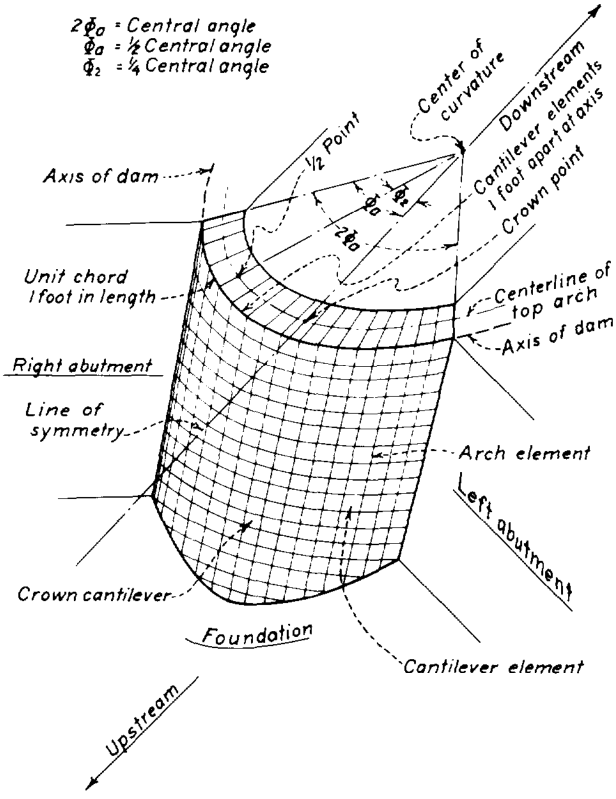


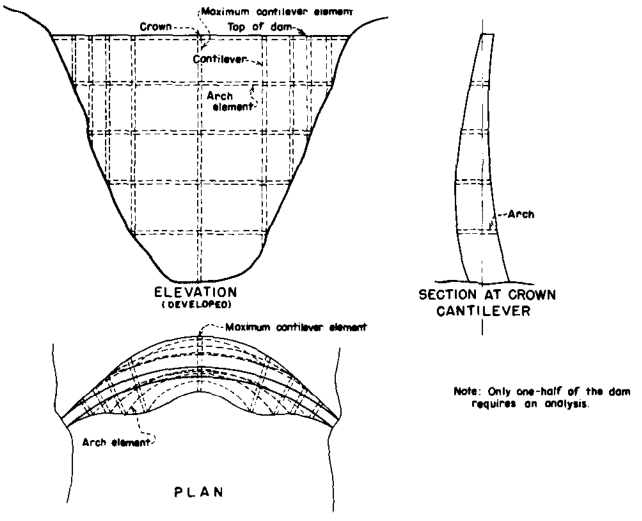
**Figure 4-10. Arch and cantilever elements of a constant-radius symmetrical arch dam. 288-D-138**

self-balancing loads applied and established by trial.

Instead of investigating a great number of vertical and horizontal elements, only a few sample arches and cantilevers are analyzed in order to perform the analysis within a reasonable length of time. If the dam is reasonably symmetrical about the maximum cantilever section, as shown on figures 4-10 and 4-11, only half of the structure need be analyzed, and five to seven arches and four to seven cantilevers may be sufficient. If the dam is nonsymmetrical, as shown on figure 4-12, both sides must be analyzed. More cantilevers, usually 9 to 11, are therefore required.

Normally, one cantilever is located at the maximum section of the dam and the others at the arch abutments, as shown on figures 4-11 and 4-12. In the event of a sharp change in abutment or foundation slope, additional cantilevers may be analyzed. This is done so that the adjustment can be obtained in this region for an accurate determination of stresses. Such a procedure is necessary for sites that contain irregularities which cannot be removed economically. If at all practicable, it is desirable to smooth out irregularities of the canyon profile in assuming excavation lines, and to adopt a design symmetrical about the crown-cantilever section. Deep holes or relatively narrow gorges in the bottom of the canyon can sometimes be plugged with concrete and treated as parts of the foundation instead of parts of the dam.

(c) *Adjustment of Arches and Cantilevers.*—The trial-load analysis is carried out in steps, or adjustments as they are called. At present, three adjustments are made: radial, tangential, and twist. These serve to bring the arch and cantilever movements into linear agreement in radial and tangential directions, and in rotational agreement for rotations about vertical and tangential axes. With the simplified structure, it is sufficient to secure coincidence of sample arches and cantilevers at their points of juncture. Such coincidence is completely achieved when there is equality of the three linear and three angular displacements of the arch with those of the cantilever. However, the adjustments previously mentioned bring only



Note: Only one-half of the dam requires an analysis.

*Figure 4-11.* **Plan, profile, and section of a symmetrical arch dam.—288-D-2680**

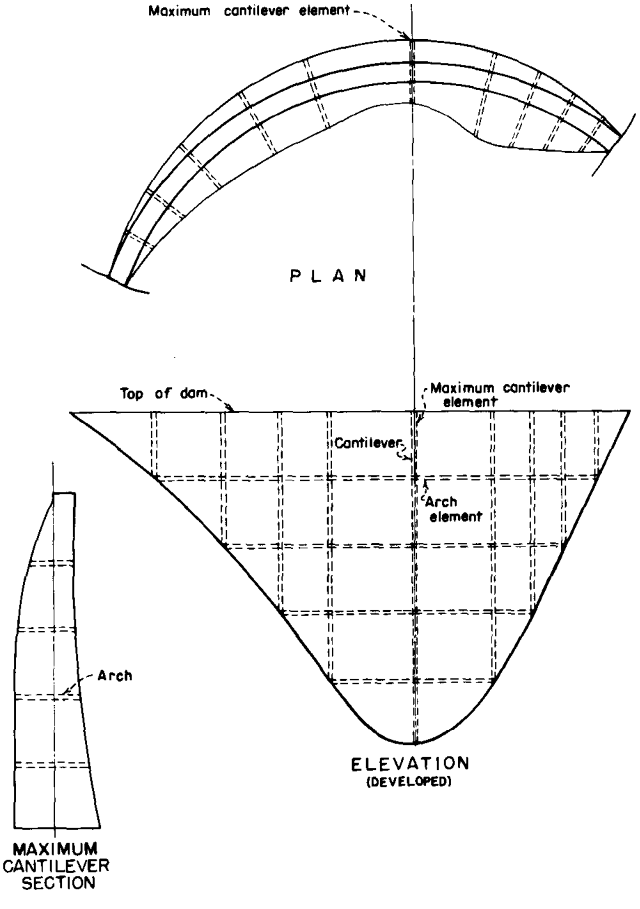


*Figure 4-12.* **Plan, profile, and section of a nonsymmetrical arch dam.—288-D-2681**

four of the six movements into agreement; namely, the radial and tangential displacements, and tangential and vertical rotations. The present method is, therefore, incomplete, since it lacks adjustments for vertical displacements and radial rotations. Of these two, the rotation about a radial axis appears to be of lesser significance and is completely neglected in the analysis.

Ordinarily, the tentative cross section of the dam is first analyzed by a simplified method. When this first analysis is completed, tentative dimensions of the structure may be modified as indicated by the resulting stress computations. Several such analyses may be made, with successive modifications in dimensions, until the final dimensions are determined. The final design can then be analyzed completely by the trial-load method for applicable loading combinations. In the final trial-load studies, it is usually desirable to analyze at least five arch elements.

4-27. ***Design Data.***—The basic considerations and assumptions upon which trial-load studies can be based are presented in sections 4-11 through 4-26. The results of field investigations and other studies such as hydrologic, geologic, climatic, laboratory, and earthquake studies should be obtained to determine or verify the design assumptions.

(a) *Constants and Structural Data.*—Constants and structural data usually required for a trial-load analysis are given in the following list:

(1) Dimensions of arches and cantilevers.

(2) Modulus of elasticity for concrete in tension and compression.

(3) Modulus of elasticity for concrete in shear.

(4) Poisson's ratio for concrete.

(5) Modulus of elasticity for abutment rock in tension and compression.

(6) Poisson's ratio for abutment rock.

(7) Dimensions of developed base of dam.

(8) Slopes of arch abutments and cantilever foundations.

(9) Unit weight of concrete.

(10) Unit weight of water.

(11) Elevation of reservoir water surface.

(12) Elevation of tailwater surface.

(13) Dynamic loading due to an assumed earthquake shock.

(14) Temperature changes at elevations of arches.

(15) Coefficient of thermal expansion for concrete.

(16) Unit weight and elevation of silt accumulations.

(17) Elevation and pressure of ice loads.

(b) *Forces Acting on the Dam.*—Forces acting on a dam include weight, temperature changes, and some combination of external loads produced by reservoir water, tailwater, uplift pressure, silt, ice, earthquake shocks, and any superstructure load.

If the dam is built in vertical sections and contraction joints are grouted after completion, effects of concrete weight are taken by the cantilevers alone. In this case, concrete weight does not affect trial-load adjustments and need not be considered until stresses are computed. If grouting is started before completion, deflections due to weights of concrete added subsequently must be included in the analysis. Likewise, when cracking of the cantilevers is assumed to take place, concrete weight becomes a factor in computing deflections of vertical elements.

Temperature data based on actual exposure conditions at the particular dam are used in the analysis. Determinations of these data are discussed in section 3-7.

Reservoir water at the upstream face and tailwater at the downstream face of a dam produce hydrostatic pressures, as discussed in sections 3-6 and 4-31 and illustrated on figure 4-26.

Uplift pressures are usually unimportant in arch dams, and any but the simplest assumptions lead to considerable difficulty in the trial-load method of analysis. On that account, the uplift assumption for arch dams has been selected from the standpoint of simplicity rather than a rigorous statement of pressure distribution. It is only used in the design of arch dams for cases involving tensile stress of such magnitudes as to imply cracking.

Silt accumulations are usually assumed to act in the same way as vertical waterloads, the unit weight being taken as the weight of saturated earth. Increases in horizontal pressure due to silt flows may be provided by using fluid pressures greater than that of water, depending on the quantity of silt.

Forces exerted on a dam by ice at the surface of the reservoir are uncertain. (See section 3-10 for a discussion of ice pressures.) Pressures from 5 to 25 tons per linear foot have been used for design, but some experiments and studies have indicated that these values are excessive. Values used recently seldom exceed 5 tons per foot.

Dynamic forces due to earthquake are calculated by means of formulas given in sections 4-55 and 4-56.

4-28. ***External Loads, Internal Loads, and Unit Loads.***—(a) *External Loads.*—In a trial-load analysis, all deflections or movements due to the adjustments are usually measured from an undeflected reference line representing the concrete-weighted position. Deflections due to concrete weight are ordinarily not calculated, but the resulting stresses are computed and added to the adjustment stresses to obtain the total stresses within the dam.

For simplicity, all the remaining external loads except horizontal waterload are assigned initially to either the arches or cantilevers, as convenient, and are not altered during subsequent adjustments. However, the application of trial loads in these subsequent adjustments redistributes the external loads appropriately between the arches and cantilevers. The vertical components of the reservoir and tailwater loads, in addition to vertical silt load and superstructure load, are usually placed on the cantilevers as an initial condition, as also is any horizontal ice component, radial component of horizontal earthquake force, and horizontal tailwater load. Vertical movements are neglected in the analysis, but initial radial cantilever deflections due to these initial loads are included in the radial adjustment.

The tangential component of any assumed horizontal earthquake force is applied as an initial tangential load on the cantilevers prior to making the tangential adjustment. Initial tangential deflections of cantilevers due to this load are added to the cantilever deflections obtained in the tangential adjustment.

The effects of temperature change in a dam are assumed to be confined to the arches. The temperature changes are determined at elevations of the arches and are assigned as initial loads on the arches. This procedure ignores the effects of temperature change on the cantilevers, which are relatively small. Initial temperature deflections of the arches must be added to arch deflections determined in the adjustment. A linear variation of temperature from upstream to downstream face will, however, affect both arches and cantilevers. Both the temperature change and linear variation of temperature from face to face may also be varied along the length of arc. These effects have been included primarily in programing for the electronic computer.

In making the first radial adjustment, the total horizontal radial reservoir load is divided between the arches and cantilevers by trial so as to have a load distribution which will give approximate agreement of radial deflections between the two systems of elements. Initial radial deflections of arches due to temperature, and of cantilevers due to initial load components, are included in the total respective radial deflections. The total horizontal reservoir waterload may include horizontal tailwater and horizontal silt load components.

In subsequent radial adjustments, a more accurate distribution of reservoir load is accomplished through application of equal and opposite radial loads on the arches and cantilevers.

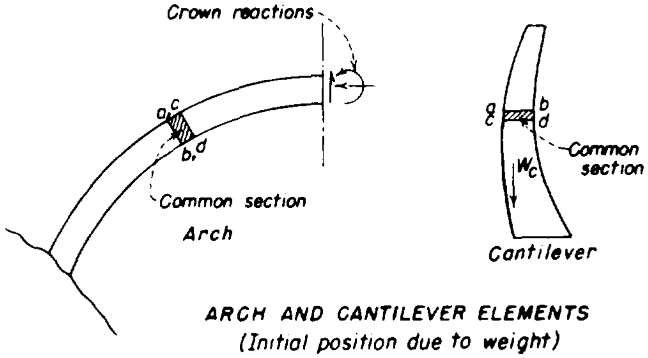
(b) *Internal Loads.*—Internal loads are commonly called self-balancing loads since they are always applied in pairs, the two loads of a pair being equal in magnitude and opposite in direction, one acting on the arch and the other acting on the cantilever. The purpose of these loads is to bring arch and cantilever deflections or movements into agreement without changing the total external loads on the structure. These loads may be freely chosen with the provision that the internal load on the cantilever must be equal and opposite to the internal load on the arch at every point. These internal loads represent, in a physical sense, forces set up by the interaction between the assumed arch and cantilever systems.

The nature and use of external and internal loads in successive steps or adjustments is illustrated diagrammatically on figures 4-13, 4-14, and 4-15. The upper portion of figure 4-13 shows a plan of a typical arch and a typical cantilever through the arch. Here it is assumed that the concrete weight has been assigned to the cantilever and that the resulting position of the *common* section (*a-b* for the cantilever, *d-c* for the arch) is the initial position from which all subsequent deflections and movements are computed.

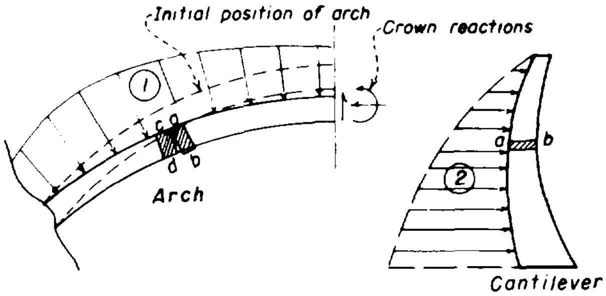
In the lower portion of figure 4-13, a temperature load has been assigned to the arch and the vertical waterload has been applied to the cantilever as an initial load. Also, the first trial distribution of the total horizontal reservoir waterload has been made between the arch and cantilever elements, the loads being shown as ① and ②, respectively. Note that the cantilever has moved radially to *a-b* and the corresponding arch section has rotated and moved tangentially to position *c-d*. Relative positions of elements are exaggerated.

In the upper part of figure 4-14, the reservoir load has been divided as indicated by ③ and ④ so that the elements *a-b* and *c-d* are in approximate radial deflection agreement. The temperature load on the arch and the horizontal earthquake and vertical reservoir waterloads on the cantilever are assumed to be acting, but are not shown.

In the lower part of figure 4-14, an internal tangential load shown by ⑤ has been applied to the cantilever at a distance of one-half the crown thickness from the upstream face and an equal and opposite tangential load shown by ⑥ applied to the arch along an arc through the center of the arch crown and concentric with the extrados,[2] so that *a-b* and *c-d* are in approximate tangential deflection agreement. The tangential component of an earthquake shock is assumed to be acting, but is omitted from the figure. This illustrates the tangential adjustment.



ARCH AND CANTILEVER ELEMENTS
(Initial position due to weight)



APPLICATION OF RESERVOIR LOAD
(Including any temperature load on the arch and any radial component of vertical waterload or horizontal earthquake on the cantilever.)

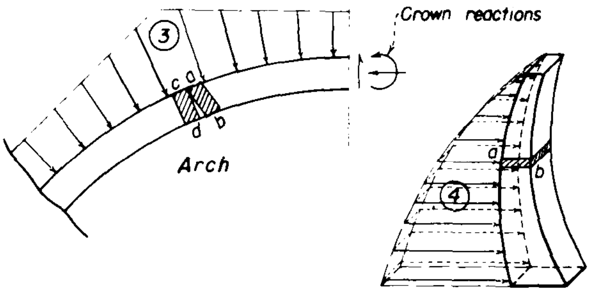
*Figure 4-13.* **Diagrammatic illustration of arch and cantilever elements and reservoir load. 288-D-2969**

Finally, on figure 4-15, equal and opposite internal twist loads have been applied to the cantilevers and arches in the same manner as discussed above for the tangential loads, so as to give approximate agreement of *a-b* and *c-d* by a rotational movement. The applied loads are shown by ⑦ and ⑧. This illustrates the twist adjustment. The loads applied in the radial and tangential adjustments are assumed to be acting as well as the initial external loads, but are omitted from the figure for clarity.

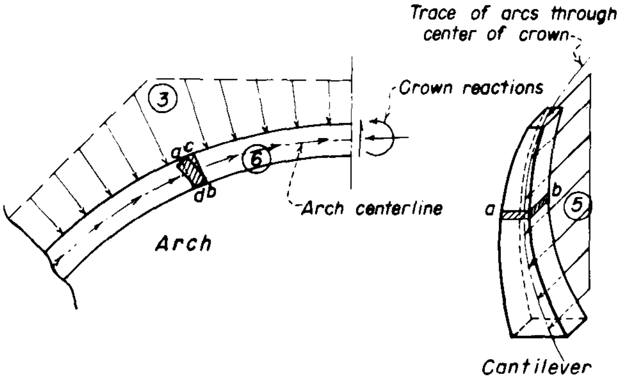
(c) *Unit Cantilever Loads.*—The unit load is a device used to simplify the application of external and internal loads and the determination of deflections. By computing unit deflections and movements for unit loads, the determination of total movements or deflections for applied loads is relatively easy.

---

[2] For a uniform-thickness arch such as illustrated on the three referenced figures and in the following discussion, this arc is coincident with the arch centerline.

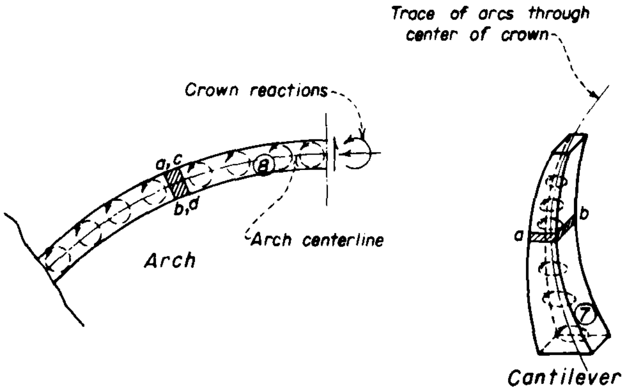


RADIAL ADJUSTMENT
(Radial load acts across entire face of element)



TANGENTIAL ADJUSTMENT
(Including tangential component of earthquake inertia on cantilever. Load ④ assumed to be acting, but omitted for clarity.)

*Figure 4-14.* **Diagrammatic illustration of radial and tangential adjustments.–288-D-2970**



TWIST ADJUSTMENT

Note: Loads ③,④,⑤, and ⑥ assumed acting, but omitted from figure for clarity

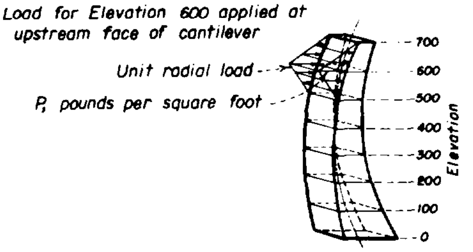
*Figure 4-15.* **Diagrammatic illustration of twist adjustment.–288-D-2971**

Unit radial loads are used in applying radial forces carried by the cantilevers. These unit loads are triangular in shape and vary from *P*, usually 1,000 pounds per square foot horizontal pressure, at one arch elevation to zero pressure at the sample arches directly above and below (see fig. 4-16). Note that the radial load acts across the entire face of the cantilever. With these loads it is possible to apply any horizontal force that varies as a straight line between successive elevations of sample arches (see sec. 31(g)). Shear and bending moments are computed for each unit load, and radial deflections due to each load are determined by the theory of flexure of beams with contributions from transverse shears included, as shown in section 4-31(h).

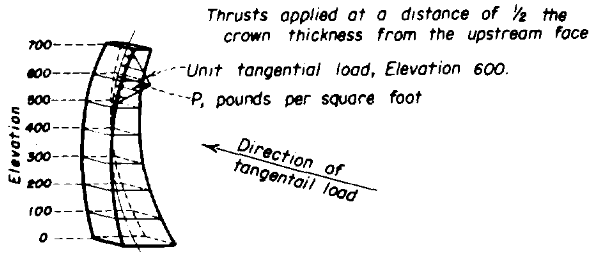
Unit tangential loads applied to cantilevers are similar to unit radial loads, except that they represent tangential shearing forces applied at the centerlines instead of radial forces applied at the faces (see fig. 4-16(b)). In computing tangential deflections, tangential shear is assumed to be distributed uniformly throughout each horizontal cantilever section. Only those deflections due to shear are evaluated. Tangential bending of cantilevers is not considered.

Unit twist loads are triangular loads which represent twisting moments applied to cantilevers as shown on figure 4-16(c). Twisting moments act on cantilevers that are elements of a continuous structure. Therefore, shears set up by these moments act in tangential directions. They are assumed to have a linear variation from the upstream face to the downstream face. With this assumption, angular movements can be calculated by the formula used for twist in a continuous slab [10].

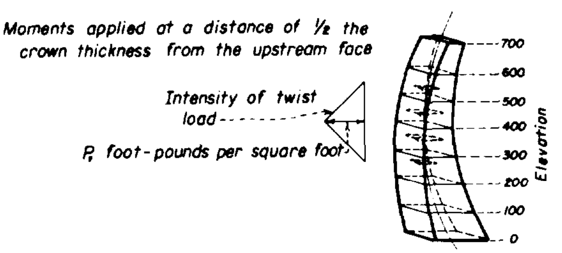
Both tangential and twist cantilever loads produce secondary movements that are generally large enough to require consideration. Tangential loads cause significant rotations in horizontal planes if the cantilever sections change considerably from the top of the dam to the base. These rotations exist because the centerlines of the cantilevers are not vertical. Twist loads cause secondary cantilever movements in radial directions, the rates of



(a) TYPICAL UNIT RADIAL LOAD



(b) TYPICAL UNIT TANGENTIAL LOAD



(c) TYPICAL UNIT TWIST LOAD

*Figure 4-16.* **Typical unit cantilever loads. 288-D-2685**

change of applied twisting moments along arch centerlines being equal to the rates of change of bending moments in the cantilevers. These secondary effects are calculated after each trial and are included in succeeding adjustments.

After deflections are calculated for unit loads, the results are tabulated for convenient use. These deflections aid in estimating trial loads since they show the flexibility of the elements. If the cantilevers are assumed to crack, unit-load deflections cannot be used because cantilever movements depend on the amounts of cracking. In this case, it is necessary to calculate total cantilever deflections each time the trial load changes.

(d) *Unit Arch Loads.*—Unit arch loads are used for the same purpose on arches as unit loads on cantilevers, the applied arch load being built up by means of unit loads. For the purpose of simplification, load constants are tabulated for typical circular arch loads so as to determine deflections at arch points. These are discussed more fully in section 4-34(g) and (h).

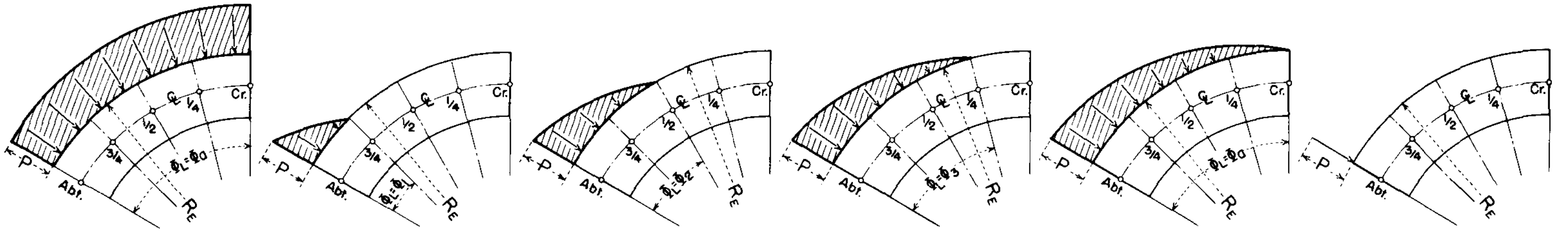
Radial arch loads include a load which is uniform over the entire arch length, and triangular loads which vary from a maximum pressure at the abutment to zero pressure at the quarter-points as shown on figure 4-17(a). It is possible, by means of these unit radial loads, to apply any radial load that varies as a straight line between quarter-points of the arches.

Unit tangential loads, consisting of tangential thrusts applied at the arch centerlines, are used in the adjustment of tangential deflections. These unit loads include a uniform load *P*, usually 1,000 pounds per square foot, and triangular loads varying from *P* pounds per square foot at the abutment to zero at the quarter-points (see fig. 4-17(b)).

Unit twist loads are also uniform and triangular in shape. The uniform unit load represents a moment of *P* foot-pounds per square foot, applied along the entire centerline of the arch. The triangular loads represent moment loads varying from *P* foot-pounds per square foot at the abutment to zero at the quarter-points, as shown on figure 4-17(c).

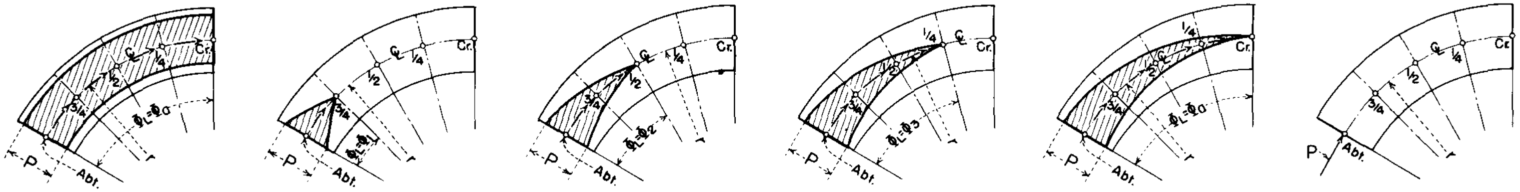
Unit loads are numbered as shown on figure 4-17 for easy reference; thus, a No. 1 load is always a uniform load, a No. 2 load begins at the abutment and terminates at the 3/4 point, and so on.

In addition to the loads just discussed, another set of unit loads is required by the assumption of elastic foundations and abutments. At the base of a symmetrical dam, the foundation is subjected to cantilever loads only. At the ends of the top arch the abutment is subjected to arch loads only. At all intermediate points the canyon wall acts both as an arch abutment and as a cantilever foundation. This is a direct corollary of the assumption that each system occupies the whole volume of the dam. Part of the arch abutment movement is due to arch loads and the balance is due to the cantilever loads. Concentrated unit loads of shear, thrust, and moment are placed on the arch abutment. These concentrated forces and moments are



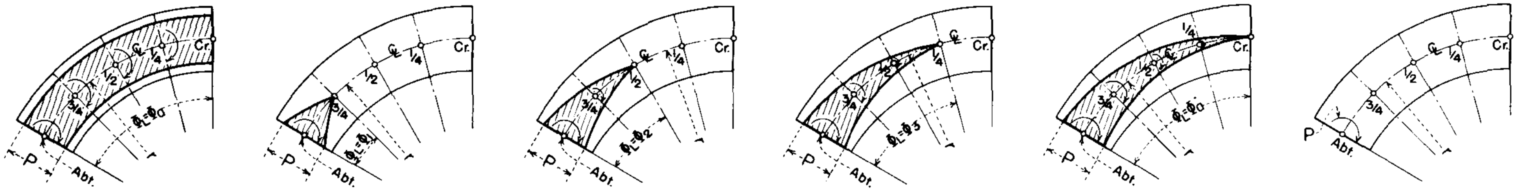
UNIT LOAD NO. 1 UNIT LOAD NO. 2 UNIT LOAD NO. 3 UNIT LOAD NO. 4 UNIT LOAD NO. 5 UNIT CONCENTRATED

(a) RADIAL LOAD PATTERNS



UNIT LOAD NO. 1 UNIT LOAD NO. 2 UNIT LOAD NO. 3 UNIT LOAD NO. 4 UNIT LOAD NO. 5 UNIT CONCENTRATED

(b) TANGENTIAL LOAD PATTERNS



UNIT LOAD NO. 1 UNIT LOAD NO. 2 UNIT LOAD NO. 3 UNIT LOAD NO. 4 UNIT LOAD NO. 5 UNIT CONCENTRATED

(c) TWIST LOAD PATTERNS

*Figure 4-17.* **Unit external load patterns showing positive application. ($\Phi_L$ equals angle under load.).–TM467** respectively, a radial shear of $P$, usually 1,000 pounds, a tangential thrust of $P$ pounds, and a twisting moment of $P$ foot-pounds, as shown by figures 4-17(a), (b), and (c), respectively.

Temperature changes in a dam may be determined at elevations of sample arches from actual temperatures observed at or near the site, and are assigned to arches as initial loads. The temperature load is not an adjustment load, but is an initial load on the arches. Unit radial, tangential, and angular movements are calculated for a unit 1° F. temperature change by means of the coefficient of expansion of concrete, as shown in section 4-34(o). Linear temperature variations assume −½° F. at the upstream face and +½° F. at the downstream face. These unit movements are multiplied by the assumed temperature change and linear variations, respectively, to give the total movement due to change of temperature within the arch.

Besides producing radial movements, radial loads on the arches cause secondary tangential and angular movements. Tangential loads cause secondary radial and angular movements, and twist loads cause secondary radial and tangential movements. In other words, each arch load produces radial, tangential, and angular movements that have to be considered in the adjustment. However, secondary movements converge rapidly with successive adjustments so that only a few readjustments are necessary.

## 3. *Preparation for Trial-Load Adjustments*

4-29. ***General Considerations.***—(a) *Preliminary Studies.*—For a complete trial-load analysis it is advisable that the proposed dam be as near to final dimensions as practicable. This would mean that preliminary studies of tentative dams by simplified methods such as the arch and crown-cantilever method, and methods based on radial adjustment only, had been previously utilized to obtain a section which is expected to be the most suitable for the given site. Owing to the comprehensive nature of the trial-load method, excavation limits should be determined as exactly as possible so that the dimensions of the dam are close to final. Arches and cantilevers are selected as indicated in section 4-26(b).

(b) *Layout of Arches.*—Each arch is drawn to a suitable scale to show dimensions, radii, and angles. Arch formulas are based on radial abutments. Actual surfaces of contact between arches and abutment rocks are so modified that when radial abutment planes are drawn through the point where the centerline of the arch intersects the abutment, the angle between the abutment and the centerline, $\Phi_A$, is taken to the nearest whole degree. This makes possible the use of tables of circular arch and load constants without interpolation. At the same time, the angles of abutment planes, $\psi$, are determined and tabulated. Arches are classed as uniform thickness or variable thickness, with or without fillets as the case may be, and symmetrical or nonsymmetrical. If symmetrical only one-half of the dam need be analyzed. If nonsymmetrical the entire dam is analyzed. If the arch is of variable thickness, or with fillets, the division into voussoirs is made as indicated in section 4-35(a) and dimensions tabulated.

(c) *Layout of Cantilevers.*—Cantilever elements are drawn to a suitable scale to show dimensions of the vertical and horizontal cross sections at each arch elevation. The cantilevers are 1 foot in width at the axis of the dam. In a variable-radius arch dam, the assumption of vertical radial planes for the sides of the cantilevers is not feasible because the radii vary in direction with a change in elevation. This variation results in warped radial surfaces which are 1 foot apart at the axis as illustrated on figure 4-18. In the summation process, however, each increment is assumed to have vertical radial sides.

(d) *Design Data.*—Constants and structural data requirements are given in section 4-27(a), and forces acting on the dam are treated in section 4-27(b). For purposes of illustration, a number of computations made for typical dams are presented in appendixes B and C.

4-30. ***Foundation Constants.***—(a) *General Discussion.*—The inclusion of yielding abutments in the trial-load analysis has the effect of increasing deflections of the cantilever

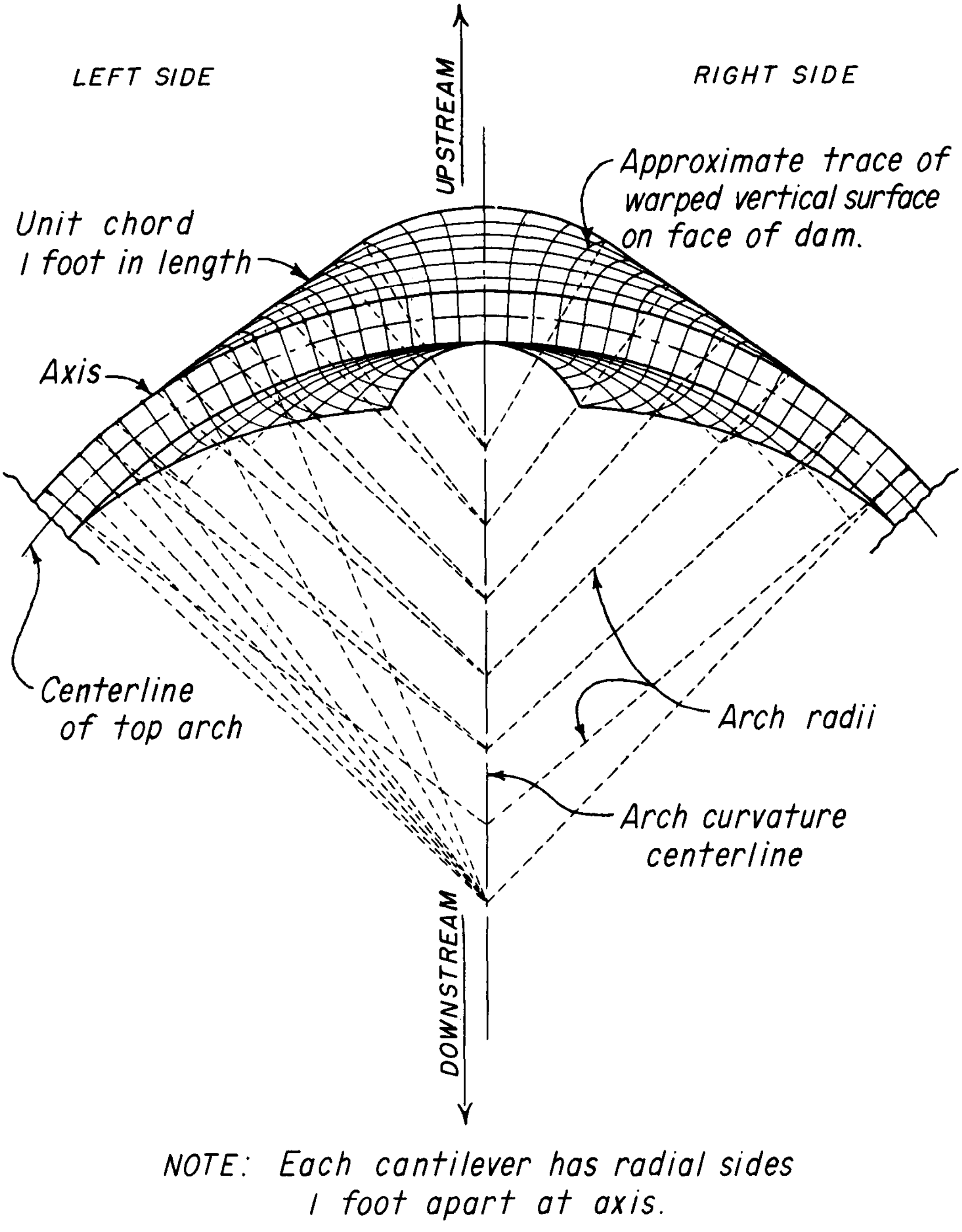


*Figure 4-18.* **Vertical elements of a variable-radius arch dam.–288-D-2738**

and arch elements. Stresses are usually decreased at the abutments and foundations, but may be increased in other parts of the structure.

Elastic deformation of the foundation causes tilting, twisting, and displacement of arch abutments and cantilever foundations. Normally, each arch abutment is coincident with a cantilever foundation, and the two must move together as a single unit. It is necessary to use the total abutment load from both members in computing foundation deformations for each trial combination of loads on the two elements. Arches and cantilevers are rotated and displaced as rigid bodies. These rigid-body movements are added directly to the elastic deformations of the elements themselves to obtain the total rotations and displacements of the arch or cantilever at any point.

(b) *Assumptions.*—The basic assumptions made for use in computing foundation deformations are given as items 1 and 2 in section 4-25. The equations by which the deformations are computed are based on the assumption of an isotropic foundation material of infinite extent in the plane of the foundation and below it. Actually the foundation is never a plane and the foundation material is never isotropic nor homogeneous. The theory makes no allowances for these variations, and consequently, the foundation analysis shown in this chapter is only approximate. The dam may be considered as being supported by a series of independent springs, the elastic constants of which are determined by reference to deformations of an infinite foundation with a plane surface.

The assumptions mentioned above are made for the purpose of utilizing the Boussinesq and Cerruti formulas for deformation of an infinite foundation with a plane surface. Basing his work on these formulas, Dr. Fredrick Vogt obtained equations for average deformations of a loaded rectangular area of the foundation surface [11]. These equations express deformation due to a bending moment in a plane normal to the surface, a force normal to the surface, and a tractive or shear force in the plane of the surface. To supplement Dr. Vogt's equations, a formula has been derived for average deformation due to a twisting moment in the plane of the surface.

(c) *Notations.*—A list of notations is given at the beginning of this chapter (sec. 4-2). The following additional items are used in this discussion of foundation constants for considering movements of unit horizontal and vertical elements. Directions are as shown on figure 4-24D. Symbols refer to the left side of the dam.

$M_x$ = moment normal to $X$ axis.
$M_y$ = moment normal to $Y$ axis.
$M_z$ = moment normal to $Z$ axis.
$H$ = force parallel to $X$ axis.
$V$ = force parallel to $Y$ axis.
$W$ = force parallel to $Z$ axis.
$\theta_x$ = angular movement normal to $X$ axis.
$\theta_y$ = angular movement normal to $Y$ axis.
$\theta_z$ = angular movement normal to $Z$ axis.
$\Delta x$ = movement parallel to $X$ axis.
$\Delta y$ = movement parallel to $Y$ axis.
$\Delta z$ = movement parallel to $Z$ axis.
$T$ = thickness of dam at abutment or foundation.
$\perp$ = perpendicular to foundation surface.
$\parallel$ = parallel to, or in plane of foundation surface.

(d) *Equations.*—Rotation and deformations of the foundation surface for moments and forces of unity, per unit length, are given by the following formulas, in which $k$ is a function of $\mu$ and $b/a$, and $T$ is equal to $a'$ (see figs. 4-19 through 4-23):

$$\alpha' = \frac{k_1}{E_r T^2} \qquad (1)$$

$$\beta' = \frac{k_2}{E_r} \qquad (2)$$

$$\gamma' = \frac{k_3}{E_r} \qquad (3)$$

$$\delta' = \frac{k_4}{E_r T^2} \qquad (4)$$

$$\alpha'' = \frac{k_5}{E_r T} \qquad (5)$$

$$\gamma'' = \frac{k_5}{E_r T} \qquad (6)$$

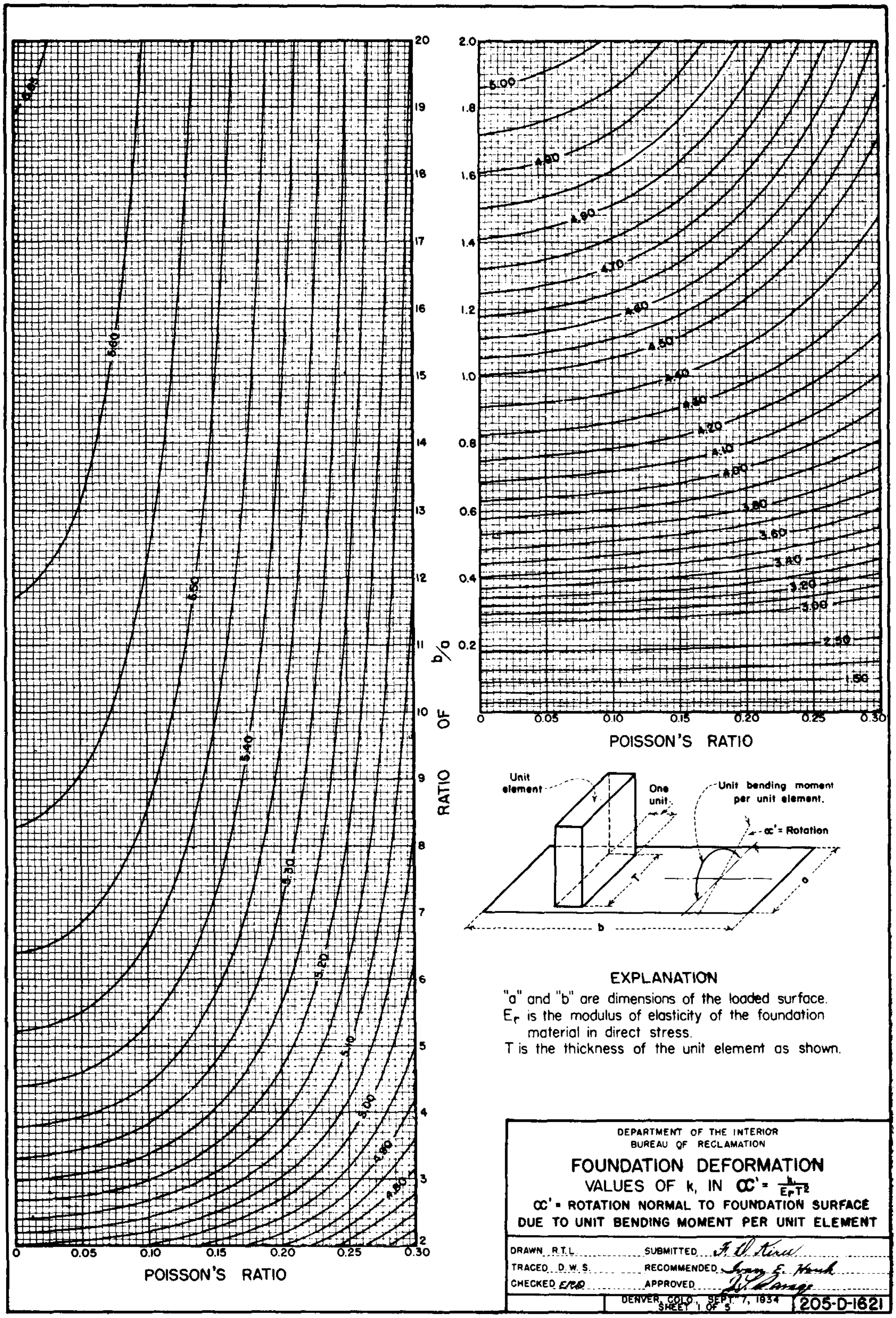
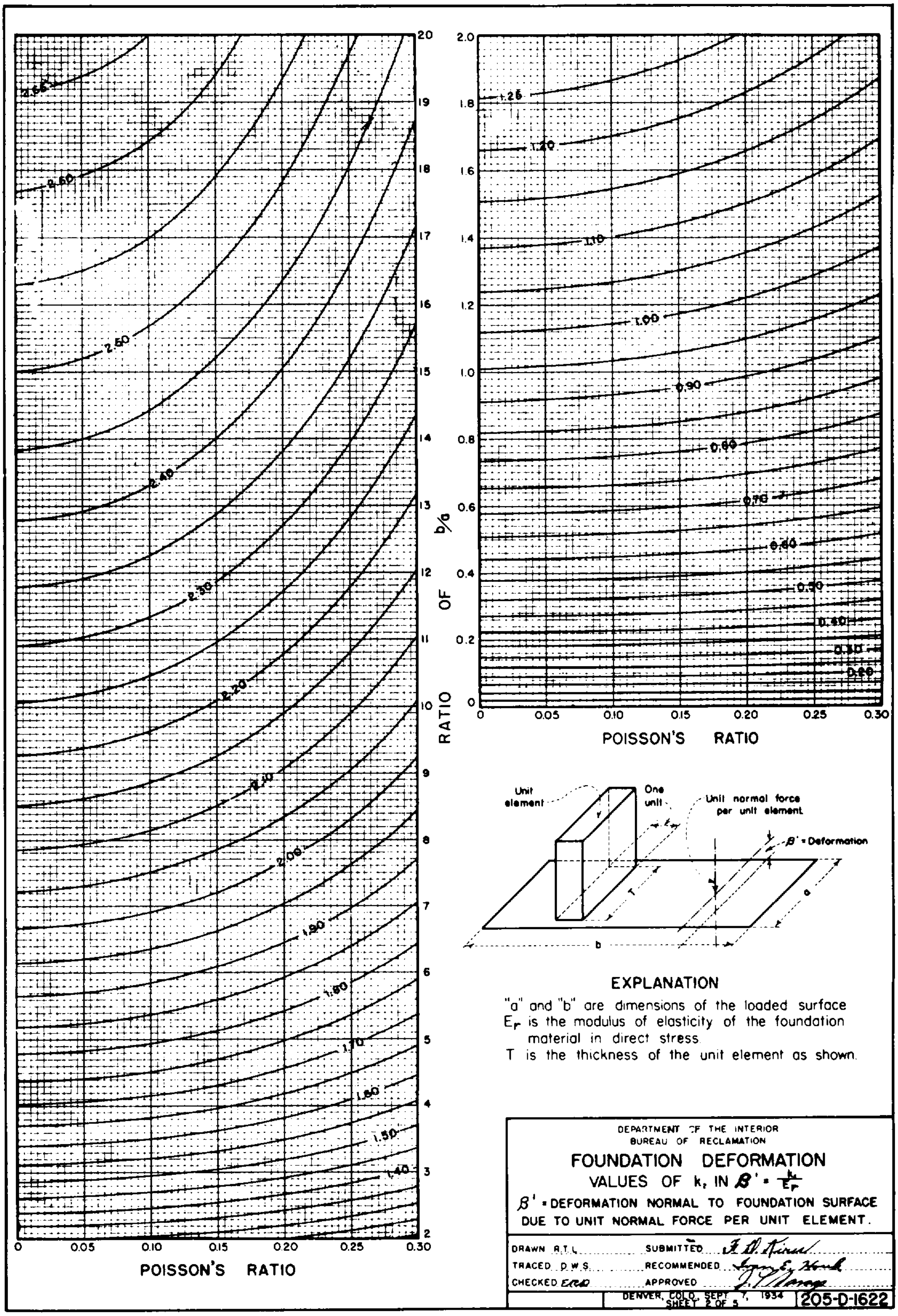


*Figure 4-19.* **Foundation deformation–values of $k_1$ in equation (1).**



EXPLANATION

"a" and "b" are dimensions of the loaded surface
$E_r$ is the modulus of elasticity of the foundation material in direct stress.
T is the thickness of the unit element as shown.

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
FOUNDATION DEFORMATION
VALUES OF $k_2$ IN $\beta' = \frac{k_2}{E_r}$
β' = DEFORMATION NORMAL TO FOUNDATION SURFACE DUE TO UNIT NORMAL FORCE PER UNIT ELEMENT.

DRAWN R.T.L. SUBMITTED
TRACED D.W.S. RECOMMENDED
CHECKED E.R.D. APPROVED
DENVER, COLO., SEPT. 7, 1934 SHEET 2 OF 5 205-D-1622

*Figure 4-20.* **Foundation deformation—values of $k_2$ in equation (2).**

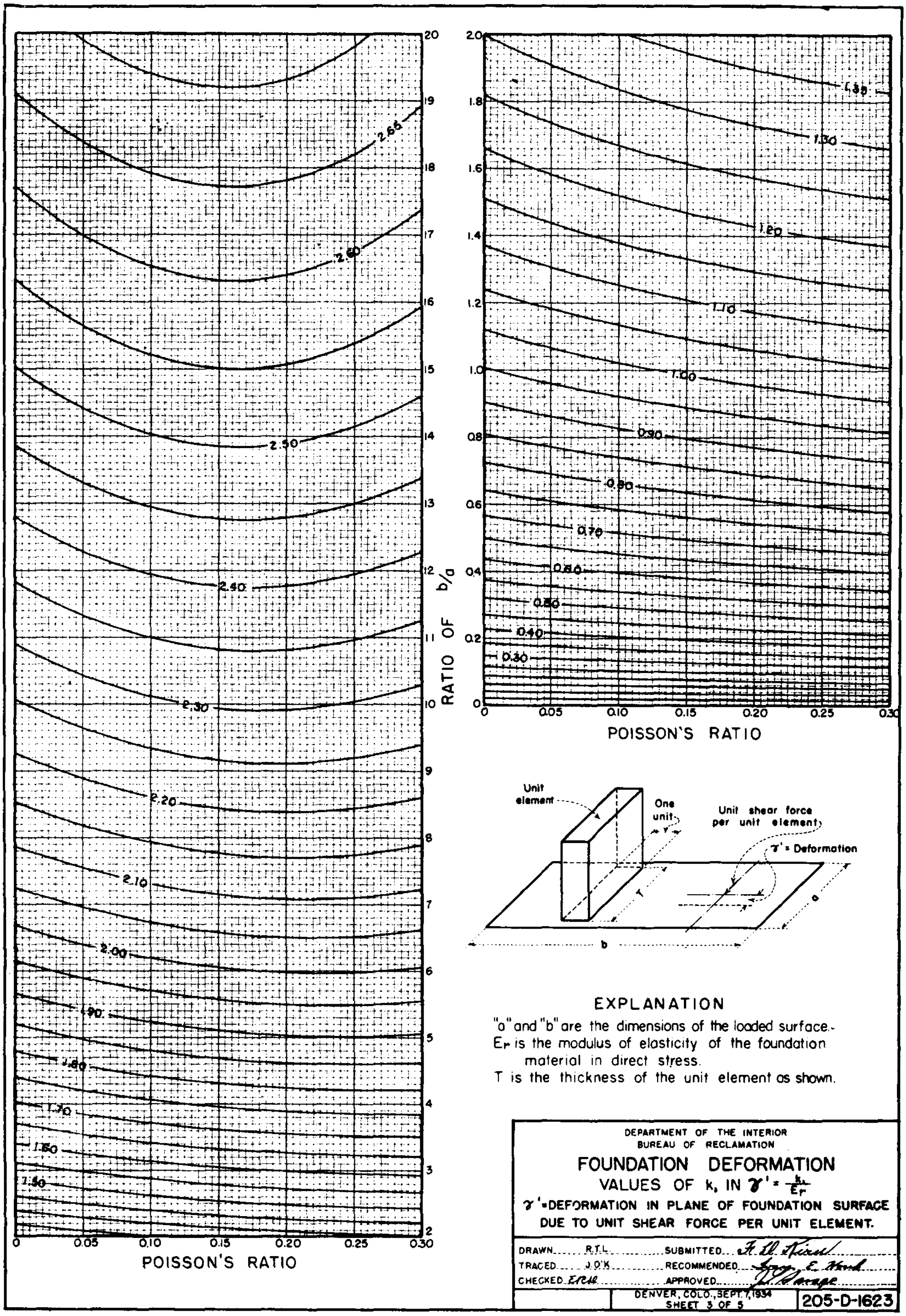
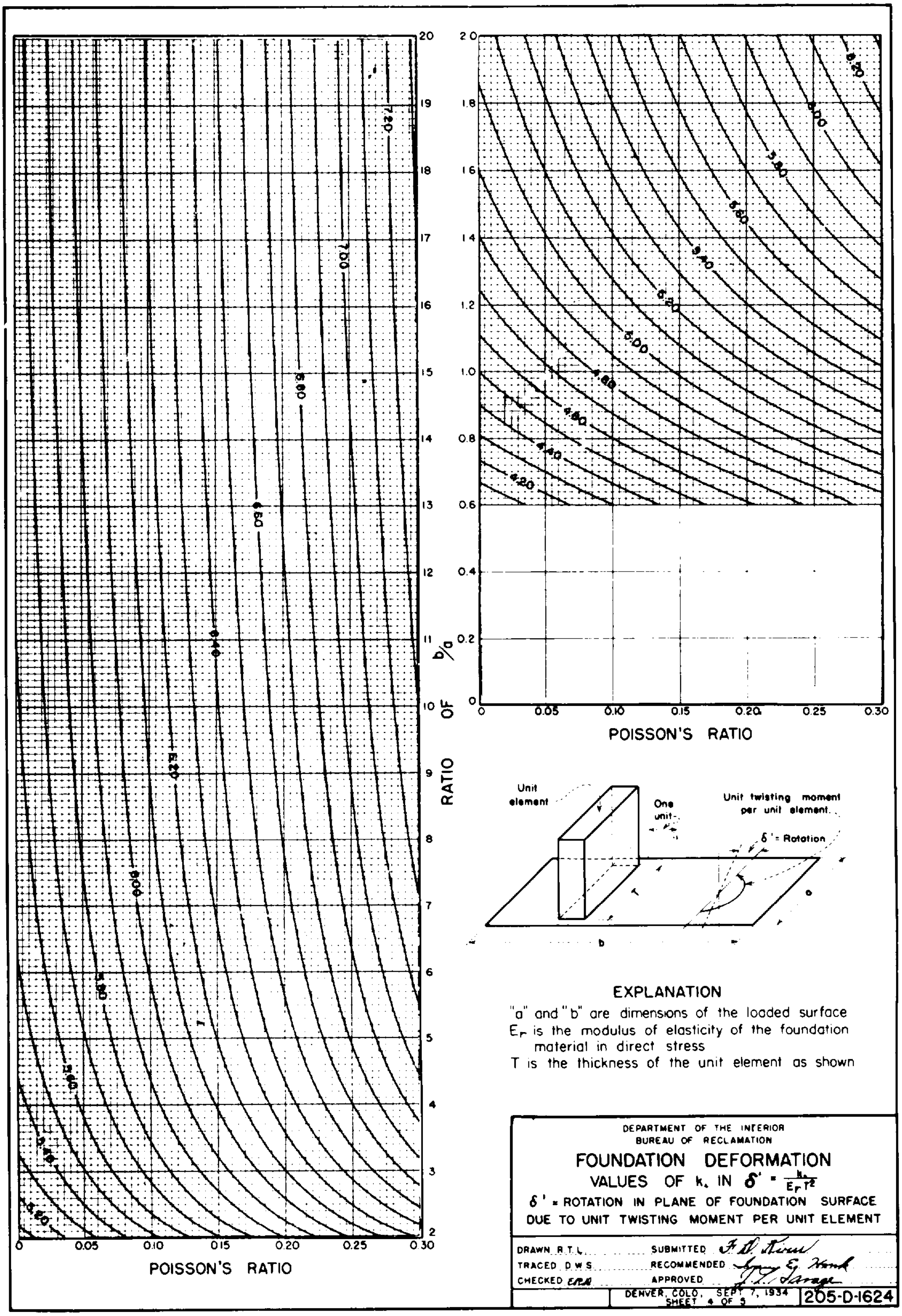


**Figure 4-21.** Foundation deformation–values of $k_3$ in equation (3).



EXPLANATION

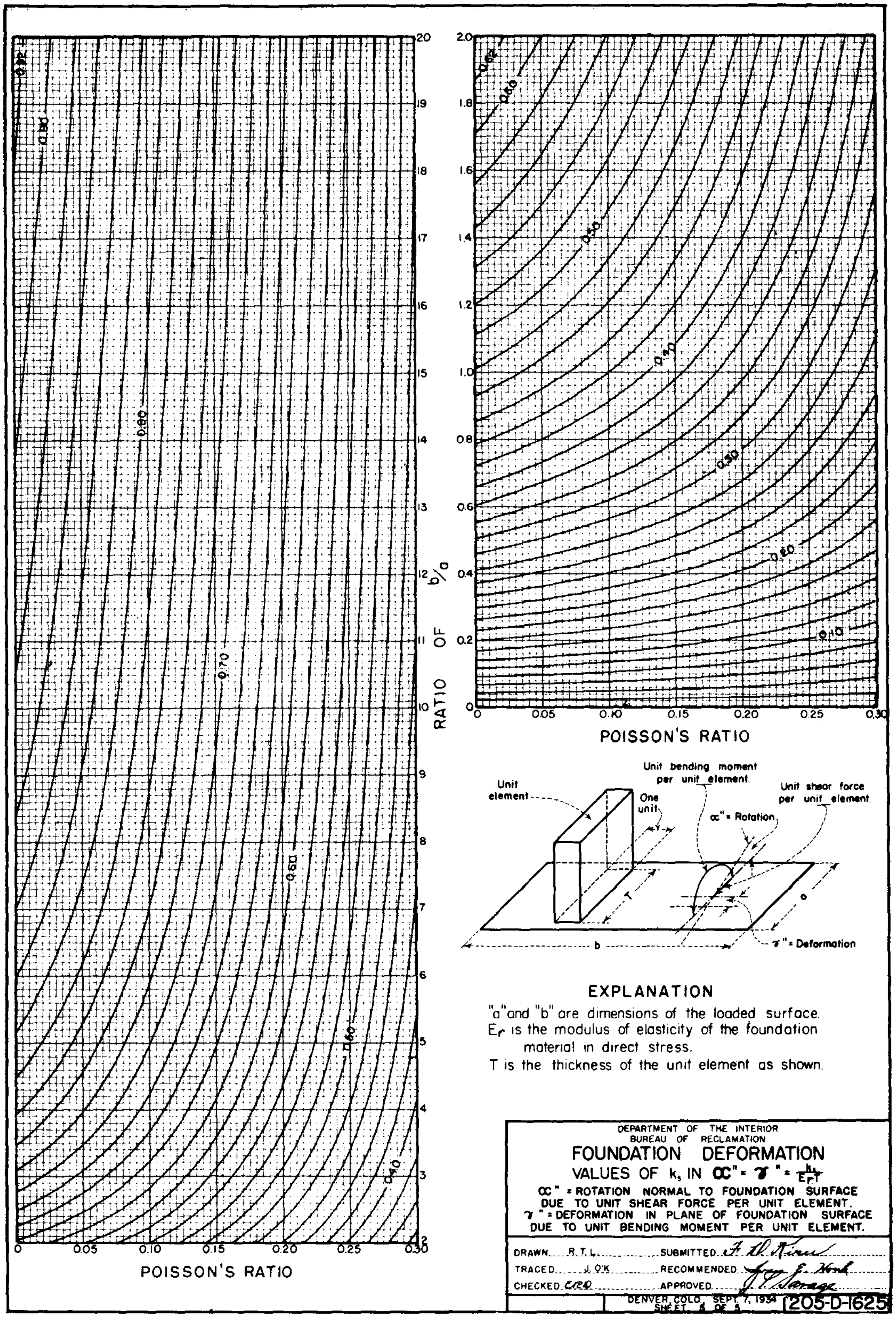
"a" and "b" are dimensions of the loaded surface
$E_r$ is the modulus of elasticity of the foundation material in direct stress
T is the thickness of the unit element as shown

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

FOUNDATION DEFORMATION
VALUES OF $k_4$ IN $\delta' = \frac{k_4}{E_r T^2}$
$\delta'$ = ROTATION IN PLANE OF FOUNDATION SURFACE DUE TO UNIT TWISTING MOMENT PER UNIT ELEMENT

DRAWN R.T.L. SUBMITTED F.D. Kirn
TRACED D.W.S. RECOMMENDED Ivan E. Houk
CHECKED E.R.B. APPROVED J.L. Savage
DENVER, COLO., SEPT. 7, 1934 SHEET 4 OF 5 205-D-1624

*Figure 4-22.* **Foundation deformation–values of $k_4$ in equation (4).**



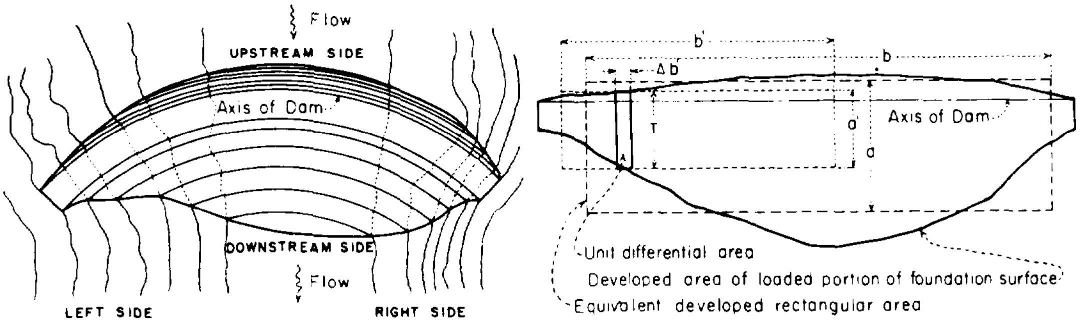
EXPLANATION

"a" and "b" are dimensions of the loaded surface.
$E_r$ is the modulus of elasticity of the foundation material in direct stress.
T is the thickness of the unit element as shown.

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
FOUNDATION DEFORMATION
VALUES OF $k_5$ IN $\alpha'' = \gamma'' = \frac{k_5}{E_r T}$
$\alpha''$ = ROTATION NORMAL TO FOUNDATION SURFACE DUE TO UNIT SHEAR FORCE PER UNIT ELEMENT.
$\gamma''$ = DEFORMATION IN PLANE OF FOUNDATION SURFACE DUE TO UNIT BENDING MOMENT PER UNIT ELEMENT.

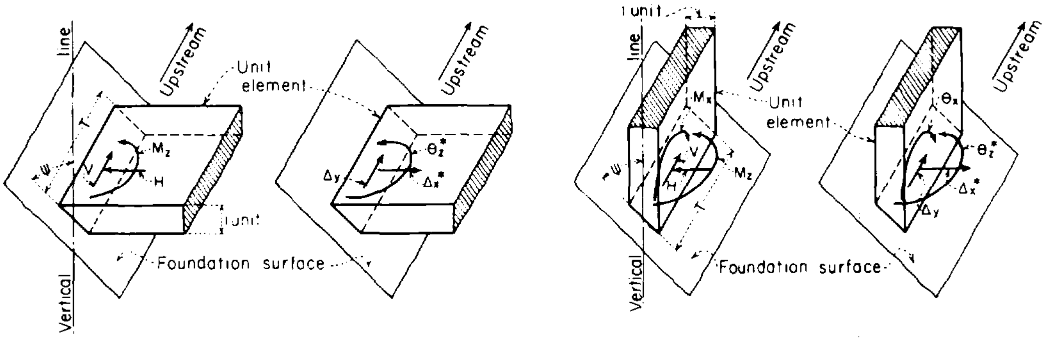
DRAWN R.T.L. SUBMITTED F. W. Kinn
TRACED J. O'K. RECOMMENDED Ivan E. Houk
CHECKED C.R.O. APPROVED J. L. Savage
DENVER, COLO., SEPT. 7, 1934
SHEET 5 OF 5
205-D-1625

*Figure 4-23.* **Foundation deformation–values of $k_5$ in equations (5) and (6).**



(a) PLAN (b) DEVELOPMENT

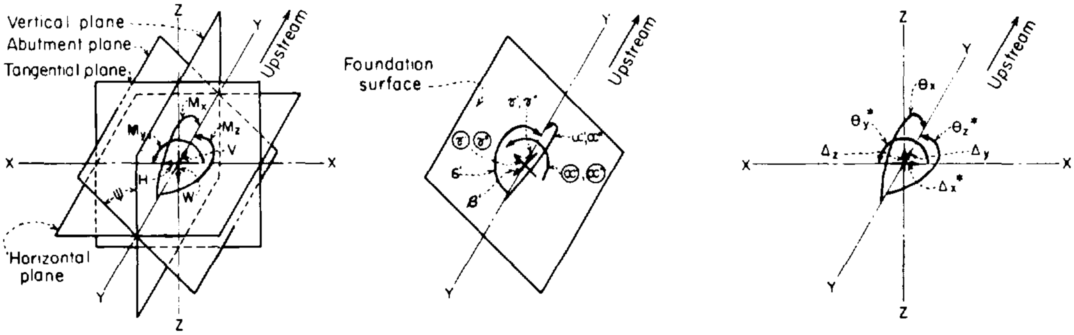
A - LOADED AREA OF FOUNDATION SURFACE



(a) FORCES AND MOMENTS (b) MOVEMENTS (a) FORCES AND MOMENTS (b) MOVEMENTS

B - UNIT HORIZONTAL ELEMENT AT LEFT ABUTMENT OF DAM

C - UNIT VERTICAL ELEMENT AT LEFT ABUTMENT OF DAM



(a) POSITIVE FORCES AND MOMENTS (b) FOUNDATION DEFORMATIONS (c) POSITIVE MOVEMENTS

D - ABUTMENT OF LEFT SIDE OF DAM

Note Positive Forces, Moments, and Movements have same direction with respect to right abutment except (*) which are opposite.

*Figure 4-24.* **Foundation surface and unit forces, moments, and movements.–288-D-2972**

Deformations $\alpha''$ and $\gamma''$ are secondary in character and are of relatively small magnitude.

The above equations contain elastic constants, $E_R$ and $\mu$, which are usually determined by direct experimental methods. The curves shown on figures 4-19 through 4-23 provide an easy means for determining values of $k_1$ to $k_5$, inclusive, after the ratio $b/a$ has been determined by means described below. Corresponding $k$ values for $\textcircled{\alpha'}$, $\textcircled{\gamma'}$, $\textcircled{\alpha''}$, and $\textcircled{\gamma''}$ are determined from the same figures as for $\alpha'$, $\gamma'$, $\alpha''$, and $\gamma''$, respectively.

It is impossible to obtain a definite value of $b/a$ for an irregular foundation surface. An approximation of some kind is necessary, and at present the following method is used. The surface of contact between the dam and foundation is developed and plotted with the axis as a straight line as shown on figure 4-24A. This surface is replaced by a rectangle of the same area and approximately the same proportions, called the equivalent developed area. The ratio of length to width of the rectangle is taken as the ratio $b/a$ for the foundation in question. The value of $b/a$ is therefore a constant for a particular dam. In computing deformations for a particular element, the width $\alpha'$ is made equal to $T$, the thickness of the dam at the element considered, making $T/b' = a/b$, or $b' = (b/a)\,T$.

Figures 4-24B, C, and D show sketches of foundation deformation movements for unit horizontal and vertical elements of a dam. Forces and moments in the dam measured with reference to radial, tangential, and horizontal planes, as shown on figure 4-24D(a), are resolved into normal and tangential components acting on the foundation, as shown for the unit elements on figures 4-24B(a) and 4-24C(a). These resolved forces and moments produce deformations and rotations of the unit differential area, both normal to and in the abutment plane, as shown on figure 4-24D(b). Deformation and rotations are resolved into components in principal planes of movements, and the summation of the components in any plane or direction is the resultant movement of the element in that plane or direction.

The final equations for the foundation movements of a unit horizontal element at either abutment of the dam are shown below. The algebraic signs are as used for the left abutment, and the asterisk (*) indicates that signs are to be reversed for movements at the right abutment.

$$*\theta_z = M_z\,\alpha + V\alpha_2 \qquad (7)$$

$$*\Delta x = -H\beta \qquad (8)$$

$$\Delta y = V\gamma + M_z\,\alpha_2 \qquad (9)$$

for which:

$$\alpha = \alpha' \cos^3\psi + \delta' \sin^2\psi \cos\psi$$

$$\alpha_2 = \alpha'' \cos^2\psi$$

$$\beta = \beta' \cos^3\psi + \textcircled{\gamma'} \sin^2\psi \cos\psi$$

$$\gamma = \gamma' \cos\psi$$

$$\alpha'' = \gamma''$$

It is customary to require that $\beta'$, $\delta'$, and $\textcircled{\gamma'}$ (see fig. 4-24A(b)), for a unit differential area on one side of the dam be average values for the equivalent developed area of that side of the dam. If the damsite is approximately symmetrical about the maximum section, dimensions of the equivalent developed area for either or both sides of the dam are $a$ and $b/2$. For this reason, ratios $\frac{b/2}{a}$ and $\frac{a}{b/2}$ are substituted for the ratio $b/a$ in some cases in obtaining values from the curves on figures 4-19 through 4-23. These substitutions are indicated below:

For $\alpha'$, $\alpha''$, $\gamma'$, and $\gamma''$ use ratio $\frac{b}{a}$

For $\beta'$ and $\delta'$ use ratio $\frac{b/2}{a}$

For $\textcircled{\gamma'}$ use ratio $\frac{a}{b/2}$ and multiply results from the graph by $\frac{b/2}{a}$ to correct the curve values for this direction.

The final equations for movements of a unit vertical element at either abutment of the dam are shown below. As before, the algebraic signs are as used for the left abutment and the asterisk (*) indicates that signs are to be reversed for movements at the right abutment.

$$\theta_x = M_x\ \alpha + V\ \alpha_2 \qquad (10)$$

$$*\theta_z = M_z\ \delta \qquad (11)$$

$$\Delta x = -\ H\ \textcircled{\gamma} \qquad (12)$$

$$\Delta y = V\ \gamma + M_x\ \alpha_2 \qquad (13)$$

for which:

$$\alpha = \alpha'\ \sin^3\ \psi + \delta'\ \sin\ \psi\ \cos^2\ \psi$$

$$\alpha_2 = \alpha''\ \sin^2\ \psi$$

$$\delta = \delta'\ \sin^3\ \psi + \alpha'\ \sin\ \psi\ \cos^2\ \psi$$

$$\textcircled{\gamma} = \textcircled{\gamma'}\ \sin^3\ \psi + \beta'\ \sin\ \psi\ \cos^2\ \psi$$

$$\gamma = \gamma'\ \sin\ \psi$$

$$\alpha'' = \gamma''$$

In order to compute abutment deformations it is necessary to introduce concentrated loads at the arch abutments representing loads transmitted by the cantilever elements resting thereon. From figure 4-25 it may be seen that at the centerline, the width of a cantilever resting on an arch of unit height is equal to tan $\psi$. Since the assumed unit cantilever used in the analysis is 1 foot wide at the axis, the radial shear at the base of the cantilever is multiplied by $\frac{R_{axis}}{r}$ tan $\psi$ to give the correct amount of concentrated radial load to be applied to the arch 1 foot high. For the same reason, tangential shears and twisting moments at the cantilever foundations are corrected by the factor $\frac{R_{axis}}{r}$ tan $\psi$ to determine, respectively, concentrated tangential and twist loads on the arches. In the adjustments, abutment movements of the arches are added algebraically to deflections of corresponding cantilevers, since cantilever foundations move with arch abutments.

Concentrated loads take care of all foundation movements except rotation in a vertical plane, $\theta_x$. Therefore, equation (10) must be used in computing movements due to unit loads for cantilevers resting on arch abutments. Computations of unit foundation movements for Monticello Dam by use of the equations listed above are shown in appendix C.

4-31. *Uncracked-Cantilever Analysis.*—(a) *Introduction.*—The purpose of this section is to show the preparation of certain essential data for uncracked cantilevers which are required before any adjustments can be made. This preparation includes the evaluation of properties of a radial-side cantilever in a concrete dam; the computation of forces and moments due to weight of concrete and water; the computation of forces and moments due to earthquake shock, unit and initial radial, tangential, and twist loads; and computation of unit and initial cantilever movements due to unit loads and initial loads. Examples of the computations discussed in this section are shown in appendix B.

The cantilever element is assumed to be composed of a series of horizontal differential elements which have circular-arc boundaries at the upstream and downstream edges and radial boundaries 1 foot apart at the axis of the dam (see fig. 4-26(b)). Also, the cantilever is assumed to be an elastic unit, set upon an elastic foundation. As previously stated all movements are measured from the concrete-weighted position, it being assumed that the cantilevers take all the weight before arch action begins, unless a construction and grouting program is included. The cantilever may have an initial radial movement due to the vertical component of waterload, silt, ice, concrete inertia, and superstructure placed upon it as initial loads; and an additional radial movement due to a radial load which represents some fractional part of the total reservoir load. Furthermore, it may have an initial tangential movement due to an initial horizontal earthquake load (see sec. 4-28). If stage construction is included, the deflections

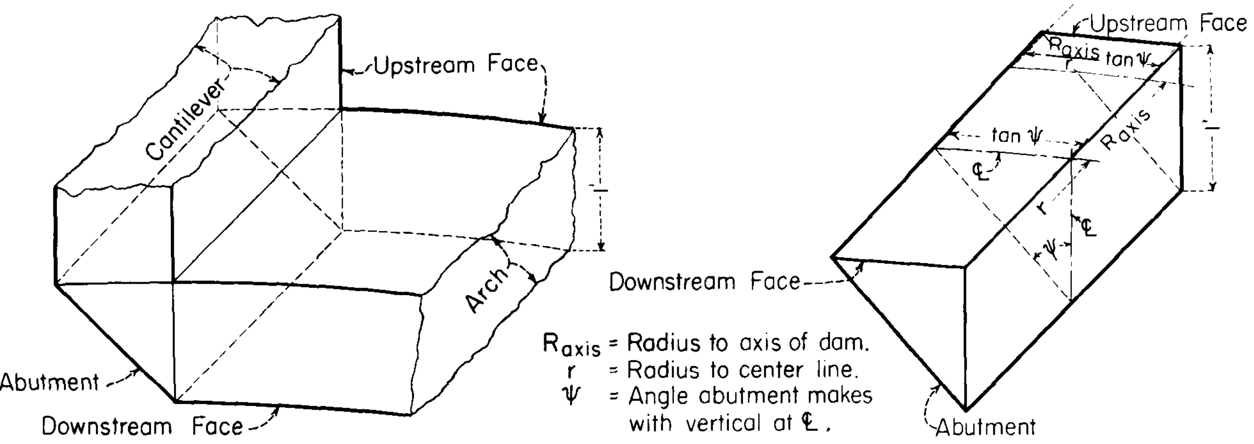
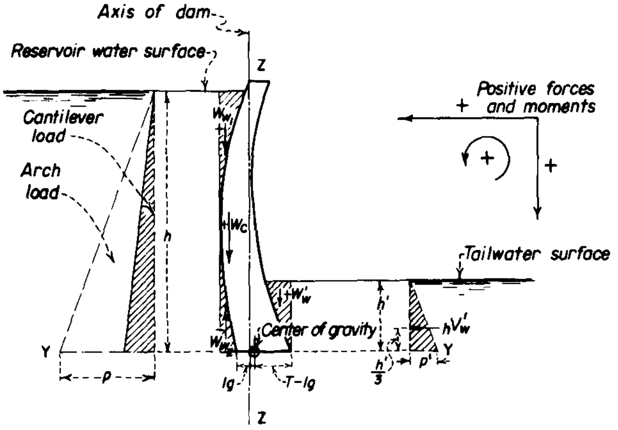
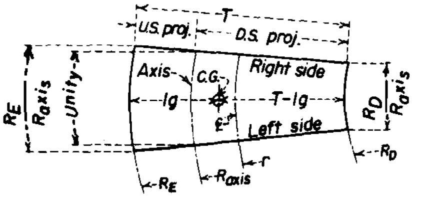


*Figure 4-25.* **Contact of arch and cantilever with abutment.–288-D-434**



*(a) VERTICAL CROSS SECTION*



*(b) HORIZONTAL CROSS SECTION*

*Figure 4-26.* **Cross section of a radial-side cantilever showing normal loading conditions.–288-D-2973**

due to the weight and moments of concrete placed after a portion of the dam is grouted are considered as part of the initial deflection for that stage.

Normal loading conditions consist of reservoir and tailwater loads, but exclude any earthquake effects. Horizontal silt load may be included in the reservoir loading. Vertical and horizontal cross sections of a typical cantilever for normal loading conditions are shown on figure 4-26. It should be noted that only a fractional part of the reservoir waterload is applied to the cantilever; that a radial force to the right is a negative force; that the weight is a positive force downward; and that positive forces give positive moments in a counterclockwise direction. The complete sign convention is shown on figure 4-29 for cantilevers to the left and right of the maximum vertical cross section of the dam, respectively. This sign convention must be carefully followed in the analysis of cantilevers and requires careful study. A prime mark (′) indicates a value for tailwater. (A list of notations is given at the beginning of this chapter (sec. 4-2).)

(b) *Properties of a Radial-Side Cantilever.*– The properties of horizontal sections of a cantilever are usually determined at elevations of sample arches. Additional sections may be analyzed at intermediate elevations if more accuracy is required. After the thickness and radii at a section have been found, the area, $A$, the moment of inertia, $I$, and the distance from the upstream face to center of gravity, $lg$, may be obtained from the equations which follow.

If the boundaries at the upstream and downstream faces in section (b) of figure 4-26 are assumed to be straight lines, the area of the horizontal cross section is:

$$A = \left[ \frac{\dfrac{R_E}{R_{axis}} + \dfrac{R_D}{R_{axis}}}{2} \right] T$$

or simplifying,

$$A = \frac{T R_E}{2 R_{axis}} \left[ 1 + \frac{R_D}{R_E} \right] \qquad (14)$$

Likewise, the equation for the distance from the upstream face to the center of gravity is:

$$lg = \frac{T}{3} \left[ \frac{\dfrac{R_E}{R_{axis}} + 2 \dfrac{R_D}{R_{axis}}}{\dfrac{R_E}{R_{axis}} + \dfrac{R_D}{R_{axis}}} \right]$$

or simplifying,

$$lg = \frac{T}{3} \left[ \frac{1 + 2 \dfrac{R_D}{R_E}}{1 + \dfrac{R_D}{R_E}} \right] \qquad (15)$$

The moment of inertia of the section about the center of gravity is:

$$I = \frac{T^3}{36} \left[ \frac{\left(\dfrac{R_E}{R_{axis}}\right)^2 + 4 \dfrac{R_E}{R_{axis}} \bullet \dfrac{R_D}{R_{axis}} + \left(\dfrac{R_D}{R_{axis}}\right)^2}{\dfrac{R_E}{R_{axis}} + \dfrac{R_D}{R_{axis}}} \right]$$

or simplifying,

$$I = \frac{T^3 R_E}{36 R_{axis}} \left[ \frac{1 + 4 \dfrac{R_D}{R_E} + \left(\dfrac{R_D}{R_E}\right)^2}{1 + \dfrac{R_D}{R_E}} \right] \qquad (16)$$

(c) *Weights and Moments Due to Concrete and Vertical Initial Loads on Cantilever.*—Weights and moments due to concrete and vertical waterloads are computed by Simpson's rule.

As stated before, initial vertical loads on the cantilever may include vertical waterload, vertical hydrodynamic load due to a horizontal earthquake shock, increased vertical waterload due to silt, and water inertia due to vertical earthquake shock. Concrete weight is not included, but the concrete inertia effect due to a vertical earthquake shock may be included as an initial vertical load.

The effect of silt in the vertical waterload can be included by using an increased density of water in computing the total weight.

(d) *Cantilever Deflections Due to Linear Temperature Effects.*—Radial deflections of cantilever elements caused by temperature variations from the upstream to the downstream face may be comparatively large. As in the arch elements, the temperature variation from the upstream to the downstream face must be assumed linear.

The curvature of cantilever elements due to linear temperature variations is defined as the rate of change of slope and is computed for a given elevation by the equation:

$$\rho = \frac{C}{T}(t) \qquad (17)$$

where:

$\rho$ = curvature,
$C$ = coefficient of thermal expansion for concrete per degree Fahrenheit,
$T$ = cantilever thickness at the elevation being considered, in feet, and
$t$ = temperature change from upstream face to downstream face in degrees Fahrenheit at the given elevation.

The total slope at any elevation on the cantilever due to linear temperature variation is obtained by integrating the curvature of the cantilever from its base to that elevation. The integration can be carried out mechanically using the following relationship:

$$\text{Slope} = \Sigma \rho \Delta z \qquad (18)$$

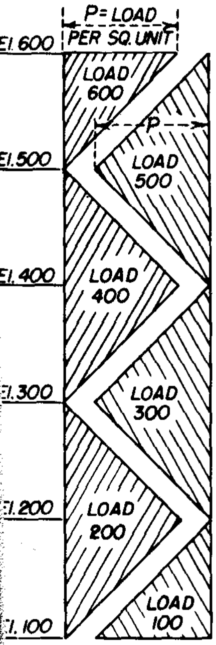
where $\Delta z$ is the incremental height.

The cantilever radial deflection, $\Delta r$, at any elevation resulting from this assumed linear temperature variation is obtained by integrating the cantilever slope from its base to the particular elevation. The following expression is in summation form used for mechanical integration:

$$\Delta r = \Sigma(\Delta z \Sigma \rho \Delta z) \qquad (19)$$

A positive temperature change is an increase in temperature from the upstream face to the downstream face. This causes an upstream $\Delta r$ which, according to the sign convention, is also positive. The $\Delta r$ computed for temperature variations as described above should be included as a part of the initial cantilever movement to be adjusted in the analysis.

(e) *Forces and Moments Due to Trial Loads on a Cantilever.*—Forces and moments at each elevation for radial loads on a cantilever are determined by means of forces and moments due to unit triangular loads, which are illustrated on figure 4-27. The triangular loads peak to a maximum value, $P$, which is generally assumed to be 1,000 pounds per square foot. Since $P$ can be considered to be a distance measured horizontally along $y$, it is spoken of as a *load ordinate*. Thus, if $P$ = 1,000 pounds per square foot, the load ordinate is equal to 1,000; or if the load is in kips, the load ordinate is unity, or one. For example, if we have an ordinate of 2 kips for a trial radial load, we can multiply a unit shear by 2 and a unit moment by 2. These resultant values must be further adjusted for the width of the face of the cantilever at each respective elevation by multiplication by the value of $R_E/R_{axis}$ for a load at the upstream face. From this discussion, it can be seen that the total reservoir load on the dam must be computed in terms of load ordinates at each respective elevation for the convenient use of trial loads



ΔV = Horizontal force of portion of load above an elevation (Area)
ΔM = Moment of ΔV about an elevation

| | Load 600 (No. 1) | Load 500 (No. 2) |
|---|---|---|
| El. 500 | $\Delta V = \frac{100P}{2} = 50P$ | $\Delta V = \frac{100P}{2} = 50P$ |
| | $\Delta M = 50P(\frac{2}{3} \cdot 100)$<br>$= 3{,}333\ P$ | $\Delta M = 50P(\frac{1}{3} \cdot 100)$<br>$= 1{,}667\ P$ |
| El. 400 | $\Delta V = 50P$ | $\Delta V = \frac{200P}{2} = 100P$ |
| | $\Delta M = 50P(66\frac{2}{3}+100)$<br>$= 8{,}333\ P$ | $\Delta M = 100P(100)$<br>$= 10{,}000\ P$ |
| El. 300 | $\Delta V = 50P$ | $\Delta V = 100P$ |
| | $\Delta M = 50P(66\frac{2}{3}+200)$<br>$= 13{,}333\ P$ | $\Delta M = 100P(200)$<br>$= 20{,}000P$ |
| El. 200 | $\Delta V = 50P$ | $\Delta V = 100P$ |
| | $\Delta M = 50P(66\frac{2}{3}+300)$ | $\Delta M = 100P(300)$ |
| El. 100 | ETC. | ETC. |

UNIT TRIANGULAR RADIAL LOADS

Loads applied at face of cantilever.
ΔV's and ΔM's for a basic unit load are multiplied by $\frac{R_E}{R_{AXIS}}$ (upstream face) for the elevation at which that basic unit load peaks.
$V$ and $M_p$ are obtained by multiplying ΔV's and ΔM's by the load ordinates at the load peak elevations.

UNIT TRIANGULAR TANGENTIAL LOADS

Loads applied at half the crown thickness from the upstream face. ΔV's for a basic unit load are multiplied by $\frac{r_0}{R_{AXIS}}$ for the elevation at which that basic unit load peaks, giving ΔH's for tangential loads.
$H$ is obtained by multiplying the ΔH's by the load ordinates at the load peak elevations.

UNIT TRIANGULAR TWIST LOADS

Loads applied at half the crown thickness from the upstream face. ΔV's for a basic unit load are multiplied by $\frac{r_0}{R_{AXIS}}$ for the elevation at which that basic unit load peaks, giving Δ~~M~~'s for twist loads.
~~M~~ is obtained by multiplying the Δ~~M~~'s by the load ordinates at the load peak elevations.

*Figure 4-27.* **Unit triangular loads for a radial-side cantilever.—288-D-2974** and unit shears and moments.

The external load to be divided by trial between the arches and cantilevers is usually due to reservoir waterload. The formula for the waterload is as follows:

$$p = wh \tag{20}$$

The units usually are pounds per square foot and feet. Any fractional part of $p$ applied to the cantilever as a trial load must be treated as described in the preceding section.

If the water surface is below the top arch to be analyzed, an inaccuracy in load occurs because of the triangular unit loads on the cantilevers (shown on fig. 4-27). The use of these triangular unit loads requires that the external pressure vary linearly from the value at the second arch to zero at the top arch. "Odd" load is the correction for the difference in water surface elevation. In the case where the water surface is above the top arch to be analyzed, the additional waterload above the top arch must be applied as an initial load.

(f) *Forces and Moments Due to Horizontal Initial Loads.*—Horizontal initial loads on the cantilever may act in a radial or tangential direction. Ice loads are usually considered as initial loads acting in a radial direction.

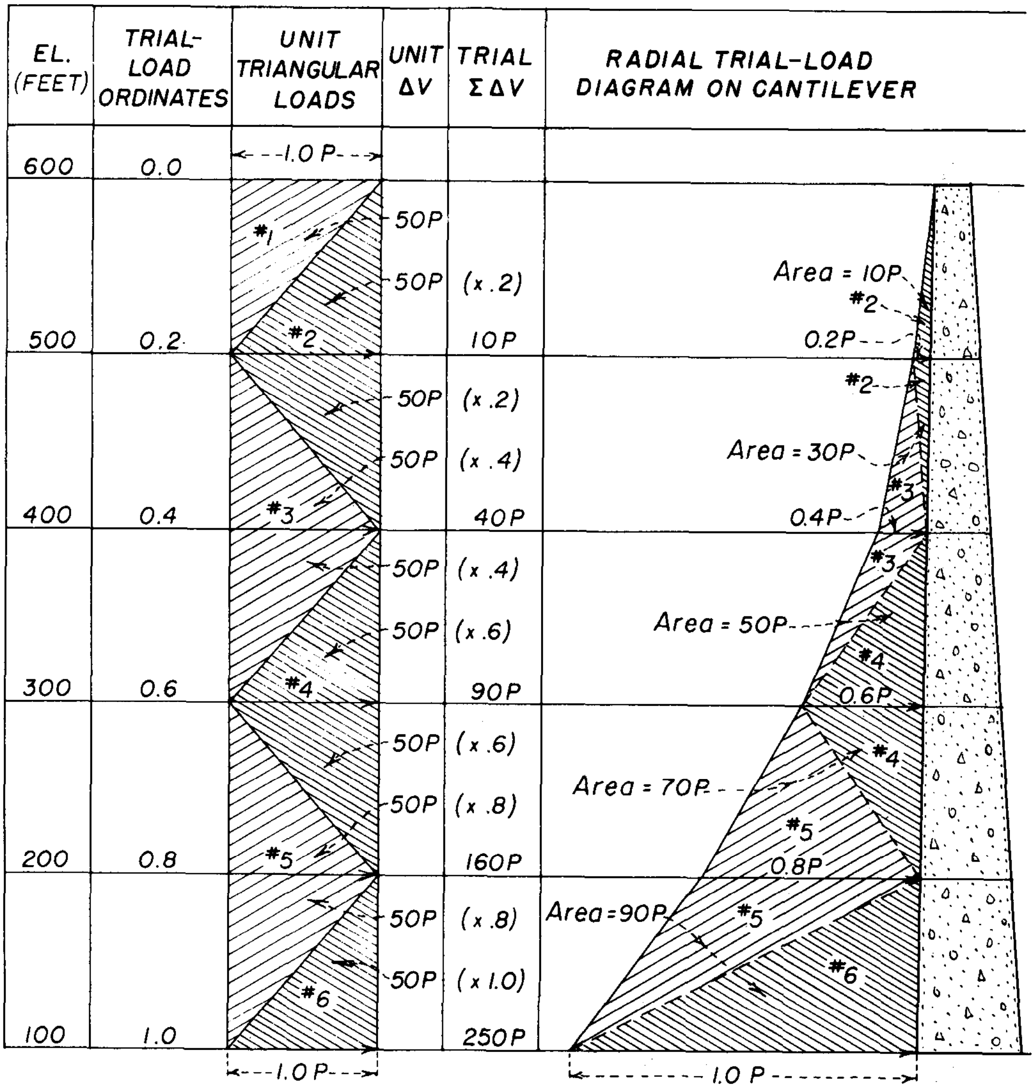
(g) *Unit Triangular Loads.*—Basic unit triangular loads are used in computing unit radial, tangential, and twist loads. These basic loads are for an area of application 1 foot in width and must be corrected in computing unit cantilever loads as described below. The basic triangular loads are shown on figure 4-27. The quantity indicated by $\Delta V$ is that portion of a basic triangular load above any given elevation, in pounds. It is equal to the total basic-load shear at a given elevation in calculating unit radial and unit tangential loads; but in calculating unit twist loads, it is equal to the total twisting couple in foot-pounds, at the given elevation, caused by the portion of a basic triangular load above that elevation. The quantity indicated by $\Delta M$ is the moment of the portion of the basic triangular load above any given elevation, about that elevation. Values of $\Delta M$ are used only in computing radial loads. Thus, on figure 4-27 it can be seen that for a certain elevation $\Delta V$ is the area of the basic-load diagram above that elevation, and $\Delta M$ is the product of $\Delta V$ and the distance of its centroid above the elevation.

For convenience, triangular loads may be designated by the elevation at which they peak; $P$ is generally assumed to be 1,000 pounds per square foot for radial and tangential loads, and 1,000 foot-pounds per square foot for twist loads.

An illustration of unit radial loads is given on figure 4-27. These loads are usually applied at the upstream face of the cantilever. Values of $\Delta V$ and $\Delta M$ for these radial loads are obtained by multiplying $\Delta V$ and $\Delta M$ of the basic unit load by $R_E/R_{axis}$ for the upstream face, for each respective elevation at which the unit load peaks. This factor adjusts the basic triangular loads, one unit in width, to the width of the face where they are applied as radial loads. Then, by multiplying these adjusted values of $\Delta V$ and $\Delta M$ by the estimated trial-load ordinates at respective elevations, the shears and moments on the cantilever due to the trial load placed upon the cantilever for the radial adjustment are quickly calculated. The method of building up a trial-load pattern is illustrated on figure 4-28. In this figure the value of $R_E/R_{axis}$ has been assumed equal to unity in order to simplify the computations. The computations for $\Sigma\Delta M$ are omitted, but are performed in a similar manner as those for $\Sigma\Delta V$.

Unit tangential shear forces are applied at a distance of one-half the crown thickness from the upstream face of the cantilever. Values for $\Delta H$ for unit tangential loads are obtained as shown on figure 4-27. The total tangential thrust is obtained by summation. Basic unit loads are corrected by $r_o/R_{axis}$ for the elevation at which the basic unit load peaks. Trial-load ordinates for tangential loads are estimated and $\Sigma H$ computed by means of unit tangential loads.

Unit triangular twist loads are twisting couples applied in horizontal planes about a point at a distance of one-half the crown thickness from the upstream face of the cantilever. The value of $P$ is assumed to be 1,000 foot-pounds per square foot. Values of



$R_E / R_{axis}$ = Unity

Total area = 250P

Note: Sketches are not to scale.

*Figure 4-28.* **Diagrammatic illustration of trial-load pattern for radial adjustment.—288-D-1396**

$\Delta\cancel{M}$ for a unit twist load are obtained by multiplying values of $\Delta V$ of the basic unit load by $r_o / R_{axis}$ for the elevation at which the basic load peaks. The twisting moment, $\cancel{M}$ is obtained by multiplying $\Delta\cancel{M}$ by the estimated trial-load ordinates at the load peak elevations; and the total twisting moment, $\Sigma\Delta\cancel{M}$, obtained by summation.

(h) *Deflections and Angular Movements of a Radial-Side Uncracked Cantilever.*—Symbols used in the notation for cantilever movements, in addition to items given at the beginning of this chapter (sec. 4-2), are listed below:

$K_6$ = ratio of detrusion due to actual shear distribution, to detrusion due to equivalent shear distributed uniformly, usually assumed to be 1.25 for $\Delta r$ and 1.00 for $\Delta s$.

$\Delta Z$ = increment of height between sections at which properties and forces are known (see fig. 4-29).

In order to determine foundation movements for a radial-side cantilever, the factor $R_{axis}/r$ is introduced in the following equations to correct for the movements at the centerline of a cantilever of unit uniform width to the movements at the axis of a radial-side cantilever one unit wide at the axis:

$$\alpha = \frac{R_{axis}}{r} [a' \sin^3 \psi + \delta' \sin \psi \cos^2 \psi] \quad (21)$$

$$\gamma = \frac{R_{axis}}{r} [\gamma' \sin \psi] \quad (22)$$

$$\textcircled{\gamma} = \frac{R_{axis}}{r} [\textcircled{\gamma'} \sin^3 \psi + \beta' \sin \psi \cos^2 \psi] \quad (23)$$

$$\delta = \frac{R_{axis}}{r} [\delta' \sin^3 \psi + \alpha' \sin \psi \cos^2 \psi] \quad (24)$$

$$\alpha^2 = \frac{R_{axis}}{r} [\alpha'' \sin^2 \psi] \quad (25)$$

Foundation movements for a radial-side cantilever are assumed to be the same as those for a parallel-side cantilever.

Radial cantilever deflections are caused by trial radial loads and initial loads. The radial deflections due to trial loads are caused by radial shear forces and bending moments, whereas the initial radial deflection due to vertical waterload is caused by a radial bending moment. For each cantilever, initial deflections due to initial loads are computed for each elevation, and radial deflections due to trial loads are calculated from unit deflections for each elevation. The total radial deflection of the cantilever at any elevation is the sum of these two and the arch abutment deformation at the base of the cantilever.

Radial deflections are determined by the common theory for cantilever beams, and include deflections due to bending, shear, and foundation yielding. The radial deflection at any horizontal section due to bending moment is the integral of the slope of the centerline from a base to the elevation of the given section. The slope of the centerline at any horizontal section is the angular foundation movement, or tilting, normal to the foundation surface, plus the integral of the curvature of the centerline from the base to the elevation of the given section, the curvature of the centerline at any point being $\frac{M}{E_c I}$, assuming the slope is small compared with unity.

The radial deflection at any horizontal section due to radial shear forces is equal to the foundation deformation plus the integral, from the base to that section, of the detrusion, or change in horizontal position, of the centerline. This detrusion of the centerline for a differential height is:

$$\left(\frac{V K_6}{A G}\right) \Delta z$$

Actual calculations for radial deflections are performed by summating the quantities noted in the preceding two paragraphs for a sufficient number of horizontal sections to render the summation reasonably accurate. Operations followed in evaluating radial deflections may be expressed by the formula,

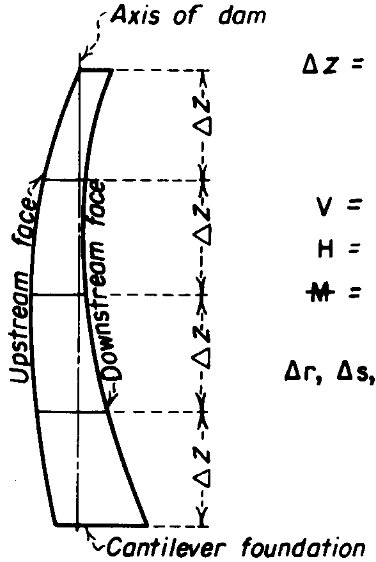
$$_{L\ and\ R}\Delta r = \sum \left(M_a\ \alpha + V_a\ \alpha_2 + \sum \frac{M}{E_c I} \Delta z\right) \Delta z + \left(V_a\ \gamma^* + M_a\ \alpha_2^* + \sum \frac{V K_6}{A G} \Delta z\right) \quad (26)$$

in which the asterisk (*) designates terms that apply only to the maximum cantilever or one that does not rest on the end of an arch. For the general case of a cantilever resting on an arch abutment, these terms are replaced by the equivalent total radial movements of the arch abutment.

Tangential shear forces produce tangential cantilever deflections by shear detrusions only. Tangential movements caused by bending are negligible. Tangential deflections are determined by the method used for radial deflections due to radial shear forces. The total tangential movement at any horizontal section equals the total tangential movement of the arch abutment on which the cantilever rests, *NOTES:*

*Radial loads are applied to the faces.*

*Tangential and twist load applied at a distance of ½ the crown thickness from the upstream face.*



*Δz = Increment of height between sections at which properties and forces are known.*

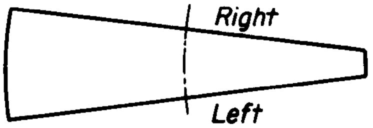
*V = Radial shear forces.*

*H = Tangential shear forces.*

*~~M~~ = Twisting moment in horizontal plane.*

*Δr, Δs, θ = Radial, tangential and angular movements of cantilever at ℄.*

*(a) VERTICAL CROSS SECTION*



*(b) HORIZONTAL CROSS SECTION*

| MAXIMUM CANTILEVER AND CANTILEVER TO LEFT OF MAXIMUM VERTICAL CROSS SECTION (LOOKING UPSTREAM) | | | | |
|---|---|---|---|---|
| DIRECTION OF POSITIVE MOVEMENTS | DIRECTION OF POSITIVE FORCES AND MOMENTS | DIRECTION OF POSITIVE LOADS | DIRECTION OF FORCES AND MOMENTS DUE TO POSITIVE LOADS | DIRECTION OF MOVEMENTS DUE TO POSITIVE LOADS |
| ALL DIRECTIONS REFER TO FIGURES | | | | |
| FIGURE (a) | | | | |
| Δr ← + | V ← +<br>M + ↺ | RADIAL ± → | V − →<br>M − ↻ | Δr − → |
| FIGURE (b) | | | | |
| Δs + ↑<br>θ + ↺ | H + ↓<br>M + ↺ | TANG. + ↑<br>TWIST + ↻ | H − ↑<br>~~M~~ − ↻<br>~~M~~ − ↻ | Δs + ↑<br>θ − ↻ |

| CANTILEVER TO RIGHT OF MAXIMUM VERTICAL CROSS SECTION (LOOKING UPSTREAM) | | | | |
|---|---|---|---|---|
| DIRECTION OF POSITIVE MOVEMENTS | DIRECTION OF POSITIVE FORCES AND MOMENTS | DIRECTION OF POSITIVE LOADS | DIRECTION OF FORCES AND MOMENTS DUE TO POSITIVE LOADS | DIRECTION OF MOVEMENTS DUE TO POSITIVE LOADS |
| ALL DIRECTIONS REFER TO FIGURES | | | | |
| FIGURE (a) | | | | |
| Δr ← + | V ← +<br>M + ↺ | RADIAL ± → | V − →<br>M − ↻ | Δr − → |
| FIGURE (b) | | | | |
| Δs + ↑<br>O + ↺ | H + ↑<br>~~M~~ + ↻ | TANG. + ↓<br>TWIST + ↺ | H − ↓<br>~~M~~ − ↺<br>~~M~~ − ↺ | Δs − ↓<br>θ + ↺ |

*Figure 4-29.* **Direction of positive movements, forces, moments, and loads; and direction of forces, moments, and movements due to positive loads.—288-D-2692**

plus the initial tangential movement of the cantilever at that section due to the assumed tangential earthquake force, plus the integral, from the base to that section, of the tangential movements of the differential heights of the cantilever. The general equations are:

$$_{L}\Delta s = -H_a \textcircled{\gamma}^* - \sum \frac{H K_6}{AG} \Delta z \qquad (27)$$

and

$$_{R}\Delta s = H_a \textcircled{\gamma}^* + \sum \frac{H K_6}{AG} \Delta z \qquad (28)$$

in which the asterisk (*) designates terms that apply only to the maximum cantilever or one that does not rest on the abutment of an arch. For the general case of a cantilever resting on an arch abutment, these terms are replaced by the equivalent movements of the arch abutment. In applying the formulas to the evaluation of tangential movements, the value of $K_6$ is taken as unity because tangential shear is assumed to be distributed uniformly at each horizontal section.

Angular movements of a cantilever are rotations about the cantilever centerline produced by twist loads. These twist loads are represented by twisting moments in horizontal planes. Angular movements due to trial loads are calculated from unit angular movements. The total angular movement at any horizontal section is the total angular rotation, in a horizontal plane, of the arch abutment on which the cantilever rests, plus the integral, from the base to that section, of the angular movements of the differential heights about the centerline. Since the summation method is used to evaluate quantities to be integrated, and since the angular movement of a differential height is $\left(\frac{\cancel{M}}{2\,G\,I}\right)\Delta z$, the formulas for the left and right sides of the dam are:

$$_{L}\theta = M_a\,\delta^* + \sum \frac{\cancel{M}}{2GI} \Delta z \qquad (29)$$

$$_{R}\theta = -M_a\,\delta^* - \sum \frac{\cancel{M}}{2GI} \Delta z \qquad (30)$$

in which the terms designated by an asterisk (*) apply only to cantilevers that do not rest on the abutment of an arch. In the general case of a cantilever resting on an arch abutment, these terms are replaced by the equivalent movements of the arch abutment. Since cantilevers are units of a continuous structure, shears set up by twisting moments are assumed to act in tangential directions and to have a linear variation from the upstream face to the downstream face of the cantilever. With these assumptions, the equations correspond to formulas for twist in a continuous slab.

Several secondary movements are produced by radial, tangential, and twist loads applied to the cantilevers in the trial-load adjustments. Most of these have been investigated, and those having small effects are usually neglected. Usually there are two important effects to consider. The first is the radial deflections due to twist loads; the second is the angular movements in horizontal planes due to tangential loads. These effects are discussed later under the tangential and twist adjustments.

(i) *Convention of Signs.*—In the application of formulas given in this chapter, the convention of signs to be used is given on figure 4-29. On the left side of the figure is shown the sign convention for a radial-side cantilever of maximum vertical cross section and for all cantilevers to the left of the maximum cantilever, looking upstream. It should be noted here that for the vertical cross section, a positive reservoir radial load is applied as a negative shear force, $V$, and produces a negative bending moment, $M$, and a negative radial movement or deflection, $\Delta r$. Also, for the horizontal cross section, a positive tangential thrust load is applied as a negative tangential force, produces a positive tangential deflection, and ordinarily produces a negative twisting moment in a clockwise direction. Furthermore, a positive twist load is applied as a negative twisting moment and produces a negative clockwise rotation for the horizontal cross section.

For a positive radial load on the vertical cross section of any cantilever at the right of the maximum cantilever, the sign convention is the same as for a cantilever at the left. However, a positive thrust load on the horizontal cross section of a right-side cantilever is applied as a negative tangential force, produces a negative tangential deflection to the left, and ordinarily produces a negative twisting moment in a counterclockwise direction. Note that this direction is opposite to that for negative twisting moment on a left-side cantilever. A positive twist load on the horizontal cross section of a right-side cantilever is applied as a negative twisting moment in a counterclockwise direction and produces a positive rotation which is counterclockwise. These sign conventions must be carefully followed throughout the trial-load analysis. The designation of right and left sides of the dam looking upstream is opposed to the convention used normally in dam engineering. However, it was adopted early in the stage of development of the trial-load analysis and no practical advantage would accrue if a great amount of labor were expended in changing it. Also, trial-load results can be shown for the dam as viewed downstream, which is with normal view and agrees with specification drawings.

Examples of computations of data for the analysis of uncracked cantilevers are given for Monticello Dam in appendix B.

4-32. ***Cantilever Data by Simpson's Rule.***—The following method is the one generally used to obtain the values of area and moment of the concrete and to compute the vertical waterload above any given elevation. The areas of horizontal sections, location of centers of gravity of these sections, and the moments of inertia can be determined very quickly by equations (14), (15), and (16). With these data known, the vertical waterload and the accumulated weights and moments due to concrete are very quickly and accurately determined from top to bottom by using Simpson's rule.

(a) *Weights and Moments Due to Concrete.*—The weight of concrete and the moment of this weight about the center of gravity of a horizontal section are determined at each elevation at which an arch element is analyzed. Between any two arch elements, the volume of the cantilever element is determined by integrating the cross-sectional areas by means of Simpson's rule. To do this, cantilever data are required at an elevation midway between the arch elements.

Referring to figure 4-30(a), let the heavy horizontal lines represent elevations at which the weights and moments are desired, that is, arch elevations. Let the broken lines represent elevations midway between the arch elements. Let $A_1$ equal the area of the cantilever element at elevation *1-1*, $A_2$ equal the area at elevation *2-2*, etc. Let $z$ equal half the difference between arch elevations, and let *1-1* be the elevation of the top of the dam.

The weight of the cantilever element between elevations *1-1* and *3-3* is then equal to:

$$(W_c)_3 = \frac{w_c z}{3}(A_1 + 4A_2 + A_3) =$$

$$z\, w_c \left[\frac{A_1}{3} + \frac{4A_2}{3} + \frac{A_3}{3}\right] \qquad (31)$$

The total weight of concrete above elevation *5-5* is equal to:

$$(W_c)_5 = (W_c)_3 + z\, w_c \left[\frac{A_3}{3} + \frac{4A_4}{3} + \frac{A_5}{3}\right] \qquad (32)$$

It can, therefore, be seen that the weight of each cantilever block is added to the accumulated weights above that block in order to give the total weight at any section.

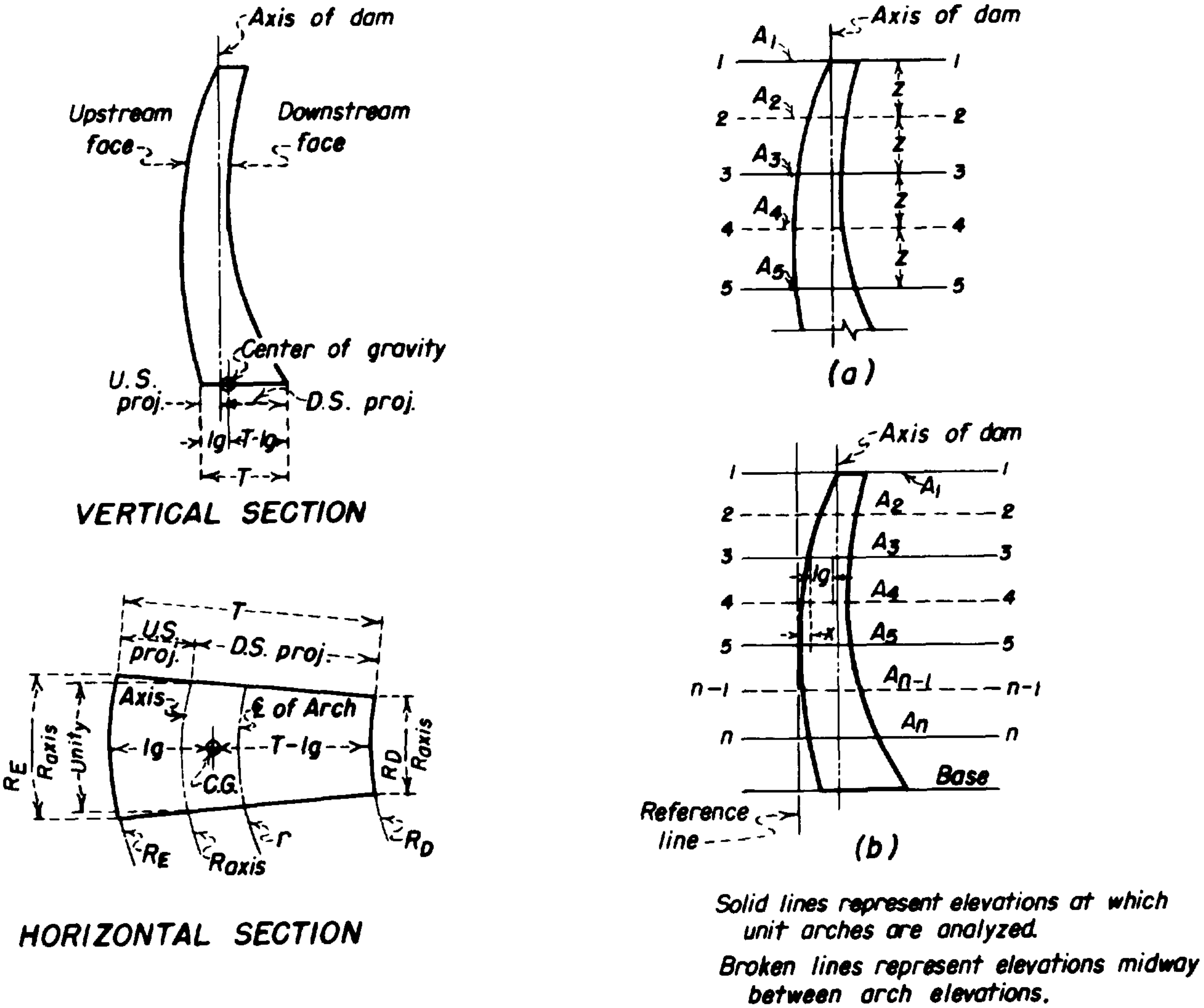
It will be noticed that at each arch elevation the area is multiplied by the constant 1/3 and at each intermediate elevation the area is multiplied by the constant 4/3. It should be noted that the $z$ term is not constant unless the arch elements occur at equal intervals.

To determine the moment of the weight of the concrete above any arch elevation about the center of gravity of the cantilever section at that elevation, it is necessary to locate the center of gravity of the concrete mass above the elevation in question. This is obtained by taking the moment of each area multiplied by the unit weight of concrete, $w_c$, about an arbitrary vertical reference line, summing these moments by Simpson's rule, and then dividing that sum by the total weight. The location of the center of gravity of the mass is then referred to the center of gravity of the section.

Referring to figure 4-30(b), let $x$ equal the distance from the arbitrary reference line to the upstream face, and $lg$ equal the distance from the upstream face to the center of gravity at any section.

The moment of the weight of concrete between elevations *1-1* and *3-3* about the arbitrary reference line is equal to:

$$({}_R M_c)_3 = \frac{z}{3}\left[A_1 \ w_c \ (lg_1 + x_1) + 4A_2 \ w_c \ (lg_2 + x_2) + A_3 \ w_c \ (lg_3 + x_3)\right] \tag{33}$$



*Solid lines represent elevations at which unit arches are analyzed.*

*Broken lines represent elevations midway between arch elevations.*

*Figure 4-30.* Cantilever elements for use in Simpson's rule.–288-D-2694 or

$$({}_RM_c)_3 = z\,w_c\left[\frac{A_1}{3}(lg_1 + x_1) + \frac{4A_2}{3}(lg_2 + x_2) + \frac{A_3}{3}(lg_3 + x_3)\right] \qquad (34)$$

The moment of the total weight of concrete above elevation *5-5* about the reference line is equal to:

$$({}_RM_c)_5 = ({}_RM_c)_3 + z\,w_c\left[\frac{A_3}{3}(lg_3 + x_3) + \frac{4A_4}{3}(lg_4 + x_4) + \frac{A_5}{3}(lg_5 + x_5)\right] \qquad (35)$$

The moment of the total weight of concrete above any elevation *n-n* about the reference line is equal to:

$$({}_RM_c)_n = ({}_RM_c)_5 + zw_c\left[\frac{A_5}{3}(lg_5 + x) + \frac{4A_{n\text{-}1}}{3}(lg + x)_{n\text{-}1} + \frac{A_n}{3}(lg_n + x_n)\right] \qquad (36)$$

The distance from the arbitrary reference line to the center of gravity of the mass above any elevation is equal to:

$$(x_o)_n = \frac{({}_RM_c)_n}{(W_c)_n} \qquad (37)$$

The moment of the total weight about the center of gravity of the section is then equal to:

$$M_c = (W_c)_n\,(lg_n + x_n - x_o) = (W_c)_n\,\mathrm{e} \qquad (38)$$

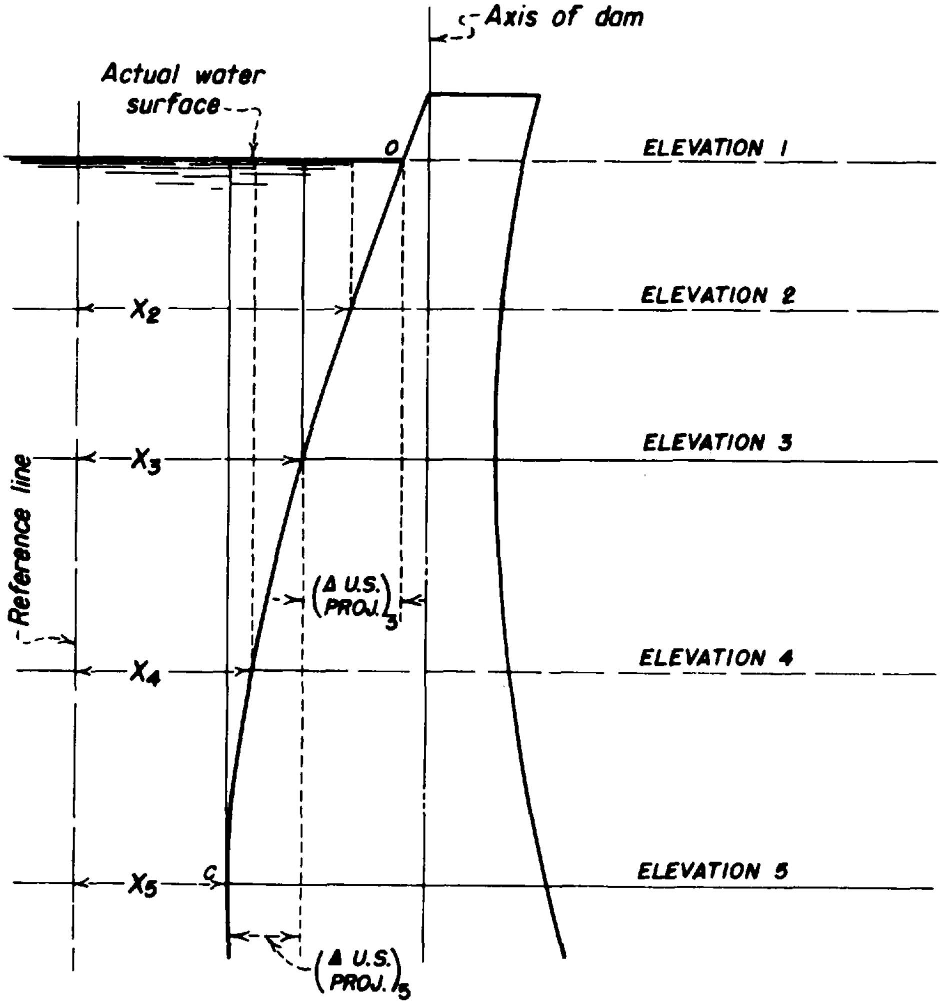
(b) *Weights and Moments Due to Vertical Upstream Waterload.*—As was noted on figure 4-30, the width of the upstream face of the cantilever element at any elevation is numerically equal to the value $\frac{R_E}{R_{axis}}$. If $h_w$ is the head of water, the differential waterload $d(W_w)$ on an increment of horizontal projection *ds* is:

$$d(W_w) = w\,h_w\left(\frac{R_E}{R_{axis}}\right)ds \qquad (39)$$

Numerically, the same waterload would be obtained if the upstream width of the above cantilever element were assumed to be unity, and the head multiplied by the ratio $\frac{R_E}{R_{axis}}$.

Applying this assumption to the vertical waterload on the upstream face of the cantilever, a new waterload will result. On figure 4-31, let the line *C-O* represent the upstream face of a cantilever element with the water surface elevation at point *O*. The ordinate of the vertical waterload at any elevation is obtained by multiplying the weight of water at that elevation by its respective value of $\frac{R_E}{R_{axis}}$. In so doing, the width of the cantilever element is assumed unity at the upstream face, and the weight of water acting on the face above any elevation is numerically equal to the area of the waterload above that elevation. To determine, as accurately as possible, the waterload above these elevations, it is necessary to determine the ordinates acting at the mid-elevations to be used in Simpson's rule.

The difference between the mid-ordinate and the average ordinate is very small. Therefore, a very close approximation of the



*Solid horizontal lines represent elevations at which arches are analyzed.*

*Broken horizontal lines represent elevations midway between arch elevations.*

*Figure 4-31.* **Cantilever section with vertical waterload.–288-D-2975**

area of the waterload may be obtained by multiplying the average ordinate by the change in horizontal projection of the cantilever face between the elevations *1* and *3* or *3* and *5* as shown on figure 4-31.

Referring to figure 4-31, let the horizontal projection of the upstream face between arch elevations be denoted by (Δ *U.S. Proj.*). Then the waterload $W_w$ acting on the face between arch elevations is equal to:

$$W_w = \begin{pmatrix}\text{Average of ordinates}\\ \text{of the waterload}\end{pmatrix} (\Delta\ U.S.\ Proj.) \qquad (40)$$

The total waterload acting above any elevation is equal to the sum of the $(W_w)$'s to that elevation.

The moment of the waterload above any elevation about the center of gravity of the cantilever section at that elevation is determined by the same procedure as for the moment due to concrete weight. The moment of the total waterload about the reference line is divided by the total water weight in order to locate the center of gravity of the water mass. The moment arms for the load ordinates are the values of $x$ used before for concrete moments. The moment arm for a mid or average ordinate would be the distance from the reference line to that ordinate. However, as was noted in the discussion above, this ordinate is slightly larger than it should be. Therefore, if it were multiplied by an arm slightly less than the distance between this ordinate and the reference line, a compensating effect would result, and the resulting moment about the reference line would be more nearly the correct value. This slightly smaller moment arm is obtained by using the $x$ value to the upstream face at the intermediate elevation.

The moment of the waterload above elevation *3* (fig. 4-31) about the reference line is then equal to:

$$({}_RM_w)_3 = \left[\frac{\Delta \text{ U.S. Proj.}}{2}\right]_3 \left[0 + \frac{4w}{3}\left(\frac{0 + h_3\, R_{E_3}}{2\, R_{axis}}\right) x_2 + \frac{1}{3} w h_3 \frac{R_{E_3}}{R_{axis}} x_3\right] \tag{41}$$

and the moment of the waterload above elevation *5* is equal to:

$$({}_RM_w)_5 = \left[\frac{\Delta \text{ U.S. Proj.}}{2}\right]_5 \cdot \left[\frac{1}{3}\left(\frac{w h_3\, R_{E_3}}{R_{axis}}\right) x_3 + \frac{4}{3}\frac{w\left(h_3\, R_{E_3} + h_5\, R_{E_5}\right)}{2\, R_{axis}} x_4 + \frac{1}{3}\frac{w h_5\, R_{E_5}}{R_{axis}} x_5\right] + ({}_RM_w)_3 \tag{42}$$

The location of the center of gravity of the water mass above elevation *3* is equal to $({}_RM_w)_3 \div (W_w)_3$, and the location of the center of gravity of the water mass above elevation *5* is equal to $({}_RM_w)_5 \div (W_w)_5$. With the location of the resultant vertical water forces known, their moments about the centers of gravity of the cantilever sections are easily determined.

(c) *Weights and Moments Due to Downstream Waterload.*—The effect of the tailwater on the downstream face of an arch dam is usually neglected. The procedure for its determination, however, is the same as for the upstream waterload. It should be noted that the width of the cantilever element at the downstream face is equal to the ratio $\frac{R_D}{R_{axis}}$.

(d) *Weights and Moments Due to Superstructure Loads.*—Owing to the eccentric application of the resultant weight of the superstructure, it is necessary to determine the bending moments due to this weight at each arch elevation.

### 4-33. *Cracked-Cantilever Analysis.*—

(a) *General Discussion.*—In thin arch dams, applied radial loads tend to produce tension in the cantilevers. Since ordinary concrete usually does possess some tensile strength, it would appear that a nominal amount of tensile stress may be allowable for good concrete. In general, when this stress is exceeded, all of the cantilevers in tension may be assumed to crack to the point of zero stress, so that the uncracked sections are entirely in compression.

The assumed cracking makes the cantilevers more flexible and transfers a greater portion of the waterload to the arch elements, thereby increasing arch stresses generally. In addition, cantilever stresses at the downstream face of the dam may be increased due to the reduced area of concrete available for transmitting cantilever forces. The assumed condition is somewhat analogous to that assumed in the reinforced concrete beam theory. The resultant of the forces above a horizontal section is equal to the total stress on the uncracked area, and the stresses are assumed to have a nonlinear distribution, ranging from zero at the end of the crack to a maximum at the uncracked face. The shape of the stress distribution diagram is assumed to vary with the depth of the crack—between linear for no cracking and parabolic for a crack depth of one-half or more of the thickness.

Actually, the cantilevers never crack at all points at which tension is indicated. Since a cantilever may crack at one of these points and not at others, cracking is assumed at the point of maximum tension only, on each face. However, if the tensions are not relieved at the other points, it may be necessary to assume cracking at additional points on that face. Of course, the dam can be modified so as to bring tensile stresses within allowable limits, thus eliminating the necessity for computation of cracked cantilevers.

If a horizontal crack occurs at the upstream face of a cantilever below the water surface, uplift pressures are assumed to be full reservoir pressure for the depth of the crack varying uniformly to tailwater pressure at the downstream face. This uplift pressure in the crack has a tendency to cause further cracking, which is resisted by the arches. The moments due to uplift forces are computed about the center of gravity of the entire section.

Since amounts of cracking and radial deflections are functions of total loads acting on a cantilever, the use of unit loads with cracked cantilevers is impossible, and it becomes necessary to calculate total radial deflections for each trial-load change.

Cracked-cantilever movements include effects of only those loads or forces capable of being distributed between the arch and cantilever elements. Loads and forces applied to a dam before grouting are resisted by the cantilevers only; hence, movements due to these loads and forces are not included in the adjustments. It is necessary to compute deflections due to concrete weight and subtract these deflections algebraically from the total cracked-cantilever movements, in order to determine the cantilever deflections due to trial loads.

The major effect of cracking is to increase radial deflections. The effects of cracking are not included in tangential and angular movements.

(b) *Notations.*—Symbols used in considering cracked cantilevers, in addition to items given at the beginning of this chapter (sec. 4-2), are listed below. These symbols refer to the base of the cantilever or to the horizontal section under consideration.

$R_o$ = radius to end of crack.

$r_1$ = radius to centerline of uncracked portion of section.

$lg_1$ = distance from upstream edge of uncracked portion of section to center of gravity of that portion.

$T_1$ = radial thickness of uncracked portion of section.

$A_1$ = area of uncracked portion of section.

$I_1$ = moment of inertia of uncracked portion of section about a circumferential line through its center of gravity.

$M_1$ = resultant moment about center of gravity of uncracked portion of section.

(c) *Cracking at Upstream Face.*—It is necessary to determine the depth to which cracking will extend in a radial direction at each location where cracking is indicated. The parabolic stress distribution is valid when the depth of cracking is one-half or more of the original thickness. When the depth of cracking is less than one-half of the thickness, values are interpolated between the parabolic stress distribution and a linear stress distribution.

From figure 4-32, the volume of the indicated stress solid, assuming the parabolic stress distribution, is:

$$\Sigma W + U = \frac{2}{9} T_1 \ \sigma_{Z_D} \left[ \frac{R_o + 2R_D}{R_{axis}} \right] \tag{43}$$

The moment of the stress solid about the original center of gravity is:

$$(\Sigma W + U)(\text{-}e) = \left(\frac{2}{3} T_1 \ \sigma_{Z_D} \ \frac{R_o}{R_{axis}}\right) \left(T - lg - \frac{2T_1}{5}\right)$$

$$- \left(\frac{4}{9} T_1 \ \sigma_{Z_D}\right) \left(\frac{R_o - R_D}{R_{axis}}\right) \left(T - lg - \frac{3T_1}{10}\right)$$

Substituting for $\Sigma W + U$ the expression given in equation (43) and simplifying,

$$\frac{R_E - lg + e}{R_D} = \frac{1}{5} \ \frac{\left[3 + 5\frac{R_D}{R_o} + 7\left(\frac{R_D}{R_o}\right)^2\right]}{\left[\frac{R_D}{R_o} + 2\left(\frac{R_D}{R_o}\right)^2\right]} \tag{44}$$

It is evident, from this equation, that the value of $\frac{R_E - lg + e}{R_D}$ determines the ratio $R_D/R_o$ for the uncracked portion of the horizontal section. If $R_D/R_o$, determined as above, is less than $R_D/R_E$, the section does not crack. If $R_D/R_o$ is unity or greater, the section is completely cracked.

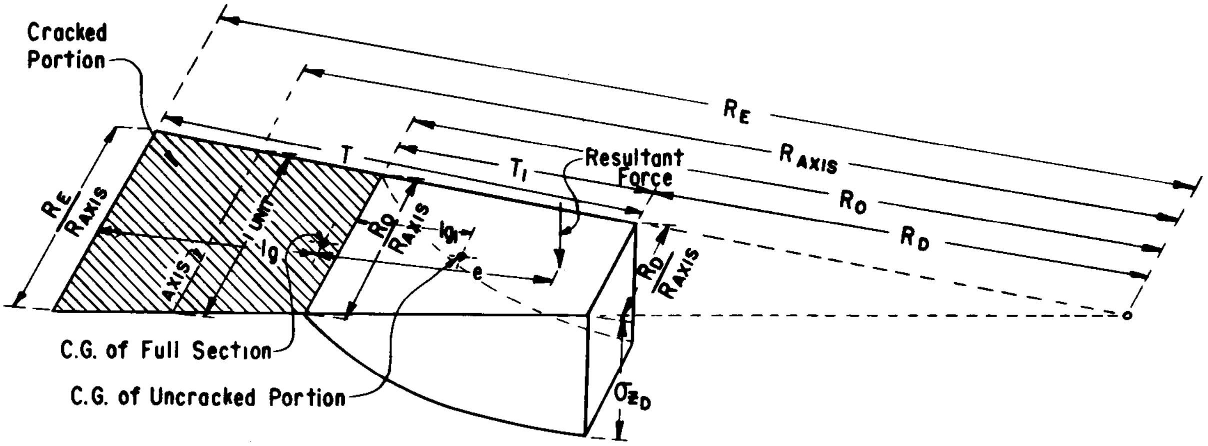


*Figure 4-32.* **Horizontal section of a cantilever cracked from the upstream face (assuming parabolic stress distribution).—288-D-2976**

$T_1$ can be evaluated by the equation,

$$T_1 = \left(\frac{R_o}{R_{axis}} - \frac{R_D}{R_{axis}}\right) R_{axis} \tag{45}$$

in which

$$\frac{R_o}{R_{axis}} = \frac{R_D}{R_{axis}} \div \frac{R_D}{R_o}$$

The revised formulas for the area of the uncracked portion, $A_1$, and the moment of inertia, $I_1$, are:

$$A_1 = \frac{T_1}{2}\left(1 + \frac{R_D}{R_o}\right)\frac{R_o}{R_{axis}} \tag{46}$$

and

$$I_1 = \frac{T_1{}^3}{36} \frac{\left[1 + 4\frac{R_D}{R_o} + \left(\frac{R_D}{R_o}\right)^2\right]}{\left[1 + \frac{R_D}{R_o}\right]} \frac{R_o}{R_{axis}} \tag{47}$$

From results obtained by Westergaard [12], the equations for rotation due to cracking and the stress at the downstream face after cracking may be derived. The constant, $K$, used in these equations can be evaluated as follows:

$$K = \frac{9}{2} \frac{(\Sigma W + U)\frac{R_{axis}}{R_o}}{T_1{}^{3/2}\left[1 + \frac{1}{5}\left(\frac{T_1}{T - T_1}\right)\right]\left(1 + 2\frac{R_D}{R_o}\right)}$$

and

$$\frac{M_1}{EI_1} = \theta_g = \frac{-8K\sqrt{T - T_1}}{3E}$$

where $E$ is the modulus of elasticity for concrete.

Substituting the above expression for $K$,

$$\frac{M_1}{EI_1} = \left[\frac{-(\Sigma W + U)}{E} \frac{R_{axis}}{R_o}\right]\left[\frac{12(T - T_1)^{1/2}}{T_1{}^{3/2}\left[1 + \frac{1}{5}\left(\frac{T_1}{T - T_1}\right)\right]\left(1 + \frac{2R_D}{R_o}\right)}\right] \tag{48}$$

The equation for stress at the downstream face is:

$$\sigma_{Z_D} = K\sqrt{T_1} \left[1 + \frac{T_1}{3(T - T_1)}\right]$$

Again substituting the previous expression for $K$, the stress in pounds per square inch is:

$$\sigma_{Z_D} = \left[\frac{5(\Sigma W + U)\, R_{axis}}{96\, T_1 (R_o + 2\, R_D)}\right] \left[\frac{3T - 2\, T_1}{5T - 4\, T_1}\right] \qquad (49)$$

If the cracking extends less than one-half the original thickness, the necessary values should be interpolated between those for a linear stress distribution and those for a parabolic stress distribution. Figure 4-33 shows a diagram of linear stress distribution for a cantilever section cracked at the upstream face. The volume of the indicated stress solid acting on the uncracked portion [13] is:

$$\Sigma W + U = \left(\frac{T_1}{2}\, \sigma_{Z_D}\, \frac{R_o}{R_{axis}}\right) - \frac{T_1}{3}\, \sigma_{Z_D} \left(\frac{R_o}{R_{axis}} - \frac{R_D}{R_{axis}}\right)$$

Simplifying,

$$\Sigma W + U = \sigma_{Z_D}\, T_1 \left(\frac{R_o + 2\, R_D}{6\, R_{axis}}\right) \qquad (50)$$

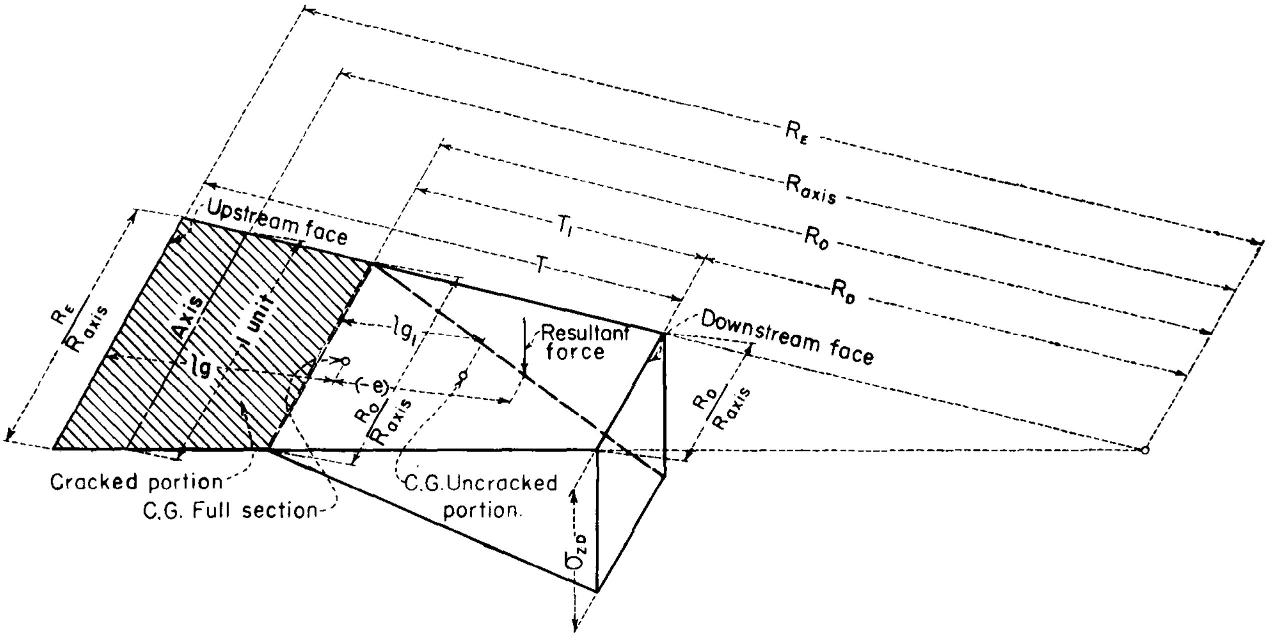


*Figure 4-33.* **Horizontal section of a cantilever cracked from the upstream face (assuming linear stress distribution).—288-D-2977**

The moment of the stress solid about the center of gravity of the original section is:

$$(\Sigma W + U)(-e) = \left(\frac{T_1}{2}\,\sigma_{Z_D}\,\frac{R_o}{R_{axis}}\right)\left(T - lg - \frac{T_1}{3}\right) - \left(\frac{T_1}{3}\,\sigma_{Z_D}\right)\left(\frac{R_o}{R_{axis}} - \frac{R_D}{R_{axis}}\right)\left(T - lg - \frac{T_1}{4}\right)$$

Substituting for $\Sigma W + U$ the expression given in equation (50) and simplifying, the equation becomes:

$$\frac{R_E - lg + e}{R_D} = \frac{1}{2}\,\frac{\left[1 + 2\frac{R_D}{R_o} + 3\left(\frac{R_D}{R_o}\right)^2\right]}{\left[\frac{R_D}{R_o} + 2\left(\frac{R_D}{R_o}\right)^2\right]} \tag{51}$$

As stated for the parabolic distribution, the value of $\frac{R_E - lg + e}{R_D}$ determines the ratio $R_D/R_o$.

$T_1$ can be evaluated by the equation,

$$T_1 = \left(\frac{R_o}{R_{axis}} - \frac{R_D}{R_{axis}}\right) R_{axis} \tag{52}$$

in which

$$\frac{R_o}{R_{axis}} = \frac{R_D}{R_{axis}} \div \frac{R_D}{R_o}$$

Equations for area, $A_1$, and moment of inertia, $I_1$, of the uncracked portion of the horizontal section are:

$$A_1 = \frac{T_1}{2}\left(1 + \frac{R_D}{R_o}\right)\left(\frac{R_o}{R_{axis}}\right) \tag{53}$$

and

$$I_1 = \frac{T_1{}^3}{36}\,\frac{\left[1 + 4\frac{R_D}{R_o} + \left(\frac{R_D}{R_o}\right)^2\right]}{\left[1 + \frac{R_D}{R_o}\right]} \bullet \frac{R_o}{R_{axis}} \tag{54}$$

The rotation at the crack is:

$$\frac{M_1}{EI_1} = \theta_g = -\frac{(\Sigma W + U)}{ET_1{}^2}\left[\frac{6}{1 + 2\frac{R_D}{R_o}}\right]\frac{R_{axis}}{R_o} \tag{55}$$

The stress at the downstream face, in pounds per square inch, is determined by the equation,

$$\sigma_{Z_D} = \frac{3(\Sigma W + U)}{144 A_1} \; \frac{\left[1 + \frac{R_D}{R_o}\right]}{\left[1 + 2\frac{R_D}{R_o}\right]} \tag{56}$$

Values should be interpolated using the ratio $\frac{T - T_1'}{0.5}$. It should be assumed that the parabolic distribution applies if the ratio is 1.0 and linear distribution applies if the ratio is zero $(T = T_1')$. $T_1'$ is the weighted average between $T_1$ for the linear assumption and $T_1$ for the parabolic assumption.

(d) *Cracking at Downstream Face.*—The depth to which the crack will extend in a radial direction from the downstream face is determined in a manner similar to that used for cracking from the upstream face.

The volume of the indicated stress solid shown on figure 4-34 is:

$$\Sigma W + U = \frac{2}{9} T_1 \, \sigma_{Z_U} \left(\frac{2 R_E + R_o}{R_{axis}}\right) \tag{57}$$

The moment of the stress solid about the original center of gravity is:

$$(\Sigma W + U)(e) = \left(\frac{2}{3} T_1 \, \sigma_{Z_U} \frac{R_o}{R_{axis}}\right) \left(lg - \frac{2}{5} T_1\right) + \left(\frac{4}{9} T_1 \, \sigma_{Z_U}\right) \left(\frac{R_E - R_o}{R_{axis}}\right) \left(lg - \frac{3}{10} T_1\right)$$

Substituting for $\Sigma W + U$ the expression given in equation (57) and simplifying, the equation becomes:

$$\frac{lg - e}{R_E} = \frac{3}{5} \; \frac{\left[1 - \left(\frac{R_o}{R_E}\right)^2\right]}{\left[\frac{R_o}{R_E} + 2\right]} \tag{58}$$

The value of $\frac{lg - e}{R_E}$ determines the ratio $\frac{R_o}{R_E}$. If the ratio is less than $\frac{R_D}{R_E}$, the section does not crack. If the ratio is unity or greater, the section is completely cracked. The following equations may be derived in the same manner as those for cracking at the upstream face:

$$T_1 = \left(\frac{R_E}{R_{axis}} - \frac{R_o}{R_{axis}}\right) R_{axis} \tag{59}$$

where

$$\frac{R_o}{R_{axis}} = \frac{R_E}{R_{axis}} \bullet \frac{R_o}{R_E}$$

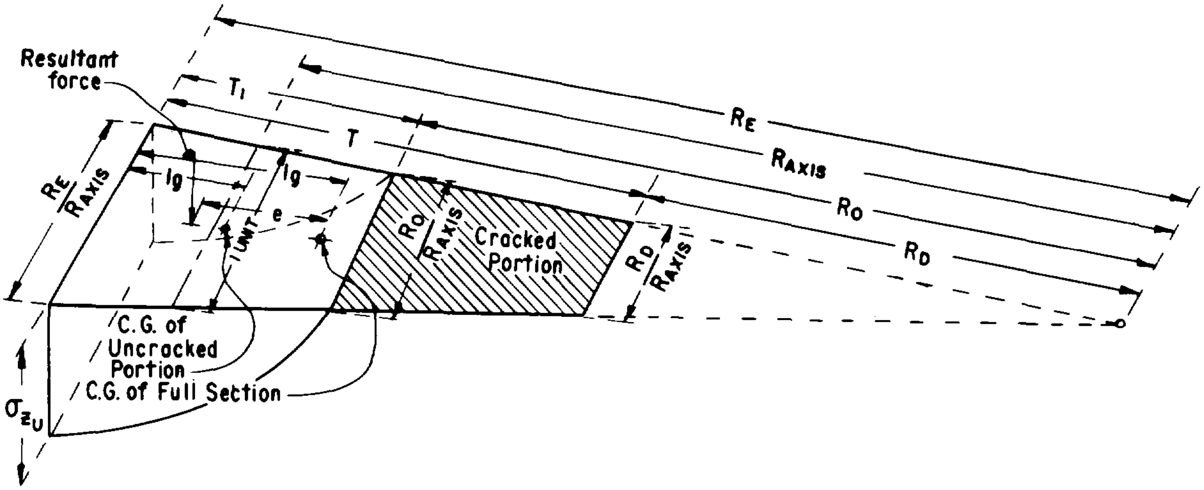


*Figure 4-34.* Horizontal section of a cantilever cracked from the downstream face (assuming parabolic stress distribution).–288-D-2978

and

$$A_1 = \frac{T_1}{2}\left(1 + \frac{R_o}{R_E}\right)\frac{R_E}{R_{axis}} \tag{60}$$

$$I_1 = \frac{T_1{}^3}{36}\;\frac{\left[1 + 4\frac{R_o}{R_E} + \left(\frac{R_o}{R_E}\right)^2\right]}{\left[1 + \frac{R_o}{R_E}\right]}\;\frac{R_E}{R_{axis}} \tag{61}$$

The constant

$$K = \frac{9}{2}\;\frac{(\Sigma W + U)\left(\frac{R_{axis}}{R_o}\right)}{T_1{}^{3/2}\left[1 + \frac{1}{5}\left(\frac{T_1}{T - T_1}\right)\right]\left(1 + \frac{R_E}{R_o}\right)}$$

Substituting this expression for $K$ in

$$\frac{M_1}{EI_1} = \theta_g = \frac{8K\sqrt{T - T_1}}{3E}$$

gives:

$$\frac{M_1}{EI_1} = \frac{\Sigma W + U}{E}\left[\frac{12(T - T_1)^{1/2}}{T_1{}^{3/2}\left[1 + \frac{1}{5}\left(\frac{T_1}{T - T_1}\right)\right]\left(1 + 2\frac{R_E}{R_o}\right)}\right]\frac{R_{axis}}{R_o} \tag{62}$$

The equation for stress at the upstream face, in pounds per square inch, is:

$$\sigma_{Z_U} = \frac{5}{96}\left[\frac{(\Sigma W + U)\, R_{axis}}{T_1\, (R_o + 2\, R_E)}\right]\left[\frac{3T - 2T_1}{5T - 4T_1}\right] \tag{63}$$

The equations for linear stress distribution to be used in interpolating when the depth of cracking is less than one-half the original thickness may be derived in the same manner as those for cracking from the upstream face. The equations for linear stress distribution on a horizontal cantilever section cracked from the downstream face as shown on figure 4-35 are:

$$\frac{lg - e}{R_E} = \frac{1}{2}\,\frac{\left[1 - \left(\frac{R_o}{R_E}\right)^2\right]}{\left[\frac{R_o}{R_E} + 2\right]} \tag{64}$$

$$T_1 = \left(\frac{R_E}{R_{axis}} - \frac{R_o}{R_{axis}}\right) R_{axis} \tag{65}$$

where

$$\frac{R_o}{R_{axis}} = \frac{R_E}{R_{axis}} \bullet \frac{R_o}{R_E}$$

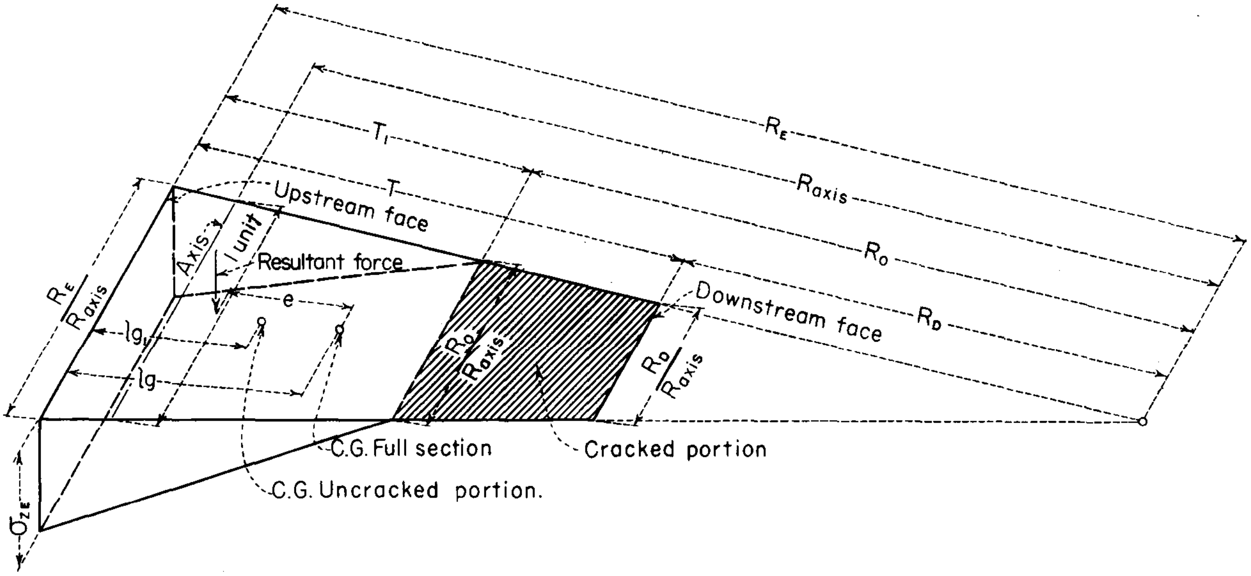


*Figure 4-35.* **Horizontal section of a cantilever cracked from the downstream face (assuming linear stress distribution).–288-D-2979**

The area of the uncracked portion is:

$$A_1 = \frac{T_1}{2}\left(1+\frac{R_o}{R_E}\right)\left(\frac{R_E}{R_{axis}}\right) \qquad (66)$$

and the moment of inertia is:

$$I_1 = \frac{T_1{}^3}{36}\;\frac{\left[1+4\frac{R_o}{R_E}+\left(\frac{R_o}{R_E}\right)^2\right]}{\left[1+\frac{R_o}{R_E}\right]}\;\frac{R_E}{R_{axis}} \qquad (67)$$

The slope at the crack is:

$$\frac{M_1}{EI_1} = \theta_g = \frac{\Sigma W + U}{ET_1{}^2}\left[\frac{6}{2+\frac{R_o}{R_E}}\right]\frac{R_{axis}}{R_E} \qquad (68)$$

and stress at the upstream face, in pounds per square inch, is:

$$\sigma_{Z_U} = \frac{3(\Sigma W + U)}{144\,A_1}\;\frac{\left[1+\frac{R_o}{R_E}\right]}{\left[\frac{R_o}{R_E}+2\right]} \qquad (69)$$

The values should be interpolated between a parabolic stress distribution and a linear stress distribution in the same manner used for cracking at the upstream face. The equations used to correct the foundation deformation constants when cracking occurs at the base of a cantilever element are the same as those used for cracking at the upstream face.

(e) *Deflections of Cracked Cantilever.*—The procedure used in computing the radial deflections of the vertical cantilever elements, including the effects of cracking, are included in calculations of deflections by assuming the cracked portion of the concrete offers no resistance to radial cantilever movements. This is done by substituting $A_1$, $I_1$, and $M_1$ in equation (26), so that we have the following equation for a cracked cantilever:

$${}_L\text{ and }{}_R\Delta r = \sum\left[{}_aM_1\,\alpha + V_a\,\alpha_2 + \Sigma\frac{M_1}{E_c\,I_1}\,\Delta z\right]\Delta z + \left[V_a\,\gamma^* + {}_aM_1\,\alpha_2^* + \Sigma\frac{V\,K_6}{A_1\,G}\Delta z\right] \qquad (70)$$

Terms indicated by the asterisk (*) apply only to the maximum cantilever or one that does not set on the end of the arch.

Cantilever foundation movements to be used if the horizontal base is cracked are:

$$\alpha = \frac{r}{r_1} \cdot \frac{T^2}{T_1{}^2} [\alpha] \qquad (71)$$

$$\gamma = \frac{r}{r_1} [\gamma] \qquad (72)$$

$$\alpha_2 = \frac{r}{r_1} \cdot \frac{T}{T_1} [\alpha_2] \qquad (73)$$

in which factors that precede the brackets correct the movements to the reduced thickness of the cantilever base. Factors within brackets are for total sections.

**4-34. *Circular Arch of Uniform Thickness.***–(a) *General.*–In the early use of the trial-load method, arches were divided into voussoirs. Total loads, moments, shears, slopes, and deflections were then calculated by summation methods. However, mathematical formulas were developed, based on circular curves at upstream and downstream edges of horizontal elements, and these are now used in analyzing arch elements. If extrados and intrados curves are not concentric, so that the arch thickness varies from crown to abutment, the half-arch is divided into four segments, a uniform thickness assumed for each segment, and the analysis made by the use of formulas especially derived for the segments.

If appreciable tensile stresses are indicated in the arch elements, so that investigation of cracked arches is necessary, analyses may be made by the original voussoir summation method. Analyses of uncracked arches have been greatly facilitated by the compilation and use of tables of arch constants. As previously stated, analyses of arch elements have also been greatly facilitated by calculating effects of unit loads of rectangular and triangular shape, covering different parts of the loaded surface, so that effects of practically any shape of arch load desired can be obtained by combining effects of unit loads already calculated.

Horizontal arch elements are usually assumed to be 1 foot high, with horizontal top and bottom faces and vertical upstream and downstream edges. The analysis of circular arch elements provides a means of solution for variable-thickness arches and arches with fillets. The analyses of circular arches of the three-centered uniform-thickness type and arches of the polycentered variable-thickness type can also be made when desired. Although ends of arch abutments are generally assumed to be radial, they may be assumed as nonradial and the analysis made on that basis.

The basis for the analysis of arch elements is found in various textbooks on concrete design under the subject of arch bridges. Since formulas deduced in such books are generally simplified to include only bending effects, the equations were amplified to include effects of rib-shortening and shear along vertical radial sections. The use of the original voussoir summation method required considerable time for the calculation of arch deflections. Therefore, completion of a trial-load analysis for an arch dam was a rather lengthy process.

The analysis of arch elements by means of integrals resulted in an appreciable saving of time, especially after effects of tangential shear and twist were included. A further saving in time was effected by computing tables of integrals needed in the calculations and tabulating functions for different types of unit loads which can be used in building up total loads carried by arch elements. As stated previously, the integral tables were prepared for circular arches of constant thickness, but they also may be used for the analysis of variable-thickness arches.

Horizontal sections of an arch dam are statically indeterminate elements with elastic abutments. A unit element is the volume of a dam intercepted by two horizontal planes one unit apart. For practical purposes and ease of computation, upstream and downstream faces of arch elements are assumed vertical. Arch elements resist radial forces applied at the upstream or downstream faces, tangential forces applied along the arcs through the crown centerlines, and twisting moments applied along the arcs through the crown centerlines. The analysis of the elements consists of calculating movements and stresses due to these forces and moments. Movements are determined by the usual theory of flexure for curved beams with effects of rib-shortening, transverse shear, and yielding abutments included. Stresses are evaluated by equations for bending and direct stress.

(b) *Arch Loads.*—Arch loads have previously been discussed in section 4-28. The various loads applied to arch elements during an analysis include radial, tangential, twist, and temperature loads. To build up these applied loads, other than temperature, a system of unit loads is used. These unit loads (see fig. 4-17) are uniform, triangular, or concentrated, and may be either symmetrical or nonsymmetrical. Symmetrical loads are the same on both left and right parts of the arch; nonsymmetrical loads are placed on one side of the arch only, the other side being unloaded. The intensity of loads, $P$, shown on figure 4-17, is usually 1,000 pounds per square foot at the upstream face. For twist loads with bending couples applied along the arch centerline the value of the unit couple, $P$, is usually 1,000 foot-pounds per square foot.

A temperature load is required in analyzing the effects of a change in mass-concrete temperature after the dam has been grouted. It is not an adjustment load, such as radial, tangential, or twist loads. It is a definite effect that is assigned to the arches as an initial load. A positive temperature load is one that denotes a rise in temperature. A convenient unit to use in computing temperature effects is a temperature change of $1^{\circ}$ F. Variations in temperature may also be included both from upstream face to downstream face and from abutment to crown along the length of the arch. This permits the inclusion of almost any possible temperature condition in the analysis.

(c) *Assumptions.*—The following assumptions are usually made in analyzing arch elements:

(1) The material in the arches is stable, homogeneous, and isotropic.

(2) Hooke's law applies, and the proportional elastic limit is not exceeded.

(3) A plane section before bending remains plane after bending.

(4) Direct stresses have a linear variation from extrados to intrados.

(5) The modulus of elasticity in direct stress is the same for both tension and compression.

(6) The ratio of the modulus of elasticity in direct stress, $E$, to the shearing modulus of elasticity, $G$, equals $2(1 + \mu)$, in which $\mu$ is Poisson's ratio. To account for nonlinear distribution of shear stress, the value of $K_6$, the ratio of detrusion caused by actual shear distribution to detrusion caused by an equivalent shear distributed uniformly, is assumed to be 1.25. With a Poisson's ratio of 0.2, the value of $K_6/G$ is $(1.25 \times 2.4)/E$, which equals $3/E$. This relationship of moduli is included in the formulas and tabulated arch and load constants.

(7) Temperature strains are proportional to temperature changes.

(8) A temperature change is generally assumed uniform throughout an arch. However, when necessary, a variable temperature from extrados to intrados face may be assumed. A temperature variation along the length of arch may also be assumed (see sec. 34(o)).

(9) Abutments of arch elements are assumed normal to the extrados for purposes of analysis.

(d) *Theory.*—The arch theory is developed for the general case of a nonsymmetrical arch with nonsymmetrical loads. Equations derived for this case are applicable to any type of arch with any type of loading. Later in the chapter the equations are modified for special use in analyzing uniform-thickness and variable-thickness circular arches. In general, the method of solution consists of cutting the loaded arch at the crown and substituting a moment, thrust, and shear at the crown to replace the influence of the other part of the arch (see fig. 4-36).

A load placed at any point on either part of the arch causes the crown point to be rotated and displaced in a horizontal plane. These movements are due partly to bending of the arch ring, partly to changes in length of the arch centerline, and partly to detrusions caused by shearing stresses. Since the arch must remain continuous, the crown point of both parts of the arch must have the same rotation and displacement. By equating the radial, tangential, and angular movements of both parts at the crown, three equations are obtained from which the unknown moment, thrust, and shear at the crown may be determined. These crown forces and moments are equal and opposite on the two parts of the arch, as shown on figure 4-36. In determining other forces and movements, each part of the arch may be considered as a curved cantilever beam. In the following discussions the left and right parts of the arch are the parts as viewed when looking upstream.

(e) *Convention of Signs.*—The convention of signs is shown on figure 4-37. Positive moments cause compression at the upstream face, and positive thrusts cause compression. In the left part of the arch, positive shears cause positive moments on the section of the arch to the left. In the right part of the arch a positive shear produces a positive moment in the section to the right, except $V_o$, which acts as shown on figure 4-37. Directions of application of positive loads are as shown on the same figure.

Positive moments, thrusts, and shears, $M_L$, $H_L$, $V_L$, or $M_R$, $H_R$, $V_R$, due to applied loads, are in the same direction as moments, thrusts, and shears of positive radial loads. Following this convention, moments, thrust, and shears of all positive triangular and concentrated loads are positive, except thrusts of tangential loads which are negative. Since the portion of the uniform tangential or twist load on the right part of the arch is applied in the same direction as the load on the left part, the $M_R$, $H_R$, and $V_R$ for these loads change sign.

Positive radial deflections are upstream, positive tangential deflections are toward the right, and positive angular movements are counterclockwise. Sign conventions for abutment movements and loads are as given on figures 4-24B, C, and D.

(f) *Arch Forces and Deformations.*—Additional nomenclature used in deriving equations for movements of the arch at the crown, equations for solution of crown forces, and equations for moment, thrust, and shear at any arch point except the crown, are shown on figure 4-38 and listed below:

$s$ = length along centerline of arch from crown to any arch point.
$\Phi$ = angle from crown to any arch point.
$\Phi_a$ = angle from crown of arch to abutment.
$x, y$ = coordinates of any arch point with origin at crown.

Consider a differential element of length $ds$, in the left part of the arch, as shown on figure 4-38. From mechanics, equations for arch movements at the crown, due to a moment, thrust, and shear acting on the differential elements, are:

$$d\theta_o = \frac{M\,ds}{E_cI} \tag{74}$$

$$d(\Delta r)_o = \frac{Mx\,ds}{E_cI} - \frac{H}{E_cA}\sin\Phi\,ds + \frac{K_6}{G_c}\frac{V}{A}\cos\Phi\,ds + ct\sin\Phi\,ds \tag{75}$$

$$d(\Delta s)_o = -\frac{My\,ds}{E_cI} - \frac{H}{E_cA}\cos\Phi\,ds - \frac{K_6}{G_c}\frac{V}{A}\sin\Phi\,ds + ct\cos\Phi\,ds \tag{76}$$

The first term in these equations gives the movement due to bending, the second term gives the movement due to rib-shortening, the third term gives the movement due to shear detrusion, and the last term gives the movement due to temperature change. If the equations are integrated from the crown to the abutment, and rock abutment movements added, the following equations result:

$$_L\theta_o = \int_0^s \frac{M\,ds}{E_cI} + M_a\,\alpha + V_a\,\alpha_2 \tag{77}$$

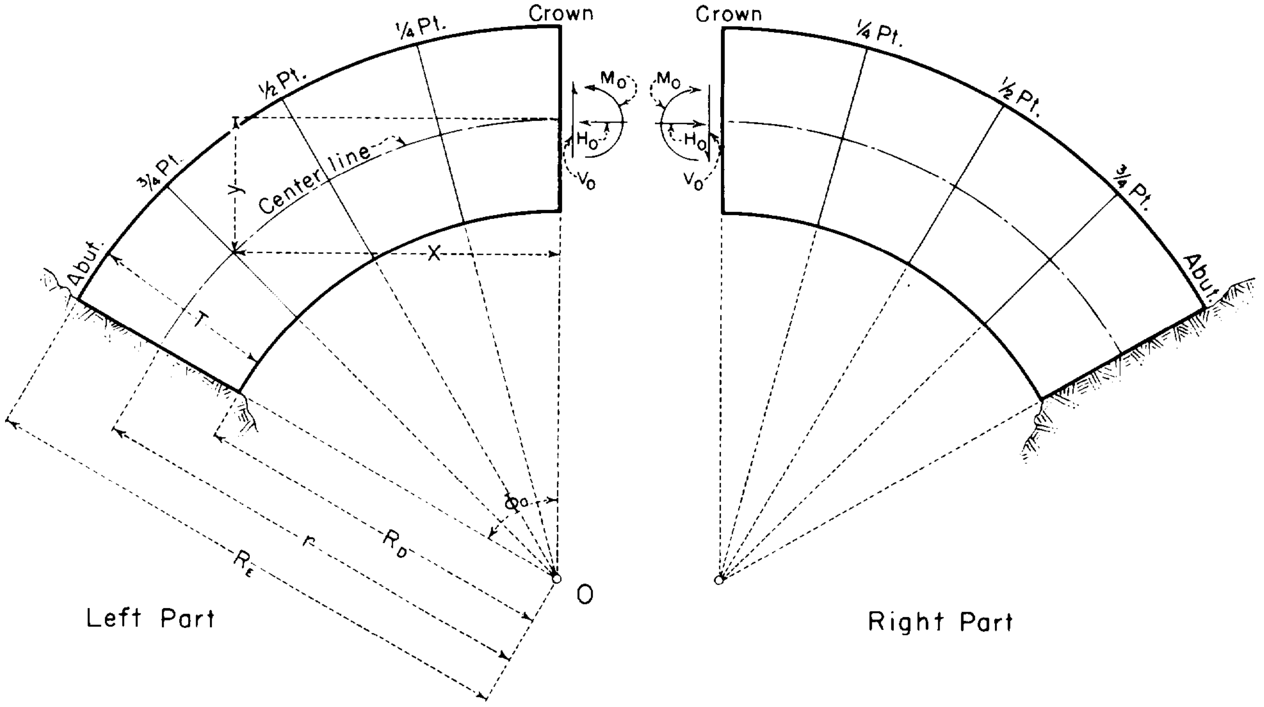
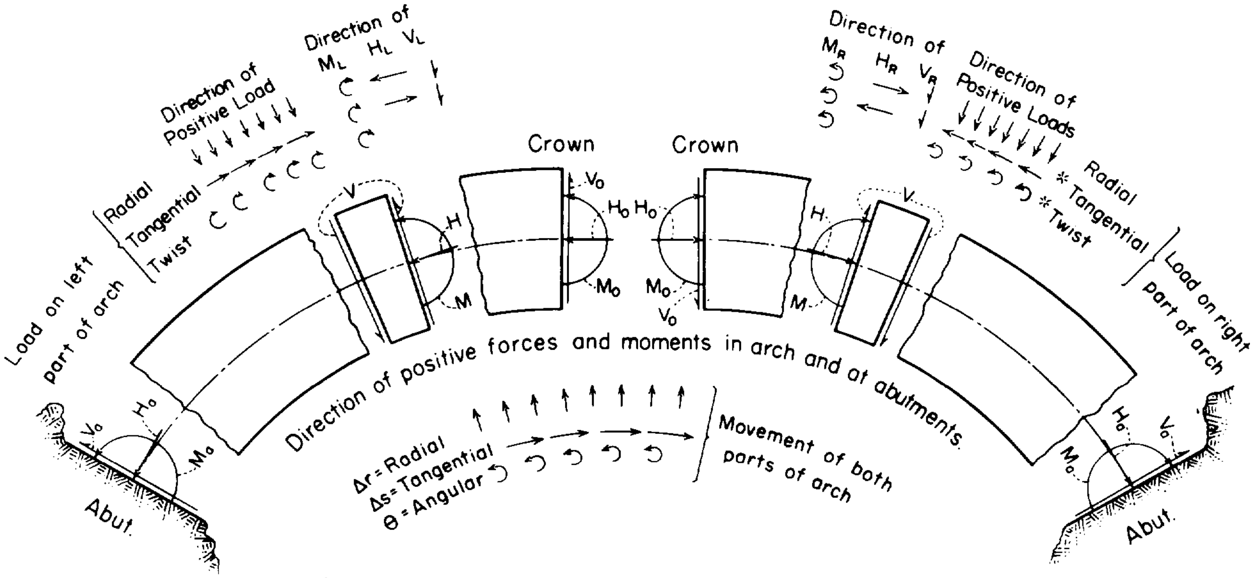


*Figure 4-36.* **Circular arch cut at crown.–288-D-392.**



*Uniform tangential and twist loads are continuous along the arch and their directions are those for load on left part of arch.

*Figure 4-37.* **Direction of positive loads, forces, moments, and movements.–288-D-393**

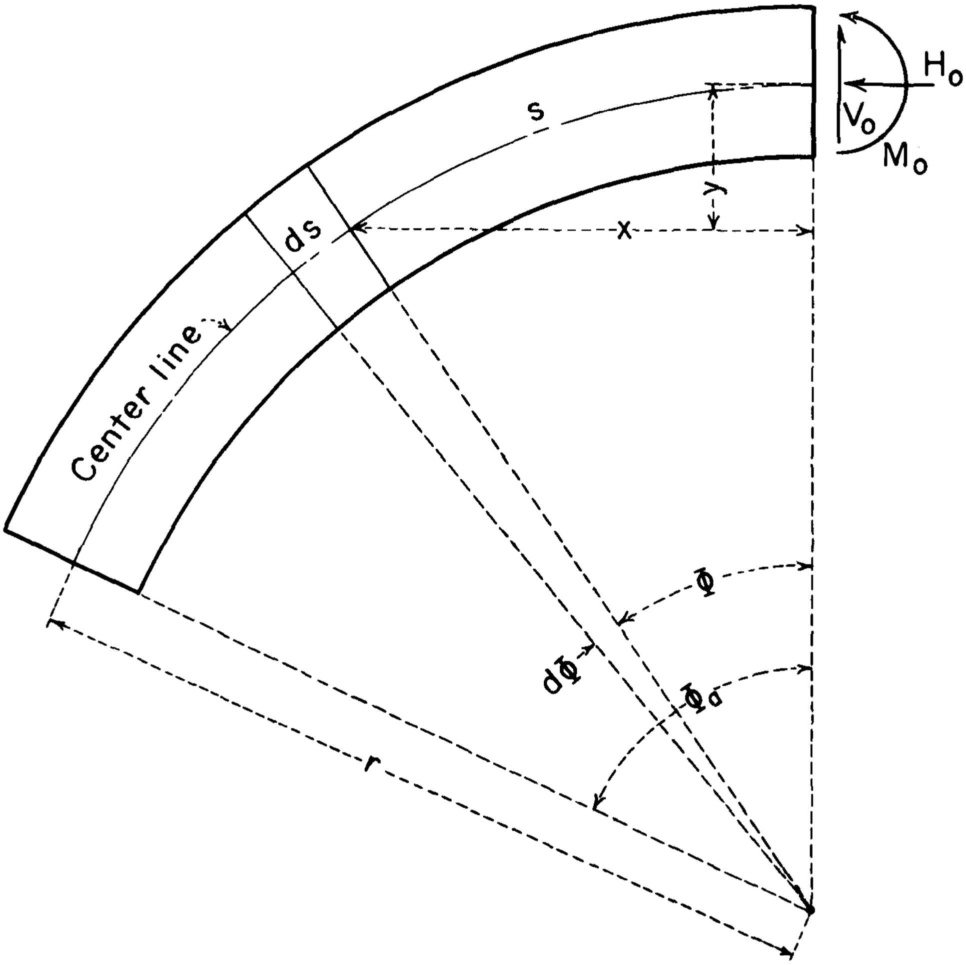


*Figure 4-38.* **Coordinates and crown forces for left part of arch.—288-D-394**

$$
{}_L\Delta r_o = \int_0^s \frac{Mx\,ds}{E_cI} - \int_0^s \frac{H}{E_cA}\sin\Phi\,ds + \int_0^s \frac{K_6}{G_c}\frac{V}{A}\cos\Phi\,ds
$$

$$
+ (\alpha x_a + \alpha_2 \cos\Phi_a)M_a - H_a\,\beta \sin\Phi_a + (\gamma\cos\Phi_a + \alpha_2\,x_a)V_a
$$

$$
+ \int_0^s ct\sin\Phi\,ds \qquad (78)
$$

$$
{}_L\Delta s_o = -\int_0^s \frac{My\,ds}{E_cI} - \int_0^s \frac{H}{E_cA}\cos\Phi\,ds - \int_0^s \frac{K_6}{G_c}\frac{V}{A}\sin\Phi\,ds
$$

$$
- (\alpha y_a + \alpha_2 \sin\Phi_a)M_a - H_a\,\beta\cos\Phi_a - (\gamma\sin\Phi_a + \alpha_2\,y_a)V_a
$$

$$
+ \int_0^s ct\cos\Phi\,ds \qquad (79)
$$

By substituting,

$$M = M_o + H_o\,y + V_o\,x - M_L$$

$$H = H_o \cos\Phi - V_o \sin\Phi + H_L$$

$$V = H_o \sin\Phi + V_o \cos\Phi - V_L$$

and using $K_6/G_c$ equal to $3/E_c$, equations (77), (78), and (79) for the left part of the arch become:

$$\begin{aligned} {}_L\theta_o = {} & M_o \left[\int_0^s \frac{ds}{E_cI} + \alpha\right] + H_o \left[\int_0^s \frac{y\,ds}{E_cI} + \alpha y_a + \alpha_2 \sin\Phi_a\right] \\ & + V_o \left[\int_0^s \frac{x\,ds}{E_cI} + \alpha x_a + \alpha_2 \cos\Phi_a\right] - \left[\int_0^s \frac{M_L\,ds}{E_cI} + {}_aM_L\,\alpha + {}_aV_L\,\alpha_2\right] \end{aligned} \tag{80}$$

$$\begin{aligned} {}_L\Delta r_o = {} & M_o \left[\int_0^s \frac{x\,ds}{E_cI} + \alpha x_a + \alpha_2 \cos\Phi_a\right] + H_o \Bigg[\int_0^s \frac{xy\,ds}{E_cI} - \int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} \\ & + 3\int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} + \alpha x_a y_a + \alpha_2 x_a \sin\Phi_a - \beta \sin\Phi_a \cos\Phi_a \\ & + \gamma \sin\Phi_a \cos\Phi_a + \alpha_2 y_a \cos\Phi_a\Bigg] + V_o \Bigg[\int_0^s \frac{x^2\,ds}{E_cI} + \int_0^s \frac{\sin^2\Phi\,ds}{E_cA} \\ & + 3\int_0^s \frac{\cos^2\Phi\,ds}{E_cA} + \alpha x_a^2 + 2\,\alpha_2 x_a \cos\Phi_a + \beta \sin^2\Phi_a + \gamma \cos^2\Phi_a\Bigg] \\ & - \Bigg[\int_0^s \frac{M_L x\,ds}{E_cI} + \int_0^s \frac{H_L \sin\Phi\,ds}{E_cA} + 3\int_0^s \frac{V_L \cos\Phi\,ds}{E_cA} - \int_0^s ct \sin\Phi\,ds \\ & + {}_aM_L\,(\alpha x_a + \alpha_2 \cos\Phi_a) + {}_aH_L\,\beta \sin\Phi_a + {}_aV_L\,(\gamma \cos\Phi_a + \alpha_2 x_a)\Bigg] \end{aligned} \tag{81}$$

$$\begin{aligned}
{}_L\Delta s_o = & -M_o \Bigg[\int_0^s \frac{y\,ds}{E_cI} + \alpha y_a + \alpha_2 \sin\Phi_a\Bigg] - H_o \Bigg[\int_0^s \frac{y^2\,ds}{E_cI} + \int_0^s \frac{\cos^2\Phi\,ds}{E_cA} \\
& + 3\int_0^s \frac{\sin^2\Phi\,ds}{E_cA} + \alpha y_a^2 + 2\alpha_2 y_a \sin\Phi_a + \beta\cos^2\Phi_a + \gamma\sin^2\Phi_a\Bigg] \\
& - V_o \Bigg[\int_0^s \frac{xy\,ds}{E_cI} - \int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} + 3\int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} + \alpha x_a y_a \\
& + \alpha_2 y_a \cos\Phi_a - \beta\sin\Phi_a\cos\Phi_a + \gamma\sin\Phi_a\cos\Phi_a + \alpha_2 x_a \sin\Phi_a\Bigg] \\
& + \Bigg[\int_0^s \frac{M_L y\,ds}{E_cI} - \int_0^s \frac{H_L\cos\Phi\,ds}{E_cA} + 3\int_0^s \frac{V_L\sin\Phi\,ds}{E_cA} \\
& + {}_aM_L(\alpha y_a + \alpha_2\sin\Phi_a) - {}_aH_L\,\beta\cos\Phi_a + {}_aV_L(\gamma\sin\Phi_a + \alpha_2 y_a) \\
& + \int_0^s ct\cos\Phi\,ds\Bigg] \qquad (82)
\end{aligned}$$

Inspection shows that some of the multipliers for $M_o$, $H_o$, and $V_o$ are duplicated in the equations. If symbols $A_1$, $B_1$, $B_2$, $B_3$, $C_1$, $C_2$, $D_1$, $D_2$, and $D_3$ are used for these multipliers and for the terms depending on loads, the equations may be written:

$${}_L\theta_o = {}_LA_1M_o + {}_LB_1H_o + {}_LC_1V_o - {}_LD_1 \qquad (83)$$

$${}_L\Delta r_o = {}_LC_1M_o + {}_LB_2H_o + {}_LC_2V_o - {}_LD_2 \qquad (84)$$

$${}_L\Delta s_o = -{}_LB_1M_o - {}_LB_3H_o - {}_LB_2V_o + {}_LD_3 \qquad (85)$$

Equations for the right part, developed in the same manner, are:

$${}_R\theta_o = -{}_RA_1M_o - {}_RB_1H_o + {}_RC_1V_o + {}_RD_1 \qquad (86)$$

$${}_R\Delta r_o = {}_RC_1M_o + {}_RB_2H_o - {}_RC_2V_o - {}_RD_2 \qquad (87)$$

$${}_R\Delta s_o = {}_RB_1M_o + {}_RB_3H_o - {}_RB_2V_o - {}_RD_3 \qquad (88)$$

Movements of each part of the arch, cut at the crown, may be determined by equations (83) through (88). Using these equations and following the method outlined in section 4-34(d), the three general equations needed to solve the unknown crown forces, $M_o$, $H_o$, and $V_o$, are:

$$[{}_LA_1 + {}_RA_1]M_o + [{}_LB_1 + {}_RB_1]H_o + [{}_LC_1 - {}_RC_1]V_o = [{}_LD_1 + {}_RD_1] \tag{89}$$

$$[{}_LC_1 - {}_RC_1]M_o + [{}_LB_2 - {}_RB_2]H_o + [{}_LC_2 + {}_RC_2]V_o = [{}_LD_2 - {}_RD_2] \tag{90}$$

$$[{}_LB_1 + {}_RB_1]M_o + [{}_LB_3 + {}_RB_3]H_o + [{}_LB_2 - {}_RB_2]V_o = [{}_LD_3 + {}_RD_3] \tag{91}$$

By combining constants in the brackets so that

$$[{}_LA_1 + {}_RA_1] = A_1, \quad [{}_LB_1 + {}_RB_1] = B_1,$$

and so forth, the equations reduce to:

$$A_1M_o + B_1H_o + C_1V_o = D_1 \tag{92}$$

$$C_1M_o + B_2H_o + C_2V_o = D_2 \tag{93}$$

$$B_1M_o + B_3H_o + B_2V_o = D_3 \tag{94}$$

The various quantities included in the $A_1$, $B_1$, $C_1$, $B_2$, $C_2$, $B_3$, $D_1$, $D_2$, and $D_3$ terms in these equations are given in table 4-4. Quantities in the table are obtained from equations (80), (81), and (82) for the left part of the arch, and from similar equations for the right part.

By solving equations (92), (93), and (94) simultaneously, formulas for $M_o$, $H_o$, and $V_o$ are derived as follows:

$$M_o = \frac{1}{K_o}\left[D_1(B_3C_2 - B_2^2) - D_3(B_1C_2 - C_1B_2) - D_2(B_3C_1 - B_1B_2)\right] \tag{95}$$

$$H_o = \frac{1}{K_o}\left[-D_1(B_1C_2 - B_2C_1) + D_3(A_1C_2 - C_1^2) + D_2(B_1C_1 - A_1B_2)\right] \tag{96}$$

$$V_o = \frac{1}{K_o}\left[-D_1(B_3C_1 - B_1B_2) + D_3(B_1C_1 - A_1B_2) + D_2(A_1B_3 - B_1^2)\right] \tag{97}$$

in which

$$K_o = A_1(B_3C_2 - B_2^2) - B_1(B_1C_2 - C_1B_2) - C_1(B_3C_1 - B_1B_2) \tag{98}$$

For a symmetrical arch, these equations reduce to:

$$M_o = \frac{1}{K_o}\left[D_1B_3 - D_3B_1\right] \tag{99}$$

$$H_o = \frac{1}{K_o}\left[-D_1B_1 + D_3A_1\right] \tag{100}$$

Table 4-4.—*Constants for solution of crown forces.*

| $A_1$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{ds}{E_cI}$ |
| | Abut. | $\alpha$ |
| LEFT | Arch | $\int_0^s \frac{ds}{E_cI}$ |
| | Abut. | $\alpha$ |

| $B_1$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{yds}{E_cI}$ |
| | Abut. | $\alpha y_a$<br>$\alpha_2 \sin \Phi_a$ |
| LEFT | Arch | $\int_0^s \frac{yds}{E_cI}$ |
| | Abut. | $\alpha y_a$<br>$\alpha_2 \sin \Phi_a$ |

| $C_1$ | | |
|---|---|---|
| RIGHT | Arch | $-\int_0^s \frac{xds}{E_cI}$ |
| | Abut. | $-\alpha x_a$<br>$-\alpha_2 \cos \Phi_a$ |
| LEFT | Arch | $\int_0^s \frac{xds}{E_cI}$ |
| | Abut. | $\alpha x_a$<br>$\alpha_2 \cos \Phi_a$ |

| $B_2$ | | |
|---|---|---|
| RIGHT | Arch | $-\int_0^s \frac{xyds}{E_cI}$<br>$\int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_cA}$<br>$-3\int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_cA}$ |
| | Abut. | $-\alpha x_a y_a$<br>$\beta \sin \Phi_a \cos \Phi_a$<br>$-\gamma \sin \phi_a \cos \Phi_a$<br>$-\alpha_2 x_a \sin \Phi_a$<br>$-\alpha_2 y_a \cos \Phi_a$ |
| LEFT | Arch | $\int_0^s \frac{xyds}{E_cI}$<br>$-\int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_cA}$ |
| | Abut. | $\alpha x_a y_a$<br>$-\beta \sin \Phi_a \cos \Phi_a$<br>$\gamma \sin \Phi_a \cos \Phi_a$<br>$\alpha_2 x_a \sin \Phi_a$<br>$\alpha_2 y_a \cos \Phi_a$ |

| $C_2$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{x^2ds}{E_cI}$<br>$\int_0^s \frac{\sin^2 \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{\cos^2 \Phi \, ds}{E_cA}$ |
| | Abut. | $\alpha x_a^2$<br>$\beta \sin^2 \Phi_a$<br>$\gamma \cos^2 \Phi_a$<br>$2\alpha_2 x_a \cos \Phi_a$ |
| LEFT | Arch | $\int_0^s \frac{x^2ds}{E_cI}$<br>$\int_0^s \frac{\sin^2 \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{\cos^2 \Phi \, ds}{E_cA}$ |
| | Abut. | $\alpha x_a^2$<br>$\beta \sin^2 \Phi_a$<br>$\gamma \cos^2 \Phi_a$<br>$2\alpha_2 x_a \cos \Phi_a$ |

| $B_3$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{y^2ds}{E_cI}$<br>$\int_0^s \frac{\cos^2 \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{\sin^2 \Phi \, ds}{E_cA}$ |
| | Abut. | $\alpha y_a^2$<br>$\beta \cos^2 \Phi_a$<br>$\gamma \sin^2 \Phi_a$<br>$2\alpha_2 y_a \sin \Phi_a$ |
| LEFT | Arch | $\int_0^s \frac{y^2ds}{E_cI}$<br>$\int_0^s \frac{\cos^2 \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{\sin^2 \Phi \, ds}{E_cA}$ |
| | Abut. | $\alpha y_a^2$<br>$\beta \cos^2 \Phi_a$<br>$\gamma \sin^2 \Phi_a$<br>$2\alpha_2 y_a \sin \Phi_a$ |

| $D_1$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{M_R ds}{E_cI}$ |
| | Abut. | ${}_aM_R \alpha$<br>${}_aV_R \alpha_2$ |
| LEFT | Arch | $\int_0^s \frac{M_L ds}{E_cI}$ |
| | Abut. | ${}_aM_L \alpha$<br>${}_aV_L \alpha_2$ |

| $D_2$ | | |
|---|---|---|
| RIGHT | Arch | $-\int_0^s \frac{M_R xds}{E_cI}$<br>$-\int_0^s \frac{H_R \sin \Phi \, ds}{E_cA}$<br>$-3\int_0^s \frac{V_R \cos \Phi \, ds}{E_cA}$ |
| | Abut. | $-{}_aM_R \alpha x_a$<br>$-{}_aH_R \beta \sin \Phi_a$<br>$-{}_aV_R \gamma \cos \Phi_a$<br>$-{}_aM_R \alpha_2 \cos \Phi_a$<br>$-{}_aV_R \alpha_2 x_a$ |
| | Temp. | $cty_a$ |
| LEFT | Arch | $\int_0^s \frac{M_L xds}{E_cI}$<br>$\int_0^s \frac{H_L \sin \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{V_L \cos \Phi \, ds}{E_cA}$ |
| | Abut. | ${}_aM_L \alpha x_a$<br>${}_aH_L \beta \sin \Phi_a$<br>${}_aV_L \gamma \cos \Phi_a$<br>${}_aM_L \alpha_2 \cos \Phi_a$<br>${}_aV_L \alpha_2 x_a$ |
| | Temp. | $-cty_a$ |

| $D_3$ | | |
|---|---|---|
| RIGHT | Arch | $\int_0^s \frac{M_R yds}{E_cI}$<br>$-\int_0^s \frac{H_R \cos \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{V_R \sin \Phi \, ds}{E_cA}$ |
| | Abut. | $M_R \alpha y_a$<br>$-{}_aH_R \beta \cos \Phi_a$<br>${}_aV_R \gamma \sin \Phi_a$<br>${}_aM_R \alpha_2 \sin \Phi_a$<br>${}_aV_R \alpha_2 y_a$ |
| | Temp. | $ctx_a$ |
| LEFT | Arch | $\int_0^s \frac{M_L yds}{E_cI}$<br>$-\int_0^s \frac{H_L \cos \Phi \, ds}{E_cA}$<br>$3\int_0^s \frac{V_L \sin \Phi \, ds}{E_cA}$ |
| | Abut. | ${}_aM_L \alpha y_a$<br>$-{}_aH_L \beta \cos \Phi_a$<br>${}_aV_L \gamma \sin \Phi_a$<br>${}_aM_L \alpha_2 \sin \Phi_a$<br>${}_aV_L \alpha_2 y_a$ |
| | Temp. | $ctx_a$ |

288-D-398

$$V_o = \frac{D_2}{C_2} \qquad (101)$$

in which

$$K_o = A_1 B_3 - B_1{}^2 \qquad (102)$$

To derive general equations for arch deflections at any arch point, the following meanings are assigned to terms used in equations (109) through (117). The origin is assumed to be at the point where the deflections are desired.

$M, H, V$ = moment, thrust, and shear at the point where deflections are desired. These values are computed from equations (103) to (105) inclusive, for the left part of the arch and equations (106) to (108) for the right part of the arch. In these equations, $x$, $y$, and $\Phi$ are measured from the crown to the point where the deflections are desired, not from the point as defined below, which refers to equations (109) to (117), inclusive.

$s$ = length along centerline of arch from point where deflections are desired to any point in the arch.

$\Phi$ = angle measured from arch point where deflections are desired to any point in the arch

$x, y$ = coordinates of any arch point with origin at the point at which deflections are desired.

$\Phi_a$ = angle from arch point at which deflections are desired to abutment.

Arch deflections at any point are obtained by considering the portion of the arch between the point and the abutment as a curved cantilever beam. The desired arch movements are the sum of the movements due to the applied load between the point considered and the abutment, and the movements due to the moment, thrust, and shear acting at the point.

It is evident that the moment, thrust, and shear at the point at which deflections are desired must be determined before deflections can be evaluated. These are easily determined after crown values have been computed. Formulas for the left part of the arch are:

$$M = M_o + H_o y + V_o x - M_L \qquad (103)$$

$$H = H_o \cos\Phi - V_o \sin\Phi + H_L \qquad (104)$$

$$V = H_o \sin\Phi + V_o \cos\Phi - V_L \qquad (105)$$

Similar formulas for the right part are:

$$M = M_o + H_o y - V_o x - M_R \qquad (106)$$

$$H = H_o \cos\Phi + V_o \sin\Phi + H_R \qquad (107)$$

$$V = H_o \sin\Phi - V_o \cos\Phi - V_R \qquad (108)$$

Equations for arch deflections at any point are derived in the same way as equations (80), (81), and (82) for the crown. However, in this case total forces and moments acting on the abutments can be calculated by equations (103) through (108) after the crown values are known, which makes it possible to consider effects of resulting abutment movements separately. Equations for deflections at any point in the left part of the arch are:

$${}_L\theta = M\left[\int_0^s \frac{ds}{E_c I}\right] + H\left[\int_0^s \frac{y\,ds}{E_c I}\right] + V\left[\int_0^s \frac{x\,ds}{E_c I}\right] - \left[\int_0^s \frac{M_L\,ds}{E_c I}\right] + M_a\alpha + V_a\alpha_2 \qquad (109)$$

$$
\begin{aligned}
{}_L\Delta r = {} & M\left[\int_0^s \frac{x\,ds}{E_cI}\right] + H\left[\int_0^s \frac{xy\,ds}{E_cI} - \int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} + 3\int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA}\right] \\
& + V\left[\int_0^s \frac{x^2\,ds}{E_cI} + \int_0^s \frac{\sin^2\Phi\,ds}{E_cA} + 3\int_0^s \frac{\cos^2\Phi\,ds}{E_cA}\right] \\
& - \left[\int_0^s \frac{M_L x\,ds}{E_cI} + \int_0^s \frac{H_L\sin\Phi\,ds}{E_cA} + 3\int_0^s \frac{V_L\cos\Phi\,ds}{E_cA} - \int_0^s ct\sin\Phi\,ds\right] \\
& + (M_a\alpha + V_a\alpha_2)x_a + (V_a\gamma + M_a\alpha_2)\cos\Phi_a - H_a\,\beta\sin\Phi_a
\end{aligned}
\tag{110}
$$

$$
\begin{aligned}
{}_L\Delta s = {} & -M\left[\int_0^s \frac{y\,ds}{E_cI}\right] - H\left[\int_0^s \frac{y^2\,ds}{E_cI} + \int_0^s \frac{\cos^2\Phi\,ds}{E_cA} + 3\int_0^s \frac{\sin^2\Phi\,ds}{E_cA}\right] \\
& - V\left[\int_0^s \frac{xy\,ds}{E_cI} - \int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA} + 3\int_0^s \frac{\sin\Phi\cos\Phi\,ds}{E_cA}\right] \\
& + \left[\int_0^s \frac{M_L y\,ds}{E_cI} - \int_0^s \frac{H_L\cos\Phi\,ds}{E_cA} + 3\int_0^s \frac{V_L\sin\Phi\,ds}{E_cA} + \int_0^s ct\cos\Phi\,ds\right] \\
& - (M_a\alpha + V_a\alpha_2)y_a - (V_a\gamma + M_a\alpha_2)\sin\Phi_a - H_a\,\beta\cos\Phi_a
\end{aligned}
\tag{111}
$$

in which $M_a$, $H_a$, and $V_a$ are the moment, thrust, and shear at the abutment.

By substituting symbols for integrals or groups of integrals in the brackets, the following equations may be written:

$$ {}_L\theta = A_1M + B_1H + C_1V - D_1 + M_a\alpha + V_a\alpha_2 \tag{112} $$

$$ {}_L\Delta r = C_1M + B_2H + C_2V - D_2 + (M_a\alpha + V_a\alpha_2)x_a - H_a\,\beta\sin\Phi_a + (V_a\gamma + M_a\alpha_2)\cos\Phi_a \tag{113} $$

$$ {}_L\Delta s = -B_1M - B_3H - B_2V + D_3 - (M_a\alpha + V_a\alpha_2)y_a - (V_a\gamma + M_a\alpha_2)\sin\Phi_a - H_a\,\beta\cos\Phi_a \tag{114} $$

Equations for deflections in the right part of the arch are:

$$ {}_R\theta = -A_1M - B_1H - C_1V + D_1 - M_a\alpha - V_a\alpha_2 \tag{115} $$

$$ {}_R\Delta r = C_1M + B_2H + C_2V - D_2 + (M_a\alpha + V_a\alpha_2)x_a - H_a\,\beta\sin\Phi_a + (V_a\gamma + M_a\alpha_2)\cos\Phi_a \tag{116} $$

$$ {}_R\Delta s = B_1M + B_3H + B_2V - D_3 + (M_a\alpha + V_a\alpha_2)y_a + (V_a\gamma + M_a\alpha_2)\sin\Phi_a + H_a\,\beta\cos\Phi_a \tag{117} $$

The quantities $A_1$, $B_1$, $B_2$, $B_3$, $C_1$, and $C_2$ consist of integrals or groups of integrals which are functions of the arch and are therefore designated “arch constants.” The quantities $D_1$, $D_2$, and $D_3$ consist of integrals or combinations of integrals which are functions of both the arch and the load and are designated "load constants." Integrals for these constants also appear in quantities required for solution of crown forces. For this reason arch and load constants may be used for the solution of crown forces as well as for calculating arch deflections. The evaluation of constants for various types of arches, together with a general discussion concerning their use, is given in subsequent sections.

At this point in the development of arch theory it is convenient to consider the arch as circular and of uniform thickness, because arch and load constants can be evaluated for this special case. In derivations of arch and load constants in the following sections, the arch centerline is used instead of the neutral axis. This introduces a small error but simplifies the analysis. Using the regular notation, $I = T^3/12$, $s = r\Phi$, and $ds = r\,d\Phi$. Since $T$ and $E_c$ remain constant throughout the arch, and since $x = r \sin \Phi$, and $y = r \operatorname{vers} \Phi$, it is evident that the arch and load constants can be integrated.

(g) *Arch Constants.*—Arch constants are deflections at an arch point due to a unit force or couple at the point. They include the following:

$A_1$ = angular movement at a point due to a unit moment at the point.
$B_1$ = angular movement at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit negative moment at the point.
$C_1$ = angular movement at a point due to a unit shear at the point; or, it is the radial deflection at a point due to a unit moment at the point.
$B_2$ = radial deflection at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit negative shear at the point.
$C_2$ = radial deflection at a point due to a unit shear at the point.
$B_3$ = tangential deflection at a point due to a unit negative thrust at the point.

The following formulas for arch constants are for any arch point and may be used for either the left or right part of the arch:

$$A_1 = \int_0^s \frac{ds}{E_c I} = \frac{12r}{E_c T^3} \int_0^{\Phi_a} d\Phi = \frac{12r}{E_c T^3} [\Phi_a] \tag{118}$$

$$B_1 = \int_0^s \frac{y\,ds}{E_c I} = \frac{12r^2}{E_c T^3} \int_0^{\Phi_a} \operatorname{vers} \Phi \, d\Phi = \frac{12r^2}{E_c T^3} [\Phi_a - \sin \Phi_a] \tag{119}$$

$$C_1 = \int_0^s \frac{x\,ds}{E_c I} = \frac{12r^2}{E_c T^3} \int_0^{\Phi_a} \sin \Phi \, d\Phi = \frac{12r^2}{E_c T^3} [\operatorname{vers} \Phi_a] \tag{120}$$

$$B_2 = \int_0^s \frac{xy\,ds}{E_c I} - \int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_c A} + 3 \int_0^s \frac{\sin \Phi \cos \Phi \, ds}{E_c A}$$

$$= \frac{12r^3}{E_c T^3} \int_0^{\Phi_a} \sin \Phi \operatorname{vers} \Phi \, d\Phi + \frac{2r}{E_c T} \int_0^{\Phi_a} \sin \Phi \cos \Phi \, d\Phi$$

$$= \frac{12r^3}{E_c T^3} \left[ \operatorname{vers} \Phi_a - \frac{\sin^2 \Phi_a}{2} \right] + \frac{r}{E_c T} [\sin^2 \Phi_a] \tag{121}$$

$$C_2 = \int_0^s \frac{x^2\,ds}{E_c\,I} + \int_0^s \frac{\sin^2\Phi\,ds}{E_c\,A} + 3\int_0^s \frac{\cos^2\Phi\,ds}{E_c\,A}$$

$$= \frac{12r^3}{E_c\,T^3}\int_0^{\Phi_a} \sin^2\Phi\,d\Phi + \frac{r}{E_c\,T}\int_0^{\Phi_a} \sin^2\Phi\,d\Phi + \frac{3r}{E_c\,T}\int_0^{\Phi_a} \cos^2\Phi\,d\Phi$$

$$= \frac{12r^3}{E_c\,T^3}\left[\frac{\Phi_a - \sin\Phi_a\cos\Phi_a}{2}\right]$$

$$+ \frac{r}{E_c\,T}\left[\frac{\Phi_a - \sin\Phi_a\cos\Phi_a}{2} + 3\,\frac{\Phi_a + \sin\Phi_a\cos\Phi_a}{2}\right] \qquad (122)$$

$$B_3 = \int_0^s \frac{y^2\,ds}{E_c\,I} + \int_0^s \frac{\cos^2\Phi\,ds}{E_c\,A} + 3\int_0^s \frac{\sin^2\Phi\,ds}{E_c\,A}$$

$$= \frac{12r^3}{E_c\,T^3}\int_0^{\Phi_a} \text{vers}^2\,\Phi\,d\Phi + \frac{r}{E_c\,T}\int_0^{\Phi_a} \cos^2\Phi\,d\Phi + \frac{3r}{E_c\,T}\int_0^{\Phi_a} \sin^2\Phi\,d\Phi$$

$$= \frac{12r^3}{E_c\,T^3}\left[\Phi_a - 2\sin\Phi_a + \frac{\Phi_a + \sin\Phi_a\cos\Phi_a}{2}\right]$$

$$+ \frac{r}{E_c\,T}\left[\frac{\Phi_a + \sin\Phi_a\cos\Phi_a}{2} + 3\,\frac{\Phi_a - \sin\Phi_a\cos\Phi_a}{2}\right] \qquad (123)$$

Inspection of equations (118) through (123) shows that quantities in brackets depend only on the arch angle. These equations have been evaluated for angles between zero and 90° and the results tabulated in the table of arch constants in appendix H.

(h) *Load Constants.*—Load constants are deflections at a point due to all loads between the point and the abutment. They include the following:

$D_1$ = angular movement at a point due to loads between the point and the abutment.
$D_2$ = radial deflection at a point due to loads between the point and the abutment.
$D_3$ = tangential deflection at a point due to loads between the point and the abutment.

Integrals for the $D$-terms for the left side of the arch are:

$$D_1 = \int_0^s \frac{M_L\,ds}{E_c\,I} \qquad (124)$$

$$D_2 = \int_0^s \frac{M_L x\,ds}{E_c I} + \int_0^s \frac{H_L \sin\Phi\,ds}{E_c A} + 3\int_0^s \frac{V_L \cos\Phi\,ds}{E_c A} - \int_0^s ct \sin\Phi\,ds \tag{125}$$

$$D_3 = \int_0^s \frac{M_L y\,ds}{E_c I} - \int_0^s \frac{H_L \cos\Phi\,ds}{E_c A} + 3\int_0^s \frac{V_L \sin\Phi\,ds}{E_c A} + \int_0^s ct \cos\Phi\,ds \tag{126}$$

These cannot be conveniently expressed until the terms $M_L$, $H_L$, and $V_L$ for the left or $M_R$, $H_R$, and $V_R$ for the right side of the arch are evaluated. This is described in the following sections which deal with the various unit loads separately. Formulas for moment, thrust, shear, and $D$-terms are derived for the left part of the arch but may be used for the right part also.

It should be remembered that $M_L$, $H_L$, and $V_L$ are the moment, thrust, and shear in the left part of the arch due to load at the left of the point. For the purpose of evaluating $D$-term integrals and deriving equations for moment, thrust, and shear ($M_L$, $H_L$, and $V_L$, or $M_R$, $H_R$, and $V_R$) at any arch point due to unit loads, the following additional notation is used (see fig. 4-39).

${}_a\Phi_1$ = for uniform loads applied on the arch, ${}_a\Phi_1$ is the angle from the point where deflections are desired to the abutment, for triangular loads, ${}_a\Phi_1$ is the angle from the point where the loading begins to the abutment.

$\Phi_0$ = angle from point where deflections are desired, for arch points not under load, to beginning of external load.

$\Psi$ = angle from point where loading begins to any differential element of the arch under load.

$\Phi_1$ = angle from point where loading begins to any arch point under load.

$P$ = intensity of applied load acting on the arch.

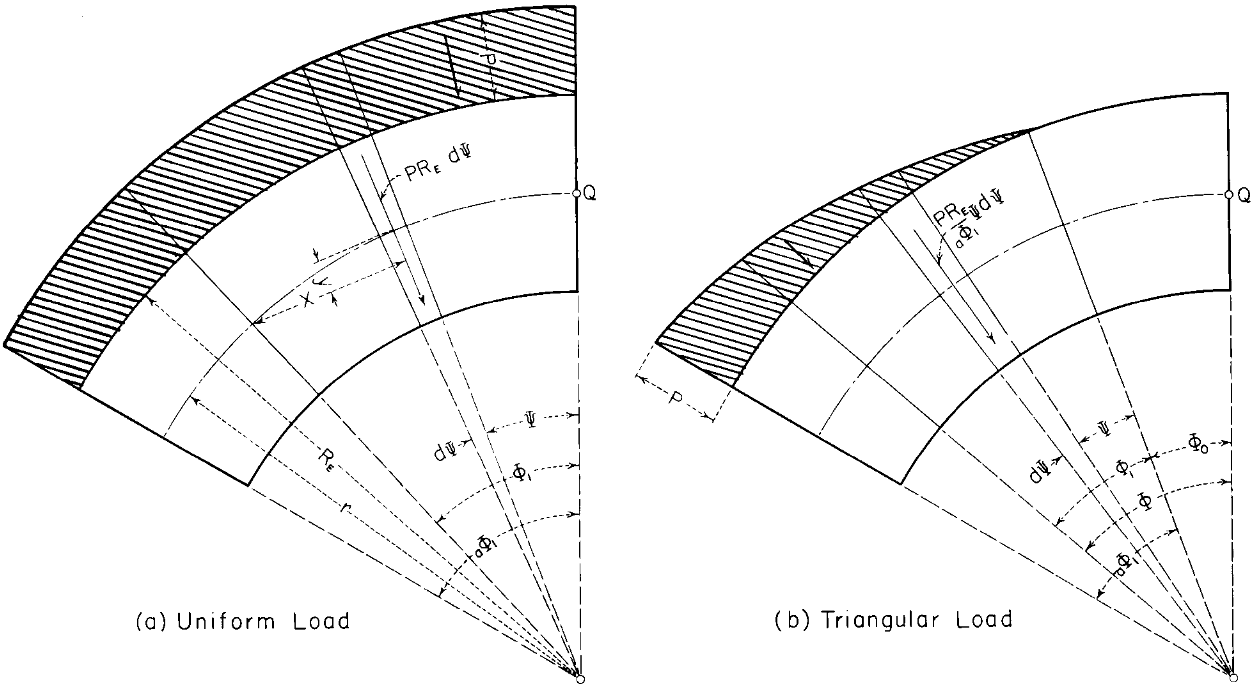
In the following equations for load constants, $D_2$ and $D_3$ values are divided into two terms. The first represents the effect of bending, and the second represents the effects of rib-shortening and shear detrusion.

(i) *Uniform Radial Load.*—Moments, thrusts, and shears due to a uniform radial load at the upstream face, as shown on figure 4-39(a), may be computed by the following equations:

$$M_L = \int_0^{\Phi_1} PR_E r \sin(\Phi_1 - \Psi)\,d\Psi = PR_E r \operatorname{vers}\Phi_1 \tag{127}$$

$$H_L = \int_0^{\Phi_1} PR_E \sin(\Phi_1 - \Psi)\,d\Psi = PR_E \operatorname{vers}\Phi_1 \tag{128}$$

$$V_L = \int_0^{\Phi_1} PR_E \cos(\Phi_1 - \Psi)\,d\Psi = PR_E \sin\Phi_1 \tag{129}$$



(a) Uniform Load

(b) Triangular Load

*Figure 4-39.* **Radial loads on arch elements.—288-D-3025**

By substituting these values in equations (124), (125), and (126), the following formulas are obtained:

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c\, I} = \frac{12\, R_E r^2}{E_c\, T^3}\, P \left[\int_0^{a\Phi_1} \operatorname{vers}\Phi_1\, d\Phi_1\right] \tag{130}$$

$$D_2\text{, 1}^{st}\text{ term} = \int_0^s \frac{M_L\, x\, ds}{E_c\, I} = \frac{12\, R_E r^3}{E_c\, T^3}\, P \left[\int_0^{a\Phi_1} \sin\Phi_1 \operatorname{vers}\Phi_1\, d\Phi_1\right] \tag{131}$$

$$D_2\text{, 2}^{nd}\text{ term} = \int_0^s \frac{H_L \sin\Phi\, ds}{E_c\, A} + 3 \int_0^s \frac{V_L \cos\Phi\, ds}{E_c\, A}$$

$$= \frac{R_E r}{E_c T} P \left[\int_0^{a\Phi_1} \sin\Phi_1 \operatorname{vers}\Phi_1\, d\Phi_1 + 3 \int_0^{a\Phi_1} \sin\Phi_1 \cos\Phi_1\, d\Phi_1\right] \tag{132}$$

$$D_3\text{, 1}^{st}\text{ term} = \int_0^s \frac{M_L y\, ds}{E_c\, I}$$

$$= \frac{12\, R_E r^3}{E_c\, T^3} P \left[\int_0^{a\Phi_1} \operatorname{vers}\Phi_1\, d\Phi_1 - \int_0^{a\Phi_1} \operatorname{vers}\Phi_1 \cos\Phi_1\, d\Phi_1\right] \tag{133}$$

$$D_3,\ 2^{nd}\ \text{term} = -\int_0^s \frac{H_L \cos\Phi\, ds}{E_c A} + 3\int_0^s \frac{V_L \sin\Phi\, ds}{E_c A}$$

$$= \frac{R_E r}{E_c T} P \left[ -\int_0^{{}_a\Phi_1} \text{vers}\,\Phi_1 \cos\Phi_1\, d\Phi_1 + 3\int_0^{{}_a\Phi_1} \sin^2\Phi_1\, d\Phi_1 \right] \qquad (134)$$

The evaluation of these and succeeding integrals is discussed in this chapter. It should be noted that the $D_2$ and $D_3$ temperature terms are omitted in equations (130) through (134), since the formulas are for uniform radial loads, which do not include temperature effects.

(j) *Triangular Radial Load.*—Moments, thrusts, and shears due to a triangular radial load at the upstream face, as shown on figure 4-39(b), may be computed by the following equations:

$$M_L = \int_0^{\Phi_1} \frac{PR_E r}{{}_a\Phi_1} \Psi \sin(\Phi_1 - \Psi)\, d\Psi = \frac{PR_E r}{{}_a\Phi_1}(\Phi_1 - \sin\Phi_1) \qquad (135)$$

$$H_L = \int_0^{\Phi_1} \frac{PR_E}{{}_a\Phi_1} \Psi \sin(\Phi_1 - \Psi)\, d\Psi = \frac{PR_E}{{}_a\Phi_1}(\Phi_1 - \sin\Phi_1) \qquad (136)$$

$$V_L = \int_0^{\Phi_1} \frac{PR_E}{{}_a\Phi_1} \Psi \cos(\Phi_1 - \Psi)\, d\Psi = \frac{PR_E}{{}_a\Phi_1}(\text{vers}\,\Phi_1) \qquad (137)$$

Load constants for a triangular radial load are:

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c I} = \frac{12 R_E r^2}{E_c T^3} \bullet \frac{P}{{}_a\Phi_1} \left[ \int_0^{{}_a\Phi_1} (\Phi_1 - \sin\Phi_1)\, d\Phi_1 \right] \qquad (138)$$

$$D_2,\ 1^{st}\ \text{term} = \int_0^s \frac{M_L x\, ds}{E_c I} = \frac{12 R_E r^3}{E_c T^3} \bullet \frac{P}{{}_a\Phi_1} \left[ \int_0^{{}_a\Phi_1} (\Phi_1 - \sin\Phi_1) \sin\Phi\, d\Phi_1 \right] \qquad (139)$$

$$D_2,\ 2^{nd}\ \text{term} = \int_0^s \frac{H_L \sin\Phi\, ds}{E_c A} + 3\int_0^s \frac{V_L \cos\Phi\, ds}{E_c A}$$

$$= \frac{R_E r}{E_c T} \cdot \frac{P}{{}_a\Phi_1} \left[ \int_0^{{}_a\Phi_1} (\Phi_1 - \sin\Phi_1) \sin\Phi\, d\Phi_1 + 3\int_0^{{}_a\Phi_1} \text{vers}\,\Phi_1 \cos\Phi\, d\Phi_1 \right] \qquad (140)$$

$$D_3\text{, 1}^{st}\text{ term} = \int_0^s \frac{M_L y\, ds}{E_c I}$$

$$= \frac{12 R_E r^3}{E_c T^3} \cdot \frac{P}{{}_a\Phi_1} \left[ \int_0^{a^{\Phi_1}} (\Phi_1 - \sin \Phi_1) d\Phi_1 \right.$$

$$\left. - \int_0^{a^{\Phi_1}} (\Phi_1 - \sin \Phi_1) \cos \Phi \; d\Phi_1 \right] \quad (141)$$

$$D_3\text{, 2}^{nd}\text{ term} = -\int_0^s \frac{H_L \cos \Phi \; ds}{E_c A} + 3 \int_0^s \frac{V_L \sin \Phi \; ds}{E_c A}$$

$$= \frac{R_E r}{E_c T} \cdot \frac{P}{{}_a\Phi_1} \left[ - \int_0^{a^{\Phi_1}} (\Phi_1 - \sin \Phi_1) \cos \Phi \; d\Phi_1 \right.$$

$$\left. + 3 \int_0^{a^{\Phi_1}} \text{vers}\, \Phi_1 \sin \Phi \; d\Phi_1 \right] \quad (142)$$

(k) *Uniform Tangential Load.*—Moments, thrusts, and shears due to a uniform tangential load applied along the centerline of an arch, as shown on figure 4-40(a), may be computed by the following equations:

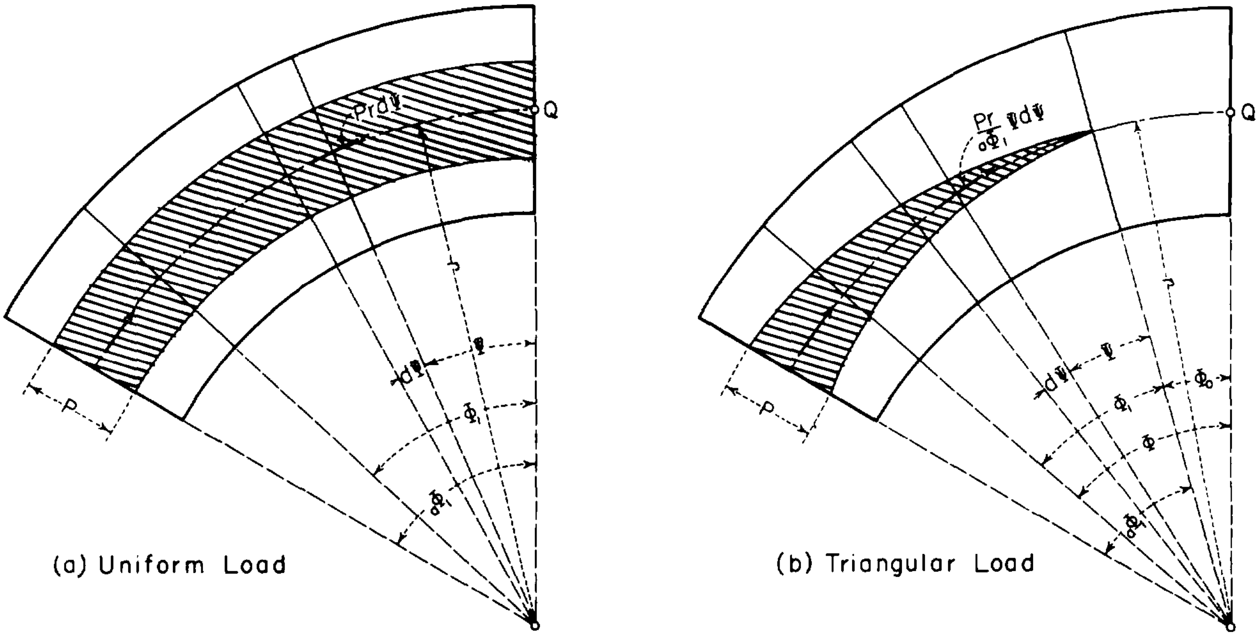
$$M_L = \int_0^{\Phi_1} Pr^2 \text{ vers} (\Phi_1 - \Psi) \, d\Psi = Pr^2 (\Phi_1 - \sin \Phi_1) \quad (143)$$

$$H_L = -\int_0^{\Phi_1} Pr \cos (\Phi_1 - \Psi) \, d\Psi = -Pr \sin \Phi_1 \quad (144)$$

$$V_L = \int_0^{\Phi_1} Pr \sin (\Phi_1 - \Psi) \, d\Psi = Pr \text{ vers } \Phi_1 \quad (145)$$

Equations for load constants, derived by substituting equations for $M_L$, $H_L$, and $V_L$ in equations (124), (125), and (126), are given below:

$$D_1 = \int_0^s \frac{M_L \, ds}{E_c I} = \frac{12 r^3}{E_c T^3} P \left[ \int_0^{a^{\Phi_1}} (\Phi_1 - \sin \Phi_1) \, d\Phi_1 \right] \quad (146)$$



(a) Uniform Load (b) Triangular Load

*Figure 4-40.* **Tangential loads on arch elements.–288-D-2980**

$$D_2\text{, }1^{st}\text{ term} = \int_0^s \frac{M_L x\, ds}{E_c I} = \frac{12\, r^4}{E_c\, T^3}\, P \left[\int_0^{a\Phi_1} (\Phi_1 - \sin\Phi_1)\sin\Phi_1\, d\Phi_1\right] \tag{147}$$

$$D_2\text{, }2^{nd}\text{ term} = \int_0^s \frac{H_L \sin\Phi_1\, ds}{E_c\, A} + 3\int_0^s \frac{V_L \cos\Phi_1\, ds}{E_c\, A}$$

$$= \frac{r^2}{E_c T} P \left[-\int_0^{a\Phi_1} \sin\Phi_1 \sin\Phi_1\, d\Phi_1 + 3\int_0^{a\Phi_1} \text{vers}\,\Phi_1 \cos\Phi_1\, d\Phi_1\right] \tag{148}$$

$$D_3\text{, }1^{st}\text{ term} = \int_0^s \frac{M_L y\, ds}{E_c I}$$

$$= \frac{12\, r^4}{E_c\, T^3}\, P \left[\int_0^{a\Phi_1} (\Phi_1 - \sin\Phi_1)\, d\Phi_1\right.$$

$$\left. - \int_0^{a\Phi_1} (\Phi_1 - \sin\Phi_1)\cos\Phi_1\, d\Phi_1\right] \tag{149}$$

$$D_3,\ 2^{nd}\ \text{term} = -\int_0^s \frac{H_L \cos\Phi_1\, ds}{E_c A} + 3\int_0^s \frac{V_L \sin\Phi_1\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} P \left[\int_0^{a\Phi_1} \sin\Phi_1 \cos\Phi_1\, d\Phi_1 + 3\int_0^{a\Phi_1} \text{vers}\,\Phi_1 \sin\Phi_1\, d\Phi_1\right] \qquad (150)$$

(l) *Triangular Tangential Load.*—Moments, thrusts, and shears due to a triangular tangential load, as shown on figure 4-40(b), may be computed by the following equations:

$$M_L = \int_0^{\Phi_1} \frac{Pr^2}{a\Phi_1}\Psi\ \text{vers}\,(\Phi_1 - \Psi)\, d\Psi = \frac{Pr^2}{a\Phi_1}\left[\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right] \qquad (151)$$

$$H_L = -\int_0^{\Phi_1} \frac{Pr}{a\Phi_1}\Psi \cos(\Phi_1 - \Psi)\, d\Psi = -\frac{Pr}{a\Phi_1}\ \text{vers}\,\Phi_1 \qquad (152)$$

$$V_L = \int_0^{\Phi_1} \frac{Pr}{a\Phi_1}\Psi \sin(\Phi_1 - \Psi)\, d\Psi = \frac{Pr}{a\Phi_1}(\Phi_1 - \sin\Phi_1) \qquad (153)$$

Load constants for a triangular tangential load are:

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c I} = \frac{12\, r^3}{E_c T^3} \bullet \frac{P}{a\Phi_1}\left[\int_0^{a\Phi_1} \left(\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right) d\Phi_1\right] \qquad (154)$$

$$D_2,\ 1^{st}\ \text{term} = \int_0^s \frac{M_L x\, ds}{E_c I} = \frac{12\, r^4}{E_c T^3} \bullet \frac{P}{a\Phi_1}\left[\int_0^{a\Phi_1} \left(\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right) \sin\Phi\, d\Phi_1\right] \qquad (155)$$

$$D_2,\ 2^{nd}\ \text{term} = \int_0^s \frac{H_L \sin\Phi\, ds}{E_c A} + 3\int_0^s \frac{V_L \cos\Phi\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} \bullet \frac{P}{a\Phi_1}\left[-\int_0^{a\Phi_1} \text{vers}\,\Phi_1 \sin\Phi\, d\Phi_1\right.$$

$$\left. + 3\int_0^{a\Phi_1} (\Phi_1 - \sin\Phi_1)\cos\Phi\, d\Phi_1\right] \qquad (156)$$

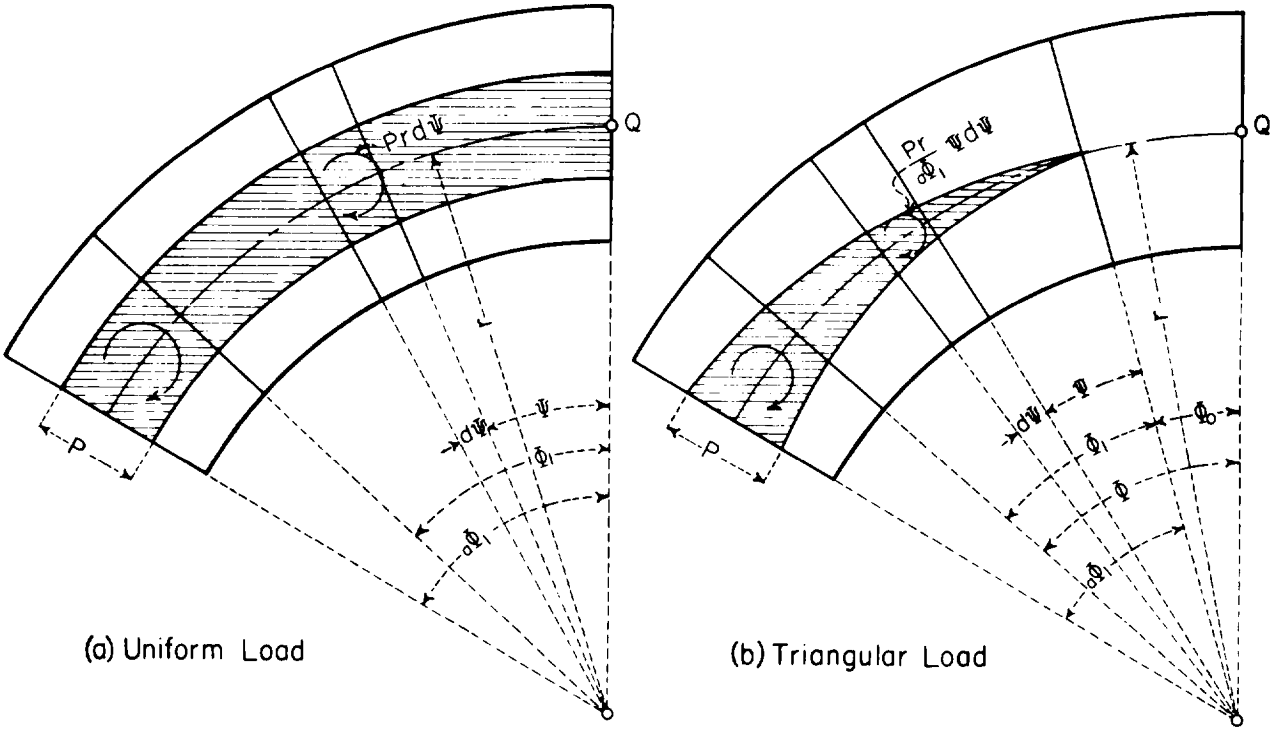
$$D_3,\ 1^{st}\ \text{term} = \int_0^s \frac{M_L y\, ds}{E_c I}$$

$$= \frac{12\, r^4}{E_c\, T^3} \bullet \frac{P}{{}_a\Phi_1} \left[ \int_0^{a^{\Phi_1}} \left( \frac{\Phi_1{}^2}{2} - \text{vers}\, \Phi_1 \right) d\Phi_1 - \int_0^{a^{\Phi_1}} \left( \frac{\Phi_1{}^2}{2} - \text{vers}\, \Phi_1 \right) \cos \Phi\, d\Phi_1 \right] \tag{157}$$

$$D_3,\ 2^{nd}\ \text{term} = - \int_0^s \frac{H_L \cos \Phi\, ds}{E_c A} + 3 \int_0^s \frac{V_L \sin \Phi\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} \bullet \frac{P}{{}_a\Phi_1} \left[ \int_0^{a^{\Phi_1}} \text{vers}\, \Phi_1 \cos \Phi\, d\Phi_1 + 3 \int_0^{a^{\Phi_1}} (\Phi_1 - \sin \Phi_1) \sin \Phi\, d\Phi_1 \right] \tag{158}$$

(m) *Uniform Twist Load.*—Twist loads are horizontal couples causing pure bending of the arches. Consequently, they do not produce thrusts or shears, and terms involving these quantities do not appear in the following formulas. The moment due to a uniform twist load applied along the centerline, as shown on figure 4-41(a), is:



(a) Uniform Load

(b) Triangular Load

*Figure 4-41.* **Twist loads on arch elements.–288-D-2981**

$$M_L = \int_0^{\Phi_1} Pr\, d\Psi = Pr\, \Phi_1 \tag{159}$$

Load constants for a uniform twist load involving this internal moment are:

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c\, I} = \frac{12\, r^2}{E_c\, T^3} P \left[\int_0^{{}_a\Phi_1} \Phi_1\, d\Phi_1\right] \tag{160}$$

$$D_2 = \int_0^s \frac{M_L x\, ds}{E_c\, I} = \frac{12\, r^3}{E_c\, T^3} P \left[\int_0^{{}_a\Phi_1} \Phi_1 \sin \Phi_1\, d\Phi_1\right] \tag{161}$$

$$D_3 = \int_0^s \frac{M_L y\, ds}{E_c\, I} = \frac{12\, r^3}{E_c\, T^3} P \left[\int_0^{{}_a\Phi_1} \Phi_1\, d\Phi_1 - \int_0^{{}_a\Phi_1} \Phi_1 \cos \Phi_1\, d\Phi_1\right] \tag{162}$$

(n) *Triangular Twist Load.*—The moment due to a triangular twist load, as shown on figure 4-41(b), is obtained by the formula,

$$M_L = \int_0^{\Phi_1} \frac{Pr}{{}_a\Phi_1} \Psi\, d\Psi = \frac{Pr}{{}_a\Phi_1} \bullet \frac{\Phi_1{}^2}{2} \tag{163}$$

and load constants are:

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c\, I} = \frac{12\, r^2}{E_c\, T^3} \bullet \frac{P}{{}_a\Phi_1} \left[\int_0^{{}_a\Phi_1} \frac{\Phi_1{}^2}{2} d\Phi_1\right] \tag{164}$$

$$D_2 = \int_0^s \frac{M_L x\, ds}{E_c\, I} = \frac{12\, r^3}{E_c\, T^3} \bullet \frac{P}{{}_a\Phi_1} \left[\int_0^{{}_a\Phi_1} \frac{\Phi_1{}^2}{2} \sin \Phi\, d\Phi_1\right] \tag{165}$$

$$D_3 = \int_0^s \frac{M_L y\, ds}{E_c\, I} = \frac{12\, r^3}{E_c\, T^3} \bullet \frac{P}{{}_a\Phi_1} \left[\int_0^{{}_a\Phi_1} \frac{\Phi_1{}^2}{2} d\Phi_1 - \int_0^{{}_a\Phi_1} \frac{\Phi_1{}^2}{2} \cos \Phi\, d\Phi_1\right] \tag{166}$$

(o) *Temperature Load.*—Load constants for a uniform temperature change ($t$) throughout an entire arch are:

$$D_1 = 0 \tag{167}$$

$$D_2 = -\int_0^s ct \sin \Phi_1\, ds = -\, ctr \int_0^{{}_a\Phi_1} \sin \Phi_1\, d\Phi_1$$

$$= -\, ctr \text{ vers }{}_a\Phi_1 = -\, ct\, y_a \tag{168}$$

$$D_3 = \int_0^s ct \cos \Phi_1 \, ds = ctr \int_0^{{}_a\Phi_1} \cos \Phi_1 \, d\Phi_1 = ctr \sin {}_a\Phi_1 = ct\, x_a \tag{169}$$

Load constants for a linear temperature variation ($t_1$) from –½° F. at the upstream face to +½° F. at the downstream face for the entire arch are:

$$D_1 = \sum \frac{ct_1 r}{T} \int_{\Phi_a}^{\Phi} d\Phi = +\sum \frac{ct_1 r}{T} [\Phi]_{\Phi_a}^{\Phi}$$

$$= -\sum \frac{ct_1 r}{T} (\Phi_a - \Phi) = -\sum \frac{ct_1 r}{T} \Delta A_1 \tag{170}$$

$$D_2 = \sum \frac{ct_1 r^2}{T} \int_{\Phi_a}^{\Phi} \sin \Phi \, d\Phi = \sum \frac{ct_1 r^2}{T} [- \cos \Phi]_{\Phi_a}^{\Phi}$$

$$= -\sum \frac{ct_1 r^2}{T} [(1 - \cos \Phi_a) - (1 - \cos \Phi)] = -\sum \frac{ct_1 r^2}{T} (\text{vers}\, \Phi_a - \text{vers}\, \Phi)$$

$$= -\sum \frac{ct_1 r^2}{T} \Delta C_1 \tag{171}$$

$$D_3 = \sum \frac{ct_1 r}{T} \int_{\Phi_a}^{\Phi} y \, d\Phi$$

$$= \sum \frac{ct_1 r^2}{T} \int_{\Phi_a}^{\Phi} d\Phi - \sum \frac{ct_1 r^2}{T} \int_{\Phi_a}^{\Phi} \cos \Phi \, d\Phi$$

$$= \sum \frac{ct_1 r^2}{T} [\Phi - \sin \Phi]_{\Phi_a}^{\Phi}$$

$$= -\sum \frac{ct_1 r^2}{T} \Delta(\Phi - \sin \Phi)$$

$$= -\sum \frac{ct_1 r^2}{T} \Delta B_1 \tag{172}$$

In some situations the temperatures applied to an arch vary along the length of the arch, and the linear variation from upstream face to downstream face may also be different for various segments of the arch. For such cases, the arch can be divided into finite lengths, each length being defined as a voussoir. Within each voussoir the linear variation from the upstream to the downstream face is assumed constant and the uniform temperature change is also constant. The variation throughout the entire arch, however, can be approximated in a stepwise pattern voussoir by voussoir.

Load constants for a temperature variation by voussoirs along the length of the arch, including both variations in temperature changes from face to face and uniform temperature across the section, are as given below. The arch is divided into four voussoirs, *0* (crown) to *1*, *1* to *2*, *2* to *3*, and *3* to *4* (abutment).

$t_2$ = temperature difference between faces = (−½° F. to +½° F.) (number of degrees variation) for each voussoir.

$t_3$ = uniform temperature load across voussoir = (+1° F.) (number of degrees of temperature change from grouting temperature).

Load constants for each voussoir may be computed for $t_2$ using equations (194), (195), and (196), and for $t_3$ using equations (197), (198), and (199). The following expressions are examples of how the load constants for uniform and linear temperature changes are combined for voussoirs (*4-3*) and (*3-2*). Note that the *D*-terms for voussoir (*3-2*) include the transferred *D*-term from voussoir (*4-3*).

Voussoir (*4-3*):

$$D_{1\,(4-3)} = -\sum \frac{ct_{2\,(4-3)}\,r_{(4-3)}}{T_{(4-3)}} \Delta A_{1\,(4-3)} + 0 \tag{173}$$

$$D_{2\,(4-3)} = -\sum \frac{ct_{2\,(4-3)}\,r^2_{\,(4-3)}}{T_{(4-3)}} \Delta C_{1\,(4-3)} - ct_{3\,(4-3)}\,y_{(4-3)} \tag{174}$$

$$D_{3\,(4-3)} = -\sum \frac{ct_{2\,(4-3)}\,r^2_{\,(4-3)}}{T_{(4-3)}} \Delta B_{1\,(4-3)} + ct_{3\,(4-3)}\,x_{(4-3)} \tag{175}$$

Voussoir (*3-2*):

Use transfer equations (206), (207), and (208).

$$D_{1\,(3-2)} = -\sum \frac{ct_{2\,(4-3)}\,r_{(4-3)}}{T_{(4-3)}} \Delta A_{1\,(4-3)} -\sum \frac{ct_{2\,(3-2)}\,r_{(3-2)}}{T_{(3-2)}} \Delta A_{1\,(3-2)} \tag{176}$$

$$\begin{aligned} D_{2\,(3-2)} = {} & \left[ -\sum \frac{ct_{2\,(4-3)}\,r_{(4-3)}}{T_{(4-3)}} \Delta A_{1\,(4-3)} \right] x_{(4-3)} \\ & + \left[ -\sum \frac{ct_{2\,(4-3)}\,r^2_{\,(4-3)}}{T_{(4-3)}} \Delta C_{1\,(4-3)} - ct_{3\,(4-3)}\,y_{(4-3)} \right] \cos \Phi_{(4-3)} \\ & - \left[ -\sum \frac{ct_{2\,(4-3)}\,r^2_{\,(4-3)}}{T_{(4-3)}} \Delta B_{1\,(4-3)} + ct_{3\,(4-3)}\,x_{(4-3)} \right] \sin \Phi_{(4-3)} \\ & + \left[ -\sum \frac{ct_{2\,(3-2)}\,r^2_{\,(3-2)}}{T_{(3-2)}} \Delta C_{1\,(3-2)} - ct_{3\,(3-2)}\,y_{(3-2)} \right] \end{aligned} \tag{177}$$

$$D_{3_{(3-2)}} = \left[-\sum \frac{ct_{2_{(4-3)}}\, r_{(4-3)}}{T_{(4-3)}}\, \Delta A_{1_{(4-3)}}\right] y_{(4-3)}$$

$$+ \left[-\sum \frac{ct_{2_{(4-3)}}\, r^{2}_{(4-3)}}{T_{(4-3)}}\, \Delta C_{1_{(4-3)}} - ct_{3_{(4-3)}}\, y_{(4-3)}\right] \sin \Phi_{(4-3)}$$

$$+ \left[-\sum \frac{ct_{2_{(4-3)}}\, r^{2}_{(4-3)}}{T_{(4-3)}}\, \Delta B_{1_{(4-3)}} + ct_{3_{(4-3)}}\, x_{(4-3)}\right] \cos \Phi_{(4-3)}$$

$$+ \left[-\sum \frac{ct_{2_{(3-2)}}\, r^{2}_{(3-2)}}{T_{(3-2)}}\, \Delta B_{1_{(3-2)}} + ct_{3_{(3-2)}}\, x_{(3-2)}\right] \tag{178}$$

(p) *Summary of Load Formulas.*—Table 4-5 gives a summary of formulas for moments, thrusts, shears, and $D$-terms for all unit loads on a circular arch. Integrals involved in the formulas are given in the left part of the table and items for which they are used comprise the remainder of the table. Columns designated "Item" give the formulas in their original form, as in equations (124), (125), and (126). Columns headed "Designation" give the convenient $D$-term notation used for the load constants. The remaining "Multiplier" columns give quantities by which the integrals must be multiplied to give desired arch movements. For example, under the headings "Radial Loads" and "Triangular,"

$$D_1 = \int_0^s \frac{M_L\, ds}{EI} = \frac{PR_E\, r^2}{\Phi_1\, EI} \int_0^{\Phi_1} (\Phi_1 - \sin \Phi_1)\, d\Phi_1$$

which agrees with equation (138) previously derived. The term $E$ in the formulas is the modulus of elasticity of concrete, $E_c$, with the subscript $c$ omitted for convenience.

The evaluation of integrals in the column "Trigonometric Part" is discussed in sections H-5 through H-7 of appendix H, which also includes tabulations of numerical values of the integrals for angles between zero and 90°. Tables of numerical values for portions of $D$-terms that are functions of arch angles only are given in sections H-8 through H-10. These include 15 unit-load patterns for all angles between 10° and 90°.

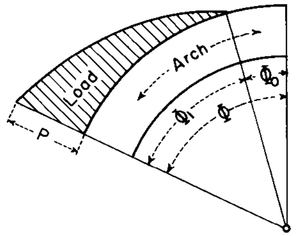
**4-35. *Circular Arch of Variable Thickness.***—(a) *General Discussion.*—Variable-thickness arches discussed herein have a constant upstream radius, and variable centerline and downstream radii, as shown on figure 4-42. General arch formulas are used in their analysis. By making certain revisions in various terms, as discussed subsequently, uniform-thickness circular-arch formulas are applicable to variable-thickness arches.

Each side of the arch is divided into voussoirs by drawing lines radial to the upstream face with each voussoir subtended by a central angle equal to one-fourth the angle from the crown to the abutment. Each voussoir is considered as a segment of a circular arch whose thickness is equal to the average thickness of the voussoir. This makes it possible to use the tabulated arch and load constants for a uniform-thickness arch.

Although this method is approximate, it is believed sufficiently accurate for a trial-load analysis. The method was checked by comparing it with an accurate analysis of an arch in which the thickness varied as the secant of the central angle, the term ($T_o$ sec $\Phi$) being integrated and substituted for $T$ in the uniform-thickness arch formulas. A relatively

Table 4-5.—*Formulas for circular arch load constants.*

| TRIGONOMETRIC PART | RADIAL LOADS | | | | | | TANGENTIAL LOADS | | | | | | TWIST LOADS | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | UNIFORM | | | TRIANGULAR | | | UNIFORM | | | TRIANGULAR | | | UNIFORM | | | TRIANGULAR | | |
| | ITEM | DESIGNATION | MULTIPLIER | ITEM | DESIGNATION | MULTIPLIER | ITEM | DESIGNATION | MULTIPLIER | ITEM | DESIGNATION | MULTIPLIER | ITEM | DESIGNATION | MULTIPLIER | ITEM | DESIGNATION | MULTIPLIER |
| $\Phi_1$ | | | | | | | | | | | | | $M_L$ | | $Pr$ | | | |
| $\sin \Phi_1$ | $V_L$ | | $PR_E$ | | | | $H_L$ | | $-Pr$ | | | | | | | | | |
| $\text{vers}\, \Phi_1$ | $H_L$ | | $PR_E$ | $V_L$ | | $\frac{PR_E}{\Phi_1}$ | $V_L$ | | $Pr$ | $H_L$ | | $-\frac{Pr}{\Phi_1}$ | | | | | | |
| | $M_L$ | | $PR_E r$ | | | | | | | | | | | | | | | |
| $\int_0^{\Phi_1} \Phi_1 d\Phi_1$ | | | | | | | | | | | | | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{Pr^2}{EI}$ | $M_L$ | | $\frac{Pr}{\Phi_1}$ |
| | | | | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Part | $\frac{Pr^3}{EI}$ | | | |
| $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} d\Phi_1$ | | | | | | | | | | | | | | | | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{Pr^2}{\Phi_1 EI}$ |
| | | | | | | | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Part | $\frac{Pr^3}{\Phi_1 EI}$ |
| $\int_0^{\Phi_1} \Phi_1 \sin \Phi \, d\Phi_1$ | | | | | | | | | | | | | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ | $\frac{Pr^3}{EI}$ | | | |
| $\int_0^{\Phi_1} \Phi_1 \cos \Phi \, d\Phi_1$ | | | | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 2nd Part | $-\frac{Pr^3}{EI}$ | | | |
| $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin \Phi \, d\Phi_1$ | | | | | | | | | | | | | | | | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ | $\frac{Pr^3}{\Phi_1 EI}$ |
| $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos \Phi \, d\Phi_1$ | | | | | | | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 2nd Part | $-\frac{Pr^3}{\Phi EI}$ |
| $\int_0^{\Phi_1} \sin \Phi_1 \sin \Phi \, d\Phi_1$ | $\int_0^s \frac{V_L \sin \Phi ds}{ET}$ | $D_3$ 2nd Term 2nd Part | $3 \frac{PR_E r}{ET}$ | | | | $\int_0^s \frac{H_L \sin \Phi ds}{ET}$ | $D_2$ 2nd Term 1st Part | $-\frac{Pr^2}{ET}$ | | | | | | | | | |
| $\int_0^{\Phi_1} \sin \Phi_1 \cos \Phi \, d\Phi_1$ | $\int_0^s \frac{V_L \cos \Phi ds}{ET}$ | $D_2$ 2nd Term 2nd Part | $3 \frac{PR_E r}{ET}$ | | | | $\int_0^s \frac{H_L \cos \Phi ds}{ET}$ | $D_3$ 2nd Term 1st Part | $\frac{Pr^2}{ET}$ | | | | | | | | | |
| $\int_0^{\Phi_1} \text{vers}\, \Phi_1 d\Phi_1$ | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{PR_E r^2}{EI}$ | $H_L$ | | $\frac{PR_E}{\Phi_1}$ | $M_L$ | | $Pr^2$ | $V_L$ | | $\frac{Pr}{\Phi_1}$ | | | | | | |
| | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 1st Part | $\frac{PR_E r^3}{EI}$ | $M_L$ | | $\frac{PR_E r}{\Phi_1}$ | | | | | | | | | | | | |
| $\int_0^{\Phi_1} \text{vers}\, \Phi_1 \sin \Phi \, d\Phi_1$ | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ 1st Term | $\frac{PR_E r^3}{EI}$ | | | | | | | | | | | | | | | |
| | $\int_0^s \frac{H_L \sin \Phi ds}{ET}$ | $D_2$ 2nd Term 1st Part | $\frac{PR_E r}{ET}$ | $\int_0^s \frac{V_L \sin \Phi ds}{ET}$ | $D_3$ 2nd Term 2nd Part | $3 \frac{PR_E r}{\Phi_1 ET}$ | $\int_0^s \frac{V_L \sin \Phi ds}{ET}$ | $D_3$ 2nd Term 2nd Part | $3 \frac{Pr^2}{ET}$ | $\int_0^s \frac{H_L \sin \Phi ds}{ET}$ | $D_2$ 2nd Term 1st Part | $-\frac{Pr^2}{\Phi_1 ET}$ | | | | | | |
| $\int_0^{\Phi_1} \text{vers}\, \Phi_1 \cos \Phi \, d\Phi_1$ | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 2nd Part | $-\frac{PR_E r^3}{EI}$ | | | | | | | | | | | | | | | |
| | $\int_0^s \frac{H_L \cos \Phi ds}{ET}$ | $D_3$ 2nd Term 1st Part | $-\frac{PR_E r}{ET}$ | $\int_0^s \frac{V_L \cos \Phi ds}{ET}$ | $D_2$ 2nd Term 2nd Part | $3 \frac{PR_E r}{\Phi_1 ET}$ | $\int_0^s \frac{V_L \cos \Phi ds}{ET}$ | $D_2$ 2nd Term 2nd Part | $3 \frac{Pr^2}{ET}$ | $\int_0^s \frac{H_L \cos \Phi ds}{ET}$ | $D_3$ 2nd Term 1st Part | $\frac{Pr^2}{\Phi_1 ET}$ | | | | | | |
| $\int_0^{\Phi_1} [\Phi_1 - \sin \Phi_1] \, d\Phi_1$ | | | | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{PR_E r^2}{\Phi_1 EI}$ | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{Pr^3}{EI}$ | $M_L$ | | $\frac{Pr^2}{\Phi_1}$ | | | | | | |
| | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 1st Part | $\frac{PR_E r^3}{\Phi_1 EI}$ | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 1st Part | $\frac{Pr^4}{EI}$ | | | | | | | | | |
| $\int_0^{\Phi_1} [\Phi_1 - \sin \Phi_1] \sin \Phi \, d\Phi_1$ | | | | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ 1st Term | $\frac{PR_E r^3}{\Phi_1 EI}$ | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ 1st Term | $\frac{Pr^4}{EI}$ | | | | | | | | | |
| | | | | $\int_0^s \frac{H_L \sin \Phi ds}{ET}$ | $D_2$ 2nd Term 1st Part | $\frac{PR_E r}{\Phi_1 ET}$ | | | | $\int_0^s \frac{V_L \sin \Phi ds}{ET}$ | $D_3$ 2nd Term 2nd Part | $3 \frac{Pr^2}{\Phi_1 ET}$ | | | | | | |
| $\int_0^{\Phi_1} [\Phi_1 - \sin \Phi_1] \cos \Phi \, d\Phi_1$ | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 2nd Part | $-\frac{PR_E r^3}{\Phi_1 EI}$ | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 2nd Part | $-\frac{Pr^4}{EI}$ | | | | | | | | | |
| | | | | $\int_0^s \frac{H_L \cos \Phi ds}{ET}$ | $D_3$ 2nd Term 1st Part | $-\frac{PR_E r}{\Phi_1 ET}$ | | | | $\int_0^s \frac{V_L \cos \Phi ds}{ET}$ | $D_2$ 2nd Term 2nd Part | $3 \frac{Pr^2}{\Phi_1 ET}$ | | | | | | |
| $\int_0^{\Phi_1} \left[\frac{\Phi_1^2}{2} - \text{vers}\, \Phi_1\right] d\Phi_1$ | | | | | | | | | | $\int_0^s \frac{M_L ds}{EI}$ | $D_1$ | $\frac{Pr^3}{\Phi_1 EI}$ | | | | | | |
| | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 1st Part | $\frac{Pr^4}{\Phi_1 EI}$ | | | | | | |
| $\int_0^{\Phi_1} \left[\frac{\Phi_1^2}{2} - \text{vers}\, \Phi_1\right] \sin \Phi \, d\Phi_1$ | | | | | | | | | | $\int_0^s \frac{M_L xds}{EI}$ | $D_2$ 1st Term | $\frac{Pr^4}{\Phi_1 EI}$ | | | | | | |
| $\int_0^{\Phi_1} \left[\frac{\Phi_1^2}{2} - \text{vers}\, \Phi_1\right] \cos \Phi \, d\Phi_1$ | | | | | | | | | | $\int_0^s \frac{M_L yds}{EI}$ | $D_3$ 1st Term 2nd Part | $-\frac{Pr^4}{\Phi_1 EI}$ | | | | | | |



**NOTATION**

$M_L$ = Moment due to external load.
$H_L$ = Thrust due to external load.
$V_L$ = Shear due to external load.
For radial load, P = lb. per sq. ft.
For tangential load, P = lb. per sq. ft.
For twist load, P = ft.-lb. per sq. ft.

**NOTE**

Formula = Trigonometric part x multiplier.

288-D-408

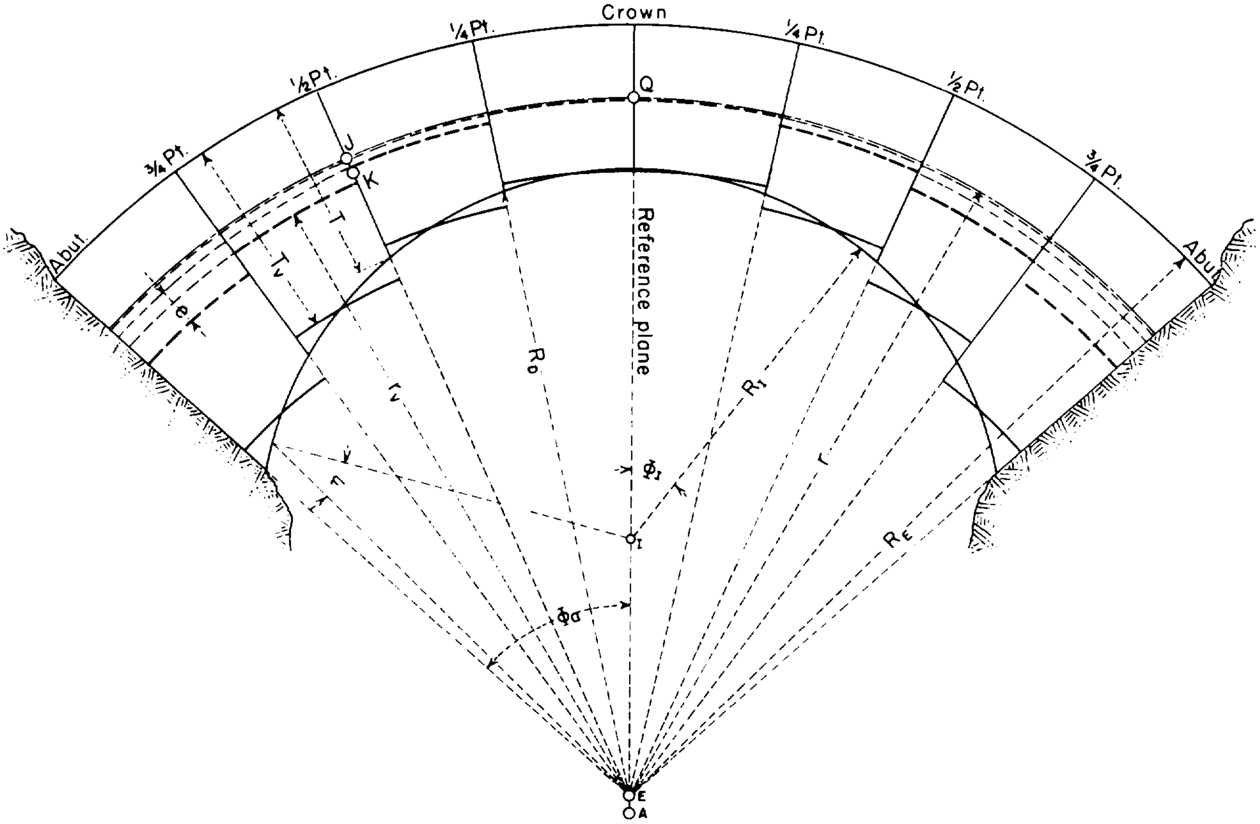


***Figure 4-42.* Layout of variable-thickness arch.—288-D-409**

close agreement, within 2 percent, was found between the results of the two analyses. The approximate method is probably within the limits of accuracy of the trial-load analysis for any variable-thickness arch likely to be encountered in dam design. A discussion of the approximate method is given in the following sections.

(b) *Notations.*—Notations needed to discuss a variable-thickness arch, in addition to those given at the beginning of this chapter (sec. 4-2), are given below:

$r_p$ = distance to arch centerline at any quarter-point = $R_E - T/2$.

$T_v$ = average thickness of voussoir, radial to upstream face.

$r_v$ = radius to centerline of voussoir = $R_E - T_v/2$.

$I_v$ = moment of inertia about centerline of voussoir = $T_v^3/12$.

$e$ = eccentricity of voussoir centerline from a circular arc drawn through the midpoint of the arch thickness at a quarter-point = $r_p - r_v$.

$e'$ = eccentricity of voussoir centerline with reference to the crown centerline arc; this is the eccentricity of the applied tangential thrust with reference to the voussoir centerline = $r_o - r_v$.

$e_p$ = eccentricity of arch points (midpoints of arch thicknesses at quarter-points) with reference to the crown centerline arc = $r_o - r_p$.

(c) *Arch Constants.*—The method of computing arch constants is apparent from a brief examination of equations (118) through

(123). Radius $r$ and thickness $T$ appear therein as constants. In the case of the variable-thickness arch, these quantities are functions of $\Phi$ and must be included within the integral sign. When the variable-thickness arch is replaced by the approximation shown on figure 4-42, integrals of equations (118) through (123) are replaced by summations of successive integrals. Constants of integration due to abrupt changes in $r$ and $T$ appear and must be included in the computations. These integrals and integration constants may be written in terms of uniform-thickness arch constants, a procedure which is followed in equations (179) through (184). The derivation of these equations from a physical standpoint is discussed below. In this discussion, constants of integration appear in terms of eccentricity of the arch centerline.

The application of arch constants for a uniform-thickness arch to the analysis of a variable-thickness arch requires the determination of the effects of movements of an assumed uniform-thickness voussoir on all arch points between it and the crown of the arch. The contribution of a voussoir to an arch constant at a point is the product of the multiplier for that voussoir, in terms of $r_v$, $T_v$, and $I_v$, and the increment of the uniform-thickness tabular values for the angles from the point considered to the limits of the voussoir. It should be kept in mind that the tables of constants contain values of the integrals only, the multipliers being computed separately. The tabular values are commonly referred to as prime (′) values. In transferring voussoir movements to a distant arch point, the fact that the centerlines of the voussoir are eccentric with respect to arch points makes it necessary to include terms involving eccentricities in the constants. For example, consider the contribution of the voussoir between the *3/4* and *1/2* points to the $B_1$ terms at the crown. By definition, $B_1$ is the angular movement at a point due to a unit thrust at the point. Applied to this case, the angular movement at $Q$ is desired for a unit thrust at $Q$. The thrust at $Q$ has an eccentricity, $e$, with respect to the voussoir considered. This results in a moment, $e$, which causes an additional angular movement at $Q$ of $\left[\frac{r_v}{E_c\, I_v}\right] \Delta A'_1 e$. Then the contribution to $B_1$, of the *3/4*-point to *1/2*-point voussoir is:

$$B_1, 3/4\ Pt.\ to\ 1/2\ Pt., = \frac{r_v^2}{E_c\, I_v}(B'_1, 3/4\ Pt. - B'_1, 1/2\ Pt.)$$

$$+\frac{r_v}{E_c\, I_v}(A'_1, 3/4\ Pt. - A'_1, 1/2\ Pt.)e$$

The contributions from other voussoirs are determined similarly and $B_1$ is computed as shown in equation (180) below. Other terms involving $e$ are determined in a similar manner.

The following equations for arch constants are for any point and represent the summation of the various voussoir contributions at the point considered:

$$A_1 = \sum \frac{r_v}{E_c\, I_v} \Delta A'_1 \qquad (179)$$

$$B_1 = \sum \frac{r_v^2}{E_c\, I_v} \Delta B'_1 + \sum \frac{r_v}{E_c\, I_v} \Delta A'_1 e \qquad (180)$$

$$C_1 = \sum \frac{r_v^2}{E_c\, I_v} \Delta C'_1 \qquad (181)$$

$$B_2 = \sum \frac{r_v^3}{E_c I_v} \Delta B_2' \text{ 1}^{st} \text{ term} + \sum \frac{r_v}{E_c T_v} \Delta B_2' \text{ 2}^{nd} \text{ term} + \sum \frac{r_v^2}{E_c I_v} \Delta C_1' e \qquad (182)$$

$$C_2 = \sum \frac{r_v^3}{E_c I_v} \Delta C_2' \text{ 1}^{st} \text{ term} + \sum \frac{r_v}{E_c T_v} \Delta C_2' \text{ 2}^{nd} \text{ term} \qquad (183)$$

$$B_3 = \sum \frac{r_v^3}{E_c I_v} \Delta B_3' \text{ 1}^{st} \text{ term} + \sum \frac{r_v}{E_c T_v} \Delta B_3' \text{ 2}^{nd} \text{ term}$$

$$+ \sum \frac{r_v}{E_c I_v} \Delta A_1' e^2 + 2 \sum \frac{r_v^2}{E_c I_v} \Delta B_1' e \qquad (184)$$

(d) *Load Constants.*–Load constants are obtained in much the same manner as arch constants. The contribution of a voussoir to a load constant at a point is the product of the multiplier for that voussoir, in terms of $R_E$, $r_v$, $T_v$, and $I_v$, and the increment of the uniform-thickness tabular values for the angles from the point considered to the limits of the voussoir. Since uniform-thickness tabular values include the effect of the load from the point to the rock abutment, fractional portions of tabular values must be used to obtain correct increments.

Equations for load constants in the following sections are for any point and represent the summation of the various voussoir contributions at the point. Separate formulas are given for radial, tangential, and twist loads because each type has different multipliers and different points of application.

Radial loads are applied normally at the upstream face. Equations for their load constants are:

$$D_1 = \sum \frac{r_v^2 R_E}{E_c I_v} \Delta D_1' \qquad (185)$$

$$D_2 = \sum \frac{r_v^3 R_E}{E_c I_v} \Delta D_2' \text{ 1}^{st} \text{ term} + \sum \frac{r_v R_E}{E_c T_v} \Delta D_2' \text{ 2}^{nd} \text{ term} \qquad (186)$$

$$D_3 = \sum \frac{r_v^3 R_E}{E_c I_v} \Delta D_3' \text{ 1}^{st} \text{ term} + \sum \frac{r_v R_E}{E_c T_v} \Delta D_3' \text{ 2}^{nd} \text{ term}$$

$$+ \sum \frac{r_v^2 R_E}{E_c I_v} \Delta D_1' e \qquad (187)$$

Tangential loads act along the arch centerline, theoretically, but for the purpose of simplification in this method of analyzing a variable-thickness arch, they are assumed to be applied along the arc through the crown centerline. Consequently, the eccentricity of this load application must be taken into account in computing $D$-terms. The moment of a tangential load at an arch point, as $K$ on figure 4-42, is $H_L e'$ greater than the moment at a point $J$ which is on the arc through the crown centerline. Terms involving this additional moment, $H_L e'$, are included in the following equations for tangential load constants:

$$D_1 = \sum \frac{r_o r_v^2}{E_c I_v} \Delta D_1' + \sum - (H_L e') \frac{r_v}{E_c I_v} \Delta A_1' \qquad (188)$$

$$D_2 = \sum \frac{r_o\, r_v^{\,3}}{E_c\, I_v}\, \Delta D_2' \ 1^{st}\ \text{term} + \sum \frac{r_o\, r_v}{E_c\, T_v} \Delta D_2' \ 2^{nd}\ \text{term}$$

$$+ \sum - (H_L\, e') \frac{r_v^{\,2}}{E_c\, I_v} \Delta C_1' \tag{189}$$

$$D_3 = \sum \frac{r_o\, r_v^{\,3}}{E_c\, I_v}\, \Delta D_3' \ 1^{st}\ \text{term} + \sum \frac{r_o\, r_v}{E_c\, T_v} \Delta D_3' \ 2^{nd}\ \text{term}$$

$$+ \sum - (H_L\, e') \frac{r_v}{E_c\, I_v} \Delta A_1'\, e + \sum - (H_L\, e') \frac{r_v^{\,2}}{E_c\, I_v}\, \Delta B_1'$$

$$+ \sum \frac{r_o\, r_v^{\,2}}{E_c\, I_v}\, \Delta D_1'\, e \tag{190}$$

Twist loads act along the centerline of the arch, theoretically, but for the purposes of evaluating *D*-terms, they are assumed to be applied along the arc through the midpoint of the crown. Application of twist load to the arc through the crown centerline produces an eccentricity with reference to other arch points and causes an additional tangential movement. This factor is included in the equation for $D_3$. Equations for total *D*-terms for twist loads are:

$$D_1 = \sum \frac{r_o\, r_v}{E_c\, I_v} \Delta D_1' \tag{191}$$

$$D_2 = \sum \frac{r_o\, r_v^{\,2}}{E_c\, I_v} \Delta D_2' \tag{192}$$

$$D_3 = \sum \frac{r_o\, r_v^{\,2}}{E_c\, I_v} \Delta D_3' + \sum \frac{r_o\, r_v}{E_c\, I_v} \Delta D_1'\, e \tag{193}$$

The equations below present the load constants for use with linear and uniform temperature changes in an arch. The dimensions used in the equations should correspond to those for the voussoir, and in the following six general equations are denoted with the subscript *v*.

Load constants for a linear temperature variation in a voussoir from −½° F. at the upstream face to +½° F. at the downstream face are:

$$D_1 = -\sum \frac{c t_{1\,v} r_v}{T_v} (\Phi_v) = -\sum \frac{c t_{1\,v} r_v}{T_v}\, \Delta A_{1\,v} \tag{194}$$

$$D_2 = -\sum \frac{c t_{1\,v} r_v^{\,2}}{T_v} \text{ vers } \Phi_v = -\sum \frac{c t_{1\,v} r_v^{\,2}}{T_v} \Delta C_{1\,v} \tag{195}$$

$$D_3 = -\sum \frac{c t_{1\,v} r_v^{\,2}}{T_v} \Delta B_{1\,v} + D_{1\,v} e \tag{196}$$

Load constants for a uniform temperature change in a voussoir are:

$$D_1 = 0 \tag{197}$$

$$D_2 = -ct_{2v}\, r_v \text{ vers } \Phi_v = -ct_{2v}\, y_v \tag{198}$$

$$D_3 = ct_{2v}\, r_v \sin \Phi_v = ct_{2v}\, x_v \tag{199}$$

The following expressions are examples of how the load constants for uniform and linear temperature change are combined for voussoirs (*4-3*) and (*3-2*). Note that the *D*-terms for voussoir (*3-2*) include the transferred *D*-term from voussoir (*4-3*).

Voussoir (*4-3*):

$$D_{1(4-3)} = -\sum \frac{ct_{1(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1(4-3)} + 0 \tag{200}$$

$$D_{2(4-3)} = -\sum \frac{ct_{1(4-3)}\, r^2_{(4-3)}}{T_{(4-3)}} \Delta C_{1(4-3)} - ct_{2(4-3)}\, y_{(4-3)} \tag{201}$$

$$\begin{aligned} D_{3(4-3)} = {} & -\sum \frac{ct_{1(4-3)}\, r^2_{(4-3)}}{T_{(4-3)}} \Delta B_{1(4-3)} - \sum \frac{ct_{1(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1(4-3)} \bullet e \\ & + ct_{2(4-3)}\, x_{(4-3)} \end{aligned} \tag{202}$$

Voussoir (*3-2*):

Use transfer equations (206), (207), and (208).

$$D_{1(3-2)} = -\sum \frac{ct_{1(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1(4-3)} - \sum \frac{ct_{1(3-2)}\, r_{(3-2)}}{T_{(3-2)}} \Delta A_{1(3-2)} \tag{203}$$

$$\begin{aligned} D_{2(3-2)} = {} & \left[ -\sum \frac{ct_{1(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1(4-3)} \right] x_{(4-3)} \\ & + \left[ -\sum \frac{ct_{1(4-3)}\, r^2_{(4-3)}}{T_{(4-3)}} \Delta C_{1(4-3)} - ct_{2(4-3)}\, y_{(4-3)} \right] \cos \Phi_{(4-3)} \\ & - \Bigg[ -\sum \frac{ct_{1(4-3)}\, r^2_{(4-3)}}{T_{(4-3)}} \Delta B_{1(4-3)} - \sum \frac{ct_{1(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1(4-3)} \bullet e \\ & + ct_{2(4-3)}\, x_{(4-3)} \Bigg] \sin \Phi_{(4-3)} \\ & + \left[ -\sum \frac{ct_{1(3-2)}\, r^2_{(3-2)}}{T_{(3-2)}} \Delta C_{1(3-2)} - ct_{2(3-2)}\, y_{(3-2)} \right] \end{aligned} \tag{204}$$

$$
\begin{aligned}
D_{3\,(3-2)} = {} & \left[ -\sum \frac{ct_{1\,(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1\,(4-3)} \right] y_{(4-3)} \\
& + \left[ -\sum \frac{ct_{1\,(4-3)}\, r^2_{\ (4-3)}}{T_{(4-3)}} \Delta C_{1\,(4-3)} - ct_{2\,(4-3)}\, y_{(4-3)} \right] \sin \Phi_{(4-3)} \\
& + \left[ -\sum \frac{ct_{1\,(4-3)}\, r^2_{\ (4-3)}}{T_{(4-3)}} \Delta B_{1\,(4-3)} - \sum \frac{ct_{1\,(4-3)}\, r_{(4-3)}}{T_{(4-3)}} \Delta A_{1\,(4-3)} \cdot e \right. \\
& \left. + ct_{2\,(4-3)}\, x_{(4-3)} \right] \cos \Phi_{(4-3)} + \left[ -\sum \frac{ct_{1\,(3-2)}\, r^2_{\ (3-2)}}{T_{(3-2)}} \Delta B_{1\,(3-2)} \right. \\
& \left. - \sum \frac{ct_{1\,(3-2)}\, r_{(3-2)}}{T_{(3-2)}} \Delta A_{1\,(3-2)} \cdot e + ct_{2\,(3-2)}\, x_{(3-2)} \right] \qquad (205)
\end{aligned}
$$

(e) *Application of Arch Formulas.*—Besides the arch and load constants, revisions of several other quantities in circular arch formulas are necessary to make them applicable to variable-thickness arches. The revisions occur in formulas for moments, thrusts, and shears due to unit loads, and in formulas for coordinates of arch points.

Formulas for $M_L$, $H_L$, and $V_L$ are the same as for uniform-thickness circular arches with the following revisions. The radius $r$, in radial load formulas is the radius to the arch point, $R_E - T/2$; and the radius $r$, in twist and tangential load for formulas is the radius to the crown centerline arc. Since the $H_L$ of the tangential load is applied along the crown centerline arc, the quantity $H_L e_p$ must be combined with the $M_L$ quantity from the load formula to give the correct value for $M_L$ at an arch point. The complete formula for $M_L$ at an arch point is $M_L$, from the load formula, minus $H_L e_p$.

The coordinates $x$ and $y$ are changed from $x = r \sin \Phi$ and $y = r \text{ vers } \Phi$ to $x = r_p \sin \Phi$ and $y = r_o - r_p \cos \Phi$. These are used in the formulas for solving crown forces. In equations (112) through (117) for arch deflections at any arch point, coordinates $x_a$ and $y_a$ for the abutment, with the arch points as origins, are $x_a = r_a \sin \Phi_a$ and $y_a = r_p - r_a \cos \Phi_a$.

## 4-36. *Arch With Intrados Fillets.*—

(a) *General Discussion.*—If arch abutment stresses are much greater than crown stresses, they can be reduced by providing fillets at the ends of the intrados curves. The fillets reduce abutment stresses because the arch thickness is increased and the resultant thrust is closer to the centerline of the arch. If it is necessary to extend the fillet beyond the *1/2* point of the arch, the arch is more conveniently designed as a variable-thickness arch. Short-radius fillets should be avoided because of indefinite excavation limits; if excavation is extended further into the canyon walls than anticipated, short-radius fillets may not intersect the rock abutments.

An arch with a fillet section, as shown on figure 4-43, is a special case of a variable-thickness arch. In the analysis of such an arch, the left and right parts are each divided into two sections, a fillet section and a uniform-thickness section. By making the fillet section and the uniform section subtend central angles of whole degrees, the uniform-thickness arch data with certain revisions can be used for the analysis.

Arch constants are obtained in the same manner as for a variable-thickness arch. Load constants are determined independently for the fillet section and the uniform-thickness section.

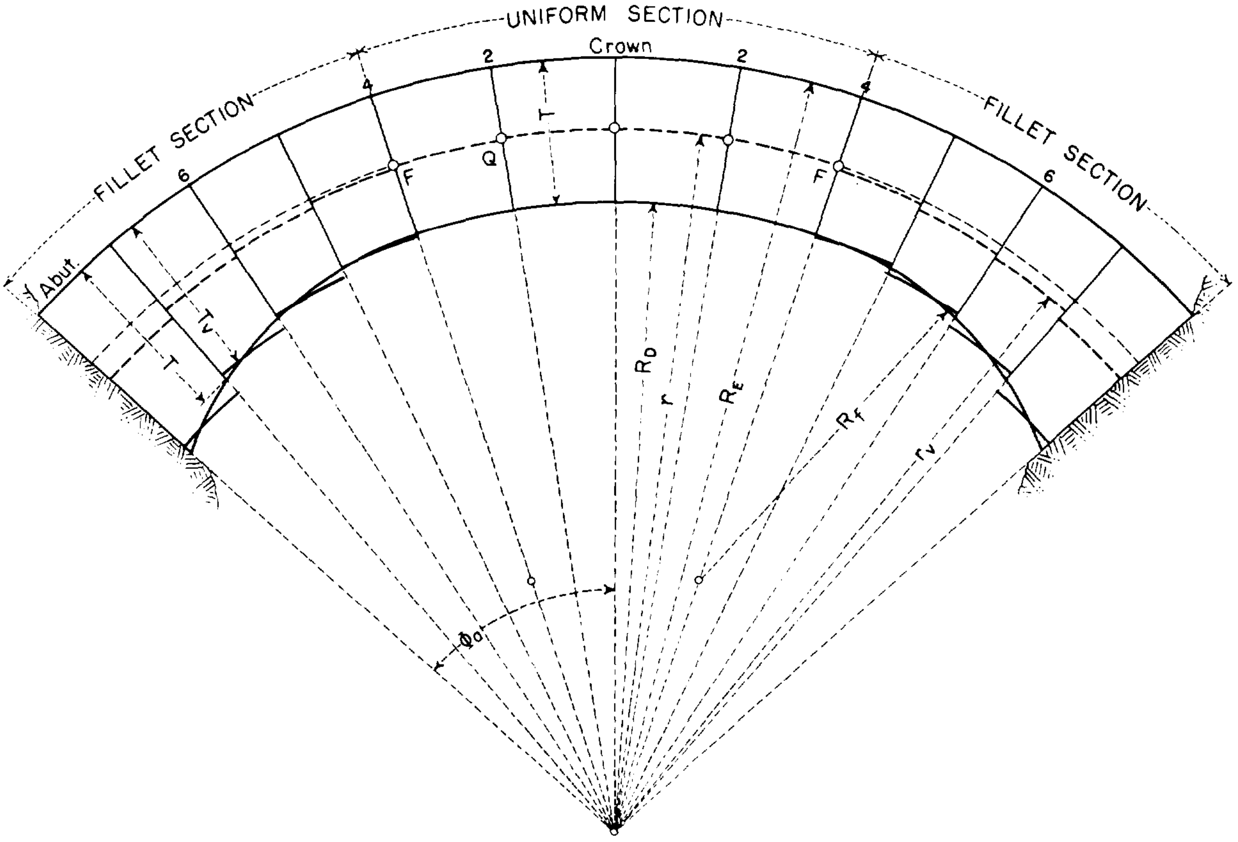


*Figure 4-43.* **An arch with fillets at downstream face.–288-D-411**

The transfer of load effects at the fillet crown, point $F$ in figure 4-43, to points in the uniform-thickness section introduces the influence of the fillet on that section.

In some cases the central section may be of variable thickness because of change in $R_E$ or $R_D$. In these instances, the arch constants as well as the load constants are determined independently for the fillet section and the variable-thickness section. Therefore, arch and load constants are transferred from the crown of the fillet section to arch points in the central variable-thickness section by means of general transfer equations, numbered (212) to (220), inclusive.

(b) *Notations.*—The notation for uniform-thickness and variable-thickness arches applies to arches with fillets, with the following additions:

$F$ = crown of fillet section.
$Q$ = arch point in uniform-thickness section.
$r$ = radius to centerline of uniform-thickness section.
$R_D$ = radius to downstream face of uniform-thickness section.
$R_f$ = radius to fillet at downstream face.

(c) *Arch Constants.*—For the case of a uniform-thickness central section as shown on figure 4-43, arch constants can be determined by equations (179) through (184), provided that the fillet section is divided into four voussoirs and the central uniform section is included as an additional voussoir, giving a total of five voussoirs.

In the case of a variable-thickness central section, it is more convenient to calculate arch constants independently for the fillet section and the variable-thickness central section. The central section can be divided into two voussoirs and the fillet section into four voussoirs. Then arch constants are calculated independently for each section and transferred from the crown of the fillet section, $F$, to arch points in the center section by means of general transfer equations, equations (212) through (217) using the notation of (′) values at $F$ and (″) values at $Q$.

(d) *Load Constants.*—Load constants at a point in the central uniform-thickness section, as $Q$ on figure 4-43, or at a point in a central variable-thickness section, as described in the preceding section, are the sum of three quantities or effects: first, the effect of load acting on the central section; second, the effect of forces and moments acting on the fillet section due to load acting on the central section; and third, the effect of applied load between $F$ and the abutment acting on the fillet section. The first quantity is evaluated in the same way as for uniform-thickness arch $D$-terms. The second quantity is obtained by transferring the deflections at $F$, products of unit-load moment, thrusts, and shears, and the proper arch constants, to $Q$. The third quantity is determined by transferring to $Q$ the $D$-terms at $F$ due to the portion of the load on the fillet section.

In the following equations for $D$-terms at $Q$ in the central section, (′) values are values at $F$ due to load on the fillet section; (″) values are values at $Q$ due to load on the central section: $M_L$, $H_L$, and $V_L$ refer to $F$; $x$ and $y$ are coordinates of $F$ with $Q$ as the origin; and $\Phi$ is the angle from $F$ to $Q$.

$$D_1 = [A'_1 M_L - B'_1 H_L + C'_1 V_L] + D'_1 + D''_1 \qquad (206)$$

$$\begin{aligned} D_2 = {} & [(A'_1 x - B'_1 \sin\Phi + C'_1 \cos\Phi) M_L \\ & - (B'_1 x - B'_3 \sin\Phi + B'_2 \cos\Phi) H_L \\ & + (C'_1 x - B'_2 \sin\Phi + C'_2 \cos\Phi) V_L ] \\ & + [D'_1 x + D'_2 \cos\Phi - D'_3 \sin\Phi] \\ & + D''_2 \end{aligned} \qquad (207)$$

$$\begin{aligned} D_3 = {} & [(A'_1 y + B'_1 \cos\Phi + C'_1 \sin\Phi) M_L \\ & - (B'_1 y + B'_3 \cos\Phi + B'_2 \sin\Phi) H_L \\ & + (C'_1 y + B'_2 \cos\Phi + C'_2 \sin\Phi) V_L ] \\ & + [D'_1 y + D'_2 \sin\Phi + D'_3 \cos\Phi] \\ & + D''_3 \end{aligned} \qquad (208)$$

Quantities $M_L$, $H_L$, and $V_L$ in these equations may be evaluated by equations for uniform-thickness arches if $Q$ is the crown point. If $Q$ is between the fillet section and the crown of the arch, the effect of the load between $Q$ and the crown must be eliminated. This has been done in the following equations which apply to radial, tangential, and twist loads. The subscripts $F$ and $Q$ preceding the terms refer to values at points $F$ and $Q$, respectively, for the entire load between $F$ or $Q$ and the crown of the arch.

$$M_L \text{ at } F = {}_F M_L - {}_Q M_L + {}_Q H_L\, y - {}_Q V_L\, x \qquad (209)$$

$$H_L \text{ at } F = {}_F H_L - {}_Q H_L \cos\Phi - {}_Q V_L \sin\Phi \qquad (210)$$

$$V_L \text{ at } F = {}_F V_L + {}_Q H_L \sin\Phi - {}_Q V_L \cos\Phi \qquad (211)$$

(e) *Application of Formulas.*—Other revisions, in addition to those given for arch and load constants, occur in formulas for moments, thrusts, and shears due to applied loads, and in formulas for coordinates of arch points. These revisions are the same as for a variable-thickness arch, given in section A-35(e).

**4-37. *Three-Centered, Variable-Thickness With Nonradial Abutments, Noncircular, and Cracked Arches.***—(a) *Three-Centered Arch.*—A three-centered arch of uniform thickness, as shown on figure 4-44, is used to make the line of thrust correspond more nearly with the

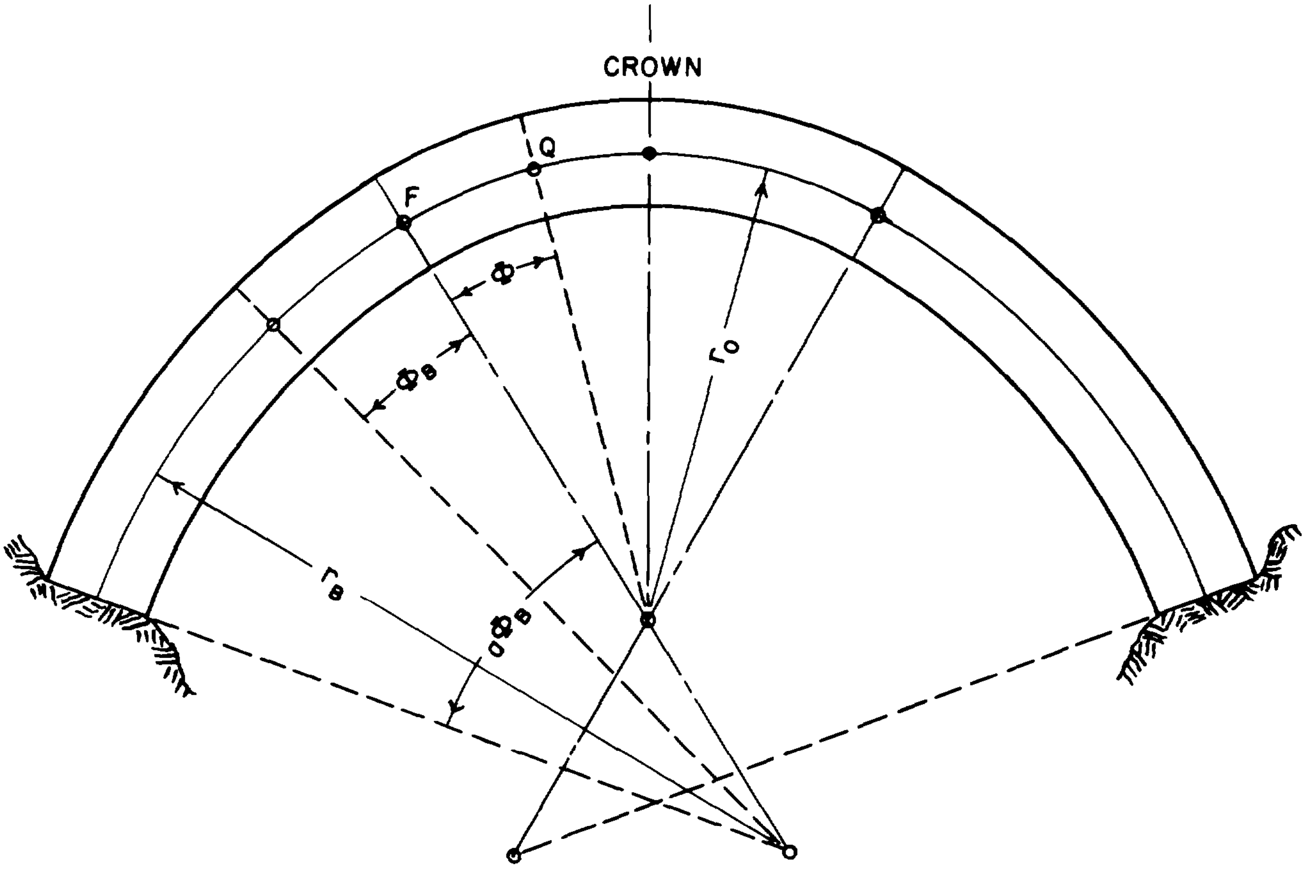


*Figure 4-44.* **A three-centered arch of uniform thickness.–288-D-2982**

centerline. This reduces bending moments at the crown and abutments, and thereby tends to eliminate the high tensile stresses that would exist at the abutments in a single-centered arch.

Formulas for a single-centered arch can be used for this case, but arch and load constants must be transferred from the crown of the abutment circular section, point $F$, to arch points in the central circular section. In the following general transfer equations for constants at a point $Q$ in the central circular section, (′) values are values at $F$ due to load on the abutment circular sections; (″) values are values at $Q$ due to load on the central section; $M_L$, $H_L$, and $V_L$ refer to point $F$; $x$ and $y$ are coordinates of point $F$ with $Q$ as the origin; and $\Phi$ is the angle from $F$ to $Q$.

$$A_1 = A_1' + A_1'' \tag{212}$$

$$B_1 = A_1' y + B_1' \cos\Phi + C_1' \sin\Phi + B_1'' \tag{213}$$

$$C_1 = A_1' x - B_1' \sin\Phi + C_1' \cos\Phi + C_1'' \tag{214}$$

$$\begin{aligned} B_2 = {} & (A_1' y + B_1' \cos\Phi + C_1' \sin\Phi)x \\ & + (C_1' \cos\Phi - B_1' \sin\Phi)y \\ & + B_2' (\cos^2\Phi - \sin^2\Phi) \\ & + (C_2' - B_3') \sin\Phi \cos\Phi + B_2'' \end{aligned} \tag{215}$$

$$\begin{aligned} B_3 = {} & (A_1' y + 2B_1' \cos\Phi + 2C_1' \sin\Phi)y \\ & + 2 B_2' \sin\Phi \cos\Phi \\ & + B_3' \cos^2\Phi + C_2' \sin^2\Phi + B_3'' \end{aligned} \tag{216}$$

$$
\begin{aligned}
C_2 = {} & (A'_1 x - 2B'_1 \sin \Phi + 2\, C'_1 \cos \Phi)x \\
& - 2\, B'_2 \sin \Phi \cos \Phi \\
& + B'_3 \sin^2 \Phi + C'_2 \cos^2 \Phi + C''_2
\end{aligned} \tag{217}
$$

$$
D_1 = A'_1 M_L - B'_1 H_L + C'_1 V_L + D'_1 + D''_1 \tag{218}
$$

$$
\begin{aligned}
D_2 = {} & (A'_1 x - B'_1 \sin \Phi + C'_1 \cos \Phi) M_L \\
& - (B'_1 x - B'_3 \sin \Phi + B'_2 \cos \Phi) H_L \\
& + (C'_1 x - B'_2 \sin \Phi + C'_2 \cos \Phi) V_L \\
& + (D'_1 x + D'_2 \cos \Phi - D'_3 \sin \Phi) \\
& + D''_2
\end{aligned} \tag{219}
$$

$$
\begin{aligned}
D_3 = {} & (A'_1 y + B'_1 \cos \Phi + C'_1 \sin \Phi) M_L \\
& - (B'_1 y + B'_3 \cos \Phi + B'_2 \sin \Phi) H_L \\
& + (C'_1 y + B'_2 \cos \Phi + C'_2 \sin \Phi) V_L \\
& + (D'_1 y + D'_2 \sin \Phi + D'_3 \cos \Phi) \\
& + D''_3
\end{aligned} \tag{220}
$$

The $M_L$, $H_L$, and $V_L$ terms in these equations are determined in the same way as corresponding values for fillet arches at the fillet crown. Equations for these quantities are given in section 4-36(d), as equations (209), (210), and (211).

In addition to equations for arch and load constants, formulas are needed for $x$ and $y$ values used in computing effect of abutment yielding on points in the central circular section. These abutment coordinates, with the origin at any point $Q$ in the central circular section, are as follows:

$$
x = r_B \sin (\Phi + {}_a\Phi_B) + (r_o - r_B) \sin \Phi \tag{221}
$$

$$
\begin{aligned}
y = {} & r_o - r_B \cos (\Phi + {}_a\Phi_B) \\
& - (r_o - r_B) \cos \Phi
\end{aligned} \tag{222}
$$

in which

$r_o$ = radius to centerline of central circular section,

$r_B$ = radius to centerline of abutment circular section,

$\Phi$ = angle from any point $Q$ to point $F$, and

${}_a\Phi_B$ = angle from point $F$ to abutment.

It is also necessary to have equations for $M_L$, $H_L$, and $V_L$ for points in the abutment circular section. These are developed by transferring the $M_L$, $H_L$, and $V_L$ at $F$ to points in the abutment circular section, giving the equations,

$$
\begin{aligned}
M_L = {} & {}_BM_L + {}_FM_L - {}_FH_L\, y_B \\
& + {}_FV_L\, x_B
\end{aligned} \tag{223}
$$

$$
\begin{aligned}
H_L = {} & {}_BH_L + {}_FH_L \cos \Phi_B \\
& + {}_FV_L \sin \Phi_B
\end{aligned} \tag{224}
$$

$$
\begin{aligned}
V_L = {} & {}_BV_L - {}_FH_L \sin \Phi_B \\
& + {}_FV_L \cos \Phi_B
\end{aligned} \tag{225}
$$

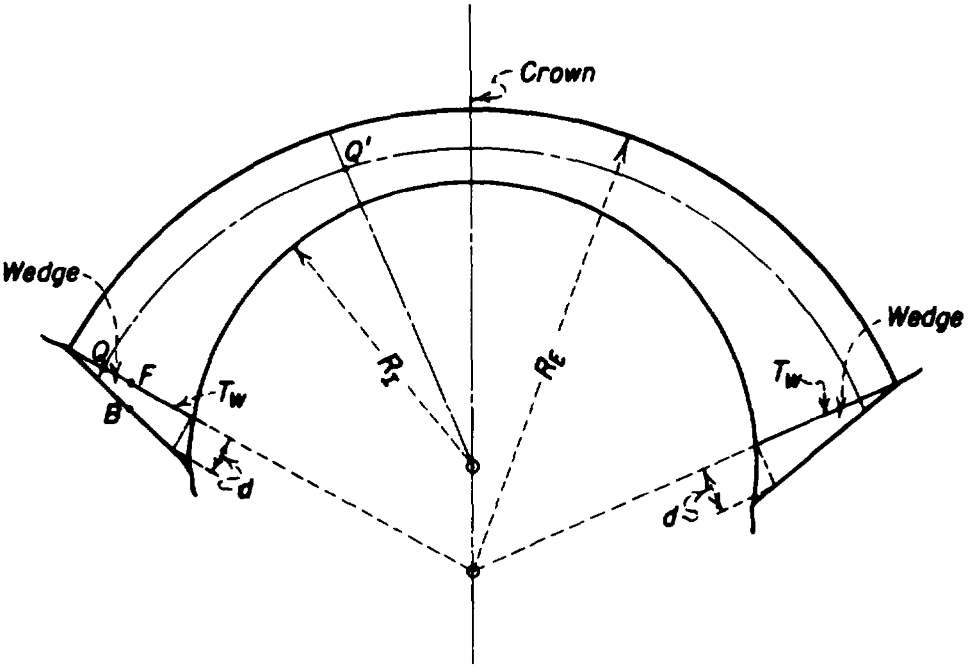
in which ${}_BM_L$, ${}_BH_L$, and ${}_BV_L$ are moments, thrusts, and shears of the load on the abutment circular section only; ${}_FM_L$, ${}_FH_L$, and ${}_FV_L$ are corresponding values at $F$; and $x_B$, $y_B$, and $\Phi$ are coordinates from $F$ to points in the abutment circular section.

(b) *Variable-Thickness Arch With Triangular Wedge Abutment.*—There are several types of nonradial abutments for arches, one of which is illustrated on figure 4-45. An example of this type is Hungry Horse Dam. Computations are given in appendix C in considerable detail.

The method of analysis is based on dividing the arch into four voussoirs or less, and a wedge section. In general, it can be said that the method of analysis outlined here is applicable to any type of variable-thickness arch. If the wedge is located at the abutment, the method is applicable as presented. If there is no wedge, the terms for the wedge drop out of the equations.

Formulas for transferring arch and load constants, given in the previous subsection as equations (212) to (220), inclusive, are applicable for the variable-thickness arch with nonradial abutments. They are utilized for transferring arch and load constants from point $F$ of the wedge section to point $Q'$ of the central section (see fig. 4-45).

The method of determining arch constants for the variable-thickness section is described in section 4-35(c). For a point in a uniform-thickness section, the method would be as outlined in section 4-34(g). However, for a wedge section located in the arch or at the abutment, the equations on the following page are required.



(Proportions exaggerated)

$R_E$ = Radius to extrados of dam.

$R_I$ = Radius to intrados of dam.

$Q$ = Arch point in wedge.

$F$ = Crown point of wedge.

$Q'$ = Arch point in central section.

$d$ = Base of right triangle formed by wedge. (assumed)

$T_W$ = Thickness of central section at wedge.

$B$ = Midway point of abutment.

*Figure 4-45.* Variable-thickness arch with triangular wedge abutments.
288-D-2696

$$A'_1 = + \frac{6d}{E_c\, T_w{}^3} \tag{226}$$

$$B'_1 = - \frac{d}{E_c\, T_w{}^2} \tag{227}$$

$$C'_1 = + \frac{12d^2}{5\, E_c\, T_w{}^3} \tag{228}$$

$$B'_2 = - \frac{d^2}{2\, E_c\, T_w{}^2} \tag{229}$$

$$C'_2 = + \frac{3d}{5\, T_w\, G} + \frac{7d^3}{5\, E_c\, T_w{}^3} \tag{230}$$

$$B'_3 = + \frac{d}{2\, E_c\, T_w} \tag{231}$$

In the above equations, $d$ is the assumed length of the base of a 90° triangle formed by the wedge, and $T_w$ is the altitude of the wedge (see fig. 4-45). The quantities $E_c$ and $G$ are the moduli of elasticity of concrete in direct stress and shear stress, respectively.

The $M_L$, $H_L$, and $V_L$ values to be used for transferring the $D$-terms to points in the central section are calculated for points in the central section, and for point $F$ for loads between points $F$ and $Q'$.

Formulas for $M_L$, $H_L$, and $V_L$ for a variable-thickness arch are the same as those for a uniform-thickness arch except that the radius $r$, in radial load formulas, is the radius to the arch point, $R_E - T/2$; and the radius $r$ in twist and tangential load formulas is the radius to the crown centerline arc, as explained in section 4-35(e). Also, the quantity $H_L e_p$ must be combined with the $M_L$ quantity from the load formula to give the correct value for $M_L$ at an arch point. The coordinates $x$ and $y$ are changed as shown in section 4-35(e).

The general equations for computing total $M_L$, $H_L$, and $V_L$ at the abutment of the triangular wedge, due to total external load to right of the point, are:

$$M_L = {}_FM_L - {}_FH_L\, y_B + {}_FV_L\, x_B \tag{232}$$

$$H_L = {}_FH_L \cos \Omega_B + {}_FV_L \sin \Omega_B \tag{233}$$

$$V_L = {}_FV_L \cos \Omega_B - {}_FH_L \sin \Omega_B \tag{234}$$

In these equations, ${}_FM_L$, ${}_FH_L$, and ${}_FV_L$ are values at $F$ due to a load between $F$ and any point $Q'$; and $x_B$, $y_B$, and $\Omega_B$ are linear or angular distances from $F$ to the abutment. The angle $\Omega$ is formed by the intersection of the radius $R_E$ passing through $F$ with the nonradial abutment. This is more clearly shown in the calculations of variable-thickness arches with triangular wedge abutments in appendix C.

The general equations for computing $D$-terms for the wedge section are as follows:

All radial load $D$-terms are zero.

Tangential loads where $P_F$ = unit load at $F$ and $P_B$ = unit load at $B$ (for $D$-terms only), ${}_BH'_L = -\frac{1}{4}(P_F + P_B)\, d$.

$$(\text{Avg.})\ {}_FH'_{L\,(B\text{-}F)} = \frac{1}{2}({}_BH'_L) = {}_FH''_{L\,(B\text{-}F)} \tag{235}$$

$${}_FD'_1 = - B'_1 \left[{}_FH''_{L\,(B\text{-}F)}\right] \tag{236}$$

$${}_FD'_2 = - B'_2 \left[{}_FH''_{L\,(B\text{-}F)}\right] \tag{237}$$

$${}_FD'_3 = - B'_3 \left[{}_FH''_{L\,(B\text{-}F)}\right] \tag{238}$$

Twist loads where $P_F$ = unit moment at $F$ and $P_B$ = unit load at $B$ (for $D$-terms only), ${}_BM'_L = \frac{1}{4}(P_F + P_B)\, d$

$${}_FD'_1 = \frac{1}{2}\, {}_BM'_L \bullet A'_1 \tag{239}$$

$${}_FD'_2 = \frac{1}{2}\, {}_BM'_L \bullet C'_1 \tag{240}$$

$${}_FD'_3 = \frac{1}{2}\, {}_BM'_L \bullet B'_1 \tag{241}$$

(c) *Noncircular Arches.*—Arches that do not have an extrados composed of circular segments can be analyzed by the voussoir summation method. Since tabulated arch and load constant cannot be used in the voussoir method, the summations for different trial loads require considerable time.

In the summation method, integrals in general arch equations are replaced by mechanical summations of the various quantities calculated for the separate voussoirs, usually 10, into which each side of the arch is divided. Otherwise, equations for solution of crown forces and calculation of arch deflections are identical with those for the integration method.

(d) *Cracked Arches.*—In the design of arch dams it is difficult to eliminate tension in the extrados at the abutments and in the intrados at the crown. Sections of the arch subjected to excessive tension are assumed to be cracked and are disregarded in the analysis. In arch analyses based on this assumption, it has been found that deflections are not greatly different from those for an uncracked arch. Therefore, analyses of cracked arches have only a small influence on trial-load adjustments of deflections. The small effect on the analysis leads to the omission of the use of cracked arches in all but special cases. Cracking of vertical cantilevers, however, does have an important effect on deflection adjustments. Consequently, cantilever cracking is usually considered in trial-load analyses of arch dams where preliminary analyses indicate the occurrence of excessive vertical tensile stresses.

If it is decided that effects of cracking should be considered in analyzing arch elements, the percentage of cracking, $n$, the rotation due to cracking, $\frac{\theta_g}{2}$, and the maximum stress on the uncracked portion, $\sigma_{xm}$, may be computed using a table and equations which have been developed. The table, equations, and drawing are shown on figure 4-46. Since the inclusion of rotation due to cracking will affect the load distribution, several readjustments will be necessary to obtain the final loading for the cracked arch.

A first estimate of $n$ is based on the percentage of tension across the arch section as follows:

$$n = \frac{\sigma_x(\text{tension})}{-\sigma_x(\text{tension}) + \sigma_x(\text{compression})} \qquad (242)$$

$$\sigma_{xm} = \frac{H}{T}\;\frac{5(2n+1)}{2(4n+1)(1-n)}$$

$$\frac{y}{T} = \frac{2}{7}\;\frac{(1-n)(6n+1)}{(4n+1)}$$

$$\left(\frac{\theta_g}{2}\right) = \frac{H}{ET}\;\frac{2}{\left(\frac{1-n}{n}\right)^{\frac{3}{2}}\left[1+\frac{1}{5}\left(\frac{1-n}{n}\right)\right]}$$

STRESSES AND ANGULAR CHANGES AT A CRACK

| $n$ | $\frac{\sigma_{xm}}{H}(T)$ | $\frac{y}{T}$ | $(\frac{\theta_g}{2})(\frac{ET}{H})$ |
|---|---|---|---|
| 0.0 | 2.0000 | 0.3333 | 0 |
| 0.1 | 2.2667 | 0.3048 | 0.2646 |
| 0.2 | 2.5334 | 0.2762 | 0.6944 |
| 0.3 | 2.8001 | 0.2476 | 1.2754 |
| 0.4 | 3.0667 | 0.2191 | 2.0936 |
| 0.5 | 3.3333 | 0.1905 | 3.3333 |
| 0.6 | 4.0441 | 0.1546 | 5.4036 |
| 0.7 | 5.2632 | 0.1173 | 9.3787 |
| 0.8 | 7.7381 | 0.0789 | 19.0476 |
| 0.9 | 15.2174 | 0.0398 | 58.7546 |
| 1.0 | ∞ | 0 | ∞ |

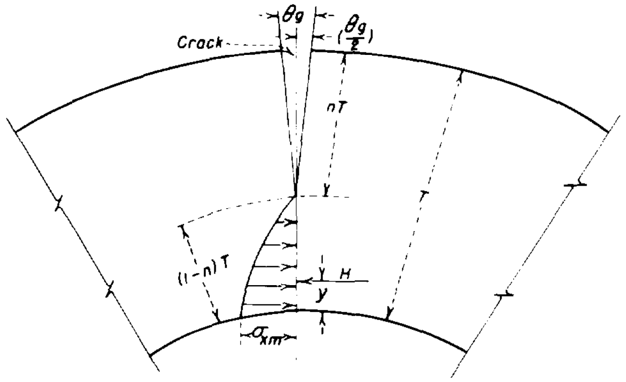


*Figure 4-46.* **Analysis of arches cracked at the upstream face.—288-D-2697**

Then, from the drawing (fig. 4-46),

$$\frac{y}{T} = \frac{0.4(1-n)T}{T} = 0.4(1-n) \qquad (243)$$

This value of $\frac{y}{T}$ may then be used to find the other terms in the table (fig. 4-46).

Terms for computing $\frac{\theta_g}{2}$ and $\sigma_{xm}$ from the table are then multiplied by the appropriate values to obtain the amount of rotation produced by cracking and the maximum compressive stress on the uncracked portion.

The effects of cracking are then introduced into the arch deflections and any necessary readjustments are made. This process continues until they are in agreement. The effects of arch cracking are also included in the tangential and twist adjustments.

## 4. Procedure for Trial-Load Adjustments

4-38. *Radial Adjustment.*—Agreement of radial deflections of arch and cantilever elements is obtained by dividing the horizontal radial load by trial between the two systems without altering the total external radial load. Prior to starting the radial adjustment, the initial deflections of arches and cantilevers due to initial loads are computed, and tables of unit deflections due to unit loads are prepared as outlined in section 4-31(h) and appendix C.

The designer, by inspection of initial and unit deflections, by comparison with load patterns obtained for similar dams, and by experience, is able to estimate the first set of trial loads for the radial adjustment. This first set of loads is plotted on a load diagram in which the cantilever loading is the difference between the total radial load and the arch load, as determined graphically by measurement on the arch loading diagram or directly by computation. Such a method of determination is necessary because cantilevers and arch points usually do not coincide, as may be seen in figure 4-47.

A typical load and deflection diagram for an arch is shown on figure 4-47. Horizontal radial loads are plotted above the developed centerline of the arch. The location of the center of each cantilever and of each arch quarter-point is indicated by full and dotted lines, respectively. Initial deflections of the arch and cantilever are plotted below the developed arch centerline, different symbols being used to distinguish between the arch and cantilever deflections. When the loading is applied, deflections are computed and replotted from the initial positions. A diagram similar to that shown is also used for cantilevers.

For the first trial-load distribution, crown-cantilever radial deflections are adjusted to arch radial deflections. Subsequently, a few other points are brought into deflection agreement, so that after a few trials all points are in approximate agreement. The degree of exactness to which this agreement is made will be dictated by experience and the purpose of the analysis.

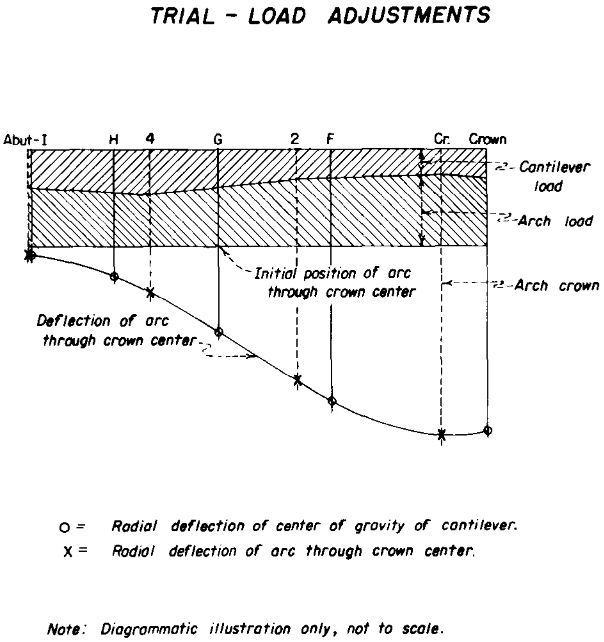


*Figure 4-47.* **Diagram of radial loads and deflections for a typical arch.—288-D-2736**

Upon completion of the radial adjustment, stresses may be computed. If excessive tensile stresses are found, the adjustment can be repeated, this time on the assumption that cracking takes place at those elevations at which excessive tension is indicated, as explained in section 4-33. However, in most instances, the dam should be redesigned to eliminate the tensions.

Subsequent readjustments of radial deflections are usually necessary because loads introduced in tangential and twist adjustments produce additional radial movements of arches and cantilevers. These movements will be discussed in a later section.

4-39. *Tangential Adjustment.*—The radial adjustment is normally followed by the tangential adjustment whereby movements of arches and cantilevers are brought into deflection agreement in the tangential direction. Usually, in the tangential adjustment, there are initial tangential movements due to earthquake and temperature, and, in addition, initial relative movements of arches and cantilevers caused by the radial adjustment. Tangential arch movements due to the first radial adjustment are obtained by calculating tangential deflections for the trial radial loads used in the first radial adjustment. The tangential cantilever foundation movement is equal to the movement of the arch abutment due to these same loads, since each cantilever rests on an arch abutment. Radial loads on the cantilevers usually produce negligible tangential movements in the cantilevers, and are therefore neglected in the tangential adjustment.

Equal and opposite tangential loads, one on the arch and the other on the cantilever, are introduced to remove relative tangential deflections. Required amounts of tangential loads are estimated and applied to the two systems in successive trials. As in the radial adjustment, unit loads and deflections are used in determining total tangential loads and deflections. Equal cantilever loads are measured graphically or computed from the arch loads along the crown centerline.

The adjustment is begun by bringing the tangential deflections of one or two cantilevers near the half-points of the arches into agreement with the arch deflections. Generally, agreement of the other cantilever and arch deflections is accomplished after only a few trials. Subsequent readjustments of the tangential deflections are required because twist adjustments and radial adjustments produce additional tangential movements.

Tangential loads and deflections are plotted in the same manner as the radial loads and deflections. In accordance with the established sign convention, tangential loads on the arches usually act toward the crown, while actual displacements are toward the abutments. In most dams, tangential loads and deflections are small compared with radial loads and deflections. However, in most cases tangential loads cause considerable change in arch stresses, since they change the arch thrusts without causing appreciable movements in the structure.

4-40. *Twist Adjustment.*—The twist adjustment is concerned only with rotations of arch and cantilever elements about vertical axes. Radial and tangential adjustments cause angular movements of both arches and cantilevers. Angular movements of arches due to these adjustments are computed. However, angular movements of cantilevers due to radial loads are negligible; hence, only those initial angular movements of cantilevers due to the eccentricity of tangential loads are calculated. The latter are obtained by summating angular movements due to applied tangential loads and combining these movements with the abutment rotation of the arch at the cantilever foundation.

Equal and opposite twist loads on the cantilevers are determined in a similar manner to that used for tangential loads. Starting with one or two cantilevers near the half-points of the arches, an approximate deflection agreement is made for the arches and cantilevers at these points. Generally, agreement at the other points is produced after only a few trials. Since radial and tangential readjustments both produce angular movements in arches and cantilevers, only experience will show what is necessary to accomplish a satisfactory agreement.

Twist loads, in addition to rotating the elements of a dam, produce radial and tangential deflections in the arches and radial deflections in the cantilevers. Being horizontal couples, they act on the arches as bending moments, thereby producing radial and tangential deflections, which may be computed from the twist loads.

From the theory of twisted structures, couples also exist in vertical radial planes. These twist the arches and bend the cantilevers. An additional adjustment is not required since rotations in vertical radial planes are in adjustment when rotations in horizontal planes are in adjustment, providing radial agreement is maintained. However, cantilever deflections due to vertical couples must be computed in order that radial agreement may be restored in the radial adjustment.

Vertical couples are computed from values of horizontal twist loads, utilizing the equality from slab theory, between twisting moments on mutually perpendicular planes. Twist loads, being distributed loads, require that this relationship be modified. For distributed loads, horizontal rates of change of horizontal twisting moments are equal to vertical rates of change of vertical twisting moments. Since cantilever bending moments of the twisted structure are equal to vertical twisting moments, the proposition may be restated as follows. Rates of change of applied twisting moments along arch centerlines are equal to rates of change of cantilever bending moments along cantilever centerlines. It should be noted that bending moments here referred to are those due to twist alone. The steps to be followed in calculating the above deflections are given below:

(1) Twisting moments are computed for all cantilevers at all arch elevations.

(2) Twisting moments are differentiated along arch crown centerlines, giving ranges of change of twisting moments along arch crown centerlines, which are equal to rates of change of radial bending moments in the cantilevers.

(3) Rates of change are integrated from the top of each cantilever to the lower elevations, thus giving bending moments in the cantilevers.

(4) A double integration of the $M/EI$ curve from the base of each cantilever gives the desired cantilever deflections due to bending in a radial direction.

Twist effects may be relatively large in some dams. Twist resistance stiffens the structure, thereby reducing movements and stress determined by the radial adjustment. Usually, twist effects are greater than effects of tangential shear, but are relatively small when compared with the results of the radial adjustment. The importance of the twist adjustment, however, is not diminished by this fact.

**4-41.** ***Readjustments.***—The usual sequence of the first cycle of adjustments is, first, the radial adjustment, followed in order by the tangential and twist adjustments; hence, radial deflections due to tangential and twist loads cannot be considered in the first cycle of adjustments, nor can tangential deflections due to twist loads. A second cycle of adjustments is therefore required in which appropriate corrections are made for deflections caused by the loads of the preceding adjustments. The adjustments of the first cycle are termed first adjustments, those of the second cycle are termed first readjustments, those of the third cycle are termed second readjustments, and so on.

The radial deflections due to first-adjustment tangential and twist loads are included in the first radial readjustment. However, the additional radial loads introduced in this readjustment cause further tangential and twist movements. Hence, the first tangential readjustment is made to correct for discrepancies introduced by this first radial readjustment, and also those caused by the first twist adjustment. Likewise, a first twist readjustment is made to correct for the first radial and tangential readjustments. This, of course, serves to unbalance the second cycle of adjustments and usually requires a third cycle of adjustments. However, the effects usually converge fairly rapidly. Normally, not more than three or four cycles are required to obtain a deflection agreement that will not be appreciably affected by further readjustments.

A complete adjustment determines the total movements of the dam and the amounts and distribution of all loads, thus enabling the designer to calculate moments, thrusts, and shears for each arch and cantilever element. With forces and moments known, stresses can be calculated by formulas given in section 4-45.

4-42. *Adjustments for Poisson's Ratio Effects.*—The effect of Poisson's ratio is to cause additional movements and stresses in arches and cantilevers. Since the movements are different for each, a new adjustment is required. Poisson's ratio adjustments made so far have shown relatively small effects. In general, it has been found that small increases in compressive stresses or decreases in tensile stresses occur at the upstream face, that there are small decreases in compressive stresses at the downstream face, and that movements in the dam are not appreciably altered. Appendix E shows the stress changes in one dam due to the inclusion of these effects. The following paragraphs describe the general procedure for adjustments for Poisson's ratio effects.

Cantilever movements due to Poisson's ratio are caused by arch stresses and radial stresses. Arch stresses can be readily calculated, but complete radial stresses require laborious solutions of long equations. However, radial stresses can be readily calculated at the upstream and downstream faces, and, by assuming a linear distribution of stress between faces, approximate intermediate radial stresses can be computed. These are considered satisfactory for this step in analysis.

Following the determination of stresses, strains due to arch and radial stresses are determined and the products of the strains and Poisson's ratio are computed. These products are strains in the cantilevers due to effects of Poisson's ratio. Horizontal radial deflections due to strains can then be determined. These are the initial cantilever movements used in the adjustments for Poisson's ratio effects.

From the theory of elasticity, the equations for unit strain are:

$$\epsilon_x = -\frac{1}{E}[\sigma x - \mu(\sigma y + \sigma z)]$$

$$\epsilon_z = -\frac{1}{E}[\sigma z - \mu(\sigma x + \sigma y)]$$

In these equations we are interested in the portion affected by Poisson's ratio. In the equations a positive sign indicates compression. The strain on the upstream face of the cantilever is:

$$\epsilon_{zE} = \frac{\mu}{E}(\sigma x_E + \sigma y_E)$$

and on the downstream face,

$$\epsilon_{zD} = \frac{\mu}{E}(\sigma x_D + \sigma y_D)$$

The difference between the upstream strain and the downstream strain, divided by the thickness of the section, gives a curvature of the cantilever,

$$\rho = \frac{\epsilon_{zE} - \epsilon_{zD}}{T} \tag{244}$$

The radial deflection in the cantilever is then:

$$\Delta r = + \int_0^z \int_0^z \rho \, dz \, dz \tag{245}$$

Movements in the arches due to Poisson's ratio are caused by cantilever stresses and radial stresses. Here again, it is expedient to use approximate radial stresses. Strains due to such stresses are determined and products of strains and Poisson's ratio calculated. The values cannot be used directly as arch strains because the arches are statically indeterminate elements. However, the arch can be cut at the crown and movements due to strains computed for each half of the arch. Then arch moments, thrusts, shears, and deflections can be calculated. Resulting deflections are initial arch movements to be used in adjustments for Poisson's ratio effects.

The strains on the arches are:

$$\epsilon_{xE} = \frac{\mu}{E}(\sigma z_E + \sigma y_E)$$

$$\epsilon_{xD} = \frac{\mu}{E}(\sigma z_D + \sigma y_D)$$

$$\epsilon_x = \frac{\epsilon_{xE} + \epsilon_{xD}}{2}$$

The horizontal curvature at an arch point is the difference between the strain at the upstream face and the downstream face divided by the thickness of the section,

$$\rho = \frac{\epsilon_{xE} - \epsilon_{xD}}{T} \tag{246}$$

The formulas for the *D*-terms in arches as applied here are:

$$D_1 = \int_0^s \rho\, ds \tag{247}$$

$$D_2 = \int_0^s \rho x\, ds - \int_0^s \epsilon_{x_{avg.}} \sin\Phi\, ds \tag{248}$$

$$D_3 = \int_0^s \rho y\, ds + \int_0^s \epsilon_{x_{avg.}} \cos\Phi\, ds \tag{249}$$

In the equations above, the following substitutions can be made:

$$\Phi = \Phi_o + \Phi_1$$

$$x = r\sin\Phi$$

$$y = r \operatorname{vers}\Phi$$

$$d\Phi = d\Phi_1$$

$$ds = r\, d\Phi = r\, d\Phi_1$$

Where $\Phi_o$ is the angle from the origin to the face of the voussoir under consideration, and $\Phi_1$ is the angle subtended by the voussoir.

The formulas then become:

$$D_1 = \int_0^{\Phi_1} \rho\, r\, d\Phi_1 \tag{250}$$

$$D_2 = \int_0^{\Phi_1} \rho\, r^2 \sin(\Phi_o + \Phi_1)\, d\Phi_1 - \int_0^{\Phi_1} \epsilon_{x_{avg.}} r \sin(\Phi_o + \Phi_1)\, d\Phi_1 \tag{251}$$

$$D_3 = \int_0^{\Phi_1} \rho\, r^2 \operatorname{vers}(\Phi_o + \Phi_1)\, d\Phi_1 + \int_0^{\Phi_1} \epsilon_{x_{avg.}} r \cos(\Phi_o + \Phi_1)\, d\Phi_1 \tag{252}$$

Integrating,

$$D_1 = \rho\, r\Phi_1 \tag{253}$$

$$D_2 = \rho\, r^2(\sin\Phi_o \sin\Phi_1 + \cos\Phi_o \operatorname{vers}\Phi_1) - \epsilon_{x_{avg.}} r(\sin\Phi_o \sin\Phi_1 + \cos\Phi_o \operatorname{vers}\Phi_1) \tag{254}$$

$$D_3 = \rho\, r^2(\Phi_1 + \sin\Phi_o \operatorname{vers}\Phi_1 - \cos\Phi_o \sin\Phi_1) + \epsilon_{x_{avg.}} r(\cos\Phi_o \sin\Phi_1 - \sin\Phi_o \operatorname{vers}\Phi_1) \tag{255}$$

Using these *D*-terms, the forces and deflections in the arches are calculated with the usual methods.

Adjustments are similar to those described in preceding sections, except that initial arch and cantilever movements due to Poisson's ratio are plotted on adjustment sheets as initial movements. Equal and opposite loads are applied by successive trials until agreement has been restored between the arch and cantilever elements.

If the cantilevers are assumed to crack, a different procedure is followed, since actual trial loads instead of unit loads must then be applied to the cracked cantilever. For this case, initial movements due to Poisson's ratio are combined with final movements as determined by the first set of adjustments. The loads needed to restore agreement between arches and cantilevers are introduced in additional readjustments until the desired agreement is secured.

Contraction joints in a dam are usually grouted after the structure has been built to final height. In this case, the weight of the structure has no Poisson's ratio effect on arch stresses, because the movements simply tend to close the open contraction joints. However, if sections of the dam are grouted before the final heights are reached, weights of concrete placed after grouting must be considered in the Poisson's ratio adjustments. This must be kept in mind at the time stresses are calculated for determining initial Poisson's ratio movements.

**4-43.** ***Adjustment for Vertical Displacement Effects.***—The effects of vertical displacement can be brought into the adjustments by computing the tangential movements of the cantilevers due to the vertical displacement. The arches and cantilevers are then readjusted in the tangential direction and the effects of this loading brought into the complete adjustment.

The vertical displacements in a dam are caused by the following:

(1) The difference in temperature between the temperature of the dam at the time it is grouted and the minimum operating temperature.

(2) The bending moments in the cantilevers due to the radial cantilever loads and to the twisting moments from the tangential and twist cantilever loads.

(3) The vertical axial force resulting from integration of the rate of change of tangential shears due to tangential cantilever loads used in the adjustments and the tangential component of concrete inertia.

The vertical displacement for each cantilever due to temperature change may be calculated by integrating the thermal expansions at each elevation, thus:

$$\Delta v = \int_0^z c\, t^\circ\, dz \qquad (256)$$

where $t^\circ$ is the change in degrees F. between the temperature of the dam at the time of grouting and the applicable operating temperature.

The vertical displacement in each cantilever due to bending moments in the cantilevers may be computed by integrating the cantilever slopes due to bending moments in a radial direction between centers of gravity, thus:

$$\Delta v = \int_0^y \int_0^z \frac{M}{EI}\, dz\, dy \qquad (257)$$

where $M$ is the bending moment in the cantilever due to radial cantilever loads and to the twisting moments from the tangential and twist cantilever loads.

Where tangential shears exist in the dam, equal shears in the vertical direction are also present. The vertical axial force resulting from double integration of the rate of change of these shears across the unit cantilever produces vertical displacements, as shown below:

$$F_v = \int_z^0 \frac{\partial\, V_{TA}}{\partial x}\, dz \qquad (258)$$

$$\Delta v = \int_0^z \frac{F_v}{AE}\, dz \qquad (259)$$

In the above equations, $F_v$ is the axial force developed in the cantilever, and $V_{TA}$ is the tangential shear due to the tangential cantilever loads.

None of the vertical displacement is assumed to be transmitted into the foundation.

The vertical displacements from these forces are then added together to give displacements at each cantilever elevation. Differences in vertical displacements between adjacent cantilever points produce slopes in the cantilevers in a tangential direction. Starting with zero deflection at the bases of the cantilevers, the slopes are integrated to give the tangential cantilever movements, thus:

$$\Delta s = \int_0^z \frac{\partial\, \Delta v}{\partial x}\, dz \qquad (260)$$

The only arch deflections due to vertical displacements result from the adjustment necessary for the vertical displacements in the cantilevers. The tangential cantilever movements are introduced into the tangential adjustment. This can be done after the first tangential adjustment or after the normal complete trial-load analysis has been finished.

An example of the computation and adjustment of vertical displacement effects is contained in appendix E.

## 5. Calculation of Stresses

4-44. *General Considerations.*—Ordinarily, arch, cantilever, and principal stresses parallel to the faces at the faces of the dam, and shearing stresses in the interior of the dam, are the only values determined in a final stress analysis. These, of course, are determined from forces and moments obtained as a result of a deflection adjustment.

Although maximum tensile and compressive stresses generally occur at the faces of the dam, assuming a linear variation of normal stress between the upstream and downstream faces, sometimes the stresses at interior points are important and should be investigated, particularly if high stresses might cause failure at some plane or zone of weakness, or if unusual conditions exist in the interior of the dam.

For design purposes, it is generally assumed that normal stresses on horizontal sections in a cantilever element vary linearly from the upstream face to the downstream face at all elevations. An approach toward a determination of normal stresses may be made by the two-dimensional finite element method, discussed in sections 4-57 through 4-64, or by other analytical or experimental methods.

Equations are given for determining stresses at the faces of, or at any point within, the dam. These include equations for inclined cantilever stresses at the face of the dam and at the abutment rock plane. In developing stress equations, ordinary stress formulas are used from textbooks on mechanics of materials whenever applicable. These are used in the derivations, with necessary changes in notation and form. Special equations for stress conditions not covered in engineering textbooks are included in the following sections. Examples of stress computations are given in appendix E.

(a) *Notations and Definitions.*—In addition to terms in the list of notations at the beginning of this chapter (sec. 4-2), the following notations and definitions are used in derivations of stress equations:

$'$ = symbols referring to coordinate axes $X'$, $Y'$, and $Z'$.
$''$ = symbols referring to coordinate axes $X''$, $Y''$, and $Z''$.
$*$ = stress or force normal to or parallel to the abutment plane.

$X, Y, Z$, or $x, y, z$ = coordinates; origin at downstream face ($X$ positive toward abutment, $Y$ positive toward upstream face, $Z$ positive downward).

The functions of the angles $\eta$ and $\Phi$ must be used with proper regard to signs, as given in the following tabulation:

| Side of dam (Looking upstream) | Face of dam | Positive $\eta$ (Looking downward) |
|---|---|---|
| Left | Upstream | Clockwise |
| Left | Downstream | Counterclockwise |
| Right | Upstream | Counterclockwise |
| Right | Downstream | Clockwise |

| Side of dam (Looking upstream) | Face of dam | Positive $\Phi$ (Looking toward abutment) |
|---|---|---|
| Left | Upstream | Counterclockwise |
| Left | Downstream | Clockwise |
| Right | Upstream | Clockwise |
| Right | Downstream | Counterclockwise |

Additional symbols are as follows:

$W$ = total vertical force on a cantilever 1 foot wide at the axis, including external vertical loads and weight.
$M$ = total moment of cantilever, 1 foot wide at the axis, about center of gravity of horizontal section, foot-pounds (positive moment causes compression at the upstream face).
$V_{CA}$ = total horizontal radial shear on a horizontal cantilever section 1 foot wide at the axis, pounds (positive shear acts upstream).
$V_{\text{℄}}$ = total horizontal radial shear on a horizontal cantilever section 1 foot wide at the centerline, pounds.
$V_A$ = total horizontal radial shear on vertical arch section 1 foot high, pounds (positive shear acts upstream).
$V_{TA}$ = total horizontal tangential shear force on horizontal cantilever section 1 foot wide at the axis, pounds (positive shear acts toward abutment).
$\rlap{-}M$ = total twisting moment on horizontal cantilever section 1 foot wide at the axis, foot-pounds (positive moment acts counterclockwise on left side of dam).
$M_A$ = total horizontal moment for an arch 1 foot high, foot-pounds (positive moment causes compression at the upstream face).
$H_A$ = total horizontal thrust for an arch 1 foot high, pounds (compression positive).
$\sigma_x$ = horizontal arch stress normal to vertical radial plane, pounds per square foot (compression positive).
$\sigma_y$ = horizontal radial stress normal to vertical tangential plane, pounds per square foot (compression positive).
$\sigma_z$ = vertical cantilever stress normal to horizontal plane, pounds per square foot (compression positive).
$\tau_{xy}$ = horizontal arch shear stress acting in radial direction on a vertical radial plane, pounds per square foot (direction of positive shear shown on drawings).
$\tau_{zx}$ = horizontal cantilever shear stress acting in tangential direction on a horizontal plane, pounds per square foot.
$\tau_{zy}$ = horizontal cantilever shear stress acting in radial direction on a horizontal plane, pounds per square foot.
$\tau_{ry}$ = horizontal shear stress acting in a radial direction along abutment rock plane, pounds per square foot.
$\sigma_P$ = principal stress at face of dam, pounds per square foot.
$\zeta$ = angle between a circumferential inclined plane and the corresponding vertical plane.
$\tau_1$ = horizontal shear stress on inclined plane, pounds per square foot.
$\tau_2$ = shear stress perpendicular to $\tau_1$ on inclined plane, pounds per square foot.
$\tau_m$ = maximum shear stress on inclined plane, pounds per square foot.
$\omega$ = angle maximum shear stress makes with $\tau_1$.
$\sigma_N$ = normal stress on inclined plane, pounds per square foot.

$j, m, n$ = direction cosines of any inclined plane referred to $X$, $Y$, and $Z$ axes, respectively.
$j', m', n'$ = direction cosines of maximum shear stress, $\tau_m$, in an inclined plane referred to $X$, $Y$, and $Z$ axes, respectively.
$S$ = total stress on an inclined plane, pounds per square foot.
$S_1, S_2, S_3$ = components of total stress on inclined plane in $X$, $Y$, and $Z$ directions, respectively, pounds per square foot.

**4-45. *Stresses at Faces of Dam.*—** (a) *Procedure.*—Since maximum stresses in a dam usually occur at the faces of the structure, an analysis ordinarily requires only the evaluation of stresses at the upstream and downstream faces. In considering stresses at the faces, the problem is simplified because shear stress is zero in the plane of the face, and one principal stress is normal to the face.

In the following derivations it is assumed that cantilever forces and moments, $W$, $M$, $V_{CA}$, $V_{TA}$, $\cancel{M}$, and arch forces and moments, $H_A$, $M_A$, $V_A$, are known. Equations are derived for the left part of the dam, looking upstream. These can also be used for the right part of the dam if signs of forces and moments follow the convention used in the trial-load analysis.

Stresses are first determined for three mutually perpendicular planes: a vertical radial plane, $YZ$; a vertical tangential plane, $XZ$; and a horizontal plane, $XY$. There are six stress components on these three planes. Three stresses, horizontal arch stress $\sigma_x$, vertical cantilever stress $\sigma_z$, and tangential cantilever shear stress $\tau_{zx}$, are assumed to have a linear variation from upstream face to downstream face. Radial cantilever shear stresses $\tau_{zy}$, radial arch shear stresses $\tau_{xy}$, and radial stresses normal to vertical tangential planes $\sigma_y$ are derived by equating forces acting along $X$, $Y$, and $Z$ axes (see fig. 4-48). Equations are derived first for the downstream face, and later for the upstream face.

(b) *Stresses at Downstream Face.*—Equations for stresses at the downstream face are given below:

Vertical cantilever stress on a horizontal plane,

$$\sigma_{zD} = \frac{W}{A_{CA}} - \frac{M}{I_{CA}}(T - lg) \tag{261}$$

Horizontal arch stress on a vertical radial plane,

$$\sigma_{xD} = \frac{H_A}{A_A} - \frac{M_A}{I_A} \cdot \frac{T}{2} \tag{262}$$

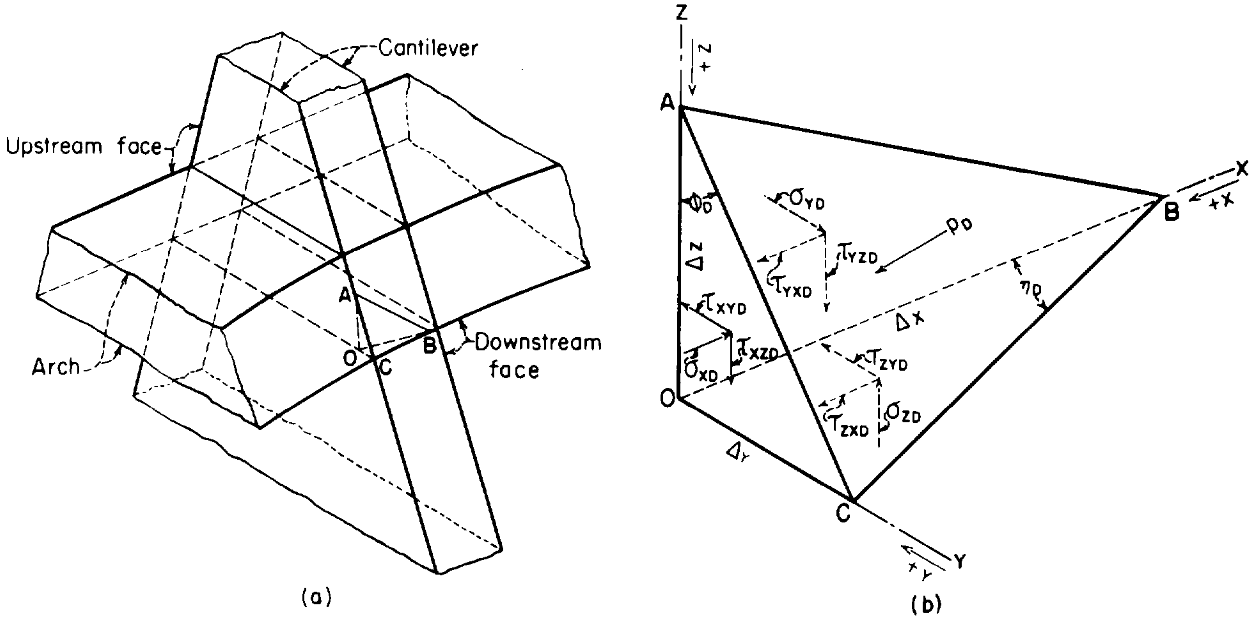
Horizontal cantilever shear stress acting in a tangential direction on a horizontal plane,

$$\tau_{zxD} = \tau_{xzD} = -\frac{V_{TA}}{A_{CA}} + \frac{\cancel{M}}{I_{CA}}(T - lg) \tag{263}$$

Consider an element at the downstream face, as shown on figure 4-48(b), with six stresses acting on three mutually perpendicular planes, and normal pressure $p_D$ acting on the face $ABC$.

Equating forces in the $X$ direction,

$$p_D \frac{\Delta z\, \Delta y}{2} = \sigma_{xD} \frac{\Delta z\, \Delta y}{2} - \tau_{yxD} \frac{\Delta z\, \Delta x}{2} - \tau_{zxD} \frac{\Delta y\, \Delta x}{2}$$



INTERSECTION OF ARCH AND CANTILEVER ELEMENTS

STRESSES AT TETRAHEDRON ABCO

*Figure 4-48.* **Diagram of stresses in arches and cantilevers.–288-D-438**

Equating forces in the $Y$ direction,

$$p_D \frac{\Delta z\,\Delta x}{2} = \sigma_{yD} \frac{\Delta z\,\Delta x}{2} - \tau_{xyD} \frac{\Delta z\,\Delta y}{2} - \tau_{zyD} \frac{\Delta y\,\Delta x}{2}$$

Equating forces in the $Z$ direction,

$$p_D \frac{\Delta x\,\Delta y}{2} = \sigma_{zD} \frac{\Delta x\,\Delta y}{2} - \tau_{xzD} \frac{\Delta z\,\Delta y}{2} - \tau_{yzD} \frac{\Delta z\,\Delta x}{2}$$

Substituting $\Delta z = \Delta y \cot \phi_D$, $\Delta x = \Delta y \cot \eta_D$, and dividing by $\frac{(\Delta y)^2}{2}$,

$$p_D \cot \phi_D = \sigma_{xD} \cot \phi_D - \tau_{yxD} \cot \phi_D \cot \eta_D - \tau_{zxD} \cot \eta_D$$

$$p_D \cot \phi_D \cot \eta_D = \sigma_{yD} \cot \phi_D \cot \eta_D - \tau_{xyD} \cot \phi_D - \tau_{zyD} \cot \eta_D$$

$$p_D \cot \eta_D = \sigma_{zD} \cot \eta_D - \tau_{xzD} \cot \phi_D - \tau_{yzD} \cot \phi_D \cot \eta_D$$

Since $\tau_{xyD} = \tau_{yxD}$, $\tau_{xzD} = \tau_{zxD}$, and $\tau_{yzD} = \tau_{zyD}$, the unknowns can be solved, giving:

$$\tau_{xyD} = \tau_{yxD} = (\sigma_{xD} - p_D) \tan \eta_D - \tau_{xzD} \tan \phi_D \tag{264}$$

$$\tau_{yzD} = \tau_{zyD} = (\sigma_{zD} - p_D) \tan \phi_D - \tau_{xzD} \tan \eta_D \tag{265}$$

$$\sigma_{yD} = p_D + \tau_{xyD} \tan \eta_D + \tau_{yzD} \tan \phi_D \tag{266}$$

The inclined cantilever stress at the downstream face of the dam may be expressed in terms of water pressure and stresses $\sigma_z$, $\sigma_x$, and $\tau_{zx}$. This stress, designated $\sigma''_{zD}$, acts normal to the plane *BOF* (see fig. 4-49(a)), which is perpendicular to the downstream face of the cantilever. Figure 4-49(b) shows the projection of the prism *BOCF* on the *YZ* plane. Equating the forces acting on this prism in the direction of the $Z''$ axis, or parallel to the line *AC*, gives the following equation:

$$\sigma''_{zD} \frac{\Delta x\ \Delta y \cos\phi_D}{2} = \sigma_{zD} \frac{\Delta x\ \Delta y \cos\phi_D}{2} + \tau_{zyD} \frac{\Delta x\ \Delta y \sin\phi_D}{2}$$

$$+ \tau_{xyD} \frac{\overline{\Delta y}^2 \sin^2\phi_D \cos\phi_D}{2} - \tau_{xzD} \frac{\overline{\Delta y}^2 \sin\phi_D \cos^2\phi_D}{2}$$

Dividing both sides by $\dfrac{\Delta x\ \Delta y \cos\phi_D}{2}$ and substituting the proper trigonometric functions, the above equation reduces to:

$$\sigma''_{zD} = \sigma_{zD} + \tau_{zyD} \tan\phi_D + \tau_{xyD} \tan\eta_D \sin^2\phi_D - \tau_{xzD} \tan\eta_D \sin\phi_D \cos\phi_D$$

Substituting equations (264) and (265) and simplifying,

$$\sigma''_{zD} = \sigma_{zD} \sec^2\phi_D - p_D \tan^2\phi_D + (\sigma_{xD} - p_D) \tan^2\eta_D \sin^2\phi_D - 2\,\tau_{xzD} \tan\eta_D \tan\phi_D \qquad (267)$$

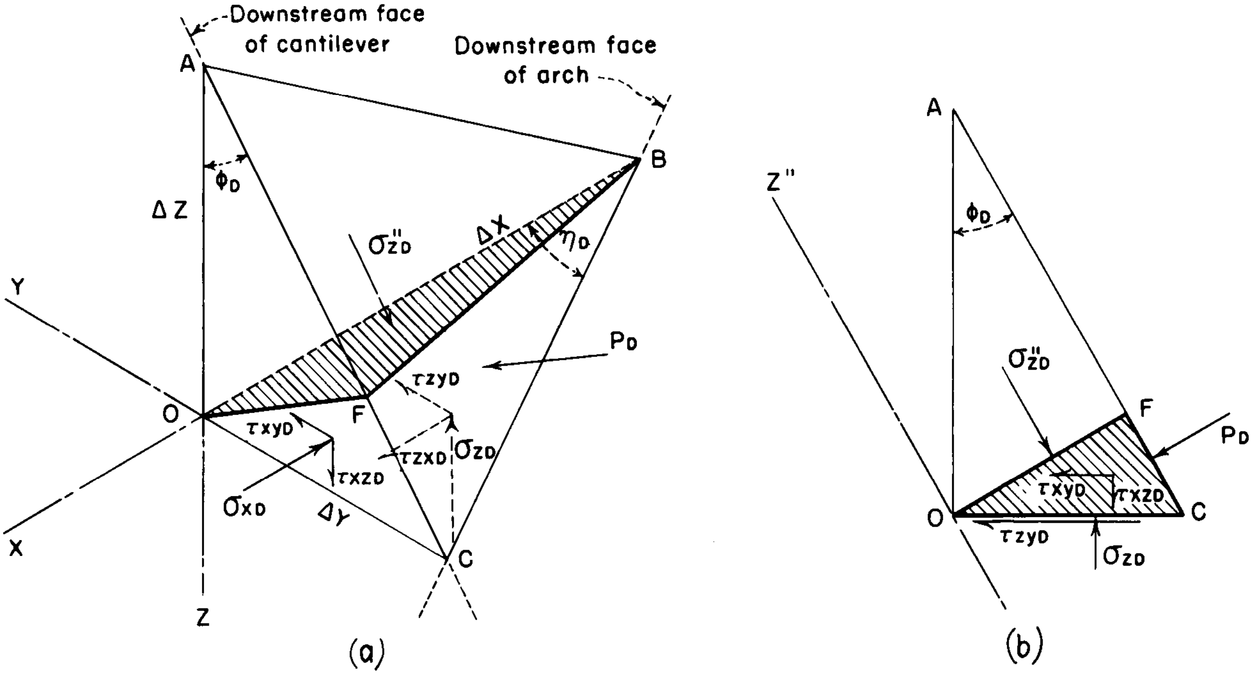


*Figure 4-49.* **Diagrams showing inclined cantilever stress at downstream face of dam.–288-D-3026**

For an arch dam with constant-thickness arches, the angle $\eta$ is equal to zero. Consequently, terms containing $\eta$ drop out.

Shear stresses acting in radial directions at the downstream face along the abutment rock plane may be calculated by using the following equation:

$$\tau_{ryD} = \tau_{yzD} \sin \psi + \tau_{xyD} \cos \psi \tag{268}$$

in which $\psi$ is the vertical angle between a vertical plane and the plane of the abutment surface.

The next step is to find the stresses referred to three new coordinate axes, $X'$, $Y'$, $Z'$, parallel to $BC$, $OE$, and $AE$, respectively (see fig. 4-50(a)). In considering the tetrahedron $ABCO$ the plane $ABC$ is the downstream face of the dam, the $YZ$ plane is rotated about the $Z$ axis until the line $AD$ is perpendicular to $BC$, and the angle the line $AD$ makes with the vertical $Z$ axis is designated by $\phi'_D$.

$$OC = \Delta z \tan \phi_D = OD \sec \eta_D$$

$$OD = \Delta z \frac{\tan \phi_D}{\sec \eta_D} = \Delta z \tan \phi'_D$$

$$\phi'_D = \tan^{-1} (\tan \phi_D \cos \eta_D) \tag{269}$$

Considering section $ADO$ on figure 4-50(b),

$$\tau'''_{yzD} = (\sigma_{zD} - p_D) \tan \phi'_D = \tau'''_{zyD}$$

$$\sigma'_{zD} \Delta z \sin \phi'_D = \sigma_{zD} \Delta z \tan \phi'_D \cos \phi'_D + \tau'''_{zyD} \Delta z \tan \phi'_D \sin \phi'_D$$

$$\sigma'_{zD} = \sigma_{zD} + (\sigma_{zD} - p_D) \tan^2 \phi'_D$$

$$\sigma'_{zD} = \sigma_{zD} \sec^2 \phi'_D - p_D \tan^2 \phi'_D \tag{270}$$

Considering plane $BCO$ on figure 4-50(c),

$$\sigma'_{xD} \Delta y_1 \sec \eta_D = \sigma_{xD} \Delta y_1 \cos \eta_D + \tau_{xyD} \Delta y_1 \tan \eta_D \cos \eta_D$$
$$+ \tau_{xyD} \Delta y_1 \sin \eta_D + \sigma_{yD} \Delta y_1 \tan \eta_D \sin \eta_D$$

$$\sigma'_{xD} = \sigma_{xD} \cos^2 \eta_D + \sigma_{yD} \sin^2 \eta_D + 2 \tau_{xyD} \sin \eta_D \cos \eta_D \tag{271}$$

Equations for computing arch stress, $\sigma'_{xD}$, parallel to the face of the arch can also be expressed in terms of $\sigma_x$, $\sigma_z$, $\tau_{zx}$, and $p_D$,

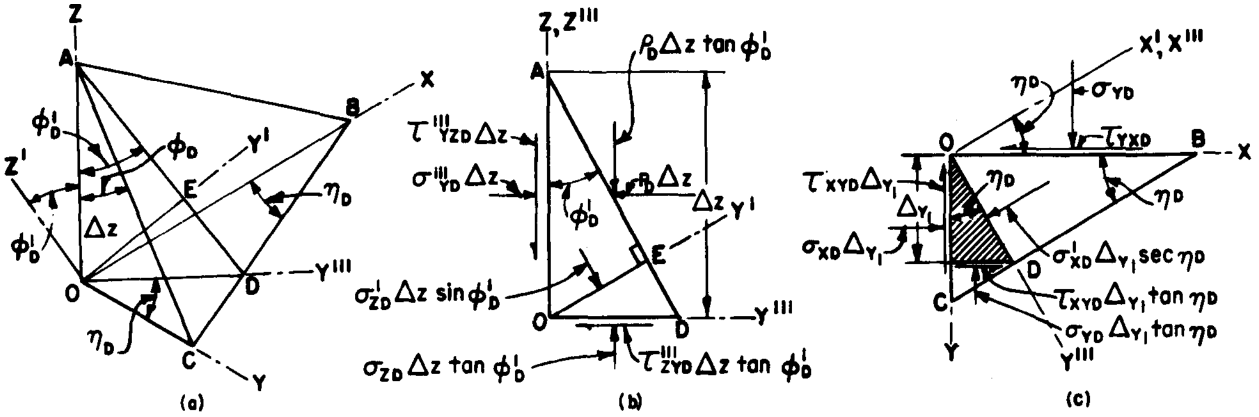
$$\sigma'_{xD} = \sigma_{xD} \sec^2 \eta_D - p_D \tan^2 \eta_D + (\sigma_{zD} - p_D) \tan^2 \phi_D \sin^2 \eta_D - 2 \tau_{xzD} \tan \phi_D \tan \eta_D \tag{272}$$

This last equation is more often used and simplifies the calculations.

Considering figure 4-51(a), the shear stress parallel to $CB$, on plane $BCO$, is:

$$\tau_{CBD} = \tau_{zxD} \cos \eta_D - \tau_{zyD} \sin \eta_D$$

Considering figure 4-51(b), the shear stress parallel to $AD$ is the resultant shear stress on $ADO$ because the shear stress perpendicular to it is zero.

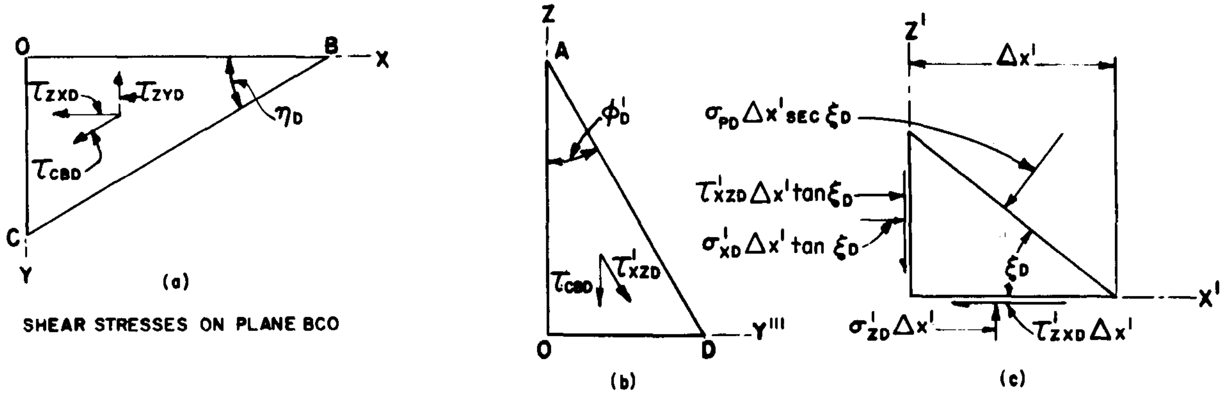


TETRAHEDRON ABCO

STRESSES AT SECTION ADO

STRESSES IN PLANE BCO

*Figure 4-50.* **Axes, angles, and stresses for tetrahedron ABCO shown on figure 4-49.–288-D-2983**



SHEAR STRESSES ON PLANE BCO

SHEAR STRESSES ON SECTION ADO

STRESSES IN X'Z' PLANE

*Figure 4-51.* **Stresses on tetrahedron ABCO shown on figure 4-49.–288-D-2984**

$$\tau'_{xzD} = \tau_{CBD} \sec \phi'_D$$

$$\tau'_{xzD} = \tau'_{zxD} = (\tau_{xzD} \cos \eta_D - \tau_{zyD} \sin \eta_D) \sec \phi'_D \qquad (273)$$

Considering figure 4-51(c), two of the principal stresses are given by the formula,

$$\sigma_{pD} = \frac{\sigma'_{zD} + \sigma'_{xD}}{2} \pm \sqrt{\left(\frac{\sigma'_{zD} - \sigma'_{xD}}{2}\right)^2 + (\tau'_{xzD})^2} \qquad (274)$$

Where $(\sigma'_{zD} - \sigma'_{xD}) > 0$, use + for radical sign.

Where $(\sigma'_{zD} - \sigma'_{xD}) < 0$, use – for radical sign.

$$\tan 2\xi_D = \frac{2\tau'_{xzD}}{\sigma'_{zD} - \sigma'_{xD}}$$

If $\tan 2\xi_D$ is +, $0 < \xi_D < 45^\circ$

If $\tan 2\xi_D$ is –, $-45^\circ < \xi_D < 0$ (275)

Positive angles are measured clockwise from the $Z'$ axis on the left part of the dam and counterclockwise on the right part of the dam, looking upstream.

The two values of $\sigma_{pD}$ given by equation (274) and the normal pressure $p_D$ are the three principal stresses at the face of the dam. The maximum shear stress acts on a plane that bisects the angle between the largest and the smallest of the principal stresses. It is equal to one-half the difference between the principal stresses.

A summary of stresses calculated at the downstream face of the dam, with symbols and equation numbers, is tabulated below:

| | Symbol | Equation |
|---|---|---|
| Vertical cantilever stress | $\sigma_{zD}$ | (261) |
| Horizontal arch stress | $\sigma_{xD}$ | (262) |
| Tangential shear on horizontal plane | $\tau_{zxD}$ | (263) |
| Radial shear on vertical plane | $\tau_{xyD}$ | (264) |
| Radial shear on horizontal plane | $\tau_{zyD}$ | (265) |
| Radial shear on abutment plane | $\tau_{ryD}$ | (268) |
| Horizontal radial stress | $\sigma_{yD}$ | (266) |
| Cantilever stress parallel to face | $\sigma''_{zD}$ | (267) |
| Cantilever stress parallel to Z' axis | $\sigma'_{zD}$ | (270) |
| Arch stress parallel to intrados | $\sigma'_{xD}$ | (271), (272) |
| Tangential shear parallel to intrados | $\tau'_{zxD}$ | (273) |
| Principal stress | $\sigma_{pD}$ | (274) |

If the extrados and intrados curves are concentric circular arches, the angle $\phi'_D$ becomes $\phi_D$ and equation (270) gives the cantilever stress parallel to the downstream face.

(c) *Stresses at Upstream Face.*—Stresses at the upstream face are derived in the same way as those at the downstream face. The resulting equations are as follows:

$$\sigma_{zE} = \frac{W}{A_{CA}} + \frac{M}{I_{CA}} \bullet lg \tag{276}$$

$$\sigma_{xE} = \frac{H_A}{A_A} + \frac{M_A}{I_A} \bullet \frac{T}{2} \tag{277}$$

$$\tau_{xzE} = \tau_{zxE} = -\frac{V_{TA}}{A_{CA}} - \frac{\cancel{M}}{I_{CA}} \bullet lg \tag{278}$$

$$\tau_{xyE} = \tau_{yxE} = -\ (\sigma_{xE} - p_E) \tan \eta_E + \tau_{xzE} \tan \phi_E \tag{279}$$

$$\tau_{yzE} = \tau_{zyE} = -\ (\sigma_{zE} - p_E) \tan \phi_E + \tau_{xzE} \tan \eta_E \tag{280}$$

$$\tau_{ryE} = \tau_{yzE} \sin \psi + \tau_{xyE} \cos \psi \tag{281}$$

$$\sigma_{yE} = p_E - \tau_{xyE} \tan \eta_E - \tau_{yzE} \tan \phi_E \tag{282}$$

$$\sigma''_{zE} = \sigma_{zE} \sec^2 \phi_E - p_E \tan^2 \phi_E + (\sigma_{xE} - p_E) \tan^2 \eta_E \sin^2 \phi_E - 2\,\tau_{xzE} \tan \eta_E \tan \phi_E \tag{283}$$

$$\phi'_E = \tan^{-1} (\tan \phi_E \cos \eta_E) \tag{284}$$

$$\sigma'_{zE} = \sigma_{zE} \sec^2 \phi_E - p_E \tan^2 \phi_E \tag{285}$$

$$\sigma'_{xE} = \sigma_{xE} \cos^2 \eta_E + \sigma_{yE} \sin^2 \eta_E - 2\, \tau_{xyE} \sin \eta_E \cos \eta_E \tag{286}$$

Also,

$$\sigma'_{xE} = \sigma_{xE} \sec^2 \eta_E - p_E \tan^2 \eta_E + (\sigma_{zE} - p_E) \tan^2 \phi_E \sin^2 \eta_E - 2\, \tau_{xzE} \tan \phi_E \tan \eta_E \tag{287}$$

$$\tau'_{xzE} = \tau'_{zxE} = (\tau_{xzE} \cos \eta_E + \tau_{yzE} \sin \eta_E) \sec \phi'_E \tag{288}$$

$$\sigma_{pE} = \frac{\sigma'_{zE} + \sigma'_{xE}}{2} \pm \sqrt{\left(\frac{\sigma'_{zE} - \sigma'_{xE}}{2}\right)^2 + (\tau'_{xzE})^2} \tag{289}$$

$$\tan 2\, \xi_E = - \frac{2\, \tau'_{xzE}}{\sigma'_{zE} - \sigma'_{xE}} \tag{290}$$

(d) *Stresses at Faces of Dam Where Abutment Wedge Occurs.*–Arch stresses, designated $\sigma^*_{xD}$, are computed at the wedge abutment using the equations for stresses normal to the upstream radius and will, in this case, be those normal to the nonradial abutment. Therefore, these stresses must be resolved through the angle of the wedge, Ω (shown on fig. 4-52), as well as through η to obtain stresses parallel to the faces. For the purpose of computation, it is assumed that the location of cantilever stresses is coincident with that of arch stresses. The equation of resolution for arch stresses at the downstream face may be stated as follows:

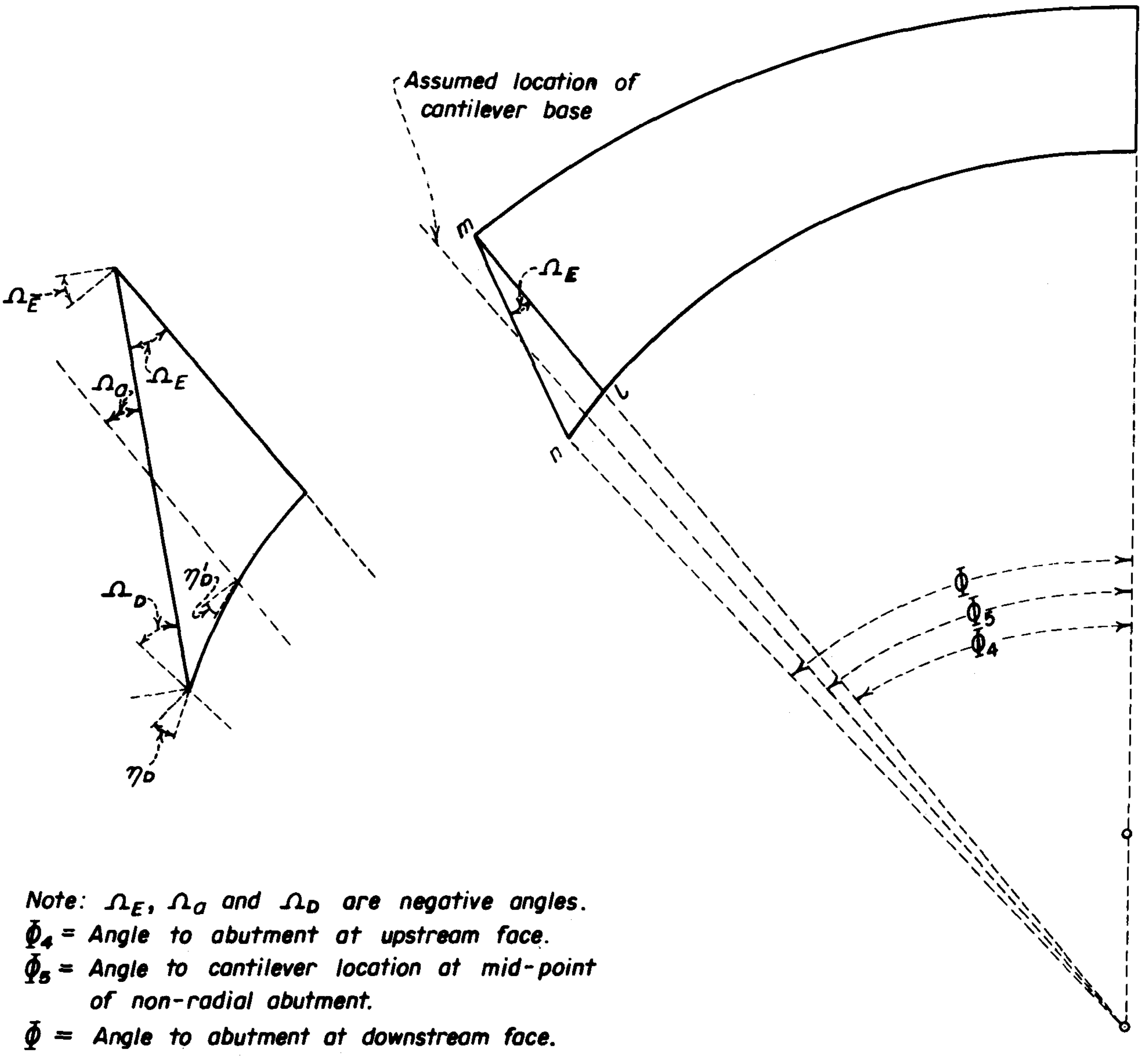
$$\begin{aligned}\sigma_{xD} = {} & \sigma^*_{xD} [\cos \eta_D \sec(\eta_D + \Omega_D) \cos \Omega_D ] \\ & + [\tau^*_{xyD} - \tau_{zxD} \tan \phi_D ] [\cos \eta_D \sec(\eta_D + \Omega_D) \sin \Omega_D ] \\ & - p_D [\sin \Omega_D \sec(\eta_D + \Omega_D) \sin \eta_D ]\end{aligned} \tag{291}$$

$\sigma'_{xD}$ is computed as in equation (272).

For shear stresses parallel to the abutment:

$$\begin{aligned}\tau^*_{xyD} = {} & (\sigma^*_{xD} - p_D) \tan (\eta_D + \Omega_D) - \tau_{xzD} \left[\frac{\tan \eta_D \sin \Omega_D}{\cos(\eta_D + \Omega_D)} + \sec \eta_D\right] \tan \phi_D \\ & - \tau_{yzD} \left[\frac{\cos \eta_D \sin \Omega_D}{\cos (\eta_D + \Omega_D)}\right] \tan \phi_D\end{aligned} \tag{292}$$

$$\tau^*_{yzD} = \tau_{zyD} \cos \Omega_D - \tau_{zxD} \sin \Omega_D \tag{293}$$



*Note:* $\Omega_E$, $\Omega_a$ *and* $\Omega_D$ *are negative angles.*
$\Phi_4$ *= Angle to abutment at upstream face.*
$\Phi_5$ *= Angle to cantilever location at mid-point of non-radial abutment.*
$\Phi$ *= Angle to abutment at downstream face.*

*Figure 4-52.* **Resolution of stress at a nonradial abutment (variable-thickness arch with a triangular abutment wedge).–288-D-2985**

For shear stresses in the plane of the abutment:

$$\tau^*_{ryD} = \tau^*_{yzD} \sin \psi + \tau^*_{xyD} \cos \psi \tag{294}$$

Stresses $\sigma_{zD}$, $\sigma'_{zD}$, $\sigma''_{zD}$, $\tau_{zyD}$, and $\tau_{xzD}$ are computed as described in section 4-45(b). These stresses are determined at the radial cantilever location and are thus not affected by the angle of the nonradial abutment. Stress $\sigma^*_{xE}$ may be computed normal to, and $\tau_{xyE}$ parallel to, a radius through point $m$ (shown on fig. 4-52). The equation for $\sigma'_{xE}$ is then applicable. Stress $\sigma^*_{xE}$ may also be computed at the same point using nonradial abutment data. Stresses $\sigma_{zE}$, $\sigma'_{zE}$, $\sigma''_{zE}$, $\tau_{zyE}$, and $\tau_{xzE}$ are computed as described in section 4-45(c). The resolution of shear stresses parallel to the nonradial abutment at the upstream face is:

$$\tau^*_{xyE} = (\sigma^*_{xE} - p_E) \tan \Omega_E + \tau_{xyE}(1 + \tan \eta_E \tan \Omega_E) + \tau_{zyE} \tan \phi_E \tan \Omega_E \tag{295}$$

$$\tau^*_{yzE} = \tau_{zyE} \cos \Omega_E - \tau_{zxE} \sin \Omega_E \tag{296}$$

and for shear stresses in the abutment plane:

$$\tau^*_{ryE} = \tau^*_{yzE} \sin \psi + \tau^*_{xyE} \cos \psi \tag{297}$$

**4-46.** ***Stresses on Horizontal and Vertical Planes.***—(a) *Procedure.*—After stresses have been evaluated for the faces of a dam, equations may be derived for stresses on horizontal planes and vertical radial and tangential planes at any interior point in the dam. Since horizontal arch stresses $\sigma_x$, vertical cantilever stresses $\sigma_z$, and tangential cantilever shear stresses $\tau_{zx}$ are assumed to have a linear variation from the upstream face to the downstream face, equations can be derived for radial cantilever shear stresses $\tau_{zy}$, radial arch shear stresses $\tau_{xy}$, and radial stresses normal to the vertical tangential planes $\sigma_y$. Since $\tau_{xy}$ equals $\tau_{yx}$, $\tau_{xz}$ equals $\tau_{zx}$, and $\tau_{yz}$ equals $\tau_{zy}$, it is evident that the six stresses, $\sigma_x, \sigma_y, \sigma_z$, $\tau_{xy}$, $\tau_{xz}$, and $\tau_{yz}$ determine all stresses acting on three mutually perpendicular planes. These planes are a vertical radial plane $YZ$, a vertical tangential plane $XZ$, and a horizontal plane $XY$.

(b) *Normal Stresses on Horizontal Plane for Cantilevers and Arches.*—Vertical cantilever stresses on a horizontal plane, shown on figure 4-53(b), may be calculated by the equation,

$$\sigma_z = \frac{W}{A_{CA}} + \frac{M}{I_{CA}} [y - (T - lg)] = \sigma_{zD} + \frac{M}{I_{CA}} \bullet y \tag{298}$$

Horizontal arch stresses on a vertical radial plane, shown on figure 4-54(b), may be calculated by the equation,

$$\sigma_x = \frac{H_A}{A_A} + \frac{M_A}{I_A} \left(y - \frac{T}{2}\right) = \sigma_{xD} + \frac{M_A}{I_A} \bullet y \tag{299}$$

(c) *Tangential and Radial Shear Stresses on Horizontal Plane for Cantilevers.*—Horizontal cantilever shear stresses, acting in tangential directions on horizontal planes, may be computed by the equation,

$$\tau_{zx} = \tau_{xz} = -\frac{V_{TA}}{A_{CA}} - \frac{\cancel{M}}{I_{CA}} [y - (T - lg)] = \tau_{xzD} - \frac{\cancel{M}}{I_{CA}} \bullet y \tag{300}$$

Formulas for horizontal cantilever shear stresses, acting in radial directions on horizontal planes, as shown on figure 4-53(b), may be derived in the following manner. Assume a parabolic distribution of shear and let

$$\tau_{zy} = \tau_{yz} = a_1 + b_1 y + c_1 y^2$$

$$\text{When } y = 0, \tau_{yzD} = a_1 \tag{301}$$

$$\text{When } y = T, \tau_{yzE} = \tau_{yzD} + b_1 T + c_1 T^2 \tag{302}$$

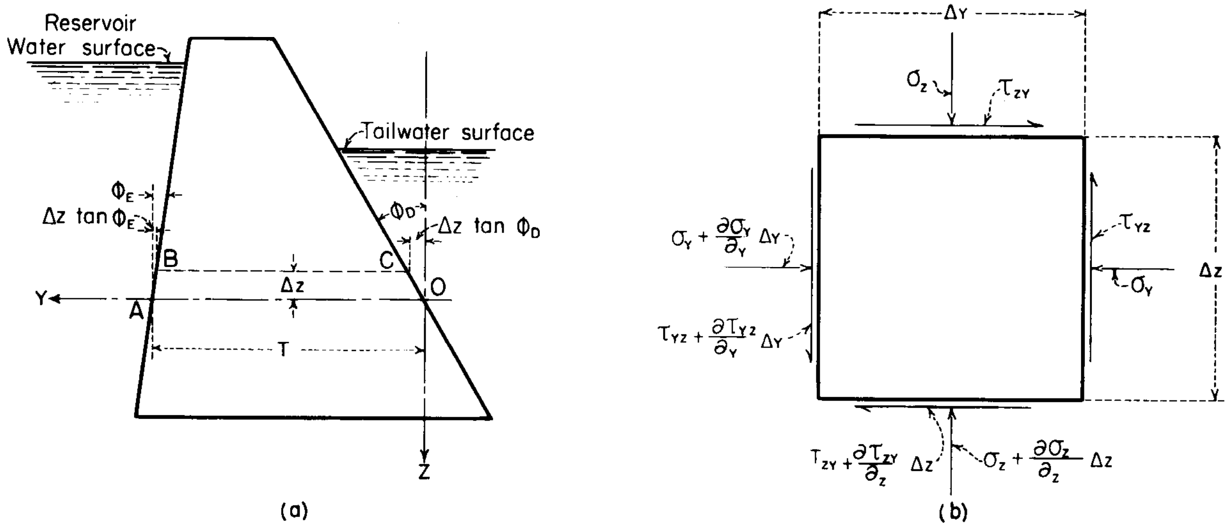


*Figure 4-53.* **Vertical cantilever element.–288-D-444**

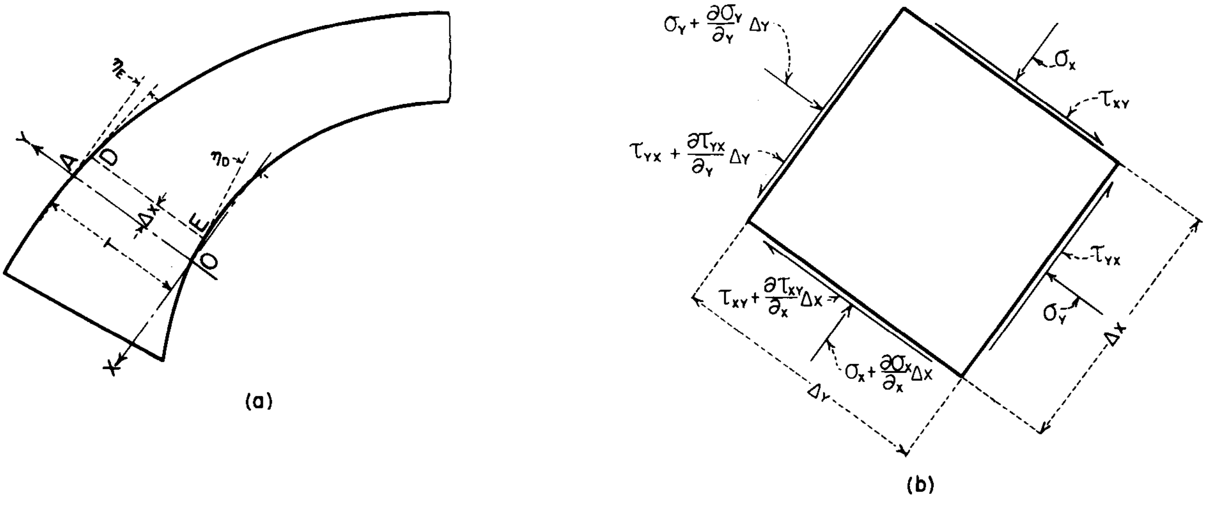


*Figure 4-54.* **Horizontal arch element.–288-D-445**

The total shear on a parallel-side horizontal plane, 1 foot wide, between the origin and any point $y$ is:

$$\int_0^y \tau_{yz}\, dy = a_1\, y \ + b_1 \frac{y^2}{2} \ + c_1 \frac{y^3}{3}$$

In considering horizontal planes of cantilevers with radial sides, it is assumed that the total shear on a plane 1 foot wide at the axis is equal to the total shear on a horizontal plane with parallel sides 1 foot apart.

$$V_{\mathcal{L}} = + \frac{R_{axis}}{r} V_{CA} \tag{303}$$

$$\int_0^T \tau_{yz}\, dy = -\, V_{\text{℄}} = \tau_{yzD}\, T + b_1 \frac{T^2}{2} + c_1 \frac{T^3}{3}$$

$$b_1 = -\frac{2\, V_{\text{℄}}}{T^2} - \frac{2\, \tau_{yzD}}{T} - \frac{2\, c_1\, T}{3} \tag{304}$$

Substituting the above value of $b_1$ in equation (302),

$$c_1 = \frac{3\, \tau_{yzE}}{T^2} + \frac{3\, \tau_{yzD}}{T^2} + \frac{6\, V_{\text{℄}}}{T^3} \tag{305}$$

Substituting the above value of $c_1$ in equation (304),

$$b_1 = -\frac{2\, V_{\text{℄}}}{T^2} - \frac{2\, \tau_{yzD}}{T} - \frac{2\, \tau_{yzE}}{T} - \frac{2\, \tau_{yzD}}{T} - \frac{4\, V_{\text{℄}}}{T^2} = -\frac{6\, V_{\text{℄}}}{T^2} - \frac{2\, \tau_{yzE}}{T} - \frac{4\, \tau_{yzD}}{T} \tag{306}$$

Therefore,

$$\tau_{yz} = \tau_{yzD} - y \left( \frac{6\, V_{\text{℄}}}{T^2} + \frac{2\, \tau_{yzE}}{T} + \frac{4\, \tau_{yzD}}{T} \right) + y^2 \left( \frac{3\, \tau_{yzE}}{T^2} + \frac{3\, \tau_{yzD}}{T^2} + \frac{6\, V_{\text{℄}}}{T^3} \right)$$

$$\begin{aligned} \tau_{zy} = \tau_{yz} = \tau_{yzD} &- y \left[ \frac{1}{T} \left( \frac{6\, V_{\text{℄}}}{T} + 2\, \tau_{yzE} + 4\, \tau_{yzD} \right) \right] \\ &+ y^2 \left[ \frac{1}{T^2} \left( 3\, \tau_{yzE} + 3\, \tau_{yzD} + \frac{6\, V_{\text{℄}}}{T} \right) \right] \end{aligned} \tag{307}$$

Values of $\tau_{yzE}$ and $\tau_{yzD}$ are given by equations (280) and (265), respectively.

The value of $y$ for maximum $\tau_{zy}$ is given by the equation,

$$y = \left[ \frac{\dfrac{6\, V_{\text{℄}}}{T} + 2\, \tau_{zyE} + 4\, \tau_{zyD}}{\dfrac{6\, V_{\text{℄}}}{T} + 3\, \tau_{zyE} + 3\, \tau_{zyD}} \right] \frac{T}{2} \tag{308}$$

When $T < y < 0$, the point of maximum shear falls outside the section, and the shear is equal to $\tau_{zyE}$ or $\tau_{zyD}$ in equation (307), whichever is the larger.

For a dam with uniform-thickness circular arches and vertical faces, maximum

$$\tau_{zy} = -\frac{3}{2} \cdot \frac{V_{\text{℄}}}{T}$$

In equation (303) the value for $V_{CA}$ is obtained by multiplying radial shears due to unit radial loads by the trial loads on the cantilever, sometimes called load factors, and adding total radial shears due to initial loads on the cantilever.

If the sign of the second derivative of equation (307) is positive, the value of $\tau_{zy}$ is a minimum on the shear curve. If the sign is negative, $\tau_{zy}$ is a maximum on the shear curve. The sign of the second derivative is the same as that of the term,

$$\left(\frac{6\,V_{\text{℄}}}{T} + 3\,\tau_{zyE} + 3\,\tau_{zyD}\right)$$

(d) *Radial Shear on Vertical Plane for Arches.*—An equation for horizontal arch shear stress acting in a radial direction on a vertical radial plane, as shown on figure 4-54(a), may be derived by the procedure followed in deriving equation (307). The formula obtained is:

$$\tau_{xy} = \tau_{yx} = \tau_{xyD} - y\left[\frac{1}{T}\left(\frac{6\,V_A}{T} + 2\,\tau_{xyE} + 4\,\tau_{xyD}\right)\right] + y^2\left[\frac{1}{T^2}\left(3\,\tau_{xyE} + 3\,\tau_{xyD} + \frac{6\,V_A}{T}\right)\right] \qquad (309)$$

Values of $\tau_{xyE}$ and $\tau_{xyD}$ are given by equations (279) and (264), respectively.

The value of $y$ for maximum $\tau_{xy}$ is given by the expression,

$$y = \frac{T}{2}\left[\frac{\dfrac{6\,V_A}{T} + 2\,\tau_{xyE} + 4\,\tau_{xyD}}{\dfrac{6\,V_A}{T} + 3\,\tau_{xyE} + 3\,\tau_{xyD}}\right] \qquad (310)$$

When $T < y < 0$, the point of maximum shear on the shear curve falls outside the section. Therefore, the shear is equal to $\tau_{xyE}$ or $\tau_{xyD}$ in equation (309), whichever is the larger.

For a dam with uniform-thickness circular arches, and vertical faces, maximum

$$\tau_{xy} = -\frac{3}{2}\cdot\frac{V_A}{T}$$

The value for $V_A$ is obtained by multiplying arch shears due to unit loads by load factors for that particular arch. The sign of the second derivative is the same as that of the term,

$$\left(\frac{6\,V_A}{T} + 3\,\tau_{zyE} + 3\,\tau_{zyD}\right)$$

(e) *Horizontal Radial Stresses on Vertical Tangential Planes.*—Horizontal radial stresses acting on vertical tangential planes may be computed by the formula,

$$\sigma_y = \int_0^y \frac{\partial \tau_{yz}}{\partial z}\,dy + \int_0^y \frac{\partial \tau_{xy}}{\partial x}\,dy + \sigma_{yD} \qquad (311)$$

Integral terms in equation (311) may be computed from the values of $\tau_{yz}$ and $\tau_{xy}$ given by equations (307) and (309), respectively. Since $y$ is measured from the downstream face, it is a function of $z$ and must be considered in differentiating $\tau_{yz}$ and $\tau_{xy}$. For example, the bracket term multiplied by $y$, equation (307), is the product of two functions of $z$ and is differentiated by the usual rule as follows:

$$\frac{\partial}{\partial z} y \left[\frac{1}{T}\left(\frac{6\,V_{℄}}{T} + 2\,\tau_{yzE} + 4\,\tau_{yzD}\right)\right]$$

$$= y \left[6\,\frac{T^2 \dfrac{\partial V_{℄}}{\partial z} - 2\,V_{℄}\,T \dfrac{\partial T}{\partial z}}{T^4} + 2\,\frac{T \dfrac{\partial \tau_{yzE}}{\partial z} - \tau_{yzE} \dfrac{\partial T}{\partial z}}{T^2} + 4\,\frac{T \dfrac{\partial \tau_{yzD}}{\partial z} - \tau_{yzD} \dfrac{\partial T}{\partial z}}{T^2}\right]$$

$$+ \frac{\partial y}{\partial z}\left[\frac{1}{T}\left(\frac{6\,V_{℄}}{T} + 2\,\tau_{yzE} + 4\tau_{yzD}\right)\right] \tag{312}$$

Following this method, $\tau_{yz}$ is first differentiated with respect to $z$.

$$\frac{\partial \tau_{yz}}{\partial z} = \frac{\partial y}{\partial z}\left[\frac{6y\,\tau_{yzE}}{T^2} + \frac{6y\,\tau_{yzD}}{T^2} + \frac{12\,y\,V_{℄}}{T^3} - \frac{2\,\tau_{yzE}}{T} - \frac{4\,\tau_{yzD}}{T} - \frac{6\,V_{℄}}{T^2}\right]$$

$$+ \frac{\partial T}{\partial z}\left[\frac{2y\,\tau_{yzE}}{T^2} + \frac{4y\,\tau_{yzD}}{T^2} + \frac{12y\,V_{℄}}{T^3} - \frac{6y^2\,\tau_{yzE}}{T^3} - \frac{6y^2\,\tau_{yzD}}{T^3} - \frac{18y^2\,V_{℄}}{T^4}\right]$$

$$+ \frac{\partial \tau_{yzE}}{\partial z}\left[\frac{3y^2}{T^2} - \frac{2y}{T}\right] + \frac{\partial \tau_{yzD}}{\partial z}\left[1 + \frac{3y^2}{T^2} - \frac{4y}{T}\right] - \frac{\partial\,V_{℄}}{\partial z}\left[\frac{6y}{T^2} - \frac{6y^2}{T^3}\right] \tag{313}$$

This expression is then integrated with respect to $y$.

$$\int_0^y \frac{\partial \tau_{yz}}{\partial z}\,dy = \frac{\partial y}{\partial z}\left[\frac{3y^2\,\tau_{yzE}}{T^2} + \frac{3y^2\,\tau_{yzD}}{T^2} + \frac{6\,y^2\,V_{℄}}{T^3} - \frac{2y\,\tau_{yzE}}{T} - \frac{4y\,\tau_{yzD}}{T} - \frac{6y\,V_{℄}}{T^2}\right]$$

$$+ \frac{\partial T}{\partial z}\left[\frac{y^2\,\tau_{yzE}}{T^2} + \frac{2y^2\,\tau_{yzD}}{T^2} + \frac{6y^2\,V_{℄}}{T^3} - \frac{2y^3\,\tau_{yzE}}{T^3} - \frac{2y^3\,\tau_{yzD}}{T^3} - \frac{6y^3\,V_{℄}}{T^4}\right]$$

$$+ \frac{\partial \tau_{yzE}}{\partial z}\left[\frac{y^3}{T^2} - \frac{y^2}{T}\right] + \frac{\partial \tau_{yzD}}{\partial z}\left[y + \frac{y^3}{T^2} - \frac{2y^2}{T}\right]$$

$$- \frac{\partial\,V_{℄}}{\partial z}\left[\frac{3y^2}{T^2} - \frac{2y^3}{T^3}\right] \tag{314}$$

$$\begin{aligned}\int_0^y \frac{\partial \tau_{yz}}{\partial z}\,dy = {} & \frac{\partial y}{\partial z}\bullet\frac{y}{T}\left[\frac{3y}{T}(\tau_{yzE}+\tau_{yzD})-\frac{6\,V_{\text{℄}}}{T}\left(1-\frac{y}{T}\right)-2\,\tau_{yzE}-4\,\tau_{yzD}\right]\\ & +\frac{\partial T}{\partial z}\bullet\frac{y^2}{T^2}\left[\tau_{yzE}+2\,\tau_{yzD}+\frac{6\,V_{\text{℄}}}{T}\left(1-\frac{y}{T}\right)-\frac{y}{T}(2\,\tau_{yzE}+2\,\tau_{yzD})\right]\\ & +\frac{\partial \tau_{yzE}}{\partial z}\bullet y\left[\frac{y^2}{T^2}-\frac{y}{T}\right]+\frac{\partial \tau_{yzD}}{\partial z}\bullet y\left[1+\frac{y^2}{T^2}-\frac{2y}{T}\right]\\ & -\frac{\partial\,V_{\text{℄}}}{\partial z}\bullet\frac{y^2}{T^2}\left[3-\frac{2y}{T}\right]\end{aligned} \tag{315}$$

$$\frac{\partial y}{\partial z}=\tan\phi_D \tag{316}$$

$$\frac{\partial T}{\partial z}=\tan\phi_E+\tan\phi_D \tag{317}$$

The terms $\frac{\partial \tau_{yzE}}{\partial z}$ and $\frac{\partial \tau_{yzD}}{\partial z}$ are evaluated by assuming a linear variation of stress between arch elevations.

$$-\frac{\partial\,V_{\text{℄}}}{\partial z}=\left[\text{Horizontal cantilever trial load at section}\right]\ \frac{R_E}{r}$$

The same procedure is followed in deriving a formula for the second integral term of equation (311), obtaining:

$$\begin{aligned}\int_0^y \frac{\partial \tau_{xy}}{\partial x}\,dy = {} & \frac{\partial y}{\partial x}\bullet\frac{y}{T}\left[\frac{3y}{T}(\tau_{xyE}+\tau_{xyD})-\frac{6\,V_A}{T}\left(1-\frac{y}{T}\right)-2\,\tau_{xyE}-4\,\tau_{xyD}\right]\\ & +\frac{\partial T}{\partial x}\bullet\frac{y^2}{T^2}\left[\tau_{xyE}+2\,\tau_{xyD}+\frac{6\,V_A}{T}\left(1-\frac{y}{T}\right)-\frac{y}{T}\left(2\,\tau_{xyE}+2\,\tau_{xyD}\right)\right]\\ & +\frac{\partial \tau_{xyE}}{\partial x}\bullet y\left[\frac{y^2}{T^2}-\frac{y}{T}\right]+\frac{\partial \tau_{xyD}}{\partial x}\bullet y\left[1+\frac{y^2}{T^2}-\frac{2y}{T}\right]\\ & -\frac{\partial\,V_A}{\partial x}\bullet\frac{y^2}{T^2}\left[3-\frac{2y}{T}\right]\end{aligned} \tag{318}$$

$$\frac{\partial y}{\partial x}=\tan\eta_D \tag{319}$$

$$\frac{\partial T}{\partial x} = \tan \eta_E + \tan \eta_D \qquad (320)$$

Values of $\frac{\partial \tau_{xyE}}{\partial x}$ and $\frac{\partial \tau_{xyD}}{\partial x}$ are determined by assuming a linear variation of stress between trial cantilevers.

$$-\frac{\partial V_A}{\partial x} = \text{horizontal arch trial load at section} \qquad (321)$$

If temperature changes are being considered in the analysis, equation (321) must be amplified to include temperature effects. The derivation of $\sigma_{yD}$ is explained in section 4-45(b), and its value is given by equation (266). When $y$ equals $T$, $\sigma_y$ becomes $\sigma_{yE}$, which is determined by equation (282).

The sum of equations (315), (318), and (266) gives equation (311).

(f) *Summary of Stresses.*—A summary of stresses calculated on horizontal and vertical planes, with symbols and equation numbers, is tabulated below.

| | Symbol | Equation |
|---|---|---|
| Vertical cantilever stress | $\sigma_z$ | (298) |
| Horizontal arch stress | $\sigma_x$ | (299) |
| Tangential shear on horizontal plane | $\tau_{zx}$ | (300) |
| Radial shear on horizontal plane, upstream face | $\tau_{zyE}$ | (280) |
| Radial shear on horizontal plane, downstream face | $\tau_{zyD}$ | (265) |
| Radial shear on horizontal plane | $\tau_{zy}$ | (307) |
| Radial shear on vertical plane, upstream face | $\tau_{xyE}$ | (279) |
| Radial shear on vertical plane, downstream face | $\tau_{xyD}$ | (264) |
| Radial shear on vertical plane | $\tau_{xy}$ | (309) |
| Radial horizontal stress on vertical plane | $\sigma_y$ | (311) |
| Radial horizontal stress, upstream face | $\sigma_{yE}$ | (282) |
| Radial horizontal stress, downstream face | $\sigma_{yD}$ | (266) |

**4-47.** ***Stresses on Other Planes.***—(a) *Stresses on Abutment Plane.*—Maximum horizontal shear stresses acting in a radial direction along the abutment rock plane are important and can be calculated by equations given below. These equations are derived in a similar manner to those given in section 4-46(c). Values for the rock plane shears, $\tau_{ryD}$ and $\tau_{ryE}$ are given by equations (268) and (281).

The maximum horizontal shear stress on rock plane acting in a radial direction can be found by the following equation:

$$\tau_{ry} = \tau_{ryD} - \frac{y}{T}\left(\frac{6\,V_r}{T} + 2\,\tau_{ryE} + 4\,\tau_{ryD}\right) + \frac{y^2}{T^2}\left(\frac{6\,V_r}{T} + 3\,\tau_{ryE} + 3\,\tau_{ryD}\right) \qquad (322)$$

when $y < T$ and $> 0$. If $y > T$ or $< 0$, the point of maximum shear falls outside of the section; and the shear is equal to $\tau_{ryE}$ or $\tau_{ryD}$ whichever is the larger value. In the above equation the value of $V_r$ is:

$$V_r = V \cos \psi$$

where:

$$V = V_A + V_{CA} \left(\frac{R_{axis}}{r}\right) \tan \psi$$

The sign of the second derivative is used in a manner similar to that shown in section 4-46(c). The value $y$ is:

$$y = \frac{T}{2}\left[\frac{\frac{6\,V_r}{T} + 2\,\tau_{ryE} + 4\,\tau_{ryD}}{\frac{6\,V_r}{T} + 3\,\tau_{ryE} + 3\,\tau_{ryD}}\right] \tag{323}$$

For a dam with uniform-thickness circular arches and vertical faces, the maximum shear is:

$$\tau_{ry} = -\frac{3}{2} \bullet \frac{V_r}{T}$$

(b) *Stresses on Inclined Circumferential Area.*—If a dam is to be increased in height, it may be necessary to add new concrete on the downstream face. As this introduces a possible plane of weakness, investigations of normal and shear stresses along the face are required.

Equations for normal and shear stresses on an inclined circumferential area, which is parallel to the $X$ axis and inclined to the $Y$ and $Z$ axes, are determined by using the six stresses, $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{xz}$, and $\tau_{yz}$ considering the elemental area as a plane. Forces acting on an element, shown on figure 4-55, are equated to derive stress equations. It is assumed that forces on planes $BFC$ and $AED$ are self-balancing. It must be remembered that $\tau_{xy}$ equals $\tau_{yx}$, $\tau_{xz}$ equals $\tau_{zx}$, and $\tau_{yz}$ equals $\tau_{zy}$.

Considering the forces in figure 4-55, the following stress equations are found:

Horizontal shear stress on inclined plane,

$$\tau_1 \sec \zeta \,\Delta x\,\Delta z = \tau_{xy}\,\Delta x\,\Delta z + \tau_{xz} \tan \zeta\,\Delta x\,\Delta z$$

$$\tau_1 = \tau_{xy} \cos \zeta + \tau_{xz} \sin \zeta \tag{324}$$

Shear stress on inclined plane, acting at right angles to horizontal shear stress,

$$\tau_2 \sec \zeta\,\Delta x\,\Delta z = \tau_{zy} \cos \zeta\,\Delta x\,\Delta z - \tau_{zy} \tan \zeta \sin \zeta\,\Delta x\,\Delta z$$

$$- \sigma_z \tan \zeta \cos \zeta\,\Delta x\,\Delta z + \sigma_y \sin \zeta\,\Delta x\,\Delta z$$

$$\tau_2 = \tau_{zy}(\cos^2 \zeta - \sin^2 \zeta) - \sin \zeta \cos \zeta(\sigma_z - \sigma_y) \tag{325}$$

Maximum shear stress in inclined plane,

$$\tau_m = \sqrt{\tau_1{}^2 + \tau_2{}^2} \tag{326}$$

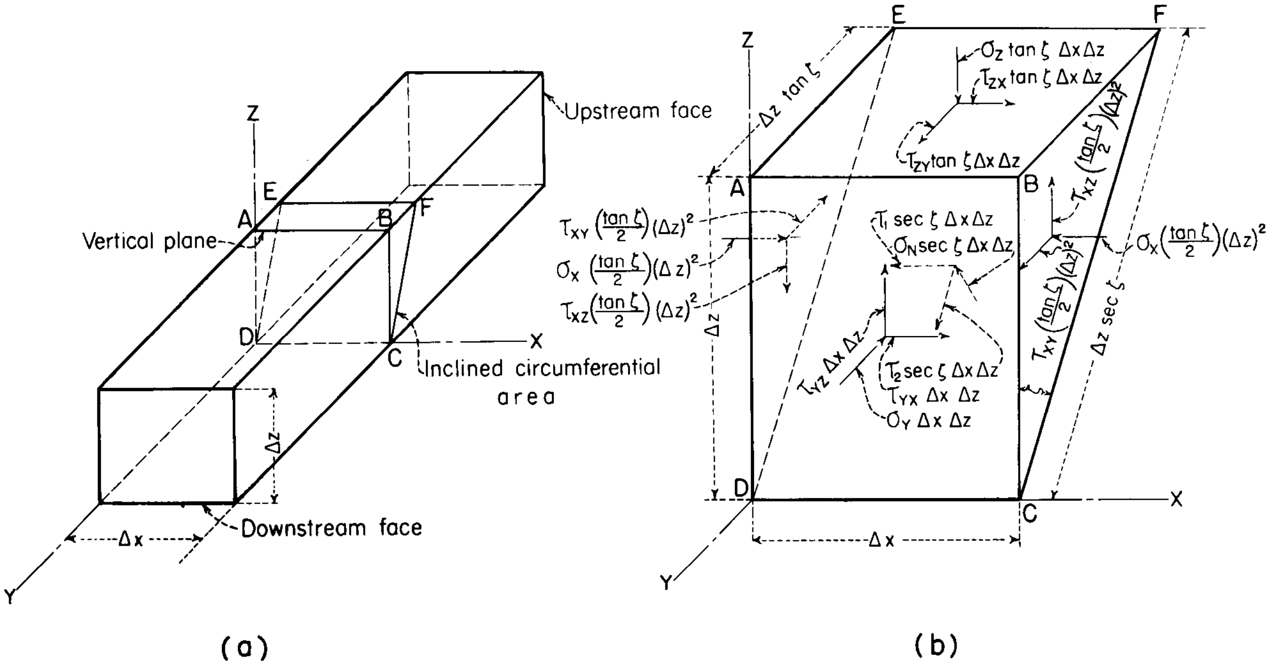


*Figure 4-55.* **Stresses on inclined circumferential area.—288-D-448**

Angle maximum shear stress makes with $\tau_1$,

$$\omega = \tan^{-1} \frac{\tau_2}{\tau_1} \tag{327}$$

Normal stress on inclined plane,

$$\sigma_N \sec \zeta \, \Delta x \, \Delta z = \sigma_z \tan \zeta \sin \zeta \, \Delta x \, \Delta z + \sigma_y \cos \zeta \, \Delta x \, \Delta z$$
$$- \tau_{zy} \, \Delta x \, \Delta z \, (\tan \zeta \cos \zeta + \sin \zeta)$$

$$\sigma_N = \sigma_z \sin^2 \zeta + \sigma_y \cos^2 \zeta - 2 \, \tau_{zy} \sin \zeta \cos \zeta \tag{328}$$

The preceding formulas for stresses on an inclined circumferential area may be checked by substituting direction cosines, given in the following section.

(c) *Stresses on Oblique Plane.*—Equations for normal and shear stresses on an oblique plane are derived so that stresses in any direction can be determined at any point in the dam.

Consider the tetrahedron *OABC* cut from the dam as shown on figure 4-56(a). The three mutually perpendicular planes are taken as coordinate planes, and the unit stresses, not the total forces, are shown on the faces. If *O* is the point where stresses on the *XY*, *XZ*, and *YZ* planes are known, stresses on the plane *ABC* approach the stresses on a plane through *O* parallel to plane *ABC* if the tetrahedron *OABC* is made infinitesimal. Since the element is made very small, body forces can be neglected, and stresses over the sides can be assumed as uniformly distributed.

Let *j*, *m*, and *n* denote the direction cosines of the oblique plane *ABC*. Then,

*j* = cosine of the angle between the normal to *ABC* and the *X* axis,
*m* = cosine of the angle between the normal to *ABC* and the *Y* axis, and
*n* = cosine of the angle between the normal to *ABC* and the *Z* axis.

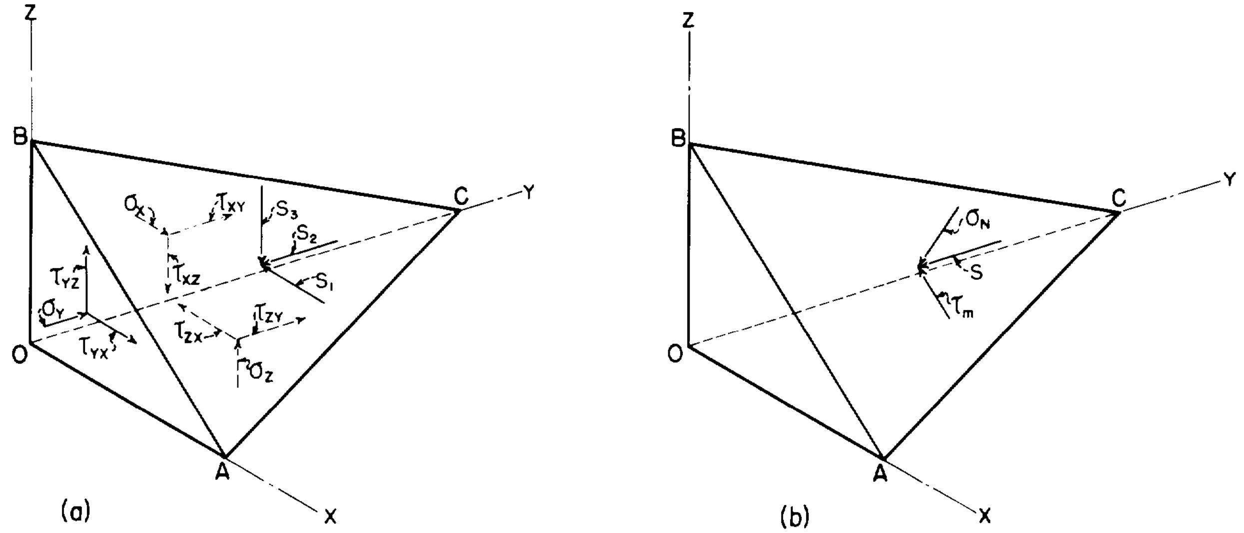


*Figure 4-56.* **Stresses in interior of dam.–288-D-450**

Since the angle between two planes is equal to the angle between normals to the planes,

$$\text{Area } OBC = j\,(\text{area } ABC)$$
$$\text{Area } OAB = m\,(\text{area } ABC)$$
$$\text{Area } OAC = n\,(\text{area } ABC)$$

By equating forces in $X$, $Y$, and $Z$ directions, the following equations can be written:

$$j\,\sigma_x + m\,\tau_{yx} - n\,\tau_{zx} = S_1 \qquad (329)$$

$$m\,\sigma_y + n\,\tau_{zy} + j\,\tau_{xy} = S_2 \qquad (330)$$

$$n\,\sigma_z - j\,\tau_{xz} + m\,\tau_{yz} = S_3 \qquad (331)$$

$S_1$, $S_2$, and $S_3$ are components of the resultant stress on $ABC$ in the $X$, $Y$, and $Z$ directions, respectively. By using these values, normal and maximum shear stresses on plane $ABC$ can be derived.

Normal stress on the oblique plane (fig. 4-56(b)) is determined by the equation,

$$\sigma_N = j\,S_1 + m\,S_2 + n\,S_3$$

$$\sigma_N = j^2\,\sigma_x + j\,m\,\tau_{yx} - j\,n\,\tau_{zx} + m^2\,\sigma_y + m\,n\,\tau_{zy}$$
$$+ j\,m\,\tau_{xy} + n^2\,\sigma_z - j\,n\,\tau_{xz} + m\,n\,\tau_{yz}$$

$$\sigma_N = j^2\,\sigma_x + m^2\,\sigma_y + n^2\,\sigma_x + 2\,j\,m\,\tau_{xy} + 2\,m\,n\,\tau_{yz} - 2\,j\,n\,\tau_{xz} \qquad (332)$$

The maximum shear stress on the oblique plane can be determined as follows: If $S$ = the total stress on plane $ABC$,

$$S^2 = S_1{}^2 + S_2{}^2 + S_3{}^2$$

Then the square of the maximum shear stress can be written,

$$\tau_m{}^2 = S^2 - \sigma_N{}^2, \text{ or}$$

$$\tau_m{}^2 = S_1{}^2 + S_2{}^2 + S_3{}^2 - (j^2 S_1{}^2 + m^2 S_2{}^2 + n^2 S_3{}^2 + 2\,j\,m\,S_1 S_2 + 2\,j\,n\,S_1 S_3 + 2\,m\,n\,S_2 S_3)$$

$$\tau_m{}^2 = (1 - j^2)S_1{}^2 + (1 - m^2)S_2{}^2 + (1 - n^2)S_3{}^2 - 2\,j\,m\,S_1 S_2 - 2\,j\,n\,S_1 S_3 - 2\,m\,n\,S_2 S_3 \qquad (333)$$

Therefore, $\tau_m$ can be determined by using equations (329), (330), (331), and (333).

Since, $S_1$, $S_2$, and $S_3$ are components of the total stress $S$, and $\sigma_N j$, $\sigma_N m$, and $\sigma_N n$ are components of the stress $\sigma_N$ in the $X$, $Y$, and $Z$ directions, respectively, direction cosines of the maximum shear stress are:

$$j' = \frac{S_1 - \sigma_N\, j}{\tau_m} \qquad (334)$$

$$m' = \frac{S_2 - \sigma_N\, m}{\tau_m} \qquad (335)$$

$$n' = \frac{S_3 - \sigma_N\, n}{\tau_m} \qquad (336)$$

(d) *Principal Stresses in Interior of Dam.*—If the complete stress distribution is required at any point in a dam, principal stresses and maximum shear stresses should be determined. Again consider the tetrahedron *OABC* (fig. 4-56(a)), and let *ABC* be a principal plane with the principal stress, $S$, acting normal to the plane. By substituting $jS$ for $S_1$, $mS$ for $S_2$, and $nS$ for $S_3$ in equations (329), (330), and (331) the following equations can be written:

$$j(\sigma_x - S) + m\,\tau_{yx} - n\,\tau_{zx} = 0 \qquad (337)$$

$$m(\sigma_y - S) + n\,\tau_{zy} + j\,\tau_{xy} = 0 \qquad (338)$$

$$n(\sigma_z - S) - j\,\tau_{xz} + m\,\tau_{yz} = 0 \qquad (339)$$

If $j$, $m$, and $n$ are eliminated and the equations are solved for $S$, the following cubic equation is derived:

$$(\sigma_x - S)(\sigma_y - S)(\sigma_z - S) - (\sigma_x - S)\tau_{yz}{}^2 - (\sigma_y - S)\tau_{xz}{}^2 - (\sigma_z - S)\tau_{xy}{}^2 - 2\tau_{xy}\,\tau_{yz}\,\tau_{xz} = 0$$

This equation may be written,

$$S^3 - (\sigma_x + \sigma_y + \sigma_z)S^2 + (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_x \sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2)S$$
$$- (\sigma_x \sigma_y \sigma_z - 2\tau_{xy} \tau_{yz} \tau_{xz} - \sigma_x \tau_{yz}^2 - \sigma_y \tau_{xz}^2 - \sigma_z \tau_{xy}^2) = 0 \qquad (340)$$

The three roots of equation (340) give the principal stresses. By substituting the stresses in equations (337), (338), and (339), and using the relation $(j^2 + m^2 + n^2) = 1$, the three sets of direction cosines for the three principal planes can be found. The maximum shear stress acts on a plane that bisects the angle between the largest and the smallest principal stresses, and is equal to one-half the difference between these two principal stresses.

Examples of stress calculations for an arch dam are shown in appendix F.

## C. ARCH DAM STRESS ANALYSIS SYSTEM

4-48. *Introduction.*—The need for an acceptably accurate, comprehensive method of analyzing arch dams resulted in the development of the trial-load method discussed in the previous subchapter. The chief limitations of the trial-load method are its complexity and the protracted computations required to make an analysis. These time-consuming computations were programed for the electronic computer and linked together to form the Arch Dam Stress Analysis System (ADSAS). This system permits the stress analysis of an arch dam to be made in a very short time and at low cost.

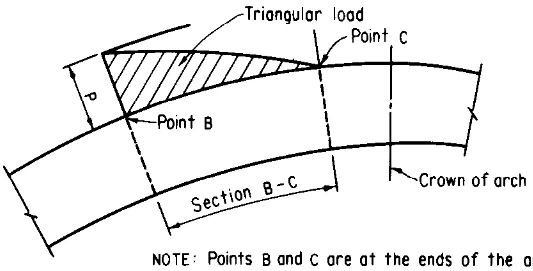
Many comparisons have been made to ensure the reliability of ADSAS. Results from the computerized analysis have been compared with measurements from actual structures [6, 7] and scale models. Recently, a comparison was made between ADSAS results and a three-dimensional finite element analysis with excellent agreement.

4-49. *Comparison With Trial-Load Method.*—In general, ADSAS follows the same procedures used in the trial-load analysis except where solution methods more adaptable to the computer were used. The significant changes from the trial-load method are discussed in the following paragraphs.

One change is in the method of application of the reservoir waterload. In a trial-load analysis the waterload is not applied directly to either arch or cantilever elements, but is divided between them as a part of the radial adjustment loading. In the computer solution, however, all of the external initial loads, including the water loading, are applied to the cantilever elements. Initial deformations of the cantilever elements are computed for these loads. Geometrical continuity is then attained by applying equal but opposite loading to arch and cantilever elements for the radial adjustment as well as for the tangential and twist adjustments. Arch quarter points are used in the trial-load method with unit loads peaking at the abutments (see sec. 4-28(d) and fig. 4-17). The computer program locates the points on the arches by using the intersections of cantilever elements with the arches. In addition, unit loads are peaked at these points and varied linearly to zero at adjacent points on each side.

These changes facilitate the development of a set of simultaneous equations for each adjustment. Matrix solutions are used to solve each set for the loading distributions required to maintain geometric continuity throughout the structure. These solutions replace the trial-and-error procedure used in the trial-load method to determine load distributions.

Unit arch loads for each section of the arch between node points are calculated in the computer program. Uniform and triangular loads are determined for radial, tangential, and twist loadings. Figure 4-57 is a sketch of the radial loads on section *B-C*. Tangential and



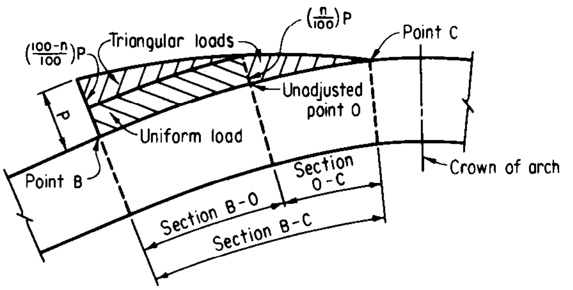
NOTE: Points B and C are at the ends of the arch section as shown. These points may be either unadjusted points used to describe geometry or cantilever-arch intersections.
P=unit radial load ordinate=1,000 pounds per foot.

*Figure 4-57.* **Unit radial loads for arch section, using ADSAS.–288-D-2986**

twist loads are obtained in a similar manner. Their locations are along the same arc used in the trial-load method (see fig. 4-17). These computations are described in section 4-34 for uniform-thickness arches and section 4-35 for variable-thickness arches. Arch constants, load constants, and forces are transferred to other arch points on the same side of the arch as described in section 4-37. Total forces and deflections due to each unit load on each arch section are then computed for the entire arch (see sec. 4-34).

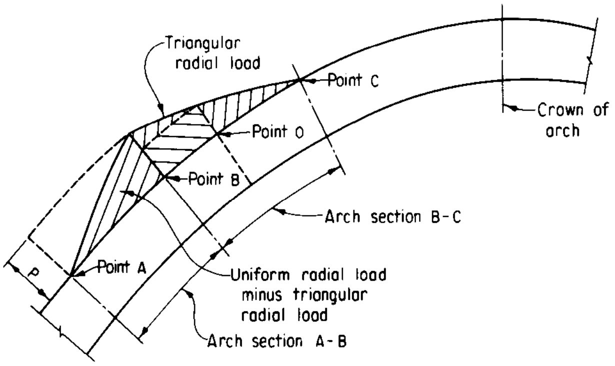
The triangular load for a compound section (see fig. 4-58) is obtained by combining the required percentage *n* of the triangular load on section *O-C* with *n* percent of the uniform load and 100-*n* percent of the triangular load on section *B-O*. The percentage of triangular load on section *O-C* is determined as shown on figure 4-58. The uniform load on a compound section is obtained by combining uniform loads on the two original sections. Unit arch loads peaking at node points and extending linearly to zero at the adjacent node points on either side, as shown on figure 4-59, are obtained by combining loads on two adjacent sections. The triangular load on the section nearest the crown (section *B-C*) is added to the uniform load minus the triangular load on the other section (section *A-B*).

**4-50.** ***Program Organization.***—Organization of ADSAS is described briefly below. The analysis system is divided into eight major parts, some of which are further subdivided as shown.



NOTE: $n = \frac{\text{(length of section O-C)}}{\text{(length of section B-C)}} \times 100$ percent
P = unit radial load ordinate=1,000 pounds per foot.
$(\frac{n}{100})$P = load ordinate at unadjusted point O.
Points B and C are adjusted points on the arch.
Point O is the point of compounding for section B-C.

*Figure 4-58.* **Unit triangular radial load for compound section, using ADSAS.–288-D-2987**



NOTE: The unit radial load peaking at point B is composed of a unit triangular radial load on section B-C plus a unit uniform radial load on section A-B minus a unit triangular radial load on section A-B.
The triangular load may be either as shown on figure 4-57 or figure 4-58.

*Figure 4-59.* **Unit radial arch load, using ADSAS.–288-D-2988**

*Part 1.*—Read card input and produce geometric and abutment data for use by the arch and cantilever programs.

(1) Read card input.

(2) Produce geometric data for one- or two-centered layouts.

(3) Produce geometric data for three-centered layouts.

(4) Compute abutment properties and produce the final output tape from part 1.

*Part 2.*—Compute forces and deflections due to unit loads on the arch elements.

*Part 3.*—Compute forces and deflections due to unit and initial loading on cantilever elements.

*Part 4.*—Compute stresses for a "Crown Adjustment" (optional).

*Part 5.*—Reorganize forces, deflections, and geometric data for the "Complete" analysis.

*Part 6.*—Generate equations for the solution program.

(1) Perform a second reorganization of the data.

(2) Generate elements for three of the nine submatrices which make up the full matrix.

(3) Invert each of the submatrices.

*Part 7.*—Solve the full matrix for the load distribution between arch and cantilever elements.

*Part 8.*—Compute stresses due to the load distribution.

**4-51.** ***Capabilities.***—Although the capabilities of ADSAS are constantly being expanded, the present choice of geometrical configuration and loading covers many of the options.

(a) *Geometry.*—The approach used for the geometrical description of the arch dam is the *solid of revolution* concept. Earlier arch dams were defined by a vertical section at the reference plane to be revolved through specific angles about a vertical *line of centers*. For the more recent arch dams, the centers are allowed to vary by elevation and the angles of revolution also vary by elevation. In the case of multicentered designs, the locations of centers of revolution may also change horizontally. The *reference plane*, used to describe the *crown* cantilever section of the dam and the loci of centers, is usually located at the maximum section which extends to the lowest point in the assumed excavation profile.

The section through the dam and the loci of centers are defined on the reference plane by combinations of circles and straight lines. No restriction is placed on the combinations of circles and straight lines which may be used, except the relatively large limit to the total number of segments per line.

Several configurations are available for handling the horizontal curvature to produce the shape of arch required. These options are listed below:

(1) *Single-centered dam.*—A configuration usually used in narrow, symmetrical sites. The loci of centers are the same for both sides of the dam and lie on the reference plane.

(2) *Two-centered dam.*—A configuration which may be used in relatively narrow sites which are too asymmetrical to permit a good single-centered design. The loci of centers differ from one side of the dam to the other, with the compounding of curvature at the reference plane. The loci of all centers lie on the reference plane.

(3) *Three-centered dam.*—A configuration which may be used to advantage in some of the wider sites. The three-centered dam has shorter radii in the central part and longer radii in the outer segments. Loci for the central portion centers lie on the reference plane. Those for the outer parts do not, although they are described on the reference plane.

Abutment pads may be used with the three-centered geometry. Several line segments can be used in describing abutment pads. The pads may be used on either or both faces of the dam.

(b) *Loading.*—ADSAS can be used to analyze the effects of most any loading which may occur on a dam. The loads which may be included in an analysis are:

(1) Reservoir water loading.

(2) Tailwater loading.

(3) Ice loading.

(4) Temperature change from the closure temperature (uniform from upstream to downstream). The temperature variation from upstream to downstream faces may also be included by using an equivalent linear temperature gradient. The temperature changes and equivalent linear temperature gradients may either be constant by elevation or vary by arch section. The gradients may also vary by cantilever and elevation.

(5) Radial and tangential loads may be applied at node points (intersections of arch and cantilever elements) to include the effects of silt or dynamic response to some selected earthquake.

(6) Dead load stresses are included with total stress and are also computed for a check of stresses due to concrete weight during placement.

(c) *Other Variables.*–In addition to these options, the construction program can be simulated by a series of studies. The results from these studies are then combined to give total stresses in the dam when a construction program is included.

The effects of variations in foundation deformation moduli both vertically and horizontally can be included in the analysis when desired.

A dam may also be analyzed assuming the upper portions of the contraction joints are open and no arch action exists in that area. Loads above the top of the arch action are carried by cantilever action only.

**4-52. *Input Requirements.***–The input of data to ADSAS is by a punched card deck. The list in table 4-6 includes the input-output control cards, the geometric description cards required, and the loading data cards for the normal setup.

Heading card 1 (item 3 in list) controls the loadings to be used. It also lists values for the properties of concrete and rock. Additional data for some of the loads are included either on the angle and control cards or by a special card setup for that particular load. Heading card 2 gives elevations for base of the dam, top of dam, reservoir water surface, tailwater surface, top of grout, and bottom of concrete (to be used in stage construction studies).

If desired, the geometric computations can be bypassed and the required geometric data can be read from punched cards for use in the arch, cantilever, and abutment segments of the system.

Examples of the forms for submitting input data to ADSAS are shown in appendix G. A printout of the input data is also included there in the output from the system.

**4-53. *Output.***–The normal output from ADSAS is a print of the following items.

(1) Input deck.

(2) Summary sheet of physical properties and loading data.

(3) Geometric properties at the reference plane as well as for arch and cantilever elements.

(4) Volume of the dam.

(5) Foundation constants.

(6) Dead load stresses during construction.

(7) Loads and movements for final tangential, twist, and radial adjustments.

(8) Arch and cantilever forces.

(9) Lists of stresses.

(10) Stress maps and summary of principal stresses along the foundation.

The normal output can be supplemented by prints of data to be used for checking. In addition, geometric data can be punched on cards to be used as input for the arch, cantilever, and abutment segments of ADSAS. These may be changed to modify the geometry when such modifications cannot be satisfactorily included in the basic geometric descriptions.

For special studies, stresses from an analysis can be stored on a tape to be used when superposition for stage construction or earthquake analysis is required.

Error message printouts are provided to cover most of the more common problems which occur because of input errors.

Examples of the output prints for ADSAS are shown in appendix G.

**4-54. *Limitations.***–Although the analysis system has a large capacity and many capabilities, there are some limitations. The possible geometrical configurations are limited to those described earlier unless input to the arch, cantilever, and abutment segments is to be by punched cards. If cards are used, almost any shape can be approximated.

At present, the analysis system does not have the capability of analyzing the effects of nonradial abutments or thrust blocks.

The maximum number of arch and cantilever node points is limited. The system will analyze a dam with up to 12 arch elements and 25 cantilever elements (one at each arch abutment plus the crown cantilever) or 10 arch elements and 25 cantilever elements (one at each arch abutment, one at the crown, and two on each side between the crown and the lowest arch abutment).

Table 4-6.–*Punched card input for ADSAS.*

PUNCHED CARD INPUT

ITEM

1. DAM NAME CARD.
2. IOC CONTROL CARD.
3. HEADING CARDS 1 AND 2.
4. COMMENT CARDS (OPTIONAL). MAXIMUM OF 4 CARDS.
5. IOC CARD FOR GEOMETRY SEGMENT. (THIS IS OPTIONAL).
6. GEOMETRIC DATA AT THE BASE OF THE DAM.
7. AUXILIARY GEOMETRIC DATA CARD (IF NEEDED).
8. DECK OF CARDS DESCRIBING THE PLANE, OR PLANES, OF CENTERS. LIMITED TO 10 SEGMENTS PER LINE, ORGANIZED AS SHOWN BELOW.
    - A. DESCRIPTION OF UPSTREAM FACE.
    - B. DESCRIPTION OF DOWNSTREAM FACE.
    - C. DESCRIPTION OF INTRADOS LINE OF CENTERS ON THE LEFT OR CENTRAL SEGMENT FOR 3-CENTERED DAM.
    - D. DESCRIPTION OF EXTRADOS LINE OF CENTERS ON THE LEFT OR CENTRAL SEGMENT FOR 3-CENTERED DAM.
    - E. DESCRIPTION OF INTRADOS LINE OF CENTERS ON THE RIGHT (IF NEEDED) OR OUTER SEGMENT FOR 3-CENTERED DAM.
    - F. DESCRIPTION OF EXTRADOS LINE OF CENTERS ON THE RIGHT (IF NEEDED) OR OUTER SEGMENT FOR 3-CENTERED DAM.
9. GEOMETRIC DATA AT THE TOP OF THE DAM.
10. ANGLE AND CONTROL CARDS ORGANIZED BY ELEVATION WITH THE LOWEST ELEVATION FIRST.
11. 3-CENTERED GEOMETRY DATA CARD. UTILIZED ONLY IF 3-CENTERED GEOMETRY LAYOUT IS BEING ANALYZED.
12. COMPOUNDING ANGLE DATA CARDS. UTILIZED ONLY IF 3-CENTERED GEOMETRY LAYOUT IS BEING ANALYZED AND THE ANGLE OF COMPOUND CURVATURE VARIES BY ELEVATION. THE NUMBER AND ORDER OF THE CARDS IS IDENTICAL TO THE NUMBER AND ORDER OF THE ANGLE AND CONTROL CARDS USED.
13. HEADING CARD FOR PAD DESCRIPTION. REQUIRED ONLY IF THE STRUCTURE HAS A PAD.
14. LINE SEGMENT DESCRIPTION CARDS FOR A PAD. REQUIRED ONLY IF THE STRUCTURE HAS A PAD. PREPARE FIRST A SET FOR THE UPSTREAM FACE AND THEN A SET FOR THE DOWNSTREAM FACE. DATA FOR THE HIGHEST LINE SEGMENT COMES FIRST.
15. PAD ELEVATION AND LENGTH CARDS. UTILIZED ONLY IF THE STRUCTURE HAS A PAD. THE NUMBER AND ORDER OF THE CARDS IS IDENTICAL TO THE NUMBER AND ORDER OF THE ANGLE AND CONTROL CARDS USED. TOP OF PAD MUST NOT PASS THROUGH AN ARCH-CANTILEVER NODE. MOVE AT LEAST 1 FOOT AWAY.
16. IOC CARD FOR THE ABUTMENT SEGMENT. (THIS IS AN OPTIONAL ITEM).

Only one combination of loading can be handled at one time, and any change in shape requires a separate analysis.

ADSAS requires a machine with a 64,000-word storage capacity for the maximum size analysis. A machine with a storage capacity of 32,000 words can be used with some reduction in maximum problem size.

## D. DYNAMIC ANALYSIS

**4-55.** ***Earthquake (Response Spectra Method).***—(a) *Introduction.*—The earthquake response spectra concept can be used to analyze the dynamic behavior of arch dams. The response spectrum is a plot of the maximum values of acceleration, velocity, or displacement experienced by a family of single-degree-of-freedom systems subjected to a time-history of ground motion. Maximum values of the parameters are expressed as a function of the natural period and damping of the system. This method uses the acceleration spectrum.

(b) *Natural Frequencies.*—If the longest natural periods of vibration of the dam are within the range of maximum spectral response, the structure is assumed to respond dynamically [20, 21].

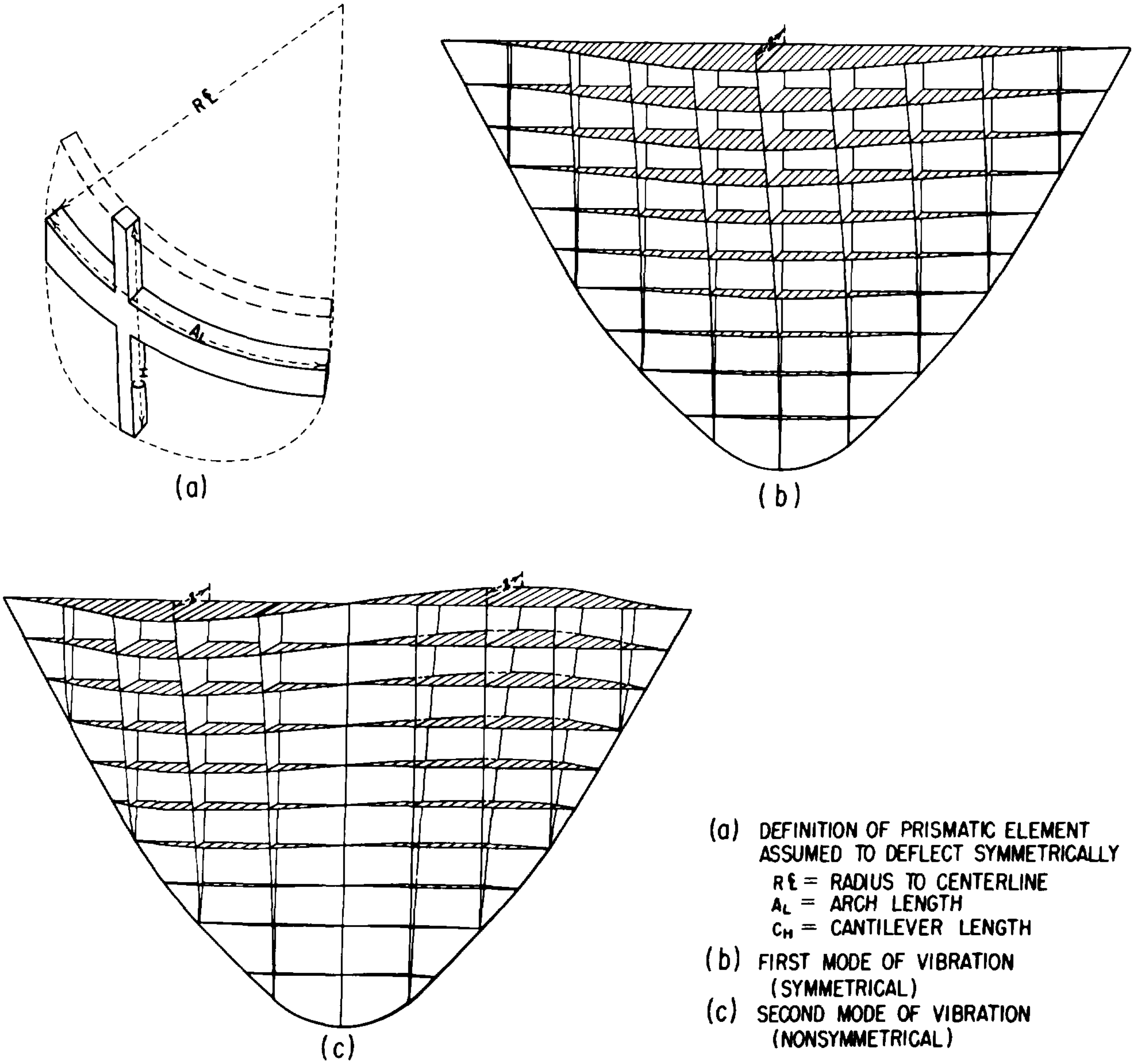
To determine the deformed shape and frequency of a vibrating arch dam, the dam is assumed to be divided into a convenient number of horizontal and vertical elements, each set of elements occupying the entire volume of the dam. The dam is assumed to deflect dynamically as a group of fixed-end arches. The deflection of the arches from the static position is assumed to be the same as that of fixed-end prismatic beams during vibration [22, 23]. In the vertical plane, the structure is assumed to take the shape of a vibrating prismatic cantilever. Thus, in the fundamental mode, the maximum deflections of the top arch and the crown cantilever will be identical. To find the deflection at any other arch location in the dam, the deflection of the crown cantilever at that elevation is computed. This becomes the maximum deflection of the arch under consideration. The deflection of any point in the arch is computed as a function of the relative distance from the crown cantilever. In the second mode, the crown cantilever is at the arch node and so does not deflect. However, the maximum relative arch deflection is assumed to vary with elevation the same as in the fundamental mode. The assumed prismatic elements and relative deflections of arches and cantilevers are shown on figure 4-60.

The square of the circular frequencies of the first two modes of a dam is estimated by equating the maximum potential energy of the structure during vibration to the maximum kinetic energy. The kinetic energy includes that due to the mass of the structure plus the effective hydrodynamic mass of reservoir water. The potential energy includes that due to bending and rib shortening of the arches and bending of the cantilevers.

The resulting equation (see reference [24]) is:

$$\frac{\omega^2}{E} = \frac{C_1 \sum_{i=1}^{n} \frac{I_i K_i^{\,2}}{L_i^{\,3}} + C_2 \sum_{i=1}^{n} \frac{A_i L_i K_i^{\,2}}{R_i^{\,2}}}{C_4 \sum_{i=1}^{n} \mu_i L_i K_i^{\,2} + C_5 \sum_{i=1}^{n} \frac{B_i W d_i L_i K_i^{\,2}}{g}}$$

$$+ \frac{C_3 \sum_{j=1}^{m} \frac{b_j K_j^{\,2}}{H_j^{\,3}} \left[a_j^{\,3} + 4(a_j c_j)^{3/2} + c_j^{\,3}\right]}{C_4 \sum_{i=1}^{n} \mu_i L_i K_i^{\,2} + C_5 \sum_{i=1}^{n} \frac{B_i W d_i L_i K_i^{\,2}}{g}} \tag{341}$$



(a) DEFINITION OF PRISMATIC ELEMENT ASSUMED TO DEFLECT SYMMETRICALLY
R℄ = RADIUS TO CENTERLINE
$A_L$ = ARCH LENGTH
$C_H$ = CANTILEVER LENGTH
(b) FIRST MODE OF VIBRATION (SYMMETRICAL)
(c) SECOND MODE OF VIBRATION (NONSYMMETRICAL)

*Figure 4-60.* **Assumed prismatic element and slopes of first and second modes.–288-D-2989**

where:

$\omega$ = circular frequency of dam, radians per second
$i$ = number of the arch, 1st, 2d, 3d, etc.
$j$ = number of the cantilever, 1st, 2d, 3d, etc.
$n$ = total number of arches
$m$ = total number of cantilevers
$E$ = modulus of elasticity, pounds per square foot
$I_i$ = moment of inertia of arch cross-section about vertical axis, feet$^4$
$A_i$ = area of arch cross-section, square feet
$K_i$ = maximum relative deflection of arch $i$, feet
$K_j$ = maximum relative deflection of cantilever $j$, feet
$L_i$ = length of arch, feet
$H_j$ = length of cantilever $j$, feet

$R_i$ = radius of curvature of centerline of arch $i$, feet
$a_j$ = thickness at base of cantilever $j$, feet
$b_j$ = width of cantilever $j$, feet
$c_j$ = thickness of top of cantilever $j$, feet
$\mu_i$ = mass per unit length of concrete in arch $i$, pound-second$^2$ per square foot
$B_i$ = horizontal dimension of effective water volume at abutment of arch $i$, feet
$d_i$ = vertical depth of arch $i$, feet
$W$ = unit weight of water, pounds per cubic foot
$g$ = acceleration of gravity, feet per second per second

$C_1$, $C_2$, $C_3$, $C_4$, $C_5$ are constants resulting from integration of the energy expressions.

For the first mode, $C_1 = 99.1$, $C_2 = 0.137$, $C_3 = 0.0278$, $C_4 = 0.198$, $C_5 = 0.198$. For the second mode, $C_1 = 875$, $C_2 = 0$, $C_3 = 0.0278$, $C_4 = 0.230$, $C_5 = 0.095$.

The natural frequency, $F$, is obtained by the equation:

$$F = \frac{\omega}{2\pi}$$

Measurements made by Okamoto and Takahashi [25] show the equivalent viscous damping in two arch dams to be about 4 to 5 percent of critical with the reservoir empty, and only slightly higher with reservoir full. Recent Bureau studies indicate a viscous damping of 3 percent of critical is more appropriate for arch dams.

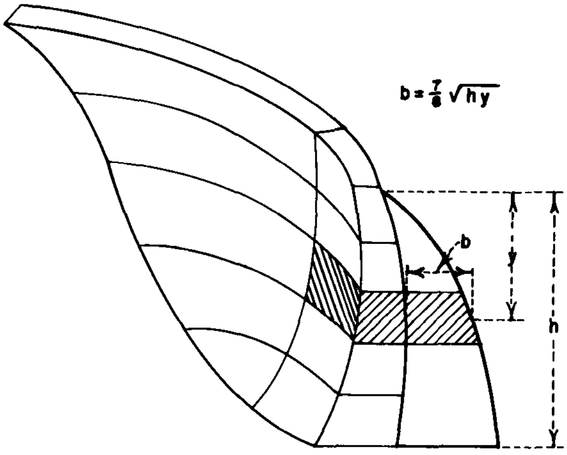
The force necessary to accelerate a dam restraining a reservoir is greater than that needed to accelerate the dam alone. Westergaard [26] found the effective volume of water accelerated with the dam for the two-dimensional case of a straight gravity dam with vertical upstream face. By electric analog methods, Zangar [27] included the effects of various slopes on the upstream face of the dam. Zienkiewicz and Nath [28] by electric analog, have extended this study to include arch dams, canyon walls, and cross-canyon motion.

Based on the above, the dimension $b$ of the volume of water accelerated with a dam in the fundamental mode is:

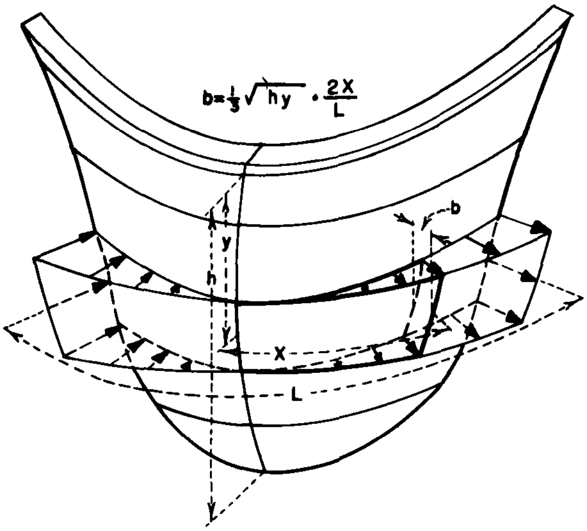
$$b = 7/8\sqrt{hy} \tag{342}$$

where $h$ is the reservoir depth at the section being studied, and $y$ is the vertical distance from the top of the water surface to any desired elevation. (See fig. 4-61.)

In the second mode of vibration, the dimension $b$ of the volume of water accelerated



(a) UPSTREAM-DOWNSTREAM MOTION



(b) CROSS-CANYON MOTION

*Figure 4-61.* Typical body of water assumed to move with dam.–288-D-2990 with the dam is given by:

$$b = 1/3\sqrt{hy} \bullet \frac{2X}{L} \qquad (343)$$

where $L$ is the length of contact of the water with the face of the dam, and $X$ is the distance measured from the crown of the arch or midpoint of the dam. (See fig. 4-61.) These equations are not perfect, but are considered adequate for an estimate of the volume of water accelerated with the dam.

(c) *Earthquake Loading With Dynamic Response.*—If a dam is assumed to be excited to resonance by earthquake, the dynamic loading should be determined using the response spectra concept.

An arch dam located in an area with no known earthquake history, or with a history of only minor earthquake activity, should be designed to resist earthquake loadings of moderate intensity. For arch dam design, moderate intensity means a moderately strong earthquake located relatively close to the structure. Quantitatively, such earthquakes are assumed to produce a ground acceleration of 0.1 $g$ (or 0.1 gravity) in sound rock foundation. A study of strong-motion earthquake response spectra indicates an average maximum acceleration magnification of about 3.5 for 5 percent critical damping. This magnification factor defines the relationship of the dynamic response of a structural system, excited by a certain ground force, to the response of the same system produced by an equivalent static force. The base shear effectiveness factor for each of the first two modes of an arch dam is approximately 0.6. Thus, the base shear produced by the assumed earthquake will be that caused by an acceleration of

$$(0.1\, g)(3.5)(0.6) \approx 0.2\, g$$

The maximum spectral acceleration response occurs in a range of periods from about 0.15 second to 0.6 second, or a frequency range of about 1.5 to 7 cycles per second. The longest periods of most arch dams will lie in this range. A dam subjected to very intense earthquake might have to sustain loadings several times as great as this value, if the structure remains elastic during the vibration.

The dynamic load at any point in a dam is directly proportional to the product of the dynamic deflection and the mass associated with that point. The associated mass includes the mass of the structure plus the mass of the volume of water accelerated with it. To determine the dynamic load in the first mode, the relative dynamic deflection at each point selected for analysis is multiplied by the effective accelerated mass associated with that point. This loading is integrated over the entire structure to obtain a total radial load. This load is then adjusted to produce a base shear considered proper for design; in the case of the assumed earthquake of moderate intensity, the base shear would be 20 percent of the effective weight of the structure and water in the mode. First mode dynamic loadings are alternately directed radially upstream then downstream at the natural frequency for that mode. In the second mode of vibration, the loads on opposite sides of the crown or midpoint of the dam are in opposite directions and change direction during every cycle of the natural frequency of the mode. The shape and magnitudes of relative loads are computed in a similar manner to that used for the first mode. The loading is integrated from the crown to either abutment, and the loads are adjusted to produce the desired base shear on one-half of the structure.

Typical earthquake loadings for the first two modes of an arch dam are shown on figure 4-62.

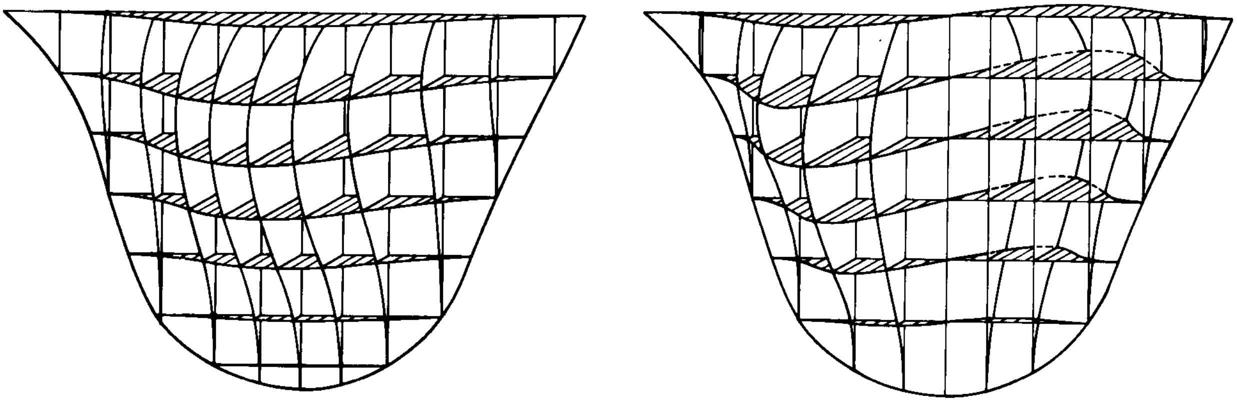
Dynamic loadings for a dam excited to resonance by earthquake should be determined as follows:

(1) Determine the dynamic shapes of the deflected structure in the first two arch modes.

(2) Compute the effective mass of the dam considered to vibrate in the two modes.

(3) Compute the effective hydrodynamic mass considered to move with the dam in the first two modes.

(4) Determine the dynamic loading at the selected points, as the product of the



(a) FIRST MODE OF VIBRATION

(b) SECOND MODE OF VIBRATION

*Figure 4-62.* **Typical earthquake intensity loads.—288-D-2991**

deflection at the point and the total effective mass associated with that point.

(5) Adjust the values of the loads to produce a base shear equal to the desired percentage of the total effective weight of dam and water in that mode.

Earthquake loadings should be applied separately for each of the first two modes, and stresses computed for each mode by the Arch Dam Stress Analysis System (ADSAS) or some equally good method of analysis. Dynamic properties of concrete should be used in the stress analysis. Stresses for the two modes may be combined at any point as:

$$\sigma_E = \sqrt{\sigma_1{}^2 + \sigma_2{}^2} \qquad (344)$$

where:

$\sigma_E$ = earthquake stress,
$\sigma_1$ = stress for mode 1, and
$\sigma_2$ = stress for mode 2.

4-56. *Earthquake (Time-History Method).*—(a) *Introduction.*—An alternative method for earthquake analysis is under development and evaluation at this time. This method may be applied in general to any structure acting elastically under the influence of an earthquake. The application to the analysis of an arch dam is a specific application of the general method. Only a brief discussion of the theory is given. For a more thorough discussion,see references [29], [30], and [31], which apply specifically to earthquake engineering. The method discussed here can be described as a lumped mass, generalized coordinate method using the principle of mode superposition.

There are four major differences in this method from that previously described:

(1) Mode shapes and natural frequencies are not assumed, but are determined from the stiffness and mass characteristics of the dam.

(2) All natural frequencies within the earthquake range are considered.

(3) Acceleration records of earthquakes are used.

(4) The effective mass of the water against the dam is considered to be the same in all modes.

The lumped mass points are selected by dividing the dam into a series of arches and cantilevers. A mass point is considered to be located at the center of the intersection of an arch and a cantilever. Mode shapes are described by relative deflections of all mass points in the dam. Inertia forces are considered to act through these mass points.

(b) *Natural Frequencies and Mode Shapes.*—Determining the natural frequencies and mode shapes is a free vibration problem. Damping forces can be neglected, since in most structures the damped frequencies are essentially the same as the undamped frequencies. A brief development of the theory for determining natural frequencies and mode shapes follows:

For a single-degree-of-freedom structure (one lumped mass), the differential equation for free vibration is:

$$M\ddot{x} + Kx = 0 \qquad (345)$$

where:

$M$ = mass,
$x$ = deflection,
$\ddot{x} = \frac{d^2x}{dt^2}$ = acceleration, and
$K$ = stiffness or restraining force per unit deflection.

The equation for number one mass of a multidegree-of-freedom system is:

$$M_1\ddot{x}_1 + K_{1i}x_i + \cdots K_{1n}x_n = 0 \quad i = 1,n \qquad (346)$$

where:

$M_1$ = mass of the number one mass,
$\ddot{x}_1$ = acceleration of the number one mass,
$K_{1i}$ = force on number one mass due to a unit deflection of another mass $i$,
$x_i$ = displacement of mass $i$, and
$n$ = total number of masses (or mass points).

A similar expression can be written for each additional mass.

If the equations for all masses are written in matrix form, the following matrix equation results:

$$[M_i]\left\{\ddot{x}_i\right\} + [K_{ij}]\left\{x_i\right\} = 0 \qquad (347)$$

where:

$[M_i]$ = the mass matrix, a diagonal matrix,
$\{\ddot{x}_i\}$ = a column matrix of accelerations,
$[K_{ij}]$ = the stiffness matrix, and
$\{x_i\}$ = a column matrix of displacements.

The mass matrix [M] is composed of all the lumped masses. Each lumped mass includes the concrete enclosed by the intersecting surfaces of the arch and the cantilever, and the corresponding mass of the water assumed to be accelerated with the dam. The water mass is estimated using a formula developed by Westergaard [32]. The radially measured dimension, $b$, of the water assumed to be moving with the dam is given by the equation,

$$b = 7/8\sqrt{hy} \qquad (348)$$

where:

$h$ = depth of water at the section studied, and
$y$ = distance of $b$ below the water surface.

The stiffness matrix [K] is equal to the inverse of the flexibility matrix [F]. The flexibility matrix is more easily constructed than the stiffness matrix, so it is used and the inversion is performed by computer as is the entire dynamic analysis. An element, $S_{ij}$, of the flexibility matrix is the deflection at $i$ due to a unit load at $j$. Both radial and tangential deflections are used for the flexibility matrix. Any of a number of methods may be used to obtain the matrix [31]. At a minimum, bending and shear deflections should be taken into account for the cantilevers and bending and rib shortening deflections for the arches.

The solution of the matrix equation is an eigenvalue problem. An in-depth discussion of eigenvalues is beyond the scope of this manual, and solutions to the problem in the form of standard computer programs are available. The input to such programs consists of the stiffness and mass matrices. The output comprises the natural frequencies $f_1$, $f_2$, etc. (eigenvalues), and for each natural frequency there will be a corresponding mode shape $(\phi_i)_1$, $(\phi_i)_2$ (eigenvector) where $i$ indicates the mass (mass point). It is possible to compute as many natural frequencies and mode shapes as lumped masses or degrees of freedom. However, only those within the earthquake range are of interest. The natural frequencies, mode shapes, mass distribution, and structural damping (see subsec. (c) below) are required for earthquake analysis.

(c) *Time Varying Response of a System to External Loads.*—In the following paragraphs a brief outline of the dynamic theory of response of a structure to external loads will be discussed. The developments will again proceed from a structure with a single degree of freedom to a multidegree-of-freedom structure. The final result is a method of determining the stress history in a dam for any earthquake.

(1) *Structure with a single degree of freedom.*—Figure 4-63 shows a single-degree-of-freedom structure with a stiffness $K$ subjected to a time-varying force $P(t)$ at the mass. Motion is resisted by a damping force, $c\dot{x}$, proportional to the velocity.

The differential equation of dynamic equilibrium is:

$$M\ddot{x} + C\dot{x} + Kx = P(t) \tag{349}$$

Figure 4-64 shows the structure subjected to a ground motion, and the equation of motion including support movement is as follows:

$$M(\ddot{x}_g + \ddot{x}) + c\dot{x} + Kx = 0 \tag{350}$$

or

$$M\ddot{x} + c\dot{x} + Kx = -M\ddot{x}_g(t) \tag{351}$$

where:

$\ddot{x}_g(t)$ = acceleration of ground as dependent on time, and all other terms are as previously defined.

The solution of equation (351) (assuming no initial displacement or velocity) is:

$$x = \frac{T}{2\pi}\int_0^t [\ddot{x}_g(\tau)]\, e^{\left[-\lambda \frac{2\pi}{T}(t-\tau)\right]} [\sin(\frac{2\pi}{T}(t-\tau))]\, d\tau \tag{352}$$

where:

$T$ = natural period of vibration $= \frac{1}{f}$, and

$\lambda$ = viscous damping factor [33]

$= \frac{c}{c_c}$ where $c$ is the critical damping factor.

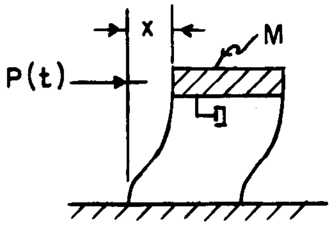


*Figure 4-63.* **Single-degree-of-freedom structure with force at the mass.—288-D-2992**

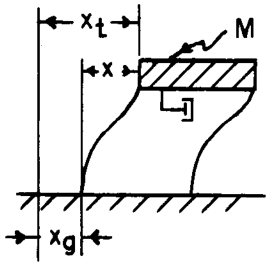


*Figure 4-64.* **Single-degree-of-freedom structure with ground motion.—288-D-2993**

The inertial force at any time is given as follows:

$$F = M\ddot{x} = -M\left(\frac{2\pi}{T}\right)^2 x \qquad (353)$$

(2) *Structure with several degrees of freedom.*—The principle of generalized orthogonality and independence of eigenvectors [34] provides keys in the development of the following equation for a multidegree-of-freedom structure.

$$F_{i_m} = \frac{M_i\,\phi_{i_m}\,\Sigma M_i\,\phi_{i_m}}{\Sigma M_i(\phi_{i_m})^2} \bullet \frac{2\pi}{T_m} \int_0^t \left[\ddot{x}_g(\tau)e^{\left[-\lambda\frac{2\pi}{T_m}(t-\tau)\right]}\right]\left[\sin\frac{2\pi}{T_m}(t-\tau)\right] d\tau \qquad (354)$$

where:

$F_{i_m}$ = inertial force on mass $i$ in mode $m$ due to ground motion $\ddot{x}_g$, and all other terms are as previously defined.

The total force at mass point $i$ is:

$$\left\{F_i\right\}_{total} = c_1\left\{L_i\right\}_1 + c_2\left\{L_i\right\}_2 + \cdots + c_m\left\{L_i\right\}_m \qquad (355)$$

where:

$\left\{F_i\right\}_{total}$ = a vector of total forces,

$\left\{L_i\right\}_m$ = a vector of effective mass, and

$c_m$ = acceleration for mode $m$.

The quantity $L_{i_m}$ is defined as:

$$L_{i_m} = M_i\,\phi_{i_m}\,\frac{\sum_i M_i\,\phi_{i_m}}{\sum_i M_i\,\phi^2_{i_m}}\,, \qquad (356)$$

and the quantity $c_m$ as:

$$c_m = \frac{2\pi}{T_m}\int_0^t \left[\ddot{x}_g(\tau)e^{\left[\frac{-\lambda 2\pi(t-\tau)}{T_m}\right]}\right]\left[\sin\frac{2\pi}{T_m}(t-\tau)\right] d\tau \qquad (357)$$

The viscous damping factor, λ, in equation (357) is an important factor in the response of the structure to an earthquake. Little data are available, however, on the damping factor for concrete dams. Measurements during the forced vibration tests at Monticello Dam [35] indicated a damping factor of from 2 to 3 percent. Other measurements of damping on monolithic concrete structures indicate a similar range of 2 to 3 percent [36, 37].

It is impractical to make a complete stress analysis for the loading pattern occurring at each interval of time. However, a unit acceleration can be used to determine the forces at each mass point due to the loads from each mode.

Any reliable method of stress analysis may then be used to determine the stresses, $\left\{S_i\right\}_m$ from the forces, $\left\{F_i\right\}_m$. These unit stress patterns are stored on magnetic tape and can be used to determine total stresses due to any earthquake.

The equation for total stress is as follows:

$$\left\{S_i\right\}_{total} = c_1 \left\{S_i\right\}_1 + c_2 \left\{S_i\right\}_2 + \cdots c_m \left\{S_i\right\}_m \tag{358}$$

where:

$\left\{S_i\right\}_m$ = a vector of unit stresses for mode $m$, and

$\left\{S_i\right\}_{total}$ = stresses at mass point $i$ due to all modes.

The $c_m$ values are computed at selected intervals of time (usually 0.01 second) from a digitized record of an earthquake. To determine the stress history for an earthquake, the $c_m$ values at each interval of time are multiplied by the corresponding $\left\{S_i\right\}_m$ values and summed.

## E. THE FINITE ELEMENT METHOD OF ANALYSIS

4-57. ***Introduction.***—The finite element method utilizes the idea that a continuous body may be considered an assemblage of distinct elements connected at their corners. This method has become a widely used and accepted means of stress analysis in the last decade. The literature of the past few years contains numerous examples of specialized uses of the finite element method. The reason for the ready acceptance and tremendous amount of use of this method is that it made possible the approximate solution of many problems which engineers had been neglecting, overdesigning, or grossly approximating. The inclusion of complex geometrical and physical property variations prior to adaption of the finite element method and the modern high-speed digital computer was simply beyond the realm of reality. The finite element method permits a very close approximation of the actual geometry and extensive variations of material properties simply and inexpensively. The formulation and theory of the finite element method are given in several publications including those by Clough [14] and Zienkiewicz [15].

Because of the ability of the method to analyze special situations, this is the area in which the most application has been made. The two-dimensional finite element method is capable of analyzing the majority of problems associated with variations in the geometry of sections of the dam. Three-dimensional effects can be approximated by making a two-dimensional analysis in more than one plane. The two-dimensional finite element method is capable of solving for stresses economically even when great detail is necessary to attain sufficient accuracy.

When the structure or loading is such that plane stress or strain conditions may not be assumed, the three-dimensional finite element method may be used. The applicability of this method to problems with great detail is limited by computer storage capacity and economics. However, the method is often used for problems with near uniform cross section or where only the general state of stress is desired. Additionally, the three-dimensional method finds application when the effect of an eccentric load or member is to be found.

Many two- and three-dimensional finite element programs with varying accuracy and capability have been written. Accuracy of an analysis also depends on the fineness of the grid system used in describing the structure. The programs used by the Bureau for analyses connected with arch dams are discussed below.

### 1. Two-Dimensional Finite Element Program

4-58. *Purpose.*—The purpose of this computer program is to determine deformations and stresses within two-dimensional plane stress structures of arbitrary shape. The structure may be loaded by concentrated forces, gravity, temperature, or by given displacements. Materials whose properties vary in compression and tension may be included by successive approximations.

4-59. *Method.*—The structure is divided into elements of arbitrary quadrilateral or triangular shape. The vertices of these shapes form nodal points. The deflections at the nodal points due to various stresses applied to each element are a function of the element geometry and material properties. The coefficient matrix relating this deflection of the element to the load applied is the individual element stiffness matrix. These stiffnesses are combined with the stiffnesses of all the other elements to form a global stiffness matrix. The loads existing at each node are determined. The deflections of each node in two directions are unknown. The same number of equations relating stiffness coefficients times unknown deflections to existing loads (right hand members) have been generated. The very large coefficient matrix is banded and symmetric. Advantage of this fact is taken into account in the storage of this matrix. The equations are solved by Gauss elimination.

In this method each unknown is progressively solved for in terms of the other unknowns existing in the equation. This value is then substituted into the next equation. The last equation then is expressible in only one unknown. The value of this unknown is determined and used in the solution of the previous equation which has only two unknowns. This process of back substitution continues until all unknowns are evaluated. The known deflections, the stiffness of the individual elements, and the equations relating strain and stress for the element are then used to calculate the stress condition for the element.

4-60. *Input.*—The problem is defined by a card input that describes the geometry and boundary conditions of the structure, the material properties, the loads, the control information for plotting, and the use of options in the program. Mesh generation, load generation, and material property generation are incorporated in the system.

4-61. *Output.*—The output of this program consists primarily of a print of the input data and the output of displacements at each node and stresses within each element. In addition, a microfilm display of the mesh and of portions of the mesh with stresses plotted on the display is available. Some punched card output is also available for special purposes of input preparation or output analysis.

4-62. *Capabilities.*—

(1) *Loading.*—External forces, temperature, and known displacements are shown, and accelerations are given as a percentage of the acceleration due to gravity in the $X$ and $Y$ directions.

(2) *Physical property variations.*—The program allows reading-in changes in modulus, density, reference temperature, and accelerations after each analysis. Stresses and displacements may then be computed with the new properties and loading without redefinition of the structure.

(3) *Plotting.*—A microfilm plot of the entire grid or details of it may be obtained. The detailed plot may be blank or can be given with principal, horizontal, vertical, and shear stresses. Either plot may also be obtained with the material number identification given within each element.

(4) *Bilinear material properties.*—The program allows for input of a modulus in compression and in tension. The tension modulus is included in successive approximations after the determination of tension in an element has been made.

(5) *Openings.*—An opening may be simulated in the structure by assigning a material number of zero to any element or by actually defining the structure with the

opening not included in the definition. The former method allows for optimum use of mesh generation and allows for considerably more flexibility.

(6) *Checking and deck preparation.*–Several options exist that allow for checking and facilitating input preparation.

(7) *Shear stiffness.*–The effect of shear stiffness in the third dimension may be included.

(8) *Units.*–The program output units match the input units. In general, these units are not shown on the output. The option exists, however, that allows units to be given on the output in feet and pounds per square inch provided that the input was in feet and kips.

(9) *Normal stress and shear stress on a plane.*–The normal stress and the shear stress on any given plane can be computed. In addition, given the angle of internal friction and the cohesion for the plane, the factor of safety against sliding can be computed.

(10) *Reference temperature.*–Temperature loads are applied with respect to a given reference temperature for the entire problem. If certain portions of the problem have different reference temperatures, these may be input on the material properties card and would override the overall reference temperature for that material only.

(11) *External forces* may be applied using boundary pressures. The program calculates concentrated loads at the nodes based on these pressures.

(12) *The input coordinates* may be prepared by digitizing a scale drawing of the problem. The actual scale can be adjusted for within the program by inputting a scale factor on the control card. The coordinates used by the program are the input coordinates times the scale factor. If no scale factor is involved, the coordinates are used as they are given.

**4-63.** ***Limitations.***–

(1) *Nodes,* 999; *elements,* 949; *materials,* 100.

(2) *Bandwidth* (maximum difference between nodes of any element) = 42.

(3) *Maximum number of rows* in a detailed plot section = 25.

**4-64.** ***Approximations.***–

(1) *Linear deflection distribution* between nodes.

(2) *Curved surface* has to be approximated by a series of straight lines.

(3) *Points of fixity* must be established on the boundaries.

(4) *Two-dimensional* plane stress.

### 2. Three-Dimensional Finite Element Program

**4-65.** ***Introduction.***–This computer program, which was developed by the University of California at Berkeley, uses the Zienkiewicz-Irons isoparametric eight-nodal-point (hexahedron) element to analyze three-dimensional elastic solids [15]. The elements use the local or natural coordinate system which is related to the *X-Y-Z* system by a set of linear interpolation functions. These local coordinates greatly simplify the stiffness formulation for the element. The displacements are also assumed to vary linearly between the nodes. Thus the same interpolation functions can be used for displacements. This common relationship of geometry and displacement is the reason for the name isoparametric element.

Once the displacement functions have been established, the element strains can be formulated. The nodal point displacements are related to the element strains in the strain-displacement relations. The element stress is related to strain using the stress-strain relations for an elastic solid. Energy considerations (either minimum potential energy or virtual work) are used to establish the relationship between nodal-point displacements and nodal-point forces. The relationship is a function of the stress-strain and the strain-displacement characteristics. This function, by definition, is the element stiffness.

The element stiffness is the key feature in the finite element solution. Each element stiffness is combined into a global stiffness matrix. In this matrix the stiffness at each node is obtained by summing the contribution from each element which contains that node. A set of equations for the entire system is obtained by equating the products of the unknown displacements times the stiffnesses to the known forces at each nodal point.

Nodal displacements are determined by solving this set of equations. Stresses are computed at the nodes of each element, using the same strain-displacement and stress-strain relations used in the formulation of the element stiffness. The stresses at a node are taken as the average of the contributions from all the elements meeting at that node.

**4-66.** ***Capabilities and Limitations.***—The program is able to analyze any three-dimensional elastic structure. The linear displacement assumption, however, limits the efficient use of the program to problems where bending is not the primary method of load resistance. Accurate modeling of bending requires the use of several elements (three have been shown to work fairly well) across the bending section. When acceptance of load is by shear and/or normal displacement along with bending, the element is capable of modeling the displacement efficiently. A comparison of the accuracy of elements by Clough [16] demonstrates this point with several sample problems.

The program capacity for a 65,000-word-storage computer is 900 elements, 2,000 nodal points, and a maximum bandwidth of 264. The bandwidth is defined as three times the maximum difference between any two node numbers on an element plus 3. On a CDC 6400 electronic computer the time for analysis in seconds is approximately:

$$0.024 \begin{bmatrix} \text{number of} \\ \text{nodal points} \end{bmatrix} + 0.45 \begin{bmatrix} \text{number of} \\ \text{elements} \end{bmatrix} + 100 \begin{bmatrix} \dfrac{\text{number of nodal points}}{775} \end{bmatrix} \bullet 3 \bullet \begin{bmatrix} \dfrac{\text{bandwidth}}{96} \end{bmatrix}^2 \qquad (359)$$

For a problem which uses the full program capacity, this is equal to:

$$0.024(2{,}000) + 0.45(900) + \frac{100(2{,}000)}{775} \cdot 3 \cdot \left(\frac{264}{96}\right)^2 = 6{,}308 \text{ seconds}$$

or about 105 minutes.

The cost of operating increases approximately as the square of the bandwidth. This economic consideration often restricts the user to a relatively coarse mesh.

Capability for use of mesh generation, concentrated loads, automatic uniform or hydrostatic load application, and varying material properties exists in the program.

The elements (see fig. 4-65) are arbitrary six-faced solids formed by connecting the appropriate nodal points by straight lines. Nonrectangular solid elements, however, require additional time for stiffness formulation because of the necessity of increased numerical integration.

**4-67.** ***Input.***—The structure to be analyzed is approximated by an assemblage of elements. The finest mesh (smallest sized elements) are located in the region of greatest stress change to allow for accurate modeling of deformations. The division is also made such that the minimum bandwidth is possible, and the nodes and elements are numbered with this consideration in mind. The program requires the following basic information:

(1) Operational data such as title, number of jobs, number of elements, maximum bandwidth, number of materials, etc.

(2) The conditions of restraint on the boundary.

(3) The material description of the elements.

(4) The accuracy of integration required for each element.

(5) The *X-Y-Z* coordinates of each nodal point and the eight nodal-point numbers forming each element (mesh generation can be used to accomplish these functions).

(6) Applied forces (it is possible to use automatic load generation).

**4-68.** ***Output.***—The program output consists of:

(1) A reprint of all input information including the information automatically generated.

(2) The displacements in the *X*, *Y*, and *Z* directions for each of the nodal points.

(3) The normal stress in the *X*, *Y*, and *Z* directions and the shear stress in the *XY*, *YZ*,

and *XZ* planes at each nodal point.

4-69. ***Application to Arch Dams.***—The general two- and three-dimensional finite element analysis programs find application to stress analysis related to arch dam design in several areas, such as:

(1) Analysis of the effects of auxiliary works located on the dam, in the dam, or structurally associated with the dam.

(2) Analysis of the effects of a rapid change in the dam geometry where detailed stress information cannot be obtained by conventional methods. (An example which illustrates the application of the finite element analysis in this area is given as appendix J.)

(3) Determination of the deformation modulus of the foundation of the dam and determination of treatment required for weak zones within the foundation.

(4) Determination of the stress distribution in concrete placed on the foundation to support the dam for either a local condition (such as fill concrete) or an extensive one (such as an abutment pad).

## 3. *Three-Dimensional Finite Element Program for Arch Dams and Their Foundations*

4-70. ***Introduction.***—The University of California has developed a specialized three-dimensional finite element program for the Bureau of Reclamation specifically for the analysis of arch dams. A specialized finite element program differs from a general program only in the standardization allowable in the input and output. The standardization greatly reduces the time required for formulation of the problem and reduces the chance for errors.

The "Arch Dam Analysis Program" (ADAP) provides several options which allow it to be used to perform analyses of varying degrees of precision. This is accomplished by varying the element type and the number of elements used in the analysis.

4-71. ***Input.***—(a) *Elements.*—The dam may be formed using either eight-noded three-dimensional elements as shown on figure 4-65 or special thick-shell elements as shown on figure 4-66. Multiple element representations through the thickness are used to determine nonlinear stress distributions and to provide more detailed presentation of results. In such cases, the eight-noded three-dimensional elements are used. A simpler, less detailed analysis can be performed using the shell elements and prescribing that each element occupies the full thickness of the section for the area of the dam it represents. The foundation is modeled using eight-noded three-dimensional elements in all cases.

(b) *Geometry.*—Single-centered, two-centered, and three-centered dams are described to the computer by an automatic mesh generator. Geometric configurations

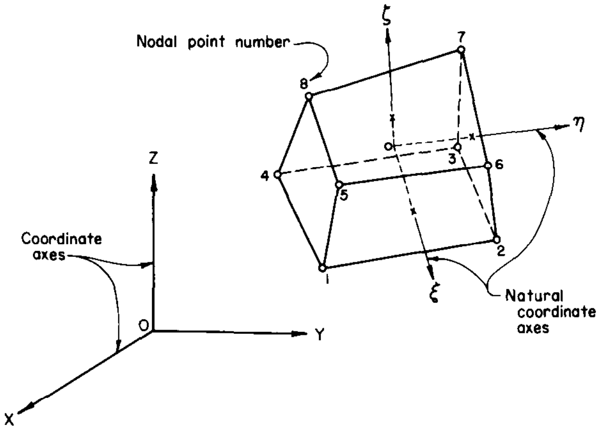


*Figure 4-65.* **A finite element with nodal point numbers and coordinate axes.—288-D-2994**

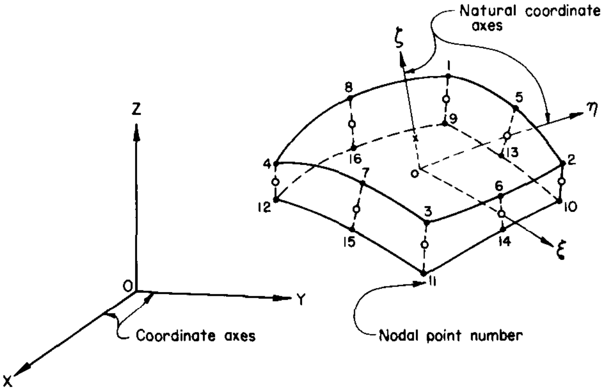


*Figure 4-66.* **A thick-shell finite element.—288-D-2995**

other than those listed must be described externally by the user.

(c) *Material Properties.*—The program allows variation in material properties through the dam and foundation. Foundation element properties can be varied along the three orthogonal axes (orthotropic material).

(d) *Loads.*—The dam may be loaded with temperature loads which vary horizontally from upstream to downstream and vertically from the base to the crest of the dam. Dead load, inertial loads, concentrated loads, uniform pressure, and hydrostatically varying pressure may be applied. Loads from earthquake, using either a response spectrum or the three components of a time-history record, can also be imposed. The program computes the mode shapes and frequencies and then performs the dynamic analysis using as many modes as desired.

4-72. ***Output.***—The output from the programs static analysis includes the deflections for each node point and the stresses within each element. The output from the dynamic analysis describes each mode shape and gives the natural frequency of each mode. The deflections and stresses from the response spectrum analysis are given if that option is selected. If the time-history analysis is used, the stresses and deflections for each increment of time are available. However, because of the excessive output generated, program options are provided whereby selected data can be printed for each time interval or complete data can be printed for selected time intervals.

4-73. ***Application.***—This program offers considerable flexibility which allows it to be used at any stage of a design. The analysis of the dam and foundation as a unit is an important feature which finds application at sites with significant variation in foundation properties.

## F. FOUNDATION ANALYSIS

4-74. ***Purpose.***—The foundation or portions of it must be analyzed for stability whenever the rock against which the dam thrusts has a configuration such that direct shear failure is possible or whenever sliding failure is possible along faults, shears, and joints. Associated with stability are problems of local overstressing in the dam due to foundation deficiencies. The presence of such weak zones can cause problems under either of two conditions: (1) when differential displacement of rock blocks occurs on either side of weak zones, and (2) when the width of a weak zone represents an excessive span for the dam to bridge over. To prevent local overstressing, the zones of weakness in the foundation must be strengthened so that the applied forces can be distributed without causing excessive differential displacements, and so that the dam is not overstressed due to bridging over the zone. Analyses can be performed to determine the geometric boundaries and extent of the necessary replacement concrete to be placed in weak zones to limit overstressing in the dam.

### 1. *Stability Analyses*

4-75. ***Methods Available.***—Methods available for stability analysis are:

(a) Two-Dimensional Methods.
   (1) Rigid section method.
   (2) Finite element method.
(b) Three-Dimensional Methods.
   (1) Rigid block method.
   (2) Partition method.
   (3) Finite element method.

Each of these analyses produces a shearing force and a normal force. The normal force can be used to determine the shearing resistance as described in section 3-4. The factor of safety against sliding is then computed by dividing the shear resistance by the shearing force.

4-76. ***Two-Dimensional Methods.***—A problem may be considered two dimensional if the geological features creating the questionable stability do not vary in cross section over a considerable length so that the end boundaries have a negligible contribution to the total resistance, or when the end boundaries are free faces offering no resistance. The representation of such a problem is shown on figure 4-67.

(a) *Rigid Section Method.*–The rigid section method offers a simple method of analysis. The assumption of no deformation of the section allows a solution according to statics and makes the method comparable to the three-dimensional rigid block method. As shown on figure 4-67, the resultant of all loads on the section of mass under investigation are resolved into a shearing force, *V*, parallel to the potential sliding plane and a normal force, *N*. The normal force is used in determining the amount of resistance as discussed in section 3-4. The factor of safety or shear friction factor is determined by dividing the resisting force by the sliding force.

This method may also be used when two or more features combine to form the potential sliding surface. For this case each feature can be assumed to form a section. Load which cannot be carried by one section is then transferred to the adjacent one as an external load. This procedure is similar to the method of slices in soil mechanics, except that the potential sliding surface may have abrupt changes in direction.

(b) *Finite Element Method.*–The finite element method, discussed in sections 4-57 through 4-73, allows deformations to occur and permits more accurate placement of loads. The analysis gives the resulting stress distribution in the section. This distribution allows the variation in normal load to be considered in determination of the resisting force and shearing force along the plane of potential sliding. The shear friction factor can then be computed along the plane to determine the stability. By having the stress distribution along the plane, a check can be made to determine if stress concentrations may cause sliding of the material in localized areas.

It should be noted that this distribution can be approximated without using the finite element method if the potential sliding mass and underlying rock are homogeneous. The finite element method is very useful if there are materials with significantly different properties in the section.

4-77. ***Three-Dimensional Methods.***–A typical three-dimensional stability problem is a four-sided wedge with two faces exposed and the other two faces offering resistance to sliding. The wedge shown on figure 4-68 is used in the discussion to illustrate the various methods.

(a) *Rigid Block Method* [17].–The following assumptions are made for this method:

(1) All forces may be combined into one resultant force.

(2) No deformation within the block mass can take place.

(3) Sliding on a single plane can occur only if the shear force on the plane is directed toward an exposed (open or free) face.

(4) Sliding on two planes can occur only in the direction of the intersection of the two planes and toward an exposed face.

(5) No transverse shear forces are

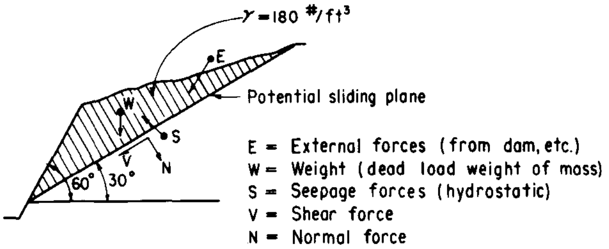


***Figure 4-67.*** **Sketch illustrating the two-dimensional stability problem.–288-D-2996**

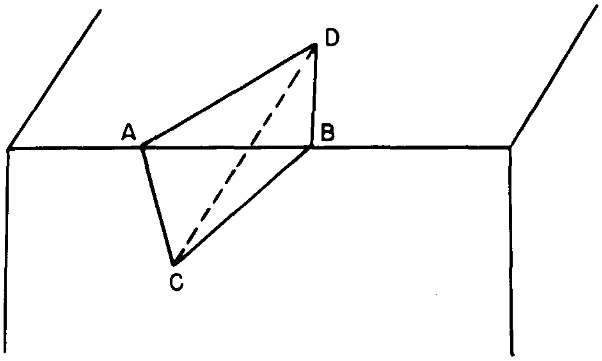


***Figure 4-68.*** **Four-sided wedge for three-dimensional sliding stability analysis.–288-D-2997**

developed (that is, there is no shear on the planes normal to the sliding direction).

The rigid block analysis proceeds in the following manner:

(1) The planes forming the block are defined.

(2) The intersections of the planes form the edges of the block.

(3) The areas of the faces of the block and the volume of the block are computed.

(4) The hydrostatic forces, if applicable, are computed normal to the faces.

(5) The resultant of all forces is computed.

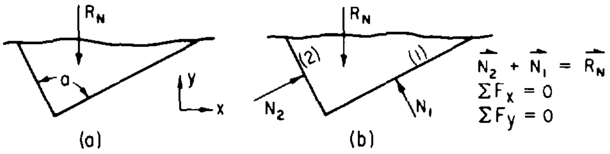
(6) The possibility for sliding on one or two planes is checked.

(7) The factor of safety against sliding is computed for all cases where sliding is possible.

To determine whether the rock mass will slide on one or two planes, a test is applied to each possible resisting plane. If the resultant vector of all forces associated with the rock mass has a component normal to and directed into a plane, it will offer resistance to sliding. If only one plane satisfies the criterion, the potential sliding surface will be one plane; and if two planes satisfy the criterion,the potential sliding surface will be the two planes.

Sliding on three planes is impossible according to the assumptions of rigid block. If an analysis of a block with many resistant faces is desired according to rigid block procedure, several blocks will need to be analyzed with any excess shear load from each block applied to the next.

The resultant force for the case of a single sliding plane is resolved into one normal and one shear force. For the case of sliding on two planes, the resultant is divided into a shear force along the intersection line and a resultant force normal to the intersection line. Forces normal to the two planes are then computed such that they are in equilibrium with the resultant normal force. As a result of assumption (5) at the beginning of this subsection, these normal loads are the maximum that can occur and the resulting shear resistance developed is a maximum. Figures 4-69(a) and 4-69(b) show a section through the potential sliding mass normal to the intersection line of the two planes with the resultant normal load balanced by normals to the two potential sliding planes.



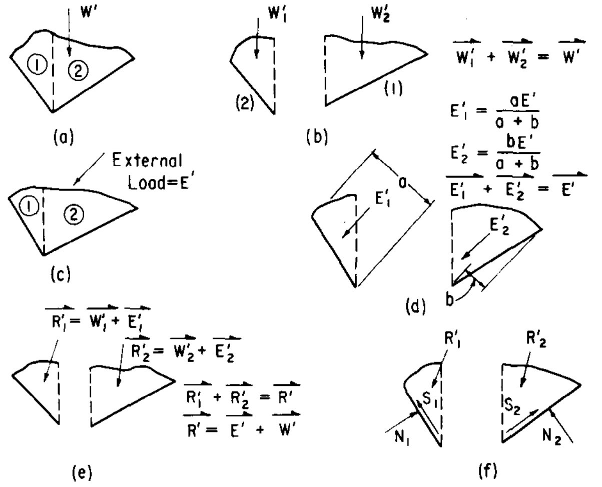
$R_N$ = The portion of the resultant normal to the direction of potential movement.

N = The normal load on the face indicated by the subscript.

*Figure 4-69.* **Section through a potential sliding mass normal to the intersection line of two planes.–288-D-2998**

The shearing resistance developed for either a single plane or two planes is computed using the normal forces acting on the planes and the methods discussed in section 3-4.

(b) *Partition Method.*–The rigid block method permits no deformation of the mass of the block. Because of this restriction no shear load is developed in the potential sliding planes transverse to the direction of sliding. The development of shear in the transverse direction decreases the normal load and consequently the developable shear resistance [18]. An approximation to the minimum developable shear resistance is made by the partition method. In this method the planes (normal to the sliding direction) are parted according to the dead load associated with each plane as shown on figures 4-70(a) and 4-70(b). The component of the external load perpendicular to the sliding direction is then proportionately assigned to each plane according to the ratio of projected areas of the planes with respect to the direction of loading as shown on figures 4-70(c) and 4-70(d). (Note: If the external load is parallel to one of the planes, the load assignment may have to be assumed differently depending on the point of load application.) All the forces on each plane are then combined to form a resultant on that plane (fig. 4-70(e)). This resultant is assumed to be balanced by a normal force and a shear



Where: W = DEAD LOAD
E = EXTERNAL LOADS
R = RESULTANT LOAD ON MASS
Subscripts refer to the appropriate portions of the mass. No subscript implies that the entire mass is being considered.
Planes are normal to the direction of potential sliding.
Loads resolved into the plane normal to direction of potential sliding are indicated with a prime.

*Figure 4-70.* **Partition method of determining shear resistance of a block.—288-D-2999**

force on the potential sliding plane (fig. 4-70(f)). The normal force is then used in determining the resistance of the block to sliding.

Although it is recognized that the developable shear force is probably less than that required to balance the resultant, the assumption that this strength is developed allows computation of the minimum developable strength. The shear resistance developed by using $N_1$ and $N_2$ (fig. 4-70(f)) is considered the minimum possible.

The shearing force tending to drive the block in the direction of sliding is determined as described for the rigid block method. The computation of the resistance according to the partition method utilizes the information obtained for the rigid block analysis, and therefore requires very little additional computation. The shear resistance determined by the rigid block method is an upper bound and that determined by the partition method is considered a lower bound. As the angle between the planes (see fig. 4-69(a)) increases, the results obtained from the two methods converge. The correct shear resistance lies between the upper and lower bounds and is a function of the deformation properties of the potential sliding mass and host mass of rock, and even more importantly, of the sliding and deformation characteristics of the joint or shear material forming the potential sliding surface. The effect of these properties on the resistance developed can be approximated by using a three-dimensional finite element program with planar weakened zone elements. This method is discussed in the next subsection.

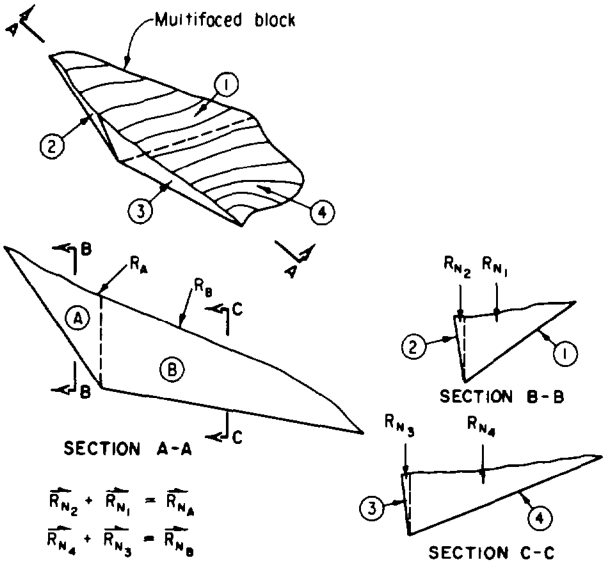
The partition method can be extended to multifaced blocks very readily. Just as the section normal to the direction of sliding is partitioned, so can a section along the direction of sliding as shown on figure 4-71. Excess shear load from one partition, *A*, must be applied to the adjacent one, *B*, as an external loading as shown on the figure.

(c) *Finite Element Method.*—A program developed by Mahtab [19] allows representation of the rock masses by three-dimensional solid elements and representation of the potential sliding surface by two-dimensional planar elements. The planar elements are given properties of deformation in compression (normal stiffness) and in shear (shear stiffness) in two directions.

The ratio of the normal stiffness to the shear stiffness influences greatly the amount of load which will be taken in the normal direction and in the transverse shear direction. If the normal stiffness is much greater than the shear stiffness, as is the case for a joint with a slick coating, the solution approaches that given by the rigid block method. However, as the shear stiffness increases with respect to the normal stiffness, more load is taken by transverse shear and the solution given by the partition method is approached.

The three-dimensional finite element method allows another important refinement in the solution of stability problems. Since deformations are allowed, the stress state on all planes of a multifaced block can be computed rather than approximated and stress concentrations located.

The refinements available in the analysis by



NOTE: Circled numbers refer to faces.
Circled letters refer to blocks.
$R_{N_1}$ = The portion of the resultant assigned to a face. The sub-subscript indicates the face number.
$R_{N_A}$ = The portion of the resultant normal to the direction of potential movement of a block. The sub-subscript refers to the block.
$R_A$ = Resultant external load acting on block A.

*Figure 4-71.* **Partition method extended to multifaced blocks.–288-D-3000**

the three-dimensional finite element method should be used when the upper and lower bounds determined by the other methods are significantly different. The method should also be used if there is considerable variation in material properties either in the potential sliding planes or in the rock masses.

A more detailed discussion of the finite element method is given in sections 4-57 through 4-73.

## 2. Other Analyses

4-78. *Differential Displacement Analysis.*—The problem of relative deflection or differential displacement of masses or blocks within the foundation arises due to variations in the foundation material. Methods that approximate or compute the displacement of masses or zones within the foundation are required to analyze problems of this nature. Typical problems that may occur are as follows:

(1) Displacement of a mass whose stability depends on sliding friction.

(2) Displacement of a mass sliding into a low modulus zone.

(3) Displacement of a mass with partial intact rock continuity.

(4) Displacement of zones with variable loading taken by competent rock in two directions but cut off from adjacent rock by weak material incapable of transmitting shear load.

The displacements may be approximated by (1) extension of shear-displacement data obtained from specimen testing in situ or in the laboratory; (2) model testing; (3) development of an analytical model which can be solved manually; or (4) two- or three-dimensional finite element methods.

Although the method used depends on the particular problem, it should be noted that the finite element method offers considerable advantage over the other procedures. The finite element method allows accurate material property representation, gives stress distribution, and permits representation of treatment necessary to obtain acceptable displacements.

4-79. *Analysis of Stress Concentrations Due to Bridging.*—A stress concentration may occur in the dam due to the presence of a low-modulus zone within the foundation as shown on figure 4-72. To minimize the buildup of stress in the dam, a portion of the weak

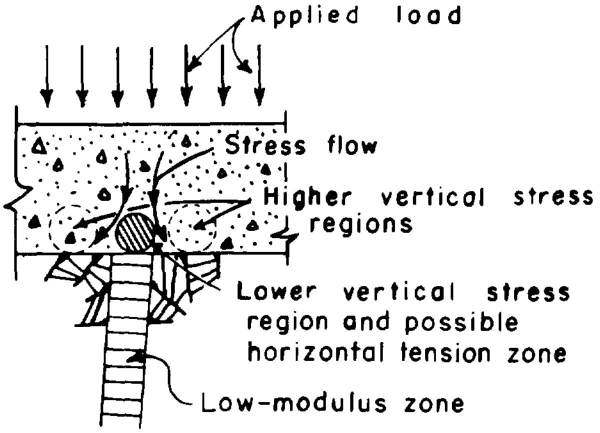


*Figure 4-72.* **Stress distribution near a low-modulus zone.–288-D-3001**

material in the low-modulus zone may be replaced with concrete. The depth of replacement required is determined as the depth at which stresses in the dam and foundation are within allowable limits. The two-dimensional finite element method, discussed in sections 4-58 through 4-64, is in excellent method for solving this problem.

## G. BIBLIOGRAPHY

### 4-80. *Bibliography.*

[1] Glover, R. E., "Fundamentals of the Trial-Load Method for the Design of Arch Dams," presented to the faculty of the Graduate College in the University of Nebraska, April 30, 1936, in partial fulfillment of requirements for the professional degree of Civil Engineer.

[2] Howell, C. H., and Jaquith, A. C., "Analysis of Arch Dams by Trial-Load Method," ASCE Proceedings, January 1928 (or Trans. ASCE, vol. 93, 1929, p. 1191).

[3] The Engineering Foundation, "Report on Arch Dam Investigations," vols. I and III, 1927 and 1933.

[4] Larned, A. I., and Merrill, V. S., "Actual Deflections and Temperatures in a Trial-Load Arch Dam," Trans. ASCE, vol. 99, 1934, pp. 897-961.

[5] Bureau of Reclamation, "Model Tests of Boulder Dam," Boulder Canyon Project Final Reports, Part V, Bulletin 3, 1939.

[6] Bureau of Reclamation, "Comparison of Analytical and Structural Behavior Results for Flaming Gorge Dams," Research Report No. 14, 1968.

[7] Kramer, M. A., and Jones, K., "Comparison of Analytical and Structural Behavior Results for Morrow Point Dam," REC-ERC-72-8, Bureau of Reclamation, 1972.

[8] Love, A.E.H., "Mathematical Theory of Elasticity," fourth edition, 1927, par. 118.

[9] Townsend, C. L., "Control of Cracking in Mass Concrete Structures," Engineering Monograph No. 34, Bureau of Reclamation, 1965.

[10] Westergaard, H. M., "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, vol. II, March 1930, pp. 1-23.

[11] Vogt, Fredrick, "Uber die Berechnung der Fundament Deformationen," Det Norske Bidenskaps, Akademi, Oslo, 1925.

[12] Westergaard, H. M., "Stresses at a Crack, Size of the Crack, and the Bending of Reinforced Concrete," Journal of the American Concrete Institute, November-December 1933.

[13] Bureau of Reclamation, "Trial-Load Method of Analyzing Arch Dams," Boulder Canyon Project Final Reports, Part V, Bulletin 1, 1938.

[14] Clough, R. W., "The Finite Element Method in Plane Stress Analysis," ASCE Conference Papers (Second Conference on Electronic Computation, September 1960).

[15] Zienkiewicz, O. C., "The Finite Element in Structural and Continuum Mechanics," McGraw-Hill, London, 1967.

[16] Clough, R. W., "Comparison of Three-Dimensional Finite Elements," Proceedings of the Symposium on the Application of Finite Element Methods in Civil Engineering, Vanderbilt University, Nashville, Tenn., November 13-14, 1969.

[17] Londe, P. (1965), "Une Methode d'Analyze o' trois dimensions de la stabilite d'une rive rocheme," Annis Ponts Chaus. No. 1 37-60.

[18] Guzina, Bosko, and Tucovic, Ignjat, "Determining the Minimum Three-Dimensional Stability of a Rock Wedge," Water Power, London, October 1969.

[19] Mahtab, M. A., and Goodman, R. E., "Three-Dimensional Finite Element Analysis of Jointed Rock Slopes," Final Report to Bureau of Reclamation, contract No. 14-06-D-6639, December 31, 1969.

[20] Copen, M. D., "Selection of Design Criteria for Concrete Dams Subjected to Siesmic Action," International Commission on Large Dams, Turkey, 1967.

[21] Crawford, C. C., and Copen, M. D., "Proposed Earthquake Loadings for the Design of Thin Arch Dams," American Society of Civil Engineers, Structural Engineering Conference, San Francisco, Calif., October 1963.

[22] Timoshenko, S., "Vibration Problems in Engineering," second edition, D. Van Nostrand Co., 1937.

[23] Love, A.E.H., "Mathematical Theory of Elasticity," Cambridge, Mass., 1934.

[24] Crawford, C. C., "Earthquake Design Loadings for Thin Arch Dams," Third World Conference on Earthquake Engineering, New Zealand, 1965.

[25] Okamoto, Shunzo, and Takahashi, "On Behavior of Arch Dams During Earthquakes," Proceedings of Second World Conference on Earthquake Engineering, vol. II, Japan, 1960.

[26] Westergaard, H. M., "Water Pressure on Dams During Earthquakes," Trans. ASCE, vol. 9, 1933.

[27] Zangar, C. N., "Hydrodynamic Pressures on Dams Due to Horizontal Earthquake Effects," Engineering Monograph No. 11, Bureau of Reclamation, 1952.

[28] Zienkiewicz, O. C., and Nath, B., "Earthquake Hydrodynamic Pressures on Arch Dams—An Electric Analogue Solution," Institute of Civil Engineers, London, Paper No. 6668, 1963.

[29] Clough, R. W., "Earthquake Response of Structures," Earthquake Engineering, Prentice-Hall, Englewood Cliffs, N.J., 1970, ch. 12.

[30] "Earthquake Engineering for Concrete and Steel Structures," Proceedings of a conference with R. W. Clough and Bureau of Reclamation staff members, Denver, Colo., March 1963.

[31] Dungar, R., and Severn, R. T., "Dynamic Analysis of Arch Dams," Paper No. 7, Symposium on Arch Dams, Institution of Civil Engineers, March 1968.

[32] Westergaard, H. M., "Water Pressures on Dams During Earthquakes," Trans. ASCE, vol. 98, 1933.

[33] Myklestad, N. O., "Vibration Analysis," McGraw Hill Book Co., Inc., New York, N.Y., 1944, p. 71.

[34] Wylie, C. R. Jr., "Advanced Engineering Mathematics," McGraw-Hill Book Co., Inc., New York, N.Y., 1960, pp. 37-43.

[35] Rouse, G. C., and Bouwkamp, J. G., "Vibration Studies of Monticello Dam," Research Report No. 9, Bureau of Reclamation, 1967.

[36] Keightly, W. O., Housner, G. W., and Hudson, D. E., "Vibration Tests of Encino Dam Intake Tower," Engineering Research Laboratory, California Institute of Technology, Pasadena, Calif., July 1961.

[37] Cozart, C. W., "The Response of an Intake Tower at Hoover Dam to Earthquakes," Bureau of Reclamation Report REC-ERC-71-50, 1971.

[38] Boggs, H. L., "Guide for Preliminary Design of Arch Dams," Engineering Monograph No. 36, Bureau of Reclamation, 1966.

## <<Chapter V

# River Diversion

## A. DIVERSION REQUIREMENTS

5-1. *General.*—The design for a dam which is to be constructed across a stream channel must consider diversion of the streamflow around or through the damsite during the construction period. The extent of the diversion problem will vary with the size and flood potential of the stream; at some damsites diversion may be costly and time-consuming and may affect the scheduling of construction activities, while at other sites it may not offer any great difficulties. However, a diversion problem will exist to some extent at all sites except those located offstream, and the selection of the most appropriate scheme for handling the flow of the stream during construction is important to obtain economy in the cost of the dam. The scheme selected ordinarily will represent a compromise between the cost of the diversion facilities and the amount of risk involved. The proper diversion plan will minimize serious potential flood damage to the work in progress at a minimum of expense. The following factors should be considered in a study to determine the best diversion scheme:

(1) Characteristics of streamflow.
(2) Size and frequency of diversion flood.
(3) Regulation by existing upstream dam.
(4) Methods of diversion.
(5) Specifications requirements.
(6) Turbidity and water pollution control.

5-2. *Characteristics of Streamflow.*—Streamflow records provide the most reliable information regarding stream characteristics, and should be consulted whenever available.

Depending upon the geographical location of the drainage area, floods on a stream may be the result of snowmelt, rain on snow, seasonal rains, or cloudbursts. Because these types of runoff have their peak flows and their periods of low flow at different times of the year, the nature of runoff will influence the selection of the diversion scheme. A site subject mainly to snowmelt or rain on snow floods will not have to be provided with elaborate measures for use later in the construction season. A site where seasonal rains may occur will require only the minimum of diversion provisions for the rest of the year. A stream subject to cloudbursts which may occur at any time is the most unpredictable and probably will require the most elaborate diversion scheme, since the contractor must be prepared to handle both the low flows and floodflows at all times during the construction period.

5-3. *Selection of Diversion Flood.*—It is not economically feasible to plan on diverting the largest flood that has ever occurred or may be expected to occur at the site, and consequently some lesser requirement must be decided upon. This, therefore, brings up the question as to how much risk to the partially completed work is involved in the diversion scheme under consideration. Each site is different and the extent of damage done by flooding is dependent upon the area of foundation and structure excavation that would be involved, and the time and cost of cleanup and reconstruction that would be required.

In selecting the flood to be used in the diversion designs, consideration should be given to the following:

(1) How long the work will be under construction, to determine the number of flood seasons which will be encountered.

(2) The cost of possible damage to work completed or still under construction if it is flooded.

(3) The cost of delay to completion of the work, including the cost of forcing the contractor's equipment to remain idle while the flood damage is being repaired.

(4) The safety of workmen and possibly the safety of downstream inhabitants in case the failure of diversion works results in unnatural flooding.

After an analysis of these factors is made, the cost of increasing the protective works to handle progressively larger floods can be compared to the cost of damages resulting if such floods occurred without the increased protective work. Judgment can then be used in determining the amount of risk that is warranted. Figure 5-1 shows a view from the right abutment of Monticello Dam with a major flood flowing over the low blocks and flooding the construction site. This flood did not damage the dam and caused only nominal damage to the contractor's plant.

The design diversion flood for each dam is dependent upon so many factors that rules cannot be established to cover every situation. Generally, however, for small dams which will be constructed in a single season, only the floods which may occur for that season need be considered. For most small dams, involving at the most two construction seasons, it should be sufficiently conservative to provide for a flood with a probability of occurrence of 20 percent. For larger dams involving more than a 2-year construction season, a design diversion flood with a probability of occurrence of anywhere between 20 and 4 percent may be established depending on the loss risk and the completion time for the individual dam.

Floods may be recurrent; therefore, if the diversion scheme involves temporary storage of cloudburst-type runoff, facilities must be provided to evacuate such storage within a reasonable period of time, usually a few days.

**5-4. *Regulation by an Existing Upstream Dam.*** —If the dam is to be built on a stream below an existing dam or other control structure, it is sometimes possible to modify the characteristics of the streamflow by planned operation of the existing structure. During the construction period, a modified program of operation of the existing structure may be used to reduce the peak of the flood outflow hydrograph and reduce the diversion requirements at the construction site. Upstream control can also be utilized to reduce flow during the construction of cofferdams, plugging of diversion systems and the removal of cofferdams.

**5-5. *Turbidity and Water Pollution Control.*** —One of the more important factors to be considered in determining the diversion

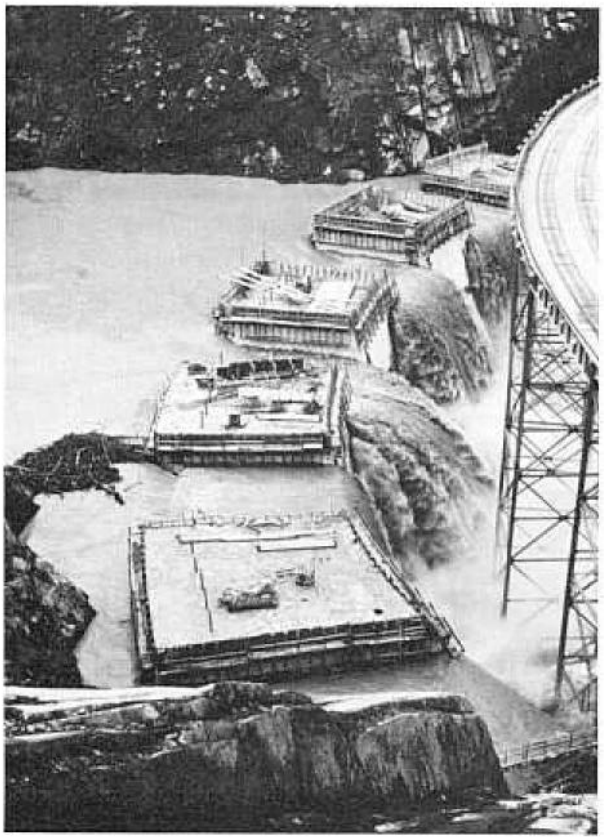


*Figure 5-1.* View from right abutment of partially completed Monticello Dam in California showing water flowing over low blocks.—SO-1446-R2

scheme is how the required construction work affects the turbidity and pollution of the stream. A scheme that limits the turbidity, present in all diversion operations, to the shortest practicable period and creates less total effect on the stream should be given much consideration. Factors which contribute to turbidity in the stream during diversion are the construction and removal of cofferdams, required earthwork in or adjacent to the stream, pile driving, and the dumping of waste material. Therefore, all diversion schemes should be reviewed for the effect of pollution and turbidity on the stream during construction and removal of the diversion works, as well as the effect on the stream during the time construction is carried on between the cofferdams. Sample specifications for the control of turbidity and pollution are shown in appendix N.

## B. METHODS OF DIVERSION

5-6. *General.*—The method or scheme of diverting floods during construction depends on the magnitude of the flood to be diverted; the physical characteristics of the site; the size and shape of dam to be constructed; the nature of the appurtenant works, such as the spillway, penstocks, and outlet works; and the probable sequence of construction operations. The objective is to select the optimum scheme considering practicability, cost, turbidity and pollution control, and the risks involved. The diversion works should be such that they may be incorporated into the overall construction program with a minimum of loss, damage, or delay.

Diverting streams during construction utilizes one or a combination of the following provisions: tunnels driven through the abutments, flumes or conduits through the dam area, or multiple-stage diversion over the tops of alternate construction blocks of the dam. On a small stream the flow may be bypassed around the site by the installation of a temporary wood or metal flume or pipeline, or the flow may be impounded behind the dam during its construction, pumps being used if necessary to control the water surface. In any case, barriers are constructed across or along the stream channel in order that the site, or portions thereof, may be unwatered and construction can proceed without interruption.

A common problem is the meeting of downstream requirements when the entire flow of the stream is stopped following closure of the diversion works. Downstream requirements may demand that a small flow be maintained at all times. In this case the contractor must provide the required flow by pumping or by other means (bypasses or siphons) until water is stored in the reservoir to a sufficient elevation so that releases may be made through the outlet works.

Figure 5-2 shows how diversion of the river was accomplished during the construction of Folsom Dam and Powerplant on the American River in California. This photograph is included because it illustrates many of the diversion principles discussed in this chapter. The river, flowing from top to bottom in the picture, is being diverted through a tunnel; "a" and "b" mark the inlet and outlet portals, respectively. Construction is proceeding in the original river channel between earthfill cofferdams "c" and "d." Discharge from pipe "e" at the lower left in the photograph is from unwatering of the foundation. Since it was impracticable to provide sufficient diversion tunnel capacity to handle the large anticipated spring floods, the contractor made provisions to minimize damage that would result from overtopping of the cofferdam. These provisions included the following:

(1) Placing concrete in alternate low blocks in the dam "f" to permit overflowing with a minimum of damage.

(2) Construction of an auxiliary rockfill and cellular steel sheet-piling cofferdam "g" to protect the powerplant excavation "h" from being flooded by overtopping of the cofferdam.

(3) Early construction of the permanent

*Figure 5-2.* **Diversion of the river during construction of Folsom Dam and Powerplant in California.–AR-1627-CV**

training wall "i" to take advantage of the protection it affords.

**5-7.** ***Tunnels.***–It is usually not feasible to do a significant amount of foundation work in a narrow canyon until the stream is diverted. If the lack of space or a planned powerplant or other feature eliminates diversion through the construction area by flume or conduit, a tunnel may prove the most feasible means of diversion. The streamflow may be bypassed around the construction area through tunnels in one or both abutments. A diversion tunnel should be of a length that it bypasses the construction area. Where suitable area required by the contractor for shops, storage, fabrication, etc., is not readily available, it may be advantageous to lengthen the tunnel to provide additional work area in the streambed. However, the tunnel should be kept as short as practicable for economic and hydraulic reasons. Figure 5-3 shows such a tunnel which was constructed at Flaming Gorge Dam site, a relatively narrow canyon, to permit diversion through the abutment.

The diversion system must be designed to bypass, possibly also contain part of, the design diversion flood. The size of the diversion tunnel will thus be dependent on the magnitude of the diversion flood, the height of the upstream cofferdam (the higher the head, the smaller the tunnel needs to be for a given discharge), and the size of the reservoir formed by the cofferdam if this is appreciable. An economic study of cofferdam height versus tunnel size may be involved to establish the most economical relationship.

The advisability of lining the diversion tunnel will be influenced by the cost of a lined tunnel compared with that of a larger unlined tunnel of equal carrying capacity; the nature of

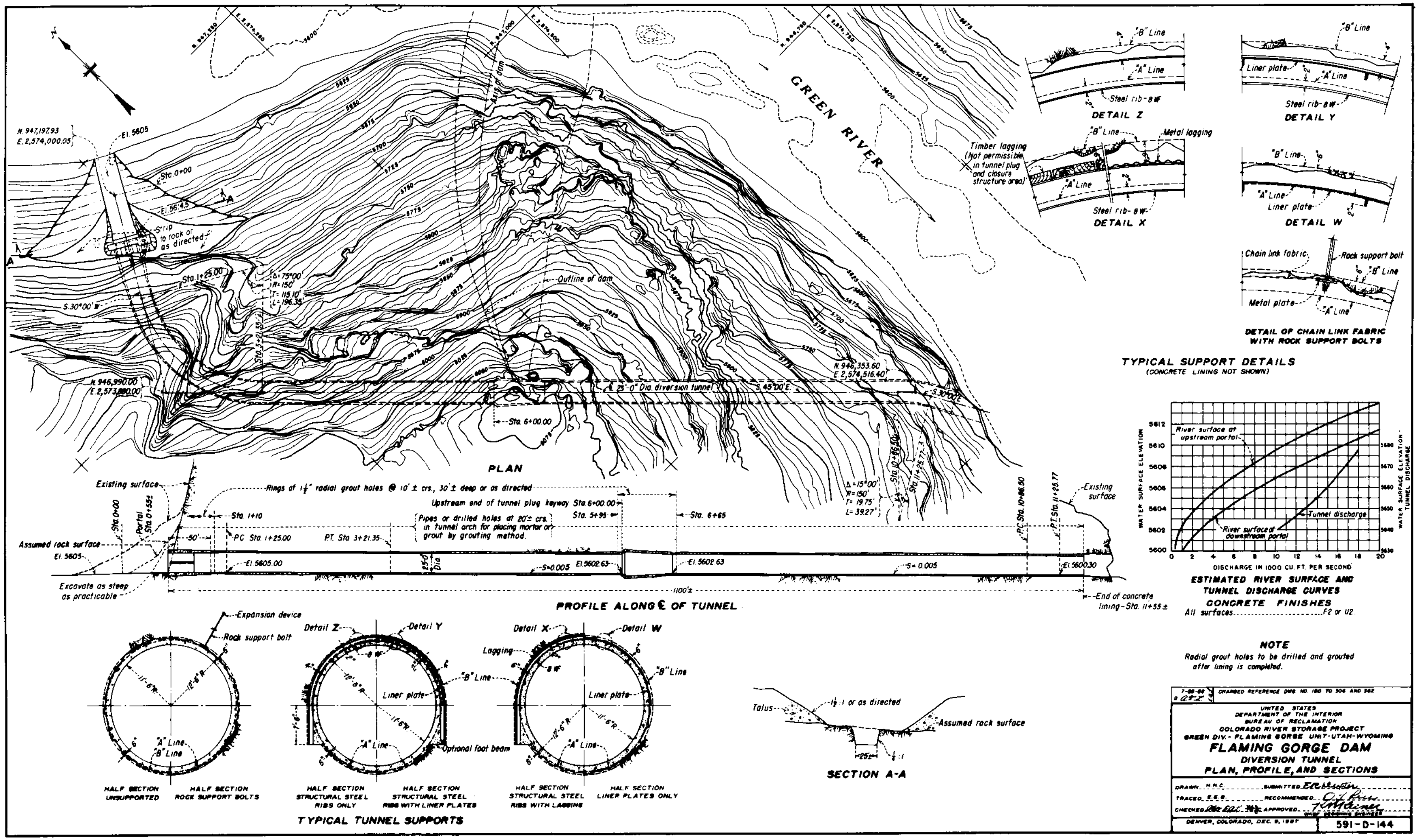


*Figure 5-3.* Diversion tunnel for Flaming Gorge Dam, a large concrete dam in Utah–plan, profile, and sections.

the rock in the tunnel, as to whether it can stand unsupported and unprotected during the passage of the diversion flows; and the permeability of the material through which the tunnel is carried, as it will affect the amount of leakage through or around the abutment.

If tunnel spillways are provided in the design, it usually proves economical to utilize them in the diversion plan. When the proposed spillway tunnel consists of a high intake and a sloping tunnel down to a near horizontal portion of tunnel close to streambed elevation, a diversion tunnel can be constructed between the near horizontal portion of tunnel and the channel elevation upstream to effect a streambed bypass. Figure 5-4 shows such a typical diversion tunnel which will permit diversion through the lower, nearly horizontal portion of the spillway tunnel. Provisions for the final plugging, such as excavation of keyways, grouting, etc., should be incorporated into the initial construction phase of the diversion tunnel.

Some means of shutting off diversion flows must be provided; in addition, some means of regulating the flow through the diversion tunnel may be necessary. Closure devices may consist of a timber, concrete, or steel bulkhead gate; a slide gate; or stoplogs. Regulation of flow to satisfy downstream needs after storage of water in the reservoir has started can be accomplished by the use of a slide gate on a temporary bypass until the water surface in the reservoir reaches the level of the outlet works intake. Figure 5-5 shows the closure structure constructed at Flaming Gorge Dam, which was incorporated in the upstream 50-foot length of the diversion tunnel. A high-pressure slide gate on a small conduit was provided in the left side of the closure structure to bypass required flows while filling the reservoir to the elevation of the river outlet.

Permanent closure of the diversion tunnel is made by placing a concrete plug in the tunnel. If the tunnel passes close to and under the dam, the plug should be located near the upstream face in line with the grout curtain cutoff or it may extend entirely under the dam, depending on the stresses from the dam and the condition of the foundation. If the diversion tunnel joins a permanent tunnel, the plug is usually located immediately upstream from the intersection as indicated in figure 5-4. Keyways may be excavated into rock or formed into the lining to insure adequate shear resistance between the plug and the rock or lining. After the plug has been placed and the concrete cooled, grout is forced through previously installed grout connections into the contact between the plug and the surrounding rock or concrete lining to insure a watertight joint.

5-8. ***Conduits Through Dam.***—Diversion conduits at stream level are sometimes provided through a dam. These conduits may be constructed solely for the purpose of diversion or they may be conduits which later will form part of the outlet works or power penstock systems. As with tunnels, some means of shutting off the flow at the end of the diversion period and a method of passing downstream water requirements during the filling of the reservoir must be incorporated into the design of the conduit. The most common procedure for closing the diversion conduit before the placement of the permanent plug is by lowering bulkheads down the upstream face of the dam which will seal against the upstream face. Figure 5-6 shows typical details of a conduit through a dam.

After serving their purpose, all diversion conduits must be filled with concrete for their entire length. This is accomplished with the bulkheads in place. The conduit should be provided with keyways, metal seals, and grouting systems within the initial construction to assure a satisfactory permanent seal. The shrinkage and temperature of the plug concrete should be controlled by the installation of a cooling system.

5-9. ***Flumes.***—In a wide canyon, an economical method of diversion may be the use of a flume to carry the streamflow around the construction area. A flume may also be used to carry the streamflow over a low block and through the construction area. The flume should be designed to accommodate the design diversion flood, or a portion thereof if the flume is used in conjunction with another method of diversion. The most economical

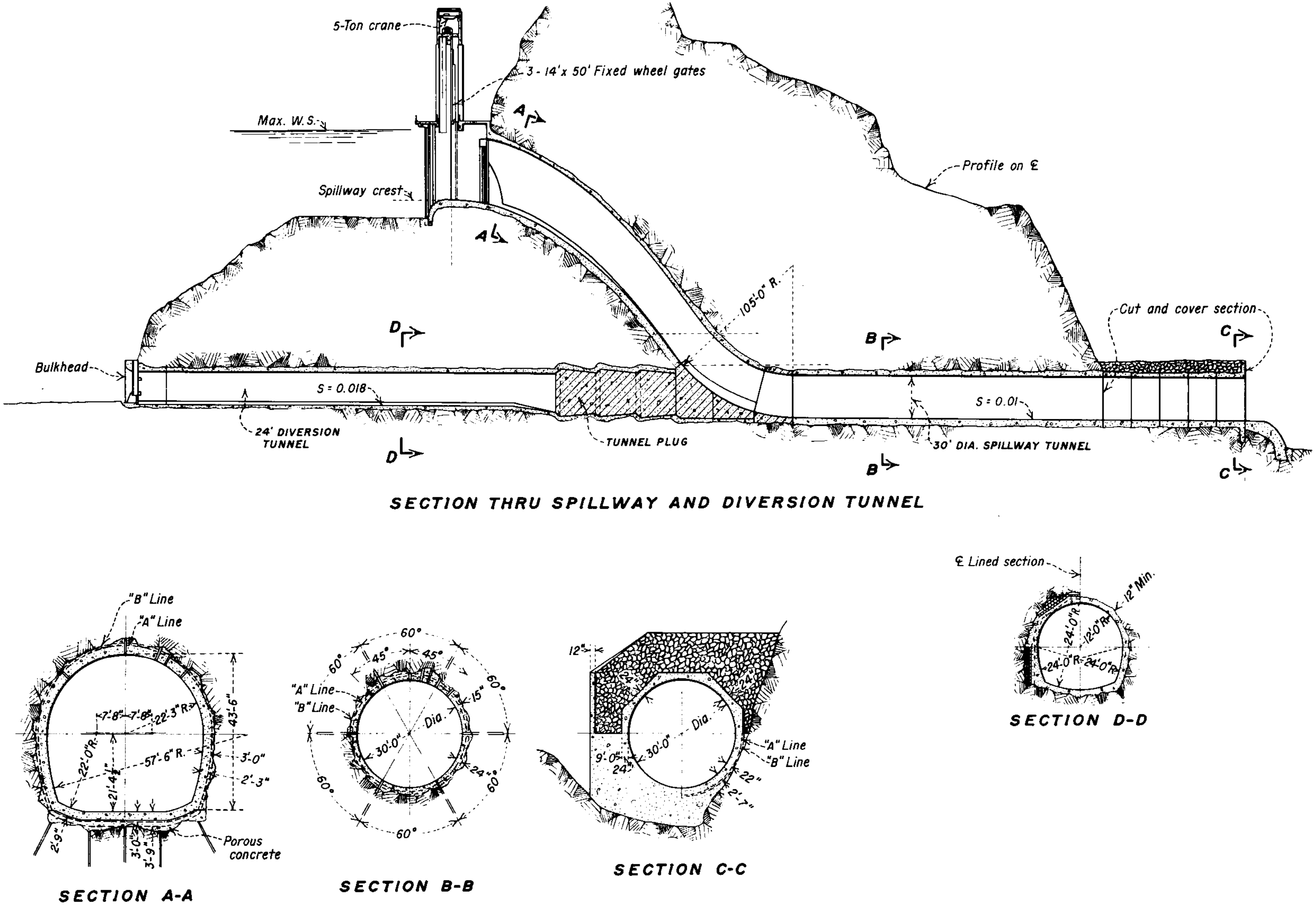


*Figure 5-4.* Typical arrangement of diversion tunnel with spillway tunnel.—288-D-3002

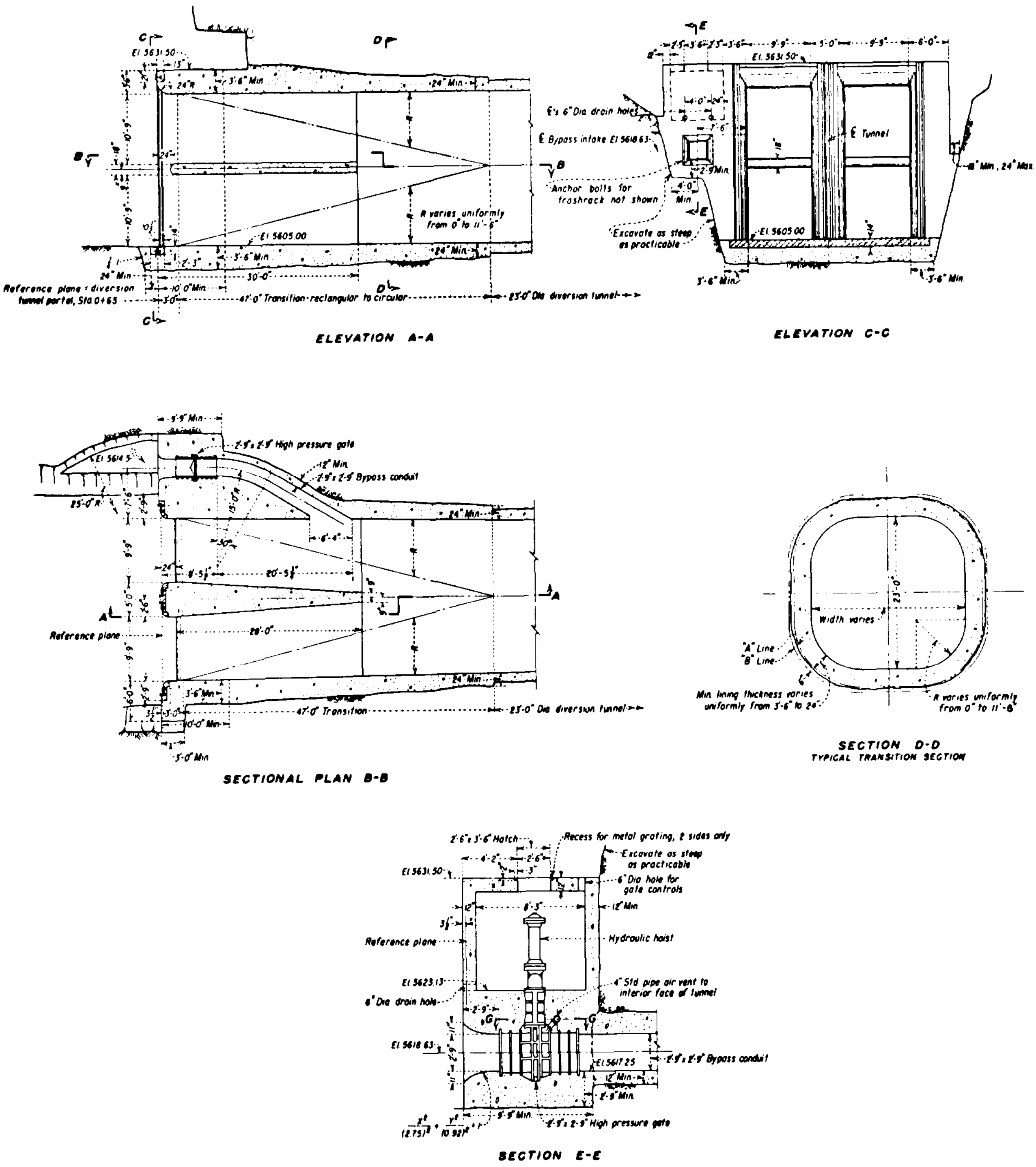
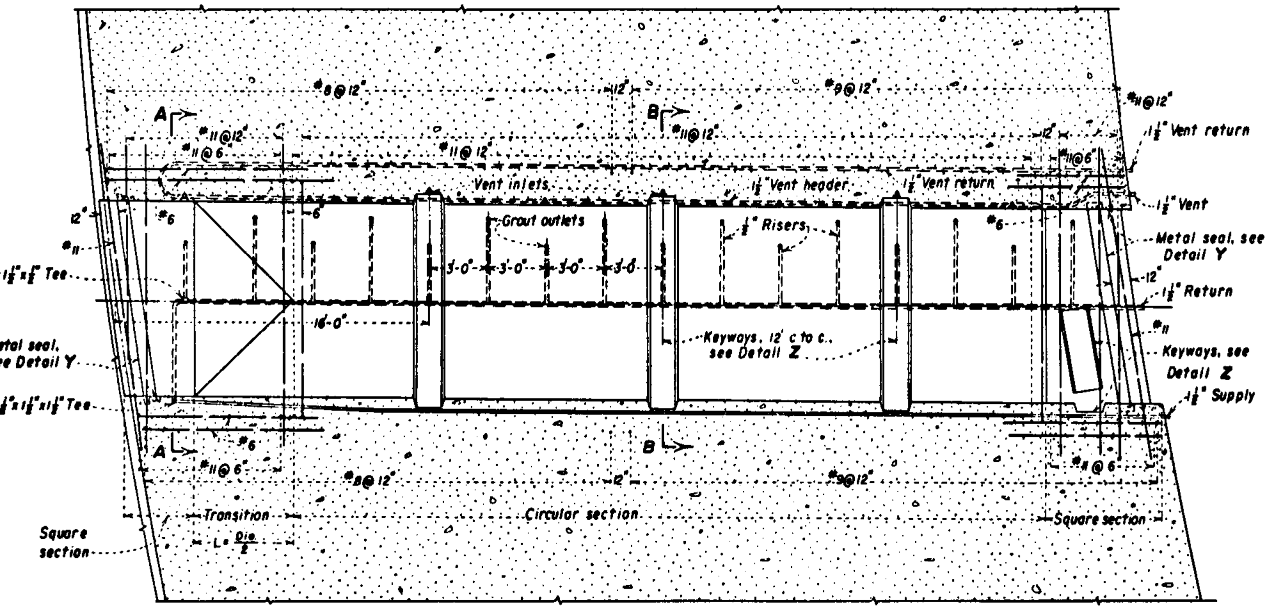
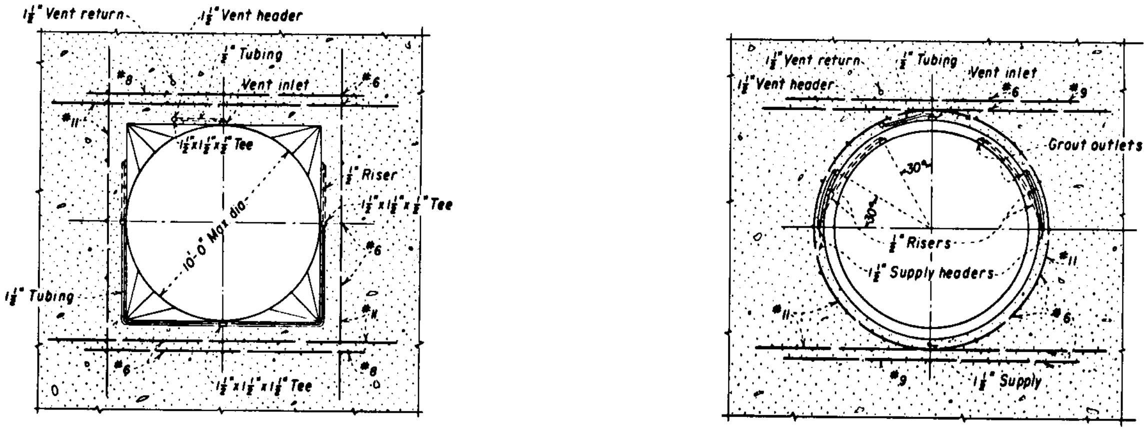


*Figure 5-5.* Diversion tunnel closure structure for a large concrete dam (Flaming Gorge Dam in Utah).–288-D-3003

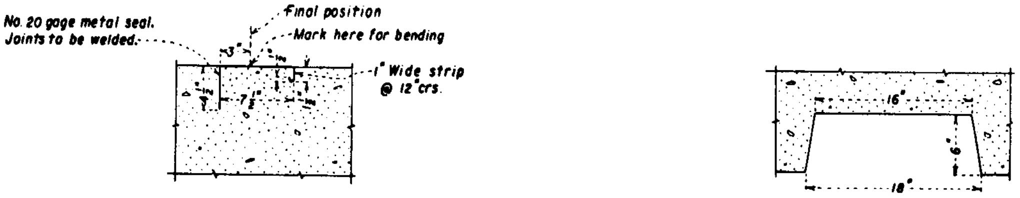


SECTION ALONG ℄ OF CONDUIT



SECTION A-A

SECTION B-B



DETAIL Y

DETAIL Z

*Figure 5-6.* Diversion conduit through Morrow Point Dam, a thin arch structure in Colorado—plan and sections.—288-D-3004 scheme can be found by comparing costs of various cofferdam heights versus the corresponding flume capacity. Large flumes may be of steel or timber frame with a timber lining, and smaller flumes may be of timber or metal construction, pipe, etc.

The flume is usually constructed around one side or the other of the damsite or over a low block. The flume can then be moved to other areas as the work progresses and stage construction can be utilized. During the construction of Canyon Ferry Dam, a steel-framed, timber-lined flume was constructed along the right bank of the river to be used as the first stage of diversion. The flume was designed for a capacity of approximately 23,000 cubic feet per second. The completed flume can be seen in figure 5-7 and a view of the flume in use can be seen in figure 5-8.

**5-10. *Multiple-Stage Diversion.***—The multiple-stage method of diversion over the tops of alternate low construction blocks or through diversion conduits in a concrete dam requires shifting of the cofferdam from one side of the river to the other during construction. During the first stage, the flow is restricted to one portion of the stream channel while the dam is constructed to a safe elevation in the remainder of the channel. In the second stage, the cofferdam is shifted and the stream is carried over low blocks or through diversion conduits in the constructed section of the dam while work proceeds on the unconstructed portion of the dam. The dam is then carried to its final height, with diversion ultimately being

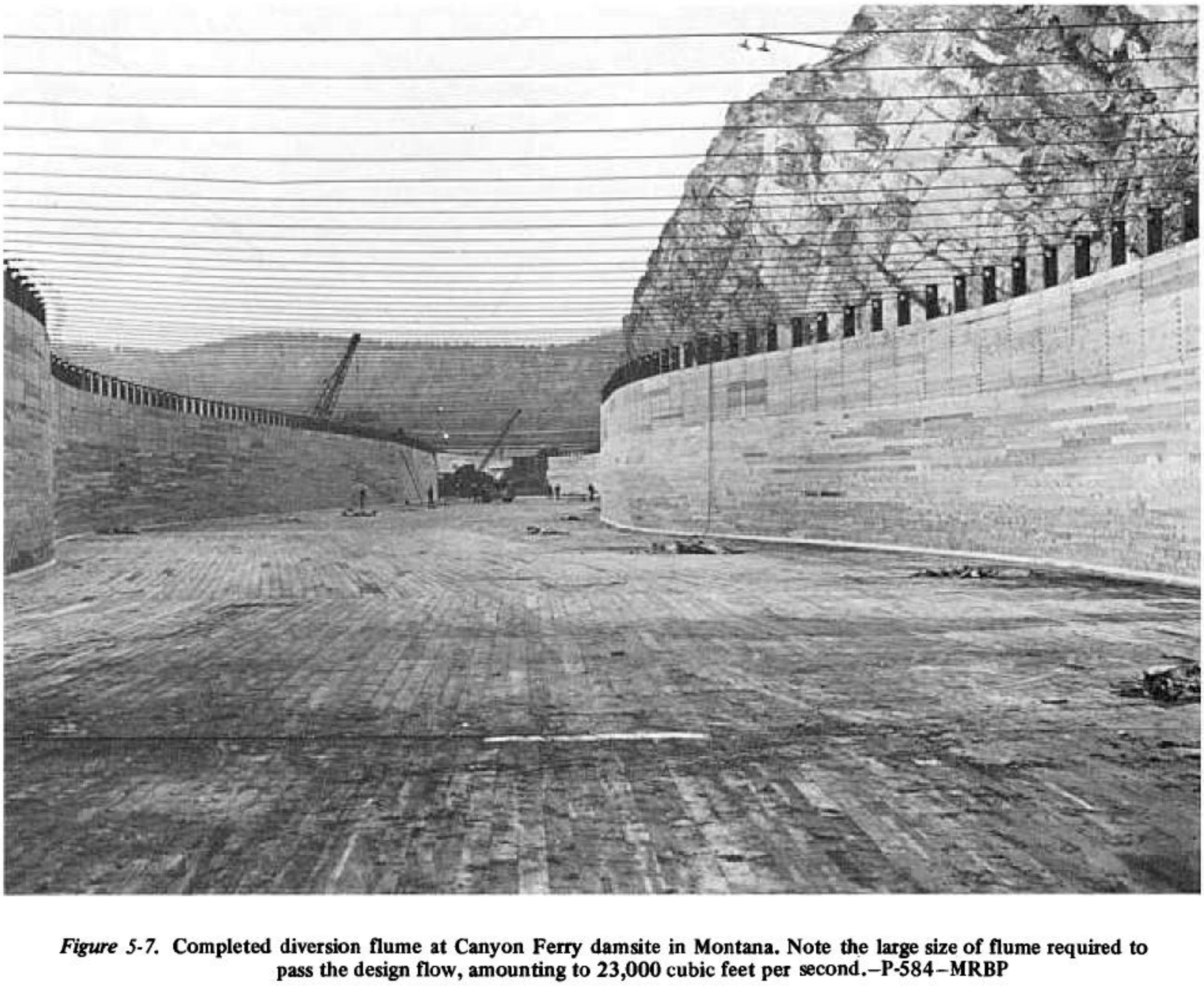


*Figure 5-7.* Completed diversion flume at Canyon Ferry damsite in Montana. Note the large size of flume required to pass the design flow, amounting to 23,000 cubic feet per second.—P-584—MRBP

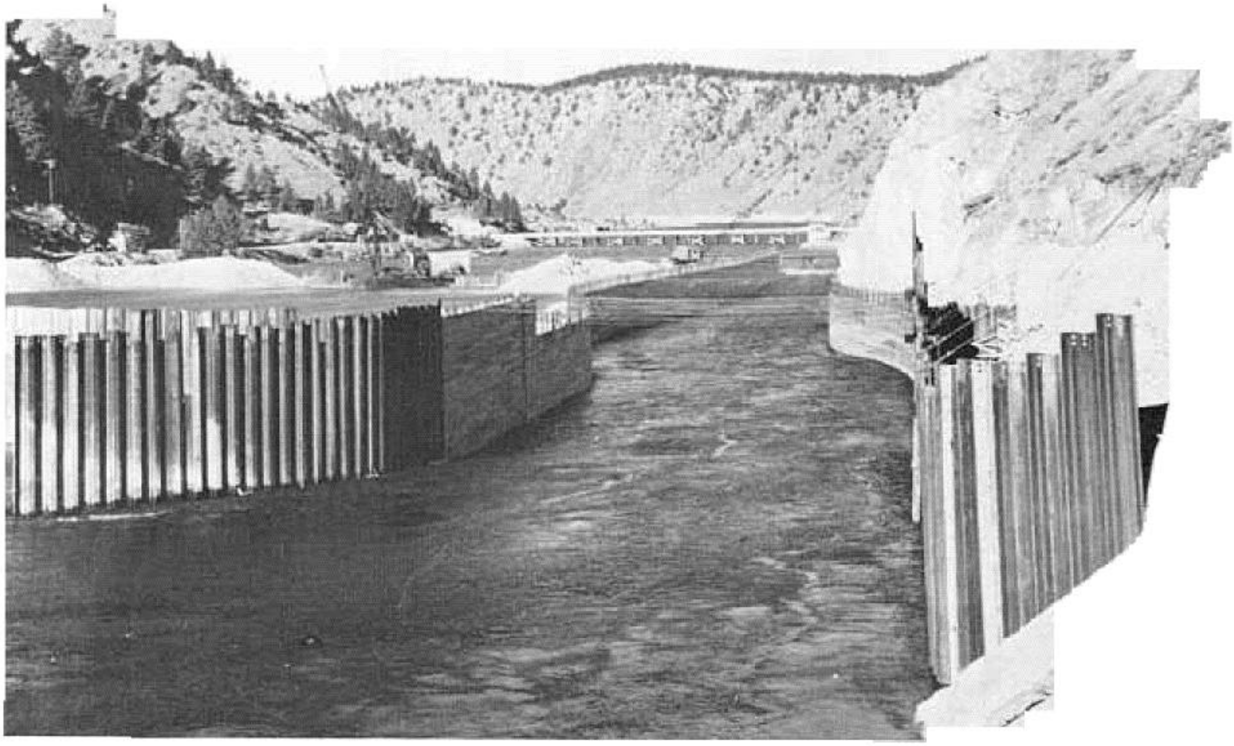


*Figure 5-8.* **Completed diversion flume at Canyon Ferry damsite in use for first-stage diversion.–P-591–MRBP**

made through the spillway, penstock, or permanent outlets. Figure 5-9 shows diversion through a conduit in a concrete dam, with excess flow over the low blocks.

**5-11.** ***Cofferdams.***–A cofferdam is a temporary dam or barrier used to divert the stream or to enclose an area during construction. The design of an adequate cofferdam involves the problem of construction economics. Where the construction is timed so that the foundation work can be executed during the low water season, use of cofferdams can be held to a minimum. Where the streamflow characteristics are such that this is not practicable, the cofferdam must be so designed that it is not only safe, but also of the optimum height. The height to which a cofferdam should be constructed involves an economic study of cofferdam height versus diversion works capacity, including routing studies of the diversion design flood. This is particularly true when the outlet works requirements are small. It should be remembered that floodwater accumulated behind the cofferdam must be evacuated in time to accommodate a recurrent storm. The maximum height to which it is feasible to construct the cofferdam without encroaching upon the area to be occupied by the dam must also be considered. Furthermore.

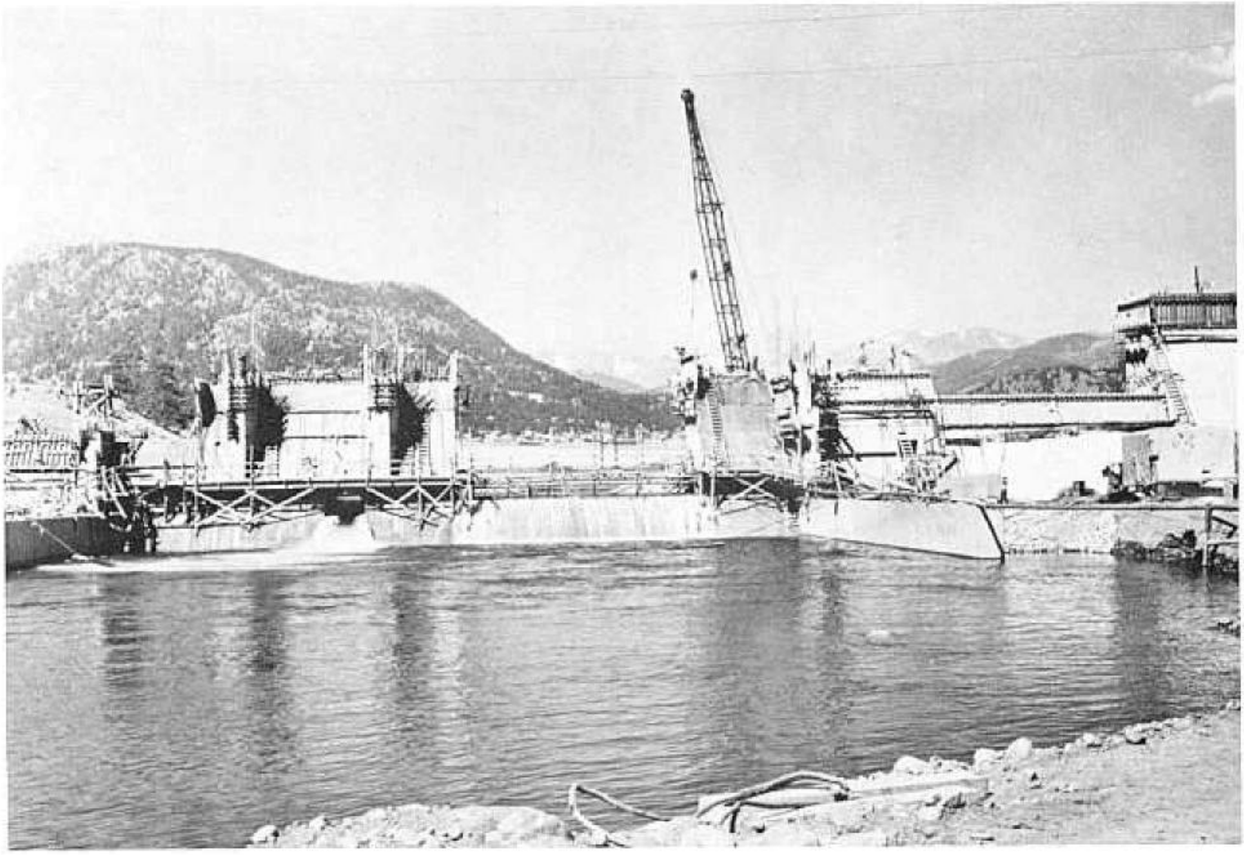


*Figure 5-9.* **Flows passing through diversion opening and over low blocks of a concrete and earth dam (Olympus Dam in Colorado).–EPA-PS-330-CBT**

the design of the cofferdam must take into consideration the effect that excavation and unwatering of the foundation of the dam will have on the cofferdam stability, and must anticipate removal, salvage, and other factors.

When determining the type and location of the cofferdams, the effects on the stream as related to water pollution and turbidity should be examined for each scheme under consideration. Unwatering work for structural foundations, constructing and removing cofferdams, and earthwork operations adjacent to or encroaching on streams or watercourses should be conducted in such a manner as to prevent muddy water and eroded materials from entering the channel. Therefore, the cofferdams should be placed in such a location that earthwork near the stream will be kept to a minimum, by containing as much of the excavation and work area within the confines of the cofferdams as practicable. During the construction and removal of the cofferdams, mechanized equipment should not be operated in flowing water except where necessary to perform the required work, and this should be restricted as much as possible.

Generally, cofferdams are constructed of materials available at the site. The two types normally used in the construction of dams are earthfill cofferdams and rockfill cofferdams, the design considerations of which closely follow those for permanent small dams of the same type. Other types, although not as common, include timber or concrete cribs filled with earth or rock, and cellular steel cofferdams filled with pervious material. Cribs and cellular steel cofferdams can be used when space for a cofferdam is limited or material is scarce. Cellular cofferdams are especially adaptable to confined areas where currents are swift and normal cofferdam construction would be difficult.

In many situations, a combination of several types of cofferdams may be used to develop the diversion scheme in the most economical and practical manner. The type of cofferdam would be determined for each location depending upon such factors as the materials available, required height, available space, swiftness of water, and ease of removal.

## C. SPECIFICATIONS REQUIREMENTS

5-12. *Contractor's Responsibilities.*—It is general practice to require the contractor to assume responsibility for the diversion of the stream during construction of the dam and appurtenant structures. The requirement should be defined by appropriate paragraphs in the specifications which describe the contractor's responsibilities and inform him as to what provisions, if any, have been incorporated in the design to facilitate construction. Usually the specifications should not prescribe the capacity of the diversion works, nor the details of the diversion method to be used; but hydrographs prepared from streamflow records, if available, should be included. Also, the specifications usually require that the contractor's diversion plan be subject to the owner's approval.

In some cases the entire diversion scheme might be left in the contractor's hands, with the expectation that the flexibility afforded to the contractor's operations by allowing him to choose the scheme of diversion will be reflected in low bids. Since various contractors will usually present different schemes, the schedule of bids in such instances should require diversion of the river to be included as a lump-sum bid. Sometimes a few pertinent paragraphs are appropriate in the specifications giving stipulations which affect the contractor's construction procedures. For example, restriction from certain diversion schemes may be specified because of safety requirements, geology, ecology, or time and space limitations. The contractor may also be required to have the dam constructed to a certain elevation or have the channel or other downstream construction completed before closure of the diversion works is permitted.

These or similar restrictions tend to guide the contractor toward a safe diversion plan. However, to further define the contractor's responsibility, other statements should be made to the effect that the contractor shall be responsible for and shall repair at his expense any damage to the foundation, structures, or any other part of the work caused by flood, water, or failure of any part of the diversion or protective works. The contractor should also be cautioned concerning the use of the hydrographs, by a statement to the effect that the contracting authority does not guarantee the reliability or accuracy of any of the hydrographs and assumes no responsibility for any deductions, conclusions, or interpretations that may be made from them.

5-13. *Designer's Responsibilities.*—For difficult and/or hazardous diversion situations, it may prove economical for the owner to assume the responsibility for the diversion plan. One reason for this is that contractors tend to increase bid prices for diversion of the stream if the specifications contain many restrictions and there is a large amount of risk involved. A definite scheme of cofferdams and tunnels might be specified where the loss of life and property damage might be heavy if a cofferdam built at the contractor's risk were to fail.

Another consideration is that many times the orderly sequence of constructing various stages of the entire project depends on a particular diversion scheme being used. If the responsibility for diversion rests on the contractor, he may pursue a different diversion scheme, with possible delay to completion of the entire project. This could result in a delay in delivery of irrigation water or in generation of power, or both, with a subsequent loss in revenue.

If the owner assumes responsibility for the diversion scheme, it is important that the diversion scheme be realistic in all respects, and compatible with the probable ability and capacity of the contractor's construction plant.

# Foundation Treatment

## A. EXCAVATION

6-1. *General.*—The entire area to be occupied by the base of an arch dam should be excavated to firm material capable of withstanding the loads imposed by the dam, reservoir, and appurtenant structures. Considerable attention must be given to blasting operations to assure that the rock foundation is not damaged by excessive blasting. All excavations should conform to the lines and dimensions shown on the construction drawings where practicable; however, it may be necessary or even desirable to vary dimensions or excavation slopes due to local conditions.

Foundations containing seams of shales, siltstones, chalks, or mudstones may require protection against air and water slaking, or in some environments against freezing. Such excavations can be protected by leaving a temporary cover of unexcavated material, by immediately covering the exposed surfaces with a minimum of 12 inches of pneumatically applied mortar, or by any other method that will prevent damage to the foundation.

6-2. *Shaping.*—Although not considered essential, a symmetrical or near symmetrical profile is desirable for an arch dam from the standpoint of stress distribution. However, asymmetrical canyons frequently have to be chosen as arch damsites. The nonsymmetry may introduce stress problems, but these can be overcome by proper design. Abutment pads between the dam and its foundation may be used to overcome some of the detrimental effects of nonsymmetry or foundation irregularities. Thrust blocks are sometimes used at nonsymmetrical sites. However, the primary use of a thrust block is not to provide symmetry, but simply to establish an artificial abutment where a natural one did not exist. Overexcavation of a site to achieve symmetry on a nonsymmetrical profile is not recommended. In all cases, the foundation should be excavated in such a way as to eliminate sharp breaks in the excavated profile, since these may cause stress concentrations in both the foundation rock and the dam. The foundation should also be excavated to about radial or part radial lines as explained in section 4-7(e).

6-3. *Dental Treatment.*—Very often the exploratory drilling or final excavation uncovers faults, seams, or shattered or inferior rock extending to such depths that it is impracticable to attempt to clear such areas out entirely. Such geologic discontinuities can affect both the stability and the deformation modulus of the foundation. From a practical viewpoint, the foundation modulus need not be known accurately if, in a quantitative sense, the ratio of the foundation modulus $E_f$ to the concrete modulus $E_c$ of the dam is known to be greater than 1:4. Moreover, canyons with extensive zones of highly deformable materials and consequently $E_f/E_c$ ratios less than 1:4 should not be arbitrarily dismissed as potential arch dam sites. The deformation modulus of such zones can be improved by removing sheared material, gouge, and/or inferior rock and replacing the same with backfill concrete. A case in point is the Auburn Dam foundation located in northeast California. Here the basic volcanic rock was interspersed with faults and shears, most of which had continuous seams of gouge that varied from paper thin to several feet in thickness, and with other rock anomalies each with individual deformation moduli. The resulting anistropy required a definitive analysis to obtain the existing foundation modulus, and then a determination of which and how much of the geologic discontinuities need be treated to obtain acceptable deformation moduli throughout the foundation.

The finite element method of analysis provides a way to combine the physical properties of different rock types and geologic discontinuities such as faults, shears, and joint sets into a value representative of the stress and deformation in a given segment of the foundation. The method also permits substitution of backfill concrete in faults, shears, and zones of weak rock, and evaluates the degree of beneficiation contributed by this "dental treatment" as it is frequently called.

Data required for the finite element method of analysis are: dimensions and composition of the lithologic bodies and geologic discontinuities, deformation moduli for each of the elements incorporated into the study, and the loading pattern imposed by the dam and reservoir. Methods for obtaining data related to the rock and discontinuities are discussed in the sections on foundation investigations in chapter II.

"Dental treatment" concrete may also be required to improve the stability of rock masses. By inputting data related to the shearing strength of faults, shears, joints, intact rock, pore water pressures induced by the reservoir and/or ground water, the weight of the rock mass, and the driving force induced by the dam and reservoir, a safety factor for a particular rock mass can be calculated.

Methods of rock stability analysis are discussed in chapter IV in the sections on finite element method and foundation analysis.

Frequently, relatively homogeneous rock foundations with only nominal faulting or shearing do not require the sophisticated analytical procedures described above. Theoretical studies developed for the foundation conditions and stresses at Shasta and Friant Dams have resulted in the development of the following approximate formulas for determining the depth of dental treatment:

$$d = 0.002\,bH + 5 \text{ for } H \geqq 150 \text{ feet}$$

$$d = 0.3\text{ b} + 5 \text{ for } H < 150 \text{ feet}$$

where:

$H$ = height of dam above general foundation level in feet,
$b$ = width of weak zone in feet, and
$d$ = depth of excavation of weak zone below surface of adjoining sound rock in feet.
(In clay gouge seams, $d$ should not be less than $0.1\,H$.)

These rules provide a means of approach to the question of how much should be excavated, but final judgment must be exercised in the field during actual excavation operations.

6-4. *Protection Against Piping.*—The approximate and analytical methods described in the preceding paragraphs will satisfy the stress, deformation, and stability requirements for a foundation, but they may not provide suitable protection against piping. Faults, shears, and seams may contain material conducive to piping and its accompanying dangers, so to mitigate this condition upstream and downstream cutoff shafts should be excavated in each seam, shear, or fault and backfilled with concrete. The dimension of the shaft perpendicular to the seam should be equal to the width of the weak zone plus a minimum of 1 foot on each end to key the concrete backfill into sound rock. The shaft dimension parallel with the seam should be at least one-half of the other dimension. In any instance a minimum shaft dimension of 5 feet each way should be used to provide working space.

The depth of cutoff shafts may be computed by constructing flow nets and computing the cutoff depths required to eliminate piping effects, or by the methods outlined by Khosla in reference [1].[1]

Other adverse foundation conditions may be due to horizontally bedded clay and shale seams, caverns, or springs. Procedures for treating these conditions will vary and will depend upon field studies of the characteristics of the particular condition to be remedied.

## B. GROUTING

6-5. *General.*—The principal objectives of grouting in a rock foundation are to establish an effective barrier to seepage under the dam and to consolidate the foundation. Spacing, length, and orientation of grout holes and the procedure to be followed in grouting a foundation are dependent on the height of the structure and the geologic characteristics of the foundation. Since the characteristics of a foundation will vary for each site, the grouting plan must be adapted to suit field conditions.

Grouting operations may be performed from the surface of the excavated foundation, from the upstream fillet of the dam, from the top of concrete placements for the dam, from galleries within the dam, and from tunnels driven into the abutments, or any combination of these locations.

The general plan for grouting the foundation rock of a dam provides for preliminary low-pressure, shallow consolidation grouting to be followed by high-pressure, deep curtain grouting. As used here, "high pressure" and "low pressure" are relative terms. The actual pressures used are usually the maximum that will result in filling the cracks and voids as completely as practicable without causing any uplift or lateral displacement of foundation rock.

6-6. *Consolidation Grouting.*—Low-pressure grouting to fill voids, fracture zones, and cracks at and below the surface of the excavated foundation is accomplished by drilling and grouting relatively shallow holes, called "B" holes. The extent of the area grouted and the depth of the holes will depend on local conditions.

Usually for structures 100 feet and more in height, a preliminary program will call for lines of holes parallel to the axis of the dam extending from the heel to the toe of the dam and spaced approximately 10 to 20 feet apart. Holes are staggered on alternate lines to provide better coverage of the area. The depths of the holes vary from 20 to 50 feet depending on local conditions and to some extent on the height of the structure. For thin arch structures and depending on local conditions, "B" hole grouting has been applied only in the area of the heel of the dam. In this case the upstream line of holes should lie at or near the heel of the dam to furnish a cutoff for leakage of grout from the high-pressure holes drilled later in the same general location. "B" holes are drilled normal to the excavated surface unless it is desired to intersect known faults, shears, fractures, joints, and cracks. Drilling is usually accomplished from the excavated surface, although in some cases drilling and grouting to consolidate steep abutments has been accomplished from the tops of concrete placements in the dam to prevent "slabbing" of the rock. In rarer cases, consolidation grouting has been performed from foundation galleries within the dam after the concrete placement has reached a certain elevation. This method of consolidation grouting requires careful control of grouting pressures and close inspection of the foundation to assure that the structure is not being disbonded from the foundation. Figure 6-1 illustrates a typical spacing and length pattern for "B" hole grouting.

In the execution of the consolidation grouting program, holes with a minimum diameter of 1½ inches are drilled and grouted 40 to 80 feet apart before split-spaced intermediate holes are drilled. The amount of grout which the intermediate holes accept determines whether any additional

---

[1] Numbers in brackets refer to items in the bibliography, sec. 6-9.

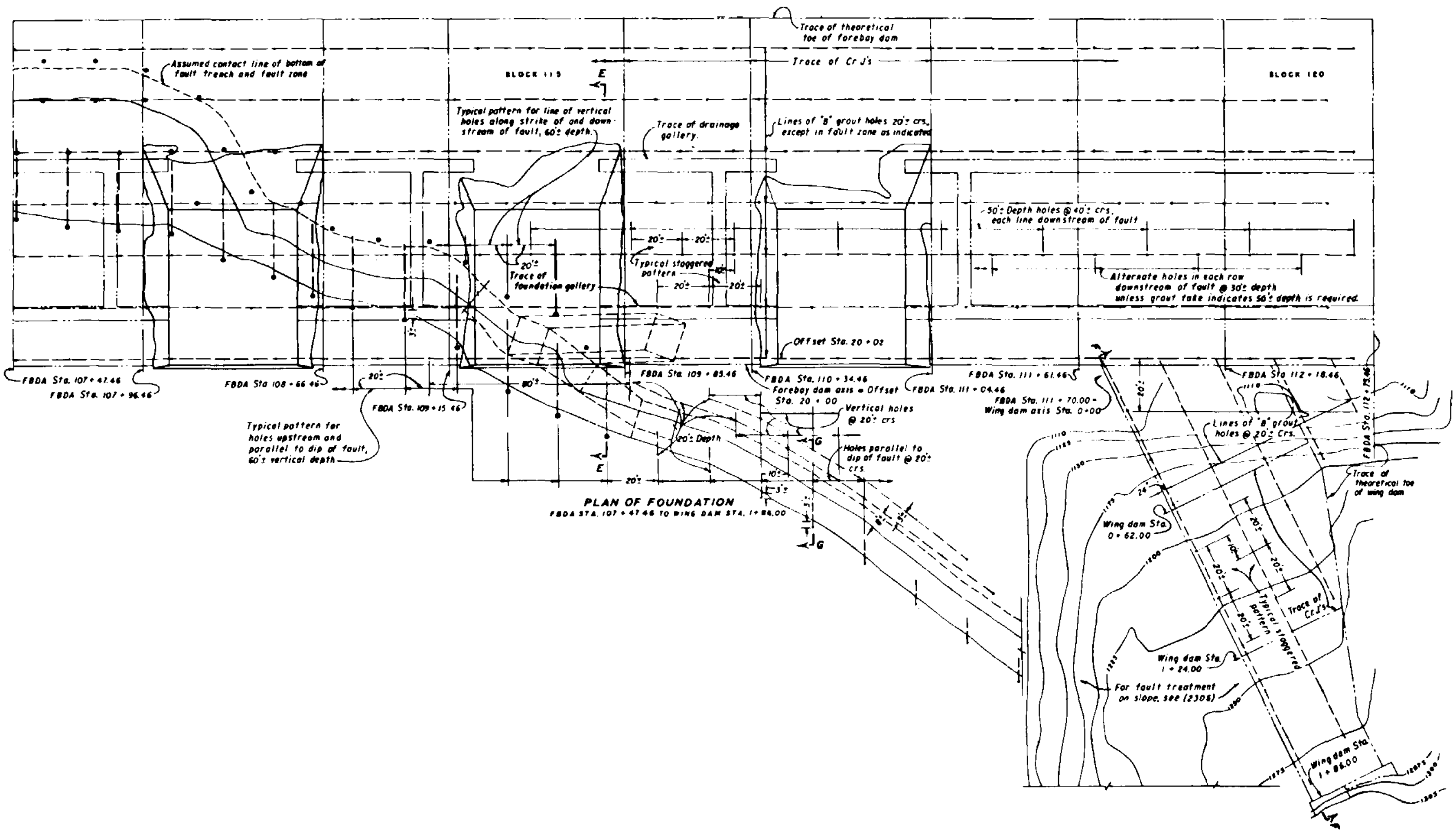
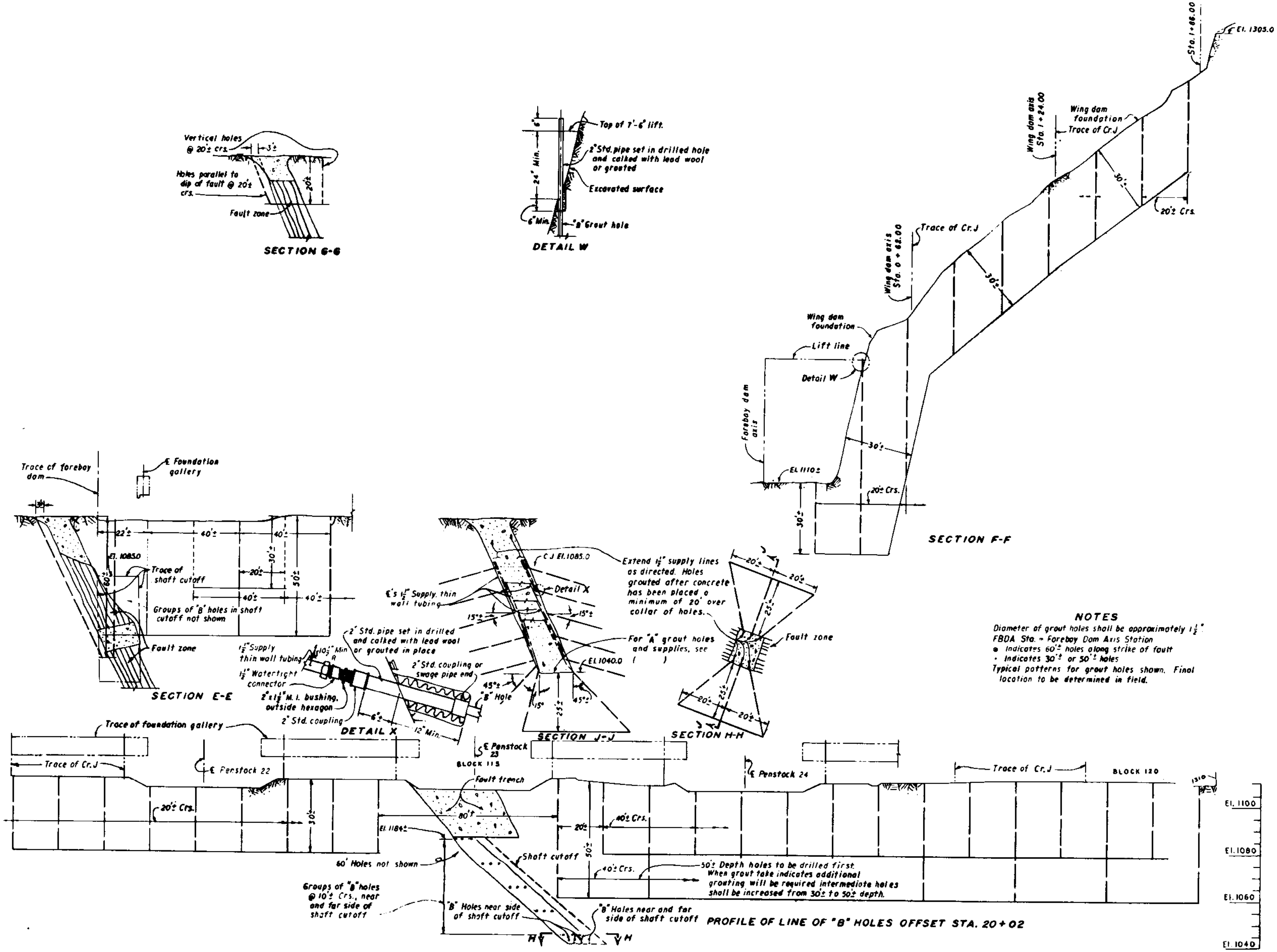


*Figure 6-1.* **Foundation treatment for Grand Coulee Forebay Dam in Washington (sheet 1 of 2).–288-D-3005(1/2)**



NOTES

Diameter of grout holes shall be approximately 1½"
FBDA Sta. = Forebay Dam Axis Station
● Indicates 60'± holes along strike of fault
· Indicates 30'± or 50'± holes
Typical patterns for grout holes shown. Final location to be determined in field.

*Figure 6-1.* Foundation treatment for Grand Coulee Forebay Dam in Washington (sheet 2 of 2).–288-D-3005(2/2) intermediate holes should be drilled. This split-spacing process is continued until grout "take" for the final closure holes is negligible and it is reasonably assured that all groutable seams, fractures, cracks, and voids have been filled.

Water-cement ratios for grout mixes may vary widely, depending on the permeability of the foundation rock. Starting water-cement ratios usually range from 8:1 to 5:1 by volume. Most foundations have an optimum mix that can be injected which should be determined by trial in the field by gradually thickening the starting mix. An admixture such as sand or clay may be added if large voids are encountered.

Consolidation grouting pressures vary widely and are dependent in part on the characteristics of the rock, i.e., its strength, tightness, joint continuity, stratification, etc.; and on the depth of rock above the stage being grouted. In general, grout pressures as high as practicable but which, as determined by trial, are safe against rock displacement are used in grouting. These pressures may vary from a low of 10 pounds per square inch to a high range of 80 to 100 pounds per square inch. A common rule of thumb is to increase the above minimum collar pressure by 1 pound per square inch per foot of depth of hole above the packer, as a trial. If the take is small, the pressure may be increased.

6-7. *Curtain Grouting.*–Construction of a deep grout curtain near the heel of the dam to control seepage is accomplished by drilling deep holes and grouting them using higher pressure. These holes are identified as "A" holes when drilled from a gallery. Tentative designs will usually specify a single line of holes drilled on 10-foot centers, although wider or closer spacing may be required depending on the rock condition. To permit application of high pressures without causing displacement in the rock or loss of grout through surface cracks, this grouting procedure is carried out subsequent to consolidation grouting and after some of the concrete has been placed. Usually, grouting will be accomplished from galleries within the dam and from tunnels driven into the abutments especially for this purpose. However, when no galleries are provided, as is the case for most thin arch dams, high-pressure grouting is done from curtain holes located in the upstream fillet of the dam before reservoir storage is started. Such grouting holes are identified as "C" holes.

The alinement of holes should be such that the base of the grout curtain will be located on the vertical projection of the heel of the dam. If drilled from a gallery that is some distance from the upstream face, the holes may be inclined as much as $15^{\circ}$ upstream from the plane of the axis. If the gallery is near the upstream face, the holes will be nearly vertical. Holes drilled from foundation tunnels may be inclined upstream or they may be vertical depending on the orientation of the tunnel with the axis of the dam. When the holes are drilled from the upstream fillet, they are usually inclined downstream. Characteristics of the foundation seams may also influence the amount of inclination.

To facilitate drilling, pipes of 2-foot minimum length are embedded in the floor of the gallery or foundation tunnel, or in the upstream fillet. When the structure has reached an elevation that is sufficient to prevent movement of concrete, the grout holes are drilled through these pipes and into the foundation. Although the tentative grouting plan may indicate holes to be drilled on 10-foot centers, the usual procedure will be first to drill and grout holes approximately 40 feet apart, or as far apart as necessary to prevent grout from one hole leaking into another drilled but ungrouted hole. Also, leakage into adjacent contraction joints must be prevented by prior grouting of the joints. Intermediate holes, located midway between the first holes, will then be drilled and grouted. Drilling and grouting of additional intermediate holes, splitting the spaces between completed holes, will continue until the desired spacing is reached or until the amount of grout accepted by the last group of intermediate holes indicates no further grouting is necessary.

The depth to which the holes are drilled will vary greatly with the characteristics of the foundation and the hydrostatic head. In a hard, dense foundation, the depth may vary from 30 to 40 percent of the head. In a poor foundation the holes will be deeper and may reach as deep as 70 percent of the head. During the progress of the grouting, local conditions may determine the actual or final depth of grouting. Supplementary grouting may also be required after the waterload has come on the dam and observations have been made of the rate of seepage and the accompanying uplift.

For high dams where foundation galleries are located at a relatively long distance from the upstream face, as at Hungry Horse Dam, "A" hole grouting may be augmented by a line of "C" holes, drilled from the upstream face of the dam and inclined downstream in order to supplement the main grout curtain. The depth of these holes is usually about 75 feet and their spacing is usually the same as for the "A" holes. The supplementary grout curtain formed by grouting this line of holes serves as an upstream barrier for subsequent "A" hole grouting, permitting higher "A" hole grout pressures with less chance of excessive upstream grout travel.

Usually the foundation will increase in density and tightness of seams as greater depths are reached, and the pressure necessary to force grout into the tight joints of the deep planes may be sufficient to cause displacements of the upper zones. Two general methods of grouting are used, each permitting the use of higher pressures in the lower zones.

(1) *Descending stage grouting* consists of drilling a hole to a limited depth or to its intersection with an open seam, grouting to that depth, cleaning out the hole after the grout has taken its initial set, and then drilling and grouting the next stage. To prevent backflow of grout during this latter operation, a packer is seated at the bottom of the previously grouted stage. This process is repeated, using higher pressures for each succeeding stage until the final depth is reached.

(2) *Ascending stage grouting* consists of drilling a hole to its final depth and grouting the deepest high-pressure stage by use of a packer which is seated at the top of this stage. The packer limits grout injection to the desired stage and prevents the grout from rising into the hole above the packer. After grouting this stage, the grout pipe is raised so that the packer is at the top of the next stage, which is subsequently grouted using somewhat lower pressure. This stage process is repeated upward until the hole is completely grouted. Ascending stage grouting is becoming more generally used, as it reduces the chances for displacement of the foundation rock, gives better control as to the zones of injection, and expedites the drilling.

The discussion in section 6-6 concerning grout pressures applies in general to curtain grouting. An exception is that higher initial collar pressures are permitted, depending on the height of concrete above the hole.

## C. DRAINAGE

6-8. ***Foundation Drainage.***—Although a well-executed grouting program may materially reduce the amount of seepage, some means must be provided to intercept the water which will percolate through and around the grout curtain, and, if not removed, may build prohibitive hydrostatic pressures on the base of the structure. Drainage is usually accomplished by drilling one or more lines of holes downstream from the high-pressure grout curtain. The size, spacing, and depth of these holes are assumed on the basis of judgment of the physical characteristics of the rock. Holes are usually 3 inches in diameter (NX size). Spacing, depth, and orientation are all influenced by the foundation conditions. Usually the holes are spaced on 10-foot centers with depths dependent on the grout curtain and reservoir depths. As a general role, hole depths vary from 20 to 40 percent of the reservoir depth and 35 to 75 percent of the deep curtain grouting depth.

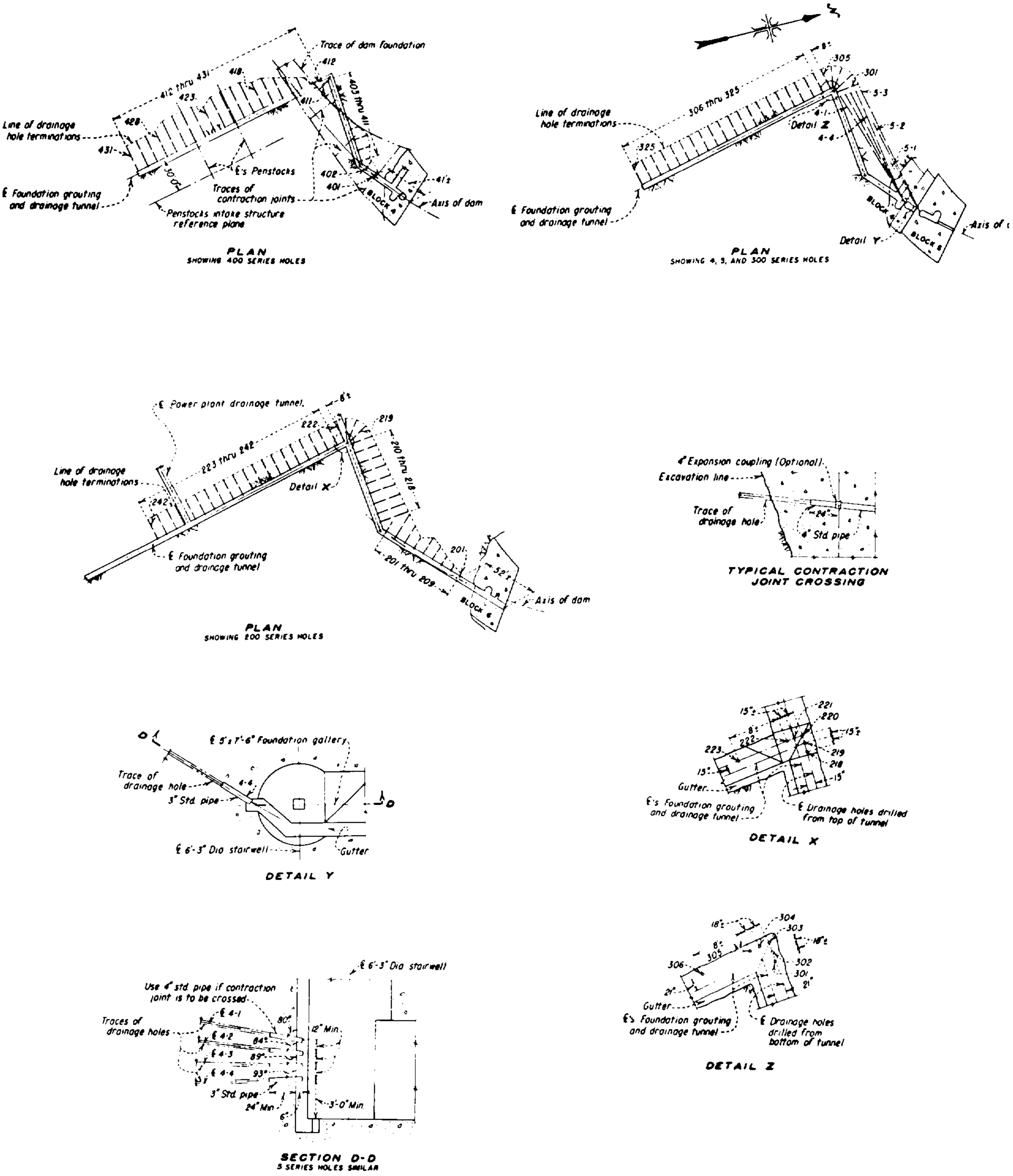
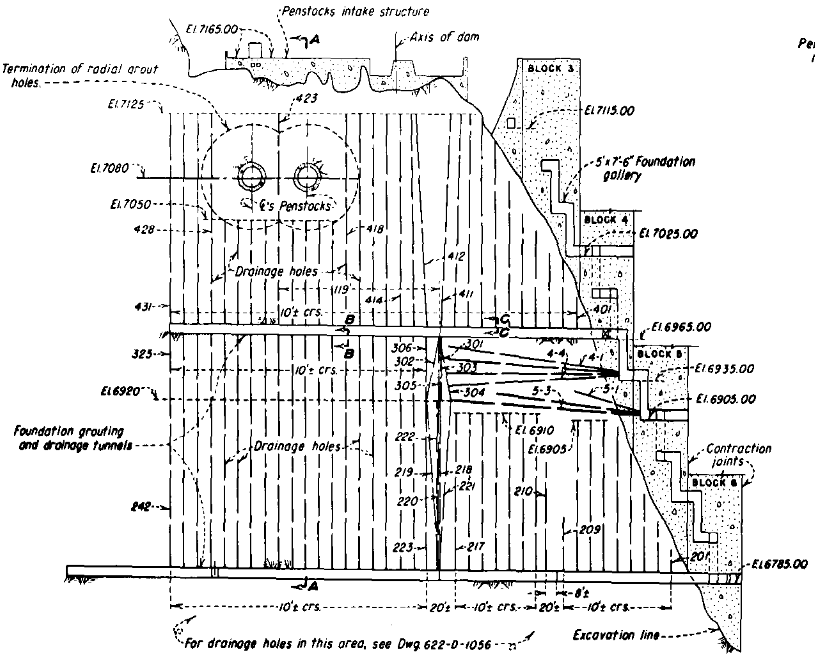
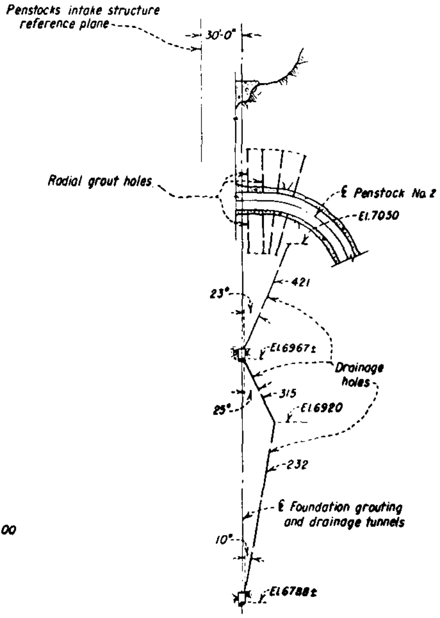


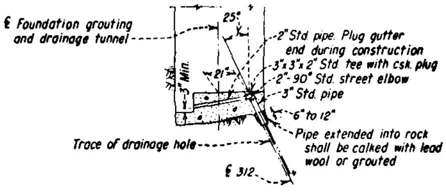
*Figure 6-2.* Foundation drainage system for Morrow Point Dam in Colorado (sheet 1 of 2).–288-D-3007(1/2)



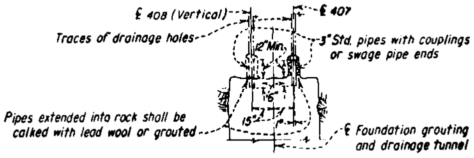
**ELEVATION - LEFT ABUTMENT**
DEVELOPED ALONG ℄'S OF FOUNDATION GROUTING AND DRAINAGE TUNNELS



**SECTION A-A**



**SECTION B-B**



**SECTION C-C**

| DRAINAGE HOLE | DEG. FROM VERTICAL | BEARING DEG. | APPROX. LGTH.-FT. | DRAINAGE HOLE | DEG. FROM VERTICAL | BEARING DEG. | APPROX. LGTH.-FT. |
|---|---|---|---|---|---|---|---|
| 401 | 10 | N.41 W. | 23 | 4-3 | 89 | S.79 W. | 115 |
| 402 | 10 | N.41 W. | 50 | 4-4 | 93 | S.81 W. | 116 |
| 403 | 10 | N.6 W. | 55 | 301 | 25 | N.6 W. | 51 |
| 404 | 10 | N.6 W. | 71 | 302 | 25 | N.30 W. | 51 |
| 405 | 10 | N.6 W. | 83 | 303 | 25 | N.54 W. | 51 |
| 406 | 5 | N.6 W. | 104 | 304 | 25 | N.78 W. | 51 |
| 407 | 1 | N.6 W. | 131 | 305-325 | 25 | S.77 W. | 52 |
| 408 | 0 | - | 145 | 201 | 10 | N.44 W. | 18 |
| 409 | 2 | S.36 E. | 152 | 202 | 10 | N.44 W. | 38 |
| 410 | 4 | S.36 E. | 152 | 203 | 10 | N.44 W. | 58 |
| 411 | 6 | S.36 E. | 152 | 204 | 10 | N.44 W. | 88 |
| 412 | 4 | S.8 E. | 152 | 205 | 10 | N.44 W. | 109 |
| 413 | 4 | S.61 W. | 152 | 206 | 7 | N.44 W. | 113 |
| 414 | 5 | S.77 W. | 152 | 207 | 10 | N.44 W. | 114 |
| 415 | 6 | S.77 W. | 152 | 208 | 13 | N.44 W. | 115 |
| 416 | 8 | S.77 W. | 153 | 209 | 17 | N.44 W. | 43 |
| 417-418 | 10 | S.77 W. | 153 | 210 | 15 | N.6 W. | 83 |
| 419-422 | 23 | S.77 W. | 82 | 211-212 | 14 | N.6 W. | 119 |
| 423 | 10 | S.77 W. | 153 | 213-214 | 13 | N.6 W. | 119 |
| 424-427 | 23 | S.77 W. | 82 | 215-216 | 12 | N.6 W. | 118 |
| 428-431 | 10 | S.77 W. | 152 | 217 | 11 | N.6 W. | 118 |
| 5-1 | 73 | S.74 W. | 57 | 218 | 10 | N.6 W. | 128 |
| 5-2 | 81 | S.77 W. | 95 | 219 | 10 | N.30 W. | 128 |
| 5-3 | 85 | S.79 W. | 129 | 220 | 10 | N.54 W. | 128 |
| 4-1 | 80 | S.74 W. | 119 | 221 | 10 | N.78 W. | 127 |
| 4-2 | 84 | S.76 W. | 117 | 222 thru 242 | 10 | S.77 W. | 127 |

**ORIENTATION OF HOLES**

**NOTES**

Diameter of drainage holes shall be approximately 3".

Unless otherwise directed, drainage holes shall not be drilled until all adjacent grout holes within a minimum distance of 250' have been drilled and grouted.

All dimensions measured on ℄'s of gallery or tunnels, unless otherwise specified.

Depth, spacing, and orientation of drainage holes shall be as shown unless otherwise directed by the contracting officer.

*Figure 6-2.* **Foundation drainage system for Morrow Point Dam in Colorado (sheet 2 of 2).—288-D-3007(2/2)**

Drain holes should be drilled after all foundation grouting has been completed in the area. They can be drilled from foundation and drainage galleries within the dam, or from the downstream face of the dam if no gallery is provided. Frequently drainage holes are drilled from foundation grouting and drainage tunnels excavated into the abutments, as shown on figure 6-2 (see previous page).

In some instances where the stability of a rock foundation may be beneficiated by reducing the hydrostatic pressure along planes of potentially unstable rock masses, drainage holes have been introduced to alleviate this condition. A collection system for such drainages should be designed so that flows can be collected and removed from the area.

## D. BIBLIOGRAPHY

### 6-9. *Bibliography.*

[1] Khosla, "Design of Dams on Permeable Foundations," Central Board of Irrigation, India, September 1936.

# <<Chapter VII

# Temperature Control of Concrete

## A. INTRODUCTION

7-1. ***Purposes.***—Temperature control measures are employed in mass concrete dams to (1) facilitate construction of the structure, (2) minimize and/or control the size and spacing of cracks in the concrete, and (3) permit completion of the structure during the construction period. The measures and degree of temperature control to be employed are determined by studies of the structure, its method of construction, and its temperature environment.

Cracking in mass concrete structures is undesirable because it affects the watertightness, internal stresses, durability, and appearance of the structures. Cracking will occur when tensile stresses are developed which exceed the tensile strength of the concrete. These tensile stresses may occur because of imposed loads on the structure, but more often occur because of restraint against volumetric change. The largest volumetric change in mass concrete results from change in temperature. The cracking tendencies which occur as a result of temperature changes and temperature differentials can be reduced to acceptable levels, in most instances, by the use of appropriate design and construction procedures.

Temperature control measures which minimize volumetric changes make possible the use of larger construction blocks, thereby resulting in a more rapid and economical construction. One of these measures, post cooling, is also necessary if contraction joint grouting is to be accomplished. Basically, to function in the manner designed, an arch dam must be a continuous structure from abutment to abutment. Further, a dam with longitudinal joints must have a monolithic section in an upstream-downstream direction. Therefore, provision for the construction of these structures with transverse and/or longitudinal contraction joints must include measures by which the concrete is cooled and contraction joints are closed by grouting before the reservoir loads are applied.

Complete temperature treatments, over and above the use of precooling measures and embedded pipe cooling systems, have been used in some structures. In these instances, reductions were made in the amount of cement used, low-heat cements were specified, and effective use was made of pozzolan to replace a part of the cement. Glen Canyon Dam, because of the size of the construction blocks and the relatively low grouting temperature, was constructed with a $50^{\circ}$ F. maximum placing temperature, embedded cooling coils, a type II cement, and a mix containing 2 sacks of cement and 1 sack of pozzolan per cubic yard of concrete.

7-2. ***Volumetric Changes.***—Mass concrete structures undergo volumetric changes which, because of the dimensions involved, are of concern to the designer. The changes in volume due to early-age temperature changes can be controlled within reasonable limits and incorporated into the design of the structure. The final state of temperature equilibrium depends upon site conditions, and little if any degree of control over the subsequent periodic volumetric changes can be effected.

The ideal condition would be simply to eliminate any temperature drop. This could be achieved by placing concrete at such a low temperature that the temperature rise due to hydration of the cement would be just sufficient to bring the concrete temperature up to its final stable state. Most measures for the prevention of temperature cracking, however, can only approach this ideal condition. The degree of success is related to site conditions, economics, and the stresses in the structure.

The volumetric changes of concern are those caused by the temperature drop from the peak temperature, occurring shortly after placement, to the final stable temperature of the structure. A degree of control over the peak temperature can be attained by limiting the placing temperature of the fresh concrete and by minimizing the temperature rise after placement. The placing temperature can be varied, within limits, by precooling measures which lower the temperatures of one or more of the ingredients of the mix before batching. The temperature rise in newly placed concrete can be restrained by use of embedded pipe cooling systems, placement in shallow lifts with delays between lifts, and the use of a concrete mix designed to limit the heat of hydration. These measures will reduce the peak temperature which otherwise would have been attained. Proportionately, this reduction in peak temperature will reduce the subsequent volumetric change and the accompanying crack-producing tendencies.

7-3. ***Factors to be Considered.***—The methods and degree of temperature control should be related to the site conditions and the structure itself. Such factors as exposure conditions during and after construction, final stable temperature of the concrete mass, seasonal temperature variations, the size and type of structure, composition of the concrete, construction methods, and rate of construction should be studied and evaluated in order to select effective, yet economical, temperature control measures. The construction schedule and design requirements must also be studied to determine those procedures necessary to produce favorable temperature conditions during construction. Such factors as thickness of lifts, time interval between lifts, height differentials between blocks, and seasonal limitations on placing of concrete should be evaluated. Study of the effect of these variables will permit the determination of the most favorable construction schedules consistent with the prevention of cracking from temperature stresses.

Some structures favor the use of a particular method of temperature control. For arch dams, contraction joint grouting will normally be required since the arches are to be monolithic and are to transfer the reservoir load to the abutments by arch action. This contraction joint grouting will normally require that an embedded pipe system be used to cool the concrete artificially, either during construction of the dam or immediately afterward, so that the contraction joints will be open and can be grouted before the reservoir is filled. In some instances, as with an extremely thin arch dam, natural cooling over a winter period can accomplish the desired effect.

Thick arch dams may require longitudinal contraction joints in the blocks. Since open longitudinal joints would prevent a block from carrying its load as a monolith, arch dams with longitudinal joints must also provide for contraction joint grouting of the longitudinal joints. This normally requires cooling by means of an embedded pipe cooling system and grouting of the joints before the reservoir load is applied. These thick arch dams may also require additional temperature control measures such as precooling of aggregates and the use of low-heat cements, reduced cement content, and pozzolans.

7-4. ***Design Data.***—The collection of design data should start at the inception of the project and should be continued through the construction period. Data primarily associated with the determination of temperature control measures include the ambient air temperatures at the site, river water temperatures, anticipated reservoir and tailwater temperatures, and the diffusivity of the concrete in the dam.

The estimate of air temperatures which will occur in the future at a given site is based on air temperatures which have occurred in the past, either at that location of one in the rear vicinity. The National Weather Service[1] has collected climatological data at a great number of locations, and long-time records from one or more of these nearby locations may be selected and adjusted to the site. For this adjustment, an increase of 250 feet in elevation can be assumed to decrease the air temperature 1° F. Similarly, an increase of 1.4° in latitude can be assumed to decrease the temperature 1° F. River water temperatures and streamflow data can be obtained from various hydrometeorological and water supply reports and papers. A program for obtaining actual maximum and minimum daily air and river water temperatures at the site should be instituted as soon as possible to verify or adjust the data assumed for early studies. Representative wet- and dry-bulb temperatures should also be obtained throughout the year.

The best estimate of the future reservoir water temperatures would be one based on water temperatures recorded at nearby reservoirs of similar depth and with similar inflow and outflow conditions. The Bureau of Reclamation has obtained reservoir water temperatures over a period of several years in a number of reservoirs. From these data, maximum ranges of temperature for the operating conditions encountered were determined. When no data are available on nearby reservoirs, the next best estimate of the reservoir temperatures can be obtained by the principle of heat continuity. This method takes into consideration the quantity and temperature of the water entering and leaving the reservoir, and the heat transfer across the reservoir surface. These heat budget computations, though accurate in themselves, are based on estimates of evaporation, conduction, absorption and reflection of solar radiation, and reradiation–which in turn are related to cloud cover, air temperatures, wind velocities, and relative humidity. Because of these variables, any forecast of temperature conditions in a reservoir based on the principle of heat continuity can only be considered as an estimate.

The diffusivity of concrete, $h^2$, is an index of the facility with which concrete will undergo temperature change. Although desirable from the heat standpoint, it is not practicable to select aggregate, sand, and cement for a concrete on the basis of heat characteristics. The thermal properties of the concrete must therefore be accepted for what they are. The value of the diffusivity of concrete is expressed in square feet per hour, and can be determined from the relationship,

$$h^2 = \frac{K}{C\rho}$$

where:

$K$ = conductivity in B.t.u. per foot per hour per ° F.,
$C$ = specific heat in B.t.u. per pound per ° F., and
$\rho$ = density in pounds per cubic foot.

Values of the diffusivity for a given concrete are determined from laboratory tests, although they must normally be estimated for early studies. As the thermal characteristics of the coarse aggregate largely govern the thermal characteristics of the concrete, the earliest of these estimates can be based upon the probable type of coarse aggregate to be used in the concrete. Table 7-1 gives the thermal properties of concretes in Bureau of Reclamation dams and representative values for several rock types.

7-5. ***Cracking.***—Temperature cracking in mass concrete occurs as tensile stresses are developed when a temperature drop takes place in the concrete and some degree of restraint exists against this volumetric change. The stresses developed are related to the amount and rate of the temperature drop, the age of the concrete when the temperature drop takes place, and the elastic and inelastic properties of the particular concrete. The restraint may be external, such as the restraint exerted by the foundation of a structure; or it may be internal, such as the restraint exerted by a mass upon its surface. Tensile stresses also occur when a nonlinear temperature variation occurs across a section of the structure. Because of the inelastic properties of concrete, the stresses developed are related to the temperature

---

[1] Official designation: U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service.

Table 7-1.–*Thermal properties of concrete for various dams.*

| Dam | Density (saturated) | Conductivity K B.t.u./ft.-hr.-° F. | | | Specific heat C B.t.u./lb.-° F. | | | Diffusivity $h^2$ ft.$^2$/hr. | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | lb./cu. ft. | 50° | 70° | 90° | 50° | 70° | 90° | 50° | 70° | 90° |
| East Canyon (predominately quartz and quartzite) | 152.9 | 2.56 | 2.53 | 2.50 | 0.208 | 0.213 | 0.217 | 0.081 | 0.078 | 0.075 |
| Glen Canyon | 148.4 | 2.02 | 2.01 | 2.01 | .211 | .216 | .222 | .065 | .063 | .061 |
| Seminoe | 155.3 | 1.994 | 1.972 | 1.951 | .204 | .213 | .222 | .063 | .060 | .057 |
| Norris | 160.6 | 2.120 | 2.105 | 2.087 | .234 | .239 | .247 | .056 | .055 | .053 |
| Wheeler | 145.5 | 1.815 | 1.800 | 1.785 | .223 | .229 | .236 | .056 | .054 | .052 |
| Flaming Gorge (limestone and sandstone) | 150.4 | 1.78 | 1.77 | 1.76 | .221 | .226 | .232 | .054 | .052 | .050 |
| Kortes mixes: | | | | | | | | | | |
| 1 bbl. cement/cu. yd. and 0.0-percent air | 157.6 | 1.736 | 1.724 | 1.711 | .210 | .215 | .221 | .052 | .051 | .049 |
| 0.85 bbl. cement/cu. yd. and 0.0-percent air | 158.1 | 1.715 | 1.710 | 1.705 | .209 | .215 | .220 | .052 | .050 | .049 |
| Hungry Horse | 150.1 | 1.72 | 1.72 | 1.71 | .217 | .223 | .229 | .053 | .051 | .050 |
| Hoover | 156.0 | 1.699 | 1.688 | 1.677 | .212 | .216 | .221 | .051 | .050 | .049 |
| Gibson | 155.2 | 1.676 | 1.667 | 1.657 | .218 | .222 | .229 | .050 | .048 | .047 |
| Canyon Ferry | 151.3 | 1.63 | 1.62 | 1.61 | .214 | .218 | .222 | .050 | .049 | .048 |
| Swift (limestone) | 158.2 | 1.82 | 1.79 | 1.76 | .237 | .242 | .246 | .049 | .047 | .041 |
| Altus | 149.7 | 1.578 | 1.579 | 1.580 | .225 | .229 | .234 | .047 | .046 | .045 |
| Monticello | 153.1 | 1.57 | 1.56 | 1.55 | .225 | .230 | .235 | .046 | .044 | .043 |
| Yellowtail | 152.8 | 1.57 | 1.56 | 1.55 | .219 | .223 | .227 | .047 | .046 | .045 |
| Angostura mixes: | | | | | | | | | | |
| 0.9 bbl. cement/cu. yd. and 3.0-percent air | 151.2 | 1.491 | 1.484 | 1.478 | .221 | .228 | .234 | .045 | .043 | .042 |
| 1.04 bbl. cement/cu. yd. and 0.0-percent air | 152.6 | 1.571 | 1.554 | 1.537 | .227 | .234 | .240 | .045 | .044 | .042 |
| Hiwassee | 155.7 | 1.505 | 1.491 | 1.478 | .218 | .225 | .233 | .044 | .042 | .041 |
| Parker | 155.1 | 1.409 | 1.402 | 1.395 | .213 | .216 | .221 | .043 | .042 | .041 |
| Owyhee | 152.1 | 1.376 | 1.373 | 1.369 | .208 | .214 | .222 | .044 | .042 | .041 |
| O'Shaughnessy | 152.8 | 1.316 | 1.338 | 1.354 | .217 | .218 | .223 | .040 | .040 | .040 |
| Friant mixes: | | | | | | | | | | |
| Portland cement | 153.6 | 1.312 | 1.312 | 1.312 | .214 | .214 | .217 | .040 | .040 | .039 |
| 20-percent pumicite | 153.8 | 1.229 | 1.232 | 1.234 | .216 | .221 | .227 | .037 | .036 | .035 |
| Shasta | 157.0 | 1.299 | 1.309 | 1.319 | .222 | .229 | .235 | .037 | .037 | .036 |
| Bartlett | 156.3 | 1.293 | 1.291 | 1.289 | .216 | .222 | .230 | .038 | .037 | .036 |
| Morris | 156.9 | 1.290 | 1.291 | 1.293 | .214 | .216 | .222 | .039 | .038 | .037 |
| Chickamauga | 156.5 | 1.287 | 1.277 | 1.266 | .225 | .229 | .233 | .037 | .036 | .035 |
| Morrow Point (andesite-basalt) | 145.5 | 0.99 | 0.97 | 0.94 | .212 | .217 | .222 | .032 | .031 | .029 |
| Grand Coulee | 158.1 | 1.075 | 1.077 | 1.079 | .219 | .222 | .227 | .031 | .031 | .030 |
| Ariel | 146.2 | 0.842 | 0.884 | 0.915 | .228 | .235 | .244 | .025 | .026 | .026 |
| Bull Run | 159.1 | 0.835 | 0.847 | 0.860 | .215 | .225 | .234 | .024 | .024 | .023 |
| Thermal Properties of Coarse Aggregate | | | | | | | | | | |
| Quartzite | 151.7 | 2.052 | 2.040 | 2.028 | .209 | .217 | .226 | .065 | .062 | .059 |
| Dolomite | 156.2 | 1.948 | 1.925 | 1.903 | .225 | .231 | .238 | .055 | .053 | .051 |
| Limestone | 152.8 | 1.871 | 1.842 | 1.815 | .221 | .224 | .230 | .055 | .054 | .052 |
| Granite | 150.9 | 1.515 | 1.511 | 1.588 | .220 | .220 | .224 | .046 | .045 | .045 |
| Basalt | 157.5 | 1.213 | 1.212 | 1.211 | .226 | .226 | .230 | .034 | .034 | .033 |
| Rhyolite | 146.3 | 1.197 | 1.203 | 1.207 | .220 | .226 | .232 | .037 | .036 | .036 |

history of the structure.

The most common cracking in mass concrete occurs when large blocks of concrete are placed on the foundation in the fall of the year, after which concreting is stopped for the winter. Under these conditions, foundation restraint is high, large drops in temperature are possible because concrete placing temperatures and peak temperatures are relatively high, and concrete temperatures will be dropping quite rapidly due to exposure conditions. For blocks not larger than 50 by 50 feet, cracking under these conditions has no particular pattern. In larger blocks, and where the length-to-width ratio is over 2, cracking under the above conditions often occurs at or near the third points of the longer side. Generally, if the blocks are not placed more than 10 or 15 feet off the foundation, cracking will start at the exposed top edge of the block and progress into the block and down the side to within a few feet of the foundation. Such cracks vary from extremely small or hairline surface cracks which penetrate only a few inches into the mass, to irregular structural cracks of varying width which completely cross the construction block. The maximum crack width is at the top edge and normally will be from 1/32 to 1/64 inch in width.

Similar cracking across the full width of a block can occur during the colder months of the year in a high block which has been constructed well off the foundation and which is 25 to 50 feet higher than the adjacent blocks. In this instance, the upper part of the block will cool at a relatively fast rate while that part of the block below the elevation of the adjacent blocks may remain at the same temperature or may possibly rise in temperature depending upon its age.

Surface cracking which occurs because of internal restraint seldom follows any particular pattern. The most general cracking is along the horizontal construction joints where the tensile strength is low. Such cracking normally occurs when wood or insulated steel forms are used and then removed when exposure temperatures are low. Upon removal of the forms, the surface is subjected to a thermal drop which sets up a severe temperature gradient between the surface and the interior. Practically all of these cracks are from hairline width to 1/64 inch in thickness. Aside from the horizontal construction joints, most other surface cracking is evidenced by vertical or near-vertical cracks associated with surface irregularities such as openings, reentrant corners, or construction discontinuities which occurred during placement. Most of these cracks do not progress beyond the one placement lift, but those that do often are the beginning of the cracks described above.

## B. METHODS OF TEMPERATURE CONTROL

7-6. ***Precooling.***—One of the most effective and positive temperature control measures is that which reduces the placing temperature of the concrete. Methods of reducing the placing temperature which would otherwise be obtained at a site can be varied from restricting concrete placement during the hotter part of the day or the hotter months of the year, to a full treatment of refrigerating the various parts of the concrete mix to obtain a predetermined, maximum concrete placing temperature.

The method or combination of methods used to reduce concrete placing temperatures will vary with the degree of cooling required and the contractor's equipment and previous experience. For some structures, sprinkling and shading of the coarse aggregate piles may be the only precooling measures required. The benefits of sprinkling depend largely on the temperature of the applied water and on the contractor's operations at the stockpile. A secondary benefit, evaporative cooling, can also be obtained but is restricted to areas with a low relative humidity. Insulating and/or painting the surfaces of the batching plant, water lines, etc., with reflective paint can also be beneficial.

Mixing water can be cooled to varying degrees, the more common temperatures being from $32^{\circ}$ to $40^{\circ}$ F. Adding slush or crushed ice to the mix is an effective method of cooling because it takes advantage of the latent heat of fusion of ice. The addition of large amounts of ice, however, may not be very practical in some instances. For example, if the coarse aggregate and sand both contain appreciable amounts of free water, the amount of water to be added to the mix may be so small that replacement of part of the added water with ice would not be appreciable.

Cooling of the coarse aggregates to about $35^{\circ}$ F. can be accomplished in several ways. One method is to chill the aggregate in large tanks of refrigerated water for a given period of time. Relatively effective cooling of coarse aggregate can also be attained by forcing refrigerated air through the aggregate while the aggregate is draining in stockpiles, while it is on a conveyor belt, and while it is passing through the bins of the batching plant. Spraying with cold water will also cool the aggregate. Sand may be cooled by passing it through vertical tubular heat exchangers. Cold air jets directed on the sand as it is transported on conveyor belts can also be used. Immersion of sand in cold water is not practical because of the difficulty in removing the free water from the sand after cooling.

Cooling of the cement is seldom practicable. Bulk cement in the quantities used for dams is almost always obtained at relatively high temperatures, generally from $140^{\circ}$ to $180^{\circ}$ F. Seldom will it cool naturally and lose a sizable portion of the excess heat before it is used.

Use of the above treatments has resulted in concrete placing temperatures of $50^{\circ}$ F. in a number of instances. Concrete placing temperatures as low as $45^{\circ}$ F. have been attained, but these can usually be achieved only at a considerable increase in cost. The temperature of the concrete at the mixing plant should be $3^{\circ}$ to $4^{\circ}$ F. lower than the desired placing temperature. This will compensate for the heat developed and absorbed by the concrete during mixing and transporting.

7-7. ***Postcooling.***—Postcooling of mass concrete in arch dams is used primarily to reduce the temperature of the concrete to the desired contraction joint grouting temperature during the construction period. Subcooling below the final stable state temperature may be accomplished, if desired, and the temperature may be varied throughout the structure at the time of grouting the joints. If a suitable temperature variation is provided, subsequent temperature changes will result in a better stress distribution during reservoir operation. The layout of embedded cooling systems used in postcooling mass concrete is described in section 7-20.

Postcooling is an effective means of crack control. Artificially cooling mass concrete by circulating cold water through embedded cooling coils on the top of each construction lift will materially reduce the peak temperature of the concrete below that which would otherwise be attained. However, these embedded coils will not actually prevent a temperature rise in the concrete, because of the high rate of heat development during the first few days after placement and the relatively low conductivity of the concrete. The use of an embedded pipe system affords flexibility in cooling through operation of the system. Any degree of cooling may be accomplished at any place at any time. This can minimize the formation of large temperature gradients from the warm interior to the colder exterior. The formation of such gradients in the fall and winter is particularly conducive to cracking.

7-8. ***Amount and Type of Cement.***—Mass concrete structures require lesser amounts of cement than the ordinary size concrete structures because of a lower strength requirement. Because of their dimensions, however, less heat is lost to the surfaces and a greater maximum temperature is attained. Since the heat generated within the concrete is directly proportional to the amount of cement used per cubic yard, the mix selected should be that one which will provide the required strength and durability with the lowest cement content. The cement content in mass concrete structures has varied in the past from 4 to 6 sacks of cement per cubic yard, but present-day structures contain as low as 2 sacks of cement plus other cementing materials. In thin arch structures, durability of the concrete also has to be considered, and the required strength plus the durability may require 3 to 4 sacks of cement.

The heat-producing characteristics of cement play an important role in the amount of temperature rise. Although cements are classified by type as type I, type II, etc., the heat generation within each type may vary widely because of the chemical compounds in the cement. Types II and IV were developed for use in mass concrete construction. Type II cement is commonly referred to as modified cement, and is used where a relatively low heat generation is desirable. Type IV cement is a low-heat cement characterized by its low rate of heat generation during early age.

Specifications for portland cement generally do not state within what limits the heat of hydration shall be for each type of cement. They do, however, place maximum percentages on certain chemical compounds in the cement. They further permit the purchaser to specifically request maximum heat of hydration requirements of 70 or 80 calories per gram at ages 7 and 28 days, respectively, for type II cement; and 60 or 70 calories per gram at ages 7 and 28 days, respectively, for type IV cement.

In most instances, type II cement will produce concrete temperatures which are acceptable. In the smaller structures, type I cement will often be entirely satisfactory. Other factors being equal, type II cement should be selected because of its better resistance to sulfate attack, better workability, and lower permeability. Type IV cement is now used only where an extreme degree of temperature control is required. For example, it would be beneficial near the base of long blocks where a high degree of restraint exists. Concrete made with type IV cement requires more curing than concrete made with other types of cement, and extra care is required at early ages to prevent damage to the concrete from freezing during cold weather. Often, the run-of-the-mill cement from a plant will meet the requirements of a type II cement, and the benefits of using this type of cement can be obtained at little or no extra cost. Type IV cement, because of its special composition, is obtained at premium prices.

**7-9. *Use of Pozzolans.***—Pozzolans are used in concrete for several reasons, one of which is to reduce the peak temperature due to heat of hydration from the cementing materials in the mix. This is possible because pozzolans develop heat of hydration at a much lower rate than do portland cements. Pozzolans can also be used as a replacement for part of the portland cement to improve workability, effect economy, and obtain a better quality concrete. The more common pozzolans used in mass concrete include calcined clays, diatomaceous earth, volcanic tuffs and pumicites, and fly ash. The actual type of pozzolan to be used is normally determined by cost and availability.

**7-10. *Miscellaneous Measures.***—(a) *Shallow Construction Lifts.*—Shallow construction or placement lifts can result in a greater percentage of the total heat generated in the lift being lost to the surface. Such a temperature benefit exists only during periods of time when the exposure temperatures are lower than the concrete temperature as described in section 7-22. Unless the site conditions are such that a sizable benefit can be obtained, shallow placement lifts are generally limited to placements over construction joints which have experienced prolonged exposure periods, or over foundation irregularities where they are helpful in the prevention of settlement cracks.

(b) *Water Curing.*—Water curing on the top and sides of each construction lift will reduce the temperature rise in concrete near the surfaces as described in section 7-29. Proper application of water to the surfaces will cause the surface temperature to approximate the curing water temperature instead of the prevailing air temperatures. In areas of low humidity, the effect of evaporative cooling may result in a slightly lower surface temperature than the temperature of the curing water.

(c) *Retarding Agents.*—Retarding agents added to the concrete mix will provide a temperature benefit when used in conjunction with pipe cooling. The retarding agents reduce the early rate of heat generation of the cement, so that the total temperature rise during the first 2 or 3 days will be $2^{\circ}$ or perhaps $3^{\circ}$ F. lower than for a similar mix without retarder. The actual benefit varies with the type and amount of retarder used. The percentage of retarder by weight of cement is generally about one-fourth to one-third of 1 percent. Percentages higher than this may give added temperature benefit but can create construction problems such as delay in form removal, increased embedment of form ties required, etc.

(d) *Surface Treatments.*—If the near-surface concrete of a mass concrete structure can be made to set at a relatively low temperature and can be maintained at this temperature during the early age of the concrete, say, for the first 2 weeks, cracking at the surface can be minimized. Under this condition, tensions at the surface are reduced or the surface may even be put into compression when the interior mass of the concrete subsequently drops in temperature. Such surface cooling can be accomplished by circulating water in closely spaced embedded cooling-pipe coils placed adjacent to and parallel with the exposed surfaces, by use of cold water sprays on noninsulated steel forms and on the exposed concrete surfaces, or by use of special refrigerated forms.

(e) *Rate of Temperature Drop.*—Temperature stresses and the resultant tendency to crack in mass concrete can be minimized by controlling the rate of temperature drop and the time when this drop occurs. In thick sections with no artificial cooling, the temperature drop will normally be slow enough to present no problem. In thin sections with artificial cooling, however, the temperature can drop quite rapidly and the drop may have to be controlled. This can be accomplished by reducing the amount of cooling water circulated through the coils or by raising the cooling water temperature. The operation of the cooling systems, and the layout of the header systems to supply cooling water to the individual cooling coils, should be such that each coil can be operated independently. No-cooling periods should also be utilized where necessary. In thin sections where no artificial cooling is employed, the temperature drop during periods of cold weather can be controlled by the use of insulated forms and insulation placed on exposed surfaces. Such measures not only reduce the rate of change, but also reduce the temperature gradients near the surface resulting in a definite reduction in cracking.

## C. TEMPERATURE STUDIES

7-11. ***General Scope of Studies.***—The measures required to obtain a monolithic structure and the measures necessary to reduce cracking tendencies to a minimum are determined by temperature control studies. In addition to the climatic conditions at the site, the design requirements of the structure and the probable construction procedures and schedules require study to determine the methods and degree of temperature control for the structure.

Early design studies and specification requirements are based on existing data and on a possible construction schedule. The ambient temperatures and probable concrete temperatures are then related to the dimensions of the structure, the conditions arising during construction, and the desired design stresses. As a result of these studies, a maximum concrete placing temperature may be determined, measures taken to limit the initial temperature rise within the concrete, and protective measures planned to alleviate cracking conditions arising during the construction period. Actual exposure conditions, water temperatures, and construction progress may vary widely from the assumed conditions, and adjustments should be made during the construction period to obtain the best structure possible consistent with economy and good construction practices.

The following discussions cover the more common temperature investigations and studies. In all of these studies, certain conditions must be assumed. Since any heat flow computation is dependent on the validity of the assumed exposure conditions and concrete properties, experience and good judgment are essential.

7-12. ***Range of Concrete Temperatures.***—In arch dam stress analysis studies, the range or amplitude of the mean concrete temperature for each of the arches or voussoirs is required. This range of mean concrete temperature is determined from the air and water temperatures at the site, as modified by the effects of solar radiation. For preliminary studies, the range of mean concrete temperature can be obtained in a short computation by applying the air and water exposure temperatures as sinusoidal waves with applicable periods of 1 day, 1 week or 2 weeks depending upon the severity of the weather to be used for the design, and 1 year. Solar radiation is then added to obtain the final range of mean concrete temperature.

For average (mean) weather conditions, the ambient air temperatures are obtained from a plotting of the mean monthly air temperatures on a year scale. For usual and extreme weather conditions, the above ambient air temperatures are adjusted for a 7-day period and a 14-day period, respectively, at the high and low points of the annual curve. The amount of the adjustment for these weather conditions is described in subsection (a) below.

The average arch thickness from abutment to abutment is normally used in computing the mean concrete temperatures. For variable-thickness arches, mean temperatures at the quarter points should be computed. The arches for which mean temperatures are determined should be the same arches used in the stress analysis.

(a) *Ambient Air Temperatures.*—When computing the range of mean concrete temperature, mean daily, mean monthly, and mean annual air temperatures are used. The theory applies the daily and annual air temperatures as sinusoidal variations of temperature, even though the cycles are not true sine waves. The annual and daily amplitudes are assumed to be the same for all weather conditions.

To account for the maximum and minimum recorded air temperatures, a third and somewhat arbitrary temperature cycle is assumed. This temperature variation is associated with the movements of barometric pressures and storms across the country. Plots throughout the western part of the United States show from one to two cycles per month. Arbitrarily, this third temperature variation is assumed as a sine wave with either a 7-day or 14-day period for usual weather conditions and extreme weather conditions, respectively. For extreme weather conditions, the amplitudes of the arbitrary cycle are assigned numerical values which, when added to the amplitudes of the daily and annual cycles, will account for the actual maximum and minimum recorded air temperatures at the site. For usual weather conditions, these amplitudes are assigned values which account for temperatures halfway between the mean monthly maximum (minimum) and the maximum (minimum) recorded. When computing the mean concrete temperature condition, no third cycle is used.

(b) *Reservoir Water Temperatures.*—The reservoir water temperatures used in determining the range of mean concrete temperature for a proposed dam are those temperatures which will occur after the reservoir is in operation. These reservoir water temperatures vary with depth, and for all practical purposes can be considered to have only an annual cycle. There will also be a time lag between the air and water temperatures, the greatest lag occurring in the lower part of the reservoir. Normally, this time lag need not be taken into consideration. However, with extremely thin arch dams, the effects of the time lag should be investigated. The Schmidt method of analysis will automatically take this lag into consideration. For preliminary studies, the range of mean concrete temperature with full reservoir is the normal condition. For final designs, stage construction should be taken into consideration and the design reservoir operation used. When the reservoir is to be filled or partially filled before concrete temperatures have reached their final stage of temperature equilibrium, further studies are needed for the particular condition.

(c) *Solar Radiation Effect.*—The downstream face of a dam, and the upstream face when not covered by water, receives an appreciable amount of radiant heat from the sun. This has the effect of warming the concrete surface above the surrounding air temperature. The amount of this temperature rise above the air temperature was recorded on the faces of several dams in the western portion of the United States. These data were then correlated with theoretical studies which took into consideration varying slopes, orientation of the exposed faces, and latitudes. The results of these studies are presented in reference [1].[1] These theoretical temperature rises due to solar radiation should be corrected by a terrain factor obtained from an east-west profile of the site terrain. This is required because the theoretical computations assumed a horizontal plane at the base of the structure, and the effect of the surrounding terrain is to block out some hours of sunshine. This terrain factor will vary for each of the arches and along each arch.

(d) *Amplitudes of Concrete Temperatures.*—The range or amplitude of concrete temperatures is determined by applying the above-described external sinusoidal air and water temperatures to the edges of a theoretical flat slab, the width of the slab being equal to the thickness of the dam at the elevation under consideration. The problem is idealized by assuming that no heat flows in a direction normal to the slab. The law of superposition is used in that the final amplitude in the concrete slab is the sum of the amplitudes obtained from the different sinusoidal variations.

To apply the theoretical heat flow in a practical manner, unit values are assumed for the several variables and a curve is drawn to show the ratio of the variation of the mean temperature of the slab to the variation of the external temperature. Figure 7-1 shows the relationship thus derived for temperature variations in flat slabs exposed to sinusoidal variations for $h^2$ = 1.00 square foot per day, a period of 1 day, and a thickness of slab of $l_1$. A correlation equation is given to take into account the actual thickness of dam, diffusivity constant, and period of time. The computations are shown in figures 7-2 and 7-3.[2] For the actual thickness of dam, $l_2$, a value of $l_1$ is obtained from the correlation equation for each of the air temperature cycles. For each value of $l_1$, a ratio of the variation of mean concrete temperature to the variation of external temperature is obtained. The sums of the products of these ratios and their respective amplitudes are algebraically added to and subtracted from the mean annual air temperature to obtain mean concrete temperatures for the condition of air on both faces. For thin arch dams the amplitude (15-day or 365-hour variation) should be decreased to reflect the thinner concrete sections. Mean concrete temperatures are then obtained in the same manner for a fictitious condition of water on both faces, and the two conditions are simply averaged together to obtain the condition of air on one face and water on the other. Solar radiation values are then added to obtain the final range of mean concrete temperatures.

7-13. ***Temperature Gradients.***—Temperature distributions in a mass where boundary conditions vary with time are easily determined by the Schmidt method. (See references [1], [2], [3], [4].) This method is generally used for temperature studies of mass concrete structures when the temperature gradient or distribution across the section is desired. The approximate date to grout a thin arch dam after a winter's exposure, the depth of freezing, and temperature distribution after placement are typical of the solutions which can be obtained by this step-by-step method. Different exposure temperatures on the two faces of the theoretical slab and the autogenous heat of hydration are easily taken into consideration.

An early objection to the Schmidt method of temperature computation was the time required to complete the step-by-step computation. This has been overcome by the use of electronic data processing machines which save many man-hours of work. Programs have been developed which will take into consideration any thickness of section, varying exposures on the two faces of the slab, variable initial temperatures, a varying heat of hydration with respect to time, and increasing

---

[1] Numbers in brackets refer to items in bibliography, sec. 7-31.

[2] These and several other figures and tables in this chapter were reprinted from Bureau of Reclamation Engineering Monograph No. 34, listed as reference [1] in the bibliography, sec. 7-31.

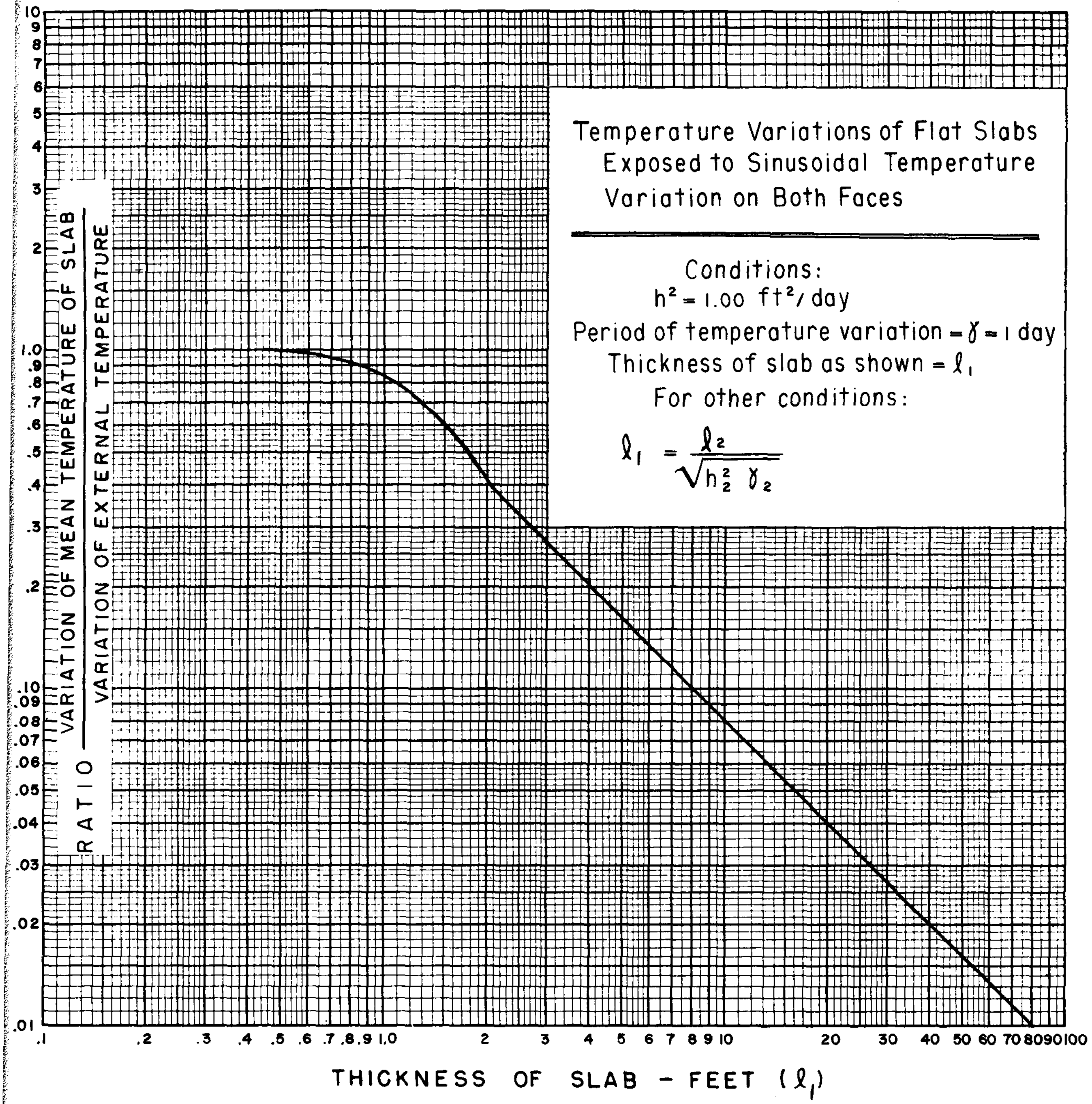


*Figure 7-1.* **Temperature variations of flat slabs exposed to sinusoidal temperature variations on both faces.–288-D-3008**

the thickness of slab at regular intervals as would occur when lifts of concrete are placed on previously placed lifts.

A second method of temperature computation in mass concrete which is particularly adaptable to thick walls and placement lifts near the rock foundation was devised by R. W. Carlson. This method is described in reference [5]. It, like the Schmidt method, is essentially a step-by-step integration which can be simplified by selection of certain variables. Conditions such as initial

## RANGE OF MEAN CONCRETE TEMPERATURES

Hungry Horse DAM

(Effect of solar radiation not included)

For yearly change $\ell_1 = \frac{\ell_2}{\sqrt{h_2^2 \gamma_2}} = \frac{\ell_2}{\sqrt{0.053 \times 8760}} = 0.0464\ \ell_2$

For 365-hr. change $\ell_1 = \frac{\ell_2}{\sqrt{0.053 \times 365}} = 0.227\ \ell_2$

For daily change $\ell_1 = \frac{\ell_2}{\sqrt{0.053 \times 24}} = 0.887\ \ell_2$

Remarks: Air temperatures taken from 34 year record at Columbia Falls, Montana. $h^2 = 0.053$ from laboratory data

| Avg. Annual Air Temperature 43.2°F. | Extreme Weather Conditions | | Usual Weather Conditions | |
|---|---|---|---|---|
| | Above Mean | Below Mean | Above Mean | Below Mean |
| | | | | |
| Yearly | 22.0 | 21.4 | 22.0 | 21.4 |
| 365-hr. | 30.6 | 49.6 | 20.4 | 25.2 |
| Daily | 7.2 | 7.2 | 7.2 | 7.2 |
| | | | | |

| Elevation | Thickness of Dam $\ell_2$ | Due to Yearly Range | | Due to 365-hr. Range | | Due to Daily Range | | Exposed to air on both surfaces | | | | | | | | Water Temperature | | | | Mean Conc. Temp. Exposed to Water on Both Faces | | | Air on D.S. Face Water on U.S. Face | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Ext. Wea. Conditions | | | | Usual Wea. Conditions | | | | | | | | | | | Ext. Wea. Conditions | | Usual Wea. Conditions | |
| | | | | | | | | Amplitude | | Temperature | | Amplitude | | Temperature | | | | | | | | | | | | |
| | | $\ell_1$ | Ratio from Curve | $\ell_1$ | Ratio from Curve | $\ell_1$ | Ratio from Curve | Above Mean | Below Mean | Max. | Min. | Above Mean | Below Mean | Max. | Min. | Max. | Min | Avg. | Amp | Amp. | Max. | Min. | Max | Min | Max. | Min. |
| 3565 | 40 | 1.86 | 0.460 | 9.08 | 0.087 | 35.48 | 0.022 | 12.9 | 14.3 | 56.1 | 28.9 | 12.1 | 12.2 | 55.3 | 31.0 | 69.0 | 32.0 | 50.5 | 18.5 | 8.5 | 59.0 | 42.0 | 57.5 | 35.5 | 57.2 | 36.5 |
| 3550 | 39 | 1.81 | .480 | 8.85 | .089 | 34.59 | .023 | 13.4 | 14.9 | 56.6 | 28.3 | 12.5 | 12.7 | 55.7 | 30.5 | 69.0 | 33.0 | 51.0 | 18.0 | 8.6 | 59.6 | 42.4 | 58.1 | 35.4 | 57.6 | 36.4 |
| 3500 | 56 | 2.60 | .310 | 12.71 | .063 | 49.67 | .016 | 8 9 | 9.9 | 52.1 | 33.3 | 8.2 | 8.3 | 51 4 | 34.9 | 54.5 | 36.8 | 45.6 | 8.8 | 2.7 | 48.3 | 42.9 | 50.2 | 38.1 | 49.9 | 38.9 |
| 3450 | 81 | 3.76 | .215 | 18.39 | .043 | 71 85 | .011 | 6.1 | 6.8 | 49.3 | 36.4 | 5.7 | 5.8 | 48.9 | 37.4 | 47.5 | 38.5 | 43.0 | 4.5 | 1.2 | 44.2 | 41.8 | 46.7 | 39.1 | 46.6 | 39.6 |
| 3400 | 111 | 5.15 | .156 | 25.20 | .032 | – | – | 4.4 | 4.9 | 47.6 | 38.3 | 4.1 | 4.1 | 47.3 | 39.1 | 45.0 | 39.0 | 42.0 | 3.0 | 0.5 | 42.5 | 41.5 | 45.0 | 39.7 | 44.9 | 40.3 |
| 3350 | 141 | 6.54 | .122 | 32.01 | .025 | | | 3,4 | 3.9 | 46.6 | 39.3 | 3.2 | 3.2 | 46.4 | 40.0 | 43.0 | 39.0 | 41.0 | 2.0 | 0.2 | 41.2 | 40.8 | 43.9 | 40.0 | 43.8 | 40.4 |
| 3300 | 171 | 7.93 | .100 | 38.82 | .021 | | | 2.8 | 3.2 | 46.0 | 40.0 | 2.6 | 2.7 | 45.8 | 40.5 | 43.0 | 39.0 | 41.0 | 2.0 | 0.2 | 41.2 | 40.8 | 43.6 | 40.4 | 43.5 | 40.6 |
| 3250 | 201 | 9.33 | .085 | 45.63 | .018 | | | 2.4 | 2.7 | 45.6 | 40.5 | 2.2 | 2.3 | 45.4 | 40.9 | 43.0 | 39.0 | 41.0 | 2.0 | 0.2 | 41.2 | 40.8 | 43.4 | 40.6 | 43.3 | 40.8 |
| 3200 | 231 | 10.72 | .075 | 52.44 | .015 | | | 2.1 | 2.3 | 45.3 | 40.9 | 2.0 | 2.0 | 45.2 | 41.2 | 43.0 | 39.0 | 41.0 | 2.0 | 0.2 | 41.2 | 40.8 | 43.2 | 40.8 | 43.2 | 41.0 |
| 3150 | 274 | 12.71 | .063 | 62.20 | .013 | | | 1.8 | 2.0 | 45.0 | 41.2 | 1.7 | 1.7 | 44.9 | 41.5 | 43.0 | 39.0 | 41.0 | 2.0 | 0.1 | 41.1 | 40.9 | 43.0 | 41.0 | 43.0 | 41.2 |
| 3100 | 292 | 13.55 | .060 | 66.28 | .012 | | | 1.7 | 1.9 | 44.9 | 41.3 | 1.6 | 1.6 | 44.8 | 41.6 | 43.0 | 39.0 | 41.0 | 2.0 | 0.1 | 41.1 | 40.9 | 43.0 | 41.1 | 42.9 | 41.3 |
| 3067 | 366 | 16.98 | .047 | 83.08 | – | | | 1.0 | 1.0 | 44.2 | 42.2 | 1.0 | 1.0 | 44.2 | 42.2 | 43.0 | 39.0 | 41.0 | 2.0 | 0.1 | 41.1 | 40.9 | 42.7 | 41.5 | 42.7 | 41.5 |

NOTES: $\ell_2$ = Thickness of dam, ft.
Curve referred to is "Temperature Variations of Flat Slabs Exposed to Sinusoidal Temperature Variations on Both Faces."

*Figure 7-2.* **Computation form, sheet 1 of 2–range of mean concrete temperatures.–288-D-3009**

temperature distributions, diffusivity, and adiabatic temperature rise must be known or assumed. Carlson's method can also be modified to take into account the flow of heat between different materials. This would be the case where insulated or partially insulated forms are used, or where concrete lifts are placed on rock foundations.

The variation in temperature in a semi-infinite solid at any particular point can also be estimated from figure 7-4. This illustration gives the ratio of the temperature range in the concrete at the particular point, to the temperature range at the surface for daily, 15-day, and annual cycles of temperature.

Stresses due to temperature gradients may be of concern not only during the construction period but during the life of the structure. Stresses across a section due to temperature gradients can be obtained from the expression

$$\sigma_y = \frac{eE}{b^3(1-\mu)}\left[b^2 \int_o^b T(x)dx + 3(2x-b)\int_o^b (2x-b)T(x)dx - b^3\,T(x)\right]$$

where:

$e$ = thermal coefficient of expansion,
$E$ = modulus of elasticity,
$\mu$ = Poisson's ratio, and
$b$ = thickness of section with a temperature distribution $T(x)$.

Where the temperature variation, $T(x)$, cannot be expressed analytically, the indicated

**RANGE OF MEAN CONCRETE TEMPERATURES**

Hungry Horse DAM

(Effect of Solar Radiation included)

Latitude 48°N

Remarks:

| | | | | | | MEAN CONCRETE TEMPERATURES | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Elevation | Thickness of Dam | Effects of Solar Radiation | | | | Exposed to air on both faces | | | | Air on D.S. Face Water on U.S. Face | | | |
| | | | | | | Ext. Wea. Conditions | | Usual Wea. Conditions | | Ext. Wea. Conditions | | Usual Wea. Conditions | |
| | | U.S. | D.S. | Avg. | ½D.S. | Max. | Min. | Max. | Min. | Max. | Min. | Max. | Min. |
| 3565 | 40 | 6.1 | 4.3 | 5.2 | 2.2 | 61.3 | 34.1 | 60.5 | 36.2 | 59.7 | 37.6 | 59.4 | 38.7 |
| 3550 | 39 | 6.0 | 5.0 | 5.5 | 2.5 | 62.1 | 33.8 | 61.2 | 36.0 | 60.6 | 37.9 | 60.1 | 38.9 |
| 3500 | 56 | 5.8 | 5.6 | 5.7 | 2.8 | 57.8 | 39.0 | 57.1 | 40.6 | 53.0 | 40.9 | 52.7 | 41.7 |
| 3450 | 81 | 5.6 | 5.6 | 5.6 | 2.8 | 54.9 | 42.0 | 54.5 | 43.0 | 49.5 | 41.9 | 49.4 | 42.4 |
| 3400 | 111 | 5.4 | 5.4 | 5.4 | 2.7 | 53.0 | 43.7 | 52.7 | 44.5 | 47.7 | 42.4 | 47.6 | 43.0 |
| 3350 | 141 | 5.2 | 5.5 | 5.3 | 2.7 | 51.9 | 44.6 | 51.7 | 45.3 | 46.6 | 42.7 | 46.5 | 43.1 |
| 3300 | 171 | 5.0 | 5.4 | 5.2 | 2.7 | 51.2 | 45.2 | 51.0 | 45.7 | 46.3 | 43.1 | 46.1 | 43.3 |
| 3250 | 201 | 5.0 | 6.0 | 5.5 | 3.0 | 51.1 | 46.0 | 50.9 | 46.4 | 46.4 | 43.6 | 46.3 | 43.8 |
| 3200 | 231 | 4.8 | 4.7 | 4.8 | 2.3 | 50.1 | 45.7 | 50.0 | 46.0 | 45.5 | 43.1 | 45.5 | 43.3 |
| 3150 | 274 | 4.6 | 4.7 | 4.6 | 2.3 | 49.6 | 45.8 | 49.5 | 46.1 | 45.3 | 43.3 | 45.3 | 43.5 |
| 3100 | 292 | 4.4 | 4.7 | 4.5 | 2.3 | 49.4 | 45.8 | 49.3 | 46.1 | 45.3 | 43.4 | 45.2 | 43.6 |
| | | | | | | | | | | | | | |

| SOLAR RADIATION VALUES | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Elevation | Terrain Factor % | 131° Point 1 49° | | | | | | 106° Point 2 74° | | | | | | 81° Point 3 99° | | | | | | Average Temp. Rise | |
| | | Upstream | | | Downstream | | | Upstream | | | Downstream | | | Upstream | | | Downstream | | | | |
| | | Normal angle | | | Normal angle | | | Normal angle | | | Normal angle | | | Normal angle | | | Normal angle | | | | |
| | | Slope | Temp. Rise | | Slope | Temp. Rise | | Slope | Temp. Rise | | Slope | Temp. Rise | | Slope | Temp. Rise | | Slope | Temp. Rise | | U.S. | D.S. |
| | | | 100% | Actual | | 100% | Actual | | 100% | Actual | | 100% | Actual | | 100% | Actual | | 100% | Actual | | |
| 3565 | 100 | 0 | 7.3 | 7.3 | 0 | 2.7 | 2.7 | 0 | 6.3 | 6.3 | 0 | 4.3 | 4.3 | 0 | 4.8 | 4.8 | 0 | 6.0 | 6.0 | 6.1 | 4.3 |
| 3550 | 97 | 0 | 7.3 | 7.1 | 0.25 | 3.4 | 3.3 | 0 | 6.3 | 6.1 | 0.25 | 5.1 | 4.9 | 0 | 4.8 | 4.7 | 0.25 | 7.1 | 6.9 | 6.0 | 5.0 |
| 3500 | 94 | 0 | 7.3 | 6.9 | .5 | 4.0 | 3.8 | 0 | 6.3 | 5.9 | .5 | 6.0 | 5.6 | 0 | 4.8 | 4.5 | .5 | 7.9 | 7.4 | 5.8 | 5.6 |
| 3450 | 91 | 0 | 7.3 | 6.6 | .6 | 4.2 | 3.8 | 0 | 6.3 | 5.7 | .6 | 6.2 | 5.6 | 0 | 4.8 | 4.4 | .6 | 8.1 | 7.4 | 5.6 | 5.6 |
| 3400 | 88 | 0 | 7.3 | 6.4 | .6 | 4.2 | 3.7 | 0 | 6.3 | 5.6 | .6 | 6.2 | 5.5 | 0 | 4.8 | 4.2 | .6 | 8.1 | 7.1 | 5.4 | 5.4 |
| 3350 | 85 | 0 | 7.3 | 6.2 | .87 | 4.9 | 4.2 | 0 | 6.3 | 5.4 | .6 | 6.2 | 5.3 | 0 | 4.8 | 4.1 | .6 | 8.1 | 6.9 | 5.2 | 5.5 |
| 3300 | 82 | 0 | 7.3 | 6.0 | .87 | 4.9 | 4.0 | 0 | 6.3 | 5.2 | .6 | 6.2 | 5.1 | 0 | 4.8 | 3.9 | .82 | 8.6 | 7.1 | 5.0 | 5.4 |
| 3250 | 79 | | | | | | | 0 | 6.3 | 5.0 | .6 | 6.2 | 4.9 | 0 | 4.8 | 3.8 | .97 | 8.8 | 7.0 | 5.0 | 6.0 |
| 3200 | 76 | | | | | | | 0 | 6.3 | 4.8 | .6 | 6.2 | 4.7 | | | | | | | 4.8 | 4.7 |
| 3150 | 73 | | | | | | | 0 | 6.3 | 4.6 | .7 | 6.5 | 4.7 | | | | | | | 4.6 | 4.7 |
| 3100 | 70 | | | | | | | 0 | 6.3 | 4.4 | .8 | 6.7 | 4.7 | | | | | | | 4.4 | 4.7 |
| | | | | | | | | | | | | | | | | | | | | | |

*Figure 7-3.* **Computation form, sheet 2 of 2—range of mean concrete temperatures.—288-D-3010**

integrations can be performed numerically by the use of Simpson's rule. For example, using $b = 30$ feet, $e = 6.0 \times 10^{-6}$, $E = 2{,}500{,}000$ pounds per square inch, $\mu = 0.20$, and an assumed $T(x)$, the stresses would be computed as shown in table 7-2.

The above expression for stress is not valid in all essentials for those temperature gradients which occur during the first few days after placement, because the extreme creep characteristics of the concrete during this age result in a highly indeterminate condition of stress. The expression is also not valid where external restraints occur such as near the foundation of a block or structure.

**7-14.** ***Temperature Rise.***—Newly placed concrete undergoes a rise in temperature due to the exothermic reaction of the cementing materials in the concrete. Early temperature rise studies may be based on past experience records with the type of cement to be used. Figure 7-5 shows typical temperature rise curves for the various types of cement. The temperature rise curves are based on 1 barrel (4 sacks) of cement per cubic yard of concrete, a diffusivity of 0.050 square foot per hour, and no embedded pipe cooling. These curves should be used only for preliminary studies because there are wide variations of heat generation within each type of cement and of diffusivity

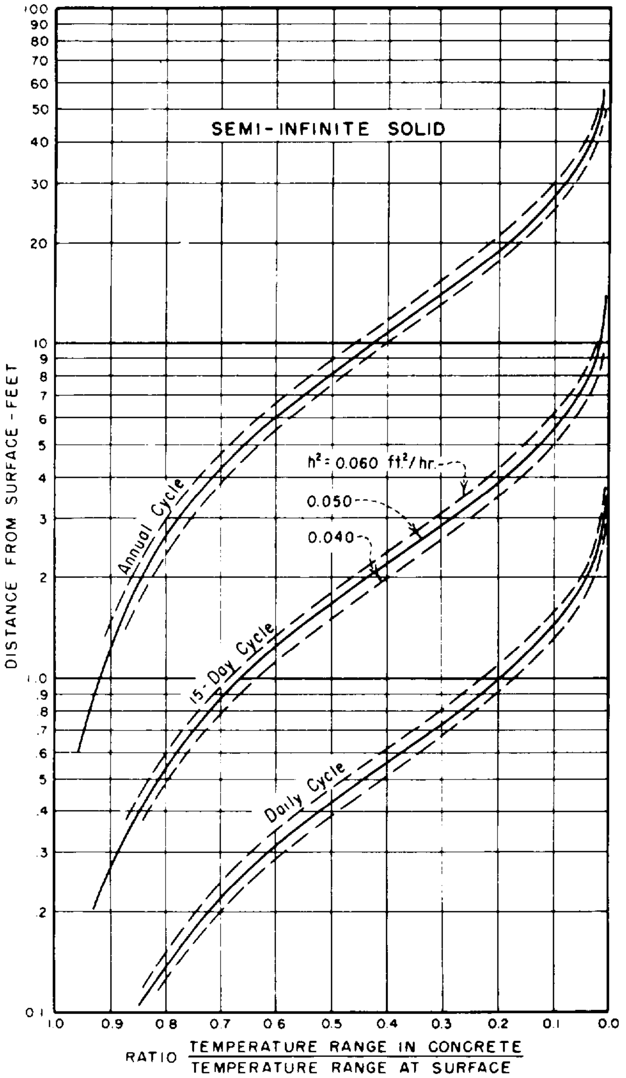
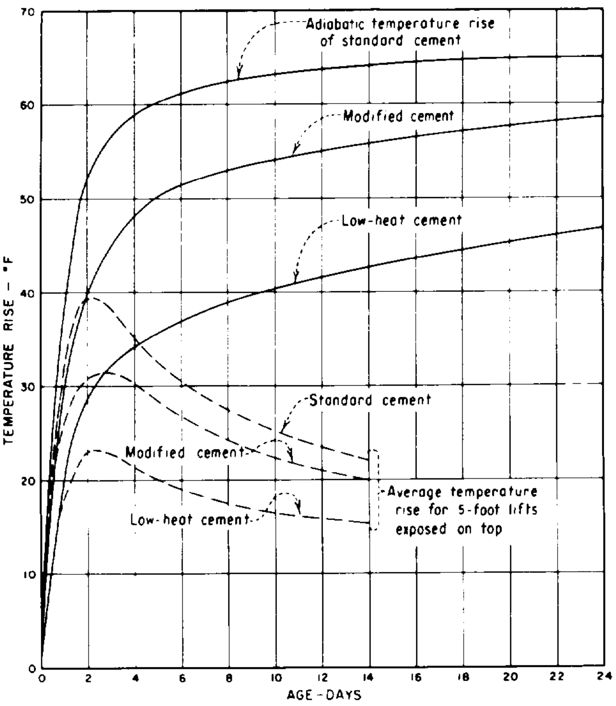


*Figure 7-4.* **Temperature variations with depth in semi-infinite solid.–288-D-3011**



Cement content - 1 bbl per cu. yd.
Diffusivity - 0.050 ft²/hr

*Figure 7-5.* **Temperature rise in mass concrete for various types of cement.–288-D-3013**

in concrete. (See reference [6].) Where less than 4 sacks of cement per cubic yard is to be used, the temperature rise can be estimated by direct proportion since the heat generation is directly proportional to the amount of cement.

As with cements, the heat-development characteristics of pozzolans vary widely. When a pozzolan is to be used to replace a part of the cement, the heat of hydration of the pozzolan, for early studies, can be assumed to be about 50 percent of that developed by an equal amount of cement. For final temperature control studies, the heat generation for a particular concrete mix should be obtained by laboratory tests using the actual cement, pozzolan, concrete mix proportions, and mass-cure temperature cycle for the concrete to be placed in the structure.

The above heat of hydration relates to the adiabatic temperature rise in the concrete. Because the surfaces of a structure are exposed or in contact with inert or near-inert bodies, a flow of heat will take place through these surfaces and the actual temperature rise in the concrete will be affected accordingly. The loss or gain of heat to the surface due to exposure conditions, and the loss or gain of heat from an underlying lift or to the foundation are illustrated in reference [7]. Schmidt's method or Carlson's method can also be used to determine the actual temperature rise.

Several difficulties are encountered in the conditions given in reference [7]. For example, the theoretical equation for the adiabatic temperature rise is given as $T = T_o\ (1 - e^{-mt})$, and $T_o$ and $m$ are selected to make the theoretical curve fit the laboratory data. Any

Table 7-2.–*Computation of temperature stress.*

| $x$ ft. | $T(x)$ °F. | $(2x-30)\,T(x)$ ft.–°F. | | $\sigma_y$ lb./ft.$^2$ | $\sigma_y$ lb./in.$^2$ |
|---|---|---|---|---|---|
| 0 | 0. 0 | 0 | For the given conditions: | 10, 584 | 74 |
| 3 | 8. 3 | −199 | | 4, 125 | 29 |
| 6 | 15. 8 | −284 | $\frac{eE}{b^3(1-\mu)}=0.1$ | −174 | −1 |
| 9 | 22. 7 | −272 | | −2, 853 | −20 |
| 12 | 29. 1 | −175 | $\sigma_y=0.1[(900)(1003.8)$ | −4, 182 | −29 |
| 15 | 35. 1 | 0 | $+3(2x-30)(8862)$ | −4, 431 | −31 |
| 18 | 40. 7 | 244 | $-(30)^3\mathrm{T}(x)]$ | −3, 600 | −25 |
| 21 | 46. 0 | 552 | | −1, 959 | −14 |
| 24 | 50. 9 | 916 | | 762 | 5 |
| 27 | 55. 6 | 1334 | Simplifying: | 4, 023 | 28 |
| 30 | 60. 0 | 1800 | $\sigma_y=5317x-2700\mathrm{T}(x)$<br>$+10{,}584$ | 8, 094 | 56 |
| $\int_0^b$ | 1003. 8 | 8862 | | | |

variance between the theoretical and actual curves will result in some error in the theoretical heat loss in the heat-generating lift. The loss from the inert lift does not take into consideration a varying surface temperature, which also introduces an error. A third error may be introduced when a new lift is placed on an older lift which is still generating heat. Depending upon the age of the older lift, the heat generated may still be enough to be considered.

**7-15.** ***Artificial Cooling.***–The design of an artificial cooling system requires a study of each structure, its environment, and the maximum temperatures which are acceptable from the standpoint of crack control. The temperature effects of various heights of placement lifts and such layout variables as size, spacing, and length of embedded coils should be investigated. Variables associated with the operation of the cooling systems, such as rate of water circulation and the temperature differential between the cooling water and the concrete being cooled, are studied concurrently. All of these factors should be considered in arriving at an economical cooling system which can achieve the desired temperature control.

The theory for the removal of heat from concrete by embedded cooling pipes was first developed for use in Hoover Dam. (See reference [7].) From these studies, a number of curves and nomographs were prepared for a vertical spacing (height of placement lift) of 5 feet. The concrete properties and a single rate of flow of water were also used as constants. Subsequent to the earlier studies, the theory was developed using dimensionless parameters. Nomographs were then prepared on the basis of a ratio of $b/a$ of 100, where $b$ is the radius of the cooled cylinder and $a$ is the radius of the cooling pipe. Actual cooling pipe spacings are nominal spacings and will seldom result in a $b/a$ ratio of 100. In order to take the actual horizontal and vertical spacings into consideration, a fictitious diffusivity constant can be used which is based on tests of concrete made with similar aggregates. Table 7-3 gives the values of $D$, $D^2$, and $h^2{}_f$ for various spacings of cooling pipe. The $b/a$ ratios of the spacings shown vary from about 34 to 135. Within these limits, the values of $h^2{}_f$ may be used with sufficient accuracy.

Figures 7-6 and 7-7 are used for pipe cooling computations. In these illustrations,

$$X=\frac{\left[\begin{array}{c}\text{Difference between mean temper-}\\ \text{ature of the concrete and temper-}\\ \text{ature of the cooling water}\end{array}\right]}{\left[\begin{array}{c}\text{Initial temperature difference}\\ \text{between the concrete and the}\\ \text{cooling water}\end{array}\right]}$$

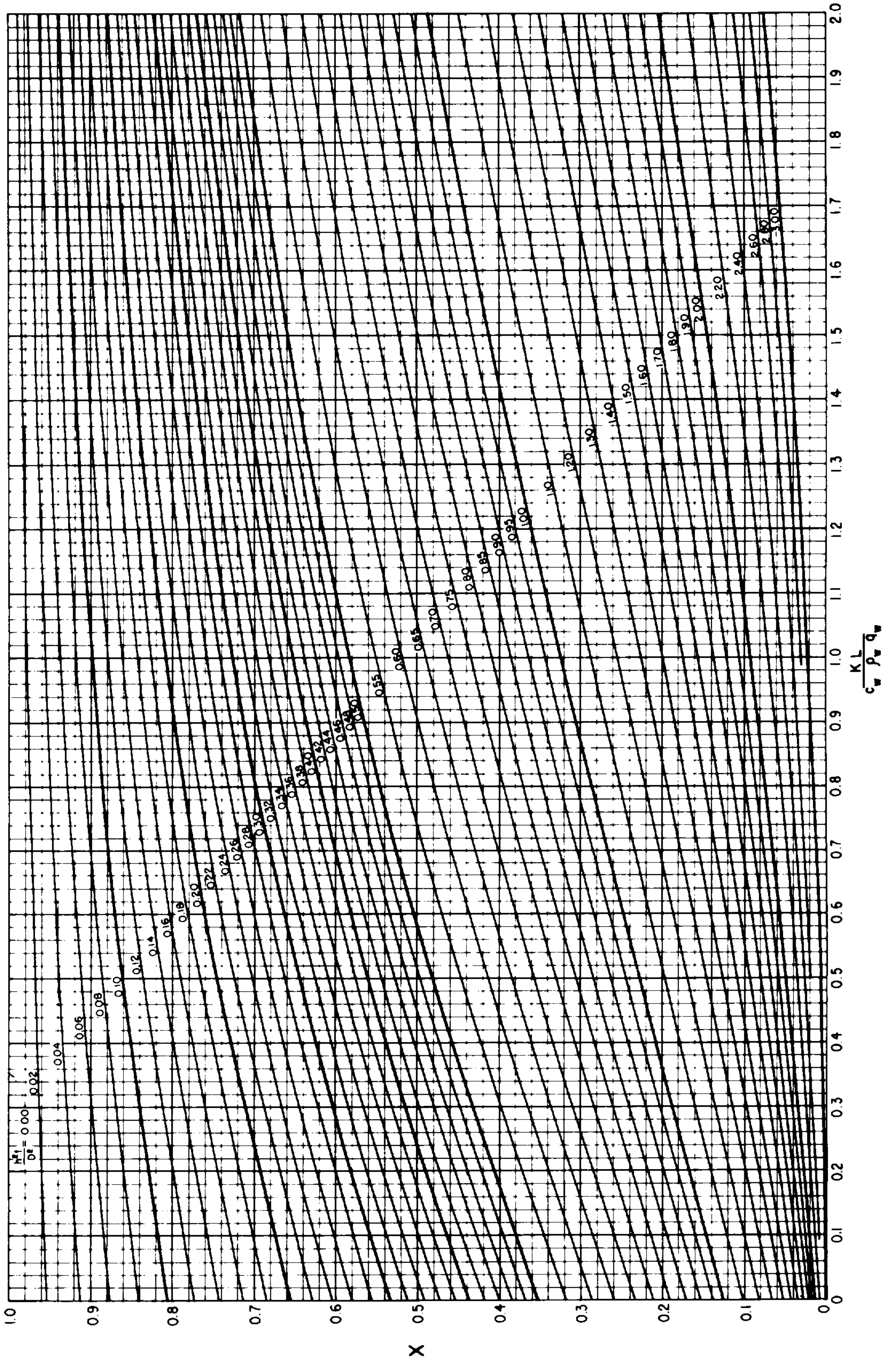


*Figure 7-6.* **Pipe cooling of concrete—values of *X*.—288-D-3015**

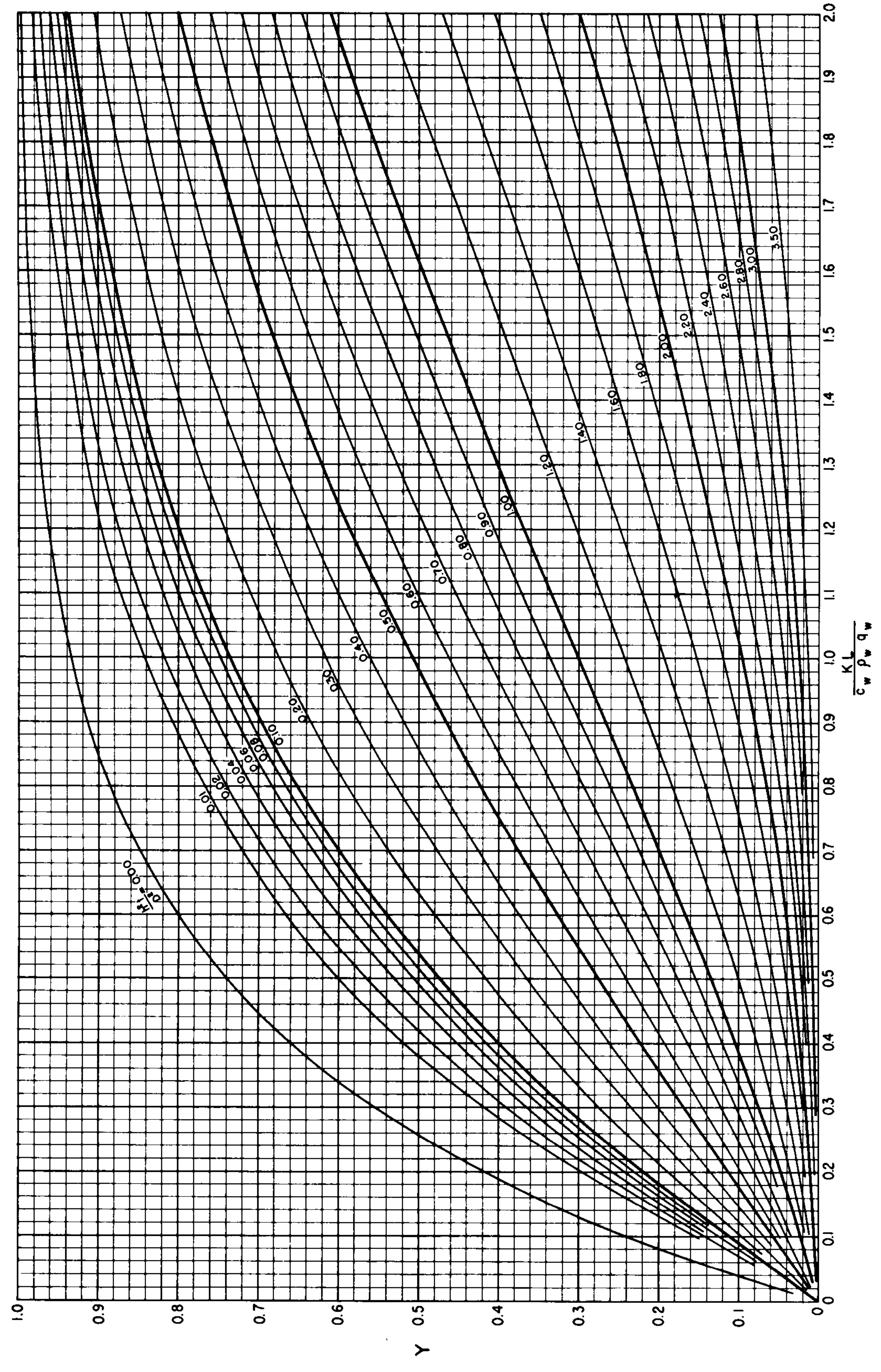


*Figure 7-7.* **Pipe cooling of concrete—values of *Y*.—288-D-3016**

Table 7-3.–*Values of D, $D^2$, and $h^2{}_f$ for pipe cooling.*

| Spacing | | D | $D^2$ | $h^2{}_f$ |
|---|---|---|---|---|
| Vertical (feet) | Horizontal (feet) | | | |
| 2½ | 2½ | 2.82 | 7.95 | $1.31h^2$ |
| 5 | 2½ | 3.99 | 15.92 | $1.19h^2$ |
| 5 | 3 | 4.35 | 18.92 | 1.16 |
| 5 | 4 | 5.02 | 25.20 | 1.12 |
| 5 | 5 | 5.64 | 31.81 | 1.09 |
| 5 | 6 | 6.18 | 38.19 | 1.07 |
| 7½ | 2½ | 4.88 | 23.81 | $1.13h^2$ |
| 7½ | 4 | 6.15 | 37.82 | 1.07 |
| 7½ | 5 | 6.86 | 47.06 | 1.04 |
| 7½ | 6 | 7.54 | 56.85 | 1.02 |
| 7½ | 7½ | 8.46 | 71.57 | 1.00 |
| 7½ | 9 | 9.26 | 85.75 | 0.98 |
| 10 | 10 | 11.284 | 127.33 | $0.94h^2$ |

$$Y = \frac{\left[\text{Temperature rise of the cooling water}\right]}{\left[\text{Initial temperature difference between the concrete and the cooling water}\right]}$$

$K$ = conductivity of the concrete,
$L$ = length of cooling coil,
$c_w$ = specific heat of water,
$\rho_w$ = density of water,
$q_w$ = volume of water flowing through the coil,
$t$ = time from start of cooling,
$D$ = diameter of the cooling cylinder, and
$h^2$ = diffusivity of the concrete.

Consistent units of time and distance must be used throughout.

The curves in figures 7-6 and 7-7 are used in a straight-forward manner as long as no appreciable heat of hydration is occurring in the concrete during the period of time under consideration. When the effect of artificial cooling is desired during the early age of the concrete, a step-by-step computation is required which takes into consideration heat increments added at uniform time intervals during the period.

Varying the temperature of the water circulated through the coil, the length of the embedded coil, and the horizontal spacing of the pipe are effective means of varying the cooling operation to obtain the desired results. Figures 7-8, 7-9, and 7-10[3] show how these variables affect the concrete temperatures. These studies were made using 4 sacks of type II cement per cubic yard, a diffusivity of 0.050 square foot per hour, a flow of 4 gallons per minute through 1-inch outside-diameter pipe, 5-foot placement lifts, and a 3-day exposure of each lift. Figures 7-9 and 7-10 were derived using the adiabatic temperature rise shown in figure 7-8. In general, cooling coil lengths of 800 to 1,200 feet are satisfactory. Spacings varying from 2½ feet on the rock foundation to 6 feet on tops of 7½-foot lifts have been used. The temperature of the cooling water has varied from a refrigerated brine at about 30° F. to river water with temperatures as high as 75° F.

Varying the size of the embedded pipe will affect the cooling results but is uneconomical as compared to the other methods of varying the cooling. The use of 1-inch outside-diameter metal pipe or tubing is common practice. Although black steel pipe is cheaper in material cost, aluminum tubing has been used in many instances because it can be furnished in coils and will result in a lower installation cost. Increasing the rate of flow through 1-inch pipe will give a marked improvement of performance up to a rate of 4 gallons per minute. However, doubling the flow to 8 gallons per minute decreases the time required for cooling by only 20 to 25 percent for average conditions, whereas it doubles the capacity requirements, increases the friction losses, and more than doubles the power costs.

7-16. ***Miscellaneous Studies.***–Solutions for idealized heat flow problems associated with the design and construction of mass concrete dams are given in reference [7]. Illustrative examples are given which demonstrate the use of the theory in practical applications.

---

[3] These three illustrations are reprinted from an article "Control of Temperature Cracking in Mass Concrete," by C. L. Townsend, published in ACI Publication SP-20, "Causes, Mechanism, and Control of Cracking in Concrete," 1968.

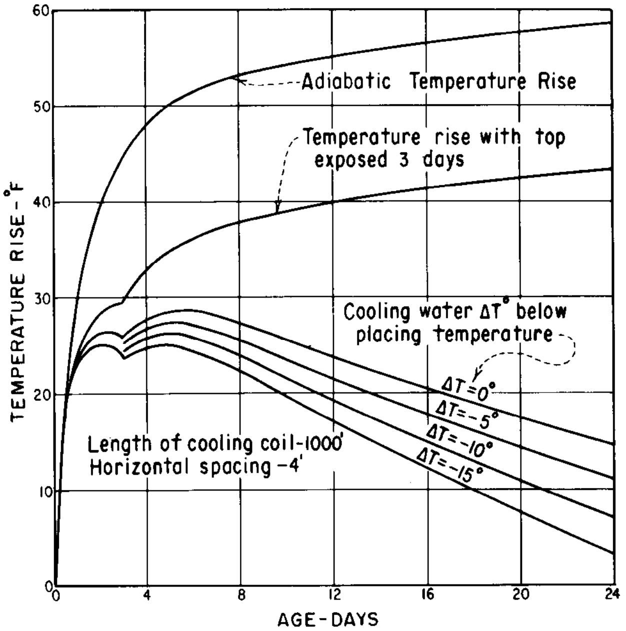


*Figure 7-8.* **Artificial cooling of concrete—effect of cooling water temperature. (From ACI Publication SP-20.)—288-D-3017**

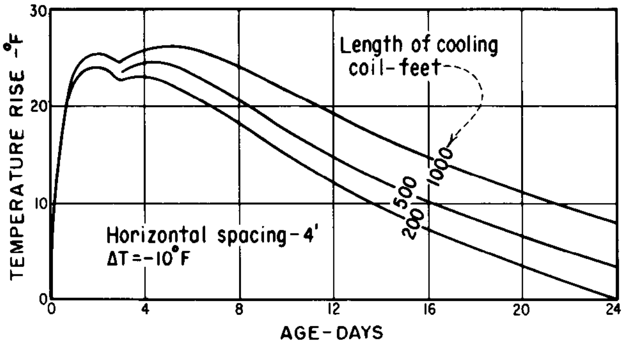


*Figure 7-9.* **Artificial cooling of concrete—effect of coil length. (From ACI Publication SP-20.)—288-D-3018**

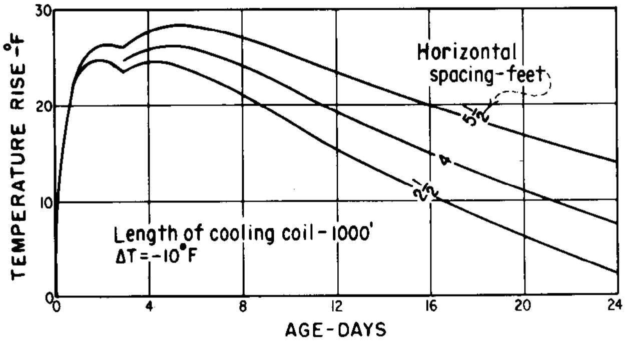


*Figure 7-10.* **Artificial cooling of concrete—effect of horizontal spacing of pipe (From ACI Publication SP-20.)—288-D-3019**

Temperature distributions and gradients in semi-infinite solids are given for both constant exposure and variable exposure temperature conditions. Natural cooling of slabs, cylinders, and spheres is discussed using initial uniform temperature distributions, uniformly varying initial temperatures, constant exposure temperatures, and variable exposure temperatures.

Studies for the insulation requirements on concrete structures as a protection against freezing and to minimize the formation of extreme temperature gradients are discussed in reference [8].

Although specific methods of cooling are normally left to the contractor, the requirements for cooling the various parts of a concrete mix to obtain a predetermined placing temperature can require a detailed study. The various considerations for such an operation are discussed in an article by F. B. Kinley in reference [9].

## D. DESIGN CONSIDERATIONS

7-17. ***Placing Temperatures.***—The maximum temperature attained in mass concrete is determined to a large extent by the temperature of the concrete as it is placed in the structure. This makes the placing temperature of the concrete of concern because (1) lower concrete temperatures will minimize temperature differentials near the surface, and (2) a measure of control over the subsequent temperature drop from the maximum concrete temperature to the grouting or final stable state temperature can be achieved.

When no special provisions are employed, concrete placing temperatures will approximate the mean monthly air temperature, ranging from 4° to 6° F. higher than the mean air temperature in the wintertime and this same amount lower than the mean air temperature in the summertime. The actual temperature of the concrete mix depends upon the temperatures, batch weights, and specific heats of the separate materials going into the concrete mix. The placing temperature of the concrete may be lowered by reducing the temperatures of one or more of the separate materials. The computation for determining the temperature of a mix, both with and without precooling measures, is illustrated in references [1] and [9].

Minimal tensile stresses of the base of a placement lift will be developed if the placing temperature of the concrete is at or slightly below the temperature of the foundation and if the temperature rise is minimized. These tensile stresses resulting after placement will be lower if successive lift placements in a block are made at regular, periodic intervals with the shortest practicable time between lifts. Form removal and lifting of forms, installation of required metalwork, and construction joint cleanup will normally require a minimum of almost 3 days between lifts.

7-18. ***Closure Temperature.***—One design consideration related to temperature control is the closure temperature of an arch dam. The lowest practicable grouting temperature may be desirable from the stress standpoint when designing for full reservoir and minimum ambient temperature. Such a grouting temperature will reduce tensile stresses in the arches and cantilevers, even to the point of obtaining compressive stresses at some points inherently in tension. The same structure, however, when investigated for minimum reservoir level and maximum temperature, a condition commonly occurring in storage dams, will often require a higher closure temperature if extreme tensions in parts of the arches and cantilevers are to be avoided. Although this last loading condition is one which will not result in failure of the structure, it can cause cracking which will lead to maintenance problems on the structure.

The closure temperature for an arch dam is normally based upon what the arch dam stress analyses show to be desirable from a stress standpoint, but the actual temperature may be influenced by practical or economic considerations. The designer often has to make a design decision whether to use only the river water available to cool the concrete, thereby losing the benefit of 2° to 5° F. additional cooling which could be obtained by artificial methods, or to obtain the desired temperature reduction by requiring mechanically refrigerated water to perform the cooling.

From the practical standpoint, it is possible to cool the concrete by means of an embedded pipe cooling system to within 4° or 5° F. of the mean temperature of the cooling water. Concrete temperatures as low as 35° F. have been obtained with a refrigerating plant using brine as the coolant. Where cooling is accomplished with river water, concrete temperatures attainable depend on the mean river water temperature. At Hungry Horse Dam, river water at 32° to 34° F. was available during the colder months of the year, and final cooling was accomplished to 38° F. with this river water. At Monticello Dam, river water was limited in quantity and was relatively warm since the stream primarily carries a surface runoff during periods of rainfall. Refrigeration of the cooling water was required in this instance to obtain the desired closure temperatures. Two closure temperatures were used at Monticello Dam, 45° F. in the lower part of the dam and 55° F. in the upper part. This was accomplished so that more load would be carried by the lower portion of the dam.

In relatively thin arch dams and where temperatures in the winter are sufficiently cold, pipe cooling may be omitted in all or part of the dam. Concrete permitted to cool naturally over the winter period will normally have different mean temperatures for each of the theoretical arches. Closure temperatures as low as 34° F. have been obtained in some dams. Schmidt's method of computation based on average ambient temperatures is used to estimate the time for grouting the contraction joints. The actual temperature at the time of grouting should be determined, however, and used in as-constructed studies.

7-19. *Size of Construction Block.*—Temperature cracking in mass concrete structures is related to the dimensions and shape of the construction blocks in the structure and to the climatic conditions occurring during the construction period. Generally, a block with a length of 50 feet or less can be placed with only a minimum of control. Likewise, blocks up to 200 feet long can be placed with normal temperature control measures and have no more than nominal cracking. The location of appurtenances generally controls the spacing between transverse contraction joints, but this spacing should be guided to some extent by the shape of the block as it progresses from the foundation to the top of the dam.

(a) *Length of Construction Block.*—For a given site and given loading conditions, the thickness of a dam is determined by arch analyses. Where this thickness is large, the section can be broken into two or more construction blocks separated by longitudinal joints, or it can be constructed as a single block by applying rigid temperature control measures. Normally, a 25° to 30° F. temperature drop can be permitted in blocks of the size commonly used before tensile stresses are developed which will be great enough to cause cracking across the block. In low temperature climates, special precautions are needed to avoid high differential temperatures caused by sudden temperature drops.

The length of a construction block is not governed by the capacity of the concrete mixing plant, since each block is first constructed to its full width and height at the downstream end of the block and then progressively placed to the upstream face. More generally, the length of block is related to the tensile stresses which tend to develop within the block between the time the block is placed and the time it reaches its final temperature. The stresses are subject to some degree of control by operations affecting the overall temperature drop from the maximum temperature to the final or closure temperature, the rate of temperature drop, the thermal coefficient of expansion, and the age of the concrete when it is subjected to the temperature change. Factors in addition to temperature which affect the stresses in the block are the effective modulus of elasticity between the block and its foundation, the elastic and inelastic properties of the concrete, and the degree of external restraint.

The actual stresses will further vary between rather wide limits because of conditions occurring during the construction period which introduce localized stress conditions. Tensile stresses and resulting cracks may occur because the larger blocks, by reason of their greater area, will have a greater number of stress concentrations arising from the physical irregularities and variable composition of the foundation. Cracks may also occur because of delays in the construction schedule and construction operations. Longer blocks are more likely to have cold joints created during placement of the concrete, and these cold joints are definite planes of weakness. A special problem exists with respect to the longer blocks at the base of the dam. These will normally be exposed for longer periods of time because concrete placement is always slow at the start of a job. Under this condition, extreme temperature gradients may form near the surfaces. The stresses caused by these steep temperature gradients may then cause cracks to form along any planes of weakness which exist as a result of construction operations.

Unlike ordinary structural members undergoing temperature change, the stresses induced in mass concrete structures by temperature changes are not capable of being defined with any high degree of accuracy. The indeterminate degree of restraint and the varying elastic and inelastic properties of the concrete, particularly during the early age of the concrete, make such an evaluation an estimate at best. Field experiences on other jobs should guide the designer to a great extent. Such experiences are reflected in table 7-4 which can be used as a guide during the early stages of design.

(b) *Width of Construction Block.*—Contraction joints are normally spaced about 50 feet apart, but may be controlled in some parts of the dam by the spacing and location of penstocks and river outlets, or by definite

Table 7-4.–*Temperature treatment versus block length.*

| Block length | Treatment | | |
|---|---|---|---|
| Over 200 feet | Use longitudinal joint. Stagger longitudinal joints in adjoining blocks by minimum of 30 feet | | |
| | Temperature drop from maximum concrete temperature to grouting temperature—°F. | | |
| | Foundation to $H=0.2L$[1] | $H=0.2L$ to $0.5L$[1] | Over $H=0.5L$[1] |
| 150 to 200 feet | 25 | 35 | 40 |
| 120 to 150 feet | 30 | 40 | 45 |
| 90 to 120 feet | 35 | 45 | No restriction |
| 60 to 90 feet | 40 | No restriction | No restriction |
| Up to 60 feet | 45 | No restriction | No restriction |

[1] $H$=height above foundation; $L$=block length.

breaks and irregularities of the foundation. Although a uniform spacing of joints is not necessary, it is desirable so that the contraction joint openings will be essentially uniform at the time of contraction joint grouting. Spacings have varied from 30 to 80 feet as measured along the axis of the dam. When the blocks are 30 feet or less in width, a larger temperature drop than would otherwise be necessary may be required to obtain a groutable opening of the contraction joint. This temperature drop should be compatible with the permissible drop for the long dimension of the block.

A further consideration is the maximum length-to-width ratio of the blocks which will exist as construction of a block progresses from its foundation to the top of the dam. If the ratio of the longer dimension to the shorter dimension is much over 2½, cracking at approximate third points of the block can be expected. Ratios of 2 to 1 or less are desirable, if practicable.

7-20. ***Concrete Cooling Systems.***–The layout of the concrete cooling systems consists of pipe or tubing placed in grid-like coils over the top surface of each lift of concrete after the concrete has hardened. Coils are formed by joining together lengths of thin-wall metal pipe or tubing. The number of coils in a block depends upon the size of the block and the horizontal spacing. Supply and return headers, with manifolds to permit individual connections to each coil, are normally placed on the downstream face of the dam. In some instances, cooling shafts, galleries, and embedded header systems can be used to advantage. Figures 7-11 and 7-12 show cooling details for Glen Canyon Dam, and figure 7-13 shows similar details for Monticello Dam.

The velocity of flow of the cooling water through the embedded coils is normally required to be not less than 2 feet per second, or about 4 gallons per minute for the commonly used 1-inch pipe or tubing. Cooling water is usually pumped through the coils, although a gravity system has at times been used. When river water is used, the warmed water is usually wasted after passing through the coils. River water having a high percentage of solids should be avoided as it can clog the cooling systems. When refrigerated water is used, the warmed water is returned to the water coolers in the refrigerating plant, recooled, and recirculated.

For control of the cooling operations, electrical resistance-type thermometers can be embedded at midlift and the electrical cable extended to a terminal board where readings can be taken whenever desired. Thermometer tubes can also be embedded in the concrete. Insert-type thermometers are inserted into these tubes when readings are desired. In many installations thermocouples have been used and are not as costly as the thermometer installations. The thermocouples are placed in the fresh concrete at midlift and at least 10 feet from an exposed face, with the lead wires from the thermocouples carried to readily accessible points on the downstream face.

Varying the length of the embedded coil, the horizontal spacing of the pipe, and the temperature of the water circulated through the coil can be done during the construction period to meet changed conditions. The effect of these variables is given in section 7-15.

Specification requirements for the installation and operation of the cooling systems should provide for the cooling systems to be water tested prior to embedment to assure the operation of each individual coil. The arrangement of the pipe headers and connections to the individual cooling coils should be such as to ensure dependable and

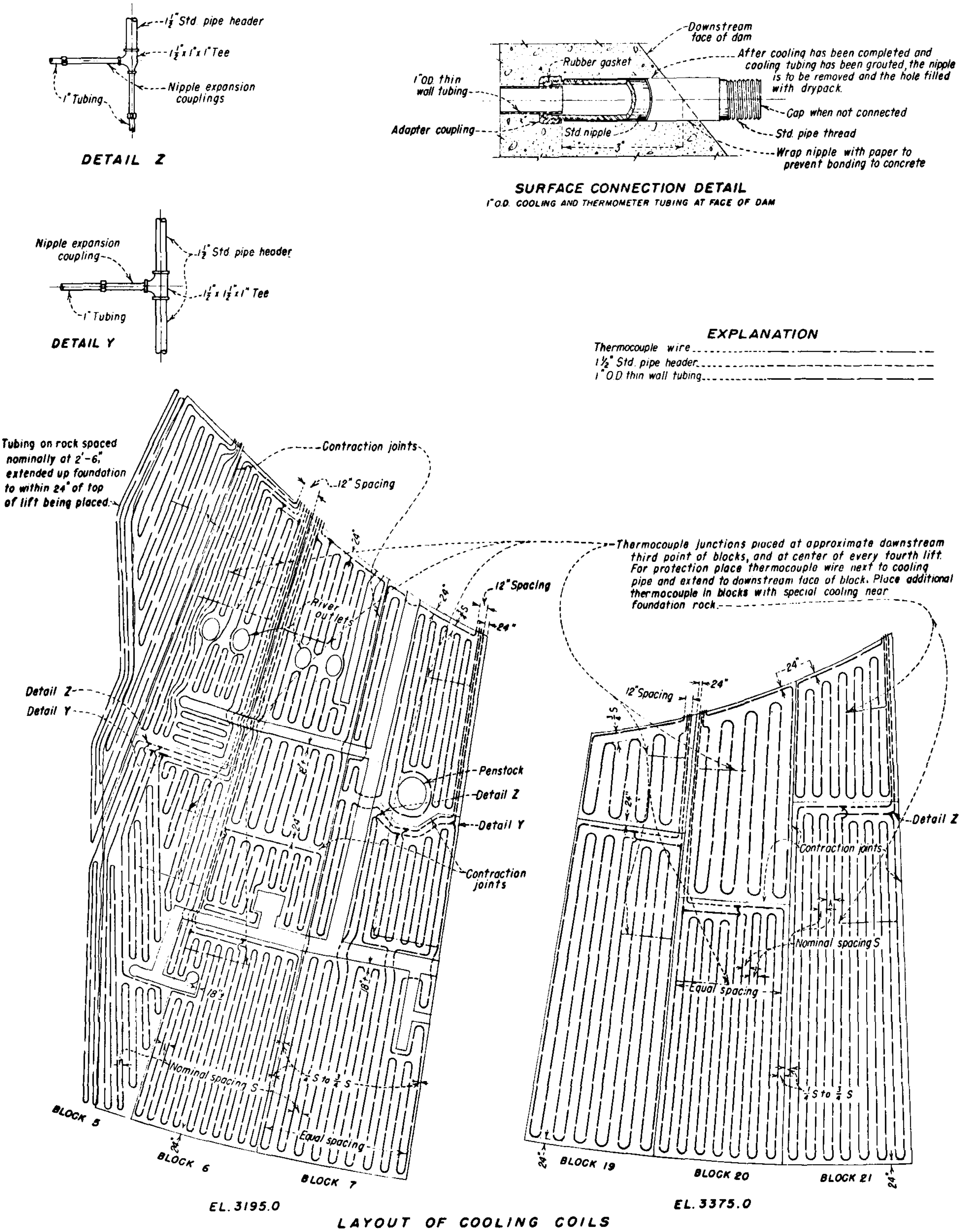
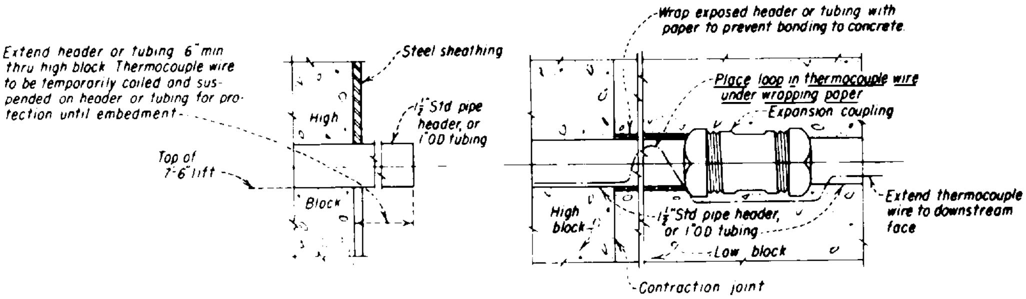


*Figure 7-11.* **Glen Canyon Dam–cooling pipe layout.–288-D-3021**



INITIAL INSTALLATION

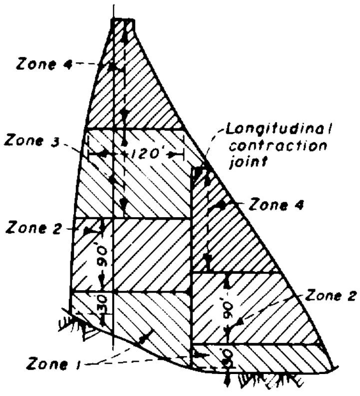
FINAL INSTALLATION

TYPICAL EXPANSION COUPLING

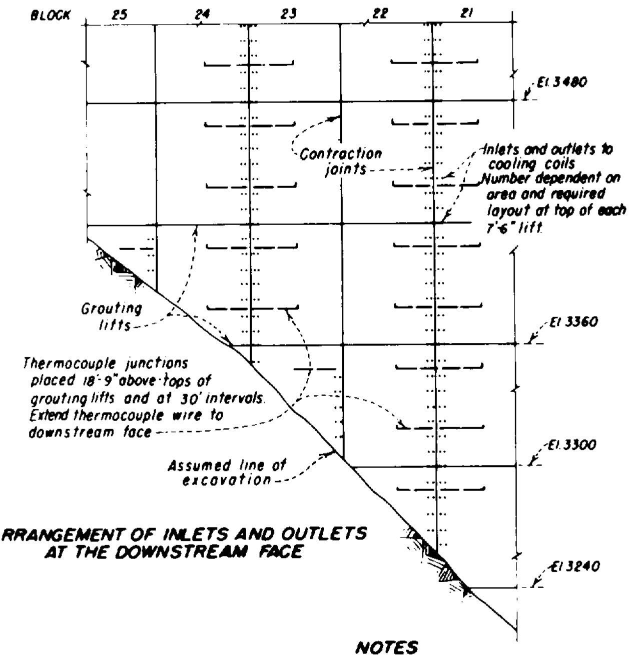
SHOWING CONTRACTION JOINT CROSSING

HORIZONTAL SPACING OF COOLING TUBING
(CRITERIA TO BE APPLIED TO EACH BLOCK)

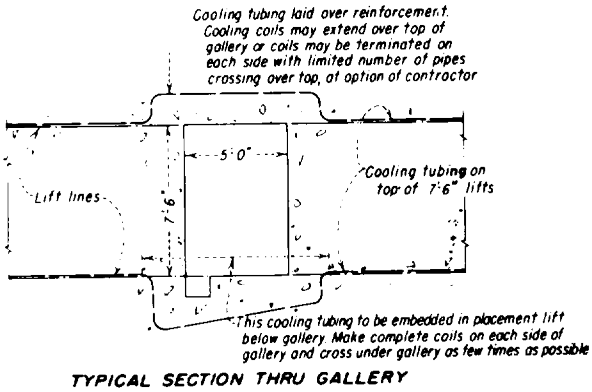
| ZONE | DESCRIPTION | MAXIMUM NOMINAL SPACING S | |
|---|---|---|---|
| | | CONCRETE PLACED | |
| | | NOV JAN DEC FEB | ALL OTHER MONTHS |
| 1 | Between the foundation and 30' above the highest point of the foundation | 3'0" on all concrete, 2'6" along foundation rock | 3'0" on all concrete, 2'6" along foundation rock |
| 2 | Between 30' and 120' above the highest point of the foundation | 5'-0" | 4'-0" |
| 3 | Greater than 120' above the highest point of the foundation and where the 7'-6" lift is longer than 120' | 6'-0" | 5'-0" |
| 4 | Greater than 120' above the highest point of the foundation and where the 7'-6" lift is 120' in length or less | 6'-0" | 6'-0" |



TYPICAL SECTION THRU DAM



ARRANGEMENT OF INLETS AND OUTLETS AT THE DOWNSTREAM FACE



TYPICAL SECTION THRU GALLERY

## NOTES

Actual required foundations may differ widely from assumed excavation lines shown

Cooling coils shall be placed on top of each 7'-6" concrete lift

Place tubing on all rock surfaces to within 24" of the top of the lift being placed

Cooling tubing to be placed to clear openings in dam a min of 18" or as directed

Expansion couplings shall be used at contraction joint crossings.

Where tubing is installed for thermometer wells, the embedded end of the tubing is to be flattened and crimped to seal against grout leakage

Arrangement of tubing may vary from that shown. The actual arrangement of the tubing in the structure shall be as directed.

Where a block is bounded by the downstream face and requires two or more coils, the contractor may elect to terminate all coils at downstream face in lieu of using 1½" headers. Upstream blocks requiring two or more coils will require headers.

Tubing placed on rock to be spaced at 2'-6", tubing at top of each 7'-6" lift to be spaced according to zones as shown in table.

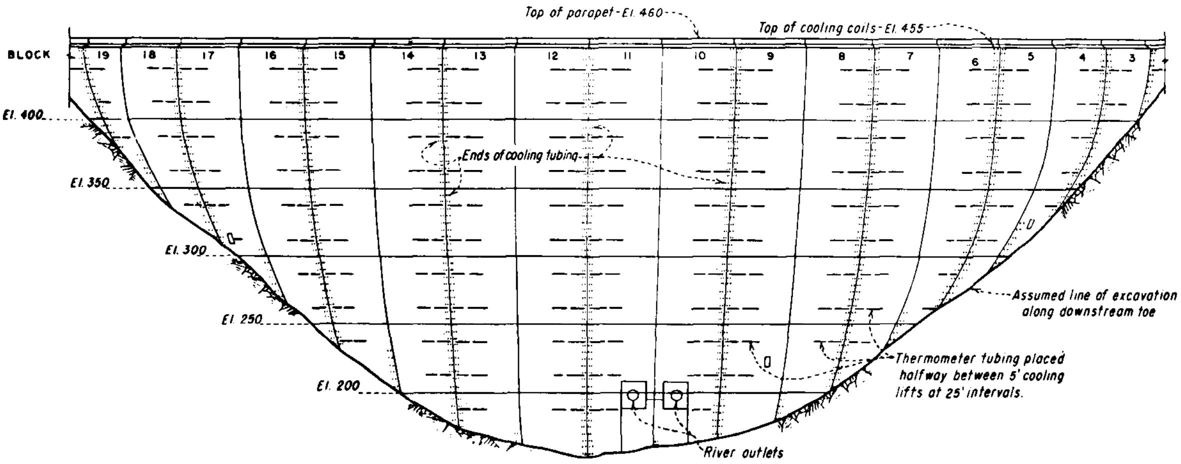
Coils placed in Zone 1 shall be approximately 800' in length with no coil longer than 900' in length; all other coils shall be approximately 1200' in length with no coil longer than 1300' in length. Adjacent coils served by the same header shall be as nearly the same length as possible.

Thermometer wells will be used to determine concrete temperatures at locations directed to supplement or replace thermocouple wire.

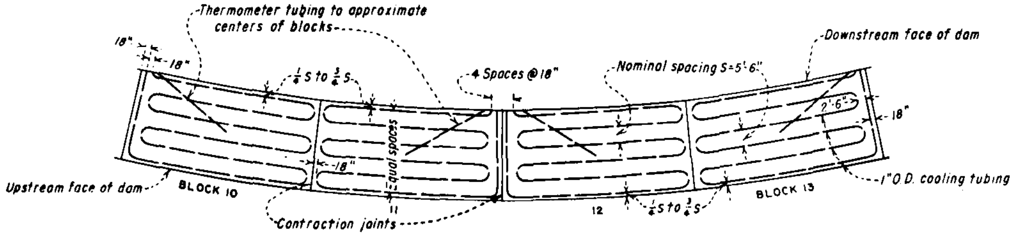
Cooling tubing placed within 25 to 30 feet of the foundation rock shall, where practicable, be placed as separate coils or with separate headers to facilitate special cooling in this area.

Each block shall have an independent cooling system at each concrete lift.

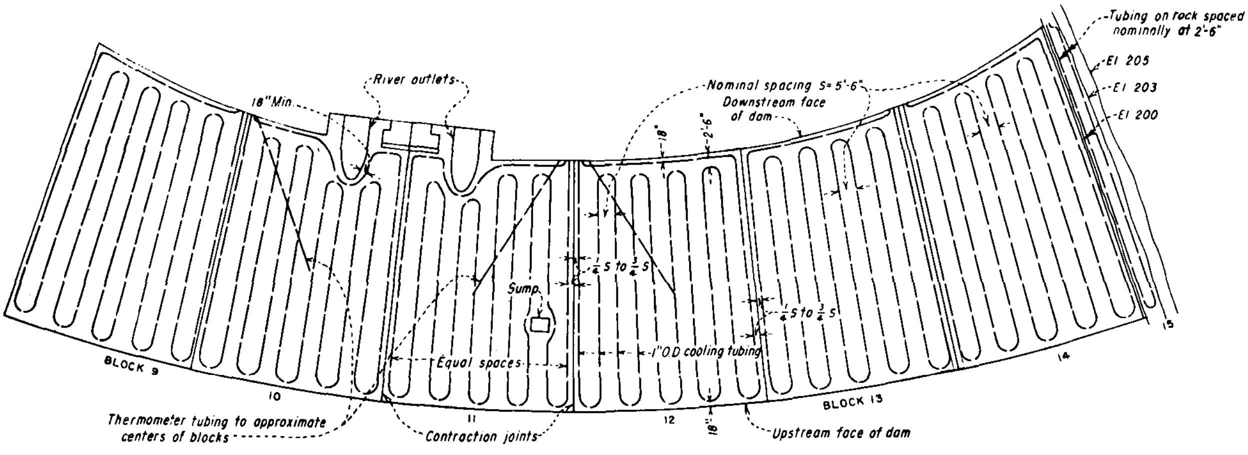
*Figure 7-12.* Glen Canyon Dam—concrete cooling details.—288-D-3022



DOWNSTREAM ELEVATION
Projected



PLAN AT EL. 400
TYPICAL LAYOUT FOR BLOCKS
WHERE WIDTH EXCEEDS LENGTH



PLAN AT EL. 200
TYPICAL LAYOUT FOR BLOCKS
WHERE LENGTH EXCEEDS WIDTH

*Figure 7-13.* **Monticello Dam—cooling pipe layout.—288-D-3023** continuous operation. Provisions should be made in the pumping or header systems for reversing the flow of water in the individual coils once each day. This is necessary to obtain a uniform cooling across the block. Because of varying construction schedules and progress and varying climatic conditions, the specifications should also provide that the times when cooling is to be performed in the individual cooling coils be as directed by the contracting officer. This will permit the operation of the cooling systems to be such as to minimize adverse conditions of temperature drops and temperature gradients which could lead to undesirable cracking.

7-21. *Height Differentials.*—A maximum height differential between adjacent blocks is normally specified in construction specifications for concrete dams. From a temperature standpoint, an even temperature distribution throughout the structure will be obtained when all blocks in the dam are placed in a uniform and continuous manner. This even temperature distribution is desirable because of the subsequent uniform pattern of contraction joint openings. Extreme temperature gradients on the exposed sides of blocks will also be lessened when each lift is exposed for a minimum length of time.

Minimizing the overall height differential between the highest and lowest blocks in the dam will cause construction of the dam to progress uniformly up from the bottom of the canyon. Contraction joints can then be grouted in advance of a rising reservoir, thus permitting storage at earlier times than would be possible if construction progress were concentrated in selected sections of the dam.

The height differential specified is a compromise between the uniform temperature conditions and construction progress desired, and the contractor's placement program. In practice, the maximum height differential between adjacent blocks is usually 25 feet when 5-foot lifts are used or 30 feet when 7½-foot lifts are used. The maximum differential between the highest block in the dam and the lowest block is usually limited to 40 feet when 5-foot lifts are used and 52.5 feet when 7½-foot lifts are used.

If cold weather is to be expected during any part of the construction period, height differentials between adjacent blocks should be limited to those needed for construction. If concrete placement is to be discontinued during winter months, the height differentials should be reduced to practical minimums before the shutdown period.

7-22. *Lift Thickness.*—Economy of construction should be considered in determining the heights of placement lifts in mass concrete. Shallow lifts not only slow up construction but result in increased construction joints which have to be cleaned and prepared for the next placement lift. Secondarily, the thickness of lift should be considered and related to the temperature control measures proposed for the structure.

When no precooling measures are used, the placing temperature of the concrete will approximate the ambient temperature at the site. With this condition, a considerable portion of the total heat of hydration in a placement lift can be lost through the top exposed surface before the next lift is placed. Shallow lifts and longer delays between placement lifts will result in the minimum temperature rise in the concrete under these conditions. The opposite condition may occur, and should be studied, when precooling measures are used. During the summer months, the ambient temperatures will normally be higher than the concrete temperatures for the first few days after placement and a heat gain will result. Under these conditions, higher placement lifts and minimum periods of time between placements would be beneficial.

7-23. *Delays Between Placements.*—Construction of mass concrete blocks by placement lifts incurs periodic time delays between lifts. Depending upon ambient temperatures, these delays can be beneficial or harmful. The minimum elapsed time between placing of successive lifts in any one block is usually restricted to 72 hours, but temperature studies should be made to relate heat loss or heat gain to the placement lifts. These studies should take into account the anticipated temperature control measures and the seasonal effects to be met during the construction period. Delays between placements, and lift thicknesses should be studied simultaneously to take these variables into consideration as discussed in section 7-22.

The size and number of construction blocks in the dam will influence the time between placement lifts. Normal construction operations will require a minimum of 2 or 3 days between lifts. On the larger dams, however, an average placement time of about 6 or 7 days between successive lifts in a block will elapse because of the large number of construction blocks and the concrete yardage involved.

7-24. ***Closure Slots.***—Closure slots are 2- to 4-foot-wide openings left in the dam between adjacent blocks during construction. Closure is made by filling the slots with concrete at a time when temperature conditions are favorable, usually during the late winter months of the construction period when the adjacent blocks are at minimum temperature. The use of closure slots will often expedite construction and will result in economy of labor and materials. In thin arch dams, for example, closure slots can be used instead of contraction joint grouting. Lower closure temperatures may also be obtained with a closure slot. If the concrete in the adjacent blocks is below 32° F., however, provision should be made to warm up the near-surfaces of the closure slot to at least 45° F. prior to placement of concrete in the slot.

## E. CONSTRUCTION OPERATIONS

7-25. ***Temperature Control Operations.***—The typical temperature history of artificially cooled concrete is shown on figure 7-13. Owing to hydration of the cement, a temperature rise will take place in the concrete after placement. After the peak temperature is reached, the temperature will decline depending upon the thickness of section, the exposure conditions, the rate and amount of continued heat of hydration, and whether or not artificial cooling is continued. The peak temperature is generally reached between ages 7 and 20 days in massive concrete sections where no artificial cooling is employed. These sections may maintain this maximum temperature for several weeks, after which the temperature will drop slowly over a period of several years. In thin structures or when artificial cooling is employed, the peak temperature is generally reached at about age 2½ to 6 days, after which the temperature can drop at a fairly rapid rate. With artificial cooling, the rate of temperature drop is usually limited to ½° to 1° F. per day, exposure conditions permitting. In thin structures exposed to very low air temperatures, the exposure conditions alone may cause temperatures to decline as much as 3° to 4° F. per day.

Initial cooling is normally accomplished with water not warmer than that obtainable from the river. Intermediate and final cooling may be accomplished with either river water or refrigerated water, depending upon the temperatures involved. River water will usually be sufficient if its temperature is 4° to 5° F. below the grouting temperature and if such a temperature persists for a minimum of about 2 months. The main objection to refrigerated water is its high cost. Advantages, however, include its availability at any time of the year and the wide range of temperatures possible.

Timely operation of the embedded cooling system will reduce the tendency of the concrete to crack during the construction period. The effects of unanticipated changes such as a change in the type or amount of cement used or the curing method employed, exposure temperatures varying from those assumed, or any other factor which influences concrete temperatures are normally taken into account by varying the period of flow and the temperature and rate of flow of the cooling water. Intermittent cooling periods can be used to lower interior temperatures prior to exposure of the concrete to cold weather. During cold weather placement, the normal period of initial cooling may be shortened considerably to prevent forcing too rapid a drop in temperature. Depending upon the dimensions of the structure and the exposures expected, insulating the exposed surfaces while artificially cooling the interior may be necessary to control temperature cracking. This is especially true for areas near corners of the construction blocks where temperatures can drop very rapidly.

(a) *Initial Cooling.*—Artificial cooling is employed for a limited period of time initially. Upon completion of this initial cooling period, temperatures within the concrete may continue to drop but at a slower rate, they may hold steady at about the same temperature, or they may start rising again. This part of the temperature history is primarily dependent upon the thickness of section and the exposure conditions existing at the time. Continued heat of hydration at this age may also affect the concrete but would be of lesser importance.

The normal initial cooling period is from 10 to 16 days. During this initial cooling period, the concrete temperatures are reduced from the maximum concrete temperature to such a value that, upon stoppage of the flow of water through the cooling system, the continued heat of hydration of the cement will not result in temperatures higher than the maximum previously obtained. The rate of cooling is controlled so that the tensions in the concrete caused by the drop in temperature will not exceed the tensile strength of the concrete for that age of concrete.

In the early spring and late fall months when exposure temperatures may be low, the length of the initial cooling period and the rate of temperature drop can be critical in thin concrete sections. In these sections, pipe cooling, combined with the low exposure temperatures, can cause the concrete temperature to drop too fast. During these seasons, artificial cooling should be stopped shortly after the peak temperature is reached and the concrete then allowed to cool in a natural manner. In structures with thicker sections, the exposure temperatures have less effect on the immediate temperature drop, and the initial cooling period can be continued with the primary purpose of controlling the differential temperature between the exposed faces and the interior.

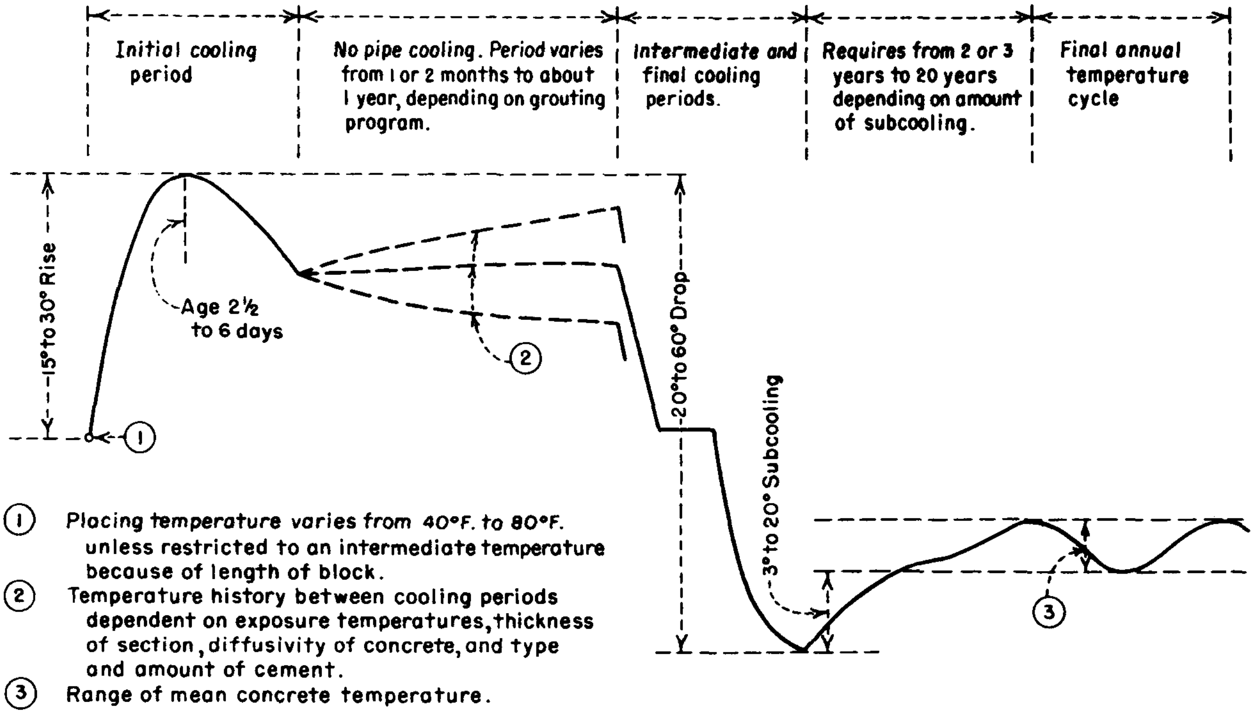
(b) *Intermediate and Final Cooling.*—Subsequent to the initial cooling period, intermediate and final cooling periods are employed to obtain desired temperature distributions or desired temperatures prior to contraction joint grouting. Final cooling for contraction joint grouting is normally accomplished just prior to grouting the contraction joints, the program of cooling being dictated by construction progress, method of cooling, season of the year, and any reservoir filling criteria.

As indicated in figure 7-14, cooling prior to grouting the contraction joints is normally started after the concrete has attained an age of 2 months to 1 year. Cooling is normally performed by grout lifts. In the smaller construction blocks, final cooling may be accomplished in a single, continuous cooling period. In the larger blocks, however, the final cooling should be performed in two steps to reduce the vertical temperature gradient between grout lifts. The first of these steps is commonly referred to as the intermediate cooling period and the second step as the final cooling period.

In practice, the intermediate cooling period for a grout lift lowers the temperature of the concrete in that lift to approximately halfway between the temperature existing at the start of the cooling period and the desired final temperature. Each grout lift, in succession, undergoes this intermediate cooling period before the final cooling of the next lower grout lift is undertaken.

Depending upon the temperature drop and final temperature to be obtained, the season of the year when this cooling is accomplished, and the temperature of the cooling water, the intermediate and final cooling periods will require a total of from 30 to 60 days. The rate of temperature drop should be held to not more than 1° F. per day, and a rate of ½° to ¾° F. per day is preferable.

It is theoretically possible to compute the required temperature drop to obtain a desired joint opening. The theoretical joint opening



① Placing temperature varies from 40°F. to 80°F. unless restricted to an intermediate temperature because of length of block.

② Temperature history between cooling periods dependent on exposure temperatures, thickness of section, diffusivity of concrete, and type and amount of cement.

③ Range of mean concrete temperature.

*Figure 7-14.* Temperature history of artificially cooled concrete.–288-D-3024

does not occur, however, because some compression is built up in the block as the temperature increases during the first few days after placement. A temperature drop of $4^{\circ}$ to $8^{\circ}$ F. from the maximum temperature, depending on the creep properties of the concrete, may be required to relieve this compression before any contraction joint opening will occur. Measured joint openings in Hungry Horse Dam averaged 75 percent of the theoretical. Other experiences with arch dams having block widths of approximately 50 feet have indicated that a minimum temperature drop of $25^{\circ}$ F. from the maximum temperature to the grouting temperature is desirable, and will result in groutable contraction joint openings of 0.06 to 0.10 inch. For the wider blocks with 70 feet or more between contraction joints, a temperature drop of $20^{\circ}$ F. will usually be sufficient.

(c) *Warming Operations.*–In relatively thin arch dams, artificial cooling may be omitted and the concrete permitted to cool naturally over the winter. Depending on the severity of the exposure conditions, it may be necessary to wait until the concrete temperatures rise above $32^{\circ}$ F. before grouting the contraction joints. Because of the varying thicknesses, concrete temperatures will reach the grouting temperature at different times in the several grout lifts. This requires close control over the contraction joint grouting program, and may require that lower portions of the dam be artificially warmed to permit an orderly grouting program to be completed before the top of the dam becomes too warm. This warming may be accomplished by circulating warm water through the embedded cooling coils.

Prolonged exposure of horizontal construction joints will often result in poor bond of the construction lifts. Horizontal leafing cracks may occur between the older and newer concretes, extending from the face of the structure into the interior. Cracks of this type quite often lead to freezing and thawing deterioration of the concrete. Preventive steps should be directed toward obtaining a better than average bond between the old concrete and the new concrete. This includes minimizing the temperature differential between the old and the new concrete. Several shallow placement lifts placed over the cold construction joint may be sufficient. For lifts exposed over a winter season, treatment may include warming the top 10 to 15 feet of the old concrete to the placing temperature of the new concrete. This will reduce the temperature gradient which will occur. The warming operation can be performed by circulating warm water through the embedded cooling coils. Warming operations should immediately precede the placement of the new concrete. If exposure temperatures are extremely low at the time placement is to be resumed, insulation should be placed over the tops of the lifts during the warming operations.

7-26. ***Foundation Irregularities.***—Although the trial-load analyses and designs assume relatively uniform foundation and abutment excavations, the final excavation may vary widely from that assumed. Faults or crush zones are often uncovered during excavation, and the excavation of the unsound rock leaves depressions or holes which must be filled with concrete. Unless this backfill concrete has undergone most of its volumetric shrinkage at the time overlying concrete is placed, cracks can occur in the overlying concrete near the boundaries of the backfill concrete as loss of support occurs due to continuing shrinkage of the backfill concrete. Where the area of such dental work is extensive, the backfill concrete should be placed and cooled before additional concrete is placed over the area.

Similar conditions exist where the foundation has abrupt changes in slope. At the break of slope, cracks often occur because of the differential movement which takes place between concrete held in place by rock, and concrete held in place by previously placed concrete which has not undergone its full volumetric shrinkage. A forced cooling of the concrete adjacent to and below the break in slope, and a delay in placement of concrete over the break in slope, can be employed to minimize cracking at these locations. If economical, the elimination of these points of high stress concentration is worthwhile. Such cracks in lifts near the abutments very often develop leakage and lead to spalling and deterioration of the concrete.

7-27. ***Openings in Dam.***—Because openings concentrate stresses at their corners, all possible means should be used to minimize stresses at the surfaces of such openings. Proper curing methods should be used at all times. The entrances to such openings should be bulkheaded and kept closed, with self-closing doors where traffic demands, to prevent the circulation of air currents through the openings. Such air currents not only tend to dry out the surfaces but can cause the formation of extreme temperature gradients during periods of cold weather.

7-28. ***Forms and Form Removal.***—The time of removal of forms from mass concrete structures is important in reducing the tendency to crack at the surface. This is especially true when wooden forms or insulated steel forms are used. If exposure temperatures are low and if the forms are left in place for several days, the temperature of the concrete adjacent to the form will be relatively high when the forms are stripped, and the concrete will be subjected to a thermal shock which may cause cracking. From the temperature standpoint, these forms should either be removed as early as practicable or should remain in place until the temperature of the mass has stabilized. In the latter case, a uniform temperature gradient will be established between the interior mass and the surface of the concrete, and removal of the forms, except in adverse exposure conditions, will have no harmful results.

When the ordinary noninsulated steel form is used, the time of form removal may or may not be important. The use of steel forms which are kept cool by continuous water sprays will tend to cause the near-surface concrete to set at a lower temperature than the interior of the mass. Form removal can then be accomplished with no detrimental effects. If, however, water sprays are not used to modify the temperature of the steel forms, the early-age temperature variation of the face concrete may be even greater than the daily cycle of air temperature because of absorbed heat from solar radiation and reradiation.

7-29. ***Curing.***—Drying shrinkage can cause, as a skin effect, hairline cracks on the surface of a mass concrete structure. The primary objection to these random hairline cracks of limited depth is that they are usually the beginning of further and more extensive cracking and spalling under adverse exposure conditions. Following the removal of forms, proper curing is important if drying shrinkage and resulting surface cracking are to be avoided. Curing compounds which prevent the loss of moisture to the air are effective in this respect, but lack the cooling benefit which can be obtained by water curing. In effect, water curing obtains a surface exposure condition more beneficial than the fluctuating daily air temperature. With water curing, the daily exposure cycle is dampened because the daily variation of the water temperature is less than that of the air temperature.

A benefit also occurs from the evaporative cooling effect of the water on the surface. The evaporative cooling effect is maximized by intermittent sprays which maintain the surface of the concrete in a wet to damp condition with some free water always available.

In general, water curing should be used instead of membrane curing on mass concrete structures. Where appearance is of prime importance, other methods of curing may be considered because water curing will often result in stains on the faces. Water curing during periods of cold weather also can be a safety problem because of icing hazards.

**7-30.** ***Insulation.***—During the fall of the year when placing temperatures are still relatively high, and during periods of cold weather, the temperature of the surface concrete tends to drop rapidly to the exposure temperature. This may occur while the interior concrete is still rising in temperature. Such conditions will cause high tensile stresses to form at the surface. Surface treatments previously described can reduce these temperature gradients, particularly when used in conjunction with artificial cooling, but the use of insulation will give greater protection. Such insulation may be obtained by measures varying from simply leaving wooden or insulated forms in place, to the use of commercial-type insulation applied to the forms or to the surfaces of the exposed concrete. Tops of blocks can be protected with sand or sawdust when an extended exposure period is anticipated.

Unless required immediately after placement to prevent surface freezing, the insulation should be placed after the maximum temperature is reached in the lift. This permits loss of heat to the surface and will cause the near-surface concrete to set at a relatively low temperature. Normally, during periods of cold weather, the insulation is removed at such time as required for placement of the next lift. Otherwise, it may be removed when the cold weather has abated or when interior temperatures have been reduced substantially below the peak temperatures.

Whatever the type of insulation, measures should be taken to exclude as much moisture from the insulation as practicable. The insulation should also be as airtight as possible. For a short period of exposure, small space heaters may be used, either by themselves or in conjunction with work enclosures. Care should be taken when using space heaters in enclosed areas to avoid drying out the concrete surfaces.

## F. BIBLIOGRAPHY

### 7-31. *Bibliography.*

[1] Control of Cracking in Mass Concrete Structures," Engineering Monograph No. 34, Water Resources Technical Publication, Bureau of Reclamation, 1965.

[2] Schack, Alfred, "Industrial Heat Transfer," John Wiley & Sons, New York, N.Y., 1933.

[3] Jakob, Max, "Heat Transfer," vol. I, pp. 373-375, John Wiley & Sons, New York, N.Y., 1949.

[4] Grinter, L. E., "Numerical Methods of Analysis in Engineering," p. 86, Macmillan Co., New York, N.Y., 1949.

[5] "A Simple Method for the Computation of Temperatures in Concrete Structures," ACI Proceedings, vol. 34, (November-December 1937 ACI Journal).

[6] "Thermal Properties of Concrete," Part VII, Bulletin No. 1, Boulder Canyon Project Final Reports, Bureau of Reclamation, 1940.

[7] "Cooling of Concrete Dams," Part VII, Bulletin No. 3, Boulder Canyon Project Final Reports, Bureau of Reclamation, 1949.

[8] "Insulation Facilitates Winter Concreting," Engineering Monograph No. 22, Bureau of Reclamation, 1955.

[9] Kinley, F. B., "Refrigeration for Cooling Concrete Mix," Air Conditioning, Heating and Ventilating, March 1955.

## <<Chapter VIII

# Joints in Structures

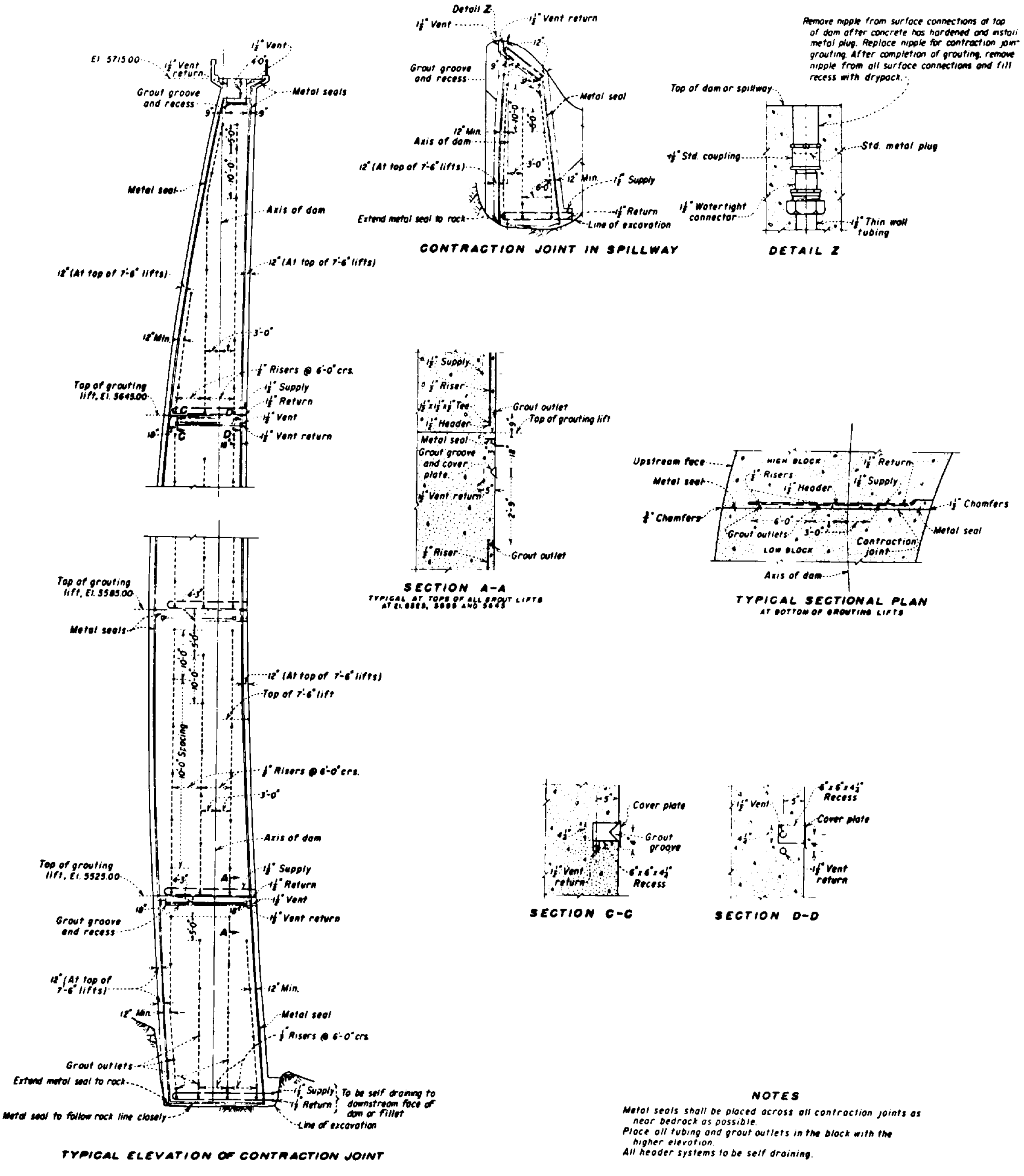
8-1. ***Purpose.***—Cracking in concrete dams is undesirable because cracking in random locations can destroy the monolithic nature of the structure, thereby impairing its serviceability and leading to an early deterioration of the concrete. Joints placed in mass concrete dams are essentially designed cracks, located where they can be controlled and treated to minimize any undesirable effects. The three principal types of joints used in concrete dams are contraction, expansion, and construction joints.

Contraction and expansion joints are provided in concrete structures to accommodate volumetric changes which occur in the structure after placement. Contraction joints are provided in a structure to prevent the formation of tensile cracks as the structure undergoes a volumetric shrinkage due to a temperature drop. Expansion joints are provided in a unit-structure to allow for the expansion (a volumetric increase due to temperature rise) of the unit in such a manner as not to change the stresses in, or the position of, an adjacent unit or structure. Construction joints are placed in concrete structures to facilitate construction, to reduce initial shrinkage stresses, to permit installation of embedded metalwork, or to allow for the subsequent placing of other concrete, including backfill and second-stage.

8-2. ***Contraction Joints.***—In order to control the formation of cracks in mass concrete dams, current practice is to construct the dam in blocks separated by transverse contraction joints. These contraction joints are vertical and normally extend from the foundation to the top of the dam. Transverse joints are radial to the axis of the dam and are continuous from the upstream face to the downstream face.

Depending upon the size of the structure, it may also be necessary to provide longitudinal contraction joints in the blocks formed by the transverse contraction joints. If longitudinal contraction joints are provided, construction of the dam will consist of placing a series of adjoining columnar blocks, each block free to undergo its own volume change without restraint from the adjoining blocks. The longitudinal contraction joints are also vertical and on chords approximately parallel with the axis of the dam. The joints are staggered a minimum of 25 feet at the transverse joints. Generally, the longitudinal joints pass entirely through the structure. As the longitudinal joint nears the sloping downstream face, either the direction of the joint is changed from the vertical to effect a perpendicular intersection with the face, with an offset of 3 to 5 feet, or the joint is terminated at the top of a lift when it is within 15 to 20 feet of the face. In the latter case, strict temperature control measures will be required to prevent cracking of the concrete directly above the termination of the joint.

Typical transverse contraction joints can be seen on figures 8-1 and 8-2, and a typical longitudinal contraction joint can be seen on figure 8-3. Contraction joints should be constructed so that no bond exists between the concrete blocks separated by the joint. Reinforcement should not extend across a contraction joint. The intersection of the joints with the faces of the dam should be chamfered to give a desirable appearance and to minimize spalling. In order to standardize block


CONTRACTION JOINT IN SPILLWAY

DETAIL Z

Remove nipple from surface connections at top of dam after concrete has hardened and install metal plug. Replace nipple for contraction joint grouting. After completion of grouting, remove nipple from all surface connections and fill recess with drypack.

SECTION A-A

TYPICAL AT TOPS OF ALL GROUT LIFTS AT EL. 5585, 5525 AND 5645

TYPICAL SECTIONAL PLAN

AT BOTTOM OF GROUTING LIFTS

SECTION C-C

SECTION D-D

TYPICAL ELEVATION OF CONTRACTION JOINT

NOTES

Metal seals shall be placed across all contraction joints as near bedrock as possible.

Place all tubing and grout outlets in the block with the higher elevation.

All header systems to be self draining.

*Figure 8-1.* Transverse contraction joint and grouting system for East Canyon Dam, a small arch dam in Utah.–288-D-3029

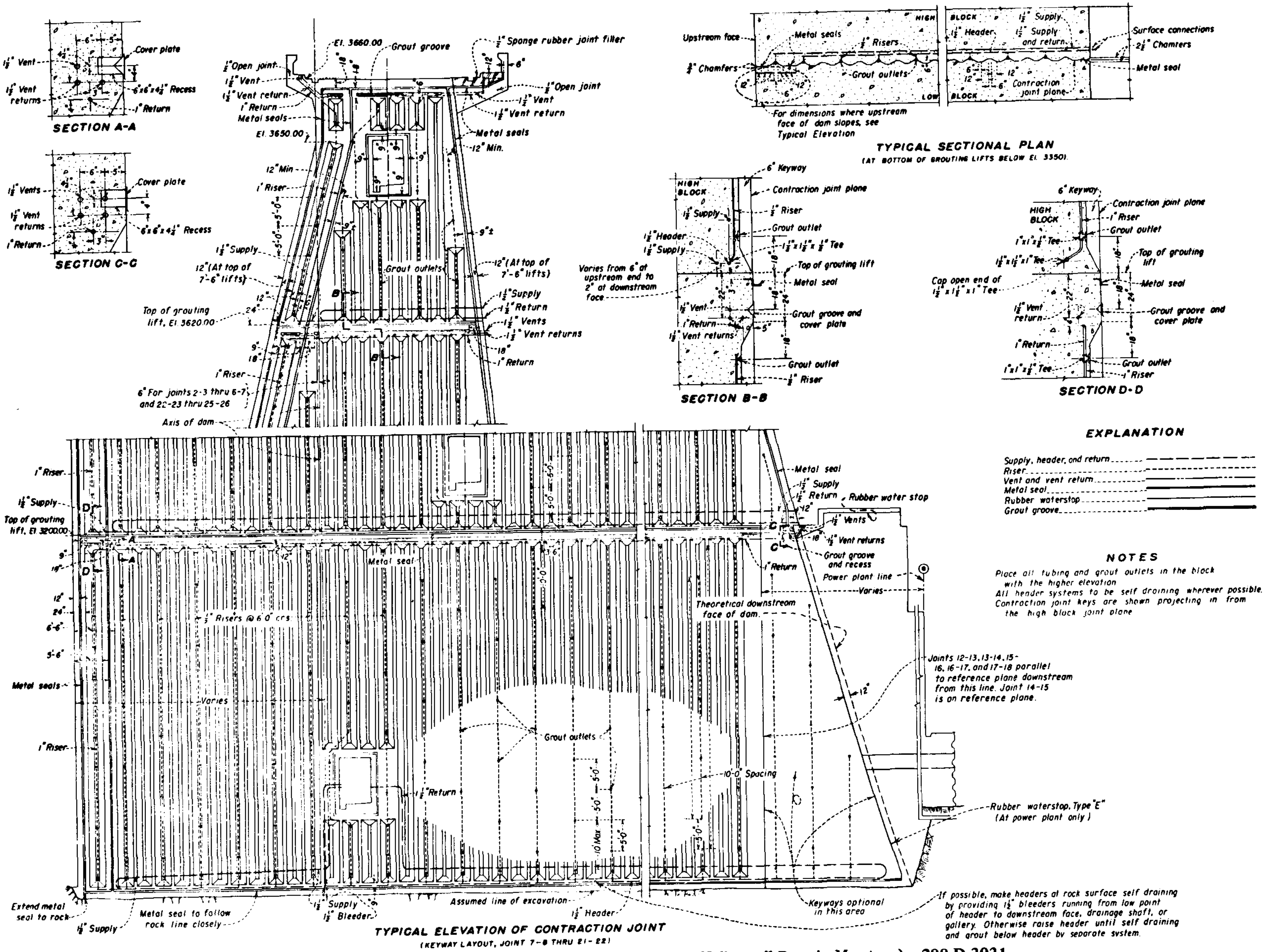
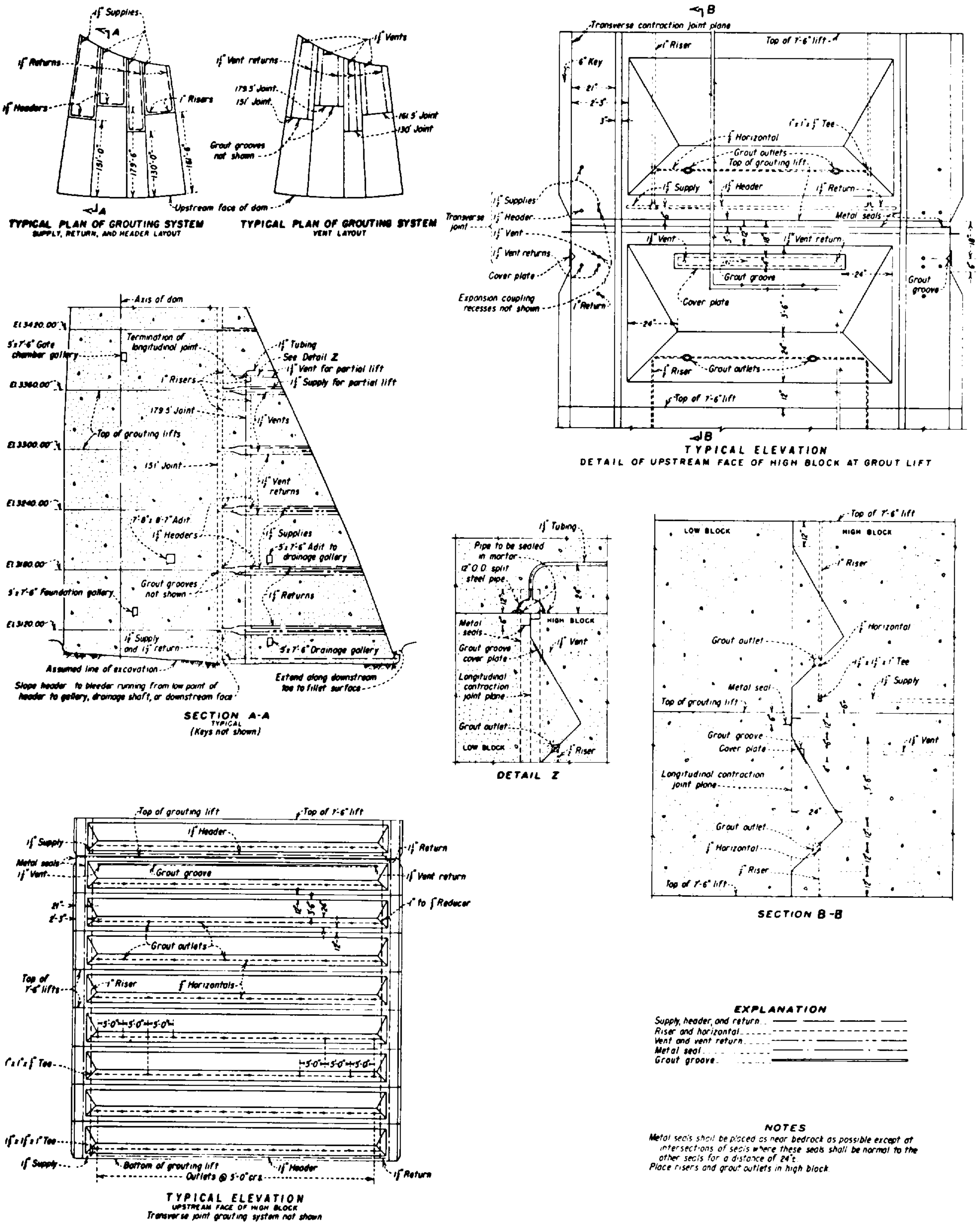


*Figure 8-2.* Standard key on a transverse contraction joint (Yellowtail Dam in Montana).—288-D-3031



NOTES

Metal seals shall be placed as near bedrock as possible except at intersections of seals where these seals shall be normal to the other seals for a distance of 24"±.

Place risers and grout outlets in high block.

*Figure 8-3.* Standard key on a longitudinal contraction joint (Glen Canyon Dam in Arizona).–288-D-3033 identification on all future dams, a criterion has recently been established which calls for the designation of blocks in the longitudinal direction by number, starting with block 1 on the right abutment (looking downstream). The blocks in each transverse row are to be designated by letter starting with the upstream block as the "A" block.

8-3. ***Expansion Joints.***—Expansion joints are provided in concrete structures primarily to accommodate volumetric change due to temperature rise. In addition, these joints frequently are installed to prevent transferal of stress from one structure to another. Notable examples are: (1) powerplants constructed adjacent to the toe of a dam, wherein the powerplant and the mass of the dam are separated by a vertical expansion joint; and (2) outlet conduits encased in concrete and extending downstream from the dam, in which case an expansion joint is constructed near the toe of the dam separating the encasement concrete from the dam.

Like contraction joints, previously discussed, expansion joints are constructed so that no bond exists between the adjacent concrete structures. A corkboard, mastic, sponge rubber, or other compressible-type filler usually separates the joint surfaces to prevent stress or load transferal. The thickness of the compressible material will depend on the magnitude of the anticipated deformation induced by the load.

8-4. ***Construction Joints.***—A construction joint in concrete is defined as the surface of previously placed concrete upon or against which new concrete is to be placed and to which the new concrete is to adhere when the previously placed concrete has attained its initial set and hardened to such an extent that the new concrete cannot be incorporated integrally with the earlier placed concrete by vibration. Although most construction joints are planned and made a part of the design of the structure, some construction joints are expedients used by a contractor to facilitate construction. Construction joints may also be required because of inadvertent delays in concrete placing operations. Treatment and preparation of construction joints are discussed in chapter XIV.

8-5. ***Spacing of Joints.***—The location and spacing of transverse contraction joints should be governed by the physical features of the damsite, details of the structures associated with the dam, results of temperature studies, placement methods, and the probable concrete mixing plant capacity.

Foundation defects and major irregularities in the rock are conducive to cracking and this can sometimes be prevented by judicious location of the joints. Although cracks may develop normal to the canyon wall, it is not practicable to form inclined joints. Consideration should be given to the canyon profile in spacing the joints so that the tendency for such cracks to develop is kept to a minimum.

Outlets, penstocks, spillway gates, or bridge piers may affect the location of joints and consequently influence their spacing. Consideration of other factors, however, may lead to a possible relocation of these appurtenances to provide a spacing of joints which is more satisfactory to the dam as a whole. Probably the most important of these considerations is the permissible spacing of the joints determined from the results of concrete temperature control studies. If the joints are too far apart, excessive shrinkage stresses will produce cracks in the blocks. On the other hand, if the joints are too close together, shrinkage may be so slight that the joints will not open enough to permit effective grouting. Data on spacing of joints as related to the degree of temperature control are discussed in chapter VII.

Contraction joints should be spaced close enough so that, with the probable placement methods, plant capacity, and the type of concrete being used, batches of concrete placed in a lift can always be covered while the concrete is still plastic. For average conditions, a spacing of 50 feet has proved to be satisfactory. In dams where pozzolan and retarders are used, spacings up to 80 feet have been acceptable. An effort should be made to keep the spacing uniform throughout the dam.

The practice of spacing longitudinal joints follows, in general, that for the transverse joints, except that the lengths of the blocks are not limited by plant capacity. Depending on the degree to which artificial temperature control is exercised, spacings of 50 to 200 feet may be employed.

8-6. *Keys.*—Vertical keys in transverse joints are used primarily to provide increased shearing resistance between blocks; thus, when the joints and keys are grouted, a monolithic structure is created which has greater rigidity and stability because of the transfer of load from one block to another through the keys. A secondary benefit of the use of keys is that they minimize water leakage through the joints. The keys increase the percolation distance through joints and, by forming a series of constrictions, are beneficial in hastening the sealing of the joints with mineral deposits.

Keys are not always needed in the transverse contraction joints of concrete arch dams. Because the requirement for keys adds to form and labor costs, the need for keys and the benefits which would be attained from their use should be investigated and determined for each dam. Foundation irregularities may be such that a bridging action over certain portions of the foundation would be desirable. Keys can lock together adjacent blocks to help accomplish this bridging action. The corresponding thrusts and shears at all sections across the length of the dam, and particularly at any thrust blocks, should be investigated, and keys should be provided if additional shear strength is needed. In double-curvature arch dams keys may be required, at least in the lower portions of the dam, to maintain alinement and/or stability of the block during construction.

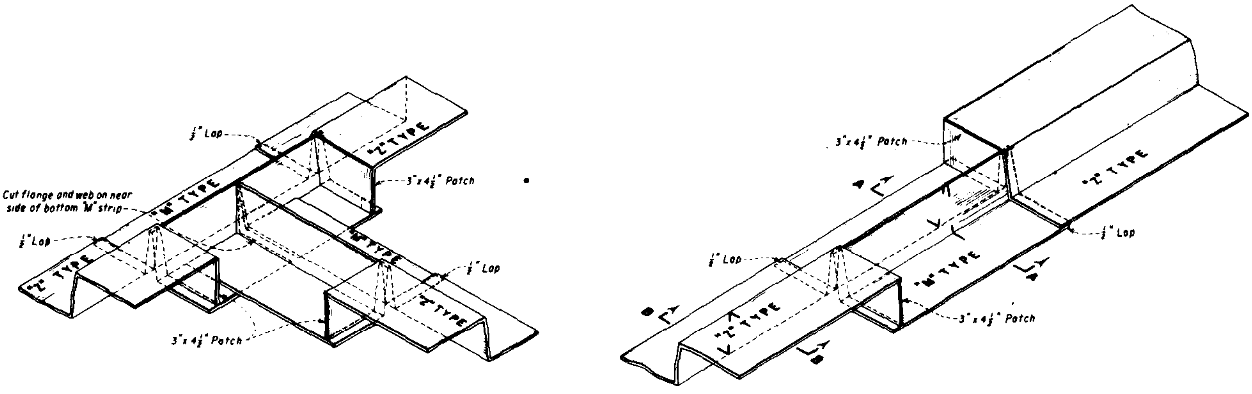
The transverse joint key developed by the Bureau has been standardized. The standard key offers minimum obstruction to the flow of grout, provides a good theoretical shear value, eliminates sharp corners which commonly crack upon removal of forms, improves the reentrant angles conducive to crack development associated with volume changes, and is well adapted to the construction of forms. Figure 8-2 shows the shape and dimensions of the standard key on the face of a typical transverse contraction joint.

Shear keys are important accessories in longitudinal contraction joints and are provided to maintain stability of the dam by increasing the resistance to vertical shear. The key faces are inclined to make them conform approximately with the lines of principal stress for full waterload. Inasmuch as the direction of principal stresses varies from the upstream face to the downstream face of the dam and from the foundation to the crest, an unlimited number of key shapes with resulting high forming costs would be required if close conformity were considered necessary. In order to simplify keyway forms, a single key shape, determined largely by the general direction of the lines of principal stress in the lower, downstream portion of the dam where the vertical shear is at a maximum, has been adopted for standard use. Details of the shape and dimensions of longitudinal keys used on Glen Canyon Dam are shown on figure 8-3. These keys are proportioned to accommodate the 7½-foot concrete placement lifts used on that dam.

8-7. *Seals.*—The opening of transverse contraction joints between construction blocks provides passages through the dam which, unless sealed, would permit the leakage of water from the reservoir to the downstream face. To prevent this leakage, seals are installed in the joints adjacent to the upstream face. Seals are also required on both transverse and longitudinal joints during grouting operations to confine the fluid grout in the joint. Figure 8-4 illustrates typical seals used in contraction joints.

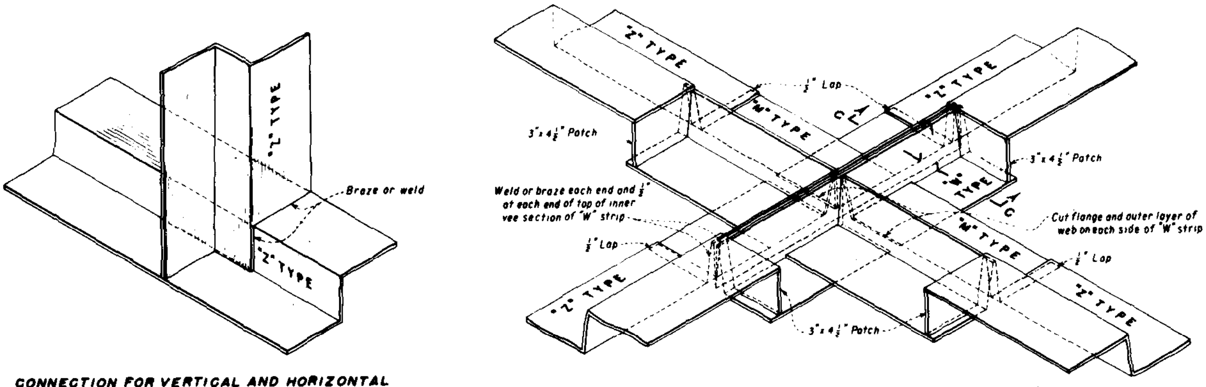
For seals to be effective in the contraction joints of concrete dams, installation is of greater importance than shape or material. Good workmanship in making connections, adequate protection to keep them from becoming torn prior to embedment, and careful placement and consolidation of the concrete around the seals are of primary importance.

(a) *Metal Seals.*—The most common type of seal used in concrete dams has been a metal seal embedded in the concrete across the joint.



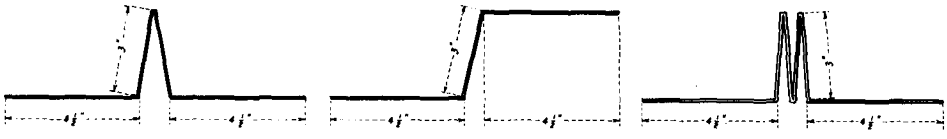
THREE-WAY FLEXIBLE CONNECTION FOR "Z" TYPE METAL SEALING STRIPS

CONNECTION FOR REVERSING "Z" TYPE METAL SEALING STRIPS



CONNECTION FOR VERTICAL AND HORIZONTAL "Z" TYPE METAL SEALING STRIPS

FOUR-WAY FLEXIBLE CONNECTION FOR "Z" TYPE METAL SEALING STRIPS



"M" TYPE
(STANDARD)
SECTION A-A

"Z" TYPE
(STANDARD)
SECTION B-B

"W" TYPE
SECTION C-C

NOTES
All metal sealing strips made of 20 gage material
All connections are to be brazed or welded

*Figure 8-4.* **Metal seals and connections at contraction joints.–288-D-3200**

Metal seals are similar in design whether used as water or grout seals. Bureau practice has standardized two shapes–the Z-type and the M-type. The Z-type seal is of simpler design, is easily installed and spliced, but will accommodate only small lateral movements. Such a seal is well adapted to joints which are to be grouted, since grouting tends to consolidate the two blocks and restrict any movement. The M-type seal is more difficult to splice, but its shape accommodates greater movement of the joint. This shape is well adapted for use as a water seal in ungrouted joints. Figure 8-4 shows the general dimensions and connections for the Z- and M-type seals.

Metal seals are made from a 12- or 15-inch strip of corrosion-resistant metal, usually copper or stainless steel. No. 20 gage United States Standard (0.0375-inch thick) stainless steel has proved satisfactory. The stainless steel is more rigid and will stay in position during embedment better than the more ductile copper. It is harder to weld, however, and is generally higher in initial cost. Copper strip can be furnished in rolls and will minimize the number of connections which have to be made.

(b) *Polyvinyl Chloride Seals.*–Recent advancements in the specifications for and manufacture of materials have resulted in the acceptance of polyvinyl chloride as a suitable material for joint seals. This material can be manufactured in a number of shapes and sizes. The 12-inch seal having a ½-inch thickness, serrations, and a center bulb is acceptable for high dams. The 9-inch similar seal is satisfactory for low dams.

(c) *Other Seals.*–Rubber seals have been used in special joints in concrete sections of dams and appurtenant works where it is desired to provide for greater movement at the joint than can be accommodated by metal seals. Rubber seals have been used successfully in contraction joints between piers and the cantilevers of drum gate crests, to permit unrestrained deflection of the cantilevers and prevent leakage from the reservoir into the drum gate chamber. They can also be used in expansion and contraction joints of thin cantilever walls in stilling basins to prevent objectionable leakage caused by unequal deflection and settlement of the walls. A similar use would be in ungrouted contraction joints of low diversion dams to prevent excessive leakage caused by differential settlement.

Asphalt seals have not proved satisfactory for sealing contraction joints in concrete dams, and they are no longer used.

8-8. ***Joint Drains.***–In arch dams, the contraction joints are normally grouted. And effective grouting, when the joints are open to their widest extent, will normally obviate any need for drainage. Since provision for open-joint drains makes effective grouting difficult, joint drains are usually omitted where contraction joint grouting is performed. However, in certain cases, a drainage system may be required and can be installed. The drains normally consist of a 5- or 6-inch-diameter formed drain constructed on the joint. These joint drains are then connected to tubing which carries the seepage water into the gallery system.

8-9. ***Grouting Systems.***–The purpose of contraction joint grouting is to bind the blocks together so that the structure will act as a monolithic mass. In order to make the individual blocks act as a monolith, a grout mixture of portland cement and water is forced into each joint under pressure. Upon setting, the mixture will form a cement mortar which fills the joint. The means of introducing grout into the joint is through an embedded pipe system. Typical pipe systems are shown on figures 8-1, 8-2, and 8-3.

In order to ensure complete grouting of a contraction joint before the grout begins to set, and to prevent excessive pressure on the seals, the joint is normally grouted in lifts 50 to 60 feet in height, although heights to about 75 feet have been used. Such a grouting lift in a transverse joint consists of an area bounded on the sides by seals adjacent to the upstream and downstream faces of the dam, and on the top and bottom by grout seals normally 50 to 60 feet apart. Since the longitudinal joints are staggered, the grouting area of a longitudinal joint is bounded by vertical seals placed close to the adjacent transverse joints and horizontal seals placed at 50- to 60-foot intervals in elevation. Each area of a transverse or longitudinal joint is sealed off from adjacent areas and has its own piping system independent of all other systems.

The layout of a piping system for transverse joints is illustrated on figure 8-2. A horizontal 1½-inch-diameter looped supply-header-return is embedded in the concrete adjacent to the lower boundary of the lift. One-half-inch-diameter embedded vertical risers take off from the header at approximately 6-foot intervals and terminate near the top of the lift or near the downstream face of the dam. Grout outlets are connected to the risers at 10-foot staggered intervals to give better coverage of the joint. The looped supply-header-return permits the delivery of grout to the various ½-inch riser pipes from either or both ends of the header, as may be desired, and provides reasonable assurance that grout will be admitted to all parts of the joint area. The top of each grout lift is vented to permit the escape of air, water, and thin grout which rises in the joint as grouting proceeds. A triangular grout groove is formed in the face of the high block and covered with a metal plate which serves as a form for the concrete when the adjacent low block is placed. Vent pipes are connected to each end of the groove, thereby providing venting in either direction which will allow venting to continue if an obstruction is formed at any one point in the groove.

The piping arrangement for longitudinal joints is illustrated on figure 8-3. A horizontal 1½-inch-diameter looped supply-header-return line from either the downstream face or the gallery system is embedded in the concrete adjacent to the lower boundary of the lift. The 1½-inch supply line conveys the grout to the piping at each longitudinal joint. At each side of the grouting lift, a 1-inch-diameter riser takes off from the header and extends nearly to the top of the lift. The return line aids in the release of entrapped air and water in the system, and may be used for grouting at the joint in the event the supply line becomes plugged. One-half-inch-diameter horizontal distribution pipes are connected between these risers spaced at 5 feet or 7 feet 6 inches, conforming to the height of the placement lifts. Grout outlets are attached to the horizontal distribution pipes at approximately staggered 10-foot intervals. As in the case of transverse joints, grout grooves or vent outlets are provided at the top of each lift and are connected to 1½-inch-diameter vent pipes which lead to the downstream face or to a gallery.

The location of the inlets and outlets of the supply-header-return and vents varies with conditions. Normally, these piping systems terminate at the downstream face of the dam. Under some conditions, these systems can be arranged to terminate in galleries. In order that the exposed ends of these systems will not be exposed after grouting operations have been completed, the pipes are terminated with a protruding pipe nipple which is wrapped with paper to prevent bonding to the concrete. This nipple is removed when no longer needed and the holes thus formed are dry-packed with mortar.

Typical grout outlets are shown on figure 8-5. The metal fitting alternative consists of two conduit boxes connected to the riser by a standard pipe tee. The blockout alternative is a blockout with a galvanized sheet steel cover. The riser goes through the blockout and a 2-inch section of the pipe is cut out. In erection, the box or blockout is placed in the high or first placed block and secured to the form. After the concrete has hardened and the forms have been removed, the cover box or sheet steel cover is placed in position and firmly held in place by wire or nails. A metal strap fastened to the cover serves as an anchor to fasten the cover to the second or low block so that the cover moves with it. When the two blocks contract upon cooling, the covers and the box or blockout are pulled apart and an opening equal to the joint opening is provided for grout injection.

The grout grooves, formed in contraction joints and used for venting air, water, and thin grout, are covered with metal cover plates which act as forms when the concrete is placed in the low block. Details of the installation of the metal cover plates and the grout groove are shown on figure 8-5. Before the cover plates are placed, the grooves are cleaned thoroughly

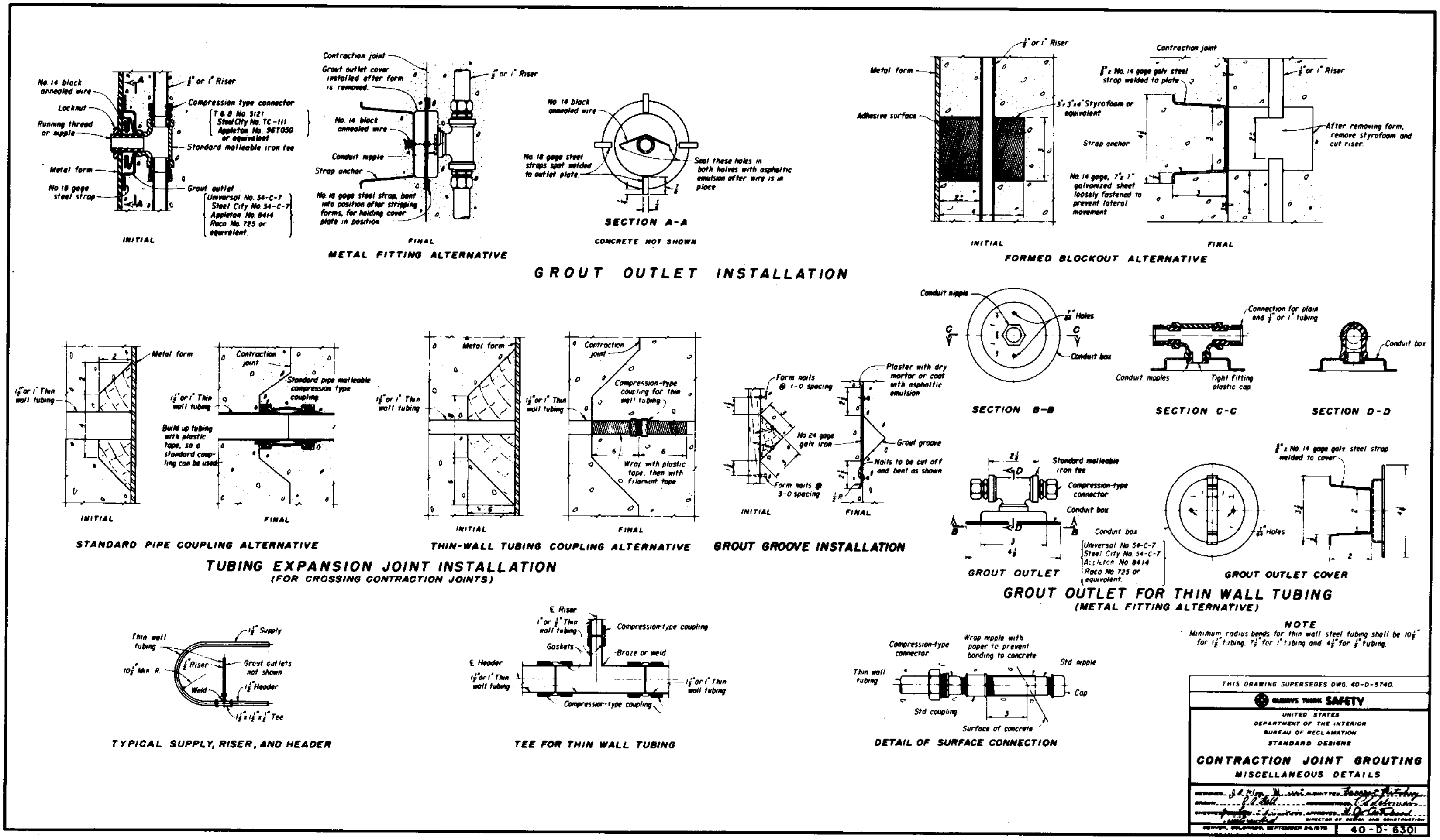


*Figure 8-5.* **Grouting system details.**

of all concrete, dirt, and other foreign substances. At the upper edges of the cover plates, the joint between the cover plate and the concrete is covered with dry cement mortar or with asphalt emulsion to prevent mortar from the concrete from plugging the groove.

8-10. ***Grouting Operations.***—Before any lift of a joint is grouted, the lift is washed thoroughly with air and water under pressure, the header and vent systems are tested to determine that they are unobstructed, and the joint is allowed to remain filled with water for a period of 24 hours. Immediately prior to being grouted, the water is drained from the joint lifts to be grouted. During the grouting operations, the lifts in two or more ungrouted adjacent joints at the same level are filled with water to the level of the top of the lift being grouted. As the grouting of the lift of the joint nears completion, the grouting lift of the joint immediately above the lift being grouted is filled with water. Immediately after a grouting operation is completed, the water is drained from the joints in the lift above, but the water is not drained from the adjacent ungrouted joint lifts at the same level until 6 hours after completion of the grouting operation.

The material used in grouting contraction joints is a mixture of cement and water, the consistency of which varies from thin to thick as the operation proceeds. Usually, a 2 to 1 mixture by volume of water and cement is used at the start of the grouting operation to assure grout travel and the filling of small cracks. As the grouting proceeds the mixture is thickened to a 1 to 1 water-cement ratio to fill the grout system and joint. If the joint is wide and accepts grout readily, grout of 0.7 or 0.8 water-cement ratio by volume may be used to finish the operation. Normally, the supply line from the grout pump is connected to the supply so that grout first enters the joint through outlets in the most remote riser pipe, thereby setting up conditions most favorable for the expulsion of air, water, and diluted grout as the grouting operations proceed. If the grout introduced in the normal way makes a ready appearance at the return, the indications are that the header system is unobstructed and the return header can be capped. Grout from the header is forced up the risers and into the joint through the grout outlets, while air and water is forced up to the vent groove above.

Grouting of contraction joints in a dam is normally done in groups and in separate successive lifts, beginning at the foundation and finishing at the top of the dam. The grout is applied in rotation from joint to joint by batches in such quantities and with such time delays as necessary to allow the grout to settle in the joint. Each joint is filled at approximately the same rate. The grouting of each joint lift is completed before the grout takes its set in the grouting system, but the lift is not grouted so rapidly that the grout will not settle in the joint. In no case is the time consumed in filling any lift of a joint less than 2 hours.

When thick grout flows from the vent outlets, injection is stopped for awhile to allow the grout to settle. After several repetitions of a showing of thick grout, the valves on the outlets are closed. The pressure on the supply line is then increased to the allowable limit for the particular joint to force grout into all small openings of the joint and to force the excess water into the pores of the concrete, leaving a grout film of lower water-cement ratio and higher density in the joint. The limiting pressure, usually from 30 to 50 pounds per square inch as measured at the vent, must be low enough to avoid deflecting the block excessively or causing opening of the grouted portion of the joint below. This maximum pressure is maintained until no more grout can be forced into the joint, and the system is then sealed off.

# <<Chapter IX

# Spillways

## A. GENERAL DESIGN CONSIDERATIONS

9-1. ***Function***.—Spillways are provided at storage and detention dams to release surplus or floodwater which cannot be contained in the allotted storage space, and at diversion dams to bypass flows exceeding those which are turned into the diversion system. Ordinarily, the excess is drawn from the top of the pool created by the dam and released through a spillway back to the river or to some natural drainage channel. Figure 9-1 shows the spillway at Morrow Point Dam in operation.

The importance of a safe spillway cannot be overemphasized; many failures of dams have been caused by improperly designed spillways or by spillways of insufficient capacity. However, concrete dams usually will be able to withstand moderate overtopping. Generally, the increase in cost of a larger spillway is not directly proportional to increase in capacity. Very often the cost of a spillway of ample capacity will be only moderately higher than that of one which is obviously too small.

In addition to providing sufficient capacity, the spillway must be hydraulically and structurally adequate and must be located so that spillway discharges will not erode or undermine the downstream toe or abutments of the dam. The spillway's flow surfaces must be erosion resistant to withstand the high scouring velocities created by the drop from the reservoir surface to tailwater, and usually some device will be required for dissipation of energy at the bottom of the drop.

The frequency of spillway use will be determined by the runoff characteristics of the drainage area and by the nature of the development. Ordinary riverflows are usually stored in the reservoir, used for power generation, diverted through headworks, or released through outlets, and the spillway is not required to function. Spillway flows will result during floods or periods of sustained high runoff when the capacities of other facilities are exceeded. Where large reservoir storage is provided, or where large outlet or diversion capacity is available, the spillway will be utilized infrequently. Where storage space is limited and outlet releases or diversions are relatively small compared to normal riverflows, the spillway will be used frequently.

9-2. ***Selection of Inflow Design Flood***.—(a) *General Considerations*.—When floods occur in an unobstructed stream channel, it is considered a natural event for which no individual or group assumes responsibility. However, when obstructions are placed across the channel, it becomes the responsibility of the sponsors either to make certain that hazards to downstream interests are not appreciably increased or to obligate themselves for damages resulting from operation or failure of such structures. Also, the loss of the facility and the loss of project revenue occasioned by a failure should be considered.

If danger to the structures alone were involved, the sponsors of many projects would prefer to rely on the improbability of an extreme flood occurrence rather than to incur the expense necessary to assure complete safety. However, when the risks involve downstream interests, including widespread

*Figure 9-1.* **Free-fall orifice-type spillway in operation at Morrow Point Dam in Colorado.–P622B-427-8886M**

damage and loss of life, a conservative attitude is required in the development of the inflow design flood. Consideration of potential damage should not be confined to conditions existing at the time of construction. Probable future development in the downstream flood plain, encroachment by farms and resorts, construction of roads and bridges, etc., should be evaluated in estimating damages and hazards to human life that would result from failure of a dam.

Dams impounding large reservoirs and built on principal rivers with high runoff potential unquestionably can be considered to be in the high-hazard category. For such developments, conservative design criteria are selected on the basis that failure cannot be tolerated because of the possible loss of life and because of the potential damages which could approach disaster proportions. However, dams built on isolated streams in rural areas where failure would neither jeopardize human life nor create damages beyond the sponsor's financial capabilities can be considered to be in a low-hazard category. For such developments design criteria may be established on a much less conservative basis. There are numerous instances, however, where failure of dams of low heights and small storage capacities have resulted in loss of life and heavy property damage. Most dams will require a reasonable conservatism in design, primarily because of the criterion that a dam failure must not present a serious hazard to human life.

(b) *Inflow Design Flood Hydrograph.*–Concrete dams are usually built on rivers from major drainage systems and impound large reservoirs. Because of the magnitude of the damage which would result from a failure of the dam, the probable maximum flood is used as the inflow design flood. The hydrograph for this flood is based on the hydrometeorological approach, which requires estimates of storm potential and the amount and distribution of runoff. The derivation of the probable maximum flood is discussed in appendix L.

The probable maximum flood is based on a rational consideration of the chances of simultaneous occurrence of the maximum of the several elements or conditions which contribute to the flood. Such a flood is the largest that reasonably can be expected and is ordinarily accepted as the inflow design flood for dams where failure of the structure would increase the danger to human life. The inflow design flood is determined by evaluating the hydrographs of the following situations to ascertain the most critical flood:

(1) A probable maximum rainstorm in conjunction with a severe, but not uncommon, antecedent condition.

(2) A probable maximum rainstorm in conjunction with a major snowmelt flood somewhat smaller than the probable maximum.

(3) A probable maximum snowmelt flood in conjunction with a major rainstorm less severe than the probable maximum for that season.

9-3. ***Relation of Surcharge Storage to Spillway Capacity.***–The inflow design flood is normally represented in the form of a hydrograph, which charts the rate of flow in relation to time. A typical hydrograph representing a storm runoff is illustrated in figure 9-2, curve A. The flow into a reservoir at

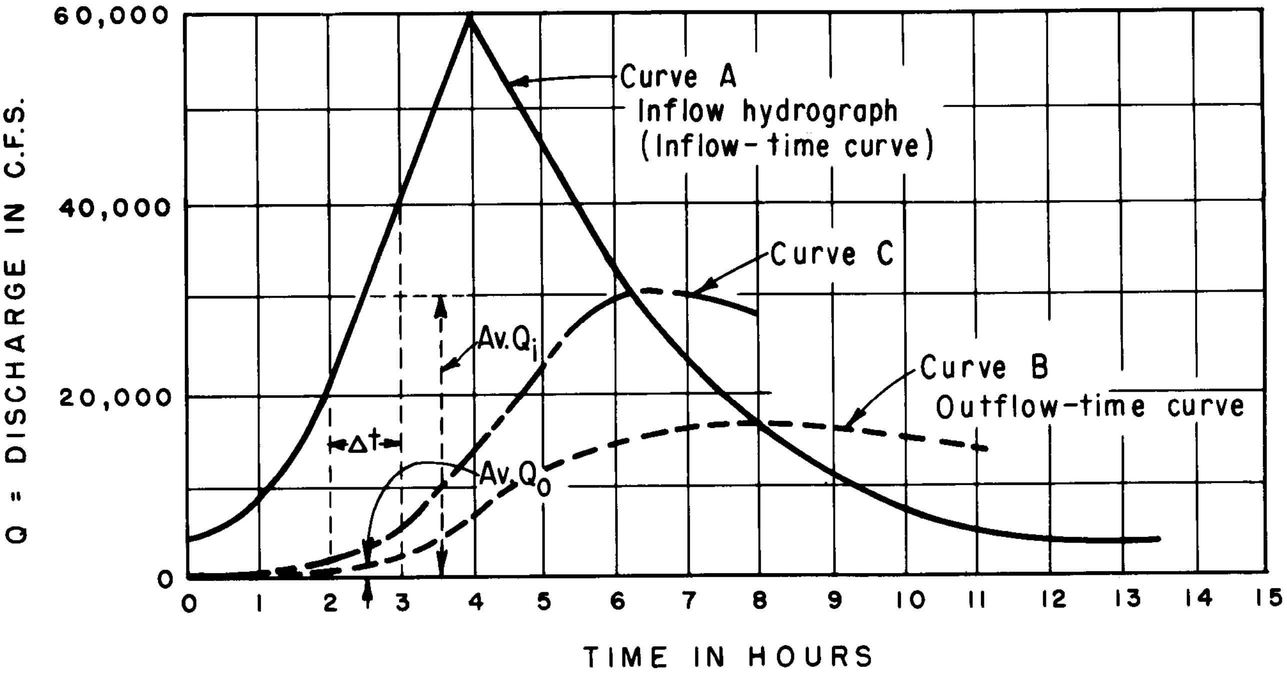


*Figure 9-2.* **Typical inflow and outflow hydrographs.–288-D-3035**

any time and the momentary peak can be read from this curve. The area under the curve is the volume of the inflow, since it represents the product of rate of flow and time.

Where no surcharge storage is allowed in the reservoir, the spillway capacity must be sufficiently large to pass the peak of the flood. The peak rate of inflow is then of primary interest and the total volume in the flood is of lesser importance. However, where a relatively large storage capacity above normal reservoir level can be made available economically by constructing a higher dam, a portion of the flood volume can be retained temporarily in reservoir surcharge space and the spillway capacity can be reduced considerably.

In many projects involving reservoirs, economic considerations will necessitate a design utilizing surcharge. The most economical combination of surcharge storage and spillway capacity requires flood routing studies and economic studies of the costs of spillway-dam combinations, subsequently described.

9-4. *Flood Routing.*—The storage accumulated in a reservoir depends on the difference between the rates of inflow and outflow. For an interval of time $\Delta t$, this relationship can be expressed by the equation:

$$\Delta S = Q_i\Delta t - Q_o\Delta t \qquad (1)$$

where:

$\Delta S$ = storage accumulated during $\Delta t$,
$Q_i$ = average rate of inflow during $\Delta t$, and
$Q_o$ = average rate of outflow during $\Delta t$.

Referring to figure 9-2, the rate of inflow at any time, $t$, is shown by the inflow design flood hydrograph; the rate of outflow may be obtained from the curve of spillway discharge versus reservoir water surface elevation; and storage is shown by the curve of reservoir capacity versus reservoir water surface elevation.

The quantity of water a spillway can discharge depends on the size and type of spillway. For a simple overflow crest the flow will vary with the head on the crest, and the surcharge will increase with an increase in spillway discharge. For a gated spillway, however, outflow can be varied with respect to reservoir head by operation of the gates. For example, one assumption for an operation of a gate-controlled spillway might be that the gates will be regulated so that inflow and outflow are equal until the gates are wide open; or an assumption can be made to open the gates at a slower rate so that surcharge storage will accumulate before the gates are wide open.

Outflows need not necessarily be limited to discharges through the spillway but might be supplemented by other releases such as through river outlets, irrigation outlets, and powerplant turbines. In all such cases the size, type, and method of operation of the spillway and other releases with reference to the storage and/or to the inflow must be predetermined in order to establish an outflow-elevation relationship.

If simple equations could be established for the inflow design flood hydrograph curve, the outflow (as may be modified by operational procedures), and the reservoir capacity curve, a solution of flood routing could be made by mathematical integration. However, simple equations usually cannot be written for these variables, and such a solution is not practical. Many techniques of flood routing have been devised, each with its advantages and disadvantages. These techniques vary from a strictly arithmetical method to an entirely graphical solution.

Electronic computers are being used to make flood routing computations. The computer programs were developed using an iteration technique. For simplicity, an arithmetical trial and error tabular method is illustrated in this manual. Data required for the routing, which is the same regardless of the method used, are as follows:

(1) Inflow hydrograph, figure 9-2.

(2) Reservoir capacity, figure 9-3.

(3) Outflow, figure 9-4. (Spillway discharge only was assumed in this illustration.)

The flood routing computations are shown in table 9-1. The procedure for making the computations is as follows:

(1) Select a time interval, $\Delta t$, column (2).

(2) Obtain column (3) from the inflow hydrograph, figure 9-2.

(3) Column (4) represents average inflow for $\Delta t$ in c.f.s. (cubic feet per second).

(4) Obtain column (5) by converting column (4) values of c.f.s. for $\Delta t$ to acre-feet (1 c.f.s. for 12 hours = 1 acre-foot).

(5) Assume trial reservoir water surface in column (6), determine the

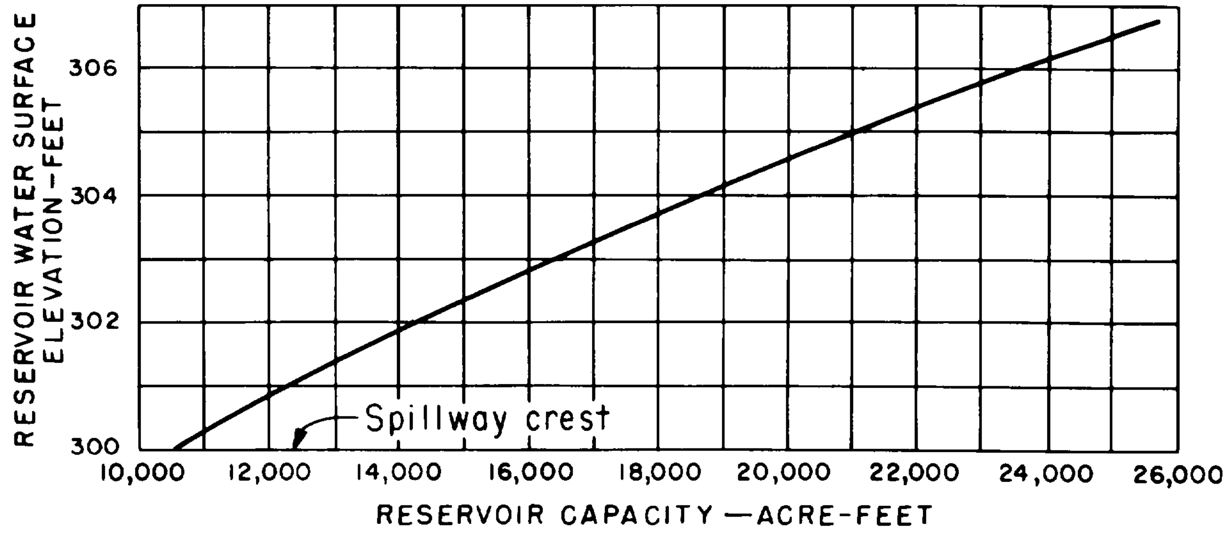


*Figure 9-3.* Typical reservoir capacity curve.—288-D-3036

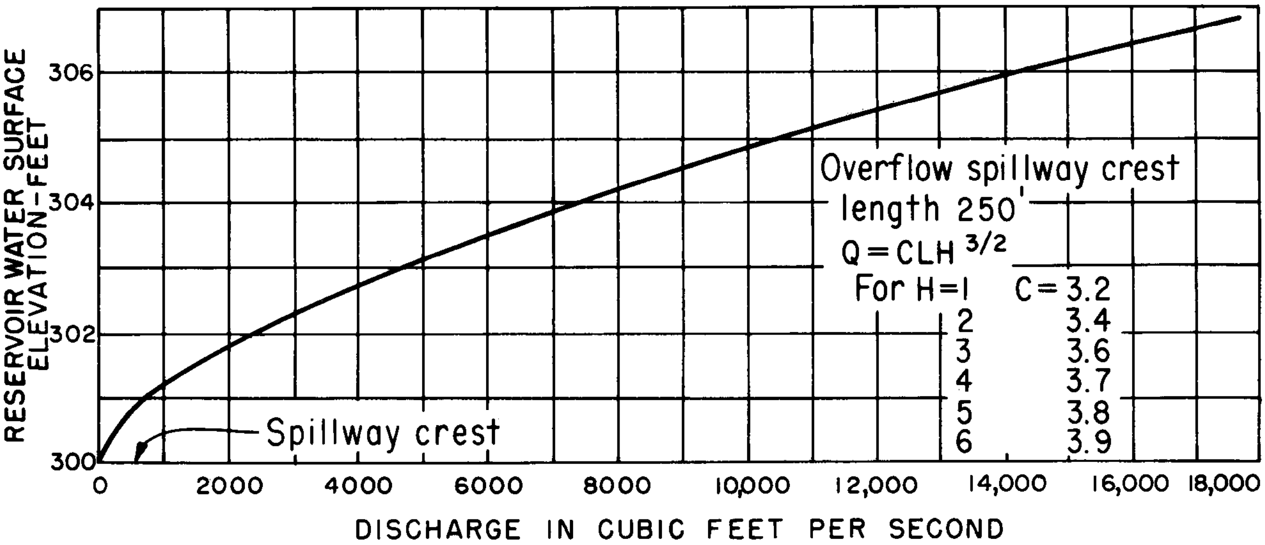


*Figure 9-4.* Typical spillway discharge curve.–288-D-3037

Table 9-1.–*Flood routing computations.*

| (1) Time t, hours | (2) Δt, hours | (3) Inflow at time, t, c.f.s. | (4) Average rate of inflow $Q_i$ for Δt, c.f.s. | (5) Inflow, acre-feet | (6) Trial reservoir storage elevation at time t, | (7) Outflow at time t, c.f.s. | (8) Average rate of outflow $Q_0$ for Δt, c.f.s. | (9) Outflow acre-feet | (10) Incremental storage Δs, acre-feet | (11) Total storage, acre-feet | (12) Reservoir elevation, end of Δt, feet | (13) Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 4,000 | | | | 0 | | | | 10,500 | 300.0 | |
| | 1 | | 6,000 | 500 | ~~300.2~~ | ~~50~~ | ~~25~~ | ~~2~~ | ~~498~~ | ~~10,998~~ | ~~300.3~~ | ~~HIGH~~ |
| 1 | | 8,000 | | | 300.3 | 100 | 50 | 4 | 496 | 10,996 | 300.3 | OK |
| | 1 | | 14,000 | 1,167 | ~~300.8~~ | ~~540~~ | ~~320~~ | ~~27~~ | ~~1,140~~ | ~~12,136~~ | ~~301.0~~ | ~~HIGH~~ |
| 2 | | 20,000 | | | 301.0 | 800 | 450 | 38 | 1,129 | 12,125 | 301.0 | OK |
| | 1 | | 30,000 | 2,500 | ~~302.3~~ | ~~3,000~~ | ~~1,900~~ | ~~158~~ | ~~2,342~~ | ~~14,467~~ | ~~302.1~~ | ~~LOW~~ |
| 3 | | 40,000 | | | 302.1 | 2,600 | 1,700 | 142 | 2,358 | 14,483 | 302.1 | OK |
| | 1 | | 50,000 | 4,167 | ~~303.9~~ | ~~7,100~~ | ~~4,850~~ | ~~404~~ | ~~3,763~~ | ~~18,246~~ | ~~303.8~~ | ~~LOW~~ |
| 4 | | 60,000 | | | 303.8 | 6,900 | 4,750 | 396 | 3,771 | 18,254 | 303.8 | OK |
| | 1 | | 53,500 | 4,458 | ~~305.0~~ | ~~10,600~~ | ~~8,750~~ | ~~729~~ | ~~3,729~~ | ~~21,983~~ | ~~305.3~~ | ~~HIGH~~ |
| 5 | | 47,000 | | | 305.3 | 11,600 | 9,250 | 771 | 3,687 | 21,941 | 305.3 | OK |
| | 1 | | 40,000 | 3,333 | ~~306.3~~ | ~~15,300~~ | ~~13,450~~ | ~~1121~~ | ~~2,212~~ | ~~24,153~~ | ~~306.2~~ | ~~LOW~~ |
| 6 | | 33,000 | | | 306.2 | 15,100 | 13,350 | 1113 | 2,220 | 24,161 | 306.2 | OK |
| | 1 | | 28,500 | 2,375 | 306.6 | 16,800 | 15,950 | 1329 | 1,046 | 25,207 | 306.6 | OK |
| 7 | | 24,000 | | | | | | | | | | |
| | 1 | | 20,000 | 1,667 | 306.7 | 17,200 | 17,000 | 1417 | 250 | 25,457 | 306.7 | OK |
| 8 | | 16,000 | | | | | | | | | | |
| | 1 | | 13,500 | 1,125 | 306.6 | 16,800 | 17,000 | 1417 | -292 | 25,165 | 306.6 | OK |
| 9 | | 11,000 | | | | | | | | | | |
| | 2 | | 8,000 | 1,333 | ~~306.0~~ | ~~14,300~~ | ~~15,550~~ | ~~2,592~~ | ~~-1259~~ | ~~23,906~~ | ~~306.1~~ | ~~HIGH~~ |
| 11 | | 5,000 | | | 306.1 | 14,700 | 15,750 | 2,625 | -1292 | 23,873 | 306.1 | OK |

corresponding rate of outflow from figure 9-4, and record in column (7).

(6) Average the rate of outflow determined in step (5) and the rate of outflow for the reservoir water surface which existed at the beginning of the period and enter in column (8).

(7) Obtain column (9) by converting column (8) values of c.f.s. for $\Delta t$ to acre-feet, similar to step (4).

(8) Column (10) = column (5) minus column (9).

(9) The initial value in column (11) represents the reservoir storage at the beginning of the inflow design flood. Determine subsequent values by adding

$\Delta S$ values from column (10) to the previous column (11) value.

(10) Determine reservoir elevation in column (12) corresponding to storage in column (11) from figure 9-3.

(11) Compare reservoir elevation in column (12) with trial reservoir elevation in column (6). If they do not agree within 0.1 foot, make a second trial elevation and repeat procedure until agreement is reached.

The outflow-time curve resulting from the flood routing shown in table 9-1 has been plotted as curve B on figure 9-2. As the area under the inflow hydrograph (curve A) indicates the volume of inflow, so will the area under the outflow hydrograph (curve B) indicate the volume of outflow. It follows then that the volume indicated by the area between the two curves will be the surcharge storage. The surcharge storage computed in table 9-1 can, therefore, be checked by comparing it with the measured area on the graph.

A rough approximation of the relationship of spillway size to surcharge volume can be obtained without making an actual flood routing, by arbitrarily assuming an approximate outflow-time curve and then measuring the area between it and the inflow hydrograph. For example, if the surcharge volume for the problem shown on figure 9-2 is sought where a 30,000-c.f.s. spillway would be provided, an assumed outflow curve represented by curve C can be drawn and the area between this curve and curve A can be planimetered. Curve C will reach its apex of 30,000 c.f.s. where it crosses curve A. The volume represented by the area between the two curves will indicate the approximate surcharge volume necessary for this capacity spillway.

9-5. ***Selection of Spillway Size and Type.***—(a) *General Considerations.*—In determining the best combination of storage and spillway capacity to accommodate the selected inflow design flood, all pertinent factors of hydrology, hydraulics, geology, topography, design requirements, cost, and benefits should be considered. These considerations involve such factors as (1) the characteristics of the flood hydrograph; (2) the damages which would result if such a flood occurred without the dam; (3) the damages which would result if such a flood occurred with the dam in place; (4) the damages which would occur if the dam or spillway should fail; (5) effects of various dam and spillway combinations on the probable increase or decrease of damages above or below the dam (as indicated by reservoir backwater curves and tailwater curves); (6) relative costs of increasing the capacity of spillways; and (7) use of combined outlet facilities to serve more than one function, such as control of releases and control or passage of floods. Other outlets, such as river outlets, irrigation outlets, and powerplant turbines, should be considered in passing part of the inflow design flood when such facilities are expected to be available in time of flood.

The outflow characteristics of a spillway depend on the particular device selected to control the discharge. These control facilities may take the form of an overflow crest or orifice. Such devices can be unregulated or they can be equipped with gates or valves to regulate the outflow.

After the overflow characteristics have been selected, the maximum spillway discharge and the maximum reservoir water level can be determined by flood routing. Other components of the spillway can then be proportioned to conform to the required capacity and to the specific site conditions, and a complete layout of the spillway can be established. Cost estimates of the spillway and dam can then be made. Estimates of various combinations of spillway capacity and dam height for an assumed spillway type, and of alternative types of spillways, will provide a basis for selection of the economical spillway type and the optimum relation of spillway capacity to height of dam. Figures 9-5 and 9-6 illustrate the results of such a study. The relationships of spillway capacities to maximum reservoir water surfaces obtained from the flood routings is shown on figure 9-5 for two spillways. Figure 9-6 illustrates the comparative costs for different combinations of spillway and dam, and indicates a

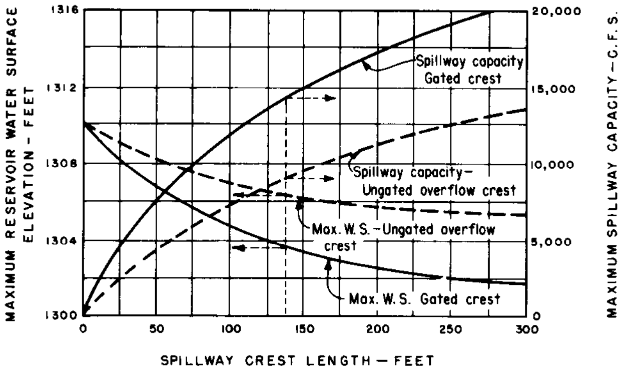


*Figure 9-5.* **Spillway capacity–surcharge relationship. –288-D-3039**

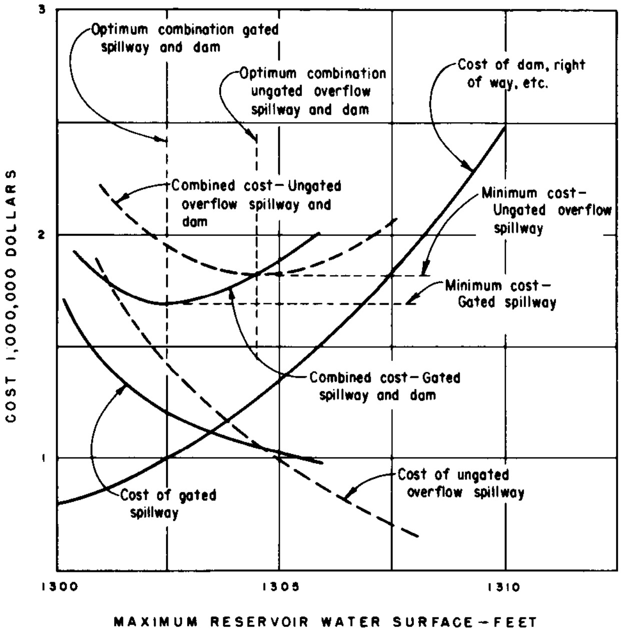


*Figure 9-6.* **Comparative cost of spillway-dam combinations.–288-D-3040**

combination which results in the least total cost.

To make such a study as illustrated requires many flood routings, spillway layouts, and spillway and dam estimates. Even then, the study is not necessarily complete since many other spillway arrangements could be considered. A comprehensive study to determine alternative optimum combinations and minimum costs may not be warranted for the design of some dams. Judgment on the part of the designer would be required to select for study only the combinations which show definite advantages, either in cost or adaptability. For example, although a gated spillway might be slightly cheaper overall than an ungated spillway, it may be desirable to adopt the latter because of its less complicated construction, its automatic and trouble-free operation, its ability to function without an attendant, and its less costly maintenance.

(b) *Combined Service and Auxiliary Spillways.*—Where site conditions are favorable, the possibility of gaining overall economy by utilizing an auxiliary spillway in conjunction with a smaller service-type structure should be considered. In such cases the service spillway should be designed to pass floods likely to occur frequently and the auxiliary spillway control set to operate only after such small floods are exceeded. In certain instances the outlet works may be made large enough to serve also as a service spillway. Conditions favorable for the adoption of an auxiliary spillway are the existence of a saddle or depression along the rim of the reservoir which leads into a natural waterway, or a gently sloping abutment where an excavated channel can be carried sufficiently beyond the dam to avoid the possibility of damage to the dam or other structures.

Because of the infrequency of use, it is not necessary to design the entire auxiliary spillway for the same degree of safety as required for other structures; however, at least the control portion must be designed to forestall failure, since its breaching would release large flows from the reservoir. For example, concrete lining may be omitted from an auxiliary spillway channel excavated in rock which is not easily eroded. Where the channel is excavated through less competent material, it might be lined but terminated above the river channel with a cantilevered lip rather than extending to a stilling basin at river level. The design of auxiliary spillways is often based on the premise that some damage to portions of the structure from passage of infrequent flows is permissible. Minor damage by scour to an unlined channel, by erosion and undermining at the downstream end of the channel, and by creation of an erosion pool downstream from the spillway might be tolerated.

An auxiliary spillway can be designed with a fixed crest control, or it can be stoplogged or gated to increase the capacity without additional surcharge head.

## B. DESCRIPTION OF SPILLWAYS

9-6. ***Selection of Spillway Layout.***—The design of a spillway, including all of its components, can be prepared by properly considering the various factors influencing the spillway size and type, and correlating alternatively selected components. Many combinations of components can be used in forming a complete spillway layout. After the hydraulic size and outflow characteristics of a spillway are determined by routing of the design flood, the general dimensions of the control can be selected. Then, a specific spillway layout can be developed by considering the topography and foundation conditions, and by fitting the control structure and the various components to the prevailing conditions.

Site conditions greatly influence the selection of location, type, and components of a spillway. Factors that must be considered in the selection are the possibility of incorporating the spillway into the dam, the steepness of the terrain that would be traversed by a chute-type spillway, the amount of excavation required and the difficulty of its disposal, the chances of scour of the flow surfaces and the need for lining the spillway channel, the permeability and bearing capacity of the foundation, the stability of the excavated slopes, and the possible use of a tunnel-type spillway.

The adoption of a particular size or arrangement for one of the spillway components may influence the selection of other components. For example, a wide control structure with the crest placed normal to the centerline of the spillway would require a long converging transition to join it to a narrow discharge channel or to a tunnel; a better alternative might be the selection of a narrower gated control structure or a side channel control arrangement. Similarly, a wide stilling basin may not be feasible for use with a cut-and-cover conduit or tunnel, because of the long, diverging transition needed.

A spillway may be an integral part of a dam such as an overflow section of a concrete dam, or it may be a separate structure. In some instances, it may be integrated into the river diversion plan for economy. Thus, the location, type, and size of other appurtenances are factors which may influence the selection of a spillway location or its arrangement. The final plan will be governed by overall economy, hydraulic sufficiency, and structural adequacy.

The components of a spillway and common types of spillways are described and discussed herein. Hydraulic design criteria and procedures are discussed in sections 9-10 through 9-29.

9-7. ***Spillway Components.***—(a) *Control Structure.*—A major component of a spillway is the control device, since it regulates and controls the outflows from the reservoir. This control limits or prevents outflows below fixed reservoir levels, and it also regulates releases when the reservoir rises above these levels. The control structure is usually located at the upstream end of the spillway and consists of some form of overflow crest or orifice. Sometimes the configuration of the spillway downstream from the control structure is such that with higher discharges the structure no longer controls the flow. For example, with the morning glory spillway, shown on figure 9-44, the tunnel rather than the crest or orifice usually controls the flow at higher discharges (see sec. 9-25).

Control structures may take various forms in both positioning and shape. In plan, overflow crests can be straight, curved, semicircular, U-shaped, or circular. Figure 9-7 shows the circular crest for the morning glory spillway at Hungry Horse Dam. Orifice controls can be

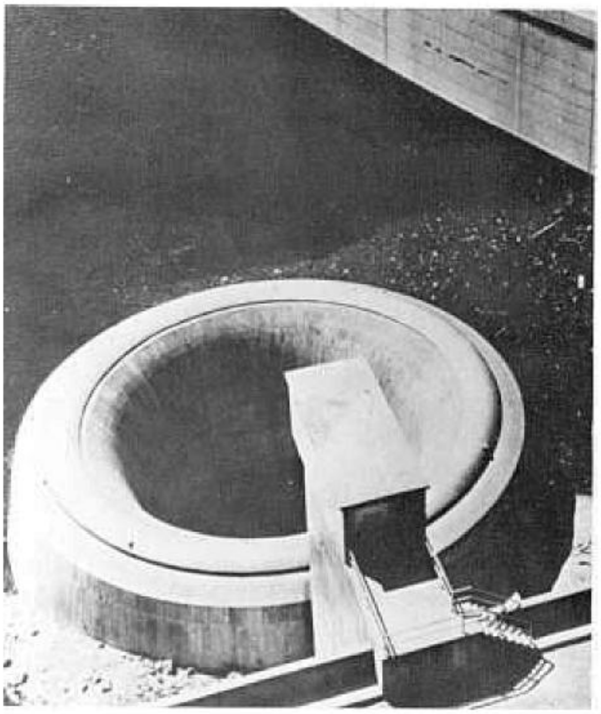


***Figure 9-7.*** **Circular crest for morning glory spillway at Hungry Horse Dam in Montana.–P447-105-5587**

placed in a horizontal, inclined, or vertical position. The orifice can be circular, square, rectangular, triangular, or varied in shape.

An overflow can be sharp crested, ogee shaped, broad crested, or of varied cross section. Orifices can be sharp edged, round edged, or bellmouth shaped, and can be placed so as to discharge with a fully contracted jet or with a suppressed jet. They may discharge freely or discharge partly or fully submerged.

(b) *Discharge Channel.*–Flow released through the control structure usually is conveyed to the streambed below the dam in a discharge channel or waterway. Exceptions are where the discharge falls free from an arch dam crest or where the flow is released directly along the abutment hillside to cascade down the abutment face. The conveyance structure may be the downstream face of a concrete dam, an open channel excavated along the ground surface on one abutment, or a tunnel excavated through an abutment. The profile may be variably flat or steep; the cross section may be variably rectangular, trapezoidal, circular, or of other shape; and the discharge channel may be wide or narrow, long or short.

Discharge channel dimensions are governed primarily by hydraulic requirements, but the selection of profile, cross-sectional shape, width, length, etc., is influenced by the geologic and topographic characteristics of the site. Open channels excavated in the abutment usually follow the ground surface profile; steep canyon walls usually make a tunnel desirable. In plan, open channels may be straight or curved, with sides parallel, convergent, divergent, or a combination of these. Discharge channels must be cut through or lined with material which is resistant to the scouring action of the high velocities, and which is structurally adequate to withstand the forces from backfill, uplift, waterloads, etc.

(c) *Terminal Structure.*–When water flows in a spillway from reservoir pool level to downstream river level, the static head is converted to kinetic energy. This energy manifests itself in the form of high velocities which if impeded result in large pressures. Means of returning the flow to the river without serious scour or erosion of the toe of the dam or damage to adjacent structures must usually be provided.

In some cases the discharge may be delivered at high velocities directly to the stream where the energy is absorbed along the streambed by impact, turbulence, and friction. Such an arrangement is satisfactory where erosion-resistant bedrock exists at shallow depths in the channel and along the abutments or where the spillway outlet is sufficiently removed from the dam or other appurtenances to avoid damage by scour, undermining, or abutment sloughing. The discharge channel may be terminated well above the streambed level or it may be continued to or below streambed.

Upturned deflectors, cantilevered extensions, or flip buckets can be provided to project the jet some distance downstream from the end of the structure. Often, erosion of the streambed in the area of impact of the jet can be minimized by fanning the jet into a thin sheet by the use of a flaring deflector.

Where severe scour at the point of jet

impingement is anticipated, a plunge pool can be excavated in the river channel and the sides and bottom lined with riprap or concrete. It may be expedient to perform a minimum of excavation and to permit the flow to erode a natural pool; protective riprapping or concrete lining may be later provided to halt the scour if necessary. In such arrangements an adequate cutoff or other protection must be provided at the end of the spillway structure to prevent it from being undermined.

Where serious erosion to the streambed is to be avoided, the high energy of the flow must be dissipated before the discharge is returned to the stream channel. This can be accomplished by the use of an energy dissipating device, such as a hydraulic jump basin, a roller bucket, an apron, a basin incorporating impact baffles and walls, or some similar energy absorber or dissipator.

(d) *Entrance and Outlet Channels.*– Entrance channels serve to draw water from the reservoir and convey it to the control structure. Where a spillway draws water immediately from the reservoir and delivers it directly back into the river, as in the case with an overflow spillway over a concrete dam, entrance and outlet channels are not required. However, in the case of spillways placed through abutments or through saddles or ridges, channels leading to the spillway control and away from the spillway terminal structure may be required.

Entrance velocities should be limited and channel curvatures and transitions should be made gradual, in order to minimize head loss through the channel (which has the effect of reducing the spillway discharge) and to obtain uniformity of flow over the spillway crest. Effects of an uneven distribution of flow in the entrance channel might persist through the spillway structure to the extent that undesirable erosion could result in the downstream river channel. Nonuniformity of head on the crest may also result in a reduction in the discharge.

The approach velocity and depth below crest level each have important influences on the discharge over an overflow crest. As discussed in section 9-11(b), a greater approach depth with the accompanying reduction in approach velocity will result in a larger discharge coefficient. Thus, for a given head over the crest, a deeper approach will permit a shorter crest length for a given discharge. Within the limits required to secure satisfactory flow conditions and nonscouring velocities, the determination of the relationship of entrance channel depth to channel width is a matter of economics. When the spillway entrance channel is excavated in material that will be eroded by the approach velocity, a zone of riprap is often provided immediately upstream from the inlet lining to prevent scour of the channel floor and side slope adjacent to the spillway concrete.

Outlet channels convey the spillway flow from the terminal structure to the river channel below the dam. An outlet channel should be excavated to an adequate size to pass the anticipated flow without forming a control which will affect the tailwater stage in the stilling device.

The outlet channel dimensions and its need for protection by lining or riprap will depend on the nature of the material through which the channel is excavated and its susceptibility to scouring. Although stilling devices are provided, it may be impossible to reduce resultant velocities below the natural velocity in the original stream; and some scouring of the riverbed, therefore, may not be avoidable. Further, under natural conditions the beds of many streams are scoured during the rising stage of a flood and filled during the falling stage by deposition of material carried by the flow. After creation of a reservoir the spillway will normally discharge clear water and the material scoured by the high velocities will not be replaced by deposition. Consequently, there will be a gradual retrogression of the downstream riverbed, which will lower the tailwater stage-discharge relationship. Conversely, scouring where only a pilot channel is provided may build up bars and islands downstream, thereby effecting an aggradation of the downstream river channel which will raise the tailwater elevation with respect to discharges. The dimensions and erosion-protective measures at the outlet channel may be influenced by these considerations.

9-8. ***Spillway Types.***—Spillways are ordinarily classified according to their most distinguishing feature, either as it pertains to the control, to the discharge channel, or to some other component. Spillways often are referred to as controlled or uncontrolled, depending on whether they are gated or ungated. Common types are the free fall, ogee (overflow), side channel, chute or open channel, tunnel, and morning glory spillways.

(a) *Free Fall Spillways.*—A free fall spillway is one in which the flow drops freely, usually into the streambed. Flows may be free discharging, as with a sharp-crested weir or orifice control, or they may be supported part way down the face of the dam and then trajected away from the dam by a flip bucket.

Where no artificial protection is provided at the base, scour will occur in some streambeds and will form a deep plunge pool. The volume and depth of the hole are related to the range of discharges, the height of the drop, and the depth of tailwater. The erosion-resistant properties of the streambed material including bedrock have little influence on the size of the hole, the only effect being the time necessary to scour the hole to its full depth. Probable depths of scour are discussed in section 9-24. Where erosion cannot be tolerated, a plunge pool can be created by constructing an auxiliary dam downstream from the main structure, or by excavating a basin which is then provided with a concrete apron or bucket.

If tailwater depths are sufficient, a hydraulic jump will form when a free fall jet falls upon a flat apron. It has been demonstrated that the momentum equation for the hydraulic jump may be applied to the flow conditions at the base of the fall to determine the elements of the jump.

(b) *Ogee (Overflow) Spillways.*—The ogee spillway has a control weir which is ogee- or S-shaped in profile. The upper curve of the ogee ordinarily is made to conform closely to the profile of the lower nappe of a ventilated sheet of water falling from a sharp-crested weir. Flow over the crest is made to adhere to the face of the profile by preventing access of air to the underside of the sheet. For discharges at designed head, the flow glides over the crest with minimum interference from the boundary surface and attains near-maximum discharge efficiency. The profile below or downstream of the upper curve of the ogee is continued tangent along a slope to support the flowing sheet on the face of the weir. A reverse curve at the bottom of the slope turns the flow onto the apron of a stilling basin, into a flip bucket, or into the spillway discharge channel.

The upper curve at the crest may be made either broader or sharper than the nappe profile. A broader shape will support the sheet and positive hydrostatic pressure will occur along the contact surface. The supported sheet thus creates a backwater effect and reduces the efficiency of discharge. For a sharper shape, the sheet tends to pull away from the crest and to produce subatmospheric pressure along the contact surface. This negative pressure effect increases the effective head, and thereby increases the discharge.

An ogee crest and apron may comprise an entire spillway, such as the overflow portion of a concrete gravity dam, or the ogee crest may be only the control structure for some other type of spillway. Because of its high discharge efficiency, the nappe-shaped profile is used for most spillway control crests.

(c) *Side Channel Spillways.*—The side channel spillway is one in which the control weir is placed along the side of and approximately parallel to the upper portion of the spillway discharge channel. Flow over the crest falls into a narrow trough behind the weir, turns an approximate right angle, and then continues into the main discharge channel. The side channel design is concerned only with the hydraulic action in the upstream reach of the discharge channel and is more or less independent of the details selected for the other spillway components. Flows from the side channel can be directed into an open discharge channel or into a closed conduit or inclined tunnel. Flow into the side channel might enter on only one side of the trough in the case of a steep hillside location, or on both sides and over the end of the trough if it is located on a knoll or gently sloping abutment. Figure 9-8 shows the Arizona spillway at

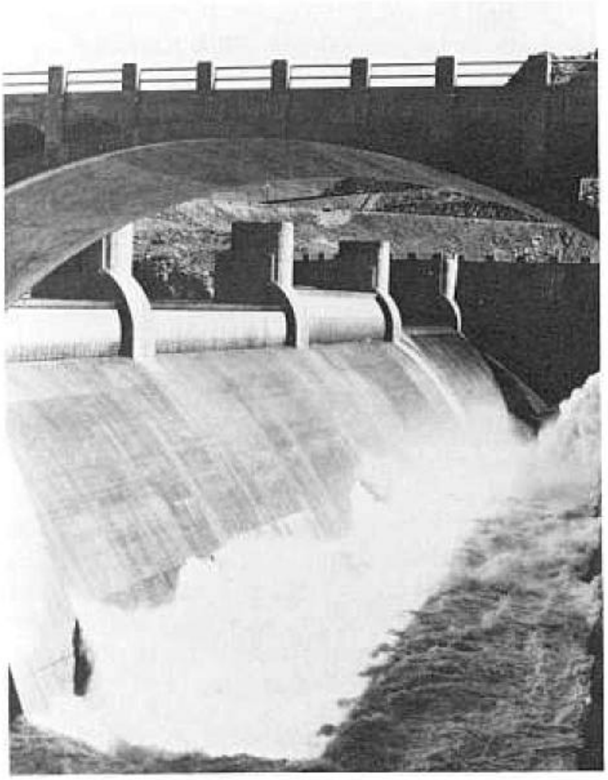


*Figure 9-8.* **Drumgate-controlled side channel spillway in operation at Hoover Dam on the Colorado River.–BC P5492**

Hoover Dam which consists of a side channel discharging into a large tunnel.

Discharge characteristics of a side channel spillway are similar to those of an ordinary overflow and are dependent on the selected profile of the weir crest. However, for maximum discharges the side channel flow may differ from that of the overflow spillway in that the flow in the trough may be restricted and may partly submerge the flow over the crest. In this case the flow characteristics will be controlled by the channel downstream from the trough.

Although the side channel is not hydraulically efficient nor inexpensive, it has advantages which make it adaptable to certain spillway layouts. Where a long overflow crest is desired in order to limit the surcharge head and the abutments are steep and precipitous, or where the control must be connected to a narrow discharge channel or tunnel, the side channel is often the best choice.

(d) *Chute Spillways.*—A spillway whose discharge is conveyed from the reservoir to the downstream river level through an open channel, placed either along a dam abutment or through a saddle, is called a chute or open channel spillway. These designations can apply regardless of the control device used to regulate the flow. Thus, a spillway having a chute-type discharge channel, though controlled by an overflow crest, a gated orifice, a side channel crest, or some other control device, might still be called a chute spillway. However, the name is most often applied when the spillway control is placed normal or nearly normal to the axis of an open channel, and where the streamlines of flow both above and below the control crest follow in the direction of the axis.

Chute spillways ordinarily consist of an entrance channel, a control structure, a discharge channel, a terminal structure, and an outlet channel. The simplest form of chute spillway has a straight centerline and is of uniform width. Often, either the axis of the entrance channel or that of the discharge channel must be curved to fit the alinement of the chute to the topography. In such cases, the curvature is confined to the entrance channel if possible, because of the low approach velocities. Where the discharge channel must be curved, its floor is sometimes superelevated to guide the high-velocity flow around the bend, thus avoiding a piling up of flow toward the outside of the chute.

Chute spillway profiles are usually influenced by the site topography and by subsurface foundation conditions. The control structure is generally placed in line with or upstream from the dam. Usually the upper portion of the discharge channel is carried at minimum grade until it "daylights" along the downstream hillside to minimize excavation. The steep portion of the discharge channel then follows the slope of the abutment.

Flows upstream from the crest are generally at subcritical velocity, with critical velocity occurring when the water passes over the control. Flows in the chute are ordinarily maintained at supercritical stage, either at constant or accelerating rates, until the terminal structure is reached. For good hydraulic performance, abrupt vertical changes or sharp convex or concave vertical curves in the chute profile should be avoided. Similarly, the convergence or divergence in plan should be gradual in order to avoid cross waves, "ride-up" on the walls, excessive turbulence, or uneven distribution of flow at the terminal structure.

Figure 9-9 shows the chute-type structure at Stewart Mountain Dam in Arizona.

(e) *Tunnel Spillways.*—Where a tunnel is used to convey the discharge around a dam, the spillway is called a tunnel spillway. The spillway tunnel usually has a vertical or inclined shaft, a large-radius elbow, and a horizontal tunnel at the downstream end. Most forms of control structures, including overflow crests, vertical or inclined orifice entrances, and side channel crests can be used with tunnel spillways.

With the exception of morning glory spillways, discussed later, tunnel spillways are designed to flow partly full throughout their length. To guarantee free flow in the tunnel, the ratio of the flow area to the total tunnel area is often limited to about 75 percent. Air vents may be provided at critical points along the tunnel to insure an adequate air supply which will avoid unsteady flow through the spillway.

Tunnel spillways may present advantages for damsites in narrow canyons with steep abutments or at sites where there is danger to

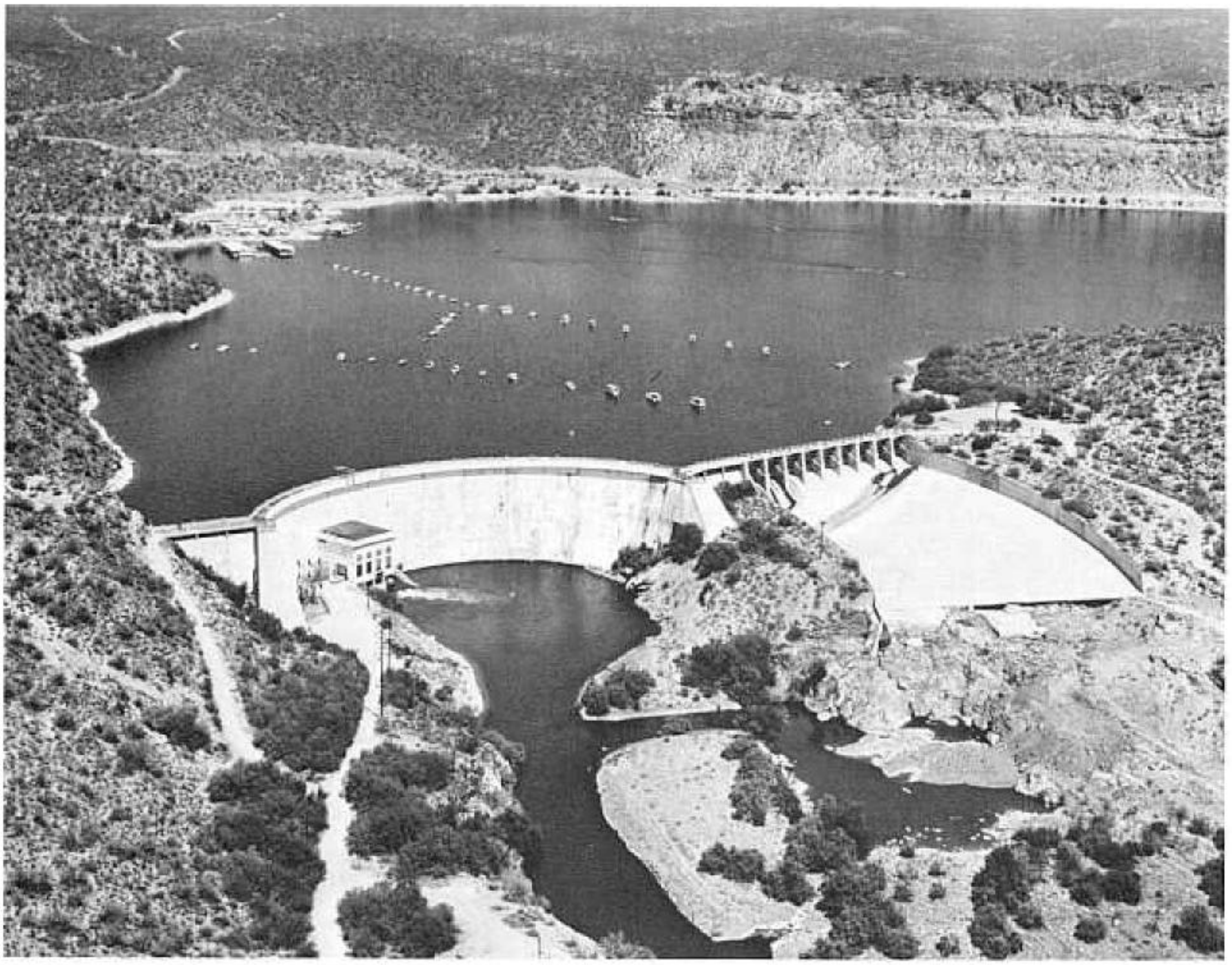


*Figure 9-9.* **Chute type spillway at Stewart Mountain Dam in Arizona.—P25-300-10882**

open channels from snow or rock slides.

(f) *Morning Glory Spillways.*—A morning glory spillway (sometimes called a drop inlet spillway) is one in which the water enters over a horizontally positioned lip, which is circular in plan, drops through a vertical or sloping shaft, and then flows to the downstream river channel through a horizontal or near horizontal tunnel. The structure may be considered as being made up of three elements; namely, an overflow control weir, an orifice control section, and a closed discharge channel.

Discharge characteristics of the morning glory spillway usually vary with the range of head. The control will shift according to the relative discharge capacities of the weir, the orifice, and the tunnel. For example, as the head increases, the control will shift from weir flow over the crest to orifice flow in the throat and then to full tunnel flow in the downstream portion of the spillway. Full tunnel flow design for spillways, except those with extremely low drops, is not recommended, as discussed in section 9-29.

A morning glory spillway can be used advantageously at damsites in narrow canyons where the abutments rise steeply or where a diversion tunnel is available for use as the downstream leg. Another advantage of this type of spillway is that near maximum capacity is attained at relatively low heads; this characteristic makes the spillway ideal for use where the maximum spillway outflow is to be limited. This characteristic also may be considered disadvantageous, in that there is little increase in capacity beyond the designed heads, should a flood larger than the selected inflow design flood occur. This would not be a disadvantage if this type of spillway were used as a service spillway in conjunction with an auxiliary spillway.

**9-9.** ***Controls for Crests.***—The simplest form of control for a spillway is the free or uncontrolled overflow crest which automatically releases water whenever the reservoir water surface rises above crest level. The advantages of the uncontrolled crest are the elimination of the need for constant attendance and regulation of the control device by an operator, and the freedom from maintenance and repairs of the device.

A regulating device or movable crest must be employed if a sufficiently long uncontrolled crest or a large enough surcharge head cannot be obtained for the required spillway capacity. Such control devices will also be required if the spillway is to release storages below the normal reservoir water surface. The type and size of the selected control device may be influenced by such conditions as discharge characteristics of a particular device, climate, frequency and nature of floods, winter storage requirements, flood control storage and outflow provisions, the need for handling ice and debris, and special operating requirements. Whether an operator will be in attendance during periods of flood, and the availability of electricity, operating mechanisms, operating bridges, etc., are other factors which will influence the type of control device employed.

Many types of crest control have been devised. The type selected for a specific installation should be based on a consideration of the factors noted above as well as economy, adaptability, reliability, and efficiency. In the classification of movable crests are such devices as flashboards and stoplogs. Regulating devices include vertical and inclined rectangular lift gates, radial gates, drum gates, and ring gates. These may be controlled manually or automatically. Automatic gates may be either mechanical or hydraulic in operation. The gates are often raised automatically to follow a rising water surface, then lowered if necessary to provide sufficient spillway capacity for larger floods.

(a) *Flashboards and Stoplogs.*—Flashboards and stoplogs provide a means of raising the reservoir storage level above a fixed spillway crest level, when the spillway is not needed for releasing floods. Flashboards usually consist of individual boards or panels supported by vertical pins or stanchions anchored to the crest; stoplogs are boards or panels spanning horizontally between grooves recessed into supporting piers. In order to provide adequate spillway capacity, the flashboards or stoplogs must be removed before the floods occur, or they must be designed or arranged so that they can be removed while being overtopped.

Various arrangements of flashboards have been devised. Some must be placed and removed manually, some are designed to fail after being overtopped, and others are arranged to drop out of position either automatically or by being manually triggered after the reservoir exceeds a certain stage. Flashboards provide a simple, economical type of movable crest device, and they have the advantage that an unobstructed crest is provided when the flashboards and their supports are removed. They have numerous disadvantages, however, which greatly limit their adaptability. Among these disadvantages are the following: (1) They present a hazard if not removed in time to pass floods, especially where the reservoir area is small and the stream is subject to flash floods; (2) they require the attendance of an operator or crew to remove them, unless they are designed to fail automatically; (3) if they are designed to fail when the water reaches certain stages their operation is uncertain, and when they fail they release sudden and undesirably large outflows; (4) ordinarily they cannot be restored to position while flow is passing over the crest; and (5) if the spillway functions frequently the repeated replacement of flashboards may be costly.

Stoplogs are individual beams or girders set one upon the other to form a bulkhead supported in grooves at each end of the span. The spacing of the supporting piers will depend on the material from which the stoplogs are constructed, the head of water acting against the stoplogs, and the handling facilities provided for installing and removing them. Stoplogs which are removed one by one as the need for increased discharge occurs are the simplest form of a crest gate.

Stoplogs may be an economical substitute for more elaborate gates where relatively close spacing of piers is not objectionable and where removal is required only infrequently. Stoplogs which must be removed or installed in flowing water may require such elaborate hoisting mechanisms that this type of installation may prove to be as costly as gates. A stoplogged spillway requires the attendance of an operating crew for removing and installing the stoplogs. Further, the arrangement may present a hazard to the safety of the dam if the reservoir is small and the stream is subject to flash floods, since the stoplogs must be removed in time to pass the flood.

(b) *Rectangular Lift Gates.*—Rectangular lift gates span horizontally between guide grooves in supporting piers. Although these gates may be made of wood or concrete, they are often made of metal (cast iron or steel). The support guides may be placed either vertically or inclined slightly downstream. The gates are raised or lowered by an overhead hoist. Water is released by undershot orifice flow for all gate openings.

For sliding gates the vertical side members of the gate frame bear directly on the guide members; sealing is effected by the contact pressure. The size of this type of installation is limited by the relatively large hoisting capacity required to operate the gate because of the sliding friction that must be overcome.

Where larger gates are needed, wheels can be mounted along each side of the rectangular lift gates to carry the load to a vertical track on the downstream side of the pier groove. The use of wheels greatly reduces the amount of friction and thereby permits the use of a smaller hoist.

(c) *Radial Gates.*—Radial gates are usually constructed of steel. They consist of a cylindrical segment which is attached to supporting bearings by radial arms. The face segment is made concentric to the supporting pins so that the entire thrust of the waterload passes through the pins; thus, only a small moment need be overcome in raising and lowering the gate. Hoisting loads then consist of the weight of the gate, the friction between the side seals and the piers, and the frictional resistance at the pins. The gate is often counterweighted to partially counterbalance the effect of its weight, which further reduces the required capacity of the hoist.

The small hoisting effort needed to operate radial gates makes hand operation practical on small installations which otherwise might require power. The small hoisting forces involved also make the radial gate more adaptable to operation by relatively simple automatic control apparatus. Where a number of gates are used on a spillway, they might be arranged to open automatically at successively increasing reservoir levels, or only one or two might be equipped with automatic controls, while the remaining gates would be operated by hand or power hoists.

(d) *Drum Gates.*—Drum gates are constructed of steel plate and, since they are hollow, are buoyant. Each gate is triangular in section and is hinged to the upstream lip of a hydraulic chamber in the weir structure, in which the gate floats. Water introduced into or drawn from the hydraulic chamber causes the gate to swing upwards or downwards. Controls governing the flow of water into and out of the hydraulic chamber are located in the piers adjacent to the chambers. Figure 9-8 shows the drum gates on the Arizona spillway at Hoover Dam, which are automatic in operation.

(e) *Ring Gates.*—A ring gate consists of a full-circle hollow steel ring with streamlined top surface which blends with the surface of a morning glory inlet structure. The bottom portion of the ring is contained within a circular hydraulic chamber. Water admitted to or drawn from the hydraulic chamber causes the ring to move up or down in the vertical direction. Figure 9-7 shows the morning glory spillway for Hungry Horse Dam with the ring gate in the closed position.

## C. CONTROL STRUCTURES

9-10. ***Shape for Uncontrolled Ogee Crest.***—Crest shapes which approximate the profile of the under nappe of a jet flowing over a sharp-crested weir provide the ideal form for obtaining optimum discharges. The shape of such a profile depends upon the head, the inclination of the upstream face of the overflow section, and the height of the overflow section above the floor of the entrance channel (which influences the velocity of approach to the crest).

A simple scheme suitable for most dams with a vertical upstream face is to shape the upstream surface (in section) to an arc of a circle and the downstream surface to a parabola. The necessary information for defining the shape is shown on figure 9-10. This method will define a crest which approximates the more refined shape discussed below. It is suitable for preliminary estimates and for final designs when a refined shape is not required.

Crest shapes have been studied extensively in the Bureau of Reclamation hydraulic laboratories, and data from which profiles for overflow crests can be obtained have been published [1].[1] For most conditions the data can be summarized according to the form shown on figure 9-11(A), where the profile is defined as it relates to axes at the apex of the crest. That portion upstream from the origin is defined as either a single curve and a tangent or as a compound circular curve. The portion downstream is defined by the equation:

$$\frac{y}{H_o} = -K\left(\frac{x}{H_o}\right)^n \qquad (2)$$

in which $K$ and $n$ are constants whose values depend on the upstream inclination and on the velocity of approach. Figure 9-11 gives values

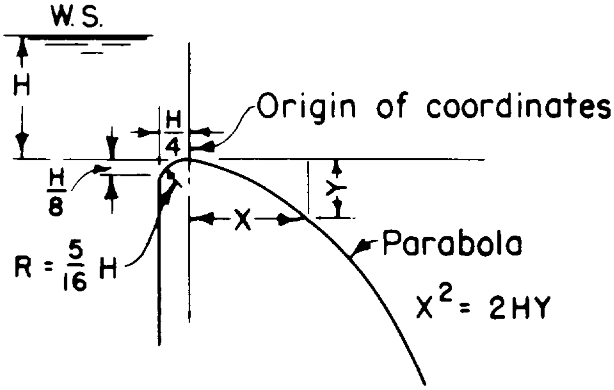
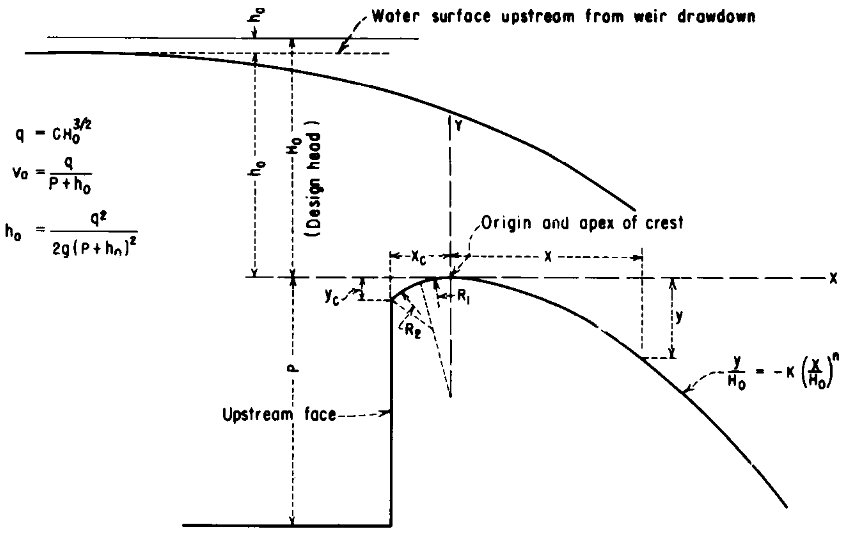


*Figure 9-10.* **A simple ogee crest shape with a vertical upstream face.—288-D-3041**

---

[1] Numbers in brackets refer to items in the bibliography, sec. 9-31.



(A) ELEMENTS OF NAPPE-SHAPED CREST PROFILES

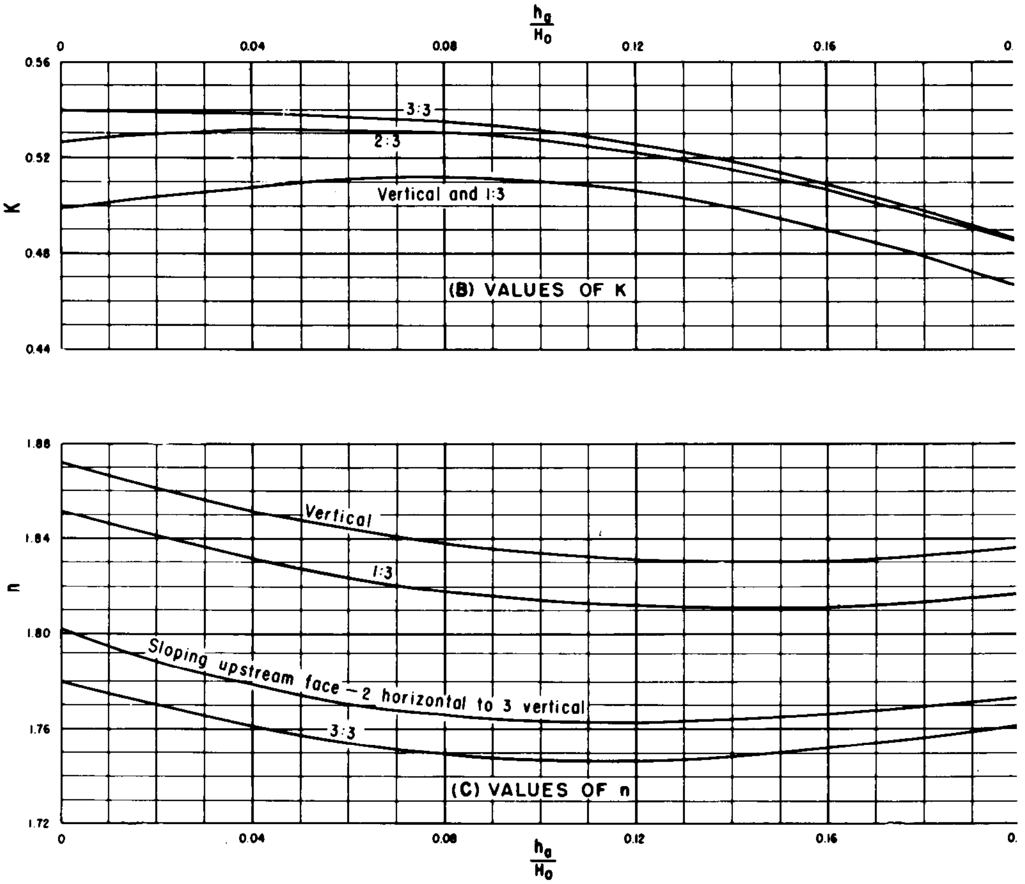


*Figure 9-11.* Factors for definition of nappe-shaped crest profiles (sheet 1 of 2).–288-D-2406

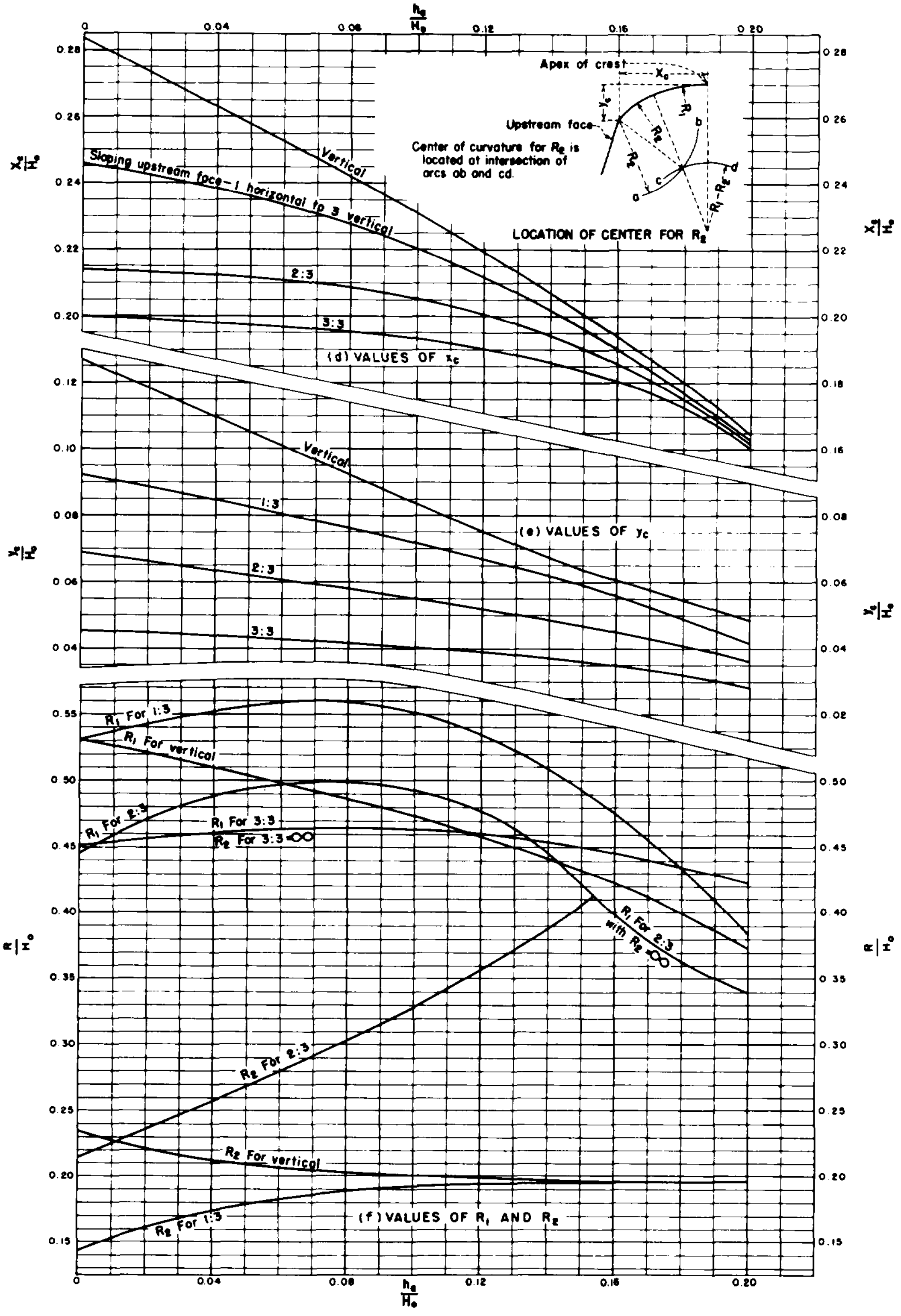


*Figure 9-11.* Factors for definition of nappe-shaped crest profiles (sheet 2 of 2).–288-D-2407

of these constants for different conditions.

The approximate profile shape for a crest with a vertical upstream face and negligible velocity of approach is shown on figure 9-12. The profile is constructed in the form of a compound circular curve with radii expressed in terms of the design head, $H_o$. This definition is simpler than that shown on figure 9-11, since it avoids the need for solving an exponential equation; further, it is presented in a form easily used by a layman for constructing forms or templates. For ordinary conditions of design of spillways where the approach height, $P$ (fig. 9-11(A)), is equal to or greater than one-half the maximum head on the crest, this profile is sufficiently accurate to avoid seriously reduced crest pressures and does not materially alter the hydraulic efficiency of the crest. When the approach height is less than one-half the maximum head on the crest, the profile should be determined from figure 9-11.

In some cases, it is necessary to use a crest shape other than that indicated by the above design. Information from model studies performed on many spillways has been accumulated and a compilation of the coefficient data has been made. This information is shown in Engineering Monograph No. 9 [2]. In this monograph, the crests are plotted in a dimensionless form with the design head, $H_o$, equal to 1. By plotting other crests to the same scale, comparisons with model-tested crest shapes can be made.

9-11. ***Discharge Over an Uncontrolled Overflow Ogee Crest.***—The discharge over an ogee crest is given by the formula:

$$Q = CLH_e^{3/2} \qquad (3)$$

where:

$Q$ = discharge,
$C$ = a variable coefficient of discharge,
$L$ = effective length of crest, and
$H_e$ = total head on the crest, including velocity of approach head, $h_a$.

The total head on the crest, $H_e$, does not include allowances for approach channel friction losses or other losses due to curvature of the upstream channel, entrance loss into the inlet section, and inlet or transition losses. Where the design of the approach channel results in appreciable losses, they must be added to $H_e$ to determine reservoir elevations corresponding to the discharges given by the above equation.

(a) *Coefficient of Discharge.*—The discharge coefficient, $C$, is influenced by a number of factors, such as (1) the depth of approach, (2) relation of the actual crest shape to the ideal nappe shape, (3) upstream face slope, (4) downstream apron interference, and (5) downstream submergence. The effect of these various factors is discussed in subsections (b) through (d). The effect of the discharge coefficient for heads other than the design head is discussed in subsection (e). The discharge coefficient for various crest profiles can be determined from Engineering Monograph No. 9 [2] or it may be approximated by finding the design shape it most nearly matches.

(b) *Effect of Depth of Approach.*—For a high sharp-crested weir placed in a channel, the velocity of approach is small and the under side of the nappe flowing over the weir attains maximum vertical contraction. As the approach depth is decreased, the velocity of approach increases and the vertical contraction diminishes. When the weir height becomes zero, the contraction is entirely suppressed and the overflow weir becomes in effect a channel or a broad-crested weir, for which the theoretical coefficient of discharge is 3.087. If the sharp-crested weir coefficients are related to the head measured from the point of maximum contraction instead of to the head above the sharp crest, coefficients applicable to ogee crests shaped to profiles of under nappes for various approach velocities can be established. The relationship of the ogee crest coefficient, $C_o$, to various values of $\frac{P}{H_o}$ is shown on figure 9-13. These coefficients are valid only

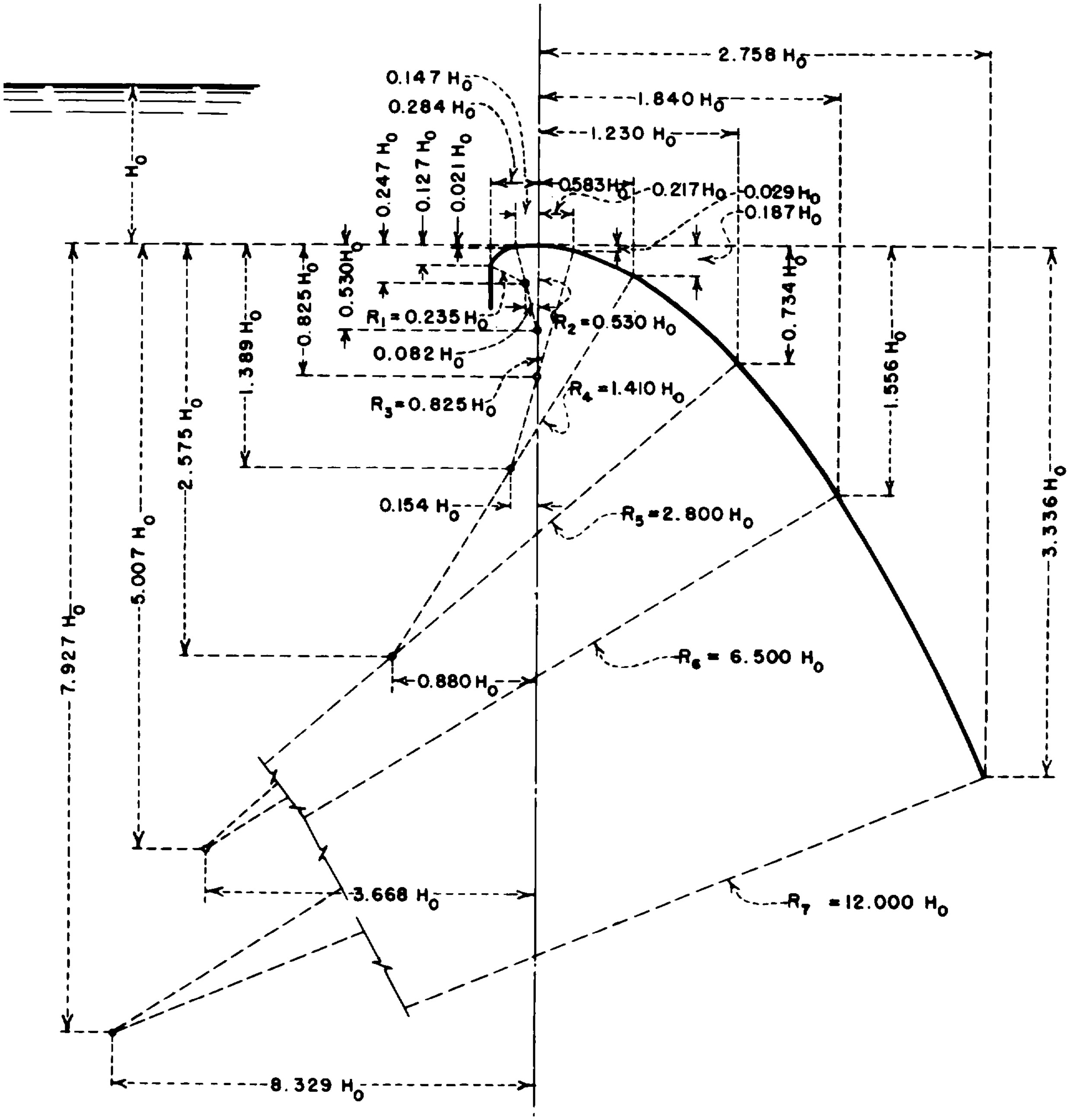


*Figure 9-12.* **Ogee crest shape defined by compound curves.–288-D-2408**

when the ogee is formed to the ideal nappe shape, that is when $\frac{H_e}{H_o} = 1$.

(c) *Effect of Upstream Face Slope.*—For small ratios of the approach depth to head on the crest, sloping the upstream face of the overflow results in an increase in the coefficient of discharge. For large ratios the effect is a decrease of the coefficient. Within the range considered in this text, the coefficient of discharge is reduced for large ratios of $\frac{P}{H_o}$ only for relatively flat upstream slopes. Figure 9-14 shows the ratio of the

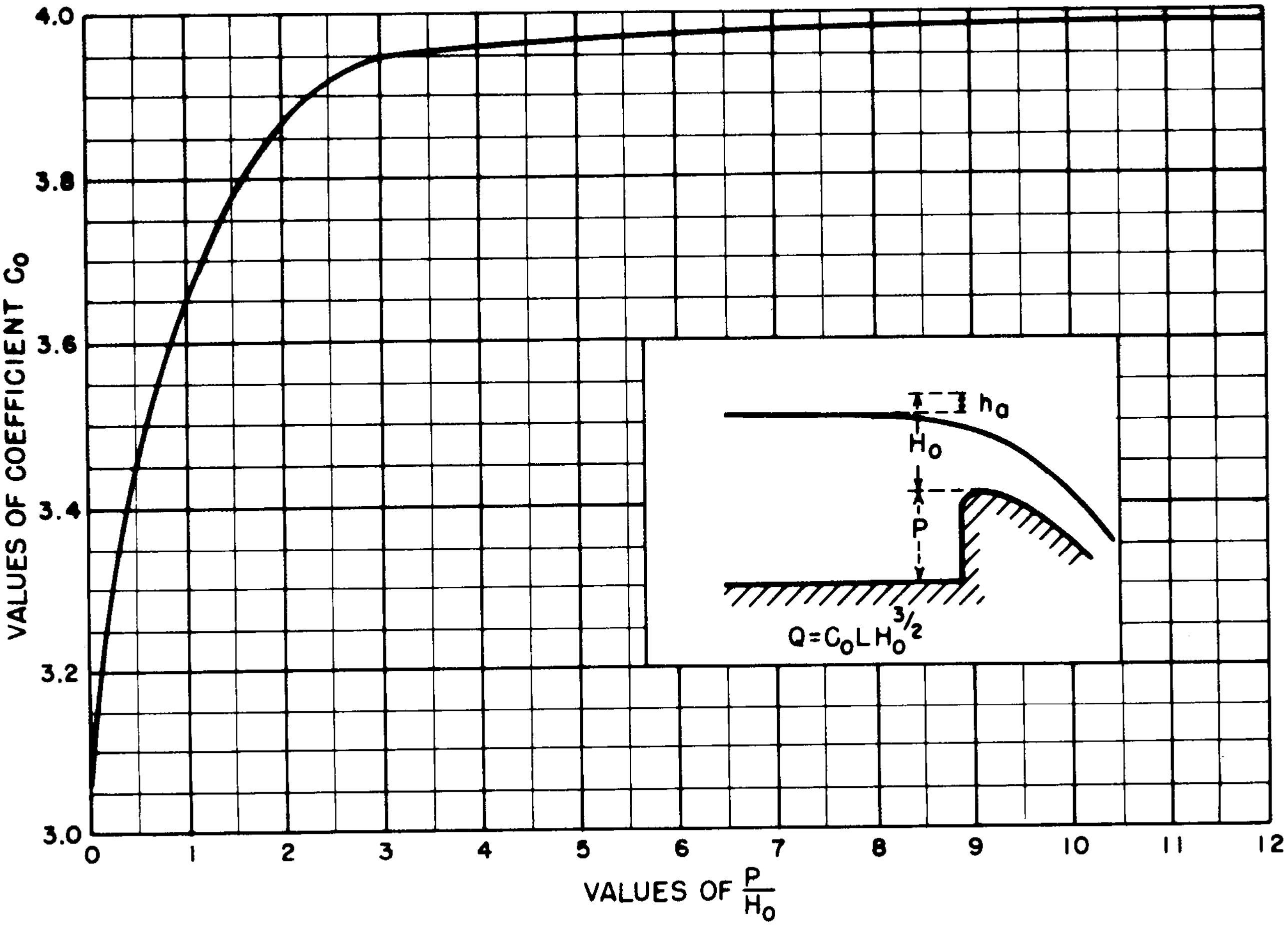


*Figure 9-13.* **Coefficient of discharge for ogee-shaped crest with vertical upstream face.–288-D-3042**

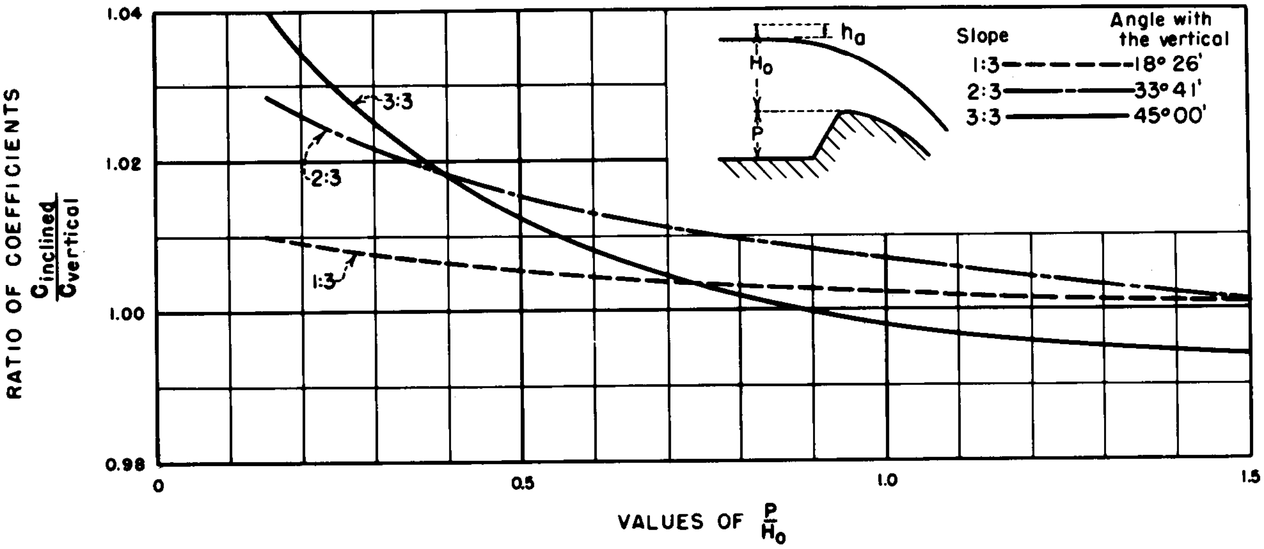


*Figure 9-14.* **Coefficient of discharge for ogee-shaped crest with sloping upstream face.–288-D-2411**

coefficient for an overflow ogee crest with a sloping face to the coefficient for a crest with a vertical upstream face as obtained from figure 9-13, as related to values of $\frac{P}{H_o}$.

(d) *Effect of Downstream Apron Interference and Downstream Submergence.*—When the water level below an overflow weir is high enough to affect the discharge, the weir is said to be submerged. The vertical distance from the crest of the overflow weir to the downstream apron and the depth of flow in the downstream channel, as it relates to the head pool level, are factors which alter the coefficient of discharge.

Five distinct characteristic flows can occur below an overflow crest, depending on the relative positions of the apron and the downstream water surface: (1) Flow will continue at supercritical stage; (2) a partial or incomplete hydraulic jump will occur immediately downstream from the crest; (3) a true hydraulic jump will occur; (4) a drowned jump will occur in which the high-velocity jet will follow the face of the overflow and then continue in an erratic and fluctuating path for a considerable distance under and through the slower water; and (5) no jump will occur—the jet will break away from the face of the overflow and ride along the surface for a short distance and then erratically intermingle with the slow-moving water underneath. Figure 9-15 shows the relationship of the floor positions and downstream submergences which produce these distinctive flows.

Where the downstream flow is at supercritical stage or where the hydraulic jump occurs, the decrease in the coefficient of discharge is due principally to the back-pressure effect of the downstream apron and is independent of any submergence effect due to tailwater. Figure 9-16 shows the effect of downstream apron conditions on the coefficient of discharge. It will be noted that this curve plots the same data represented by the vertical dashed lines on figure 9-15 in a slightly different form. As the downstream apron level nears the crest of the overflow $\left(\frac{h_d + d}{H_e} \text{ approaches } 1.0\right)$, the coefficient of discharge is about 77 percent of that for unretarded flow. On the basis of a coefficient of 3.98 for unretarded flow over a high weir, the coefficient when the weir is submerged will be about 3.08, which is virtually the coefficient for a broad-crested weir.

From figure 9-16 it can be seen that when the $\frac{h_d + d}{H_e}$ values exceed about 1.7, the downstream floor position has little effect on the coefficient, but there is a decrease in the coefficient caused by tailwater submergence. Figure 9-17 shows the ratio of the coefficient of discharge where affected by tailwater conditions, to the coefficient for free flow conditions. This curve plots the data represented by the horizontal dashed lines on figure 9-15 in a slightly different form. Where the dashed lines on figure 9-15 are curved, the decrease in the coefficient is the result of a combination of tailwater effects and downstream apron position.

(e) *Effect of Heads Differing from Design Head.*—When the crest has been shaped for a head larger or smaller than the one under consideration, the coefficient of discharge, $C$, will differ from that shown on figure 9-13. A widened shape will result in positive pressures along the crest contact surface, thereby reducing the discharge; with a narrower crest shape negative pressures along the contact surface will occur, resulting in an increased discharge. Figure 9-18 shows the variation of the coefficient as related to values of $\frac{H_e}{H_o}$, where $H_e$ is the actual head being considered. The adjusted coefficient can be used for preparing a discharge-head relationship.

(f) *Pier and Abutment Effects.*—Where crest piers and abutments are shaped to cause side contractions of the overflow, the effective length, $L$, will be less than the net length of the crest. The effect of the end contractions may be taken into account by reducing the net crest

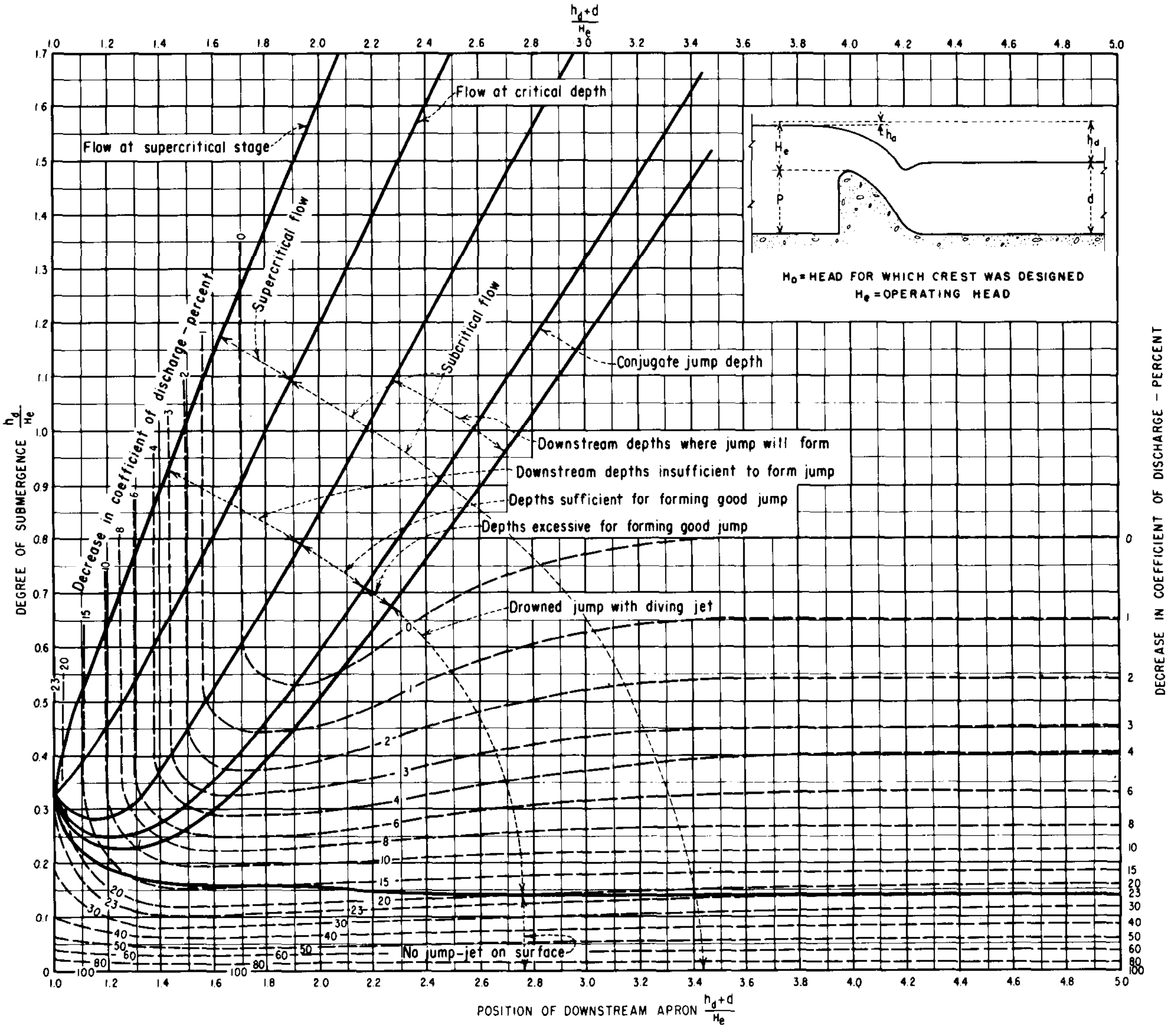


*Figure 9-15.* **Effects of downstream influences on flow over weir crests.–288-D-2412**

length as follows:

$$L = L' - 2(NK_p + K_a)H_e \qquad (4)$$

where:

$L$ = effective length of crest,
$L'$ = net length of crest,
$N$ = number of piers,
$K_p$ = pier contraction coefficient,
$K_a$ = abutment contraction coefficient, and
$H_e$ = total head on crest.

The pier contraction coefficient, $K_p$, is affected by the shape and location of the pier nose, the thickness of the pier, the head in relation to the design head, and the approach velocity. For conditions of design head, $H_o$, average pier contraction coefficients may be assumed as follows:

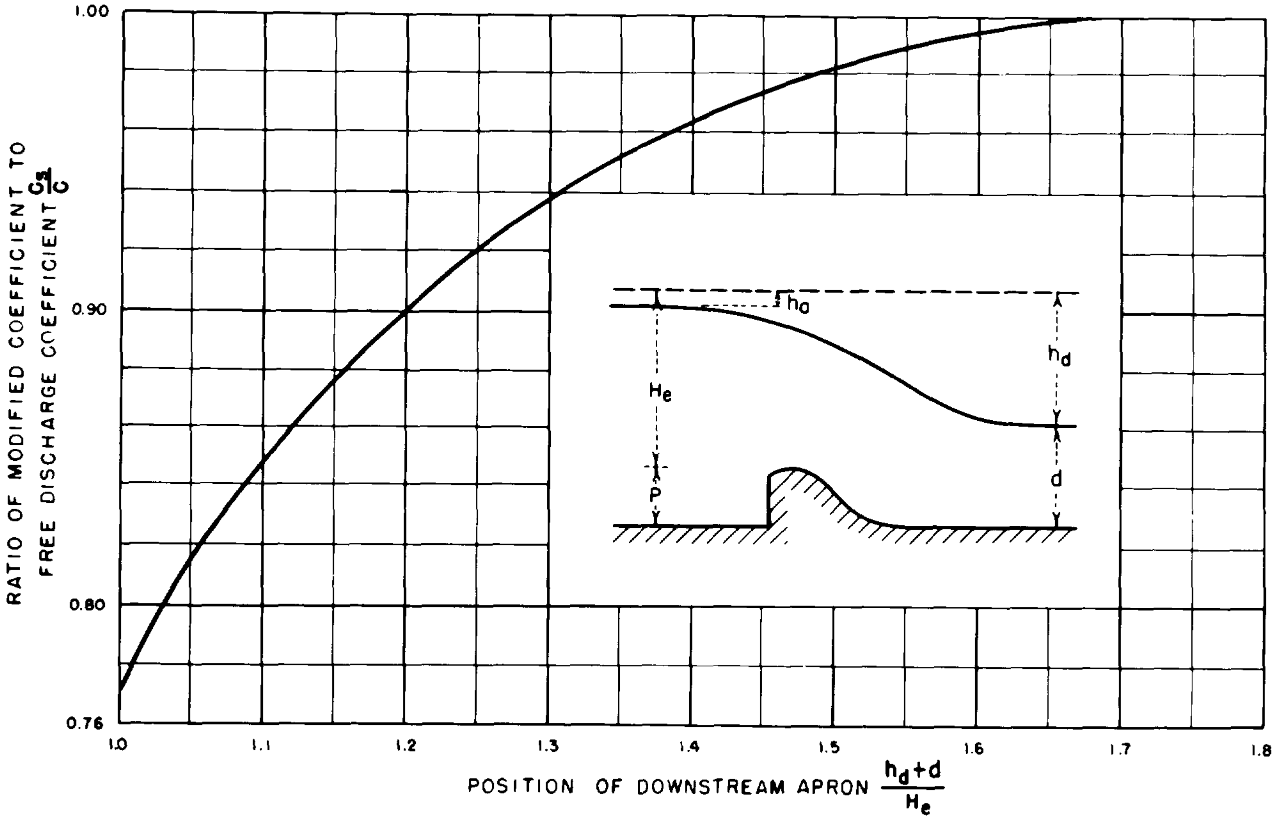


*Figure 9-16.* **Ratio of discharge coefficients due to apron effect.–288-D-2413**

| | $K_p$ |
|---|---|
| For square-nosed piers with corners rounded on a radius equal to about 0.1 of the pier thickness | 0.02 |
| For round-nosed piers | 0.01 |
| For pointed-nose piers | 0 |

The abutment contraction coefficient is affected by the shape of the abutment, the angle between the upstream approach wall and the axis of flow, the head in relation to the design head, and the approach velocity. For conditions of design head, $H_o$, average coefficients may be assumed as follows:

| | $K_a$ |
|---|---|
| For square abutments with headwall at 90° to direction of flow | 0.20 |
| For rounded abutments with headwall at 90° to direction of flow, when $0.5H_o \geqq r \geqq 0.15H_o$ | 0.10 |
| For rounded abutments where $r > 0.5H_o$ and headwall is placed not more than 45° to direction of flow | 0 |

where $r$ = radius of abutment rounding.

9-12. *Uncontrolled Ogee Crests Designed for Less Than Maximum Head.*—Economy in

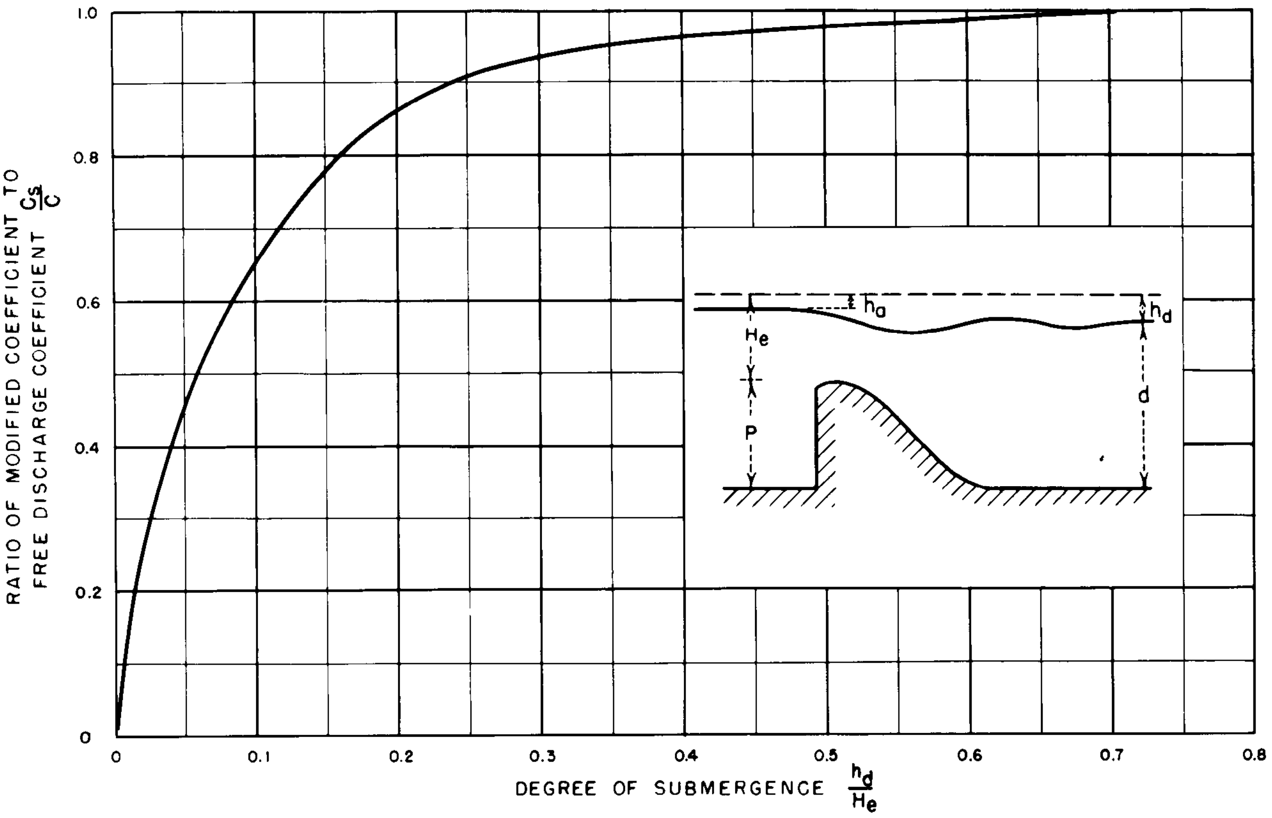


*Figure 9-17.* **Ratio of discharge coefficients due to tailwater effect.—288-D-2414**

the design of an ogee crest may sometimes be effected by using a design head less than the maximum expected for determining the ogee profile. Use of a smaller head for design results in increased discharges for the full range of heads. The increase in capacity makes it possible to achieve economy by reducing either the crest length or the maximum surcharge head.

Tests have shown that the subatmospheric pressures on a nappe-shaped crest do not exceed about one-half the design head when the design head is not less than about 75 percent of the maximum head. As long as these subatmospheric pressures do not approach pressures which might induce cavitation, they can be tolerated. Care must be taken, however, in forming the surface of the crest where these negative pressures will occur, since unevenness caused by abrupt offsets, depressions, or projections will amplify the negative pressures to a magnitude where cavitation conditions can develop.

The negative pressure on the crest may be resolved into a system of forces acting both upward and downstream. These forces should be considered in analyzing the structural stability of the crest structure.

An approximate force diagram of the subatmospheric pressures when the design head used to determine the crest shape is 75 percent of the maximum head, is shown on figure 9-19. These data are based on average results of tests made on ideal shaped weirs with negligible velocities of approach. Pressures for intermediate head ratios can be assumed to vary linearly, considering that no subatmospheric pressure prevails when $\frac{H_o}{H_e}$ is equal to 1.

9-13. *Gate-Controlled Ogee Crests.*—Releases for partial gate openings for gated

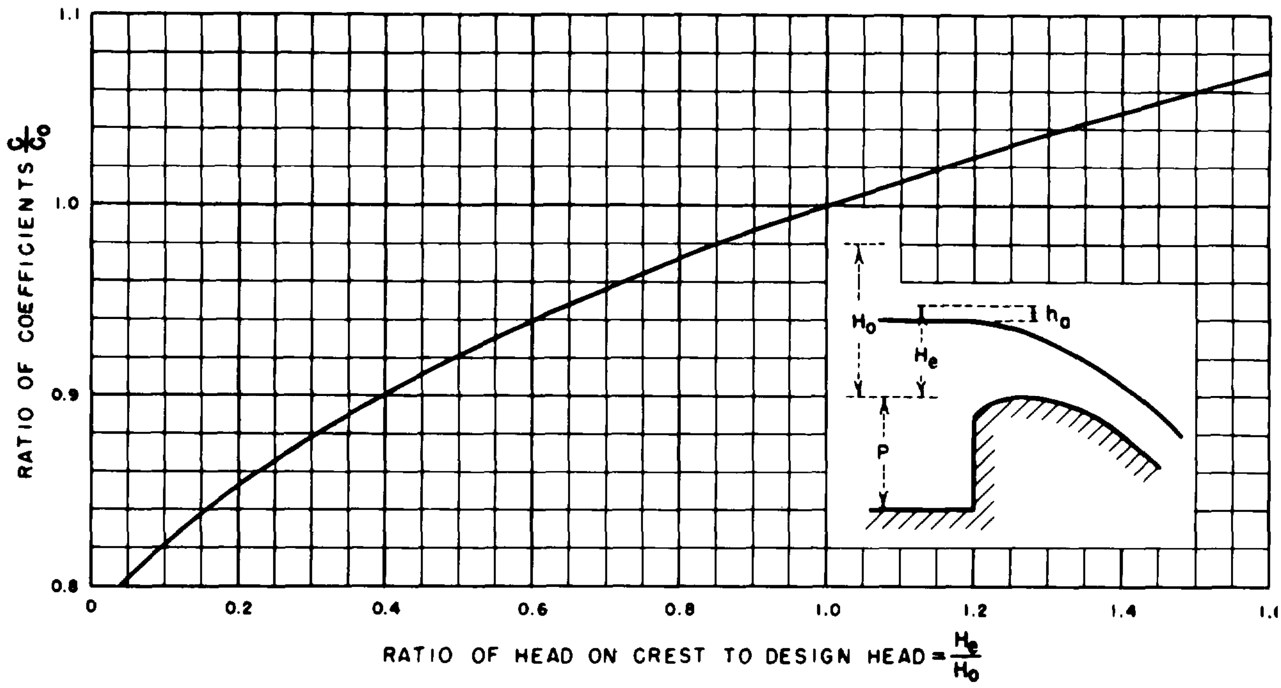


*Figure 9-18.* Coefficient of discharge for other than the design head.–288-D-2410

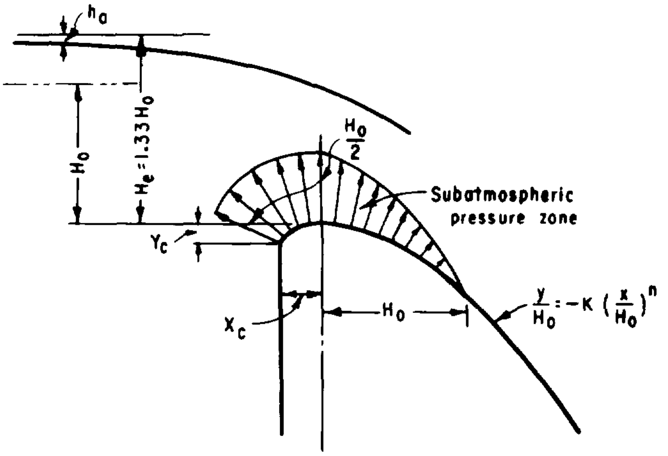


*Figure 9-19.* Subatmospheric crest pressures for a 0.75 ratio of $H_o$ to $H_e$.–288-D-3043

crests will occur as orifice flow. With full head on the gate and with the gate opened a small amount, a free discharging trajectory will follow the path of a jet issuing from an orifice. For a vertical orifice the path of the jet can be expressed by the parabolic equation:

$$-y = \frac{x^2}{4H} \qquad (5)$$

where $H$ is the head on the center of the opening. For an orifice inclined an angle of $\theta$ from the vertical, the equation will be:

$$-y = x \tan \theta + \frac{x^2}{4H \cos^2 \theta} \qquad (6)$$

If subatmospheric pressures are to be avoided along the crest contact, the shape of the ogee downstream from the gate sill must conform to the trajectory profile.

Gates operated with small openings under high heads produce negative pressures along the crest in the region immediately below the gate if the ogee profile drops below the trajectory profile. Tests have shown that subatmospheric pressures would be equal to about one-tenth of the head when the gate is operated at small opening and the ogee is shaped to the nappe profile as defined by equation (2) for maximum head $H_o$. The force diagram for this condition is shown on figure 9-20.

The adoption of a trajectory profile rather than a nappe profile downstream from the gate sill will result in a wider ogee, and reduced discharge efficiency for full gate opening.

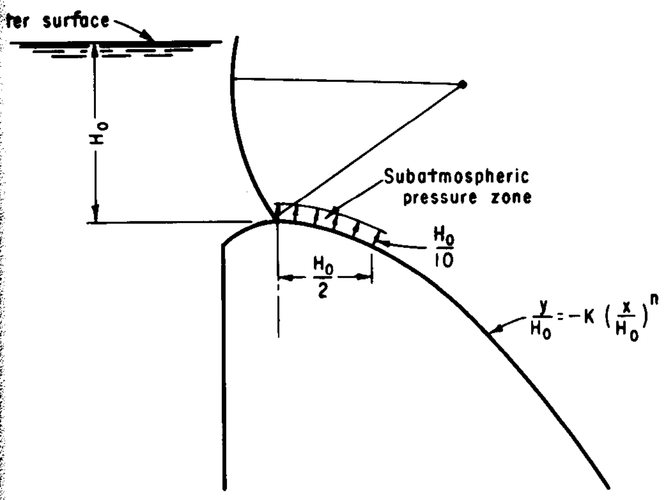


*Figure 9-20.* **Subatmospheric crest pressures for undershot gate flow.–288-D-3044**

Where the discharge efficiency is unimportant and where a wider ogee shape is needed for structural stability, the trajectory profile may be adopted to avoid subatmospheric pressure zones along the crest. Where the ogee is shaped to the ideal nappe profile for maximum head, the subatmospheric pressure area can be minimized by placing the gate sill downstream from the crest of the ogee. This will provide an orifice which is inclined downstream for small gate openings, and thus will result in a steeper trajectory more nearly conforming to the nappe-shaped profile.

9-14. ***Discharge Over Gate-Controlled Ogee Crests.***–The discharge for a gated ogee crest at partial gate openings will be similar to flow through an orifice and may be computed by the equation:

$$Q = \frac{2}{3}\sqrt{2g}\, CL\left(H_1{}^{3/2} - H_2{}^{3/2}\right) \qquad (7)$$

where $H_1$ and $H_2$ are the total heads (including the velocity head of approach) to the bottom and top of the orifice, respectively. The coefficient, $C$, will differ with different gate and crest arrangements; it is influenced by the approach and downstream conditions as they affect the jet contractions. Thus, the top contraction for a vertical leaf gate will differ from that for a curved, inclined radial gate; the upstream floor profile will affect the bottom contraction of the issuing jet; and the downstream profile will affect the back pressure and consequently the effective head. Figure 9-21 shows coefficients of discharge for orifice flows for various ratios of gate opening to total head. The curve represents averages determined for the various approach and downstream conditions described and is sufficiently reliable for determining discharges for most spillway structures. The curve is for a gate at the apex of the ogee crest, and so long as the bottom of the gate when closed is less than 0.03 $H_o$ vertically from the apex the coefficient should not change significantly.

9-15. ***Orifice Control Structures.***–Orifice control structures are often incorporated into a concrete arch dam, one or more orifices being formed through the dam. If the invert of the orifice is below normal water surface the orifice must be gated. If the invert is at or above normal water surface the orifice may be either gated or ungated. Figure 9-22 shows typical orifice control structures.

(a) *Shape.*–The entrance to the orifice must be streamlined to eliminate negative pressures. Portions of ellipses are used to streamline the entrances. The major axis of the ellipse is equal to the height or width of the orifice *H* or *W* in figure 9-22, and the minor axis is one-third of this amount. Orifices may be horizontal or they may be inclined downward to change the location of the impingement area in the case of a free fall spillway, or to provide improved alinement into a discharge channel. If inclined downward, the bottom of the orifice should be shaped similar to an ogee crest to eliminate negative pressures. The top should be made parallel to or slightly converging with the bottom.

(b) *Hydraulics.*–The discharge characteristics of an orifice flowing partially full with the upper nappe not in contact with the orifice are similar to those of an ogee crest, and the discharge can be computed by use of equation (3). The discharge coefficient, *C*, can be determined as described in section 9-10 for an overflow crest. Where practicable, a model study should be made to confirm the value of the coefficient. An orifice flowing full will function similar to a river outlet, and the discharge can be determined using the same

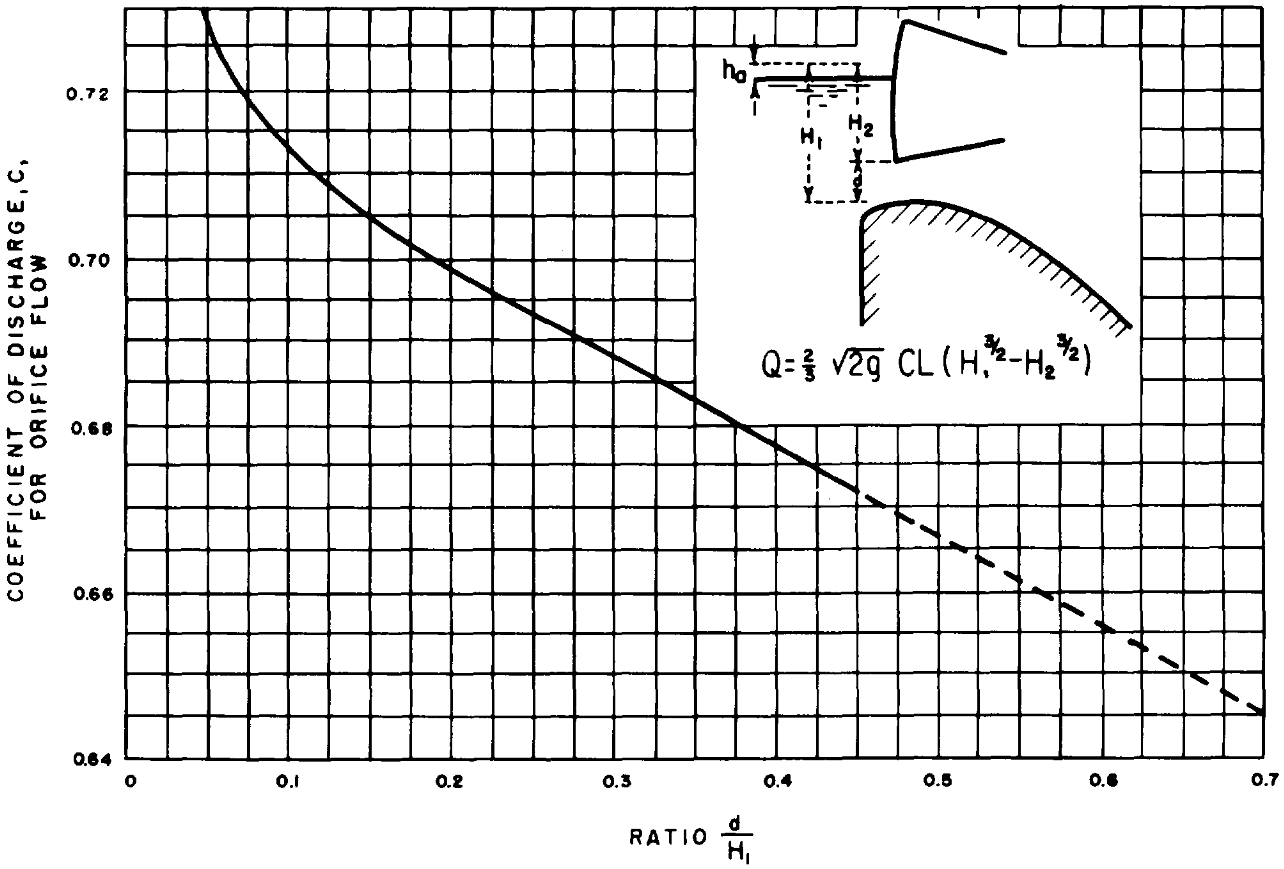


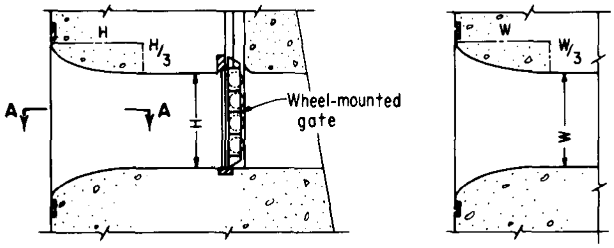
*Figure 9-21.* **Coefficient of discharge for flow under a gate (orifice flow).–288-D-3045**

procedures as for river outlets discussed in chapter X.

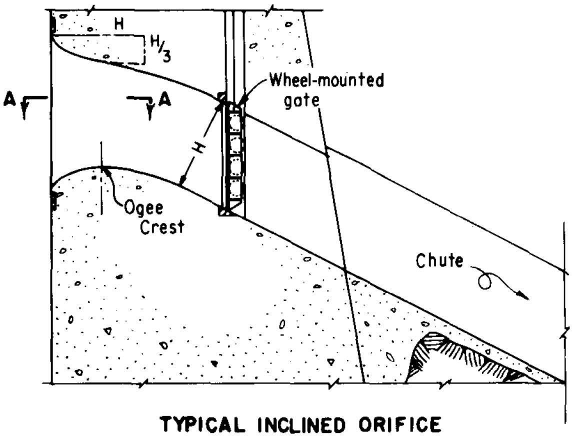
9-16. *Side Channel Control Structures.*–The side channel control structure consists of an ogee crest to control releases from the reservoir, and a channel immediately downstream of and parallel to the crest to carry the water to the discharge channel.

(a) *Layout.*–The ogee crest is designed by the methods in section 9-10 if the crest is uncontrolled or section 9-13 if it is controlled.

The cross-sectional shape of the side channel trough will be influenced by the overflow crest on the one side and by the bank conditions on the opposite side. Because of turbulences and vibrations inherent in side channel flow, a side channel design is ordinarily not considered except where a competent foundation such as rock exists. The channel sides will, therefore, usually be a concrete lining placed on a slope and anchored directly to the rock. A trapezoidal cross section is the one most often employed for the side channel trough. The width of such a channel in relation to the depth should be considered. If the width to depth ratio is large, the depth of flow in the channel will be shallow, similar to that depicted by the cross section *abfg* on figure 9-23. It is evident that for this condition a poor diffusion of the incoming flow with the channel flow will result. A cross section with a minimum width-depth ratio will provide the best hydraulic performance, indicating that a cross section approaching that depicted as *adj* on the figure would be the ideal choice both from the standpoint of hydraulics and economy. Minimum bottom widths are required, however, to avoid construction difficulties due to confined working space. Furthermore, the stability of the structure and the hillside which might be jeopardized by an extremely deep cut in the abutment must also



TYPICAL HORIZONTAL ORIFICE SEC. A-A



TYPICAL INCLINED ORIFICE

*Figure 9-22.* Typical orifice control structures.–288-D-3046

be considered. Therefore, a minimum bottom width must be selected which is commensurate with both the practical and structural aspects of the problem.

The slope of the channel profile is arbitrary; however, a relatively flat slope will provide greater depths and slower velocities and consequently will ensure better intermingling of flows at the upstream end of the channel and avoid the possibility of accelerating or supercritical flows occurring in the channel for smaller discharges.

A control section is usually constructed downstream from the side channel trough. It is achieved by constricting the channel sides or elevating the channel bottom to produce a point of critical flow. Flows upstream from the control will be at the subcritical stage and will provide a maximum of depth in the side channel trough. The side channel bottom and control dimensions are then selected so that flow in the trough immediately downstream from the crest will be at the greatest depth possible without excessively submerging the flow over the crest. Flow in the discharge channel downstream from the control will be the same as that in an ordinary channel or chute type spillway. If a control section is not provided, the depth of water and its velocity in the side channel will depend upon either the slope of the side channel trough floor or the backwater effect of the discharge channel.

Figure 9-24(A) illustrates the effect of a control section and the slope of the side channel trough floor on the water surface profile. When the bottom of the side channel trough is selected so that its depth below the hydraulic gradient is greater than the minimum specific energy depth, flow will be either at the subcritical or supercritical stage, depending on the relation of the bottom profile to critical slope or on the influences of a downstream control section. If the slope of the bottom is greater than critical and a control section is not established below the side channel trough, supercritical flow will prevail throughout the length of the channel. For this stage, velocities will be high and water depths will be shallow, resulting in a relatively high fall from the reservoir water level to the water surface in the trough. This flow condition is illustrated by profile B′ on figure 9-24(A). Conversely, if a control section is established downstream from the side channel trough to increase the upstream depths, the channel can be made to flow at the subcritical stage. Velocities at this stage will be less than critical and the greater depths will result in a smaller drop from the reservoir water surface to the side channel water surface profile. The condition of flow for subcritical depths is illustrated on figure 9-24(A) by water surface profile A′.

The effect of the fall distance from the reservoir to the channel water surface for each type of flow is depicted on figure 9-24(B). It can be seen that for the subcritical stage, the incoming flow will not develop high transverse velocities because of the low drop before it meets the channel flow, thus effecting a good diffusion with the water bulk in the trough. Since both the incoming velocities and the channel velocities will be relatively slow, a fairly complete intermingling of the flows will

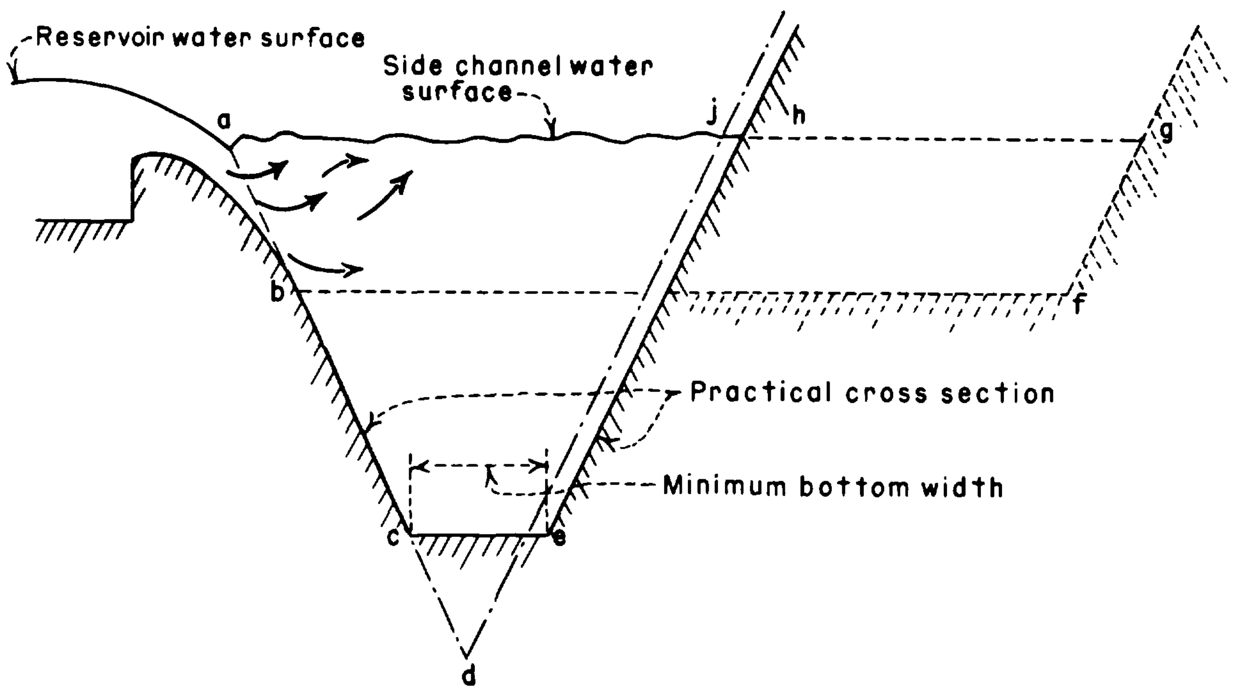


*Figure 9-23.* **Comparison of side channel cross sections.—288-D-2419**

take place, thereby producing a comparatively smooth flow in the side channel. Where the channel flow is at the supercritical stage, the channel velocities will be high, and the intermixing of the high-energy transverse flow with the channel stream will be rough and turbulent. The transverse flows will tend to sweep the channel flow to the far side of the channel, producing violent wave action with attendant vibrations. It is thus evident that flows should be maintained at subcritical stage for good hydraulic performance. This can be achieved by establishing a control section downstream from the side channel trough.

Variations in the design can be made by assuming different bottom widths, different channel slopes, and varying control sections. A proper and economical design can usually be achieved after comparing several alternatives.

(b) *Hydraulics.*—The theory of flow in a side channel [3] is based principally on the law of conservation of linear momentum, assuming that the only forces producing motion in the channel result from the fall in the water surface in the direction of the axis. This premise assumes that the entire energy of the flow over the crest is dissipated through its intermingling with the channel flow and is therefore of no assistance in moving the water along the channel. Axial velocity is produced only after the incoming water particles join the channel stream.

For any short reach of channel $\Delta x$, the change in water surface, $\Delta y$, can be determined by either of the following equations:

$$\Delta y = \frac{Q_1}{g} \bullet \frac{(v_1 + v_2)}{(Q_1 + Q_2)} \left[ (v_2 - v_1) + \frac{v_2 (Q_2 - Q_1)}{Q_1} \right] \quad (8)$$

$$\Delta y = \frac{Q_2}{g} \bullet \frac{(v_1 + v_2)}{(Q_1 + Q_2)} \left[ (v_2 - v_1) + \frac{v_1 (Q_2 - Q_1)}{Q_2} \right] \quad (9)$$

where $Q_1$ and $v_1$ are values at the beginning of the reach and $Q_2$ and $v_2$ are the values at the end of the reach. The derivation of these formulas can be found in reference [3].

By use of equation (8) or (9), the water surface profile can be determined for any particular side channel by assuming successive short reaches of channel once a starting point is found. The solution of equation (8) or (9) is obtained by a trial-and-error procedure. For a reach of length $\Delta x$ in a specific location, $Q_1$

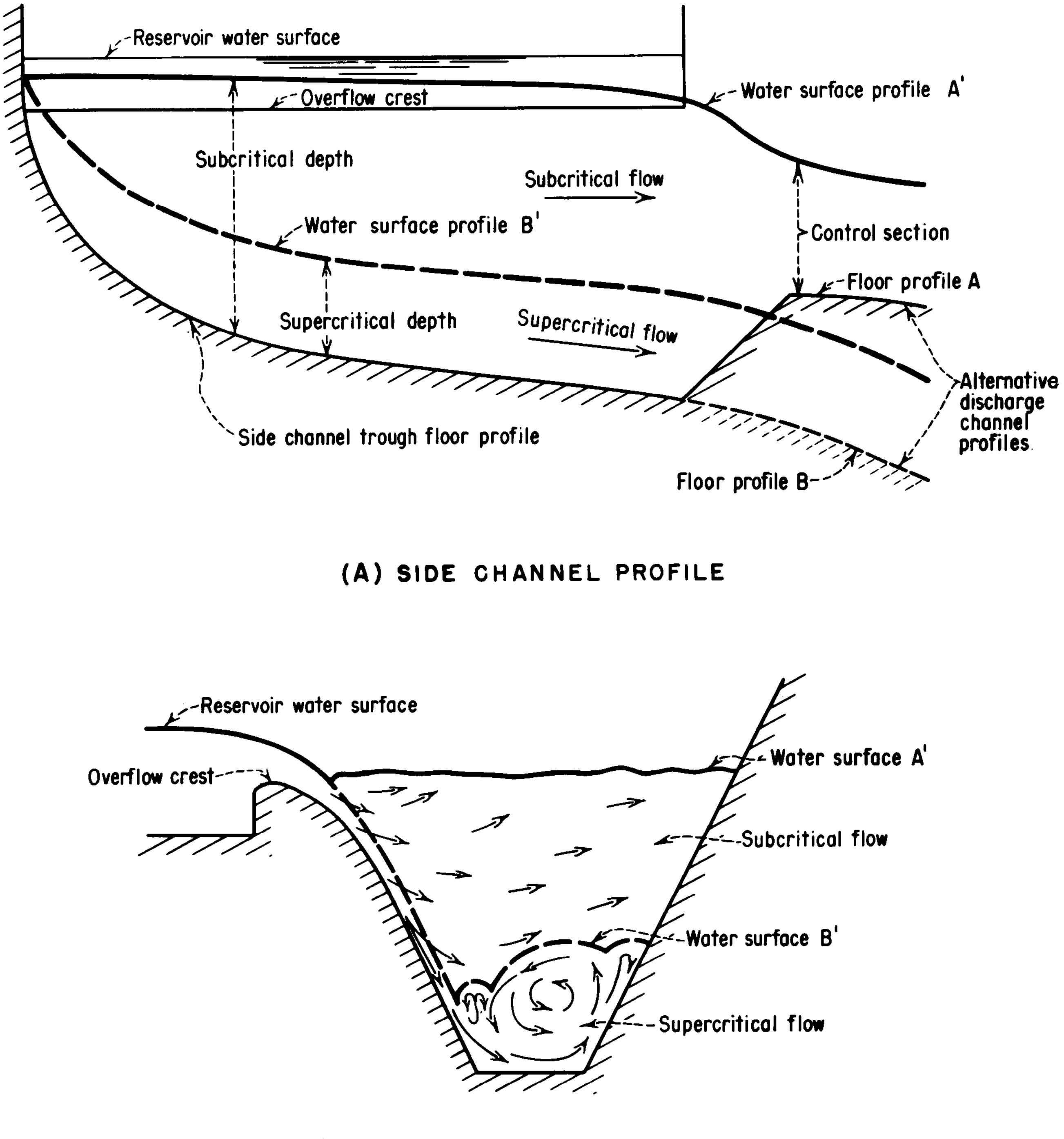


*Figure 9-24.* Side channel flow characteristics.–288-D-2418

and $Q_2$ will be known.

As in other water surface profile determinations, the depth of flow and the hydraulic characteristics of the flow will be affected by backwater influences from some control point, or by critical conditions along the reach of the channel under consideration. A control section is usually constructed at the downstream end of the side channel. After determining the depth of water at the control section, the water surface at the downstream end of the side channel can be determined by routing the water between the two points. With the depth of water at the downstream point known, depths for successive short reaches can be computed as previously described. It is assumed that a maximum of two-thirds submergence of the crest can be tolerated without affecting the water surface profile.

## D. HYDRAULICS OF DISCHARGE CHANNELS

9-17. *General.*—Discharge generally passes through the critical stage in the spillway control structure and enters the discharge channel as supercritical or shooting flow. To avoid a hydraulic jump below the control, the flow must remain at the supercritical stage throughout the length of the channel. The flow in the channel may be uniform or it may be accelerated or decelerated, depending on the slopes and dimensions of the channel and on the total drop. Where it is desired to minimize the grade to reduce excavation at the upstream end of a channel, the flow might be uniform or decelerating, followed by accelerating flow in the steep drop leading to the downstream river level. Flow at any point along the channel will depend upon the specific energy, $(d + h_v)$, available at that point. This energy will equal the total drop from the reservoir water level to the floor of the channel at the point under consideration, less the head losses accumulated to that point. The velocities and depths of flow along the channel can be fixed by selecting the grade and the cross-sectional dimensions of the channel.

The velocities and depths of free surface flow in a channel, whether an open channel or a tunnel, conform to the principle of the conservation of energy as expressed by the Bernoulli's theorem, which states: "The absolute energy of flow at any cross section is equal to the absolute energy at a downstream section plus intervening losses of energy." As applied to figure 9-25 this relationship can be expressed as follows:

$$\Delta Z + d_1 + h_{v_1} = d_2 + h_{v_2} + \Delta h_L \quad (10)$$

When the channel grades are not too steep, for practical purposes the normal depth $d_n$ can be considered equal to the vertical depth $d$, and $\Delta L$ can be considered to be the horizontal distance. The term $\Delta h_L$ includes all losses which occur in the reach of channel, such as friction, turbulence, impact, and transition losses. Since in most channels changes are made gradually, ordinarily all losses except those due to friction can be neglected. The friction loss can then be expressed as:

$$\Delta h_L = s \bullet \Delta L \quad (11)$$

where $s$ is the average friction slope expressed by either the Chezy or the Manning formula. For the reach $\Delta L$, the head loss can be expressed as $\Delta h_L = \left(\frac{s_1 + s_2}{2}\right) \Delta L$. From the Manning formula, as given in section K-2(c) of appendix K,

$$s = \left(\frac{vn}{1.486r^{2/3}}\right)^2$$

The coefficient of roughness, $n$, will depend on the nature of the channel surface. For conservative design the frictional loss should be maximized when evaluating depths of flow and minimized when evaluating the energy content of the flow. For a concrete-lined channel, a conservative value of $n$, varying from 0.014 for a channel with good alinement and a smooth finish to 0.018 for a channel with poor alinement and some unevenness in the finish, should be used in estimating the depth of flow. For determining specific energies of flow needed for designing the dissipating device, a value of $n$ of about 0.008 should be assumed.

Where only rough approximations of depths

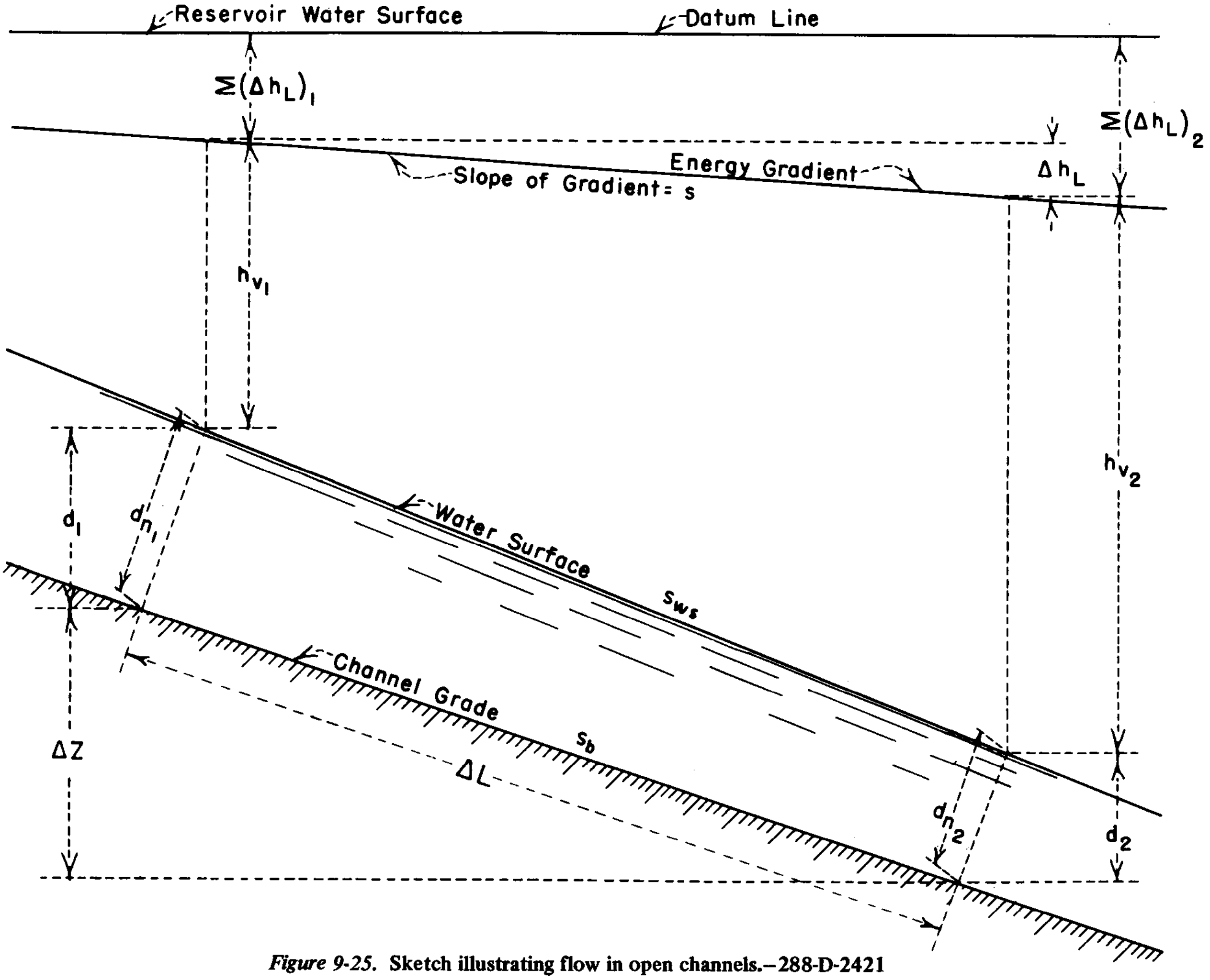
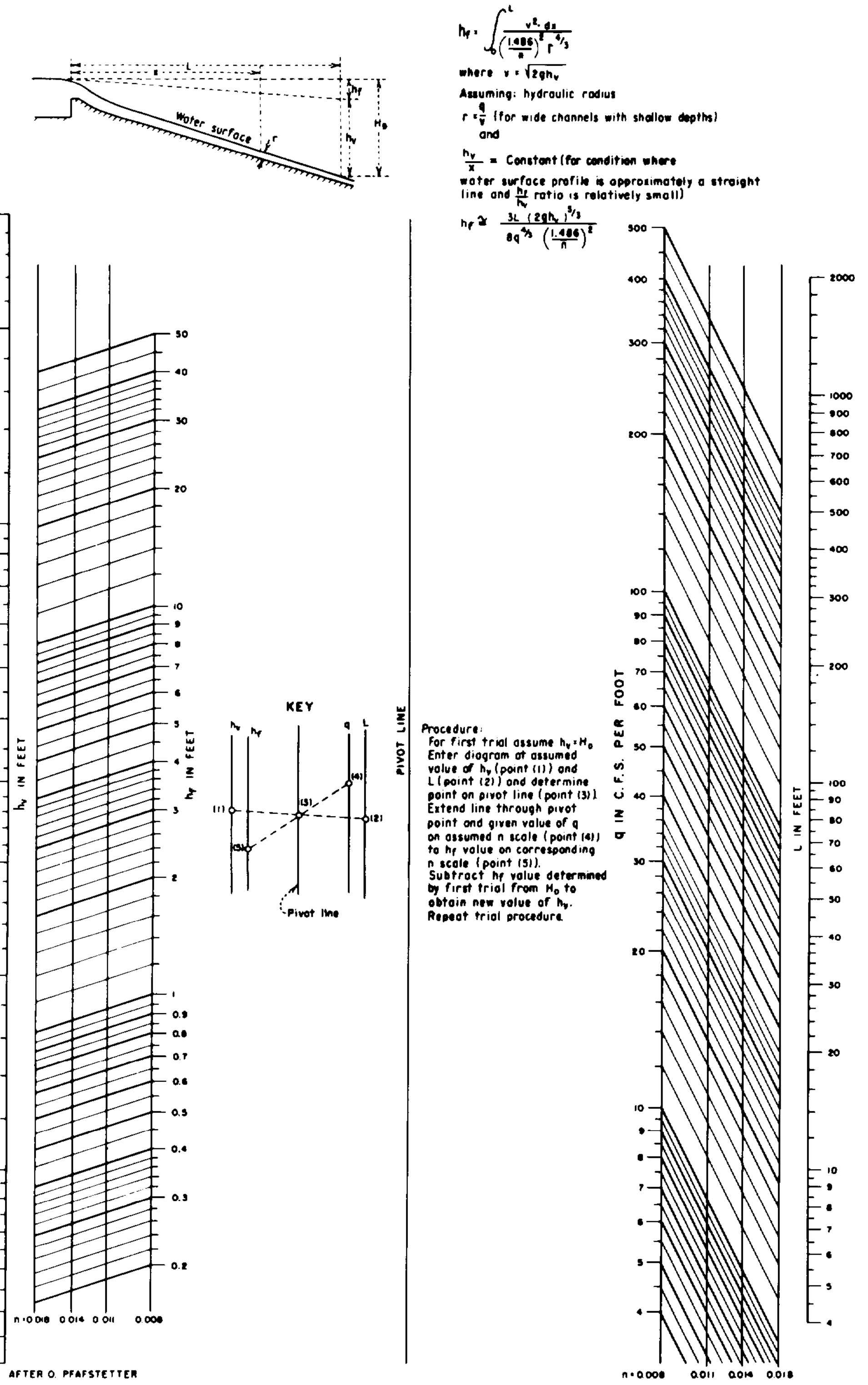


*Figure 9-25.* **Sketch illustrating flow in open channels.—288-D-2421**

and velocities of flow in a discharge channel are desired, the total head loss $\Sigma(\Delta h_L)$ to any point along the channel might be expressed in terms of the velocity head. Thus, at any section the relationship can be stated: Reservoir water surface elevation minus floor grade elevation = $d + h_v + Kh_v$. For preliminary spillway layouts, $K$ can be assumed as approximately 0.2 for determining depths of flow and 0.1 or less for evaluating the energy of flow. Rough approximations of losses can also be obtained from figure 9-26. The assumptions used in determining the losses in figure 9-26 are discussed in section K-2(f) of appendix K.

9-18. ***Open Channels.***—(a) *Profile*—The profile of an open channel is usually selected to conform to topographic and geologic site conditions. It is generally defined as straight reaches joined by vertical curves. Sharp convex and concave vertical curves should be avoided to prevent unsatisfactory flows in the channel. Convex curves should be flat enough to maintain positive pressures and thus avoid the tendency for separation of the flow from the floor. Concave curves should have a sufficiently long radius of curvature to minimize the dynamic forces on the floor brought about by the centrifugal force which results from a change in the direction of flow.

To avoid the tendency for the water to spring away from the floor and thereby reduce the surface contact pressure, the floor shape for convex curvature should be made substantially flatter than the trajectory of a free-discharging jet issuing under a head equal



$$h_f = \int_0^L \frac{v^2 \cdot dx}{\left(\frac{1.486}{n}\right)^2 r^{4/3}}$$

where $v = \sqrt{2gh_v}$

Assuming: hydraulic radius

$r = \frac{q}{v}$ (for wide channels with shallow depths)

and

$\frac{h_v}{x}$ = Constant (for condition where water surface profile is approximately a straight line and $\frac{h_f}{h_v}$ ratio is relatively small)

$$h_f \cong \frac{3L\ (2gh_v)^{3/2}}{8q^{4/3}\ \left(\frac{1.486}{n}\right)^2}$$

Procedure:

For first trial assume $h_v = H_o$. Enter diagram at assumed value of $h_v$ (point (1)) and L (point (2)) and determine point on pivot line (point (3)). Extend line through pivot point and given value of q on assumed n scale (point (4)) to $h_f$ value on corresponding n scale (point (5)). Subtract $h_f$ value determined by first trial from $H_o$ to obtain new value of $h_v$. Repeat trial procedure.

*Figure 9-26.* **Approximate losses in chutes for various values of water surface drop and channel length.–288-D-3047** to the specific energy of flow as it enters the curve. The curvature should approximate a shape defined by the equation:

$$-y = x \tan \theta + \frac{x^2}{K[4(d + h_v) \cos^2 \theta]} \qquad (12)$$

where $\theta$ is the slope angle of the floor upstream from the curve. Except for the factor $K$, the equation is that of a free-discharging trajectory issuing from an inclined orifice. To assure positive pressure along the entire contact surface of the curve, $K$ should be equal to or greater than 1.5.

For the concave curvature, the pressure exerted upon the floor surface by the centrifugal force of the flow will vary directly with the energy of the flow and inversely with the radius of curvature. An approximate relationship of these criteria can be expressed in the equations:

$$R = \frac{2qv}{p} \text{ or } R = \frac{2dv^2}{p} \qquad (13)$$

where:

$R$ = the minimum radius of curvature measured in feet,

$q$ = the discharge in c.f.s. per foot of width,

$v$ = the velocity in feet per second,

$d$ = the depth of flow in feet, and

$p$ = the normal dynamic pressure exerted on the floor, in pounds per square foot.

An assumed value of $p$ = 100 will normally produce an acceptable radius; however, a minimum radius of $10d$ is usually used. For the reverse curve at the lower end of the ogee crest, radii of not less than $5d$ have been found acceptable.

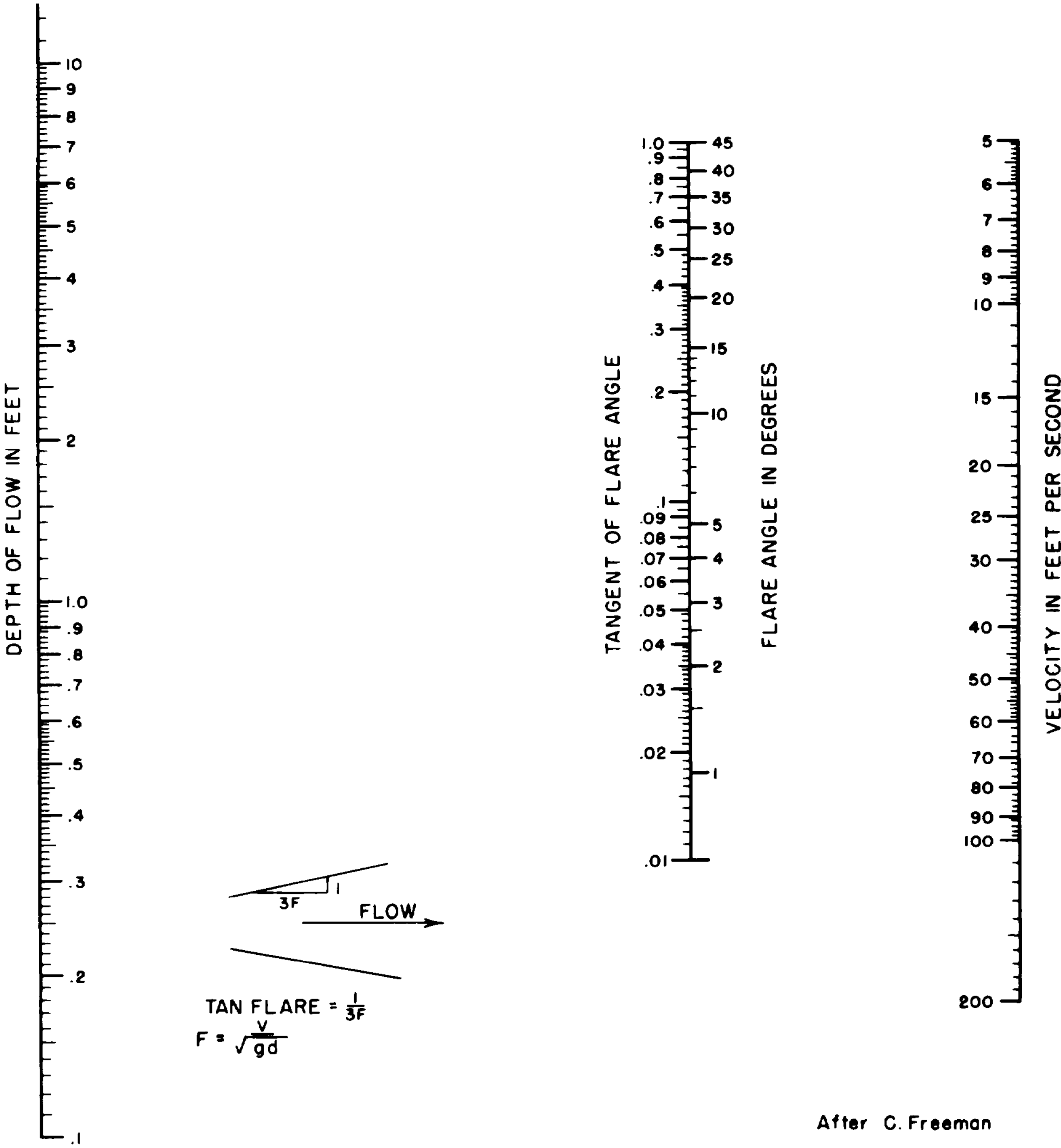
(b) *Convergence and Divergence.*—The best hydraulic performance in a discharge channel is obtained when the confining sidewalls are parallel and the distribution of flow across the channel is maintained uniform. However, economy may dictate a channel section narrower or wider than either the crest or the terminal structure, thus requiring converging or diverging transitions to fit the various components together. Sidewall convergence must be made gradual to avoid cross waves, "ride ups" on the walls, and uneven distribution of flow across the channel. Similarly, the rate of divergence of the sidewalls must be limited or else the flow will not spread to occupy the entire width of the channel uniformly, which may result in undesirable flow conditions at the terminal structure.

The inertial and gravitational forces of streamlined kinetic flow in a channel can be expressed by the Froude number parameter, $\frac{v}{\sqrt{gd}}$. Variations from streamlined flow due to outside interferences which cause an expansion or a contraction of the flow also can be related to this parameter. Experiments have shown that an angular variation of the flow boundaries not exceeding that produced by the equation,

$$\tan \alpha = \frac{1}{3F} \qquad (14)$$

will provide an acceptable transition for either a contracting or an expanding channel. In this equation, $F = \frac{v}{\sqrt{gd}}$ and $\alpha$ is the angular variation of the sidewall with respect to the channel centerline; $v$ and $d$ are the averages of the velocities and depths at the beginning and at the end of the transition. Figure 9-27 is a nomograph from which the tangent of the flare angle or the flare angle in degrees may be obtained for known values of depth and velocity of flow.

(c) *Channel Freeboard.*—In addition to using a conservative value for $n$ in determining the depth of water, a freeboard of 3 to 6 feet is usually provided to allow for air bulking, wave action, etc. When the channel is constructed on the downstream face of the dam and some overtopping of the wall will not cause damage, a minimal freeboard can be permitted. Where damage can occur, such as when the channel is



After C. Freeman

*Figure 9-27.* **Flare angle for divergent or convergent channels.–288-D-2422**

located on an abutment, the higher freeboard is needed for safety. Engineering judgment should be used in setting the height of freeboard by comparing the cost of additional wall height against the possible damage due to overtopping of the channel walls. Wherever practicable, a hydraulic model should be used in determining the wall height.

In some cases, a minimum wall height of about 10 feet is used since there is very little increase in cost of a 10-foot wall over a lower wall. Also, the fill behind the wall provides a berm for catching material sloughing off the excavation slope, thus preventing it from getting into the channel.

**9-19.** ***Tunnel Channels.***—(a) *Profile.*—Figure 9-28 shows a typical tunnel spillway channel. The profile at the upper end is curved to coincide with the profile of the control structure. The inclined portion is usually sloped at 55° from the horizontal. Steeper slopes increase the total length of the tunnel. On flatter slopes the blasted rock tends to stay on the slope during excavation rather than falling to the bottom where it can be easily removed from the tunnel.

The radius of the elbow at the invert may be determined by using equation (13); however, a radius of about 10 tunnel diameters is usually satisfactory. From the elbow, the tunnel is usually excavated on a slight downslope to the downstream portal.

(b) *Tunnel Cross Section.*—In the transition, the cross section changes from that required at the control structure to that required for the tunnel downstream from the elbow. This transition may be accomplished in one or more stages and is usually completed upstream of the elbow. Because a circular shape better resists the external loadings, it is usually desirable to attain a circular shape as soon as practicable.

The transition should be designed so that a uniform flow pattern is maintained and no negative pressures are developed which could lead to cavitation damage. No criteria have been established for determining the shape of the transition. Preliminary layouts are made using experience gained from previous tunnels. The layout should be checked using equation (10) so that no portion of the transition will flow more than 75 percent full (in area). This will allow for air bulking of the water and avoid complete filling of the tunnel. If the tunnel were to flow full, the control could move from the control structure and cause surging in the tunnel.

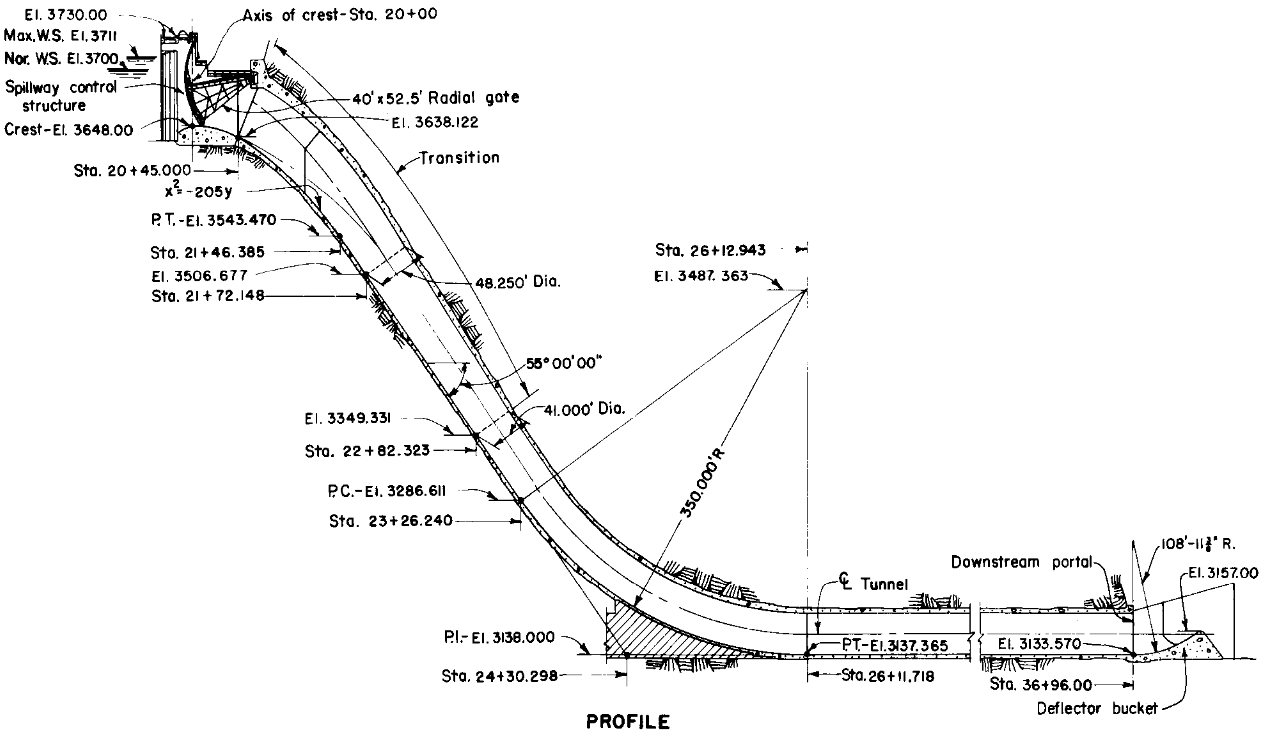


*Figure 9-28.* **Profile of typical tunnel spillway channel.—288-D-3048**

Downstream of the elbow, generally the slope of the energy gradient (equivalent to the friction slope, $s$) is greater than the slope of the tunnel invert (see fig. 9-25). This condition causes the velocity of the water to decrease and the depth of the water to increase. Usually it is not economically feasible to change the tunnel size downstream of the elbow, and therefore the conditions at the downstream end of the tunnel determine the size of the tunnel. This portion of the tunnel is frequently used as a part of the diversion scheme. If diversion flows are large, it may be economical to make the tunnel larger than required for the spillway flows. Because proper function of the spillway is essential, consideration should be given in these instances to checking of the final layout in a hydraulic model.

9-20. ***Cavitation Erosion of Concrete Surfaces.***—Concrete surfaces adjacent to high-velocity flow must be protected from cavitation erosion. Cavitation will occur when, due to some irregularities in the geometry of the flow surface, the pressure in the flowing water is reduced to the vapor pressure, about 0.363 pound per square inch absolute at 70° F. As the vapor cavities move with the flowing water into a region of higher pressure, the cavities collapse causing instantaneous positive water pressures of many thousands of pounds per square inch. These extremely high localized pressures will cause damage to any flow surface adjacent to the collapsing cavities (reference [4]).

Protection against cavitation damage may include: (1) use of surface finishes and alinements devoid of irregularities which might produce cavitation, (2) use of construction materials which are resistant to cavitation damage, or (3) admission of air into the flowing water to cushion the damaging high pressures of collapsing cavities. (See reference [5].)

## E. HYDRAULICS OF TERMINAL STRUCTURES

9-21. ***Hydraulic Jump Stilling Basins.***—Where the energy of flow in a spillway must be dissipated before the discharge is returned to the downstream river channel, the hydraulic jump basin is an effective device for reducing the exit velocity to a tranquil state. Figure 9-29 shows a hydraulic-jump stilling basin in operation at Shasta Dam in California.

The jump which will occur in a stilling basin has distinctive characteristics and assumes a definite form, depending on the energy of flow which must be dissipated in relation to the depth of the flow. Comprehensive tests have been performed by the Bureau of Reclamation [6] in connection with the hydraulic jump. The jump form and the flow characteristics can be related to the Froude number parameter, $\frac{v}{\sqrt{gd}}$. In this context $v$ and $d$ are the velocity and depth, respectively, before the hydraulic jump occurs, and $g$ is the acceleration due to gravity. Forms of the hydraulic jump phenomena for various ranges of the Froude number are illustrated on figure 9-30. The depth $d_2$, shown on the figure, is the downstream conjugate depth, or the minimum tailwater depth required for the formation of a hydraulic jump. The actual tailwater depth may be somewhat greater than this, as discussed in subsection (d).

When the Froude number of the incoming flow is equal to 1.0, the flow is at critical depth and a hydraulic jump cannot form. For Froude numbers from 1.0 up to about 1.7, the incoming flow is only slightly below critical depth, and the change from this low stage to the high stage flow is gradual and manifests itself only by a slightly ruffled water surface. As the Froude number approaches 1.7, a series of small rollers begin to develop on the surface, which become more intense with increasingly higher values of the number. Other than the surface roller phenomena, relatively smooth flows prevail throughout the Froude number range up to about 2.5. Stilling action for the range of Froude numbers from 1.7 to 2.5 is

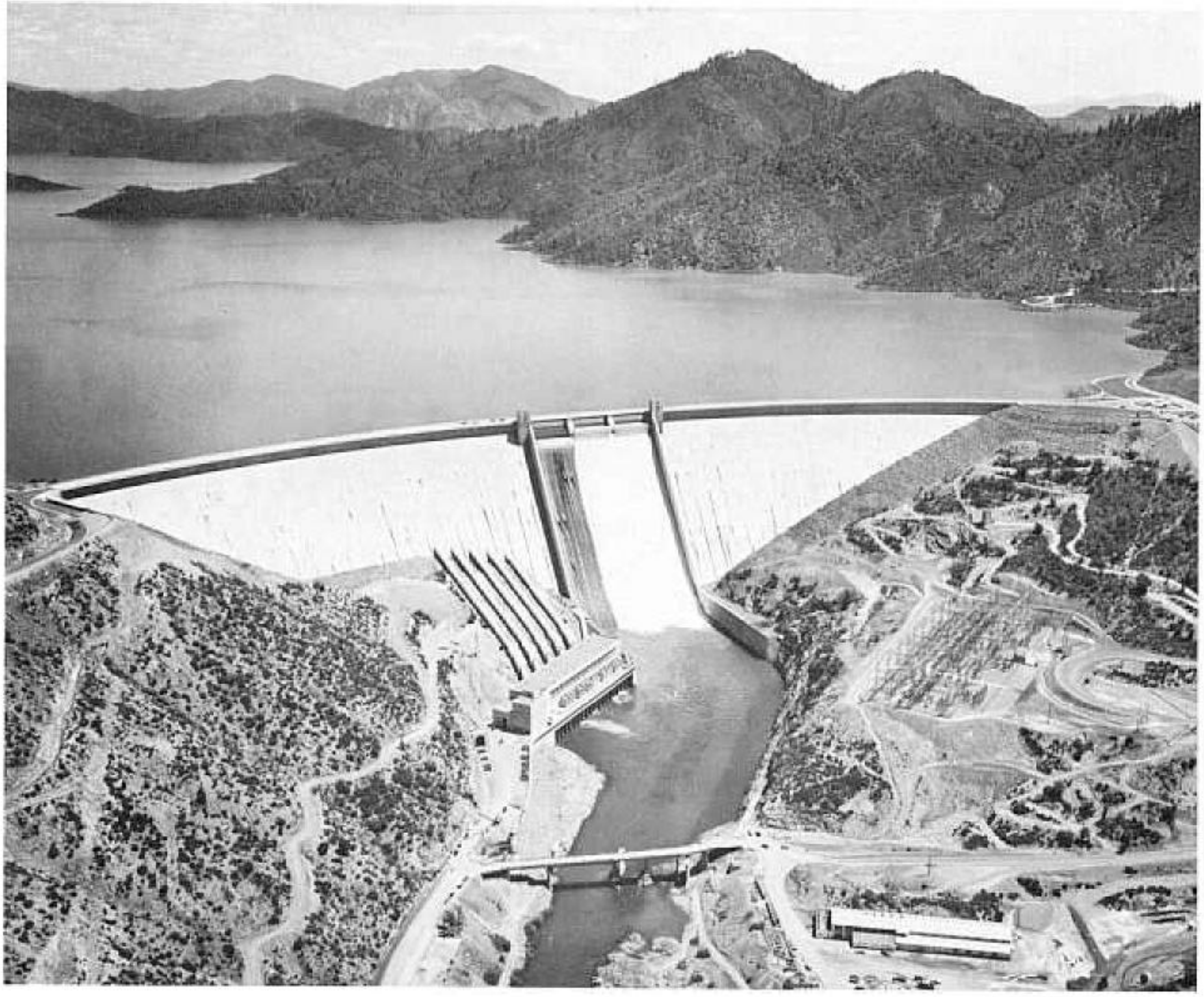


***Figure 9-29.*** **Hydraulic jump stilling basin in operation at Shasta Dam in California.—K1007-CV**

designated as form A on figure 9-30.

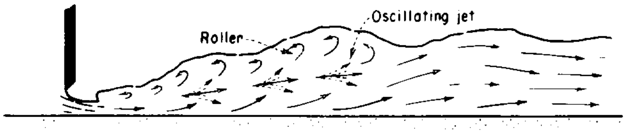
For Froude numbers between 2.5 and 4.5 an oscillating form of jump occurs, the entering jet intermittently flowing near the bottom and then along the surface of the downstream channel. This oscillating flow causes objectionable surface waves which carry considerably beyond the end of the basin. The action represented through this range of flows is designated as form B on figure 9-30.

For the range of Froude numbers for the incoming flow between 4.5 and 9, a stable and well-balanced jump occurs. Turbulence is confined to the main body of the jump, and the water surface downstream is comparatively smooth. As the Froude number increases above 9, the turbulence within the jump and the surface roller becomes increasingly active, resulting in a rough water surface with strong surface waves downstream from the jump. Stilling action for the range of Froude numbers between 4.5 and 9 is designated as form C on figure 9-30 and that above 9 is designated as form D.

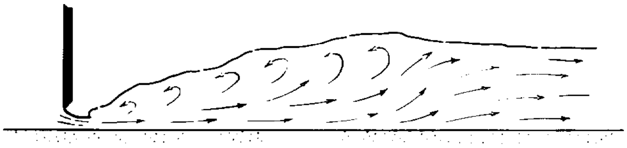
Figure 9-31 plots relationships of conjugate depths and velocities for the hydraulic jump in a rectangular channel or basin. Also indicated on the figure are the ranges for the various forms of hydraulic jump described above.

(a) *Hydraulic Design of Stilling Basins.*—Stilling basins are designed to provide suitable stilling action for the various forms of

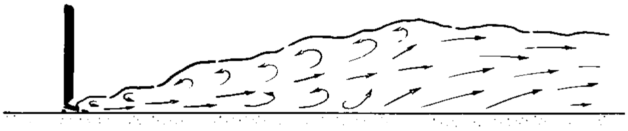
$F_1$ BETWEEN 1.7 AND 2.5
FORM A - PRE-JUMP STAGE



$F_1$ BETWEEN 2.5 AND 4.5
FORM B - TRANSITION STAGE



$F_1$ BETWEEN 4.5 AND 9.0
FORM C-RANGE OF WELL BALANCED JUMPS



$F_1$ HIGHER THAN 9.0
FORM D-EFFECTIVE JUMP BUT
ROUGH SURFACE DOWNSTREAM

*Figure 9-30.* **Characteristic forms of hydraulic jump related to the Froude number.–288-D-2423**

hydraulic jump previously discussed. Type I basin, shown on figure 9-32, is a rectangular channel without any accessories such as baffles or sills and is designed to confine the entire length of the hydraulic jump. Seldom are stilling basins of this type designed since it is possible to reduce the length and consequently the cost of the basin by the installation of baffles and sills, as discussed later for types II, III, and IV basins. The type of basin best suited for a particular situation will depend upon the Froude number.

(1) *Basins for Froude numbers less than 1.7.*–For a Froude number of 1.7 the conjugate depth $d_2$ is about twice the incoming depth, or about 40 percent greater than the critical depth. The exit velocity $v_2$ is about one-half the incoming velocity, or 30 percent less than the critical velocity. No special stilling basin is needed to still flows where the incoming flow Froude factor is less than 1.7, except that the channel lengths beyond the point where the depth starts to change should be not less than about $4d_2$. No baffles or other dissipating devices are needed.

(2) *Basins for Froude numbers between 1.7 and 2.5.*–Flow phenomena for basins where the incoming flow factors are in the Froude number range between 1.7 and 2.5 will be in the form designated as the prejump stage, as illustrated on figure 9-30. Since such flows are not attended by active turbulence, baffles or sills are not required. The basin should be a type I basin as shown on figure 9-32 and it should be sufficiently long and deep to contain the flow prism while it is undergoing retardation. Depths and lengths shown on figure 9-32 will provide acceptable basins.

(3) *Basins for Froude numbers between 2.5 and 4.5.*–Jump phenomena where the incoming flow factors are in the Froude number range between 2.5 and 4.5 are designated as transition flow stage, since a true hydraulic jump does not fully develop. Stilling basins to accommodate these flows are the least effective in providing satisfactory dissipation, since the attendant wave action ordinarily cannot be controlled by the usual basin devices. Waves generated by the flow phenomena will persist beyond the end of the basin and must often be dampened by means of wave suppressors.

Where a stilling device must be provided to dissipate flows for this range of Froude number, the basin shown on figure 9-33 which is designated as type IV basin, has proved to be relatively effective for dissipating the bulk of the energy of flow. However, the wave action propagated by the oscillating flow cannot be entirely dampened. Auxiliary wave dampeners or wave suppressors must sometimes be employed to provide smooth surface flow downstream. Because of the tendency of the jump to sweep out and as an aid in suppressing wave action, the water depths in the basin should be about 10 percent greater than the computed conjugate depth.

Often the need for utilizing the type IV basin in design can be avoided by selecting stilling basin dimensions which will provide

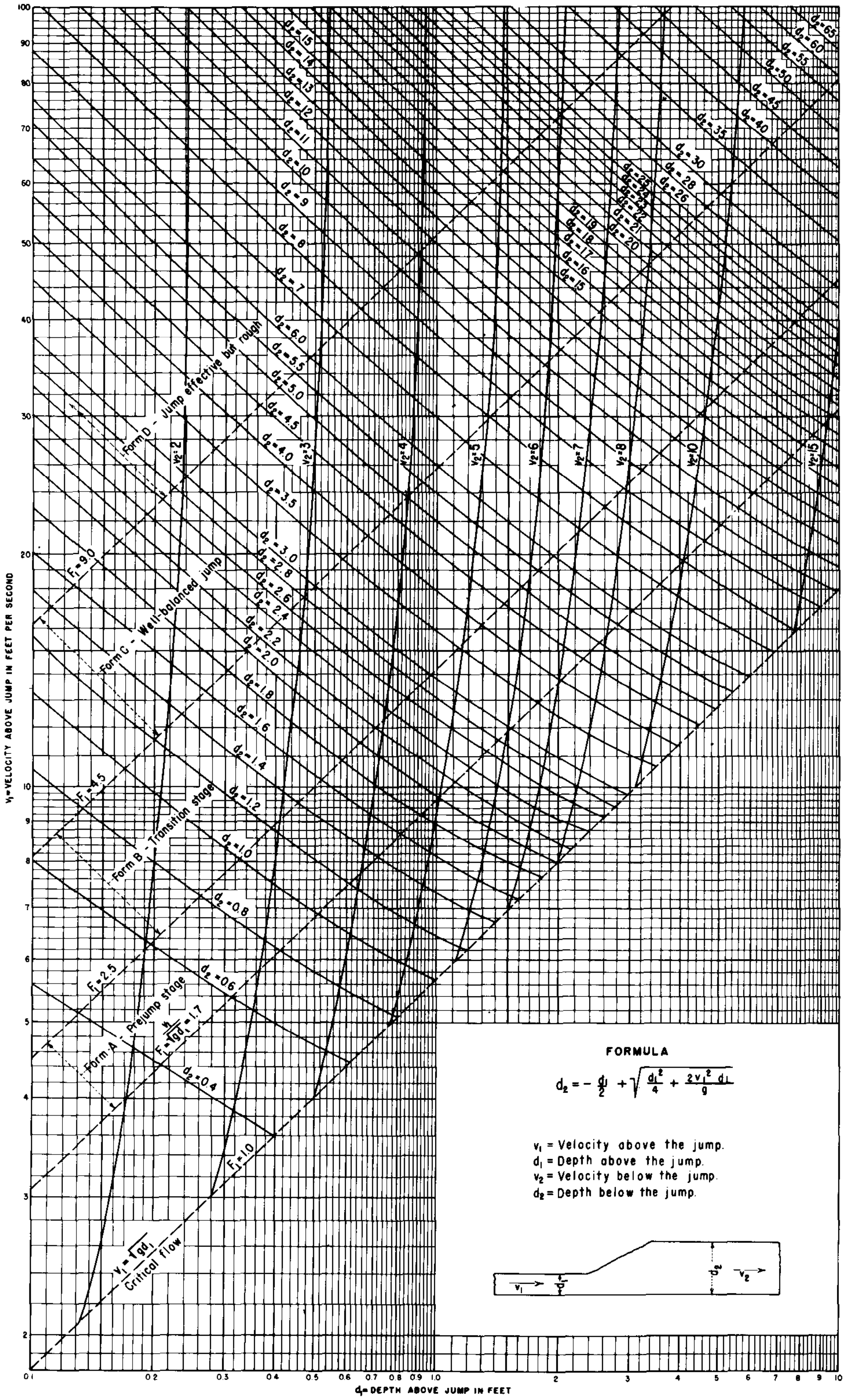
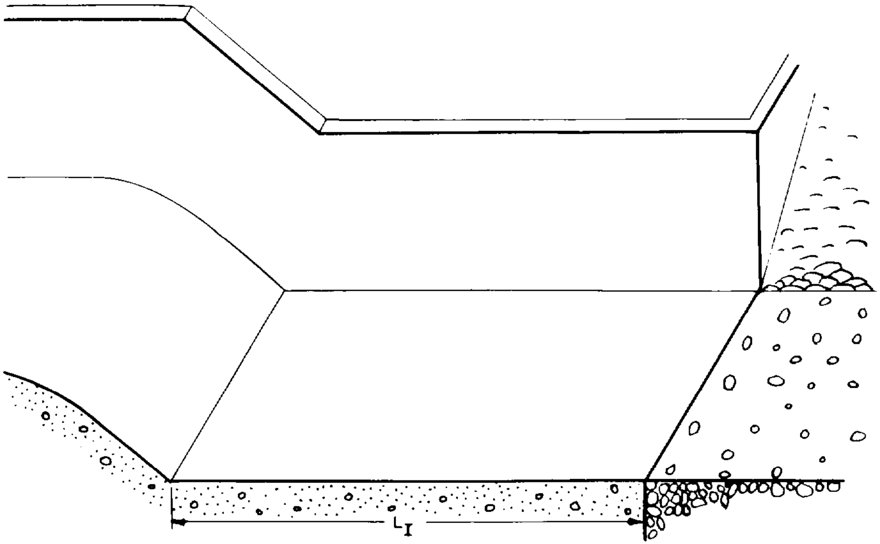


*Figure 9-31.* Relations between variables in hydraulic jumps for rectangular channels.—288-D-2424



(A) TYPE I BASIN DIMENSIONS

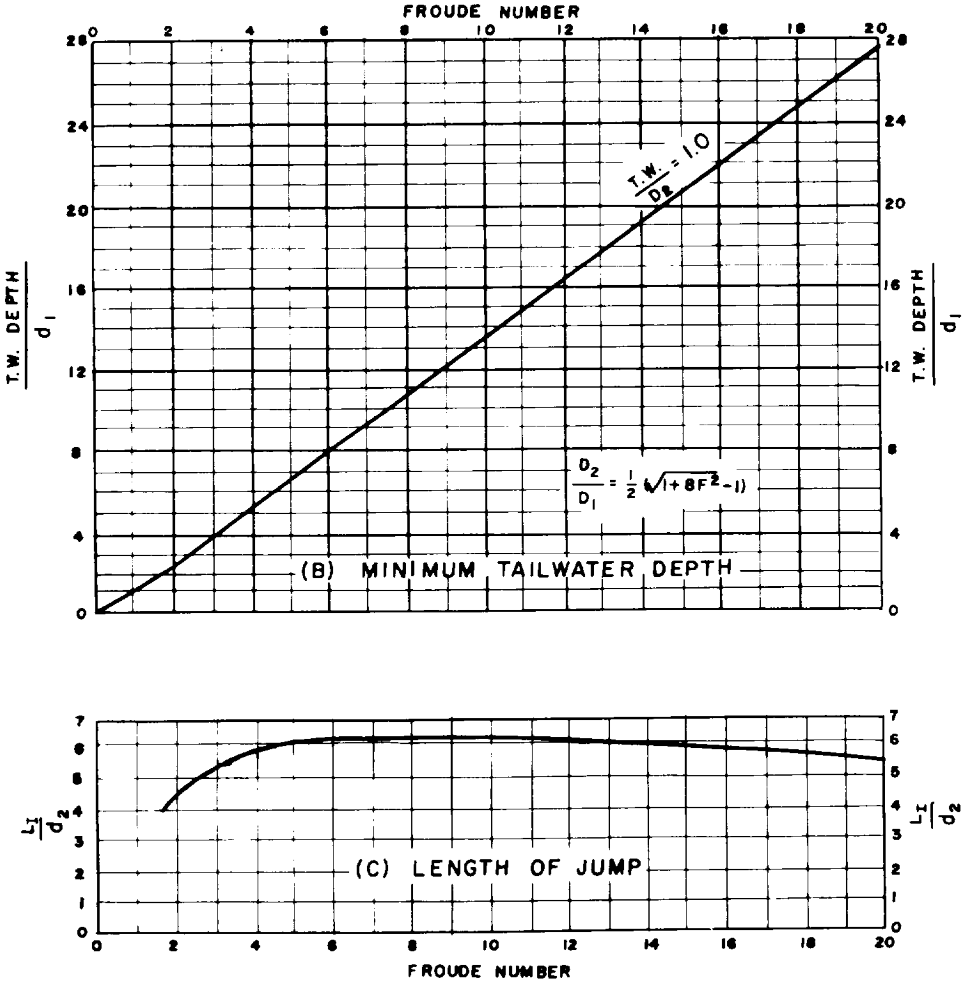
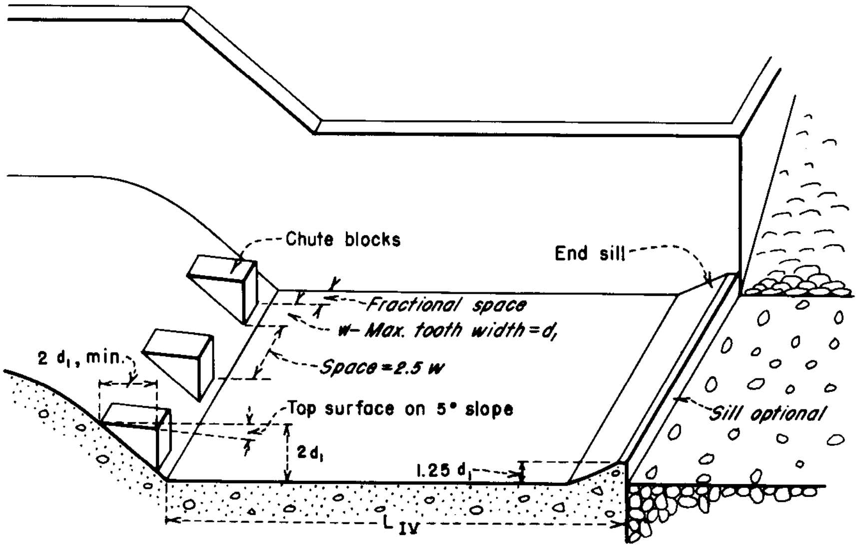


*Figure 9-32.* Type I stilling basin characteristics.–288-D-3049



(A) TYPE IV BASIN DIMENSIONS

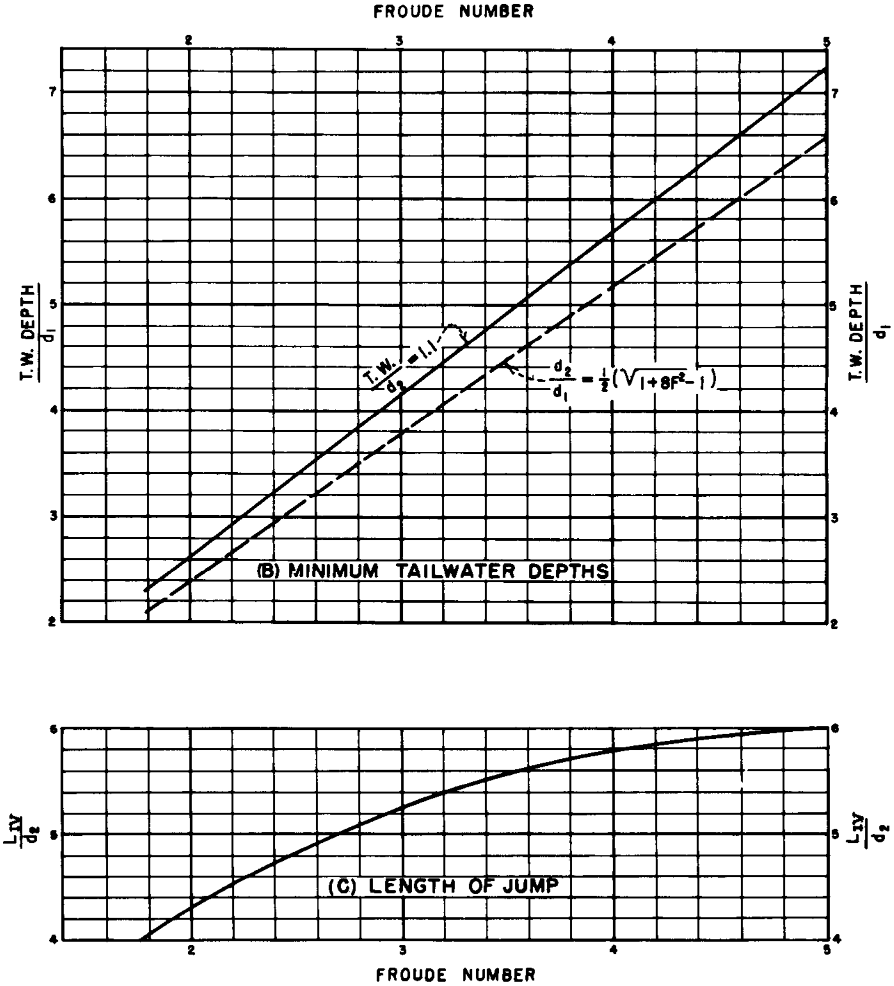


*Figure 9-33.* **Stilling basin characteristics for Froude numbers between 2.5 and 4.5.—288-D-3050**

flow conditions which fall outside the range of transition flow. For example, with an 800-c.f.s. capacity spillway where the specific energy at the upstream end of the basin is about 15 feet and the velocity into the basin is about 30 feet per second, the Froude number will be 3.2 for a basin width of 10 feet. The Froude number can be raised to 4.6 by widening the basin to 20 feet. The selection of basin width then becomes a matter of economics as well as hydraulic performance.

(4) *Basins for Froude numbers higher than 4.5.*—For basins where the Froude number value of the incoming flow is higher than 4.5, a true hydraulic jump will form. The installation of accessory devices such as blocks, baffles, and sills along the floor of the basin produces a stabilizing effect on the jump, which permits shortening the basin and provides a factor of safety against sweep-out due to inadequate tailwater depth.

The basin shown on figure 9-34, which is designated as a type III basin, can be adopted where incoming velocities do not exceed 50 feet per second. This basin utilizes chute blocks, impact baffle blocks, and an end sill to shorten the jump length and to dissipate the high-velocity flow within the shortened basin length. This basin relies on dissipation of energy by the impact blocks and also on the turbulence of the jump phenomena for its effectiveness. Because of the large impact forces to which the baffles are subjected by the impingement of high incoming velocities and because of the possibility of cavitation along the surfaces of the blocks and floor, the use of this basin must be limited to heads where the velocity does not exceed 50 feet per second.

Cognizance must be taken of the added loads placed upon the structure floor by the dynamic force brought against the upstream face of the baffle blocks. This dynamic force will approximate that of a jet impinging upon a plane normal to the direction of flow. The force, in pounds, may be expressed by the formulas:

$$\text{Force} = 2wA(d_1 + h_{v_1}) \qquad (15)$$

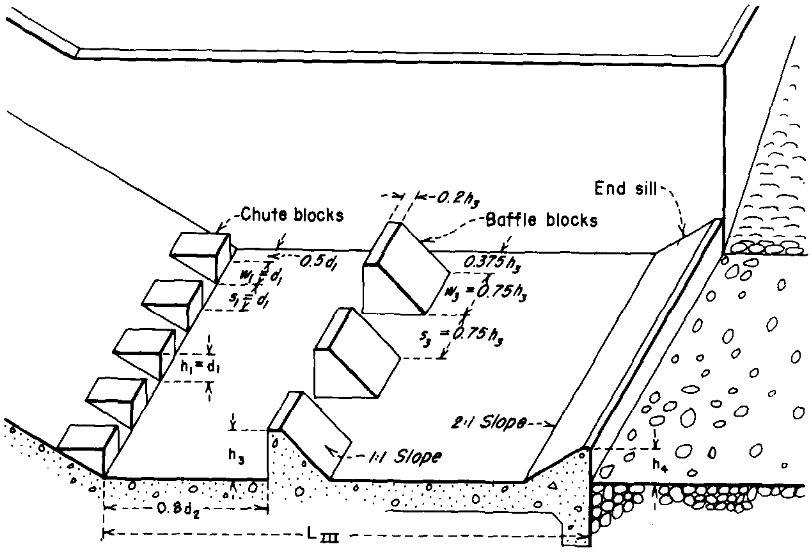
where:

$w$ = the unit weight of water,
$A$ = the area of the upstream face of the block, and
$(d_1 + h_{v_1})$ = the specific energy of the flow entering the basin.

Negative pressure on the back face of the blocks will further increase the total load. However, since the baffle blocks are placed a distance equal to $0.8d_2$ beyond the start of the jump, there will be some cushioning effect by the time the incoming jet reaches the blocks and the force will be less than that indicated by the above equation. If the full force computed by equation (15) is used, the negative pressure force may be neglected.

Where incoming velocities exceed 50 feet per second, or where impact baffle blocks are not employed, the basin designated as type II on figure 9-35 can be adopted. Because the dissipation is accomplished primarily by hydraulic jump action, the basin length will be greater than that indicated for the type III basin. However, the chute blocks and dentated end sill will still be effective in reducing the length from that which would be necessary if they were not used. Because of the reduced margin of safety against sweep-out, the water depth in the basin should be about 5 percent greater than the computed conjugate depth.

(b) *Rectangular Versus Trapezoidal Stilling Basin.*—The utilization of a trapezoidal stilling basin in lieu of a rectangular basin may often be proposed where economy favors sloped side lining over vertical wall construction. Model tests have shown, however, that the hydraulic jump action in a trapezoidal basin is much less complete and less stable than it is in the rectangular basin. In the trapezoidal basin the water in the triangular areas along the sides of the basin adjacent to the jump is not opposed by the incoming high-velocity jet. The jump, which tends to occur vertically, cannot spread



(A) TYPE III BASIN DIMENSIONS

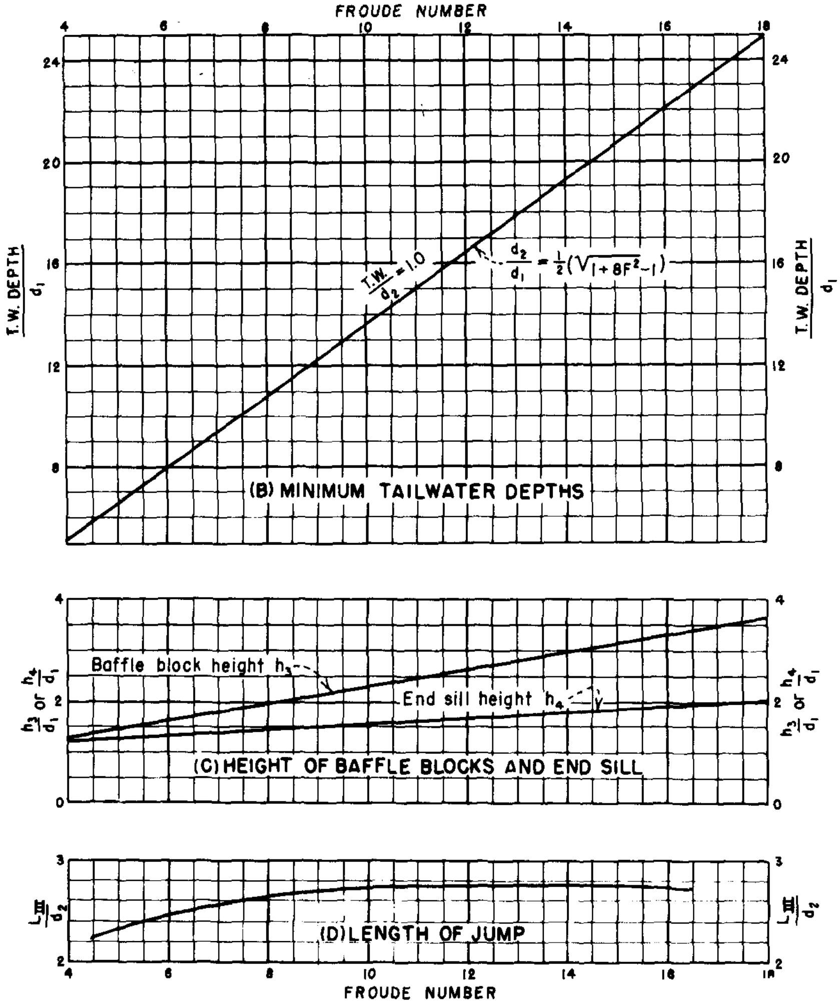


*Figure 9-34.* **Stilling basin characteristics for Froude numbers above 4.5 where incoming velocity does not exceed 50 feet per second.–288-D-3051**

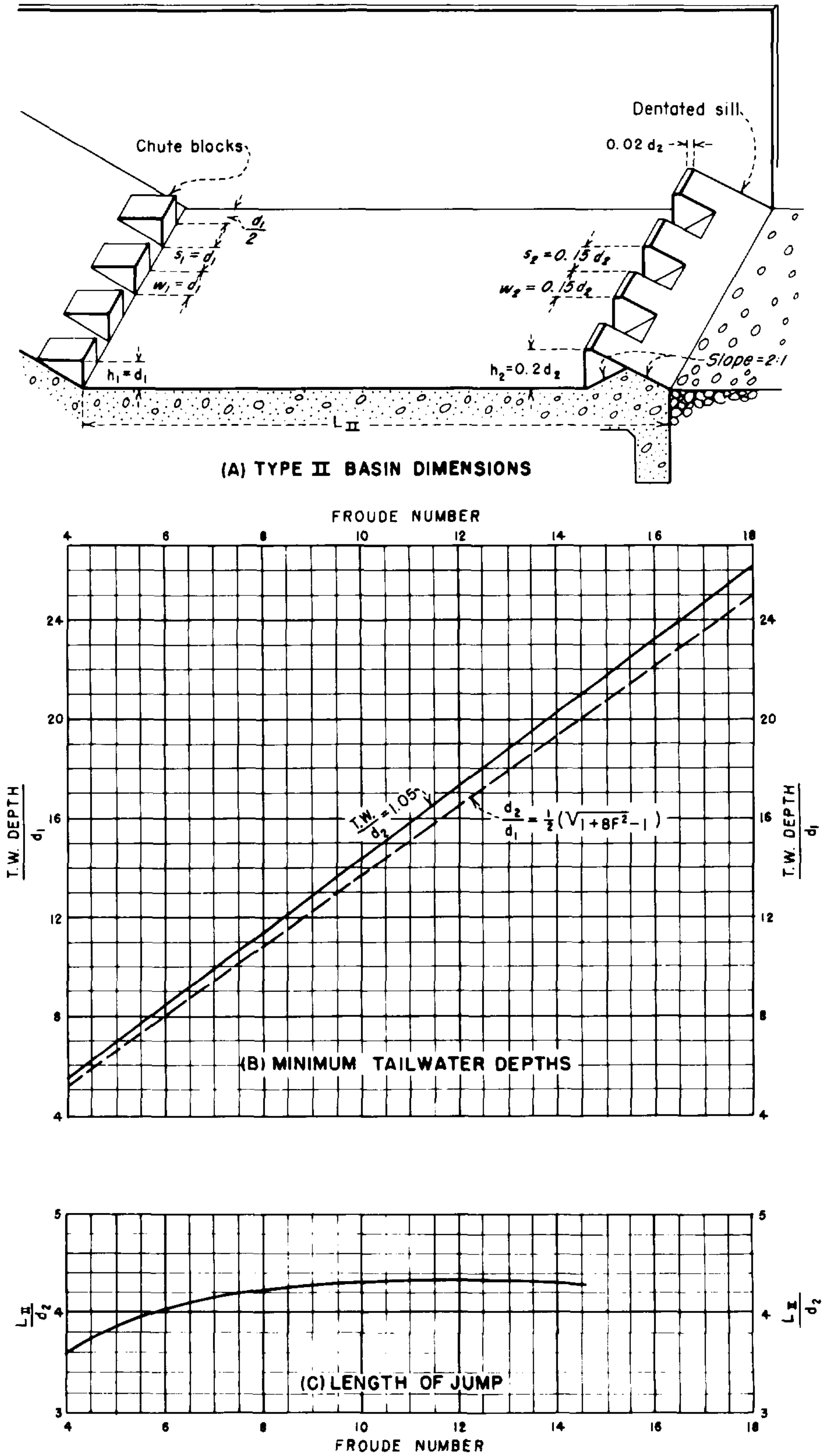


*Figure 9-35.* **Stilling basin characteristics for Froude numbers above 4.5.—288-D-3052**

sufficiently to occupy the side areas. Consequently, the jump will form only in the central portion of the basin, while areas along the outside will be occupied by upstream-moving flows which ravel off the jump or come from the lower end of the basin. The eddy or horizontal roller action resulting from this phenomenon tends to interfere and interrupt the jump action to the extent that there is incomplete dissipation of the energy and severe scouring can occur beyond the basin. For good hydraulic performance, the sidewalls of a stilling basin should be vertical or as near vertical as is practicable.

(c) *Basin Depths by Approximate Methods.*–The nomograph shown on figure 9-36 will aid in determining approximate basin depths for various basin widths and for various differences between reservoir and tailwater levels. Plottings are shown for the condition of no loss of head to the upstream end of the stilling basin, and for 10, 20, and 30 percent loss. (These plottings are shown on the nomographs as scales A, B, C, and D, respectively.) The required conjugate depths, $d_2$, will depend on the specific energy available at the entrance of the basin, as determined by the procedure discussed in section 9-17. Where only a rough determination of basin depths is needed, the choice of the loss to be applied for various spillway designs may be generalized as follows:

(1) For a design of an overflow spillway where the basin is directly downstream from the crest, or where the chute is not longer than the hydraulic head, consider no loss of head.

(2) For a design of a channel spillway where the channel length is between one and five times the hydraulic head, consider 10 percent loss of head.

(3) For a design of a spillway where the channel length exceeds five times the hydraulic head, consider 20 percent loss of head.

The nomograph on figure 9-36 gives values of $d_2$, the conjugate depth for the hydraulic jump. Tailwater depths for the various types of basin described in subsection (a) above should be increased as noted in that subsection.

(d) *Tailwater Considerations.*–The tailwater rating curve, which gives the stage-discharge relationship of the natural stream below the dam, is dependent on the natural conditions along the stream and ordinarily cannot be altered by the spillway design or by the release characteristics. As discussed in section 9-7(d), retrogression or aggradation of the river below the dam, which will affect the ultimate stage-discharge conditions, must be recognized in selecting the tailwater rating curve to be used for stilling basin design. Usually riverflows which approach the maximum design discharges have never occurred, and an estimate of the tailwater rating curve must either be extrapolated from known conditions or computed on the basis of assumed or empirical criteria. Thus, the tailwater rating curve at best is only approximate, and factors of safety to compensate for variations in tailwater must be included in the design.

For a given stilling basin design, the tailwater depth for each discharge seldom corresponds to the conjugate depth needed to form a perfect jump. The basin floor level must therefore be selected to provide tailwater depths which most nearly agree with the conjugate depths. Thus, the relative shapes and relationships of the tailwater curve to the conjugate depth curve will determine the required minimum depth to the basin floor. This is illustrated on figure 9-37. The tailwater rating curve is shown in (A) as curve 1, and a conjugate depth versus discharge curve for a basin of a certain width, *W*, is represented by curve 3. Since the basin must be made deep enough to provide for conjugate depth (or some greater depth to include a factor of safety) at the maximum spillway design discharge, the curves will intersect at point D. For lesser discharges the tailwater depth will be greater than the conjugate depth, thus providing an excess of tailwater which is conducive to the formation of a so-called drowned jump. (With the drowned jump condition, instead of achieving good jump-type dissipation by the intermingling of the upstream and downstream flows, the incoming jet plunges to the bottom and carries along the entire length of the basin floor at high velocity.) If the basin floor is

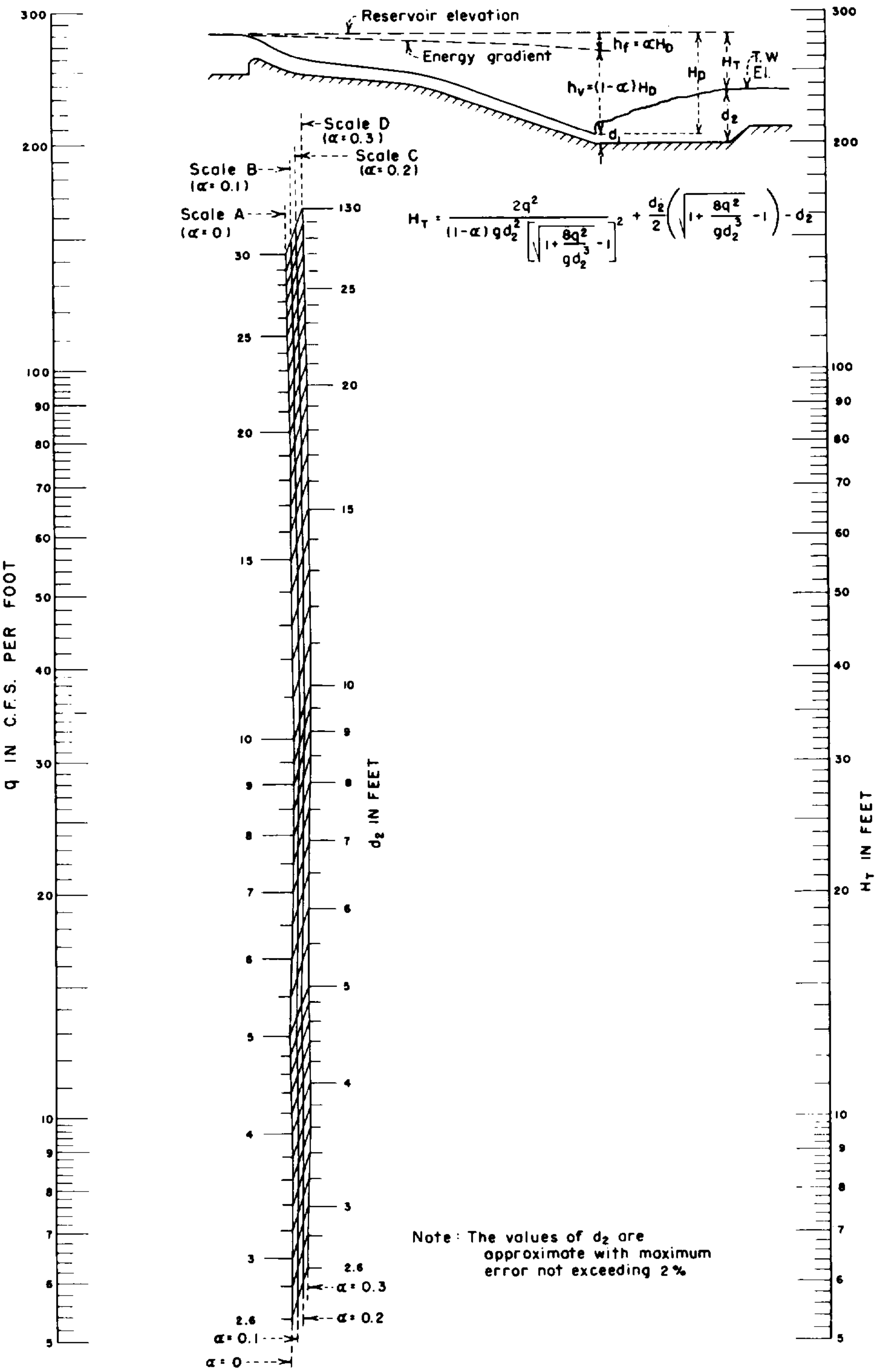


*Figure 9-36.* Stilling basin depths versus hydraulic heads for various channel losses.—288-D-3053

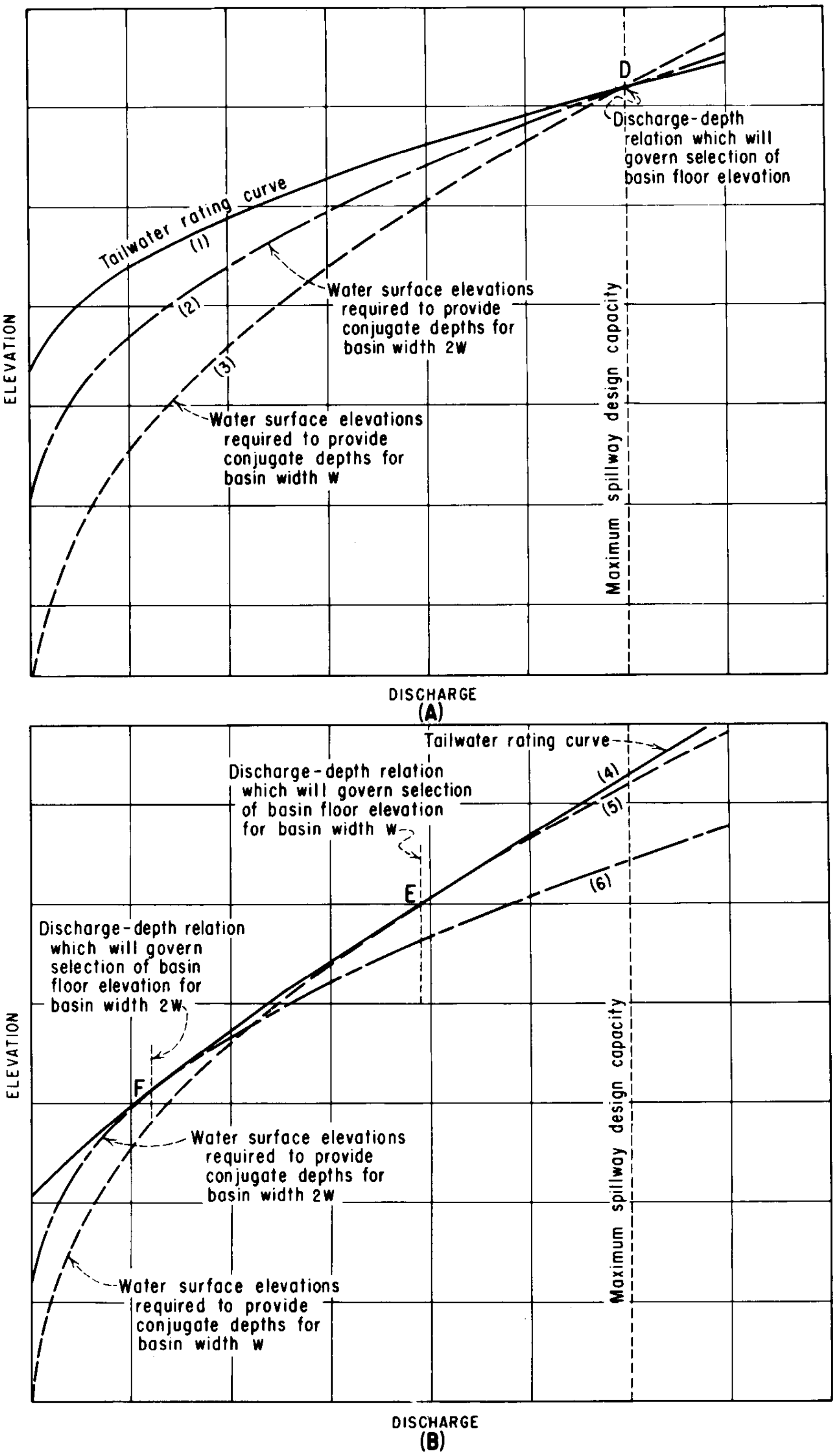


*Figure 9-37.* **Relationships of conjugate depth curves to tailwater rating curves.–288-D-2439**

made higher than indicated by the position of curve 3 on the figure, the depth curve and tailwater rating curve will intersect to the left of point D, thus indicating an excess of tailwater for smaller discharges and a deficiency of tailwater for higher discharges.

As an alternative to the selected basin which is represented by curve 3, a wider basin might be considered for which the conjugate depth curve 2 will apply. This design will provide a shallower basin, in which the conjugate depths will more nearly match the tailwater depths for all discharges. The choice of basin widths, of course, involves consideration of economics, as well as hydraulic performance.

Where a tailwater rating curve shaped similar to that represented by curve 4 on figure 9-37(B) is encountered, the level of the stilling basin floor must be determined for some discharge other than the maximum design capacity. If the tailwater rating curve were made to intersect the required water surface elevation at the maximum design capacity, as in figure 9-37(A), there would be insufficient tailwater depth for most smaller discharges. In this case the basin floor elevation is selected so that there will be sufficient tailwater depth for all discharges. For the basin of width $W$ whose required tailwater depth is represented by curve 5, the position of the floor would be selected so that the two curves would coincide at the discharge represented by point E on the figure. For all other discharges the tailwater depth will be in excess of that needed for forming a satisfactory jump. Similarly, if a basin width of $2W$ were considered, the basin floor level would be selected so that curve 6 would intersect the tailwater rating curve at point F. Here also, the selection of basin widths should be based on economic aspects as well as hydraulic performance.

Where exact conjugate depth conditions for forming the jump cannot be attained, the question of the relative desirability of having insufficient tailwater depth as compared to having excessive tailwater depth should be considered. With insufficient tailwater the back pressure will be deficient and sweep-out of the basin will occur. With an excess of tailwater the jump will be formed and energy dissipation within the basin will be quite complete until the drowned jump phenomenon becomes critical. Chute blocks, baffles, and end sills will further assist in energy dissipation, even with a drowned jump.

(e) *Stilling Basin Freeboard.*—A freeboard of 5 to 10 feet is usually provided to allow for surging and wave action in the stilling basin. For smaller, low-head basins, the required freeboard will be nearer the lower value, whereas the higher value will normally be used for larger, high-head spillways. A minimum freeboard may be used if overtopping by the waves will not cause significant damage. Engineering judgment should be used in setting the height of freeboard by comparing the cost of additional wall height against possible damage caused by overflow of the stilling basin walls. Wherever practical, a hydraulic model should be used in determining the amount of freeboard.

**9-22. *Deflector Buckets.***—Where the spillway discharge may be safely delivered directly to the river without providing an energy dissipating or stilling device, the jet is often projected beyond the structure by a deflector bucket or lip. Flow from these deflectors leaves the structure as a free-discharging upturned jet and falls into the stream channel some distance from the end of the spillway. The path the jet assumes depends on the energy of flow available at the lip and the angle at which the jet leaves the bucket.

With the origin of the coordinates taken at the end of the lip, the path of the trajectory is given by the equation:

$$y = x \tan \theta - \frac{x^2}{K[4(d + h_v) \cos^2 \theta]} \qquad (16)$$

where:

$\theta$ = the angle between the curve of the bucket at the lip and the horizontal (or lip angle), and

$K$ = a factor, equal to 1 for the theoretical jet.

To compensate for loss of energy and velocity reduction due to the effect of air resistance, internal turbulences, and disintegration of the jet, a value for $K$ of about 0.85 should be assumed.

The horizontal range of the jet at the level of the lip is obtained by making $y$ in equation (16) equal to zero. Then:

$$\begin{aligned} x &= 4K\,(d + h_v) \tan\theta \cos^2\theta \\ &= 2K\,(d + h_v) \sin 2\theta \end{aligned} \qquad (17)$$

The maximum value of $x$ will be equal to $2K\,(d + h_v)$ when $\theta$ is 45°. The lip angle is influenced by the bucket radius and the height of the lip above the bucket invert. It usually varies from 20° to 45°, with 30° being the preferred angle.

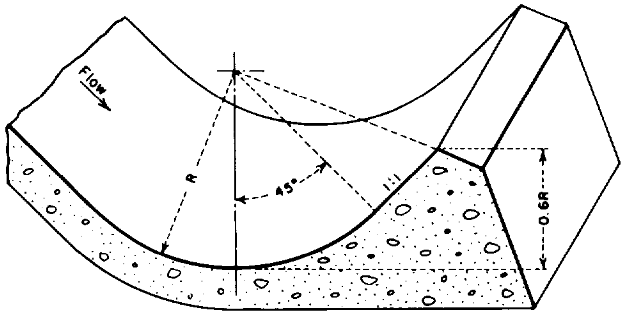
The bucket radius should be made long enough to maintain concentric flow as the water moves around the curve. The rate of curvature must be limited similar to that of a vertical curve in a discharge channel (sec. 9-18), so that the floor pressures will not alter the streamline distribution of the flow. The minimum radius of curvature can be determined from equation (13), except that values of $p$ not exceeding 500 pounds per square foot will produce values of the radius which have proved satisfactory in practice. However, the radius should not be less than five times the depth of water. Structurally, the cantilever bucket must be of sufficient strength to withstand this normal dynamic force in addition to the other applied forces.

Figure 9-38 shows the deflector at the end of the spillway tunnel at Hungry Horse Dam in operation.

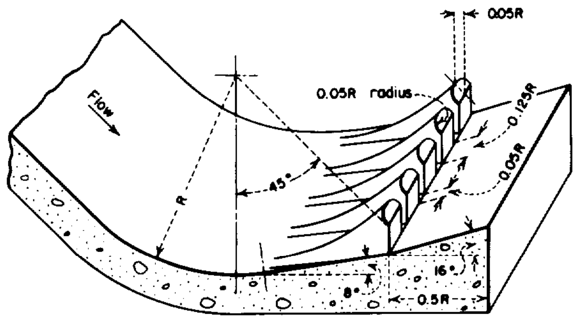
**9-23. *Submerged Bucket Energy Dissipators.***—When the tailwater depth is too great for the formation of a hydraulic jump, dissipation of the high energy of flow can be effected by the use of a submerged bucket deflector. The hydraulic behavior in this type of dissipator is manifested primarily by the formation of two rollers; one is on the surface moving counterclockwise (if flow is to the right) and is contained within the region above the curved bucket, and the other is a ground roller moving in a clockwise direction and is situated downstream from the bucket. The movements of the rollers, along with the intermingling of the incoming flows, effectively dissipate the high energy of the water and prevent excessive scouring downstream from the bucket.

*Figure 9-38.* **Deflector bucket in operation for the spillway at Hungry Horse Dam in Montana.--P447-105-5924**

Two types of roller bucket have been developed and model tested [6]. Their shape and dimensional arrangements are shown on figure 9-39. The general nature of the dissipating action for each type is represented on figure 9-40. Hydraulic action of the two buckets has the same characteristics, but distinctive features of the flow differ to the extent that each has certain limitations. The high-velocity flow leaving the deflector lip of the solid bucket is directed upward. This creates a high boil on the water surface and a violent ground roller moving clockwise downstream from the bucket. This ground roller continuously pulls loose material back towards the lip of the bucket and keeps some of the intermingling material in a constant state of agitation. In the slotted bucket the high-velocity jet leaves the lip at a flatter angle, and only a part of the high-velocity flow finds

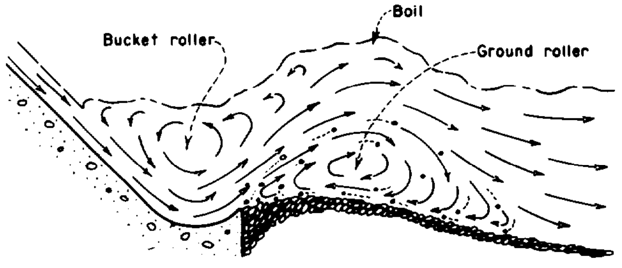


(A) SOLID BUCKET

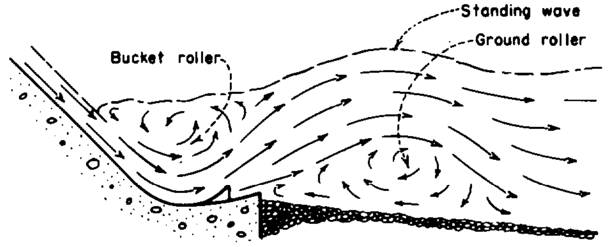


(B) SLOTTED BUCKET

*Figure 9-39.* Submerged bucket energy dissipators.–288-D-2430



(A) SOLID TYPE BUCKET



(B) SLOTTED TYPE BUCKET

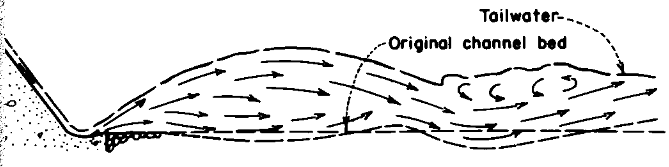
*Figure 9-40.* Hydraulic action in solid and slotted buckets.–288-D-2431

its way to the surface. Thus, a less violent surface boil occurs and there is a better dispersion of flow in the region above the ground roller which results in less concentration of high-energy flow throughout the bucket and a smoother downstream flow.

Use of a solid bucket dissipator may be objectionable because of the abrasion on the concrete surfaces caused by material which is swept back along the lip of the deflector by the ground roller. In addition, the more turbulent surface roughness induced by the severe surface boil carries farther down the river, causing objectionable eddy currents which contribute to riverbank sloughing. Although the slotted bucket provides better energy dissipation with less severe surface and streambed disturbances, it is more sensitive to sweep-out at lower tailwaters and is conducive to a diving and scouring action at excessive tailwaters. This is not the case with the solid bucket. Thus, the tailwater range which will provide good performance with the slotted bucket is much narrower than that of the solid bucket. A solid bucket dissipator should not be used wherever the tailwater limitations of the slotted bucket can be met. Therefore, only the design of the slotted bucket will be discussed.

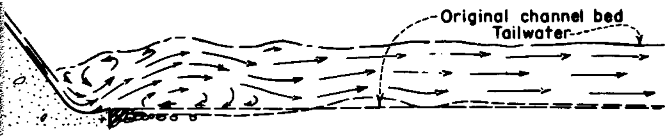
Flow characteristics of the slotted bucket are illustrated on figure 9-41. For deficient tailwater depths the incoming jet will sweep the surface roller out of the bucket and will produce a high-velocity flow downstream, both along the water surface and along the riverbed. This action is depicted as stage (A) on figure 9-41. As the tailwater depth is increased, there will be a depth at which instability of flow will occur, where sweep-out and submergence will alternately prevail. To obtain continuous operation at the submerged stage, the minimum tailwater depth must be above this instable state. Flow action within the acceptable operating stage is depicted as stage (B) on figure 9-41.

When the tailwater becomes excessively deep, the phenomenon designated as diving flow will occur. At this stage the jet issuing from the lip of the bucket will no longer rise and continue along the surface but intermittently will become depressed and dive



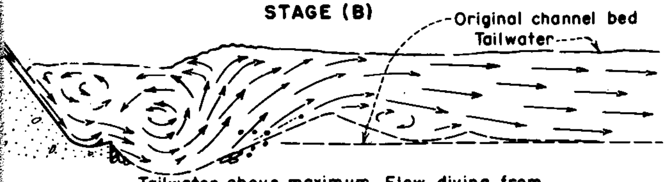
Tailwater below minimum. Flow sweeps out.

STAGE (A)



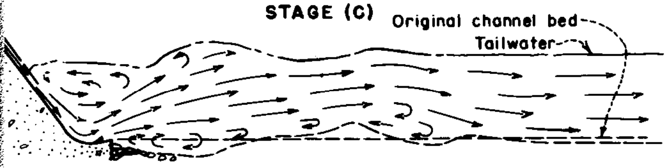
Tailwater below average but above minimum. Within normal operating range.

STAGE (B)



Tailwater above maximum. Flow diving from apron scours channel

STAGE (C)



Tailwater same as in C. Diving jet is lifted by ground roller. Scour hole backfills similar to B. Cycle repeats.

STAGE (D)

*Figure 9-41.* **Flow characteristics in a slotted bucket.–288-D-2432**

to the riverbed. The position of the downstream roller will change with the change in position of the jet. It will occur at the surface when the jet dives and will form along the river bottom as a ground roller when the jet rides the surface. Scour will occur in the streambed at the point of impingement when the jet dives but will be filled in by the ground roller when the jet rides. The characteristic flow pattern for the diving stage is depicted in (C) and (D) of figure 9-41. Maximum tailwater depths must be limited to forestall the diving flow phenomenon.

The design of the slotted bucket involves determination of the radius of curvature of the bucket and the allowable range of tailwater depths. These criteria, as determined from experimental results, are plotted on figure 9-42 in relation to the Froude number parameter. The Froude number values are for flows at the point where the incoming jet enters the bucket. Symbols are defined on figure 9-43.

9-24. *Plunge Pools.*—When a free-falling overflow nappe drops almost vertically into a pool in a riverbed, a plunge pool will be scoured to a depth which is related to the height of the fall, the depth of tailwater, and the concentration of the flow [7]. Depths of scour are influenced initially by the erodibility of the stream material or the bedrock and by the size or the gradation of sizes of any armoring material in the pool. However, the armoring or protective surfaces of the pool will be progressively reduced by the abrading action of the churning material to a size which will be scoured out, and the ultimate scour depth will, for all practical considerations, stabilize at a limiting depth irrespective of the material size. An empirical approximation based on experimental data has been developed by Veronese [8] for limiting scour depths, as follows:

$$d_s = 1.32 H_T^{0.225} q^{0.54} \qquad (18)$$

where:

$d_s$ = the maximum depth of scour below tailwater level in feet,

$H_T$ = the head from reservoir level to tailwater level in feet, and

$q$ = the discharge in c.f.s. per foot of width.

Three Bureau of Reclamation dams which have plunge pools for energy dissipators have been tested in hydraulic models. Reports of the results of these tests are given in references [9], [10], and [11].

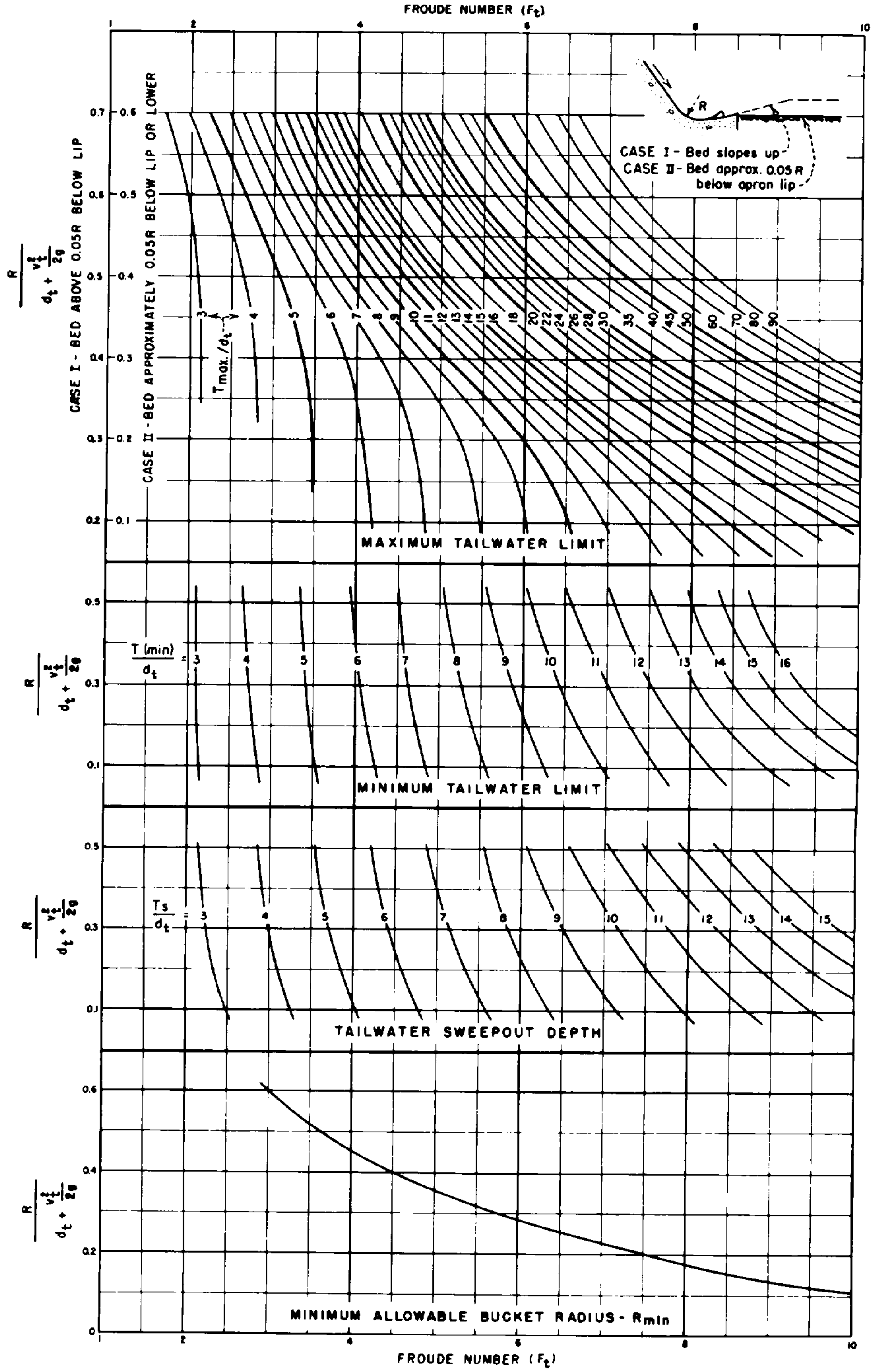


*Figure 9-42.* Limiting criteria for slotted bucket design.–288-D-2433

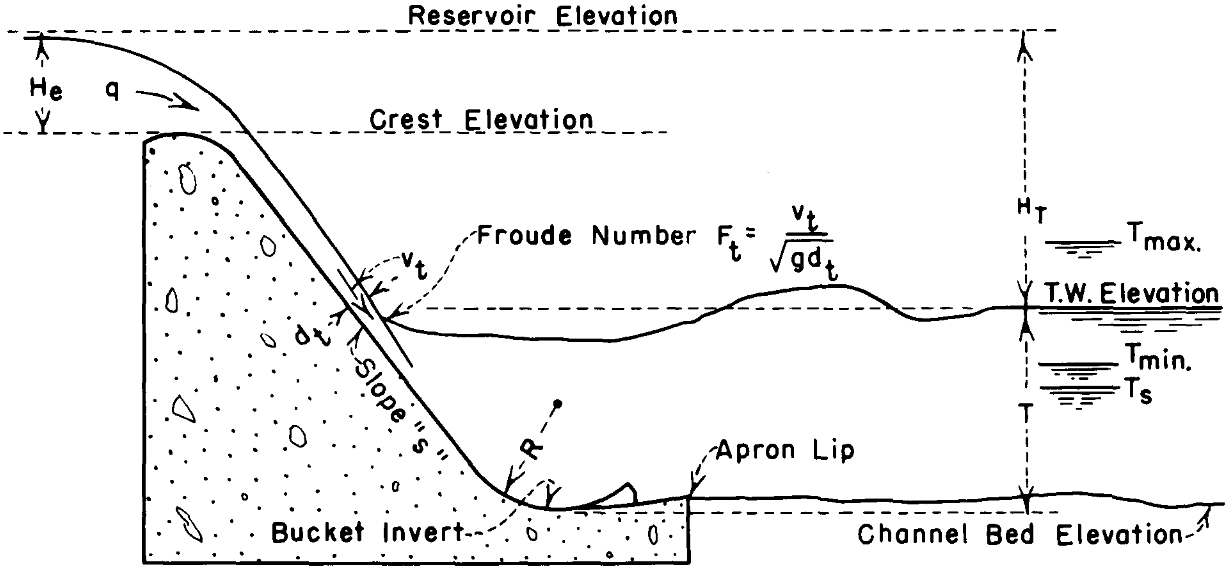


*Figure 9-43.* Definition of symbols–submerged bucket.–288-D-2434

## F. HYDRAULICS OF MORNING GLORY (DROP INLET) SPILLWAYS

9-25. *General Characteristics.*–The flow conditions and discharge characteristics of a morning glory spillway are unique in the respect that, in normal operation, the control changes as the head changes. As brought out in the following discussion, at low heads the crest is the control and the orifice and tunnel serve only as the discharge channel; whereas at progressively higher heads the orifice and then the tunnel serves as the control. Because of this uniqueness the hydraulics of morning glory spillways are discussed separately from other spillway components.

Typical flow conditions and discharge characteristics of a morning glory spillway are represented on figure 9-44. As illustrated on the discharge curve, crest control (condition 1) will prevail for heads between the ordinates of *a* and *g*; orifice control (condition 2) will govern for heads between the ordinates of *g* and *h*; and the spillway tunnel will flow full for heads above the ordinate of *h* (represented as condition 3).

Flow characteristics of a morning glory spillway will vary according to the proportional sizes of the different elements. Changing the diameter of the crest will change the curve *ab* on figure 9-44 so that the ordinate of *g* on curve *cd* will be either higher or lower. For a larger diameter crest, greater flows can be discharged over the crest at low heads and orifice control will occur with a lesser head on the crest, tending to fill up the transition above the orifice. Similarly, by altering the size of the orifice, the position of curve *cd* will shift, changing the head above which orifice control will prevail. If the orifice is made of sufficient size that curve *cd* is moved to coincide with or lie to the right of point *j*, the control will shift directly from the crest to the downstream end of the tunnel. The details of the hydraulic flow characteristics are discussed in following sections.

9-26. *Crest Discharge.*–For small heads, flow over the morning glory spillway is governed by the characteristics of crest discharge. The throat, or orifice, will flow partly full and the flow will cling to the sides of the shaft. As the discharge over the crest increases, the overflowing annular nappe will become thicker, and eventually the nappe flow will converge into a solid vertical jet. The point

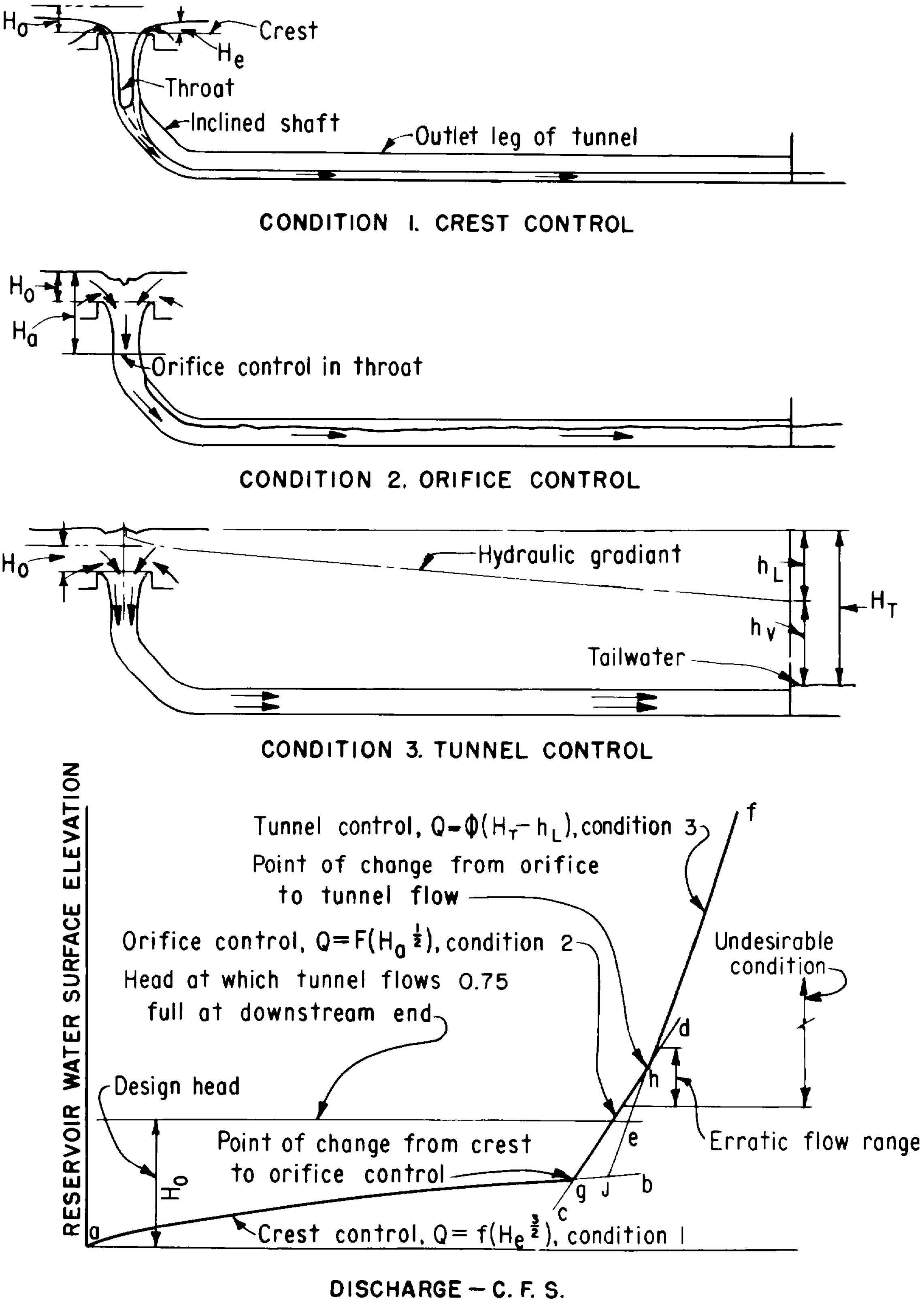


*Figure 9-44.* Flow and discharge characteristics of a morning glory spillway.—288-D-3054

where the annular nappe joins the solid jet is called the crotch. After the solid jet forms, a "boil" will occupy the region above the crotch: both the crotch and the top of the boil become progressively higher with larger discharges. For high heads the crotch and boil may almost flood out, showing only a slight depression and eddy at the surface.

Until such time as the nappe converges to form a solid jet, free-discharging weir flow prevails. After the crotch and boil form, submergence begins to affect the weir flow and ultimately the crest will drown out. Flow is then governed either by the nature of the contracted jet which is formed by the overflow entrance, or by the shape and size of the throat as determined by the crest profile if it does not conform to the jet shape. Vortex action must be minimized to maintain converging flow into the inlet. Guide piers are often employed along the crest for this purpose [12, 13, 14].

If the crest profile conforms to the shape of the lower nappe of a jet flowing over a sharp-crested circular weir, the discharge characteristics for flow over the crest and through the throat can be expressed as:

$$Q = CL\,H^{3/2} \qquad (3)$$

where $H$ is the head measured either to the apex of the under nappe of the overflow, to the spring point of the circular sharp-crested weir, or to some other established point on the overflow. Similarly, the choice of the length $L$ is related to some specific point of measurement such as the length of the circle at the apex, along the periphery at the upstream face of the crest, or along some other chosen reference line. The value of $C$ will change with different definitions of $L$ and $H$. If $L$ is taken at the outside periphery of the overflow crest (the origin of the coordinates in figure 9-45) and if the head is measured to the apex of the overflow shape, equation (3) can be written:

$$Q = C_o(2\pi R_s)H_o{}^{3/2} \qquad (19)$$

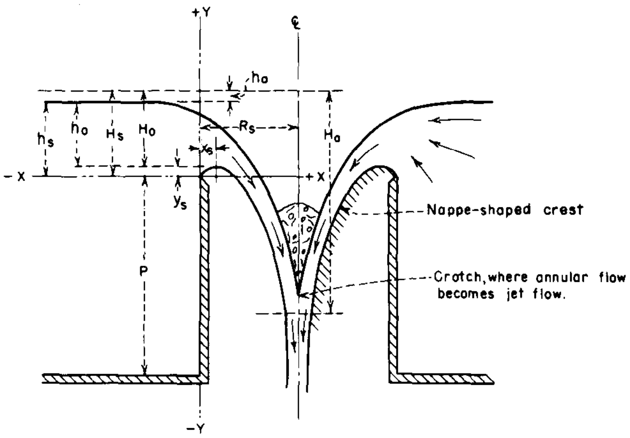


*Figure 9-45.* **Elements of nappe-shaped profile for a circular crest.–288-D-2440**

It will be apparent that the coefficient of discharge for a circular crest differs from that for a straight crest because of the effects of submergence and back pressure incident to the joining of the converging flows. Thus, $C_o$ must be related to both $H_o$ and $R_s$, and can be expressed in terms of $\frac{H_o}{R_s}$. The relationship of $C_o$, as determined from model tests [15], to values of $\frac{H_o}{R_s}$ for three conditions of approach depth is plotted on figure 9-46. These coefficients are valid only if the crest profile conforms to that of the jet flowing over a sharp-crested circular weir at $H_o$ head and if aeration is provided so that subatmospheric pressures do not exist along the lower nappe surface contact.

When the crest profile conforms to the profile of the under nappe shape for an $H_o$ head over the crest, free flow prevails for $\frac{H_o}{R_s}$ ratios up to approximately 0.45, and crest control governs. As the $\frac{H_o}{R_s}$ ratio increases above 0.45 the crest partly submerges and flow showing characteristics of a submerged crest is the controlling condition. When the $\frac{H_o}{R_s}$ ratio approaches 1.0 the water surface above the crest is completely submerged. For this and higher stages of $\frac{H_o}{R_s}$ the flow phenomena is that of orifice flow. The weir formula, $Q = CLH^{3/2}$,

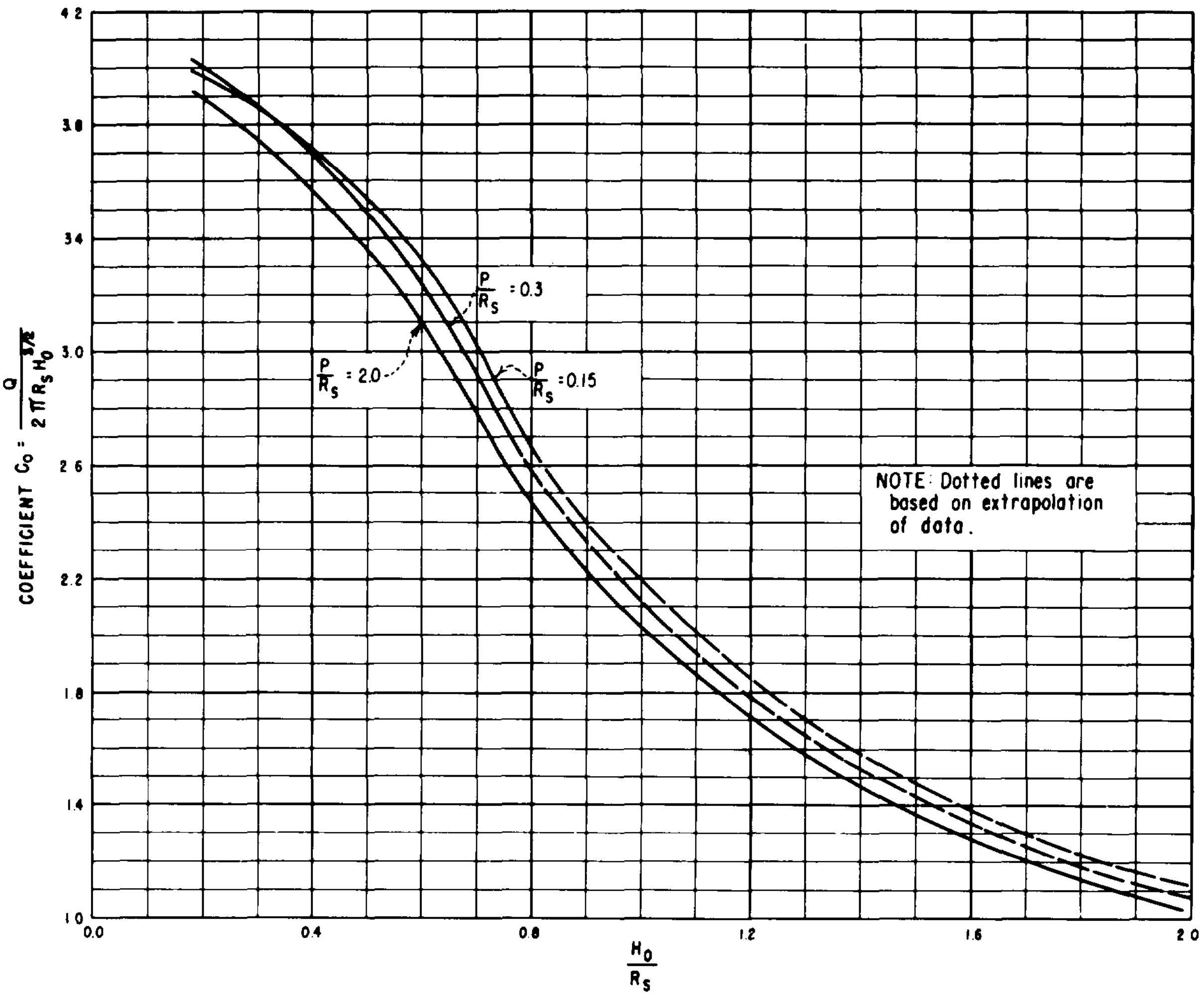


*Figure 9-46.* **Relationship of circular crest coefficient $C_o$ to $\frac{H_o}{R_s}$ for different approach depths (aerated nappe).–288-D-2441**

is used as the measure of flow over the crest regardless of the submergence, by using a coefficient which reflects the flow conditions through the various $\frac{H}{R_s}$ ranges. Thus, from figure 9-46 it will be seen that the crest coefficient is only slightly changed from that normally indicated for values of $\frac{H_o}{R_s}$ less than 0.45, but reduces rapidly for the higher $\frac{H_o}{R_s}$ ratios.

It will be noted that for most conditions of flow over a circular crest the coefficient of discharge increases with a reduction of the approach depth, whereas the opposite is true for a straight crest. For both crests a shallower approach lessens the upward vertical velocity component and consequently suppresses the contraction of the nappe. However, for the circular crest the submergence effect is reduced because of a depressed upper nappe surface, giving the jet a quicker downward impetus, which lowers the position of the crotch and increases the discharge.

Coefficients for partial heads of $H_e$ on the crest can be determined from figure 9-47 to prepare a discharge-head relationship. The designer must be cautious in applying the above criteria, since subatmospheric pressure or submergence effects may alter the flow conditions differently for variously shaped

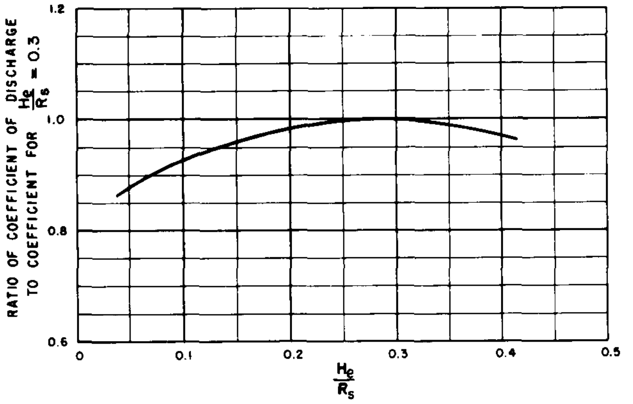


*Figure 9-47.* **Circular crest coefficient of discharge for other than design head.—288-D-2446**

profiles. These criteria, therefore, should not be applied for flow conditions where $\frac{H_e}{R_s}$ exceeds 0.4.

9-27. ***Crest Profiles.***—In this discussion, the crest profile is considered to extend from the crest to the orifice control, and forms the transition to the orifice. Values of coordinates to define the shape of the lower surface of a nappe flowing over an aerated sharp-crested circular weir for various conditions of $\frac{P}{R_s}$ and $\frac{H_s}{R_s}$ are shown in tables 9-2, 9-3, and 9-4. These data are based on experimental tests [15] conducted by the Bureau of Reclamation. The relationships of $H_s$ to $H_o$ are shown on figure 9-48. Typical upper and lower nappe profiles for various values of $\frac{H_s}{R_s}$ are plotted on figure 9-49 in terms of $\frac{x}{H_s}$ and $\frac{y}{H_s}$ for the condition of $\frac{P}{R_s} = 2.0$.

Illustrated on figure 9-50 are typical lower nappe profiles, plotted for various values of $H_s$ for a given value of $R_s$. In contrast to the straight crest where the nappe springs farther from the crest as the head increases, it will be seen from figure 9-50 that the lower nappe profile for the circular crest springs farther only in the region of the high point of the profile, and then only for $\frac{H_s}{R_s}$ values up to about 0.5. The profiles become increasingly suppressed for larger $\frac{H_s}{R_s}$ values. Below the high point of the profile, the paths cross and the shapes for the higher heads fall inside those for the lower heads. Thus, if the crest profile is designed for heads where $\frac{H_s}{R_s}$ exceeds about 0.25 to 0.3, it appears that subatmospheric pressure will occur along some portion of the profile when heads are less than the designed maximum. If subatmospheric pressures are to be avoided along the crest profile, the crest shape should be selected so that it will give support to the overflow nappe for the smaller $\frac{H_e}{R_s}$ ratios. Figure 9-51 shows the approximate increase in radius required to minimize subatmospheric pressures on the crest. The crest shape for the enlarged crest radius is then based on a $\frac{H_s'}{R_s'}$ ratio of 0.3.

9-28. ***Orifice Control.***—The diameter of a jet issuing from a horizontal orifice can be determined for any point below the water surface if it is assumed that the continuity equation, $Q = av$, is valid and if friction and other losses are neglected.

For a circular jet the area is equal to $\pi R^2$. The discharge is equal to $av = \pi R^2 \sqrt{2gh}$. Solving for $R$, $R = \frac{Q_a{}^{1/2}}{5H_a{}^{1/4}}$ where $H_a$ is equal to the difference between the water surface and the elevation under consideration. The diameter of the jet thus decreases indefinitely with the distance of the vertical fall for normal design applications.

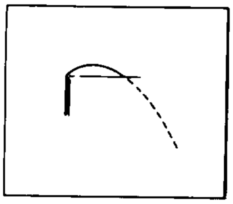
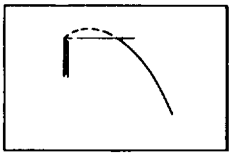
If an assumed total loss (to allow for jet contraction losses, friction losses, velocity losses due to direction change, etc.) is taken as $0.1H_a$, the equation for determining the approximate radius of the circular jet can be written:

$$R = 0.204\frac{Q^{1/2}}{H_a{}^{1/4}} \tag{20}$$

Since this equation is for the shape of the

Table 9-2.–*Coordinates of lower nappe surface for different values of* $\frac{H_s}{R}$ *when* $\frac{P}{R} = 2$.

[Negligible approach velocity and aerated nappe]

| $\frac{H_s}{R}$ | 0.00 | 0.10* | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 | 1.00 | 1.20 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{X}{H_s}$ | $\frac{Y}{H_s}$ For portion of the profile above the weir crest | | | | | | | | | | | | | | |
| 0.000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| .010 | .0150 | .0145 | .0133 | .0130 | .0128 | .0125 | .0122 | .0119 | .0116 | .0112 | .0104 | .0095 | .0086 | .0077 | .0070 |
| .020 | .0280 | .0265 | .0250 | .0243 | .0236 | .0231 | .0225 | .0220 | .0213 | .0202 | .0180 | .0159 | .0140 | .0115 | .0090 |
| .030 | .0395 | .0365 | .0350 | .0337 | .0327 | .0317 | .0308 | .0299 | .0289 | .0270 | .0231 | .0198 | .0168 | .0126 | .0085 |
| .040 | .0490 | .0460 | .0435 | .0417 | .0403 | .0389 | .0377 | .0363 | .0351 | .0324 | .0268 | .0220 | .0176 | .0117 | .0050 |
| .050 | .0575 | .0535 | .0506 | .0487 | .0471 | .0454 | .0436 | .0420 | .0402 | .0368 | .0292 | .0226 | .0168 | .0092 | |
| .060 | .0650 | .0605 | .0570 | .0550 | .0531 | .0510 | .0489 | .0470 | .0448 | .0404 | .0305 | .0220 | .0147 | .0053 | |
| .070 | .0710 | .0665 | .0627 | .0605 | .0584 | .0560 | .0537 | .0514 | .0487 | .0432 | .0308 | .0201 | .0114 | .0001 | |
| .080 | .0765 | .0710 | .0677 | .0655 | .0630 | .0603 | .0578 | .0550 | .0521 | .0455 | .0301 | .0172 | .0070 | | |
| .090 | .0820 | .0765 | .0722 | .0696 | .0670 | .0640 | .0613 | .0581 | .0549 | .0471 | .0287 | .0135 | .0018 | | |
| .100 | .0860 | .0810 | .0762 | .0734 | .0705 | .0672 | .0642 | .0606 | .0570 | .0482 | .0264 | .0089 | | | |
| .120 | .0940 | .0880 | .0826 | .0790 | .0758 | .0720 | .0683 | .0640 | .0596 | .0483 | .0195 | | | | |
| .140 | .1000 | .0935 | .0872 | .0829 | .0792 | .0750 | .0 05 | .0654 | .0599 | .0460 | .0101 | | | | |
| .160 | .1045 | .0980 | .0905 | .0855 | .0812 | .076[illegible] | .0710 | .0651 | .0585 | .0418 | | | | | |
| .180 | .1080 | .1010 | .0927 | .0872 | .0820 | .0766 | 0705 | .0637 | .0559 | .0361 | | | | | |
| .200 | .1105 | .1025 | .0938 | .0877 | .0819 | .0756 | .0688 | .0611 | .0521 | .0292 | |  | | | |
| .250 | .1120 | .1035 | .0926 | .0850 | .0773 | .0683 | .0596 | .0495 | .0380 | .0068 | | | | | |
| .300 | .1105 | .1000 | .0850 | .0764 | .0668 | .0559 | .0446 | .0327 | .0174 | | | | | | |
| .350 | .1060 | .0930 | .0750 | .0650 | .0540 | .0410 | .0280 | .0125 | | | | | | | |
| .400 | .0970 | .0830 | .0620 | .0500 | .0365 | .0220 | .0060 | | | | | | | | |
| .450 | .0845 | .0700 | .0450 | .0310 | .0170 | .000 | | | | | | | | | |
| .500 | .0700 | .0520 | .0250 | .0100 | | | | | | | | | | | |
| .550 | .0520 | .0320 | .0020 | | | | | | | | | | | | |
| .600 | .0320 | .0080 | | | | | | | | | | | | | |
| .650 | .0090 | | | | | | | | | | | | | | |
| $\frac{Y}{H_s}$ | $\frac{X}{H_s}$ For portion of the profile below the weir crest | | | | | | | | | | | | | | |
| 0.000 | 0.668 | 0.615 | 0.554 | 0.520 | 0.487 | 0.450 | 0.413 | 0.376 | 0 334 | 0.262 | 0.158 | 0.116 | 0.093 | 0.070 | 0.048 |
| −.020 | .705 | .652 | .592 | .560 | .526 | .488 | .452 | .414 | .369 | .293 | .185 | .145 | .120 | .096 | .074 |
| −.040 | .742 | .688 | .627 | .596 | .563 | .524 | .487 | .448 | .400 | .320 | .212 | .165 | .140 | .115 | .088 |
| −.060 | .777 | .720 | .660 | .630 | .596 | .557 | .519 | .478 | .428 | .342 | .232 | .182 | .155 | .129 | .100 |
| −.080 | .808 | .752 | .692 | .662 | .628 | .589 | .549 | .506 | .454 | .363 | .250 | .197 | .169 | .140 | .110 |
| −.100 | .838 | .784 | .722 | .692 | .657 | .618 | .577 | .532 | .478 | .381 | .266 | .210 | .180 | .150 | .118 |
| −.150 | .913 | .857 | .793 | .762 | .725 | .686 | .641 | .589 | .531 | .423 | .299 | .238 | .204 | .170 | .132 |
| −.200 | .978 | .925 | .860 | .826 | 790 | .745 | .698 | .640 | .575 | .459 | .326 | .260 | .224 | .184 | .144 |
| −.250 | 1.040 | .985 | .919 | .883 | .847 | .801 | .750 | .683 | .613 | .490 | .348 | .280 | .239 | .196 | .153 |
| −.300 | 1.100 | 1.043 | .976 | .941 | .900 | .852 | .797 | .722 | .648 | .518 | .368 | .296 | .251 | .206 | .160 |
| −.400 | 1.207 | 1.150 | 1.079 | 1.041 | 1.000 | .944 | .880 | .791 | .706 | .562 | .400 | .322 | .271 | .220 | .168 |
| −.500 | 1.308 | 1.246 | 1.172 | 1.131 | 1.087 | 1.027 | .951 | .849 | .753 | .598 | .427 | .342 | .287 | .232 | .173 |
| −.600 | 1.397 | 1.335 | 1.260 | 1.215 | 1.167 | 1.102 | 1.012 | .898 | .793 | .627 | .449 | .359 | .300 | .240 | .179 |
| −.800 | 1.563 | 1.500 | 1.422 | 1.369 | 1.312 | 1.231 | 1.112 | .974 | .854 | .673 | .482 | .384 | .320 | .253 | .184 |
| −1.000 | 1.713 | 1.646 | 1.564 | 1.508 | 1.440 | 1.337 | 1.189 | 1.030 | .899 | .710 | .508 | .402 | .332 | .260 | .188 |
| −1.200 | 1.846 | 1.780 | 1.691 | 1.635 | 1.553 | 1.422 | 1.248 | 1.074 | .933 | .739 | .528 | .417 | .340 | .266 | |
| −1.400 | 1.970 | 1.903 | 1.808 | 1.748 | 1.653 | 1.492 | 1.293 | 1.108 | .963 | .760 | .542 | .423 | .344 | | |
| −1.600 | 2.085 | 2.020 | 1.918 | 1.855 | 1.742 | 1.548 | 1.330 | 1.133 | .988 | .780 | .553 | .430 | | | |
| −1.800 | 2.196 | 2.130 | 2.024 | 1.957 | 1.821 | 1.591 | 1.358 | 1.158 | 1.008 | .797 | .563 | .433 | | | |
| −2.000 | 2.302 | 2.234 | 2.126 | 2.053 | 1.891 | 1.630 | 1.381 | 1.180 | 1.025 | .810 | .572 | | | | |
| −2.500 | 2.557 | 2.475 | 2.354 | 2.266 | 2.027 | 1.701 | 1.430 | 1.221 | 1.059 | .838 | .588 |  | | | |
| −3.000 | 2.778 | 2.700 | 2.559 | 2.428 | 2.119 | 1.748 | 1.468 | 1.252 | 1.086 | .853 | | | | | |
| −3.500 | ...... | 2.916 | 2.749 | 2.541 | 2.171 | 1.777 | 1.489 | 1.267 | 1.102 | | | | | | |
| −4.000 | ...... | 3.114 | 2.914 | 2.620 | 2.201 | 1.796 | 1.500 | 1.280 | | | | | | | |
| −4.500 | ...... | 3.306 | 3.053 | 2.682 | 2.220 | 1.806 | 1.509 | | | | | | | | |
| −5.000 | ...... | 3.488 | 3.178 | 2.734 | 2.227 | 1.811 | | | | | | | | | |
| −5.500 | ...... | 3.653 | 3.294 | 2.779 | 2.229 | | | | | | | | | | |
| −6.000 | ...... | 3.820 | 3.405 | 2.812 | 2.232 | | | | | | | | | | |
| $\frac{H_s}{R}$ | 0.00 | 0.10 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 | 1.00 | 1.20 | 1.50 | 2.00 |

*The tabulation for $\frac{H_s}{R}$ = 0.10 was obtained by interpolation between $\frac{H_s}{R}$ = 0 and 0.20.

After Wagner

Table 9-3.—*Coordinates of lower nappe surface for different values of* $\frac{H_s}{R}$ *when* $\frac{P}{R}$ *= 0.30.*

| $\frac{H_s}{R}$ | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{X}{H_s}$ | $\frac{Y}{H_s}$ For portion of the profile above the weir crest | | | | | | | | |
| 0.000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| .010 | .0130 | .0130 | .0130 | .0125 | .0120 | .0120 | .0115 | .0110 | .0100 |
| .020 | .0245 | .0242 | .0240 | .0235 | .0225 | .0210 | .0195 | .0180 | .0170 |
| .030 | .0340 | .0335 | .0330 | .0320 | .0300 | .0290 | .0270 | .0240 | .0210 |
| .040 | .0415 | .0411 | .0390 | .0380 | .0365 | .0350 | .0320 | .0285 | .0240 |
| .050 | .0495 | .0470 | .0455 | .0440 | .0420 | .0395 | .0370 | .0325 | .0245 |
| .060 | .0560 | .0530 | .0505 | .0490 | .0460 | .0440 | .0405 | .0350 | .0250 |
| .070 | .0610 | .0575 | .0550 | .0530 | .0500 | .0470 | .0440 | .0370 | .0245 |
| .080 | .0660 | .0620 | .0590 | .0565 | .0530 | .0500 | .0460 | .0385 | .0235 |
| .090 | .0705 | .0660 | .0625 | .0595 | .0550 | .0520 | .0480 | .0390 | .0215 |
| .100 | .0740 | .0690 | .0660 | .0620 | .0575 | .0540 | .0500 | .0395 | .0190 |
| .120 | .0800 | .0750 | .0705 | .0650 | .0600 | .0560 | .0510 | .0380 | .0120 |
| .140 | .0840 | .0790 | .0735 | .0670 | .0615 | .0560 | .0515 | .0355 | .0020 |
| .160 | .0870 | .0810 | .0750 | .0675 | .0610 | .0550 | .0500 | .0310 | |
| .180 | .0885 | .0820 | .0755 | .0675 | .0600 | .0535 | .0475 | .0250 | |
| .200 | .0885 | .0820 | .0745 | .0660 | .0575 | .0505 | .0435 | .0180 | |
| .250 | .0855 | .0765 | .0685 | .0590 | .0480 | .0390 | .0270 | | |
| .300 | .0780 | .0670 | .0580 | .0460 | .0340 | .0220 | .0050 | | |
| .350 | .0660 | .0540 | .0425 | .0295 | .0150 | | | | |
| .400 | .0495 | .0370 | .0240 | .0100 | | | | | |
| .450 | .0300 | .0170 | .0025 | | | | | | |
| .500 | .0090 | −.0060 | | | | | | | |
| .550 | | | | | | | | | |
| $\frac{Y}{H_s}$ | $\frac{X}{H_s}$ For portion of the profile below the weir crest | | | | | | | | |
| −0.000 | 0.519 | 0.488 | 0.455 | 0.422 | 0.384 | 0.349 | 0.310 | 0.238 | 0.144 |
| −.020 | .560 | .528 | .495 | .462 | .423 | .387 | .345 | .272 | .174 |
| −.040 | .598 | .566 | .532 | .498 | .458 | .420 | .376 | .300 | .198 |
| −.060 | .632 | .601 | .567 | .532 | .491 | .451 | .406 | .324 | .220 |
| −.080 | .664 | .634 | .600 | .564 | .522 | .480 | .432 | .348 | .238 |
| −.100 | .693 | .664 | .631 | .594 | .552 | .508 | .456 | .368 | .254 |
| −.150 | .760 | .734 | .701 | .661 | .618 | .569 | .510 | .412 | .290 |
| −.200 | .831 | .799 | .763 | .723 | .677 | .622 | .558 | .451 | .317 |
| −.250 | .893 | .860 | .826 | .781 | .729 | .667 | .599 | .483 | .341 |
| −.300 | .953 | .918 | .880 | .832 | .779 | .708 | .634 | .510 | .362 |
| −.400 | 1.060 | 1.024 | .981 | .932 | .867 | .780 | .692 | .556 | .396 |
| −.500 | 1.156 | 1.119 | 1.072 | 1.020 | .938 | .841 | .745 | .595 | .424 |
| −.600 | 1.242 | 1.203 | 1.153 | 1.098 | 1.000 | .891 | .780 | .627 | .446 |
| −.800 | 1.403 | 1.359 | 1.301 | 1.227 | 1.101 | .970 | .845 | .672 | .478 |
| −1.000 | 1.549 | 1.498 | 1.430 | 1.333 | 1.180 | 1.028 | .892 | .707 | .504 |
| −1.200 | 1.680 | 1.622 | 1.543 | 1.419 | 1.240 | 1.070 | .930 | .733 | .524 |
| −1.400 | 1.800 | 1.739 | 1.647 | 1.489 | 1.287 | 1.106 | .959 | .757 | .540 |
| −1.600 | 1.912 | 1.849 | 1.740 | 1.546 | 1.323 | 1.131 | .983 | .778 | .551 |
| −1.800 | 2.018 | 1.951 | 1.821 | 1.590 | 1.353 | 1.155 | 1.005 | .797 | .560 |
| −2.000 | 2.120 | 2.049 | 1.892 | 1.627 | 1.380 | 1.175 | 1.022 | .810 | .569 |
| −2.500 | 2.351 | 2.261 | 2.027 | 1.697 | 1.428 | 1.218 | 1.059 | .837 | |
| −3.000 | 2.557 | 2.423 | 2.113 | 1.747 | 1.464 | 1.247 | 1.081 | .852 | |
| −3.500 | 2.748 | 2.536 | 2.167 | 1.778 | 1.489 | 1.263 | 1.099 | | |
| −4.000 | 2.911 | 2.617 | 2.200 | 1.796 | 1.499 | 1.274 | | | |
| −4.500 | 3.052 | 2.677 | 2.217 | 1.805 | 1.507 | | | | |
| −5.000 | 3.173 | 2.731 | 2.223 | 1.810 | | | | | |
| −5.500 | 3.290 | 2.773 | 2.228 | | | | | | |
| −6.000 | 3.400 | 2.808 | | | | | | | |
| $\frac{H_s}{R}$ | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 |

After Wagner

Table 9-4.—*Coordinates of lower nappe surface for different values of* $\frac{H_s}{R}$ *when* $\frac{P}{R}$ *= 0.15.*

| $\frac{H_s}{R}$ | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{X}{H_s}$ | $\frac{Y}{H_s}$ For portion of the profile above the weir crest | | | | | | | | |
| 0.000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| .010 | .0120 | .0120 | .0115 | .0115 | .0110 | .0110 | .0105 | .0100 | .0090 |
| .020 | .0210 | .0200 | .0195 | .0190 | .0185 | .0180 | .0170 | .0160 | .0140 |
| .030 | .0285 | .0270 | .0265 | .0260 | .0250 | .0235 | .0225 | .0200 | .0165 |
| .040 | .0345 | .0335 | .0325 | .0310 | .0300 | .0285 | .0265 | .0230 | .0170 |
| .050 | .0405 | .0385 | .0375 | .0360 | .0345 | .0320 | .0300 | .0250 | .0170 |
| .060 | .0450 | .0430 | .0420 | .0400 | .0380 | .0355 | .0330 | .0265 | .0165 |
| .070 | .0495 | .0470 | .0455 | .0430 | .0410 | .0380 | .0350 | .0270 | .0150 |
| .080 | .0525 | .0500 | .0485 | .0460 | .0435 | .0400 | .0365 | .0270 | .0130 |
| .090 | .0560 | .0530 | .0510 | .0480 | .0455 | .0420 | .0370 | .0265 | .0100 |
| .100 | .0590 | .0560 | .0535 | .0500 | .0465 | .0425 | .0375 | .0255 | .0065 |
| .120 | .0630 | .0600 | .0570 | .0520 | .0480 | .0435 | .0365 | .0220 | |
| .140 | .0660 | .0620 | .0585 | .0525 | .0475 | .0425 | .0345 | .0175 | |
| .160 | .0670 | .0635 | .0590 | .0520 | .0460 | .0400 | .0305 | .0110 | |
| .180 | .0675 | .0635 | .0580 | .0500 | .0435 | .0365 | .0260 | .0040 | |
| .200 | .0670 | .0625 | .0560 | .0465 | .0395 | .0320 | .0200 | | |
| .250 | .0615 | .0560 | .0470 | .0360 | .0265 | .0160 | .0015 | | |
| .300 | .0520 | .0440 | .0330 | .0210 | .0100 | | | | |
| .350 | .0380 | .0285 | .0165 | .0030 | | | | | |
| .400 | .0210 | .0090 | | | | | | | |
| .450 | .0015 | | | | | | | | |
| .500 | | | | | | | | | |
| .550 | | | | | | | | | |
| $\frac{Y}{H_s}$ | $\frac{X}{H_s}$ For portion of the profile below the weir crest | | | | | | | | |
| −0.000 | 0.454 | 0.422 | 0.392 | 0.358 | 0.325 | 0.288 | 0.253 | 0.189 | 0.116 |
| −.020 | .499 | .467 | .437 | .404 | .369 | .330 | .292 | .228 | .149 |
| −.040 | .540 | .509 | .478 | .444 | .407 | .368 | .328 | .259 | .174 |
| −.060 | .579 | .547 | .516 | .482 | .443 | .402 | .358 | .286 | .195 |
| −.060 | .615 | .583 | .550 | .516 | .476 | .434 | .386 | .310 | .213 |
| −.100 | .650 | .616 | .584 | .547 | .506 | .462 | .412 | .331 | .228 |
| −.150 | .726 | .691 | .660 | .620 | .577 | .526 | .468 | .376 | .263 |
| −.200 | .795 | .760 | .729 | .685 | .639 | .580 | .516 | .413 | .293 |
| −.250 | .862 | .827 | .790 | .743 | .692 | .627 | .557 | .445 | .319 |
| −.300 | .922 | .883 | .843 | .797 | .741 | .671 | .594 | .474 | .342 |
| −.400 | 1.029 | .988 | .947 | .893 | .828 | .749 | .656 | .523 | .381 |
| −.500 | 1.128 | 1.086 | 1.040 | .980 | .902 | .816 | .710 | .567 | .413 |
| −.600 | 1.220 | 1.177 | 1.129 | 1.061 | .967 | .869 | .753 | .601 | .439 |
| −.800 | 1.380 | 1.337 | 1.285 | 1.202 | 1.080 | .953 | .827 | .655 | .473 |
| −1.000 | 1.525 | 1.481 | 1.420 | 1.317 | 1.164 | 1.014 | .878 | .696 | .498 |
| −1.200 | 1.659 | 1.610 | 1.537 | 1.411 | 1.228 | 1.059 | .917 | .725 | .517 |
| −1.400 | 1.780 | 1.731 | 1.639 | 1.480 | 1.276 | 1.096 | .949 | .750 | .531 |
| −1.600 | 1.897 | 1.843 | 1.729 | 1.533 | 1.316 | 1.123 | .973 | .770 | .544 |
| −1.800 | 2.003 | 1.947 | 1.809 | 1.580 | 1.347 | 1.147 | .997 | .787 | .553 |
| −2.000 | 2.104 | 2.042 | 1.879 | 1.619 | 1.372 | 1.167 | 1.013 | .801 | .560 |
| −2.500 | 2.340 | 2.251 | 2.017 | 1.690 | 1.423 | 1.210 | 1.049 | .827 | |
| −3.000 | 2.550 | 2.414 | 2.105 | 1.738 | 1.457 | 1.240 | 1.073 | .840 | |
| −3.500 | 2.740 | 2.530 | 2.153 | 1.768 | 1.475 | 1.252 | 1.088 | | |
| −4.000 | 2.904 | 2.609 | 2.180 | 1.780 | 1.487 | 1.263 | | | |
| −4.500 | 3.048 | 2.671 | 2.198 | 1.790 | 1.491 | | | | |
| −5.000 | 3.169 | 2.727 | 2.207 | 1.793 | | | | | |
| −5.500 | 3.286 | 2.769 | 2.210 | | | | | | |
| −6.000 | 3.396 | 2.800 | | | | | | | |
| $\frac{H_s}{R}$ | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.60 | 0.80 |

After Wagner

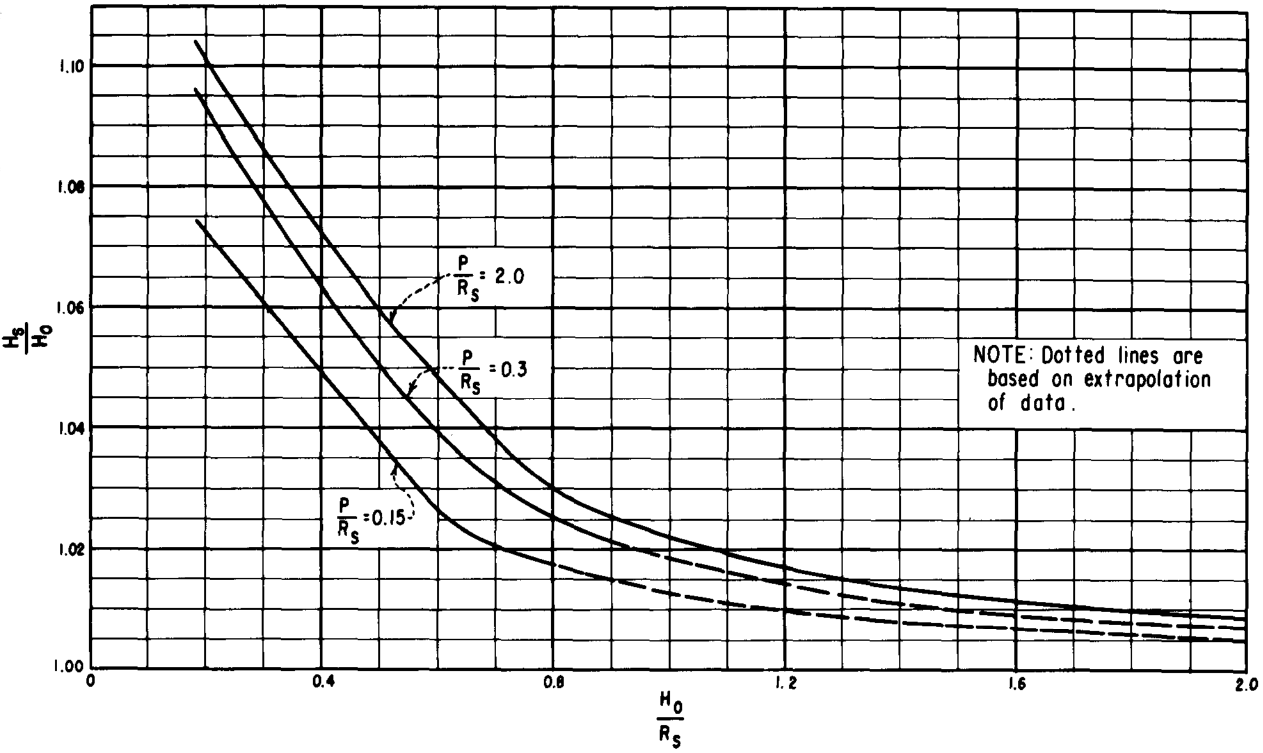


*Figure 9-48.* Relationship of $\frac{H_s}{H_o}$ to $\frac{H_o}{R_s}$ for circular sharp-crested weirs.—244-D-2443

jet, its use for determining the theoretical size and shape of a shaft in the area of the orifice would result in the minimum size shaft which would not restrict the flow and would not develop pressures along the sides of the shaft.

A theoretical jet profile or shaft as determined by equation (20) is shown by the dot-dash lines *abc* on figure 9-52. Superimposed on the jet of that figure is an overflow crest with a radius $R_s$, which serves as an entrance to the shaft. If both the crest and the shaft are designed for the same water surface elevation, the crest profile shape will be the same as the undernappe of the weir discharge, the shaft will flow full at section A-A, and there will be no pressure on the crest or in the shaft for the design head. For higher heads, section A-A will act as an orifice control. The shaft above section A-A will flow full and under pressure. Below section A-A, it will flow full but will not be under pressure. For lower heads, the crest will control and the shaft will flow partially full. Assuming the same losses, equation (20) can be rewritten, as follows, to determine the orifice discharge:

$$Q = 23.90 R^2 H_a^{1/2} \tag{21}$$

If the profile is modified to enlarge the shaft as shown by the solid lines *be* and aeration is provided, the shaft will not flow full. Neglecting losses, the jet below section A-A will then occupy an equivalent area indicated by the lines *bc*.

Aeration is usually provided at the orifice control, either through introduction of air at a sudden enlargement of the shaft or at the installation of a deflector to ensure free flow below the control section A-A. Waterway sizes and slopes must be such that free flow is maintained below the section of control. Failure to provide adequate aeration at the section of control may induce cavitation.

For submerged flow at the crest, the corresponding nappe shape as determined from section 9-27 for a design head $H_o$ will be such that along its lower levels it will closely follow the profile determined from equation (20) if

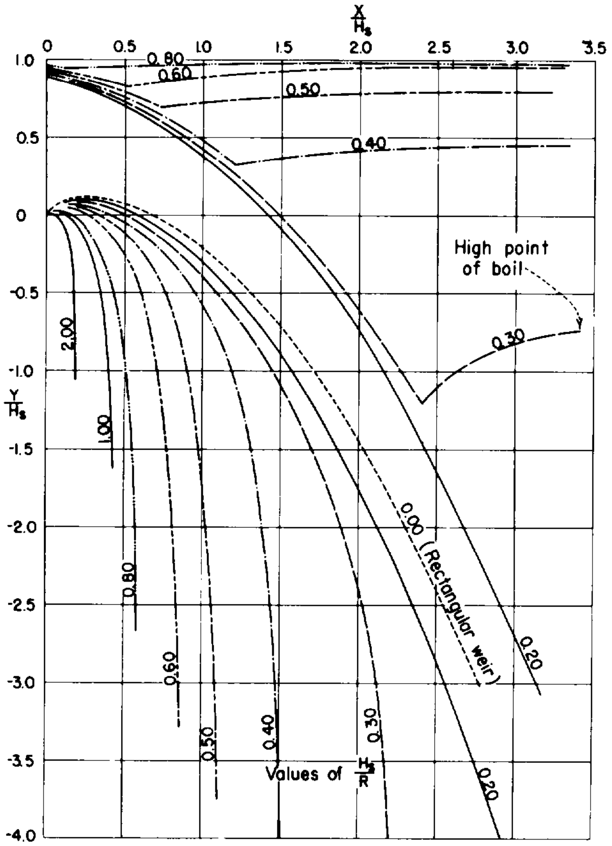


*Figure 9-49.* **Upper and lower nappe profiles for a circular weir (aerated nappe and negligible approach velocity).–288-D-2444**

$H_e$ approximates $H_0$. It must be remembered that on the basis of the losses assumed in equation (20), profile *abc* will be the minimum shaft size which will accommodate the required flow and that no part of the crest shape should be permitted to project inside this profile. As has been noted in section 9-12, small subatmospheric crest pressures can be tolerated if proper precautions are taken to obtain a smooth surface and if the negative pressure forces are recognized in the structural design. The choice of the minimum crest and orifice control shapes in preference to some wider shape then becomes a matter of economics, structural arrangement, and layout adaptability.

Where the orifice control profile corresponds to the continuation of the crest shape as determined by tables 9-2, 9-3, and 9-4, the discharge can be computed from equation (19)

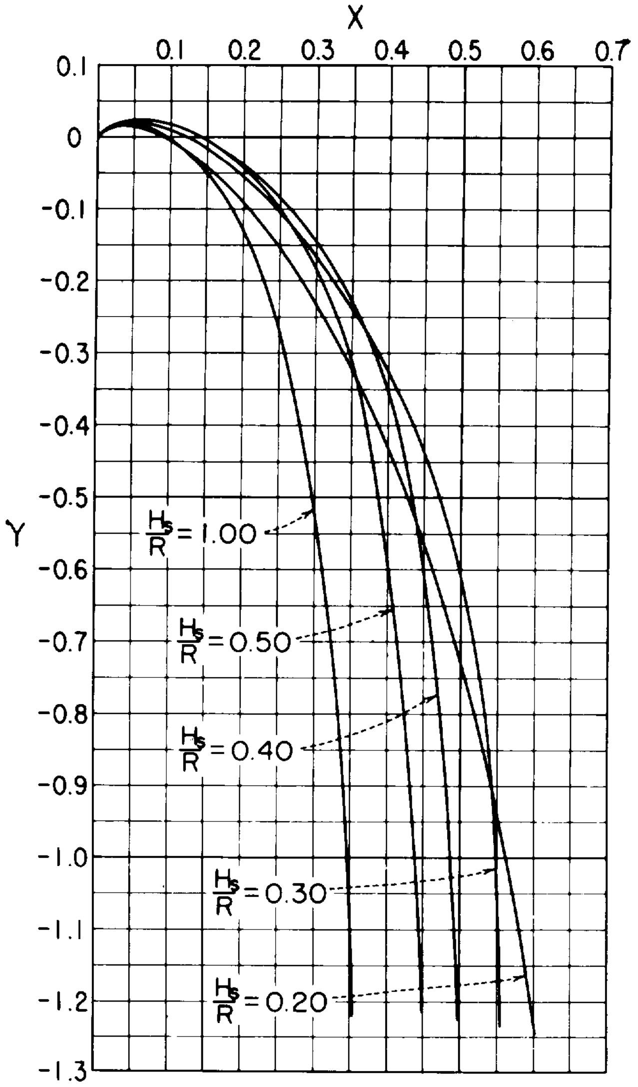


*Figure 9-50.* **Comparison of lower nappe shapes for a circular weir for different heads.–288-D-2445**

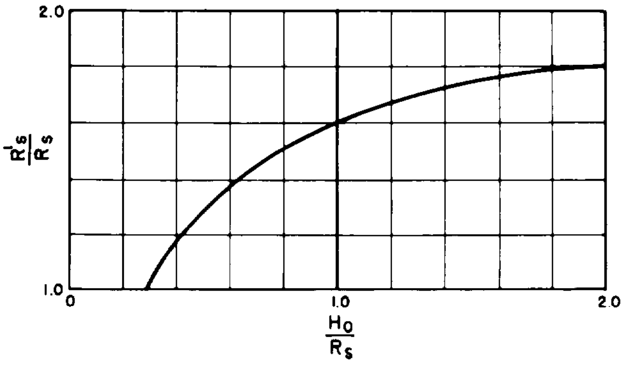


*Figure 9-51.* **Increased circular crest radius needed to minimize subatmospheric pressure along crest. 288-D-2446**

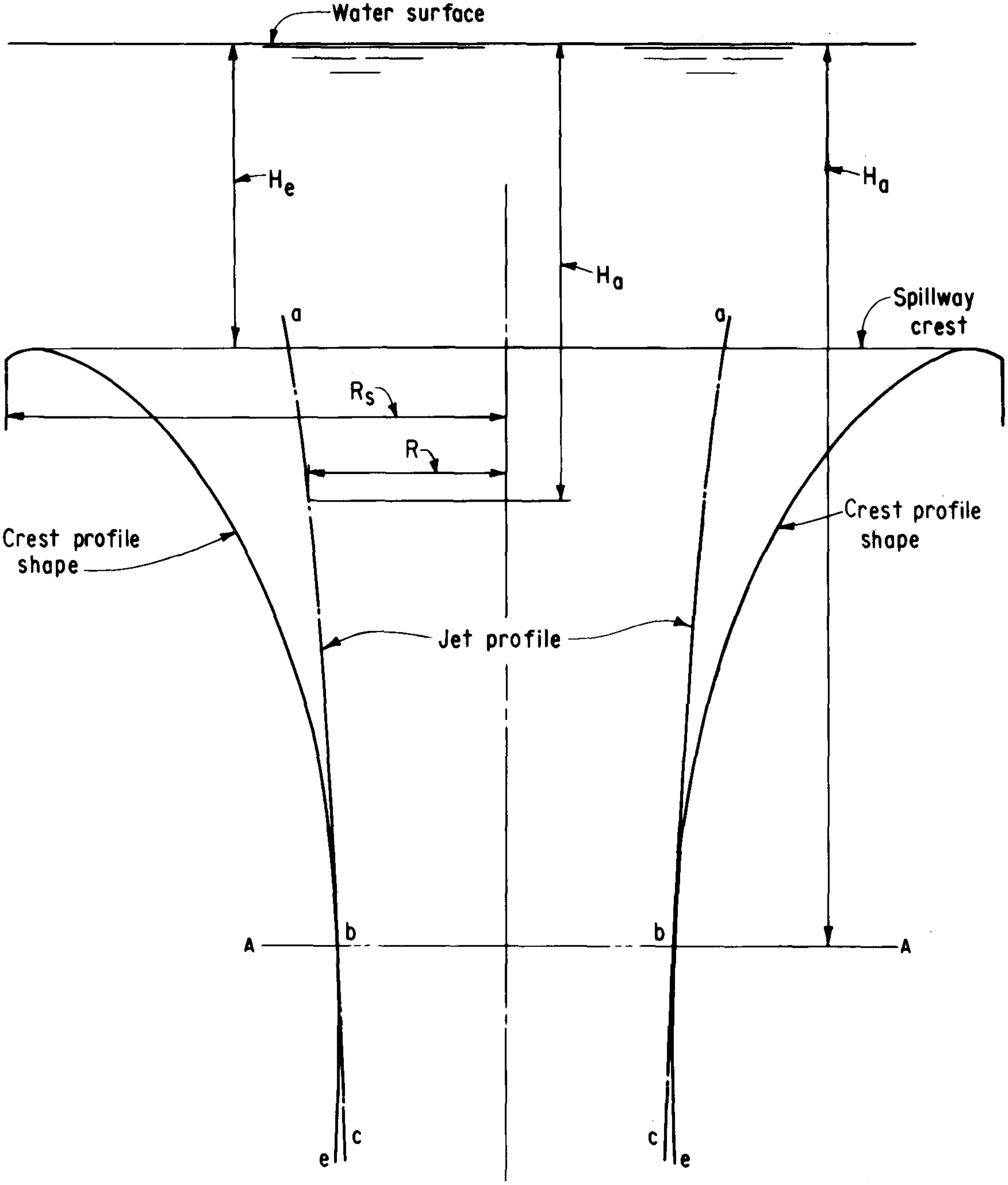


*Figure 9-52.* **Comparison of crest profile shape with theoretical jet profile.–288-D-3058**

using a coefficient from figure 9-46. Where the orifice control profile differs from the crest shape profile so that a constricted control section is established, the discharge must be determined from equation (20). On figure 9-44 the discharge head relationship curve *ag* will then be computed from the coefficients determined from figure 9-47 while the discharge head relationship curve *gh* will be based on equation (20).

9-29. ***Tunnel Design.***–If, for a designated discharge, the tunnel of a morning glory spillway were to flow full without being under pressure, the required size would vary along its length. So long as the slope of the hydraulic gradient which is dictated by the hydraulic losses is flatter than the slope of the tunnel, the flow will accelerate and the tunnel could decrease in size. When the tunnel slope becomes flatter than the slope of the hydraulic gradient, flow will decelerate and a larger tunnel may be required. All points along the tunnel will act simultaneously to control the rate of flow. For heads in excess of that used to proportion the tunnel, it will flow under pressure with the control at the downstream end; for heads less than that used to determine the size, the tunnel will flow partly full for its entire length and the control will remain upstream. On figure 9-44 the head at which the tunnel just flows full is represented by point *h*. At heads above point *h* the tunnel flows full under pressure; at heads less than *h* the tunnel flows partly full with controlling conditions dictated by the crest or orifice control design.

Because it is impractical to build a tunnel with a varying diameter, it is ordinarily made of a constant diameter. Thus the tunnel from the control point to the downstream end will have an excess of area. If atmospheric pressure can be maintained along the portion of the tunnel flowing partly full, the tunnel will continue to flow at that stage even though the downstream end fills. Progressively greater discharges will not alter the part full flow condition in the upper part of the tunnel, but full flow conditions under pressure will occupy increasing lengths of the downstream end of the tunnel. At the discharge represented by point *h* on figure 9-44, the full flow condition has moved back to the throat control section and the tunnel will flow full for its entire length.

If the tunnel flows at such a stage that the downstream end flows full, both the inlet and outlet will be sealed. To forestall siphon action by the withdrawal of air from the tunnel would require an adequate venting system. Unless venting is effected over the entire length of tunnel, it may prove inadequate to prevent subatmospheric pressures along some portion of the length because of the possibility of sealing at any point by surging, wave action, or eddy turbulences. Thus, if no venting is provided or if the venting is inadequate, a make-and-break siphon action will attend the flow in the range of discharges approaching full flow conditions. This action is accompanied by erratic discharges, by thumping and vibrations, and by surges at the entrance and outlet of the spillway. This is an undesirable condition and should be avoided.

To avoid the possibility of siphonic flow, the downstream tunnel size for ordinary designs (and especially for those for higher heads) is chosen so that the tunnel will never flow full beyond the throat. To allow for air bulking, surging, etc., the tunnel is selected of such a size that its area will not flow more than 75 percent full at the downstream end at maximum discharges. Under this limitation, air ordinarily will be able to pass up the tunnel from the downstream portal and thus prevent the formation of subatmospheric pressure along the tunnel length. Precautions must be taken, however, in selecting vertical or horizontal curvature of the tunnel profile and alinement to prevent sealing along some portion by surging or wave action.

## G. STRUCTURAL DESIGN

9-30. *General.*—The structural design of a spillway and the selection of specific structural details follow the determination of the spillway type and arrangement of components and the completion of the hydraulic design. The design criteria for each component part should be established for any condition which may exist at any time during the life of the structure. Design loads are different for each type of spillway. Each component should be carefully analyzed for loads that can be applied to it.

Structures in or on the dam should be designed for the stresses in the dam due to external loadings and temperatures, as well as the hydraulic load and other loads applied directly to the structure. Slabs, walls, and ogee crests should be designed for dead load and hydraulic pressures plus any other loads such as fill, surcharge, and control or operating equipment. Appurtenant structures not built on the dam which are subject to uplift due to the reservoir water and tailwater should be designed accordingly.

Because of the velocities involved, dynamic water pressures should be considered in addition to the static water pressures in all cases. Wherever practicable, laboratory model tests should be used to determine hydraulic loads, particularly dynamic loads.

Normal methods of design should be used for walls, slabs, etc. Where special design problems are encountered, the finite element method of analysis (appendix K) can be used to determine the stresses.

## H. BIBLIOGRAPHY

### 9-31. *Bibliography.*

[1] Bureau of Reclamation, "Studies of Crests of Overfall Dams," Bulletin 3, Part VI, Hydraulic Investigations, Boulder Canyon Project Final Reports, 1948.

[2] Bureau of Reclamation, "Discharge Coefficients For Irregular Overfall Spillways," Engineering Monograph No. 9, 1952.

[3] Hinds, Julian, "Side Channel Spillways," Trans. ASCE, vol. 89, 1926, p. 881.

[4] Ball, J. W., "Construction Finishes and High-Velocity Flow," Journal of the Construction Division, ASCE Proceedings, September 1963.

[5] Colgate, D. M., "Hydraulic Model Studies of Aeration Devices For Yellowtail Dam Spillway Tunnel," Pick-Sloan Missouri River Basin Program, Montana, REC-ERC-71-47, 1971.

[6] Bureau of Reclamation, "Hydraulic Design of Stilling Basin and Bucket Energy Dissipators," Engineering Monograph No. 25, 1964.

[7] Doddiah, D., Albertson, M. L., and Thomas, R. A., "Scour From Jets," Proceedings, Minnesota International Association for Hydraulic Research and Hydraulics Division, ASCE, Minneapolis, Minn., August 1953, p. 161.

[8] Scimemi, Ettore, "Discussion of Paper 'Model Study of Brown Canyon Debris Barrier' by Bermeal and Sanks," Trans. ASCE, vol. 112, 1947, p. 1016.

[9] Bureau of Reclamation, "Hydraulic Model Studies of Morrow Point Dam Spillway, Outlet Works and Powerplant Tailrace," Report No. HYD-557, 1966.

[10] Bureau of Reclamation, "Hydraulic Model Studies of the Pueblo Dam Spillway and Plunge Basin," REC-ERC-71-18, 1971.

[11] Bureau of Reclamation, "Hydraulic Model Studies of Crystal Dam Spillway and Outlet Works," REC-ERC-72-01, 1972.

[12] Peterka, A. J., "Morning-Glory Shaft Spillways," Trans. ASCE, vol. 121, 1956, p. 385.

[13] Bradley, J. N., "Morning-Glory Shaft Spillways: Prototype Behavior," Trans. ASCE, vol. 121, 1956, p. 312.

[14] Blaisdell, F. W., "Hydraulics of Closed Conduit Spillways–Parts II through VII–Results of Tests on Several Forms of the Spillway," University of Minnesota, Saint Anthony Falls Hydraulic Laboratory, Technical Paper No. 18, series B, March 1958.

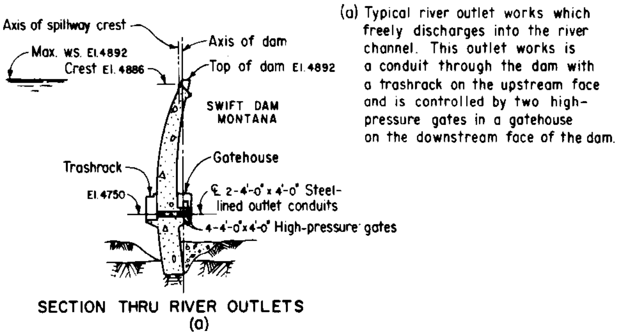
[15] Wagner, W. E., "Morning Glory Shaft Spillways: Determination of Pressure-Controlled Profiles," Trans. ASCE, vol. 121, 1956, p. 345.

# Outlet Works and Power Outlets

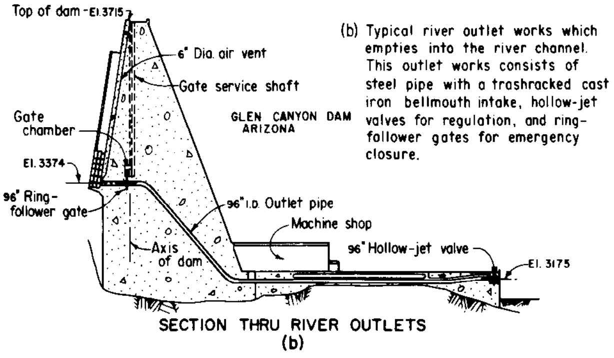
## A. INTRODUCTION

10-1. *Types and Purposes.*—An outlet works is a combination of structures and equipment required for the safe operation and control of water released from a reservoir to serve various purposes. Outlet works are usually classified according to their purpose such as river outlets, which serve to regulate flows to the river and control the reservoir elevation; irrigation or municipal water supply outlets, which control the flow of water into a canal, pipeline, or river to satisfy specified needs; or power outlets which provide passage of water to the turbines for power generation. Each damsite has its own requirements as to the type and size of outlet works needed. The outlet works may be designed to satisfy a single requirement or a combination of multipurpose requirements. Typical outlet works installations are shown on figures 10-1 and 10-2.

Downstream water requirements, preservation of aquatic life, abatement of stream pollution, and emergency evacuation of the reservoir are some of the factors that influence the design of a river outlet. In certain instances, the river outlet works may be used to increase the flow past the dam in conjunction with the normal spillway discharge. It may also act as a flood control regulator to release waters temporarily stored in flood control storage space or to evacuate storage in anticipation of flood inflows. Further, the river outlet works may serve to empty the reservoir to permit inspection, to make needed repairs, or to maintain the upstream face of the dam or other structures normally inundated.



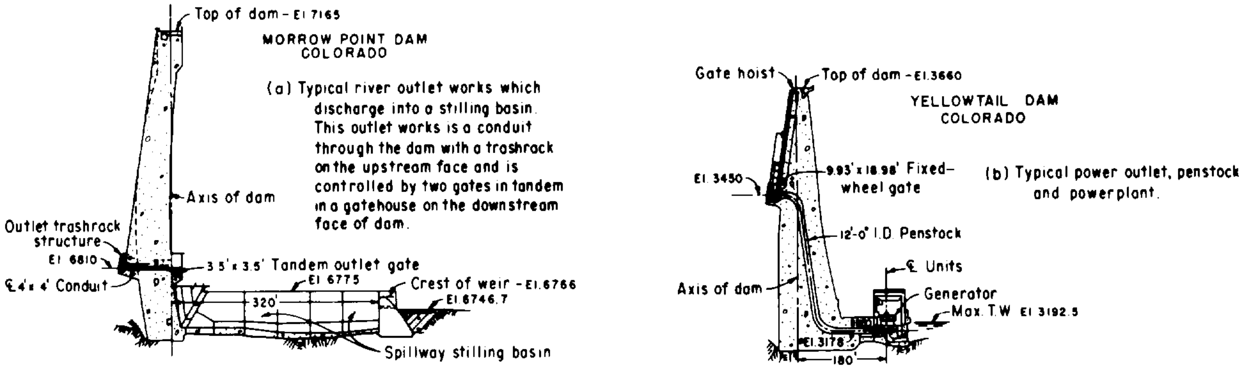
(a) Typical river outlet works which freely discharges into the river channel. This outlet works is a conduit through the dam with a trashrack on the upstream face and is controlled by two high-pressure gates in a gatehouse on the downstream face of the dam.



(b) Typical river outlet works which empties into the river channel. This outlet works consists of steel pipe with a trashracked cast iron bellmouth intake, hollow-jet valves for regulation, and ring-follower gates for emergency closure.

***Figure 10-1.*** **Typical river outlet works without a stilling basin. —288-D-3059**

The general details of operation and design of irrigation or municipal and industrial outlets are similar to those for river outlets. The quantity of irrigation water is determined from project or agricultural needs and is related to the anticipated use and to any special water requirements of the irrigation system. The quality and quantity of water for domestic use is determined from the commercial, industrial, and residential water needs of the area served.



SECTION THRU OUTLET WORKS AND STILLING BASIN
(a)

SECTION THRU PENSTOCK AND POWER PLANT
(b)

*Figure 10-2.* **Typical river outlet works and power outlet.** -288-D-3061

The number and size of irrigation and municipal and industrial outlet works will depend on the capacity requirements with the reservoir at a predetermined elevation, and on the amount of control required as the elevation of the reservoir fluctuates.

Power outlets provide for the passage of water to the powerplant; therefore, they should be designed to minimize hydraulic losses and to obtain the maximum economy in construction and operation. If the powerplant can be located at the toe of the dam, a layout with the penstocks embedded through the dam usually is most economical. Where the powerplant must be located away from the toe of the dam, the penstocks can be located in tunnels or embedded in the dam in the upper portion of their length and run exposed down the abutment to the powerplant.

## B. OUTLET WORKS OTHER THAN POWER OUTLETS

10-2. *General.*—An outlet works consists of the equipment and structures which together release the required water for a given purpose or combination of purposes. The flows through river outlets and canal or pipeline outlets vary throughout the year and may involve a wide range of discharges under varying heads. The accuracy and ease of control are major considerations, and a great amount of planning may be justified in determining the type of control devices that can be best utilized.

Ordinarily in a concrete dam, the most economical outlet works consists of an intake structure, a conduit or series of conduits through the dam, discharge flow control devices, and an energy dissipating device where required downstream of the dam. The intake structure includes a trashrack, an entrance transition, and stoplogs or an emergency gate. The control device can be placed (1) at the intake on the upstream face, (2) at some point along the conduit and be regulated from galleries inside the dam, or (3) at the downstream end of the conduit with the operating controls placed in a gatehouse on the downstream face of the dam. When there is a powerplant or other structure near the face of the dam, the outlet conduits can be extended further downstream to discharge into the river channel beyond these features. In this case, a control valve may be placed in a gate structure at the end of the conduit.

10-3. *Layout.*—The layout of a particular outlet works will be influenced by many conditions relating to the hydraulic requirements, the height and shape of dam, the site adaptability, and the relationship of the outlet works to the construction procedures and to other appurtenances of the development. An outlet works leading to a high-level canal or into a closed pipeline will differ from one emptying into the river. Similarly, a scheme in which the outlet works is used for diversion may vary from one where diversion is effected by other methods. In certain instances, the proximity of the spillway may permit combining some of the outlet works and spillway components into a single structure. As an example, the spillway and outlet works layout might be arranged so that discharges from both structures will empty into a common stilling basin.

The topography and geology of a site will have a great influence on the layout. The downstream location of the channel, the nearby location of any steep cliffs, and the width of the canyon are all factors affecting the selection of the most suitable type and location of outlet works. The river outlets should be located close to the river channel to minimize the downstream excavation. Geology, such as the location, type, and strength of bedrock, is also an important factor to consider when making the layout of an outlet works. An unfavorable foundation such as deep overburden or inferior foundation rock requires special consideration when selecting an impact area; with a weak foundation, a stilling basin may be required to avoid erosion and damage to the channel.

An outlet works may be used for diverting the riverflow or portion thereof during a phase of the construction period, thus avoiding the necessity for supplementary installations for that purpose. The outlet structure size dictated by this use rather than the size indicated for ordinary outlet requirements may determine the final outlet works capacity.

The establishment of the intake level is influenced by several considerations such as maintaining the required discharge at the minimum reservoir operating elevation, establishing a silt retention space, and allowing selective withdrawal to achieve suitable water temperature and/or quality. Dams which will impound waters for irrigation, domestic use, or other conservation purposes must have the outlet works intake low enough to be able to draw the water down to the bottom of the allocated storage space. Further, if the outlets are to be used to evacuate the reservoir for inspection or repair of the dam, they should be placed as low as practicable. However, it is usual practice to make an allowance in a reservoir for inactive storage for silt deposition, fish and wildlife conservation, and recreation.

Reservoirs become thermally stratified and taste and odor vary between elevations; therefore, the outlet intake should be established at the best elevation to achieve satisfactory water quality for the purpose intended. Downstream fish and wildlife requirements may determine the temperature at which the outlet releases should be made. Municipal and industrial water use increases the emphasis on water quality and requires the water to be drawn from the reservoir at the elevation which produces the most satisfactory combination of odor, taste, and temperature. Mineral concentrations, algae growth, and temperature are factors which influence the quality of the water and should be taken into consideration when establishing the intake elevation. Water supply releases can be made through separate outlet works at different elevations if the requirements for the individual water uses are not the same and the reservoir is stratified in temperature and quality of water.

Downstream water requirements may change throughout the year and the stratifications of water temperature and quality may fluctuate within the reservoir; therefore, the elevation at which the water should be drawn from the reservoir may vary. Selective withdrawal can be accomplished by a multilevel outlet arrangement in which the stratum of water that is most desirable can be released through the outlet works. Two schemes of multilevel outlet works are common. The first consists of a series of river outlet conduits through the dam at various elevations, and the second consists of a single outlet through the dam with a shutter arrangement on the trashrack structure. The shutters can be adjusted to allow selective withdrawal from the desired reservoir elevation. Figure 15-1 in chapter XV shows an example of a multilevel outlet works consisting of four outlet conduit intakes at different elevations, and figure 15-2 shows a typical example of a shutter arrangement on a trashrack structure.

Another factor to consider in determining a layout for an outlet works is the effect of a particular scheme on construction progress. A scheme which slows down or interferes with the normal construction progress of the concrete dam should be avoided if possible. Usually a horizontal conduit through the dam has the least effect on construction progress; however, sometimes other conditions restrict its use. Generally speaking, the fewer conduit or other outlet works components that must be installed within the mass concrete, the more rapid the rate of construction.

10-4. ***Intake Structures.***—In addition to forming the entrance into the outlet works, an intake structure may accommodate control devices. It also supports necessary auxiliary appurtenances (such as trashracks, fish screens, and bypass devices), and it may include temporary diversion openings and provisions for installation of bulkhead or stoplog closure devices.

An intake structure may take one of many forms, depending on the functions it must serve, the range in reservoir head under which it must operate, the discharge it must handle, the frequency of reservoir drawdown, the trash conditions in the reservoir, the reservoir ice conditions, and other considerations.

An intake structure for a concrete dam usually consists of a submerged structure on the upstream face of the dam; however, intake towers in the reservoir have been used in some instances. The most common intake structure consists of a bellmouth intake, a transition between the bellmouth and conduit if required, a trashrack structure on the upstream face of the dam, and guides to be used with a bulkhead gate or stoplogs to seal off the conduit for maintenance and repair.

If the upstream face of a concrete dam is curved in vertical section, the face may be modified in the area of the intake structure to facilitate the use of stoplogs or a bulkhead gate. This can be accomplished by making the face at the intake structure planar or curved gradually so that the guides can be embedded and the stoplogs or bulkhead gate can operate satisfactorily. The bulkhead gate or stoplogs are usually installed and removed by use of either a gantry or a mobile crane operating on top of the dam or from a barge in the reservoir.

(a) *Trashrack.*—A trashrack is used to keep trash and other debris from entering the outlet conduit and causing damage or fouling of the control device. Two basic types of trashracks are used for outlet works. One type is a concrete or metal frame structure on which metal trashracks are placed, and the other is an all-concrete structure that consists of relatively large openings formed in the concrete and is without metal racks. The metal trashrack type of structure provides for the screening of small debris when protection is needed to prevent damage to the conduit or control devices. Metal trashracks usually consist of relatively thin, flat steel bars which are placed on edge from 2 to 9 inches apart and assembled in rack sections. The required area of the trashrack is fixed by a limiting velocity through the rack, which in turn depends on the nature of the trash which must be excluded. Where the trashracks are inaccessible for cleaning, the velocity through the racks ordinarily should not exceed 2 feet per second. A velocity up to approximately 5 feet per second may be tolerated for racks which are accessible for cleaning.

An example of a concrete trashrack structure with metal racks is shown on figure 10-3. The concrete frame structure consists of a base cantilevered from the upstream face of the dam on which the trashrack structure is supported, a series of columns placed in a semicircle around the centerline of the intake, and a series of horizontal ribs spaced along the full height of the structure. The spacing between columns is dependent upon the structural requirements for the head differential that may be applied to the trashracks and the size of the metal rack section that can conveniently be fabricated and shipped. The vertical height of the trashrack structure is divided into a series of bays by arch-shaped ribs that are attached to the face of the dam and give lateral support to the columns. A solid concrete slab is usually constructed as a top for the structure with a slot formed, where required, to allow for placement and removal of the stoplogs or bulkhead gate. Grooves are formed into the vertical columns to hold the metal trashracks which are lowered into position from the top. When the intakes are deeply submerged, it may be desirable to remove and install the metal trashracks from the reservoir water surface. Guides can be supported on a curved concrete wall or "silo" which will facilitate the removal and installation of the trashrack sections.

An all-metal trashrack structure contains horizontal steel arches spaced along the height of the structure with vertical steel supports between the arches. The structure can be constructed so that the racks slide into the metal frame similar to the system used with the concrete frame, or the frame and trashracks can be fabricated into composite units and these arch-trashrack sections assembled to create the final structure. The top of the all-metal structure usually consists of trashrack bars supported as required and containing the slot required for placement and removal of the stoplogs or bulkhead gate.

When small trash is of no consequence and can be washed through the outlet works without damage to the conduit or control device, an all-concrete structure having only formed openings in the concrete can be used. The height and size of this trashrack structure, as well as the size of the formed openings, are dependent upon the desired discharge, the velocity at the intake, and the size and amount of debris in the reservoir. The openings for this type of trashrack usually range from 12 inches to 3 feet. The shape of the trashrack structure in plan can be rectangular, circular, or built in chords for ease of construction as shown on figure 10-4.

The frame used to support metal trashracks requires considerable construction time when formed of concrete; therefore, the use of a metal frame is often desirable because of the shorter construction time required for installation. This type also interferes least with the rapid placement of concrete in a dam.

Where winter reservoir storage is maintained in cold climates, the effect of possible icing conditions on the intake structure must be considered. Where reservoir surface ice can freeze around an intake structure, there is danger to the structure not only from the ice pressure acting laterally, but also from the uplift forces if a filling reservoir lifts the ice mass vertically. These effects should be considered in the design of the trashrack and the inlet structure, and may be a factor in determining the height of the trashrack structure. If practicable, the structure should be submerged at all times. However, if the structure will likely be above the reservoir water surface at times and ice loadings will present a hazard, an air bubbling system can be installed around the structure to circulate the warmer water from lower in the reservoir which will keep the surface area adjacent to the structure free of ice. Such a system will require a constant supply of compressed air and must be operated continuously during the winter months.

(b) *Entrance and Transition.*—The entrance to a conduit should be streamlined and provide smooth, gradual changes in the direction of flow to minimize head losses and to avoid zones where cavitation pressures can develop. Any abrupt change in the cross section of a conduit or any projection into the conduit, such as a gate frame, creates turbulence in the flow which increases in intensity as the velocity increases. These effects can be minimized by shaping the entrance to conform to the shape of a jet issuing from a standard orifice. These bellmouth entrances, as they are called, are discussed in section 10-11. Any time that a change in cross section of the outlet works is required, such as where the outlet changes from the size and shape of the entrance to that of the conduit, a smooth gradual transition should be utilized.

10-5. ***Conduits.***—The outlet conduits through a concrete dam are the passageways that carry the water from the reservoir downstream to the river, canal, or pipeline. A conduit may consist of a formed opening through or a steel liner embedded in the mass concrete. The shape may be rectangular or round, or it may transition from one shape to the other depending on the shape of the intake

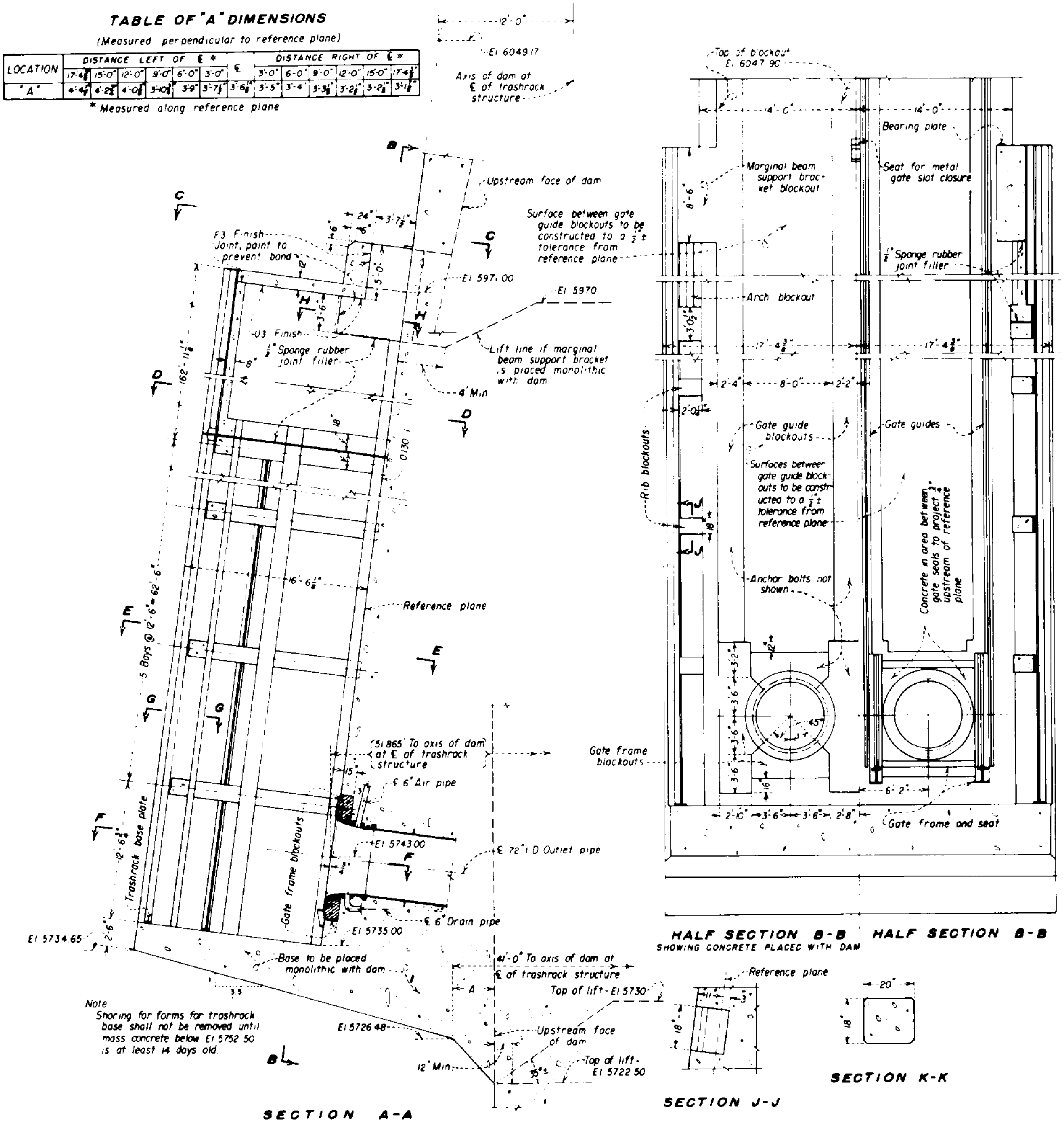


TABLE OF "A" DIMENSIONS

(Measured perpendicular to reference plane)

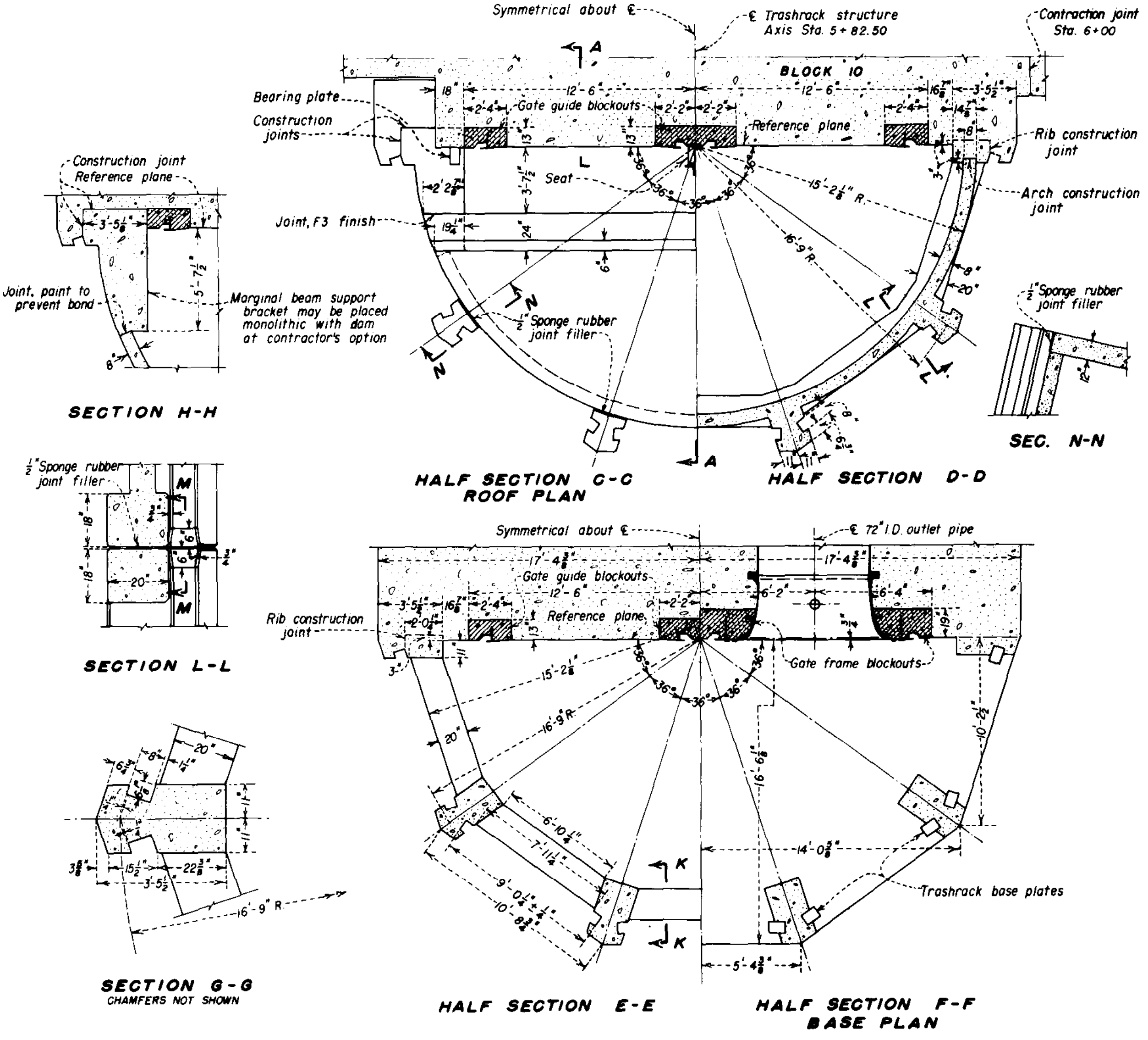
| LOCATION | DISTANCE LEFT OF ℄ * | | | | | | ℄ | DISTANCE RIGHT OF ℄ * | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 17'-4⅝" | 15'-0" | 12'-0" | 9'-0" | 6'-0" | 3'-0" | | 3'-0" | 6'-0" | 9'-0" | 12'-0" | 15'-0" | 17'-4⅝" |
| "A" | 4'-4⅞" | 4'-2⅜" | 4'-0⅞" | 3'-10⅜" | 3'-9" | 3'-7⅛" | 3'-6⅛" | 3'-5" | 3'-4" | 3'-3⅛" | 3'-2⅛" | 3'-2⅛" | 3'-1⅞" |

* Measured along reference plane

*Figure 10-3.* **River outlets trashrack structure–plans and sections (sheet 1 of 2). –288-D-3063 (1/2)**

entrance and on the type and location of the control equipment. The outlet works may contain one or more conduits depending on the discharge requirements for a predetermined reservoir water surface elevation. Two smaller conduits are preferable to one larger one, so that one outlet can be operated while the other is shut down for inspection and maintenance.

The design of the conduits required to pass a given discharge through a concrete dam is based upon the head, velocity of flow, type of control, length of conduit, and the associated

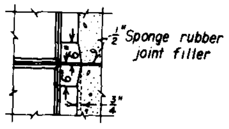


**CONCRETE REQUIREMENTS**

FINISHES:
All surfaces except as noted..................F2 or U2
STRENGTH:
Design of concrete, other than mass, is based on a compressive strength of 3000 p.s.i. at 28 days

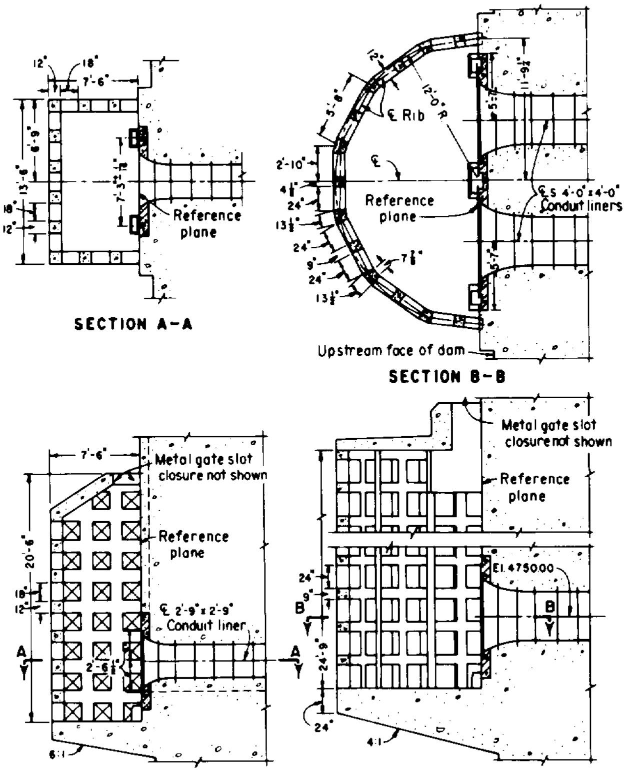
**NOTES**

The reference plane is 12'-0" upstream from the axis at Sta. 5+82.50 and at El. 6049.17 with slope of 0.130:1 horizontal to vertical.
Chamfer ¾" or tool all exposed corners unless otherwise specified.
Reinforcement required but not shown.
Ice prevention air system not shown.



SECTION M-M

*Figure 10-3.* River outlets trashrack structure—plans and sections (sheet 2 of 2). –288-D-3063 (2/2)



TYPICAL SECTION THRU CONDUIT (RECTANGULAR TRASHRACK)

TYPICAL SECTION THRU CONDUIT (CIRCULAR TRASHRACK)

*Figure 10-4.* **Typical trashrack installation. –288-D-3064**

economic considerations. Generally, the most economical conduit for an outlet is one that is horizontal and passes through the narrowest portion of the dam; however, most outlet works require that the conduit inlet and outlet be at different elevations to meet controlling requirements upstream and downstream. The number of bends required in an outlet conduit should be minimized and all the radii should be made as long as practicable to reduce head loss.

10-6. ***Gates and Outlet Controls.***—The discharges from a reservoir outlet works vary throughout the year depending upon downstream water needs and reservoir flood control requirements. Therefore, the impounded water must be released at specific regulated rates. To achieve this discharge control, gates or valves must be installed at some point along the conduit.

Control devices for outlet works are categorized according to their function in the structure. Operating gates and regulating valves are used to control and regulate the outlet works flow and are designed to operate in any position from closed to fully open. Guard or emergency gates are designed to effect closure in the event of failure of the operating gates, or where unwatering is desired either to inspect the conduit below the guard gates or to inspect and repair the operating gates.

Guides may be provided at the conduit entrance to accommodate stoplogs or bulkheads so that the conduit can be closed during an emergency period or for maintenance. For such installations, guard gates may or may not be provided, depending on whether the stoplogs can be readily installed if an emergency arises during normal reservoir operating periods.

Standard commercial gates and valves are available and may be adequate for low-head installations involving relatively small discharges. High-head installations, however, usually require specially designed equipment. The type of control device should be utilized that least affects flow in the conduit. For example, if possible, control and emergency gates or valves should be used that will not require transitions from one size and shape of conduit to another because these transitions are costly and can contribute greatly to the head loss through the conduit.

(a) *Location of Control Devices.*—The control gate for an outlet works can be placed at the upstream end of the conduit, at an intermediate point along its length, or at the outlet end of the conduit. Where flow from a control gate is released directly into the open as free discharge, only that portion of the conduit upstream from the gate will be under pressure. Where a control gate or valve discharges into a closed pressure pipe, the control will serve only to regulate the releases; full pipe flow will occur in the conduit both upstream and downstream from the control gate. For the pressure-pipe type, the location of the gate or valve will have little influence on the design insofar as internal pressures are concerned. However, where a control discharges into a free-flowing conduit, the location of the control gate becomes an important consideration in the design of the outlet.

Factors that should be considered in locating the control devices to be used on an outlet works include the size of the conduits required, the type of dam, the downstream structures, and the topography. The use of gates at the upstream or downstream face of the dam may be precluded if a satisfactory location for the gate and operating equipment or access is not available due to the layout of the dam or to the surrounding topography. The use of gate chambers within the dam is possible only if the thickness of the dam is great enough to safely contain the required chamber. When the outlet works discharges onto a spillway apron, the control device may, of necessity, have to be located either at a chamber within the dam or at the upstream face of the dam.

The most desirable location for the control device is usually at the downstream end of the conduit. This location permits most of the energy to be dissipated outside of the conduit, removing a possible cause of cavitation and vibration from the conduit. By eliminating gate operation at the entrance and within the conduit, better flow conditions can be maintained throughout the entire conduit length. Also, the size of the intake structure can sometimes be reduced if the control gate is not incorporated into the structure, and this may give the downstream location an additional advantage of economy.

(b) *Types of Gates and Valves.*—Many types of valves and gates are available for the control of outlet works. Each outlet works plan requires gates or valves that are well suited for the operating conditions and the characteristics of that plan. The location of the control device along the conduit, the amount of head applied, and the size and shape of conduit are all factors used in determining the type of control device considered likely to be most serviceable. Some types of gates and valves operate well at any opening, thus can be used as control gates, while others operate satisfactorily only at full open and can be used only as emergency or guard gates.

Where the control device is located at the outlet works intake and is to be operated under low head, the most commonly used device would be a slide gate. If the control is at an intermediate point along the conduit, control devices such as high-pressure slide gates, butterfly valves, or fixed-wheel gates can be used for the discharge control. Control at the downstream end of the outlet conduit may be accomplished by the use of a high-pressure slide gate, a jet-flow gate, or a hollow-jet valve discharging into the channel or stilling device. These are control devices that are commonly used; other types of gates or valves can be utilized if found to be more suitable for a particular situation.

Emergency or guard gates or valves are installed in the outlets upstream from the control device, to provide an emergency means of closing the conduit. These emergency devices may consist of a fixed-wheel gate to close the entrance to the conduit, a duplicate of the control gate or valve in tandem and operated from a chamber or gallery in the dam or in a control house on the downstream face, or a gate such as a ring-follower gate in tandem with the control gate. A ring-follower gate is well suited to serve as an emergency or guard gate (which operates either fully open or fully closed), since the ring-follower gate when fully open is the same size and shape as the conduit and causes little disturbance to the flow.

Stoplogs or a bulkhead gate on the face of the dam can be used to permit unwatering of the entire waterway and both are usually designed to operate under balanced pressure. Either device is lowered into place over the entrance with the control gate or an emergency gate closed and the conduit is then unwatered. A means of bypassing water from the reservoir into the conduit to balance the pressure on both sides of the stoplogs or bulkhead gate before they are raised must be provided. Adequate air passageways should be provided immediately downstream from the stoplogs or bulkhead gate, to prevent air from being trapped and compressed when the water is admitted to the conduit through the filling bypasses, and to reduce or eliminate negative pressure during unwatering.

10-7. *Energy Dissipating Devices.*—The discharge from an outlet, whether through gates, valves, or free-flow conduits, will emerge at a high velocity, usually in a near horizontal direction. The discharge may be released directly into the channel or riverbed if downstream structures are not endangered by the high-velocity flow and if the geology and topography are such that excessive erosion will not occur. However, if scouring and erosion are likely to be present, some means of dissipating the energy of the flow should be incorporated in the design. This may be accomplished by the construction of a stilling basin or other energy dissipating structure immediately downstream of the outlet.

The two types of energy dissipating devices most commonly used in conjunction with outlet works on concrete dams are hydraulic jump stilling basins and plunge pools. On some dams, it is possible to arrange the outlet works in conjunction with the spillway to utilize the spillway stilling device for dissipating the energy of the water discharging from the river outlets. Energy dissipating devices for free-flow conduit outlet works are essentially the same as those for spillways, discussed in chapter IX. The design of devices to dissipate jet flow is discussed in section 10-12.

## *1. Hydraulic Design of Outlet Works*

10-8. *General Considerations.*—The hydraulics of outlet works involves either one or both of two conditions of flow—open channel (or free) flow and full conduit (or pressure) flow. Analysis of open channel flow in outlet works, either in an open waterway or in a partly full conduit, is based on the principle of steady nonuniform flow conforming to the law of conservation of energy. Full pipe flow in closed conduits is based on pressure flow, which involves a study of hydraulic losses to determine the total heads needed to produce the required discharges.

Hydraulic jump basins, plunge pools, or other stilling devices can be employed to dissipate the energy of flow at the end of the outlet works if the conditions warrant their use.

10-9. *Pressure Flow in Outlet Conduits.*—Most outlet works for concrete dams have submerged entrance conditions and flow under pressure with a control device on the downstream end.

For flow in a closed pipe system, as shown on figure 10-5, Bernoulli's equation can be written as follows:

$$H_T = h_L + h_{v_1} \tag{1}$$

where:

$H_T$ = the total head needed to overcome the various head losses to produce discharge,
$h_L$ = the cumulative losses of the system, and
$h_{v_1}$ = the velocity head exit loss at the outlet.

Equation (1) can be expanded to list each loss, as follows:

$$\begin{aligned} H_T = h_t + h_e + h_{f_{(8)}} + h_{f_{(7)}} + h_{b_{(7)}} \\ + h_{f_{(6)}} + h_{f_{(5)}} + h_{b_{(5)}} + h_{f_{(4)}} \\ + h_{c_{(4\text{-}3)}} + h_{g_{(3)}} + h_{ex_{(3\text{-}2)}} \\ + h_{f_{(2)}} + h_{b_{(2)}} + h_{c_{(2\text{-}1)}} \\ + h_{g_{(1)}} + h_{v_{(1)}} \end{aligned} \tag{2}$$

where:

$h_t$ = trashrack losses,
$h_e$ = entrance losses,
$h_b$ = bend losses,
$h_c$ = contraction losses,
$h_{ex}$ = expansion losses,
$h_g$ = gate or valve losses, and
$h_f$ = friction losses.

In equation (2) the number subscripts refer to the various components, transitions, and reaches to which head losses apply.

For a free-discharging outlet, $H_T$ is measured from the reservoir water surface to the center of the outlet gate or the outlet opening. If the outflowing jet is supported on a downstream floor, the head is measured to the top of the emerging jet at the point of greatest contraction; if the outlet portal is submerged the head is measured to the tailwater level.

The various losses are related to the velocity head in the individual components, and equation (2) can be written:

$$
\begin{aligned}
H_T = {} & K_t\left(\frac{v_9{}^2}{2g}\right) + K_e\left(\frac{v_8{}^2}{2g}\right) + \frac{fL_8}{D_8}\left(\frac{v_8{}^2}{2g}\right) \\
& + \frac{fL_7}{D_7}\left(\frac{v_7{}^2}{2g}\right) + K_{b_7}\left(\frac{v_7{}^2}{2g}\right) + \frac{fL_6}{D_6}\left(\frac{v_6{}^2}{2g}\right) \\
& + \frac{fL_5}{D_5}\left(\frac{v_5{}^2}{2g}\right) + K_{b_5}\left(\frac{v_5{}^2}{2g}\right) + \frac{fL_4}{D_4}\left(\frac{v_4{}^2}{2g}\right) \\
& + K_{c(4\text{-}3)}\left(\frac{v_3{}^2}{2g} - \frac{v_4{}^2}{2g}\right) + K_{g_3}\left(\frac{v_3{}^2}{2g}\right) \\
& + K_{ex(3\text{-}2)}\left(\frac{v_3{}^2}{2g} - \frac{v_2{}^2}{2g}\right) + \frac{fL_2}{D_2}\left(\frac{v_2{}^2}{2g}\right) \\
& + K_{b_2}\left(\frac{v_2{}^2}{2g}\right) + K_{c(2\text{-}1)}\left(\frac{v_1{}^2}{2g} - \frac{v_2{}^2}{2g}\right) \\
& + K_{g_1}\left(\frac{v_1{}^2}{2g}\right) + K_v\left(\frac{v_1{}^2}{2g}\right)
\end{aligned}
\tag{3}
$$

where:

$D$ = diameter of conduit,
$g$ = acceleration due to force of gravity,
$L$ = length of conduit,
$v$ = velocity,
$K_t$ = trashrack loss coefficient,
$K_e$ = entrance loss coefficient,
$K_b$ = bend loss coefficient,
$f$ = friction factor in the Darcy-Weisbach equation for pipe flow,
$K_{ex}$ = expansion loss coefficient,
$K_c$ = contraction loss coefficient,
$K_g$ = gate loss coefficient, and
$K_v$ = exit velocity head coefficient at the outlet.

Equation (3) can be simplified by expressing the individual losses in terms of an arbitrarily chosen velocity head. This velocity head is usually selected as that in a significant section of the system. If the various velocity heads for the system shown on figure 10-5 are related to that in the downstream conduit, with an area (2), the conversion for any area ($x$) is found as shown below.

By the principle of continuity,

$$Q = av = a_2 v_2 = a_x v_x$$

where:

$Q$ = discharge,
$a$ = cross-sectional area of conduit, and
$v$ = velocity.

Then:

$$a_2{}^2 v_2{}^2 = a_x{}^2 v_x{}^2, \text{ and}$$

$$\frac{a_2{}^2 v_2{}^2}{2g} = \frac{a_x{}^2 v_x{}^2}{2g}$$

from which:

$$\frac{v_x{}^2}{2g} = \left(\frac{a_2}{a_x}\right)^2 \frac{v_2{}^2}{2g}$$

Equation (3) then can be written:

$$
\begin{aligned}
H_T = \frac{v_2{}^2}{2g}\Bigg[ & \left(\frac{a_2}{a_9}\right)^2\left(K_t\right) + \left(\frac{a_2}{a_8}\right)^2\left(K_e + \frac{fL_8}{D_8}\right) \\
& + \left(\frac{a_2}{a_7}\right)^2\left(\frac{fL_7}{D_7} + K_{b_7}\right) + \left(\frac{a_2}{a_6}\right)^2\left(\frac{fL_6}{D_6}\right) \\
& + \left(\frac{a_2}{a_5}\right)^2\left(\frac{fL_5}{D_5} + K_{b_5}\right) + \left(\frac{a_2}{a_4}\right)^2\left(\frac{fL_4}{D_4} - K_{c(4\text{-}3)}\right) \\
& + \left(\frac{a_2}{a_3}\right)^2\left(K_{c(4\text{-}3)} + K_{g_3} + K_{ex(3\text{-}2)}\right) \\
& + \left(\frac{fL_2}{D_2} - K_{ex(3\text{-}2)} + K_{b_2} - K_{c(2\text{-}1)}\right) \\
& + \left(\frac{a_2}{a_1}\right)^2\left(K_{c(2\text{-}1)} + K_{g_1} + K_v\right)\Bigg]
\end{aligned}
\tag{4}
$$

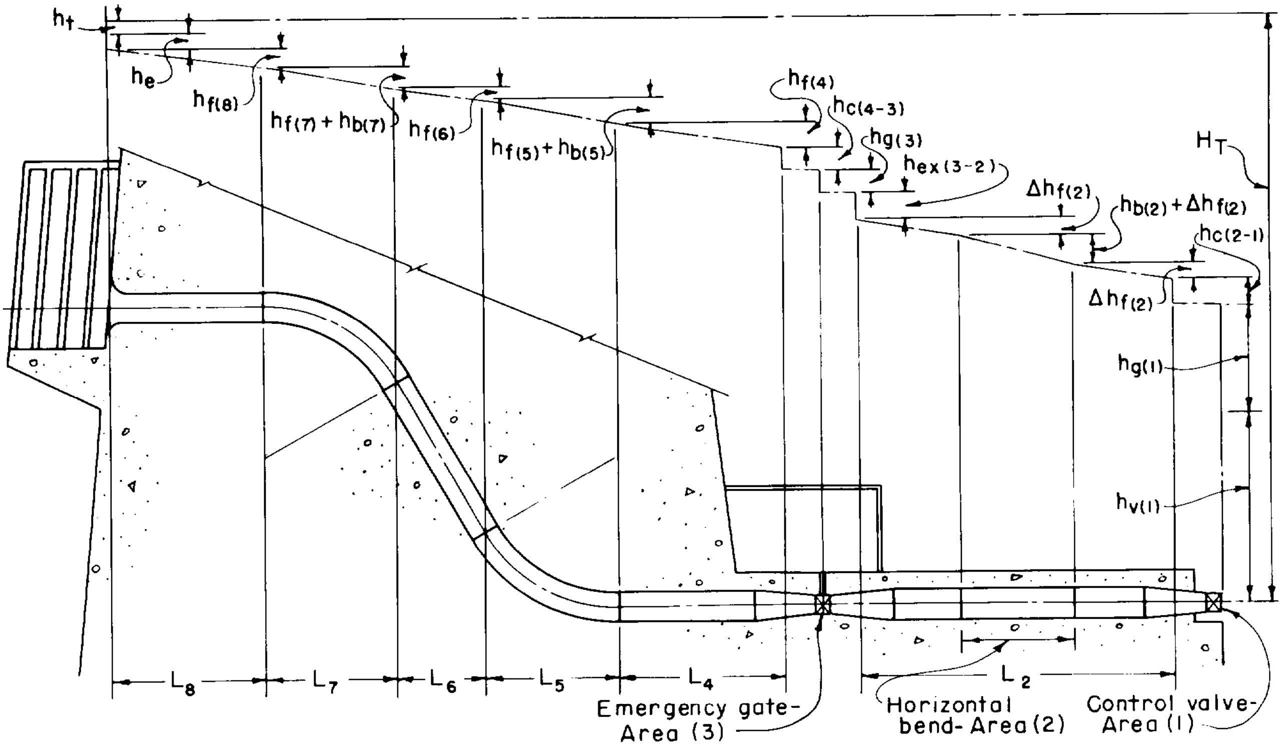


*Figure 10-5.* **Pictorial representation of typical head losses in outlet under pressure. –288-D-3065**

If the bracketed part of the expression is represented by $K_L$, the equation can be written:

$$H_T = K_L \frac{v_2^{\,2}}{2g} \tag{5}$$

Then:

$$Q = a_2 \sqrt{\frac{2gH_T}{K_L}} \tag{6}$$

10-10. ***Pressure Flow Losses in Conduits.***—Head losses in outlet works conduits are caused primarily by the frictional resistance to flow along the conduit sidewalls. Additional losses result from trashrack interferences, entrance contractions, contractions and expansions at gate installations, bends, gate and valve constrictions, and other interferences in the conduit. For a conservative design, greater than average loss coefficients should be assumed for computing required conduit and component sizes, and smaller loss coefficients should be used for computing energies of flow at the outlet. The major contributing losses of a conduit or pipe system are discussed in the remainder of this section.

(a) *Friction Losses.*—For flow in large pipes, the Darcy-Weisbach formula is most often employed to determine the energy losses due to frictional resistances of the conduit. The loss of head is stated by the equation:

$$h_f = \frac{fL}{D}\,\frac{v^2}{2g} \tag{7}$$

where $f$ is the friction loss coefficient and other symbols are as previously defined. This coefficient varies with the conduit surface roughness and with the Reynolds number. The latter is a function of the diameter of the pipe and the velocity, viscosity, and density of the fluid flowing through it. Data and procedures for evaluating the loss coefficient are presented in Engineering Monograph No. 7[1].[1] Since $f$

---

[1] Numbers in brackets refer to items in the bibliography, sec. 10-26.

is not a fixed value, many engineers are unfamiliar with its variations and would rather use Manning's coefficient of roughness, $n$, which has been more widely defined. If the influence of the Reynolds number is neglected, and if the roughness factor in relation to the pipe size is assumed constant, the relation of $f$ in the Darcy-Weisbach equation to $n$ in the Manning equation will be:

$$f = \frac{116.5n^2}{r^{1/3}} = \frac{185n^2}{D^{1/3}} \qquad (8)$$

where:

$r$ = hydraulic radius, and
$D$ = conduit diameter.

Relationships between the Darcy-Weisbach and Manning's coefficients can be determined graphically from figure 10-6.

Where the conduit cross section is rectangular in shape, the Darcy-Weisbach formula does not apply because it is for circular pipes, and the Manning equation may be used to compute the friction losses. Manning's equation (see sec. K-2(c) in appendix K) as applied to closed conduit flow is:

$$h_f = 29.1\, n^2 \frac{L}{r^{4/3}} \frac{v^2}{2g} \qquad (9)$$

Maximum and minimum values of $n$ which may be used to determine the conduit size and the energy of flow are as follows:

| Conduit material | Maximum $n$ | Minimum $n$ |
|---|---|---|
| Concrete pipe or cast-in-place conduit | 0.014 | 0.008 |
| Steel pipe with welded joints | .012 | .008 |

(b) *Trashrack Losses.*—Trashrack structures which consist of widely spaced structural members without rack bars will cause very little head loss, and trashrack losses in such a case might be neglected in computing conduit losses. When the trash structure consists of racks of bars, the loss will depend on the bar thickness, depth, and spacing. As shown in reference [2], an average approximation can be obtained from the equation:

$$\text{Loss} = K_t \frac{v_n{}^2}{2g} \qquad (10)$$

where:

$$K_t = 1.45 - 0.45\frac{a_n}{a_g} - \left(\frac{a_n}{a_g}\right)^2$$

In the above:

$K_t$ = the trashrack loss coefficient (empirical),
$a_n$ = the net area through the rack bars,
$a_g$ = the gross area of the racks and supports, and,
$v_n$ = the velocity through the net trashrack area.

Where maximum loss values are desired, assume that 50 percent of the net rack area is clogged. This will result in twice the velocity through the trashrack. For minimum trashrack losses, assume no clogging of the openings when computing the loss coefficient, or neglect the loss entirely.

(c) *Entrance Losses.*—The loss of head at the entrance of a conduit is comparable to the loss in a short tube or in a sluice. If $H$ is the head producing the discharge, $C$ is the coefficient of discharge, and $a$ is the area, the discharge is

$$Q = Ca\sqrt{2gH}$$

and the velocity is

$$v = C\sqrt{2gH}$$

or

$$H = \frac{1}{C^2}\frac{v^2}{2g} \qquad (11)$$

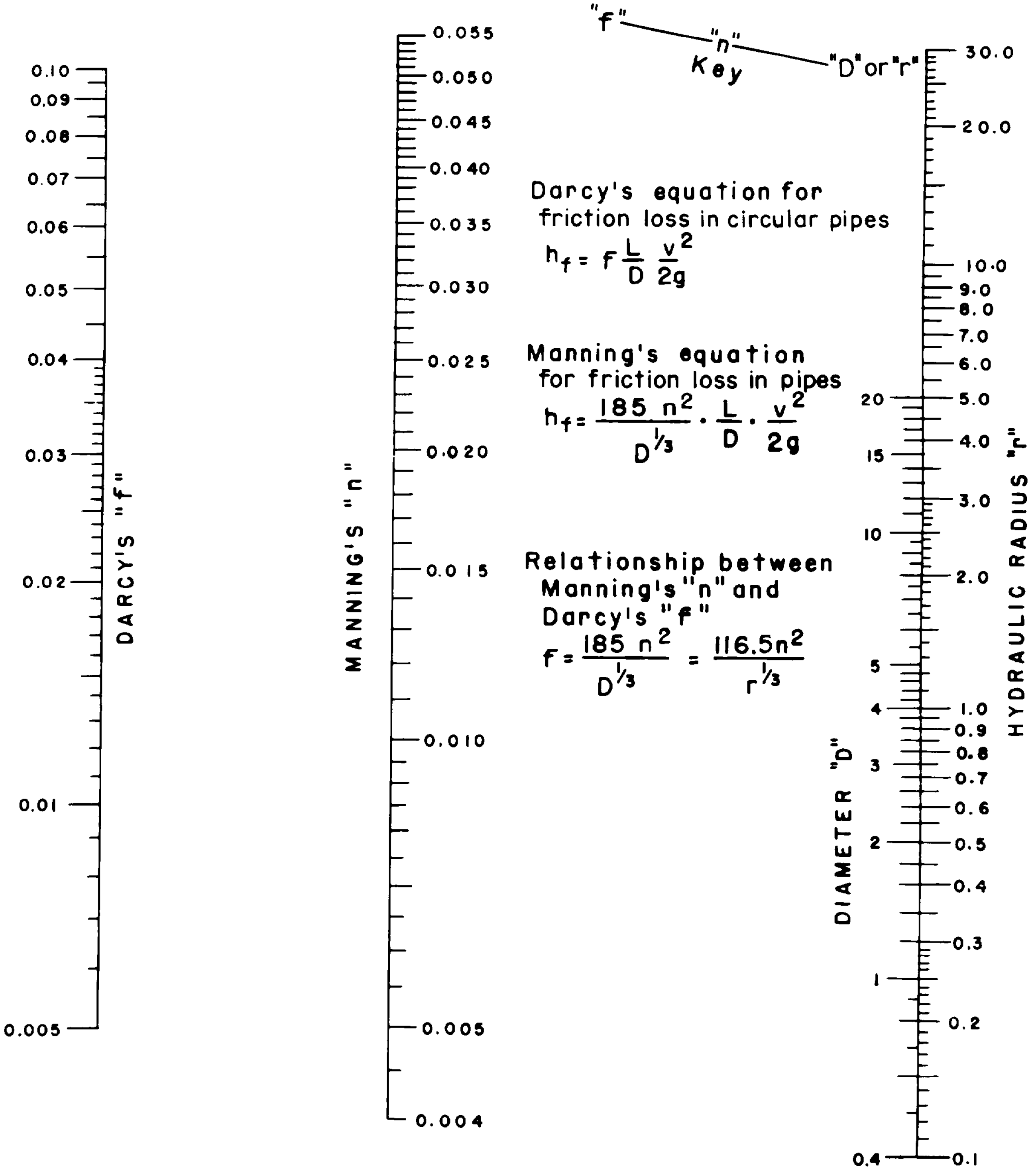


*Figure 10-6.* Relationship between Darcy's *f* and Manning's *n* for flow in pipes. –288-D-3066

Since $H$ is the sum of the velocity head $h_v$ and the head loss at the entrance $h_e$, equation (11) may be written:

$$\frac{v^2}{2g} + h_e = \frac{1}{C^2}\frac{v^2}{2g}$$

or

$$h_e = \left(\frac{1}{C^2} - 1\right)\frac{v^2}{2g}$$

Then:

$$K_e = \frac{1}{C^2} - 1 \qquad (12)$$

Coefficients of discharge and loss coefficients for typical entrances for conduits, as given in various texts and technical papers, are listed in table 10-1.

(d) *Bend Losses.*—Bend losses in closed conduits in excess of those due to friction loss through the length of the bend are a function of the bend radius, the pipe diameter, and the angle through which the bend turns.

Graphs taken in part from reference [3] giving $K_b$ as a function of these parameters are shown on figure 10-7. Figure 10-7(b) shows the coefficients for 90° bends for various ratios of radius of bend to diameter of pipe. Figure 10-7(c) indicates the coefficients for other than 90° bends. The value of the loss coefficient, $K_b$, for various values of $\frac{R_b}{D}$ can be applied directly for circular conduits; for rectangular conduits $D$ is taken as the height of the section in the plane of the bend.

(e) *Transition Losses.*—Head losses in gradual contractions or expansions in a conduit can be considered in relation to the increase or decrease in velocity head, and will vary according to the rate of change of the area and the length of the transition. For contractions the loss of head, $h_c$ will be approximately equal to $K_c\left(\frac{v_2^{\,2}}{2g} - \frac{v_1^{\,2}}{2g}\right)$, where $K_c$ varies from 0.1 for gradual contractions to 0.5 for abrupt contractions. Where the flare angle does not exceed that indicated in section 10-11, the loss coefficient can be assumed as 0.1. For greater flare angles, the loss coefficient can be assumed to vary in a straight-line relationship to a maximum of 0.5 for a right angle contraction.

For expansions, the loss of head, $h_{ex}$, will be approximately equal to $K_{ex}\left(\frac{v_1^{\,2}}{2g} - \frac{v_2^{\,2}}{2g}\right)$, where $K_{ex}$ is as follows:

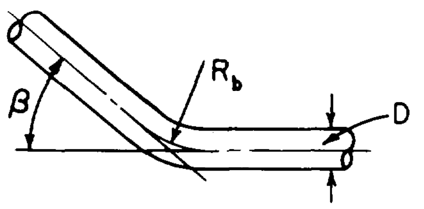
| Flare angle α | 2° | 5° | 10° | 12° | 15° | 20° |
|---|---|---|---|---|---|---|
| $K_{ex}$ [4] | 0.03 | 0.04 | 0.08 | 0.10 | 0.16 | 0.31 |
| $K_{ex}$ [5] | .02 | .12 | .16 | – | .27 | .40 |

| Flare angle α | 25° | 30° | 40° | 50° | 60° |
|---|---|---|---|---|---|
| $K_{ex}$ [4] | 0.40 | 0.49 | 0.60 | 0.67 | 0.72 |
| $K_{ex}$ [5] | .55 | .66 | .90 | 1.00 | – |

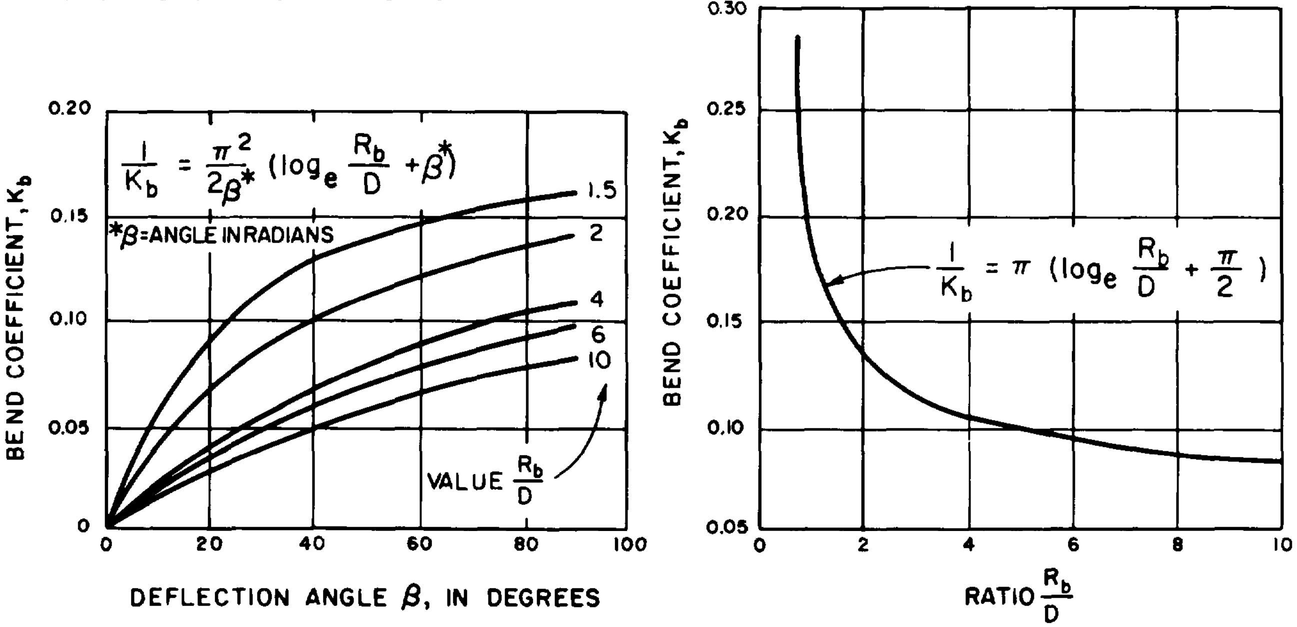
(f) *Gate and Valve Losses.*—No gate loss need be assumed where a gate is mounted at the entrance to the conduit so that when wide open it does not interfere with the entrance flow conditions. Also, emergency gates that are of the same size and shape as the conduit, such

Table 10-1.—*Coefficients of discharge and loss coefficients for conduit entrances.*

| Type of entrance | Coefficient $C$ | | | Loss coefficient $K_e$ | | |
|---|---|---|---|---|---|---|
| | Maximum | Minimum | Average | Maximum | Minimum | Average |
| (1) Square-cornered | 0.85 | 0.77 | 0.82 | 0.70 | 0.40 | 0.50 |
| (2) Slightly rounded | .92 | .79 | .90 | .60 | .18 | .23 |
| (3) Fully rounded<br>$\frac{r}{D} \geq 0.15$ | .96 | .88 | .95 | .27 | .08 | .10 |
| (4) Circular bellmouth | .98 | .95 | .98 | .10 | .04 | .05 |
| (5) Square bellmouth | .97 | .91 | .93 | .20 | .07 | .16 |
| (6) Inward projecting | .80 | .72 | .75 | .93 | .56 | .80 |



(a) DEFINITION SKETCH



(c) $K_b$ VS DEFLECTION ANGLE

(b) $K_b$ VS $\frac{R_b}{D}$ FOR 90° BENDS

*Figure 10-7.* Coefficient for bend losses in a closed conduit. –288-D-3067

as ring-follower gates in a circular conduit, do not affect the flow and their associated losses are negligible. Emergency gates such as wheel-mounted or roller-mounted gates, although only operated at full open, have a $K_g$ of not exceeding 0.1 due to the effect of the slot.

For control gates, as with emergency gates, mounted in a conduit so that the floor, sides, and roof, both upstream and downstream, are continuous with the gate opening, only the losses due to the slot will need to be considered, for which a value of $K_g$ not exceeding 0.1 might be assumed. For partly open gates, the coefficient of loss will depend on the top contraction.

The loss and discharge coefficients for the individual control gates and valves vary with each type and design; therefore, the actual coefficients used in design should be acquired from the manufacturer or from tests performed in a laboratory. As stated above, the $K_g$ also varies for partial openings of the gate or valve.

(g) *Exit Losses.*–No recovery of velocity head will occur where the release from a pressure conduit discharges freely, or is submerged or supported on a downstream floor. The velocity head loss coefficient, $K_v$, in these instances is equal to 1.0. When a diverging tube is provided at the end of a conduit, recovery of a portion of the velocity head will be obtained if the tube expands gradually and if the end of the tube is submerged. The velocity head loss coefficient will then be reduced from the value of 1.0 by the degree of velocity head recovery. If $a_1$ is the area at the beginning of the diverging tube and $a_2$ is the area at the end of the tube, $K_v$ is equal to $\left(\frac{a_1}{a_2}\right)^2$

10-11. *Transition Shapes.*—(a) *Entrances.*—To minimize head losses and to avoid zones where cavitation pressures can develop, the entrance to a pressure conduit should be streamlined to provide smooth, gradual changes in the flow. To obtain the best inlet efficiency, the shape of the entrance should simulate that of a jet discharging into air. As with the nappe-shaped weir, the entrance shape should guide and support the jet with minimum interference until it is contracted to the dimensions of the conduit. If the entrance curve is too sharp or too short, subatmospheric pressure areas may develop which can induce cavitation. A bellmouth entrance which conforms to or slightly encroaches upon the free-jet profile will provide the best entrance shape. For a circular entrance, this shape can be approximated by an elliptical entrance curve represented by the equation:

$$\frac{x^2}{(0.5D)^2}+\frac{y^2}{(0.15D)^2}=1 \qquad (13)$$

where $x$ and $y$ are coordinates whose $x$-$x$ axis is parallel to and $0.65D$ from the conduit centerline and whose $y$-$y$ axis is normal to the conduit centerline and $0.5D$ downstream from the entrance face. The factor $D$ is the diameter of the conduit at the end of the entrance transition.

The jet issuing from a square or rectangular opening is not as easily defined as one issuing from a circular opening; the top and bottom curves may differ from the side curves both in length and curvature. Consequently, it is more difficult to determine a transition for a square or rectangular opening which will eliminate subatmospheric pressures. An elliptical curved entrance which will tend to minimize the negative pressure effects is defined by the equation:

$$\frac{x^2}{D^2}+\frac{y^2}{(0.33D)^2}=1 \qquad (14)$$

where $D$ is the vertical height of the conduit for defining the top and bottom curves, and is the horizontal width of the conduit for defining the side curves. The major and minor axes are positioned similarly to those indicated for the circular bellmouth.

(b) *Contractions and Expansions.*—To minimize head losses and to avoid cavitation tendencies along the conduit surfaces, contraction and expansion transitions to and from gate control sections in a pressure conduit should be gradual. For contractions, the maximum convergent angle should not exceed that indicated by the relationship:

$$\tan a=\frac{1}{U} \qquad (15)$$

where:

$a$ = the angle of the conduit wall surfaces with respect to its centerline, and
$U$ = an arbitrary parameter = $\frac{v}{\sqrt{gD}}$

The values of $v$ and $D$ are the averages of the velocities and diameters at the beginning and end of the transition.

Expansions should be more gradual than contractions because of the danger of cavitation where sharp changes in the sidewalls occur. Furthermore, as has been indicated in section 10-10(e), loss coefficients for expansions increase rapidly after the flare angle exceeds about 10°. Expansions should be based on the relationship:

$$\tan a=\frac{1}{2U} \qquad (16)$$

The notations are the same as for equation (15). For usual installations, the flare angle should not exceed about 10°.

The criteria for establishing maximum contraction and expansion angles for conduits flowing partly full are the same as those for open channel flow, as given in section 9-18(b) of chapter IX.

10-12. *Energy Dissipating Devices.*—Whenever practicable, the outlet works should be located so that the spillway energy dissipating structures can also be used to still the flow of the outlet works. Deflector buckets and hydraulic jump basins are commonly designed for stilling both outlet works and spillway flows when the outlet works flow can be directed into the spillway stilling basin. The hydraulic design for free-flow spillways and outlet works is discussed in chapter IX. Plunge pools and hydraulic jump stilling basins designed only for outlet works are discussed below.

(a) *Hydraulic Jump Basins.*—Where the outlet works discharge consists of jet flow, the open-channel flow hydraulic jump stilling basins mentioned above are not applicable. The jet flow either has to be directed onto the transition floor approaching the basin so it will become uniformly distributed, thus establishing open-channel flow conditions at the basin, or a special basin has to be designed.

The design of a basin that will work well at all discharges is difficult using theoretical calculations, and model tests should be conducted to finalize all designs if practicable. The Bureau of Reclamation hydraulic laboratory has developed generalized designs of several kinds of basins based upon previously run model tests. General design rules are presented so that the necessary dimensions for a particular structure may be easily and quickly determined. One such example is the design of a hydraulic jump basin to still the jet flow from a hollow-jet valve. This basin is about 50 percent shorter than a conventional basin. The stilling basin is designed to take advantage of the hollow-jet shape, so solid jets cannot be used. The general design procedure can be found in Engineering Monograph No. 25 [6].

(b) *Plunge Pools.*—Where the flow of an outlet conduit issues from a downstream control valve or freely discharging pipe, a riprap- or concrete-lined plunge pool might be utilized. Such a pool should be employed only where the jet discharges into the air and then plunges downward into the pool.

When a free-falling overflow nappe drops vertically into a pool in a riverbed, a plunge pool will be scoured to a depth which is related to the height of the fall, the depth of tailwater, and the concentration of the flow [7]. Depths of scour are influenced initially by the erodibility of the stream material or the bedrock and by the size or the gradation of sizes of any armoring material in the pool. However, the armoring or protective surfaces of the pool will be progressively reduced by the abrading action of the churning material to a size which will be scoured out and the ultimate scour depth will, for all practical considerations, stabilize at a limiting depth irrespective of the material size. An empirical approximation based on experimental data has been developed by Veronese [8] for limiting scour depths, as follows:

$$d_s = 1.32\, H_T^{0.225}\, q^{0.54} \tag{17}$$

where:

$d_s$ = the maximum depth of scour below tailwater level in feet,
$H_T$ = the head from the reservoir to tailwater levels in feet, and
$q$ = the discharge in cubic feet per second per foot of width. (The width used for a circular valve or discharge pipe should be the diameter.)

Plunge pools used as energy dissipators should be tested in hydraulic models or, if possible, compared with similar designs in use or previously tested in a hydraulic model.

**10-13.** ***Open Channel Flow in Outlet Works.***—If the outlet control gate or valve is at the upstream end or at some point along the conduit, open channel flow may exist downstream of the control; however, upstream of the control the flow is under pressure and the analysis is similar to that discussed in previous sections. The conduit downstream of the control may be enlarged or flared to assure nonpressure conditions, if desired. When open channel flow conditions exist, the design procedures are similar to those for open channel spillway flow discussed in chapter IX. An example of an outlet works with open channel flow downstream of the control gate is shown on figure 10-8.

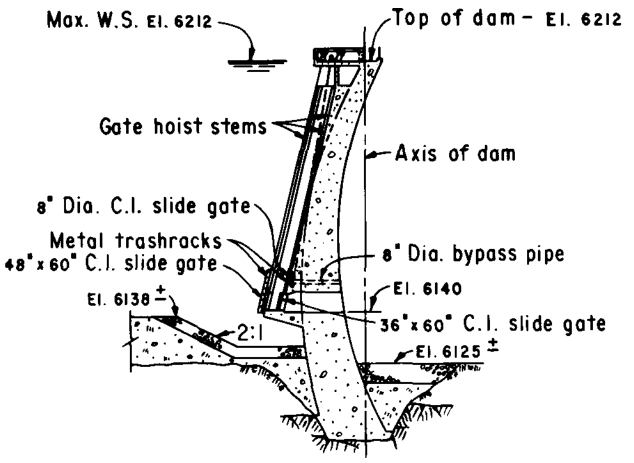


*Figure 10-8.* **A river outlet works with open channel flow. —288-D-3068**

## 2. *Structural Design of Outlet Works*

10-14. *General.*—The structural design of an outlet works is dependent upon the actual characteristics of that feature, the head, where the outlet works are incorporated in the dam, the stresses in the dam due to external loadings and temperature. The design criteria for each component of the outlet works should be established for the conditions which exist or may be expected to exist at any time during the life of the structure.

10-15. *Trashrack.*—A trashrack structure, regardless of the type, should be designed for a head differential due to the possible clogging of the rack with trash. This head differential will depend upon the location of the trashrack and its susceptibility to possible clogging, but should be a minimum of 5 feet. Temperature loads during construction should also be investigated in the design. If the trashrack will sometimes be exposed or partially exposed above the reservoir in areas subject to freezing, lateral loads from ice should be considered. In these instances, ice loads due to vertical expansion and the vertical load applied to the structure as ice forms on the members should also be included in the final analysis.

10-16. *Conduit.*—The outlet works conduit through a concrete dam may either be lined or unlined, but when the conduit is lined it may be assumed that a portion of the stress is being taken by the liner and not all is being transferred to the surrounding concrete. The temperature differential between the relatively cool water passing through the conduit and the relatively warm concrete mass will produce tensile stresses in the concrete in the immediate vicinity of the conduit. Also, the opening through the dam formed by the conduit will alter the distribution of stress in the dam in the vicinity of the conduit, tending to produce tensile stresses in the concrete at the periphery of the conduit. In addition, the bursting effect from hydrostatic pressures will cause tensile stresses at the periphery of the conduit. The above tensile stresses and possible propagation of concrete cracking usually extend only a short distance from the opening of the conduit, so it is common practice to reinforce only the concrete adjacent to the opening. The most useful method for determining the stresses in the concrete surrounding the outlet conduit is the finite element method of analysis, discussed in appendix J and in subchapter E of chapter IV.

10-17. *Valve or Gate House.*—The design of a control house depends upon the location and size of the structure, the operating and control equipment required, and the conditions of operation. The loadings and temperature conditions used in the design should be established to meet any situation which may be expected to occur during construction or during operation of the outlet works. The basic design approach should be the same as that for any commercial building.

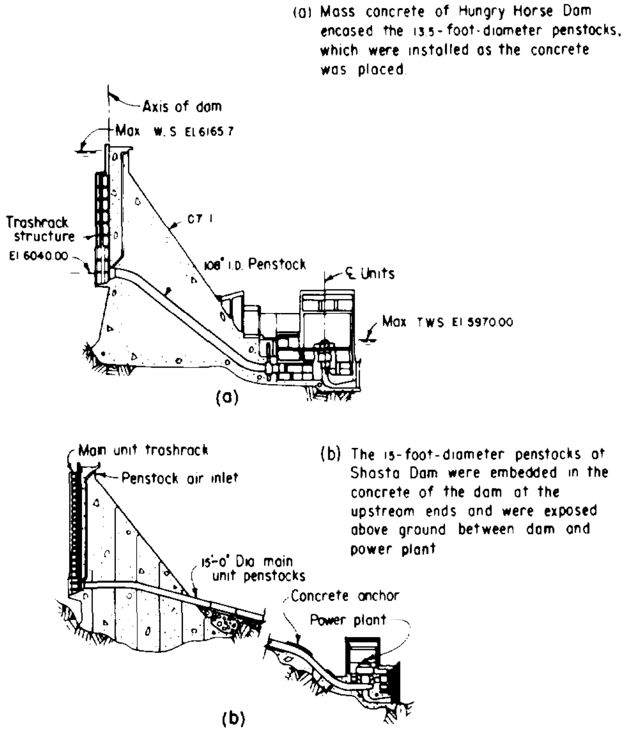
10-18. *Energy Dissipating Devices.*—The structural design of an energy dissipating device is accomplished by usual methods of analysis for walls, slabs, and other structural members. Because each type of outlet works usually requires a different type of energy dissipator, the design loads depend upon the type of basin used, and have to be determined for the characteristics of the particular outlet works. Because of the dynamic pressures exerted on the structure from the hydraulic stilling process, laboratory tests or other means are usually required to establish the actual design loadings.

## C. POWER OUTLETS

10-19. *General.*—Power outlets provide a passage for water from the reservoir to the turbines within a powerplant. The power outlets consist of: (1) an intake structure which normally includes the emergency gates, a bulkhead gate or stoplog slots and guides, a trashrack structure on the face of the dam, and a bellmouth intake entrance; (2) a transition of the circular shape at the upstream end of the penstock; and (3) a penstock. The penstock acts as a pressure conduit between the turbine scroll case and the intake structure. The power outlets should be as hydraulically efficient as practicable to conserve available head; moreover, the intake structure should be designed to satisfactorily perform all of the tasks for which it was intended.

10-20. *Layout.*—The location and arrangement of the power outlets will be influenced by the size and shape of the concrete dam, the location of the river outlet works and the spillway, the relative location of the dam and powerplant, and the possibility of incorporating the power outlets with a diversion tunnel or the river outlets. For low-head concrete dams, penstocks may be formed in the concrete of the dam; however, a steel lining is desirable to insure watertightness. The penstocks may be completely embedded within the mass concrete of the concrete dam as shown on figure 10-9(a), embedded through the dam while the downstream portions between the dam and powerplant are above ground as shown on figure 10-9(b), or in an abutment tunnel as shown on figure 10-10.

When a powerplant has two or more turbines, the question arises whether to use an individual penstock for each turbine or a single penstock with a header system to serve all units. Considering only the economics of the layout, the single penstock with a header system will usually be less in initial cost; however, the cost of this item alone should not dictate the design. Flexibility of operation should be given consideration, because with a single penstock system the inspection or repair of the penstock will require shutting down the


(a) Mass concrete of Hungry Horse Dam encased the 13.5-foot-diameter penstocks, which were installed as the concrete was placed.

(b) The 15-foot-diameter penstocks at Shasta Dam were embedded in the concrete of the dam at the upstream ends and were exposed above ground between dam and power plant

*Figure 10-9.* **Typical penstock installations.—288-D-3070**

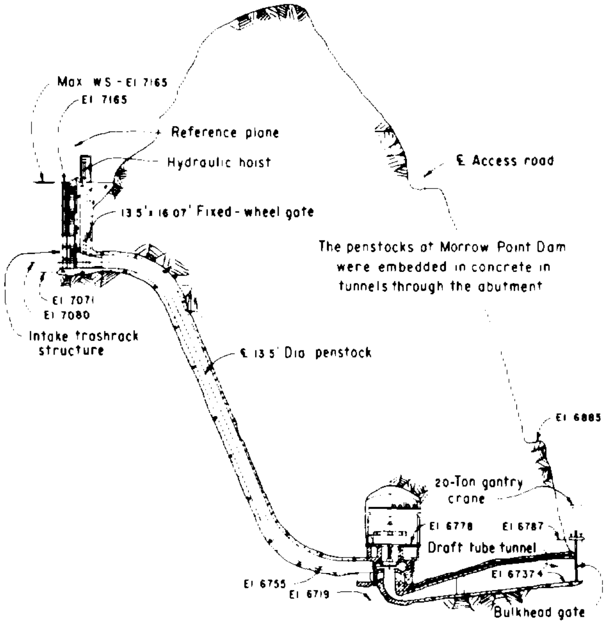

The penstocks at Morrow Point Dam were embedded in concrete in tunnels through the abutment

*Figure 10-10.* **Embedded penstock in abutment tunnel.—288-D-3072** entire plant. Further, a single penstock with a header system requires complicated branch connections and a valve to isolate each turbine. Also, the bulkhead gates will be larger, requiring heavier handling equipment. In concrete dams, it is desirable to have all openings as small as possible. The decision as to the penstock arrangement must be made considering all factors of operation, design, and overall cost of the entire installation.

Proper location of the penstock intake is important. The intake is usually located on the upstream face of the dam, which facilitates operation and maintenance of the intake gates. However, other structures or topographic conditions may influence the arrangement, and the penstock intake may best be situated in an independent structure located in the reservoir. Regardless of the arrangement, the intake should be placed at an elevation sufficiently below the low reservoir level and above the anticipated silt level to allow an uninterrupted flow of water under all conditions. Each intake opening should be protected against floating trash and debris by means of a trashrack structure.

Bends increase head loss and can cause the development of a partial vacuum during certain operating conditions. Therefore, penstock profiles from intake to turbine should, whenever practicable, be laid on a continuous slope. When vertical or horizontal bends are required in a penstock, their effect should be kept to a minimum by using as long a radius and as small a central angle as practicable.

10-21. ***Intake Structures.***—The intake structure consists of several components, each designed to accomplish a specific purpose. A trashrack is incorporated to keep trash from entering the penstocks and causing damage to the turbines; a bellmouth intake is used to establish flow lines at the entrance which minimize the amount of head loss; a transition, from the entrance size and shape to the circular diameter of the penstock, is established to least affect the flow and to minimize head loss. Also, the emergency gates can be incorporated into the intake structure to close off the flow through the penstock. Stoplogs are provided upstream of the emergency gates to unwater the entrance area and the emergency gate seats and guides for inspection and maintenance.

The velocity of flow in power intakes is usually much less than that in high-velocity river outlet works. For this reason, a smaller and less costly entrance structure can usually be designed for a power intake than for a river outlet works of equivalent physical size.

(a) *Trashracks.*—The trashrack structures for power intakes are similar to those required for other outlet works. However, because of the possible damage to the turbine and other hydraulic machinery, metal trashracks consisting of closely spaced bars are almost always required on power outlets to prevent the passage of even small trash and debris. With the lower velocity of flow through power outlets, large bellmouth openings at the intakes are not needed, and the length that the trashrack structure is required to span may be less than that for a high-velocity outlet works of equivalent physical size. The structure on which the trashracks are placed may consist of structural steel or of reinforced concrete as shown on figure 10-11. The determination of the type of trashrack structure depends not only upon the comparison of costs between the various structures but also upon the influence on the total time of construction for each scheme. Construction time may be reduced in some instances by using an all-metal or precast concrete structure instead of a cast-in-place structure.

Submerged trashracks should be used, if at all possible, because fully submerged racks normally require less maintenance than those which are alternately wet and dry. Experience has shown that steel will last longer if fully submerged. However, by bolting the all-metal trashrack structure to the concrete with stainless steel bolts, the racks can be replaced by divers if necessary.

When the reservoir surface fluctuates above and below the top of the trashrack structure, trash can accumulate on top of the structure and create a continuous maintenance problem. Normally, in large reservoirs submerged trashracks do not have to be raked as a result of trash accumulations, except during the

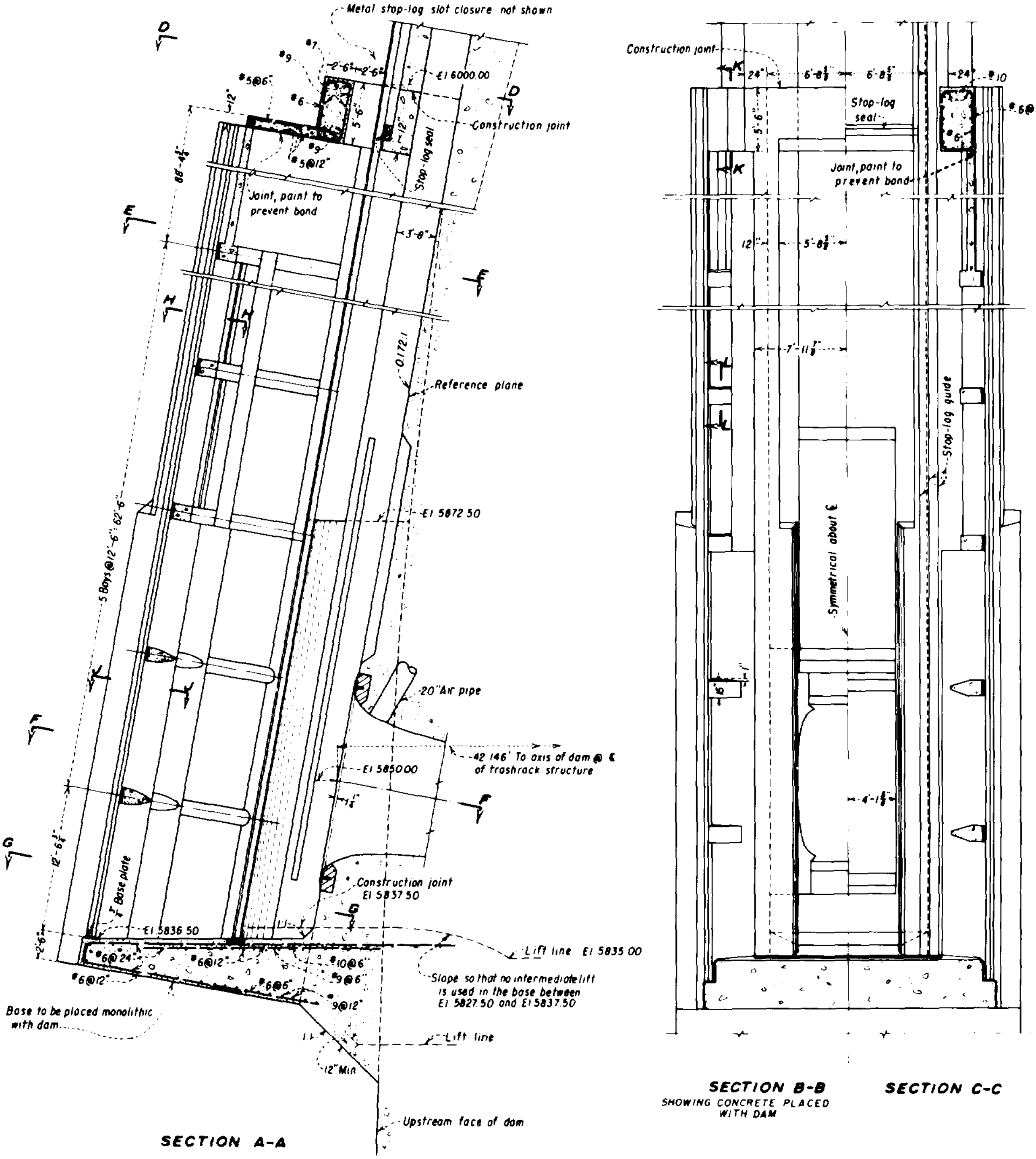
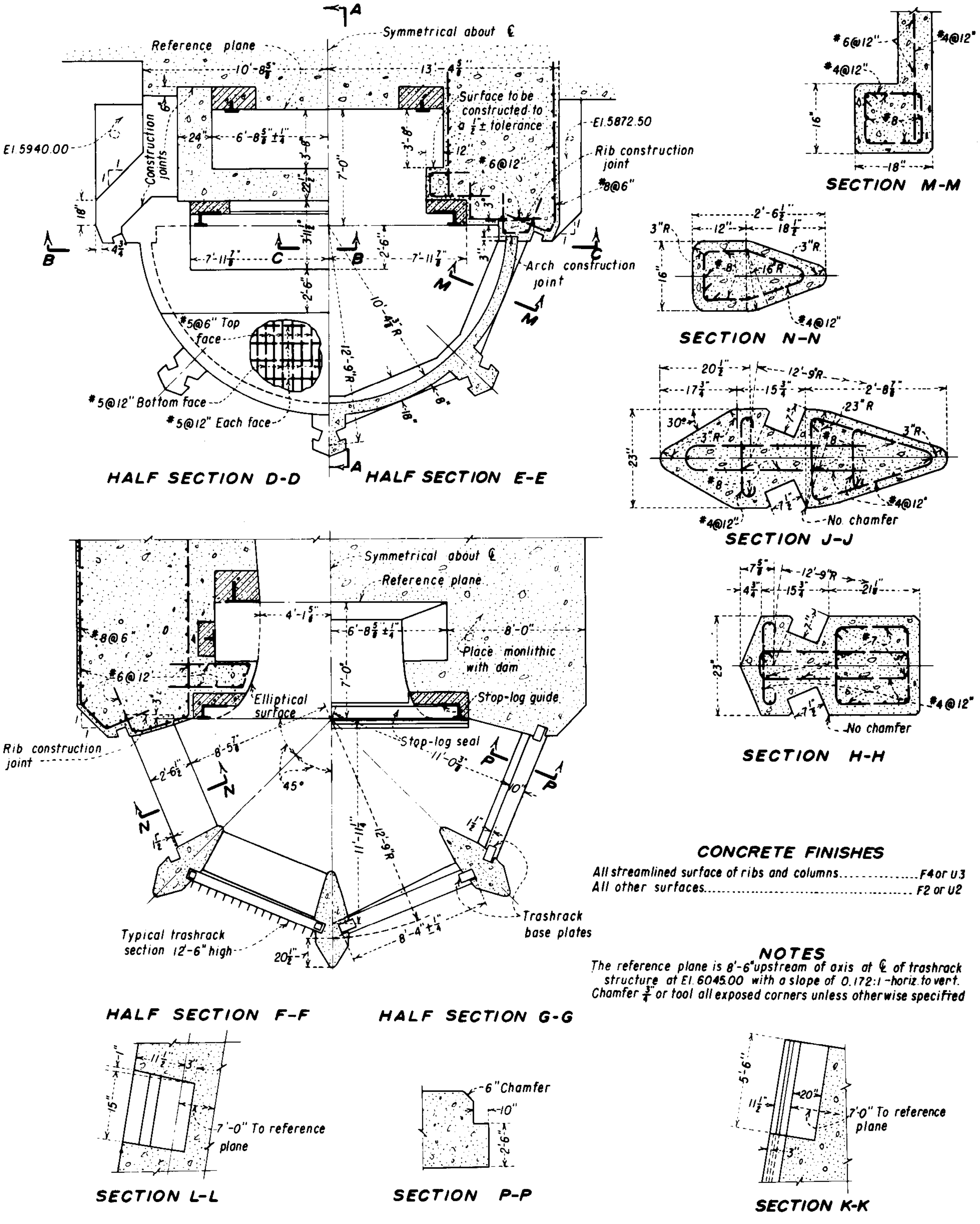


*Figure 10-11.* Typical concrete trashrack structure for a penstock (sheet 1 of 2). –288-D-3074 (1/2)



## CONCRETE FINISHES

| | |
|---|---|
| All streamlined surface of ribs and columns | F4 or U3 |
| All other surfaces | F2 or U2 |

## NOTES

The reference plane is 8'-6" upstream of axis at ℄ of trashrack structure at El. 6045.00 with a slope of 0.172:1 -horiz. to vert.

Chamfer ¾" or tool all exposed corners unless otherwise specified

*Figure 10-11.* **Typical concrete trashrack structure for a penstock (sheet 2 of 2). –288-D-3074 (2/2)** initial filling. Ice loads must be considered if the trashrack structure is above the reservoir at times during cold winters. Ice loadings may be prevented by the installation of an air bubbling system around the structure. This system circulates the warmer water from lower in the reservoir around the structure to keep the members ice free.

The trash bars usually consist of relatively thin, flat steel bars which are placed on edge from 2 to 9 inches apart and assembled in rack sections. The spacing between the bars is related to the size of trash in the reservoir and the size of trash that can safely be passed through the turbines without damage. The required area of the trashrack is fixed by a limiting velocity through the rack, which in turn depends on the nature of the trash which must be excluded. Where the trashracks are inaccessible for cleaning, the velocity should not exceed approximately 2 feet per second; however, a velocity up to approximately 5 feet per second may be tolerated for racks accessible for cleaning.

(b) *Bellmouth Entrance.*—It was brought out in section 10-11 that the entrance to a river outlet should be streamlined to provide smooth, gradual changes in the flow, thus minimizing head losses and avoiding disturbances of the flow in the conduit. This is also true for power outlets; however, because the velocities in penstocks are considerably lower, the bellmouths do not have to be as streamlined as those designed for the high-velocity river outlets. Experience on hydraulic models has shown that relatively simple rounding of corners eliminates most of the entrance losses when velocities are low. With the low velocities, pressure gradients in the bellmouth area are less critical.

(c) *Transition.*—Like the bellmouth entrance, the transition for the power outlets does not need to be as gradual as does the transition for the high-velocity river outlet works. The area throughout the transition can remain approximately the same, changing only from the shape of the gate to that of the penstock, with the gate area nearly equal to that of the penstock.

10-22. ***Penstocks.***—The penstock is the pressure conduit which carries the water from the reservoir to the powerplant. The penstock for a low-head concrete dam may be formed in the mass concrete; however, a steel shell or lining is normally used to assure watertightness and prevent leakage into a gallery or chamber or to the downstream face. In large concrete dams under a high-head condition, steel penstock liners are always used to provide the required watertightness in the concrete. Penstocks can be embedded in concrete dams, encased in concrete, or installed in tunnels and backfilled with concrete. The penstocks should be as short as practicable and should be designed hydraulically to keep head loss to a minimum. The size of the penstock is determined from economic and engineering studies that determine the most efficient diameter for overall operation.

10-23. ***Gates or Valves.***—Emergency gates or valves are used only to completely shut off the flow in the penstocks for repair, inspection, maintenance, or emergency closure. The wicket gates of the turbines act to throttle the flow in normal operation. The gates or valves, then, need to be designed only for full open operation. Many types of gates or valves can be utilized in the power outlets. Common emergency gates used in a concrete dam are fixed-wheel gates either at the face of the dam and controlled from the top of dam (see fig. 10-12), or in a gate slot in the dam and controlled from a chamber beneath the roadway.

An in-line control device, such as a butterfly valve, can be used anywhere along the length of the penstock and can be controlled from a chamber or control house. Also, in-line controls should be used on each individual penstock if more than one penstock branches off the main power outlet header, to permit the closure of each penstock without interfering with the flow of the others. In addition to butterfly valves, other types of in-line control devices that can be used to close off the flow include gate valves, ring-follower gates, and sphere valves. A determination of the type of valve or gate to be used is influenced by many factors such as the size of penstocks, the location best suited for controls

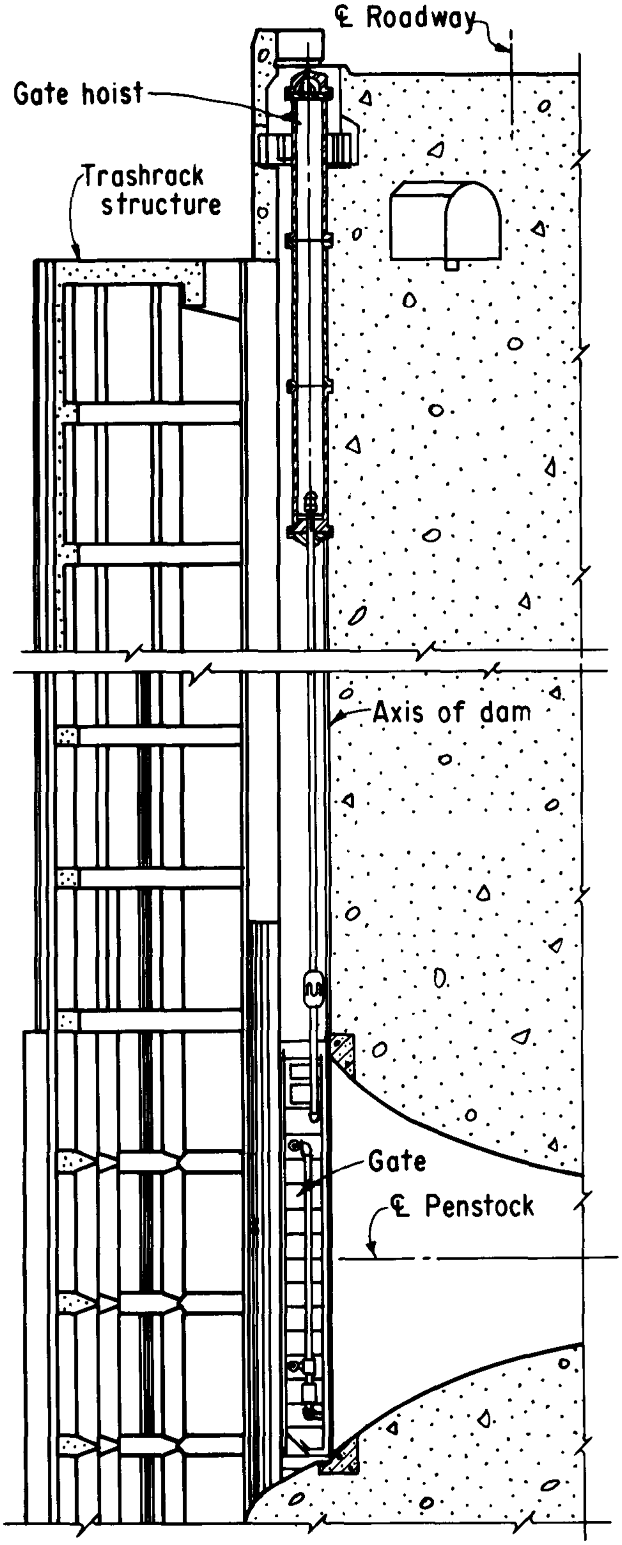


*Figure 10-12.* Typical fixed-wheel gate installation at upstream face of dam.—288-D-3075

and operators, the operating head, and the general layout of the power outlets. Another factor to consider in determining the control device to be used is the amount of head loss through each alternative type of gate or valve.

**10-24.** ***Hydraulic Design of Power Outlets.***—The hydraulics of power outlets involves pressure flow through a closed conduit. The methods of hydraulic analysis are similar to those required for other outlet works. A power outlet is designed to carry water to a turbine with the least loss of head consistent with the overall economy of installation. An economic study will size a penstock from a monetary standpoint, but the final diameter should be determined from combined engineering and monetary considerations.

(a) *Size Determination of Penstock.*—A method for determining the economic diameter of a penstock is given in Engineering Monograph No. 3 [9]. All the variables used in this economic study must be obtained from the most reliable sources available, so as to predict as accurately as possible the average variables for the life of the project. The designer must assure himself that all related costs of construction are considered during the economic study.

The head losses used in the economic study for the power outlet are similar to the losses in other outlet works. Because of the lower velocities, these losses are usually small. But over a long period, even a small loss of head can mean a sizable loss of power revenue. The various head losses which occur between reservoir and turbine are as follows:

(1) Trashrack losses.
(2) Entrance losses.
(3) Losses due to pipe friction.
(4) Bend losses.
(5) Contraction losses (if applicable).
(6) Losses in gate or valve.

Engineering Monograph No. 3 gives a complete discussion of these losses and how they should be used in the determination of the economic size of a penstock.

(b) *Intake Structure.*—As stated in earlier sections, the lower velocity through a power outlet requires less streamlining of the intake structure to achieve economically acceptable hydraulic head losses. The gate can be made smaller, the bellmouths can be designed with sharper curvature, and the transition need not be made as gradual as for a high-velocity river outlet works. The design of the trashrack structure is similar to that for the river outlet works, discussed in section 10-4(a).

10-25. ***Structural Design of Power Outlets.***—The structural design of a power outlet is dependent upon the actual characteristics of the power outlet works; the head; and where applicable, the stresses within the dam, due to temperature, gravity, and external loads. The design criteria for power outlet works should be established for the conditions which exist or may be expected to exist at any time during the operation or life of the structure.

(a) *Trashrack.*—The design of a trashrack structure for a power outlet should be based on a head differential of 5 feet due to partial clogging of trash. This small head differential minimizes power loss and is sufficient for the low velocities at which the power outlets operate. Ice loads should be applied in cold climates if the trashrack is exposed or partially exposed above the reservoir. Temperature loads during construction should also be investigated in the design procedures.

(b) *Penstocks.*—The penstocks through a concrete dam are usually lined with a steel shell; however, for low heads, penstocks may be simply a formed opening through the dam. Most penstock linings begin downstream from the transition. Therefore, when designing reinforcement around the penstocks, two conditions, lined and unlined, are usually present. In the area through which the penstocks are lined, a reasonable portion of the stresses may be assumed to be taken by the liner and not transferred to the surrounding concrete. Hydrostatic bursting pressures, concentration of the stresses within the dam, and temperature differentials between the water in the penstock and the mass concrete all may create tensile stresses in the concrete at the periphery of the penstock. Reinforcement is therefore placed around the penstock within the areas of possible tensile stress. A common method of analysis to determine the stresses in the concrete is a finite element study using a computer for the computations. Bursting pressures, dam loadings, and temperature variations can all be incorporated into this analysis to design the required reinforcement.

## D. BIBLIOGRAPHY

### 10-26. *Bibliography.*

[1] Bradley, J. N., and Thompson, L. R., "Friction Factors for Large Conduits Flowing Full," Engineering Monograph No. 7, Bureau of Reclamation, March 1951.
[2] Creager, W. P., and Justin, J. D., "Hydroelectric Handbook," second edition, John Wiley & Sons, Inc., New York, N. Y., 1954.
[3] "Hydraulic Design Criteria, Sheet 228-1, Bend Loss Coefficients," Waterways Experiment Station, U.S. Army Engineers, Vicksburg, Miss., April 1, 1952.
[4] King, W. H., "Handbook of Hydraulics," fourth edition, McGraw Hill Book Co., Inc., New York, N. Y., 1954.
[5] Rouse, Hunter, "Engineering Hydraulics," John Wiley & Sons, Inc., New York, N. Y., 1950.
[6] "Hydraulic Design of Stilling Basins and Energy Dissipators," Engineering Monograph No. 25, Bureau of Reclamation, 1964.
[7] Doddiah, D., Albertson, M. L., and Thomas, R. A., "Scour From Jets," Proceedings, Minnesota International Hydraulics Convention (Joint Meeting of International Association for Hydraulic Research and Hydraulics Division, ASCE), Minneapolis, Minn., August 1953, p. 161.
[8] Scimemi, Ettore, "Discussion of Paper 'Model Study of Brown Canyon Debris Barrier' by Bermeal and Sanks," Trans. ASCE, vol. 112, 1947, p. 1016.
[9] "Welded Steel Penstocks," Engineering Monograph No. 3, Bureau of Reclamation, 1967.

# Galleries and Adits

11-1. *General.*—A gallery is a formed opening within the dam to provide access into or through the dam. Galleries may run either transversely or longitudinally and may be either horizontal or on a slope. Where used as a connecting passageway between other galleries or to other features such as powerplants, elevators, and pump chambers, the gallery is usually called an adit. Where a gallery is enlarged to permit the installation of equipment, it is called a chamber or vault.

11-2. ***Purpose.***—The need for galleries varies from dam to dam. Some of the more common uses or purposes of galleries are:

(1) To provide a drainageway for water percolating through the upstream face or seeping through the foundation.

(2) To provide space for drilling and grouting the foundation.

(3) To provide space for headers and equipment used in artificially cooling the concrete blocks and grouting contraction joints.

(4) To provide access to the interior of the structure for observing its behavior after completion.

(5) To provide access to, and room for, mechanical and electrical equipment such as that used for the operation of gates in the spillways and outlet works.

(6) To provide access through the dam for control cables and/or power cables.

(7) To provide access routes for visitors.

Other galleries may be required in a particular dam to fulfill a special requirement.

Galleries are named to describe their location or use in the dam; for example, the foundation gallery is the gallery that follows the foundation of the dam, and the gate gallery is the gallery for servicing the gates. A typical gallery layout is shown on figures 11-1, 11-2, and 11-3.

11-3. ***Location and Size.***—The location and size of a gallery will depend upon its intended use or purpose. Some of the more common types of galleries are:

(a) *Foundation Gallery.*—The foundation gallery generally extends the length of the dam near the foundation rock surface, conforming in elevation to the transverse profile of the canyon; in plan it is near the upstream face and approximately parallel to the axis of the dam. It is from this gallery that the holes for the main grout curtain are drilled and grouted and from which the foundation drain holes are drilled. Its size, normally 5 feet wide by 7½ feet high, is sufficient to accommodate a drill rig. There should be a minimum of 5 feet of concrete between the floor of the gallery and the foundation rock.

(b) *Drainage Gallery.*—In high dams a supplementary drainage gallery is sometimes located further downstream, about two-thirds of the base width from the upstream face, for the purpose of draining the downstream portion of the foundation. This gallery usually extends only through the deepest portion of the dam. Drainage holes may be drilled from this gallery, so the 5- by 7½-foot size is usually adopted.

(c) *Gate Galleries and Chambers.*—Gate galleries and chambers are placed in dams to provide access to, and room for, the mechanical and electrical equipment required

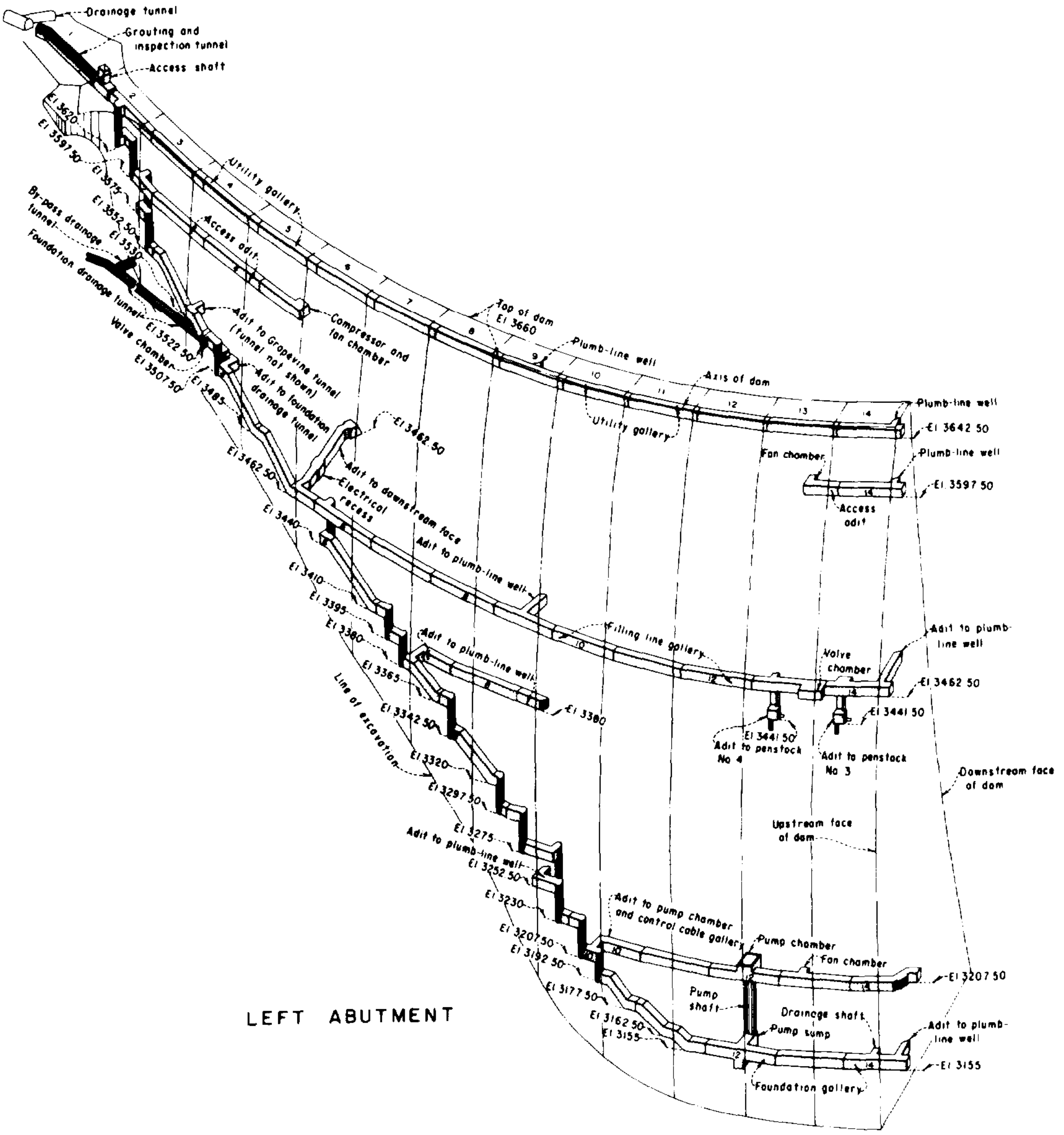


*Figure 11-1.* Gallery system in Yellowtail Dam—left abutment. 288-D-3076

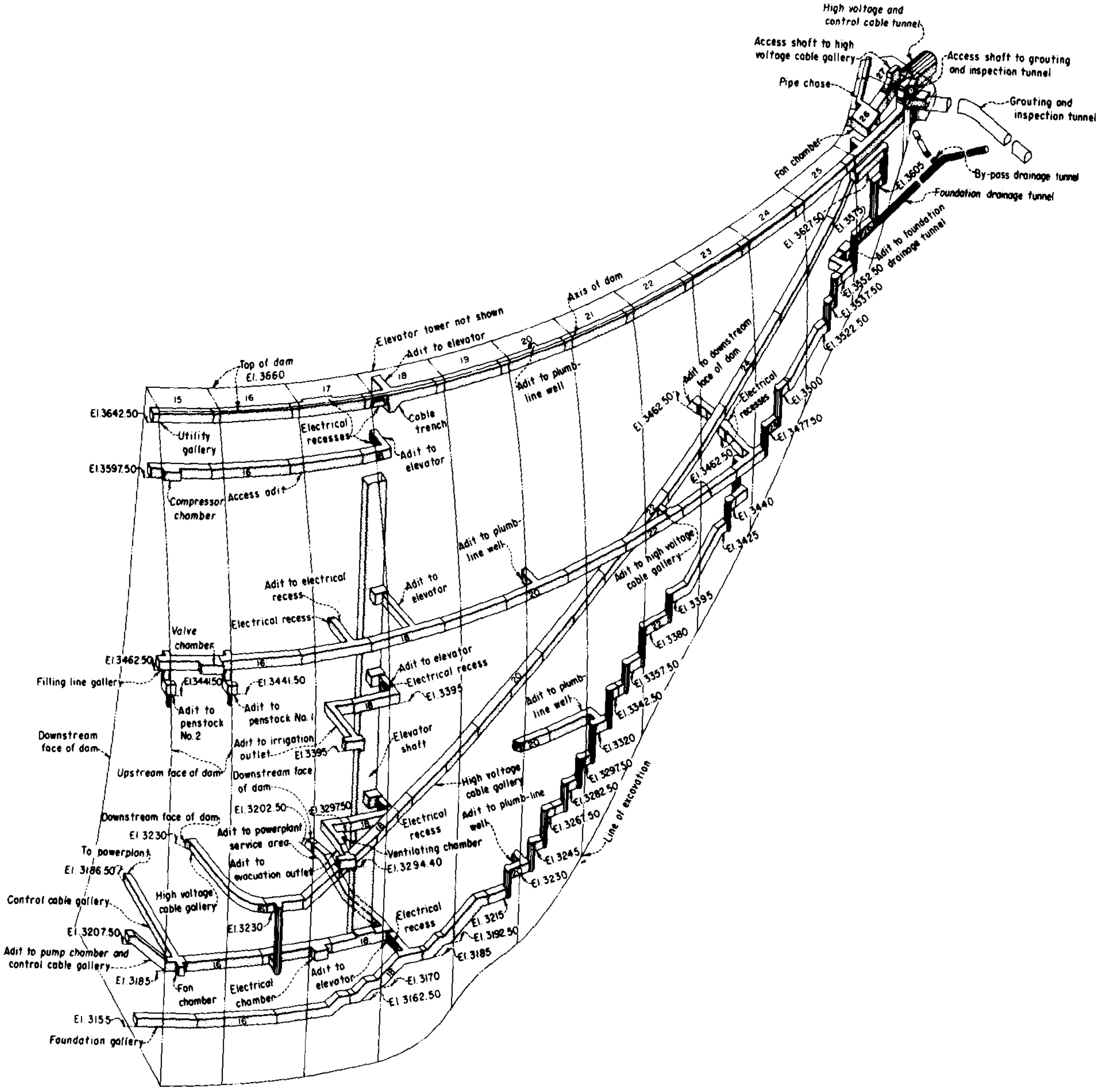


*Figure 11-2.* Gallery system in Yellowtail Dam–right abutment. 288-D-3078

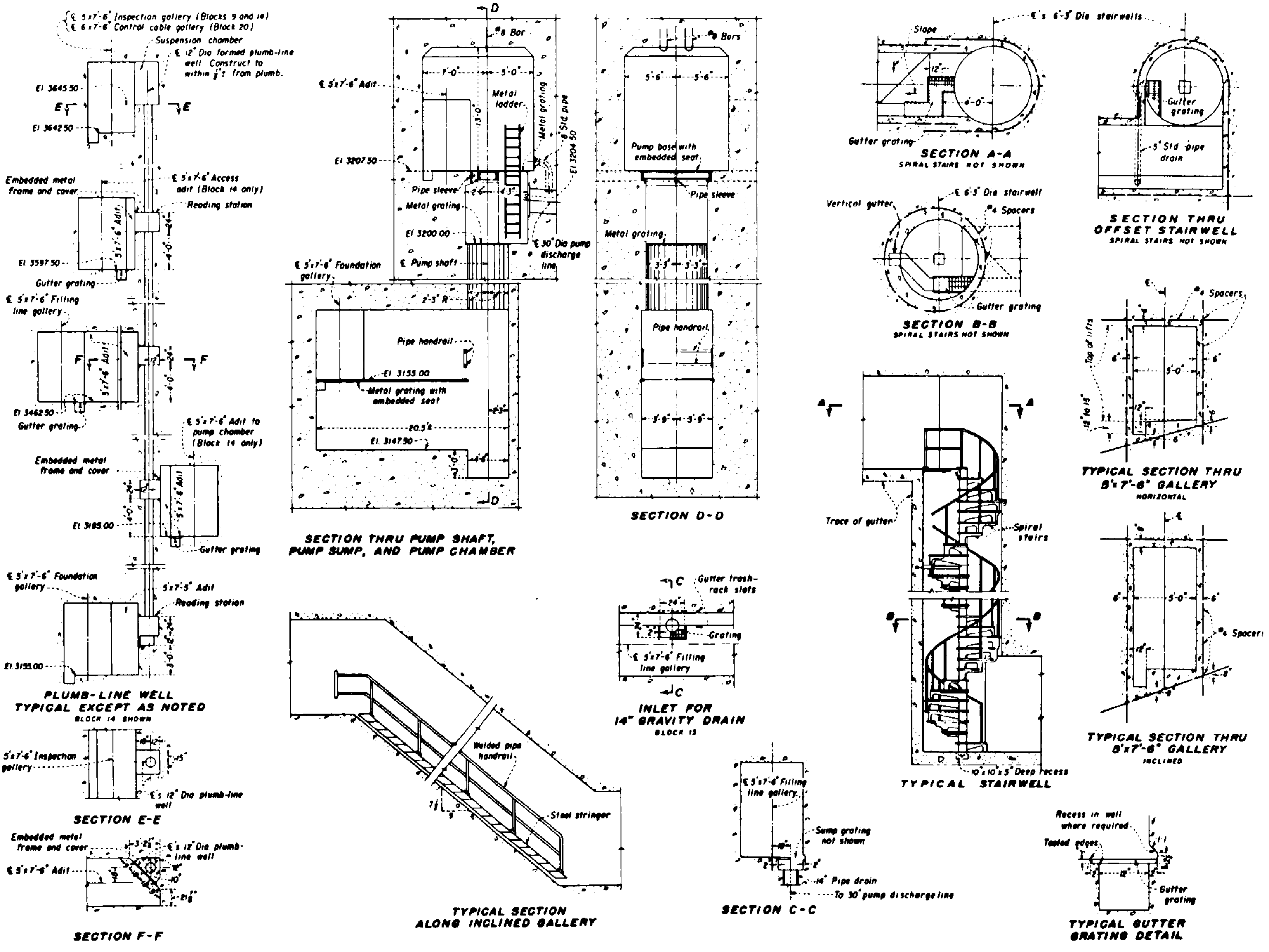


*Figure 11-3.* Galleries and shafts in Yellowtail Dam. 288-D-3080

for the operation of gates for outlets, power penstocks, or the spillway. Their size will depend on the size of the gates to be served.

(d) *Grouting Galleries.*—If it is impracticable to grout contraction joints from the face of the dam, the grout-piping system should be arranged so as to locate the supply, return, and vent headers in galleries placed near the top of each grout lift. The piping system for artificial cooling of the blocks may also be arranged to terminate in these galleries.

Transverse galleries or adits may be required for foundation consolidation grouting.

(e) *Visitors' Galleries.*—Visitors' galleries are provided to allow visitors into points of interest or as part of a tour route between visitors' facilities and the powerplant. The size would depend upon the anticipated number of visitors.

(f) *Cable Galleries.*—Galleries may be utilized, in conjunction with tunnels, cut and cover sections or overhead lines, as a means to carry control cables or power cables from the powerplant to the switchyard or spreader yard. The size of the gallery will depend upon the number of cables, the space required for each cable, and the space required for related equipment.

(g) *Inspection Galleries.*—Inspection galleries are located in a dam to provide access to the interior of the mass in order to inspect the structure and take measurements which are used to monitor the structural behavior of the dam after completion. All the galleries discussed above, which are located primarily for other specific purposes, also serve as inspection galleries.

As mentioned previously, galleries are usually made rectangular and 5 feet wide by 7½ feet high with a 12-inch-wide gutter along the upstream face for drainage. The 4-foot width is a comfortable width for walking and the 7½-foot height corresponds with the 7½-foot placement lift in mass concrete. Experience has shown that this size of gallery will provide adequate work area and access for equipment for normal maintenance except where special equipment is required such as at gate chambers. Galleries as narrow as 2 feet have been used; however, a minimum of 3 feet is recommended.

11-4. ***Drainage Gutter.***—All galleries should have gutters to carry away any seepage which gets into the gallery. On horizontal runs, the depth of gutter may vary from 9 to 15 inches to provide a drainage slope. Pipes should collect the water at low points in the gutter and take it to lower elevations where it will eventually go to the pump sump or drain directly to the downstream face by gravity.

11-5. ***Formed Drains.***—Five-inch-diameter drains are formed in the mass concrete to intercept water which may be seeping into the dam along joints or through the concrete. By intercepting the water, the drains minimize the hydrostatic pressure which could develop within the dam. They also minimize the amount of water that could leak through the dam to the downstream face where it would create an unsightly appearance.

The drains are usually located about 10 feet from the upstream face and are parallel to it. They are spaced at approximately 10-foot centers along the axis of the dam. The lower ends of the drains extend to the gallery, or are connected to the downstream face near the fillet through a horizontal drain pipe or header system if there are no galleries. The tops of the drains are usually located in the crest of the dam to facilitate cleaning when required. Where the top of the dam is thin, the drains may be terminated at about the level of the normal reservoir water surface. A 1½-inch pipe then connects the top of the drain with the crest of the dam and can be used to flush the drains.

11-6. ***Reinforcement.***—Reinforcement is usually required around galleries in a dam only where high tensile stresses are produced, such as around large openings, openings whose configuration produces high tensile stress concentrations, and openings which are located in areas where the surrounding concrete is in tension due to loads on the dam or temperature or shrinkage. Reinforcement should also be utilized where conditions are such that a crack could begin at the gallery and propagate through the dam to the reservoir.

Stresses around openings can be determined using the finite element method for various loading assumptions such as dam stresses, temperature, and shrinkage loads. Reinforcement is usually not required if the tensile stresses in the concrete around the opening are less than 5 percent of the compressive strength of the concrete. If tensile stresses are higher than 5 percent of the compressive strength, reinforcement should be placed in these areas to limit cracking. Each gallery should be studied individually using the appropriate dam section and loads.

In areas of high stress or where the stresses are such that a crack once started could propagate, reinforcement should be used. If unreinforced, such a crack could propagate to the surface where it would be unsightly and/or admit water to the gallery. It could also threaten the structure safety. The stresses determined by the finite element analysis can be used to determine the amount of reinforcement required around the opening to control the cracking.

In some cases, reshaping or relocating the gallery can reduce or eliminate the tensile stresses.

11-7. ***Services and Utilities.***—Service lines, such as air and water lines, can be installed in the gallery to facilitate operation and maintenance after the dam has been completed. To supply these lines, utility pipe should be embedded vertically between the galleries and from the top gallery to the top of the dam. This will enable the pipe at the top of the dam to be connected with an air compressor, for example, and deliver compressed air to any gallery. The number and size of the utility piping would depend upon anticipated usage.

Galleries should have adequate lighting and ventilation so as not to present a safety hazard to persons working in the galleries. The ventilation system should be designed to prevent pockets of stale air from accumulating.

Telephones should be installed at appropriate locations in the gallery for use in an emergency and for use of operations and maintenance personnel.

The temperature of the air in the gallery should be about the same as that of the surrounding mass concrete to minimize temperature stresses. This may require heating of incoming fresh air, particularly in colder climates. Galleries used for high-voltage power cables may require cooling since the cables give off considerable heat.

11-8. ***Miscellaneous Details.***—Horizontal runs of galleries, where practicable, should be set with the floor at the top of a placement lift in the dam for ease of construction. Galleries on a slope should provide a comfortable slope for walking on stairs. A 7½ to 10 slope is reasonable for stairs, yet is steep enough to follow most abutments. A slope of 7½ to 9 has been used on steeper abutments. Ramp slopes may be used where small or gradual changes in elevation are required. Ramp slopes should be less than 10° but can be up to 15° if special nonslip surfaces and handrails are provided.

Spiral stairs in a vertical shaft are used where the abutments are steeper than can be followed by sloping galleries. These shafts are usually made 6 feet 3 inches in diameter to accommodate commercially available metal stairs.

To minimize the possibility of a crack developing between the upstream face of the dam and a gallery which would leak water, galleries are usually located a minimum distance of 5 percent of the reservoir head on the gallery from the upstream face. A minimum of 5 feet clear distance should be used between galleries and the faces of the dam and contraction joints, to allow room for placement of mass concrete and to minimize stress concentrations.

## <<Chapter XII

# Miscellaneous Appurtenances

12-1. *Elevator Tower and Shaft.*—Elevators are placed in concrete dams to provide access between the top of the dam and the gallery system, equipment and control chambers, and powerplant. The elevators can also be used by the visiting public for tours through the dam. The elevator structure consists of an elevator shaft that is formed within the mass concrete, and a tower at the crest of the dam. The shaft should have connecting adits which provide access into the gallery system and into operation and maintenance chambers. These adits should be located to provide access to the various galleries and to all locations at which monitoring and inspection of the dam or maintenance and control of equipment may be required. Stairways and/or emergency adits to the gallery system should be incorporated between elevator stops to provide an emergency exit.

The tower provides a sheltered entrance at the top of the dam and houses the elevator operating machinery and equipment. Moreover, the tower may be designed to provide space for utilities, storage, and offices. Tourist concessions and information space may also be provided in the tower at the top of the dam, if the project is expected to have a large tourist volume. The height of the tower above the roadway is dependent upon the number of floors needed to fulfill the space requirements of the various functions for which the tower is intended. On large dams more than one elevator may be incorporated into the design to make access more available. Moreover, separate elevators may be constructed for visitors other than the elevators provided for operation and maintenance. Since the towers provide the entrance to the interior of the structure and are used by most visitors, they are a focal point of interest and their architectural considerations should be an important factor in their design and arrangement. The architectural objective should be simplicity and effectiveness blending with the massiveness of the dam to present a pleasing and finished appearance to the structure.

The machinery and equipment areas should include sufficient space for the required equipment and adequate additional space for maintenance and operation activities. Electrical, telephone, water, air, and any other services which may be required should be provided to the appropriate areas. Restrooms for visitors as well as those for maintenance personnel may also be included in the layout of the tower. Stairways, either concrete or metal, are usually included for access to machinery and equipment floors to facilitate maintenance and repair. Stairways can also be provided as emergency access between levels. An example of the layout of a typical elevator shaft and tower can be seen on figure 12-1.

(a) *Design of Shaft.*—The design of reinforcement around a shaft can be accomplished by the use of finite element studies, with the appropriate loads applied to the structure. The stresses within the dam near the shaft and any appropriate temperature loads should be analyzed to determine if tension can develop at the shaft and be of such magnitude that reinforcement would be required. A nominal amount of reinforcement should be placed around the shaft if it is near any waterway or the upstream face of the dam to minimize any chance of leakage through any

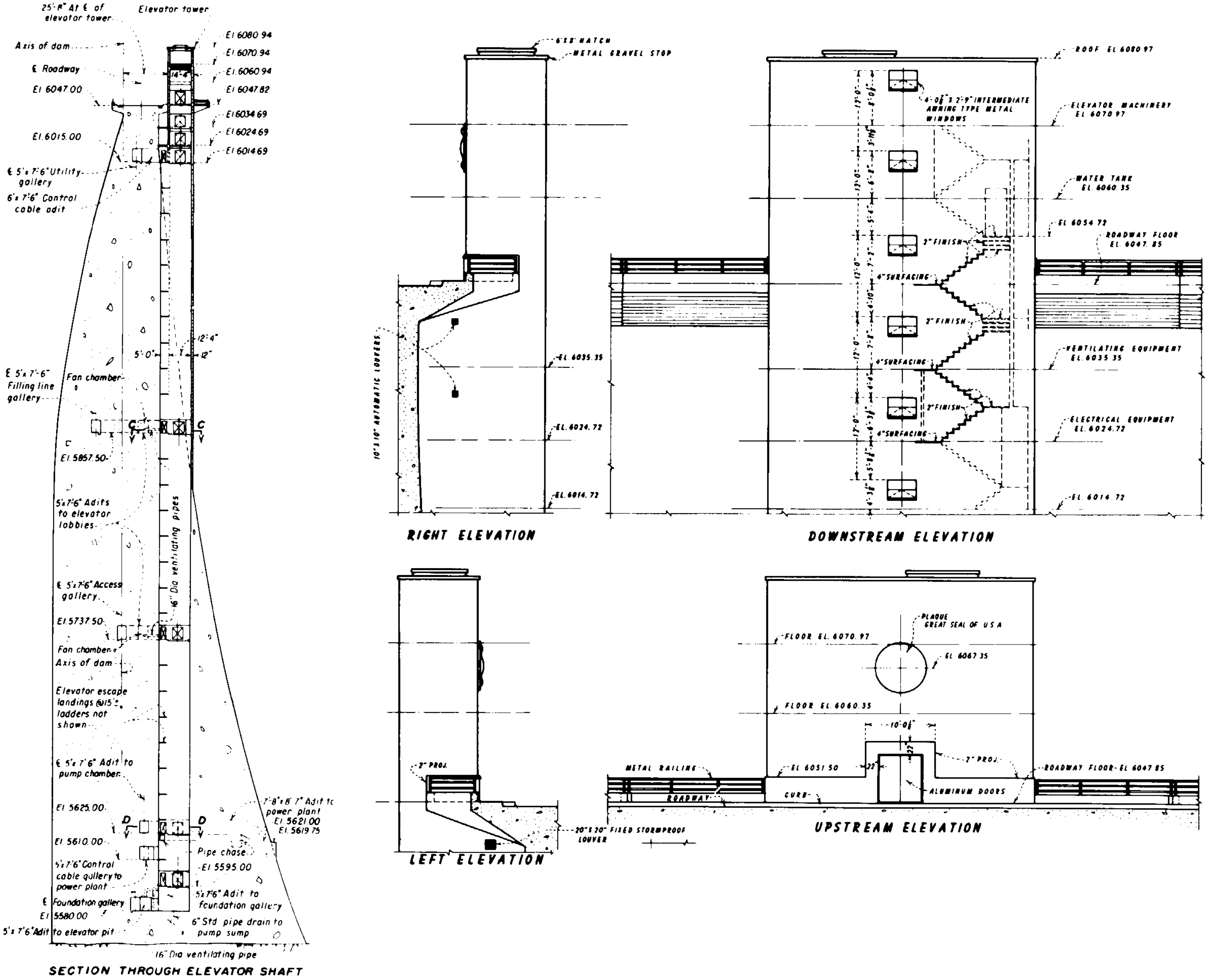
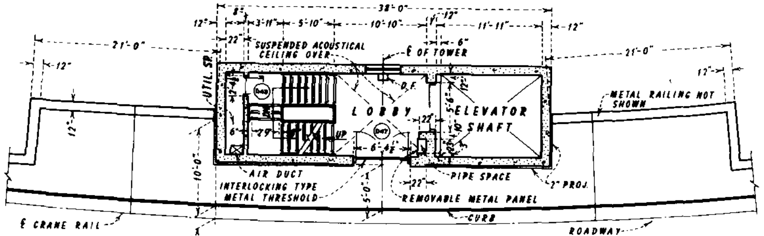
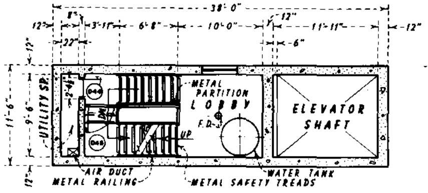


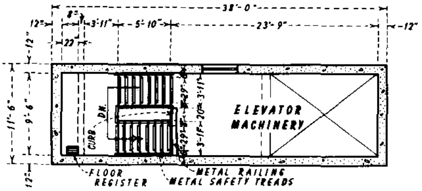
*Figure 12-1.* **Structural and architectural layout of elevator shaft and tower in Flaming Gorge Dam (sheet 1 of 2).—288-D-3081 (1/2)**



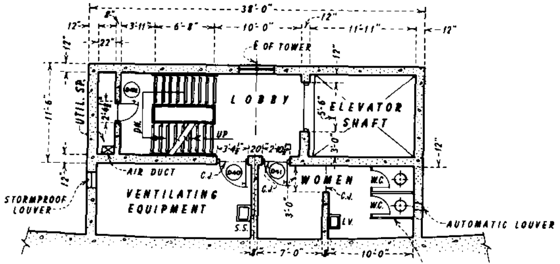
PLAN - EL.6047.85



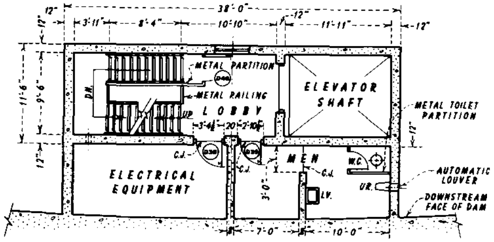
PLAN - EL.6060.35



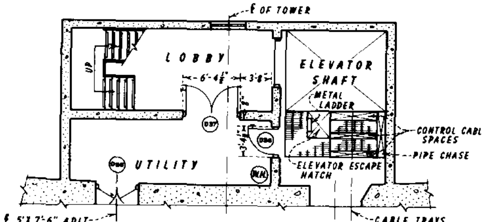
PLAN - EL.6070.97



PLAN - EL.6035.35



PLAN - EL.6024.72



PLAN - EL.6014.72

5 0 5 10 15 20

SCALE OF FEET

PLUMBING SCHEDULE

| SYMBOL | FIXTURE |
|---|---|
| W.C. | WATER CLOSET |
| UR. | URINAL |
| LV. | LAVATORY |
| S.S. | SERVICE SINK |
| D.F. | DRINKING FOUNTAIN |
| W.H. | WATER HEATER |
| F.D. | FLOOR DRAIN |

FINISH SCHEDULE

| FLOOR ELEV. | ROOM | FLOOR FINISH | BASE | WALL FINISH | CEILING |  | REMARKS |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  | HT. | MATERIAL |  |
| EL.6014.72 | UTIL. | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6014.72 | LOBBY | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6024.72 | ELEC. EQUIP. | *4" SURF. |  | CONC. F2 |  | CONC. F2 |  |
| EL.6024.72 | TOILET AREA | *4" SURF. | * 5" HIGH SEE DETAIL | CONC. F3 |  | CONC. F3 | D.S.F. OF DAM F2 |
| EL.6024.72 | LOBBY | *4" SURF. | * 5" HIGH SEE DETAIL | CONC. F3 |  | CONC. F3 | D.S.F OF DAM F2 |
| EL.6035.35 | LOBBY | *4" SURF. | * 5" HIGH SEE DETAIL | CONC. F3 |  | CONC. F3 | D.S.F. OF DAM F2 |
| EL.6035.35 | TOILET AREA | *4" SURF. | * 5" HIGH SEE DETAIL | CONC. F3 |  | CONC. F3 | D.S.F. OF DAM F2 |
| EL.6035.35 | VENT. EQUIP. | *4" SURF. |  | CONC. F2 |  | CONC. F2 |  |
| EL.6035.35 | UTIL. SPACE | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6047.85 | LOBBY | *4" SURF. | * 5" HIGH SEE DETAIL | CONC. F3 | 9'-0" | *SUSPENDED ACOUSTICAL |  |
| EL.6047.85 | UTIL. SPACE | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6060.35 | LOBBY | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6060.35 | UTIL. SPACE- | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
| EL.6070.97 | ELEV. MACH. | CONC. U3 |  | CONC. F2 |  | CONC. F2 |  |
|  |  |  |  |  |  |  |  |
| EL.6014.72<br>EL.6024.72 | STAIR | LANDING CONC. U3 |  | CONC. F2 |  | RAKES CONC. F2 | TREADS & RISERS-CONC. U3 & F3. METAL SAFETY TREADS |
| EL.6024.72<br>EL.6054.72 | STAIR | *LANDINGS 2" FINISH | SEE DETAIL | CONC. F3 |  | RAKES CONC. F3 | TREADS & RISERS * 2" FIN. *METAL SAFETY TREADS |
| EL.6054.72<br>EL.6070.97 | STAIR | LANDING CONC. U3 |  | CONC. F2 |  | RAKES CONC. F3 | TREADS & RISERS-CONC. U3 & F3. METAL SAFETY TREADS |
|  |  |  |  |  |  |  |  |
| EL.5580.00<br>EL.6070.97 | ELEV. SHAFT | CONC. U2 |  | CONC. F2 |  | CONC. F2 |  |
|  |  |  |  |  |  |  |  |

* INDICATES MATERIAL TO BE FURNISHED AND INSTALLED UNDER SPEC. NO. DC-5700

CONCRETE TO RECEIVE 4" SURFACING TO HAVE U1 FINISH
CONCRETE TO RECEIVE 2" FINISH TO HAVE F1 OR U1 FINISH.

*Figure 12-1.* Structural and architectural layout of elevator shaft and tower in Flaming Gorge Dam (sheet 2 of 2). — 288-D-3081 (2/2) cracks which may open. Reinforcement should also be placed around the periphery of the shaft as it approaches the downstream face of the dam, where tensile stresses due to temperature loadings become more likely to occur.

(b) *Design of Tower.*—The structural design of the elevator tower above mass concrete should be accomplished by using standard design procedures and the appropriate loads that can be associated with the structure. Live loads, dead loads, temperature loads, wind loads, and earthquake loads should all be included in the design criteria. The magnitude of earthquake load on the tower (see (2)b below) may be increased substantially by the resonance within the structure and must be determined by actual studies. Reinforcement to be placed in the structure at all the various components should be designed with respect to the characteristics of the structure and the requirements of the reinforced concrete code.

Dead loads and live loads usually used in the design of an elevator tower are as follows:

(1) *Dead loads:*

Reinforced concrete—150 pounds per cubic foot

Roofing—varies with type of material

(2) *Live loads:*

a. Uniformly distributed floor loads, pounds per square foot.

| | |
|---|---|
| Lobby | 150 |
| Office space | 100 |
| Roof (includes snow) | 50 |
| Toilets | 100 |
| Stairways | 100 |
| Elevator—machinery floor | *250 |
| Storage space—heavy | 250 |
| Storage space—light | 125 |

---

*Concentrated loads from the elevator machinery may control the design instead of the uniform load given.

b. Other loads:

| | |
|---|---|
| Wind loads | 30 pounds per square foot on vertical projection |
| Earthquake loads: | |
| Horizontal | 0.1 gravity |
| Vertical | 0.05 gravity |

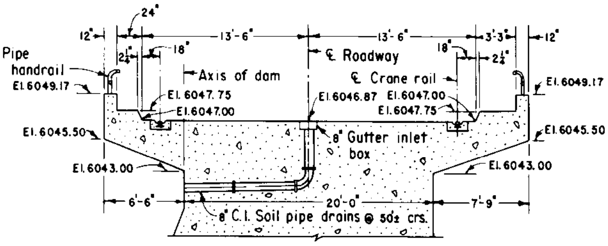
12-2. ***Bridges.***—Bridges may be required on the top of the dam to carry a highway over the spillway or to provide roadway access to the top of the dam at some point other than at the end of the dam. A bridge may also be provided over a spillway when bulkhead gates for river outlets or spillway crest gates require the use of a traveling crane for their operation or maintenance. Where there is no highway across the dam and no crane operations are required, a spillway bridge designed only to facilitate operation and maintenance may be constructed. When a bridge is to be used for a highway or to act as a visitors' access route, architectural treatment should be undertaken to give the structure a pleasing appearance. This architectural treatment should be based on the size of dam, the size and type of other appurtenant structures, local topography, and a type of bridge structure which blends pleasingly with the entire feature.

Design criteria for highway bridges usually conform to the standard specifications adopted by the American Association of State Highway Officials, modified to satisfy local conditions and any particular requirement of the project. The width of roadway for two-way traffic should be a minimum of 24 feet curb to curb plus sidewalk widths as required. However, with new highway regulations requiring greater widths, both Federal and local codes should be consulted to establish a final width. The structural members can consist of reinforced concrete, structural steel, or a combination of both types of materials. The bridge structure can be one of many types such as barrel-arch, slab and girder, or slab, depending on the required architecture, loads, and span. The structure should be designed to carry the class of traffic which is to use the bridge; however, the traffic design load used should generally not be less than the HS-20 classification.

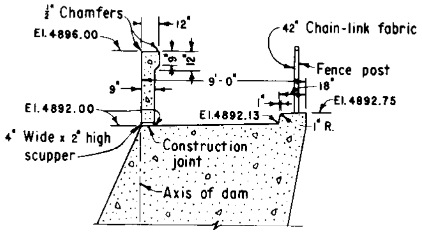
Special heavy loads during the construction period, such as powerplant equipment hauled on specially constructed trailers, may produce stresses far in excess of those produced by the normal highway traffic and these should be considered in the design criteria. If the bridge deck is to be used for servicing gates or other mechanical equipment, the loading imposed by the weight of the crane, the force necessary to lift the gate or equipment, as well as the normal traffic loads should all be included in the design. Sidewalk and pedestrian bridge design loads should be a minimum live load of 85 pounds per square foot. Other considerations which should be covered in the design are camber, crown of roadway slab, storm drainage, and roadway lighting.

12-3. ***Top of Dam.***—The top of the dam may contain a highway, maintenance road, or walkway depending upon the requirements at the site. If a roadway is to be built across the dam, the normal top of the dam can be widened by the use of cantilevers from the upstream and downstream faces of the dam. Operation and maintenance areas, and where conditions warrant visitors' parking, may also be provided on the top of the dam by further enlarging the cantilevers to the required size. The width of the roadway on the top of the dam is dependent upon the type and size of roadway, sidewalks, and maintenance and operation spaces needed to accomplish the tasks required. The minimum width for a two-lane roadway is 24 feet between curbs; however, the actual width should be established by the class of roadway crossing the dam. For highways, the roadway between curbs should be made the width required by the American Association of State Highway Officials or stipulated by local considerations. The sidewalks should be a minimum of 18 inches wide; however, the actual width should be determined by the proposed usage and the overall layout and space required for operation and maintenance. Two typical examples of the top of an arch dam, one containing a two-lane roadway and the other including only a walkway, can be seen in figure 12-2.

When a highway is not to be taken across the dam, the top width should be established to meet the requirement for operation and maintenance. A width can be established which allows a truck to be taken out on the dam if operation requires it, or a walkway may be all that is needed for normal operation and maintenance. If only a walkway is required, the minimum width should be no less than the actual top width minus the width required for handrails and/or parapets. Widened areas for service decks can be constructed, where required, to facilitate operation of outlet works, power outlets, and spillways.



SECTION THRU TOP OF DAM
FLAMING GORGE DAM
(SHOWING ROADWAY AND CRANE RAILS)



SECTION THRU TOP OF DAM
EAST CANYON DAM
(SHOWING WALKWAY)

*Figure 12-2.* **Typical sections at the top of an arch dam.–288-D-3083**

Parapets or handrails are required both upstream and downstream on the top of the dam and should be designed not only to meet the safety requirements but also to blend into the architectural scheme. On dams where a large tourist traffic is expected, extreme care should be taken to assure the safety of the public. Therefore, the parapets should be of a height sufficient to keep anyone from falling over the side. The minimum height of parapet above the sidewalk should be 3 feet 6 inches; however, the minimum height may be more on some dams because of local conditions. When a handrail is used, chain-link fabric may be used to prevent a child falling or crawling between the rails. A solid upstream parapet may be used to increase the freeboard above the top of dam if additional height is needed.

Adequate drainage and lighting should be provided along the top of the dam. Service lines such as electricity, water, and air should also be provided as required. Crane rails may be embedded in the top of the dam if a gantry crane will be used for operation and maintenance (see fig. 12-2).

The design of the reinforcement for the top of the dam involves determining the amount of reinforcement required for the live and dead loadings on the roadway cantilevers and any temperature stresses which may develop. If a highway is to cross the dam, the cantilevers should be designed for a minimum AASHO loading of HS-20; however, special heavy loads which could occur during the construction period should also be investigated. Crane loads should also be included in the design criteria if a crane is to be used for operation and maintenance. A sidewalk live load of 85 pounds per square foot should be used in the design. Concrete parapets should be designed for a transverse force of 10,000 pounds spread over a longitudinal length of 5 feet; moreover, the parapets should be designed to withstand the appropriate waterload if the parapet is expected to create additional freeboard.

The temperature reinforcement requirement at the top of the dam is dependent upon the configuration and size of the area and the temperature conditions which may occur at the site. Many dams have a gallery or chamber below the roadway, which complicates the analysis and increases the amount of reinforcement needed to resist stresses caused by variations between the outside air temperature and the temperature within the opening in the dam. All temperature studies should be based on historic temperature data from that area and the temperatures occurring in galleries or chambers within the dam. After the temperature distributions are determined by studies, the temperature stresses that occur can be analyzed by the use of finite element methods.

12-4. ***Fishways.***—The magnitude of the fishing industry in various localities has resulted in Federal, State, and local regulations controlling construction activities which interfere with the upstream migration and natural spawning of anadromous fish. All dams constructed on rivers subject to fish runs must be equipped with facilities enabling the adult fish to pass the obstruction on their way upstream, or other methods of fish conservation must be substituted. Since it is required that all facilities for fish protection designed by Federal agencies be approved by the U.S. Fish and Wildlife Service, this agency and similar State or local agencies should be consulted prior to the final design stage.

Low dams offer little difficulty in providing adequate means for handling fish. High dams, however, create difficulties not only in providing passage for adult fish on their way upstream, but also in providing safe passage for the young fish on their journey downstream. Fish ladders for high dams may require such length and size as to become impracticable. Large reservoirs created by high dams may cause flooding of the spawning areas. The velocity and turbulence of the flow over the spillway or the sudden change in pressure in passing through the outlet works may result in heavy mortality for the young fish. These difficulties often necessitate the substitution of artificial propagation of fish in lieu of installation of fishways.

Several types of fishways have been developed, the most common of which is the fish ladder. In its simplest form, it consists of an inclined flume in which vertical baffles are constructed to form a series of weirs and pools. The slope of the flume is usually 10 horizontal to 1 vertical. The difference in elevation of successive pools and the depth of water flowing over the weirs are made such that the fish are induced to swim rather than leap from pool to pool, thereby insuring that the fish will stay in the ladder for its entire length. The size of the structure is influenced by the size of the river, height of dam, size of fish, and magnitude of the run.

Another type of fishway in common use is the fish lock. This structure consists of a vertical water chamber, gate-controlled entrance and exit, and a system of valves for alternately filling and draining the chamber. Fish locks are usually provided with a horizontal screen which can be elevated, thereby forcing the fish to rise in the chamber to the exit elevation.

12-5. ***Restrooms.***—Restrooms should be placed throughout a dam and its appurtenant works at convenient locations. The number required depends on the size of dam, ease of access from all locations, and the estimated amount of usage. At least one restroom should be provided at all dams for the use of operation and maintenance personnel. Separate restrooms should be provided for tourists at dams which may attract visitors. In larger dams, restrooms should be placed at convenient locations throughout the gallery system as well as in appurtenant structures such as elevator towers and gate houses.

12-6. ***Service Installations.***–Various utilities, equipment, and services are required for the operation and maintenance of mechanical and electrical features of the dam, outlet works, spillway, and other appurtenant structures. Other utilities and services are required for the convenience of operating personnel and visitors. The amount and type of services to be provided will vary with the requirements imposed by the size, complexity, and function of the various appurtenant structures. The elaborateness of installations for personal convenience will depend on the size of the operating forces and the number of tourists attracted to the project.

(a) *Electrical Services.*–Electrical services to be installed include such features as the power supply lines to gate operating equipment, drainage pumps, elevators, crane hoists, and all lighting systems. Adequate lighting should be installed along the top of the dam, at all service and maintenance yards, and internally in the galleries, tunnels, and appurtenant structures. Power outlet receptacles should be provided at the top of the dam, in all appurtenant structures, throughout the gallery system and at any location which may require a power source.

(b) *Mechanical Services.*–Mechanical installations and equipment that may be required include such features as overhead traveling cranes in gate or valve houses, gantry cranes on top of the dam for gate operation and trashrack servicing, hoisting equipment for accessories located inside the dam, and the elevator equipment. Compressed air lines should be run into the gallery system, into service and maintenance chambers, into appurtenant structures, and anywhere else where compressed air could be utilized.

(c) *Other Service Installations.*–Chambers or recesses in the dam may be provided for the storage of bulkhead gates when these are not in use. Adequate storage areas should be provided throughout the dam such as in the gallery system, elevator towers, gate or valve house, and other appurtenant structures for maintenance and operation supplies and equipment. If gantry cranes are to be installed at a dam, recesses in the canyon walls may be provided for housing them when they are not in use. The gallery system and all appurtenant structures should be supplied with a heating and ventilating system where required.

A telephone or other communication system should be established at most concrete dams for use in emergency and for normal operation and maintenance communication. The complexity of the system will depend on the size of the dam, the size of the operating force, and the amount of mechanical control equipment. Telephones are usually placed throughout the gallery system for ease of access and safety in case of an emergency such as flooding or power failure. Telephones are also placed near mechanical equipment such as in gate or valve houses, elevator towers, machinery rooms, and other areas in which maintenance may be required. Telephones should also be placed at convenient locations along the top of the dam.

Water lines should be installed to provide a water source throughout the dam and the appurtenant structures. Water for operation and maintenance should be taken into the gallery system at the various levels of the galleries and into the appurtenant structures where required. The water for operation and maintenance can come from the river or reservoir but water for restrooms and drinking fountains requires a potable water source. Drinking fountains should be placed at convenient locations that are readily accessible to both maintenance personnel and tourist traffic.

## «Chapter XIII

# Structural Behavior Measurements

13-1. *Scope and Purpose.*—Knowledge of the behavior of a concrete arch dam and its foundation may be gained by studying the service action of the dam and the foundation, using measurements of an external and an internal nature. Of primary importance is the information by which a continuing assurance of the structural safety of the dam can be gaged. Of secondary importance is information on structural behavior and the properties of concrete that may be used to give added criteria for use in the design of future concrete arch dams.

In order to determine the manner in which a dam and its foundation behave during the periods of construction, reservoir filling, and service operations, measurements are made on the structure and on the foundation to obtain actual values of behavior criteria in terms of strain, temperature, stress, deflection, and deformation of the foundation. Properties of the concrete from which the dam is constructed, such as temperature coefficient, modulus of elasticity, Poisson's ratio, and creep, are determined in the laboratory.

(a) *Development of Methods.*—The investigations of the behavior of concrete dams began at least 50 years ago, and have included scale model and prototype structures. Reports on the investigations are available in references [1], [2], and [3].[1] Along with the development of instruments [4] to use for measurements and the instrumentation programs, there was the development of a suitable method for converting strain, as determined in the concrete which creeps under load [5], to stresses that are caused by the measured deformation [6]. The basic method, which departs from simple Hooke's law relationships obtained for elastic materials, has been presented in reference [7] with later refinements presented in other publications [8, 9]. As analyses of the behavioral data from dams were completed, reports on the results of the investigations became available [10, 11].

Similarly, reports on the results of investigations of foundation behavior have become available [12, 13].

(b) *Two General Methods.*—At a major concrete dam, two general methods of measurement are used to gain the essential behavioral information, each method having a separate function in the overall program.

The first method of measurement involves several types of instruments that are embedded in the mass concrete of the structure and on features of the dam and appurtenances to the dam. Certain types of instruments are installed at the rock-concrete interfaces on the abutments and at the base of the dam for measuring deformation of the foundation. Others are installed on the steel liners of penstocks for measuring deformation from which stress is determined, and at the outer surface of the penstocks for measuring hydrostatic head near the conduit. This type of instrumentation may also be used with rock bolts in walls of underground openings such as a powerplant or tunnel and in reinforcement steel around penstocks and spillway openings to measure deformation from which stress is determined.

The second method involves several types of precise surveying measurements which are

---

[1] Numbers in brackets refer to items in the bibliography sec. 13-11.

made using targets on the downstream face of a dam, through galleries and vertical wells in a dam, in tunnels, on the abutments, and with targets on the top of a dam.

13-2. ***Planning.***—From the modest programs for measurements provided at the earliest dams, there have evolved the extensive programs which are presently in operation in recently constructed Bureau of Reclamation dams. The formulation of programs for the installation of structural behavior instruments and measurement systems in dams has required careful and logical planning and coordination with the various phases of design and of construction.

Plans for a measurement program for a dam should be initiated at the time the feasibility plans are prepared for the structure. The layout should include both the embedded instrument system and systems for external measurements. Appropriate details must be included with those layouts to provide sufficient information for preparing a cost estimate of items needed for the program.

The information which a behavioral measurement system is to furnish is usually somewhat evident from the analytical design investigations which have been made for the dam and from a study of past experience with behavioral measurements at the other dams. This information includes temperature, strain, stress, hydrostatic pressure, contraction joint behavoir, deformation of foundation, and deformation of the structure, all as influenced by the loading which is imposed on the structure with respect to time.

The cost of a program is contingent on the size of structure, the number of segments which make up the program, the types of instruments to be used, and the number of instruments of the various types needed to obtain the desired information.

13-3. ***Measurement Systems.***—Measurement systems, their layouts, and the locations and use of the various devices embedded in the mass concrete of dams for determining volumetric changes are discussed in the following sections. Measurement systems which employ surveying methods for determining deformation changes in a dam are discussed separately.

The locations of the instruments to be installed in an arch dam are shown on the elevation and sections of figures 13-1 and 13-2.

(a) *Embedded Instrument Measurements.*—Embedded instruments in a concrete arch dam usually consist of those which measure length change (strain), stress, contraction joint opening, temperature, concrete pore pressure, and foundation deformation. Instruments to measure stress may be installed at locations in reinforcing steel such as around a spillway opening or other opening in the dam and on the steel liners of penstocks. All instruments are connected through electrical cables to terminal boards located at appropriate reading stations in the gallery system of the dam. At those stations readings from the instruments are obtained by portable readout units. Mechanical-type deformation gages which utilize invar-type tapes, and a micrometer-type reading head may be installed vertically in cased wells which extend from the foundation gallery into the foundation to any desired depth. They may also be installed horizontally in tunnels in the abutments.

In the arch dam shown on figure 13-1, instrumentation is sufficiently extensive to obtain measurements in several arch and cantilever elements of the structure. For investigation of the dam's behavior, instrumentation to determine temperature, stress, and deformation is required.

The extensive scope of the instrumentation in an arch dam is necessary to monitor the complex configuration of the dam, which is of relatively thin cross section and possibly of double curvature in its plan and section. Stress and joint movement are determined at a number of locations. An arch dam may appear to be symmetrical or nearly so in plan and elevation, and in the interest of economy, consideration might be given in some cases to the instrumentation of but half of such a structure. That consideration can be misleading and should absolutely be avoided. Experience has proved that instrumentation should be made for the entire structure, since results obtained near one abutment may differ significantly from results near the other abutment.

The layout of instruments in an arch dam should be made as shown by figures 13-1 and 13-2 so that the gagelines of instruments will be compatible with the locations of stress determinations in the arch and cantilever elements of the analytical stress solution. The resulting stress distributions in the arch and cantilever elements of both studies then can be compared. The usual instrument arrangement provides for instrumentation of four or five arch elements, one near the top of the dam and the remainder at uniform spacing between the top and the base of the maximum section. Also, the base of the maximum section is instrumented.

In each arch element, radial gagelines are located at or near the crown, near each abutment, and at intermediate locations. In the arch dam shown in the example, the instruments are installed at several locations on various radial gagelines in clusters of 12 instruments each. These are designated as groups, for determination of multidimensional stress at the cluster locations. From these configurations, stress distributions normal to vertical and to horizontal planes at the gagelines may be determined as well as shear stresses and principal stresses. Duplicate instruments are installed on the three major orthogonal axes in each cluster. Eleven instruments of each cluster are supported by a holding device or spider. The twelfth instrument is placed vertically beside the cluster. Clusters are located along each radial gageline near each face and at midpoint between. An additional cluster usually is located between the interior cluster and the one near each face.

The patterns of radial lines of instrument groups are generally located on the centerlines of blocks and are selected to define a system of horizontal and vertical planes that represent approximate arch and cantilever elements on which stress information is obtained by the design analysis of the dam. Stress information at a number of arch-cantilever intersections common to both methods of investigation is thus available. At two locations in the maximum block of the dam, radial lines of instrument groups are installed near each contraction joint for elevations containing instrument groups along centerlines. They are included for determining stress distribution within the block.

Vertical and horizontal stresses are determined at the base of the maximum section where maximum vertical stresses are to be expected. Similarly, the stresses are determined at sections in a number of arches which are about uniformly spaced through the elevation of the dam and at elevations where the distributions of horizontal and vertical stresses are desired. To obtain stress distribution in each of the sections, except for the areas adjacent to spillway openings and near the base of the maximum section, the instrument groups are placed at three locations between the upstream and downstream faces of the dam. Near the base of the maximum section and at the mid-elevation of that section, five or more instrument groups are placed between the faces of the dam.

In the example, the instrument groups nearest the base are located at elevation 6710.0 on the reference plane of the dam. At the higher elevations the instruments are located at three to five sections in arches at elevations 6800.0, 6920.0, 7040.0, and 7152.5. At these elevations the locations of investigation are on the reference plane of the dam, near the arch abutments, and at intermediate locations about midway between the center and each abutment. The instrument groups indicate length changes which are used to compute true structural stresses. The stress measuring instruments indicate stress conditions from which true stresses are obtained with a minimum of computation.

Stress measuring instruments usually are installed at several locations in the arch of the topmost instrumented elevation, except for the maximum section where strain measuring instruments are installed. These latter instruments are located on radial lines in blocks which contain gagelines of instruments at lower elevations. The stress measuring instruments are installed to measure stress parallel to the

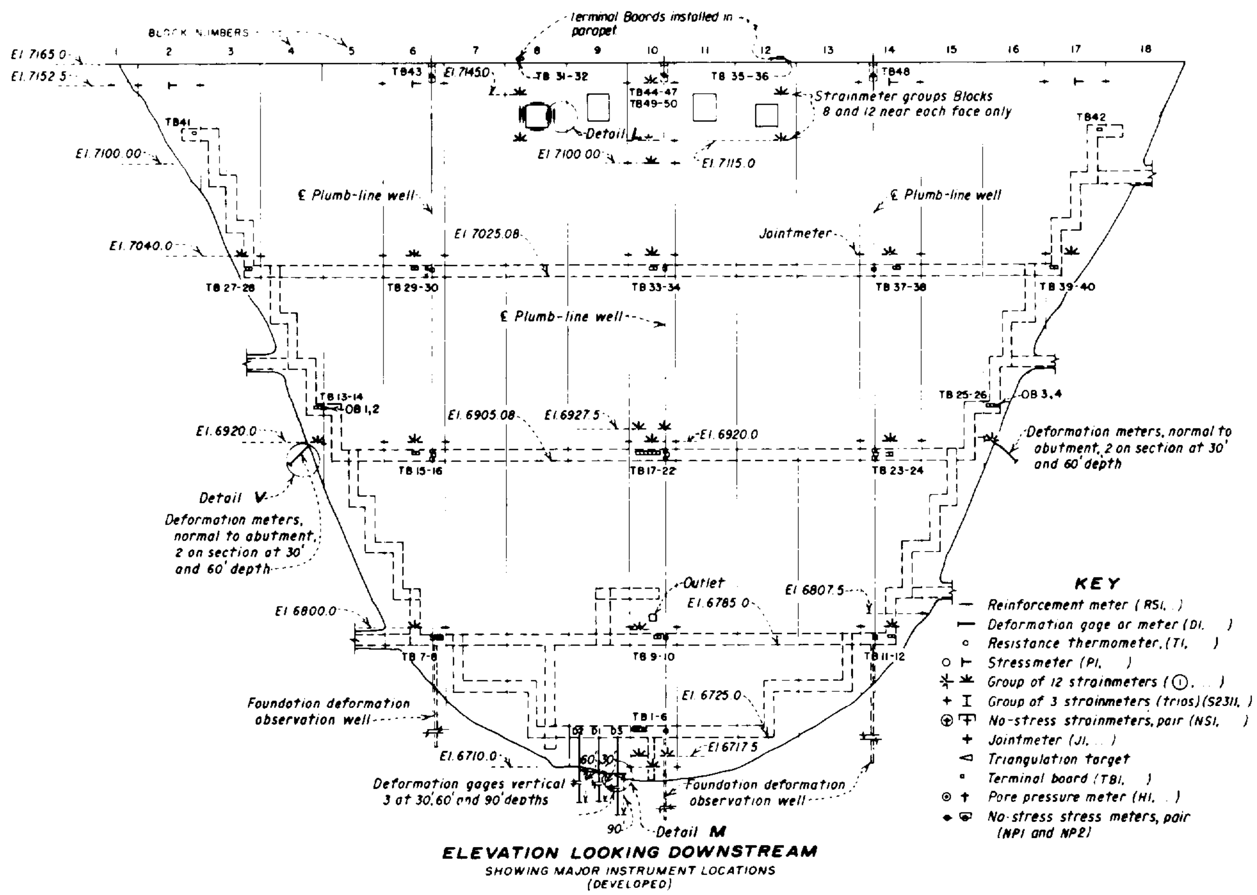


*Figure 13-1.* **Locations of instrumentation installed in an arch dam–developed elevation. (Referenced details are not included in this illustration.)–288-D-3086**

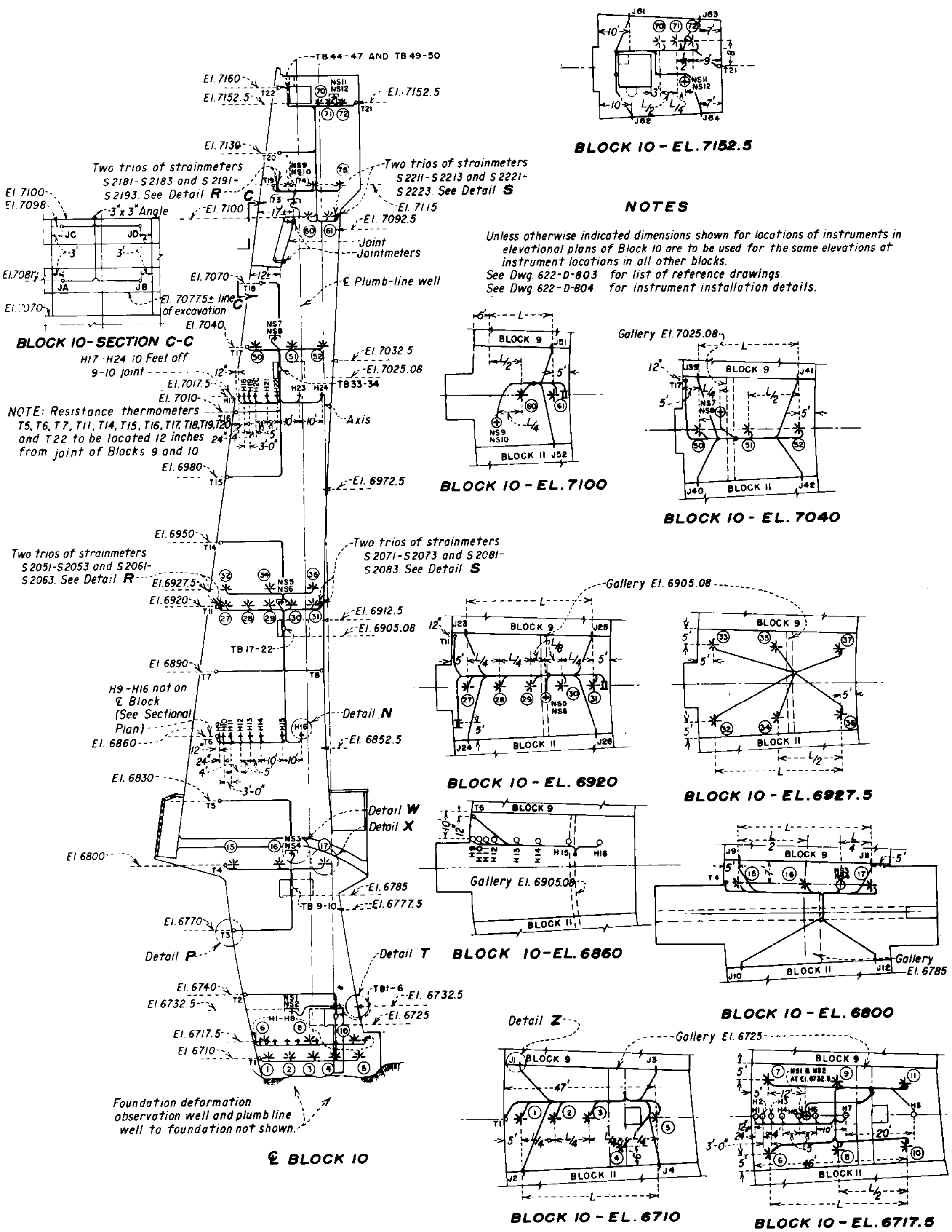
direction of thrust in the arch.

A pair of instruments, one vertical and one horizontal, placed in the concrete under a supported surface, is usually installed near the centrally located cluster of instruments. This pair of instruments is needed to determine stress-free behavior of the mass concrete. Instruments in various arrays may be installed near the faces or near contraction joints to determine conditions of special interest in the concrete, or in structural elements. Data are obtained from all instruments at frequent intervals so that time lapse variations of stress will be available for study during the entire period of observation, usually several years.

Instruments are installed across the contraction joints bounding the blocks containing the instrument clusters. These instruments provide a means of monitoring the behavior of the joints to determine the beginning and extent of joint opening due to cooling of the mass concrete. They serve as indicators of maximum joint opening to indicate when grouting should be performed. The instruments also give an indication of the effectiveness of grouting and show whether any movement in the joint occurs after grouting.

Several deformation measuring instruments are installed at selected locations in the foundation below the concrete of the maximum section and other sections of an arch dam.

A pattern of temperature-sensing devices is included in the maximum section of the dam. In a structure of unusual size, similar installations could be made in additional sections when deemed desirable to determine the manner in which heat of hydration from



**NOTES**

Unless otherwise indicated dimensions shown for locations of instruments in elevational plans of Block 10 are to be used for the same elevations at instrument locations in all other blocks.
See Dwg. 622-D-803 for list of reference drawings.
See Dwg. 622-D-804 for instrument installation details.

*Figure 13-2.* **Locations of instrumentation installed in an arch dam—maximum section, sectional plans. (Referenced details are not included in this illustration.)—288-D-3088** the mass concrete is generated and dissipated. These instruments should be located on gagelines at several elevations in a section. They are not located near the instrument clusters, as the stress instrumentation also senses temperature.

An installation of instruments, when required for investigation of stress in the steel liner of a penstock, consists of instruments attached circumferentially to the penstock by supporting brackets. Instruments to detect pore pressure are placed on the outer surface of the penstock when it is embedded in concrete or when it extends through rock. The instruments are connected by electrical cables to terminal boards at appropriately located reading stations.

Instruments for measuring stress are sometimes installed in reinforcing steel which surrounds openings through a dam such as penstocks, spillway openings, or galleries. Similarly, instruments may be installed in rock bolts used to stabilize rock masses. Temperature-sensing devices installed on a grid pattern in the maximum section or in several sections of a dam have been used to determine the distribution of temperature. This is of great importance because the volume change caused by temperature fluctuation is one of the factors which contributes significantly to stress and deflection. Temperature-sensing devices are also used for control in the cooling operations. Another extensive use of these devices has been the development of concrete temperature histories to study the heat of hydration which is generated and dissipated, and to evaluate conditions which contribute to or accompany the formation of thermal cracking in mass concrete.

Concrete surface temperatures of dams are obtained by temperature-sensing devices embedded at various random locations on the downstream faces and embedded at uniform vertical intervals between the base and crest on the upstream faces. The latter installations furnish information on temperature due to the thermal variations in the reservoir.

Measurements of strain obtained by extensometers used with appropriate gage point anchors have been made on the faces of dams and on gallery walls. These furnish records of change in strain due to change in surface stress. Similar measurements which have been made across contraction joints and across cracks in concrete have furnished records of the joint or crack opening or closing as variations occur with time.

(b) *Deformation Measurements.*—The deformation measuring systems for a dam usually contain provisions for determining horizontal structural deformation between its base and top elevation. An additional system is needed to determine horizontal deformations with respect to references located on the abutments. Both systems employ methods of surveying to obtain the required information.

Plumblines are installed in arch dams to determine horizontal deformation of the dam which occurs between its top and the base. They are located in vertical wells usually formed in the maximum section and in sections about midway between the top arch abutments and the maximum section. Each plumbline consists of a wire with a weight hung on it at the lowest accessible elevation, or the wire is anchored at the bottom of the well and suspended by a float in a tank of liquid at the top. Access to the plumblines for measurements is from stations at the several elevations where galleries are located in the dam. Figure 13-3 shows the layout of a typical plumbline well with reading stations at several elevations.

Horizontal deformation of the structure which occurs at its top elevation with respect to off-dam reference stations is determined by collimation measurements normal to the axis at several locations. These measurements are made between the stations at the top of the dam and sight lines between the off-dam reference stations. The measurements are made using a movable reference on the dam, the on-line position being indicated by an operator with a sighting instrument at one off-dam reference station. The horizontal deformation is obtained from differences between successive measurements.

To determine vertical deformations of the structure, a line of leveling across the top of the dam is used. Stations for measurements are

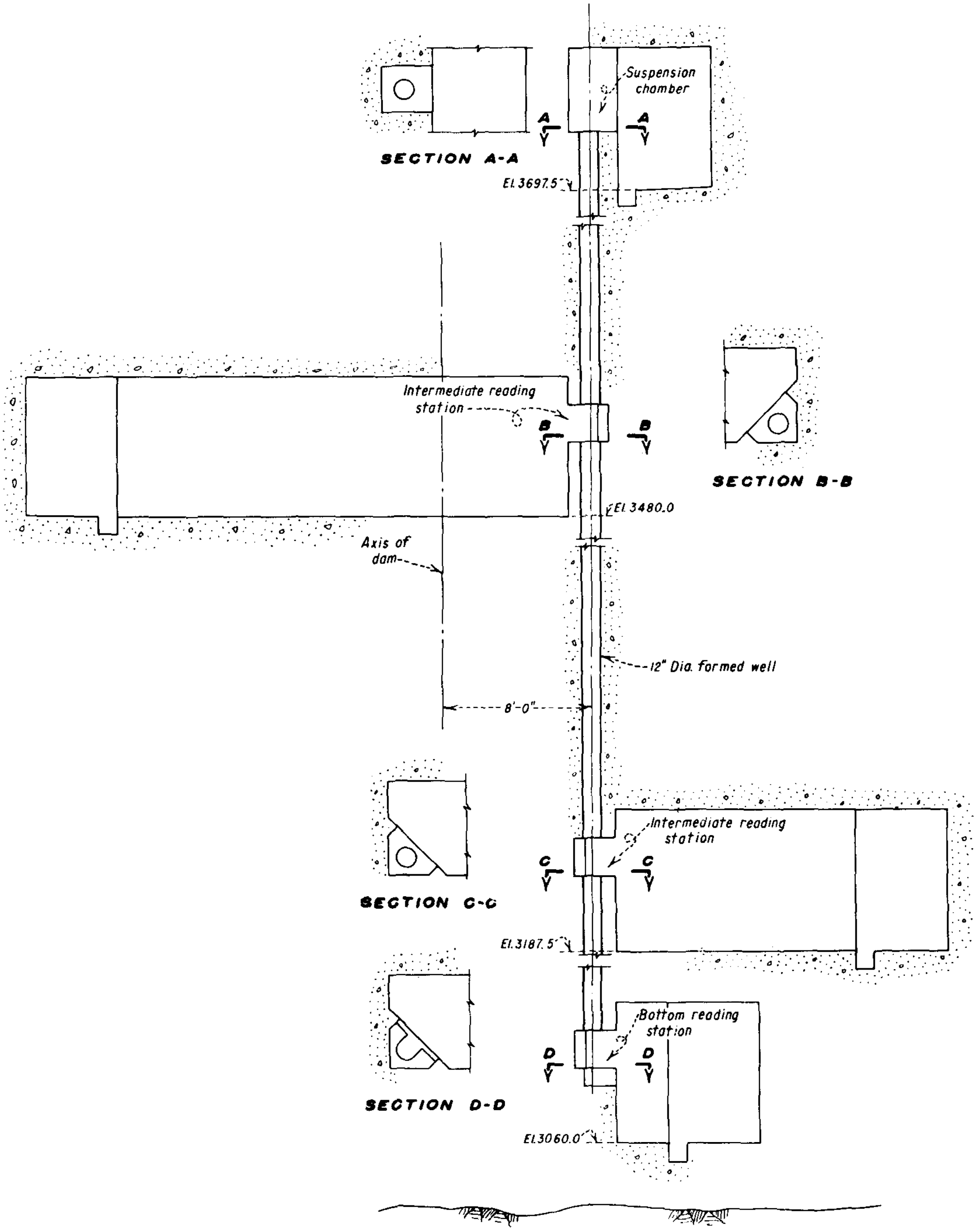


*Figure 13-3.* **Typical plumbline well in a concrete dam with reading stations at several elevations.–288-D-3090**

located on several blocks. The leveling should begin and end at locations sufficiently distant from the dam to avoid locations which would be materially affected by vertical displacement of the dam.

Similarly, leveling measurements are made in other locations such as in powerplants and on gate structures to detect settlement or tipping of large machine units and appurtenant features of a dam.

13-4. *Embedded Instrumentation.*–The instruments to be used for the embedded measurements in a concrete dam may be selected from several types presently available on the commercial market. One type, the Carlson elastic wire instrument [14], is available in patterns suited to most purposes. They are dual-purpose instruments and measure temperature as well as the function for which designed. These instruments have proved reliable and stable for measurements which extend over long periods of time. Installations have been made in many Bureau of Reclamation dams and experience with the instruments covers a period of many years. The description of the instruments, their operation, and the manner in which they have been installed appear in other publications [4, 15, 16]. Foreign made instruments have been used occasionally, as they were more applicable to particular installations than the Carlson-type instruments. Satisfactory results have been furnished by those instruments.

For locations where only temperature measurements which are a part of the behavior program are desired, resistance thermometers are used. Temperature measurements of a special nature and of short duration such as for concrete cooling operations are made with thermocouples.

The instruments which are used for determining stress in an arch or other type concrete dam are strain meters in groups of 12. Eleven strain meters are supported by a framework, or "spider," and installed in a cluster as shown on figure 13-4. The twelfth strain meter is placed vertically adjacent to the cluster.

Stress meters as shown on figure 13-5 are used for some special applications such as determining vertical stress at the base of a section for comparison and checks of results from strain meters, and in arches for determining horizontal stress normal to the direction of thrust in the thinner arch elements which are near the top of a dam. Contraction joint openings are measured by joint meters as shown on figure 13-6. Temperatures are measured by resistance thermometers, and foundation deformation by a special joint meter. Investigation of hydrostatic pressure is made by means of pore pressure meters. Other applications of meters may be made as conditions to be investigated require.

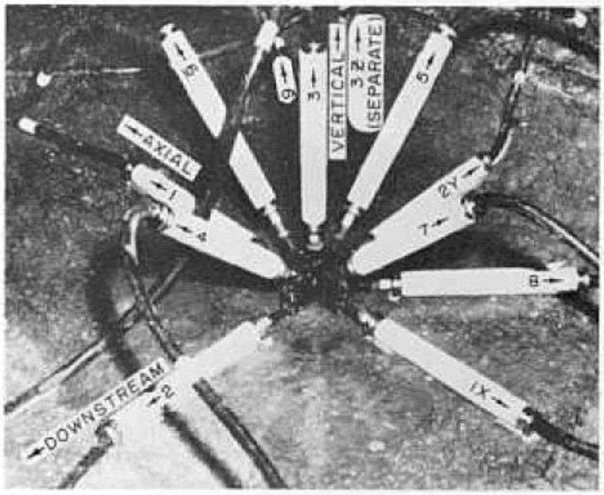


*Figure 13-4.* **A cluster of strain meters supported on a "spider" and ready for embedment in concrete.–P557-420-05870**

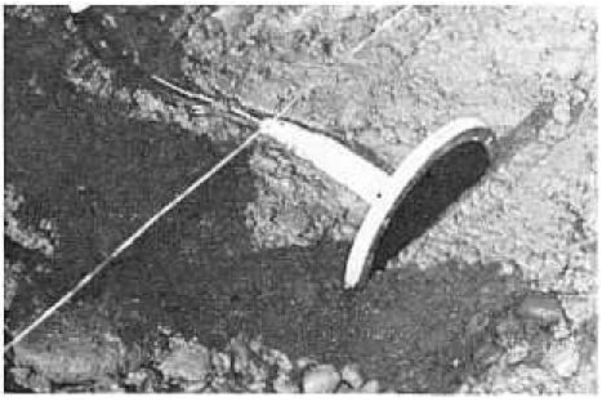


*Figure 13-5.* **A stress meter partially embedded in concrete.–PX-D-74011**

The meters are terminated through electrical cables which connect the instruments to terminal boards as shown on figure 13-7, located at appropriate reading stations in the system of galleries throughout the dam. At each station, readings from the instruments are obtained with special type portable wheatstone bridge test sets shown on figure 13-8.

Mechanical deformation gages which utilize an invar tape and a micrometer reading head may be installed vertically in each of several cased wells which extend from the foundation gallery to distances of 30, 60, 90 feet, or more

*Figure 13-6.* A joint meter in position at a contraction joint.—P591-421-3321

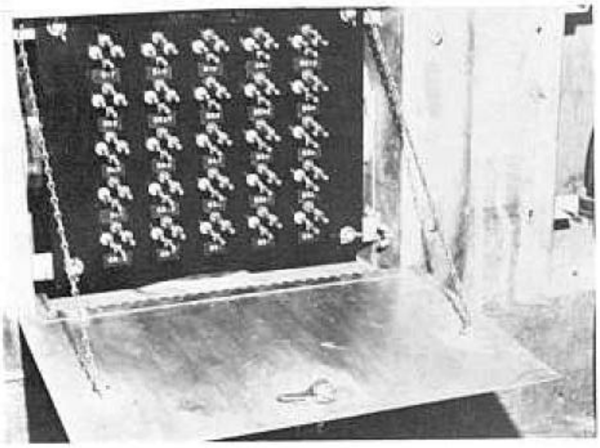


*Figure 13-7.* An instrument terminal board and cover box.—PX-D-74012

below the base of the dam, usually at locations in the maximum section. The locations of the instruments are shown on figures 13-1 and 13-2.

*Figure 13-8.* A special portable wheatstone bridge test set for reading strain meters.—C-8343-2

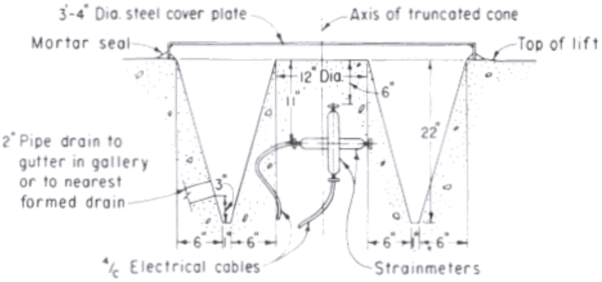
Groups of strain meters in multidimensional configuration as shown on figure 13-4 are embedded in the mass concrete on the gagelines through the dam (fig. 13-2) to measure volume changes from which the stresses can be computed. The strain meters also measure temperature. The gagelines of strain meter groups usually are identical to the centerlines of the construction blocks. In a maximum section of an arch dam, gagelines of meter groups in addition to the gageline at the block centerline may be installed near each contraction joint at the elevation of the meter groups on the block centerline. The three radial gagelines of meter groups permit more extensive determination of stress distributions within the block than those resulting from a single gageline.

Vertical and horizontal stresses are determined at the base of the maximum section where maximum cantilever stresses may be expected. Vertical and horizontal stresses are also determined at other locations in the dam.

Data regarding the volume changes in the concrete that take place in the absence of loading are required for analysis of stress. "No-stress" strain meters as shown on figure 13-9(a) are installed to supply this information. A pair of "no-stress" strain meters are installed near each gageline of strain meter groups on the block centerline. These strain meters are installed in a truncated cone of mass concrete as shown on figure 13-9(b) under a free surface at the interior of the dam so that the instruments are not affected by vertical or horizontal loads.

In some instrumentation layouts, stress meters may be installed companion to each strain meter of a selected strain meter group as shown on figure 13-10. Strain meters in groups indicate length changes which are used to compute structural stresses. The stress meters indicate stress conditions from which stresses are obtained with only a minimum of computation. These serve as a check on results from the strain meters.

Trios of mutually perpendicular strain meters are sometimes installed as shown on figure 13-11 near the upstream and the downstream face of a dam to determine strain gradients near the surfaces. The trios of strain meters are located on the gagelines of strain meter groups, and are installed at distances of 2, 4, and 6 feet from the upstream and downstream faces of a dam.



(a) STRAIN METER LAYOUT.–288-D-3091

(b) TRUNCATED CONE OF MASS CONCRETE CONTAINING STRAIN METERS.–P622-427-3434NA

*Figure 13-9.* "No-stress" strain meter installation.

Gagelines of strain meter groups may be installed near large openings which extend through a dam such as a spillway. Each gageline usually contains two meter groups from which the stress distribution near those openings is determined.

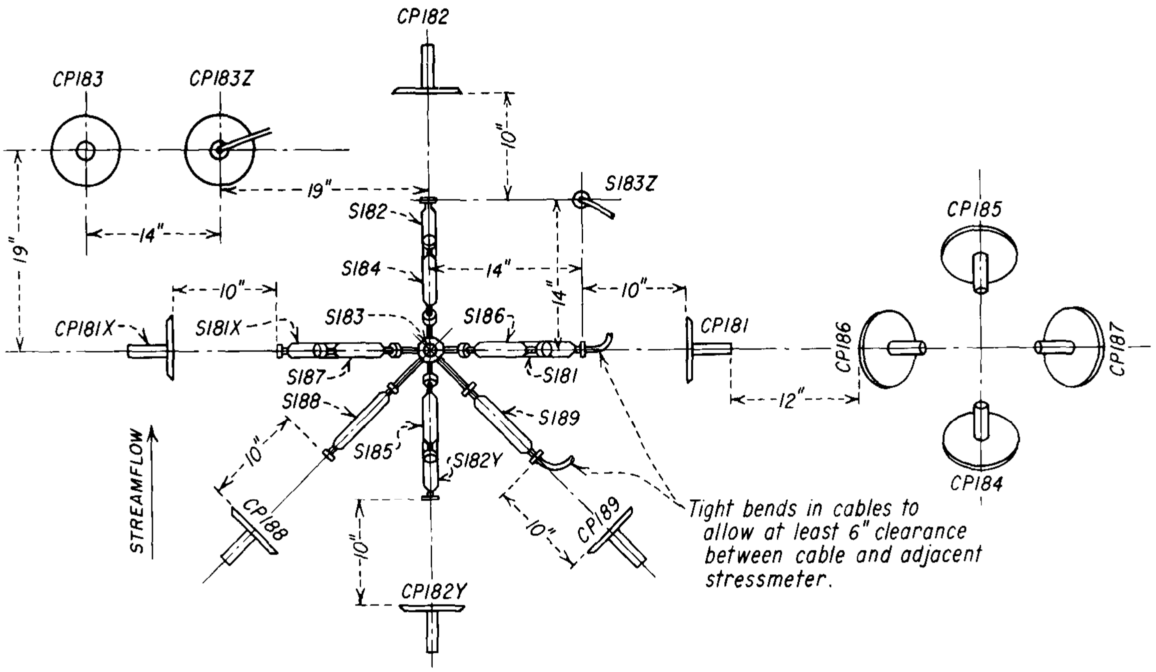
In conjunction with the installations of strain meter groups and stress meter arrays at the various locations throughout a dam, joint meters as shown on figure 13-6 are placed on the radial contraction joints at the same elevations as the groups of strain meters and the stress meters.

In designs where stress in reinforcement steel is to be investigated, reinforcement meters as shown on figure 13-12 are installed in the reinforcement placed around a penstock, spillway, or other opening. The instruments are placed on at least one bar of each row of reinforcement at selected locations around the opening to measure deformation in the reinforcement from which stress is determined. Installations of reinforcement meters will be noted on figure 13-1, near the top of block 8.

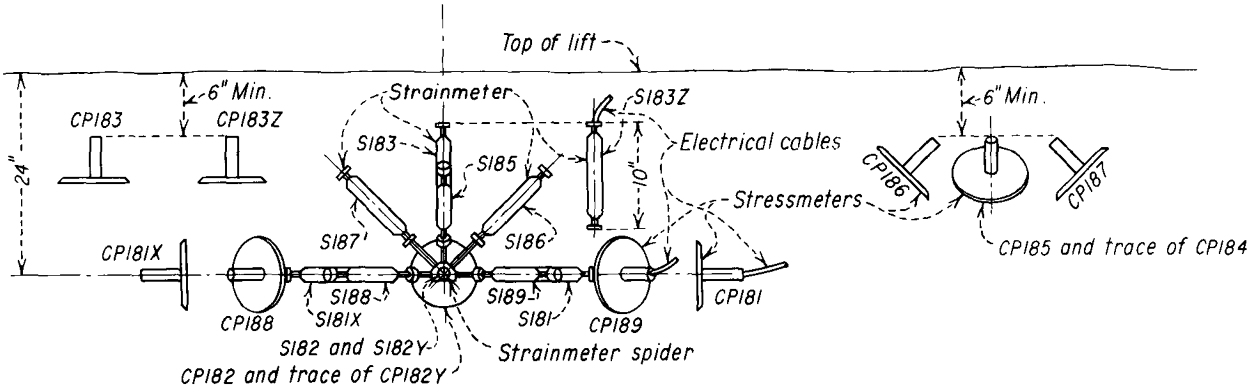
Where stress in the steel liners of penstocks is to be investigated, strain meters are attached to the outer surface of a penstock by supporting brackets also shown on figure 13-12. The instruments are installed at each of three equally spaced circumferential locations and at two or more elevations on a steel liner.

At each location of a penstock strain meter installation, pore pressure meters shown on figure 13-13 are installed at the outer surface of the steel liner to measure possible hydrostatic pressure which may develop between the liner and the surrounding concrete. The pore pressure meters are particularly useful in cases where backfill concrete is placed around a penstock in a tunnel.

Pore pressure meters as shown on figure 13-14 are sometimes placed at several locations at the same elevation in the concrete on the centerline of a block near the base of the maximum section to measure hydrostatic pressure in pores if it develops. The meters usually are spaced 1, 3, 6, 10, 15, 20, 30, and



PLAN



ELEVATION

*Figure 13-10.* **Meter group comprising strain meters and stress meters.–288-D-3092**

40 to 50 feet from the upstream face of the dam.

Resistance thermometers as shown on figure 13-15, spaced at equal vertical intervals between the base and top elevations of the dam at the upstream face, are installed in the maximum section to record reservoir water temperature at various depths. Resistance thermometers usually are installed at two elevations at the downstream face of a dam in the maximum section to record temperature of the concrete caused by solar heat.

At several general locations between the foundation and the mass concrete at the base of a dam, deformation meters as shown on figure 13-16 are installed. These meters employ a joint meter as the measuring device and are installed in cased holes to detect deformation

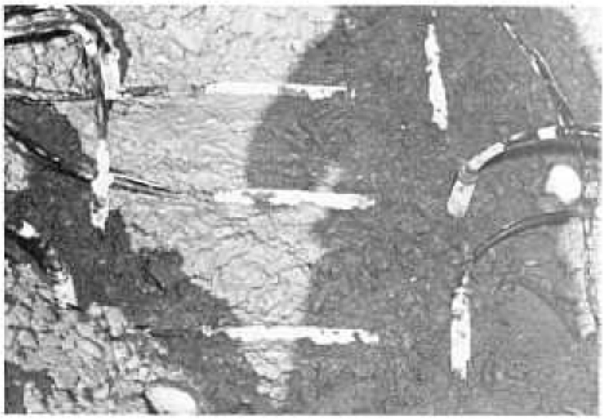


*Figure 13-11.* **Trios of mutually perpendicular strain meters installed near face of dam.–P557-420-7933**

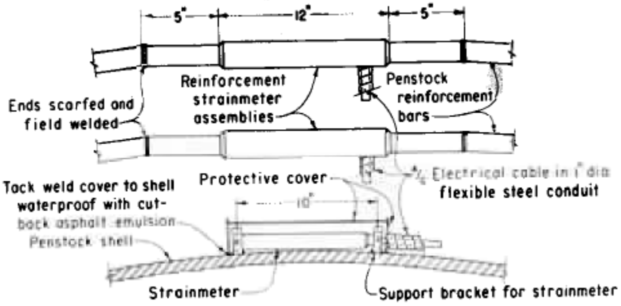


*Figure 13-12.* **Penstock and reinforcement strain meters.–288-D-3093**

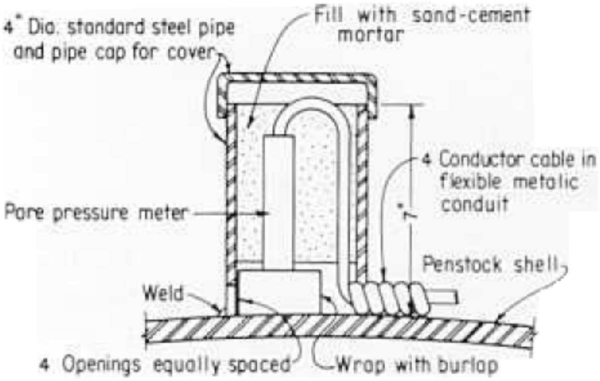


*Figure 13-13.* **Pore pressure meter installed on a penstock.–288-D-3094**

*Figure 13-14.* **Pore pressure meters installed in mass concrete.–HH2653**

*Figure 13-15.* **Resistance thermometer installed at upstream face of a dam.–3PXI 3/10/71-3**

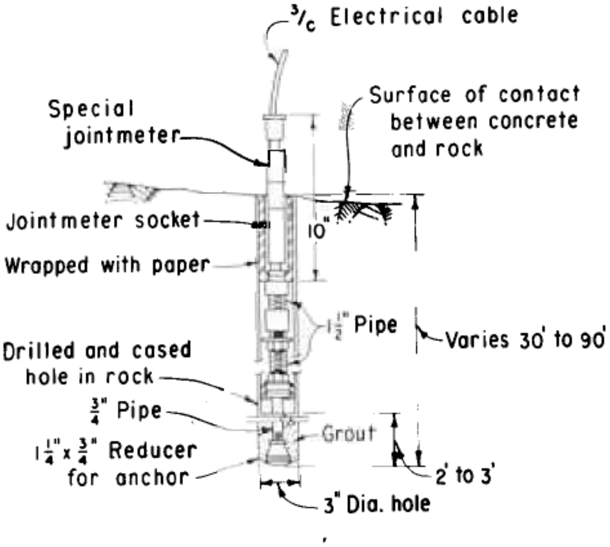


***Figure 13-16.*** **Deformation meter installed in cased well under dam to measure deformation of foundation rock.–288-D-3095**

of the foundation rock, usually over depths of 30 to 90 feet below the rock-concrete contact surface.[2]

Ordinarily, two deformation meters are installed between the upstream and downstream boundaries of the blocks. In areas such as beside foundation and other galleries in the base of the dam, where access is available at a blockout on a gallery wall or floor location, a mechanical-type deformation gage is installed in place of a deformation meter.

The deformation gages, which utilize invar-type tapes and micrometer-type reading heads as shown on figure 13-17, are installed vertically in cased wells in the base of the dam in the maximum section. These gages extend below the surface of contact between the rock and the concrete from appropriate reading stations in the foundation gallery. The gages show length change over their depths into the rock in the same manner as the deformation meter shows the amount of vertical compressive deformation caused by the weight of the dam and by the loading on the dam.

Similar deformation gages may be installed horizontally in tunnels which have been excavated into the abutment formations of a dam. Figure 13-18 shows the micrometer-type reading head of one portion of a horizontal installation which is comprised of several 100-foot sections.

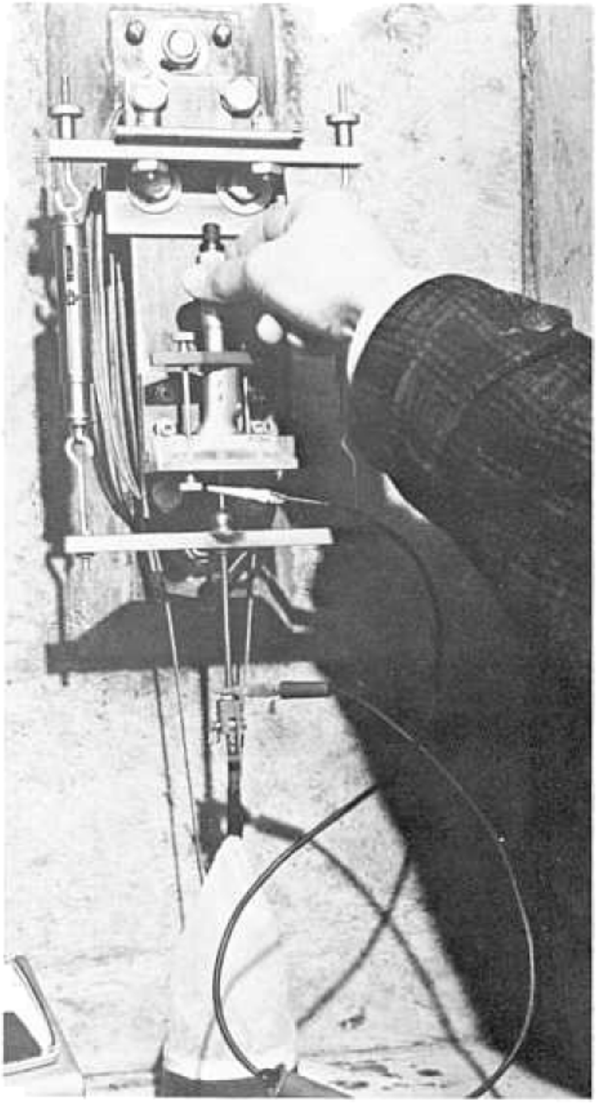


***Figure 13-17.*** **Micrometer-type reading head for use with foundation deformation gage.–P622B-427-3916NA**

The strain, stress, pore pressure, foundation deformation, and reinforcement meters, and the resistance thermometers embedded in the mass concrete of a dam will furnish data over a long period of time for determining the stress behavior of the structure and conditions of stress which develop in features that have been

---

[2] Depths of 200 feet are planned for the deformation meters to be installed at Crystal Dam, currently under construction in Colorado.

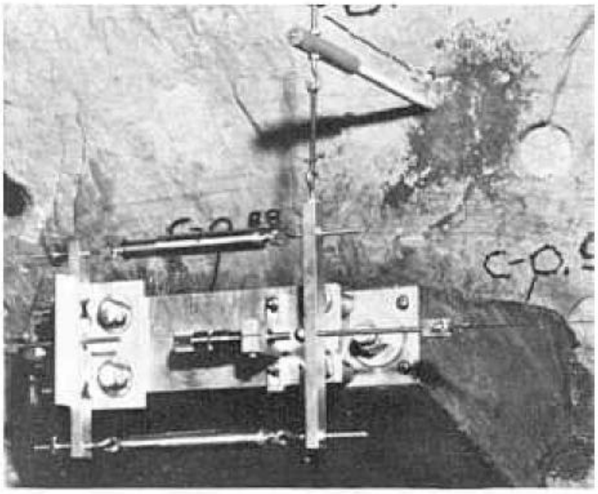


*Figure 13-18.* **Micrometer reading head and invar tape used with horizontal tape gage in abutment tunnel.–P557-420-9328NA**

instrumented. The joint meters detect the amount of contraction joint opening for information during joint grouting.

All of the above-mentioned instruments except the deformation gages employ a wheatstone bridge measuring circuit, and the same portable resistance bridge as shown on figure 13-8 can be used in common with all instruments. Also, the same bridge is used for obtaining temperature from resistance thermometers.

Data supplied by the strain meters, stress meters, joint meters, pore pressure meters, reinforcement meters, and deformation meters are in terms of total ohmic resistance and in terms of the ratio of the resistance of the two coils contained in the meter. Data supplied by the resistance thermometers are in terms of ohmic resistance of the coil of the thermometer. All data are recorded on appropriate data sheets. Computations of stress, temperature, hydrostatic pressure, joint opening, and foundation deformation are made from the field data by computer. Results from the computations are plotted as functions of time by an electronic plotter. Distributions of stress and temperature on gagelines of the instruments are then prepared for various loading conditions on the dam and presented in report form.

**13-5.** ***Supplementary Laboratory Tests.***—The determination of stress in the mass concrete of a dam or other large structure requires a knowledge of the concrete from which the structure is built. Accordingly, after the concrete mix for the structure is determined in the laboratory, and when practicable, prior to the beginning of construction at the site, a testing program for that specific concrete is developed and expedited. The results of the concrete properties and creep tests are an important part of a behavior program, as that information is needed for the solution of stress from the clusters of strain meters which are embedded in a dam.

The program includes creep tests, test cylinders for which are shown on figure 13-19, as well as the usual concrete strength tests and tests for determining elastic modulus, Poisson's ratio, thermal coefficient, autogeneous growth and drying shrinkage. All these tests are made on specimens which are fabricated in the laboratory and utilize materials from which the structure will be built. The materials, which are shipped to the laboratory from the damsite, are mixed in the same proportions as the mix for the structure, and cast into appropriate cylinders. The cylinders are stored and tested under controlled environmental conditions. Reports are available on the methods of testing and on the creep tests (see references [17], [18], [19], [20], and [21]).

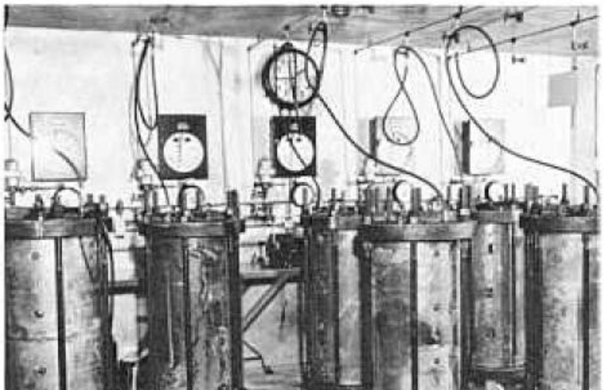


*Figure 13-19.* **Creep tests in progress on 18- by 36-inch mass concrete cylinders.–P557-D-34369**

13-6. ***Deformation Instrumentation.***—Of equal importance to the measurements made by embedded instruments are the measurements which are made with surveying instruments and by mechanical devices using precise surveying methods. These measurements involve plumblines, tangent line collimation, precise leveling, and triangulation deflection targets on the face of a dam. Over a period of several years, results from these measurements show the range of deformation of a structure during the cyclic loading conditions of temperature and water to which a dam is subjected.

Plumblines provide a convenient and relatively simple way to measure the manner in which a dam deforms due to the waterload and temperature change. In early Bureau of Reclamation dams where elevator shafts were provided in the structures, plumblines were sometimes contained in these shafts. This proved generally to be unsatifactory and, at present, plumblines are suspended in vertically formed wells which extend from the top of the dam to near the foundation at three or more locations in the dam. Wherever feasible, reading stations are located at intermediate elevations, as well as at the lowest possible elevation to measure the deflected position of the section over the full height of the structure. A typical well with reading stations is shown on figure 13-3.

The wells are usually 1 foot in diameter and maintained to withstand one-half inch of plumb as the dam increases in elevation. In some dams, pipe or casing has been used and left in place for forming the well, while in other dams the wells have been formed using slip-forms. The reading stations on a plumbline are located at galleries in the dam. A door frame is set in the concrete of the gallery wall at each reading station, and doors seating against sponge rubber seals are provided as closures. The doors of the reading stations are kept locked except when readings are being made, to prevent the plumbline being disturbed. Reading stations are oriented so that measurements may be made in the directions of anticipated movements hence avoiding the need for trigonometric resolution. In the older dams orientation of the reading stations requires that measurements be made at 45° to the directions of dam movements, thus necessitating computation. Measurements of deformation are made with a micrometer slide device having either a peep sight or a microscope for viewing. The measured movements indicate deformation of the structure with respect to the plumbline.

Plumbline installations of two types have been used. These are the weighted plumbline and the float-suspended plumbline. For the weight-supporting plumbline the installation consists of a weight near the base of the dam suspended by a wire from near the top of the dam. The suspension is located in a manhole at the roadway or when practicable in a utility gallery near the top. The components of equipment for the installation are shown on figure 13-20. Recent plumbline installations are float suspended, using antifreeze in a tank at the top of the dam with a float holding the wire. Figure 13-21 shows one type of float and tank. When the lower end of the plumbline is at a gallery reading station, the wire is fixed at the bottom location. In other cases, where a pipe well is extended below the foundation, the wire is attached to a weight which is lowered into that well from the lowest reading station. Figure 13-22 shows the weight and weight support. The support is attached to the plate which closes the lower end of the pipe well prior to lowering the pipe well into the hole in the foundation. Figure 13-23 shows a typical reading station and reading devices.

In conjuction with the plumbline installations in dams, reference monuments have been established in cased wells below the foundation near the base elevation of the plumblines. These are used to determine whether horizontal movement of the dam occurs. The locations of these monuments are periodically determined with respect to the top elevation of the well to determine whether movement at the elevation of the measurement location has occurred. Figure 13-24 shows the optical plummet and the reference grid that are used for measurements at the gallery elevation of a well which extends into the foundation of a dam.

Tangent line, or collimation measurements,

*Figure 13-20.* Components of equipment for weighted plumbline installation.–PX-D-74010

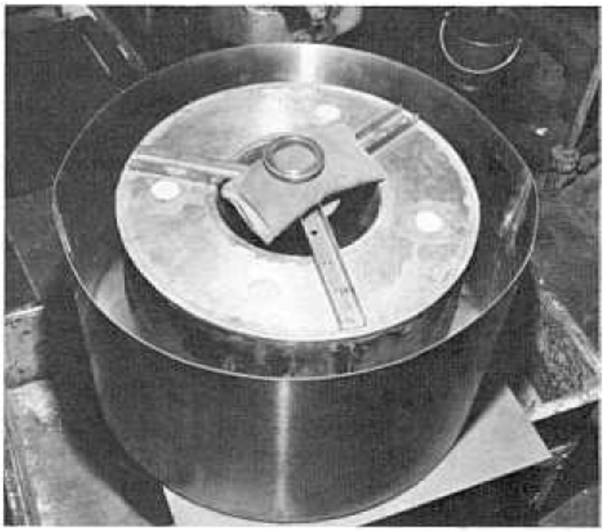
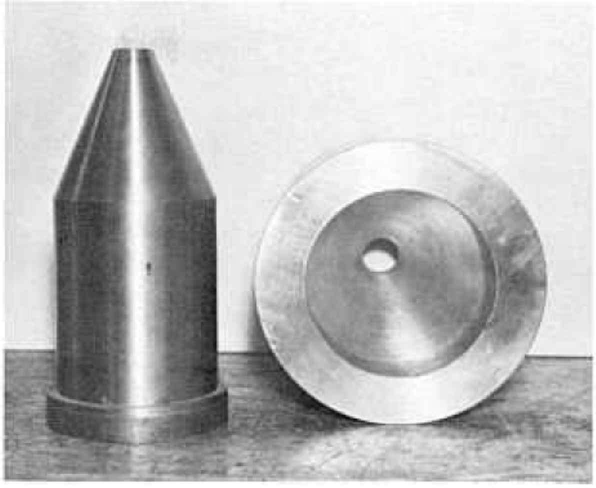
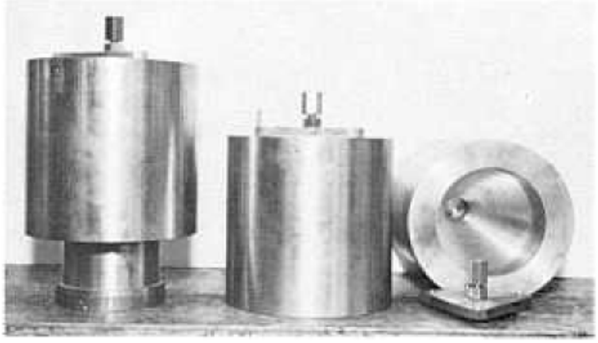


*Figure 13-21.* Tank and float for use with float-suspended plumbline.–C-8163-1NA



(a) SUPPORT AND WEIGHT.



(b) WEIGHT RESTING ON SUPPORT, AND OTHER WEIGHTS.

*Figure 13-22.* Anchorage for float-suspended plumbline.–C8170-2NA, C-8170-1NA

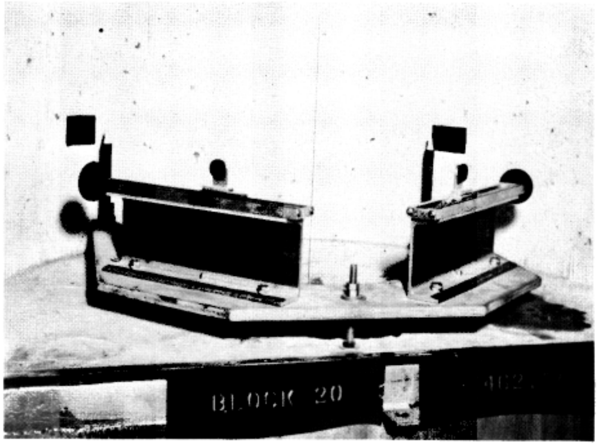


*Figure 13-23.* Typical plumbline reading station and reading devices.–P459-640-3593NA

*Figure 13-24.* Foundation deformation well, optical plummet, and reference grid.–P459-640-4221

*Figure 13-25.* An instrument pier for use with collimation or triangulation systems.–P526-400-7877

are a useful means for determining the deformation of a dam at its top elevation with respect to off-dam references. This method has been used for measuring the deflection at the top of some Bureau of Reclamation arch and gravity dams. It is also used at major structures and is convenient for measurement at small dams that have no assigned survey personnel, since the survey can usually be conducted by two persons.

Collimation measurements are made with a theodolite or jig-transit. An instrument pier as shown on figure 13-25 is constructed on one reservoir bank upstream and at a higher elevation than the dam. A reference target as shown on figure 13-26 is installed on the opposite reservoir bank and at about the same elevation as the pier. The target and pier locations are selected so that a sight line between them will be tangent to the axis or to some reference plane at the location of a movable measuring target as shown on figure 13-27 on the top of the dam. Progressive differences in the position of the movable target from the sight line indicate the deformation change in fractions of an inch at the measurement station. Usually three or four stations on a dam are sufficient to obtain the desired information. The results are correlated with plumbline measurements to provide sufficient data for charting the deformation behavior of the structure. A typical layout for a collimation system and locations of the items of equipment are shown on figure 13-28.

*Figure 13-26.* **A reference sighting target for use in obtaining collimation measurements.–P526-400-7852**

*Figure 13-27.* **A movable collimation target at a measuring station on top of a dam.–PX-D-74009**

A more elaborate installation, and one that requires experienced and trained personnel, is that of triangulation targets on the face of a dam from off-dam references. The layout of targets on the face of the dam is made in terms of horizontal and vertical elements of the structure compatible with the layout of the embedded instruments, as well as with that of the analytical design investigations. Targets are located on the gagelines of instruments, and on the locations of plumbline reading stations projected radially from the plane of the axis.

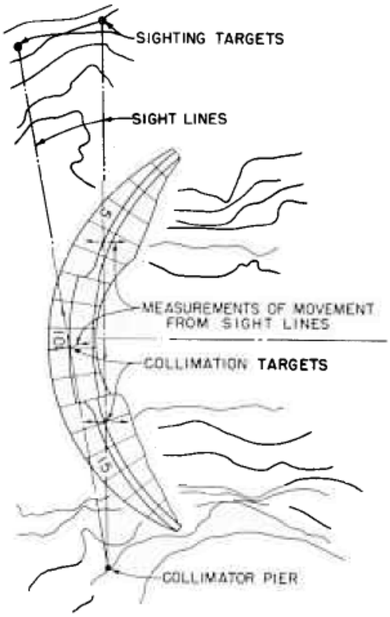


*Figure 13-28.* **A collimation system layout for an arch dam.–288-D-3097**

This system requires a net of instrument piers and a baseline downstream from the dam. The configuration is laid out to provide the greatest strength of the geometrical figures [22] and to afford sight lines to each target from as many instrument piers as is feasible. The nature of the terrain and the topography of the area are governing factors in the size of the net layout. The measurements are made using first-order equipment, methods, and procedures insofar as feasible. The results from these measurements show deformation of a dam with respect to off-dam references and deformation of the canyon downstream from a dam in the streamwise and cross-stream directions. The layout of a system and locations of items of equipment are shown on figure 13-29. Figure 13-25 shows a pier suitable for theodolite stations. Figure 13-30 shows the

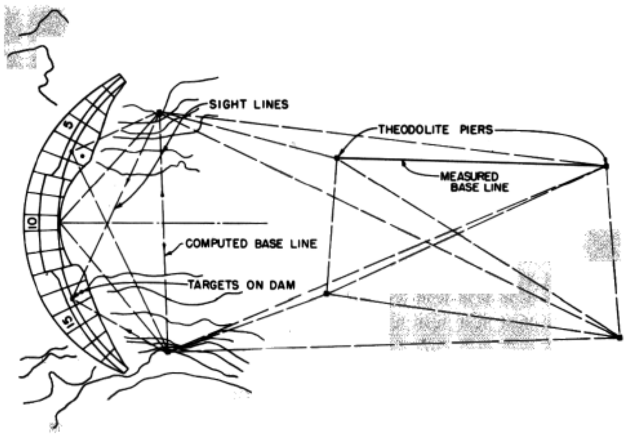


*Figure 13-29.* **A triangulation system layout for an arch dam.–288-D-3099**

*Figure 13-30.* **A tensioning device used with a tape for precise baseline measurements.–P557-D-58714**

tensioning device used with the tape for precise baseline measurements, and figure 13-31 shows targets used on the face of a dam and on the theodolite piers.

Leveling measurements are used to determine vertical displacements of a structure with respect to off-dam references. These measurements employ first-order equipment and procedures [23]. Base references for the measurements should be far enough from the dam to assure that they are unaffected by vertical displacement caused by the dam and reservoir.

Combinations of the several precise surveying-method measurements are included in the behavior measurement layouts for new dams. Except for plumbline and deformation well measurements, all are readily adaptable to older dams, should monitoring of behavior become desirable.

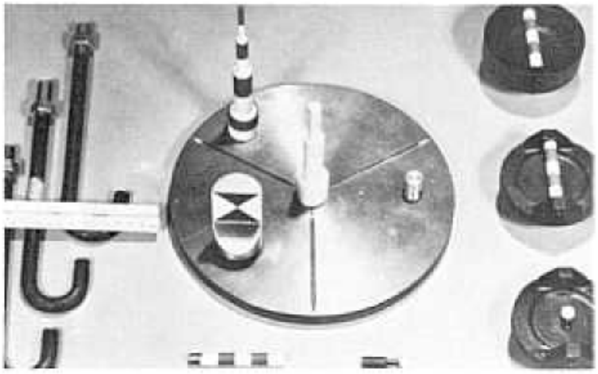


*Figure 13-31.* **A pier plate, pier targets, and dam deformation targets.–P557-D-58717**

13-7. ***Other Measurements.***–Under this general category may be included types of measurements which are related to and have an influence on the structural behavior measurements, but which are not included as a part of the program for those measurements. The measurements of primary interest are those of air temperature as recorded at an official weather station, air temperature as may be recorded at certain locations on a project, river water temperature, concrete temperature during the construction of a dam, reservoir water elevation, uplift pressure under a dam, and flow of water from drainage systems. The latter two items are discussed more fully.

(a) *Uplift Pressure Measurements.*–A system of piping is usually installed in several blocks at the contact between the foundation rock and the concrete of an arch dam as shown on figure 13-32. Although uplift pressure is not a critical factor in the design of arch dams, piping is installed to determine whether any hydraulic underpressures may be present at the base of the dam due to percolation or seepage of water along underlying foundation seams or joint systems after filling the reservoir. Measured values of uplift pressure also may indicate the effectiveness of foundation grouting and of drainage. The uplift pressure gradient through the section of a dam used for

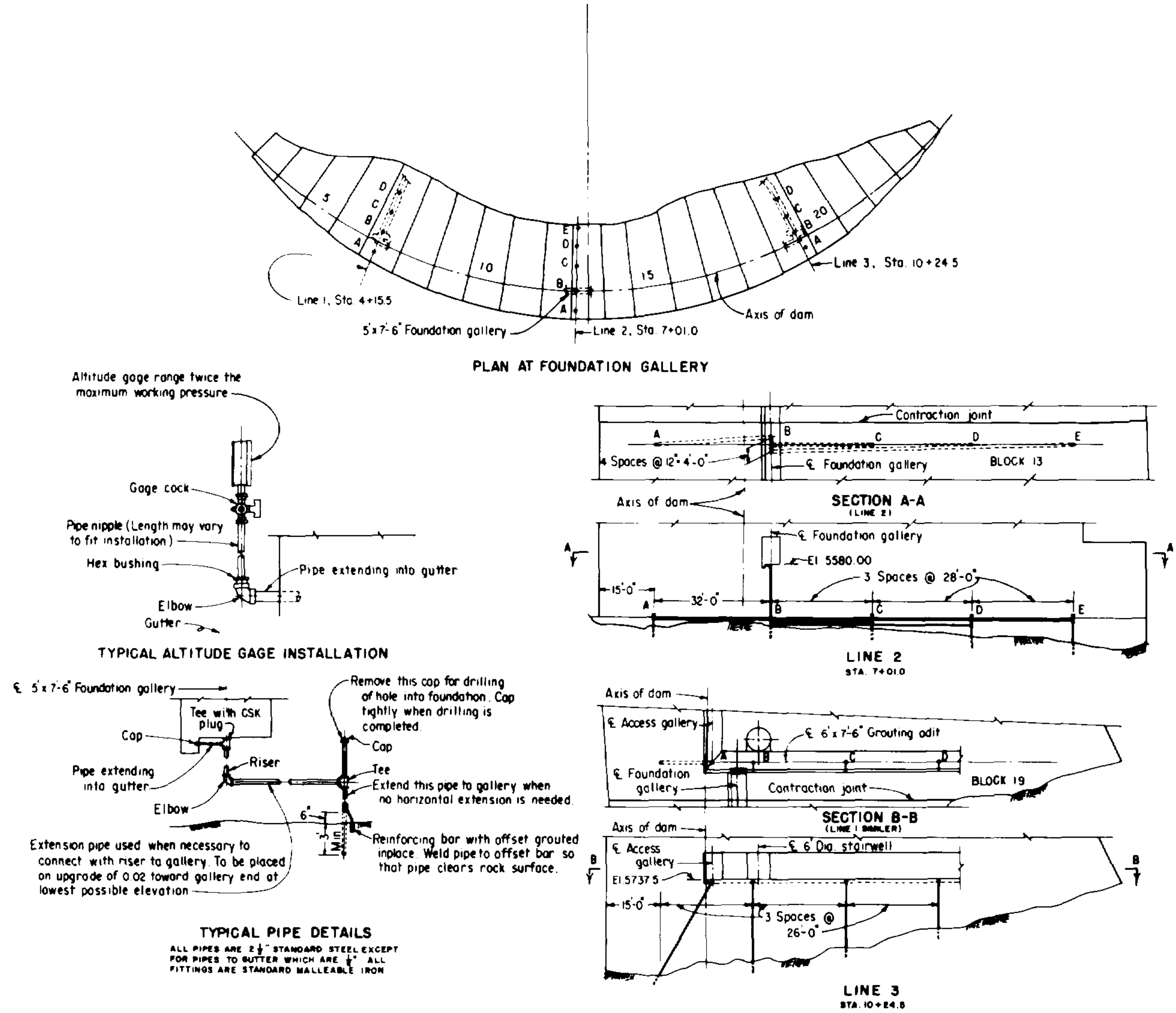


*Figure 13-32.* **An uplift pressure measurement system for an arch dam.–288-D-3101**

design is an assumed variation between the upstream and downstream faces of a dam as shown in reference [24] and in section 3-8 of this manual.

Uplift pressures are determined by pressure gages or by soundings. When a pipe is under pressure, the pressure is measured by a Bourdon-type pressure gage calibrated in feet, attached through a gage cock to the pipe. When zero pressure is indicated at a pipe, the water level in the pipe is determined by sounding.

Another system for measuring uplift pressure at the base of a structure where no galleries are included near the foundation into which a piping system may be routed is to install pore pressure cells at the locations to be investigated. Electrical cables may be routed from the cell locations to appropriate reading stations on the downstream face or top of the structure where measurements can be obtained. The installation of pore pressure cells is particularly applicable to installations beneath concrete apron slabs downstream from an overflow section of a dam, spillway training walls, and powerhouse structures. A typical pressure cell installation is shown on figure 13-33. Details on figure 13-34 show the manner in which contraction joints can be crossed by electrical instrument cable which is encased in electrical conduit.

(b) *Drainage Flow Measurements.*—A system of foundation drains is installed during the construction of an arch dam. The drainage system usually consists of 3- or 4-inch-diameter pipes placed on approximately 10-foot centers in the axis direction, in the floors of the foundation gallery and foundation tunnels. Periodic measurements of flow from the individual drains should be made and recorded. When flows from drains are minimal, measurements may be made using any suitable container of known volume and noting the number of containers filled per minute. When flows are too great to measure by that method, measuring weirs may be installed as needed in the drainage gutters of the galleries and adits of a dam. The locations of weirs should be limited to specific zones in a dam.

When flows of drainage water are sufficient to be measured by weirs, the measurements are usually made on a monthly schedule and records maintained on appropriate data sheets. Any sudden increase or decrease in drainage should be noted and correlated with the reservoir water surface elevation and any change in the percolating conditions of the drains. All drains should be protected against obstructions and should be kept free-flowing.

**13-8.** ***Measurement Program Management.***—The overall planning, execution, and control of a measurement program must be under the supervision of the central design office to expedite the various phases of the entire program. Control of the program starts with the installation of the various instruments and measurement systems during the construction period.

Cooperation between the central design office, the project construction office, the contractor's organization, and later with the operations and maintenance organization is important and necessary in obtaining reliable installations of instruments and reliable information from the various phases of the measurement program.

A schedule for installation of instrumentation and for obtaining readings at a new dam begins almost with the placement of the first bucket of concrete, continues through the construction period, and then extends into the operating stage, possibly for an undetermined period of time.

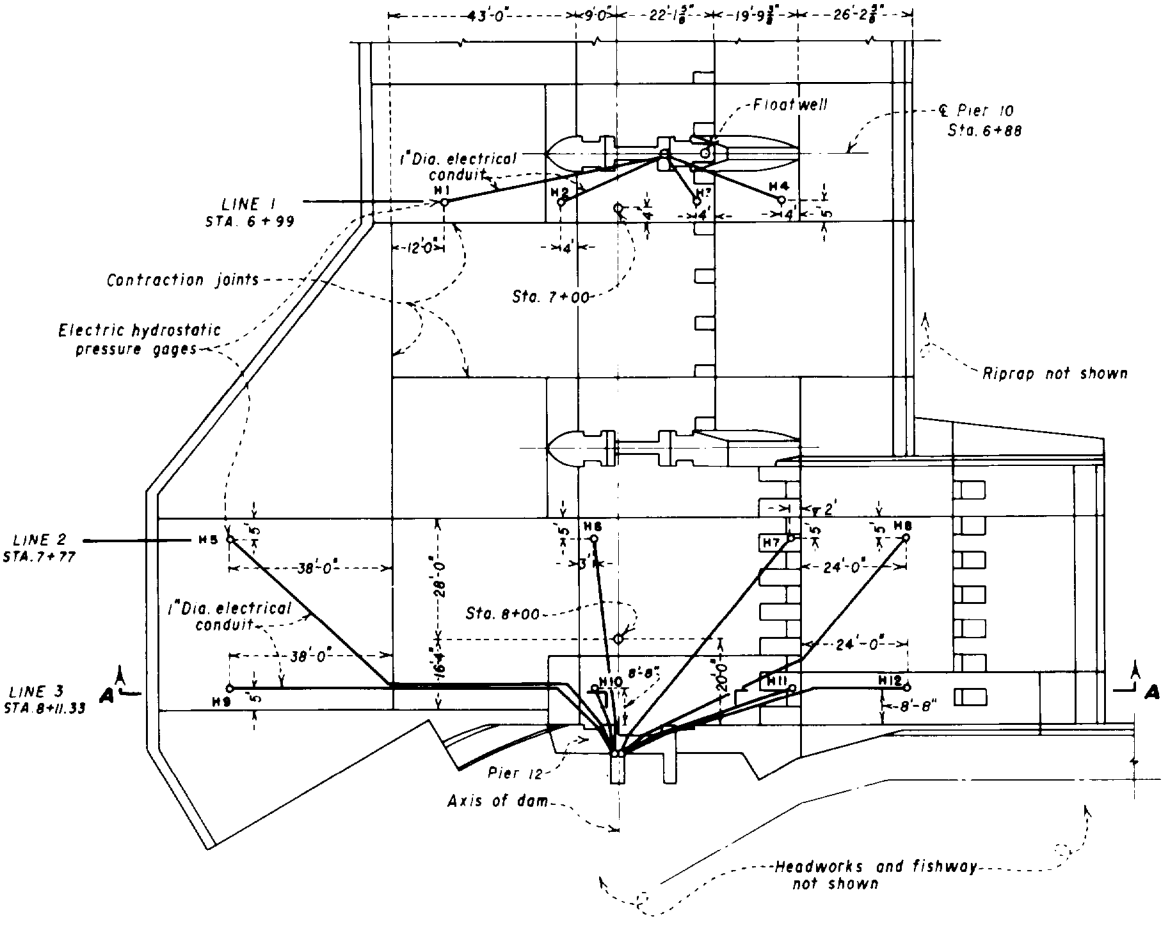
The information which is obtained is forwarded to the design office in the form of a report prepared at monthly intervals as explained in reference [25]. It includes all tabulations of instrument readings obtained during the prior month and other pertinent information, such as daily records of air and water temperature, reservoir and tailwater elevations when the operating period is reached, any other data which may have an effect on the structural action of a dam, and comments concerning the operation of instruments or measurement devices. Photographs and sketches should be used freely to convey information.

The schedules for obtaining data from structural behavior installations are somewhat varied. Embedded instrument readings are required more frequently immediately after embedment than at later periods. The reading frequency is usually weekly or every 10 days during construction and semimonthly after construction.

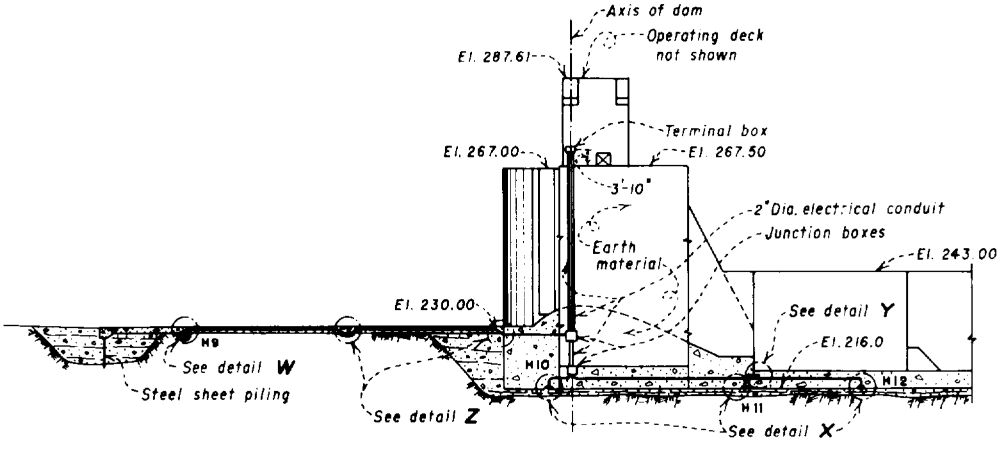
In some cases, instrument readings at monthly intervals can be allowed. Although the wider spread of intervals is not desirable for strain meters, it is satisfactory for stress meters, reinforcement meters, joint meters, pressure cells, and thermometers. During periods of reservoir filling or rapid drawdown, readings at more frequent intervals are preferred. In this case, schedules for reading may be accelerated for the periods of time involved.

Data from deflection measuring devices such as plumblines and collimation are preferred weekly. During events of special interest, such as a rapidly rising or falling reservoir, readings at closer intervals may be desired.

Data from uplift pressure measurement systems may be obtained monthly except during the initial filling of a reservoir when data are obtained at weekly or 10-day intervals. Pore pressure gages may be read monthly. At dams where drain flow is of a sufficient

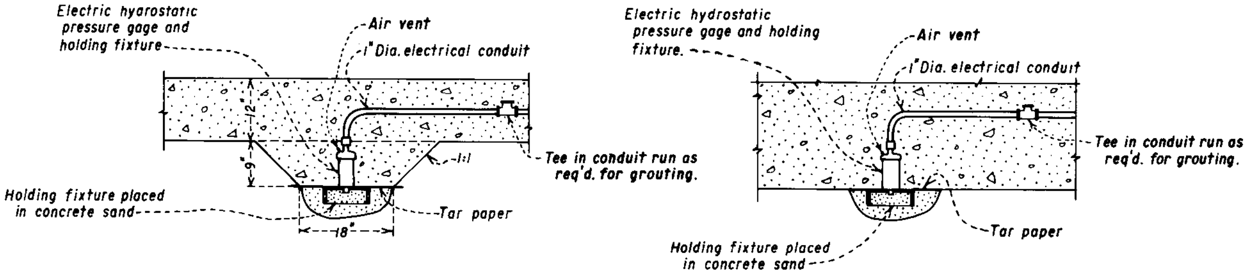


PLAN



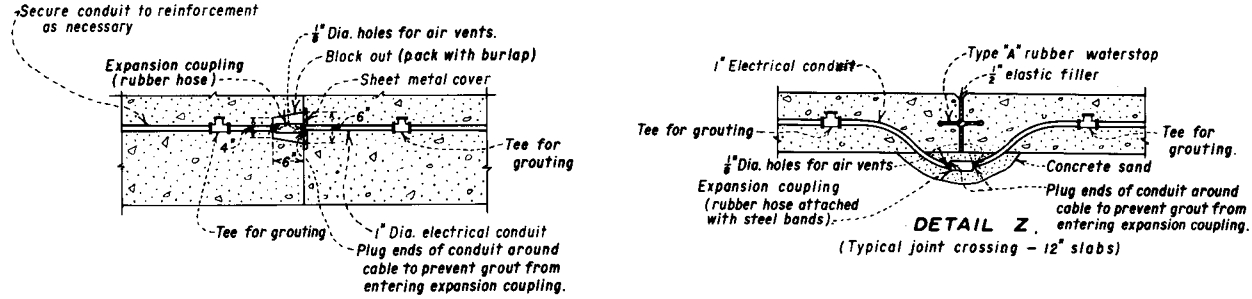
SECTION A-A
LINE 3 GAGES
STA. 8+11.33

*Figure 13-33.* A pore pressure meter installation for determining uplift pressure.–288-D-3102



DETAIL W
(Typical installation – 12" slab)

DETAIL X
Typical installation – slabs over 12")



DETAIL Z
(Typical joint crossing – 12" slabs)

DETAIL Y
(Typical joint crossing – slabs over 12")

*Figure 13-34.* **Details of pore pressure meter installation illustrated on figure 13-33.—288-D-3103**

quantity to be measured, these data should be obtained at monthly intervals.

The target deflection and pier net triangulation measurements should be conducted at least semiannually during the periods of minimum and maximum air temperature so as to obtain the furthest downstream and upstream deformed positions of a dam. During the early stages of reservoir filling and operation, additional measurements are desirable and are made approximately midway between those of minimum and maximum air temperature conditions. These latter data are useful in noting deformation trends and for correlating collimation and plumbline information.

Periodic leveling should be conducted in the vicinity of and across the top of a dam to detect possible vertical displacement of the structure.

The planned program for measurements should cover a time period which will include a full reservoir plus two cycles of reservoir operation, after which a major portion of the measurements are suspended. After the suspension of a major portion of readings, some types of measurements, such as those from plumblines, collimation, foundation deformation meters and gages, and from certain clusters of embedded meters which are considered essential for long-time structural surveillance, are continued indefinitely. For these measurements, the intervals between successive readings may be lengthened.

13-9. *Data Processing.*—The installations of instruments and measurement systems in dams and the associated gathering of quantities of data require that a program for processing be planned in advance. This requires definitely established schedules and adherence to the processing plan. Otherwise, seemingly endless masses of data can accumulate from behavior instrumentation and become overwhelming with no apparent end point in sight. Careful planning with provisions for the execution of such a program, possibly during a period of several years, cannot be too strongly emphasized.

For some measurements, computations and plots can be made and used to advantage by construction or operating personnel at a damsite. Under this category are data from resistance thermometers, joint meters, extensometers, Bourdon pressure gages, and the less complex systems for measuring deformation, such as collimation and plumblines. X-Y hand plotting of these data can be maintained with relative ease, as required.

For measurements from the other instruments such as strain meters, stress meters, and reinforcement meters, the obtaining of results is complex and time consuming.

The methods and details for computing which are used to reduce the instrument data to temperature, stress, and deflection are completely described in separate reports (see references [6], [8], [26], and [27]). The results of the Bureau's laboratory creep test program, which covers a period of more than 20 years, are described in references [5], [18], [28], and [29].

The processing of large masses of raw data is efficiently and economically handled by computer methods. The instrument and deformation data are processed in the Bureau's E&R Center in Denver. Processing of the majority of these data is presently done by punched cards, magnetic tape, and electronic computers, using programs of reference [26] for the computing which have been devised for the specific purposes. Plotted results are from output material which is fed into an electronic X-Y plotter. Reports are prepared from these results.

13-10. ***Results.***—The interpretation of data and compiled results includes the careful examination of the measurements portion of the program as well as examination of other influencing effects, such as reservoir operation, air temperature, precipitation, drain flow and leakage around a structure, contraction joint grouting, concrete placement schedule, seasonal shutdown during construction, concrete testing data, and periodic instrument evaluations. All of these effects must be reviewed and, when applicable, fitted into the interpretation. The presentation of results, both tabular and graphical, must be simple, forceful, and readily understood.

The interpretation of the measurement results, as shown in references [27], [30], and [31], progresses along with the processing of the gathered data. Progress reports usually cover the findings which are noted during the periods of construction and initial reservoir filling stages for a dam. The resume of findings, as a final report, is usually not forthcoming until several years after completion of the structure, since the factors of a full reservoir, its seasonal operating cycle, the seasonal range of concrete temperature, and local effects of temperature on concrete are all time-governed.

## 13-11. *Bibliography.*

[1] "Arch Dam Investigation," vol. I, Engineering Foundation Committee on Arch Dam Investigation, ASCE, 1927.

[2] "Arch Dam Investigation," vol. II, Committee on Arch Dam Investigation, The Engineering Foundation, 1934.

[3] "Arch Dam Investigation," vol. III, Committee on Arch Dam Investigation, The Engineering Foundation, 1933.

[4] Raphael, J. M., and Carlson, R. H., "Measurement of Structural Action in Dams," James J. Gillick and Co., Berkeley, Calif., 1956.

[5] McHenry, Douglas, "A New Aspect of Creep in Concrete and its Application to Design," Proc. ASTM, vol. 43, pp. 1069-1087, 1943.

[6] Jones, Keith, "Calculations of Stress from Strain in Concrete," Engineering Monograph No. 25, Bureau of Reclamation, October 29, 1961.

[7] Raphael, J. M., "Determination of Stress from Measurements in Concrete Dams," Question No. 9, Report 54, Third Congress on Large Dams, ICOLD, Stockholm, Sweden, 1948.

[8] Roehm, L. H., and Jones, Keith, "Structural Behavior Analysis of Monticello Dam for the Period September 1955 to September 1963," Technical Memorandum No. 622, with Appendixes I and II, Bureau of Reclamation, September 1964.

[9] Carlson, R. W., "Manual for the Use of Stress Meters, Strain Meters, and Joint Meters in Mass Concrete," second edition, 1958, R. W. Carlson, 55 Maryland Avenue, Berkeley, Calif.

[10] Raphael, J. M., "The Development of Stress in Shasta Dam," Trans. ASCE, vol. 118, pp. 289-321, 1953.

[11] Copen, M. D., and Richardson, J. T., "Comparison of the Measured and the Computed Behavior of Monticello (Arch) Dam," Question No. 29, Report 5, 8th Congress on Large Dams, ICOLD, Edinburgh, Scotland, 1964.

[12] Rice, O. L., "In Situ Testing of Foundation and Abutment Rock on Large Dams," Question No. 28, Report 5, 8th Congress on Large Dams, ICOLD, Edinburgh, Scotland, 1964.

[13] Rouse, G. C., Richardson, J. T., and Misterek, D. L., "Measurement of Rock Deformations in Foundations on Mass Concrete Dams," ASTM Symposium, Instrumentation and Apparatus for Soil and Rock, 68th Annual Meeting, Purdue University, 1965.

[14] Technical Bulletin Series, Bulletins 16 through 23, Terrametrics Division of Earth Sciences, Teledyne Co., Golden, Colo., 1972.

[15] Technical Record of Design and Construction, "Glen Canyon Dam and Powerplant," Bureau of Reclamation, pp. 117-138 and 449-453, and p. 464, December 1970.

[16] Technical Record of Design and Construction, "Flaming Gorge Dam and Powerplant," Bureau of Reclamation, 1968.

[17] Hickey, K. B., "Effect of Stress Level on Creep and Creep Recovery of Lean Mass Concrete," Report REC-OCE-69-6, Bureau of Reclamation, December 1969.

[18] "A Loading System for Compressive Creep Studies on Concrete Cylinders," Concrete Laboratory Report No. C-1033, Bureau of Reclamation, June 1962.

[19] Best, C. H., Pirtz, D., and Polivka, M., "A Loading System for Creep Studies of Concrete," ASTM Bulletin No. 224, pp. 44-47, September 1957.

[20] "A 10-Year Study of Creep Properties of Concrete," Concrete Laboratory Report No. SP-38, Bureau of Reclamation, July 1953.

[21] Hickey, K. B., "Stress Studies of Carlson Stress Meters in Concrete," Report REC-ERC-71-19, Bureau of Reclamation, April 1971.

[22] "Manual of Geodetic Triangulation," Special Publication No. 247, Coast and Geodetic Survey, Department of Commerce, Washington, D.C., 1950.

[23] "Manual of Geodetic Leveling," Special Publication No. 239, Coast and Geodetic Survey, Department of Commerce, Washington, D.C., 1948.

[24] "Design Criteria for Concrete Arch and Gravity Dams," Engineering Monograph No. 19, Bureau of Reclamation, February 1977.

[25] Reclamation Instructions, Part 175, Reports of Construction and Structural Behavior (L-21 Report) Bureau of Reclamation, 1972.

[26] "Calculations of Deflections Obtained by Plumblines," Electronic Computer Description No. C-114, Bureau of Reclamation, 1961.

[27] Roehm, L. H., "Investigation of Temperature Stresses and Deflections in Flaming Gorge Dam," Technical Memorandum 667, Bureau of Reclamation, 1967.

[28] "Twenty-Year Creep Test Results on Shasta Dam Concrete," Laboratory Report No. C-805A, Bureau of Reclamation, February 1962.

[29] "Properties of Mass Concrete in Bureau of Reclamation Dams," Laboratory Report No. C-1009, Bureau of Reclamation, December 1961.

[30] Roehm, L. H., "Deformation Measurements of Flaming Gorge Dam," Proc. ASCE, Journal of the Surveying and Mapping Division, vol. 94, No. SU1, pp. 37-48, January 1968.

[31] Richardson, J. T., "Measured Deformation Behavior of Glen Canyon Dam," Proc. ASCE, Journal of the Surveying and Mapping Division, vol. 94, No. SU2, pp. 149-168, September 1968.

# Concrete Construction

14-1. *General.*—Concrete control and concrete construction operations are of vital concern to the designer of a concrete structure. The ideal situation would be to have the engineer responsible for the design of a structure go to the site and personally supervise the construction to assure its intended performance. Since this is not practicable, it falls on the construction engineer and his inspection staff, the design engineer's closest contact with the work, to assure that the concrete meets the requirements of the design.

The safety of any structure is related to certain design criteria which include factors of safety. Only when all concrete control and construction operations are of high quality will the factors of safety be valid for the completed structure. Whereas steel used for structures can be tested for material requirements and structural properties, with the full knowledge that another piece of that same steel will react in the same manner, concrete is mixed and placed under varying conditions. Concrete is placed in the structure knowing what it has done in the past under similar circumstances. From experience, we know what concrete *can do*. Time alone will tell if it *will do this*. A high assurance that it *will* can be obtained by the concrete inspector by making certain that the concrete is mixed and placed, and the structure is completed, in full compliance with the specifications.

Appendix M covers those specifications paragraphs relating to concrete that are normally required for construction of concrete dams.

14-2. *Design Requirements.*—Basically, a concrete structure must be capable of performing its intended use for what may be an unknown but usually long period of time. To serve its purpose, the concrete in the structure must be of such strength and have such physical properties as are necessary to carry the design loads in a safe and efficient manner. The concrete throughout the structure must be of uniform quality because a structure is only as strong as its weakest part. The concrete must be durable and resistant to weathering, chemical attack, and erosion. The structure must be relatively free of surface and structural cracks. Because of increasing environmental demands, the final completed structure must be pleasing in appearance. And, last but not least, the construction processes and procedures should reflect an economical design and use of materials, manpower, and construction effort.

A number of the above design requirements are the responsibility of the designer. These include the determination of the configuration and dimensions of the structure, the sizes and positioning of reinforcing bars, and the finishes necessary to minimize erosion and cavitation on the surfaces of the structure. Additional design requirements are determined from field investigations of the site conditions, including such items as the type and condition of the foundation for the structure, and the availability of sand and coarse aggregates. Other design requirements may be obtained from concrete laboratory investigations on the concrete mix, from hydraulic laboratory model studies, and from environmental studies on the desired appearance of the structure. The fulfillment of all design requirements is dependent upon actual construction processes and procedures. A continuing effort must therefore be made by all inspection personnel to assure the satisfactory construction of the desired structure.

Aggregates for use in concrete should be of good quality and reasonably well graded. Usually, an aggregate source has been selected and tested during preconstruction investigations. Also, in some cases, concrete mix design studies have been made as part of the preconstruction investigations using the aggregates from the deposit concerned. When good quality natural sand and coarse aggregate are available, use of crushed sand and/or coarse aggregate is generally limited to that needed to make up deficiencies in the natural materials, as crushing generally increases the cost of the aggregates and resulting concrete. In these instances, crushing is usually restricted to crushing of oversize materials and/or the excess of any of the individual sizes of coarse aggregate. Where little or no natural coarse aggregate is available in a deposit, it may be necessary to use crushed coarse aggregate from a good quality quarry rock.

14-3. ***Composition of Concrete.***—The concrete for a specific concrete structure is proportioned to obtain a given strength and durability. Concrete with a higher strength than required could be designed by adding more cement, and perhaps admixtures, but this higher strength concrete is not desirable from the standpoint of economy of design. In addition, for an arch dam, the higher strength concrete may be a disadvantage with respect to temperature loads because of the higher modulus which will cause higher stresses for the same temperature change. On the larger and more important Bureau of Reclamation structures, trial mixes are made in the laboratories at the Engineering and Research Center not only to obtain an economical and workable mix but to assure that the required strength and durability can be obtained with the cement and aggregates proposed for the construction.

Adjustments in the field are sometimes necessary to obtain a workable mix. These may be occasioned by variations in aggregate characteristics within the deposit being worked, or by a change in the characteristics of the cement being used.

The amount of cement to be used per cubic yard of concrete is determined by mix investigations which are primarily directed toward obtaining the desired strength and durability of the concrete. The type of cement, however, may be determined by other design considerations.

Considerable bad experience has been encountered where alkali reactive aggregates are used in concrete. Where field and laboratory investigations of aggregate sources indicate that alkali reactive aggregates will be encountered, a low-alkali cement is normally required to protect against disruptive expansion of the concrete which may occur due to alkali-aggregate reaction (a chemical reaction between alkalies in the cement and the reactive aggregates). Another means of controlling alkali-reactive aggregates is by use of a suitable fly ash or natural pozzolan. If a highly reactive aggregate is to be used, it may be necessary to use both low-alkali cement and a pozzolan.

Another design consideration is the type of cement to be used. Type II cement is normally used by the Bureau of Reclamation in mass concrete dams. Limitations on the heat of hydration of this cement are specified when determined necessary to minimize cracking in the concrete structure. Use of a type II cement will generally reduce the heat of hydration to an acceptable level, particularly since type II cement is usually used in conjunction with other methods of heat reduction. These include use of lower cement contents, inclusion of a pozzolan as part of the cementitious material, use of a pipe cooling system, and use of a specified maximum placing temperature of the concrete, which may be as low as 50° F. Use of all or some of these methods will usually reduce or eliminate the need for stringent limitations on the heat of hydration of the cement. However, a limitation of 58 percent on the tricalcium aluminate plus tricalcium silicate ($C_3A + C_3S$) content of the type II cement may be required where heat of hydration of cement must be kept low. Further limitation on the heat of hydration, if more stringent control of heat is needed, can be obtained with a type II cement by providing a maximum limitation on the cement of 70 calories per gram at 7 days or 80 calories per gram at 28 days, or both.

If the above measures are insufficient, use of type IV cement, an extremely low heat of hydration cement, may be specified. This type of cement, referred to as low-heat cement, was developed many years ago for mass concrete when thick, very massive, high-cement-content concrete dams were being built. Maximum limitations on heat of hydration of type IV cement are 60 calories per gram at 7 days and 70 calories per gram at 28 days. The amount and type of cement used must be compatible with strength, durability, and temperature requirements.

Admixtures are incorporated into the mix design as needed to obtain economy, workability, or certain other desired objectives such as permitting placement over extended periods of time. Admixtures have varying effects on concretes, and should be employed only after a thorough evaluation of their effects. Most commonly used admixtures are accelerators; air-entraining agents; water-reducing, set-controlling admixtures (WRA); and pozzolans. Calcium chloride should not be used as an accelerator where aluminum or galvanized metalwork is embedded. When accelerators are used, added care will be necessary to prevent cold joints during concrete placing operations. Air-entraining agents should be used to increase the durability of the concrete, especially if the structure will be exposed to cycles of freezing and thawing. Use of a WRA will expedite the placing of concrete under difficult conditions, such as for large concrete placements in hot weather. Also, use of a WRA will aid in achieving economy by producing higher strengths with a given cement content.

Good quality pozzolans can be used as a replacement for cement in the concrete without sacrificing later-age strength. Pozzolan is generally less expensive than cement and will, as previously indicated, aid in reducing heat of hydration. Since the properties of pozzolans vary widely, if a pozzolan is to be used in a concrete dam it is necessary to obtain one that will not introduce adverse qualities into the concrete. Pozzolan, if used in face concrete of the dam, must provide adequate durability to the exposed surfaces. Concrete containing pozzolan requires thorough curing to assure good resistance to freezing and thawing.

The water used in the concrete mix should be reasonably free of silt, organic matter, alkali, salts, and other impurities. Water containing objectionable amounts of chlorides or sulfates is particularly undesirable, because these salts prevent the full development of the desired strength.

**14-4.** ***Batching and Mixing.***–Inherently, concrete is not a homogeneous material. An approach to a "homogeneous" concrete is made by careful and constant control of batching and mixing operations which will result in a concrete of uniform quality throughout the structure. Because of its effect on strength, the amount of water in the mix must be carefully controlled. This control should start in the stockpiles of aggregate where an effort must be made to obtain a uniform and stable moisture content. Water should be added to the mix by some method which will assure that the correct amount of water is added to each batch.

Close control of the mixing operation is required to obtain the desired uniform mix. Sand, rock, and cement pockets will result in a structure weaker in some sections than in others. A nonuniform concrete mix will also result in stress concentrations which cause a redistribution of stresses within the structure. These redistributed stresses may or may not be detrimental depending on where the stresses occur.

Segregation of sand and coarse aggregates can also result in surface defects such as rock pockets, surface scaling and crazing, and sand streaks. These are not only unsightly but are the beginning of surface deterioration in structures subjected to severe weathering.

**14-5.** ***Preparations for Placing.***–The integrity of a concrete structure is dependent to a large extent on the proper preparation of construction joints before placing fresh concrete upon the construction joint surfaces. Bond is desired between the old and new concretes and every effort must be directed toward obtaining this bond. All laitance and inferior surface concrete must be removed from the old surface with air and water jets and wet sandblasting as necessary. All surfaces should be washed thoroughly prior to placing the new concrete, but should be surface dry at the time they are covered with the fresh concrete. Rock surfaces to be covered with concrete must be sound and free of loose material and should also be saturated, but surface dry, when covered with fresh concrete or mortar. Mortar should be placed only on those rock surfaces which are highly porous or are horizontal or nearly horizontal absorptive surfaces.

14-6. ***Placing.***—Mass concrete placement can result in a nonuniform concrete when the concrete is dropped too great a distance or in the wrong manner. The same effect will occur when vibrators are used to move the concrete into its final position. All discharge and succeeding handling methods should therefore be carefully watched to see that the uniformity obtained in mixing will not be destroyed by separation.

Thorough vibration and revibration is necessary to obtain the dense concrete desired for structures. Mass concrete is usually placed in 5- or 7½-foot lifts and each of these lifts is made up of 18- to 20-inch layers. Each successive layer must be placed while the next lower layer is still plastic. The vibrators must penetrate through each layer and revibrate the concrete in the upper portion of the underlying layer to obtain a dense monolithic concrete throughout the lift. Such a procedure will also prevent cold joints within the placement lift.

14-7. ***Curing and Protection.***—One of the major causes of variation in attained concrete strength is the lack of proper curing. Laboratory tests show that strength of poorly cured concrete can be as much as one-third less than that of well-cured concrete. This variance is more for some cements than for others. Curing of concrete is therefore important if high quality is to be obtained. The full effectiveness of water curing requires that it be a continuous, not intermittent, operation. Curing compounds, if used, must be applied as soon as the forms are stripped, and must be applied to completely cover all exposed surfaces.

Poor curing often results in the formation of surface cracking. These cracks affect the durability of the structure by permitting weathering and freeze-thaw actions to cause deterioration of the surface. The larger structural cracks often begin with the cracks caused by poor curing.

Protection of the newly placed concrete against freezing is important to the designer, since inadequate protective measures will be reflected by lower strength and durability of the concrete. Protective measures include addition of calcium chloride to the mix and maintaining a minimum $40^{\circ}$ F. placement temperature. Although calcium chloride in a quantity of not to exceed 1 percent, by weight of cement, is normally required when weather conditions in the area of the work will permit a drop in temperature to freezing, its use should not preclude the application of more positive means to assure that early age concrete will not freeze. When freezing temperatures may occur, enclosures and surface insulation should also be required. One of the most important factors associated with protection of concrete is advance preparation for the placement of concrete in cold weather. Arrangements for covering, insulating, or otherwise protecting newly placed concrete must be made in advance of placement and should be adequate to maintain the temperature and moisture conditions recommended for good curing.

14-8. ***Finishes and Finishing.***—Suitable finishing of concrete surfaces is of particular concern to the designer. Some surfaces of concrete, because of their intended function, can be rough and of varying texture and evenness; whereas, others in varying degree must be smooth and uniform, some necessitating stringent allowable irregularity limits. The Bureau of Reclamation uses a letter-number system to differentiate between the different types of finishes, using F1, F2, F3, and F4 for formed surfaces and U1, U2, and U3 for unformed surfaces. Each finish is defined as to allowable abrupt and gradual irregularities. For formed surfaces, the particular forming materials permitted are also related to the letter-number system.

Finish F1 applies to formed surfaces that will be covered by fill material or concrete, which includes vertical construction and contraction joints, and upstream faces of mass concrete dams that are below the minimum water pool. Finish F2 applies to formed surfaces that will be permanently exposed to view but which do not require any special architectural appearance or treatment, or which do not involve surfaces that are subject to high-velocity waterflow. Finish F3 is used for formed surfaces for which, because of prominent exposure to public view, an aesthetic appearance from an architectural standpoint is considered desirable. Finish F4 is for formed flow surfaces of hydraulic structures where accurate alinement and evenness of surfaces are required to eliminate destructive effects of high-velocity water.

Finish U1 applies to unformed surfaces that will be covered by fill material or concrete. This is a screeded surface where considerable roughness can be tolerated. Screeding of an unformed surface is preliminary to the application of a U2 finish. The U2 finish is a wood-floated finish. This finish applies to all exposed unformed surfaces, and is a preliminary to applying a U3 finish, which requires steel troweling. A U3 finish is required on high-velocity flow surfaces of spillway tunnels and elsewhere where a steel-troweled surface is considered desirable.

When finishing the surfaces of newly placed concrete, overtroweling is to be avoided in all instances. Surfaces which are overtroweled are susceptible to weathering and erosion and usually result in a requirement for early repair measures on the structure.

14-9. ***Tolerances.***—The prescribed tolerances on all structures should be maintained at all times. Some of these tolerances are placed in the specifications to control the overall construction and are necessary if the structure is to be completed as designed. Some tolerances are for appearances and others are to minimize future maintenance of the structure. For example, near-horizontal surfaces with very slight slopes are hard to finish without leaving depressions in the surface. These depressions collect moisture and usually begin weathering at an early age.

14-10. ***Repair of Concrete.***—Repair of concrete covers not only the patching of holes remaining after construction operations but also the repair of cracks and damaged concrete. Repair of concrete in Bureau of Reclamation structures is required to conform to the Bureau's "Standard Specifications for Repair of Concrete." These specifications generally provide for concrete to be repaired with concrete, dry pack or portland cement mortar, or, at the option of the contractor, with epoxy-bonded concrete or epoxy-bonded epoxy mortar, where and as permitted by the specifications for the particular repair to be made. Repairs to high-velocity flow surfaces of concrete in hydraulic structures are required to be made with concrete, epoxy-bonded concrete, or epoxy-bonded epoxy mortar. Concrete is used for areas of extensive repair which exceed 6 inches in depth, while epoxy-bonded concrete is used for areas having depths of 1½ to 6 inches. Epoxy-bonded epoxy mortar is used for shallow surface repairs for depths ranging from 1½ inches to featheredges.

Before making any repair, all deteriorated and defective concrete must be removed. Unsound or questionable concrete may negate the successful repair of any concrete. Removal of the defective concrete should be followed by a thorough washing of the surfaces. A surface-dry condition should exist at the time replacement concrete is placed.

Cracks should not be repaired until all evidences indicate that the crack has stablized. The cause of the crack should also be investigated and, if possible, corrective measures initiated so that the crack will not reopen. All repairs should be thoroughly cured to minimize drying shrinkage in the repair concrete. Except where repairs are made with epoxy-bonded epoxy mortar, featheredges should be avoided in all repair of concrete.

<<Chapter XV

# Ecological and Environmental Considerations

## A. INTRODUCTION

15-1. *General Considerations.*—The rapid increase in world population and the increasing demands this population has made on the planet's natural resources have called into question the long-term effect of man upon his environment. The realization that man is an integral part of nature, and that his interaction with the fragile ecological systems which surround him is of paramount importance to his continued survival, is prompting a reevaluation of the functional relationships that exist between the environment, its ecology, and man.

Of increasing concern is the effect which man's structures have upon the ecosystems in which they are placed, and especially on the fish, wildlife, and human inhabitants adjacent to these structures. The need to store water for use through periods of drought, to supply industry and agriculture with water for material goods and foodstuffs, to provide recreational opportunity in ever-increasing amounts, and to meet the skyrocketing electric power demand has required the development of water resources projects involving the construction and use of dams and other related structures. These structures help man and yet at the same time cause problems in the environment and in the ecosystems into which they are placed. Many of these problems are exceedingly complex, and few answers which encompass the total effect of a structure on its environment are readily available.

Included in the answer to these problems must be the development and protection of a quality environment which serves both the demands of nature for ecological balance and the demands of man for social and psychological balance. The present challenge is to develop and implement new methods of design and construction which minimize environmental disturbances, while also creating aesthetic and culturally pleasing conditions under which man can develop his most desirable potentialities. This challenge can only be answered by the reasoned, pragmatic approach of sensitive, knowledgeable human beings.

The purpose of this chapter is to provide practical solutions to some of the environmental and ecological problems which confront the designer. This discussion is not exhaustive and it is hoped that the reader will consult the references at the end of this chapter (and numerous others available on this topic) for a more extensive coverage. The amount of scientific data concerning the environment and man's relation to it is expanding rapidly, and new design methods should become available soon. The practical information presented here can provide a useful introduction to the designer and a basis for maximizing the project's benefits and minimizing its negative environmental and ecological effects.

Recognizing the importance of man's environment, the 91st Congress passed the National Environmental Policy Act of 1969. This act established a three-member Council on Environmental Quality in the Executive Office of the President. Before beginning construction of a project, an Environmental Impact

Statement must be prepared by the agency having jurisdiction over project planning and submitted through proper channels to appropriate governmental agencies and interested private entities for review and comments.

The term environment is meant here to include the earth resources of land, water, air, and vegetation and manmade structures which surround or are directly related to the proposed structure. The term ecology is meant to encompass the pattern of relationships that exist between organisms (plant, animal, and human) and their environment.

15-2. *Planning Operations.*—One of the most important aspects of dealing correctly and completely with the ecological and environmental impact of any structure is proper planning. If possible, an environmental team should be formed consisting of representatives from groups who will be affected by the structure and experts from various scientific fields who can contribute their ideas and experience. The team approach will help assure that environmental considerations are placed in proper perspective with other vital issues such as reliability, cost, and safety, and that the relative advantages and disadvantages of each proposal are carefully weighed. It should also assure that the project is compatible with the natural environment. A suggested list of participants is given below:

(1) Concerned local and community officials.
(2) Design personnel.
(3) Environment and ecology experts.
(4) Fish biologists and wildlife experts.
(5) Building architects.
(6) Landscape architects.
(7) Recreational consultants.

This team should be responsible for the submittal of an ecological and environmental report to the designers with a list of criteria which the designs should encompass. Some of the topics which should be discussed in the report are covered briefly in this chapter. Since each site will present unique problems, only a general outline of the most important considerations is provided herein.

## B. FISH AND WILDLIFE CONSIDERATIONS

15-3. *General.*—The placement of a dam and its reservoir within the environment should be done with due consideration to the effects on the fish and wildlife populations of the specific area. These considerations often involve complex problems of feeding patterns and mobility, and where possible an expert in this field should be consulted. The Fish and Wildlife Service of the Department of the Interior, the Forest Service of the Department of Agriculture, and appropriate State agencies can supply considerable expertise on the environmental impact of a proposed structure. It should be remembered that dams and reservoirs can be highly advantageous in that they provide a year-round supply of drinking water for wildlife, breeding grounds for waterfowl, and spawning areas for fish. At the time of design, as many benefits as practicable to fish, wildlife, and waterfowl populations should be included and provisions should be made for the future protection of these populations. The following sections discuss some of the items which affect fish and wildlife and outline what can be done to aid them.

15-4. *Ecological and Environmental Considerations for Fish.*—Critically important to the survival of fish population are three items: (1) water quality, (2) water temperature, and (3) mobility. Water quality is obviously important to the survival of fish, and an effort should be made to see that the quantity of pollutants which enter the stream during construction and the reservoir after completion is kept to the minimum. Strict regulations concerning pollutants should be instituted and enforced. Quantities of degradable, soluble, or toxic pollutants should not be left within the reservoir area after construction. Heavy pesticide runoffs can cause fish kills, and some means such as a holding pond or contour ditches should be used to reduce their presence and steps taken to eventually eliminate them. Substances that can cloud or darken the water interfere with the ability of sight-feeding fish to forage, and should not be allowed to enter the water.

In mining areas where heavy erosion often occurs, careful consideration must be given to the effects of siltation which may rapidly reduce the reservoir capacity. Consideration must also be given to the acidic character of the water since it can cause fish kills. Control dams may be the solution, and in one case the Bureau of Reclamation has constructed an off-reservoir dam to reduce the rapid sedimentation of the main reservoir and to limit the amount of acidic inflow to an acceptable level.

The temperature of the water controls timing of migration, breeding, and hatching and affects the appetite, growth, rate of heartbeat, and oxygen requirements of all fish. Each species of fish has an optimum temperature range within which it can survive, and consideration must be given to the temperature range which will exist both in the reservoir and in the stream below the dam due to the reservoir releases. For example, if a low dam is constructed in a mountainous area, the cool water entering the shallow reservoir can be warmed by the sun during the summer months to an undesirable extent. The warm temperatures of the shallow water within the lake and also of the downstream releases could then prevent the spawning of cold water species of fish such as trout.

To remedy this problem, care must be taken to provide sufficiently deep reservoir areas where cold water will remain, and to use an outlet works which is capable of selectively withdrawing the colder water from the lower reservoir depths. Federal and State fish and wildlife agencies should be consulted as to the correct depth for the outlets in a specific area. Figure 15-1 shows the selective withdrawal outlet works to be used at Pueblo Dam in Colorado. Figure 15-2 shows a selective withdrawal outlet works used at Folsom Dam in California. Movable shutters were placed upstream from the trashracks. By manipulation of the shutters water may be drawn from the desired level. The Bureau of Reclamation has used selective withdrawal outlet works at several locations to create favorable temperatures for fish spawning downstream of the dam. Further information concerning these structures is available in references [1] and [2].[1] Another reason for providing adequate reservoir depth, in addition to creating favorable conditions for spawning, is to prevent fish kill in the winter due to extreme cold. However, shallow reservoir areas are sometimes required to develop a warm water fishery or for waterfowl habitats.

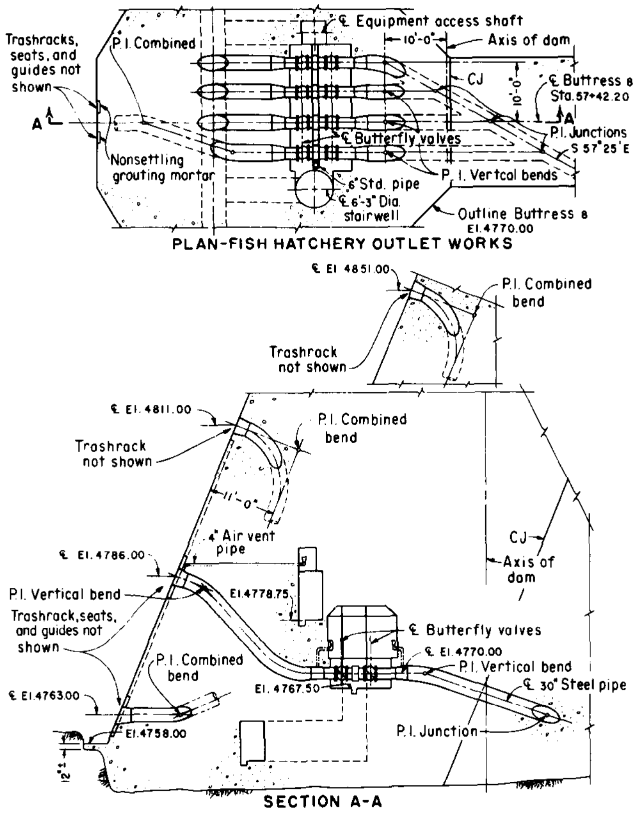


*Figure 15-1.* **Selective withdrawal outlet at Pueblo Dam in Colorado. Water can be withdrawn at any of four levels.—288-D-3104**

Although salmon are commonly thought of as the only migrators, other species of fish such

---

[1] Numbers in brackets refer to items in the bibliography, section 15-12.

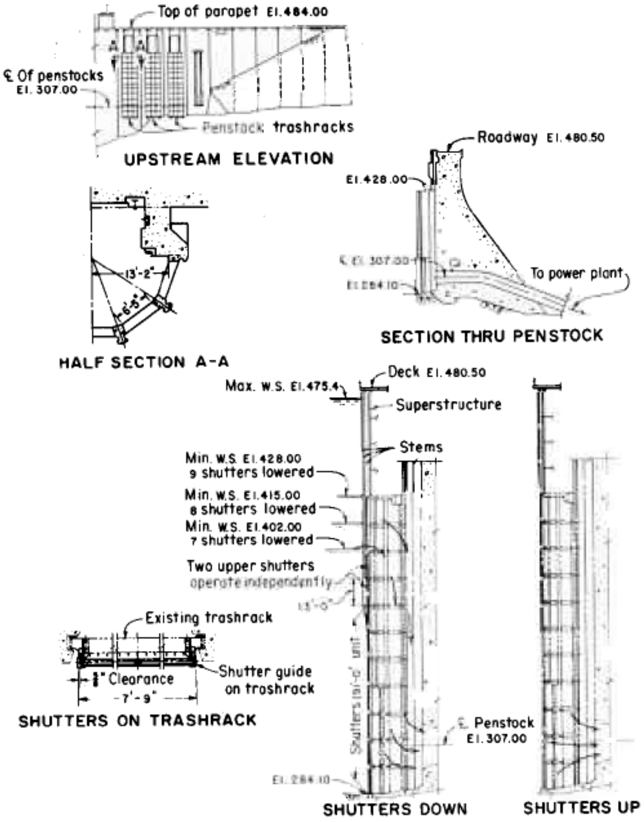


*Figure 15-2.* **Selective withdrawal outlet at Folsom Dam in California. This outlet makes use of an adjustable shutter arrangement.–288-D-3105**

as shad, steelhead trout, and other trout also require mobility considerations. The most common method for allowing fish to pass by a dam is use of the fish ladder. Figure 15-3 shows the fish ladder used by the Bureau of Reclamation on the Red Bluff Diversion Dam in California. Specific design requirements for fish ladders may be obtained from the Forest Service, the Fish and Wildlife Service, or appropriate State agencies. Where practicable, fish should be prohibited from entering spillways, outlet pipes, penstocks, and other restricted areas by use of fish screens.

Where fish populations are concerned, care should be taken to avoid the destruction of vegetation in the reservoir area since this becomes a food source after the reservoir is filled. Certain amounts of standing trees or tree debris left in the reservoir area can provide a habitat for several species of fish as can brush piles, which are staked down to prevent them from being washed away. Figure 15-4 shows a reservoir in which trees have been left standing to benefit the fish population.

*Figure 15-3.* **Fish ladder used on the left abutment of Red Bluff Diversion Dam in California.–P602-200-4543 NA**

Although certain aquatic plants are desirable for water birds, such as ducks, coots, and wading birds, they can be detrimental to fish production and should be controlled when necessary. Shallow shorelines in the inlet portions of the reservoir can be deepened to eliminate the growth of any plant life found not useful.

In newly constructed reservoirs, arrangements should be made for stocking with the appropriate type or types of fish. Consultation with a fisheries expert is recommended to determine the correct type of fish and the proper time for stocking them.

The oxygen content of some reservoirs can decrease with time and an examination of available reservoir reaeration devices may prove helpful. The oxygen content of the reservoir water may be increased during release from the reservoir by the use of reaeration devices such as the U-tube [3]. Reaeration is also aided by increasing the contact of the water released in the spillways and outlet works with air. A bibliography of reaeration devices compiled by

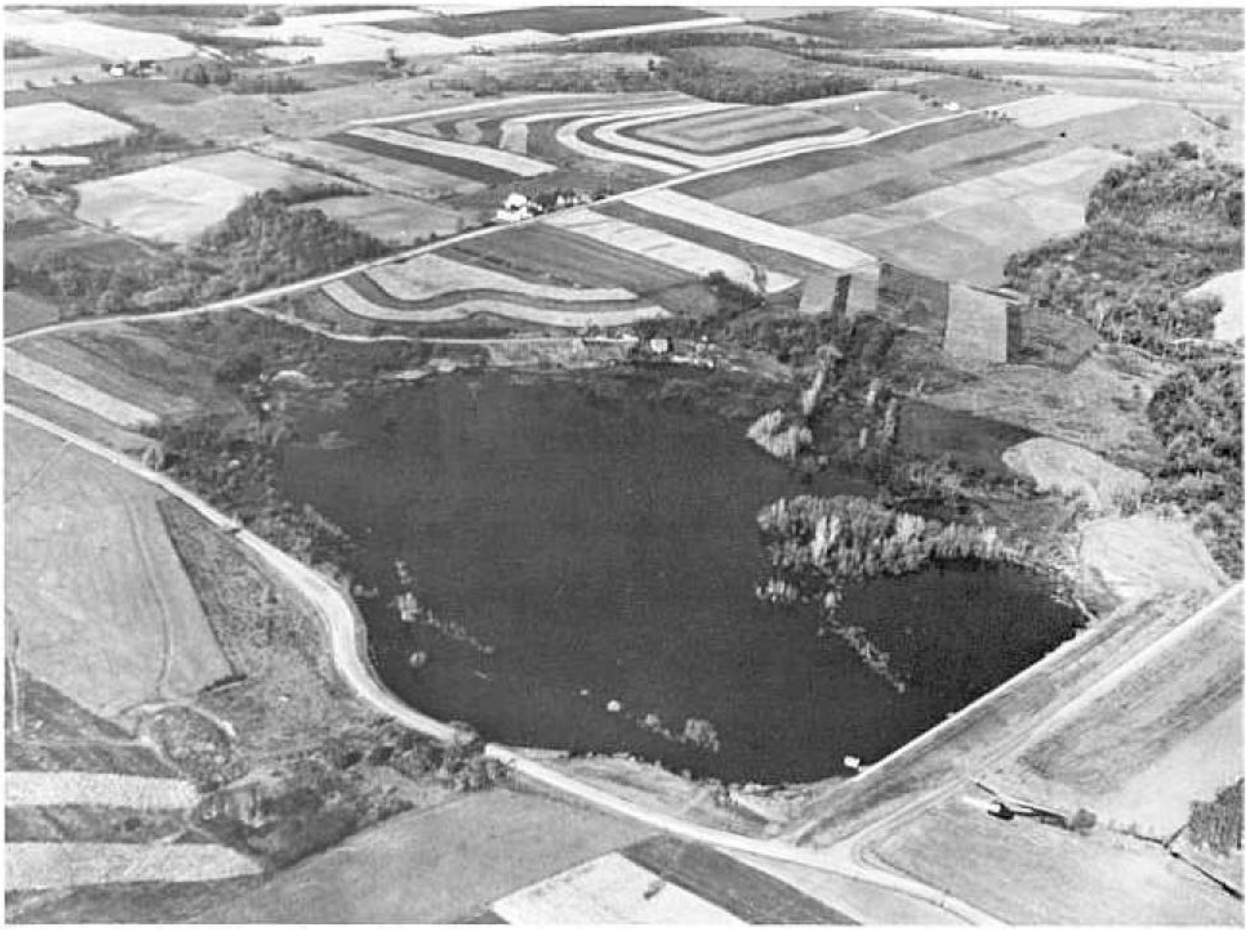


***Figure 15-4.*** **An aerial view of a small reservoir with trees left at the water's edge to provide a fish habitat.–288-D-2869**

the Bureau of Reclamation is contained in reference [4].

In some cases, fish hatcheries can be built in conjunction with the dam. Figure 15-5 shows the hatchery below Nimbus Dam in California. Canals also may provide spawning areas for fish, although considerable cost and special equipment may be required. Figure 15-6 shows an artist's conception of the "gravel cleaner" which will be provided at a salmon spawning area in the Tehama-Colusa Canal in California. Special gravel and special gravel sizes were required in the canal bottom to facilitate spawning.

**15-5.** ***Environmental Considerations for Wildlife.***—Three common detrimental effects of reservoirs on wildlife involve: (1) removal of feeding areas, (2) loss of habitat, and (3) limitation of mobility. The severity of each of these effects can be significantly reduced.

When reservoirs inundate wildlife feeding areas, new areas should be planted to lessen the impact and, if possible, new types of grasses which are suitable and which provide more food per unit area should be planted. In addition, the new feeding areas can sometimes be irrigated with reservoir water to cause rapid, heavy growth. If the reservoir water is not immediately needed for irrigation, the water level can be left below the normal water surface to allow sufficient time for the feeding areas which are to be flooded to be replaced by areas of new growth.

Where flooding of the homes of a large number of smaller animals such as muskrat and beaver will occur, consideration should be

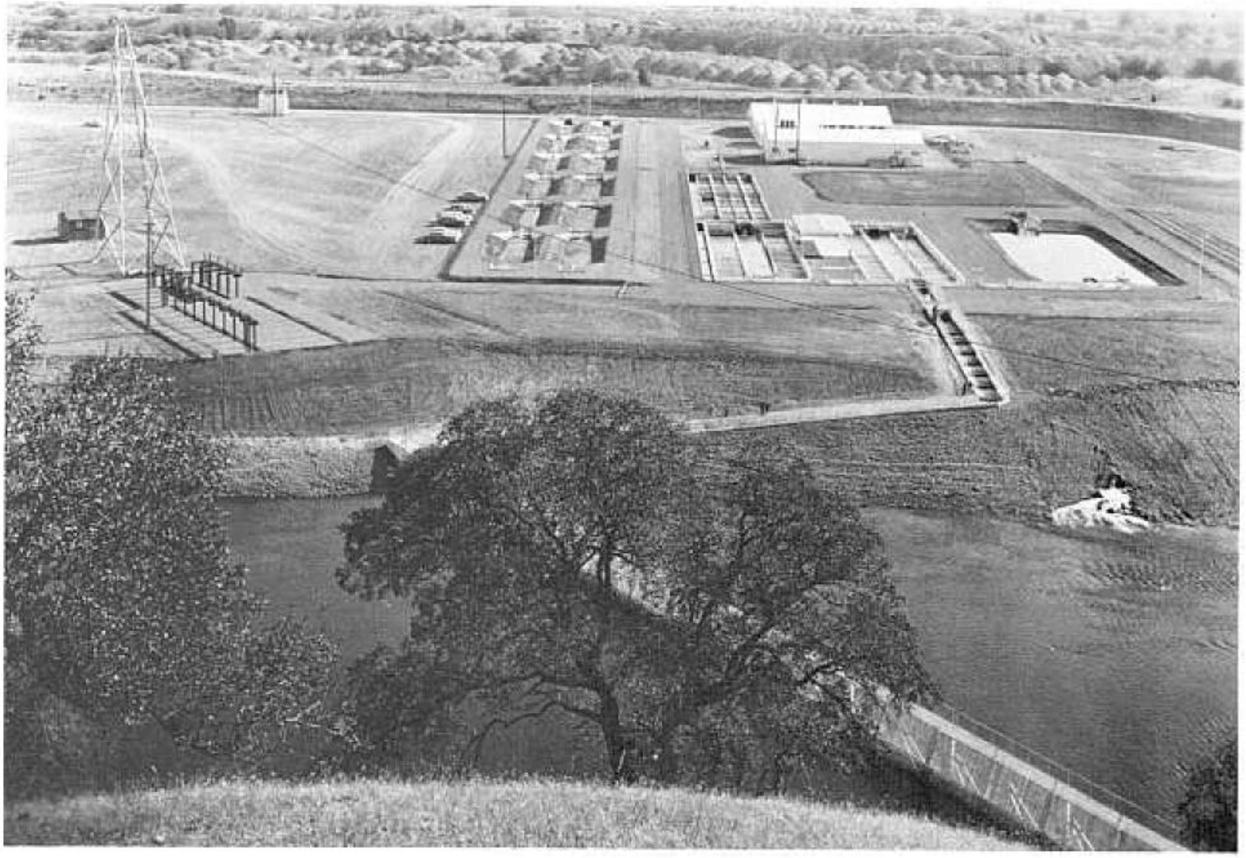


*Figure 15-5.* **Fish hatchery at Nimbus Dam in California.–AR2964-CV**

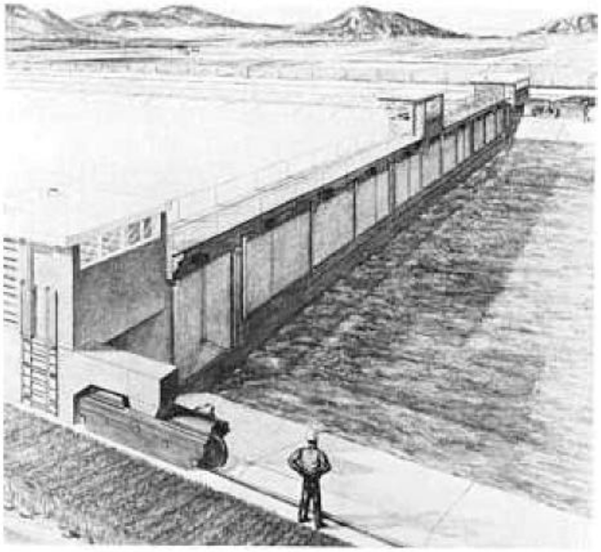


*Figure 15-6.* **An artist's conception of the gravel cleaner to be used at a salmon spawning area on the Tehama-Colusa Canal in California.–P602-D-54534-520**

given to adjusting the required excavation, reducing the reservoir water levels, or relocating the dam so that the number of animals affected will be minimized. It may also be possible to provide special dikes and drainage conditions which can lessen the effect. Problems in this area are difficult to solve and the advice of a specialist should be sought.

Provisions for ducks, geese, and other waterfowl at reservoirs can be made by planting vegetation beneficial to nesting and by leaving areas of dense grass and weeds at the water's edge. If suitable areas already exist at the damsite, an effort should be made to selectively excavate to leave the habitat in place. Assistance concerning the appropriate reservoir treatment can be obtained from the Fish and Wildlife Service of the Department of the Interior, the Forest Service of the Department of Agriculture, and appropriate State agencies.

## C. RECREATIONAL CONSIDERATIONS

15-6. *General.*—The nation's increase in population, the decrease in working hours, and the great mobility of large numbers of people have caused a significant increase in the use of reservoirs for recreational activities. These activities include fishing, boating, water skiing, swimming, scuba diving, camping, picnicking, and just simply enjoying the outdoor experience of the reservoir setting. Many of the reservoirs constructed in past years have become the recreation centers of the present, and this will undoubtedly be repeated in the future. Provisions should be made to obtain the maximum recreational benefits from the completed reservoir, and a future development plan should provide for area modifications as the recreation use increases.

15-7. *Recreational Development.*—Considerations for recreational development should start when project planning is begun and should be integrated into the total site plan. Areas of significant natural beauty should be left intact if possible, and recreational facilities should be developed around them. Boat ramps and boat docking facilities are beneficial to most reservoir areas and should be constructed at the same time as the dam. Figure 15-7 shows the docking facilities at the Bureau of Reclamation's Canyon Ferry Reservoir in Montana. Camping facilities for truck campers, trailers, and tenters, and picnicking areas can often be provided at reasonably low costs.

Trash facilities should be provided at convenient locations to help in litter control, and the excessive use of signs and billboards near the reservoir area should be prohibited. The signs which are used should be blended with the surroundings. Toilet facilities should be available at all camping grounds and proper sewage disposal facilities should be installed, especially where the possibility of reservoir pollution exists.

If the reservoir is near a population center it may prove advantageous to provide bicycle paths, equestrian paths, and foot paths for public use. At the damsite or nearby, a reservoir viewing location and possibly a visitors' center should be built. Exhibits showing the history of the project, local history, or other appropriate exhibits can enhance the visitors' enjoyment of the reservoir. These centers should be aesthetically designed to fit the location. Figure 15-8 shows a viewing area at Glen Canyon Dam.

Buildings adjacent to the reservoir should be of low profile and blend with the reservoir surroundings; however, in some cases it may be desirable to contrast the buildings with their surroundings.

Fishing benefits can be maximized by stocking the reservoir with several types of fish and by replenishing these stocks yearly.

Proper maintenance requirements for the recreation areas should be instituted after completion of the dam and reservoir complex, and should include repair of broken and damaged equipment, repainting, and rebuilding to meet expanded facility demands. Trash should be removed from the campgrounds and adjacent recreation facilities at regular intervals, and the possibility of recycling aluminum and other metal products should be explored. Recreational areas which are overused should be rotated to prevent their deterioration, and single areas which receive exceptionally heavy use should be fenced off completely for short intervals to prevent their ruination. Protection of the reservoir banks from sloughing may be required for steep slopes, and excessive erosion at any part of the site should be prevented.

## D. DESIGN CONSIDERATIONS

15-8. *General.*—Design requirements should be devoted to the accomplishment of three goals: (1) keeping the natural beauty of the surrounding area intact, (2) creating

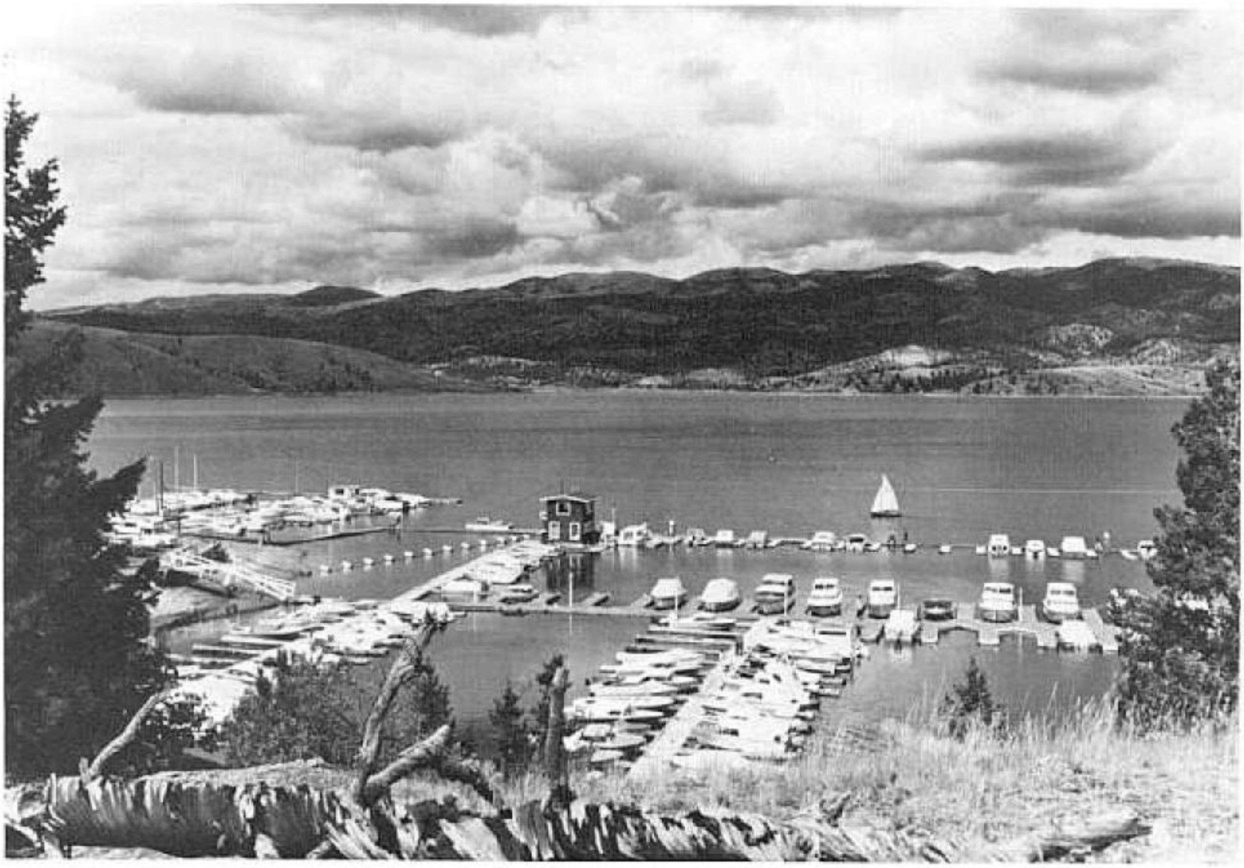


*Figure 15-7.* **Boat docking facilities at Canyon Ferry Reservoir in Montana.–P296-600-949**

aesthetically satisfying structures and landscapes, and (3) causing minimal disturbance to the area ecology. Designers should try to accomplish these goals in the most economical way. The following paragraphs discuss some items to be considered during design and will provide some practical suggestions for designers. Many of the items discussed here should be considered during the project planning stages and the critical decisions made at that time.

If it is necessary to excavate rock abutments above the crest of the dam, consideration should be given to the use of presplitting techniques since they leave clean, aesthetic surfaces. As discussed in sections 15-6 and 15-7, a scenic overlook should be provided for viewing the dam and reservoir. The overlook should have adequate parking and, if practicable, a visitors' center.

The diversion schemes (see ch. V), should be such that excessive silt created during construction will not find its way into the downstream water. Materials from excavations should be placed in the reservoir area upstream of the dam to prevent unsightly waste areas in the downstream approaches. In some cases, boat ramps, picnic areas, or view locations can be constructed with excavated material. Where foundation conditions permit it, spillway structures of a type which minimizes the required surface excavations on the dam abutments should be used. (See ch. IX.)

If a section of canal is constructed in connection with a dam, spoil piles should be shaped to the natural landscape slopes along the canal length; this material can also be used to construct recreation areas where appropriate. Pipelines should be buried as should electrical wiring; where this is not possible, the pipelines and electrical apparatus should be painted to blend with their

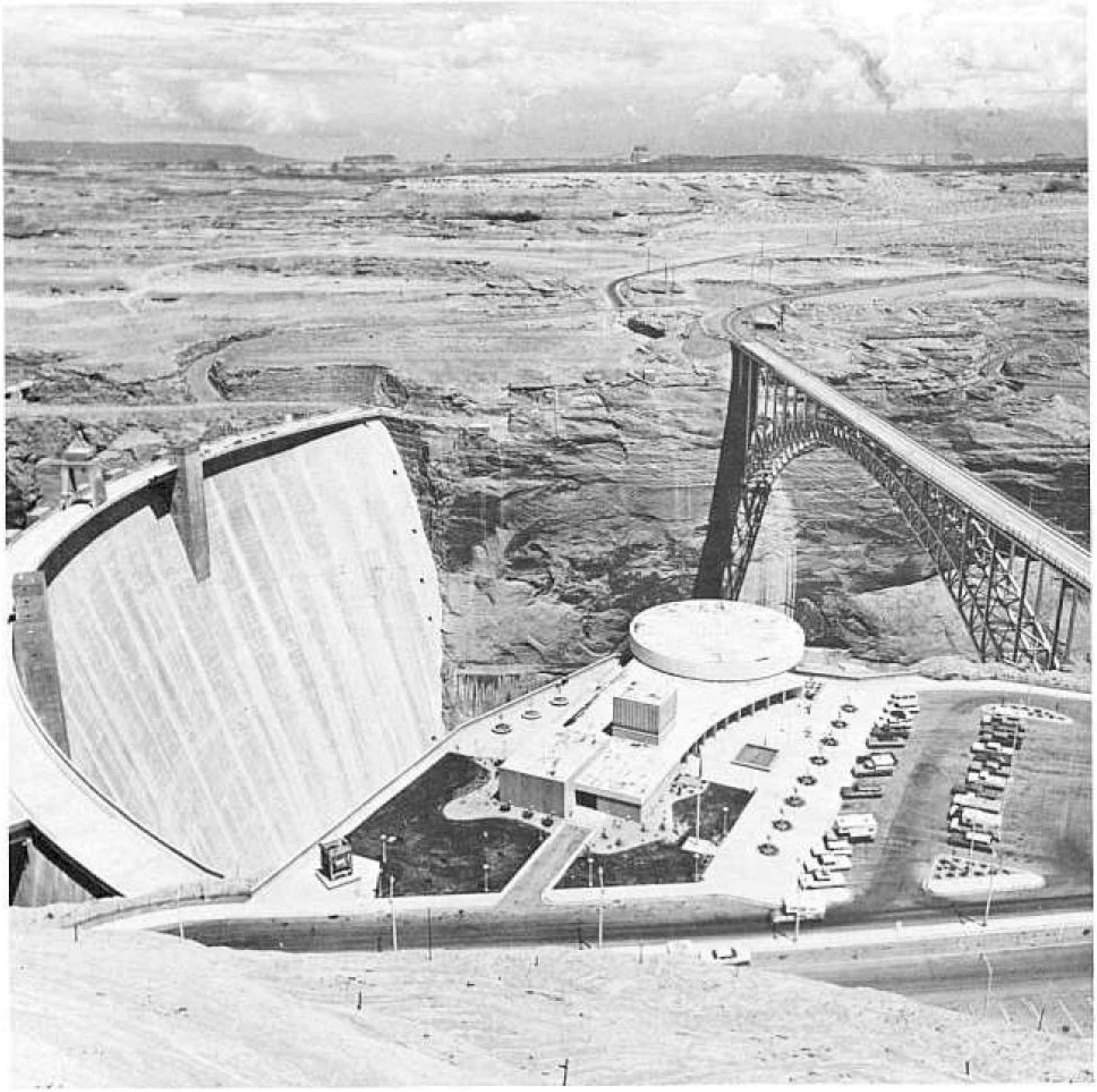


*Figure 15-8.* Viewing area at Glen Canyon Dam in Arizona.—P557-400-1133

background. Protective railings used on the dam crest and on bridges near the site should be low enough so that the reservoir may be seen from a passenger car.

15-9. ***Landscape Considerations.***—As much natural vegetation as possible should be left in place. If significant areas of natural beauty exist near the project, every effort should be made to utilize or preserve them, and additional right-of-way should be obtained to include any such adjacent areas.

Access roads to the damsite and roads used by the contractor during construction should be kept to a minimum, and those roads not planned for use after completion of the dam should be obliterated and replanted with grass or other natural vegetation. Access roads subject to excessive erosion should have protective surfacing. Roads which are required for maintenance of the dam or appurtenant works should be protectively fenced if excessive visitor usage will cause erosion.

Erosion control should be started at the beginning of the job. Roads and cut slopes should be provided with terraces, berms, or other check structures if excessive erosion is likely. Exploratory trenches which are adjacent to the damsite should be refilled and reseeded.

Quarry operations and rock excavations should be performed with care. The minimum amount of material should be removed, correct blasting techniques should be used, unsightly waste areas should not be left, and final rock slopes for the completed excavation should be shaped to have a pleasing appearance. Presplitting and/or controlled blasting should be considered for final slope cutting to permit a clean, pleasing view.

Road relocations near the dam can often eliminate deep cuts in hillsides, allow scenic alinements, and provide reservoir viewing locations. Adequate road drainage should be used and slopes should be cut so that reseeding operations will be convenient. For projects where power transmission lines will be required the publications "Environmental Criteria for Electric Transmission Systems" [5] and "Environmental Considerations in Design of Transmission Lines" [6] will provide many helpful suggestions which will lessen their environmental impact.

15-10. *Protective Considerations.*—At locations where accidents are most likely to occur, protective devices and warning systems should be installed. The most dangerous locations at a damsite are near the spillway (especially if it is a chute type), the outlet works intake tower, and the stilling basins of both the spillway and outlet works. Any portion of the spillway and outlet works stilling basins which might prove hazardous should be fenced off and marked by warning signs.

Canals with steep side slopes which prevent a person or an animal from climbing out are extremely dangerous, as also are siphons. Considerable information concerning canal safety is contained in the Bureau's publication "Reducing Hazards to People and Animals on Reclamation Canals" [7].

When the project encompasses the generation of electricity, the problems of providing adequate safety precautions are considerably increased and the advice of an expert in that field should be sought.

15-11. *Construction Considerations.*—The environmental and ecological design requirements presented in the specifications are converted from an abstraction into a reality by the builder. The contractor and his personnel should be informed that this is a most important step in the planning, design, and construction sequence. In this regard, a preconstruction conference may be invaluable in assuring an understanding of the job requirements by the builder and in enlisting his cooperation. The owner should insure compliance by having competent inspectors and by having specifications which clearly spell out the construction requirements. Excessive air and water pollution during construction should be prevented, and specifications covering these items are included in appendix N; they should provide a framework for the inclusion of other important environmental provisions. The builder should also institute safety precautions during construction, and the publication "Safety and Health Regulations for Construction" [8] will provide helpful information. The builder should be encouraged to bring forward any obvious defects in the environmental considerations which he encounters during construction and to suggest improvements.

Construction campsites should be placed within the reservoir area below normal water level. All trees, shrubs, and grassland areas which are to be protected should be staked or roped off. Any operations which would affect a large wildlife population should be moved to a different location if at all possible. Large volumes of water should not be taken from the stream if there are prior downstream commitments, and the water going downstream should be muddied as little as possible and kept pollution free; siltation ponds may be needed in extreme cases. The builder should be required to remove or bury all trash and debris collected during the construction period and to remove all temporary buildings. Every opportunity should be taken to use the timber in the reservoir area for commercial operations. The burning of trees cleared within the reservoir area should be prevented if excessive air pollution will result or if State laws prevent it. At Pueblo Dam in Colorado, the Bureau required that all brush and timber smaller than 7 inches in diameter be chipped into mulch and stockpiled for future use on the reseeded areas. The chipping operation at Pueblo Dam is shown on figure 15-9. Slightly larger timber can be cut into firewood for use at camping and recreation areas, and still larger timber can be channeled into some commercial use such as production of lumber, wallboard, or boxes.

A temporary viewing site for the project, having signs which show the completed project and explain its purpose, is helpful in promoting good community relations.

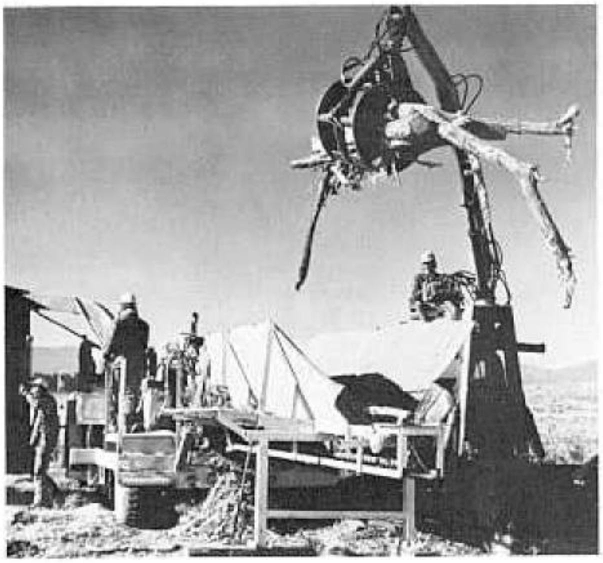


*Figure 15-9.* **Chipping operations at Pueblo Dam in Colorado. All brush and timber smaller than 7 inches in diameter are chipped and stored for use as a mulch on reseeded areas.–P382-700-790 NA**

## E. BIBLIOGRAPHY

### 15-12. *Bibliography.*

[1] "Register of Selective Withdrawal Works in United States," Task Committee on Outlet Works, Committee on Hydraulic Structures, Journal of the Hydraulics Division, ASCE, vol. 96, No. HY9, September 1970, pp. 1841-1872.

[2] Austin, G. H., Gray, D. A., and Swain, D. G., "Multilevel Outlet Works at Four Existing Reservoirs," Journal of the Hydraulics Division, ASCE, vol. 95, No. HY6, November 1970, pp. 1793-1808.

[3] Speece, R. E., and Orosco, R., "Design of U-Tube Aeration Systems," Journal of the Sanitary Engineering Division, ASCE, vol. 96, No. SA3, June 1970, pp. 715-725.

[4] King, D. L., "Reaeration of Streams and Reservoirs–Analysis and Bibliography," REC-OCE-70-55, Bureau of Reclamation, December 1970.

[5] U.S. Department of the Interior and U.S. Department of Agriculture, "Environmental Criteria for Electric Transmission Systems," Government Printing Office, Washington, D.C., 1970.

[6] Brenman, H., and Covington, D. A., "Environmental Considerations in Design of Transmission Lines," ASCE National Meeting on Transportation Engineering, Washington, D.C., July 1969.

[7] Latham, H. S., and Verzuh, J. M., "Reducing Hazards to People and Animals on Reclamation Canals," REC-OCE-70-2, Bureau of Reclamation, January 1970.

[8] Bureau of Reclamation, "Safety and Health Regulations for Construction," latest edition.

*———, "Environmental Quality–Preservation and Enhancement," Reclamation Instructions, Series 350, Part 376, 1969.

U.S. Department of the Interior, "Man–An Enlargered Species," Government Printing Office, Washington, D.C., 1968.

———, "River of Life, Water: The Environmental Challenge," Government Printing Office, Washington D.C., 1970.

———, "The Population Challenge–What It Means to America," Government Printing Office, Washington, D.C., 1966.

———, "The Third Wave," Government Printing Office, Washington, D.C., 1967.

Benson, N. G. (editor), "A Century of Fisheries in North America," American Fisheries Society, Washington, D.C., 1970.

Clawson, M., and Knetsch, J. L., "Economics of Outdoor Recreation," The John Hopkins Press, Baltimore, Md., 1966.

Dasmann, R. E., "Environmental Conservation," John Wiley & Sons, Inc., New York, N.Y., 1968.

Dober, R. P., "Environmental Design," Van Nostrand Reinhold Co., New York, N.Y., 1969.

"Environmental Quality," First annual report of the Council on Environmental Quality, Government Printing Office, Washington, D.C., August 1970.

Flawn, P. T., "Environmental Geology: Conservation, Land-Use Planning, and Resources Management," Harper & Row, New York, N.Y., 1970.

---

***References without numbers are not mentioned in text.**

McCullough, C. A., and Nicklen, R. R., "Control of Water Pollution during Dam Construction," Journal of the Sanitary Engineering Division, ASCE, vol. 97, No. SA1, February 1971, pp. 81-89.

Prokopovich, N. P., "Siltation and Pollution Problems in Spring Creek, Shasta County, California," Journal of American Water Works Association, vol. 57, No. 8, August 1965, pp. 986-995.

Reid, G. K., "Ecology of Inland Water and Estuaries," Van Nostrand Reinhold Co., New York, N.Y., 1961.

"Report of The Committee on Water Quality Criteria," U.S. Department of the Interior, Federal Water Pollution Control Administration, April 1, 1968.

Seaman, E. A., "Small Fish Pond Problem-Management Chart," Technical Publication No. 2, West Virginia Conservation Commission, Charleston, W. Va.

Smith, G. (editor), "Conservation of Natural Resources," third edition, John Wiley & Sons, Inc., New York, N.Y., 1965.

"Transactions," American Fisheries Society, Washington, D.C.

U.S. Department of the Interior, "Quest for Quality," Government Printing Office, Washington, D.C., 1965.

Vernberg, J. F., and Vernberg, W. B., "The Animal and the Environment," Holt, Reinhart & Winston, Inc., New York, N.Y. 1970.

Watt, K. E., "Ecology and Resources Management; A Qualitative Approach," McGraw-Hill, New York, N.Y., 1968.

Wing, L. W., "Practice of Wildlife Conservation," John Wiley & Sons, New York, N.Y., 1951.

## <<Appendix A

# Preliminary Layout and Volume Nomographs

A-1. *General.*—The nomographs in this appendix were developed from empirical formulas to assist designers and planners in estimating volumes and basic dimensions for arch dams. The information is to be used only as a guide for the preliminary layout of arch dams. The procedures for using each of the nomographs are given on the figures.

A typical damsite is shown on figure A-1 from which were measured the dimensions $L_1$ (550 feet) and $L_2$ (160 feet). The estimated structural height is $H$ (290 feet). For preliminary layouts, $R_{axis}$ is assumed to be $0.6L_1$ or 330 feet.

The nomographs, figures A-2 through A-5, can be used to determine thicknesses and projections for the crown cantilever and total volume, $V$, of the dam. These values, for the typical dam shown on figure A-1, are as follows:

| | |
|---|---|
| $T_c$ | 9.5 feet |
| $T_B$ | 28.9 feet |
| $T_{0.45H}$ | 27.5 feet |
| $USP_c$ | 0.0 foot |
| $DSP_c$ | 9.5 feet |
| $USP_B$ | 19.4 feet |
| $DSP_B$ | 9.5 feet |
| $USP_{0.45H}$ | 27.5 feet |
| $DSP_{0.45H}$ | 0.0 foot |
| $V_1$ | 36,800 cubic yards |
| $V_2$ | 57,100 cubic yards |
| $V$ | 93,900 cubic yards |

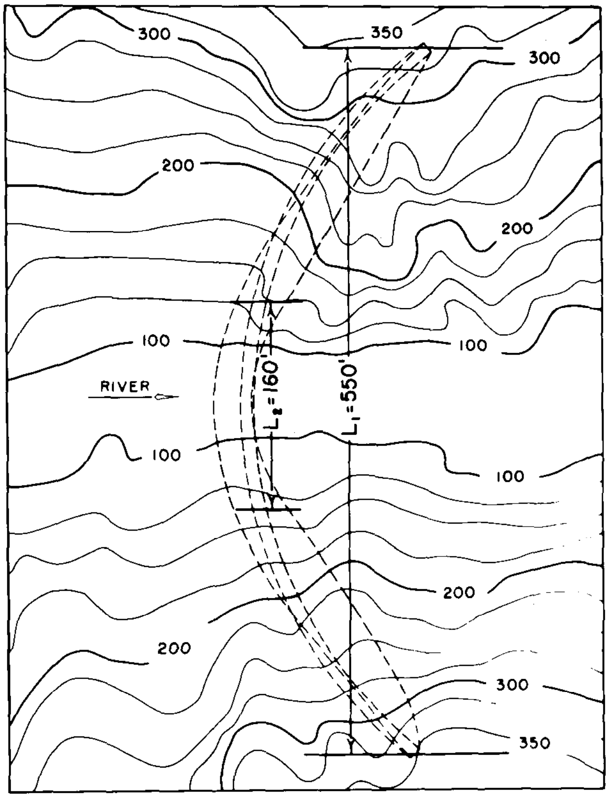
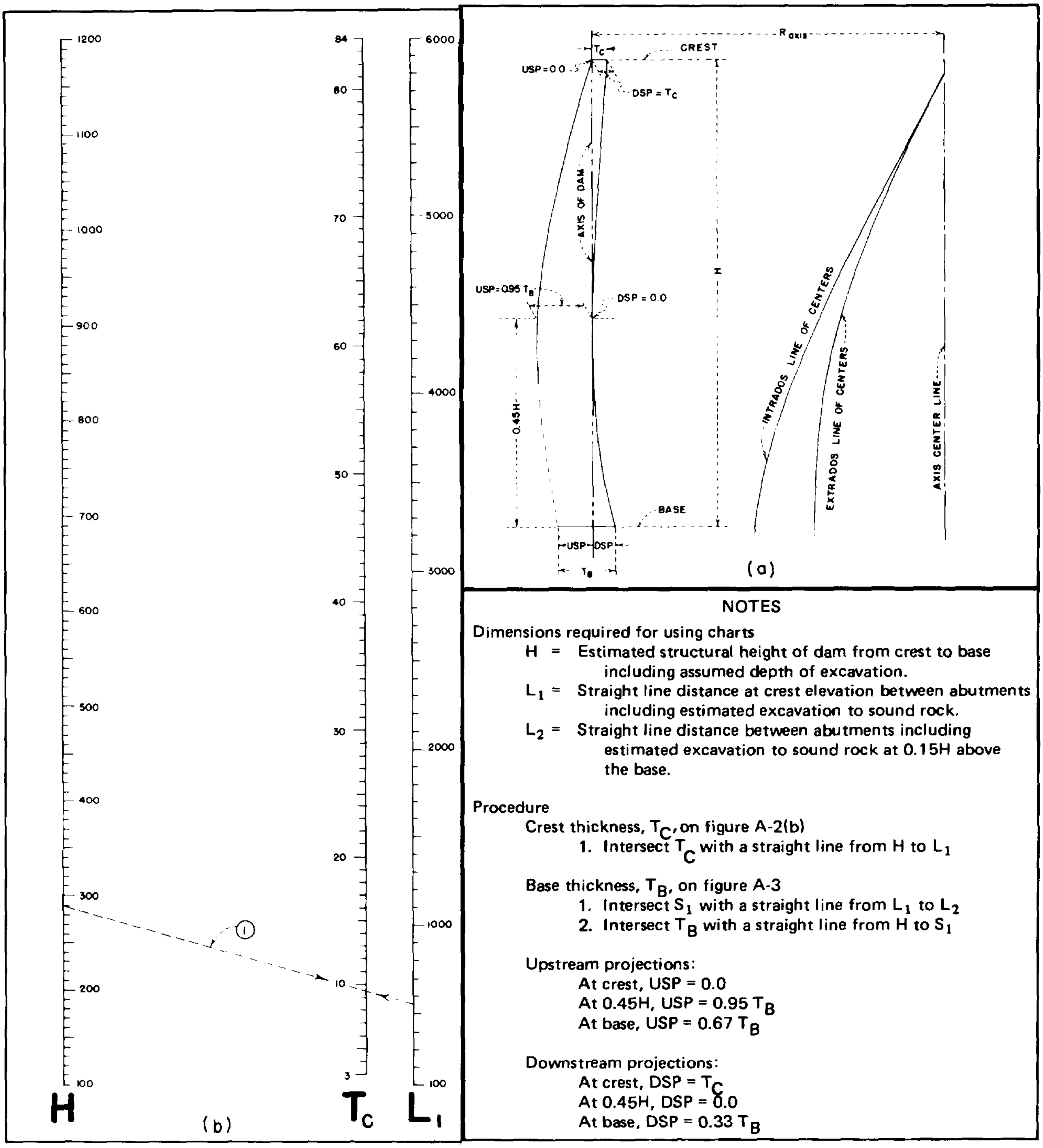


*Figure A-1.* **Dimensions for nomographs and layout of dam.—288-D-3106**



NOTES

Dimensions required for using charts

H = Estimated structural height of dam from crest to base including assumed depth of excavation.

$L_1$ = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.

$L_2$ = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.

Procedure

Crest thickness, $T_C$, on figure A-2(b)

1. Intersect $T_C$ with a straight line from H to $L_1$

Base thickness, $T_B$, on figure A-3

1. Intersect $S_1$ with a straight line from $L_1$ to $L_2$
2. Intersect $T_B$ with a straight line from H to $S_1$

Upstream projections:

At crest, USP = 0.0
At 0.45H, USP = 0.95 $T_B$
At base, USP = 0.67 $T_B$

Downstream projections:

At crest, DSP = $T_C$
At 0.45H, DSP = 0.0
At base, DSP = 0.33 $T_B$

*Figure A-2.* **Nomograph for obtaining crest thickness and projections on crown cantilever.–288-D-3107**

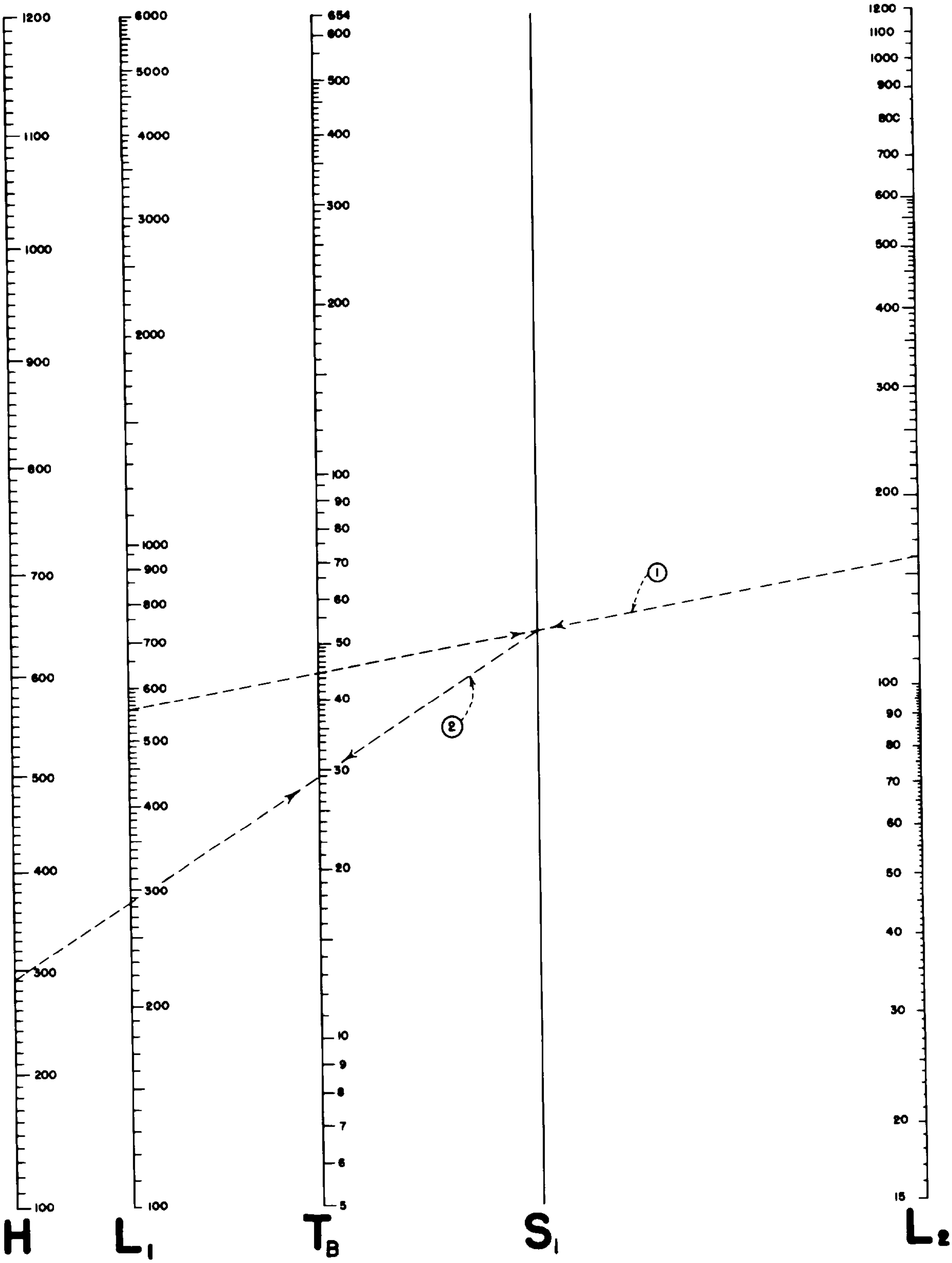
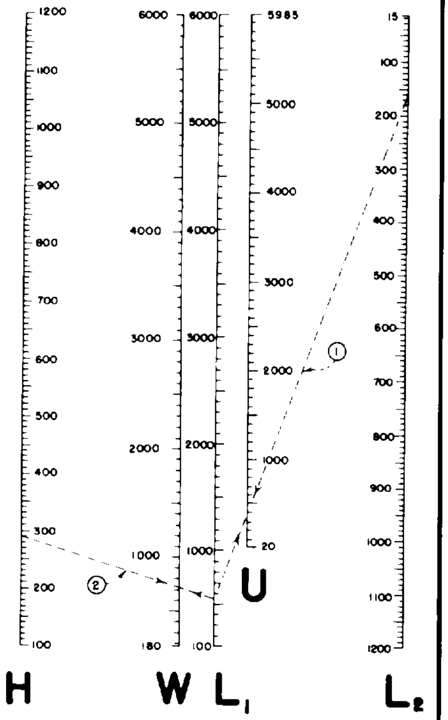


*Figure A-3.* Nomograph for obtaining base thickness and projections on crown cantilever. −288-D-3108



(a)

NOTES

Dimensions required for using charts.

H = Estimated structural height of dam from crest to base including assumed depth of excavation.

$L_1$ = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.

$L_2$ = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.

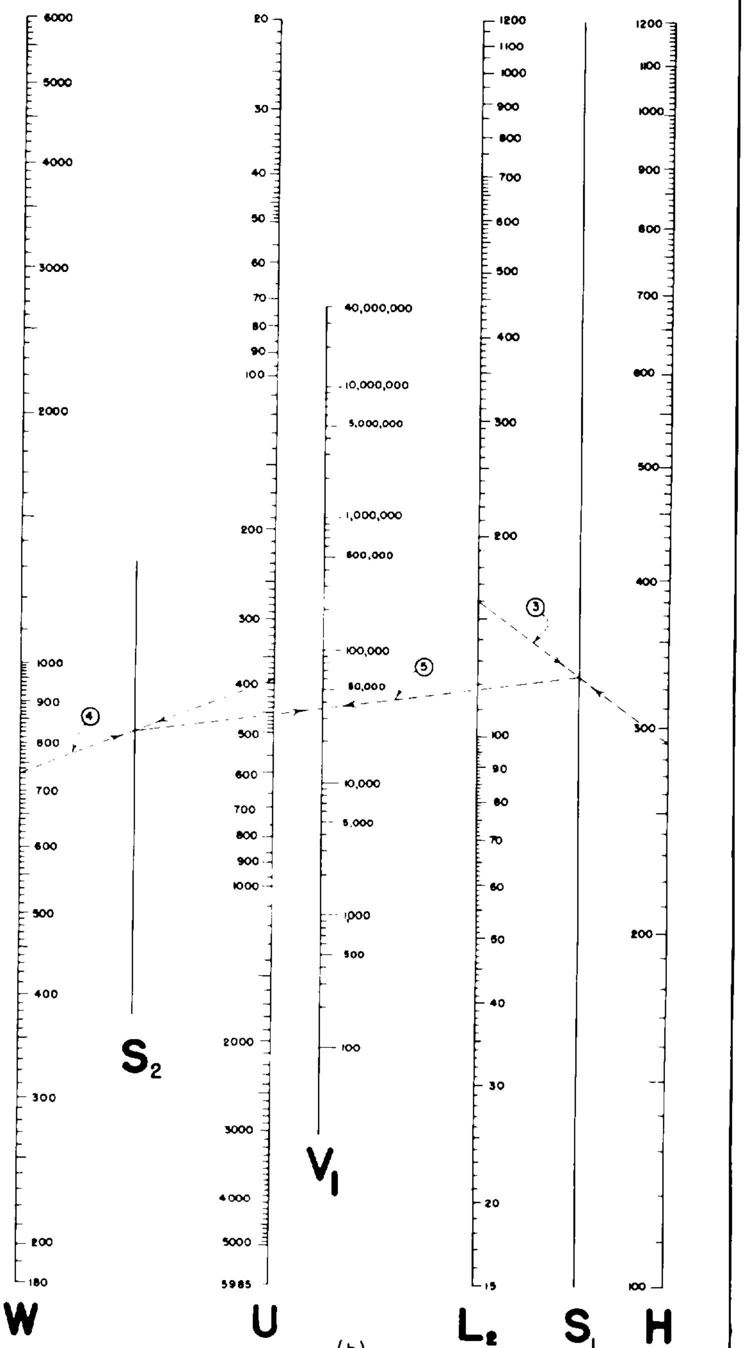
Procedure for using charts

On figure A-4(a)

1. Intersect U with a straight line from $L_1$ to $L_2$
2. Intersect W with a straight line from H to $L_1$

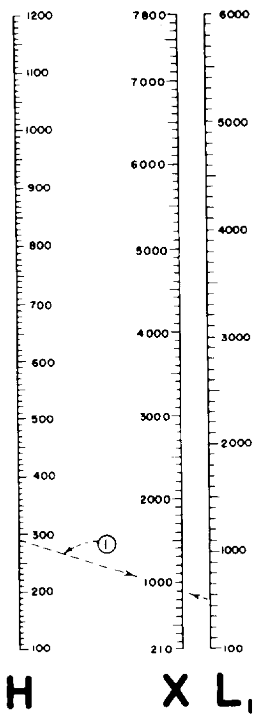
On figure A-4(b)

3. Intersect $S_1$ with a straight line from H to $L_2$
4. Intersect $S_2$ with a straight line from U to W (Values for U and W are obtained from figure A-4(a))
5. Intersect $V_1$ with a straight line from $S_1$ to $S_2$



(b)

*Figure A-4.* Nomograph for obtaining $V_1$. (*Note*: $V = V_1 + V_2$.) –288-D-3109



(a)

NOTES

Dimensions required for using charts.

H = Estimated structural height of dam from crest to base including assumed depth of excavation.

$L_1$ = Straight line distance at crest elevation between abutments including estimated excavation to sound rock.

$L_2$ = Straight line distance between abutments including estimated excavation to sound rock at 0.15H above the base.

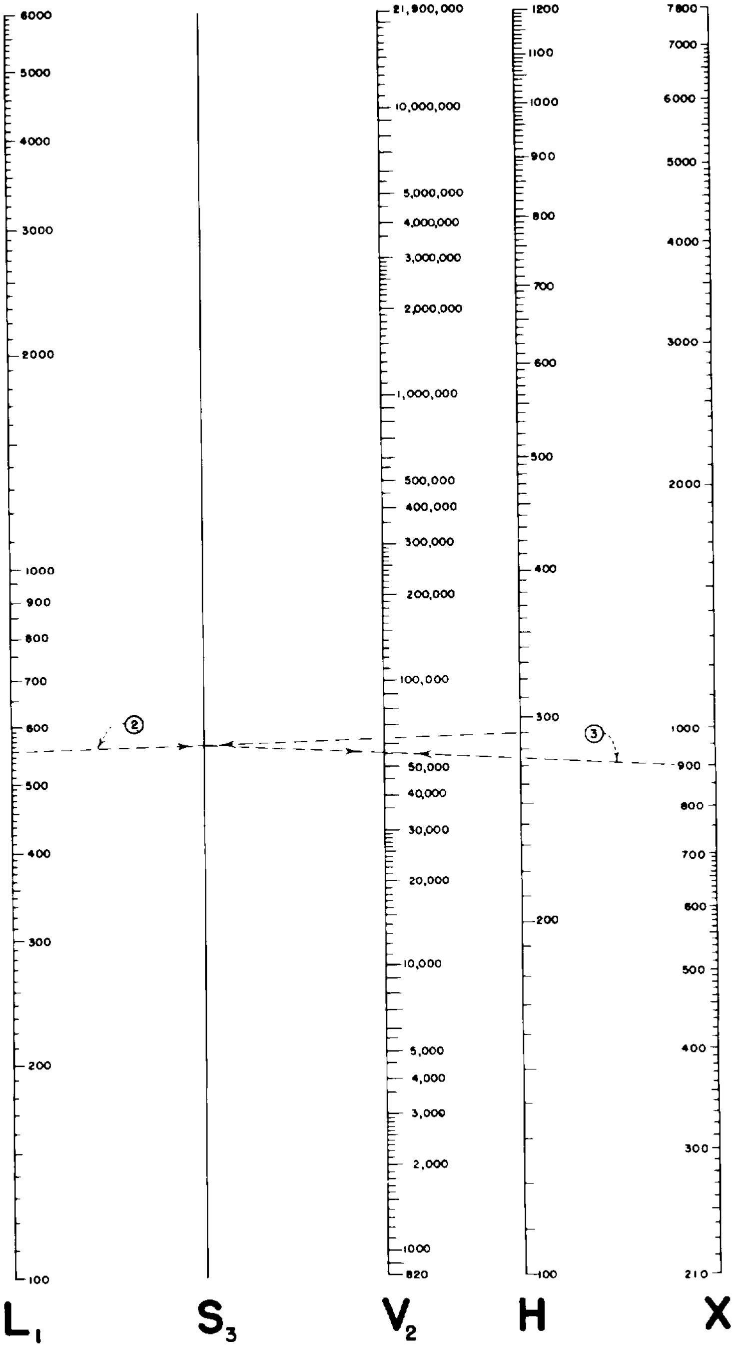
Procedure for using charts

On figure A-5(a)

1. Intersect X with a straight line from H to $L_1$

On figure A-5(b)

2. Intersect $S_3$ with a straight line from H to $L_1$
3. Intersect $V_2$ with a straight line from $S_3$ to X (Value for X is obtained from figure A-5(a))



(b)

*Figure A-5.* Nomograph for obtaining $V_2$. (*Note*: $V = V_1 + V_2$.) –288-D-3110

# Cantilever Computations for Initial and Unit Loads

B-1. *Design Assumptions for Monticello Dam.*—Monticello Dam (fig. B-1) was completed by the Bureau of Reclamation in 1957. It is chosen for illustrative purposes because it has uniform-thickness circular arches with fillets (fig. B-2).

The loading condition analyzed includes horizontal earthquake shock and normal reservoir water surface. Vertical earthquake effects were neglected in the study. The computations given in this appendix are taken from the complete analysis, and include the effects of horizontal earthquake and radial, tangential, and twist movements.

The assumptions made in this analysis are as given in section 4-25 supplemented or modified by the following:

(1) Tailwater effects are omitted from the analysis, since these effects would be small and their omission is on the side of safety.

(2) A construction and grouting program was included as follows:

*Stage I*

a. Dam was first to be built to elevation 300.

b. Contraction joints from foundation to elevation 300 were assumed to be grouted when the concrete had been cooled to a temperature of 45° F. with the reservoir empty.

c. It was assumed that concrete would be placed to the top of the dam, elevation 456, and the water surface raised to elevation 250 after the contraction joints were grouted below elevation 300.

d. A radial deflection analysis was made for the portion of the dam below elevation 300, including the effects of the reservoir waterload to elevation 250 and the weight of concrete above elevation 300. A temperature rise of 10° F. was assumed in the concrete after the contraction joints were grouted to elevation 300.

*Stage II*

a. It was assumed that contraction joints would be grouted from elevation 300 to elevation 456 with the reservoir water surface at elevation 250 and the concrete cooled to the required closure temperature of 55° F.

b. Reservoir water surface raised to elevation 440.

c. Arch and cantilever action assumed throughout entire dam.

d. A complete trial-load analysis was made for the entire dam including the effects of tangential shear and twist. The loading conditions included raising the water surface from elevation 250 to elevation 440 and horizontal earthquake effects.

(3) The maximum effect of horizontal earthquake during full-reservoir condition occurs when the maximum horizontal foundation acceleration is acting in an upstream direction. At this time the inertia force of the dam caused by the earthquake is acting in a downstream direction, the same as the waterload. Vertical earthquake effects are neglected.

(4) No additional vertical or horizontal

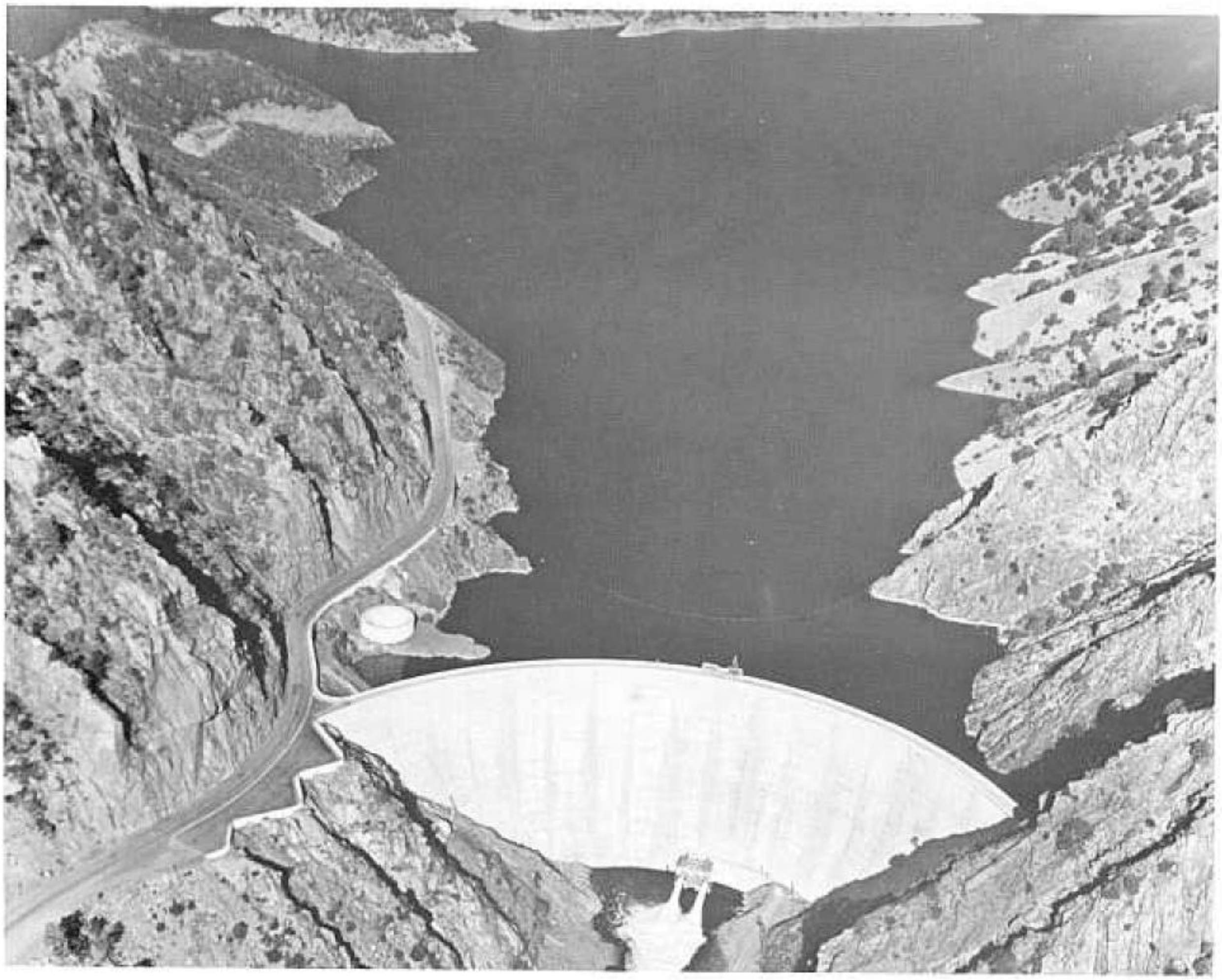


*Figure B-1.* **Monticello Dam.–SO-3994-R2**

loads due to possible accumulations of sand or silt are assumed to be imposed at the upstream face.

(5) Vertical cantilever elements are assumed to have radial sides as determined by vertical planes which are normal to the upstream face and 1 foot apart at the axis of the dam.

(6) Uplift pressure at the foundation rock and the horizontal joints in the mass concrete is assumed to have no detrimental effect on the stresses and deflections in the dam.

(7) The concrete is able to withstand whatever tensile stresses are imposed, and cracking does not occur in either the arch or cantilever elements.

(8) Normal arch and cantilever stresses have a linear variation from the upstream face to the downstream face at all elevations.

(9) Tangential cantilever shearing stresses have a linear variation from the upstream face to the downstream face at all elevations.

(10) The arch and cantilever stresses at the faces of the dam are acting in directions parallel to the faces; also the principal stresses are acting in the planes of the faces.

(11) The horizontal arch elements were assumed to be nonsymmetrical and nonsymmetrically loaded.

(12) An appropriate reduction in the

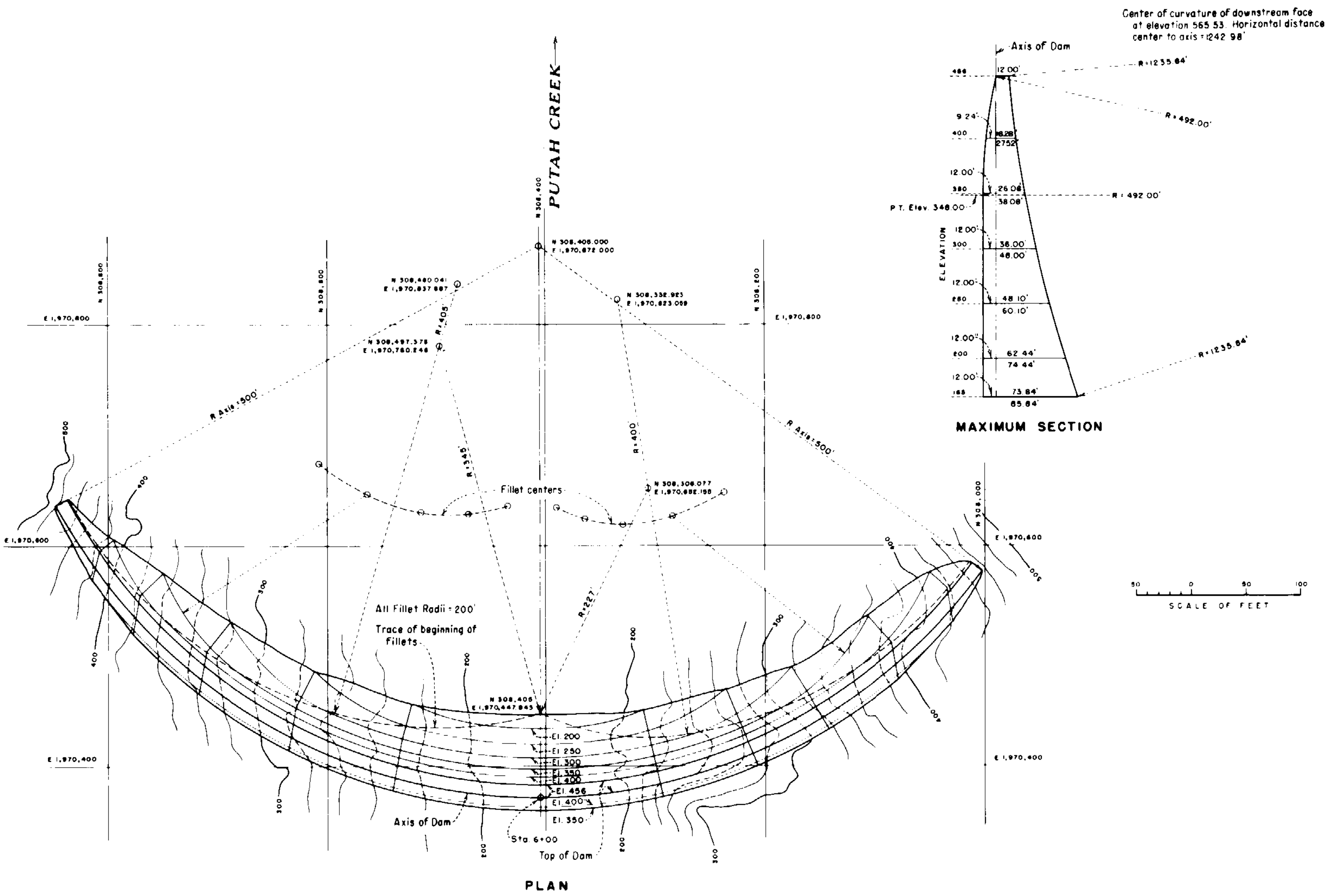


*Figure B-2.* **Layout of Monticello Dam.–413-D-101** modulus of elasticity of concrete in shear will compensate for the nonlinear variation of shearing stresses between the upstream and downstream faces of the dam.

The following additional constants were used in the analysis of Monticello Dam:

(1) Top of dam, elevation 456.

(2) Normal reservoir water surface, elevation 440.

(3) Unit weight of water, 62.5 pounds per cubic foot.

(4) Unit weight of concrete, 150 pounds per cubic foot.

(5) Modulus of elasticity of concrete in tension and compression, 2,500,000 pounds per square inch.

(6) Modulus of elasticity of concrete in shear, 1,041,700 pounds per square inch. This value was reduced to 833,330 pounds per square inch in calculating the detrusions caused by radial shears so as to allow for the nonlinear distribution of shearing stresses between the upstream and downstream faces of the dam.

(7) Modulus of elasticity of foundation and abutment rock in tension and compression, 1,300,000 pounds per square inch.

(8) Poisson's ratio for concrete, 0.20.

(9) Poisson's ratio for foundation and abutment rock, 0.03.

(10) Coefficient of thermal expansion of concrete, 0.000,005,6 per degree F.

**B-2. *Computations for Uncracked Cantilevers.***—The computation of reservoir pressures is shown on figure B-3. Following the usual procedure, the pressures were calculated in pounds per square foot to facilitate the use of basic unit triangular loads. Computations shown on figure B-3 include those for static pressure, *wh*; and for hydrodynamic pressure due to a horizontal earthquake, obtained from the equation shown on the figure. (This method of evaluating earthquake effects is no longer used by the Bureau of Reclamation.)

Concrete weights and moments were computed as indicated on figure B-4. The weights and moments of vertical waterload on the upstream face were computed by Simpson's rule as shown on the figure. On figure B-5 the horizontal concrete inertia effect of an assumed earthquake was computed omitting the angular functions. The concrete inertia effect was resolved into radial and tangential components as shown on figure B-6.

MONTICELLO DAM

W.S. Elevation = 440.00

$a$ = 0.1 Maximum acceleration Period = 1.0 sec.

$C_m$ = 0.735* $h$ = Distance below W.S. Elev.

$Z$ = Distance from W.S. Elev. to base of max section = 275'

$$C = \frac{C_m}{2}\left[\frac{h}{Z}\left(2-\frac{h}{Z}\right)+\sqrt{\frac{h}{Z}\left(2-\frac{h}{Z}\right)}\right]$$

$awZ$ = 1,719 $p_e = CawZ$

| ELEV. | $h$, ft. | $wh$, lb/ft.$^2$ | $\frac{h}{Z}(2-\frac{h}{Z})$ | $\sqrt{\frac{h}{Z}(2-\frac{h}{Z})}$ | $C$ | $p_e$, lb./ft.$^2$ | $wh+p_e$, lb/ft.$^2$ |
|---|---|---|---|---|---|---|---|
| 456 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 400 | 40 | 2,500 | .269,753 | .519,378 | .290,006 | 499 | 2,999 |
| 350 | 90 | 5,625 | .547,438 | .739,891 | .473,093 | 813 | 6,438 |
| 300 | 140 | 8,750 | .759,008 | .871,211 | .599,105 | 1,030 | 9,780 |
| 250 | 190 | 11,875 | .904,463 | .951,033 | .681,896 | 1,172 | 13,047 |
| 200 | 240 | 15,000 | .983,802 | .991,868 | .726,059 | 1,248 | 16,248 |
| 165 | 275 | 17,188 | 1.000,000 | 1.000,000 | .735,000 | 1,263 | 18,451 |

*Value obtained from figure 4-18 of "Design of Gravity Dams," Bureau of Reclamation, 1976.

*Figure B-3.* **Monticello Dam study—static pressure plus hydrodynamic effect of earthquake.—288-D-3111**

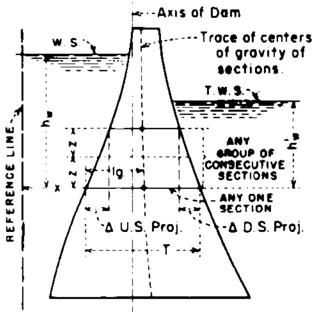
Computations for "odd" load are shown on figure B-7. Shears and moments due to unit loads have been calculated on figures B-8 and B-9 for construction stages I and II, respectively. Unit cantilever deflections due to unit radial loads are shown on figure B-10. It should be noted on figure B-10 that for obtaining the summation of $\left(\frac{M}{EI}\right)\left(\frac{\Delta Z}{2}\right)$, the values $M\alpha$ plus $V\alpha_2$ and $V\gamma$ plus $M\alpha_2$ are included at the base of the crown cantilever. The method of making the summation is explained on figure B-11. Radial crown-cantilever deflections due to unit radial loads at every elevation are tabulated on figure B-12.

Unit rotations due to unit twist loads, and unit tangential deflections due to unit tangential loads, are shown on figure B-13 for

| CONCRETE MASS | | | | | | | | | | | VERTICAL UPSTREAM WATER LOAD | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) Elev | (2) x | (3) x + lg | (4) K | (5) KA | (6) $W_c$ $W_c \cdot z \cdot \Sigma(5)$ | (7) (5)·(3) | (8) ${}_RM_c$ $W_c \cdot z \cdot \Sigma(7)$ | (9) $\frac{{}_RM_c}{W_c}$ | (10) $e_c$ (3)-(9) | (11) $M_c$ (6)·(10) | (12) $\frac{R_E h_w}{R_{AXIS}}$ | (12) $\left[\frac{R_E h_w}{R_{AXIS}}\right]$ AVERAGE | (13) ΔU.S. Proj. | (14) $\Delta W_w$ $W_w(13)\cdot(12_{AVE})$ | (15) $W_w$ | (16) k (2)·(12) | (17) ${}_RM_w$ $W_w\frac{(13)}{2}\Sigma(16)$ | (18) $\frac{{}_RM_w}{W_w}$ | (19) $e_w$ (3)-(18) | (20) $M_w$ (15)·(19) |
| 456 | 12.00 | 17.976 | $\frac{1}{3}$ | 3,952.0 | 0 | 71.041 | 0 | | | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | | | 0 |
| 428 | 6.56 | 16.621 | $\frac{4}{3}$ | 26.760 | | 444.78 | | | | | | 916.3 | | | | 8,014.6 | | | | |
| 400 | 2.76 | 16.393 | $\frac{1}{3}$ | 9.090.3 | 167,170 | 149.02 | 2,792,300 | | | -51,880 | 3,054.4 | | 5.91 | 5,415.3 | 5,415.3 | 2,810.0 | 31,987 | | | 56,786 |
| 375 | 0.74 | 17.145 | $\frac{4}{3}$ | 43.768 | | 750.40 | | | | | | 4,823.4 | | | | 4,759.1 | | | | |
| 350 | 0 | 18.795 | $\frac{1}{3}$ | 12.515 | 412,320 | 235.22 | 7,047,200 | | | 702,350 | 6,592.5 | | 2.76 | 13,313 | 18,728 | 0 | 42,492 | | | 309,560 |
| 325 | 0 | 21.079 | $\frac{4}{3}$ | 55.969 | | 1,179.8 | | | | | | 8,903.8 | | | | 0 | | | | |
| 300 | 0 | 23.607 | $\frac{1}{3}$ | 15.616 | 727,690 | 368.65 | 13,736,000 | | | 3,442,600 | 10,015 | | 0 | 0 | 18,728 | 0 | 42,432 | | | 399,680 |
| 275 | 0 | 26.470 | $\frac{4}{3}$ | 69.767 | | 1,846.7 | | | | | | | | | | 0 | | | | |
| 250 | 0 | 29.665 | $\frac{1}{3}$ | 19.461 | 1,120,900 | 577.31 | 24,208,000 | | | 9,043,500 | | | 0 | 0 | 18,728 | 0 | 42,432 | | | 513,130 |
| 225 | 0 | 33.707 | $\frac{4}{3}$ | 87.955 | | 2,964.7 | | | | | | | | | | 0 | | | | |
| 200 | 0 | 38.588 | $\frac{1}{3}$ | 25.000 | 1,617,400 | 964.70 | 41,109,000 | | | 21,303,000 | | | 0 | 0 | 18,728 | 0 | 42,432 | | | 680,240 |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | | | | | | |

| VERTICAL TAILWATER LOAD | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (21) Elev | (22) x | (23) x + lg | (24) K | (25) x + T | (26) $\frac{R_D}{R_A}$ | (27) $\frac{R_D h'_w}{R_A}$ | (27) $\left[\frac{R_D h'_w}{R_{AXIS}}\right]$ AVERAGE | (28) ΔD.S. Proj | (29) $\Delta W'_w$ $W_w(28)\cdot(27_{AVE})$ | (30) $W'_w$ | (31) k (25)·(27) | (32) ${}_RM'_w$ $W_w\frac{(28)}{2}\Sigma(31)$ | (33) $\frac{{}_RM'_w}{W'_w}$ | (34) (23)-(33) | (35) $M'_w$ (30)·(34) |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{4}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |
| | | | $\frac{1}{3}$ | | | | | | | | | | | | |

Tailwater, if any, not included



NOTES

(') Indicates values for tailwater
Subscripts c, w denotes concrete and water
w = Unit weight
W = Total weight above any section
M = Moment about center of gravity of section
${}_RM$ = Moment about reference line.
T = Thickness of section.

e = Eccentricity
x = Distance from reference line to upstream face
z = Difference in elevation between sections
$h_w$ = Head of water at any section
lg, A, $R_E$, $R_D$, $R_{AXIS}$, U.S. projection, and D.S. projection taken from computation Dwg. 205-D-1493

CANTILEVER COMPUTATION FORM
RADIAL OR PARALLEL SIDES
WEIGHTS AND MOMENTS
CONCRETE AND VERTICAL UPSTREAM
AND DOWNSTREAM WATER LOAD
PROJECT
DAM MONTICELLO
STUDY No. A-11 CANT. No. E
COMPUTED BY DATE
CHECKED BY DATE

*Figure B-4.* Monticello Dam study—weights and moments for cantilever *E* due to concrete and vertical upstream and downstream waterload.—DS2-1(35)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

CONCRETE INERTIA (Parallel to plane of centers)

CANTILEVER E By Date

| ELEV. | | | $z$ | $KAz$ | $^-0.1\,W_c = V_{ci}$ | $\Delta M_{ci}$ | $M_{ci}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 456 | | | 56 | 221.31 | 0 | 0 | 0 | | | | | | |
| | | | 28 | 749.28 | | | | | | | | | |
| 400 | | | 50 | 454.52 | -16,717 | -407,650 | -407,650 | | | | | | |
| | | | 25 | 1,094.2 | | | | | | | | | |
| 350 | | | 50 | 625.75 | -41,232 | -580,770 | -1,824,300 | | | | | | |
| | | | 25 | 1,399.2 | | | | | | | | | |
| 300 | | | 50 | 780.80 | -72,769 | -759,360 | -4,645,200 | | | | | | |
| | | | 25 | 1,744.2 | | | | | | | | | |
| 250 | | | 50 | 973.05 | -112,090 | -946,880 | -9,230,500 | | | | | | |
| | | | 25 | 2,198.9 | | | | | | | | | |
| 200 | | | | | -161,740 | -1,189,500 | -16,025,000 | | | | | | |

$-\lambda\, \omega_c \cdot z/2 = {}^-0.1 \times 150 \times 28 = -420$

$-\lambda\, \omega_c \cdot z/2 = {}^-0.1 \times 150 \times 25 = -375$

*Figure B-5.* Monticello Dam study–concrete inertia for cantilever *E*.–DS2-1(36)

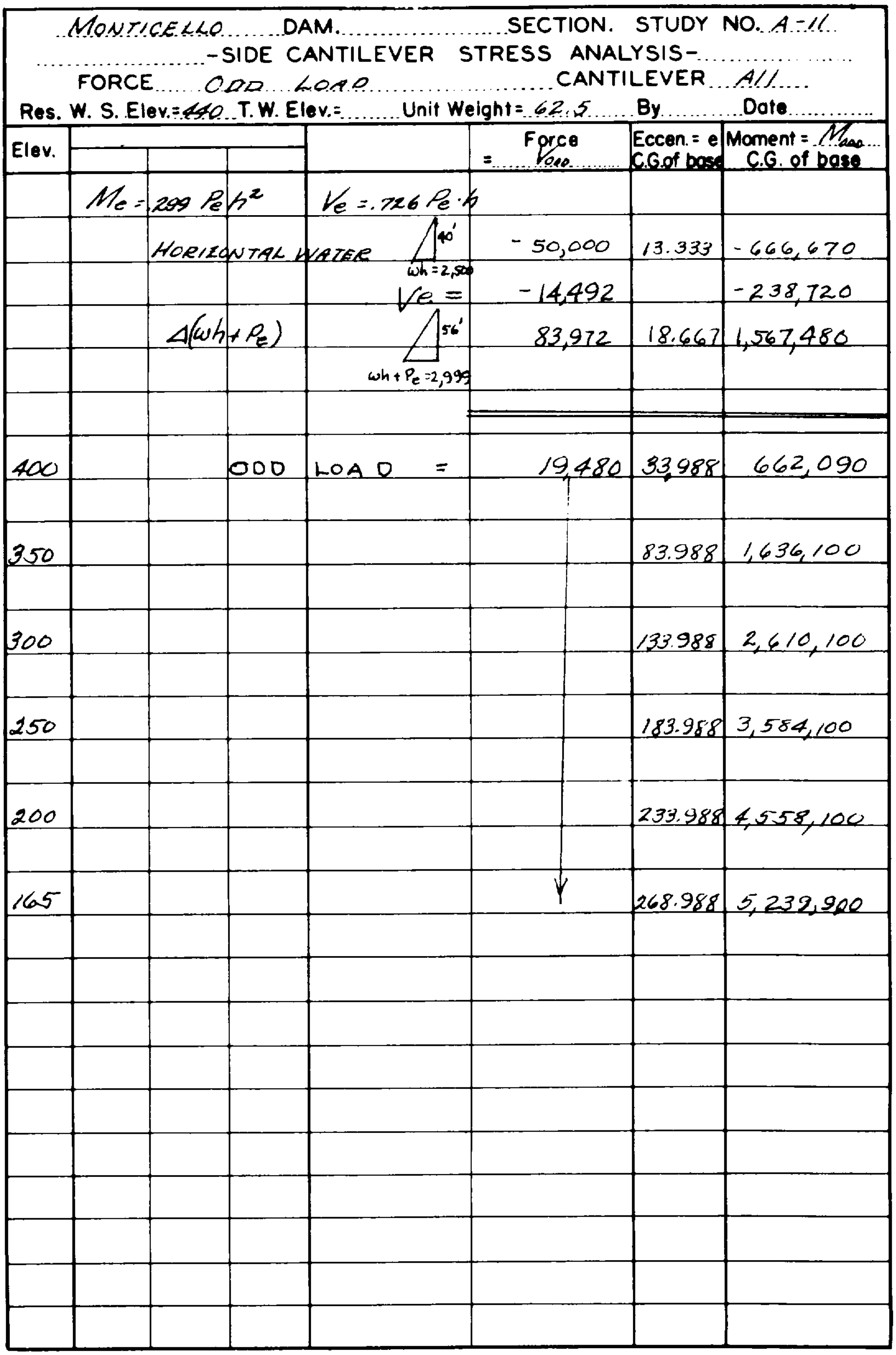
MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

RADIAL AND TANGENTIAL COMPANENTS OF CONCRETE INERTIA AND RADIAL BENDING MOMENTS DUE TO CONCRETE INERTIA FOR CANTILEVER E By Date

| | | RADIAL COMPONENT | | | | | | | TANGENTIAL COMPONENT | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ELEV. | | $\Phi_E$ | COS. $\Phi_E$ | $V_{ci}$ | $M_{ci}$ | | | | SIN $\Phi_E$ | $H_{ci}$ | | | |
| 456 | | 12° 00' | .978,147,6 | 0 | 0 | | | | .207,911,7 | 0 | | | |
| 400 | | | | -16,352 | -398,740 | | | | | +3,476 | | | |
| 350 | | | | -40,331 | -1,784,400 | | | | | +8,573 | | | |
| 300 | | | | -71,179 | -4,543,700 | | | | | +15,130 | | | |
| 250 | | | | -109,640 | -9,028,800 | | | | | +23,305 | | | |
| 200 | | ↓ | ↓ | -158,210 | -15,675,000 | | | | ↓ | +33,628 | | | |

Figure B-6. Monticello Dam study—components of concrete inertia for cantilever *E*.—DS2-1(37)



MONTICELLO DAM. SECTION. STUDY NO. A-11

-SIDE CANTILEVER STRESS ANALYSIS-

FORCE ODD LOAD CANTILEVER All

Res. W. S. Elev.= 440 T. W. Elev.= Unit Weight= 62.5 By Date

| Elev. | | | | | Force = $V_{odd}$ | Eccen. = e C.G. of base | Moment = $M_{odd}$ C.G. of base |
|---|---|---|---|---|---|---|---|
| | $M_e = .299 P_e h^2$ | | | $V_e = .726 P_e \cdot h$ | | | |
| | | HORIZONTAL WATER | | 40'<br>wh = 2,500 | - 50,000 | 13.333 | - 666,670 |
| | | | | $V_e =$ | - 14,492 | | - 238,720 |
| | | $\Delta(wh + P_e)$ | | 56' | 83,972 | 18.667 | 1,567,480 |
| | | | | $wh + P_e = 2,999$ | | | |
| | | | | | | | |
| 400 | | | ODD | LOAD = | 19,480 | 33.988 | 662,090 |
| | | | | | | | |
| 350 | | | | | | 83.988 | 1,636,100 |
| | | | | | | | |
| 300 | | | | | | 133.988 | 2,610,100 |
| | | | | | | | |
| 250 | | | | | | 183.988 | 3,584,100 |
| | | | | | | | |
| 200 | | | | | | 233.988 | 4,558,100 |
| | | | | | | | |
| 165 | | | | | | 268.988 | 5,239,900 |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

*Figure B-7.* Monticello Dam study–odd load.–DS2-1(38)

MONTICELLO DAM. SECTION. STUDY NO. A-11

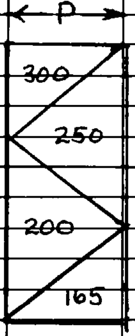
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

SHEARS AND MOMENTS DUE TO UNIT RADIAL LOADS FOR STAGE I

CROWN CANTILEVER By Date

| ELEV / LOAD |  | $R_E / R_{AXIS}$ |  | 250 | 200 | 165 | ELEV. |
|---|---|---|---|---|---|---|---|
| 300 |  | 1.024,00 | V | -25,600 | -25,600 | -25,600 | 300 |
|  |  |  | M | -853,330 | -2,133,300 | -3,029,300 |  |
| 250 |  | 1.024,00 | V | -25,600 | -51,200 | -51,200 | 250 |
|  |  |  | M | -426,670 | -2,560,000 | -4,352,000 |  |
| 200 |  | 1.024,00 | V |  | -25,600 | -43,520 | 200 |
|  |  |  | M |  | -426,670 | -1,740,800 |  |
| 165 |  | 1.024,00 | V |  |  | -17,920 | 165 |
|  |  |  | M |  |  | -209,070 |  |

$P \, \text{lb/ft}^2 = 1000 \, \text{lb/ft}^2 \cdot \frac{R_E}{R_{Axis}}$



Note: Moments and shears are computed for only that part of each individual load above the elevation under consideration. Shears are in pounds, moments in foot-pounds. Unit pressure P = 1000 pounds per square foot × $R_E/R_{Axis}$ to compensate for the change in area from the axis to the upstream face.

*Figure B-8.* Monticello Dam study—shears and moments in crown cantilever due to radial loads for stage I.—DS2-1(39)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

SHEARS AND MOMENTS DUE TO UNIT RADIAL LOADS FOR STAGE II

CROWN CANTILEVER By Date

| LOAD \ ELEV. | | $\frac{R_E}{R_{AXIS}}$ | | 400 | 350 | 300 | 250 | 200 | 165 | ELEV | P lb./ft.² = 1000 lb./ft.² · $\frac{R_E}{R_{AXIS}}$ ELEV. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ~~456~~ | | ~~1.0~~ | V | -28,000 | -28,000 | -28,000 | -28,000 | -28,000 | -28,000 | ~~456~~ | | | |
| | | | M | -1,045,300 | -2,445,300 | -3,845,300 | -5,245,300 | -6,645,300 | -7,625,300 | | | | |
| ~~400~~ | | ~~1.018,48~~ | V | -28,517 | -53,979 | -53,979 | -53,979 | -53,979 | -53,979 | ~~400~~ | | | |
| | | | M | -532,340 | -2,806,900 | -5,505,900 | -8,204,900 | -10,904,000 | -12,793,000 | | | | |
| ~~350~~ | | ~~1.024,00~~ | V | | -25,600 | -51,200 | -51,200 | -51,200 | -51,200 | ~~350~~ | | | |
| | | | M | | -426,670 | -2,560,000 | -5,120,000 | -7,680,000 | -9,472,000 | | | | |
| ~~300~~ | | ~~1.024,00~~ | V | | | -25,600 | -51,200 | -51,200 | -51,200 | ~~300~~ | | | |
| | | | M | | | -426,670 | -2,560,000 | -5,120,000 | -6,912,000 | | | | |
| ~~250~~ | | ~~1.024,00~~ | V | | | | -25,600 | -51,200 | -51,200 | ~~250~~ | | | |
| | | | M | | | | -426,670 | -2,560,000 | -4,352,000 | | | | |
| ~~200~~ | | ~~1.024,00~~ | V | | | | | -25,600 | -43,520 | ~~200~~ | | | |
| | | | M | | | | | -426,670 | -1,740,800 | | | | |
| ~~165~~ | | ~~1.024,00~~ | V | | | | | | -17,920 | ~~16~~ | | | |
| | | | M | | | | | | -209,070 | | | | |

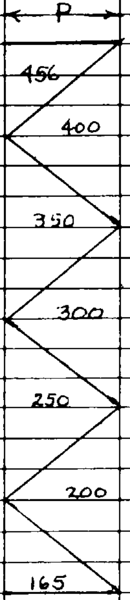


Figure B-9. Monticello Dam study—shears and moments in crown cantilever due to unit radial loads for stage II.—DS2-1(40)

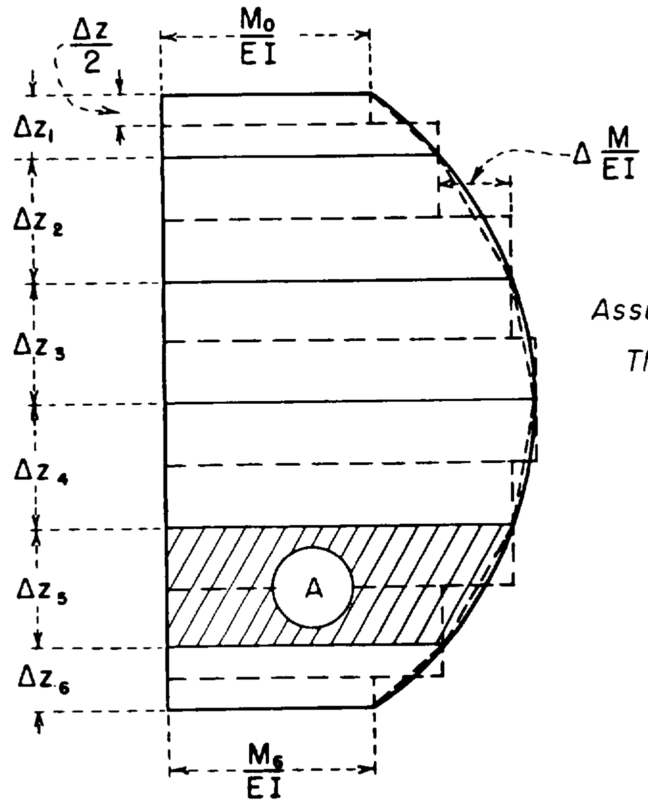
# CANTILEVER STRESS ANALYSIS

$\alpha = .0^{9}004,342,841$
$\alpha_2 = .0^{9}058,372,74$
$\gamma = .0^{6}014,291,93$
$\textcircled{\gamma} =$ ______ $\delta =$ ______

MONTICELLO DAM ______ SECTION STUDY NO. A-11
RADIAL SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
RADIAL DEFLECTION UNIT RADIAL LOAD NO. 400 CANTILEVER CROWN
$\frac{1}{E} = .0^{6}002,777,777,8$ $\frac{K_G}{G} = \frac{1.25}{G} = .0^{6}008,333,333$ By ______ Date ______

| Elev. | Deflection due to ______ | | | | | | Deflection due to ______ | | | | | Total Deflection |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Moment = M | $\frac{1}{EI}$ | $\frac{M}{EI}$ | $\frac{\Delta z}{2}$ | $\Sigma\left(\frac{M}{EI}\right)\frac{\Delta z}{2}$ | $\Sigma\frac{\Delta z}{2}\left(\Sigma\frac{M\Delta z}{EI\,2}\right)$ | Horizontal Force = V | $\frac{1}{A}$ | $\frac{V}{A}$ | $\left(\frac{K_G}{G}\right)\frac{\Delta z}{2}$ | $\Sigma\left(\frac{V}{A}\right)\left(\frac{K_G\Delta z}{G\,2}\right)$ | |
| 456 | 0 | | | | $-.0^{3}392,88$ | -.068,635 | 0 | | | | -.003,816 | -.072,451 |
| | | | | 28.0 | | | | | | $.0^{6}233,33$ | | |
| 400 | -532,340 | $.0^{9}001,614,3$ | $-.0^{6}859,36$ | | $-.0^{3}368,82$ | -.047,307 | -28,517 | .036,669 | -1,045.7 | | -.003,572 | -.050,879 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 350 | -2,806,900 | $.0^{12}612,58$ | $-.0^{3}001,719,5$ | | $-.0^{3}304,35$ | -.030,478 | -53,979 | .026,635 | -1,437.7 | | -.003,054 | -.033,532 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 300 | -5,505,900 | $.0^{12}309,06$ | $-.0^{3}001,701,7$ | | $-.0^{3}218,82$ | -.017,399 | -53,979 | .021,346 | -1,152.2 | | -.002,515 | -.019,914 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 250 | -8,204,900 | $.0^{12}159,51$ | $-.0^{3}001,308,8$ | | $-.0^{3}143,56$ | -.008,339 | -53,979 | .017,262 | -931.79 | | -.002,081 | -.010,420 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 200 | -10,904,000 | $.0^{12}085,275$ | $-.0^{6}929,84$ | | $-.0^{3}087,592$ | -.002,560 | -53,979 | .014,147 | -763.64 | | -.001,727 | -.004,287 |
| | | | | 17.5 | | | | | | $.0^{6}145,83$ | | |
| 165 | -12,793,000 | $.0^{12}056,331$ | $-.0^{6}720,64$ | | $-.0^{3}058,709$ | 0 | -53,979 | .012,418 | -670.31 | | -.001,518 | -.001,518 |
| | | | | | $M\alpha + V\alpha_2$ | | | | | | $M\alpha_2 + V\gamma$ | |

Figure B-10. Monticello Dam study—radial deflection of crown cantilever due to a unit radial load.—DS2-1(41)



*Assumptions:*

*This method assumes that $\frac{M}{EI}$ varies as a straight line between elevations and is an approximate method. For greater accuracy use a larger number of elements.*

$$\textit{Area, } Ⓐ = \left[\left(\frac{M_5}{EI}\right)\bullet\left(\frac{\Delta z_5}{2}\right) + \left(\frac{M_4}{EI}\right)\bullet\left(\frac{\Delta z_5}{2}\right)\right]$$

$$\textit{Total area} = \sum_{i=1}^{n}\left[\left(\frac{M}{EI}\right)_i + \left(\frac{M}{EI}\right)_{i-1}\right]\frac{\Delta z_i}{2} \quad n = \text{No. of elements}$$

*Note: The ordinate, $\frac{M}{EI}$, may be replaced by the ordinate $\left[\left(\frac{M}{EI}\right)\left(\frac{\Delta z}{2}\right)\right]$ and the area, which equals deflection, is obtained by summation as above.*

*Figure B-11.* **Explanation of mechanical integration.–288-D-3112**

the crown cantilever of Monticello Dam. Following the usual procedure, basic unit triangular loads were multiplied by the adjustment factor $r_o/R_{axis}$.

Shears and moments due to initial loads on cantilever *E* for stage II are listed on figure B-14. Because the upstream face of Monticello Dam is not vertical, there is a vertical waterload. Radial deflections of cantilever *E* due to initial stage II loads are calculated on figure B-15. Figure B-16 shows the initial tangential deflections for cantilever *E*.

Radial shears at the base of the cantilevers were tabulated as on figure B-17, so that

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
RADIAL CANTILEVER DEFLECTIONS DUE TO UNIT RADIAL LOADS
CROWN CANTILEVER By Date

| ELEV. \ LOAD | 456 | 400 | 350 | 300 | 250 | 200 | 165 |
|---|---|---|---|---|---|---|---|
| 456 | -.050,969 | -.033,464 | -.020,852 | -.011,999 | -.006,145 | -.002,475 | $-.0^{3}845$ |
| 400 | -.072,451 | -.050,879 | -.033,532 | -.019,914 | -.010,420 | -.004,287 | -.001,518 |
| 350 | -.041,021 | -.031,234 | -.022,518 | -.014,395 | -.007,892 | -.003,391 | -.001,285 |
| 300 | -.023,394 | -.018,612 | -.014,343 | -.010,043 | -.005,978 | -.002,742 | -.001,135 |
| 250 | -.012,069 | -.009,892 | -.007,949 | -.006,006 | -.004,014 | -.002,092 | $-.0^{3}986$ |
| 200 | -.004,724 | -.003,976 | -.003,307 | -.002,639 | -.001,971 | -.001,250 | $-.0^{3}724$ |
| 165 | $-.0^{3}926$ | $-.0^{3}805$ | $-.0^{3}697$ | $-.0^{3}589$ | $-.0^{3}481$ | $-.0^{3}373$ | $-.0^{3}268$ |

***Figure B-12.*** **Monticello Dam study—radial deflections of crown cantilever due to unit radial loads.—DS2-1(42)**

# CANTILEVER STRESS ANALYSIS

α = ..........

$\alpha_2$ = ..........

γ = ..........

MONTICELLO DAM LEFT SECTION STUDY NO. A-11

ROTATION AND RADIAL SIDE CANTILEVER STRESS ANALYSIS TRIAL LOAD

TANGENTIAL DEFLECTION TWIST AND TANGENTIAL LOAD NO. 400 CANTILEVER CROWN

Ⓣ = $.0^{6}001,594,318$ δ = $.0^{9}004,481,936$ $\frac{1}{E}$ = .......... $\frac{K}{G} = \frac{10}{G} = .0^{6}006,666,666,7$ By .......... Date ..........

| Elev. | Deflection due to .......... | | | | | | Deflection due to .......... | | | | | Total Deflection |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Moment = ~~M~~ | 1 / 2GI | ~~M~~ / 2GI | Δz / 2 | Σ(~~M~~ / 2GI)(Δz / 2) | ~~Σ(Δz / 2)(~~M~~Δz / EI2)~~ | Horizontal Force = H | 1 / A | H / A | (K / G)(Δz / 2) | Σ(H / A)(KΔz / G 2) | |
| | | | | | | | | | | | | |
| 456 | 0 | | | | $-.0^{3}006,781,7$ | | 0 | | | | .001,872,1 | |
| | | | | 28.0 | | | | | | $.0^{6}186,67$ | | |
| 400 | -27,747 | $.0^{9}001,937,2$ | $-.0^{6}053,751$ | | $-.0^{3}005,276,7$ | | -27,747 | .036,669 | -1,017.5 | | .001,682,1 | |
| | | | | 25.0 | | | | | | $.0^{6}166,67$ | | |
| 350 | -52,521 | $.0^{12}735,10$ | $-.0^{6}038,608$ | | $-.0^{3}002,967,7$ | | -52,521 | .026,635 | -1,398.9 | | .001,279,4 | |
| | | | | 25.0 | | | | | | $.0^{6}166,67$ | | |
| 300 | -52,521 | $.0^{12}370,87$ | $-.0^{6}019,478$ | | $-.0^{3}001,515,5$ | | -52,521 | .021,346 | -1,121.1 | | $.0^{3}859,4$ | |
| | | | | 25.0 | | | | | | $.0^{6}166,67$ | | |
| 250 | -52,521 | $.0^{12}191,41$ | $-.0^{6}010,053$ | | $-.0^{6}777,3$ | | -52,521 | .017,262 | -906.62 | | $.0^{3}521,4$ | |
| | | | | 25.0 | | | | | | $.0^{6}166,67$ | | |
| 200 | -52,521 | $.0^{12}102,93$ | $-.0^{6}005,374,5$ | | $-.0^{6}391,6$ | | -52,521 | .014,147 | -743.01 | | $.0^{3}246,5$ | |
| | | | | 17.5 | | | | | | $.0^{6}116,67$ | | |
| 165 | -52,521 | $.0^{12}067,597$ | $-.0^{6}003,550,3$ | | $-.0^{6}235,4$ | | -52,521 | .012,418 | -652.21 | | $.0^{3}083,7$ | |
| | | | | | | | | | | | | |
| | | | | | ~~M~~ δ | | | | | | −H Ⓣ | |

*Figure B-13.* Monticello Dam study–rotational and tangential movements of crown cantilever due to unit cantilever loads.–DS2-1(43)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

RADIAL SHEARS AND MOMENTS DUE TO INITIAL LOADS FOR STAGE II

CANTILEVER E By Date

| ELEV. | | | $M_{ci}$ | * $M_{YW}$ | $M_{ODD}$ | STAGE II $M_i$ | | | | $V_{ci}$ | $V_{ODD}$ | STAGE II $V_i$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | | 0 | 0 | 0 | 0 | | | | 0 | 0 | 0 | |
| 400 | | | -398,740 | 56,786 | 662,090 | 320,140 | | | | -16,352 | 19,480 | 3,128 | |
| 350 | | | -1,784,400 | 309,560 | 1,636,100 | 161,260 | | | | -40,331 | ↓ | -20,851 | |
| 300 | | | -4,543,700 | 399,680 | 2,610,100 | -1,533,900 | | | | -71,179 | ↓ | -51,699 | |
| 250 | | | -9,028,800 | 513,130 | 3,584,100 | -4,931,600 | | | | -109,640 | ↓ | -90,160 | |
| 200 | | | -15,675,000 | 680,240 | 4,558,100 | -10,437,000 | | | | -158,210 | ↓ | -138,730 | |

*NOTE: INCLUDES THE VERTICAL EFFECTS OF HORIZONTAL EARTHQUAKE

*Figure B-14.* Monticello Dam study–radial shears and moments in cantilever *E* due to initial loads for stage II.–DS2-1(45)

# CANTILEVER STRESS ANALYSIS

$\alpha = .0^{9}004,432,138$
$\alpha_2 = .0^{9}047,712,84$
$\gamma =$
$\textcircled{T} =$ $\delta =$

MONTICELLO DAM SECTION STUDY NO. A-11
RADIAL - SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
RADIAL DEFLECTION INITIAL RAD. LOAD NO. STAGE II CANTILEVER E
$\frac{1}{E} = .0^{6}002,777,777,8$ $\frac{K_o}{G} = \frac{1.25}{G} = .0^{6}008,333,333$ By Date

| Elev. | Deflection due to | | | | | | Deflection due to | | | | | Total Deflection |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Moment = M | $\frac{1}{EI}$ | $\frac{M}{EI}$ | $\frac{\Delta z}{2}$ | $\Sigma\left(\frac{M}{EI}\right)\frac{\Delta z}{2}$ | $\Sigma\frac{\Delta z}{2}\left(\Sigma\frac{M}{EI}\frac{\Delta z}{2}\right)$ | Horizontal Force = V | $\frac{1}{A}$ | $\frac{V}{A}$ | $\left(\frac{K_o}{G}\right)\frac{\Delta z}{2}$ | $\Sigma\left(\frac{V}{A}\right)\left(\frac{K_o}{G}\frac{\Delta z}{2}\right)$ | |
| | | | | | | | | | | | | |
| 456 | 0 | | 0 | | $-.0^{3}101,08$ | -.027,426 | 0 | | 0 | | -.001,669 | -.029,095 |
| | | | | 28.0 | | | | | | $.0^{6}233,33$ | | |
| 400 | 320,140 | $.0^{9}001,614,3$ | $.0^{6}516,80$ | | $-.0^{3}115,55$ | -.021,361 | 3,128 | .036,669 | 114.70 | | -.001,696 | -.023,057 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 350 | 161,260 | $.0^{12}612,58$ | $.0^{6}098,785$ | | $-.0^{3}130,94$ | -.015,198 | -20,851 | .026,635 | -555.37 | | -.001,604 | -.016,802 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 300 | -1,533,900 | $.0^{12}309,06$ | $-.0^{6}474,07$ | | $-.0^{3}121,55$ | -.008,886 | -51,699 | .021,346 | -1,103.6 | | -.001,259 | -.010,145 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 250 | -4,931,600 | $.0^{12}155,68$ | $-.0^{6}767,75$ | | $-.0^{3}090,509$ | -.003,585 | -90,160 | .017,129 | -1,544.4 | | $-.0^{3}707$ | -.004,292 |
| | | | | 25.0 | | | | | | $.0^{6}208,33$ | | |
| 200 | -10,437,000 | $.0^{12}070,664$ | $-.0^{6}737,52$ | | $-.0^{3}052,877$ | 0 | -138,730 | .013,333 | -1,849.7 | | 0 | 0 |
| | | | | | | | | | | | | |
| | | | | | $M\alpha + V\alpha_2$ | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |

*Figure B-15.* Monticello Dam study—radial deflections of cantilever *E* due to initial radial loads for stage II.—DS2-1(46)

# CANTILEVER STRESS ANALYSIS

$\alpha$ =

$\alpha_2$ =

$\gamma$ =

Ⓣ =

$\delta$ =

MONTICELLO DAM LEFT SECTION STUDY NO. A-11

RADIAL SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

TANGENTIAL DEFLECTION INITIAL TANGENTIAL LOAD ~~NO~~ CANTILEVER E

$\frac{1}{E} = .0^{6}002,777,777,8$ $\frac{K_s}{G} = \frac{1.0}{G} = .0^{6}006,666,666,7$ By Date

| Elev. | Deflection due to | | | | | | Deflection due to | | | | | Total Deflection |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ~~Moment = M~~ | ~~$\frac{I}{EI}$~~ | ~~$\frac{M}{EI}$~~ | ~~$\frac{\Delta z}{2}$~~ | ~~$\Sigma\left(\frac{M}{EI}\right)\frac{\Delta z}{2}$~~ | ~~$\Sigma\frac{\Delta z}{2}\left(\Sigma\frac{M\Delta z}{EI\,2}\right)$~~ | Horizontal Force = H | $\frac{1}{A}$ | $\frac{H}{A}$ | $\left(\frac{K_s}{G}\right)\frac{\Delta z}{2}$ | $\Sigma\left(\frac{H}{A}\right)\left(\frac{K_s\Delta z}{G\,2}\right)$ | |
| 456 | | | | | | | 0 | | | | $-.0^{3}437$ | |
| | | | | | | | | | | $.0^{6}186,67$ | | |
| 400 | | | | | | | 3,476 | .036,669 | 127.46 | | $-.0^{3}413$ | |
| | | | | | | | | | | $.0^{6}166,67$ | | |
| 350 | | | | | | | 8,573 | .026,635 | 228.34 | | $-.0^{3}354$ | |
| | | | | | | | | | | $.0^{6}166,67$ | | |
| 300 | | | | | | | 15,130 | .021,346 | 322.96 | | $-.0^{3}262$ | |
| | | | | | | | | | | $.0^{6}166,67$ | | |
| 250 | | | | | | | 23,305 | .017,129 | 399.19 | | $-.0^{3}141$ | |
| | | | | | | | | | | $.0^{6}166,67$ | | |
| 200 | | | | | | | 33,628 | .013,333 | 448.36 | | 0 | |

*Figure B-16.* Monticello Dam study–tangential movements of cantilever *E* due to initial tangential loads.–DS2-1(47)

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD STAGE II

RADIAL SHEARS AT BASES OF CANTILEVERS FOR COMPUTATION OF CONCENTRATED RADIAL ARCH LOADS (INITIAL AND UNIT RADIAL LOADS) By Date

| CANT. LOAD. | $\frac{R_{AXIS}}{r_a}\tan\psi$ | 1.126,739 | 1.656,935 | 1.755,675 | 1.733,735 | 1.890,354 | | 2.560,196 | 2.087,987 | 1.846,834 | 1.473,867 | 1.036,197 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | CROWN | F | G | H | I | J |
| INITIAL | | +6,469 | -27,517 | -84,875 | -154,810 | -262,250 | | -348,290 | -173,620 | -77,589 | -17,053 | +8,898 |
| 456 | | -31,549 | -46,394 | -49,159 | -48,545 | -52,930 | | -71,685 | -58,464 | -51,711 | -41,268 | -29,014 |
| 400 | | -32,131 | -89,440 | -94,770 | -93,585 | -102,040 | NOT NECESSARY | -138,200 | -112,710 | -99,690 | -79,558 | -29,549 |
| 350 | | | -42,418 | -89,891 | -88,767 | -96,786 | | -131,080 | -106,900 | -94,558 | -37,731 | |
| 300 | | | | -44,945 | -88,767 | -96,786 | | -131,080 | -106,900 | -47,279 | | |
| 250 | | | | | -44,384 | -96,786 | | -131,080 | -53,452 | | | |
| 200 | | | | | | -48,393 | | -65,541 | | | | |
| 165 | | | | | | | | | | | | |

Note; Stage I computed similarly

Figure B-17. Monticello Dam study–radial shears at bases of cantilevers for stage II.–DS2-1(48) concentrated radial arch loads could be calculated by means of the values of $\frac{R_{axis}}{r_a} \tan \psi$, which are also shown. This is explained in the last two paragraphs of section 4-30(d). Values of shears shown were obtained from figures B-9 and B-14.

Tangential shears and twisting moments at the base of the cantilevers are shown on figure B-18. These were obtained as indicated by figure B-6 and were used to compute concentrated tangential and twist loads on the arches.

Cantilever load ordinates in kips are given on figure B-19. By using these ordinates, an equivalent cantilever load due to a triangular arch load is easily computed. For a uniform arch load, of course, all ordinates are equal to unity.

B-3. *Design Assumptions for Clear Creek Dam.*—The cracked cantilever analysis of Clear Creek Dam is used as a numerical example. Dimensions of the dam are shown on figure B-20. In the analysis, six sample arches and six cantilevers were studied.

The analysis was made for one set of load conditions. Stresses were computed for the complete adjustment including effects of tangential shear and twist. The loading conditions analyzed were those of an extreme flood without earthquake shock but with an assumed uplift. Cracked cantilevers and cracked arches were analyzed. The assumptions made in this analysis are as given in section 4-25 supplemented or modified by the following:

(1) Temperature changes are uniform throughout each arch.

(2) Uplift pressures vary as a straight line from full reservoir pressure at the upstream face of the dam to zero or tailwater pressure at the downstream face.

(3) Full uplift pressures act over the horizontal cracked area of the base, varying to zero or tailwater pressure at the downstream face.

(4) Reservoir water surface is 3 feet above the crest, or at elevation 3018.

(5) Effects of silt, tailwater, ice load, and earthquake, if any, are not included.

(6) Vertical cantilever elements are assumed to have sides radial to the upstream face 1 foot apart at the axis of the dam.

(7) Arch computations are based on the assumption that the concrete cracks, and total compressive stresses are computed for cracked arches.

(8) Tension is assumed to be relieved in the cantilevers by cracking of the concrete to the point of zero stress at the point of maximum tension. The resultant redistribution of load reduces the tensions at other points. The compressive cantilever stresses have a linear variation from the point of zero stress to a maximum stress at the uncracked face.

(9) At the upstream and downstream faces, the arch and cantilever stresses are acting in directions parallel to the face.

(10) Normal arch and cantilever stresses and tangential cantilever shearing stresses have a linear variation at all elevations.

(11) All arches are circular, or are composed of circular arcs, and are subject to analysis as symmetrical arches.

(12) Arch action begins immediately since there are no vertical joints.

The following constants were used in the analysis of Clear Creek Dam:

(1) Unit weight of water, 62.5 pounds per cubic foot.

(2) Unit weight of concrete, 150 pounds per cubic foot.

(3) Modulus of elasticity of concrete in tension and compression, 3,000,000 pounds per square inch.

(4) Modulus of elasticity of foundation and abutment rock in tension and compression, 2,000,000 pounds per square inch.

(5) Poisson's ratio for concrete, 0.20.

(6) Poisson's ratio for foundation and abutment rock, 0.05.

(7) Coefficient of thermal expansion of concrete, 0.000,005,6 per degree F.

B-4. *Computations for Cracked Cantilever.*—Only those computations for a cracked cantilever which differ from those for

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD STAGE II
TANGENTIAL SHEARS AND TWISTING MOMENTS AT BASES OF CANTILEVERS
FOR COMPUTATION OF CONCENTRATED TANGENTIAL AND TWIST LOADS ON ARCHES By.......... Date..........

| LOAD \ CANT. | $\frac{R_{AXIS}}{r_a}$ tan ψ | 1.126,739 | 1.656,935 | 1.755,675 | 1.733,735 | 1.890,354 | | 2.560,196 | 2.087,987 | 1.846,834 | 1.473,867 | 1.036,197 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | CROWN | F | G | H | I | J |
| INITIAL | (TANG.) | +17,808 | +51,978 | +71,547 | +76,189 | +63,569 | | +114,170 | +113,940 | +91,965 | +50,825 | +16,120 |
| 456 | | -31,170 | -45,837 | -48,569 | -47,962 | -52,295 | | -70,825 | -57,762 | -51,091 | -40,773 | -28,665 |
| 400 | | -30,825 | -86,910 | -92,210 | -91,057 | -99,283 | NOT NECESSARY | -134,460 | -109,660 | -96,998 | -77,368 | -28,493 |
| 350 | | | -40,008 | -86,393 | -85,466 | -93,187 | | -126,210 | -102,930 | -90,890 | -35,703 | |
| 300 | | | | -42,020 | -84,395 | -92,249 | | -124,940 | -101,790 | -44,187 | | |
| 250 | | | | | -41,199 | -91,058 | | -123,390 | -49,598 | | | |
| 200 | | | | | | -44,641 | | -60,354 | | | | |
| 165 | | | | | | | | | | | | |

*Figure B-18.* Monticello Dam study–tangential shears and twisting moments at bases of cantilevers for stage II.–DS2-1(49)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD STAGE II
ORDINATES AT CANTILEVER POINTS FOR TRIANGULAR ARCH LOADS
(IN KIPS) By Date

| ELEV. | LOAD | | A | B | C | D | E | CROWN | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | 2 | | .649,12 | .087,72 | 0 | | | | | | 0 | .157,89 | .649,12 |
| | 3 | | .824,56 | .543,86 | .192,98 | 0 | | | | 0 | .263,16 | .578,95 | .824,56 |
| | 4 | | .883,04 | .695,91 | .461,99 | .251,46 | .017,54 | 0 | 0 | .251,46 | .508,77 | .719,30 | .883,04 |
| | 5 | | .912,28 | .771,93 | .596,49 | .438,60 | .263,16 | .052,63 | .228,07 | .438,60 | .631,58 | .789,47 | .912,28 |
| 400 | 2 | | 1.0 | .384,62 | 0 | | | | | | 0 | .461,54 | 1.0 |
| | 3 | | 1.0 | .692,31 | .307,69 | 0 | | | | 0 | .384,62 | .730,77 | 1.0 |
| | 4 | | 1.0 | .794,87 | .538,46 | .307,69 | .051,28 | 0 | 0 | .307,69 | .589,74 | .820,51 | 1.0 |
| | 5 | | 1.0 | .846,15 | .653,85 | .480,77 | .288,46 | .057,69 | .250,00 | .480,77 | .692,31 | .865,38 | 1.0 |
| 350 | 3 | | | 1.0 | .285,71 | 0 | | | | 0 | .307,69 | 1.0 | |
| | 4 | | | 1.0 | .661,02 | .355,93 | .016,95 | 0 | 0 | .298,25 | .684,21 | 1.0 | |
| | 5 | | | 1.0 | .777,78 | .577,78 | .355,56 | .088,89 | .272,73 | .545,45 | .795,45 | 1.0 | |
| 300 | 3 | | | | 1.0 | .485,71 | 0 | | 0 | .371,43 | 1.0 | | |
| | 4 | | | | 1.0 | .657,14 | .276,19 | 0 | .123,81 | .580,95 | 1.0 | | |
| | 5 | | | | 1.0 | .742,86 | .457,14 | .114,29 | .342,86 | .685,71 | 1.0 | | |
| 250 | 3 | | | | | 1.0 | .200,00 | 0 | .040,000 | 1.0 | | | |
| | 5 | | | | | 1.0 | .600,00 | .120,00 | .520,00 | 1.0 | | | |
| 200 | 5 | | | | | | 1.0 | .142,86 | 1.0 | | | | |

*Figure B-19.* Monticello Dam study–ordinates at cantilever points for triangular arch loads.–DS2-1(50)

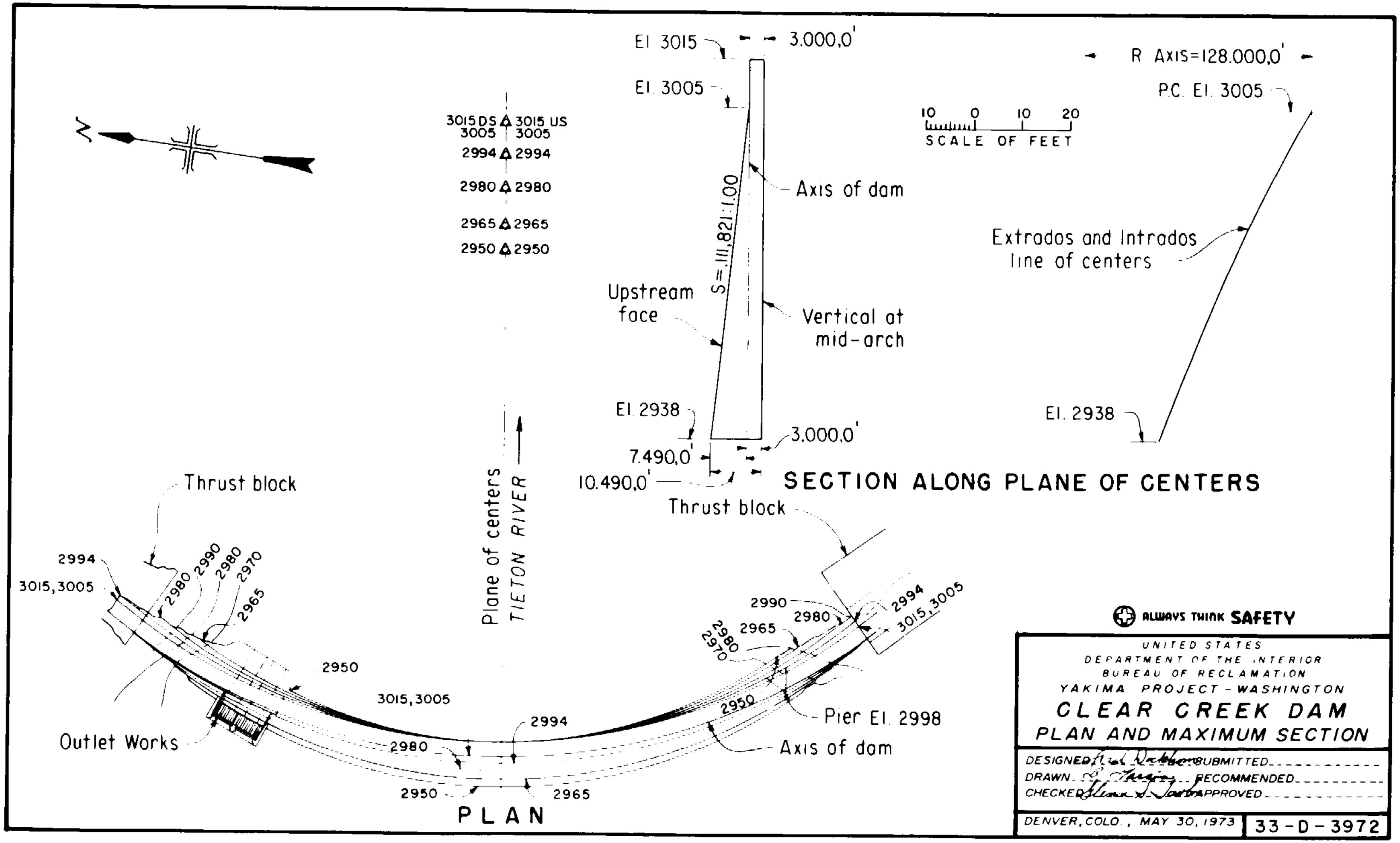


*Figure B-20.* Layout of Clear Creek Dam.

an uncracked cantilever are given on figures B-21 through B-23. These are computations for cantilever *E* in Clear Creek Dam. Since $\frac{R_D}{R_o}$ is greater than $\frac{R_D}{R_{axis}}$, cracking is assumed at the upstream face. It is also assumed that cracking of the upstream face occurs only at elevation 2940 and that other possible tensions on that face will be relieved by the redistribution of load resulting from the inclusion of the effects of this crack.

The necessary data for the above computations are given on figures B-24 to B-27, inclusive. The values of $V$ shown in column 3 of figure B-21 are shears due to load factors multiplied by the unit shears obtained from figure B-27 plus the initial shear on the cantilever. The values of $(\Sigma M + M_u)$ are obtained by adding the moment due to cantilever loads to the values for $M_i$ on figure B-26. Since the moment due to uplift at the cracked section varies, the $M_i$ is obtained by adding the moment due to changing uplift to $M_c$ and $M_{vw}$. The cantilever properties in figure B-21 are computed for the uncracked portion.

Radial deflections and stresses for cantilever *E* are shown on figures B-21 to B-23. Values of $\alpha$ and $\alpha_2$ are calculated as indicated in the top left-hand corner of the figure. $(M_1\ \alpha + V\alpha_2)$ at elevation 2940 is calculated and included in the summation of $\left[\frac{M_1}{E_c\ I_1}\right]\ \left[\frac{\Delta z}{2}\right]$ as shown. Since the total deflections include the deflection due to concrete weight and cracking, deflections due to concrete weight should normally be subtracted from the total values shown to obtain the net deflection due to the trial loads. However, in this case, deflections due to concrete weight were included in the adjustment since the dam was constructed without vertical contraction joints and arch action began as soon as concrete was placed.

Computations for $U$ and $M_u$ are given on figures B-25 and B-26 as illustrations, since they were not shown previously.

CLEAR CREEK DAM STUDY NO. 6

RADIAL-SIDE CRACKED CANTILEVER STRESS ANALYSIS

HORIZONTAL FORCES, RESULTANT MOMENTS, AND PROPERTIES. CANTILEVER E

Trial load No.____________ By__________ Date____________

| | Loads, Forces, Moments, and Eccentricities | | | | | Properties of Cracked Cantilever at Horizontal Sections | | | | | | | | | Resultant Moment | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cracked at U.S. Face | | V | $M_p$ | $\Sigma M + M_u$ | $e = \frac{\Sigma M + M_u}{\Sigma W + U}$ | $\frac{R_s}{R_{axis}}$ | $\frac{R_E - l_s + e}{R_D}$ | $\frac{R_D}{R_0}$ | $\frac{R_D}{R_{axis}} \div \frac{R_D}{R_0}$ | $\left(\frac{R_0}{R_{axis}} - \frac{R_D}{R_{axis}}\right)_{R_{axis}}$ | $\frac{A_1}{T_1(R_s/R_{axis})}$ | | $\frac{I_1}{T_1^3(R_s/R_{axis})}$ | | $\frac{M_1}{(\Sigma W + U)T_1}$ | |
| Cracked at D S Face | | V | $M_p$ | $\Sigma M + M_u$ | $e = \frac{\Sigma M + M_u}{\Sigma W + U}$ | $\frac{R_E}{R_{axis}}$ | $\frac{l_s - e}{R_E}$ | $\frac{R_0}{R_E}$ | $\frac{R_E}{R_{axis}} \times \frac{R_0}{R_E}$ | $\left(\frac{R_E}{R_{axis}} - \frac{R_0}{R_{axis}}\right)_{R_{axis}}$ | $\frac{A_1}{T_1(R_E/R_{axis})}$ | | $\frac{I_1}{T_1^3(R_E/R_{axis})}$ | | $\frac{M_1}{(\Sigma W + U)T_1}$ | |
| Elev | Load fact | Horizontal force | Moment of hor force | Total bending moment | Eccentricity | | F | | $R_o/R_{axis}$ | $T_1$ | | $A_1$ | | $I_1$ | | $M_1$ |
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 3015 | | 0 | | 0 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 3005 | | 2,855 | | +18,900 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 2994 | | 2,230 | | +39,688 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 2980 | | 4,039 | | +78,453 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 2965 | | 4,010 | | +174,141 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 2950 | | -24,513 | | +150,294 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| 2940 | | -71,289 | | -199,571 | -4.775 | .964,56 | 1.004,72 | .986,08 | .978,18 | 1.337,09 | .993,05 | 1.298,82 | .082,752 | .193,499 | -.167,06 | -9,337 |

FINAL RADIAL

SHEET 1 OF 2

*Figure B-21.* Clear Creek Dam study, radial-side cracked cantilever stress analysis–horizontal forces, resultant moments, and properties for cantilever *E*.–DS2-1(55)

For crack at U.S Face, $\frac{r}{r_1} = \frac{r}{R_0 + T_1/2}$
For crack at D.S Face, $\frac{r}{r_1} = \frac{r}{R_0 - T_1/2}$
$\left(\frac{r}{r_1} \cdot \frac{T_1^2}{T_1^4}\right)\alpha$ = .0$^{6}$,021,266,74
$\left(\frac{r}{r_1} \cdot \frac{T}{T_1}\right)\alpha_2$ = .0$^{6}$,003,295,532$\left(\frac{r}{r_1}\right)\gamma$ =

CLEAR CREEK DAM STUDY NO. 6
RADIAL-SIDE CRACKED CANTILEVER STRESS ANALYSIS
RADIAL DEFLECTIONS AND STRESSES. TRIAL LOAD NO. 2 CANTILEVER E
$\frac{1}{E_c}$ = .0$^{6}$,002,314,814,8 $\frac{K_s}{G}\frac{1.25}{G}$ = .0$^{6}$,006,944,444,4 By Date

| ELEV. | Radial Deflection | | | | | | | | Stresses at Uncracked Face Pounds per Square Inch | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Due to Bending | | | | Due to Shear | | | Total | | | | | |
| | $\frac{M_1}{E_cI_1}$ | $\frac{\Delta z}{2}$ | $\Sigma\left(\frac{M_1}{E_cI_1}\right)\frac{\Delta z}{2}$ | $\Sigma\frac{\Delta z}{2}\left(\Sigma\frac{M_1}{E_cI_1}\frac{\Delta z}{2}\right)$ | $\frac{V}{A_1}$ | $\left(\frac{K}{G}\right)\frac{\Delta z}{2}$ | $\Sigma\left(\frac{V}{A_1}\right)\left(\frac{K_s}{G}\frac{\Delta z}{2}\right)$ | | $\frac{\sigma_z}{(\Sigma W+U)/A_1}$ | Vertical | Tan $\phi$ | Sec$^2$ $\phi$ | Parallel to Face |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 3015 | 0 | | ‾.0$^{3}$,198,52 | .046,658 | 0 | | ‾.001,893 | ‾.048,55 | | | | | |
| | | 5.0 | | | | .0$^{6}$,034,722 | | | | | | | |
| 3005 | .0$^{3}$,019,675 | | ‾.0$^{3}$,296,90 | ‾.044,181 | 963 | | ‾.001,927 | ‾.046,11 | | | | | |
| | | 5.5 | | | | .0$^{6}$,038,194 | | | | | | | |
| 2994 | .0$^{3}$,014,682 | | ‾.0$^{3}$,485,86 | ‾.039,876 | 531 | | ‾.001,984 | ‾.041,86 | | | | | |
| | | 7.0 | | | | .0$^{6}$,048,611 | | | | | | | |
| 2980 | .0$^{3}$,011,224 | | ‾.0$^{3}$,667,21 | ‾.031,805 | 699 | | ‾.002,044 | ‾.033,85 | | | | | |
| | | 7.5 | | | | .0$^{6}$,052,083 | | | | | | | |
| 2965 | .0$^{3}$,011,548 | | ‾.0$^{3}$,838,00 | ‾.020,516 | 535 | | ‾.002,108 | ‾.022,62 | | | | | |
| | | 7.5 | | | | .0$^{6}$,052,083 | | | | | | | |
| 2950 | .0$^{3}$,005,391,0 | | ‾.0$^{3}$,965,04 | ‾.006,993 | ‾2648 | | ‾.001,998 | ‾.008,99 | | | | | |
| | | 5.0 | | | | .0$^{6}$,034,722 | | | | | | | |
| 2940 | ‾.0$^{3}$,111,698 | | ‾.0$^{3}$,433,50 | 0 | ‾54,888 | | 0 | 0 | .013,921 | 448.0 | | | |
| | | | | | | | | | | | | | |
| | For El. 2940, | | $M\alpha + V\alpha_2$ | | ARCH ABUTMENT MOVEMENT TO BE ADDED AT EACH ELEVATION | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | FINAL RADIAL | | | | | | | |

SHEET 2 OF 2

Figure B-22. Clear Creek Dam study, radial-side cracked cantilever stress analysis—radial deflections and stresses for cantilever E.—DS2-1(56)

# CANTILEVER STRESS ANALYSIS

CLEAR CREEK DAM SECTION STUDY NO. 6
-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD (CRACKED CANT.)
STRESS CALCULATIONS. CANTILEVER E
Res. W. S. Elev. 3018 Tailwater Elev. By Date

| Elev. | $\frac{I}{A}$ | Upstr. $\frac{c}{I}=\frac{lg}{I}$ | Downstr. $\frac{c'}{I}=\frac{T-lg}{I}$ | $\frac{W}{A}$ | $\frac{Mc}{I}$ | $\frac{Mc'}{I}$ | Stresses - Pounds per Square Foot: Upstream Face: External Pressure | Vertical $\frac{W}{A}+\frac{Mc}{I}$ | Tan $\phi_E$ | $\sec^2\phi_E$ | Parallel to Face | Downstream Face: External Pressure | Vertical $\frac{W}{A}-\frac{Mc'}{I}$ | Tan $\phi_D$ | $\sec^2\phi_D$ | Parallel to Face |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3015 | .337,28 | .671,85 | .677,25 | 0 | 0 | 0 | 188 | 0 | 0 | 1,0 | 0 | | 0 | 0 | 1.0 | 0 |
| 3005 | .337,28 | .671,85 | .677,25 | 1,500 | 13,098 | 13,203 | 812 | 14,598 | 0 | 1.0 | 14,598 | | ¯11,703 | 0 | 1.0 | ¯11,703 |
| 2994 | .238,28 | .336,08 | .339,92 | 2,787 | 14,071 | 14,232 | 1,500 | 16,858 | .104,62 | 1.010,95 | 17,026 | | ¯11,445 | .007,215,6 | 1.000,05 | ¯11,446 |
| 2980 | .172,97 | .177,57 | .180,66 | 4,334 | 15,078 | 15,340 | 2,375 | 19,412 | .104,48 | 1.010,92 | 19,598 | | ¯11,006 | .007,318,2 | 1.000,05 | ¯11,007 |
| 2965 | .133,27 | .105,83 | .108,26 | 5,943 | 20,011 | 20,471 | 3,312 | 25,954 | .104,33 | 1.010,88 | 26,200 | | ¯14,528 | .007,533,3 | 1.000,06 | ¯14,529 |
| 2950 | .108,05 | .069,840 | .071,948 | 7,525 | 12,498 | 12,875 | 4,250 | 20,023 | .104,23 | 1.010,86 | 20,194 | | ¯5,350 | .007,546,7 | 1.000,06 | ¯5,350 |
| 2940 | .095,773 | .055,047 | .056,967 | CRACKED AT THIS ELEVATION | | | | | | | 0 | | 64,512 | .007,600 | 1.000,06 | 64,516 |

*Figure B-23.* Clear Creek Dam study, cracked cantilever stress analysis–stresses for cantilever *E*.–DS2-1(57)

# CANTILEVER STRESS ANALYSIS

CLEAR CREEK DAM. STUDY NO. 6

RADIAL-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

DESCRIPTION OF CANTILEVER E

By .......... Date ..........

Properties of Cantilever at Horizontal Sections

| Elev. | T | U.S. Proj. | D.S. Proj. | $R_{axis}$ | $R_E$ | r | $R_D$ | $\frac{R_E}{R_{axis}}$ | $\frac{R_D}{R_U}$ | $\frac{A}{T(\frac{R_E}{R_{axis}})}$ | A | $\frac{I_g}{T}$ | $I_g$ | $T-I_g$ | $\frac{I}{T^3(\frac{R_E}{R_{axis}})}$ | I | $\frac{1}{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3015 | 3.000 | 0 | 3.000 | 128.000 | 128.000 | | 125.000 | 1.0 | | | 2,964,9 | | 1.494,1 | 1.505,9 | | 2.223,71 | .449,70 |
| | 3.000 | 0 | 3.000 | | | | | | | | | | | | | | |
| 3005 | 3.000 | 0 | 3.000 | 128.000 | 128.000 | | 125.000 | 1.0 | | | 2.964,9 | | 1.494,1 | 1.505,9 | | 2.223,71 | .449,70 |
| | 3.615 | 0.577 | 3.038 | | | | | | | | | | | | | | |
| 2994 | 4.230 | 1.152 | 3.078 | 121.971 | 123.123 | | 118.893 | 1.009,45 | | | 4.196,7 | | 2.102,7 | 2.127,3 | | 6.257,28 | .159,81 |
| | 5.013 | 1.885 | 3.128 | | | | | | | | | | | | | | |
| 2980 | 5.796 | 2.616 | 3.180 | 114.907 | 117.523 | | 111.727 | 1.022,77 | | | 5.781,3 | | 2.873,4 | 2.922,6 | | 16.179,0 | .061,808 |
| | 6.634 | 3.400 | 3.234 | | | | | | | | | | | | | | |
| 2965 | 7.473 | 4.182 | 3.291 | 108.054 | 112.236 | | 104.763 | 1.038,71 | | | 7.503,7 | | 3.693,5 | 3.779,5 | | 34.905,8 | .028,649 |
| | 8.312 | 4.965 | 3.347 | | | | | | | | | | | | | | |
| 2950 | 9.150 | 5.747 | 3.403 | 101.901 | 107.648 | | 98.498 | 1.056,39 | | | 9.265,3 | | 4.507,4 | 4.642,6 | | 64.533,5 | .015,496 |
| | 9.709 | 6.268 | 3.441 | | | | | | | | | | | | | | |
| 2940 | 10.268 | 6.789 | 3.479 | 98.178 | 104.967 | | 94.699 | 1.069,15 | | | 10.441,4 | | 5.046,2 | 5.221,8 | | 91.664,3 | .010,909 |

*Figure B-24.* Clear Creek Dam study, cracked cantilever stress analysis–description of cantilever *E*.–DS2-1(58)

## CANTILEVER STRESS ANALYSIS

$e_u = I_g - T/3$

$U = wh/2 \cdot A$

$M_u = U \cdot e$

CLEAR CREEK DAM. SECTION. STUDY NO. 6

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

Uplift and Moment at sections where cracking does not occur

Cantilever E By Date

| ELEV. | T | wh | A | U | e | $M_u$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3015 | | Top of | dam | | | | | | | | | | |
| 3005 | 3.000 | 812 | 2.965 | -1,203.8 | .494 | | | | | | | | |
| 2994 | 4.230 | 1,500 | 4.197 | -3,147.8 | .693 | | | | | | | | |
| 2980 | 5.796 | 2,375 | 5.781 | -6,864.9 | .941 | | | | | | | | |
| 2965 | 7.473 | 3,312 | 7.504 | -12,426.6 | 1.203 | | | | | | | | |
| 2950 | 9.150 | 4,250 | 9.255 | -19,666.9 | 1.457 | | | | | | | | |
| 2940 | 10.268 | 4,875 | 10.441 | -Cracking at base- | | | | | | | | | |
| | | | Uplift and Moment at base using final cracked section | | | | | | | | | | |
| | wh | $T-T_1$ | $U_1$ | $e_1$ | $M_{u1}$ | $I_g$ | $T_1$ | $U_2$ | $e_2$ | $M_{u2}$ | | $\Sigma U$ | $\Sigma M_u$ |
| 2940 | 4,875 | 8.930,9 | -44,568 | .646,9 | -28,831 | 5.046,2 | 1.337,09 | -3,173.3 | -4.329,4 | -13,738 | | -47,741 | -30,830 |

*Figure B-25.* Clear Creek Dam study, cracked cantilever stress analysis—uplift and moment in cantilever *E* due to uplift pressure.—DS2-1(59)

# CANTILEVER STRESS ANALYSIS

CLEAR CREEK DAM. SECTION. STUDY NO. 6

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

Weights and Moments

Cantilever E By. Date

| ELEV. | | $\Sigma W$ | $U$ | $\Sigma W+U$ | | | $M_c$ | $M_{vw}$ | $M_u$ | $M_i$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3015 | | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | | | |
| 3005 | | 4,447 | -1,204 | 3,243 | | | 0 | 0 | -595 | -595 | | | |
| 2994 | | 11,695 | -3,148 | 8,547 | | | -3,933 | 2,124 | -2,181 | -3,990 | | | |
| 2980 | | 25,055 | -6,865 | 18,190 | | | -14,551 | 7,540 | -6,460 | -13,471 | | | |
| 2965 | | 44,591 | -12,427 | 32,164 | | | -35,418 | 17,972 | -14,949 | -32,395 | | | |
| 2950 | | 69,641 | -19,667 | 49,974 | | | -69,090 | 34,671 | -28,655 | -63,074 | | | |
| 2940 | | 89,469 | — Varies | with cracking | | | -100,251 | 50,051 | — Varies | with cracking | | | |

*Figure B-26.* Clear Creek Dam study, cracked cantilever stress analysis—weights and moments for cantilever *E*.—DS2-1(60)

## CANTILEVER STRESS ANALYSIS

CLEAR CREEK DAM. SECTION. STUDY NO. 6

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

Cantilever Shears and Moments due to Unit Radial Loads

CANTILEVER E By Date

| ELEV.<br>LOAD | | | 3005 | 2994 | 2980 | 2965 | 2950 | 2940 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | V | -5,000 | -5,000 | -5,000 | -5,000 | -5,000 | -5,000 | | | | | |
| 3015 | | M | -33,333 | -88,333 | -158,333 | -233,333 | -308,333 | -358,333 | | | | | |
| | | V | -5,000 | -10,500 | -10,500 | -10,500 | -10,500 | -10,500 | | | | | |
| 3005 | | M | -16,666 | -112,000 | -259,000 | -416,500 | -574,000 | -679,000 | | | | | |
| | | | V | -5,552 | -12,618 | -12,618 | -12,618 | -12,618 | | | | | |
| 2994 | | | M | -20,357 | -164,035 | -353,307 | -542,579 | -668,760 | | | | | |
| | | | | V | -7,159 | -14,830 | -14,830 | -14,830 | | | | | |
| 2980 | | | | M | -33,411 | -217,509 | -439,962 | -588,264 | | | | | |
| | | | | | V | -7,790 | -15,581 | -15,581 | | | | | |
| 2965 | | | | | M | -38,951 | -233,708 | -389,515 | | | | | |
| | | | | | | V | -7,923 | -13,205 | | | | | |
| 2950 | | | | | | M | -39,615 | -154,057 | | | | | |
| | | | | | | | V | -5,346 | | | | | |
| 2940 | | | | | | | M | -17,819 | | | | | |

*Figure B-27.* Clear Creek Dam study, cracked cantilever stress analysis—shears and moments in cantilever *E* due to unit radial loads.–DS2-1(61).

# <<Appendix C

# Arch Computations for Initial and Unit Loads

## A. UNIFORM-THICKNESS CIRCULAR ARCH

C-1. *Introduction.*—These sections discuss the application of computation forms in analyzing a uniform-thickness circular arch. The general procedure outlined is applicable to any arch analysis which may be undertaken. Calculations given as examples are taken from analyses of Monticello Dam.

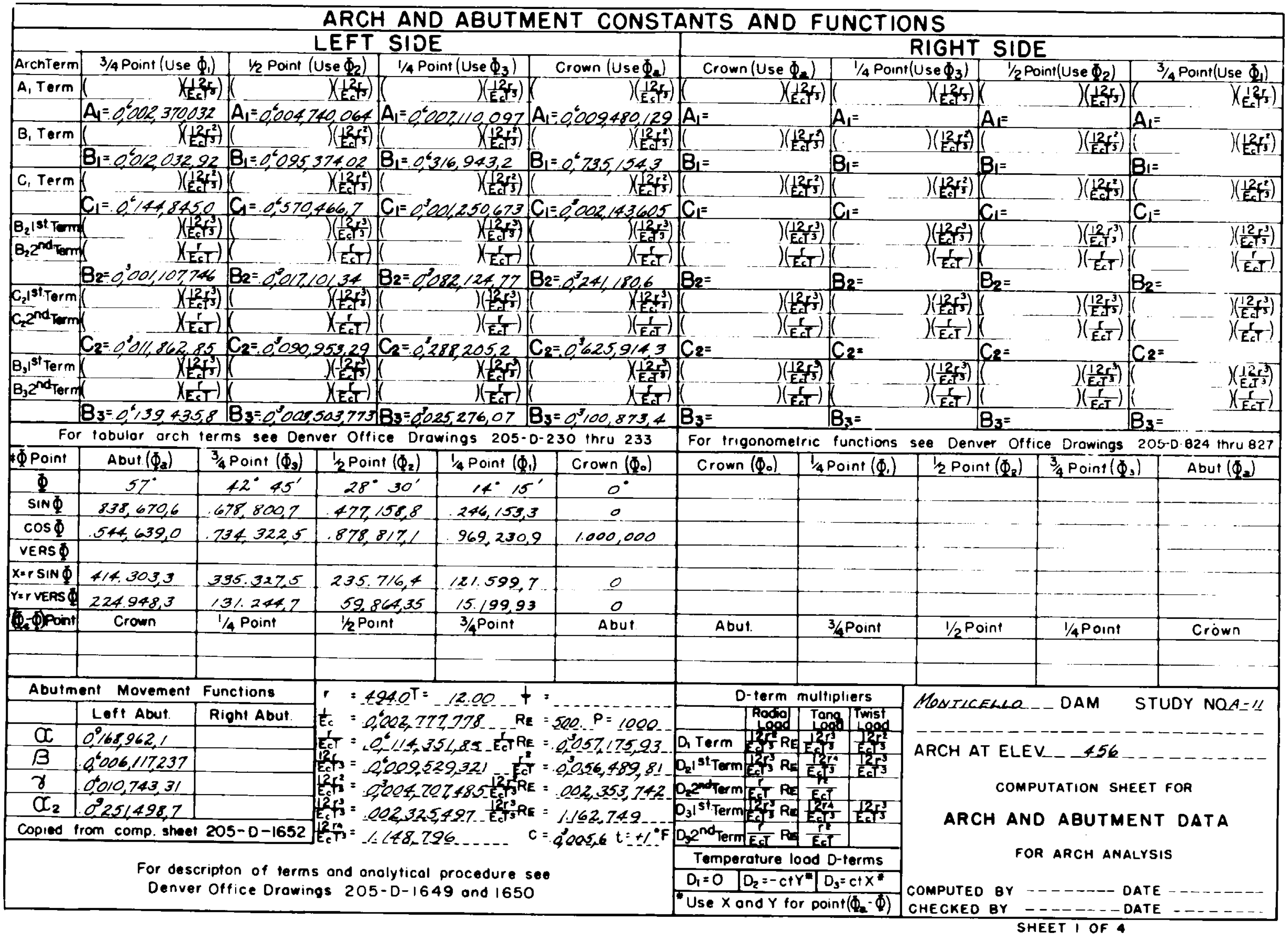
C-2. *Computation Forms.*—General computation forms consist of four sheets as shown on figures C-1 to C-4, inclusive. These are adaptable to the solution of any circular arch carrying either symmetrical or nonsymmetrical unit loads. The forms are directly applicable to a nonsymmetrical circular arch with nonsymmetrical loading. For a symmetrical arch or a symmetrical load, certain quantities for the left and right sides of the arch are eliminated or duplicated, obviating the use of certain portions of the forms and simplifying the computations. Analyses of variable-thickness and fillet arches require supplementary computation forms for evaluating arch and load constants, as given later. The following notation, in addition to items shown in section 4-2, is used on the forms:

$\Phi_1$ = angle from crown to ¼ point.
$\Phi_2$ = angle from crown to ½ point.
$\Phi_3$ = angle from crown to ¾ point.

The angles $\Phi_L$ and $\Phi_R$ are measured from the reference plane, or maximum cantilever section to the left or right abutments, respectively. These are used as $\Phi_a$ angles in analyzing nonsymmetrical arches. The value of $\Phi_a$ for analyzing symmetrical arches is the average of $\Phi_L$ and $\Phi_R$. Values of $\Phi$ must be in whole degrees to utilize the tables for arch analysis in appendix H.

C-3. *Design Data, Properties, and Dimensions of Arches.*—Design data for Monticello and Clear Creek Dams have been given in appendix B. The equivalent loaded area for Monticello Dam is shown on figure C-5. Constants for $\alpha'$, $\alpha''$, $\gamma'$, $\beta'$, $\gamma'$, and $\delta'$ are shown on figure C-6.

Monticello Dam has two types of arches. The top arch at elevation 456 is a uniform-thickness circular arch. Arches at all other elevations are fillet arches. Thickness and angles are shown on figure C-7. The properties of the arch shown in the upper half of the figure refer to the crown of the arch, only. For a uniform-thickness arch these properties are constant for all points of the arch, but for a variable-thickness arch they hold only for the crown. The tabulation of arch properties at other points will be considered later in the analysis of variable-thickness arches. On figure C-7, $L_L$ and $L_R$ are circular lengths of the arch centerline, and are not needed for analyzing the arches. They are used only for the layout of the adjustment drawings. The terms $\frac{R_{axis}}{r}$



## ARCH AND ABUTMENT CONSTANTS AND FUNCTIONS

| | LEFT SIDE | | | | RIGHT SIDE | | | |
|---|---|---|---|---|---|---|---|---|
| ArchTerm | 3/4 Point (Use $\Phi_1$) | 1/2 Point (Use $\Phi_2$) | 1/4 Point (Use $\Phi_3$) | Crown (Use $\Phi_a$) | Crown (Use $\Phi_a$) | 1/4 Point (Use $\Phi_3$) | 1/2 Point (Use $\Phi_2$) | 3/4 Point (Use $\Phi_1$) |
| $A_1$ Term | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ | $( \quad )(\frac{12r}{E_cT^3})$ |
| | $A_1 = 0,^{6}002,370032$ | $A_1 = 0,^{6}004,740,064$ | $A_1 = 0^{6}007,110,097$ | $A_1 = 0^{6}009,480,129$ | $A_1 =$ | $A_1 =$ | $A_1 =$ | $A_1 =$ |
| $B_1$ Term | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ |
| | $B_1 = 0^{6}012,032,92$ | $B_1 = 0^{6}095,374,02$ | $B_1 = 0^{6}316,943,2$ | $B_1 = 0^{6}735,154,3$ | $B_1 =$ | $B_1 =$ | $B_1 =$ | $B_1 =$ |
| $C_1$ Term | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ | $( \quad )(\frac{12r^2}{E_cT^3})$ |
| | $C_1 = 0^{6}144,845,0$ | $C_1 = 0^{6}570,466,7$ | $C_1 = 0^{3}001,250,673$ | $C_1 = 0^{3}002,143,605$ | $C_1 =$ | $C_1 =$ | $C_1 =$ | $C_1 =$ |
| $B_2$ 1st Term | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ |
| $B_2$ 2nd Term | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ |
| | $B_2 = 0^{3}001,107,746$ | $B_2 = 0^{3}017,101,34$ | $B_2 = 0^{3}082,124,77$ | $B_2 = 0^{3}241,180,6$ | $B_2 =$ | $B_2 =$ | $B_2 =$ | $B_2 =$ |
| $C_2$ 1st Term | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ |
| $C_2$ 2nd Term | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ |
| | $C_2 = 0^{3}011,862,85$ | $C_2 = 0^{3}090,953,29$ | $C_2 = 0^{3}288,205,2$ | $C_2 = 0^{3}625,914,3$ | $C_2 =$ | $C_2 =$ | $C_2 =$ | $C_2 =$ |
| $B_3$ 1st Term | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ | $( \quad )(\frac{12r^3}{E_cT^3})$ |
| $B_3$ 2nd Term | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ | $( \quad )(\frac{r}{E_cT})$ |
| | $B_3 = 0^{6}139,435,8$ | $B_3 = 0^{3}008,503,773$ | $B_3 = 0^{3}025,276,07$ | $B_3 = 0^{3}100,873,4$ | $B_3 =$ | $B_3 =$ | $B_3 =$ | $B_3 =$ |

For tabular arch terms see Denver Office Drawings 205-D-230 thru 233

For trigonometric functions see Denver Office Drawings 205-D-824 thru 827

| *$\Phi$ Point | Abut ($\Phi_a$) | 3/4 Point ($\Phi_3$) | 1/2 Point ($\Phi_2$) | 1/4 Point ($\Phi_1$) | Crown ($\Phi_o$) | Crown ($\Phi_o$) | 1/4 Point ($\Phi_1$) | 1/2 Point ($\Phi_2$) | 3/4 Point ($\Phi_3$) | Abut ($\Phi_a$) |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Phi$ | 57° | 42° 45′ | 28° 30′ | 14° 15′ | 0° | | | | | |
| SIN $\Phi$ | 838,670,6 | .678,800,7 | .477,158,8 | .246,153,3 | 0 | | | | | |
| COS $\Phi$ | .544,639,0 | .734,322,5 | .878,817,1 | .969,230,9 | 1.000,000 | | | | | |
| VERS $\Phi$ | | | | | | | | | | |
| X=r SIN $\Phi$ | 414.303,3 | 335.327,5 | 235.716,4 | 121.599,7 | 0 | | | | | |
| Y=r VERS $\Phi$ | 224.948,3 | 131.244,7 | 59.864,35 | 15.199,93 | 0 | | | | | |
| ($\Phi_a$-$\Phi$) Point | Crown | 1/4 Point | 1/2 Point | 3/4 Point | Abut | Abut. | 3/4 Point | 1/2 Point | 1/4 Point | Crown |
| | | | | | | | | | | |
| | | | | | | | | | | |

**Abutment Movement Functions**

| | Left Abut. | Right Abut. |
|---|---|---|
| $\alpha$ | $0^{9}168,962,1$ | |
| $\beta$ | $0^{6}006,117237$ | |
| $\gamma$ | $0^{6}010,743,31$ | |
| $\alpha_2$ | $0^{9}251,498,7$ | |

Copied from comp. sheet 205-D-1652

$r = 494.0$ $T = 12.00$ $\frac{1}{} =$

$\frac{1}{E_c} = 0,^{6}002,777,778$ $R_E = 500.$ $P = 1000$

$\frac{r}{E_cT} = 0^{6}114,351,85$ $\frac{r}{E_cT}R_E = 0^{3}057,175,93$

$\frac{12r}{E_cT^3} = 0^{6}009,529,321$ $\frac{r^2}{E_cT} = 0^{3}056,489,81$

$\frac{12r^2}{E_cT^3} = 0^{3}004,707,485$ $\frac{12r^2}{E_cT^3}R_E = .002,353,742$

$\frac{12r^3}{E_cT^3} = .002,325,497$ $\frac{12r^3}{E_cT^3}R_E = 1.162,749$

$\frac{12r^4}{E_cT^3} = 1.148,796$ $c = 0^{3}005,6$ $t = +1$ °F

For descripton of terms and analytical procedure see Denver Office Drawings 205-D-1649 and 1650

**D-term multipliers**

| | Radial Load | Tang. Load | Twist Load |
|---|---|---|---|
| $D_1$ Term | $\frac{12r^2}{E_cT^3}R_E$ | $\frac{12r^3}{E_cT^3}$ | $\frac{12r^2}{E_cT^3}$ |
| $D_2$ 1st Term | $\frac{12r^3}{E_cT^3}R_E$ | $\frac{12r^4}{E_cT^3}$ | $\frac{12r^3}{E_cT^3}$ |
| $D_2$ 2nd Term | $\frac{r}{E_cT}R_E$ | $\frac{r^2}{E_cT}$ | |
| $D_3$ 1st Term | $\frac{12r^3}{E_cT^3}R_E$ | $\frac{12r^4}{E_cT^3}$ | $\frac{12r^3}{E_cT^3}$ |
| $D_3$ 2nd Term | $\frac{r}{E_cT}R_E$ | $\frac{r^2}{E_cT}$ | |

**Temperature load D-terms**

| | | |
|---|---|---|
| $D_1 = 0$ | $D_2 = -ctY$* | $D_3 = ctX$* |

*Use X and Y for point($\Phi_a$-$\Phi$)

MONTICELLO DAM STUDY NO. A-11

ARCH AT ELEV. 456

COMPUTATION SHEET FOR

**ARCH AND ABUTMENT DATA**

FOR ARCH ANALYSIS

COMPUTED BY -------- DATE --------

CHECKED BY -------- DATE --------

SHEET 1 OF 4

*Figure C-1.* **Monticello Dam study–arch and abutment data.–DS2-1(75)**

# CONSTANTS FOR, AND SOLUTION OF CROWN FORCES

| | | | |
|---|---|---|---|
| Right | Arch | $A_1$ | = |
| | Abut. | $\alpha$ | = |
| Left | Arch | $A_1$ | = $.0^{6}009,480,129$ |
| | Abut. | $\alpha$ | = $.0^{9}168,962$ |
| | | $2A_1$ | = $.0^{6}019,298,182$ |

| | | | |
|---|---|---|---|
| Right | Arch | $B_1$ | = |
| | Abut. | $\alpha Y_a$ | = |
| | | $\alpha_2 \sin \Phi_a$ | = |
| Left | Arch | $B_1$ | = $.0^{6}735,154,3$ |
| | Abut | $\alpha Y_a$ | = $.0^{6}038,0077$ |
| | | $\alpha_2 \sin \Phi_a$ | = $.0^{9}210,9$ |
| | | $2B_1$ | = $.0^{3}001,546,745,8$ |

| | | | |
|---|---|---|---|
| Right | Arch | $-C_1$ | = − |
| | Abut. | $-\alpha X_a$ | = − |
| | | $-\alpha_2 \cos \Phi_a$ | = − |
| Left | Arch | $C_1$ | = |
| | Abut. | $\alpha X_a$ | = |
| | | $\alpha_2 \cos \Phi_a$ | = |
| | | $C_1$ | = |

| | | | |
|---|---|---|---|
| Right | Arch | $-B_2$ | = − |
| | Abut | $-\alpha X_a Y_a$ | = − |
| | | $\beta \sin \Phi_a \cos \Phi_a$ | = |
| | | $-\gamma \sin \Phi_a \cos \Phi_a$ | = − |
| | | $-\alpha_2 X_a \sin \Phi_a$ | = − |
| | | $-\alpha_2 Y_a \cos \Phi_a$ | = − |
| Left | Arch | $B_2$ | = |
| | Abut. | $\alpha X_a Y_a$ | = |
| | | $-\beta \sin \Phi_a \cos \Phi_a$ | = − |
| | | $\gamma \sin \Phi_a \cos \Phi_a$ | = |
| | | $\alpha_2 X_a \sin \Phi_a$ | = |
| | | $\alpha_2 Y_a \cos \Phi_a$ | = |
| | | $B_2$ | = |

| | | | |
|---|---|---|---|
| Right | Arch | $C_2$ | = |
| | Abut. | $\alpha X_a^2$ | = |
| | | $\beta \sin^2 \Phi_a$ | = |
| | | $\gamma \cos^2 \Phi_a$ | = |
| | | $2\alpha_2 X_a \cos \Phi_a$ | = |
| Left | Arch | $C_2$ | = $.0^{3}625,914,3$ |
| | Abut. | $\alpha X_a^2$ | = $.0^{3}029,001,9$ |
| | | $\beta \sin^2 \Phi_a$ | = $.0^{6}004,3$ |
| | | $\gamma \cos^2 \Phi_a$ | = $.0^{6}003,2$ |
| | | $2\alpha_2 X_a \cos \Phi_a$ | = $.0^{6}113,5$ |
| | | $2C_2$ | = $.001,310,074,4$ |

| | | | |
|---|---|---|---|
| Right | Arch | $B_3$ | = |
| | Abut. | $\alpha Y_a^2$ | = |
| | | $\beta \cos^2 \Phi_a$ | = |
| | | $\gamma \sin^2 \Phi_a$ | = |
| | | $2\alpha_2 Y_a \sin \Phi_a$ | = |
| Left | Arch | $B_3$ | = $.0^{3}100,873,4$ |
| | Abut. | $\alpha Y_a^2$ | = $.0^{3}008,549,8$ |
| | | $\beta \cos^2 \Phi_a$ | = $.0^{6}001,8$ |
| | | $\gamma \sin^2 \Phi_a$ | = $.0^{6}007,6$ |
| | | $2\alpha_2 Y_a \sin \Phi_a$ | = $.0^{6}094,9$ |
| | | $2B_3$ | = $.0^{3}219,055,0$ |

| | | | |
|---|---|---|---|
| Right | Arch | $D_1$ | = 0 |
| | Abut. | ${}_aM_R\alpha$ | = 0 |
| | | ${}_aV_R\alpha_2$ | = 0 |
| Left | Arch | $D_1$ | = $.0^{3}048,531,35$ |
| | Abut. | ${}_aM_L\alpha$ | = $.0^{3}010,379,55$ |
| | | ${}_aV_L\alpha_2$ | = 0 |
| | | $D_1$ | = $.0^{3}058,910,90$ |

| | | | |
|---|---|---|---|
| Right | Arch | $-D_2$ | = − 0 |
| | Abut | ${}_{\bar a}M_R\alpha X_a$ | = − |
| | | ${}_{\bar a}H_R\beta \sin \Phi_a$ | = − |
| | | ${}_{\bar a}V_R\gamma \cos \Phi_a$ | = − |
| | | ${}_{\bar a}M_R\alpha_2 \cos \Phi_a$ | = − |
| | | ${}_{\bar a}V_R\alpha_2 X_a$ | = − |
| Left | Arch | $D_2$ | = $.019,234,39$ |
| | Abut | ${}_aM_L\alpha X_a$ | = $.004,300,28$ |
| | | ${}_aH_L\beta \sin \Phi_a$ | = 0 |
| | | ${}_aV_L\gamma \cos \Phi_a$ | = 0 |
| | | ${}_aM_L\alpha_2 \cos \Phi_a$ | = $.0^{3}008,41$ |
| | | ${}_aV_L\alpha_2 X_a$ | = 0 |
| | | $D_2$ | = $.023,543,08$ |

| | | | |
|---|---|---|---|
| Right | Arch | $D_3$ | = |
| | Abut. | ${}_aM_R\alpha Y_a$ | = |
| | | ${}_{\bar a}H_R\beta \cos \Phi_a$ | = − |
| | | ${}_aV_R\gamma \sin \Phi_a$ | = |
| | | ${}_aM_R\alpha_2 \sin \Phi_a$ | = |
| | | ${}_aV_R\alpha_2 Y_a$ | = |
| Left | Arch | $D_3$ | = $.009,709,761$ |
| | Abut. | ${}_aM_L\alpha Y_a$ | = $.002,334,862$ |
| | | ${}_{\bar a}H_L\beta \cos \Phi_a$ | = − 0 |
| | | ${}_aV_L\gamma \sin \Phi_a$ | = 0 |
| | | ${}_aM_L\alpha_2 \sin \Phi_a$ | = $.0^{3}012,957$ |
| | | ${}_aV_L\alpha_2 Y_a$ | = 0 |
| | | $D_3$ | = $.012,057,580$ |

## EQUATIONS OF EQUILIBRIUM

$$A_1M_0 + B_1H_0 + C_1V_0 = D_1$$

$$B_1M_0 + B_3H_0 + B_2V_0 = D_3$$

$$C_1M_0 + B_2H_0 + C_2V_0 = D_2$$

## SOLUTION FOR CROWN FORCES

$$K_0 = A_1(B_3C_2 - B_2^2) - B_1(B_1C_2 - C_1B_2) - C_1(B_3C_1 - B_1B_2)$$

$= .0^{9}001,834,940,7$

$\frac{1}{K_0} =$

$$M_0 = \left[D_1(B_3C_2 - B_2^2) - D_3(B_1C_2 - C_1B_2) - D_2(B_3C_1 - B_1B_2)\right]\frac{1}{K_0}$$

$=$

$\times \frac{1}{K_0} = -3,131.046,2$

$$H_0 = \left[-D_1(B_1C_2 - B_2C_1) + D_3(A_1C_2 - C_1^2) + D_2(B_1C_1 - A_1B_2)\right]\frac{1}{K_0}$$

$=$

$\times \frac{1}{K_0} = +77.151,913$

$$V_0 = \left[-D_1(B_3C_1 - B_1B_2) + D_3(B_1C_1 - A_1B_2) + D_2(A_1B_3 - B_1^2)\right]\frac{1}{K_0}$$

$=$

$\times \frac{1}{K_0} = +17.970,80$

### NOTE

These crown forces are acting on the left side of the arch, assumed to be cut at the crown.

MONTICELLO DAM STUDY NO. A-1L

ARCH AT ELEV. 456

COMPUTATION SHEET FOR

CROWN CONSTANTS AND FORCES

TWIST LOAD NO. 2L Symmetrical Arch

FOR ARCH ANALYSIS

COMPUTED BY ______ DATE ______

CHECKED BY ______ DATE ______

SHEET 2 OF 4

For arch parts of constants use crown terms.

For load on one side of arch only, use only the D term for that side.

*Figure C-2.* Monticello Dam study—crown constants and forces.—DS2-1(76)

For $M_{L-R}$, $H_{L-R}$, and $V_{L-R}$, see External Load Formulae on Denver Office Drawings 205-D-189 thru 191 Also 205-D-824 thru 827

| | MOMENTS | | | | | THRUSTS | | | | SHEARS | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Point | $M_R$ | $H_0Y$‡ | $V_0X$‡ | $M=M_0+H_0Y-V_0X-M_R$ | $H_R$ | $H_0\cos\Phi$‡ | $V_0\sin\Phi$‡ | $H=H_0\cos\Phi+V_0\sin\Phi+H_R$ | $V_R$ | $H_0\sin\Phi$‡ | $V_0\cos\Phi$‡ | $V=H_0\sin\Phi-V_0\cos\Phi-V_R$ |
| RIGHT SIDE | Abt. $\Phi_a$ | 0 | + 17,355 192 | + 7,445.362 | + 6,778,784 | 0 | + 42.019,94 | + 15.071,58 | + 57.091,52 | 0 | + 64.705,04 | + 9.787,60 | + 54.917,44 |
| | 3/4 $\Phi_3$ | | + 10,125.78 | + 6,026.10 | + 968.63 | | + 56.654,39 | + 12.198,59 | + 68.852,98 | | + 52.370,77 | + 13.196,36 | + 39.174,41 |
| | 1/2 $\Phi_2$ | | + 4,618.649 | + 4,236.012 | − 2,748.409 | | + 67.802,42 | + 8.574,92 | + 76.377,34 | | + 36.813,71 | + 15.793,05 | + 21.020,66 |
| | 1/4 $\Phi_1$ | | + 1,172.704 | + 2,185.244 | − 4,143.586 | | + 74.778,02 | + 4.423,57 | + 79.201,59 | | + 18.991,20 | + 17.417,85 | + 1.573,35 |
| CR | $\Phi_0$ | | 0 | 0 | − 3,131.046,2 | | + 77.151,913 | 0 | + 77.151,913 | | 0 | 17.970,80 | ∓ 17.970,80 |
| LEFT SIDE | 1/4 $\Phi_1$ | | + 1,172.704 | + 2,185.244 | + 226.902 | | + 74.778,02 | + 4.423,57 | + 70.354,45 | | + 18.991,20 | + 17.417,85 | + 36.409,05 |
| | 1/2 $\Phi_2$ | | + 4,618.649 | + 4,236.012 | + 5,723.615 | | + 67.802,42 | + 8.574,92 | + 59.227,50 | | + 36.813,71 | + 15.793,05 | + 52.606,76 |
| | 3/4 $\Phi_3$ | | + 10,125.78 | + 6,026.10 | + 13,020.83 | | + 56.654,39 | + 12.198,59 | + 44.455,80 | | + 52.370,77 | + 13.196,36 | + 65.567,13 |
| | Abt $\Phi_a$ | + 61,431 23 | + 17,355.19 | + 7,445.36 | − 39,761.73 | | + 42.019,94 | + 15.071,58 | + 26.948,36 | | + 64.705,04 | + 9.787,60 | + 74.492,64 |
| | Point | $M_L$ | $H_0Y$‡ | $V_0X$‡ | $M=M_0+H_0Y+V_0X-M_L$ | $H_L$ | $H_0\cos\Phi$‡ | $V_0\sin\Phi$‡ | $H=H_0\cos\Phi-V_0\sin\Phi+H_L$ | $V_L$ | $H_0\sin\Phi$‡ | $V_0\cos\Phi$‡ | $V=H_0\sin\Phi+V_0\cos\Phi-V_L$ |

‡ Use coordinates and trigonometric functions for points $\Phi$, comp. sheet 1 of 4 (205-D-1653)

## RADIAL DEFLECTIONS

Signs of $D_2$ below operate on sign of D-Term as calculated

| | ABUTMENT MOVEMENTS | | | | 3/4 Point | 1/2 Point | 1/4 Point | Crown |
|---|---|---|---|---|---|---|---|---|
| RIGHT SIDE | $M_a\alpha+V_a\alpha_2=\theta$<br>$= +.0^{3}001,159,169,2$ | ARCH | $-D_2$ | = | − 0 | − 0 | − 0 | − 0 |
| | | | $C_1M$ | = | + $.0^{3}140,30$ | − .001,567,88 | − .005,182,27 | − .006,711,73 |
| | | | $B_2H$ | = | + $.0^{3}076,27$ | + .001,306,15 | + .006,504,41 | + .018,607,54 |
| | | | $C_2V$ | = | + $.0^{3}464,72$ | + .001,911,90 | + $.0^{3}453,45$ | − .011,248,18 |
| | $-H_a\beta=\Delta s$<br>$= -.0^{6}349,242,4$ | | Sub-total | = | | | | |
| | | ABUT. | $(M_a\alpha+V_a\alpha_2)x$* | = | + $.0^{3}140,95$ | + $.0^{3}273,24$ | + $.0^{3}388,70$ | + $.0^{3}480,25$ |
| | | | $(-H_a\beta)\sin\Phi$* | = | − $.0^{6}09$ | − $.0^{6}17$ | − $.0^{6}24$ | − $.0^{6}29$ |
| | $V_a\gamma+M_a\alpha_2=\Delta r$<br>$= +.0^{3}002,294,850$ | | $(V_a\gamma+M_a\alpha_2)\cos\Phi$* | = | + $.0^{3}002,22$ | + $.0^{3}002,02$ | + $.0^{3}001,69$ | + $.0^{3}001,25$ |
| | | | Total $\Delta r$ | = | + $.0^{3}824,37$ | + .001,925,26 | + .002,165,74 | + .001,128,84 |
| LEFT SIDE | $M_a\alpha+V_a\alpha_2=\theta$<br>$= -.0^{3}006,699,491$ | ARCH | $-D_2$ | = | − .004,441,35 | − .010,096,98 | − .015,131,26 | − .019,234,39 |
| | | | $C_1M$ | = | + .001,886,00 | + .003,265,13 | + $.0^{3}283,78$ | − .006,711,73 |
| | | | $B_2H$ | = | + $.0^{3}049,25$ | + .001,012,87 | + .005,777,84 | + .018,607,54 |
| | | | $C_2V$ | = | + $.0^{3}777,81$ | + .004,784,76 | + .010,493,28 | + .011,248,18 |
| | $-H_a\beta=\Delta s$<br>$= -.0^{6}164,849,5$ | | Sub-total | = | | | | |
| | | ABUT. | $(M_a\alpha+V_a\alpha_2)x$* | = | − $.0^{3}814,66$ | − .001,579,18 | − .002,246,52 | − .002,775,62 |
| | | | $(-H_a\beta)\sin\Phi$* | = | − $.0^{6}04$ | − $.0^{6}08$ | − $.0^{6}11$ | − $.0^{6}14$ |
| | $V_a\gamma+M_a\alpha_2=\Delta r$<br>$= -.0^{3}009,199,726$ | | $(V_a\gamma+M_a\alpha_2)\cos\Phi$* | = | − $.0^{3}008,92$ | − $.0^{3}008,08$ | − $.0^{3}006,76$ | − $.0^{3}005,01$ |
| | | | Total $\Delta r$ | = | − .002,551,91 | − .002,621,56 | − $.0^{3}829,75$ | + .001,128,83 |

* Use coordinates and trigonometric functions for points $(\Phi_a-\Phi)$, comp. sheet 1 of 4 (205-D 1653)

| D-TERM | | LEFT SIDE | | RIGHT SIDE | |
|---|---|---|---|---|---|
| From tables D.O.D.'s 205-D-393 thru 418 | | | | | |
| Cr | $D_1$ | ( ) | For multipliers see sheet 1 of 4 | ( ) | For multipliers see sheet 1 of 4 |
| Cr | $D_3$ 1ST TRM | ( ) | | ( ) | |
| | $D_3$ 2ND TRM | ( ) | | ( ) | |
| Cr | $D_2$ 1ST TRM | ( ) | | ( ) | |
| | $D_2$ 2ND TRM | ( ) | | ( ) | |
| 1/4 | $D_2$ 1ST TRM | ( ) | | ( ) | |
| | $D_2$ 2ND TRM | ( ) | | ( ) | |
| 1/2 | $D_2$ 1ST TRM | ( ) | | ( ) | |
| | $D_2$ 2ND TRM | ( ) | | ( ) | |
| 3/4 | $D_2$ 1ST TRM | ( ) | | ( ) | |
| | $D_2$ 2ND TRM | ( ) | | ( ) | |

MONTICELLO DAM STUDY NO. A-11

ARCH AT ELEV. 456

COMPUTATION SHEET FOR
MOMENTS, THRUSTS, AND SHEARS
RADIAL DEFLECTIONS
TWIST LOAD NO. 22 Symmetrical Arch
FOR ARCH ANALYSIS

COMPUTED BY ____ DATE ____
CHECKED BY ____ DATE ____

SHEET 3 OF 4

*Figure C-3.* **Monticello Dam study—moments, thrusts, shears, and radial deflections.—DS2-1(77)**

## TANGENTIAL DEFLECTIONS

|  | ABUTMENT MOVEMENTS |  |  | ¾ Point | ½ Point | ¼ Point | Crown |
|---|---|---|---|---|---|---|---|
| RIGHT SIDE |  | ARCH | $-D_3$ = | − 0 | − 0 | − 0 | − 0 |
|  | $M_a\alpha + V_a\alpha_2 = \theta$ = |  | $B_1M$ = | + $0^{3}$011,655 | − .$0^{3}$262,127 | − .001,313,281 | − .002,301,802 |
|  |  |  | $B_3H$ = | + $0^{3}$009,601 | + .$0^{3}$267,609 | + .002,001,905 | + .007,782,576 |
|  |  |  | $B_2V$ = | + .$0^{3}$043,395 | + .$0^{3}$359,481 | + $0^{3}$129,211 | − .004,334,208 |
|  | $V_a\gamma + M_a\alpha_2 = \Delta r$ = |  | Sub-total = |  |  |  |  |
|  |  | ABUT. | $(M_a\alpha + V_a\alpha_2)Y^*$ = | + $0^{3}$017,619 | + $0^{3}$069,393 | + $0^{3}$152,135 | + .$0^{3}$260,753 |
|  |  |  | $(V_a\gamma + M_a\alpha_2)\sin\Phi^*$ = | + .$0^{6}$565 | + .$0^{3}$001,095 | + $0^{3}$001,558 | + .$0^{3}$001,925 |
|  | $H_a\beta = \Delta s$ |  | $(H_a\beta)\cos\Phi^*$ = | + $0^{6}$338 | + .$0^{6}$307 | + .$0^{6}$256 | + .$0^{6}$190 |
|  | = + $0^{6}$349,242,4 |  | Total Δs = | + .$0^{3}$083,173 | + .$0^{3}$435,758 | + .$0^{3}$971,784 | + .001,409,434 |
| LEFT SIDE |  | ARCH | $D_3$ = | + .$0^{3}$443,257 | + .002,260,543 | + .005,414,064 | + .009,709,761 |
|  | $-M_a\alpha - V_a\alpha_2 = \theta$ = |  | $-B_1M$ = | − .$0^{3}$156,679 | − $0^{3}$545,884 | − .$0^{3}$071,915 | + .002,301,802 |
|  |  |  | $-B_3H$ = | − .$0^{3}$006,199 | − $0^{3}$207,520 | − .001,778,284 | − .007,782,576 |
|  |  |  | $-B_2V$ = | − .$0^{3}$072,632 | − .$0^{3}$899,646 | − .002,990,085 | − .004,334,208 |
|  | $-V_a\gamma - M_a\alpha_2 = \Delta r$ = |  | Sub-total = |  |  |  |  |
|  |  | ABUT. | $(-M_a\alpha - V_a\alpha_2)Y^*$ = | + .$0^{3}$101,832 | + .$0^{3}$401,061 | + $0^{3}$879,273 | + .001,507,039 |
|  |  |  | $(-V_a\gamma - M_a\alpha_2)\sin\Phi^*$ = | + .$0^{3}$002,265 | + .$0^{3}$004,390 | + .$0^{3}$006,245 | + .$0^{3}$007,716 |
|  | $-H_a\beta = \Delta s$ |  | $(-H_a\beta)\cos\Phi^*$ = | − $0^{6}$160 | − .$0^{6}$145 | − .$0^{6}$121 | − $0^{6}$090 |
|  | = − $0^{6}$164,849,5 |  | Total Δs = | + $0^{3}$311,684 | + .001,012,799 | + .001,459,177 | + .001,409,444 |

*Use coordinates and trigonometric functions for points $(\Phi_a - \Phi)$, comp. sheet 1 of 4 (205-D-1653).

## ANGULAR MOVEMENTS

|  | ABUTMENT MOVEMENTS |  |  | ¾ Point | ½ Point | ¼ Point | Crown |
|---|---|---|---|---|---|---|---|
| RIGHT SIDE |  | ARCH | $D_1$ = | + 0 | + 0 | + 0 | + 0 |
|  |  |  | $-A_1M$ = | − $0^{3}$002,295,68 | + .$0^{3}$013,027,63 | + .$0^{3}$029,461,30 | + .$0^{3}$029,692,72 |
|  |  |  | $-B_1H$ = | − .$0^{6}$828,50 | − .$0^{3}$007,284,41 | − .$0^{3}$025,102,41 | − .$0^{3}$056,718,56 |
|  |  |  | $-C_1V$ = | − .$0^{3}$005,674,22 | − .$0^{3}$011,991,59 | − .$0^{3}$001,967,75 | + .$0^{3}$038,522,30 |
|  |  |  | Sub-total = |  |  |  |  |
|  | $-M_a\alpha - V_a\alpha_2 = \theta$ | ABT. | $(-M_a\alpha - V_a\alpha_2)$ = | − .$0^{3}$001,159,17 | − .$0^{3}$001,159,17 | − .$0^{3}$001,159,17 | − .$0^{3}$001,159,17 |
|  | = − .$0^{3}$001,159,169 |  | Total θ = | − .$0^{3}$009,957,57 | − .$0^{3}$007,407,54 | + .$0^{3}$001,231,97 | + .$0^{3}$010,327,29 |
| LEFT SIDE |  | ARCH | $-D_1$ = | − .$0^{3}$048,531,35 | − .$0^{3}$048,531,35 | − .$0^{3}$048,531,35 | − .$0^{3}$048,531,35 |
|  |  |  | $A_1M$ = | + .$0^{3}$030,859,78 | + .$0^{3}$027,130,30 | + .$0^{3}$001,613,30 | − .$0^{3}$029,682,72 |
|  |  |  | $B_1H$ = | + .$0^{6}$534,93 | + .$0^{3}$005,648,76 | + .$0^{3}$022,298,36 | + .$0^{3}$056,718,56 |
|  |  |  | $C_1V$ = | + .$0^{3}$009,497,07 | + .$0^{3}$030,010,40 | + .$0^{3}$045,535,82 | + .$0^{3}$038,522,30 |
|  |  |  | Sub-total = |  |  |  |  |
|  | $M_a\alpha + V_a\alpha_2 = \theta$ | ABT. | $(M_a\alpha + V_a\alpha_2)$ = | − $0^{3}$006,699,49 | − .$0^{3}$006,699,49 | − .$0^{3}$006,699,49 | − .$0^{3}$006,699,49 |
|  | = − .$0^{3}$006,699,491 |  | Total θ = | − .$0^{3}$014,339,06 | + .$0^{3}$007,558,62 | + .$0^{3}$014,216,64 | + .$0^{3}$010,327,30 |

| D TERM |  | LEFT SIDE | RIGHT SIDE |
|---|---|---|---|
| Cr. | $D_3$ 1ST TRM | ( )( ) | ( )( ) |
|  | $D_3$ 2ND TRM | ( )( ) | ( )( ) |
| ¼ | $D_3$ 1ST TRM | ( )( ) | ( )( ) |
|  | $D_3$ 2ND TRM | ( )( ) | ( )( ) |
| ½ | $D_3$ 1ST TRM | ( )( ) | ( )( ) |
|  | $D_3$ 2ND TRM | ( )( ) | ( )( ) |
| ¾ | $D_3$ 1ST TRM | ( )( ) | ( )( ) |
|  | $D_3$ 2ND TRM | ( )( ) | ( )( ) |
| Cr. | $D_1$ | ( )( ) | ( )( ) |
| ¼ | $D_1$ | ( )( ) | ( )( ) |
| ½ | $D_1$ | ( )( ) | ( )( ) |
| ¾ | $D_1$ | ( )( ) | ( )( ) |

For multipliers see sheet 1 of 4

For multipliers see sheet 1 of 4

For tabular D terms see D. O. D.'s 205-D-393 thru 418.

MONTICELLO DAM STUDY NO. A-11

ARCH AT ELEV. 456

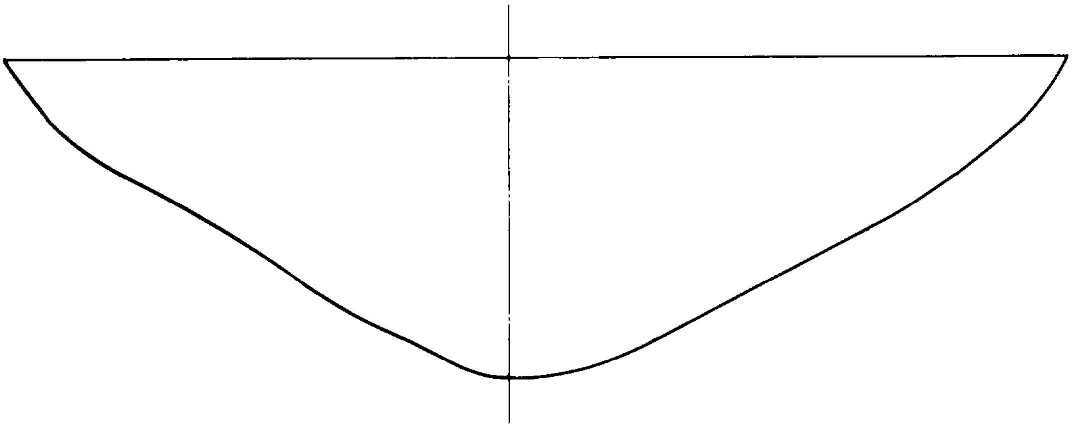
COMPUTATION SHEET FOR
TANGENTIAL DEFLECTIONS
ANGULAR MOVEMENTS
TWIST LOAD NO. 24 Symmetrical Arch
FOR ARCH ANALYSIS

COMPUTED BY ________ DATE ________
CHECKED BY ________ DATE ________

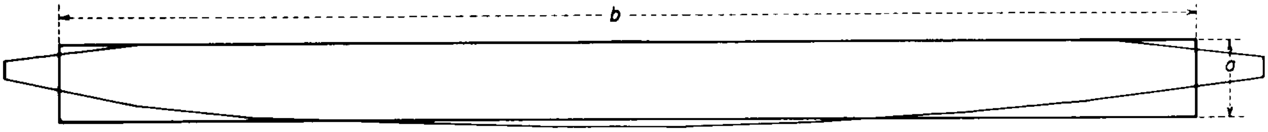
SHEET 4 OF 4

*Figure C-4.* **Monticello Dam study—tangential deflections and angular movements.—DS2-1(78)**

**MONTICELLO DAM**



**PROFILE**



**LONG AND SHORT DIMENSIONS OF EQUIVALENT LOADED AREA**

*Figure C-5.* Monticello Dam study–loaded area of foundation.–288-D-2686

$\tan \psi_L$, $\frac{R_{axis}}{r} \tan \psi_R$, and the bottom part of the forms are not needed for analyzing arches.

Figure C-8 shows the temperature data and temperature loads.

C-4. *Arch and Foundation Data.*–Computations of arch data are normally placed on four computation sheets referred to as arch sheets 1 to 4, inclusive. These sheets, as completed for twist load No. 2 on the arch at elevation 456 in Monticello Dam, are presented as figures C-1 to C-4, respectively. For convenience, they will generally be referred to in this and the next few sections as sheets 1 to 4, inclusive.

A convenient computation form has been developed for calculating unit foundation movements. This is shown on figure C-6 with illustrative computations for Monticello Dam. The design data for arches and cantilevers are shown on figures C-7, C-9, and C-10.

For a nonsymmetrical dam such as Monticello, abutment values must be computed for the entire dam. If the dam is only approximately symmetrical, abutment angles, $\psi$, are sometimes averaged for the left and right abutments of each arch in analyzing a complete arch. This simplifies the computations and gives reasonably accurate results.

The ratio $b/a$ was determined as shown on figure C-5 by the method indicated on figure 4-24A. Foundation constants for all arches 456, 400, 350, 300, 250, and 200 and for all cantilevers on the left side, including the crown cantilever, are shown on figure C-6. Computations were made by means of

MONTICELLO DAM STUDY NO. A-11

ARCH ABUTMENT AND CANTILEVER FOUNDATION DEFORMATION FUNCTIONS LEFT SIDE

Constants – See D.O.D.s 205-D-1621 thru 1625

$\mu = 0.03$ $\frac{b}{a} = 15.0$ $\alpha' = \frac{5.62}{E_rT^2}$ $\alpha'' = \frac{0.88}{E_rT}$ $\gamma' = \frac{2.51}{E_r}$ $\frac{1}{E_r} = .\overset{6}{0}005,341,880$

$\frac{b/2}{a} = 7.5$ $\beta' = \frac{2.07}{E_r}$ $\delta' = \frac{3.80}{E_rT^2}$ $\frac{a}{b/2} = 0.133$ $\textcircled{\gamma} = \frac{0.28}{E_r}$ BY ........ DATE ........

| ELEV. | $T_a$ | $\alpha'$ | $\alpha''$ | $\beta'$ | $\gamma'$ | $\textcircled{\gamma}$ | $\delta'$ | $\psi$ | SIN $\psi$ | $SIN^2\psi$ | $SIN^3\psi$ | COS $\psi$ | $COS^2\psi$ | $COS^3\psi$ | SIN $\psi$ $COS^2\psi$ | $SIN^2\psi$ COS $\psi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | 12.00 | .$\overset{9}{0}$208,481,7 | .$\overset{9}{0}$391,737,8 | For all elevations .$\overset{6}{0}$011,057,69 | For all elevations .$\overset{6}{0}$013,408,12 | For all elevations .$\overset{6}{0}$001,495,726 | .$\overset{9}{0}$215,159,1 | 36°45' | .598,324,6 | .357,992,3 | .214,195,6 | .801,253,8 | .642,007,7 | .514,411,1 | .384,129,0 | 286,842,7 |
| 400 | 41.40 | .$\overset{9}{0}$017,515,79 | .$\overset{9}{0}$113,547,2 | | | | .$\overset{9}{0}$018,076,80 | 47°45' | .740,218,1 | .547,922,9 | .405,582,4 | .672,366,8 | .452,077,1 | .303,961,6 | .334,635,7 | .368,405,2 |
| 350 | 58.16 | .$\overset{9}{0}$008,875,271 | .$\overset{9}{0}$080,826,24 | | | | .$\overset{9}{0}$009,159,532 | 58°00' | .848,048,1 | .719,185,6 | .609,904,0 | .529,919,3 | .280,814,4 | .148,809,0 | .238,144,1 | .381,110,3 |
| 300 | 66.62 | .$\overset{9}{0}$006,764,275 | .$\overset{9}{0}$070,562,20 | | | | .$\overset{9}{0}$006,980,924 | 59°15' | .859,406,4 | .738,579,4 | .634,739,9 | .511,293,1 | .261,420,6 | .133,662,5 | .224,666,5 | .377,630,6 |
| 250 | 73.48 | .$\overset{9}{0}$005,560,223 | .$\overset{9}{0}$063,974,61 | | | | .$\overset{9}{0}$005,738,308 | 58°45' | .854,911,9 | .730,874,8 | .624,833,1 | .518,773,3 | .269,125,7 | .139,615,2 | .230,078,8 | .379,158,1 |
| 200 | 79.40 | .$\overset{9}{0}$004,762,001 | .$\overset{9}{0}$059,204,72 | | | | .$\overset{9}{0}$004,914,520 | 60°45' | .872,496,0 | .761,249,3 | .664,187,0 | .488,621,2 | .238,750,7 | .116,658,7 | .208,309,0 | .371,962,5 |
| 165 | 85.84 | .$\overset{9}{0}$004,074,281 | .$\overset{9}{0}$054,762,98 | | | | .$\overset{9}{0}$004,204,774 | 90°00' | 1.0 | 1.0 | 1.0 | 0 | 0 | 0 | 0 | 0 |

**ARCH** (Ref. Dwg. 205-D-1649)

$\alpha = \alpha' COS^3\psi + \delta' SIN^2\psi COS\psi$ $\alpha_2 = \alpha'' COS^2\psi$

$\beta = \beta' COS^3\psi + \textcircled{\gamma} SIN^2\psi COS\psi$ $\gamma = \gamma' COS\psi$

**CANTILEVER** (Ref. Dwgs: Par.-side, 205-D-1647; Rad.-side, 205-D-1726)

$*\alpha = \alpha' SIN^3\psi + \delta' SIN\psi COS^2\psi$ $*\alpha_2 = \alpha'' SIN^2\psi$ $*\gamma = \gamma' SIN\psi$

$*\textcircled{\gamma} = \textcircled{\gamma} SIN^3\psi + \beta' SIN\psi COS^2\psi$ $*\delta = \delta' SIN^3\psi + \alpha' SIN\psi COS^2\psi$

| ELEV. | $\alpha$ | $\beta$ | $\gamma$ | $\alpha_2$ | CANT. | $R_{AXIS}/r$ | $*\alpha$ | $*\gamma$ | $*\delta$ | $*\alpha_2$ | $*\textcircled{\gamma}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | .$\overset{9}{0}$168,962,1 | .$\overset{6}{0}$006,117,237 | .$\overset{6}{0}$010,743,31 | .$\overset{9}{0}$251,498,7 | | | | | | | |
| 400 | .$\overset{9}{0}$011,983,71 | .$\overset{6}{0}$003,912,146 | .$\overset{6}{0}$009,015,175 | .$\overset{9}{0}$051,332,09 | A | 1.023,458 | .$\overset{9}{0}$013,461,79 | | | .$\overset{9}{0}$063,674,55 | |
| 350 | .$\overset{9}{0}$004,811,512 | .$\overset{6}{0}$002,215,520 | .$\overset{6}{0}$007,105,222 | .$\overset{9}{0}$022,697,17 | B | 1.035,368 | .$\overset{9}{0}$007,862,949 | | | .$\overset{9}{0}$060,184,98 | |
| 300 | .$\overset{9}{0}$003,540,340 | .$\overset{6}{0}$002,042,830 | .$\overset{6}{0}$006,855,479 | .$\overset{9}{0}$018,446,41 | C | 1.044,517 | .$\overset{9}{0}$006,122,891 | | | .$\overset{9}{0}$054,435,89 | |
| 250 | .$\overset{9}{0}$002,952,018 | .$\overset{6}{0}$002,110,938 | .$\overset{6}{0}$006,955,775 | .$\overset{9}{0}$017,217,21 | D | 1.052,056 | .$\overset{9}{0}$005,044,056 | | | .$\overset{9}{0}$049,191,40 | |
| 200 | .$\overset{9}{0}$002,383,546 | .$\overset{6}{0}$001,846,330 | .$\overset{6}{0}$006,551,492 | .$\overset{9}{0}$014,135,17 | E | 1.058,649 | .$\overset{9}{0}$004,432,138 | | | .$\overset{9}{0}$047,712,84 | |
| 165 | | | | | Crown | 1.065,916 | .$\overset{9}{0}$004,342,841 | .$\overset{6}{0}$014,291,93 | .$\overset{9}{0}$004,481,936 | .$\overset{9}{0}$058,372,74 | .$\overset{6}{0}$001,594,318 |

* Functions multiplied by $R_{AXIS}/r$ for radial-side cantilever

*Figure C-6.* **Monticello Dam study—foundation deformation functions.—DS2-1(20)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM    STUDY NO. A-11

DESCRIPTION OF ARCHES AND LOCATION OF CANTILEVERS FROM LINE OF CENTERS

$E_c$= 2,500,000 lb./in.² G= $E_r$= 1,300,000 lb./in.² $\mu_c$= 0.20 $\mu_r$= 0.03

b= a= By Date

| Elev. | Properties of Arch at LINE OF CENTERS | | | | | | | Dimensions of Arch- ALONG ARC THROUGH CENTER OF CROWN | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | T | U.S. Proj. | D.S. Proj. | $R_{axis}$ | $R_E$ | $r_o$ | $R_I$ | $\Phi_L$ | $\Phi_A$ | $\Phi_R$ | $L_L$ | $L_R$ | $\psi_L$ | $\psi_R$ | Tan $\psi_L$ | Tan $\psi_R$ |
| 456 | 12.00 | 0 | 12.00 | 500.00 | 500.00 | 494.00 | 488.00 | 54° | 57° | 60° | 465.58 | 517.32 | 36°45′ | 36°45′ | | |
| 400 | 27.52 | 9.24 | 18.28 | | 509.24 | 495.48 | 481.72 | 49° | 52° | 55° | 423.74 | 475.63 | 47°45′ | 45°30′ | | |
| 350 | 38.08 | 12.00 | 26.08 | | 512.00 | 492.96 | 473.92 | 41° | 45° 44° | 48° | 352.75 | 412.98 | 58°00′ | 55°00′ | | |
| 300 | 48.00 | 12.00 | 36.00 | | | 488.00 | 464.00 | 31° | 35° | 39° | 264.03 | 332.17 | 59°15′ | 60°30′ | | |
| 250 | 60.10 | 12.00 | 48.10 | | | 481.95 | 451.90 | 22° | 25° | 28° | 185.06 | 235.53 | 58°45′ | 63°15′ | | |
| 200 | 74.44 | 12.00 | 62.44 | | | 474.78 | 437.56 | 12° | 14° | 16° | 99.44 | 132.58 | 66°45′ | 67°30′ | | |
| 165 | 85.84 | | | | | | | | | | | | 90°00′ | | | |

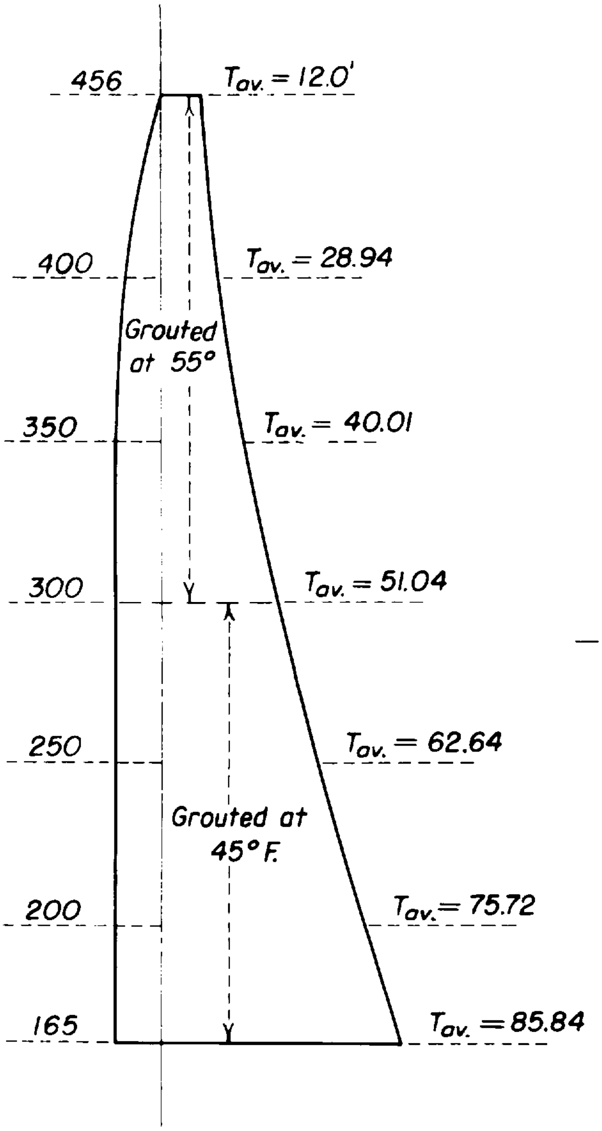
LENGTHS ALONG ARCS THROUGH CENTERS OF CROWNS (T/2 FROM UPSTREAM FACE)

| Elev. | Cant. A | | Cant. B | | Cant. C | | Cant. D | | Cant. E | | Cant. CROWN. | | Cant. F | | Cant. G | | Cant. H | | Cant. I | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L | $\Phi$ | L |
| 456 | 49° | 422.47 | 41° | 353.50 | 31° | 267.28 | 22° | 189.68 | 12° | 103.46 | 0° | 0 | 16° | 137.95 | 28° | 241.41 | 39° | 336.26 | 48° | 413.85 |
| 400 | 49° | 423.74 | 41° | 354.56 | 31° | 268.08 | 22° | 190.25 | 12° | 103.17 | 0° | 0 | 16° | 138.36 | 28° | 242.14 | 39° | 337.26 | 48° | 415.09 |
| 350 | | | 41° | 352.75 | 31° | 266.72 | 22° | 189.28 | 12° | 103.25 | 0° | 0 | 16° | 137.66 | 28° | 240.91 | 39° | 335.55 | 48° | 412.98 |
| 300 | | | | | 31° | 264.03 | 22° | 187.38 | 12° | 102.21 | 0° | 0 | 16° | 136.28 | 28° | 238.48 | 39° | 332.17 | | |
| 250 | | | | | | | 22° | 185.06 | 12° | 100.94 | 0° | 0 | 16° | 134.59 | 28° | 235.53 | | | | |
| 200 | | | | | | | | | 12° | 99.44 | 0° | 0 | 16° | 132.58 | | | | | | |
| 165 | | | | | | | | | | | 0° | 0 | | | | | | | | |

| | Cant. J | |
|---|---|---|
| | $\Phi$ | L |
| 456 | 55° | 474.21 |
| 400 | 55° | 475.63 |

*Figure C-7.* Monticello Dam study–description of arches and location of cantilevers.–DS2-1(23)

MONTICELLO DAM
TEMPERATURE DATA



TEMPERATURE LOAD

| ELEV. | *TEMP | STAGE I TEMP | STAGE II TEMP |
|---|---|---|---|
| 456 | +48.4 | | -6.6 |
| 400 | +54.6 | | -.4 |
| 350 | +57.2 | | +2.2 |
| 300 | +58.0 | + 10 | +3.0 |
| 250 | +58.2 | + 10 | +3.2 |
| 200 | +58.2 | + 10 | +3.2 |

* = Minimum operating temperature

*Figure C-8.* **Monticello Dam study–temperature data.–288-D-2698**

equations shown at the top of the form. For the crown cantilever the abutment angle has been assumed to be equal to 90°.

Abutment movement functions for the arch at elevation 456 are obtained from figure C-6 for use on the arch and abutment data sheet, sheet 1, figure C-1, which is common for all loads. *D*-term multipliers are calculated using constants given on figures C-6 and C-7. Since the arch is symmetrical and of uniform thickness, only the left side of the form need be filled out. Trigonometric functions of angles are obtained from the tables for arch analysis in appendix H. Arch constants $A_1$, $B_1$, and $C_1$ are each the product of a tabular value, secured from the table of arch constants for the value of $\Phi$ at the point under consideration, and a multiplier which is evaluated at the bottom of

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
Distance along Axis of dam
By. Date.

| | Cantilevers | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ELEV. | 456-L | A | B | C | D | E | Crown | F | G | H | I | J | 456-R |
| | | | | | | | | | | | | | |
| 456 | 471.24 | 427.61 | 357.79 | 270.53 | 191.99 | 104.72 | 0 | 139.63 | 244.35 | 340.34 | 418.88 | 479.97 | 523.60 |
| | | | | | | | | | | | | | |
| 400 | | ↓ | | | | | | | | | | ↓ | |
| | | | | | | | | | | | | | |
| 350 | | | | | | | | | | | ↓ | | |
| | | | ↓ | | | | | | | | | | |
| 300 | | | | ↓ | | | | | | ↓ | | | |
| | | | | | | | | | | | | | |
| 250 | | | | | ↓ | | | | ↓ | | | | |
| | | | | | | | | | | | | | |
| 200 | | | | | | ↓ | | ↓ | | | | | |
| | | | | | | | | | | | | | |
| 165 | | | | | | | ↓ | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |

*Figure C-9.* Monticello Dam study—distance along axis of dam to cantilevers.—DS2-1(21)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

DISTANCES BETWEEN CANTILEVERS ALONG ARCS THROUGH MID-POINTS OF CROWNS

By ........ Date ........

| Elev. | 456L | A | B | C | aDb | E | CROWN | F | G | bHa | I | J | 456R |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ΔL | 43.11 | 68.97 | 86.22 | 77.60 | 86.22 | 103.46 | 137.95 | 103.46 | 94.85 | 77.59 | 60.36 | 43.11 |
| 456 | a,b | 21.555 | 34.485 | 43.11 | 38.80 | 43.11 | 51.73 | 68.975 | 51.73 | 47.425 | 38.795 | 30.18 | 21.555 |
| | a+b | 56.040 | 77.595 | 81.91 | 81.91 | 94.84 | 120.705 | 120.705 | 99.155 | 86.22 | 68.915 | 51.735 | |
| | a/a+b | .384,64 | .444,42 | .526,31 | .473,69 | .454,56 | .428,57 | .571,43 | .428,57 | .478,29 | .449,95 | .437,55 | .416,64 |
| | | ΔL | 69.18 | 86.48 | 77.83 | 86.48 | 103.77 | 138.36 | 103.78 | 95.12 | 77.83 | 60.54 | |
| 400 | | a/a+b | .444,42 | .526,31 | .473,69 | .454,56 | .428,57 | .571,43 | .428,57 | .478,29 | .449,95 | .437,55 | |
| | | | ΔL | 86.03 | 77.44 | 86.03 | 103.25 | 137.66 | 103.25 | 94.64 | 77.43 | | |
| 350 | | | a/a+b | .526,31 | .473,69 | .454,56 | .428,57 | .571,43 | .428,57 | .478,29 | .449,95 | | |
| | | | | ΔL | 76.65 | 85.17 | 102.21 | 136.28 | 102.20 | 93.69 | | | |
| 300 | | | | a/a+b | .473,69 | .454,56 | .428,57 | .571,48 | .428,57 | .478,29 | | | |
| | | | | | ΔL | 84.12 | 100.94 | 134.59 | 100.94 | | | | |
| 250 | | | | | a/a+b | .454,56 | .428,57 | .571,43 | .428,57 | | | | |
| | | | | | | ΔL | 99.44 | 132.58 | | | | | |
| 200 | | | | | | a/a+b | .428,57 | .571,43 | | | | | |

*Figure C-10.* Monticello Dam study—distances between cantilevers.—DS2-1(22) the sheet. These multipliers are shown in parenthesis on the upper half of sheet 1, opposite their related arch terms. Arch constants $B_2$, $C_2$, and $B_3$ are each the sum of two products as indicated.

C-5. ***Crown Constants and Forces.***–The evaluation of crown constants and forces for the arch at elevation 456 of Monticello Dam is shown on sheet 2, figure C-2, for a twist load No. 2 on the left side.

The arch parts of $A_1$, $B_1$, and $B_3$ are taken from sheet 1, for the crown point. If the arch is symmetrically loaded, the summations of $C_1$, $B_2$, and $D_2$ are zero and therefore the value for $C_2$ is not needed. (See the third equation of equilibrium.) Arch and abutment load values taken from sheet 1 were doubled to obtain the constants on sheet 2 since the arch is symmetrical.

The arch parts of $D_1$ and $D_3$ are, respectively, the product of a tabular term and its multiplier and the sum of two such products. These products are secured from their component parts, for the crown point, shown on the right central portion of sheet 3, figure C-3. Abutment parts are composed of products of values found on sheets 1 and 3. The ${}_aM_L$, ${}_aH_L$, and ${}_aV_L$ values for these terms are obtained from sheet 3. Total values of $D$-terms are sums of arch and abutment parts. For concentrated loads, $D$-terms include only the effects of yielding abutments. Temperature $D$-terms are evaluated by formulas given on sheet 1; these are total $D$-terms.

Since the shear at the crown is equal to zero, values of $M_o$ and $H_o$, only, are computed by the equations shown. Experience has shown that to be assured of sufficient accuracy in the results of the computations, it is necessary to carry at least seven significant figures in all calculations. Values of $M_o$, $H_o$, and $V_o$ should be checked by substituting them in the equations of equilibrium.

C-6. ***Moments, Thrusts, Shears, and Radial Deflections.***–Moments, thrusts, and shears at arch points for a twist load No. 2 are computed in the upper part of sheet 3, figure C-3, by means of equations shown on figures C-11, C-12, and C-13. Radial deflections are computed in the lower part of the sheet.

In order to complete sheet 2 it is necessary to enter on sheet 3 tabular values of $D$-terms for the crown point and values of $M_L$, $H_L$, and $V_L$ for the abutment point. However, at the time these values are determined, it is expedient to fill in values for other points between the crown and abutment. Formulas for $M_L$, $H_L$, and $V_L$ for uniform and triangular radial, tangential, and twist loads are given on figures C-11, C-12, and C-13. Radial, tangential, and twist concentrated loads are, respectively, a shear ${}_aV_L = P$, a thrust ${}_aH_L = P$, and a moment ${}_aM_L = P$, $P$ being negative in the case of thrust.

Tabular $D$-term constants at the right end of sheet 3, figure C-3, are copied from the $D$-term tables accompanying appendix H. Products of the tabular values and their proper multipliers are $D$-terms, or $D$-constants, used in computing deflections. Crown $D$-terms are used on sheet 2 in computing crown forces.

Using the values of $M_o$, $H_o$, and $V_o$ from sheet 2, figure C-2, and the coordinate and trigonometric functions for the arch points from sheet 1, the values of $M$, $H$, and $V$ for the remaining arch points are computed in the columns at the top of sheet 3. After this has been done, and before any further work is performed, the applicable checks described in section C-8 are made.

All quantities for radial deflections are now calculated. The portion of the deflection at a point due to the elastic movement of the arch is the algebraic sum of the products of the $D$-terms on sheet 3 or the arch constants from sheet 1 and their appropriate moments, thrusts, and shears on sheet 3. The portion of the deflection due to elastic deformation of the yielding abutment is obtained by multiplying $M_a$, $H_a$, and $V_a$ by the proper abutment constants $\alpha$, $\alpha_2$, $\beta$, and $\gamma$, on sheet 1, transferred to arch points by use of functions of the angle $\Phi_a - \Phi$ on sheet 1.

Since the arch is used in a nonsymmetrical analysis, moments, thrusts, shears, $D$-terms and radial deflections must be included for the right part of the arch. Loads for the right side of the arch are the same as for the left side except that values for left and right points are reversed. Deflections indicated on sheet 3 are

| | POINT | $M_L$ | $H_L$ | $V_L$ |
|---|---|---|---|---|
| UNIT LOAD NO. 1 | 1/4 | $PR_E r \text{ vers } \Phi_1$ | $PR_E \text{ vers } \Phi_1$ | $PR_E \sin \Phi_1$ |
| | 1/2 | $PR_E r \text{ vers } \Phi_2$ | $PR_E \text{ vers } \Phi_2$ | $PR_E \sin \Phi_2$ |
| | 3/4 | $PR_E r \text{ vers } \Phi_3$ | $PR_E \text{ vers } \Phi_3$ | $PR_E \sin \Phi_3$ |
| | ABUT. | $PR_E r \text{ vers } \Phi_a$ | $PR_E \text{ vers } \Phi_a$ | $PR_E \sin \Phi_a$ |
| UNIT LOAD NO. 2 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | | | |
| | ABUT. | $\frac{PR_E r}{\Phi_1}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_1}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_1} \text{ vers } \Phi_1$ |
| UNIT LOAD NO. 3 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | $\frac{PR_E r}{\Phi_2}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_2}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_2} \text{ vers } \Phi_1$ |
| | ABUT. | $\frac{PR_E r}{\Phi_2}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_2}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_2} \text{ vers } \Phi_2$ |
| UNIT LOAD NO. 4 | 1/4 | | | |
| | 1/2 | $\frac{PR_E r}{\Phi_3}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_3}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_3} \text{ vers } \Phi_1$ |
| | 3/4 | $\frac{PR_E r}{\Phi_3}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_3}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_3} \text{ vers } \Phi_2$ |
| | ABUT. | $\frac{PR_E r}{\Phi_3}(\Phi_3 - \sin\Phi_3)$ | $\frac{PR_E}{\Phi_3}(\Phi_3 - \sin\Phi_3)$ | $\frac{PR_E}{\Phi_3} \text{ vers } \Phi_3$ |
| UNIT LOAD NO. 5 | 1/4 | $\frac{PR_E r}{\Phi_a}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_a}(\Phi_1 - \sin\Phi_1)$ | $\frac{PR_E}{\Phi_a} \text{ vers } \Phi_1$ |
| | 1/2 | $\frac{PR_E r}{\Phi_a}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_a}(\Phi_2 - \sin\Phi_2)$ | $\frac{PR_E}{\Phi_a} \text{ vers } \Phi_2$ |
| | 3/4 | $\frac{PR_E r}{\Phi_a}(\Phi_3 - \sin\Phi_3)$ | $\frac{PR_E}{\Phi_a}(\Phi_3 - \sin\Phi_3)$ | $\frac{PR_E}{\Phi_a} \text{ vers } \Phi_3$ |
| | ABUT | $\frac{PR_E r}{\Phi_a}(\Phi - \sin\Phi_a)$ | $\frac{PR_E}{\Phi_a}(\Phi_a - \sin\Phi)$ | $\frac{PR_E}{\Phi_a} \text{ vers } \Phi_a$ |

See appendix H for tabulation of trigonometic functions

*Figure C-11.* **Formulas for moments, thrusts, and shears of radial loads.–288-D-3114**

| | POINT | $M_L$ | $H_L$ | $V_L$ |
|---|---|---|---|---|
| UNIT LOAD NO. 1 | 1/4 | $Pr^2 (\Phi_1 - \sin \Phi_1)$ | $-Pr \sin \Phi_1$ | $Pr \text{ vers } \Phi_1$ |
| | 1/2 | $Pr^2 (\Phi_2 - \sin \Phi_2)$ | $-Pr \sin \Phi_2$ | $Pr \text{ vers } \Phi_2$ |
| | 3/4 | $Pr^2 (\Phi_3 - \sin \Phi_3)$ | $-Pr \sin \Phi_3$ | $Pr \text{ vers } \Phi_3$ |
| | ABUT. | $Pr^2 (\Phi_a - \sin \Phi_a)$ | $-Pr \sin \Phi_a$ | $Pr \text{ vers } \Phi_a$ |
| UNIT LOAD NO. 2 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | | | |
| | ABUT | $\frac{Pr^2}{\Phi_1}\left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right)$ | $-\frac{Pr}{\Phi_1} \text{ vers } \Phi_1$ | $\frac{Pr}{\Phi_1}(\Phi_1 - \sin \Phi_1)$ |
| UNIT LOAD NO. 3 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | $\frac{Pr^2}{\Phi_2}\left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right)$ | $-\frac{Pr}{\Phi_2} \text{ vers } \Phi_1$ | $\frac{Pr}{\Phi_2}(\Phi_1 - \sin \Phi_1)$ |
| | ABUT | $\frac{Pr^2}{\Phi_2}\left(\frac{\Phi_2^2}{2} - \text{vers } \Phi_2\right)$ | $-\frac{Pr}{\Phi_2} \text{ vers } \Phi_2$ | $\frac{Pr}{\Phi_2}(\Phi_2 - \sin \Phi_2)$ |
| UNIT LOAD NO. 4 | 1/4 | | | |
| | 1/2 | $\frac{Pr^2}{\Phi_3}\left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right)$ | $-\frac{Pr}{\Phi_3} \text{ vers } \Phi_1$ | $\frac{Pr}{\Phi_3}(\Phi_1 - \sin \Phi_1)$ |
| | 3/4 | $\frac{Pr^2}{\Phi_3}\left(\frac{\Phi_2^2}{2} - \text{vers } \Phi_2\right)$ | $-\frac{Pr}{\Phi_3} \text{ vers } \Phi_2$ | $\frac{Pr}{\Phi_3}(\Phi_2 - \sin \Phi_2)$ |
| | ABUT. | $\frac{Pr^2}{\Phi_3}\left(\frac{\Phi_3^2}{2} - \text{vers } \Phi_3\right)$ | $-\frac{Pr}{\Phi_3} \text{ vers } \Phi_3$ | $\frac{Pr}{\Phi_3}(\Phi_3 - \sin \Phi_3)$ |
| UNIT LOAD NO. 5 | 1/4 | $\frac{Pr^2}{\Phi_a}\left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right)$ | $-\frac{Pr}{\Phi_a} \text{ vers } \Phi_1$ | $\frac{Pr}{\Phi_a}(\Phi_1 - \sin \Phi_1)$ |
| | 1/2 | $\frac{Pr^2}{\Phi_a}\left(\frac{\Phi_2^2}{2} - \text{vers } \Phi_2\right)$ | $-\frac{Pr}{\Phi_a} \text{ vers } \Phi_2$ | $\frac{Pr}{\Phi_a}(\Phi_2 - \sin \Phi_2)$ |
| | 3/4 | $\frac{Pr^2}{\Phi_a}\left(\frac{\Phi_3^2}{2} - \text{vers } \Phi_3\right)$ | $-\frac{Pr}{\Phi_a} \text{ vers } \Phi_3$ | $\frac{Pr}{\Phi_a}(\Phi_3 - \sin \Phi_3)$ |
| | ABUT. | $\frac{Pr^2}{\Phi_a}\left(\frac{\Phi_a^2}{2} - \text{vers } \Phi_a\right)$ | $-\frac{Pr}{\Phi_a} \text{ vers } \Phi_a$ | $\frac{Pr}{\Phi_a}(\Phi_a - \sin \Phi_a)$ |

See appendix H for tabulation of trigonometic functions

*Figure C-12.* **Formulas for moments, thrusts, and shears of tangential loads.–288-D-3115**

| | POINT | $M_L$ | $H_L$ | $V_L$ |
|---|---|---|---|---|
| UNIT LOAD NO. 1 | 1/4 | $Pr\,\Phi_1$ | 0 | 0 |
| | 1/2 | $Pr\,\Phi_2$ | 0 | 0 |
| | 3/4 | $Pr\,\Phi_3$ | 0 | 0 |
| | ABUT. | $Pr\,\Phi_a$ | 0 | 0 |
| UNIT LOAD NO. 2 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | | | |
| | ABUT. | $\frac{Pr}{\Phi_1}\,\frac{\Phi_1^2}{2}$ | 0 | 0 |
| UNIT LOAD NO. 3 | 1/4 | | | |
| | 1/2 | | | |
| | 3/4 | $\frac{Pr}{\Phi_2}\,\frac{\Phi_1^2}{2}$ | 0 | 0 |
| | ABUT. | $\frac{Pr}{\Phi_2}\,\frac{\Phi_2^2}{2}$ | 0 | 0 |
| UNIT LOAD NO. 4 | 1/4 | | | |
| | 1/2 | $\frac{Pr}{\Phi_3}\,\frac{\Phi_1^2}{2}$ | 0 | 0 |
| | 3/4 | $\frac{Pr}{\Phi_3}\,\frac{\Phi_2^2}{2}$ | 0 | 0 |
| | ABUT. | $\frac{Pr}{\Phi_3}\,\frac{\Phi_3^2}{2}$ | 0 | 0 |
| UNIT LOAD NO. 5 | 1/4 | $\frac{Pr}{\Phi_a}\,\frac{\Phi_1^2}{2}$ | 0 | 0 |
| | 1/2 | $\frac{Pr}{\Phi_a}\,\frac{\Phi_2^2}{2}$ | 0 | 0 |
| | 3/4 | $\frac{Pr}{\Phi_a}\,\frac{\Phi_3^2}{2}$ | 0 | 0 |
| | ABUT. | $\frac{Pr}{\Phi_a}\,\frac{\Phi_a^2}{2}$ | 0 | 0 |

See appendix H for tabulation of trigonometic functions

*Figure C-13.* Formulas for moments, thrusts, and shears of twist loads.—288-D-3116 plotted for comparison with typical curves given on figures C-14 through C-19. Columns for $V_o$ $x$, $V_o$ sin $\Phi$, and $V_o$ cos $\Phi$ for the left side of the arch can be omitted, since $V_o$ is zero.

C-7. *Tangential Deflections and Angular Movements.*—Figure C-4, or sheet 4, shows calculations for tangential deflections and angular movements for twist load No. 2. Tabular *D*-term constants at the upper right corner of the sheet are copied from *D*-term tables accompanying appendix H. Total tangential deflections and total angular movements are computed by performing the indicated operations, which are similar to those outlined for computing radial deflections on sheet 3. Deflections are plotted to a smooth curve for comparison with typical curves given on figures C-14 to C-19, inclusive.

C-8. *Use of General Forms.*—Since deflection summations are sometimes multiplied by large factors and significant figures in the summations are often reduced by algebraic additions, the use of seven significant figures is necessary in all computations. Computations on sheet 1 should be independently checked, because a small error may produce large errors in subsequent calculations. Total values of arch constants on sheet 2 should be checked, as well as total *D*-terms.

On the upper part of sheet 3, figure C-3, only the total values of *M*, *H*, and *V* at the crown and abutment points should be calculated at first. With these and other values already calculated, radial deflections on sheet 3 and tangential and angular movements on sheet 4 may be determined at the crown of the arch, and the following checks and procedures applied:

(1) Tangential deflections and angular movements for symmetrical loads are zero at the crown.

(2) Radial deflections, tangential deflections, and angular movements for the two sides of the arch are equal at the crown for nonsymmetrical loads.

(3) If crown movements satisfy these conditions, calculations of quantities for other arch points are resumed and carried to completion.

(4) Inspection of final plotted deflection curves will reveal any appreciable errors at the arch quarter-points.

In using formulas given on figures C-11, C-12, and C-13, trigonometric data can be obtained from the trigonometric tables in appendix H.

For the lower arches of a dam, all unit loads are not usually required. A No. 1 (uniform) load, a No. 5 load, and possibly a No. 3 load are sufficient. After a little experience, the loads required may be estimated from the number of cantilevers intersecting the arch. Difficulties in adjustment may require additional loads which can be computed as required. For a symmetrical arch with symmetrical loads, uniform tangential and twist loads are unnecessary in the trial-load analysis.

C-9. *Symmetrical Arch With Nonsymmetrical Loads.*—An arch is nonsymmetrical if the physical properties or dimensions are not identical for both parts of the arch as divided by the reference plane or maximum cross section. The slopes of the abutments, if they disagree to any extent, render an arch nonsymmetrical although the arch structure itself may be symmetrical. If a nonsymmetrical analysis is being made, an arch must be nonsymmetrically loaded even though the arch is symmetrical. The arch at elevation 456 of Monticello Dam is such an example. For this case, the unbalanced loadings required on the left and right parts of the arch necessitate the use of nonsymmetrical loads.

A load placed on the right side will give the same deflections for the corresponding points as though placed on the left side. For example, the radial deflections at the left ¾ point for a load on the left side is numerically equal to the radial deflection at the right ¾ point for a load on the right side, and has the same sign, as shown on figure 4-37.

If the uniform radial load and the temperature-change load are worked in conjunction with nonsymmetrical loads, it is more convenient to load both halves of the arch and use the total arch and load constants from sheet 2, figure C-2. These constants include values for both sides of the arch and

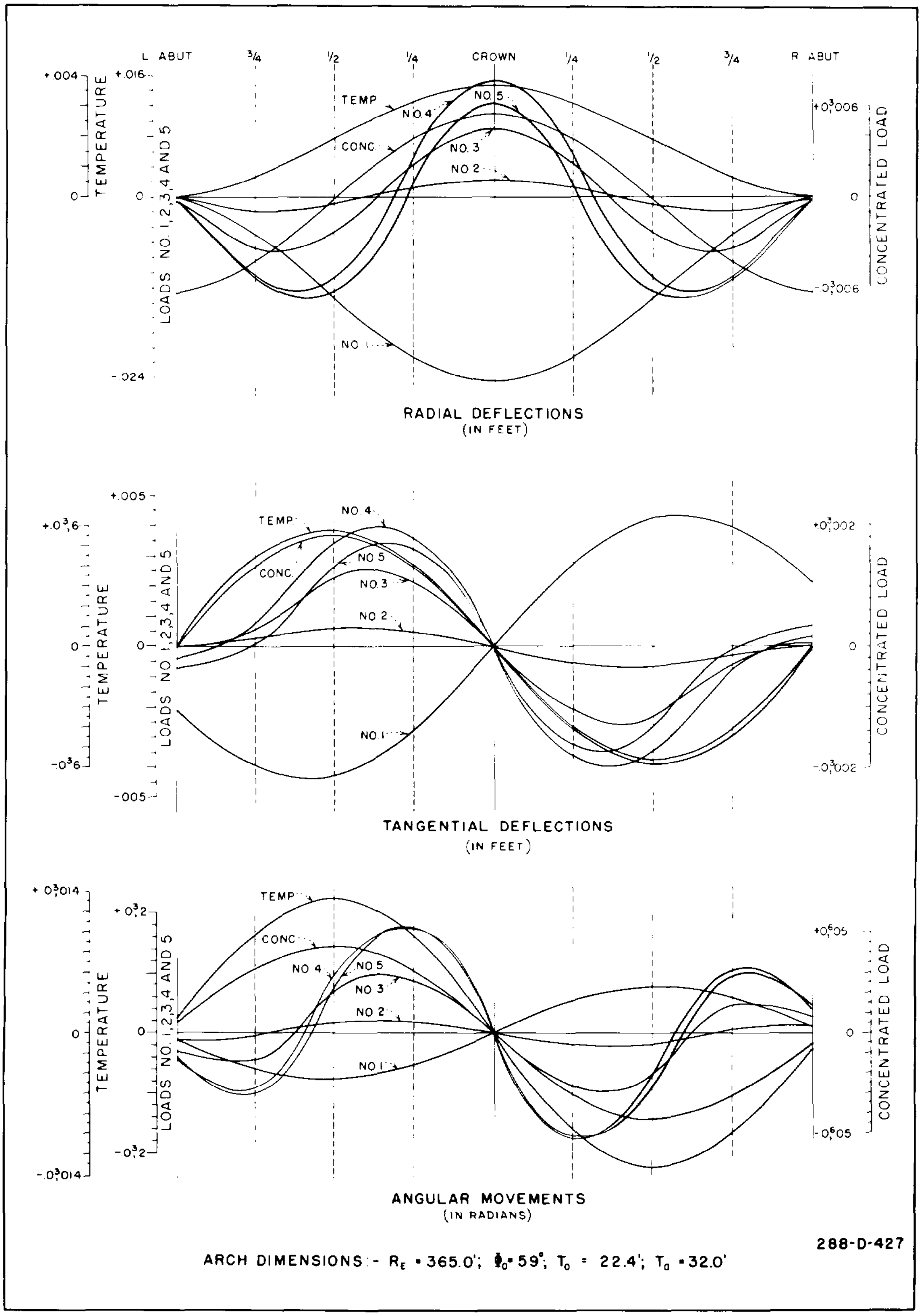


*Figure C-14.* **Typical arch deflection curves for symmetrical unit radial and temperature loads.**

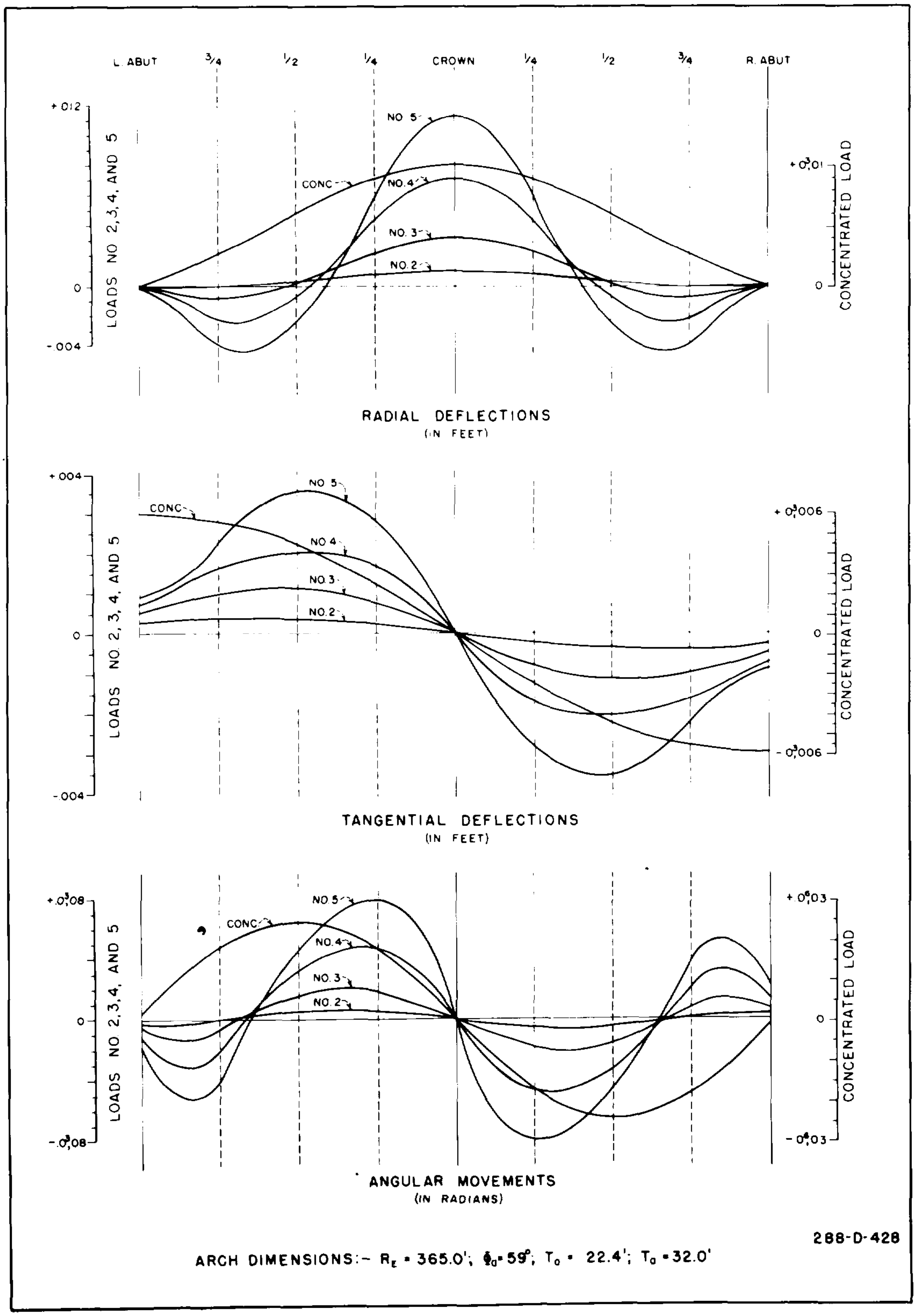


*Figure C-15.* **Typical arch deflections for symmetrical unit tangential loads.**

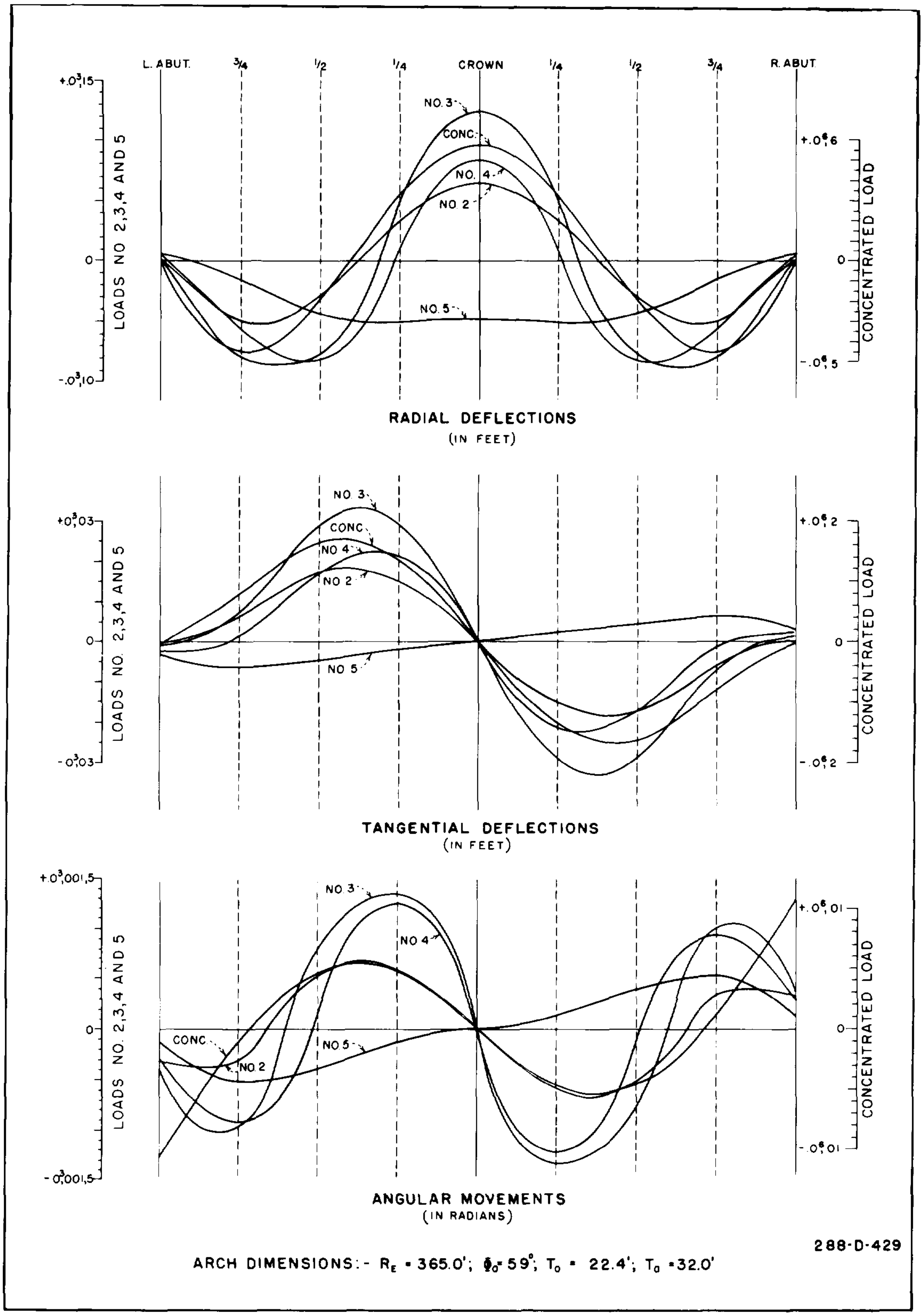


*Figure C-16.* Typical arch deflections for symmetrical unit twist loads.

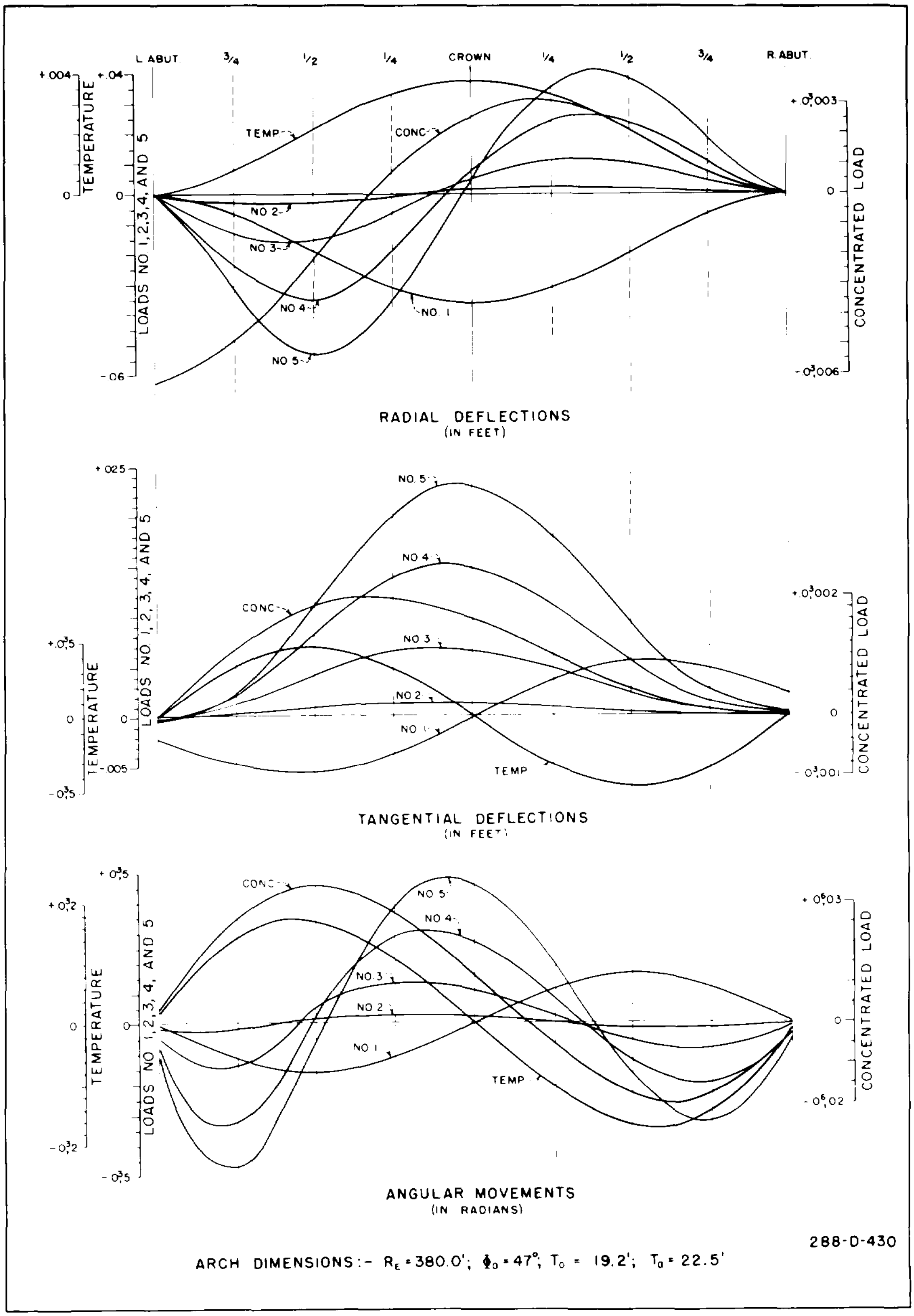
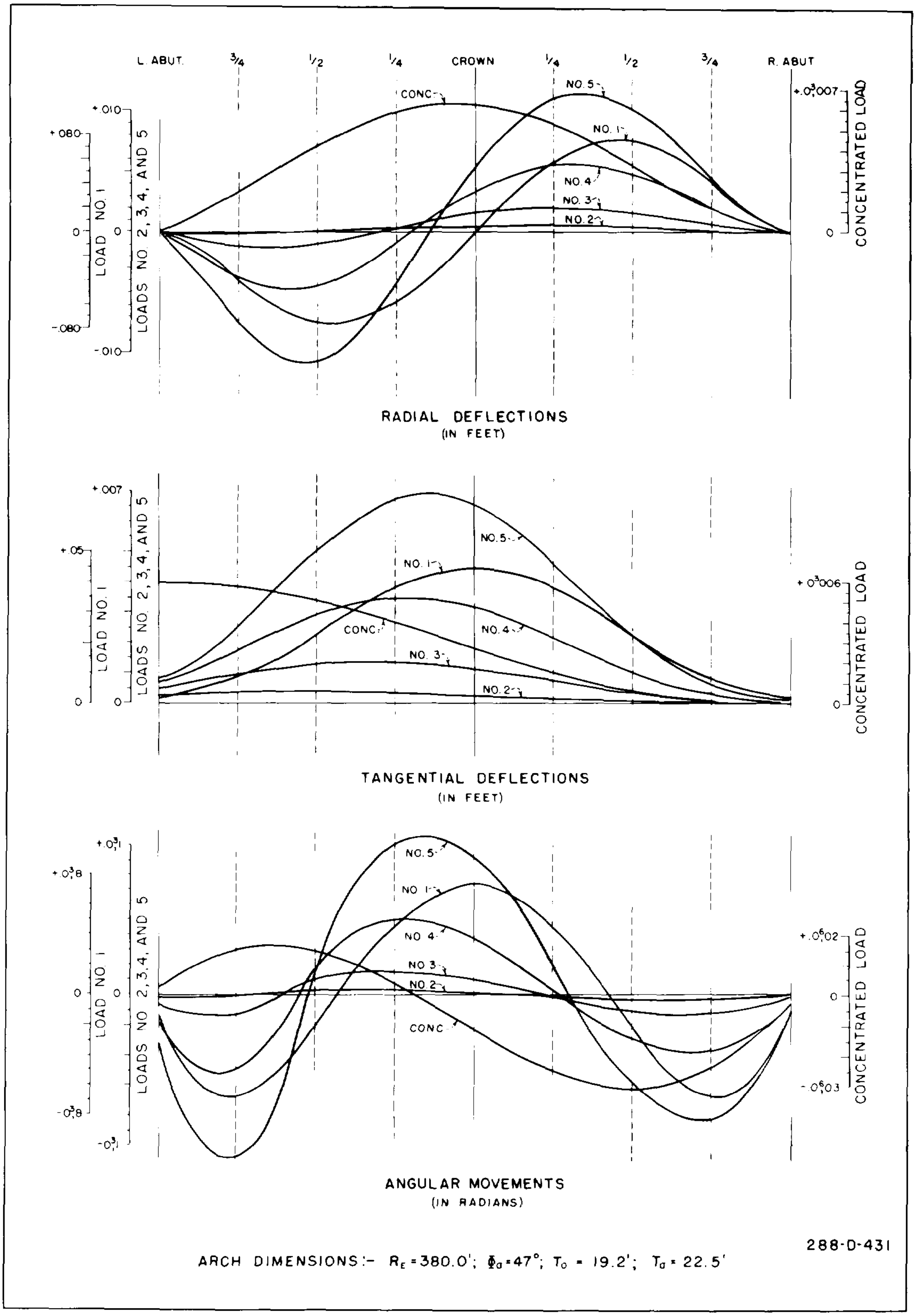


*Figure C-17.* Typical arch deflections for nonsymmetrical radial and temperature loads.



RADIAL DEFLECTIONS
(IN FEET)

TANGENTIAL DEFLECTIONS
(IN FEET)

ANGULAR MOVEMENTS
(IN RADIANS)

ARCH DIMENSIONS:– $R_E = 380.0'$; $\Phi_a = 47°$; $T_0 = 19.2'$; $T_a = 22.5'$

*Figure C-18.* Typical arch deflections for nonsymmetrical tangential loads.

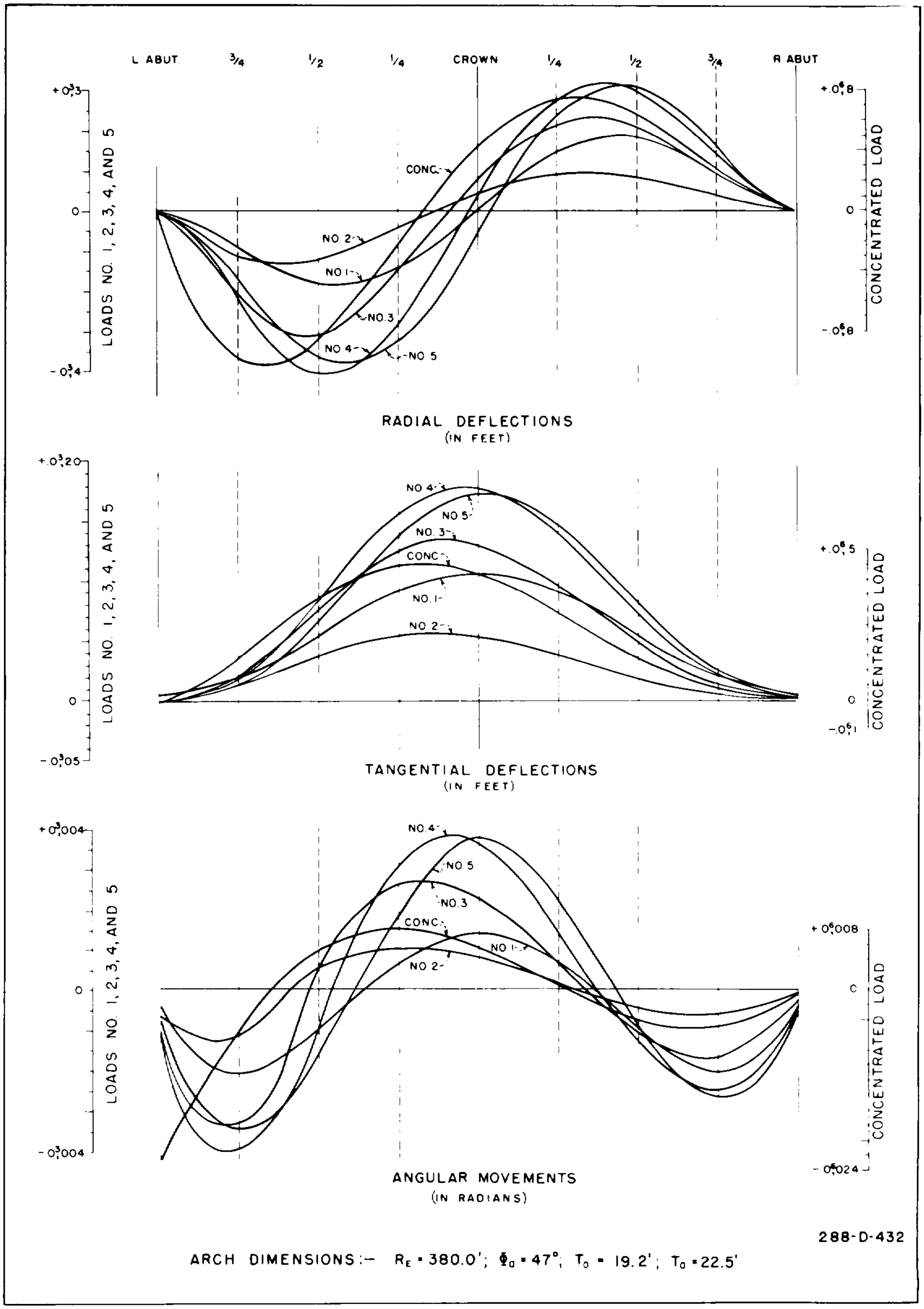


*Figure C-19.* **Typical arch deflections for nonsymmetrical twist loads.**

are double the values for symmetrical loading. The advantage of this operation is that a single set of arch constants can be used for the computation of these loads and the nonsymmetrical loads.

In using computation forms for the analysis of nonsymmetrical triangular and concentrated loads on symmetrical arches, many computations are omitted. These are briefly listed as follows:

Sheet 1–All arch constants, trigonometric functions, coordinates, and abutment movement functions for the right side of the arch, because they are the same as for the left side.

Sheet 2–All computations for $C_1$ and $B_2$, because the summations are zero. All values for $D_1$, $D_2$, and $D_3$ for the right side of the arch, because the load is absent from that side of the arch.

Sheet 3–$D$-terms and $M_R$, $H_R$, and $V_R$ for the right side of the arch, because the load is absent from that side of the arch.

Sheet 4–$D$-terms for the right side of the arch, because the load is absent from that side of the arch.

The omissions on sheet 2 reduce the equations of equilibrium to:

$$A_1 M_o + B_1 H_o = D_1$$
$$B_1 M_o + B_3 H_o = D_3$$
$$C_2 V_o = D_2$$

the simultaneous solution of which gives:

$$K_o = A_1 B_3 - B_1{}^2$$
$$M_o = (D_1 B_3 - D_3 B_1)\frac{1}{K_o}$$
$$H_o = (-D_1 B_1 + D_3 A_1)\frac{1}{K_o}$$
$$V_o = \frac{D_2}{C_2}$$

for the solution of crown forces.

It will be noted that $V_o$, the resultant crown shear, has both a positive and a negative sign in the table at the top of sheet 3, figure C-3. The positive sign is used for computing deflections on the left half of the arch, as shown on sheet 2, and the negative sign for computing deflections on the right half.

A nonsymmetrical analysis requires uniform tangential and twist loads so that loadings may be applied at the crown of the arch. To be continuous, these loads must act in the same direction on both sides of the arch. This amounts to placing a negative uniform load on the right side of the arch, and requires a reversal of algebraic sign of the following quantities for the portion of the uniform load on the right side of the arch: $M_R$, $H_R$, $V_R$; all right-side parts of the total $D_1$, $D_2$, and $D_3$ terms used for the solution of crown forces; and the $D_1$, $D_2$, and $D_3$ terms used in computing deflections on the right side of the arch. These changes cause the total $D_1$ and $D_3$ terms and values of $M_o$ and $H_o$ on sheet 2 to become zero.

C-10. *Nonsymmetrical Arch.*—A nonsymmetrical arch of uniform thickness is encountered only if it is desirable to have the crown of the arch coincide with the maximum cantilever section, or if abutment yielding conditions are different on the two sides of the canyon. Otherwise the crown is placed at the middle and the arch is analyzed as symmetrical.

A nonsymmetrical arch will obviously have nonsymmetrical loads, requiring separate calculation of loads on both sides of the arch. The general computation forms with no omissions are used for their analyses.

C-11. *Curves and Tabulations.*—After analyses have been made for all unit loads on an arch, curves are plotted showing resulting radial, tangential, and angular movements. Tabulations are also prepared showing resulting forces, moments, and deflections.

As stated previously, curves of deflections are convenient for checking the accuracy of the computations. Errors in calculations of deflections are indicated by pronounced variations from smooth curves. Studies of curves for various unit loads on an arch, together with considerations of corresponding curves for other arches in the same dam, often establish the general trend of deflection curves and may indicate other errors.

Another check is possible with deflection curves, due to the interesting relation between radial deflection and angular movement curves for the same load. The angular movement at any point approximately represents the slope of the radial deflection curve at that point. Thus the maximum angular movement and the maximum slope of the radial deflection curve occur at the same point. Similarly, the maximum radial deflection occurs at the point of zero angular movement. These relations are particularly evident if the arch is long and thin, so that the greater part of the movement is due to bending. In short and thick arches these relations do not hold, because large parts of the radial deflections are produced by shear detrusions.

Typical deflection curves for symmetrical unit loads on an arch element are shown on figures C-14, C-15, and C-16. The dimensions of the elements are shown below the curves. Although the curves are for a variable-thickness arch, they are typical for uniform-thickness arches and fillet arches.

Figures C-17, C-18, and C-19 show typical deflection curves for nonsymmetrical unit loads on a symmetrical arch. These curves are for a variable-thickness arch with dimensions similar to those for the arch represented on figures C-14, C-15, and C-16. The curves for unit triangular and concentrated loads are for loads on the left part of the arch only. If the curves are reversed, so that the left abutment becomes the right abutment and vice versa, and if the changes of sign indicated on figure 4-37 are applied, the curves will be those for unit triangular and concentrated loads on the right side of the arch. The deflections for uniform loads remain the same.

Moments, thrusts, and shears at the crown and abutment points caused by unit loads are tabulated for use in calculating stresses. Values at other arch points are obtained if stresses are desired at such points. Tabulations of arch-point deflections are also made for all unit loads.

## B. UNIFORM-THICKNESS ARCH WITH FILLETS

C-12. ***Introduction.***—The computation forms shown on figures C-1 through C-4 can be used for analyzing a fillet arch, but arch and load constants must be evaluated prior to their use by means of supplementary calculations as illustrated in the following sections. These constants are inserted in the general forms of figures C-1 to C-4, inclusive, and the analysis completed as described for a uniform-thickness arch, with the changes discussed in section 4-35(e) and shown in the following illustrations. Eccentricities and multipliers for arch and load constants are calculated as shown on figures C-22 and C-23, after the thickness and centerline radius at the arch quarter-points have been determined.

For the purpose of illustration, computations are shown for the uniform-thickness arch, with fillet, at elevation 350 for Monticello Dam (fig. C-20). Data for this arch are given on figures C-7 and C-21.

C-13. ***Arch Constants for Fillet Arch.***- An arch with abutment fillets at the downstream face is a special case of a variable-thickness arch. Since the computations are similar to those for a variable-thickness arch, the computations may be made on the same forms using formulas mentioned in section 4-36(c). The general form for computation of arch constants for variable-thickness arches is shown on figure C-24. For convenience, all summations involving Δ-values of any constant are worked on one sheet, and those summations which are parts of other constants are transferred to the proper sheets for combination with the other principal parts. The computation for arch constant $A_1$ is shown, and parts of the computations for $B_1$ and $B_3$ are shown on the same sheet.

The computation of total arch constants is shown on figure C-25. These constants are computed as described in section 4-36(c). The

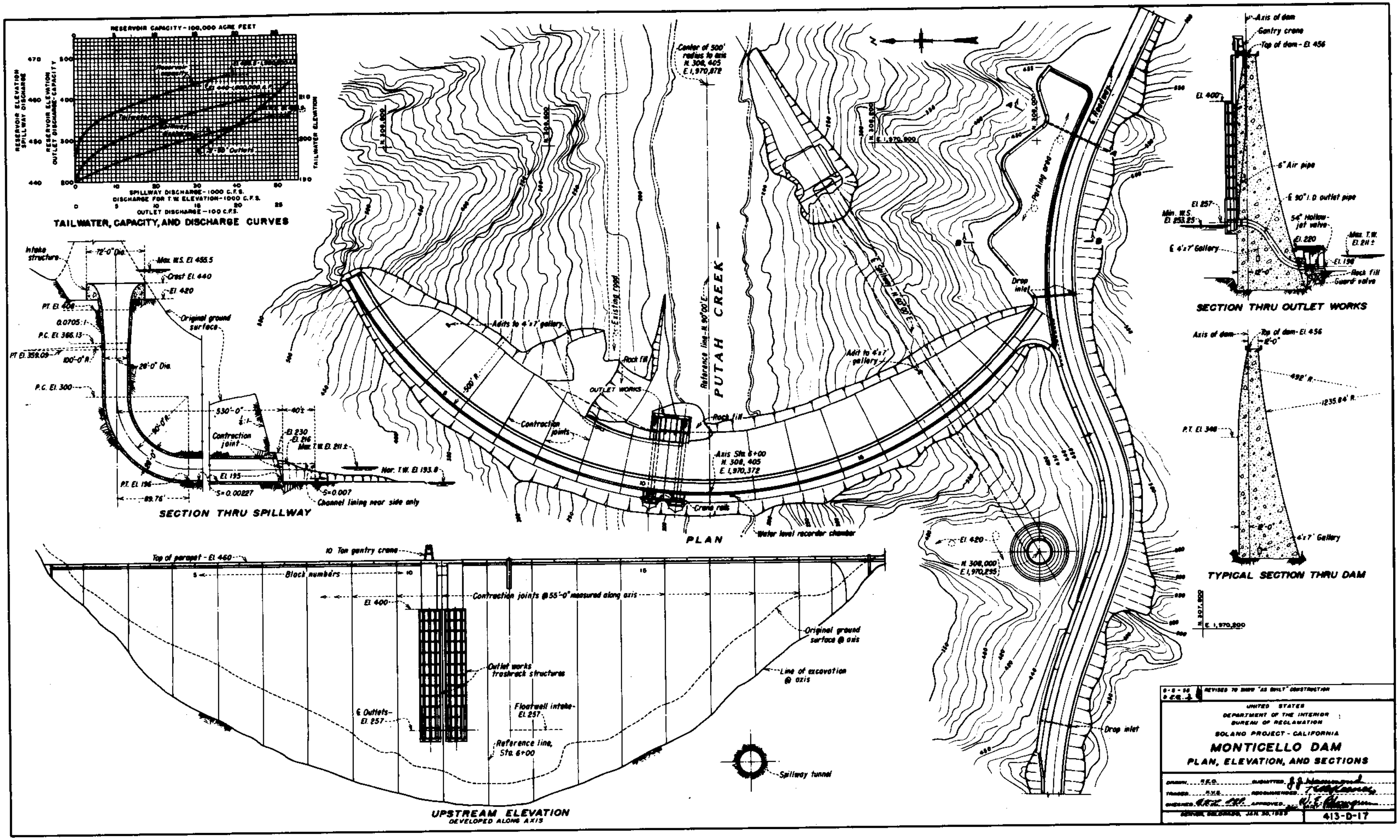


*Figure C-20.* Monticello Dam—plan, elevation, and sections.

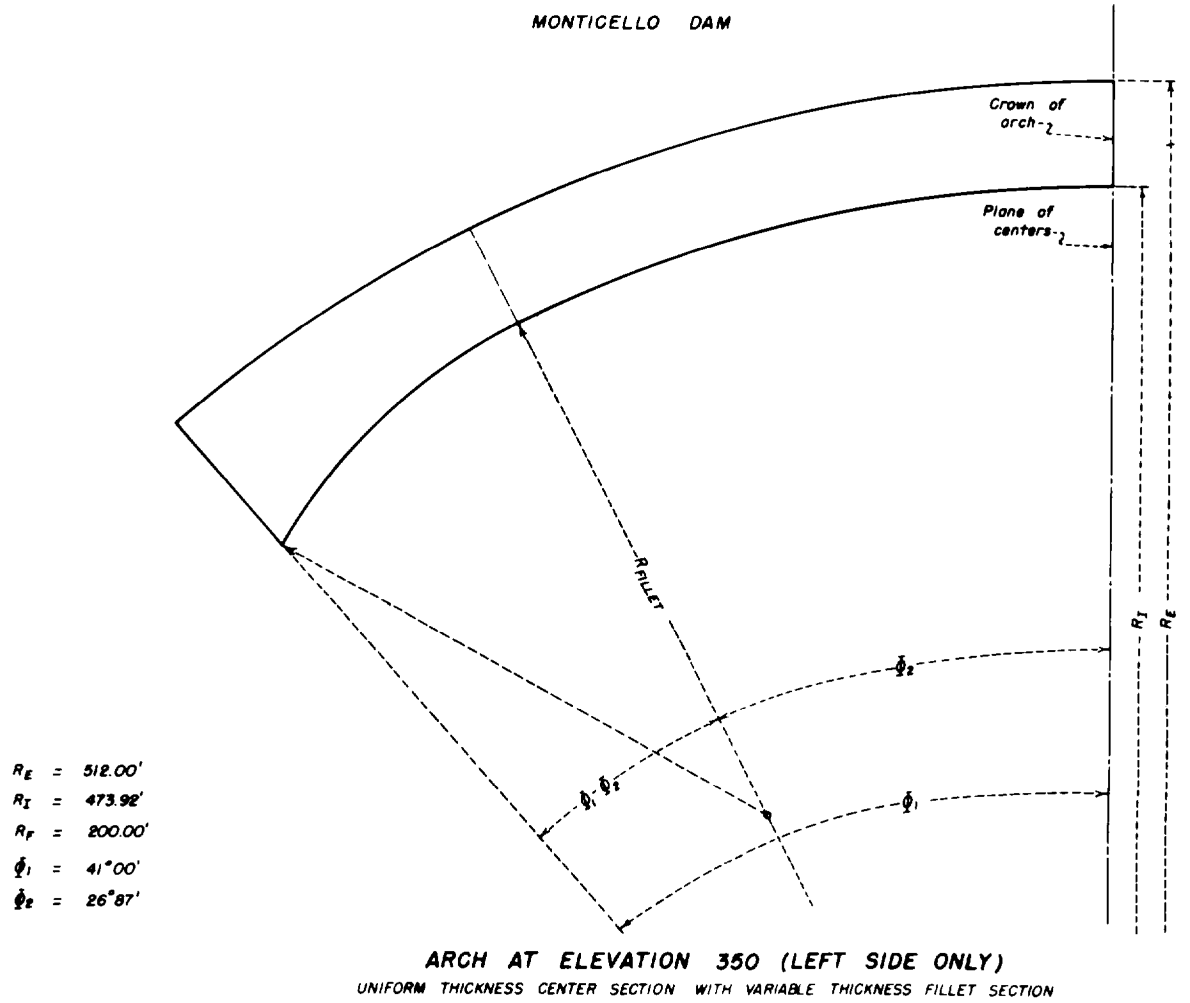


*Figure C-21.* **Monticello Dam study–left side of arch with fillet section.–288-D-2699**

calculation of $A_1$ is taken from figures C-22 and C-24; that is, ${}_4A_1' = .0^9, 048, 030, 4$ and ${}_0A_1'' = .0^9, 161, 004, 8$, their sum being the value for ${}_0A_1 = .0^9, 209, 035, 2$. Values of other total arch constants are calculated in a similar manner, as indicated by figure C-25. Calculations for $x$ and $y$ are shown on figure C-26.

C-14. *Load Constants for Fillet Arch.*–Computations for $D'$ and $D''$ values for $D$-terms $D_1$, $D_2$, and $D_3$ for a radial load No. 1 are given on figures C-27 to C-30, inclusive. $D$-terms for the uniform-thickness central section are computed as shown in sections 4-34(g) through 4-34(p). In the fillet section, tabular values for $D$-terms in the first column of each sheet are taken from the load constant tables accompanying appendix H, using figure C-31 as a guide. This figure gives fractions of circular arch $D$-terms to use for prime values. The formulas in figure C-31 are clarified by figure C-32, which illustrates the method followed in determining formulas for radial load No. 5 at the ½ point. Since $D$-terms are deflections due to load at the left of the point considered, it can be seen that the tabular $D$-term at the ½ point, for the load between the ½ point and the abutment, is the tabular value at the ½ point for load 5 for angle $\Phi_a$ (see fig. C-32(a)). Likewise, the tabular $D$-terms at the ½ point for the load between the ½ and ¾ points, $\Phi_2$ to $\Phi_3$, is equal to ¾ of the $D$-term

# MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### ARCH CONSTANTS FOR UNIFORM THICKNESS
### CENTRAL SECTION 0-4

$R_E = 512.00; \quad r_0 = r_2 = r_4 = 492.96; \quad T = 38.08$

$\frac{1}{E_c} = .0^{6}_{,}002,777,8$

$\frac{r}{E_cT} = .0^{6}_{,}035,959,384$

$\frac{12r}{E_cT^3} = .0^{9}_{,}297,577,17$

$\frac{12r^2}{E_cT^3} = .0^{6}_{,}146,693,64$

$\frac{12r^3}{E_cT^3} = .0^{3}_{,}072,314,097$

$\frac{12r^4}{E_cT^3} = .035,647,957$

$\frac{r}{E_cT} R_E = .0^{3}_{,}018,411,205$

$\frac{r^2}{E_cT} = .0^{3}_{,}017,726,538$

$\frac{12r^2}{E_cT^3} R_E = .0^{3}_{,}075,107,144$

$\frac{12r^3}{E_cT^3} R_E = .037,024,818$

POINT 0 (CROWN) $\phi_{0-4} = 31°$

${}_0A_1'' = .541,052,1 \cdot \frac{12r}{E_cT^3} = .0^{9}_{,}161,004,8$

${}_0B_1'' = .026,013,99 \cdot \frac{12r^2}{E_cT^3} = .0^{6}_{,}003,816,087$

${}_0C_1'' = .142,832,7 \cdot \frac{12r^2}{E_cT^3} = .0^{6}_{,}020,952,65$

$B_2$ - 1st Term = $.010,200,59 \cdot \frac{12r^3}{E_cT^3} = 0^{6}_{,}737,646,5$

$B_2$ - 2nd Term = $.265,264,2 \cdot \frac{r}{E_cT} = .0^{6}_{,}009,538,7$

${}_0B_2'' = .0^{6}_{,}747,185,2$

$B_3$ - 1st Term = $002,238,851 \cdot \frac{12r^3}{E_cT^3} = .0^{6}_{,}161,900,5$

$B_3$ - 2nd Term = $.640,630,3 \cdot \frac{r}{E_cT} = .0^{6}_{,}023,036,7$

${}_0B_3'' = .0^{6}_{,}184,937,2$

288-D-2700

*Figure C-22.* **Monticello Dam study—arch constants for uniform-thickness section.**

....*MONTICELLO*....... DAM STUDY NO....*A-11*..........

VARIABLE THICKNESS ARCH WITH CONSTANT UPSTREAM RADIUS

*FILLET SECTION 4-8*

ECCENTRICITIES AND MULTIPLIERS FOR ARCH AND LOAD CONSTANTS

ARCH AT ELEV....*350 L*.........

COMPUTED BY.......DATE..........

CHECKED BY..........DATE...........

| POINT | $\Phi$ | VOUSSOUR | AT POINT | | FOR VOUSSOIR | | VOUSSOIR TO POINT | | | | $e_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $T$ | $r_p$ | $T_v$ | $r_v$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | |
| | | | | | | | | | | | |
| 4 | 0° | | 38.08 | 492.96 | | | | | | | |
| | | 4 to 5 | | | 38.71 | 492.645 | .315 | | | | |
| 5 | 3° 30' | | 39.34 | 492.33 | | | | | | | |
| | | 5 to 6 | | | 41.21 | 491.395 | 1.565 | | | | |
| 6 | 7° 00' | | 43.08 | 490.46 | | | | | | | |
| | | 6 to 7 | | | 46.22 | 488.89 | 4.07 | | | | |
| 7 | 10° 30' | | 49.36 | 487.32 | | | | | | | |
| | | 7 to 8 | | | 53.76 | 485.12 | 7.84 | | | | |
| 8 | 14° 00' | | 58.16 | 482.92 | | | | | | | 10.04 |
| | | | | | | | | | | | |

$E_C = \text{..........} \frac{1}{E_c} = .\overset{6}{0}002,777,778$

$R_E = 512.0$

$r = R_E - \frac{T}{2}$ $\quad I = \frac{T_v^3}{12}$

$r_v = R_E - \frac{T_v}{2}$

$e_{4,5,6,7} = r_p - r_v = e$, for transfer of arch and load const's.

$e_4 = e'$, for tangential load eccentricity.

$e_p$ = eccentricity of arch point with reference to arc of center line through crown.

| VOUSSOIR | $\frac{1}{E_c T_v}$ | $\frac{r_v}{E_c T_v}$ | $\frac{r_v^2}{E_c T_v}$ | $\frac{r_v}{E_c I_v}$ | $\frac{r_v^2}{E_c I_v}$ | $\frac{r_v^3}{E_c I_v}$ | $\frac{r_v^4}{E_c I_v}$ | $\frac{r_v R_E}{E_c T_v}$ | $\frac{r_v^2 R_E}{E_c I_v}$ | $\frac{r_v^3 R_E}{E_c I_v}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |
| 4 to 5 | $.\overset{9}{0}071,758,661$ | $.\overset{6}{0}035,351,546$ | $.\overset{3}{0}017,415,762$ | $.\overset{9}{0}283,102,25$ | $.\overset{6}{0}139,468,91$ | $.\overset{3}{0}068,708,661$ | .033,848,978 | $.\overset{3}{0}018,099,992$ | $.\overset{3}{0}071,408,082$ | .035,178,834 |
| 5 to 6 | $.\overset{9}{0}067,405,430$ | $.\overset{6}{0}033,122,691$ | $.\overset{3}{0}016,276,325$ | $.\overset{9}{0}234,046,22$ | $.\overset{6}{0}115,009,14$ | $.\overset{3}{0}056,514,916$ | .027,771,147 | $.\overset{3}{0}016,958,817$ | $.\overset{3}{0}058,884,680$ | .028,935,637 |
| 6 to 7 | $.\overset{9}{0}060,099,043$ | $.\overset{6}{0}029,381,821$ | $.\overset{3}{0}014,364,478$ | $.\overset{9}{0}165,044,13$ | $.\overset{6}{0}080,688,43$ | $.\overset{3}{0}039,447,767$ | .019,285,619 | $.\overset{3}{0}015,043,492$ | $.\overset{3}{0}041,312,476$ | .020,197,257 |
| 7 to 8 | $.\overset{9}{0}051,669973$ | $.\overset{6}{0}025,066,137$ | $.\overset{3}{0}012,160,084$ | $.\overset{9}{0}104,075,90$ | $.\overset{6}{0}050,489,30$ | $.\overset{3}{0}024,493,369$ | .011,882,223 | $.\overset{3}{0}012,833,862$ | $.\overset{3}{0}025,850,522$ | .012,540,605 |
| | | | | | | | | | | |

*Figure C-23.* **Monticello Dam study–eccentricities and multipliers for arch and load constants.–DS2-1(87)**

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

MONTICELLO......DAM
STUDY NO...A-11...
FILLET SECTION 4-8 ARCH AT ELEV...350 L...(Fillet) COMPUTED BY......
$A_1$ DATE..................

| PT. | $\Phi$ | $A_1$ | $\Delta A_1$ | $\frac{r}{EI}\Delta A_1$ | | Part of $B_1$ $\frac{r}{EI}\Delta A_1 e$ | | Part of $B_3$ $\frac{r}{EI}\Delta A_1 e^2$ |
|---|---|---|---|---|---|---|---|---|
| 4 | | | | | | | | |
| | | | .061,086,525 | $.\overset{9}{0},017,293,73$ | | $.\overset{9}{0},005,447,52$ | | |
| | | | .061,086,525 | $.\overset{9}{0},014,297,07$ | | $.\overset{9}{0},022,374,91$ | | |
| | | | .061,086,525 | $.\overset{9}{0},010,081,97$ | | $.\overset{9}{0},041,033,62$ | | |
| | | | .061,086,525 | $.\overset{9}{0},006,357,635$ | | $.\overset{9}{0},049,843,86$ | | |
| | 14° | .244,346,1 | | | | | | |
| | | | $_{+}A_1 = \Sigma = .\overset{9}{0},048,030,40$ | | | $\Sigma = .\overset{9}{0},1.18,699,9$ | | $\Sigma = .\overset{9}{0},594,515,4$ |

*Figure C-24.* Monticello Dam study–arch constants for fillet section.–DS2-1(88)

# MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### VALUES FOR TOTAL ARCH CONSTANTS

POINT O (CROWN)

$${}_0A_1 = {}_4A_1' + {}_0A'' = 0\overset{9}{,}209{,}035{,}2$$

$${}_0B_1 = {}_4A_1' y_{(4-0)} + {}_4B_1' \cos\phi_{(4-0)} + {}_4C_1' \sin\phi_{(4-0)} + {}_0B_1'' = 0\overset{6}{,}008{,}628{,}976$$

$${}_0C_1 = {}_4A_1' x_{(4-0)} - {}_4B_1' \sin\phi_{(4-0)} + {}_4C_1' \cos\phi_{(4-0)} + {}_0C_1'' = 0\overset{6}{,}034{,}978{,}67$$

$${}_0B_2 = x_{(4-0)}\left({}_4A_1' y_{(4-0)} + {}_4B_1' \cos\phi_{(4-0)} + {}_4C_1' \sin\phi_{(4-0)}\right) + y_{(4-0)}\left({}_4C_1' \cos\phi_{(4-0)} - {}_4B_1' \sin\phi_{(4-0)}\right)$$

$$+ {}_4B_2' \left(\cos^2\phi_{(4-0)} - \sin^2\phi_{(4-0)}\right) + \left({}_4C_2' - {}_4B_3'\right)\sin\phi_{(4-0)} \cos\phi_{(4-0)} + {}_0B_2'' = 0\overset{3}{,}001{,}221{,}960$$

$$+ 0\overset{6}{,}128{,}952{,}7 + 0\overset{6}{,}011{,}427{,}3 + 0\overset{6}{,}075{,}234{,}7 + {}_0B_2'' = 0\overset{3}{,}002{,}184{,}760$$

$${}_0C_2 = x_{(4-0)}\left({}_4A_1' x_{(4-0)} - 2\,{}_4B_1' \sin\phi_{(4-0)} + 2\,{}_4C_1' \cos\phi_{(4-0)}\right) - 2\,{}_4B_2' \sin\phi_{(4-0)} \cos\phi_{(4-0)}$$

$$+ {}_4B_3' \sin^2\phi_{(4-0)} + {}_4C_2' \cos^2\phi_{(4-0)} + {}_0C_2'' = 0\overset{3}{,}004{,}026{,}098 - 0\overset{6}{,}021{,}491{,}6 + 0\overset{6}{,}002{,}993{,}3$$

$$+ 0\overset{6}{,}133{,}502{,}4 + {}_0C_2'' = 0\overset{3}{,}007{,}796{,}346$$

$${}_0B_3 = y_{(4-0)}\left({}_4A_1' y_{(4-0)} + 2\,{}_4B_1' \cos\phi_{(4-0)} + 2\,{}_4C_1' \sin\phi_{(4-0)}\right) + 2\,{}_4B_2' \sin\phi_{(4-0)} \cos\phi_{(4-0)}$$

$$+ {}_4B_3' \cos^2\phi_{(4-0)} + {}_4C_2' \sin^2\phi_{(4-0)} + {}_0B_3'' = 0\overset{6}{,}439{,}639{,}4 + 0\overset{6}{,}021{,}491{,}60 + 0\overset{6}{,}008{,}290{,}85$$

$$+ 0\overset{6}{,}048{,}198{,}82 + {}_0B_3'' = 0\overset{6}{,}702{,}557{,}9$$

288-D-2701

*Figure C-25.* **Monticello Dam study—values for total arch constants.**

## MONTICELLO DAM

STUDY No. A-11
ARCH AT El. 350 - Left

### CALCULATIONS FOR x AND y

### TRANSFERRING ARCH CONSTANTS AND D-TERMS

POINT 4 TO POINT 2 15° 30'

$\sin\Phi_{4-2} = .267,238,4$ $x = r_4 \sin\Phi_{4-2} = 131.737,8$

$\cos\Phi_{4-2} = .963,630,5$ $y = r_2 - r_4 \cos\Phi_{4-2} = 17.928,73$

POINT 4 TO POINT 0 31°

$\sin\Phi_{4-0} = .515,038,1$ $x = 253.893,2$

$\cos\Phi_{4-0} = .857,167,3$ $y = 70.410,81$

### TRANSFERRING $M_L$, $H_L$, AND $V_L$

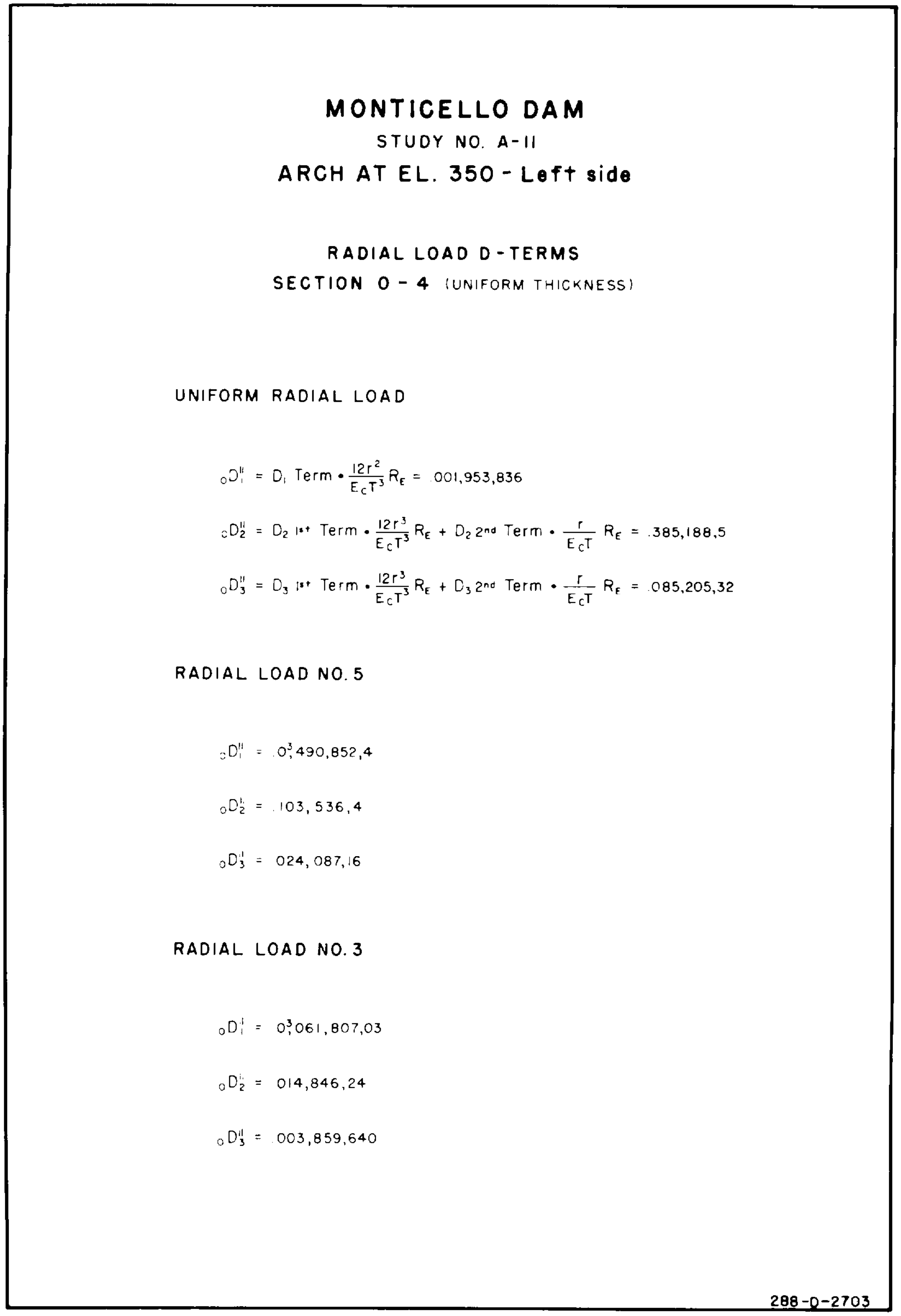
POINT 4 TO POINT 8 14°

$\sin\Phi_{4-8} = .241,921,9$ $x = 116.828,9$

$\cos\Phi_{4-8} = .970,295,7$ $y = 24.384,80$

288-D-2702

*Figure C-26.* **Monticello Dam study—*x* and *y* distances used in transferring arch constants and *D*-terms.**



## MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### RADIAL LOAD D-TERMS

### SECTION 0 - 4 (UNIFORM THICKNESS)

UNIFORM RADIAL LOAD

$$ {}_{0}D_1'' = D_1 \text{ Term} \bullet \frac{12r^2}{E_cT^3} R_E = .001{,}953{,}836 $$

$$ {}_{0}D_2'' = D_2 \text{ 1st Term} \bullet \frac{12r^3}{E_cT^3} R_E + D_2 \text{ 2nd Term} \bullet \frac{r}{E_cT} R_E = .385{,}188{,}5 $$

$$ {}_{0}D_3'' = D_3 \text{ 1st Term} \bullet \frac{12r^3}{E_cT^3} R_E + D_3 \text{ 2nd Term} \bullet \frac{r}{E_cT} R_E = .085{,}205{,}32 $$

RADIAL LOAD NO. 5

$ {}_{0}D_1'' = .0^{3}{,}490{,}852{,}4 $

$ {}_{0}D_2'' = .103{,}536{,}4 $

$ {}_{0}D_3'' = 024{,}087{,}16 $

RADIAL LOAD NO. 3

$ {}_{0}D_1'' = 0^{3}{,}061{,}807{,}03 $

$ {}_{0}D_2'' = 014{,}846{,}24 $

$ {}_{0}D_3'' = .003{,}859{,}640 $

288-D-2703

*Figure C-27.* **Monticello Dam study–radial load *D*-terms.**

# COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

MONTICELLO DAM
STUDY NO. A-11
FILLET SECTION 4-8 ARCH AT ELEV. 350 - LEFT
COMPUTED BY........
RADIAL LOAD NO. 1 - $D_1'$
DATE........

| PT. | Φ | $D_1$ | $\Delta D_1$ (FOR VOUSSOIR) | $\frac{r^2R_E}{EI}\Delta D_1$ |  | PART OF $D_3$ $\frac{r^2R_E}{EI}\Delta D_1 \cdot e$ |  |  |
|---|---|---|---|---|---|---|---|---|
| 4 |  | 0 |  |  |  |  |  |  |
|  |  |  | .037,984,3 | $.0^3002,712,39$ |  |  |  |  |
|  |  | .037,984,3 |  |  |  |  |  |  |
|  |  |  | .265,719,9 | $.0^3015,646,83$ |  |  |  |  |
|  |  | .303,704,2 |  |  |  |  |  |  |
|  |  |  | .720,341,8 | $.0^3029,759,10$ |  |  |  |  |
|  |  | 1.024,046 |  |  |  |  |  |  |
|  |  |  | 1.400,154 | $.0^3036,194,71$ |  |  |  |  |
|  | 14° | 2.424,200 |  |  |  |  |  |  |
|  |  |  |  | ${}_4D_1' = \Sigma = .0^3084,313,03$ |  | $\Sigma = .0^3430,227,8$ |  |  |

*Figure C-28.* **Monticello Dam study–$D_1'$-term for fillet section.–DS2-1(92)**

# COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Monticello DAM
STUDY NO. A-11
ARCH AT ELEV. 350 - Left
Fillet Section 4-8
Radial Load No. 1 - $D_2'$

COMPUTED BY........
DATE........

| PT. | $\Phi$ | $D_2$ 1st Term | $\Delta D_2$ 1st Term (for voussoir) | $\frac{r^3R_E}{EI}\Delta D_2$ 1st Term | $D_2$ 2nd Term | $\Delta D_2$ 2nd Term (for voussoir) | $\frac{rR_E}{ET}\Delta D_2$ 2nd Term | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | | | 0 | | | |
| | | | .001,739,49 | | | 5.592,126 | | |
| | | .001,739,49 | | | 5.592,126 | | | |
| | | | .026,040,44 | | | 16.713,864 | | |
| 4 | | .027,779,93 | | | 22.305,99 | | | |
| | | | .112,419,17 | | | 27.648,89 | | |
| | | .140,199,1 | | | 49.954,88 | | | |
| | | | .300,972,8 | | | 38.275,60 | | |
| | 14° | .441,171,9 | | | 88.230,48 | | | |
| $_4D_2' = .008,151,454$ | | | | $\Sigma = .006,859,630$ | | | $\Sigma = .001,291,824$ | |

Figure C-29. Monticello Dam study–$D_2'$-term for fillet section.–DS2-1(93)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Monticello DAM
Fillet Section 4-8 STUDY NO. A-11
ARCH AT ELEV. 350 - Left
Radial Load No. 1 - $D_3'$

COMPUTED BY........
DATE........

| PT. | $\Phi$ | $D_3$ 1st Term | $\Delta D_3$ 1st Term (for voussoir) | $\frac{r^3R_E}{EI}\Delta D_3$ 1st Term | $D_3$ 2nd Term | $\Delta D_3$ 2nd Term (for voussoir) | $\frac{rR_E}{ET}\Delta D_3$ 2nd Term | |
|---|---|---|---|---|---|---|---|---|
| 4 | | 0 | | | 0 | | | |
| | | | $.0^3042,511$ | | | .189,836 | | |
| | | $.0^3042,511$ | | | .189,836 | | | |
| | | | .001,316,029 | | | 1.325,968 | | |
| | | .001,358,54 | | | 1.515,804 | | | |
| | | | .008,935,06 | | | 3.583,839 | | |
| | | .010,293,6 | | | 5.099,643 | | | |
| | | | .032,948,8 | | | 6.934,867 | | |
| | 14° | .043,242,4 | | | 12.034,51 | | | |
| ${}_4D_3' = .001,232,302$ | | | | $\Sigma = .0^3633,237,2$ | | | $\Sigma = .0^3168,837,5$ | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

Figure C-30. Monticello Dam study—$D_3'$-term for fillet section.—DS2-1(94)

| PT. | $\Phi$ | D VALUE AT ARCH POINT. FOR LOAD PATTERN FROM ARCH POINT THROUGH ANGLE $\Phi$ | | | | |
|---|---|---|---|---|---|---|
| | | NO. 1 LOAD | NO. 2 LOAD | NO. 3 LOAD | NO. 4 LOAD | NO. 5 LOAD |
| CROWN | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\Phi_1$ | D, Load No 1, 3/4 Pt. | 0 | 0 | 0 | 1/4 D, Load No 2, 3/4 Pt. |
| | $\Phi_2$ | D, Load No 1, 1/2 Pt. | 0 | 0 | 1/3 D, Load No 2, 1/2 Pt. | 1/2 D, Load No. 3, 1/2 Pt. |
| | $\Phi_3$ | D, Load No 1, 1/4 Pt. | 0 | 1/2 D, Load No. 2, 1/4 Pt. | 2/3 D, Load No. 3, 1/4 Pt | 3/4 D, Load No. 4, 1/4 Pt. |
| | $\Phi_a$ | D, Load No 1, Cr. | D, Load No 2, Cr. | D, Load No 3, Cr. | D, Load No 4, Cr. | D, Load No. 5, Cr. |
| | | | | | | |
| 1/4 | $\Phi_1$ | 0 | 0 | 0 | 0 | 0 |
| | $\Phi_2$ | D, Load No 1, 3/4 Pt | 0 | 0 | 1/3 D, Load No. 2, 3/4 Pt | 1/2 D, Load No 3, 3/4 Pt |
| | $\Phi_3$ | D, Load No 1, 1/2 Pt | 0 | 1/2 D, Load No 2, 1/2 Pt | 2/3 D, Load No 3, 1/2 Pt. | 3/4 D, Load No 4, 1/2 Pt |
| | $\Phi_a$ | D, Load No 1, 1/4 Pt | D, Load No 2, 1/4 Pt | D, Load No 3, 1/4 Pt | D, Load No 4, 1/4 Pt. | D, Load No. 5, 1/4 Pt. |
| | | | | | | |
| 1/2 | $\Phi_2$ | 0 | 0 | 0 | 0 | 0 |
| | $\Phi_3$ | D, Load No 1, 3/4 Pt | 0 | 1/2 D, Load No 2, 3/4 Pt. | 2/3 D, Load No. 3, 3/4 Pt. | 3/4 D, Load No 4, 3/4 Pt. |
| | $\Phi_a$ | D, Load No 1, 1/2 Pt | D, Load No 2, 1/2 Pt. | D, Load No 3, 1/2 Pt | D, Load No 4, 1/2 Pt | D, Load No 5, 1/2 Pt. |
| | | | | | | |
| 3/4 | $\Phi_3$ | 0 | 0 | 0 | 0 | 0 |
| | $\Phi_a$ | D, Load No 1, 3/4 Pt | D, Load No. 2, 3/4 Pt | D, Load No 3, 3/4 Pt | D, Load No 4, 3/4 Pt | D, Load No 5, 3/4 Pt |

*Figure C-31.* Circular arch *D*-terms used for computing voussoir *D*-terms.–288-D-424

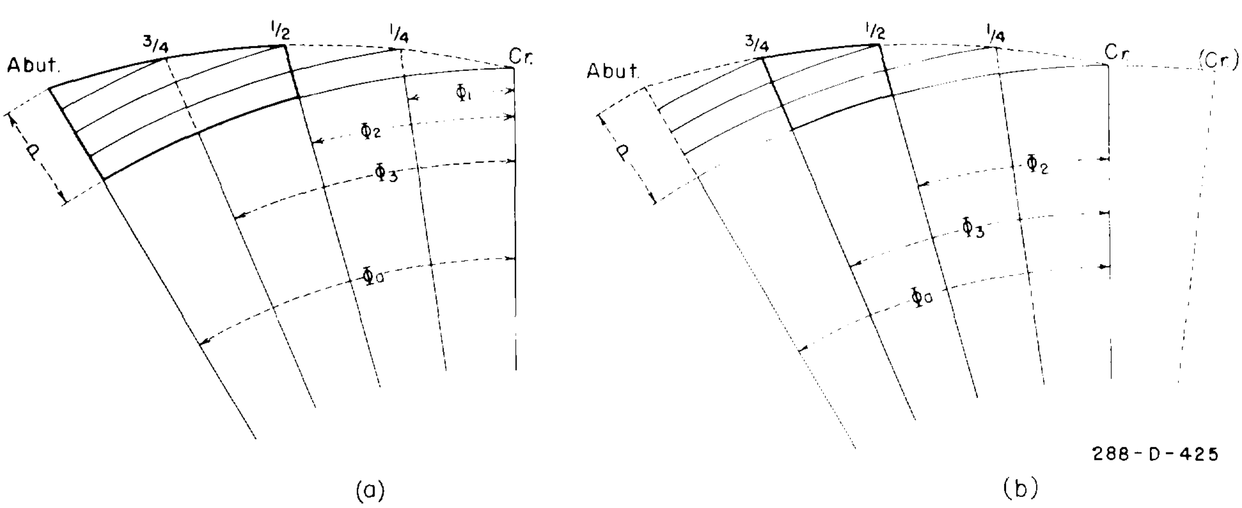


*Figure C-32.* **Sketch for determination of formulas for load No. 5 (see fig. C-12).**

at the ¾ point for load 4 for angle $\Phi_a$. This statement is evident if it is assumed that the half-arch is moved to the right, with the load stationary; so that it extends from the ¾ point to the crown. This assumption makes the load ordinate at the ¾ point equal to ¾ of $P$, and the true ½ point corresponds to the ¾ point of the assumed position of the arch. Then the tabular $D$-term for voussoir $\Phi_a$ to $\Phi_3$ is:

$$D', \Phi_a \text{ to } \Phi_3 = (D \text{ for load at } 1/2 \text{ point}) - (3/4\, D \text{ for load 4 at } 3/4 \text{ point})$$

and the tabular $D$-term for voussoir $\Phi_3$ to $\Phi_2$ is:

$$D', \Phi_3 \text{ to } \Phi_2 = 3/4\, D \text{ for load 4 at } 3/4 \text{ point.}$$

Figures C-31 and C-32 use the notation of a variable-thickness arch and may be used for fillet computations by substituting the corresponding angles and points as shown on figure C-35.

These values may now be inserted directly in equations (185) through (193) of section 4-35(d). Then, by following similar procedures, formulas can be determined for $D$-term values, or prime values, at other arch points for other unit loads.

In order to calculate total $D$-terms, values for $M_L$, $H_L$, and $V_L$ must be determined as explained in section 4-36(d), using equations (209) to (211), inclusive. $M_L$, $H_L$, and $V_L$ values for radial loads 1 and 5 on the arch are shown on figure C-33. These were calculated by means of equations (127) to (129) and (135) to (137), using data given on figures C-22, C-23, and C-7 and trigonometric tables accompanying appendix H. Values for other loads are calculated by equations given in sections 4-34(k) through 4-34(n), except that the moments due to eccentricity of tangential loads require calculations for $H_L e'$ quantities on the computation form shown on figure C-34. The computations shown on that figure are for tangential load No. 1 on the fillet. Values for other tangential loads on the fillet and on the central section are computed in a similar manner. The $H_L$ terms at the bottom of figure C-34 are the tangential thrusts at the center of the voussoirs; each $H_L$ is the thrust due to the tangential load between the center of the voussoir and the edge of the previous voussoir. Thus, ${}_8H_L$ is the tangential thrust at the 5/8 point, due to the load between that point and the 1/2 point (see fig. C-35). In the upper part of figure C-34 is a tabulation of formulas for the $H_L$ terms for various loads. These equations are based on equations (144) and (152) of sections 4-34(k) and (l). Calculations for $M'_L$, $H'_L$, and $V'_L$ values for tangential loads 1 and 5 on the fillet are shown on figure C-36. The equations used are tabulated on figure C-12.

## MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### $M_L$, $H_L$, AND $V_L$ VALUES FOR ARCH ANGLES

| RADIAL LOAD NO. 1 | RADIAL LOAD NO. 5 |
|---|---|
| ${}_{0}M_L = 0$ | ${}_{0}M_L = 0$ |
| ${}_{0}H_L = 0$ | ${}_{0}H_L = 0$ |
| ${}_{0}V_L = 0$ | ${}_{0}V_L = 0$ |
| ${}_{2}M_L = 9,179,512$ | ${}_{2}M_L = 1,056,522.$ |
| ${}_{2}H_L = 18,621.21$ | ${}_{2}H_L = 2,143.219$ |
| ${}_{2}V_L = 136,826.1$ | ${}_{2}V_L = 23,709.26$ |
| ${}_{4}M_L = 36,050,332$ | ${}_{4}M_L = 8,359,854.$ |
| ${}_{4}H_L = 73,130.34$ | ${}_{4}H_L = 16,958.48$ |
| ${}_{4}V_L = 263,699.5$ | ${}_{4}V_L = 93,112.44$ |
| ${}_{8}M_L = 72,419,321$ | ${}_{8}M_L = 24,647,291.$ |
| ${}_{8}H_L = 149,961.3$ | ${}_{8}H_L = 51,038.04$ |
| ${}_{8}V_L = 362,038.7$ | ${}_{8}V_L = 190,936.7$ |

288-D-2704

*Figure C-33.* **Monticello Dam study—$M_L$, $H_L$, and $V_L$ values for arch angles.**

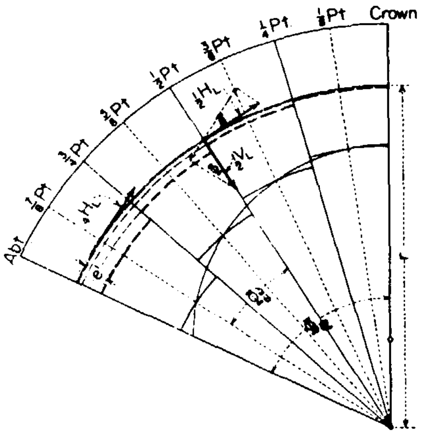
MONTICELLO DAM STUDY NO. A-11
VARIABLE THICKNESS ARCH WITH CONSTANT UPSTREAM RADIUS
**MOMENTS (He') DUE TO TANGENTIAL LOAD ECCENTRICITY**
FILLET SECTION (8-4) ARCH AT ELEV. 350 LEFT SIDE
TANG. LOAD NO. As Indicated Computed by ...... Date ......

## FORMULAE FOR COMPUTING ECCENTRIC THRUSTS

| | Tang. Load No. 1 | Tang. Load No. 2 | Tang. Load No 3 | Tang. Load No 4 | Tang. Load No. 5 |
|---|---|---|---|---|---|
| $_1H_L$ | $-Pr \sin \Phi_{\frac{1}{8}}$ | 0 | 0 | 0 | $-\frac{Pr}{\Phi_4} \text{vers } \Phi_{\frac{1}{8}}$ |
| $_2H_L$ | $-Pr \sin \Phi_{\frac{3}{8}}$ | 0 | 0 | $-\frac{Pr}{\Phi_3} \text{vers } \Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_4} \text{vers } \Phi_{\frac{3}{8}}$ |
| $_3H_L$ | $-Pr \sin \Phi_{\frac{5}{8}}$ | 0 | $-\frac{Pr}{\Phi_2} \text{vers } \Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_3} \text{vers } \Phi_{\frac{3}{8}}$ | $-\frac{Pr}{\Phi_4} \text{vers } \Phi_{\frac{5}{8}}$ |
| $_4H_L$ | $-Pr \sin \Phi_{\frac{7}{8}}$ | $-\frac{Pr}{\Phi_1} \text{vers } \Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_2} \text{vers } \Phi_{\frac{3}{8}}$ | $-\frac{Pr}{\Phi_3} \text{vers } \Phi_{\frac{5}{8}}$ | $-\frac{Pr}{\Phi_4} \text{vers } \Phi_{\frac{7}{8}}$ |
| $_5H_L$ | $_2H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ | 0 | 0 | $_2H_L$ | $_2H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ |
| $_6H_L$ | $_3H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ | 0 | $_3H_L$ | $_3H_L$ | $_3H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ |
| $_7H_L$ | $_4H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L$ | $_4H_L$ | $_4H_L$ | $_4H_L - \frac{1}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{4}V_L \sin \Phi_{\frac{1}{8}}$ |
| $_8H_L$ | $_3H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ | 0 | $_3H_L$ | $_3H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ | $_3H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ |
| * $_9H_L$ | $_4H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L$ | $_4H_L$ | $_4H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L - \frac{1}{2}H_L \cos \Phi_{\frac{1}{8}} - \frac{1}{2}V_L \sin \Phi_{\frac{1}{8}}$ |
| $_{10}H_L$ | $_4H_L - \frac{3}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{3}{4}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L$ | $_4H_L - \frac{3}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{3}{4}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L - \frac{3}{4}H \cos \Phi_{\frac{1}{8}} - \frac{3}{4}V_L \sin \Phi_{\frac{1}{8}}$ | $_4H_L - \frac{3}{4}H_L \cos \Phi_{\frac{1}{8}} - \frac{3}{4}V_L \sin \Phi_{\frac{1}{8}}$ |

## TRIGONOMETRIC FUNCTIONS

| POINT | $\Phi$ | SIN $\Phi$ | COS $\Phi$ | VERS $\Phi$ |
|---|---|---|---|---|
| $\frac{1}{8}$ | 1°45' | .030,538,51 | .999,533,6 | .0$^3$466,409,2 |
| $\frac{3}{8}$ | 5°15' | .091,501,62 | .995,804,9 | .004,195,072 |
| $\frac{5}{8}$ | 8°45' | .152,123,40 | .988,361,5 | .011,638,49 |
| $\frac{7}{8}$ | 12°15' | .212,177,70 | .977,231,1 | .022,768,89 |



—·— = Arc through crown ℄ - radius (r).
——— = Arc through mid-point of voussoir.
— — — = Voussoir mid-point arc extended.

Tangential load applied from left to right along arc through crown ℄

e' = eccentricity of thrust with reference to arc through mid-point of voussoir (Dwg 205-D-1810)

$\Phi$ = angle from crown to point

$\Phi_{\frac{1}{8}} = \frac{1}{8}$ of $\Phi_a$, $\Phi_{\frac{3}{8}} = \frac{3}{8}$ of $\Phi_a$, etc.

$H_L$ and $V_L$ at $\frac{1}{4}$ points are evaluated by formulae on drawing 205-D-190

$_1H_L$, $_2H_L$, etc are thrusts accruing along arc of crown ℄ between $\frac{1}{4}$ points of arch and center of voussoirs

*ILLUSTRATIVE SKETCH FOR EVALUATING $_9H_L$

## MOMENTS ($H_Le'$) DUE TO ECCENTRIC THRUSTS

| POINT | Fillet Crown (Point 4) #1 load | | | | Fillet Crown (Point 4) #5 load | | | |
|---|---|---|---|---|---|---|---|---|
| | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ |
| $\frac{1}{8}$ | -15,054.26 $_1H_L$ | -4,742.09 | | | -940.96 | -296.40 | | |
| $\frac{3}{8}$ | -45,106.64 $_2H_L$ | -70,591.89 | $_5H_L$ | | -8,463.42 | -13,245.25 | | |
| $\frac{5}{8}$ | -74,990.75 $_3H_L$ | -305,212.4 | $_6H_L$ | | -23,480.26 $_8H_L$ | -95,564.66 | | |
| $\frac{7}{8}$ | -104,595.1 $_4H_L$ | -820,025.6 | $_7H_L$ | | -45,935.47 $_9H_L$ | -360,134.1 | $_{10}H_L$ | |

*Figure C-34.* **Monticello Dam study–moments due to tangential load eccentricity, arch at elevation 350.–DS2-1(97)**

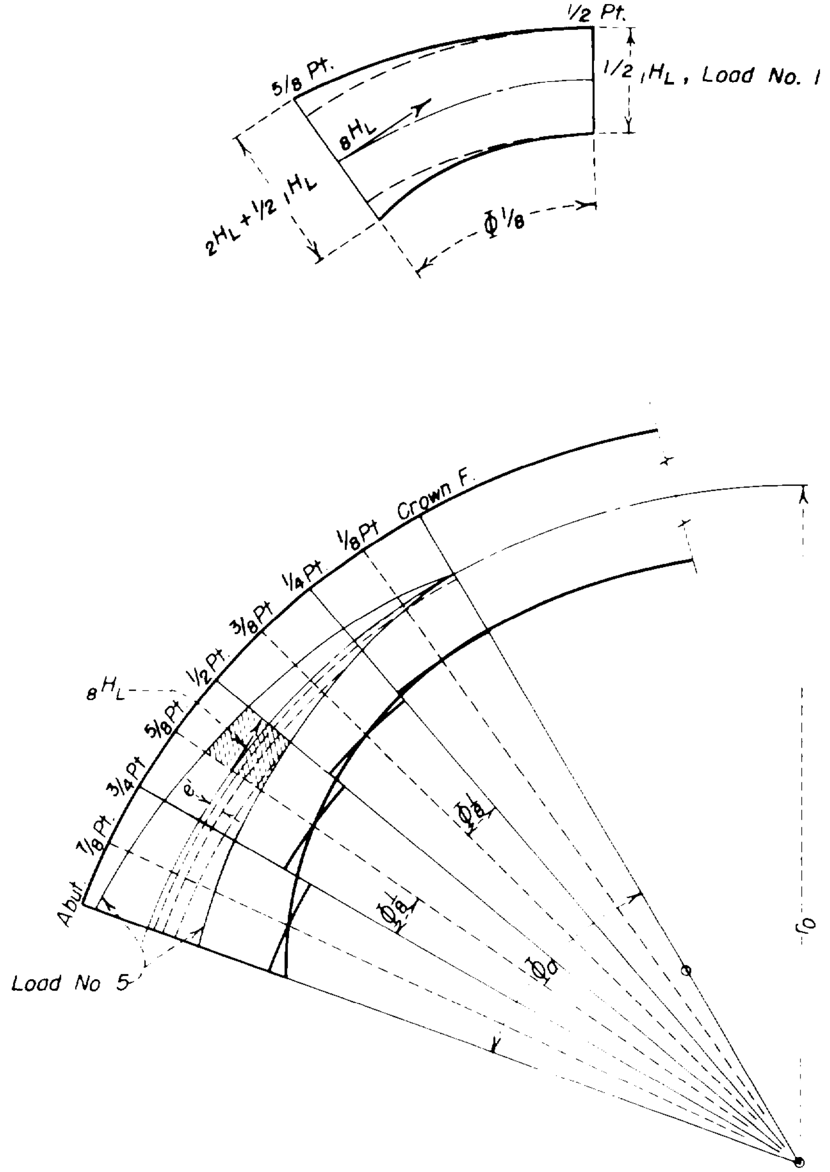


*Figure C-35.* **Illustrative sketch for evaluating $_8H_L$ for tangential load No. 5 on fillet section.–288-D-2705**

Figure C-37 shows computations for $M_L$, $H_L$, and $V_L$ for the transfer of $D$-terms using equations (209) to (211) in section 4-36(d). Values of $x$ and $y$ are given on figure C-26 and $M_L$, $H_L$, and $V_L$ values are given on figure C-33.

Computations for total $D$-terms due to radial load No. 1 are shown on figure C-38. These terms are computed by means of equations (206) to (208), shown in section 4-36(d). Data for $D_1''$ and $D_1'$ are shown on figures C-27 and C-28, and $A_1'$ values are given on figure C-24. Calculations for arch constants $B_1'$ and $C_1'$ are not shown.

C-15. ***Other Calculations.***–After arch and load constants have been calculated, general forms shown on figures C-2 to C-4 can be used provided that the following calculations are made. Formulas for $x$ and $y$ must be changed to $x = r_p \sin \Phi$ and $y = r_o - r_p \cos \Phi$ as shown

# MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### $M_L'$, $H_L'$, AND $V_L'$ VALUES FOR TANGENTIAL LOADS

TANGENTIAL LOAD NO. 1 ON FILLET

$${}_8M_L' = Pr_4^2\left(\phi_{(4-8)} - \sin\phi_{(4-8)}\right) - {}_8H_L' \cdot e = 1{,}786{,}452.1$$

$${}_8H_L' = Pr_4 \cdot \sin\phi_{(4-8)} = -119{,}257.8$$

$${}_8V_L' = Pr_4 \cdot \text{vers}\,\phi_{(4-8)} = 14{,}643.02$$

TANGENTIAL LOAD NO. 5 ON FILLET

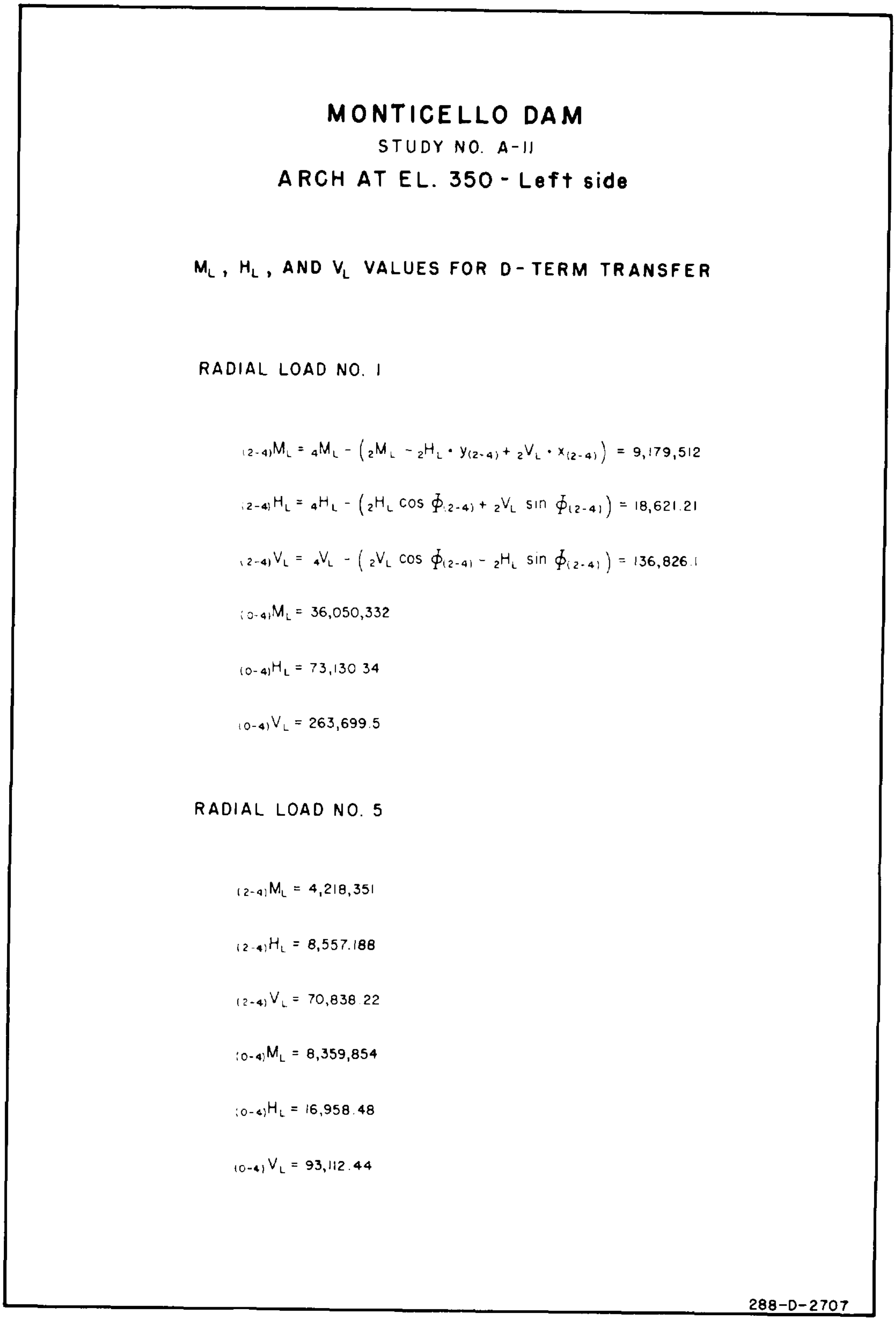
$${}_8M_L' = \frac{Pr_4^2}{\phi_{(4-8)}}\left(\frac{\phi_{(4-8)}^2}{2} - \text{vers}\,\phi_{(4-8)}\right) - {}_8H_L' e = 749{,}093.3$$

$${}_8H_L' = -\frac{Pr_4}{\phi_{(4-8)}}\left(\text{vers}\,\phi_{(4-8)}\right) = -59{,}927.36$$

$${}_8V_L' = \frac{Pr_4}{\phi_{(4-8)}}\left(\phi_{(4-8)} - \sin\phi_{(4-8)}\right) = 4{,}890.741$$

288-D-2706

*Figure C-36.* Monticello Dam study—$M_L'$, $H_L'$, and $V_L'$ values for tangential loads.



## MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### $M_L$, $H_L$, AND $V_L$ VALUES FOR D-TERM TRANSFER

RADIAL LOAD NO. 1

$${}_{(2-4)}M_L = {}_4M_L - \left({}_2M_L - {}_2H_L \cdot y_{(2-4)} + {}_2V_L \cdot x_{(2-4)}\right) = 9,179,512$$

$${}_{(2-4)}H_L = {}_4H_L - \left({}_2H_L \cos \Phi_{(2-4)} + {}_2V_L \sin \Phi_{(2-4)}\right) = 18,621.21$$

$${}_{(2-4)}V_L = {}_4V_L - \left({}_2V_L \cos \Phi_{(2-4)} - {}_2H_L \sin \Phi_{(2-4)}\right) = 136,826.1$$

${}_{(0-4)}M_L = 36,050,332$

${}_{(0-4)}H_L = 73,130.34$

${}_{(0-4)}V_L = 263,699.5$

RADIAL LOAD NO. 5

${}_{(2-4)}M_L = 4,218,351$

${}_{(2-4)}H_L = 8,557.188$

${}_{(2-4)}V_L = 70,838.22$

${}_{(0-4)}M_L = 8,359,854$

${}_{(0-4)}H_L = 16,958.48$

${}_{(0-4)}V_L = 93,112.44$

288-D-2707

*Figure C-37.* **Monticello Dam study—$M_L$, $H_L$, and $V_L$ values for $D$-term transfer.**

## MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### TOTAL D-TERMS FOR RADIAL LOAD NO. 1

${}_4D = .0^{3},084,313,03$

${}_4D'_2 = .008,151,454$

${}_4D'_3 = .001,232,302$

${}_2D_1 = {}_4A'_{1(2-4)}M_L - {}_4B_{1(2-4)}H_L + {}_4C'_{1(2-4)}V_L + {}_4D'_1 + {}_2D'' = .001,082,500$

$$
\begin{aligned}
{}_2D_2 = {} & \left[{}_4A'_1x_{(2-4)} + {}_4B_1 \sin\phi_{(2-4)} + {}_4C'_1 \cos\phi_{(2-4)}\right]_{(2-4)}M_L - \left[{}_4B'x_{(2-4)} + {}_4B'_3 \sin\phi_{(2-4)} + {}_4B'_2 \cos\phi_{(2-4)}\right]_{(2-4)}H_L \\
& + \left[{}_4C'_1x_{(2-4)} - {}_4B'_2 \sin\phi_{(2-4)} + {}_4C'_2 \cos\phi_{(2-4)}\right]_{(2-4)}V_L + \left[{}_4D'_1x_{(2-4)} + {}_4D'_2 \cos\phi_{(2-4)} - {}_4D'_3 \sin\phi_{(2-4)}\right] + {}_2D''_2 \\
& = .186,471,2
\end{aligned}
$$

$$
\begin{aligned}
{}_2D_3 = {} & \left[{}_4A'_1y_{(2-4)} + {}_4B'_1 \cos\phi_{(2-4)} + {}_4C_1 \sin\phi_{(2-4)}\right]_{(2-4)}M_L - \left[{}_4B'_1y_{(2-4)} + {}_4B'_3 \cos\phi_{(2-4)} + {}_4B'_2 \sin\phi_{(2-4)}\right]_{(2-4)}H_L \\
& + \left[{}_4C'_1y_{(2-4)} + {}_4B_2 \cos\phi_{(2-4)} + {}_4C'_2 \sin\phi_{(2-4)}\right]_{(2-4)}V_L + \left[{}_4D'_1y_{(2-4)} + {}_4D'_2 \sin\phi_{(2-4)} + {}_4D_3 \cos\phi_{(2-4)}\right] + {}_2D''_3 \\
& = .039,000,99
\end{aligned}
$$

${}_0D_1 = .004,357,261$

${}_0D_2 = 1.104,441,8$

${}_0D_3 = 339,831,6$

288-D-2708

*Figure C-38.* Monticello Dam study—total *D*-terms for radial load No. 1.

by calculations in figure C-39. These are the arch point coordinates with the crown as the origin. They are used on arch sheet 2, figure C-2, in solving the crown forces, and on sheet 3, figure C-3, in computing total moments. In addition, values of $x_a$ and $y_a$ for coordinates of the abutment with arch points as origins are required. These may be computed on sheet 1, figure C-1, on the two vacant lines under point $\Phi_a - \Phi$. These formulas are $x_a = r_a \sin(\Phi_a - \Phi)$ and $y_a = r_p - r_a \cos(\Phi_a - \Phi)$ in which $\Phi_a$ is the angle from the crown of the abutment and $\Phi$ is the angle from the crown to the point considered. These coordinates are used in calculating deflections and temperature *D*-terms. Another change to be made in use of the general forms is in the values of $M_L$, $H_L$, and $V_L$ of sheet 3, figure C-3. Also, inasmuch as calculations of arch and load constants are made on supplementary forms, values of tabular parts and multipliers are not placed on the general forms.

Arch forces and deflections are calculated for the arch crown, the point midway between the arch crown and fillet crown, the fillet crown, the ½ point of the fillet section, and the abutment. This procedure applies to the usual case where the fillet starts near the ½ point. If the fillet begins near the ¾ point of the half-arch a similar arrangement of points can be used.

## C. VARIABLE-THICKNESS ARCH WITH TRIANGULAR WEDGE ABUTMENTS

C-16. *Introduction.*—These sections illustrate a step-by-step analysis of a variable-thickness arch having triangular wedge abutments. The general analysis of a variable-thickness arch with nonradial abutments is described in section 4-37(b). The equations given in that discussion can be applied to any arch with variable-thickness sections.

The illustrations and computations shown in these paragraphs are taken from the trial-load analysis of Hungry Horse Dam (fig. C-40). Hungry Horse Dam was analyzed by the trial-load method using nonsymmetrical arches with nonsymmetrical loading. Nonradial abutments were included where applicable. The loading conditions analyzed included normal reservoir water surface, earthquake effects, and an assumed ice pressure. Only those data and assumptions which affect the computation of arch deflections due to unit loads are listed below:

(1) Crest of dam, elevation 3565.

(2) Base of crown cantilever, elevation 3050.

(3) Sustained modulus of elasticity of concrete in tension and compression, 3,940,000 pounds per square inch.

(4) Sustained modulus of elasticity of foundation and abutment rock, 4,000,000 pounds per square inch.

(5) Poisson's ratio for concrete and abutment rock, 0.20.

(6) Coefficient of thermal expansion of concrete, 0.000,005,3 per degree F.

(7) Arch 3300 is nonsymmetrical with nonradial abutments and is nonsymmetrically loaded.

(8) Foundation and abutment rock formations at the site have adequate strength to safely carry the loads transmitted by the dam.

(9) The concrete in the dam is homogeneous, uniformly elastic in all directions, and strong enough to carry the applied loads with stresses well below the elastic limit.

(10) The dam is thoroughly keyed into the foundation and abutment rock throughout its contact with the canyon profile, so that arches may be considered as fixed with relation to the abutment, and cantilevers as fixed with relation to the foundation.

(11) Contraction joints were thoroughly grouted according to the grouting schedule, and it is assumed that these joints will remain grouted

# MONTICELLO DAM

STUDY NO. A-11

## ARCH AT EL. 350 - Left side

### CALCULATIONS FOR x AND y FOR ARCH POINTS

CROWN TO POINT 2

$x = r_2 \sin \phi_{(0-2)} = 131.737,8$

$y = r_0 - r_2 \cos \phi_{(0-2)} = 17.928,73$

CROWN TO POINT 4

$x = r_4 \sin \phi_{(0-4)} = 253.893,2$

$y = r_0 - r_4 \cos \phi_{(0-4)} = 70.410,81$

CROWN TO ABUT. (POINT 8)
$\phi_{(0-8)} = 45°$

$x = r_8 \sin \phi_{(0-8)} = 341.476.0$

$y = r_0 - r_8 \cos \phi_{(0-8)} = 151.484,0$

ABUT. TO POINT 4

$x = r_8 \sin \phi_{(8-4)} = 116.828,9$

$y = r_4 - r_8 \cos \phi_{(8-4)} = 24.384,80$

ABUT. TO POINT 2

$x = r_8 \sin \phi_{(8-2)} = 237.801,2$

$y = r_2 - r_8 \cos \phi_{(8-2)} = 72.647,83$

ABUT. TO POINT 0 (CROWN)

Same as Crown to Abutment

288-D-2709

*Figure C-39.* **Monticello Dam study—*x* and *y* distances for arch points.**

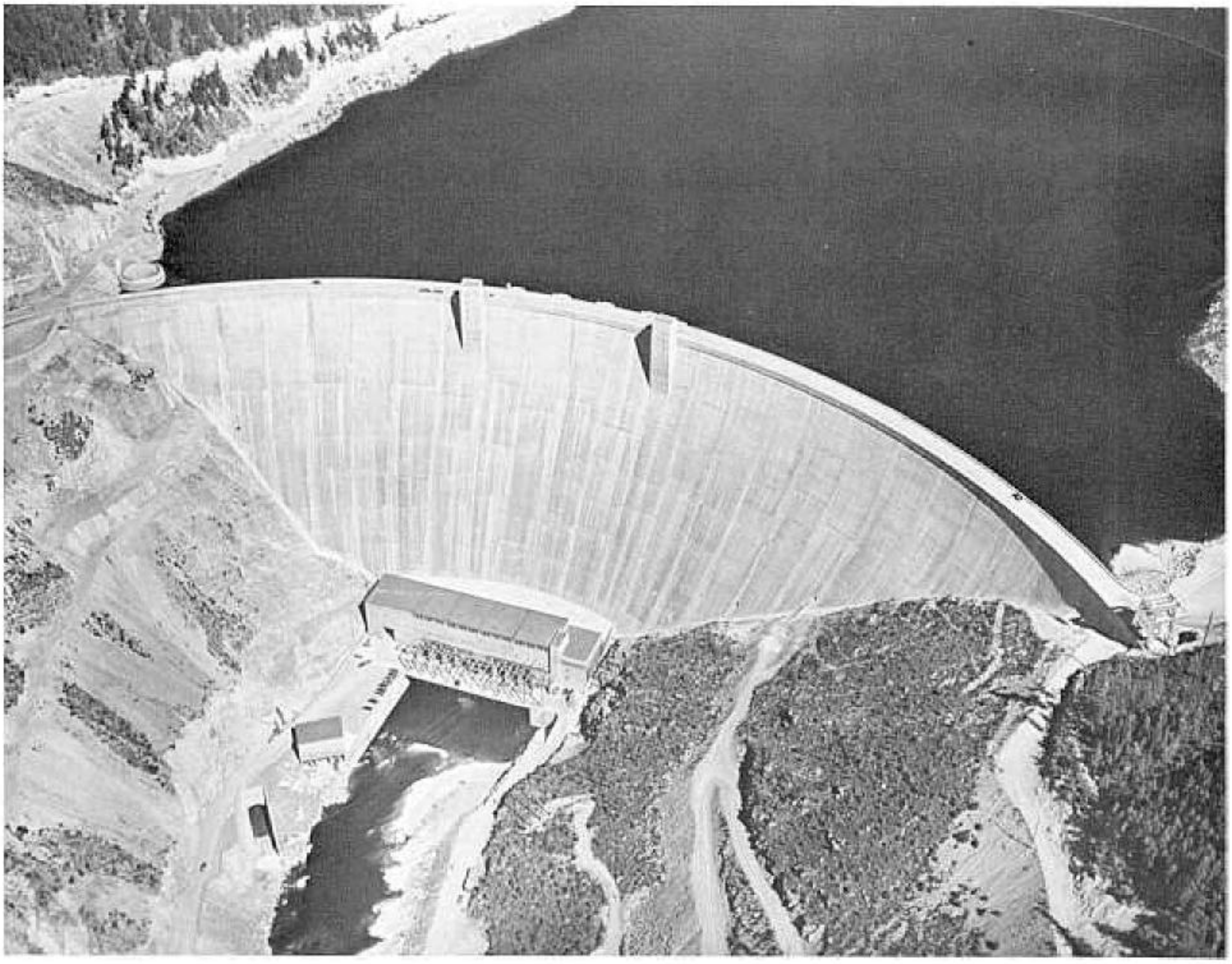


*Figure C-40.* **Hungry Horse Dam.–P447-105-5617**

throughout the life of the structure.

The arch selected for illustrations is at elevation 3300 and is shown on figure C-41. This arch has a variable thickness with a wedge section at the abutments as shown on the figure.

C-17. ***Properties of Arch.***–The properties of the arch are determined from a layout of the arch to a suitable scale as indicated by figure C-41. Voussoirs subtending equal angles for each section are laid out and thicknesses, radii, and angles at arch points are tabulated for convenient use.

C-18. ***Eccentricities and Multipliers for Arch and Load Constants.***–Eccentricities and multipliers are computed in the manner described in sections 4-35(c) and (d) and shown on figure C-42.

C-19. ***Arch Constants for Variable-Thickness Central Section and Wedge Section.***–Arch constants are computed for each section of the arch as a preliminary step prior to determining total arch constants at each given point. For a variable-thickness section, arch constants are represented by double prime (″) marks. Single prime (′) marks designate arch constants for the triangular wedge section.

Arch constants for the variable-thickness sections are determined by the method illustrated in section C-13. Computations for arch constant $A_1''$, at arch points 0, 1, 2, and 3 due to the variable-thickness section are given on figure C-43. Complete calculations would include all arch constants at points 0, 1, 2, and 3.

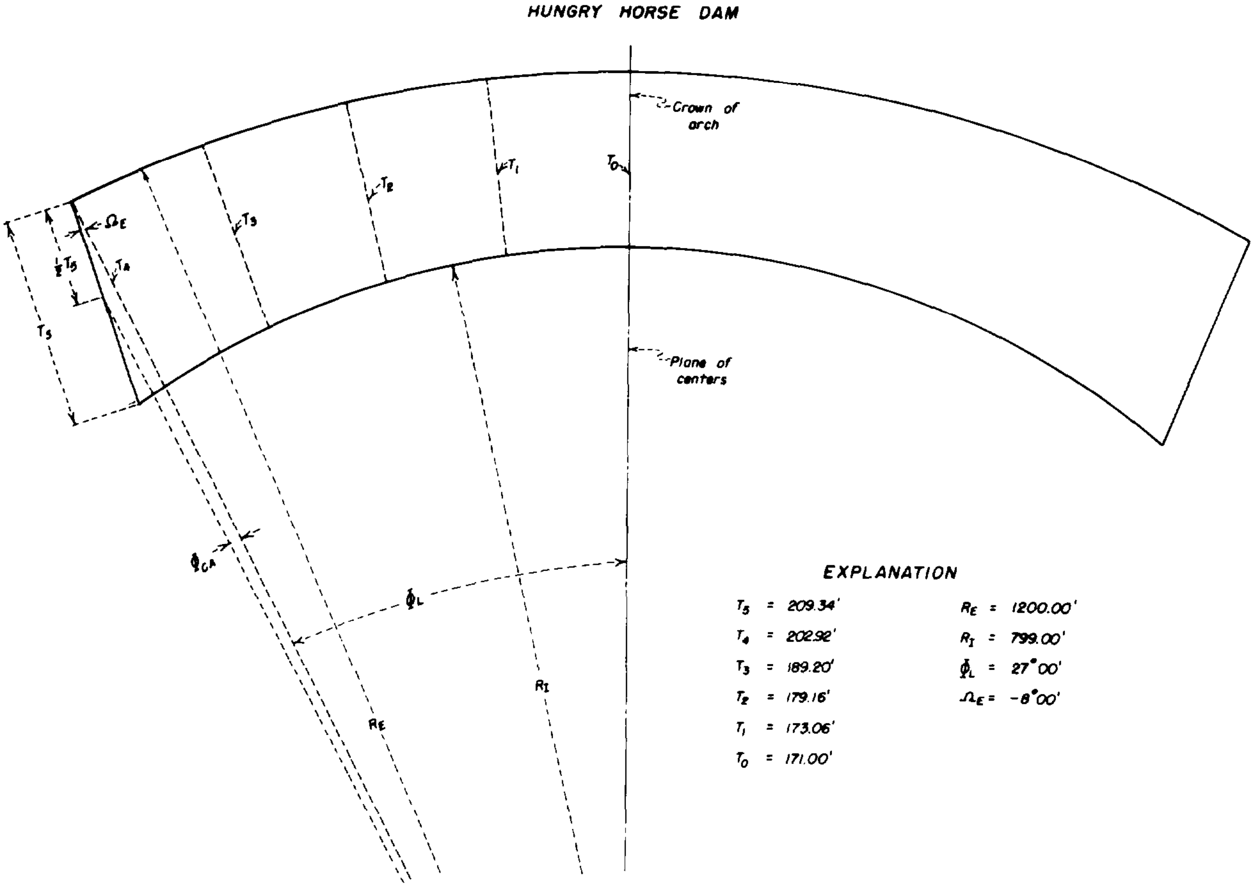


*Figure C-41.* **Hungry Horse Dam study–properties of variable-thickness arch at elevation 3300 (left side only).–288-D-2710**

For the triangular wedge section, arch constants are calculated at point 4 by means of equations (226) to (231), inclusive. Data and computations for these constants are given on figure C-44.

C-20. ***Coordinates of Arch Points.*** –The $x$ and $y$ distances to each arch point from the crown, and also from the abutment, are required in the analysis for the computation of crown forces and abutment deformations. Certain values of $x$ and $y$ are also required in the transfer equations. In order to establish these distances it is necessary to compute the angles by which they are intercepted. Then, using trigonometric relationships, $x$ and $y$ distances can be calculated. The calculation of coordinates to arch points is shown on figure C-45. The calculation of coordinates for transferring constants is shown on figure C-46.

C-21. ***Total Arch Constants at Arch Points.***–To obtain total arch constants at points 3, 2, 1, and 0, arch constants for the wedge must be transferred from point 4 to the desired points. This is accomplished by means of the general transfer equations described in section 4-37(a) numbered (212) to (217), inclusive. Each total arch constant at points 3, 2, 1, or 0 contains a (″) value and a (′) value.

Constants used for the transfer of both arch terms and $D$-terms have been combined and tabulated for convenient use on figure C-47. Figure C-48 shows the calculation of total arch constants at point 3. Total arch constants for point 2 are determined on figure C-49.

## HUNGRY HORSE DAM STUDY NO. 3

### VARIABLE THICKNESS ARCH WITH CONSTANT UPSTREAM RADIUS
AND NON-RADIAL ABUTMENT

### ECCENTRICITIES AND MULTIPLIERS FOR ARCH AND LOAD CONSTANTS

ARCH AT ELEV. 3300 - LEFT SIDE

COMPUTED BY........DATE..........

CHECKED BY........DATE..........

| POINT | $\Phi$ | VOUSSOIR | AT POINT | | FOR VOUSSOIR | | VOUSSOIR TO POINT | | | | $e_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $T$ | $r_p$ | $T_v$ | $r_v$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ | |
| 0 | 0° | | 171.0 | 1114.5 | | | | | | | 0 |
| | | 0 to 1 | | | 172.04 | 1113.98 | 0.52 | | | | |
| 1 | 6°45′ | | 173.06 | 1113.46 | | | | | | | 1.04 |
| | | 1 to 2 | | | 176.11 | 1111.945 | 2.555 | 1.515 | | | |
| 2 | 13°30′ | | 179.16 | 1110.43 | | | | | | | 4.67 |
| | | 2 to 3 | | | 184.17 | 1107.915 | 6.585 | 5.545 | 2.515 | | |
| 3 | 20°15′ | | 189.20 | 1105.40 | | | | | | | 9.10 |
| | | 3 to 4 | | | 196.07 | 1101.965 | 12.535 | 11.495 | 8.465 | 3.435 | |
| 4 | 27°00′ | | 202.92 | 1098.53 | | | | | | | 15.97 |

$E_c = 432 \times 10^6$ $\frac{1}{E_c} = .0^{6}002,314,814,8$

$R_E = 1200.00$

$r = R_E - \frac{T}{2}$ $\quad I = \frac{T_v^3}{12}$

$r_v = R_E - \frac{T_v}{2}$

$e_{0,1,2,3} = r_p - r_v = e$, for transfer of arch and load const's.

$e_0 = e'$, for tangential load eccentricity.

$e_p$ = eccentricity of arch point with reference to arc of center line through crown.

| VOUSSOIR | $\frac{1}{E_c T_v}$ | $\frac{r_v}{E_c T_v}$ | $\frac{r_v^2}{E_c T_v}$ | $\frac{r_v}{E_c I_v}$ | $\frac{r_v^2}{E_c I_v}$ | $\frac{r_v^3}{E_c I_v}$ | $\frac{r_v^4}{E_c I_v}$ | $\frac{r_v R_E}{E_c T_v}$ | $\frac{r_v^2 R_E}{E_c I_v}$ | $\frac{r_v^3 R_E}{E_c I_v}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 to 1 | $.0^{9}013,455,096$ | $.0^{6}014,988,708$ | $0^{3}016,697,121$ | $.0^{9}006,076,962,9$ | $.0^{6}006,769,615,1$ | $.0^{3}007,541,215,8$ | .008,400,763,6 | $.0^{3}017,986,449$ | $.0^{3}008,123,538,2$ | 009,049,459,0 |
| 1 to 2 | $.0^{9}013,144,142$ | $.0^{6}014,615,563$ | $.0^{3}016,251,702$ | $.0^{9}005,654,949,9$ | $.0^{6}006,287,993,3$ | $.0^{3}006,991,902,7$ | .007,774,611,2 | $.0^{3}017,538,675$ | $.0^{3}007,545,591,9$ | .008,390,283,2 |
| 2 to 3 | $.0^{9}012,568,903$ | $.0^{6}013,925,276$ | $0^{3}015,428,022$ | $.0^{9}004,926,599,7$ | $.0^{6}005,458,253,7$ | $.0^{3}006,047,281,1$ | 006,699,873,5 | $.0^{3}016,710,331$ | $.0^{3}006,549,904,4$ | .007,256,737,3 |
| 3 to 4 | $.0^{9}011,806,063$ | $0^{6}013,009,848$ | $.0^{3}014,336,420$ | $.0^{9}004,060,989,2$ | $.0^{6}004,475,068,0$ | $0^{3}004,931,368,3$ | .005,434,195,3 | $.0^{3}015,611,842$ | $.0^{3}005,370,081,7$ | .005,917,642,0 |

Figure C-42. Hungry Horse Dam study—eccentricities and multipliers for arch and load constants.—DS2-1(105)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D—TERMS FOR NON-UNIFORM THICKNESS ARCHES

Hungry Horse DAM

STUDY NO. 3

ARCH AT ELEV. 3300-LEFT

Arch Constant $A_1''$

COMPUTED BY..........

DATE..........

| PT. | $\Phi$ | $A_1$ | $\Delta A_1$ | $\frac{r}{EI_v}\Delta A_1$ |  | PART OF $B_1$<br>$\frac{r}{EI_v}\Delta A_1 \cdot e$ | PART OF $B_3$<br>$\frac{r}{EI_v}\Delta A \cdot e^2$ |  |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  | 6°45' | .117,809,72 |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  | 13°30' | .235,619,44 |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  | 20°15' | .353,429,16 |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  | 27° | .471,238,88 |  |  |  |  |  |  |
|  |  |  | $_0A_1'' = \Sigma = .0^{9}_{,}002,440,959$ |  |  | $\Sigma = .0^{9}_{,}011,893,43$ | $\Sigma = .0^{9}_{,}104,883,1$ |  |
| 1/4 |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | $_1A_1'' = \Sigma = .0^{9}_{,}001,725,033$ |  |  | $\Sigma = .0^{9}_{,}009,727,114$ | $\Sigma = .0^{9}_{,}082,591,28$ |  |
| 1/2 |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | $_2A_1'' = \Sigma = .0^{9}_{,}001,058,825$ |  |  | $\Sigma = .0^{9}005,509,568$ | $\Sigma = .0^{9}_{,}037,953,225$ |  |
| 3/4 |  |  |  |  |  |  |  |  |
|  |  |  | .117,809,72 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | $_3A_1'' = \Sigma = .0^{12}_{,}478,424,0$ |  |  | $\Sigma = .0^{9}_{,}001,643,386$ | $\Sigma = .0^{9}_{,}005,645,031$ |  |

*Figure C-43.* Hungry Horse Dam study–arch constants for nonuniform-thickness arch.–DS2-1(106)

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL.3300 - Left side

### ARCH CONSTANTS FOR TRIANGULAR WEDGE SECTION 4-5

${}_{L}\Omega_{E} = -8°$

$\tan 8° = -\frac{\sin {}_{L}\Omega_{E}}{\cos {}_{L}\Omega_{E}} = +\ 140,540,8$

$T_4 = 202\ 92$

$\frac{1}{G} = .0^{6}_{,}005,555,555,6$

$T_4^2 = 41,176.526$

$d = T_4 \tan {}_{L}\Omega_{E} = 28\ 518,54$

$T_4^3 = 8,355,540.7$

$d^2 = 813.307,$

$\frac{1}{E_c} = .0^{6}_{,}002,314,814,8$

$d^3 = 23,194.33$

${}_4A' = \frac{6d}{E_c T_4^3} = .0^{2}_{,}047,404,57$

${}_4B_1' = -\frac{d}{E_c T_4^2} = -\ 0^{9}_{,}001,603,223$

${}_4C_1' = \frac{12d^2}{5E_c T_4^3} = 0^{12}_{,}540,763,7$

${}_4B_2' = -\frac{d^2}{2E_c T_4^2} = -\ 0^{9}_{,}022,860,78$

${}_4C_2' = \frac{3d}{5GT_4} + \frac{7d^3}{5E_c T_4^3} = .0^{9}_{,}477,465,4$

${}_4B_3' = \frac{d}{2E_c T_4} = 0^{9}_{,}162,663,0$

288-D-2711

*Figure C-44.* Hungry Horse Dam study–arch constants for triangular wedge section 4-5.

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### COORDINATES TO ARCH POINTS

### X AND y DISTANCES FROM CROWN

**CROWN TO POINT 1**

$x_{(0-1)} = r_0 \sin \phi_{(0-1)} = 130.873,2$

$y_{(0-1)} = r_0 - r_1 \cos \phi_{(0-1)} = 8.757,948$

**CROWN TO POINT 2**

$x_{(0-2)} = r_0 \sin \phi_{(0-2)} = 259.224,8$

$y_{(0-2)} = r_0 - r_2 \cos \phi_{(0-2)} = 34\ 751,29$

**CROWN TO POINT 3**

$x_{(0-3)} = 382\ 597,8$

$y_{(0-3)} = 77.423,34$

**CROWN TO ABUTMENT**

$x_{(0-5)} = 510.711,4$

$y_{(0-5)} = 144\ 259,6$

### X AND y DISTANCES FROM ABUTMENT

**ABUTMENT TO CROWN**

Same as Crown to Abutment

**ABUTMENT TO POINT 1**

$x_{(1-5)} = r_5 \sin \phi_{(5-7)} = 393.131,9$

$y_{(1-5)} = r_1 - r_5 \cos \phi_{(5-7)} = 89.927,17$

**ABUTMENT TO POINT 2**

$x_{(2-5)} = 270.102,3$

$y_{(2-5)} = 47\ 764,22$

**ABUTMENT TO POINT 3**

$x_{(3-5)} = 143.328,2$

$y_{(3-5)} = 18.362,94$

288-D-2712

*Figure C-45.* Hungry Horse Dam study—coordinates to arch points.

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### COORDINATES FOR TRANSFERRING ARCH AND LOAD CONSTANTS

POINT 4 TO 0; $\phi_{(4-0)} = 27°$; $r_0 = 1114.5$

$\sin \phi_{(4-0)} = .453,990,5$

$\cos \phi_{(4-0)} = .891,006,5$

$\sin^2 \phi_{(4-0)} = .206,107,4$

$\cos^2 \phi_{(4-0)} = .793,892,6$

$\sin \phi_{(4-0)} \cos \phi_{(4-0)} = .404,508,5$

$x_{(4-0)} = 498.722,2$

$y_{(4-0)} = 135.702,6$

POINT 4 TO 1; $\phi_{(4-1)} = 20°15'$; $r_1 = 1113.46$

$\sin \phi_{(4-1)} = .346,117,1$

$\cos \phi_{(4-1)} = .938,191,3$

$\sin^2 \phi_{(4-1)} = .119,797,0$

$\cos^2 \phi_{(4-1)} = .880,203,0$

$\sin \phi_{(4-1)} \cos \phi_{(4-1)} = .324,724,1$

$x_{(4-1)} = 380.220,0$

$y_{(4-1)} = 82.828,71$

POINT 4 TO 2; $\phi_{(4-2)} = 13°30'$; $r_2 = 1110.43$

$\sin \phi_{(4-2)} = .233,445,4$

$\cos \phi_{(4-2)} = .972,369,9$

$\sin^2 \phi_{(4-2)} = .054,496,74$

$\cos^2 \phi_{(4-2)} = .945,503,3$

$\sin \phi_{(4-2)} \cos \phi_{(4-2)} = .226,995,3$

$x_{(4-2)} = 256.446,8$

$y_{(4-2)} = 42.252,49$

POINT 4 TO 3; $\phi_{(4-3)} = 6° 45'$; $r_3 = 1105.40$

$\sin \phi_{(4-3)} = .117,537,4$

$\cos \phi_{(4-3)} = .993,068,5$

$\sin^2 \phi_{(4-3)} = .013,815,04$

$\cos^2 \phi_{(4-3)} = .986,185,0$

$\sin \phi_{(4-3)} \cos \phi_{(4-3)} = .116,722,7$

$x_{(4-3)} = 129.118,4$

$y_{(4-3)} = 14.484,46$

POINT 5 TO 4; ${}_L\Omega_U = -8°$

$\sin {}_L\Omega_E = -.139,173,10$

$\cos {}_L\Omega_E = +.990,268,07$

$x_{(5-4)} = \frac{T_5}{2} \sin {}_L\Omega_E = 14.567,25$

$y_{(5-4)} = \frac{T_5}{2} \cos {}_L\Omega_E - \frac{T_4}{2} = 2.191,359$

$\frac{T_5}{2} = 104.67$

$\frac{T_4}{2} = 101.46$

288-D-2713

*Figure C-46.* **Hungry Horse Dam study—coordinates for transferring arch and load constants.**

## HUNGRY HORSE DAM

STUDY NO.3

## ARCH AT EL.3300 - Left side

### CONSTANTS FOR TRANSFERRING D-TERMS

POINT 4 TO POINT 3

For $D_2$ $J = \left({}_4A_1 x_{(4-3)} - {}_4B_1 \sin\phi_{(4-3)} + {}_4C_1 \cos\phi_{(4-3)}\right) = 0_{,}^{9}006{,}846{,}257$

$K = \left({}_4B_1 x_{(4-3)} - {}_4B_3 \sin\phi_{(4-3)} + {}_4B_2 \cos\phi_{(4-3)}\right) = -0_{,}^{9}248{,}826{,}9$

$L = \left({}_4C_1 x_{(4-3)} - {}_4B_2 \sin\phi_{(4-3)} + {}_4C_2 \cos\phi_{(4-3)}\right) = 0_{,}^{9}546{,}665{,}4$

For $D_3$ $Q = \left({}_4A_1 y_{(4-3)} + {}_4B_1 \cos\phi_{(4-3)} + {}_4C_1 \sin\phi_{(4-3)}\right) = -0_{,}^{12}841{,}920{,}7$

$R = \left({}_4B_1 y_{(4-3)} + {}_4B_3 \cos\phi_{(4-3)} + {}_4B_2 \sin\phi_{(4-3)}\right) = 0_{,}^{9}135{,}626{,}7$

$S = \left({}_4C_1 y_{(4-3)} + {}_4B_2 \cos\phi_{(4-3)} + {}_4C_2 \sin\phi_{(4-3)}\right) = 0_{,}^{9}041{,}250{,}39$

POINT 4 TO POINT 2

For $D_2$ $J = 0_{,}^{9}013{,}056{,}84$

$K = -0_{,}^{9}471{,}343{,}5$

$L = 0_{,}^{9}608{,}286{,}8$

For $D_3$ $Q = 0_{,}^{12}570{,}274{,}1$

$R = 0_{,}^{9}085{,}091{,}70$

$S = 0_{,}^{9}112{,}081{,}6$

POINT 4 TO POINT 1

For $D_2$ $J = 0_{,}^{9}019{,}086{,}41$

$K = -0_{,}^{9}687{,}325{,}7$

$L = 0_{,}^{9}661{,}475{,}5$

For $D_3$ $Q = 0_{,}^{9}002{,}609{,}497$

$R = 0_{,}^{9}011{,}903{,}61$

$S = 0_{,}^{9}188{,}601{,}9$

POINT 4 TO POINT 0 (CROWN)

For $D_2$ $J = 0_{,}^{9}024{,}851{,}38$

$K = -0_{,}^{9}893{,}779{,}5$

$L = 0_{,}^{9}705{,}494{,}2$

For $D_3$ $Q = 0_{,}^{9}005{,}249{,}943$

$R = -0_{,}^{9}083{,}006{,}32$

$S = 0_{,}^{9}269{,}778{,}7$

288-D-2718

*Figure C-47.* **Hungry Horse Dam study—constants for transferring *D*-terms.**

## HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### TOTAL ARCH CONSTANTS AT ARCH POINT 3

POINT 3

$_3A_1 = .0^{12}_{,}047,404,57 + .0^{12}_{,}478,424,0 = .0^{12}_{,}525,826,6$

$_3B = -\ 0^{12}_{,}841,920,7 + 0^{9}_{,}002,862,068 = 0^{9}_{,}002,020,147$

$_3C_1 = .0^{9}_{,}006,846,257 + .0^{9}_{,}031,019,13 = 0^{9}_{,}037,865,39$

$_3B_2 = -\ .0^{9}_{,}108,707,5 + 0^{9}_{,}010,507,8 - .0^{9}_{,}022,229,1 + .0^{9}_{,}036,744,6$

$+\ .0^{9}_{,}404,749,4 = .0^{9}_{,}321,065,2$

$_3C_2 = .0^{9}_{,}977,647,2 + .0^{9}_{,}005,336,744 = 0^{9}_{,}002,247,196$

$+\ 0^{9}_{,}470,869,22 + .0^{6}_{,}007,264,230 = .0^{6}_{,}008,720,330$

$_3B_3 = -\ 0^{9}_{,}034,334,99 - .0^{9}_{,}005,336,74 + .0^{9}_{,}160,415,81 + 0^{9}_{,}006,596,20$

$+\ 0^{6}_{,}001,566,436 = .0^{6}_{,}001,693,776$

288-D-2714

*Figure C-48.* Hungry Horse Dam study—total arch constants at arch point 3.

# HUNGRY HORSE DAM

## STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### TOTAL ARCH CONSTANTS AT ARCH POINT 2

POINT 2

$$_{2}A_{1} = 0^{12}_{,}047,404,57 + .0^{9}_{,}001,058,825 = .0^{9}_{,}001,106,230$$

$$_{2}B_{1} = 0^{12}_{,}570,274,1 + .0^{9}_{,}015,506,49 = .0^{9}_{,}016,076,76$$

$$_{2}C_{1} = .0^{9}_{,}013,056,84 + 0^{9}_{,}130,461,5 = .0^{9}_{,}143,518,3$$

$$_{2}B_{2} = .0^{9}_{,}146,245,0 + .0^{9}_{,}038,030,9 - .0^{9}_{,}020,369,1 + .0^{9}_{,}071,458,7$$

$$+ .0^{6}_{,}003,510,049 = .0^{6}_{,}003,745,414$$

$$_{2}C_{2} = .0^{6}_{,}003,579,210 + .0^{9}_{,}010,378,58 + .0^{9}_{,}008,864,60$$

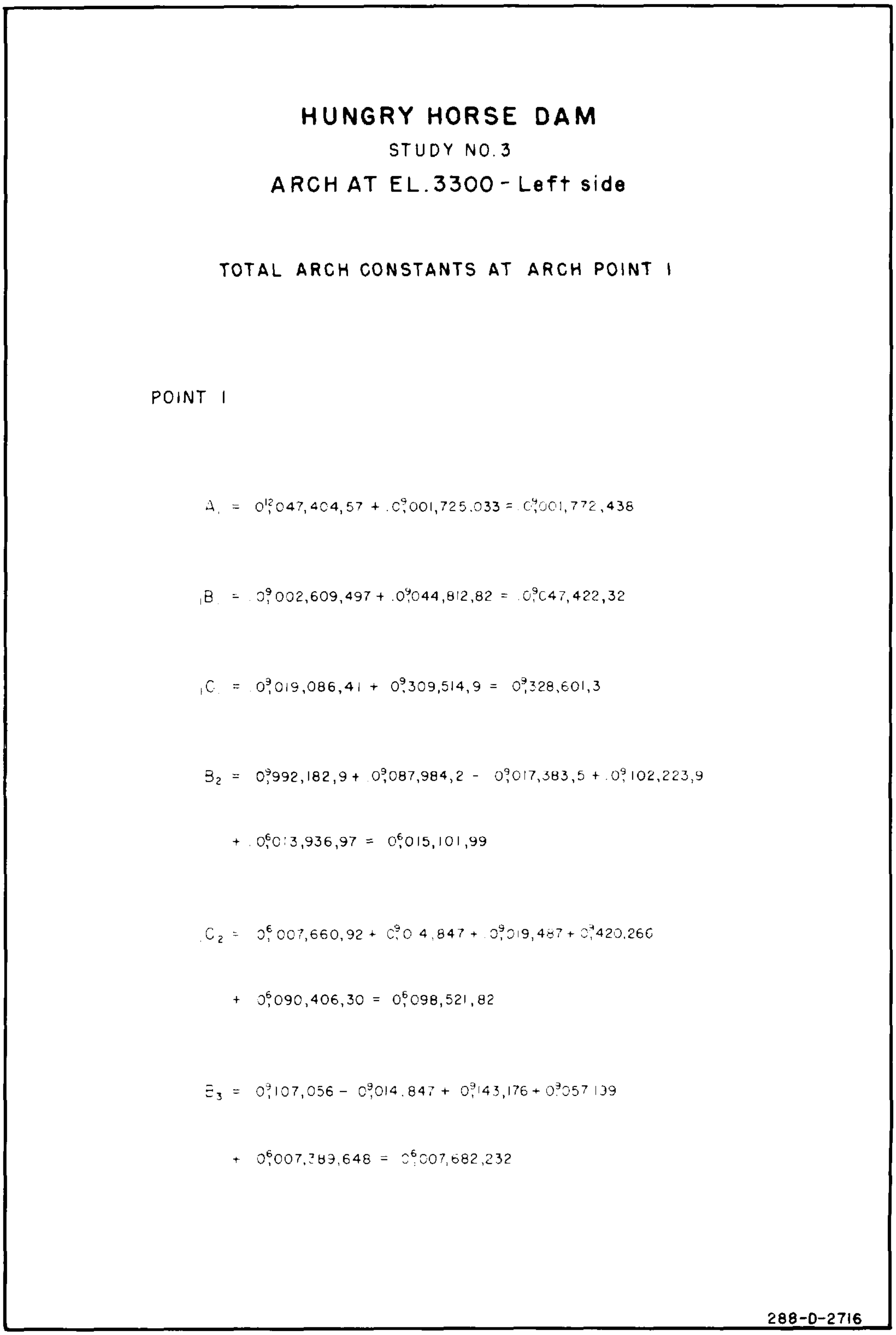
$$+ .0^{9}_{,}451,445,11 + 0^{6}_{,}031,277,55 = 0^{6}_{,}035,327,449$$

$$_{2}B_{3} = - .0^{9}_{,}036,439,09 - 0^{9}_{,}010,378,58 + 0^{9}_{,}153,798,40 + .0^{9}_{,}026,020,31$$

$$+ .0^{6}_{,}003,655,072 = .0^{6}_{,}003,788,073$$

288-D-2715

*Figure C-49.* **Hungry Horse Dam study—total arch constants at arch point 2.**



HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT EL. 3300 - Left side

TOTAL ARCH CONSTANTS AT ARCH POINT 1

POINT 1

$A_1 = 0\overset{12}{,}047,404,57 + 0\overset{9}{,}001,725.033 = 0\overset{9}{,}001,772,438$

$B_1 = 0\overset{9}{,}002,609,497 + 0\overset{9}{,}044,812,82 = 0\overset{9}{,}047,422,32$

$C_1 = 0\overset{9}{,}019,086,41 + 0\overset{9}{,}309,514,9 = 0\overset{9}{,}328,601,3$

$B_2 = 0\overset{9}{,}992,182,9 + 0\overset{9}{,}087,984,2 - 0\overset{9}{,}017,383,5 + 0\overset{9}{,}102,223,9$

$+ 0\overset{6}{,}013,936,97 = 0\overset{6}{,}015,101,99$

$C_2 = 0\overset{6}{,}007,660,92 + 0\overset{9}{,}0\ 4,847 + 0\overset{9}{,}019,487 + 0\overset{9}{,}420.266$

$+ 0\overset{6}{,}090,406,30 = 0\overset{6}{,}098,521,82$

$B_3 = 0\overset{9}{,}107,056 - 0\overset{9}{,}014.847 + 0\overset{9}{,}143,176 + 0\overset{9}{,}057\ 139$

$+ 0\overset{6}{,}007,389,648 = 0\overset{6}{,}007,682,232$

288-D-2716

*Figure C-50.* **Hungry Horse Dam study–total arch constants at arch point 1.**

## HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### TOTAL ARCH CONSTANTS AT ARCH POINT 0 (CROWN)

POINT 0

$${}_0A_1 = .0^{12}047{,}404{,}57 + 0_{,}^{9}002{,}440{,}959 = .0_{,}^{9}002{,}488{,}364$$

$${}_0B_1 = 0_{,}^{9}005{,}249{,}943 + .0_{,}^{9}098{,}205{,}45 = .0_{,}^{9}103{,}455{,}4$$

$${}_0C_1 = 0_{,}^{9}024{,}851{,}38 + .0_{,}^{9}574{,}787{,}0 = .0_{,}^{9}599{,}638{,}4$$

$${}_0B_2 = .0_{,}^{6}002{,}618{,}26 + .0_{,}^{9}164{,}16 - .0_{,}^{9}013{,}44 + 0_{,}^{9}127{,}34$$

$$+ .0_{,}^{6}038{,}862{,}73 = .0_{,}^{6}041{,}759{,}05$$

$${}_0C_2 = 0_{,}^{6}0{,}2{,}997{,}2 + .0_{,}^{9}018{,}5 + .0_{,}^{9}033{,}5 + .0_{,}^{9}379{,}1$$

$$+ .0_{,}^{6}204{,}006{,}1 = .0_{,}^{6}217{,}434{,}4$$

$${}_0B_3 = 0_{,}^{9}551{,}90 - .0_{,}^{9}018{,}49 + .0_{,}^{9}129{,}14 + .0_{,}^{9}098{,}41$$

$$+ .0_{,}^{6}015{,}131{,}85 = .0_{,}^{6}015{,}892{,}81$$

288-D-2717

*Figure C-51.* **Hungry Horse Dam study—total arch constants at arch point 0.**

Computations on figure C-50 indicate total arch constants for point 1. For point 0, total arch constants are illustrated on figure C-51.

C-22. ***Calculation of $M_L$, $H_L$, and $V_L$ Due to Radial Loads.***—Forces and moments used in the transfer of *D*-terms to points in the central section require calculations for total external moment, thrust, and shear at points in the variable-thickness central section of the arch (see figs. C-11, C-12, and C-13); for moment, thrust, and shear at point 4 for load between point 4 and points in the central section see section 4-36(d) and equations (209) to (211), inclusive; and for moment, thrust, and shear in the wedge abutment section see section 4-37(b) and equations (232) to (234), inclusive.

Computations for $M_L$, $H_L$, and $V_L$ for radial load No. 1 at points 0, 1, 2, 3, 4, and 5 are given on figure C-52. Since there is no load at the crown, the values at that point are zero. For point 1, an examination of figure C-11 shows that ${}_1M_L = PR_{E_1}$ ($r_1$ vers $\Phi_{0\text{-}1}$), and for point 2 we note that ${}_2H_L = PR_{E_2}$ (vers $\Phi_{0\text{-}2}$). For point 5, at the abutment, the equations given in section 4-37(b), numbered (232) to (234), inclusive, must be used. Then for point 5, ${}_5H_L = {}_4H_L \cos \Omega + {}_4V_L \sin \Omega$. In the equation, subscript 4 refers to point 4, and subscript 5 refers to point 5.

Following the calculations of $M_L$, $H_L$, and $V_L$ at arch points, as indicated above for all radial loads, it is necessary to compute $M_L$, $H_L$, and $V_L$ for transferring *D*-terms. These values are due to radial external loads between point 4 and points 3, 2, 1, and 0. The values are calculated by means of equations (209) to (211). From these general equations, it can be seen that $H_{L_{4\text{-}3}} = {}_4H_L - {}_3H_L \cos \Phi_{3\text{-}4} - {}_3V_L \sin \Phi_{3\text{-}4}$, where the subscript 4 refers to point 4, subscript 3 refers to point 3, and subscript 3-4 refers to distance measured from point 4 to point 3. These latter coordinates are given on figure C-45, and other values have already been computed on figure C-52. Values for $M_{L_{4\text{-}0}}$, $H_{L_{4\text{-}0}}$, and $V_{L_{4\text{-}0}}$ are the same as external values at point 4. All of these calculations are tabulated on figure C-53.

C-23. ***Calculation of D-Terms Due to Radial Loads.***—For the variable-thickness section, *D*-terms are determined in a similar manner to the method used in section C-14. These calculations are shown for a No. 1 load for $D_1$, $D_2$, and $D_3$ on figures C-54, C-55, and C-56. Radial *D*-terms for the wedge are zero.

Following the computation of *D*-terms for the central section and for the abutment section as described above, *D*-terms must be transferred from point 4 to points 3, 2, 1, and 0. Computations for *D*-term transfers are made by means of equations (218) to (220) as described in section 4-37. These computations of total *D*-terms are given on figure C-57 for a No. 1 load. Thus, it can be seen that in transferring *D*-terms from point 4 to point 3,

$${}_3D_1 = {}_4A'_1\, M_{L_{4\text{-}3}} - {}_4B'_1\, H_{L_{4\text{-}3}} + {}_4C'_1\, V_{L_{4\text{-}3}} + {}_4D'_1 + D''_1$$

and so on for the other *D*-terms. It should be noted that $\Omega_{5\text{-}4}$ is a negative angle and therefore the sine of $\Omega_{5\text{-}4}$ is negative. Similar equations to those shown above are developed and used in transferring *D*-terms for the uniform radial load, load No. 1, from point 4 to points 2, 1, and 0. These calculations are also shown on figure C-57.

*D*-term transfers for triangular radial loads are made using the same method. *D*-term transferring for radial load No. 5 is shown on figure C-58.

C-24. ***Calculation of $M_L$, $H_L$, and $V_L$ Due to Tangential Loads.***—The tangential load is assumed to be applied along the arc through the crown centerline. For the analysis of a variable-thickness arch, as shown on figure C-60, a circular arc of radius $R_E - \frac{T_0}{2} = r_0$ at the crown is assumed to pass from point 0 of the central section of variable thickness, to the face *l*, *m* of the wedge section; then from midpoint 4 along a line perpendicular to *l*, *m* to the face *n*, *m* of the wedge section.

This assumption causes a break in the tangential load at the face *l*, *m* of the wedge section, but introduces negligible error in the analysis since the eccentricities of the load to the right of points 4 and 5 are considered in the equations for moment, thrust, and shear at points 4 and 5.

In order to determine *D*-terms for the variable-thickness arch from point 0 to point 4,

## HUNGRY HORSE DAM

STUDY NO. 3

### ARCH AT EL. 3300 - Left side

$M_L$, $H_L$ AND $V_L$ AT ARCH POINTS DUE TO RADIAL LOAD NO. 1

$_0M_L$ = C

$_0H_L$ = 0

$_0V_L$ = 0

$_1M_L$ = 9,261,595.

$_1H_L$ = 8,317.851,7

$_1V_L$ = 141,044.9

$_2M_L$ = 36,817,520.

$_2H_L$ = 33,156.096

$_2V_L$ = 280,134.5

$_3M_L$ = 81,987,960.

$_3H_L$ = 74,170.397

$_3V_L$ = 415,340.5

$_4M_L$ = 143,679,100.

$_4H_L$ = 130,792.17

$_4V_L$ = 544,788.6

$r_1$ = 1113.46

$r_2$ = 1110.43

$r_3$ = 1105.40

$r_4$ = 1098.53

P = 1000

$R_E$ = 1200

POINT 5 (ROCK ABUTMENT)

$_5M_L$ = 151,328,600.

$_5H_L$ = 53,699.40

$_5V_L$ = 557,689.5

288-D-2719

*Figure C-52.* Hungry Horse Dam study—$M_L$, $H_L$, and $V_L$ at arch points due to radial load No. 1.

## HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### $M_L$, $H_L$ AND $V_L$ FOR TRANSFERRING D-TERMS

### RADIAL LOAD NO. 1

POINT 3

$M_L$ = 9,137,357.

$H_L$ = 8,317.843

$V_L$ = 141,044.8

POINT 2

$M_L$ = 36,422,912.

$H_L$ = 33,156.070

$V_L$ = 280,134.4

POINT 1

$M_L$ = 81,478,370.

$H_L$ = 74,170.38

$V_L$ = 415,340.5

POINT 0 (CROWN)

$M_L$ = 143,679,100.

$H_L$ = 130,792.17

$V_L$ = 544,788.6

### RADIAL LOAD NO. 5

POINT 3

$M_L$ = 7,614,858.

$H_L$ = 6,931.867

$V_L$ = 123,434.7

POINT 2

$M_L$ = 24,293,230.

$H_L$ = 22,114.3

$V_L$ = 210,426.6

POINT 1

$M_L$ = 40,824,400.

$H_L$ = 37,162.75

$V_L$ = 261,229.6

POINT 0

$M_L$ = 48,250,393.

$H_L$ = 43,922.69

$V_L$ = 277,549.61

### RADIAL LOAD NO. 3

POINT 3

$M_L$ = 6,092,310.

$H_L$ = 5,545.876

$V_L$ = 105,824.5

POINTS 2, 1 AND 0

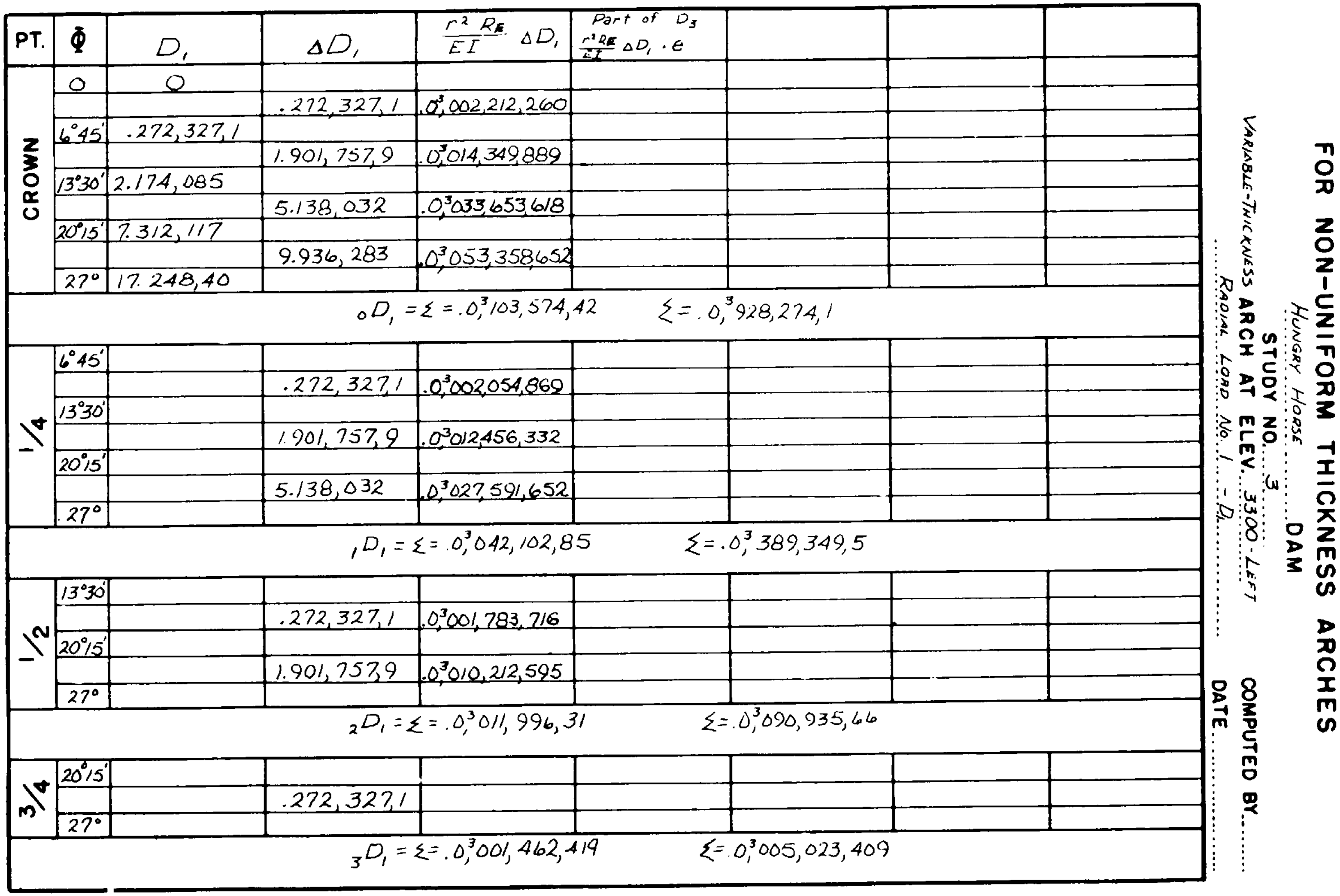
$M_L$ = 12,163,501.

$H_L$ = 11,072.525

$V_L$ = 140,718.8

288-D-2720

*Figure C-53.* Hungry Horse Dam study–$M_L$, $H_L$, and $V_L$ for transferring *D*-terms.



## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Hungry Horse DAM
STUDY NO. 3
Variable-Thickness ARCH AT ELEV. 3300-Left
Radial Load No. 1 - $D_1$
COMPUTED BY.........
DATE.........

| PT. | $\Phi$ | $D_1$ | $\Delta D_1$ | $\frac{r^2 R_E}{EI} \Delta D_1$ | Part of $D_3$ <br> $\frac{r^2 R_E}{EI} \Delta D_1 \cdot e$ | | | |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 | | | | | | |
| | | | .272,327,1 | $.0^{3}_{,}002{,}212{,}260$ | | | | |
| | 6°45′ | .272,327,1 | | | | | | |
| | | | 1.901,757,9 | $.0^{3}_{,}014{,}349{,}889$ | | | | |
| | 13°30′ | 2.174,085 | | | | | | |
| | | | 5.138,032 | $.0^{3}_{,}033{,}653{,}618$ | | | | |
| | 20°15′ | 7.312,117 | | | | | | |
| | | | 9.936,283 | $.0^{3}_{,}053{,}358{,}652$ | | | | |
| | 27° | 17.248,40 | | | | | | |
| | | | $_0D_1 = \Sigma = .0^{3}_{,}103{,}574{,}42$ | | $\Sigma = .0^{3}_{,}928{,}274{,}1$ | | | |
| 1/4 | 6°45′ | | | | | | | |
| | | | .272,327,1 | $.0^{3}_{,}002{,}054{,}869$ | | | | |
| | 13°30′ | | | | | | | |
| | | | 1.901,757,9 | $.0^{3}_{,}012{,}456{,}332$ | | | | |
| | 20°15′ | | | | | | | |
| | | | 5.138,032 | $.0^{3}_{,}027{,}591{,}652$ | | | | |
| | 27° | | | | | | | |
| | | | $_1D_1 = \Sigma = .0^{3}_{,}042{,}102{,}85$ | | $\Sigma = .0^{3}_{,}389{,}349{,}5$ | | | |
| 1/2 | 13°30′ | | | | | | | |
| | | | .272,327,1 | $.0^{3}_{,}001{,}783{,}716$ | | | | |
| | 20°15′ | | | | | | | |
| | | | 1.901,757,9 | $.0^{3}_{,}010{,}212{,}595$ | | | | |
| | 27° | | | | | | | |
| | | | $_2D_1 = \Sigma = .0^{3}_{,}011{,}996{,}31$ | | $\Sigma = .0^{3}_{,}090{,}935{,}66$ | | | |
| 3/4 | 20°15′ | | | | | | | |
| | | | .272,327,1 | | | | | |
| | 27° | | | | | | | |
| | | | $_3D_1 = \Sigma = .0^{3}_{,}001{,}462{,}419$ | | $\Sigma = .0^{3}_{,}005{,}023{,}409$ | | | |

*Figure C-54.* Hungry Horse Dam study—$D_1$-term for variable-thickness arch.—DS2-1(117)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D–TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
VARIABLE - THICKNESS ARCH AT ELEV. 3300-LEFT
RADIAL LOAD No. 1 - $D_2$

COMPUTED BY..........
DATE..........

| PT. | $\Phi$ | $D_2$ - 1st Term | $\Delta D_2$ - 1st Term | $\frac{r_v^3 R_E}{EI_v} \Delta D_2$ 1st Term | $D_2$ 2nd Term | $\Delta D_2$ 2nd Term | $\frac{r_v R_E}{ET_v} \Delta D_2$ 2nd Term | |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 | | | 0 | | | |
| | | | .024,023,14 | | | 20.746,58 | | |
| | 6°45' | .024,023,14 | | | 20.746,58 | | | |
| | | | .357,687,56 | | | 61.380,24 | | |
| | 13°30' | .381,710,7 | | | 82.126,82 | | | |
| | | | 1.528,445,3 | | | 99.478,88 | | |
| | 20°15' | 1.910,156 | | | 181.605,7 | | | |
| | | | 4.029,633 | | | 133.495,2 | | |
| | 27° | 5.939,789 | | | 315.100,9 | | | |
| ${}_0D_2$ = .043,352,07 | | | | Σ = .038,155,95 | | | Σ = .005,196,116 | |
| 1/4 | 6°45' | | | | | | | |
| | | | .024,023,14 | | | 20.746,58 | | |
| | 13°30' | | | | | | | |
| | | | .357,687,56 | | | 61.380,24 | | |
| | 20°15' | | | | | | | |
| | | | 1.528,445,3 | | | 99.478,88 | | |
| | 27° | | | | | | | |
| ${}_1D_2$ = .014,784,60 | | | | Σ = .011,842,00 | | | Σ = .002,942,600 | |
| 1/2 | 13°30' | | | | | | | |
| | | | .024,023,14 | | | 20.746,58 | | |
| | 20°15' | | | | | | | |
| | | | .357,687,56 | | | 61.380,24 | | |
| | 27° | | | | | | | |
| ${}_2D_2$ = .003,595,938 | | | | Σ = .002,290,997 | | | Σ = .001,304,941 | |
| 3/4 | 20°15' | | | | | | | |
| | | | .024,023,14 | | | 20.746,58 | | |
| | 27° | | | | | | | |
| ${}_3D_2$ = .0$^3$466,052,6 | | | | Σ = .0$^3$142,160,3 | | | Σ = 0$^3$323,892,3 | |

*Figure C-55.* **Hungry Horse Dam study–$D_2$-term for variable-thickness arch.–DS2-1(118)**

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
VARIABLE-THICKNESS ARCH AT ELEV. 3300 LEFT
RADIAL LOAD No. 1 - $D_3$

COMPUTED BY ........
DATE ........

| PT. | $\Phi$ | $D_3$ 1st Term | $\Delta D_3$ 1st Term | $\frac{r_v^3 P_E}{EI_v} \Delta D_3$ 1st Term | $D_3$ 2nd Term | $\Delta D_3$ 2nd Term | $\frac{r_v R_E}{ET_v} \Delta D_3$ 2nd Term | |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | | | | 0 | | | |
| | | | .001,132,81 | | | 1.359,370 | | |
| | 6°45' | .001,132,81 | | | 1.359,370 | | | |
| | | | .034,937,99 | | | 9.438,91 | | |
| | 13°30' | .036,070,8 | | | 10.798,28 | | | |
| | | | .235,587,5 | | | 25.218,99 | | |
| | 20°15' | .271,658,3 | | | 36.017,27 | | | |
| | | | .859,937,7 | | | 47.961,53 | | |
| | 27° | 1.131,596 | | | 83.978,80 | | | |
| $_0D_3$ = .009,390,247 | | | | Σ = .007,101,791 | | | Σ = .001,360,182 | |
| 1/4 | | | | | | | | |
| | | | .001,132,81 | | | 1.359,370 | | |
| | | | | | | | | |
| | | | .034,937,99 | | | 9.438,91 | | |
| | | | | | | | | |
| | | | .235,587,5 | | | 25.218,99 | | |
| | | | | | | | | |
| $_1D_3$ = .002,621,796 | | | | Σ = .001,657,163 | | | Σ = .0$^3$575,283,7 | |
| 1/2 | | | | | | | | |
| | | | .001,132,81 | | | 1.359,370 | | |
| | | | | | | | | |
| | | | .034,937,99 | | | 9.438,91 | | |
| | | | | | | | | |
| $_2D_3$ = .0$^3$475,981,0 | | | | Σ = .0$^3$214,971,0 | | | Σ = .0$^3$170,074,3 | |
| 3/4 | | | | | | | | |
| | | | .001,132,81 | | | 1.359,370 | | |
| | | | | | | | | |
| $_3D_3$ = .0$^3$032,949,24 | | | | Σ = .0$^3$006,703,564 | | | Σ = .0$^3$021,222,27 | |

Figure C-56. Hungry Horse Dam study—$D_3$-term for variable-thickness arch.—DS2-1(119)

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### D - TERMS FOR RADIAL LOAD NO. 1

$${}_3D = 0^{6}_{,}522,759,7 + .0^{3}_{,}001,462,419 = .0^{3}_{,}001,985,179$$

$${}_3D_2 = .0^{3}_{,}062,556,69 + .0^{3}_{,}002,069,70 + .0^{3}_{,}077,104,31 + 0^{3}_{,}466,052,6 = .0^{3}_{,}607,783,3$$

$${}_3D_3 = -0^{3}_{,}007,692.93 - .0^{3}_{,}001,128,12 + 0^{3}_{,}005,818,15 + .0^{3}_{,}032,949,24 = 0^{3}_{,}029,946,34$$

$${}_2D = 0^{3}_{,}001,931,256 + .0^{3}_{,}011,996,31 = 0^{3}_{,}013,927,57$$

$${}_2D_2 = .0^{3}_{,}475,568,1 + .0^{3}_{,}015,627,9 + .0^{3}_{,}170,402,1 + .003,595,938 = .004,257,536$$

$${}_2D_3 = .0^{3}_{,}020,771,0 - .0^{3}_{,}002,821,3 + .0^{3}_{,}031,397,9 + .0^{3}_{,}475,981,0 = .0^{3}_{,}525,328,6$$

$${}_1D_1 = .0^{3}_{,}004,205,960 + .0^{3}_{,}042,102,85 = .0^{3}_{,}046,308,81$$

$$D_2 = 001,555,130 + .0^{3}_{,}050,979 + .0^{3}_{,}274,738 + .014,784,60 = .016,665,45$$

$${}_1D_3 = .0^{3}_{,}212,617,6 - .0^{6}_{,}882,9 + .0^{3}_{,}078,334,0 + 002,621,796 = .002,911,865$$

$${}_0D_1 = .0^{3}_{,}007.3\ 5,34 + .0^{3}_{,}103,574,42 = 0^{3}_{,}110,889,8$$

$${}_0D_2 = 003.570,62 + .0^{3}_{,}116,90 + .0^{3}_{,}384,35 + .043,352.07 = .047,423,94$$

$${}_0D_3 = 0^{3}_{,}754,307 + .0^{3}_{,}010,857 + .0^{3}_{,}146,972 + 009,390,247 = .010,302,38$$

288-D-2721

*Figure C-57.* Hungry Horse Dam study—*D*-terms for radial load No. 1.

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### D - TERMS FOR RADIAL LOAD NO. 5

$${}_3D_1 = 0^{6}_{,}438{,}841 + 0^{3}_{,}001{,}188{,}236 = .0^{3}_{,}001{,}627{,}077$$

$${}_3D_2 = 0^{3}_{,}052{,}133{,}27 + .0^{3}_{,}001{,}724{,}83 + .0^{3}_{,}067{,}477{,}48 + .0^{3}_{,}385{,}997{,}0 = 0^{3}_{,}507{,}332{,}6$$

$${}_3D_3 = -.0^{3}_{,}006.411{,}11 - .0^{6}_{,}940{,}15 + .0^{3}_{,}005{,}091{,}73 + .0^{3}_{,}027{,}614{,}70 = .0^{3}_{,}025{,}355{,}17$$

$${}_2D_1 = .0^{3}_{,}001{,}300{,}855 + .0^{3}_{,}007{,}478{,}971 = .0^{3}_{,}008{,}779{,}826$$

$${}_2D_2 = 0^{3}_{,}317{,}193 + .0^{3}_{,}010{,}423 + .0^{3}_{,}128{,}000 + .002{,}316{,}675 = .002{,}772{,}291$$

$${}_2D_3 = 0^{3}_{,}013{,}853{,}8 - .0^{3}_{,}001{,}881{,}7 + .0^{3}_{,}023{,}585{,}0 + .0^{3}_{,}313{,}563{,}7 = .0^{3}_{,}349{,}120{,}8$$

$${}_1D_1 = .0^{3}_{,}002{,}136{,}11 + .0^{3}_{,}018{,}241{,}44 = .0^{3}_{,}020{,}377{,}55$$

$${}_1D_2 = .0^{3}_{,}779{,}191 + .0^{3}_{,}025{,}543 + .0^{3}_{,}172{,}797 + .006{,}752{,}376 = 007{,}729{,}907$$

$${}_1D_3 = .0^{3}_{,}106{,}531 - .0^{6}_{,}442 + .0^{3}_{,}049{,}268 + .001{,}247{,}208 = .001{,}402{,}565$$

$${}_0D_1 = .0^{3}_{,}002{,}507{,}80 + .0^{3}_{,}025{,}143{,}92 = .0^{3}_{,}027{,}651{,}72$$

$${}_0D_2 = .001{,}199{,}09 + 0^{3}_{,}039{,}26 + .0^{3}_{,}195{,}81 + .011{,}657{,}18 = .013{,}091{,}34$$

$${}_0D_3 = 0^{3}_{,}253{,}312 + .0^{3}_{,}003{,}646 + .0^{3}_{,}074{,}877 + .002{,}742{,}536 = .003{,}074{,}371$$

288-D-2722

*Figure C-58.* **Hungry Horse Dam study—*D*-terms for radial load No. 5.**

it is necessary to compute the moments due to tangential load eccentricity, using the standard form shown on figure C-59. Trigonometric functions are computed for one-eighth points as shown, and values of $H_L$ calculated by means of the equations at the top of the form. The value for $e'$ is the same as that shown for $e_0$ on figure C-42. Values for $H_L$ and $V_L$ at the quarter-points are computed by means of formulas given on figure C-12. In this connection, it should be noted that values for $H_L$ at quarter-points are negative as well as values of ${}_1H_L$, etc., when substituted in the formulas for eccentric thrusts. The above procedure is followed for any variable-thickness section.

In the equations for ${}_1H_L$, ${}_2H_L$, etc., at the bottom of figure C-59, the thrust ${}_1H_L$ is the total thrust at the 1/8 point due to the tangential load between the 1/8 point and the crown point. The quantity ${}_2H_L$ is the total thrust at the 3/8 point due to load between the 3/8 point and the crown point, 0. The point at which $D$-terms are being computed is always considered to be the origin for that computation. For example, for $D$-terms computed for the 1/4 point, the total thrust at the 3/8 point is due to load between the 3/8 point and 1/4 point, and the total thrust at the 5/8 point is due to the load between the 5/8 point and 1/4 point, and so on. The moment due to the eccentricity is $H_Le'$, where $e'$ is the eccentricity of the thrust computed at the 1/8, 3/8, 5/8 points, etc. Thus, $e' = r_0 - r_v$, where $r_0$ is the radius to the crown point and $r_v$ is the radius to the center of the voussoir in question; for example, to the 1/8, 3/8, 5/8, or 7/8 point. Computations for tangential load No. 5 are shown on figure C-59; calculations for tangential loads No. 1 and 3 are not shown. The calculation of $H_Le'$ for the variable-thickness section provides the moments necessary to compute $D$-terms for the variable-thickness section, only.

In order to compute $D$-terms for the wedge section, it is necessary to find moments, thrusts, and shears due to tangential loads on the wedge. Figure C-60 shows a tangential load No. 5 on the wedge section. It is assumed that $P'$ is the ordinate of the load at point 4, and $P$ the ordinate at point 5. It is further assumed that $P'$ is the ordinate of a No. 2, 3, or 4 tangential load at the abutment of the variable-thickness section. The expressions for $P$ and $P'$ for tangential load No. 5 are shown on the figure.

Figure C-61 shows calculations for the properties of the wedge section, the eccentricity of point 4 with respect to point 5, and the load ordinates $P$ and $P'$ for tangential loads No. 2, 3, 4, and 5. The algebraic sign of $\Omega$ is disregarded in determining properties of the wedge section.

From the definition already given of the line of action of the tangential load, it can be seen that ${}_4H_{L_{5\text{-}4}}$ due to the load on the wedge section acts at point 4, along $d'$, and that it has an eccentricity about point 4 of $e'_5$ (see fig. C-61). Then, referring to figure C-60, the total value of the thrust assumed to act on the wedge due to tangential load No. 5, or also No. 4, is:

$$H'_{L_{5\text{-}4}} = -\frac{1}{2}(P + P')\,d'$$

The average value of the thrust assumed to act along $r_0$ at point 4 due to the tangential load is assumed to be equal to:

$${}_4H^D_{L_{5\text{-}4}} = \frac{1}{2}H'_{L_{5\text{-}4}}$$

in which $H'_{L_{5\text{-}4}}$ is negative. These latter values of ${}_4H^D_{L_{5\text{-}4}}$ are required in the computation of ${}_4D'$-terms for the wedge and are to be used only for this purpose. Computations for $H'_{L_{5\text{-}4}}$ and $H^D_{L_{5\text{-}4}}$ due to tangential loads No. 1, 3, and 5 are shown on figure C-62.

The determination of $M_L$, $H_L$, and $V_L$ at point 5 due to tangential load between points 4 and 5 is shown on figure C-63. The equations used are shown for the computations due to tangential load No. 1.

The determination of final $M_L$, $H_L$, and $V_L$ values at all arch points is the next step in the analysis. For tangential load No. 5, we have,

$${}_1M_L = \frac{P'r_0^2}{\Phi_{0\text{-}2}}\left[\frac{(\Phi_{0\text{-}1})^2}{2} - \text{vers}\,\Phi_{0\text{-}1}\right] - H_Le_p$$

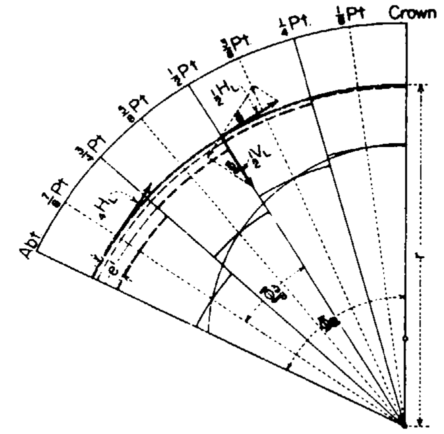
HUNGRY HORSE DAM STUDY NO. 5
VARIABLE THICKNESS ARCH WITH CONSTANT UPSTREAM RADIUS
MOMENTS (He') DUE TO TANGENTIAL LOAD ECCENTRICITY
ARCH AT ELEV. 3300 LEFT
TANG. LOAD NO. 5 Computed by.......... Date..........

### FORMULAE FOR COMPUTING ECCENTRIC THRUSTS

| | Tang. Load No. 1 | Tang. Load No. 2 | Tang. Load No. 3 | Tang. Load No. 4 | Tang. Load No. 5 |
|---|---|---|---|---|---|
| $_1H_L$ | $-Pr\sin\Phi_{\frac{1}{8}}$ | 0 | 0 | 0 | $-\frac{Pr}{\Phi_a}\text{vers}\,\Phi_{\frac{1}{8}}$ |
| $_2H_L$ | $-Pr\sin\Phi_{\frac{3}{8}}$ | 0 | 0 | $-\frac{Pr}{\Phi_3}\text{vers}\,\Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_a}\text{vers}\,\Phi_{\frac{3}{8}}$ |
| $_3H_L$ | $-Pr\sin\Phi_{\frac{5}{8}}$ | 0 | $-\frac{Pr}{\Phi_2}\text{vers}\,\Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_3}\text{vers}\,\Phi_{\frac{3}{8}}$ | $-\frac{Pr}{\Phi_a}\text{vers}\,\Phi_{\frac{5}{8}}$ |
| $_4H_L$ | $-Pr\sin\Phi_{\frac{7}{8}}$ | $-\frac{Pr}{\Phi_1}\text{vers}\,\Phi_{\frac{1}{8}}$ | $-\frac{Pr}{\Phi_2}\text{vers}\,\Phi_{\frac{3}{8}}$ | $-\frac{Pr}{\Phi_3}\text{vers}\,\Phi_{\frac{5}{8}}$ | $-\frac{Pr}{\Phi_a}\text{vers}\,\Phi_{\frac{7}{8}}$ |
| $_5H_L$ | $_2H_L-\frac{1}{4}H_L\cos\Phi_{\frac{1}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{1}{8}}$ | 0 | 0 | $_2H_L$ | $_2H_L-\frac{1}{4}H_L\cos\Phi_{\frac{1}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{1}{8}}$ |
| $_6H_L$ | $_3H_L-\frac{1}{4}H_L\cos\Phi_{\frac{3}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{3}{8}}$ | 0 | $_3H_L$ | $_3H_L$ | $_3H_L-\frac{1}{4}H_L\cos\Phi_{\frac{3}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{3}{8}}$ |
| $_7H_L$ | $_4H_L-\frac{1}{4}H_L\cos\Phi_{\frac{5}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{5}{8}}$ | $_4H_L$ | $_4H_L$ | $_4H_L$ | $_4H_L-\frac{1}{4}H_L\cos\Phi_{\frac{5}{8}}-\frac{1}{4}V_L\sin\Phi_{\frac{5}{8}}$ |
| $_8H_L$ | $_3H_L-\frac{1}{2}H_L\cos\Phi_{\frac{1}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{1}{8}}$ | 0 | $_3H_L$ | $_3H_L-\frac{1}{2}H_L\cos\Phi_{\frac{1}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{1}{8}}$ | $_3H_L-\frac{1}{2}H_L\cos\Phi_{\frac{1}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{1}{8}}$ |
| * $_9H_L$ | $_4H_L-\frac{1}{2}H_L\cos\Phi_{\frac{3}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{3}{8}}$ | $_4H_L$ | $_4H_L$ | $_4H_L-\frac{1}{2}H_L\cos\Phi_{\frac{3}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{3}{8}}$ | $_4H_L-\frac{1}{2}H_L\cos\Phi_{\frac{3}{8}}-\frac{1}{2}V_L\sin\Phi_{\frac{3}{8}}$ |
| $_{10}H_L$ | $_4H_L-\frac{3}{4}H_L\cos\Phi_{\frac{1}{8}}-\frac{3}{4}V_L\sin\Phi_{\frac{1}{8}}$ | $_4H_L$ | $_4H_L-\frac{3}{4}H_L\cos\Phi_{\frac{1}{8}}-\frac{3}{4}V_L\sin\Phi_{\frac{1}{8}}$ | $_4H_L-\frac{3}{4}H\cos\Phi_{\frac{1}{8}}-\frac{3}{4}V_L\sin\Phi_{\frac{1}{8}}$ | $_4H_L-\frac{3}{4}H_L\cos\Phi_{\frac{1}{8}}-\frac{3}{4}V_L\sin\Phi_{\frac{1}{8}}$ |

### TRIGONOMETRIC FUNCTIONS

| POINT | $\Phi$ | SIN $\Phi$ | COS $\Phi$ | VERS $\Phi$ |
|---|---|---|---|---|
| $\frac{1}{8}$ | 3° 22′ 30″ | .058,870,80 | .998,265,61 | .001,734,39 |
| $\frac{3}{8}$ | 10° 07′ 30″ | .175,796,28 | .984,426,57 | .015,573,43 |
| $\frac{5}{8}$ | 16° 52′ 30″ | .290,284,68 | .956,940,34 | .043,059,66 |
| $\frac{7}{8}$ | 23° 37′ 30″ | .400,748,83 | .916,187,96 | .083,812,04 |



—·— = Arc through crown ℄ - radius (r).
——— = Arc through mid-point of voussoir.
— — — = Voussoir mid-point arc extended.

Tangential load applied from left to right along arc through crown ℄.

e' = eccentricity of thrust with reference to arc through mid-point of voussoir (Dwg. 205-D-1810)

$\Phi$ = angle from crown to point.

$\Phi_{\frac{1}{8}} = \frac{1}{8}$ of $\Phi_a$, $\Phi_{\frac{3}{8}} = \frac{3}{8}$ of $\Phi_a$, etc.

$H_L$ and $V_L$ at $\frac{1}{4}$ points are evaluated by formulae on drawing 205-D-190

$_1H_L$, $_2H_L$, etc are thrusts accruing along arc of crown ℄ between $\frac{1}{4}$ points of arch and center of voussoirs.

*ILLUSTRATIVE SKETCH FOR EVALUATING $_9H_L$

### MOMENTS ($H_Le'$) DUE TO ECCENTRIC THRUSTS

| POINT | CROWN | | $\frac{1}{4}$ POINT | | $\frac{1}{2}$ POINT | | $\frac{3}{4}$ POINT | |
|---|---|---|---|---|---|---|---|---|
| | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ | $H_L$ | $H_Le'$ |
| $\frac{1}{8}$ | -4,101.9 $_1H_L$ | -2,133.0 | | | | | | |
| $\frac{3}{8}$ | -36,831.9 $_2H_L$ | -94,105.5 | -20,504.9 $_5H_L$ | -52,390.0 | | | | |
| $\frac{5}{8}$ | -101,838.0 $_3H_L$ | -670,603.2 | -85,813.1 $_6H_L$ | -565,079.3 | -36,907.7 $_8H_L$ | -243,037.2 | | |
| $\frac{7}{8}$ | -198,219.1 $_4H_L$ | -2,484,676. | -182,718.6 $_7H_L$ | -2,290,378. | -134,794.4 $_9H_L$ | -1,689,648. | -53,310.6 $_{10}H_L$ | -668,248.4 |

*Figure C-59.* **Hungry Horse Dam study–moments due to tangential load eccentricity, tangential load No. 5.–DS2-1(122).**

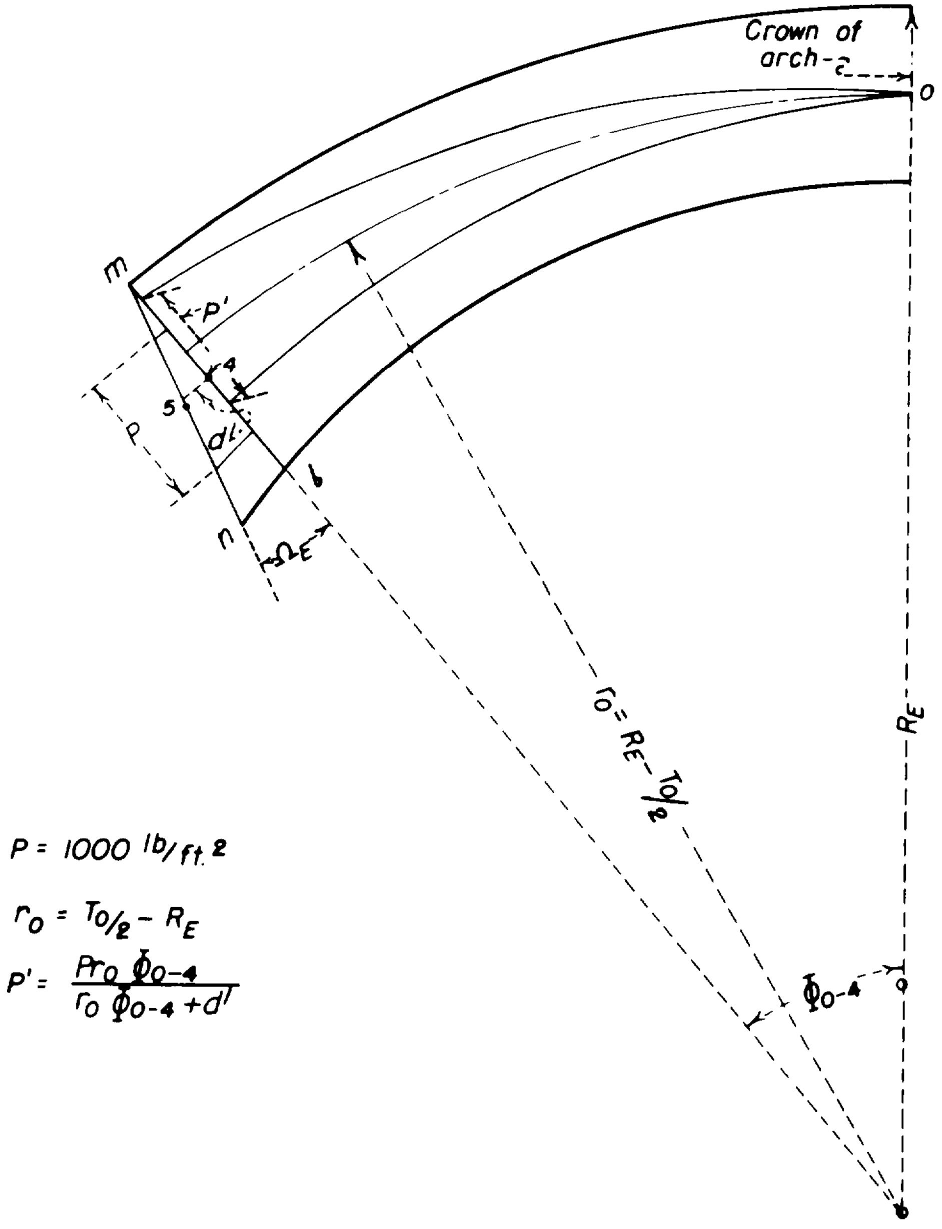


*Figure C-60.* **Tangential load No. 5 on wedge of a variable-thickness arch.–288-D-2723**

and so on for ${}_1H_L$ and ${}_1V_L$, as well as for points 2, 3, and 4.

For point 5, the general equations given in section 4-37(b), numbered from (232) to (234), inclusive, are applicable, but in this case for the wedge section the angle is equal to $\Omega$. Thus we have,

$$ {}_5M_L = {}_4M_L - {}_4H_L y_{5\text{-}4} + {}_4V_L x_{5\text{-}4} + M_{L_{5\text{-}4}} $$

$$ {}_5H_L = {}_4H_L \cos\Omega + {}_4V_L \sin\Omega + H_{L_{5\text{-}4}} $$

and so on. In these equations, algebraic signs of ${}_1H_L$, ${}_2H_L$, ${}_3H_L$, ${}_4H_L$, $H_{L_{5\text{-}4}}$, $V_{L_{5\text{-}4}}$, and sin

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

WEDGE SECTION 4-5

### COMPUTATION OF LOAD ORDINATES FOR TANGENTIAL LOADS

Disregard sign of $\Omega$ in these computations

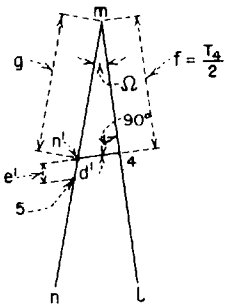
$\Omega = -8°00'$ $\qquad \frac{T_4}{2} = 101.46$ $\qquad \frac{T_5}{2} = 104.67$

$\tan 8° = .140{,}540{,}8 \quad \cos. 8° = .990{,}268{,}1$

$g = \frac{T_4}{2} \cdot \frac{1}{\cos \Omega} = 102.46 \qquad d' = f \tan \Omega = 14.259{,}27$

$\overline{n'5} = \frac{T_5}{2} - g = 104.67 - 102.46 = 2.21$

$e_5' = (\overline{n'5}) \cos 8° = 2.188{,}492 = y_{(5-4)}$



### TANGENTIAL LOAD NO. 3

$\phi_{(2-4)} = 13°30' = .235{,}619{,}45$ radians

$L_{(2-4)} = \phi_{(2-4)} \cdot r_0 = 262.597{,}88$

$P' = \frac{L_{(2-4)}P}{L_{(2-4)} + d'} = 948.495{,}93$

$P - P' = 51.504{,}07$

### TANGENTIAL LOAD NO. 5

$\phi_{(0-4)} = 27° = .471{,}238{,}90$ radians

$L_{(0-4)} = \phi_{(0-4)}\ r_0 = 525.195{,}75$

$P' = \frac{L_{(0-4)} \cdot 1000}{L_{(0-4)} + d'} \doteq 973.567{,}27$

$P - P' = 26.432{,}73$

In which $P'$ = ordinate of tangential load at Point 4

$P = 1000$ = ordinate of tangential load at Point 5

288-D-2724

*Figure C-61.* **Hungry Horse Dam study—tangential load ordinates for wedge section 4-5.**

## HUNGRY HORSE DAM

STUDY NO.3

### ARCH AT EL.3300 - Left side

COMPUTATION OF THRUSTS AT POINT 4
DUE TO TANGENTIAL LOAD ACTING ON WEDGE

TANGENTIAL LOAD NO. 1

$$H'_{L(5-4)} = -Pd' = -14,259.27$$

TANGENTIAL LOAD NO. 3

$$H_{L(5-4)} = -\frac{1}{2}(P+P')d' = -13,892.06$$

TANGENTIAL LOAD NO. 5

$$H_{L(5-4)} = \frac{1}{2}(P+P')d' = -14,070.81$$

AVERAGE VALUE OF $H_{L(5-4)}$ (FOR COMPUTING ${}_4D''$-TERMS ONLY)

TANGENTIAL LOAD NO. 1

$${}_4H^{D}_{L(5-4)} = \frac{1}{2}(-14,259.27) = -7,129.635$$

TANGENTIAL LOAD NO. 3

$${}_4H^{D}_{L(5-4)} = \frac{1}{2}(-13,892.06) = -6,946.030$$

TANGENTIAL LOAD NO. 5

$${}_4H^{D}_{L(5-4)} = \frac{1}{2}(-14,070.81) = -7,035.405$$

288-D-2725

*Figure C-62.* **Hungry Horse Dam study–thrusts at point 4 due to tangential load acting on wedge.**

# HUNGRY HORSE DAM

STUDY NO.3

## ARCH AT EL.3300 - Left side

### COMPUTATION OF $M_L$, $H_L$ AND $V_L$ AT POINT 5 DUE TO TANGENTIAL LOAD BETWEEN POINTS 4 AND 5

### TANGENTIAL LOAD NO. 1

$y_{(5-4)} = 2.188,49$

${}_5M'_{L(5-4)} = -H'_{L(5-4)} \cdot y_{(5-4)} = 31,206.270$

$\cos \Omega = .990,268,07$

${}_5H'_{L(5-4)} = H'_{L(5-4)} \cos \Omega = -14,120.500$

$\sin \Omega = -.139,173,10$

${}_5V_{L(5-4)} = -H'_{L(5-4)} \sin \Omega = -1,984.507$

### TANGENTIAL LOAD NO. 3

${}_5M'_{L(5-4)} = 30,402.634$

${}_5H'_{L(5-4)} = -13,756.863$

${}_5V'_{L(5-4)} = -1,933.401$

### TANGENTIAL LOAD NO. 5

${}_5M'_{L(5-4)} = 30,793.827$

${}_5H'_{L(5-4)} = -13,933.874$

${}_5V'_{L(5-4)} = -1,958.278$

288-D-2726

*Figure C-63.* Hungry Horse Dam study—$M_L$, $H_L$, and $V_L$ at point 5 due to tangential load between points 4 and 5.

$\Omega$ must be considered when making substitutions; these terms are all negative. Computations for tangential load No. 5 at points 1, 2, 3, 4, and 5 are given on figure C-64. Final values for $M_L$, $H_L$, and $V_L$ for point 5 must include the (′) values of $M_L$, $H_L$, and $V_L$ as determined for point 5 (see figure C-63). These values are equal to the respective ${}_BM_L$, ${}_BH_L$, and ${}_BV_L$ values shown in equations (223) to (225) of section 4-37(a). Thus we have,

$$ {}_5M_L = {}_4M_L - {}_4H_L\, y_{4\text{-}5} + {}_4V_L\, x_{4\text{-}5} + M'_{L_{4\text{-}5}} $$

Values of coordinates are given on figure C-46 and computations are shown on figure C-64. In all of the above equations the algebraic signs must be considered, as stated above. Values of $M_L$, $H_L$, and $V_L$ are zero at the crown for all loads; at point 1 for load No. 4; and at points 1 and 2 for load No. 3. At point 5 for the load on the wedge as shown on figure C-63, we have,

$$ {}_5M'_{L(5\text{-}4)} = -H'_{L(5\text{-}4)}\, y_{(5\text{-}4)} $$

$$ {}_5H'_{L(5\text{-}4)} = H'_{L(5\text{-}4)} \cos\Omega $$

$$ {}_5V'_{L(5\text{-}4)} = -H'_{L(5\text{-}4)} \sin\Omega $$

Certain values of $M_L$, $H_L$, and $V_L$ are also required for transferring $D$-terms from point 4 to points 3, 2, 1, and 0, for tangential loading. These are retabulated and are shown for load No. 5 on figure C-65. For transfer from point 4 to points 3, 2, 1, or 0, equations (209) to (211) are applicable (see sec. 4-36(d)). Thus we have, for point 4 to point 1,

$$ M_{L_{4\text{-}1}} = {}_4M_L - {}_1M_L + {}_1H_L\, y_{1\text{-}4} - {}_1V_L\, x_{1\text{-}4} $$

and so on for $H_{L_{4\text{-}1}}$ and $V_{L_{4\text{-}1}}$. For point 4 to point 0 we have,

$$ M_{L_{0\text{-}4}} = {}_4M_L\,;\; H_{L_{0\text{-}4}} = {}_4H_L\,;\; V_{L_{0\text{-}4}} = {}_4V_L $$

C-25. *Calculation of D-Terms Due to Tangential Loads.*—The $D$-terms at point 4 due to the tangential load between points 5 and 4 are computed from the values of ${}_4H^D_{L_{5\text{-}4}}$, as shown on figure C-66, by means of the expressions given below and defined and explained in sections 4-34(g) and (h).

$$ {}_4M_{L_{5\text{-}4}} = 0 \text{ and } {}_4V_{L_{5\text{-}4}} = 0 $$

$$ {}_4D'_1 = -B_1 \bullet {}_4H_{L_{5\text{-}4}} $$

$$ {}_4D'_2 = -B_2 \bullet {}_4H_{L_{5\text{-}4}} $$

$$ {}_4D'_3 = -B_3 \bullet {}_4H_{L_{5\text{-}4}} $$

For the above, arch constants for the wedge are given on figure C-44, and algebraic signs must be considered in the substitutions, as shown on figure C-66 for tangential loads No. 1, 3, and 5.

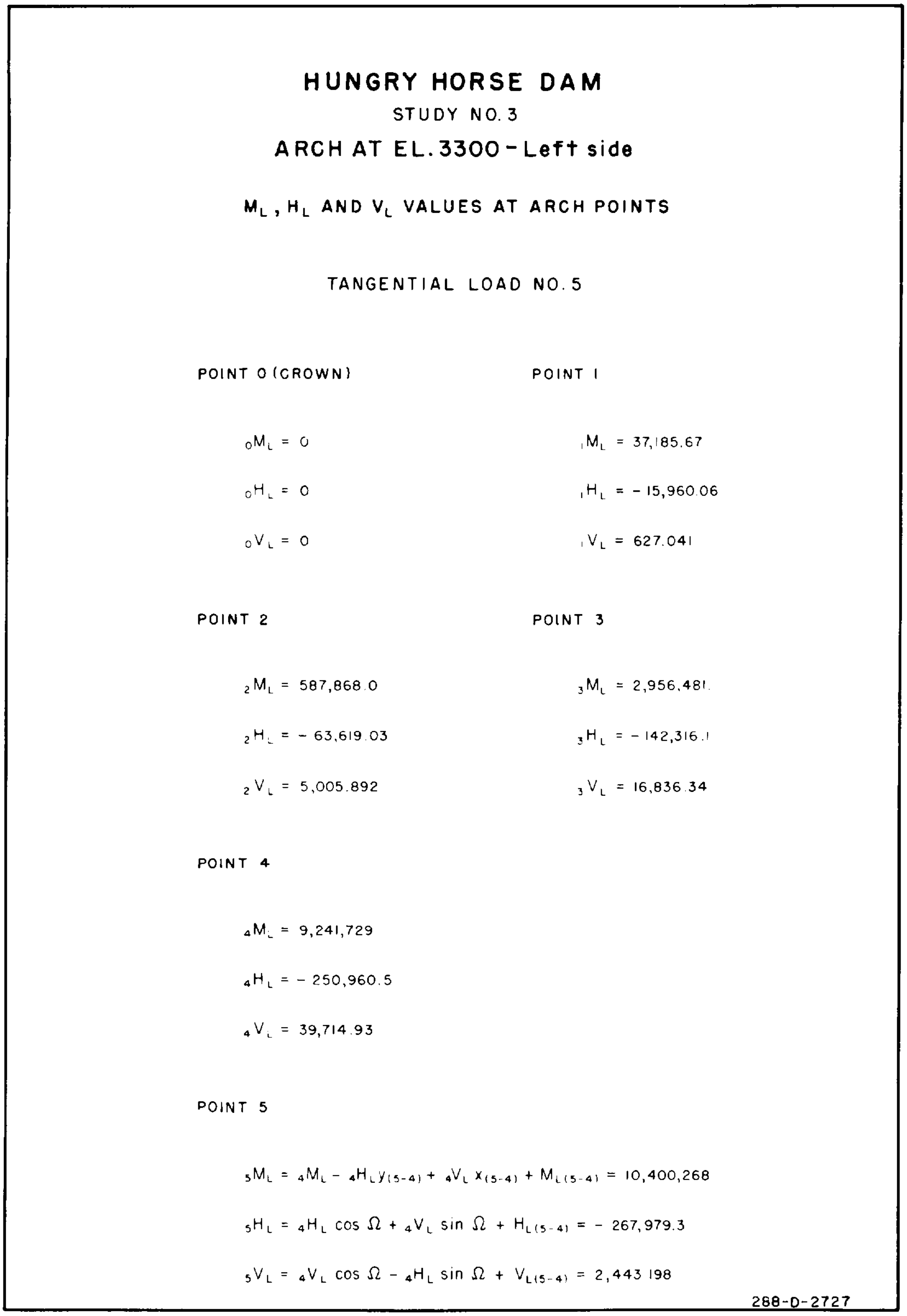
In order to determine final values of $D$-terms it is also necessary to find values of $D$-terms for loads on the central section (see fig. C-60). Computations for $D$-terms for tangential load No. 5, on the variable-thickness section, designated by (″), are given on figures C-67, C-68, and C-69.

Final $D_1$, $D_2$, and $D_3$ terms at all arch points are shown on figure C-70. For points 3, 2, 1, and 0, it is necessary to transfer $D$-terms from point 4 to point 3, point 4 to point 2, point 4 to point 1, and also from point 4 to point 0, by means of general transfer equations given in section 4-37(a), numbered (218) to (220). Thus for $D_1$-terms at point 3, we have

$$ {}_3D_1 = {}_4A_1\, M_{L_{4\text{-}3}} - {}_4B_1\, H_{L_{4\text{-}3}} + {}_4C_1\, V_{L_{4\text{-}3}} + {}_4D'_1 + \frac{P'}{1{,}000}\, {}_3D''_1 $$

at point 2, we have

$$ {}_2D_1 = {}_4A_1\, M_{L_{4\text{-}2}} - {}_4B_1\, H_{L_{4\text{-}2}} + {}_4C_1\, V_{L_{4\text{-}2}} + {}_4D'_1 + \frac{P'}{1{,}000}\, {}_2D''_1 $$



## HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

$M_L$, $H_L$ AND $V_L$ VALUES AT ARCH POINTS

TANGENTIAL LOAD NO. 5

POINT 0 (CROWN)

$_0M_L = 0$

$_0H_L = 0$

$_0V_L = 0$

POINT 1

$_1M_L = 37,185.67$

$_1H_L = -15,960.06$

$_1V_L = 627.041$

POINT 2

$_2M_L = 587,868.0$

$_2H_L = -63,619.03$

$_2V_L = 5,005.892$

POINT 3

$_3M_L = 2,956,481.$

$_3H_L = -142,316.1$

$_3V_L = 16,836.34$

POINT 4

$_4M_L = 9,241,729$

$_4H_L = -250,960.5$

$_4V_L = 39,714.93$

POINT 5

$$_5M_L = {_4M_L} - {_4H_L}\,y_{(5-4)} + {_4V_L}\,x_{(5-4)} + M_{L(5-4)} = 10,400,268$$

$$_5H_L = {_4H_L}\cos\Omega + {_4V_L}\sin\Omega + H_{L(5-4)} = -267,979.3$$

$$_5V_L = {_4V_L}\cos\Omega - {_4H_L}\sin\Omega + V_{L(5-4)} = 2,443\ 198$$

288-D-2727

*Figure C-64.* **Hungry Horse Dam study—$M_L$, $H_L$, and $V_L$ values at arch points.**

## HUNGRY HORSE DAM

STUDY NO.3

## ARCH AT EL.3300 - Left side

### $M_L$, $H_L$ AND $V_L$ FOR TRANSFER OF TANGENTIAL LOAD D-TERMS

TANGENTIAL LOAD NO. 5

POINT 4 TO 3

$M_{L(4-3)} = 2{,}049{,}993$

$H_{L(4-3)} = -111{,}609.8$

$V_{L(4-3)} = 6{,}267.820$

POINT 4 TO 2

$M_{L(4-2)} = 4{,}682{,}054$

$H_{L(4-2)} = -190{,}267.9$

$V_{L(4-2)} = 19{,}995.778$

POINT 4 TO 1

$M_{L(4-1)} = 7{,}644{,}178$

$H_{L(4-1)} = -236{,}204.0$

$V_{L(4-1)} = 33{,}602.59$

POINT 4 TO 0

$M_{L(4-0)} = 9{,}241{,}729$

$H_{L(4-0)} = -250{,}960.5$

$V_{L(4-0)} = 39{,}714.93$

288-D-2728

*Figure C-65.* Hungry Horse Dam study–$M_L$, $H_L$, and $V_L$ for transfer of tangential load *D*-terms.

# HUNGRY HORSE DAM

STUDY NO.3

## ARCH AT EL. 3300 - Left side

### D-TERMS FOR WEDGE SECTION DUE TO TANGENTIAL LOAD ON WEDGE SECTION

TANGENTIAL LOAD NO. 1

$${}_4D'_1 = -B_1\left({}_4H_{L(5-4)}\right) = -\left(-.0^{9}_{,}001,603,223\right)\left(-14,259.27\right) = -.0^{6}_{,}022,860,790$$

$${}_4D'_2 = -B_2\left({}_4H_{L(5-4)}\right) = -\left(-.0^{9}_{,}022,860,78\right)\left(-14,259.27\right) = -.0^{6}_{,}325,978,03$$

$${}_4D'_3 = -B_3\left({}_4H_{L(5-4)}\right) = -\left(+.0^{9}_{,}162,663,0\right)\left(-14,259.27\right) = +0^{3}_{,}002,319,455,6$$

TANGENTIAL LOAD NO. 3

$${}_4D'_1 = -.0^{6}_{,}022,272,070$$

$${}_4D'_2 = -0^{6}_{,}317,583,33$$

$${}_4D'_3 = .0^{3}_{,}002,259,724,2$$

TANGENTIAL LOAD NO. 5

$${}_4D'_1 = -0^{6}_{,}022,558,646$$

$${}_4D'_2 = -0^{6}_{,}321,669,69$$

$${}_4D'_3 = .0^{3}_{,}002,288,800,2$$

288-D-2729

*Figure C-66.* Hungry Horse Dam study–*D*-terms for wedge section due to tangential load on wedge section.

## COMPUTATION FORM FOR ARCH CONSTANTS AND D–TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT ELEV. 3300 LEFT
TANGENTIAL LOAD NO. 5 – $D_1$"
COMPUTED BY........
DATE........

| | | | | | PART OF $D_1$ | ← PART OF $D_3$ | | → |
|---|---|---|---|---|---|---|---|---|
| PT. | $\Phi$ | $D_1$ | $\Delta D_1$ | $\frac{r^3}{EI}\Delta D_1$ | $-(H_L e')\frac{r}{EI}\Delta A_1$ | $\frac{r^3}{EI}\Delta D_1 \cdot e$ | $-(H_L e')\frac{r}{EI}\Delta A_1 \cdot e$ | $-(H_L e')\frac{r^2}{EI}\Delta B_1$ |
| CROWN | 0 | 0 | | | | | | |
| | | | $.0^{3}401,180$ | $.0^{6}003,025,38$ | | | | |
| | 6°45' | $.0^{3}401,180$ | | | | | | |
| | | | .012,423,875 | $.0^{6}086,866,53$ | | | | |
| | 13°30' | .012,825,055 | | | | | | |
| | | | .084,404,495 | $.0^{6}510,417,7$ | | | | |
| | 20°15' | .097,229,55 | | | | | | |
| | | | .311,548,95 | $.0^{3}001,536,362,6$ | | | | |
| | 27° | .408,778,5 | | | | | | |
| $_0D_1 = .0^{3}003,778,841$ | | | | $\Sigma = 0^{3}002,136,672$ | $\Sigma = .0^{3}001,642,169$ | $\Sigma = .0^{3}022,842,92$ | $\Sigma = .0^{3}017,624,70$ | $\Sigma = .0^{3}130,418,6$ |
| 1/4 | 0 | 0 | | | | | | |
| | | | .002,406,818 | $.0^{6}016,828,2$ | | | | |
| | 6°45' | .002,406,818 | | | | | | |
| | | | .042,463,932 | $.0^{6}256,791,3$ | | | | |
| | 13°30' | .044,870,75 | | | | | | |
| | | | .214,215,35 | $.0^{3}001,056,3748$ | | | | |
| | 20°15' | .259,086,1 | | | | | | |
| $_1D_1 = .0^{3}002,788,641$ | | | | $\Sigma = .0^{3}001,329,994$ | $\Sigma = .0^{3}001,458,647$ | $\Sigma = .0^{3}013,592,43$ | $\Sigma = .0^{3}014,467,38$ | $\Sigma = .0^{3}058,618,15$ |
| 1/2 | 0 | 0 | | | | | | |
| | | | .004,412,456 | $.0^{6}026,683,36$ | | | | |
| | 6°45' | .004,412,456 | | | | | | |
| | | | .072,503,994 | $.0^{6}357,543,90$ | | | | |
| | 13°30' | .076,916,45 | | | | | | |
| $_2D_1 = .0^{3}001,333,655$ | | | | $\Sigma = .0^{6}384,227,8$ | $\Sigma = .0^{6}949,427,3$ | $\Sigma = .0^{3}003,093,718$ | $\Sigma = .0^{3}007,197,600$ | $\Sigma = .0^{3}014,741,00$ |
| 3/4 | 0 | 0 | | | | | | |
| | | | .006,418,093 | $.0^{6}031,649,98$ | | | | |
| | 6°45' | .006,418,093 | | | | | | |
| $_3D_1 = .0^{6}351,356,1$ | | | | $\Sigma = .0^{6}031,649,98$ | $\Sigma = .0^{6}319,706,1$ | $\Sigma = .0^{6}108,717,7$ | $\Sigma = .0^{3}001,098,190$ | $\Sigma = .0^{6}814,382,3$ |

*Figure C-67.* Hungry Horse Dam study – $D_1$-terms for tangential load No. 5.–DS2-1(130)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT ELEV. 3300 - LEFT "
TANGENTIAL LOAD No. 5 - $D_2$"
COMPUTED BY........
DATE........

| PT. | $\Phi$ | $D_2$ 1st Term | $\Delta D_2$ 1st Term | $\frac{r^4}{EI}\Delta D_2$ 1st Term | $D_2$ 2nd Term | $\Delta D_2$ 2nd Term | $\frac{r^2}{ET}\Delta D_2$ 2nd Term | $-(H_Le')\frac{r^2}{EI}\Delta C_1$ |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 | | | 0 | | | |
| | | | $.0^{3}039,316,8$ | | | $-.0^{3}141,625$ | | |
| | 6°45' | $.0^{3}039,316,8$ | | | $-.0^{3}141,625$ | | | |
| | | | $.002,461,277,2$ | | | $-.008,882,075$ | | |
| | 13°30' | $.002,500,594$ | | | $-.009,023,70$ | | | |
| | | | $.025,686,866$ | | | $-.092,999,55$ | | |
| | 20°15' | $.028,187,46$ | | | $-.102,023,25$ | | | |
| | | | $.127,883,24$ | | | $-.465,264,75$ | | |
| | 27° | $.156,070,7$ | | | $-.567,288$ | | | |
| | | $_0D_2=.001,540,361$ | | $\Sigma=.0^{3}886,507,0$ | | | $\Sigma=-.0^{3}008,251,74$ | $\Sigma=.0^{3}662,105,5$ |
| 1/4 | 0 | 0 | | | 0 | | | |
| | | | $.0^{3}228,025,4$ | | | $.067,373,75$ | | |
| | 6°45' | $.0^{3}228,025,4$ | | | $.067,373,75$ | | | |
| | | | $.008,272,423$ | | | $.449,088,55$ | | |
| | 13°30' | $.008,500,448$ | | | $.516,462,3$ | | | |
| | | | $.064,760,502$ | | | $1.073,714,7$ | | |
| | 20°15' | $.073,260,95$ | | | $1.590,177$ | | | |
| | | $_1D_2=.0^{3}848,976,6$ | | $\Sigma=.0^{3}409,118,2$ | | | $\Sigma=.0^{3}023,416,71$ | $\Sigma=.0^{3}416,441,7$ |
| 1/2 | 0 | .0 | | | 0 | | | |
| | | | $.0^{3}416,734$ | | | $.134,889,1$ | | |
| | 6°45' | $.0^{3}416,734$ | | | $.134,889,1$ | | | |
| | | | $.014,083,556$ | | | $.907,058,9$ | | |
| | 13°30' | $.014,500,29$ | | | $1.041,948$ | | | |
| | | $_2D_2=.0^{3}260,112,6$ | | $\Sigma=.0^{3}079,324,86$ | | | $\Sigma=.0^{3}015,085,05$ | $\Sigma=.0^{3}165,702,7$ |
| 3/4 | 0 | 0 | | | 0 | | | |
| | | | $.0^{3}605,442,3$ | | | $.202,404,5$ | | |
| | 6°45' | $.0^{3}605442,3$ | | | $.202,404,5$ | | | |
| | | $_3D_2=.0^{3}026,920,33$ | | $\Sigma=.0^{3}003,290,092$ | | | $\Sigma=-0^{3}002,901,756$ | $\Sigma=.0^{3}020,728,48$ |

*Figure C-68.* Hungry Horse Dam study–$D_2$-terms for tangential load No. 5.–DS2-1(131)

# COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT ELEV. 3300 - LEFT
TANGENTIAL LOAD No. 5 - $D_3''$

COMPUTED BY........
DATE........

| PT. | $\Phi$ | $D_3$ 1st Term | $\Delta D_3$ 1st Term | $\frac{r^4}{EI}\Delta D_3$ 1st Term | $D_3$ 2nd Term | $\Delta D_3$ 2nd Term | $\frac{r^2}{ET}\Delta D_3$ 2nd Term | |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 | | | 0 | | | |
| | | | .0$^{3}$001,986,74 | | | .580,297,5 | | |
| | 6°45' | .0$^{3}$001,986,74 | | | .580,297,5 | | | |
| | | | .0$^{3}$251,356,26 | | | 4.109,494,5 | | |
| | 13°30' | .0$^{3}$253,343 | | | 4.689,792 | | | |
| | | | .004,048,087 | | | 11.398,308 | | |
| | 20°15' | .004,301,43 | | | 16.088,10 | | | |
| | | | .027,639,84 | | | 22.877,07 | | |
| | 27° | .031,941,27 | | | 38.965,17 | | | |
| | | $_0D_3$ = .0$^{3}$930,483,2 | | Σ = .0$^{3}$179,292,8 | | | Σ = .0$^{3}$580,304,2 | |
| 1/4 | 0 | 0 | | | 0 | | | |
| | | | .0$^{3}$011,257 | | | 2.325,195 | | |
| | 6°45' | .0$^{3}$011,257 | | | 2.325,195 | | | |
| | | | .0$^{3}$832,921 | | | 9.462,975 | | |
| | 13°30' | .0$^{3}$844,178 | | | 11.788,17 | | | |
| | | | .010,140,632 | | | 20.707,18 | | |
| | 20°15' | .010,984,81 | | | 32.495,35 | | | |
| | | $_1D_3$ = .0$^{3}$628,102,3 | | Σ = .0$^{3}$060,774,16 | | | Σ = .0$^{3}$480,650,2 | |
| 1/2 | 0 | 0 | | | 0 | | | |
| | | | .0$^{3}$020,527,3 | | | 4.070,092 | | |
| | 6°45' | .0$^{3}$020,527,3 | | | 4.070,092 | | | |
| | | | .001,414,479,7 | | | 14.816,448 | | |
| | 13°30' | .001,435,007 | | | 18.886,54 | | | |
| | | $_2D_3$ = .0$^{3}$308,064,7 | | Σ = .0$^{3}$007,824,089 | | | Σ = .0$^{3}$275,208,3 | |
| 3/4 | 0 | 0 | | | 0 | | | |
| | | | .0$^{3}$029,797,6 | | | 5.814,990 | | |
| | 6°45' | .0$^{3}$029,797,6 | | | 5.814,990 | | | |
| | | $_3D_3$ = .0$^{3}$085,549,36 | | Σ = .0$^{6}$161,926,0 | | | Σ = .0$^{3}$083,366,14 | |

*Figure C-69.* **Hungry Horse Dam study–$D_3$-terms for tangential load No. 5.–DS2-1(132)**

## HUNGRY HORSE DAM

STUDY NO.3

## ARCH AT EL. 3300 - Left side

### TOTAL D-TERMS AT ARCH POINTS FOR TANGENTIAL LOAD NO. 5

POINT 0 (CROWN)

$_0D_1 = .0^{6}_{,}438,100,2 - .0^{6}_{,}402,345,6 + .0^{6}_{,}021,476,4 - .0^{6}_{,}022,558,6$

$+ .0^{3}_{,}003,678,955,9 = .0^{3}_{,}003,713,628$

$_0D_2 = .0^{3}_{,}229,669,7 - .0^{3}_{,}224,303,4 + .0^{3}_{,}028,018,7 - .0^{3}_{,}012,576,2$

$+ .001,499,645,1 = .001,520,454$

$_0D_3 = .0^{3}_{,}048,518,6 - .0^{3}_{,}020,831,3 + .0^{3}_{,}010,714,2 - .0^{3}_{,}001,168,0$

$+ .0^{3}_{,}905,888,0 = .0^{3}_{,}943,121,5$

POINT 1

$_1D_1 = .0^{3}_{,}002,694,223$

$_1D_2 = .0^{3}_{,}822,642,7$

$_1D_3 = .0^{3}_{,}640,764,0$

POINT 2

$_2D_1 = .0^{3}_{,}001,203,566$

$_2D_2 = .0^{3}_{,}230,219,4$

$_2D_3 = .0^{3}_{,}322,220,4$

POINT 3

$_3D_1 = .0^{6}_{,}241,143,2$

$_3D_2 = .0^{3}_{,}012,397,21$

$_3D_3 = .0^{3}_{,}098,866,33$

288-D-2730

*Figure C-70.* Hungry Horse Dam study—total *D*-terms at arch points for tangential load No. 5.

at point 1, we have

$$
{}_1D_1 = {}_4A_1\, M_{L_{4\text{-}1}} - {}_4B_1\, H_{L_{4\text{-}1}} + {}_4C_1\, V_{L_{4\text{-}1}} + {}_4D_1' + \frac{P'}{1{,}000}\,{}_1D_1''
$$

and at point 0, we have

$$
{}_0D_1 = {}_4A_1\, M_{L_{4\text{-}0}} - {}_4B_1\, H_{L_{4\text{-}0}} + {}_4C_1\, V_{L_{4\text{-}0}} + {}_4D_1' + \frac{P'}{1{,}000}\,{}_0D_1''
$$

On figure C-70, values for tangential loads No. 1 and 3 are not shown. Complete computations include $D$-terms at all arch points for tangential loads No. 1, 3, and 5.

C-26. ***Calculation of $M_L$ and D-Terms Due to Twist Loads.***—Twist loads are horizontal couples causing pure bending of the arch. They do not produce thrust or shears. Theoretically, twist loads act along the centerline of the arch, but for purposes of evaluating $D$-terms, they are assumed to act along the centerline of the voussoirs. This latter centerline is assumed to be the line traced in section C-24 and shown on figure C-60. Application of twist load to this centerline produces an eccentricity with reference to other arch points and causes an additional tangential movement. This factor is included in calculations for $D_3$. Values for $M_L$ are determined by assuming: that $P'$ is the ordinate of twist load at point 4 of the wedge and that $P$ is the load ordinate at the abutment for a No. 3, 4, or 5 twist load, as described in section C-24 and shown on figure C-60. Thus it can be seen that values of $P$ and $P'$ for twist loads are obtained in the same manner as for tangential loads, except that $P$ = 1,000 foot-pounds per square foot.

Equations for $M_L$ due to twist loads between points 5 and 4 are obtained in the manner described for ${}_4H^D_{L_{5\text{-}4}}$ in section C-24 by assuming that the average moment at point 4 is equal to one-half the total moment. This is the moment used for computing ${}_4D'$-terms at point 4 due to the twist load between points 5 and 4. Thus we have ${}_4M^D_{L_{5\text{-}4}} = Pd'$ for twist load No. 1; and ${}_4M^D_{L_{5\text{-}4}} = \left(\frac{P'+P}{2}\right) d'$ for twist load No. 2, 3, 4, or 5.

The ${}_4D'$-terms at point 4 due to the twist load between points 5 and 4 are determined in the same manner as for tangential loads. Thus we have,

$$
{}_4D_1' = A_1 \bullet \frac{1}{2} \bullet M^D_{L_{5\text{-}4}}
$$

$$
{}_4D_2' = C_1 \bullet \frac{1}{2} \bullet M^D_{L_{5\text{-}4}}
$$

$$
{}_4D_3' = B_1 \bullet \frac{1}{2} \bullet M^D_{L_{5\text{-}4}}
$$

In these equations the arch constants are determined as before by means of equations (226), (227), and (228) of section 4-37(b) (see fig. C-44). Moments and ${}_4D'$-terms for twist loads No. 1, 3, and 5 between points 5 and 4 are calculated and shown on figure C-71.

Moments due to twist loads on the variable-thickness section can be determined by means of general formulas given on figure C-13, section C-6, since there is no eccentricity involved. Thus, for a uniform twist load No. 1, at point 1, we have,

$$
{}_1M_{L_{0\text{-}1}} = P r_0 \bullet \Phi_{0\text{-}1}\, ; \; {}_1H_L = 0; \; {}_1V_L = 0
$$

For twist load No. 5, at point 1, we have,

$$
{}_1M_{L_{0\text{-}1}} = \frac{P r_0}{\Phi_{0\text{-}1}} \bullet \frac{(\Phi_{0\text{-}1})^2}{2}
$$

and for point 2,

$$
{}_2M_{L_{0\text{-}2}} = \frac{P r_0}{\Phi_{0\text{-}2}} \bullet \frac{(\Phi_{0\text{-}2})^2}{2}
$$

Computations for twist loads No. 1, 3, and 5 at all arch points are given on figure C-72. For point 5, equation (232) of section 4-37(b) is used in which terms for $H_L$ and $V_L$ are zero, so that for twist loads at point 5, we have,

$$
{}_5M_L = {}_4M_L + M_{L_{5\text{-}4}}
$$

## HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### MOMENTS AND D-TERMS FOR WEDGE DUE TO TWIST LOAD ON WEDGE

### ${}_4M_{L(5-4)}$ DUE TO TWIST LOAD BETWEEN POINTS 5 AND 4

TWIST LOAD NO. 1 P = 1000 d' = 14.259,27

$${}_4M^D_{L(5-4)} = Pd' = 14,259.27$$

TWIST LOAD NO. 3 P' = 948.495,93

$${}_4M^D_{L(5-4)} = \left(\frac{P'+P}{2}\right)d' = 13,892.06$$

TWIST LOAD NO. 5 P' = 973.567,27

$${}_4M^D_{L(5-4)} = \left(\frac{P'+P}{2}\right)d' = 14,070.81$$

### ${}_4D'$ - LOAD TERMS FOR WEDGE

TWIST LOAD NO. 1

$${}_4D'_1 = A''_1 \frac{{}_4M^D_{L(5-4)}}{2} = .0^{9}_{,}337,977,28$$

$${}_4D'_2 = C''_1 \frac{{}_4M^D_{L(5-4)}}{2} = .0^{6}_{,}003,855,447,8$$

$${}_4D'_3 = B''_1 \frac{{}_4M^D_{L(5-4)}}{2} = -.0^{6}_{,}011,430,395$$

| TWIST LOAD NO. 3 | TWIST LOAD NO. 5 |
|---|---|
| ${}_4D'_1 = .0^{9}_{,}329,273,57$ | ${}_4D'_1 = .0^{9}_{,}333,510,35$ |
| ${}_4D'_2 = .0^{6}_{,}003,756,160,9$ | ${}_4D'_2 = .0^{6}_{,}003,804,491,6$ |
| ${}_4D'_3 = -.0^{6}_{,}011,136,035$ | ${}_4D'_3 = -.0^{6}_{,}011,279,323$ |

288-D-2731

*Figure C-71.* **Hungry Horse Dam study—moments and *D*-terms for wedge due to twist load on wedge.**

# HUNGRY HORSE DAM

STUDY NO. 3

## ARCH AT EL. 3300 - Left side

### $M_L$ AT ARCH POINTS DUE TO TWIST LOADS

#### TWIST LOAD NO. 1 $r_0 = 1114.5$

POINT 0 (CROWN)

$_0M_L = 0$

POINT 1 $\phi_{(0-1)} = 6°45'$

$_1M_L = Pr_0\,\phi_{(0-1)} = 131,298.9$

POINT 2

$_2M_L = 262,597.9$

POINT 3

$_3M_L = 393,896.8$

POINT 4

$_4M_L = 525,195.8$

POINT 5

$_5M_L = 539,455.1$

#### TWIST LOAD NO. 3

POINT 0

$_0M_L = 0$

POINT 1

$_1M_L = 0$

POINT 2

$_2M_L = 0$

POINT 3

$_3M_L = 31,134.13$

POINT 4

$_4M_L = 124,536.5$

POINT 5

$_5M_L = 138,428.6$

#### TWIST LOAD NO. 5

POINT 0

$_0M_L = 0$

POINT 1

$_1M_L = 15,978.54$

POINT 2

$_2M_L = 63,914.17$

POINT 3

$_3M_L = 143,806.9$

POINT 4

$_4M_L = 255,656.6$

POINT 5

$_5M_L = 269,727.4$

288-D-2732

*Figure C-72.* **Hungry Horse Dam study—$M_L$ at arch points due to twist load.**

Values of moments for use in transferring *D*-terms are required in the next step of the analysis. Values for $M_{L_{5\text{-}4}}$ have been calculated as shown on figure C-71. Values for $M_{L_{4\text{-}1}}$ for transfer from point 4 to point 1 are obtained by subtracting ${}_1M_L$ from ${}_4M_L$. Values for transfer from point 4 to point 0 are equal to ${}_4M_L$ (see equation (209) in sec. 4-36(d)). All of these values for transferring *D*-terms are given on figure C-73.

Final *D*-terms for the arch are obtained by transfer in a manner similar to the procedure used in section C-2. Values of ${}_4D'$ are given on figure C-71. Computations for $D_1''$, $D_2''$, and $D_3''$ are shown on figures C-74 and C-75. *D*-terms at all arch points due to twist load No. 5 are shown on figure C-76.

Equations for *D*-terms at point 1 are as follows:

$$ {}_1D_1 = {}_4A_1\ M_{L_{4\text{-}1}} + {}_4D_1' + \frac{P'}{1{,}000} \bullet {}_1D_1'' $$

and

$$ \begin{aligned} {}_1D_2 = {} & ({}_4A_1\ x_{4\text{-}1} - {}_4B_1\ \sin\Phi_{1\text{-}4} \\ & + {}_4C_1\ \cos\Phi_{1\text{-}4})M_{L_{1\text{-}4}} \\ & + ({}_4D_1'\ x_{4\text{-}1} + {}_4D_2'\ \cos\Phi_{1\text{-}4} \\ & - {}_4D_3'\ \sin\Phi_{1\text{-}4}) + \frac{P'}{1{,}000} \bullet {}_1D_2'' \end{aligned} $$

and so on for ${}_1D_3$. *D*-terms at points 0, 2, and 3 are determined in a similar manner.

This completes the computation of arch constants and *D*-terms for the arch. General forms shown on figures C-2 through C-4 can be used, provided that coordinates and arch angles are correctly used as indicated in section C-15. The calculation of coordinates has already been made and is shown on figure C-45. Using these data, the general procedure outlined for uniform-thickness arches is applied with the exception that arch constants and *D*-terms have already been calculated by the method outlined in this section.

Owing to the fact that $\Delta r_a$ and $\Delta s_a$ are respectively parallel and normal to the nonradial abutment as shown on figure C-77, the movements at the abutment must be resolved through the angle $\Omega'$ so that the directions will be in agreement with those at the other points both in the arches and cantilevers.

$$ \Delta r_5 = \Delta r_a \cos\Omega' - \Delta s_a \sin\Omega' $$

$$ \Delta s_5 = \Delta r_a \sin\Omega' + \Delta s_a \cos\Omega' $$

in which $\Omega' = \Omega + (\Phi_5 - \Phi_4)$ as shown on figure C-77.

## HUNGRY HORSE DAM

STUDY NO. 3

### ARCH AT EL. 3300 - Left side

### MOMENTS FOR TRANSFER OF D-TERMS

#### TWIST LOAD NO. 1

$M_{L(4-0)} = {}_4M_L = 525{,}195.8$

$M_{L(4-1)} = {}_4M_L - {}_1M_L = 393{,}896.9$

$M_{L(4-2)} = {}_4M_L - {}_2M_L = 262{,}597.9$

$M_{L(4-3)} = {}_4M_L - {}_3M_L = 131{,}299.0$

#### TWIST LOAD NO. 3

$M_{L(4-0)} = 124{,}536.5$

$M_{L(4-1)} = 124{,}536.5$

$M_{L(4-2)} = 124{,}536.5$

$M_{L(4-3)} = 93{,}402.37$

#### TWIST LOAD NO. 5

$M_{L(4-0)} = 255{,}656.6$

$M_{L(4-1)} = 239{,}678.1$

$M_{L(4-2)} = 191{,}742.4$

$M_{L(4-3)} = 111{,}849.7$

288-D-2733

*Figure C-73.* **Hungry Horse Dam study—moments for transfer of *D*-terms.**

## COMPUTATION FORM FOR ARCH CONSTANTS AND D–TERMS FOR NON–UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT ELEV. 3300 LEFT
TWIST LOAD No. 5 – $D_1$"

COMPUTED BY..........
DATE..........

| PT. | Φ | $D_1$ | $\Delta D_1$ | $\frac{r^2}{EI}\Delta D_1$ |  | PART OF $D_3$<br>$\frac{r^2}{EI}\Delta D_1 \cdot e$ |  |  |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 |  |  |  |  |  |  |
|  |  |  | .578,297,25 | .0$^6_,$009,914,85 |  |  |  |  |
|  | 6°45' | .578,297,25 |  |  |  |  |  |  |
|  |  |  | 4.048,079,8 | .0$^6_,$025,454,30 |  |  |  |  |
|  | 13°30' | 4.626,377 |  |  |  |  |  |  |
|  |  |  | 10.987,648 | .0$^6_,$059,973,37 |  |  |  |  |
|  | 20°15' | 15.614,025 |  |  |  |  |  |  |
|  |  |  | 21.397,00 | .0$^6_,$095,753,03 |  |  |  |  |
|  | 27° | 37.011,02 |  |  |  |  |  |  |
|  |  |  |  | $_0D_1$ = Σ = .0$^6_,$185,095,55 |  | Σ = .0$^3_,$001,662,260,3 |  |  |
| 1/4 | 0 | 0 |  |  |  |  |  |  |
|  |  |  | 2.313,188,5 | .0$^6_,$014,545,32 |  |  |  |  |
|  | 6°45' | 2.313,188,5 |  |  |  |  |  |  |
|  |  |  | 9.252,757 | .0$^6_,$050,503,90 |  |  |  |  |
|  | 13°30' | 11.565,945 |  |  |  |  |  |  |
|  |  |  | 19.662,105 | .0$^6_,$087,989,26 |  |  |  |  |
|  | 20°15' | 31.228,05 |  |  |  |  |  |  |
|  |  |  |  | $_1D_1$ = Σ = .0$^6_,$153,038,48 |  | Σ = .0$^3_,$001,313,516,8 |  |  |
| 1/2 | 0 | 0 |  |  |  |  |  |  |
|  |  |  | 4.048,080 | .0$^6_,$022,095,45 |  |  |  |  |
|  | 6°45' | 4.048,080 |  |  |  |  |  |  |
|  |  |  | 14.457,43 | .0$^6_,$064,697,98 |  |  |  |  |
|  | 13°30' | 18.505,51 |  |  |  |  |  |  |
|  |  |  |  | $_2D_1$ = Σ = .0$^6_,$086,793,43 |  | Σ = .0$^6_,$603,238,46 |  |  |
| 3/4 | 0 | 0 |  |  |  |  |  |  |
|  |  |  | 5.782,971 | .0$^6_,$025,879,19 |  |  |  |  |
|  | 6°45' | 5.782,971 |  |  |  |  |  |  |
|  |  |  |  | $_3D_1$ = Σ = .0$^6_,$025,879,19 |  | Σ = .0$^6_,$088,895,018 |  |  |

*Figure C-74.* Hungry Horse Dam study–$D_1$-terms for twist load No. 5.–DS2-1(137)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

HUNGRY HORSE DAM
STUDY NO. 3
ARCH AT ELEV. 3300 LEFT
TWIST LOAD NO. 5 - $D_2$" AND $D_3$"

COMPUTED BY........
DATE..................

| PT. | $\phi$ | $D_2$ | $\Delta D_2$ | $\frac{r^3}{EI}\Delta D_2$ | | $D_3$ | $\Delta D_3$ | $\frac{r^3}{EI}\Delta D_3$ |
|---|---|---|---|---|---|---|---|---|
| CROWN | 0 | 0 | | | | 0 | | |
| | | | .051,018,02 | | | | .002,405,9 | |
| | 6°45' | .051,018,02 | | | | .002,405,9 | | |
| | | | .761,498,0 | | | | .074,392,1 | |
| | 13°30' | .812,516 | | | | .076,798 | | |
| | | | 3.269,148 | | | | .503,980 | |
| | 20°15' | 4.081,664 | | | | .580,778 | | |
| | | | 8.679,026 | | | | 1.852,484 | |
| | 27° | 12.760,69 | | | | 2.433,262 | | |
| | | | | $_0D_2 = \Sigma = .0^3068,277,988$ | | | $_0D_3 = .0^3014,383,536$ | $\Sigma = .0^3012,721,275$ |
| 1/4 | 0 | 0 | | | | 0 | | |
| | | | .187,087,05 | | | | .008,420,95 | |
| | 6°45' | .187,087,05 | | | | .008,420,95 | | |
| | | | 1.709,454 | | | | .164,395,55 | |
| | 13°30' | 1.896,541 | | | | .172,816,5 | | |
| | | | 5.818,340 | | | | .892,183,5 | |
| | 20°15' | 7.714,881 | | | | 1.065,000 | | |
| | | | | $_1D_2 = \Sigma = .0^3040,338,020$ | | | $_1D_3 = .0^3006,766,226,8$ | $\Sigma = .0^3005,452,710$ |
| 1/2 | 0 | 0 | | | | 0 | | |
| | | | .323,156,1 | | | | .014,436,0 | |
| | 6°45' | .323,156,1 | | | | .014,436,0 | | |
| | | | 2.657,409 | | | | .254,399,0 | |
| | 13°30' | 2.980,565 | | | | .268,835 | | |
| | | | | $_2D_2 = \Sigma = .0^3015,058,878$ | | | $_2D_3 = .0^3001,945,072,2$ | $\Sigma = .0^3001,341,833,7$ |
| 3/4 | 0 | 0 | | | | 0 | | |
| | | | .459,225,1 | | | | .020,451,0 | |
| | 6°45' | .459,225,1 | | | | .020,451,0 | | |
| | | | | $_3D_2 = \Sigma = .0^3002,264,608,1$ | | | $_3D_3 = .0^6189,746,43$ | $\Sigma = .0^6100,851,41$ |

Figure C-75. Hungry Horse Dam study—$D_2$- and $D_3$-terms for twist load No. 5.—DS2-1(138)

# HUNGRY HORSE DAM

## STUDY NO. 3

## ARCH AT EL. 3300 – Left side

### TOTAL D-TERMS AT ARCH POINTS FOR TWIST LOAD NO. 5

$P' = 973.567,27$

**POINT O (CROWN)**

$${}_{0}D_{1} = .0_{,}^{6}012,119,291 + .0_{,}^{9}333,510,35 + .0_{,}^{6}180,202,969 = .0_{,}^{6}192,655,77$$

$${}_{0}D_{2} = .0_{,}^{3}006,353,419,3 + .0_{,}^{6}174,839,5 + .0_{,}^{3}066,473,214,4 = .0_{,}^{3}073,001,473$$

$${}_{0}D_{3} = .0_{,}^{3}001,342,182,6 + .0_{,}^{6}036,935,5 + .0_{,}^{3}014,003,339,9 = .0_{,}^{3}015,382,458$$

**POINT 1**

$${}_{1}D_{1} = .0_{,}^{6}011,361,837, + .0_{,}^{9}333,510,35 + .0_{,}^{6}148,993,26 = .0_{,}^{6}160,688,61$$

$${}_{1}D_{2} = .0_{,}^{3}004,574,594,5 + .0_{,}^{6}134,280,6 + .0_{,}^{3}039,271,776,0 = .0_{,}^{3}043,980,651$$

$${}_{1}D_{3} = .0_{,}^{6}625,439,28 + .0_{,}^{6}018,358,87 + .0_{,}^{3}006,587,376,95 = .0_{,}^{3}007,231,175,1$$

**POINT 2**

$${}_{2}D_{1} = .0_{,}^{6}009,089,466,0 + .0_{,}^{9}333,510,35 + .0_{,}^{6}084,499,242,7 = .0_{,}^{6}093,922,219$$

$${}_{2}D_{2} = .0_{,}^{3}002,503,549,8 + .0_{,}^{6}091,860,1 + .0_{,}^{3}014,660,830,7 = .0_{,}^{3}017,256,241$$

$${}_{2}D_{3} = .0_{,}^{6}109,345,7 + .0_{,}^{6}004,012,1 + .0_{,}^{3}001,893,658,6 = .0_{,}^{3}002,007,016,4$$

**POINT 3**

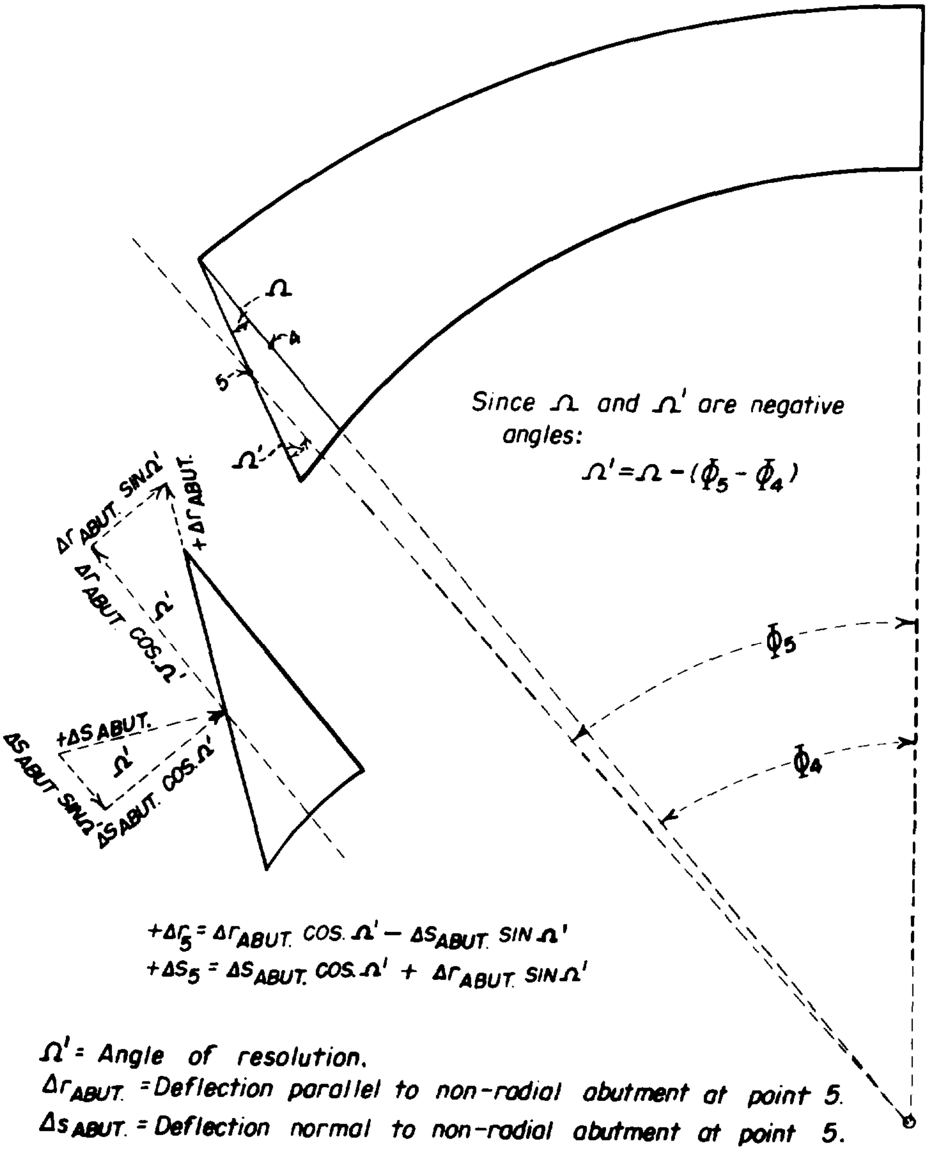
$${}_{3}D_{1} = .0_{,}^{6}005,302,186,9 + .0_{,}^{9}333,510,35 + .0_{,}^{6}025,195,132,4 = .0_{,}^{6}030,830,830$$

$${}_{3}D_{2} = .0_{,}^{6}765,751,79 + .0_{,}^{6}048,166,18 + .0_{,}^{3}002,204,748,33 = .0_{,}^{3}003,018,666,3$$

$${}_{3}D_{3} = -.0_{,}^{6}094,168,578, - .0_{,}^{6}005,923,253 + .0_{,}^{6}184,730,914 = .0_{,}^{6}084,639,083$$

288-D-2734

*Figure C-76.* **Hungry Horse Dam study–total *D*-terms at arch points for twist load No. 5.**



Since $\Omega$ and $\Omega'$ are negative angles:

$$\Omega' = \Omega - (\Phi_5 - \Phi_4)$$

$$+\Delta r_5 = \Delta r_{ABUT.} \text{ COS.}\,\Omega' - \Delta s_{ABUT.} \text{ SIN}\,\Omega'$$

$$+\Delta s_5 = \Delta s_{ABUT.} \text{ COS.}\,\Omega' + \Delta r_{ABUT.} \text{ SIN}\,\Omega'$$

$\Omega'$ = Angle of resolution.

$\Delta r_{ABUT.}$ = Deflection parallel to non-radial abutment at point 5.

$\Delta s_{ABUT.}$ = Deflection normal to non-radial abutment at point 5.

$\Delta r_5$ = Resolved radial deflection at abutment point 5.

$\Delta s_5$ = Resolved tangential deflection at abutment point 5.

*Figure C-77.* Resolution of deflections at a nonradial abutment of a variable-thickness arch with a triangular abutment wedge.–288-D-2735

# Radial, Tangential, and Twist Adjustments

D-1. *Introduction.*—Adjustments for Monticello Dam were selected to show the procedure.

D-2. *Radial Adjustment.*—The loads for the first radial adjustment, which of course includes no tangential or twist load effects, are shown on figures D-1 and D-2. The arch loads are tabulated in kips on figure D-1, in designated amounts of Nos. 1, 3, 4, 5, and concentrated loads. The temperature load is tabulated in degrees F. Cantilever loads are tabulated on figure D-2 and were computed using the load ordinates at cantilever points.

The initial radial deflections in the study under discussion include the effects of concrete inertia, vertical waterload, vertical component of hydrodynamic effects, and odd load on the cantilevers. Figures D-3 and D-4 show the radial deflections due to radial trial loads including initial radial loads on the cantilevers. The deflections due to radial trial loads are calculated using the previously computed deflections due to unit loads for both arches and cantilevers. The radial arch deflections and radial cantilever deflections, including initial radial deflections, are plotted on the load and deflection diagrams. Arch and cantilever stresses for this radial analysis are shown on figure D-5.

The loads for the first radial adjustment plus modifying loads, shown on figures D-6 and D-7, include modifying loads based on previous studies to compensate for the anticipated effects of tangential and twist loads.

D-3. *Tangential Adjustment.*—The loads for the first tangential adjustment, including the effects of the first radial adjustment, are given on figures D-8 and D-9. This adjustment was accomplished in a similar manner to the first radial adjustment.

Before the tangential adjustment is started, tangential deflections due to the first radial adjustment are computed by means of the first-adjustment radial loads and the previously computed tangential deflections due to unit radial loads; these, together with initial tangential cantilever deflections due to earthquake, are shown on figures D-10 and D-11. The tangential arch and cantilever deflections are then brought into agreement by means of trial tangential loads and the tangential deflections due to unit loads. The final trial loads are those shown on figures D-8 and D-9, and the deflections due to these loads are shown on figures D-12 and D-13. The total tangential deflections are plotted on load and deflection diagrams. It should be noted on figures D-12 and D-13 that cantilever foundation movements are equal to corresponding arch abutment movements.

The first tangential adjustment loads were modified in the same manner as those for the first radial adjustment and are shown on figures D-14 and D-15.

D-4. *Twist Adjustment.*—Loads for the first twist adjustment are given on figures D-16 and D-17. Before the twist adjustment is started, angular arch movements due to the first radial and tangential loads are calculated by means of unit angular movements due to radial and tangential loads, respectively (see fig. D-18). Angular movements are then brought into

MONTICELLO DAM. SECTION. STUDY NO. A-11
-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD - STAGE II
ARCH LOADS FOR FIRST RADIAL ADJUSTMENT
(NON-SYMMMETRICAL) By Date

| ELEV. |  | °F TEMP. | No. 1 UNIF. | No. 2-L | No. 3-L | No. 4-L | No. 5-L | CONC.-L | CONC.-R | No. 5-R | No. 4R | No. 3-R | No. 2-R |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  | -6.6 | +1.23 |  | -.17 | +.49 | -.5 | — | — | -.5 | +.5 | -.18 |  |
| 400 |  | -.4 | +3.25 |  |  | +1.4 | -2.3 | -19 | -21 | -2.4 | +1.5 |  |  |
| 350 |  | +2.2 | +4.55 |  | -1.0 |  | -.9 | +190 | +140 | -1.3 |  |  |  |
| 300 |  | +3.0 | +6.20 |  | -.5 |  | -.8 | +555 | +636 | -2.5 |  |  |  |
| 250 |  | +3.2 | +7.60 |  |  |  |  | +943 | +1,372 | -2.5 |  |  |  |
| 200 |  | +3.2 | +11.0 |  |  |  |  | +1,367 | +2,085 |  |  |  |  |

*Figure D-1.* Monticello Dam study—arch loads for first radial adjustment.—DS2-1(143)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD - STAGE II

CANTILEVER LOADS FOR FIRST RADIAL ADJUSTMENT

(NON-SYMMETRICAL) By Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -1.066 | -1.093 | -1.125 | -1.134 | -1.107 | -1.204 | -1.116 | -1.136 | -1.121 | -1.091 | -1.067 | |
| 400 | | +.649 | +.582 | +.499 | +.424 | +.341 | -.118 | +.349 | +.441 | +.526 | +.595 | +.649 | |
| 350 | | | +3.788 | +2.874 | +2.408 | +2.208 | +1.968 | +2.243 | +2.597 | +2.922 | +3.188 | | |
| 300 | | | | +4.880 | +4.417 | +3.946 | +3.671 | +4.437 | +5.294 | +6.080 | | | |
| 250 | | | | | +4.448 | +4.448 | +4.448 | +5.748 | +6.948 | | | | |
| 200 | | | | | | +2.124 | +2.124 | +2.124 | | | | | |
| 165 | | | | | | | +.500 | | | | | | |

*Figure D-2.* Monticello Dam study—cantilever loads for first radial adjustment.—DS2-1(144)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD STAGE II

RADIAL ARCH DEFLECTIONS DUE TO FIRST RADIAL ADJUSTMENT

(NON-SYMMETRICAL) By Date

| ARCH POINT<br>ELEV. | | ABUT.-L | 3/4 | 1/2 | 1/4 | CR. | 1/4 | 1/2 | 3/4 | ABUT.-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $+.0^{3},18$ | +.006,70 | -.055,84 | -.168,07 | -.217,38 | -.177,18 | -.067,74 | $+.0^{3},62$ | $+.0^{3},16$ | | | |
| 400 | | $+.0^{3},54$ | +.002,33 | -.047,80 | -.138,54 | -.187,05 | -.143,94 | -.057,03 | -.004,54 | $+.0^{3},55$ | | | |
| 350 | | -.001,53 | — | -.022,18 | -.104,83 | -.146,80 | -.095,33 | -.016,78 | — | -.001,23 | | | |
| 300 | | -.006,45 | -.027,48 | -.063,41 | -.096,24 | -.106,01 | -.087,34 | -.051,70 | -.020,42 | -.005,64 | | | |
| 250 | | -.011,52 | — | -.045,98 | — | -.065,63 | — | -.043,45 | — | -.012,77 | | | |
| 200 | | -.015,77 | — | -.027,30 | — | -.032,37 | — | -.027,19 | — | -.016,21 | | | |

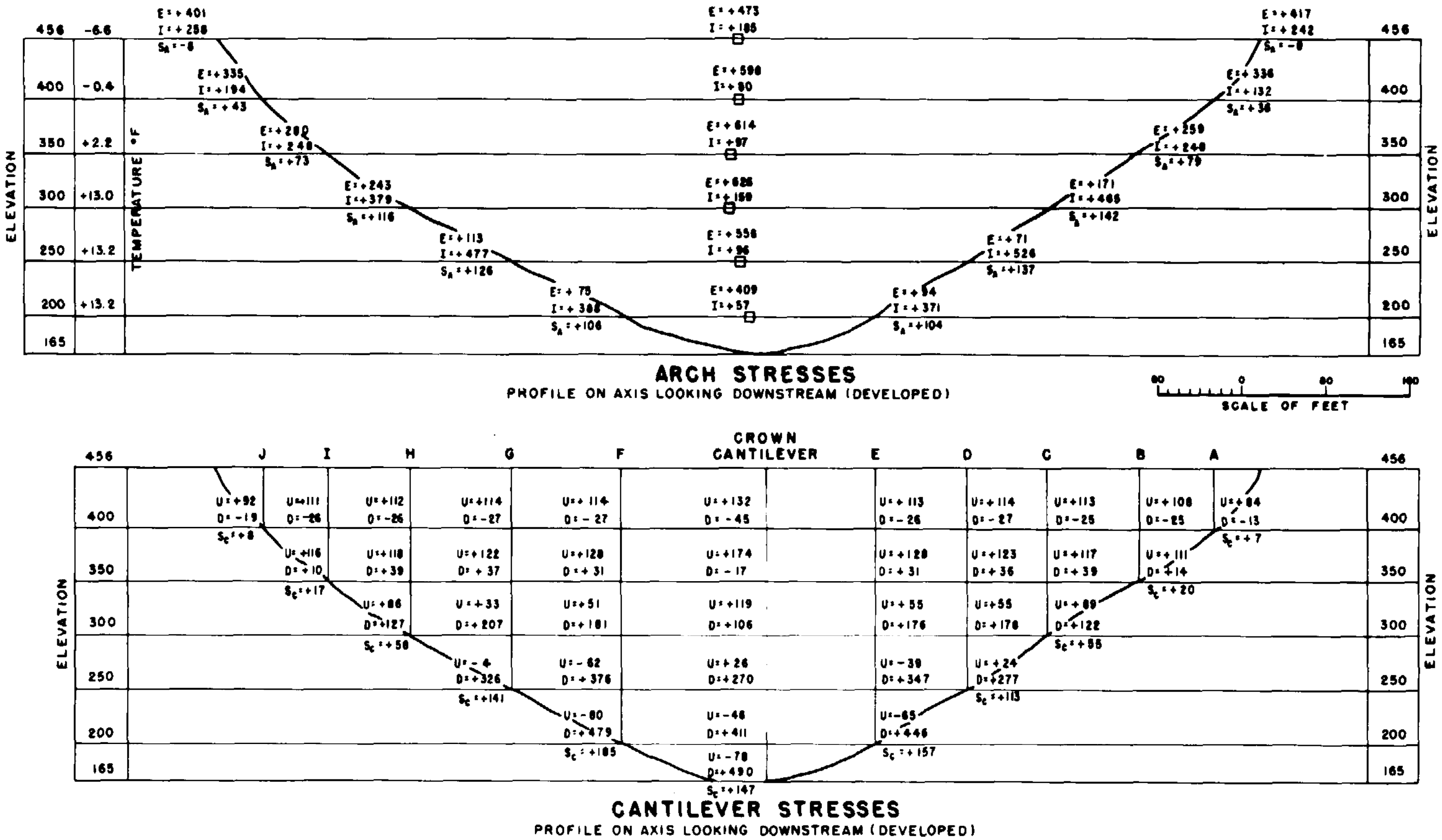
*Figure D-3.* Monticello Dam study–radial arch deflections due to first radial adjustment.–DS2-1(145)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD STAGE II
RADIAL CANTILEVER DEFLECTIONS DUE TO FIRST RADIAL ADJUSTMENT
By. Date.

| CANT. ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | +.002,07 | +.002,67 | -.015,18 | -.067,70 | -.166,66 | -.215,11 | -.183,12 | -.081,14 | -.015,08 | +.004,21 | +.002,63 | |
| 400 | | +.0$^{3}$,54 | -.001,01 | -.015,25 | -.055,74 | -.135,03 | -.180,56 | -.147,89 | -.045,72 | -.014,97 | -.0$^{3}$,25 | +.0$^{3}$,55 | |
| 350 | | | -.001,53 | -.011,95 | -.041,57 | -.103,28 | -.145,07 | -.112,89 | -.048,44 | -.011,46 | -.001,23 | | |
| 300 | | | | -.006,45 | -.025,33 | -.069,43 | -.105,32 | -.075,76 | -.029,19 | -.005,64 | | | |
| 250 | | | | | -.011,52 | -.038,72 | -.066,19 | -.041,87 | -.012,77 | | | | |
| 200 | | | | | | -.015,77 | -.032,95 | -.016,21 | | | | | |
| 165 | | | | | | | -.015,08 | | | | | | |

*Figure D-4.* Monticello Dam study—radial cantilever deflections due to first radial adjustment.—DS2-1(146)



**LOADING CONDITIONS**

Reservoir water surface elevation 440.0.
Top of dam elevation 456.0
Temperature of concrete at time of grouting contraction joints below elevation 300 assumed to be 45° Fahrenheit, above elevation 300, to be 55° Fahrenheit.
Earthquake Assumptions:
  Dam moves up and downstream horizontally in the direction of the line of centers.
  Increased water pressure acts equally on all cantilevers.
  Period of vibration = 1.0 second.
  Acceleration = 0.1 gravity.
  Effects of vertical acceleration not considered.
  Effects of tailwater, uplift, and ice load not considered.

Modulus of elasticity of concrete = 2,500,000 pounds per square inch.
Modulus of elasticity of abutment rock = 1,300,000 pounds per square inch.
Poisson's ratio of concrete = 0.2.
Poisson's ratio of abutment rock = 0.03.
Unit weight of concrete = 150 pounds per cu. ft.
Coefficient of thermal expansion for concrete = 0.000,005,6 foot per foot per degree Fahrenheit.
Arch stresses are acting in horizontal directions parallel to the edges of the arches.
E = Stress at extrados of arch.
I = Stress at intrados of arch.
$S_A$ = Maximum arch shear stress.
  (+) indicates downstream shear.

Cantilever stresses are acting parallel to the edges of the cantilevers.
U = Stress at upstream edge of cantilever.
D = Stress at downstream edge of cantilever.
$S_C$ = Maximum cantilever shear stress.
  (+) Indicates downstream shear.

All stresses are in pounds per square inch.
  + = Compression    - = Tension
□ = Crown of arch.

FINAL DESIGN A-11

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
SOLANO PROJECT - CALIFORNIA
MONTICELLO DAM
RADIAL TRIAL LOAD ANALYSIS
ARCH AND CANTILEVER STRESSES
EFFECTS OF CONSTRUCTION PROGRAM INCLUDED

DRAWN.....G.W.J.......SUBMITTED... L. G. Puls
TRACED.....V.M.M.......RECOMMENDED... J. J. Hammond
CHECKED.......APPROVED.......
CHIEF DESIGNING ENGINEER
DENVER, COLORADO, JUNE 28, 1954    413-D-105

*Figure D-5.* Monticello Dam study–arch and cantilever stresses from radial trial load analysis.

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD STAGE II

RADIAL ARCH LOADS FOR FIRST RADIAL ADJUSTMENT plus MODIFYING LOADS

By Date

| ELEV. | | °F TEMP. | No. 1 UNIF. | No.2-L | No.3-L | No.4-L | No.5-L | CONC.-L | CONC.-R | No.5-R | No.4-R | No.3-R | No.2-R |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -6.6 | +1.443 | -.02 | +.382 | -.965 | -.259 | — | — | -.249 | -.959 | +.378 | -.02 |
| 400 | | -.4 | +3.45 | | -.20 | -.98 | -1.75 | +52 | +42 | -2.15 | -.18 | -.50 | |
| 350 | | +2.2 | +5.27 | | -3.0 | -1.7 | -.9 | +453 | +367 | -1.4 | -1.5 | -2.0 | |
| 300 | | +3.0 | +6.45 | | -1.5 | +.5 | -2.8 | +828 | +882 | -4.5 | +.5 | | |
| 250 | | +3.2 | +7.50 | | | | -1.0 | +1,192 | +1,521 | -3.5 | | | |
| 200 | | +3.2 | +9.0 | | | | | +1,478 | +2,200 | | | | |

*Figure D-6.* **Monticello Dam study—modified arch loads for first radial adjustment.—DS2-1(148)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD STAGE II

RADIAL CANTILEVER LOADS FOR FIRST RADIAL ADJUSTMENT plus MODIFYING LOADS

By. Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.657 | -.778 | -.916 | -1.087 | -1.358 | -1.429 | -1.386 | -1.093 | -.897 | -.772 | -.668 | |
| 400 | | +2.479 | +1.947 | +1.282 | +.692 | +.104 | -.350 | +.086 | +.638 | +1.336 | +1.923 | +2.379 | |
| 350 | | | +6.768 | +3.849 | +2.293 | +1.517 | +1.248 | +1.550 | +2.379 | +3.923 | +6.068 | | |
| 300 | | | | +7.130 | +5.810 | +4.472 | +3.650 | +4.811 | +6.125 | +7.330 | | | |
| 250 | | | | | +5.548 | +5.148 | +4.668 | +6.368 | +8.048 | | | | |
| 200 | | | | | | +4.124 | +4.124 | +4.124 | | | | | |
| 165 | | | | | | | +3.7 | | | | | | |

*Figure D-7.* Monticello Dam study—modified cantilever loads for first radial adjustment.—DS2-1(149)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

TANGENTIAL ARCH LOADS FOR FIRST TANGENTIAL ADJUSTMENT

By Date

| ELEV. |  | No. 1 UNIF. | No. 2-L | No. 3-L | No. 4-L | No. 5-L | CONC.-L | CONC.-R | No. 5-R | No. 4-R | No. 3-R | No. 2-R |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  |  |  | +5.71 | -2.31 | +.60 | — | — | +.54 | -2.20 | +5.66 |  |  |
| 400 |  |  | +2.5 | -2.7 | +2.2 | +2.7 | -263 | -187 | +2.5 | +2.25 | -2.0 |  |  |
| 350 |  |  |  | -3.0 | -.4 | +6.3 | -530 | -424 | +7.8 | -2.3 | -3.0 |  |  |
| 300 |  |  |  |  | -16.0 | +11.7 | -432 | -599 | +12.5 | -14.5 |  |  |  |
| 250 |  |  |  |  |  | -2.0 | -303 | -289 | +2.0 |  | -10.0 |  |  |
| 200 |  | +1.0 |  |  |  |  | -387 | -805 |  |  |  |  |  |

*Figure D-8.* Monticello Dam study—arch loads for first tangential adjustment.—DS2-1(150)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TANGENTIAL CANTILEVER LOADS FOR FIRST TANGENTIAL ADJUSTMENT
By Date

| ELEV. |  | A | B | C | D | E | CROWN | F | G | H | I | J |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  | -3.216 | -1.961 | -.393 | +.318 | -.117 | -.031 | -.123 | +.316 | -.711 | -2.121 | -3.217 |  |
| 400 |  | -4.700 | -3.126 | -2.119 | -1.975 | -.892 | -.156 | -.625 | -1.894 | -2.288 | -2.548 | -2.750 |  |
| 350 |  |  | -2.900 | -3.778 | -3.498 | -2.233 | -.560 | -2.127 | -3.569 | -3.708 | -2.500 |  |  |
| 300 |  |  |  | +4.300 | +1.823 | -.929 | -1.337 | -2.491 | -.148 | +2.000 |  |  |  |
| 250 |  |  |  |  | +2.000 | +1.200 | +.240 | -.640 | +8.000 |  |  |  |  |
| 200 |  |  |  |  |  | -1.000 | -1.000 | +1.000 |  |  |  |  |  |
| 165 |  |  |  |  |  |  | -1.9 |  |  |  |  |  |  |

*Figure D-9.* Monticello Dam study–cantilever loads for first tangential adjustment.–DS2-1(151)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
INITIAL TANGENTIAL ARCH DEFLECTIONS FOR FIRST TANGENTIAL ADJUSTMENT
(EFFECTS FROM RADIAL ADJUSTMENTS FOR STAGE I & STAGE II PLUS MODIFYING LOADS) By Date

| ELEV. | | ABUT.-L | 3/4 | 1/2 | 1/4 | CR. | 1/4 | 1/2 | 3/4 | ABUT.-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.003,483 | -.025,767 | -.042,060 | -.035,317 | -.003490 | +.029,535 | +.039,077 | +.025,157 | +.003,484 | | | |
| 400 | | -.005,328 | -.018,397 | -.030,252 | -.024,728 | -.001,736 | +.021,875 | +.029,135 | +.019,184 | +.005,900 | | | |
| 350 | | -.004,439 | — | -.015,515 | -.016,287 | +.002,418 | +.019,604 | +.016,498 | — | +.005,315 | | | |
| 300 | | -.005,532 | -.006,818 | -.007,253 | -.003,957 | +.001,651 | +.007,565 | +.009,423 | +.007,632 | +.005,499 | | | |
| 250 | | -.005,404 | — | -.004,473 | — | +$.0^{3}$,213 | — | +.004,641 | — | +.005,383 | | | |
| 200 | | -.004,811 | — | -.003,247 | — | -$.0^{3}$,991 | — | +.001,259 | — | +.002,901 | | | |

*Figure D-10.* Monticello Dam study—initial tangential arch deflections for first tangential adjustment.—DS2-1(152)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TANGENTIAL CANTILEVER DEFLECTIONS DUE TO TANGENTIAL COMPONENT
OF CONCRETE INERTIA By Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $-0^{3}_{,}073$ | $-0^{3}_{,}230$ | $-0^{3}_{,}399$ | $-0^{3}_{,}510$ | $-0^{3}_{,}437$ | 0 | $+0^{3}_{,}578$ | $+0^{3}_{,}642$ | $+0^{3}_{,}488$ | $+0^{3}_{,}267$ | $+0^{3}_{,}081$ | |
| 400 | | 0 | $-0^{3}_{,}158$ | $-0^{3}_{,}341$ | $-0^{3}_{,}467$ | $-0^{3}_{,}413$ | 0 | $+0^{3}_{,}547$ | $+0^{3}_{,}588$ | $+0^{3}_{,}416$ | $+0^{3}_{,}183$ | 0 | |
| 350 | | | 0 | $-0^{3}_{,}197$ | $-0^{3}_{,}360$ | $-0^{3}_{,}354$ | 0 | $+0^{3}_{,}468$ | $+0^{3}_{,}454$ | $+0^{3}_{,}241$ | 0 | | |
| 300 | | | | 0 | $-0^{3}_{,}198$ | $-0^{3}_{,}262$ | 0 | $+0^{3}_{,}346$ | $+0^{3}_{,}249$ | 0 | | | |
| 250 | | | | | 0 | $-0^{3}_{,}141$ | 0 | $+0^{3}_{,}186$ | 0 | | | | |
| 200 | | | | | | 0 | 0 | 0 | | | | | |
| 165 | | | | | | | 0 | | | | | | |

*Figure D-11.* Monticello Dam study—initial tangential cantilever deflections for first tangential adjustment.—DS2-1(153)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD

TANGENTIAL ARCH DEFLECTIONS FOR FIRST TANGENTIAL ADJUSTMENT

By. Date.

| ELEV. |  | ABUT.-L | 3/4 | 1/2 | 1/4 | C.R. | 1/4 | 1/2 | 3/4 | ABUT.-R |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  | $-.0^{3}964$ | -.007,199 | -.010,492 | -.008,884 | +.001,178 | +.010,743 | +.011,347 | +.007,308 | $+.0^{3}968$ |  |  |  |
| 400 |  | -.003,651 | -.007,940 | -.011,278 | -.008,546 | $-.0^{3}092$ | +.008,528 | +.011,726 | +.009,229 | +.004,133 |  |  |  |
| 350 |  | -.003,872 | — | -.008,774 | -.009,089 | $+.0^{3}272$ | +.009,158 | +.008,646 | — | +.003,794 |  |  |  |
| 300 |  | -.006,741 | -.007,834 | -.007,775 | -.004,366 | $+.0^{3}277$ | +.005,426 | +.007,901 | +.007,392 | +.006,476 |  |  |  |
| 250 |  | -.006,216 | — | -.005,370 | — | $-.0^{3}023$ | — | +.005,299 | — | +.006,365 |  |  |  |
| 200 |  | -.005,113 | — | -.003,070 | — | $-.0^{3}425$ | — | +.002,065 | — | +.003,803 |  |  |  |

*Figure D-12.* **Monticello Dam study—tangential arch deflections for first tangential adjustment.—DS2-1(154)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

TANGENTIAL CANTILEVER DEFLECTIONS FOR FIRST TANGENTIAL ADJUSTMENT

By. Date.

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.004,727 | -.006,702 | -.010,408 | -.010,757 | -.009,535 | -.002,266 | +.009,062 | +.011,248 | +.010,725 | +.006,497 | +.005,073 | |
| 400 | | -.003,651 | -.005,708 | -.009,873 | -.010,399 | -.009,320 | -.002,231 | +.008,889 | +.010,894 | +.010,084 | +.005,544 | +.004,133 | |
| 350 | | | -.003,872 | -.008,389 | -.009,207 | -.008,624 | -.002,097 | +.008,289 | +.009,699 | +.008,416 | +.003,794 | | |
| 300 | | | | -.006,741 | -.007,484 | -.007,414 | -.001,749 | +.007,056 | +.007,757 | +.006,476 | | | |
| 250 | | | | | -.006,216 | -.006,139 | -.001,226 | +.005,377 | +.006,365 | | | | |
| 200 | | | | | | -.005,113 | $-.0^{3},681$ | +.003,803 | | | | | |
| 165 | | | | | | | $-.0^{3},258$ | | | | | | |

*Figure D-13.* **Monticello Dam study–tangential cantilever deflections for first tangential adjustment.–DS2-1(155)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TANGENTIAL ARCH LOADS FOR FIRST TANGENTIAL ADJUSTMENT
PLUS MODIFYING LOADS By Date

| ELEV. | | No.1 UNIF. | No.2-L | No.3-L | No.4-L | No.5-L | CONC.-L | CONC.-R | No.5-R | No.4-R | No.3-R | No.2-R | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | | -.6 | +1.199 | -4.594 | +6.091 | — | — | +5.994 | -4.496 | +1.259 | -1.75 | |
| 400 | | | +4.0 | -8.10 | +3.20 | +5.24 | -217 | -131 | +4.89 | +3.25 | -7.00 | +1.50 | |
| 350 | | | | -3.0 | -5.0 | +9.4 | -457 | -361 | +10.5 | -6.4 | -3.0 | | |
| 300 | | | | -1.0 | -6.0 | +1.4 | -438 | -504 | +1.7 | -3.5 | -3.0 | | |
| 250 | | | | | | -4.5 | -309 | -263 | | | -10.0 | | |
| 200 | | +1.0 | | | | | -318 | -646 | | | | | |

*Figure D-14.* Monticello Dam study—modified tangential arch loads for first tangential adjustment.—DS2-1(156)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
TANGENTIAL CANTILEVER LOADS FOR FIRST TANGENTIAL ADJUSTMENT
PLUS MODIFYING LOADS By Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -2.099 | -2.104 | -1.742 | -1.516 | -1.522 | -.321 | -1.367 | -1.498 | -1.830 | -1.951 | -1.400 | |
| 400 | | -4.340 | -2.908 | -2.657 | -3.504 | -1.676 | -.302 | -1.222 | -3.351 | -2.610 | -2.475 | -2.640 | |
| 350 | | | -1.400 | -3.149 | -3.651 | -3.258 | -.836 | -2.864 | -3.818 | -3.050 | -1.100 | | |
| 300 | | | | +5.600 | +3.389 | +1.017 | -.160 | -.150 | +1.982 | +4.800 | | | |
| 250 | | | | | +4.500 | +2.700 | +.540 | +.400 | +10.000 | | | | |
| 200 | | | | | | -1.000 | -1.000 | +1.000 | | | | | |
| 165 | | | | | | | -2.1 | | | | | | |

Figure D-15. Monticello Dam study—modified tangential cantilever loads for first tangential adjustment.—DS2-1(157)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TWIST ARCH LOADS FOR FIRST TWIST ADJUSTMENT
By Date

| ELEV. | | No. 1 UNIF. | No. 2-L | No. 3-L | No. 4-L | No. 5-L | CONC-L | CONC.-R | No. 5-R | No. 4-R | No. 3-R | No. 2-R | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | | -35 | -25.4 | -39.3 | +8.0 | — | — | +8.0 | -40.3 | -22.4 | -36.0 | |
| 400 | | | -10 | +68. | +15. | -43 | +1,291 | +1,297 | -33 | +15 | +53 | -10 | |
| 350 | | | | -50 | -25 | -230 | +12,315 | +10,653 | -270 | +30 | -50 | | |
| 300 | | | | +210 | -140 | -80 | +19,343 | +18,192 | -150 | -100 | +280 | | |
| 250 | | | | +30 | | -170 | +23,829 | +32,062 | -240 | | +30 | | |
| 200 | | -30 | | | | -100 | +30,220 | +37,711 | -100 | | | | |

*Figure D-16.* Monticello Dam study—twist arch loads for first twist adjustment.—DS2-1(158)

## CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD
TWIST CANTILEVER LOADS FOR FIRST TWIST ADJUSTMENT
By Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | +71.1 | +38.1 | +18.3 | +6.4 | -1.4 | -0.42 | -1.8 | +6.6 | +21.3 | +41.3 | +70.1 | |
| 400 | | -30.0 | -18.8 | -0.9 | +16.1 | +11.6 | +2.48 | +8.2 | +11.2 | -6.4 | -17.9 | -25.0 | |
| 350 | | | +305.0 | +209.7 | +141.8 | +82.2 | +20.44 | +73.6 | +138.3 | +209.6 | +290.0 | | |
| 300 | | | | +10.0 | +49.4 | +75.2 | +9.14 | +63.8 | +57.0 | -30.0 | | | |
| 250 | | | | | +140.0 | +96.0 | +20.40 | +123.6 | +210.0 | | | | |
| 200 | | | | | | +130.0 | +44.29 | +70.0 | | | | | |
| 165 | | | | | | | +61.00 | | | | | | |

*Figure D-17.* Monticello Dam study–twist cantilever loads for first twist adjustment.–DS2-1(159)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD

INITIAL ANGULAR MOVEMENT OF ARCH FOR FIRST TWIST ADJUSTMENT

(DUE TO FIRST RADIAL ADJUSTMENT PLUS MODIFYING LOADS AND FIRST TANGENTIAL ADJ. PLUS MODIFYING LOAD) By Date

| ELEV. | | ABUT-L | 3/4 | 1/2 | 1/4 | CR. | 1/4 | 1/2 | 3/4 | ABUT-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $-.0^{3}477,47$ | $-.001,302,36$ | $+.001,139,91$ | $+.001,755,76$ | $+.0^{3}022,57$ | $-.001,724,24$ | $-.001,148,85$ | $+.001,253,75$ | $+.0^{3}455,34$ | | | |
| 400 | | $+.0^{3}016,12$ | $-.0^{3}076,37$ | $-.0^{3}597,81$ | $-.0^{3}557,90$ | $+.0^{3}008,43$ | $+.0^{3}594,11$ | $+.0^{3}557,37$ | $+.0^{3}127,76$ | $-.0^{3}015,26$ | | | |
| 350 | | $-.0^{3}008,46$ | — | $-.0^{3}417,45$ | $-.0^{3}508,93$ | $+.0^{3}026,05$ | $+.0^{3}516,41$ | $+.0^{3}384,03$ | — | $+.0^{3}015,78$ | | | |
| 300 | | $-.0^{3}031,48$ | $-.0^{3}183,66$ | $-.0^{3}376,78$ | $-.0^{3}275,46$ | $+.0^{3}031,08$ | $+.0^{3}313,70$ | $+.0^{3}362,15$ | $+.0^{3}149,98$ | $+.0^{3}020,89$ | | | |
| 250 | | $-.0^{3}088,16$ | — | $-.0^{3}226,70$ | — | $+.0^{3}008,84$ | — | $+.0^{3}221,81$ | — | $+.0^{3}077,49$ | | | |
| 200 | | $-.0^{3}065,05$ | — | $-.0^{3}071,67$ | — | $+.0^{3}002,16$ | — | $+.0^{3}069,74$ | — | $+.0^{3}053,42$ | | | |

*Figure D-18.* Monticello Dam study—initial angular movement of arch for first twist adjustment.—DS2-1(160) agreement by means of trial twist loads and the angular movements due to unit twist loads. The final trial loads are those shown on figures D-16 and D-17, and the angular movements due to these twist loads are shown on figures D-19 and D-20. The total angular movements are plotted on load and deflection diagrams.

D-5. ***Radial Readjustment.***—The first radial readjustment is the second-cycle radial adjustment. In addition to computations as described for the first cycle, calculations are required to determine radial deflections due to tangential and twist loads applied in the first cycle. Before the first radial readjustment is started, radial arch and cantilever deflections due to loads of the first tangential and twist adjustments are calculated as described below.

The cantilever deflections caused by twist loads will be discussed first. To begin, twisting moments in the cantilevers are obtained by multiplying the twisting moments due to unit twist loads by the cantilever twist loads (see fig. D-21). Average rate of change of twisting and bending moments, shown on figures D-22 and D-23, are then obtained by differentiating twisting moments along arch crown centerlines (see fig. C-10). Bending moments in the cantilevers due to twist loads are next obtained, as shown on figure D-24, by integrating rates of change of bending moments in the cantilevers from the top to each lower elevation. Finally, radial deflections of the cantilevers due to these bending moments are calculated by double integration, as shown on figure D-25 for the crown cantilever and cantilever *A*. Deflections for all of the cantilevers are obtained in the same manner.

Radial arch deflections due to the first tangential and twist adjustments are computed by using first-adjustment tangential and twist loads and radial deflections due to unit tangential and twist loads, respectively.

After the above radial arch and cantilever deflections have been obtained, deflections are brought into agreement by means of trial radial loads and radial deflections due to unit radial loads. Radial deflections are summated and plotted on load and deflection diagrams, which completes the first radial readjustment.

D-6. ***Tangential Readjustment.***—In making the second-cycle tangential adjustment, or first tangential readjustment, the tangential deflections produced by the first twist adjustment as well as those caused by the first radial readjustment are considered. Arch and cantilever loads for the first tangential readjustment include the effects of the first twist adjustment and first radial readjustment. Before the first tangential readjustment is started, tangential arch deflections due to the first twist adjustment and first radial readjustment are calculated as described for the other adjustments. The deflections are then brought into agreement by means of trial tangential loads in the manner previously described. Tangential deflections are computed and plotted on the diagrams, which completes the first tangential readjustment.

D-7. ***Twist Readjustment.***The second-cycle twist adjustment or first twist readjustment includes the effects of the first radial readjustment and first tangential readjustment. After the above effects have been obtained, the angular movements are again brought into agreement and plotted on the diagrams.

D-8. ***Final Radial, Tangential, and Twist Readjustments.***—For the study under discussion, second radial, tangential, and twist readjustments were found necessary to bring the deflections into complete agreement. Total radial loads for stage II are shown on figures D-26 and D-27. Radial arch and cantilever deflections due to these loads are shown on figures D-28 and D-29.

Total tangential loads are shown on figures D-30 and D-31 and tangential deflections are shown on figures D-32 and D-33. Total twist loads and angular movements are shown on figures D-34 to D-37, inclusive.

Final radial, tangential, and twist loads and deflections are plotted on the diagrams shown on figures D-38 to D-41, inclusive. After the adjustments have been completed, stresses may be calculated as described beginning with section 4-44.

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD

ANGULAR ARCH MOVEMENTS FOR FIRST TWIST ADJUSTMENT

By. Date.

| ELEV. | | ABUT.-L | 3/4 | 1/2 | 1/4 | C.R. | 1/4 | 1/2 | 3/4 | ABUT.-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $-.0,^{3}012,79$ | $-.0,^{3}148,15$ | $-.0,^{3}504,42$ | $-.0,^{3}472,60$ | $-.0,^{3}009,60$ | $+.0,^{3}451,69$ | $+.0,^{3}494,27$ | $+.0,^{3}194,73$ | $+.0,^{3}005,65$ | | | |
| 400 | | $-.0,^{3}022,57$ | $-.0,^{3}151,30$ | $-.0,^{3}483,62$ | $-.0,^{3}425,08$ | $-.0,^{3}001,60$ | $+.0,^{3}398,42$ | $+.0,^{3}456,90$ | $+.0,^{3}202,78$ | $+.0,^{3}030,00$ | | | |
| 350 | | $-.0,^{3}037,26$ | — | $-.0,^{3}297,44$ | $-.0,^{3}397,59$ | $+.0,^{3}025,40$ | $+.0,^{3}390,62$ | $+.0,^{3}274,17$ | — | $+.0,^{3}047,65$ | | | |
| 300 | | $-.0,^{3}083,05$ | $-.0,^{3}220,36$ | $-.0,^{3}301,45$ | $-.0,^{3}219,92$ | $+.0,^{3}027,57$ | $+.0,^{3}248,33$ | $+.0,^{3}282,63$ | $+.0,^{3}220,26$ | $+.0,^{3}075,48$ | | | |
| 250 | | $-.0,^{3}121,18$ | — | $-.0,^{3}189,72$ | — | $+.0,^{3}007,52$ | — | $+.0,^{3}177,57$ | — | $+.0,^{3}122,54$ | | | |
| 200 | | $-.0,^{3}110,98$ | — | $-.0,^{3}084,48$ | — | $+.0,^{3}007,23$ | — | $+.0,^{3}091,31$ | — | $+.0,^{3}103,25$ | | | |

*Figure D-19.* Monticello Dam study—angular arch movements for first twist adjustment.—DS2-1(161)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

ANGULAR CANTILEVER MOVEMENTS FOR FIRST TWIST ADJUSTMENT

By Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $-.0^{3}041,09$ | $-.0^{3}124,13$ | $-.0^{3}352,12$ | $-.0^{3}535,44$ | $-.0^{3}488,18$ | $-.0^{3}119,48$ | $+.0^{3}431,39$ | $+.0^{3}523,94$ | $+.0^{3}327,63$ | $+.0^{3}155,25$ | $+.0^{3}059,57$ | |
| 400 | | $-.0^{3}022,57$ | $-.0^{3}098,92$ | $-.0^{3}326,01$ | $-.0^{3}501,61$ | $-.0^{3}472,82$ | $-.0^{3}116,38$ | $+.0^{3}421,75$ | $+.0^{3}497,18$ | $+.0^{3}305,30$ | $+.0^{3}122,10$ | $+.0^{3}030,00$ | |
| 350 | | | $-.0^{3}037,26$ | $-.0^{3}212,37$ | $-.0^{3}388,37$ | $-.0^{3}411,39$ | $-.0^{3}102,17$ | $+.0^{3}372,81$ | $+.0^{3}396,47$ | $+.0^{3}197,58$ | $+.0^{3}047,65$ | | |
| 300 | | | | $-.0^{3}083,05$ | $-.0^{3}231,52$ | $-.0^{3}303,80$ | $-.0^{3}078,22$ | $+.0^{3}280,87$ | $+.0^{3}241,23$ | $+.0^{3}075,48$ | | | |
| 250 | | | | | $-.0^{3}121,18$ | $-.0^{3}194,40$ | $-.0^{3}055,84$ | $+.0^{3}180,97$ | $+.0^{3}122,54$ | | | | |
| 200 | | | | | | $-.0^{3}110,98$ | $-.0^{3}036,73$ | $+.0^{3}103,25$ | | | | | |
| 165 | | | | | | | $-.0^{3}023,95$ | | | | | | |

*Figure D-20.* Monticello Dam study—angular cantilever movements for first twist adjustment.—DS2-1(162)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
TWISTING MOMENTS IN CANTILEVERS THROUGH FIRST ADJUSTMENTS
By. Date.

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 400 | | -1,146,200 | -533,050 | -481,280 | -623,780 | -283,140 | -57,190 | -177,730 | -493,350 | -411,660 | -646,120 | -1,251,800 | |
| 350 | | | -7,432,400 | -5,618,400 | -4,517,700 | -2,596,600 | -622,400 | -2,195,000 | -4,179,600 | -5,410,700 | -7,227,900 | | |
| 300 | | | | -11,017,200 | -9,215,200 | -6,457,500 | -1,349,300 | -5,565,800 | -8,977,900 | -9,850,600 | | | |
| 250 | | | | | -13,744,300 | -10,604,600 | -2,063,900 | -10,100,900 | -15,355,600 | | | | |
| 200 | | | | | | -15,986,700 | -3,606,900 | -14,729,500 | | | | | |
| 165 | | | | | | | -5,344,300 | | | | | | |

*Figure D-21.* Monticello Dam study—twisting moments in cantilevers through first adjustments.—DS2-1(163)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD

AVERAGE RATES OF CHANGE OF TWISTING MOMENTS THROUGH FIRST ADJUSTMENTS

By. Date

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 400 | | -8,863.1 | -598.6 | +1,830.9 | -3,938.9 | -2,177.4 | -871.2 | -3,041.2 | +858.8 | -3,012.5 | -10,004.6 | | |
| 350 | | | -21,086 | -14,214 | -22,331 | -19,121 | -11,424 | -19,221 | -13,008 | -23,469 | | | |
| 300 | | | | -23,509 | -32,379 | -49,977 | -30,940 | -33,386 | -9,315 | | | | |
| 250 | | | | | -37,324 | -84,612 | -59,715 | -52,058 | | | | | |
| 200 | | | | | | -124,495 | -83,893 | | | | | | |

*Figure D-22.* Monticello Dam study—average rates of change of twisting moments through first adjustments.—DS2-1(164)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL - SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
AVERAGE RATES OF CHANGE OF BENDING MOMENTS THROUGH FIRST ADJUSTMENTS
By Date

| ELEV. | $\frac{\Delta z}{2}$ | A | B | C | D | E | CROWN | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 28.0 | | | | | | | | | | | |
| 400 | | +12,536 | +5,190.2 | -680.1 | +902.2 | +3,138.2 | +1,617.6 | +2,111.2 | +1,006.5 | +1,270.6 | +6,945.2 | +13,064 |
| | 25.0 | | | | | | | | | | | |
| 350 | | | +24,703 | +17,469 | +18,059 | +20,872 | +15,822 | +15,879 | +15,980 | +18,762 | +28,176 | |
| | 25.0 | | | | | | | | | | | |
| 300 | | | | +19,307 | +27,711 | +40,378 | +41,818 | +32,338 | +20,828 | -2,198 | | |
| | 25.0 | | | | | | | | | | | |
| 250 | | | | | +15,829 | +58,819 | +73,942 | +55,340 | +48,776 | | | |
| | 25.0 | | | | | | | | | | | |
| 200 | | | | | | +141,896 | +107094 | +60,692 | | | | |
| | 17.5 | | | | | | | | | | | |
| 165 | | | | | | | +130,600 | | | | | |

*Figure D-23.* Monticello Dam study—average rates of change of bending moments through first adjustments.—DS2-1(165)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

BENDING MOMENTS THROUGH FIRST ADJUSTMENTS

(FROM TWISTING MOMENTS) By. Date.

| ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 400 | | +351,010 | +145,330 | -19,043 | +25,262 | +87,870 | +45,293 | +59,114 | +28,182 | +35,577 | +194,470 | +365,790 | |
| 350 | | | +892,660 | +400,680 | +499,290 | +688,120 | +481,280 | +508,870 | +452,840 | +536,390 | +1,072,500 | | |
| 300 | | | | +1,320,100 | +1,643,500 | +2,219,400 | +1,922,300 | +1,714,300 | +1,373,000 | +950,490 | | | |
| 250 | | | | | +2,732,000 | +4,699,300 | +4,816,300 | +3,906,200 | +3,113,100 | | | | |
| 200 | | | | | | +9,717,200 | +9,342,200 | +6,807,000 | | | | | |
| 165 | | | | | | | +13,501,800 | | | | | | |

*Figure D-24.* Monticello Dam study–bending moments in cantilevers through first adjustments.–DS2-1(166)

# CANTILEVER STRESS ANALYSIS

$\alpha$ =
$\alpha_2$ =
$\gamma$ =
Ⓡ = δ =

MONTICELLO DAM SECTION STUDY NO. A-11
RADIAL SIDE CANTILEVER STRESS ANALYSIS TRIAL LOAD
RADIAL DEFLECTION BENDING M LOAD NO. FIRST ADJS CANTILEVERS CROWN & A
$\frac{1}{E}$ = $\frac{K_6}{G}$ = $\frac{\ }{G}$ = By Date

| | Deflection due to | | | | | | Deflection due to | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Elev. | Moment = M | $\frac{1}{EI}$ | $\frac{M}{EI}$ | $\frac{\Delta z}{2}$ | $\sum\left(\frac{M}{EI}\right)\frac{\Delta z}{2}$ | $\sum\frac{\Delta z}{2}\left(\sum\frac{M}{EI}\frac{\Delta z}{2}\right)$ | Horizontal Force = V | $\frac{1}{A}$ | $\frac{V}{A}$ | $\left(\frac{K}{G}\right)\frac{\Delta z}{2}$ | $\sum\left(\frac{V}{A}\right)\left(\frac{K}{G}\frac{\Delta z}{2}\right)$ | Total Deflection |
| | CROWN CANTILEVER | | | | | | | | | | | |
| | $\alpha = .0^{9}004,342,841$ $\alpha_2 = .0^{9}058,372,74$ | | | | | | | | | | | |
| | | | | | | | | | | | | |
| 456 | 0 | — | — | | $+.0^{3}192,54$ | $+.044,22$ | | | | | | |
| | | | | 28.0 | | | | | | | | |
| 400 | + 45,293 | $.0^{9}001,614,3$ | $+.0^{6}073,116$ | | $+.0^{3}190,49$ | $+.033,50$ | | | | | | |
| | | | | 25.0 | | | | | | | | |
| 350 | +481,280 | $.0^{12}612,58$ | $+.0^{6}294,82$ | | $+.0^{3}181,29$ | $+.024,20$ | | | | | | |
| | | | | 25.0 | | | | | | | | |
| 300 | +1,922,300 | $.0^{12}309,06$ | $+.0^{6}594,11$ | | $+.0^{3}159,07$ | $+.015,69$ | | | | | | |
| | | | | 25.0 | | | | | | | | |
| 250 | +4,816,300 | $.0^{12}159,51$ | $+.0^{6}768,25$ | | $+.0^{3}125,01$ | $+.008,59$ | | | | | | |
| | | | | 25.0 | | | | | | | | |
| 200 | +9,342,200 | $.0^{12}085,275$ | $+.0^{6}796,66$ | | $+.0^{3}085,888$ | $+.003,32$ | | | | | | |
| | | | | 17.5 | | | | | | | | |
| 165 | +13,501,800 | $.0^{12}056,331$ | $+.0^{6}760,57$ | | $+.0^{3}058,636$ | $+.0^{3}79$ | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | CANTILEVER A | | | | | | | | | | |
| | | $\alpha = .0^{9}013,461,79$ | | | | | | | | | | |
| | | | | | | | | | | | | |
| 456 | 0 | — | — | | $-.0^{3}004,241,0$ | $-.0^{3}37$ | | | | | | |
| | | | | 28.0 | | | | | | | | |
| 400 | +351,010 | $.0^{12}481,06$ | $.0^{6}168,86$ | | $-.0^{3}008,969,1$ | 0 | $*M = \text{BENDING MOMENT} + \frac{\text{TWISTING MOMENT}}{\frac{R_{AXIS}}{r_a}} \tan.\psi$ | | | | | |
| *M = | -666,260 | | | | | | | | | | | |
| | | | | | $*M\alpha$ | | | | | | | |

*Figure D-25.* Monticello Dam study—radial cantilever deflection due to bending moments through first adjustments.—DS2-1(167)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

TOTAL RADIAL ARCH LOADS FOR STAGE II

By Date

| ARCH LOAD / ELEV. | | °F TEMP. | No. 1 UNIF. | NO. 2-L | No. 3-L | No. 4-L | No. 5-L | CONC.-L | CONC.-R | No. 5-R | No. 4-R | No. 3-R | No. 2-R |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -6.6 | +.583 | +.050 | -1.522 | -2.685 | +3.043 | — | — | +3.056 | -2.679 | -1.526 | +.030 |
| 400 | | -.4 | +3.430 | | +1.200 | -1.330 | -1.950 | +54 | +47 | -2.150 | -.830 | +.900 | |
| 350 | | +2.2 | +4.820 | | -.700 | +.600 | -1.950 | +301 | +235 | -2.600 | +1.000 | +.300 | |
| 300 | | +3.0 | +6.300 | | +.500 | +.500 | -.800 | +567 | +598 | -2.500 | +.500 | +2.000 | |
| 250 | | +3.2 | +7.800 | | | | -1.000 | +949 | +1,349 | -3.500 | | | |
| 200 | | +3.2 | +9.000 | | | | | +1,462 | +2,202 | | | | |

*Figure D-26.* Monticello Dam study–total radial arch loads.–DS2-1(168)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TOTAL RADIAL CANTILEVER LOADS FOR STAGE II
By Date

| CANT. ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | +.234 | -,240 | -.864 | -1,242 | -1,337 | -,743 | -1.280 | -1.250 | -.749 | -.190 | +.234 | |
| 400 | | +1.649 | +1.445 | +1.191 | +,916 | +.200 | -.319 | +.106 | +.858 | +1.201 | +1.453 | +1.649 | |
| 350 | | | +3.668 | +2.938 | +2.531 | +2.301 | +1.791 | +2.327 | +2.738 | +2.910 | +2.918 | | |
| 300 | | | | +3.280 | +3.503 | +3.708 | +3.571 | +4.275 | +4.161 | +3.480 | | | |
| 250 | | | | | +5.248 | +4.848 | +4,368 | +6.068 | +7.748 | | | | |
| 200 | | | | | | +4,124 | +4.124 | +4.124 | | | | | |
| 165 | | | | | | | +4.000 | | | | | | |

*Figure D-27.* Monticello Dam study—total radial cantilever loads.—DS2-1(169)

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
RADIAL ARCH DEFLECTIONS DUE TO TOTAL LOADS FOR STAGE II
By. Date

| ELEV. |  | ABUT.-L | 3/4 | 1/2 | 1/4 | CR. | 1/4 | 1/2 | 3/4 | ABUT.-R |
|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  | -.0$^{3}$,56 | -.013,21 | -.056,03 | -.126,88 | -.164,60 | -.129,56 | -.061,32 | -.015,74 | -.0$^{3}$,49 |
| 400 |  | -.0$^{3}$,44 | -.016,26 | -.059,62 | -.112,14 | -.135,89 | -.111,89 | -.062,79 | -.019,79 | -.0$^{3}$,45 |
| 350 |  | -.004,67 | — | -.029,58 | -.089,05 | -.115,76 | -.083,74 | -.025,74 | — | -.004,73 |
| 300 |  | -.006,48 | -.025,93 | -.055,61 | -.083,33 | -.092,88 | -.078,22 | -.049,66 | -.022,95 | -.006,15 |
| 250 |  | -.011,80 | — | -.042,45 | — | -.059,05 | — | -.040,67 | — | -.013,18 |
| 200 |  | -.015,73 | — | -.026,79 | — | -.031,19 | — | -.026,53 | — | -.016,08 |

*Figure D-28.* Monticello Dam study—radial arch deflections due to total loads.—DS2-1(170)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

RADIAL CANTILEVER DEFLECTIONS DUE TO TOTAL LOADS FOR STAGE II

By ........ Date ........

| CANT. / ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.001,42 | -.012,08 | -.031,06 | -.072,17 | -.116,92 | -.155,90 | -.137,28 | -.081,25 | -.028,46 | -.011,52 | -.001,51 | |
| 400 | | -.0$^3$,44 | -.008,70 | -.023,82 | -.059,54 | -.103,37 | -.137,27 | -.118,52 | -.066,98 | -.022,24 | -.008,60 | -.0$^3$,45 | |
| 350 | | | -.004,67 | -.015,39 | -.044,62 | -.085,70 | -.116,09 | -.096,42 | -.050,47 | -.015,21 | -.004,73 | | |
| 300 | | | | -.006,48 | -.027,30 | -.062,09 | -.089,09 | -.068,75 | -.031,44 | -.006,15 | | | |
| 250 | | | | | -.011,80 | -.036,98 | -.059,37 | -.040,08 | -.013,18 | | | | |
| 200 | | | | | | -.015,73 | -.031,91 | -.016,08 | | | | | |
| 165 | | | | | | | -.016,14 | | | | | | |

*Figure D-29.* Monticello Dam study—radial cantilever deflections due to total loads.—DS2-1(171)

# CANTILEVER STRESS ANALYSIS

Monticello DAM. SECTION. STUDY NO. A-11

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Total Tangential Arch Loads through Final Tangential Readjustment

By........ Date..........

| Arch Load / Elev. | | No. 1 Unif. | No. 2-L | No. 3-L | No. 4-L | No. 5-L | Conc.-L | Conc.-R | No. 5-R | No. 4-R | No. 3-R | No. 2-R | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | | -4.560 | -2.351 | +4.909 | +.726 | — | — | +.629 | +5.007 | -2.291 | -5.710 | |
| 400 | | | +4.000 | -4.366 | -1.822 | +7.479 | -184 | -82 | +6.579 | -.112 | -5.046 | +1.500 | |
| 350 | | | | -3.000 | -7.200 | +10.400 | -460 | -325 | +11.500 | -8.600 | -3.000 | | |
| 300 | | | | -1.000 | -5.000 | +1.400 | -438 | -490 | +1.700 | -2.800 | -3.000 | | |
| 250 | | | | | | -4.500 | -308 | -271 | | | -10.000 | | |
| 200 | | +2.000 | | | | -1.000 | -347 | -601 | | | | | |

*Figure D-30.* Monticello Dam study–total tangential arch loads.–DS2-1(172)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TOTAL TANGENTIAL CANTILEVER LOADS THROUGH FINAL TANGENTIAL READJUSIMENT
By Date

| CANT / ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.099 | -2.298 | -2.247 | -1.553 | -.277 | -.038 | -.143 | -1.535 | -2.342 | -1.870 | +.600 | |
| 400 | | -5.291 | -3.396 | -2.566 | -3.035 | -2.064 | -.431 | -1.645 | -3.129 | -2.548 | -2.606 | -2.921 | |
| 350 | | | -.200 | -2.472 | -3.446 | -3.576 | -.924 | -3.136 | -3.708 | -2.340 | +.100 | | |
| 300 | | | | +4.600 | +2.731 | +.741 | -.160 | -.236 | +1.575 | +4.100 | | | |
| 250 | | | | | +4.500 | +2.700 | +.540 | +.400 | +10.000 | | | | |
| 200 | | | | | | -1.000 | -1.857 | +2.000 | | | | | |
| 165 | | | | | | | -3.500 | | | | | | |

*Figure D-31.* Monticello Dam study—total tangential cantilever loads.—DS2-1(173)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

TANGENTIAL ARCH DEFLECTIONS DUE TO TOTAL LOADS

By........ Date..........

| ELEV. | | ABUT.-L | 3/4 | 1/2 | 1/4 | CR. | 1/4 | 1/2 | 3/4 | ABUT.-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $-.0^{3}703$ | -.007,204 | -.013,229 | -.011,050 | $-.0^{3}630$ | +.010,110 | +.013,227 | +.008,219 | +.001,125 | | | |
| 400 | | -.002,771 | -.007,694 | -.012,259 | -.009,599 | $-.0^{3}141$ | +.009,596 | +.012,604 | +.008,765 | +.003,170 | | | |
| 350 | | -.003,343 | — | -.009,301 | -.009,023 | $+.0^{3}830$ | +.009,772 | +.009,325 | — | +.004,532 | | | |
| 300 | | -.006,579 | -.007,728 | -.007,891 | -.004,865 | $+.0^{3}412$ | +.005,410 | +.007,843 | +.007,421 | +.006,378 | | | |
| 250 | | -.006,189 | — | -.005,273 | — | $-.0^{3}197$ | — | +.004,891 | — | +.006,214 | | | |
| 200 | | -.004,790 | — | -.002,664 | — | $-.0^{3}156$ | — | +.002,069 | — | +.003,622 | | | |

*Figure D-32.* Monticello Dam study—tangential arch deflections due to total loads.—DS2-1(174)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
TANGENTIAL CANTILEVER DEFLECTIONS DUE TO TOTAL LOADS
By. Date.

| CANT. / ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -.003,524 | -.006,258 | -.011,443 | -.013,289 | -.011,256 | -.002,390 | +.009,951 | +.013,808 | +.011,366 | +.006,958 | +.003,584 | |
| 400 | | -.002,771 | -.005,154 | -.010,471 | -.012,376 | -.010,789 | -.002,301 | +.009,581 | +.012,870 | +.010,367 | +.006,041 | +.003,170 | |
| 350 | | | -.003,343 | -.008,416 | -.010,216 | -.009,426 | -.002,016 | +.008,453 | +.010,623 | +.008,272 | +.004,532 | | |
| 300 | | | | -.006,579 | -.007,741 | -.007,452 | -.001,550 | +.006,695 | +.007,869 | +.006,378 | | | |
| 250 | | | | | -.006,189 | -.005,810 | -.001,104 | +.004,934 | +.006,214 | | | | |
| 200 | | | | | | -.004,790 | $-.0^{3}695$ | +.003,622 | | | | | |
| 165 | | | | | | | $-.0^{3}282$ | | | | | | |

Figure D-33. Monticello Dam study—tangential cantilever deflections due to total loads.—DS2-1(175)

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

TOTAL TWIST ARCH LOADS THROUGH FINAL TWIST READJUSTMENT

By. Date.

| LOAD<br>ELEV. |  | No. 1 Unif. | No. 2-L | No. 3-L | No. 4-L | No. 5-L | Conc-L | Conc-R | No. 5-R | No. 4-R | No. 3-R | No. 2-R |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 |  |  | +13.7 | +13.7 | -225.8 | +129.7 | — | – | +129.7 | -226.8 | +16.7 | +12.7 |  |
| 400 |  |  | -60 | +68 | +131 | -120 | +1,313 | +1,318 | -100 | +121 | +53 | -60 |  |
| 350 |  |  |  | -50 | -75 | -210 | +13,309 | +12,657 | -250 | -20 | -80 |  |  |
| 300 |  |  |  | +170 | -140 |  | +18,349 | +17,328 | -50 | -100 | +230 |  |  |
| 250 |  |  |  | +30 |  | -220 | +22,845 | +27,957 | -260 |  | +30 |  |  |
| 200 |  |  |  |  |  | -100 | +27,771 | +36,128 | -100 |  |  |  |  |

Figure D-34. Monticello Dam study—total twist arch loads.—DS2-1(176)

# CANTILEVER STRESS ANALYSIS

Monticello DAM. SECTION. STUDY NO. A-11
Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load
Total Twist Cantilever Loads through Final Twist Readjustment
By. Date.

| CANT. / ELEV. | | A | B | C | D | E | Crown | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | +60.9 | +48.4 | +24.3 | -0.1 | -30.2 | -6.8 | -29.6 | +0.1 | +29.1 | +49.1 | +59.9 | |
| 400 | | -19.0 | -26.6 | -13.0 | +17.4 | +27.9 | +6.9 | +25.0 | +10.8 | -22.5 | -23.8 | -14.0 | |
| 350 | | | +335.0 | +227.2 | +148.0 | +75.9 | +18.7 | +68.2 | +142.3 | +237.2 | +350.0 | | |
| 300 | | | | -30.0 | +9.4 | +38.7 | 0 | +29.5 | +7.0 | -80.0 | | | |
| 250 | | | | | +190.0 | +126.0 | +26.4 | +134.0 | +230.0 | | | | |
| 200 | | | | | | +100.0 | +14.3 | +100.0 | | | | | |
| 165 | | | | | | | +11.0 | | | | | | |

*Figure D-35.* Monticello Dam study—total twist cantilever loads.—DS2-1(177)

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

ANGULAR ARCH MOVEMENTS DUE TO TOTAL LOADS

By Date

| ELEV. | | ABUT.-L | 3/4 | 1/2 | 1/4 | CROWN | 1/4 | 1/2 | 3/4 | ABUT-R | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | $+.0^{3}008,39$ | $-.0^{3}162,11$ | $-.0^{3}460,33$ | $-.0^{3}390,76$ | $-.0^{3}009,60$ | $+.0^{3}369,85$ | $+.0^{3}450,42$ | $+.0^{3}208,69$ | $-.0^{3}015,53$ | | | |
| 400 | | $-.0^{3}038,93$ | $-.0^{3}190,19$ | $-.0^{3}469,53$ | $-.0^{3}401,50$ | $+.0^{6}96$ | $+.0^{3}377,85$ | $+.0^{3}434,93$ | $+.0^{3}239,59$ | $+.0^{3}045,68$ | | | |
| 350 | | $-.0^{3}063,18$ | — | $-.0^{3}294,87$ | $-.0^{3}358,14$ | $+.0^{3}030,81$ | $+.0^{3}356,79$ | $+.0^{3}262,61$ | — | $+.0^{3}072,92$ | | | |
| 300 | | $-.0^{3}090,84$ | $-.0^{3}199,89$ | $-.0^{3}266,11$ | $-.0^{3}190,59$ | $+.0^{3}025,37$ | $+.0^{3}219,08$ | $+.0^{3}251,50$ | $+.0^{3}197,12$ | $+.0^{3}083,44$ | | | |
| 250 | | $-.0^{3}112,87$ | — | $-.0^{3}175,74$ | — | $+.0^{3}007,41$ | — | $+.0^{3}166,06$ | — | $+.0^{3}110,06$ | | | |
| 200 | | $-.0^{3}109,13$ | — | $-.0^{3}086,29$ | — | $+.0^{3}002,58$ | — | $+.0^{3}084,91$ | — | $+.0^{3}099,18$ | | | |

Figure D-36. Monticello Dam study—angular arch movements due to total loads.—DS2-1(178)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

ANGULAR CANTILEVER MOVEMENTS DUE TO TOTAL LOADS

By. Date.

| CANT. / ELEV. | | A | B | C | D | E | CROWN | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | -0,$^{3}$057,76 | -0,$^{3}$158,92 | -0,$^{3}$338,66 | -0,$^{3}$499,20 | -0,$^{3}$410,56 | -0,$^{3}$090,18 | +0,$^{3}$360,08 | +0,$^{3}$460,63 | +0,$^{3}$310,90 | +0,$^{3}$193,41 | +0,$^{3}$075,75 | |
| 400 | | -0,$^{3}$038,93 | -0,$^{3}$130,67 | -0,$^{3}$321,94 | -0,$^{3}$473,37 | -0,$^{3}$414,05 | -0,$^{3}$089,95 | +0,$^{3}$367,02 | +0,$^{3}$444,47 | +0,$^{3}$301,40 | +0,$^{3}$157,88 | +0,$^{3}$045,68 | |
| 350 | | | -0,$^{3}$063,18 | -0,$^{3}$217,41 | -0,$^{3}$366,62 | -0,$^{3}$371,30 | -0,$^{3}$078,06 | +0,$^{3}$333,36 | +0,$^{3}$355,27 | +0,$^{3}$204,50 | +0,$^{3}$072,92 | | |
| 300 | | | | -0,$^{3}$090,84 | -0,$^{3}$215,41 | -0,$^{3}$276,23 | -0,$^{3}$056,20 | +0,$^{3}$251,16 | +0,$^{3}$212,82 | +0,$^{3}$083,44 | | | |
| 250 | | | | | -0,$^{3}$112,87 | -0,$^{3}$183,63 | -0,$^{3}$037,77 | +0,$^{3}$168,09 | +0,$^{3}$110,06 | | | | |
| 200 | | | | | | -0,$^{3}$109,13 | -0,$^{3}$022,55 | +0,$^{3}$099,18 | | | | | |
| 165 | | | | | | | -0,$^{3}$014,01 | | | | | | |

*Figure D-37.* Monticello Dam study—angular cantilever movements due to total loads.—DS2-1(179)

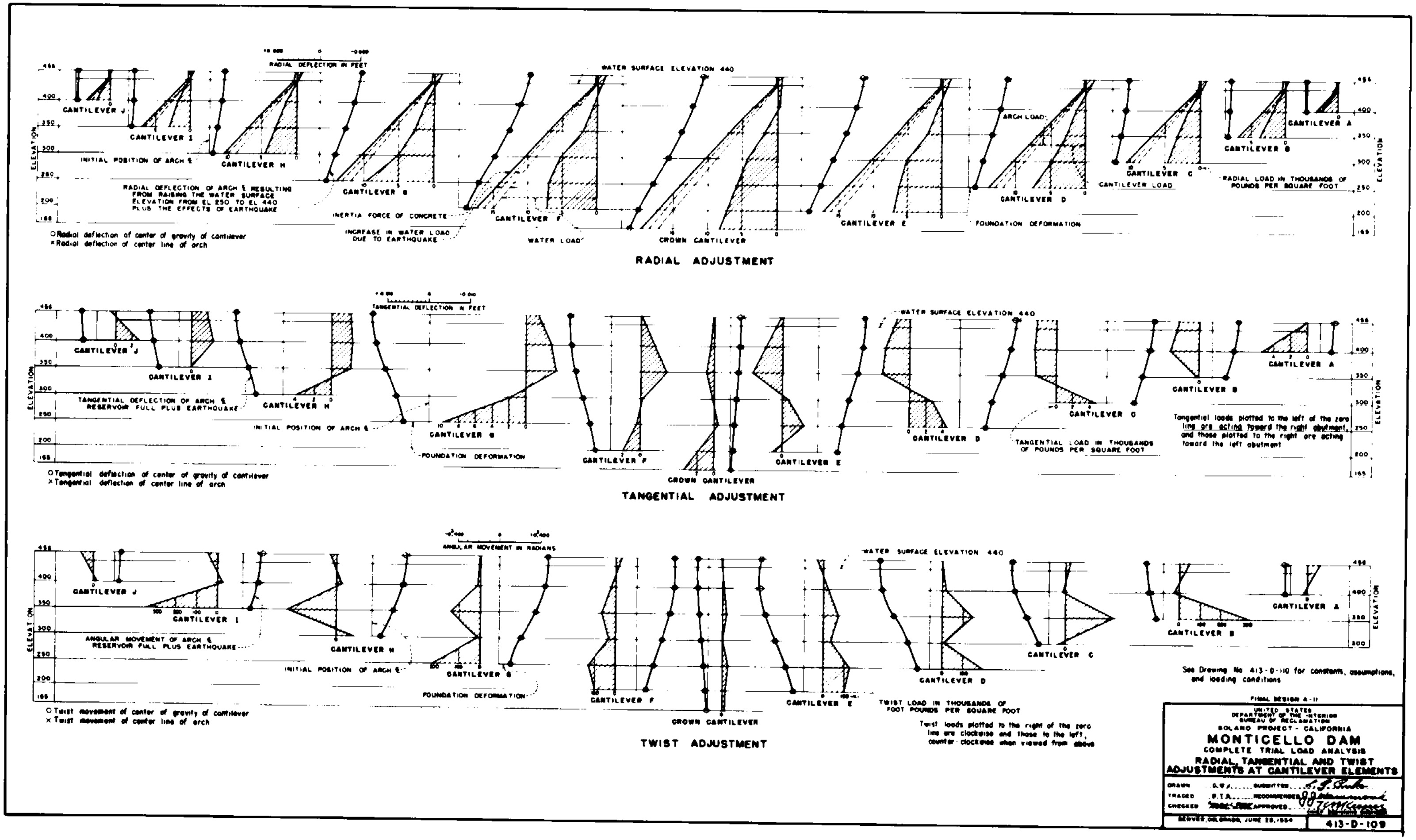


*Figure D-38.* **Monticello Dam study—radial, tangential, and twist adjustments at cantilever elements.**

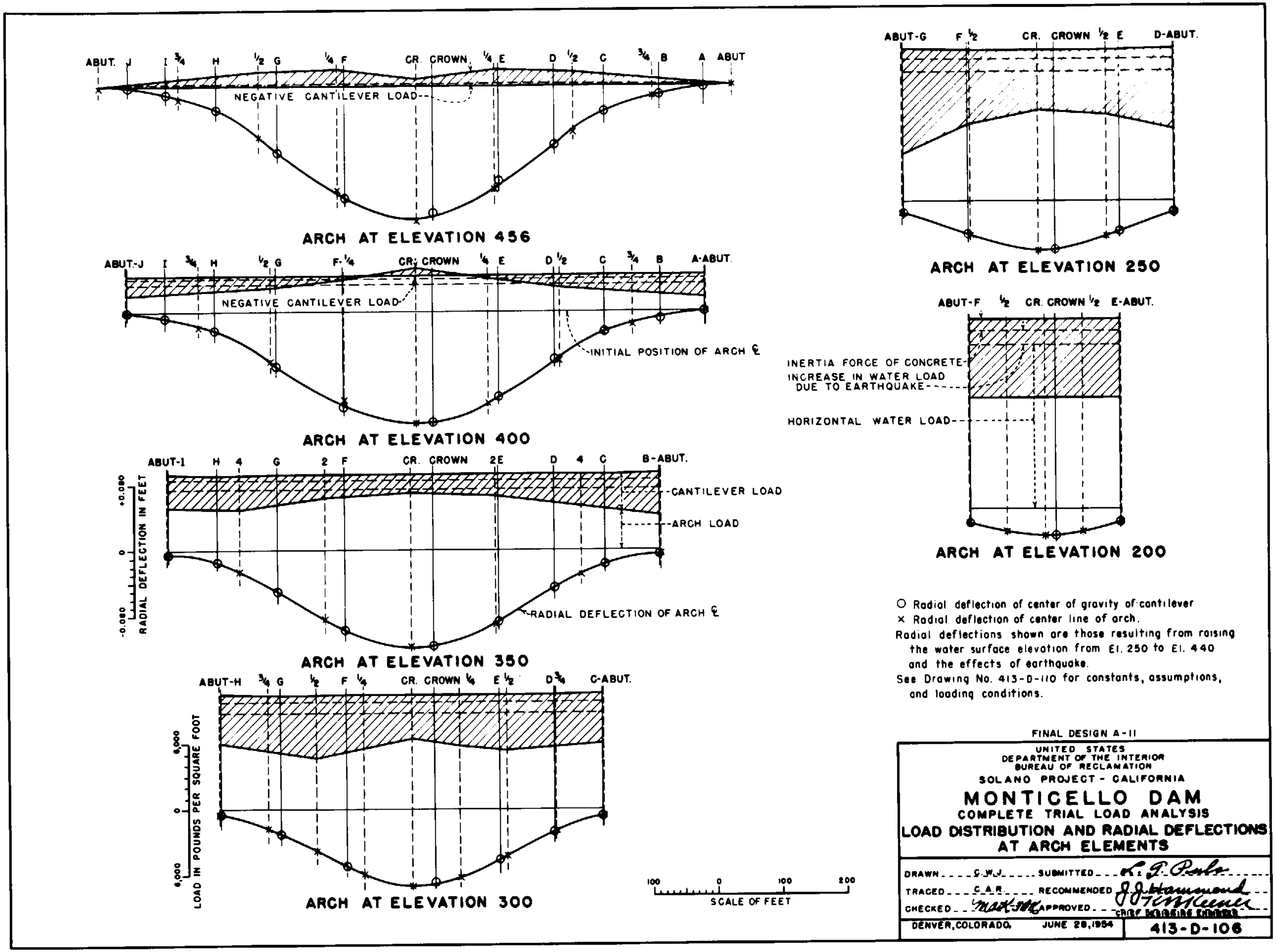


*Figure D-39.* Monticello Dam study—load distribution and radial deflections at arch elements.

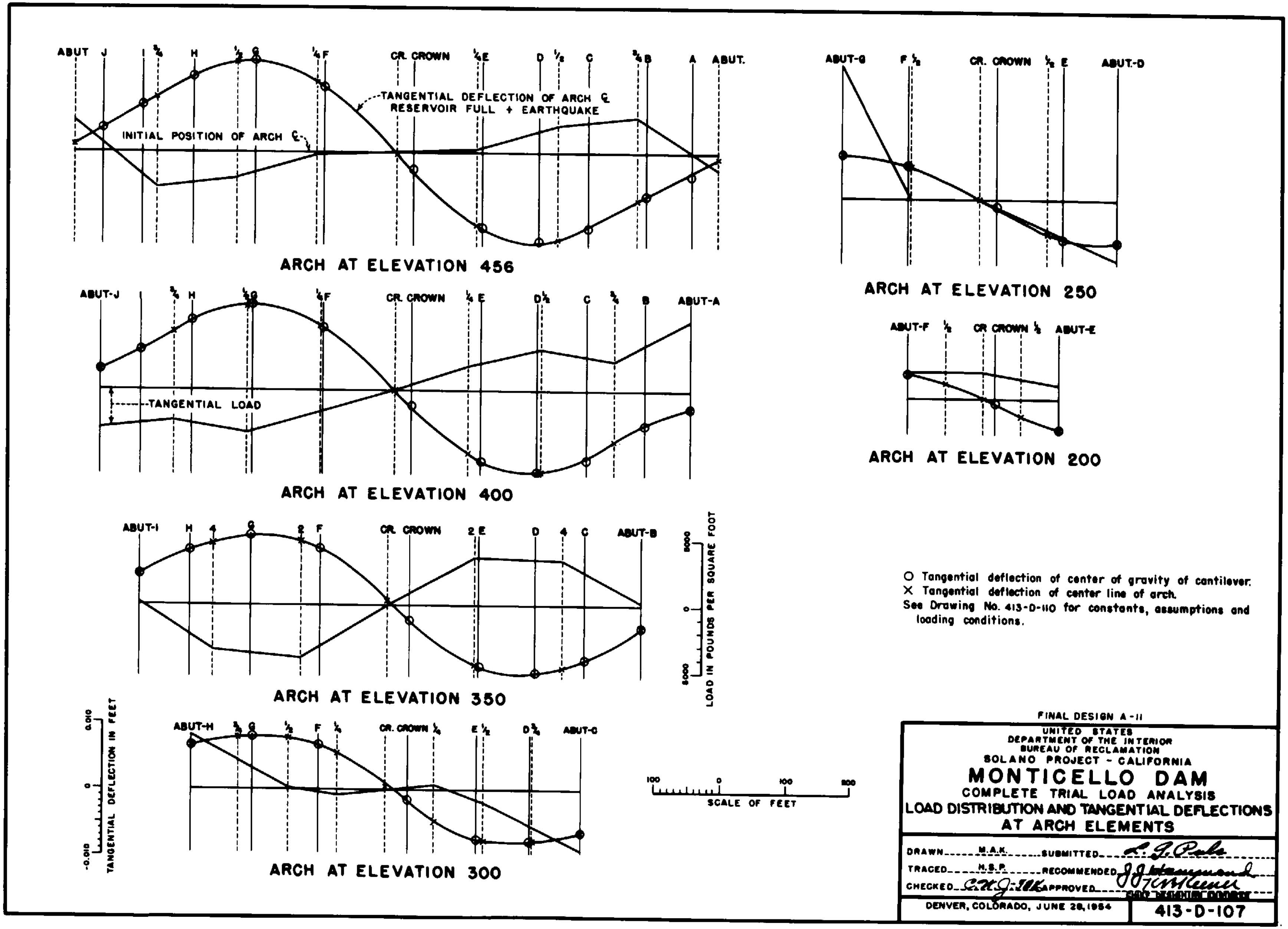


*Figure D-40.* Monticello Dam study—load distribution and tangential deflections at arch elements.

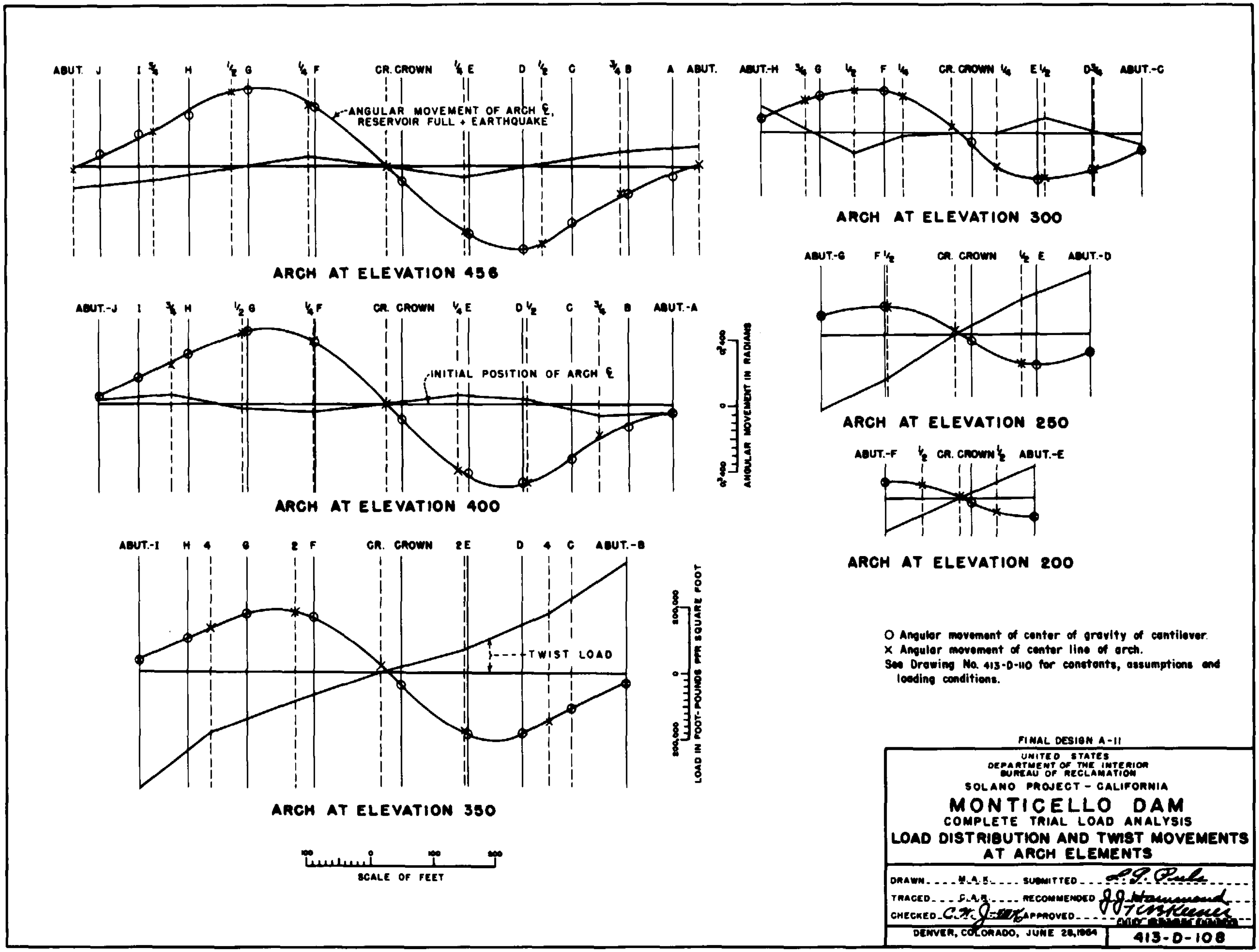


*Figure D-41.* Monticello Dam study—load distribution and twist movements at arch elements.

## «Appendix E

# Effects of Poisson's Ratio and Vertical Displacement

E-1. *Introduction.*—An analysis of Flaming Gorge Dam was selected to illustrate the adjustments necessary when the effects of Poisson's ratio and vertical displacement are included in a trial-load analysis. Figure E-1 shows Flaming Gorge and Powerplant.

E-2. *Adjustment for Poisson's Ratio Effect.*—Logically, the Poisson's ratio effect should be included in each round of adjustments. This analysis, however, was performed as a separate study; therefore, the complete adjustment stresses of a previous study were used as a starting point. Because cantilever elements and arch quarter points do not usually coincide, graphical interpolations of stresses at the faces are necessary. Calculations for a typical cantilever element (cantilever $D$) and a typical arch element (elevation 5900) are presented.

After $\sigma x$ and $\sigma y$ stresses have been determined for both cantilever faces, the vertical strains are computed for both faces. The curvature, $\rho$, of each cantilever is then calculated. Deflections are found by double summation as shown on figure E-2.

For the arches, strains are determined in the same manner as for the cantilevers. Because stage construction had been included in the original study, cantilever stresses must first be modified to account for the weight of concrete placed after grouting of the lower grout lifts and before the dam was completed. Required data are tabulated on figure E-3. Values for $\rho$ are listed on the $D$-term sheets. Values of $D_1$, $D_2$, and $D_3$ must now be computed in the usual fashion (see figs. E-4 through E-7). In these computations,

$$D_1 = \rho_{avg}\, r_v\, D_1'$$

where

$$D_1' = \Phi_1$$

$$D_2 = \rho_{avg}\, r_v^2\, D_2' - \epsilon_{x_{avg}}\, r_v\, D_2''$$

where

$$D_2' = D_2'' = \sin \Phi_o \sin \Phi_1 + \cos \Phi_o \operatorname{vers} \Phi_1$$

$$D_3 = \rho_{avg}\, r_v^2\, D_3' + \epsilon_{x_{avg}}\, r_v\, D_3'' + \rho_{avg}\, r_v\, D_1'\, e$$

where

$$D_3' = \Phi_1 + \sin \Phi_o \operatorname{vers} \Phi_1 - \cos \Phi_o \sin \Phi_1$$

and

$$D_3'' = \cos \Phi_o \sin \Phi_1 - \sin \Phi_o \operatorname{vers} \Phi_1$$

Note that because the arch is of variable thickness, the function $D_1 e$ is included in the formula for $D_3$. Arch deflections can now be determined as indicated earlier in section 4-34.

Once arch and cantilever deflections due to the effects of Poisson's ratio have been calculated, the arches and cantilevers must be brought back into adjustment in the usual manner. Upon completion of the necessary adjustments, stresses due to total loads

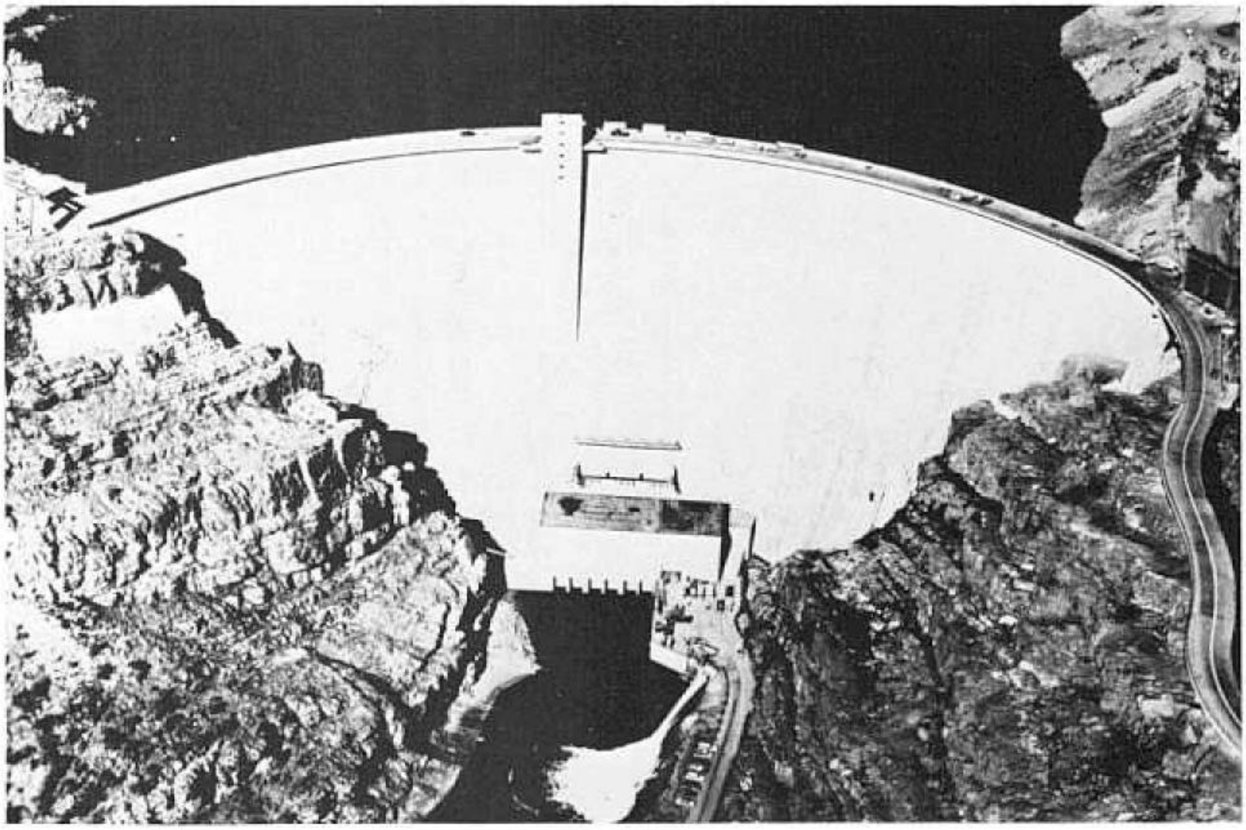


*Figure E-1.* Aerial view of Flaming Gorge Dam and Powerplant.–P-591-421-5058

including Poisson's ratio effect can be computed. Figures E-8 and E-9 show the magnitude of Poisson's ratio effects for the Flaming Gorge Dam study.

E-3. ***Vertical Displacement Adjustment.***—Vertical deflections due to the total bending moments are computed by double summation, as illustrated on figure E-10. The vertical shears are determined from the tangential shears in the same way that bending moments are determined from twisting moments (see sec. D-5). This is illustrated on figures E-11, E-12, and E-13. Vertical movements due to vertical shears can now be calculated by summation, as shown on figure E-14. Figure E-15 presents the computations for vertical movements due to uniform temperature change, again by summation.

After summing the total vertical displacements on figure E-16, the tangential movement due to these displacements must be determined. The technique described in section D-5 is used again and is illustrated on figures E-17, E-18, and E-19.

Now the tangential arch and cantilever deflections must be brought back into adjustment and the usual sequence of adjustments continued. After each regular tangential readjustment, however, the vertical effects of the additional loads must be calculated.

Figures E-20 and E-21 show the change in arch and cantilever stresses due to this vertical displacement effects adjustment.

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6b
Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load
Poisson's Ratio Effect - Cantilever Δr
Cantilever D By Date

| Elev. | | $\sigma x_u$ | $\sigma x_D$ | $\sigma y_u$ | $\sigma y_D$ | $\varepsilon_{zu}$ | $\varepsilon_{zD}$ | | $\rho$ | $\frac{\Delta z}{2}$ | $-\sum \rho \frac{\Delta z}{2}$ | $\sum\left(\rho \frac{\Delta z}{2}\right)\frac{\Delta z}{2}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | +58,800 | +34,700 | 0 | 0 | $+.0^{3}032,667$ | $+.0^{3}019,278$ | | $+.0^{6}669,45$ | | $-.0^{3}224,23$ | $-.047,46$ | |
| | | | | | | | | | | 23.5 | | | |
| 6000 | | +72,000 | +59,900 | +3,119 | | $+.0^{3}041,733$ | $+.0^{3}033,278$ | | $+.0^{6}245,36$ | | $-.0^{3}202,73$ | $-.037,43$ | |
| | | | | | | | | | | 15.0 | | | |
| 5970 | | +87,100 | +46,600 | +5,250 | | $+.0^{3}051,306$ | $+.0^{3}025,889$ | | $+.0^{6}569,63$ | | $-.0^{3}190,50$ | $-.031,53$ | |
| | | | | | | | | | | 35.0 | | | |
| 5900 | | +105,100 | +38,000 | +10,083 | | $+.0^{3}063,991$ | $+.0^{3}021,111$ | | $+.0^{6}671,26$ | | $-.0^{3}147,07$ | $-.019,71$ | |
| | | | | | | | | | | 35.0 | | | |
| 5830 | | +101,000 | +32,000 | +14,798 | | $+.0^{3}064,332$ | $+.0^{3}017,778$ | | $+.0^{6}601,78$ | | $-.0^{3}102,52$ | $-.010,98$ | |
| | | | | | | | | | | 35.0 | | | |
| 5760 | | +87,000 | +27,000 | +19,425 | | $+.0^{3}059,125$ | $+.0^{3}015,000$ | | $+.0^{6}499,38$ | | $-.0^{3}063,975$ | $-.005,15$ | |
| | | | | | | | | | | 35.0 | | | |
| 5690 | | +64,900 | +20,500 | +23,973 | | $+.0^{3}049,374$ | $+.0^{3}011,389$ | | $+.0^{6}370,51$ | | $-.0^{3}033,528$ | $-.001,74$ | |
| | | | | | | | | | | 35.0 | | | |
| 5620 | | +45,000 | +2,000 | +28,448 | 0 | $+.0^{3}040,804$ | $+.0^{3}001,111$ | | $+.0^{6}328,97$ | | $-.0^{3}009,046,7$ | $-.0^{3}25$ | |
| | | | | | | | | | | 27.5 | | | |
| 5565 | | | | | | | | | 0 | | 0 | 0 | |

*Figure E-2.* Flaming Gorge Dam study—Poisson's ratio effect, radial cantilever deflections, cantilever *D*.—DS2-1(˙85)

## CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6b
Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load
Poisson's Ratio Effect - Arch Data
Arch 5900 By Date

| Point | | $\sigma y_U$ | $\sigma y_D$ | $\sigma z_U$ | $\sigma z_D$ | $\mathcal{E}x_U$ | $\mathcal{E}x_D$ | $\Phi_0$ | $\Phi_1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cr | | +10,083 | 0 | +9,600 | -5,050 | $+.0^{3}010,935$ | $-.0^{3}002,805,6$ | | | | | | |
| | | | | | | | | 0° | 11° | | | | |
| 1/4 | | | | +7,420 | -2,900 | $+.0^{3}009,723,9$ | $-.0^{3}001,611,1$ | | | | | | |
| | | | | | | | | 11° | | | | | |
| 1/2 | | | | +7,250 | -4,000 | $+.0^{3}009,629,4$ | $-.0^{3}002,222,2$ | | | | | | |
| | | | | | | | | 22° | | | | | |
| 3/4 | | | | +5,810 | -3,000 | $+.0^{3}008,829,4$ | $-.0^{3}001,666,7$ | | | | | | |
| | | | | | | | | 33° | 11° | | | | |
| abut | | +10,083 | 0 | +8,740 | -7,880 | $+.0^{3}010,457$ | $-.0^{3}004,377,8$ | | | | | | |

*Figure E-3.* Flaming Gorge Dam study—Poisson's ratio effect, arch data, arch at elevation 5900.—DS2-1(186)

## .....Flaming Gorge....DAM STUDY NO. A-6b..........

VARIABLE THICKNESS ARCH WITH CONSTANT UPSTREAM RADIUS

## ECCENTRICITIES AND MULTIPLIERS FOR ARCH AND LOAD CONSTANTS

ARCH AT ELEV. 5900...............

COMPUTED BY........DATE..........

CHECKED BY........DATE..........

| POINT | $\Phi$ | VOUSSOIR | AT POINT | | FOR VOUSSOIR | | VOUSSOIR TO POINT | | | | $e_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | T | $r_p$ | $r_v$ | $r_v^2$ | $e_0$ | $e_1$ | $e_2$ | $e_3$ | |
| | | | | | | | | | | | |
| 0 | | | 63.22 | 568.29 | | | | | | | |
| | | 0 to 1 | | | 568.065 | 322,697.8 | .225 | | | | |
| 1 | | | 64.12 | 567.84 | | | | | | | |
| | | 1 to 2 | | | 567.180 | 321,693.2 | 1.110 | .660 | | | |
| 2 | | | 66.76 | 566.52 | | | | | | | |
| | | 2 to 3 | | | 565.455 | 319,739.4 | 2.835 | 2.385 | 1.065 | | |
| 3 | | | 71.02 | 564.39 | | | | | | | |
| | | 3 to 4 | | | 562.960 | 316,924.0 | 5.330 | 4.880 | 3.560 | 1.430 | |
| 4 | | | 76.74 | 561.53 | | | | | | | |
| | | | | | | | | | | | |

$E_c = 360 \cdot 10^6$ $\quad \frac{1}{E_c} = .0^{6}002,777,778$

$R_E = 599.90$

$r = R_E - \frac{T}{2}$ $\qquad I = \frac{T_v^3}{12}$

$r_v = R_E - \frac{T_v}{2}$

$e_{0,1,2,3} = r_p - r_v = e$, for transfer of arch and load const's.

$e_0 = e'$, for tangential load eccentricity.

$e_p$ = eccentricity of arch point with reference to arc of center line through crown.

| VOUSSOIR | $\frac{1}{E_cT_v}$ | $\frac{r_v}{E_cT_v}$ | $\frac{r_v^2}{E_cT_v}$ | $\frac{r_v}{E_cI_v}$ | $\frac{r_v^2}{E_cI_v}$ | $\frac{r_v^3}{E_cI_v}$ | $\frac{r_v^4}{E_cI_v}$ | $\frac{r_vR_E}{E_cT_v}$ | $\frac{r_v^2R_E}{E_cI_v}$ | $\frac{r_v^3R_E}{E_cI_v}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |
| 0 to 1 | | | | | | | | | | |
| | | | | | | | | | | |
| 1 to 2 | | | | | | | | | | |
| | | | | | | | | | | |
| 2 to 3 | | | | | | | | | | |
| | | | | | | | | | | |
| 3 to 4 | | | | | | | | | | |
| | | | | | | | | | | |

*Figure E-4.* Flaming Gorge Dam study—eccentricities for load constants, arch at elevation 5900.—DS2-1(187)

# COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Flaming Gorge DAM
STUDY NO. A 16b
ARCH AT ELEV. 5900
$D_1$

COMPUTED BY........
DATE........

for $D_3$

| PT. | Ø | ρ | ρav. | $r_v D_1'$ | $\rho_{av.} r_v D_1'$ |  | $\rho_{av.} r_v D_1' e$ |  |
|---|---|---|---|---|---|---|---|---|
| CROWN | Cr | $^{+}.0^{6},217,35$ |  |  |  |  |  |  |
|  |  |  | $+.0^{6},197,065$ | $+109.060,5$ | $^{+}.0^{3},021,492,01$ |  |  |  |
|  | 1 | $^{+}.0^{6},176,78$ |  |  |  |  |  |  |
|  |  |  | $^{+}.0^{6},177,15$ | $+108.890,6$ | $^{+}.0^{3},019,289,97$ |  |  |  |
|  | 2 | $^{+}.0^{6},177,52$ |  |  |  |  |  |  |
|  |  |  | $^{+}.0^{6},162,66$ | $+108.559,4$ | $^{+}.0^{3},017,658,23$ |  |  |  |
|  | 3 | $^{+}.0^{6},147,80$ |  |  |  |  |  |  |
|  |  |  | $^{+}.0^{6},170,555$ | $+108.080,4$ | $^{+}.0^{3},018,433,65$ |  |  |  |
|  | 4 | $^{+}.0^{6},193,31$ |  |  |  |  |  |  |
| $_{cr.}D_1 = {}^{+}.0^{3},076,873,86$ |  |  |  |  | $\Sigma = {}^{+}.0^{3},076,873,86$ |  | $\Sigma = {}^{+}.0^{3},174,560,0$ |  |
| 1/4 | 1 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 2 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 3 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 4 |  |  |  |  |  |  |  |
| $1/4\, D_1 = {}^{+}.0^{3},055,381,85$ |  |  |  |  |  |  | $\Sigma = {}^{+}.0^{3},144,802,5$ |  |
| 1/2 | 2 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 3 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 4 |  |  |  |  |  |  |  |
| $1/2\, D_1 = {}^{+}.0^{3},036,091,88$ |  |  |  |  |  |  | $\Sigma = {}^{+}.0^{3},084,429,8$ |  |
| 3/4 | 3 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 4 |  |  |  |  |  |  |  |
| $3/4\, D_1 = {}^{+}.0^{3},018,433,65$ |  |  |  |  |  |  | $\Sigma = {}^{+}.0^{3},026,360,12$ |  |

*Figure E-5.* Flaming Gorge Dam study—$D_1$-terms due to Poisson's ratio effect, arch at elevation 5900.—DS2-1(188)

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Flaming Gorge DAM
STUDY No. A-6b
ARCH AT ELEV. 5900
$D_2$

COMPUTED BY........
DATE........

| PT. | $\phi$ | $\rho_{av.}$ | $r_v^2 D_2'$ | $\rho_{av.} r_v^2 D_2'$ | $\varepsilon_x$ | $\varepsilon_{x_{av}}$ | $r_v D_2''$ | $\varepsilon_{x'_{av}} r_v D_2''$ |
|---|---|---|---|---|---|---|---|---|
| **CROWN** | cr | | | | $^{+}.0^{3}$,004,064,7 | | | |
| | | $^{+}.0^{6}$,197,065 | +5,928.927 | | | $^{+}.0^{3}$,004,060,55 | +10.437,06 | |
| | 1 | | | | $^{+}.0^{3}$,004,056,4 | | | |
| | | $^{+}.0^{6}$,177,15 | +17,514.26 | | | $^{+}.0^{3}$,003,880,0 | +30.879,55 | |
| | 2 | | | | $^{+}.0^{3}$,003,703,6 | | | |
| | | $^{+}.0^{6}$,162,66 | +28,301,413 | | | $^{+}.0^{3}$,003,642,5 | +50.050,68 | |
| | 3 | | | | $^{+}.0^{3}$,003,581,4 | | | |
| | | $^{+}.0^{6}$,170,555 | +37,818.86 | | | $^{+}.0^{3}$,003,310,5 | +67.178,58 | |
| | 4 | | | | $^{+}.0^{3}$,003,039,6 | | | |
| cr $D_2$ = $^{+}.0$ 14,757,84 | | | | $\Sigma = {}^{+}.0$15,324,74 | | | | $\Sigma = {}^{+}.0^{3}$,566,897,1 |
| **1/4** | 1 | | | | | | | |
| | | | +5,910.469 | | | | +10.420,80 | |
| | 2 | | | | | | | |
| | | | +17,407.89 | | | | +30.785,63 | |
| | 3 | | | | | | | |
| | | | +28,052.21 | | | | +49.829,84 | |
| | 4 | | | | | | | |
| 1/4 $D_2$ = $^{+}.$008,345,521 | | | | $\Sigma = {}^{+}.$008,663,052 | | | | $\Sigma = {}^{+}.0^{3}$,317,531 |
| **1/2** | 2 | | | | | | | |
| | | | +5,874.572 | | | | +10.389,10 | |
| | 3 | | | | | | | |
| | | | +17,254.61 | | | | +30.649,79 | |
| | 4 | | | | | | | |
| 1/2 $D_2$ = $^{+}.$003,759,110 | | | | $\Sigma = {}^{+}.$003,898,418 | | | | $\Sigma = {}^{+}.0^{3}$,139,308,4 |
| **3/4** | 3 | | | | | | | |
| | | | +5,822.845 | | | | +10.343,26 | |
| | 4 | | | | | | | |
| 3/4 $D_2$ = $^{+}.0^{3}$,958,873,9 | | | | $\Sigma = {}^{+}.0^{3}$,993,115,3 | | | | $\Sigma = {}^{+}.0^{3}$034,241,4 |

*Figure E-6.* **Flaming Gorge Dam study—$D_2$-terms due to Poisson's ratio effect, arch at elevation 5900.—DS2-1(189)**

## COMPUTATION FORM FOR ARCH CONSTANTS AND D-TERMS FOR NON-UNIFORM THICKNESS ARCHES

Flaming Gorge DAM
STUDY NO. A-6b
ARCH AT ELEV. 5900
$D_3$

COMPUTED BY.........
DATE.........

| PT. | $\phi$ | $\rho_{av.}$ | $r_v^2 D_3'$ | $\rho_{av} r_v^2 D_3'$ | | $\varepsilon x_{av.}$ | $r_v D_3''$ | $\varepsilon x_{av} r_v D_3''$ |
|---|---|---|---|---|---|---|---|---|
| CROWN | cr | | | | | | | |
| | | $^+.0^6,197,065$ | +379.815,3 | | | $^+.0^3,004,060,55$ | +108.391,9 | |
| | 1 | | | | | | | |
| | | $^+.0^6,177,15$ | +2,634.185 | | | $^+.0^3,003,880,0$ | +104.246,5 | |
| | 2 | | | | | | | |
| | | $^+.0^6,162,66$ | +7,019.559 | | | $^+.0^3,003,642,5$ | +96.145,44 | |
| | 3 | | | | | | | |
| | | $^+.0^6,170,555$ | +13,300.35 | | | $^+.0^3,003,310,5$ | +84.454,70 | |
| | 4 | | | | | | | |
| $crD_3 = ^+.005,600,701$ | | | $\Sigma = ^+.003,951,737$ | | | | $\Sigma = ^+.001,474,404$ | |
| 1/4 | 1 | | | | | | | |
| | | | +378.632,9 | | | | +108.223,0 | |
| | 2 | | | | | | | |
| | | | +2,618.186 | | | | +103.929,5 | |
| | 3 | | | | | | | |
| | | | +6,957.749 | | | | +95.721,21 | |
| | 4 | | | | | | | |
| $1/4\ D_3 = ^+.002,939,784$ | | | $\Sigma = ^+.001,679,628$ | | | | $\Sigma = ^+.001,115,354$ | |
| 1/2 | 2 | | | | | | | |
| | | | +376.333,3 | | | | +107.893,9 | |
| | 3 | | | | | | | |
| | | | +2,595.132 | | | | +103.470,9 | |
| | 4 | | | | | | | |
| $1/2\ D_3 = ^+.001,323,801$ | | | $\Sigma = ^+.0^3,503,827,1$ | | | | $\Sigma = ^+.0^3,735,543,9$ | |
| 3/4 | 3 | | | | | | | |
| | | | +373.019,5 | | | | +107.417,8 | |
| | 4 | | | | | | | |
| $3/4\ D_3 = ^+.0^3,445,587,1$ | | | $\Sigma = ^+.0^3,063,620,34$ | | | | $\Sigma = .0^3,355,606,6$ | |

*Figure E-7.* Flaming Gorge Dam study—$D_3$-terms due to Poisson's ratio effect, arch at elevation 5900.—DS2-1(190)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6b
-SIDE CANTILEVER STRESS ANALYSIS-
Stress Changes Due to Poisson's Ratio Effects
Arch Stresses Parallel to Faces (in PSI) By. Date

| Elev. | | | | | Abut. | 3/4 | 1/2 | 1/4 | Cr. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | E | +104 | -33 | -28 | +2 | +36 | | | | |
| 6047 | | | | I | -63 | -3. | +8 | +8 | -27 | | | | |
| | | | | $S_A$ | +29 | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | E | +23 | -49 | -47 | +57 | +52 | | | | |
| 6000 | | | | I | -96 | -22 | +9 | -56 | -38 | | | | |
| | | | | $S_A$ | -32 | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | E | +67 | -40 | -12 | +15 | +18 | | | | |
| 5970 | | | | I | -43 | -9 | -24 | -28 | -23 | | | | |
| | | | | $S_A$ | -30 | | | | | | | | |
| | | | | E | +15 | +11 | +7 | +4 | +3 | | | | |
| 5900 | | | | I | -26 | -20 | -16 | -14 | -13 | | | | |
| | | | | $S_A$ | -12 | | | | | | | | |
| | | | | E | +44 | +13 | -1 | -4 | -2 | | | | |
| 5830 | | | | I | -24 | -13 | -20 | -30 | -38 | | | | |
| | | | | $S_A$ | -8 | | | | | | | | |
| | | | | E | +38 | +25 | +19 | +18 | +18 | | | | |
| 5760 | | | | I | -16 | -13 | -17 | -22 | -24 | | | | |
| | | | | $S_A$ | -4 | | | | | | | | |
| | | | | E | +41 | +29 | +17 | +10 | +8 | | | | |
| 5690 | | | | I | -11 | -8 | -13 | -23 | -23 | | | | |
| | | | | $S_A$ | -3 | | | | | | | | |
| | | | | E | +28 | | +30 | | +24 | | | | |
| 5620 | | | | I | -13 | | +17 | | +32 | | | | |
| | | | | $S_A$ | -8 | | | | | | | | |

*Figure E-8.* Flaming Gorge Dam study—changes in arch stresses due to Poisson's ratio effect.—DS2-1(191)

## CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6b
-SIDE CANTILEVER STRESS ANALYSIS-
Stress Changes Due to Poisson's Ratio Effects
Cantilever Stresses Parallel to Faces (in PSI) By Date

| Elev | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | U | -16 | | -13 | | -9 | | +4 | +10 | +11 | | |
| 6000 | | D | +17 | | +14 | | +9 | | -4 | -10 | -11 | | |
| | | Sc | +7 | | | | | | | | | | |
| | | | U | -35 | -31 | | -25 | | +3 | +15 | +16 | | |
| 5970 | | | D | +40 | +35 | | +26 | | -3 | -15 | -16 | | |
| | | | Sc | +12 | | | | | | | | | |
| | | | | U | -52 | | -50 | | +3 | +24 | +25 | | |
| 5900 | | | | D | +65 | | +57 | | -3 | -25 | -25 | | |
| | | | | Sc | +22 | | | | | | | | |
| | | | | | U | -46 | -50 | | +3 | +29 | +31 | | |
| 5830 | | | | | D | +60 | +61 | | -3 | -31 | -32 | | |
| | | | | | Sc | +22 | | | | | | | |
| | | | | | | U | -45 | | 0 | +28 | +30 | | |
| 5760 | | | | | | D | +58 | | 0 | -31 | -32 | | |
| | | | | | | Sc | +21 | | | | | | |
| | | | | | | | U | -26 | 0 | +26 | +27 | | |
| 5690 | | | | | | | D | +36 | 0 | -29 | -29 | | |
| | | | | | | | Sc | +14 | | | | | |
| | | | | | | | | U | +1 | +25 | +26 | | |
| 5620 | | | | | | | | D | +1 | -29 | -29 | | |
| | | | | | | | | Sc | +3 | | | | |
| | | | | | | | | | U | +24 | — | | |
| 5565 | | | | | | | | | D | -27 | — | | |
| | | | | | | | | | Sc | -6 | +27 | U | |
| | | | | | | | | | | | -32 | D | |
| 5555 | | | | | | | | | | | -9 | Sc | |

*Figure E-9.* **Flaming Gorge Dam study—changes in cantilever stresses due to Poisson's ratio effect.—DS2-1(192)**

# CANTILEVER STRESS ANALYSIS

$\alpha = 0^{9}004,063,427$

$\alpha_2 = 0^{9}016,549,03$

$\nu =$

⊕ =

$\delta =$

Flaming Gorge DAM SECTION STUDY NO. A-6c

Radial SIDE CANTILEVER STRESS ANALYSIS Trial Load

Vertical DEFLECTION Total Bend. Mom. LOAD NO. CANTILEVER D

$\frac{1}{E} = 0^{6}002,777,778$ $\frac{K}{G} = \frac{}{G} =$ By Date

| Elev. | Deflection due to | | | | | | Deflection due to | | | | | Total Deflection |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Moment = M | $\frac{1}{EI}$ | $\frac{M}{EI}$ | $\frac{\Delta z}{2}$ | $\Sigma\left(\frac{M}{EI}\right)\frac{\Delta z}{2}$ | $\Sigma\frac{\Delta Y}{2}\left(\Sigma\frac{M\Delta z}{EI\,2}\right)$ | Horizontal Force = V | $\frac{\Delta Y}{2}$ | $\frac{V}{A}$ | $\left(\frac{K}{G}\right)\frac{\Delta z}{2}$ | $\Sigma\left(\frac{V}{A}\right)\left(\frac{K}{G}\frac{\Delta z}{2}\right)$ | |
| 6047 | 0 | — | 0 | | $-.0^{3}217,18$ | $-.006,750,0$ | | | | | | |
| | | | | 23.5 | | | | $-2.519$ | | | | |
| 6000 | +1,347,900 | $0^{12}821,47$ | $+.0^{3}001,107,3$ | | $-.0^{3}243,21$ | $-.007,909,8$ | | | | | | |
| | | | | 15 | | | | $-1.268,5$ | | | | |
| 5970 | +2,810,600 | $0^{12}377,06$ | $+.0^{3}001,059,8$ | | $-.0^{3}275,71$ | $-.008,568,0$ | | | | | | |
| | | | | 35 | | | | $-1.626$ | | | | |
| 5900 | +6,190,000 | $0^{12}127,94$ | $+.0^{6}791,95$ | | $-.0^{3}340,52$ | $-.009,570,0$ | | | | | | |
| | | | | 35 | | | | $+.228$ | | | | |
| 5830 | +7,650,400 | $0^{12}072,181$ | $+.0^{6}552,21$ | | $-.0^{3}387,57$ | $-.009,404,0$ | | | | | | |
| | | | | 35 | | | | $+2.315,5$ | | | | |
| 5760 | +1,599,900 | $0^{12}048,903$ | $+.0^{6}078,240$ | | $-.0^{3}409,63$ | $-.007,558,1$ | | | | | | |
| | | | | 35 | | | | $+3.323$ | | | | |
| 5690 | −22,834,700 | $0^{12}031,748$ | $-.0^{6}724,96$ | | $-.0^{3}387,00$ | $-.004,910,9$ | | | | | | |
| | | | | 35 | | | | $+4.202,5$ | | | | |
| 5620 | −80,225,500 | $0^{12}019,833$ | $-.0^{3}001,591,1$ | | $-.0^{3}305,94$ | $-.001,998,8$ | | | | | | |
| | | | | 27.5 | | | | $+3.921$ | | | | |
| 5565 | −156,670,000 | $0^{12}013,543$ | $-.0^{3}002,121,8$ | | $-.0^{3}203,83$ | 0 | −2,249,400 | | | | | |

*Figure E-10.* Flaming Gorge Dam study—vertical cantilever movement due to bending moments, cantilever *D*.—DS2-1(193)

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS - Trial Load

Average Rate of Change of Tangential Shears

By. Date.

| Cant. / Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | |
| 6000 | | | +294.98 | +320.08 | +330.34 | +337.40 | +332.83 | +195.06 | +209.41 | +262.72 | | | |
| 5970 | | | | +336.41 | +353.67 | +364.73 | +348.71 | +405.45 | +468.91 | +595.61 | | | |
| 5900 | | | | | -187.29 | -167.29 | -167.45 | +1,078.9 | +1,469.1 | +1,662.9 | | | |
| 5830 | | | | | | -106.47 | -109.88 | +1,651.3 | +2,696.4 | +2,830.9 | | | |
| 5760 | | | | | | | +629.82 | +2,392.1 | +3,793.2 | +3,854.0 | | | |
| 5690 | | | | | | | | +3,432.7 | +4,802.0 | +4,832.9 | | | |
| 5620 | | | | | | | | | +5,574.4 | +5,543.2 | | | |

*Figure E-11.* Flaming Gorge Dam study—average rate of change of tangential shears in cantilevers.—DS2-1(194)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Average Rate of Change of Vertical Shears

By Date

| Cant.<br>Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | $\frac{\Delta Z}{2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | |
| | | | | | | | | | | | | | 23.5 |
| 6000 | | | -291.23 | -298.73 | -324.62 | -333.89 | -334.91 | -266.48 | -200.82 | -236.14 | -262.76 | | |
| | | | | | | | | | | | | | 15 |
| 5970 | | | | -328.78 | -344.04 | -359.22 | -356.03 | -376.01 | -430.91 | -532.36 | -595.61 | | |
| | | | | | | | | | | | | | 35 |
| 5900 | | | | | +197.29 | +177.29 | +167.38 | -431.43 | -1,235.1 | -1,566.1 | -1,662.9 | | |
| | | | | | | | | | | | | | 35 |
| 5830 | | | | | | +104.62 | +108.52 | -735.47 | -2,069.4 | -2,763.6 | -2,830.9 | | |
| | | | | | | | | | | | | | 35 |
| 5760 | | | | | | | +216.09 | -1,475.7 | -2,952.5 | -3,823.6 | -3,854.0 | | |
| | | | | | | | | | | | | | 35 |
| 5690 | | | | | | | | -2,885.0 | -3,980.4 | -4,817.4 | -4,832.9 | | |
| | | | | | | | | | | | | | 35 |
| 5620 | | | | | | | | | -5,590.0 | -5,558.8 | -5,543.2 | | |
| | | | | | | | | | | | | | 27.5 |
| 5565 | | | | | | | | | | -6,141.3 | — | | |
| | | | | | | | | | | | | | 32.5 |
| 5555 | | | | | | | | | | | -6,202.8 | | |

*Figure E-12.* Flaming Gorge Dam study—average rate of change of vertical shears in cantilevers.—DS2-1(195)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Vertical Shears (vV)

By Date

| Cant. Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | |
| 6000 | | | -6,844 | -7,020 | -7,629 | -7,846 | -7,870 | -6,262 | -4,719 | -5,549 | -6,175 | | |
| 5970 | | | | -16,433 | -17,658 | -18,243 | -18,234 | -15,900 | -14,195 | -17,077 | -19,050 | | |
| 5900 | | | | | -22,795 | -24,611 | -24,837 | -44,160 | -72,506 | -90,521 | -98,098 | | |
| 5830 | | | | | | -14,744 | -15,188 | -85,002 | -188,163 | -242,060 | -255,380 | | |
| 5760 | | | | | | | -3,833 | -162,392 | -363,930 | -472,610 | -489,350 | | |
| 5690 | | | | | | | | -315,017 | -606,581 | -775,050 | -793,390 | | |
| 5620 | | | | | | | | | -941,545 | -1,138,210 | -1,156,560 | | |
| 5565 | | | | | | | | | | -1,459,970 | — | | |
| 5555 | | | | | | | | | | | -1,538,300 | | |

*Figure E-13.* **Flaming Gorge Dam study—vertical shears in cantilevers.—DS2-1(196)**

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Vertical Movements (ΔV) Due to Vertical Shears

Cantilever D By Date

| Elev | | | | | $\nu V$ | $\frac{1}{A}$ | $\frac{\nu V}{A}$ | $\frac{\Delta z}{2E}$ | $\Sigma \frac{\nu V \Delta z}{2AE}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 6047 | | | | | 0 | — | 0 | | -.006,069,7 | | | | |
| | | | | | | | | $.0^{6}065,278$ | | | | | |
| 6000 | | | | | -5,549 | .029,258 | -162.35 | | -.006,059,1 | | | | |
| | | | | | | | | $.0^{6}041,667$ | | | | | |
| 5970 | | | | | -17,077 | .022,510 | -384.04 | | -.006,036,3 | | | | |
| | | | | | | | | $.0^{6}097,222$ | | | | | |
| 5900 | | | | | -90,521 | .015,647 | -1,416.4 | | -.005,861,2 | | | | |
| | | | | | | | | $.0^{6}097,222$ | | | | | |
| 5830 | | | | | -242,060 | .012,938 | -3,131.8 | | -.005,419,1 | | | | |
| | | | | | | | | $.0^{6}097,222$ | | | | | |
| 5760 | | | | | -472,610 | .011,429 | -5,401.5 | | -.004,589,4 | | | | |
| | | | | | | | | $.0^{6}097,222$ | | | | | |
| 5690 | | | | | -775,050 | .009,980,1 | -7,735.1 | | -.003,312,3 | | | | |
| | | | | | | | | $.0^{6}097,222$ | | | | | |
| 5620 | | | | | -1,138,210 | .008,625,0 | -9,817.1 | | -.001,605,8 | | | | |
| | | | | | | | | $.0^{6}076,389$ | | | | | |
| 5565 | | | | | -1,459,970 | .007,674,4 | -11,204 | | 0 | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |

*Figure E-14.* Flaming Gorge Dam study—vertical cantilever movements due to vertical shears, cantilever *D*.—DS2-1(197)

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS-Trial Load

Vertical Movement (ΔV) Due to Uniform Temperature Change

$C = .0^{3}005,6/°F$ By Date

| Elev | t | ct | $\frac{\Delta z}{2}$ | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | -8.1 | $-.0^{3}045,360$ | | | +.001,039,6 | $+.0^{3}787,6$ | $-.0^{3}702,0$ | -.002,681,6 | -.004,857,2 | -.007,150,4 | -.009,482,8 | -.011,330,8 | -.011,666,8 |
| | | | 23.5 | | | | | | | | | | |
| 6000 | +0.2 | $+.0^{3}001,120,0$ | | | | $-.0^{3}252,0$ | -.001,741,6 | -.003,721,2 | -.005,896,8 | -.008,190,0 | -.010,522,4 | -.012,370,4 | -.012,706,4 |
| | | | 15 | | | | | | | | | | |
| 5970 | +2.8 | $+.0^{3}015,680$ | | | | 0 | -.001,489,6 | -.003,469,2 | -.005,644,8 | -.007,938,0 | -.010,270,4 | -.012,118,4 | -.012,454,4 |
| | | | 35 | | | | | | | | | | |
| 5900 | +4.8 | $+.0^{3}026,880$ | | | | | 0 | -.001,979,6 | -.004,155,2 | -.006,448,4 | -.008,780,8 | -.010,628,8 | -.010,964,8 |
| | | | 35 | | | | | | | | | | |
| 5830 | +5.3 | $+.0^{3}029,680$ | | | | | | 0 | -.002,175,6 | -.004,468,8 | -.006,801,2 | -.008,649,2 | -.008,985,2 |
| | | | 35 | | | | | | | | | | |
| 5760 | +5.8 | $+.0^{3}032,480$ | | | | | | | 0 | -.002,293,2 | -.004,625,6 | -.006,473,6 | -.006,809,6 |
| | | | 35 | | | | | | | | | | |
| 5690 | +5.9 | $+.0^{3}033,040$ | | | | | | | | 0 | -.002,332,4 | -.004,180,4 | -.004,616,4 |
| | | | 35 | | | | | | | | | | |
| 5620 | +6.0 | $+.0^{3}033,600$ | | | | | | | | | 0 | -.001,848,0 | -.002,184,0 |
| | | | 27.5 | | | | | | | | | | |
| 5565 | +6.0 | $+.0^{3}033,600$ | | | | | | | | | | 0 | — |
| | | | 32.5 | | | | | | | | | | |
| 5555 | +6.0 | $+.0^{3}033,600$ | | | | | | | | | | | 0 |

*Figure E-15.* Flaming Gorge Dam study—vertical movements of cantilevers due to uniform temperature change.—DS2-1(198)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c
Radial -SIDE CANTILEVER STRESS ANALYSIS - Trial Load
Total Vertical Displacement (ΔV)
By. Date

| Cant. / Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | +.001,060,7 | +.0$^3$934,7 | -.0$^3$665,7 | -.004,298,5 | -.007,515,9 | -.011,891,0 | -.018,283,3 | -.024,150,5 | -.025,179,4 | | |
| 6000 | | | 0 | -.0$^3$196,6 | -.001,746,2 | -.005,347,6 | -.008,647,9 | -.013,028,9 | -.019,781,9 | -.026,339,3 | -.027,178,8 | | |
| 5970 | | | | 0 | -.001,527,1 | -.004,979,2 | -.008,372,2 | -.012,817,3 | -.019,762,1 | -.026,722,7 | -.027,505,5 | | |
| 5900 | | | | | 0 | -.002,820,3 | -.006,391,4 | -.011,088,6 | -.018,329,7 | -.026,060,0 | -.026,953,7 | | |
| 5830 | | | | | | 0 | -.003,291,6 | -.008,012,4 | -.015,250,6 | -.023,472,3 | -.024,671,1 | | |
| 5760 | | | | | | | 0 | -.004,048,1 | -.010,647,5 | .018,621,1 | -.020,027,2 | | |
| 5690 | | | | | | | | 0 | -.005,344,1 | .012,403,6 | -.013,820,4 | | |
| 5620 | | | | | | | | | 0 | -.005,452,6 | -.006,690,7 | | |
| 5565 | | | | | | | | | | 0 | — | | |
| 5555 | | | | | | | | | | | 0 | | |

*Figure E-16.* Flaming Gorge Dam study – total vertical displacements of cantilevers. – DS2-1(199)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Av. Slope of Vertical Displacements $(\partial \Delta V / \partial x)$

By Date

| Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | $.0^3$012,883 | $.0^3$028,666 | $.0^3$051,831 | $.0^3$046,441 | $.0^3$075,472 | $.0^3$102,621 | $.0^3$063,286 | $.0^3$011,156 | | | |
| 6000 | | | $.0^3$020,143 | $.0^3$027,876 | $.0^3$051,471 | $.0^3$047,596 | $.0^3$075,314 | $.0^3$107,858 | $.0^3$070,260 | $.0^3$009,033,7 | | | |
| 5970 | | | | $.0^3$027,680 | $.0^3$049,564 | $.0^3$049,025 | $.0^3$076,403 | $.0^3$110,727 | $.0^3$074,357 | $.0^3$008,389,2 | | | |
| 5900 | | | | | $.0^3$040,963 | $.0^3$051,921 | $.0^3$080,903 | $.0^3$115,359 | $.0^3$082,246 | $.0^3$009,519,6 | | | |
| 5830 | | | | | | $.0^3$048,158 | $.0^3$081,661 | $.0^3$115,571 | $.0^3$087,549 | $.0^3$012,765 | | | |
| 5760 | | | | | | | $.0^3$070,549 | $.0^3$106,168 | $.0^3$085,517 | $.0^3$015,080 | | | |
| 5690 | | | | | | | | $.0^3$086,924 | $.0^3$076,551 | $.0^3$015,363 | | | |
| 5620 | | | | | | | | | $.0^3$059,998 | $.0^3$013,623 | | | |

*Figure E-17.* Flaming Gorge Dam study–average slope of vertical displacements of cantilevers.–DS2-1(200)

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

Radial -SIDE CANTILEVER STRESS ANALYSIS- Trial Load

Av. Slope of Tangential Movements ($\partial \Delta s / \partial z = - \partial V / \partial x$)

By Date

| Cant / Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | $\frac{\Delta z}{2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 6047 | | | $-.0^3 010,530$ | $-.0^3 015,236$ | $-.0^3 038,937$ | $-.0^3 049,120$ | $-.0^3 062,247$ | $-.0^3 088,559$ | $-.0^3 086,813$ | $-.0^3 037,153$ | 0 | | |
| | | | | | | | | | | | | | 23.5 |
| 6000 | | | $-.0^3 018,988$ | $-.0^3 021,298$ | $-.0^3 038,322$ | $-.0^3 049,525$ | $-.0^3 062,669$ | $-.0^3 090,988$ | $-.0^3 092,762$ | $-.0^3 059,581$ | 0 | | |
| | | | | | | | | | | | | | 15 |
| 5970 | | | | $-.0^3 018,007$ | $-.0^3 037,353$ | $-.0^3 049,294$ | $-.0^3 063,899$ | $-.0^3 092,920$ | $-.0^3 096,135$ | $-.0^3 041,320$ | 0 | | |
| | | | | | | | | | | | | | 35 |
| 5900 | | | | | $-.0^3 035,481$ | $-.0^3 046,445$ | $.0^3 067,637$ | $-.0^3 097,459$ | $-.0^3 102,100$ | $-.0^3 045,862$ | 0 | | |
| | | | | | | | | | | | | | 35 |
| 5830 | | | | | | $-.0^3 030,007$ | $-.0^3 066,309$ | $-.0^3 097,937$ | $-.0^3 104,360$ | $-.0^3 050,157$ | 0 | | |
| | | | | | | | | | | | | | 35 |
| 5760 | | | | | | | $-.0^3 053,452$ | $-.0^3 087,646$ | $-.0^3 097,908$ | $-.0^3 050,298$ | 0 | | |
| | | | | | | | | | | | | | 35 |
| 5690 | | | | | | | | $-.0^3 091,073$ | $-.0^3 082,775$ | $-.0^3 045,957$ | 0 | | |
| | | | | | | | | | | | | | 35 |
| 5620 | | | | | | | | | $-.0^3 083,186$ | $-.0^3 036,810$ | 0 | | |
| | | | | | | | | | | | | | 27.5 |
| 5565 | | | | | | | | | | $-.0^3 029,623$ | — | | |
| | | | | | | | | | | | | | 32.5 |
| 5555 | | | | | | | | | | | 0 | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |

*Figure E-18.* Flaming Gorge Dam study—average slope of tangential movements of cantilevers.—DS2-1(201)

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

-SIDE CANTILEVER STRESS ANALYSIS-

Tangential Movements ($\Delta S$) Due to Vertical Displacements

By Date

| Cant. Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6047 | | | -.0$^3$,694 | -.001,448 | -.005,500 | -.009,827 | -.018,318 | -.033,231 | -.040,430 | -.021,037 | 0 | | |
| 6000 | | | 0 | -.0$^3$,590 | -.003,684 | -.007,509 | -.015,382 | -.029,011 | -.036,210 | -.019,234 | 0 | | |
| 5970 | | | | 0 | -.002,549 | -.006,027 | -.013,484 | -.026,253 | -.033,376 | -.018,021 | 0 | | |
| 5900 | | | | | 0 | -.002,676 | -.008,880 | -.019,589 | -.026,438 | -.014,969 | 0 | | |
| 5830 | | | | | | 0 | -.004,192 | -.012,751 | -.019,212 | -.011,609 | 0 | | |
| 5760 | | | | | | | 0 | -.006,255 | -.012,133 | -.008,093 | 0 | | |
| 5690 | | | | | | | | 0 | -.005,809 | -.004,724 | 0 | | |
| 5620 | | | | | | | | | 0 | -.001,827 | 0 | | |
| 5565 | | | | | | | | | | 0 | — | | |
| 5555 | | | | | | | | | | | 0 | | |

*Figure E-19.* **Flaming Gorge Dam study—tangential movements of cantilevers due to vertical displacements.—DS2-1(202)**

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

-SIDE CANTILEVER STRESS ANALYSIS-

Stress Changes Due to Vertical Displacement Effects

Arch Stresses Parallel to Faces (in PSI) By Date

| Elev. | | Abut. | 3/4 | 1/2 | 1/4 | Cr. |
|---|---|---|---|---|---|---|
| 6047 | E | +150 | +51 | +11 | +7 | +25 |
| | I | +122 | +53 | +45 | +11 | -23 |
| | $S_A$ | -29 | | | | |
| 6000 | E | +113 | +18 | -37 | -23 | -18 |
| | I | +47 | +66 | +25 | -53 | -82 |
| | $S_A$ | -5 | | | | |
| 5970 | E | +128 | +14 | +14 | +11 | -25 |
| | I | +86 | +104 | +17 | -44 | -26 |
| | $S_A$ | -15 | | | | |
| 5900 | E | +45 | +25 | -5 | -13 | -4 |
| | I | +36 | +57 | +33 | -25 | -60 |
| | $S_A$ | -15 | | | | |
| 5830 | E | +50 | +4 | -14 | -8 | +2 |
| | I | +35 | +55 | +15 | -33 | -59 |
| | $S_A$ | -12 | | | | |
| 5760 | E | +45 | 0 | -2 | +12 | +19 |
| | I | +24 | +25 | 0 | -27 | -41 |
| | $S_A$ | -8 | | | | |
| 5690 | E | +32 | +18 | +24 | +38 | +42 |
| | I | +38 | +31 | +9 | -15 | -22 |
| | $S_A$ | -6 | | | | |
| 5620 | E | +35 | | +29 | | +33 |
| | I | +28 | | +17 | | +18 |
| | $S_A$ | +5 | | | | |

*Figure E-20.* **Flaming Gorge Dam study—arch stresses due to vertical displacement effects.—DS2-1(203)**

# CANTILEVER STRESS ANALYSIS

Flaming Gorge DAM. SECTION. STUDY NO. A-6c

-SIDE CANTILEVER STRESS ANALYSIS-

Stress Changes Due to Vertical Displacement Effects

Cantilever Stresses Parallel to Faces (in PSI) By Date

| Elev. | | | 6000-L | 5970-L | A | 5830-L | B | 5690-L | C | D | Crown | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | U | +1 | | -4 | | -2 | | +4 | +1 | -2 | | |
| 6000 | | D | -1 | | +4 | | +2 | | -4 | -1 | +2 | | |
| | | Sc | -5 | | | | | | | | | | |
| | | | U | +11 | +1 | | -5 | | +3 | +1 | -4 | | |
| 5970 | | | D | -13 | -1 | | +5 | | -3 | -1 | +4 | | |
| | | | Sc | -11 | | | | | | | | | |
| | | | | U | +15 | | -14 | | -3 | +3 | -3 | | |
| 5900 | | | | D | -21 | | +15 | | +3 | -3 | +3 | | |
| | | | | Sc | -10 | | | | | | | | |
| | | | | | U | +16 | -19 | | -9 | +3 | +3 | | |
| 5830 | | | | | D | +16 | +19 | | +9 | -3 | -3 | | |
| | | | | | Sc | +3 | | | | | | | |
| | | | | | | U | -17 | | -15 | -4 | +5 | | |
| 5760 | | | | | | D | +12 | | +13 | +4 | -6 | | |
| | | | | | | Sc | -4 | | | | | | |
| | | | | | | | U | -4 | -14 | -18 | -8 | | |
| 5690 | | | | | | | D | -10' | +7 | +19 | +9 | | |
| | | | | | | | Sc | -17 | | | | | |
| | | | | | | | | U | +1 | -19 | -24 | | |
| 5620 | | | | | | | | D | -18 | +19 | +27 | | |
| | | | | | | | | Sc | -22 | | | | |
| | | | | | | | | | U | -6 | — | | |
| 5565 | | | | | | | | | D | +1 | — | | |
| | | | | | | | | | Sc | -7 | -27 | U | |
| | | | | | | | | | | | +32 | D | |
| 5555 | | | | | | | | | | | +9 | Sc | |

*Figure E-21.* Flaming Gorge Dam study—cantilever stresses due to vertical displacement effects.—DS2-1(204)

# Stress Computations

F-1. *Introduction.*—Stress computations from a complete trial-load analysis of Monticello Dam are presented in this appendix as an example.

F-2. *Weight and Bending Moments in Cantilevers.*—Weights and bending moments for the crown cantilever are given on figure F-1 for Monticello Dam. Weights and moments due to concrete and vertical waterload are shown on figure B-4 of appendix B. Moments due to the trial radial load on the cantilever, moments due to initial loads on the cantilevers, moments due to weight of concrete, and bending moments due to twist, are added together to determine final total moments. The effect of horizontal hydrodynamic loading is included in $M_p$.

F-3. *Vertical and Inclined Stresses at Face of Cantilever.*—Final vertical cantilever stresses at the faces of the dam, $\sigma_{zE}$ and $\sigma_{zD}$, and inclined stresses parallel to the faces of the dam, $\sigma''_{zE}$ and $\sigma''_{zD}$ are given on figure F-2. These are calculated by means of equations (261) and (276) for the vertical stresses, and equations (267) and (283) for inclined stresses (see sec. 4-45). Also, the term $(\sigma_x - p) \tan^2 \eta \sin^2 \phi$ is usually omitted when it is a very small value.

It should be noted that the lower part of the upstream face of the dam is vertical and that consequently the tangent of $\phi_E$ is zero in this portion; also, that at the downstream face there is no water pressure.

Inclined cantilever stresses parallel to the face are tabulated in pounds per square inch on the lower profile of figure F-3. These stresses act in inclined directions parallel to the face of the cantilevers. Stresses at the upstream face are indicated by *U*, and those at the downstream face by *D*.

F-4. *Normal Horizontal Arch Stress and Horizontal Arch Stress Parallel to Intrados.*—Final arch stresses normal to a vertical radial plane, $\sigma_{xE}$ and $\sigma_{xD}$, and stresses parallel to the upstream face in a horizontal plane, are shown for arch 350 on figure F-4. These are calculated by means of equations (262), (277), (272), and (287) for the abutment and the crown (see sec. 4-45). In equation (262), it can be seen that

$$I_A = \frac{T^3}{12}; \ \frac{C}{I_A} = \frac{6}{T^2}; \text{ and } A_A = T$$

Values are obtained for $M_A$ by summating the respective products of the arch moments due to unit loads and the various trial loads for the arch. In these computations care must be taken that, at the arch abutment points, values of $M_a$ for unit concentrated twist load do not contain $M_L$; that is,

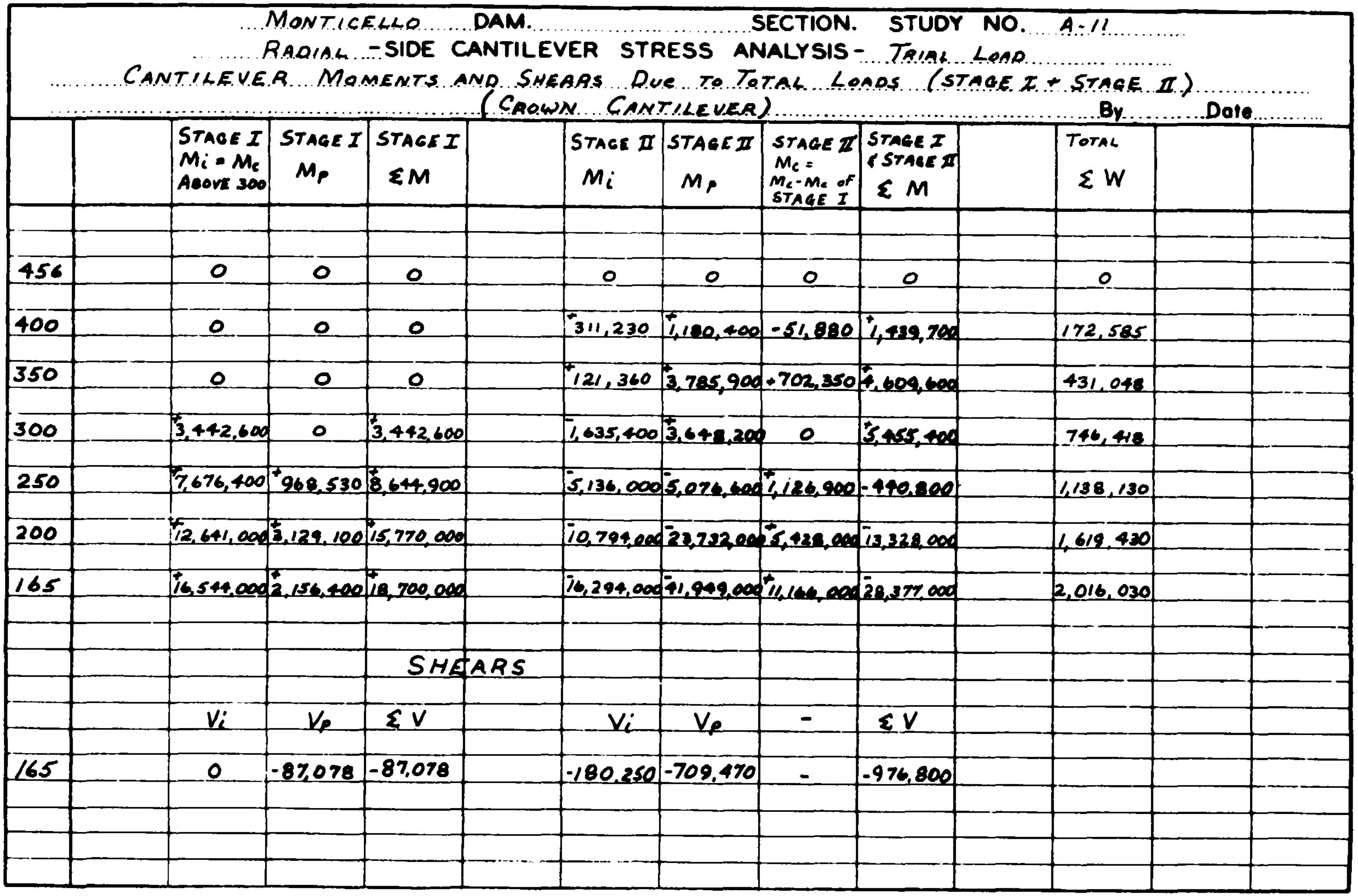
$$M_a = M_o + H_o\, y_a + V_o\, x_a$$

Values for $H_A$ are calculated by summating the respective products of the arch thrusts due to unit loads and the various trial loads for the arch. Here again, for abutment points, $H_a$ should not contain $H_L$ for a unit concentrated tangential load; that is,

$$H_a = H_o \cos \Phi_a - V_o \sin \Phi_a$$

Values for $V_A$ are obtained by summating the respective products of arch shears due to unit loads and the various trial loads for the arch. In this case $V_a$ should not contain $V_L$; that is $V_a = H_o \sin \Phi_a + V_o \cos \Phi_a$ for a unit

# CANTILEVER STRESS ANALYSIS



MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
CANTILEVER MOMENTS AND SHEARS DUE TO TOTAL LOADS (STAGE I + STAGE II)
(CROWN CANTILEVER) By Date

| | | STAGE I $M_i = M_c$ ABOVE 300 | STAGE I $M_p$ | STAGE I $\Sigma M$ | | STAGE II $M_i$ | STAGE II $M_p$ | STAGE II $M_c = M_c - M_c$ of STAGE I | STAGE I & STAGE II $\Sigma M$ | | TOTAL $\Sigma W$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | | 0 | | |
| 400 | | 0 | 0 | 0 | | +311,230 | +1,180,400 | -51,880 | +1,439,700 | | 172,585 | | |
| 350 | | 0 | 0 | 0 | | +121,360 | +3,785,900 | +702,350 | +4,609,600 | | 431,048 | | |
| 300 | | +3,442,600 | 0 | +3,442,600 | | −1,635,400 | +3,648,200 | 0 | +5,455,400 | | 746,418 | | |
| 250 | | +7,676,400 | +968,530 | +8,644,900 | | −5,136,000 | −5,076,600 | +1,126,900 | -440,800 | | 1,138,130 | | |
| 200 | | +12,641,000 | +3,129,100 | +15,770,000 | | −10,794,000 | −23,732,000 | +5,428,000 | −13,328,000 | | 1,619,430 | | |
| 165 | | +16,544,000 | +2,156,400 | +18,700,000 | | −16,294,000 | −41,949,000 | +11,166,000 | −28,377,000 | | 2,016,030 | | |
| | | | | SHEARS | | | | | | | | | |
| | | $V_i$ | $V_p$ | $\Sigma V$ | | $V_i$ | $V_p$ | - | $\Sigma V$ | | | | |
| 165 | | 0 | -87,078 | -87,078 | | -180,250 | -709,470 | - | -976,800 | | | | |

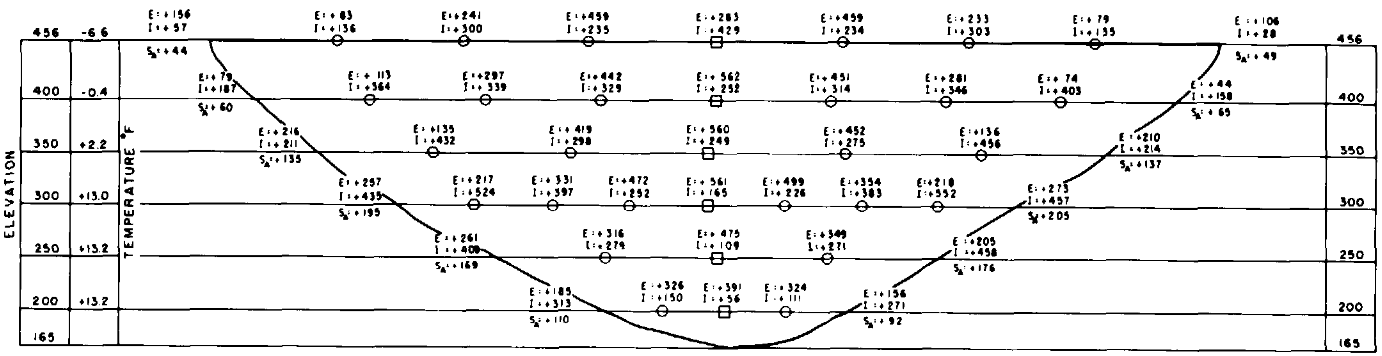
*Figure F-1.* Monticello Dam study—moments and shears at crown cantilever due to total loads. —288-D-3117

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM SECTION STUDY NO. A-11
-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
STRESS CALCULATIONS. CANTILEVER CROWN
Res. W. S. Elev. 440 Tailwater Elev. By Date

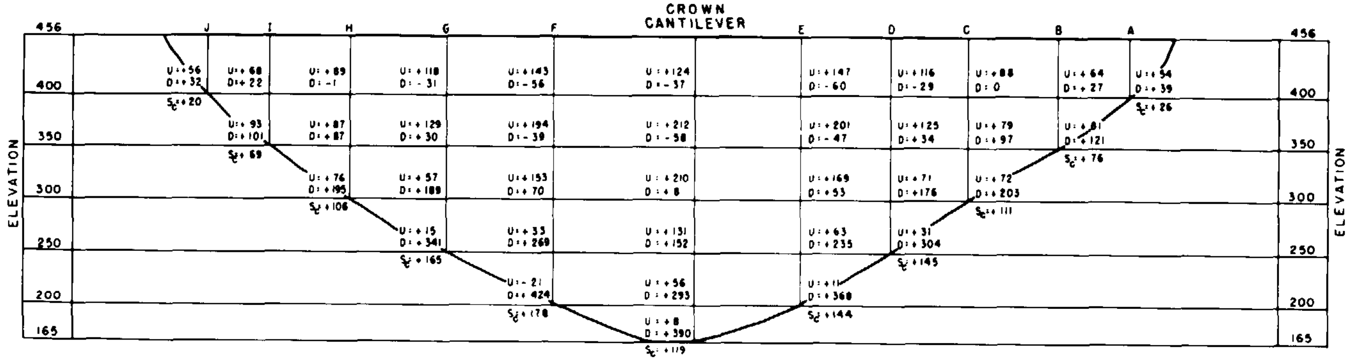
| Elev. | $\frac{I}{A}$ | Upstr. $\frac{c}{I}=\frac{Ig}{I}$ | Downstr. $\frac{c'}{I}=\frac{T-Ig}{I}$ | $\frac{W}{A}$ | $\frac{Mc}{I}$ | $\frac{Mc'}{I}$ | Stresses - Pounds per Square Foot | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Upstream Face | | | | | Downstream Face | | | | |
| | | | | | | | External Pressure | Vertical $\frac{W}{A}+\frac{Mc}{I}$ | Tan $\phi_E$ | $Sec^2\phi_E$ | Parallel to Face | External Pressure | Vertical $\frac{W}{A}-\frac{Mc'}{I}$ | Tan $\phi_D$ | $Sec^2\phi_D$ | Parallel to Face |
| 456 | — | — | — | — | — | — | — | 0 | — | — | 0 | | 0 | — | — | 0 |
| 400 | .036,669 | .007,9230 | .008,070,6 | 6,329 | 11,407 | 11,619 | 2,999 | +17,736 | .106,29 | 1.011,30 | +17,903 | | -5,290 | .135,16 | 1.018,27 | -5,387 |
| 350 | .026,635 | .004,1449 | .004,2529 | 11,481 | 19,106 | 19,604 | 6,438 | +30,587 | .004,065 | 1.000,02 | +30,587 | | -8,123 | .177,12 | 1.031,37 | -8,378 |
| 300 | .021,346 | .002,626,5 | .002,714,0 | 15,933 | 14,329 | 14,806 | | +30,262 | 0 | 1.000,00 | +30,262 | | +1,127 | .220,00 | 1.048,40 | +1,182 |
| 250 | .017,262 | .001,689,7 | .001,761,5 | 19,646 | -745 | -776 | | +18,901 | ↓ | ↓ | +18,901 | | +20,422 | .264,00 | 1.069,70 | +21,845 |
| 200 | .014,147 | .001,112,7 | .001,172,5 | 22,910 | 74,830 | -15,627 | | +8,080 | ↓ | ↓ | +8,080 | | +38,537 | .309,76 | 1.095,95 | +42,235 |
| 165 | .012,418 | $.0^{3}843,83$ | $.0^{3}896,92$ | 25,035 | -23,945 | 25,452 | | +1,090 | 0 | 1.000,00 | +1,090 | | +50,487 | .333,71 | 1.111,36 | +56,109 |

*Figure F-2.* **Monticello Dam study–crown cantilever stresses parallel to faces of dam. –DS2-1(215)**



ARCH STRESSES

PROFILE ON AXIS LOOKING DOWNSTREAM (DEVELOPED)



CANTILEVER STRESSES

PROFILE ON AXIS LOOKING DOWNSTREAM (DEVELOPED)

**LOADING CONDITIONS:**

Reservoir water surface elevation 440.0

Top of dam elevation 456.0

Temperature of concrete at time of grouting contraction joints below elevation 300 assumed to be 45° Fahrenheit; above elevation 300, to be 55° Fahrenheit

Earthquake Assumptions:

Dam moves up and downstream horizontally in the direction of the line of centers

Increased water pressure acts equally on all cantilevers.

Period of vibration = 1.0 second.

Acceleration = 0.1 gravity.

Effects of vertical acceleration not considered

Effects of tailwater, uplift, and ice load not considered.

Modulus of elasticity of concrete = 2,500,000 pounds per square inch.

Modulus of elasticity of abutment rock = 1,300,000 pounds per square inch.

Poissons ratio of concrete = 0.2

Poissons ratio of abutment rock = 0.03.

Unit weight of concrete = 150 pounds per cu ft.

Coefficient of thermal expansion of concrete = 0.000,005,6 foot per foot per degree Fahrenheit

Arch stresses are acting in horizontal directions parallel to the edges of the arches

E = Stress at extrados of arch

I = Stress at intrados of arch.

$S_A$ = Maximum arch shear stress (+) indicates downstream shear.

Cantilever stresses are acting parallel to the edges of the cantilevers.

U = Stress at upstream edge of cantilever.

D = Stress at downstream edge of cantilever.

$S_c$ = Maximum cantilever shear stress. (+) indicates downstream shear.

All stresses are in pounds per square inch

+ = Compression  - = Tension

□ = Crown of arch

○ = Intermediate arch points

FINAL DESIGN A-11

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
SOLANO PROJECT - CALIFORNIA

MONTICELLO DAM
COMPLETE TRIAL LOAD ANALYSIS
ARCH AND CANTILEVER STRESSES
EFFECTS OF CONSTRUCTION PROGRAM INCLUDED

DRAWN C.W.L. SUBMITTED L. G. Puls
TRACED C.A.R. RECOMMENDED J. J. Hammond
CHECKED Max Fox APPROVED
CHIEF DESIGNING ENGINEER

DENVER, COLORADO, JUNE 28, 1954
413-D-110

*Figure F-3.* **Monticello Dam study–loading conditions and arch and cantilever stresses.**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS-TRIAL LOAD

ARCH STRESSES PARALLEL TO FACES OF DAM

ARCH 350 By. Date.

| ARCH POINT | $\frac{1}{A}$ | $\frac{c}{I}$ | $^*M_A$ | $^*H_A$ | $^*V_A$ | $\frac{^*H_A}{A}$ | $\frac{^*M_A c}{I}$ | STRESS | lb./ft.² | STRESS | lb./ft.² | STRESS | lb./in.² |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | NORMAL TO | U.S. RADIUS | PARALLEL | TO FACES | PARALLEL | TO FACES |
| | | | | | | | | $\sigma x_E$ | $\sigma x_D$ | $\sigma x'_E$ | $\sigma x'_D$ | $\sigma x'_E$ | $\sigma'_{xD}$ |
| ABUT.-L | .017,194 | .001,773,8 | +3095,600 | +1,440,400 | -348,370 | +24,766 | +5,491 | +30,257 | +19,275 | +30,257 | +30,872 | +210 | +214 |
| 4 | .026,266 | .004,139,4 | -5296,000 | +1,580,500 | | +41,513 | -21,922 | +19,591 | +63,435 | +19,591 | +65,614 | +136 | +456 |
| 2 | .026,266 | .004,139,4 | +3081,900 | +1,993,000 | | +52,348 | +12,757 | +65,105 | +39,591 | +65,105 | +39,591 | +452 | +275 |
| CR. | .026,266 | .004,139,4 | +5,395,700 | +2,217,400 | | +58,242 | +22,335 | +80,577 | +35,907 | +80,577 | +35,907 | +560 | +249 |
| 2 | .026,266 | .004,139,4 | +2,101,400 | +1,966,000 | | +51,639 | +8,699 | +60,338 | +42,940 | +60,338 | +42,940 | +419 | +298 |
| 4 | .026,266 | .004,139,4 | -4,945,400 | +1,519,000 | | +99,898 | -20,471 | +19,427 | +60,369 | +19,427 | +62,223 | +135 | +432 |
| ABUT.-R | .018,175 | .101,982,0 | +2,864,400 | +1,402,100 | -367,630 | +25,483 | +5,677 | +31,160 | +19,806 | +31,160 | +30,326 | +216 | +211 |

* MOMENTS, THRUSTS AND SHEARS AT ARCH POINTS DUE TO TOTAL RADIAL, TANGENTIAL AND TWIST LOADS ON ARCH

*Figure F-4.* Monticello Dam study—arch stresses parallel to faces of dam, arch at elevation 350. —DS2-1(216) concentrated radial load. These changes do not apply to computations for arch deflections, but are made to facilitate calculation of arch stresses at the abutment only.

Horizontal arch stresses parallel to the faces of the arches are tabulated in pounds per square inch on the upper profile of figure F-3.

F-5. *Tangential Cantilever Shear Stresses on Horizontal Planes.*—Horizontal cantilever shear stresses acting in tangential directions at the faces of the dam, $\tau_{xzE}$ and $\tau_{xzD}$, are calculated for cantilevers $E$, crown, and $F$ on figure F-5, by means of equations (263) and (278) in section 4-45. In the above equations, the value of $V_{TA}$ at any elevation in a cantilever is the total tangential shear at that elevation. It is equal to the summation of the respective products of the final tangential trial loads and the tangential shears due to unit loads, plus the tangential shear component of earthquake inertia. (See fig. B-6 of app. B.)

The value of $\cancel{M}$ at any elevation in a cantilever is the total twisting moment at that elevation. It is equal to the summation of the respective products of the final twist trial loads and the twisting moments due to unit loads, plus the secondary twist effects due to tangential trial loads.

F-6. *Shear Stresses on Abutment Planes.*—Shear stresses in the rock planes acting in directions parallel to the abutments at the faces of the dam, $\tau_{ryE}$ and $\tau_{ryD}$, are given on figure F-6. These are calculated by means of equations (268) and (281) in section 4-45.

Since there is no tailwater, $p_D$ is equal to zero. Stresses $\tau_{yz}$ and $\tau_{xy}$ are calculated by means of equations (265), (280), (264), and (279).

F-7. *Maximum Horizontal Shear Stresses on Rock Planes.*—The maximum horizontal shear stresses on the rock planes, $\tau_{ry}$, are given on figure F-7. These are calculated by means of equation (322). Values of $\tau_{ryE}$ and $\tau_{ryD}$ for use in that equation are taken from figure F-6, and the values of $V_r$ and $y$ are obtained as described in section 4-47.

Maximum shear stresses on rock planes are tabulated on figure F-12.

F-8. *Maximum Horizontal Arch Shear Stresses.*—Maximum horizontal arch shear stresses $\tau_{xy}$, acting in directions parallel to the abutments, are shown on figure F-8. These stresses are calculated by means of equation (309) in section 4-46.

Values for $V_A$ for use in the above equation have already been tabulated on figure F-4, and $\tau_{xyE}$ and $\tau_{xyD}$ have been calculated on figure F-6. The value of $y$ is calculated by means of equation (310), using $V_A$. The rules for determining the sign of the second derivative and the effect of the value of $y$ with respect to $T$, must be observed as previously indicated in section 4-46. The maximum horizontal arch shear stresses, parallel to the abutments, are tabulated on the upper profile of figure F-3.

F-9. *Maximum Horizontal Radial Cantilever Shear Stresses.*—Figure F-9 shows the calculations for maximum horizontal cantilever shear stresses acting in a radial direction, $\tau_{zy}$, obtained by using equation (307) in section 4-46. These stresses are at points along the foundation of the dam at the bases of the cantilevers and are tabulated on the lower profile of figure F-3.

The value of $V_{CA}$ for use in equation (307) is obtained by summating the respective products of unit radial shears in the cantilever and the radial cantilever trial loads, and adding the total radial shears due to initial loads on the cantilevers. The quantity $V_{℄}$ is equal to $V_{CA}$ multiplied by $\frac{R_{axis}}{r}$. Stresses $\tau_{zyE}$ and $\tau_{zyD}$ have already been calculated. The value of $y$ is calculated by means of equation (308) using $V_{℄}$. Summations of $V_{℄}$ are carried from the top downward.

F-10. *Principal Stresses Parallel to Upstream Face.*—Calculations for principal stresses parallel to the upstream face, $\sigma_{p1}$ and $\sigma_{p2}$, are shown on figure F-10. These are calculated by means of equation (289) in section 4-45.

Values of $\sigma'_{xE}$ and $\tau'_{xzE}$ for use in the above equation have already been calculated on figures F-4 and F-5. Values of $\sigma'_{zE}$ and $\tau'_{xzE}$ must be calculated on a separate sheet, using equations (285) and (288). Values of tan 2 $\xi_E$ are obtained by using equation (290). Two values of principal stress are obtained using the plus and minus sign in front of the square root.

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD

HORIZONTAL SHEAR STRESSES AT FACES OF CANTILEVERS

(ACTING IN TANGENTIAL DIRECTIONS) By Date

| ELEV. | $\frac{1}{A_{CA}}$ | $\frac{Ig}{I_{CA}}$ | $\frac{T-Ig}{I_{CA}}$ | * $V_{TA}$ | * M | * $\frac{V_{TA}}{A_{CA}}$ | * $\frac{M\,Ig}{I_{CA}}$ | * $\frac{M(T-Ig)}{I_{CA}}$ | SHEAR STRESSES lb./ft.$^2$ $\tau_{xz_E}=\tau_{zx_E}$ | $\tau_{xz_D}=\tau_{zx_D}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | CANTILEVER E | | | | | |
| 250 | +.017,129 | +.001,662,5 | +.001,733,7 | +214,460 | -9,294,700 | +3,673 | -15,452 | -16,114 | +11,779 | -19,787 |
| 200 | +.013,333 | $+.0^{3}981,64$ | +.001,038,2 | +183,370 | -14,690,900 | +2,445 | -14,421 | -15,252 | +11,976 | -17,697 |
| | | | | | CROWN CANTILEVER | | | | | |
| 165 | +.012,418 | $+.0^{3}843,83$ | $+.0^{3}896,92$ | +183,420 | -3,126,200 | +2,278 | -2,638 | -2,804 | +360 | -5,082 |
| | | | | | CANTILEVER F | | | | | |
| 250 | +.017,257 | +.001,688,6 | +.001,760,3 | +277,680 | -8,524,800 | +4,792 | -14,395 | -15,006 | +9,603 | -19,798 |
| 200 | +.013,089 | $+.0^{3}943,89$ | $+.0^{3}999,54$ | +234,630 | -14,111,200 | +3,071 | -13,319 | -14,105 | +10,248 | -17,176 |

* TANGENTIAL SHEAR AND TWISTING MOMENT IN CANTILEVER ELEMENTS DUE TO TOTAL TANGENTIAL AND TWIST LOADS

*Figure F-5.* Monticello Dam study–horizontal shear stresses at faces of cantilevers.–DS2-1(217)

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
SHEAR STRESSES ON ROCK PLANES AT FACES OF DAM (LEFT SIDE)
(PARALLEL TO ABUTMENT PLANE) By Date

| ELEV. | $P_E$ | $P_O$ | $\tan \phi_E$ | $\tan \phi_O$ | $\tau_{yz_E}$ | $\tau_{yz_O}$ | $\tau_{xy_E}$ | $\tau_{xy_O}$ | $\sin \psi$ | $\cos \psi$ | $\tau_{ry_E}$ | $\tau_{ry_O}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | 0 | 0 | | | | | | | .598,32 | .801,25 | | | |
| 400 | 2,999 | ↓ | .106,29 | .462,14 | -507 | +3,722 | + 9 | +9,380 | .740,22 | .672,37 | - 369 | +9,062 | |
| 350 | 6,438 | ↓ | .004,065 | .638,40 | - 21 | +10,905 | +39 | +19,667 | .848,05 | .529,92 | +3 | +19,670 | |
| 300 | 9,780 | ↓ | 0 | .624,80 | 0 | +15,947 | 0 | +29,480 | .859,41 | .511,29 | 0 | +28,778 | |
| 250 | 13,047 | ↓ | ↓ | .519,20 | ↓ | +20,905 | ↓ | +25,343 | .854,91 | .518,77 | ↓ | +31,019 | |
| 200 | 16,248 | ↓ | ↓ | .412,80 | ↓ | +20,766 | ↓ | +13,222 | .872,50 | .488,62 | ↓ | +24,579 | |
| 165 | 18,451 | ↓ | ↓ | .333,71 | ↓ | +16,848 | ↓ | + 1,696 | 1.000,00 | 0 | ↓ | +16,848 | |

*Figure F-6.* **Monticello Dam study—shear stresses on rock planes at faces of dam.—DS2-1(218)**

## CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
MAXIMUM HORIZONTAL SHEAR STRESSES ON ROCK PLANES (LEFT SIDE)
(PARALLEL TO ABUTMENT PLANE) By Date

| ELEV. | $\frac{6}{T}$ | V | $V_r$ | $\frac{6V_r}{T}$ | $\tau_{ry_E}$ | $\tau_{ry_O}$ | $\frac{6V_r}{T}+2\tau_{ry_E}+4\tau_{ry_O}$ | $\frac{6V_r}{T}+3\tau_{ry_E}+3\tau_{ry_O}$ | Y (for max. $\tau_{ry}$) | $\frac{Y}{T}$ | $\frac{Y^2}{T^2}$ | $\tau_{ry}$ (Max. lb./ft.²) | $\tau_{ry}$ (Max. lb./in.²) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | — | -56,147 | -44,988 | — | $-\frac{3}{2}V_r/T$ | — | — | — | — | — | — | +5,623 | +39 |
| 400 | .144,93 | -41,320 | -27,782 | -4,026 | -369 | +9,062 | +31,484 | +22,053 | 29.522 | .713,82 | .509,53 | +9,062 | +63 |
| 350 | .103,16 | -621,370 | -329,280 | -33,969 | +3 | +19,670 | +50,711 | +25,050 | Y > T (OUTSIDE OF SECTION) | | | +19,670 | +137 |
| 300 | .090,06 | -872,060 | -445,880 | -40,156 | 0 | +28,778 | +74,956 | +46,178 | 54.069 | .811,60 | .658,69 | +28,778 | +200 |
| 250 | .081,655 | -1,612,990 | -836,770 | -68,326 | ↓ | +31,019 | +55,750 | +24,731 | Y > T (OUTSIDE OF SECTION) | | | +31,019 | +215 |
| 200 | .075,567 | -2,238,480 | -1,093,770 | -82,653 | ↓ | +24,579 | +15,663 | -8,916 | - Y < 0 (OUTSIDE OF SECTION) | | | +24,579 | +171 |
| 165 | .069,897 | | 0 | 0 | ↓ | +16,848 | +67,392 | +50,544 | 57.227 | .666,67 | .444,44 | +16,848 | +117 |

*Figure F-7.* **Monticello Dam study–maximum horizontal shear stresses on rock planes.–DS2-1(219)**

MONTICELLO DAM. SECTION. STUDY NO. A - 11
RADIAL - SIDE CANTILEVER STRESS ANALYSIS - TRIAL LOAD
MAXIMUM HORIZONTAL ARCH SHEAR STRESSES (LEFT SIDE)
(PARALLEL TO ABUTMENT PLANE) By. Date

| ELEV. | T | $\frac{6}{T}$ | $V_A$ | $\frac{6V_A}{T}$ | $\tau_{xy_E}$ | $\tau_{xy_O}$ | $\frac{6V_a}{T} + 2\tau_{xy_E} + 4\tau_{xy_O}$ | $\frac{6V_a}{T} + 3\tau_{xy_E} + 3\tau_{xy_O}$ | Y (for Max. $\tau_{xy}$) | $\frac{Y}{T}$ | $\frac{Y^2}{T^2}$ | $\tau_{xy}$ (Max. in lb./ft.$^2$) | $\tau_{xy}$ (Max. in lb./in.$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | 12.00 | .500,00 | -56,147 | -28,074 | | $-\frac{3}{2} V_a / T$ | | | | | | +7,018 | +49 |
| 400 | 41.40 | .144,93 | +2,680 | +388 | +9 | +9,380 | +37,926 | +28,555 | — | | | +9,380 | +65 |
| 350 | 58.16 | .103,16 | -348,370 | -35,938 | +39 | +19,667 | +42,808 | +23,180 | — | | | +19,667 | +137 |
| 300 | 66.62 | .090,063 | -277,060 | -24,953 | 0 | +29,480 | +92,967 | +63,487 | — | | | +29,480 | +205 |
| 250 | 73.48 | .081,655 | -617,990 | -50,462 | 0 | +25,343 | +50,910 | +25,567 | — | | | +25,343 | +176 |
| 200 | 79.40 | .075,567 | -731,480 | -55,276 | 0 | +13,222 | -2,388 | -15,610 | +6.073,26 | .076,489 | .005,8596 | +13,313 | +92 |

*Figure F-8.* **Monticello Dam study–maximum horizontal arch shear stresses. –DS2-1(220)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11

RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD

MAXIMUM SHEAR STRESSES IN HORIZONTAL PLANES OF CANTILEVER ELEMENTS

(ACTING IN RADIAL DIRECTION) By Date

| ELEV. | $\frac{6}{T}$ | $V_{CA}$ | $V_{℄}$ | $\frac{6V_{℄}}{T}$ | $\tau_{zy_E}$ | $\tau_{zy_D}$ | $\frac{6V_{℄}}{T}$ + $2\tau_{zy_E}+4\tau_{zy_D}$ | $\frac{6V_{℄}}{T}$ + $3\tau_{zy_E}+3\tau_{zy_D}$ | Y (for Max. $\tau_{zy}$) | $\frac{Y}{T}$ | $\frac{Y^2}{T^2}$ | $\tau_{zy}$ (Max. in lb./ft.$^2$) | $\tau_{zy}$ (Max. in lb./in.$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 456 | | 0 | 0 | 0 | 0 | 0 | | | | | | 0 | 0 |
| 400 | .144,93 | -38,800 | -39,710 | -5,755 | -507 | +3,722 | +8,119 | +3,890 | — | | | +3,722 | +26 |
| 350 | .103,16 | -164,710 | -170,540 | -17,593 | -21 | +10,905 | +25,985 | +15,059 | — | | | +10,905 | +76 |
| 300 | .090,06 | -338,900 | -353,990 | -31,880 | 0 | +15,947 | +31,908 | +15,961 | — | | | +15,947 | +111 |
| 250 | .081,655 | -574,100 | -603,990 | -41,319 | 0 | +20,905 | +42,301 | +21,396 | — | | | +20,905 | +145 |
| 200 | .075,567 | -797,320 | -844,080 | -63,785 | 0 | +20,766 | +19,279 | -1,487 | Y<0 | — | — | +20,766 | +144 |
| 165 | .069,897 | -976,800 | -1,041,190 | -72,776 | 0 | +16,848 | -5,384 | -22,232 | 10.394 | .121,09 | .014,663 | +17,174 | +119 |

*Figure F-9.* **Monticello Dam study–maximum shear stresses in horizontal planes of cantilever elements. –DS2-1(221)**

# CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL -SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
PRINCIPAL STRESSES PARALLEL TO UPSTREAM FACE
(LEFT SIDE) By Date

| ELEV. | $\sigma'_{z_E}$ | $\sigma'_{x_E}$ | $\tau'_{xz_E}$ | $\frac{\sigma'_{z_E}+\sigma'_{x_E}}{2}$ | $\frac{\sigma'_{z_E}-\sigma'_{x_E}}{2}$ | $\tan 2\xi$ | $2\xi$ | $\xi$ | $\sqrt{\left(\frac{\sigma'_{z_E}-\sigma'_{x_E}}{2}\right)^2+(\tau'_{xz_E})^2}$ | PRINCIPAL STRESSES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | POUNDS PER FT.$^2$ | | POUNDS PER IN.$^2$ | |
| | | | | | | | | | | $\sigma_{P_1}$ | $\sigma_{P_2}$ | $\sigma_{P_1}$ | $\sigma_{P_2}$ |
| 456 | 0 | +15,216 | 0 | +7,608 | -7,608 | 0 | 0 | 0 | 7,608 | 0 | +15,216 | 0 | +106 |
| 400 | +7,823 | +6,393 | +80 | +7,108 | +715 | -.111,89 | -6° 24' | -3° 12' | 719 | +7,827 | +6,389 | +54 | +44 |
| 350 | +11,699 | +30,252 | +9,526 | +20,978 | -9,279 | +1.026,62 | +45° 46' | +22° 53' | 13,298 | +7,680 | +34,276 | +53 | +238 |
| 300 | +10,348 | +39,281 | +10,532 | +24,814 | -14,466 | +.728,05 | +36° 04' | +18° 02' | 17,894 | +6,920 | +42,708 | +48 | +297 |
| 250 | +4,483 | +29,510 | +12,518 | +16,996 | -12,514 | +1.000,32 | +45° 00' | +22° 30' | 17,700 | -704 | +34,696 | -5 | +241 |
| 200 | +1,608 | +22,515 | +11,976 | +12,062 | -10,454 | +1.145,59 | +48° 52' | +24° 26' | 15,897 | -3,835 | +27,959 | -27 | +194 |
| 165 | +1,090 | — | +360 | +545 | +545 | -.660,55 | -33° 26' | -16° 43' | 653 | +1,198 | -108 | +8 | -1 |

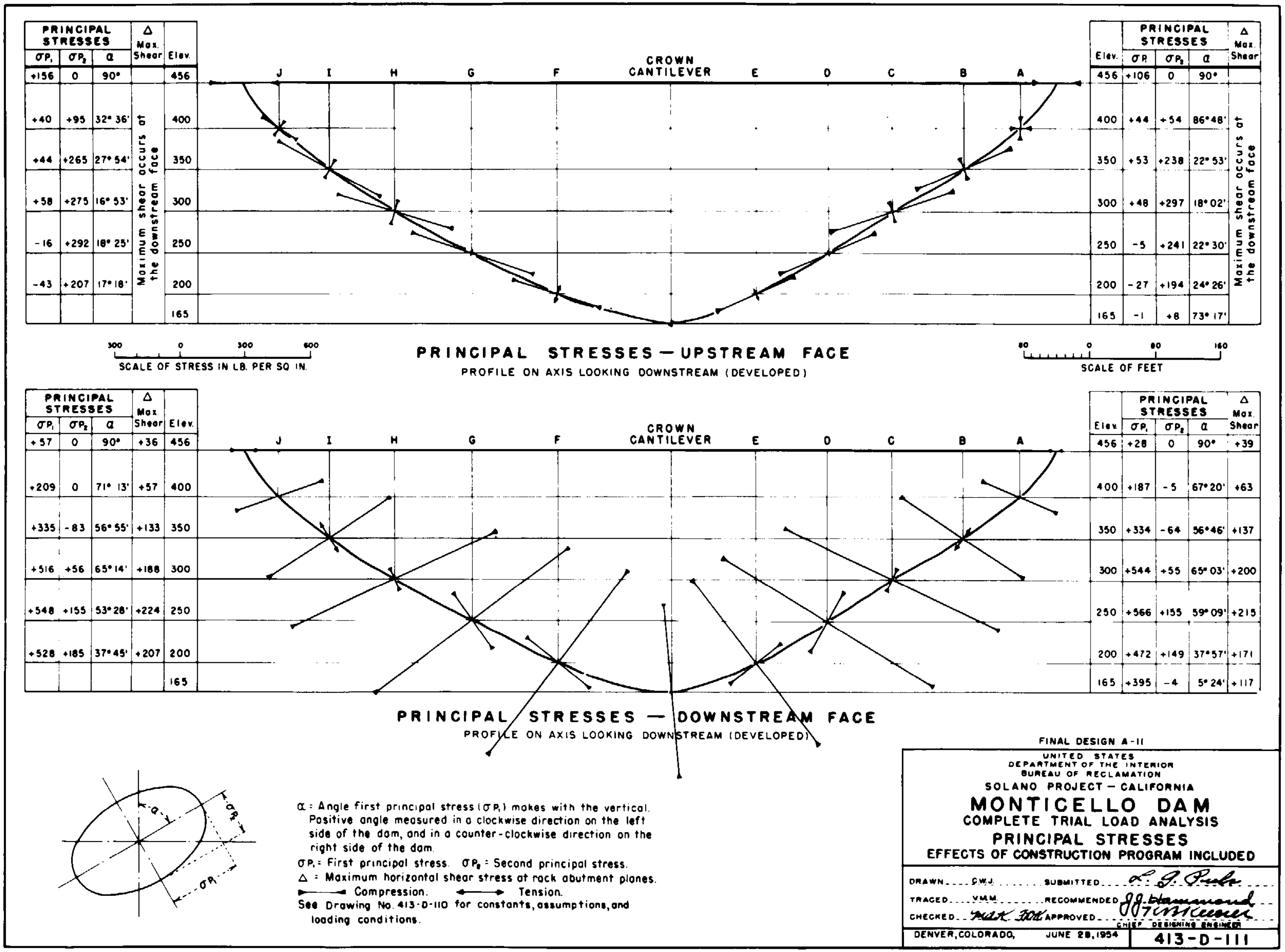
*Figure F-10.* **Monticello Dam study—principal stresses parallel to upstream face. —DS2-1(222)**

These are designated $\sigma_{p1}$ and $\sigma_{p2}$, respectively.

Final principal stresses at the upstream face of Monticello Dam are tabulated on the upper profile of figure F-11. The direction and magnitude of these stresses at each elevation are also shown along the upper profile. In the tabulations, the angle $\xi$ is the angle the first principal stress, $\sigma_{p1}$, makes with the vertical, a positive angle being counterclockwise on the right side of the dam. Compression and tension are as indicated by the directional arrowheads.

**F-11.** ***Principal Stresses Parallel to Downstream Face.***—These stresses are given on figure F-12. They are determined by using equation (274) in section 4-45, and the computations are similar to those for the upstream face. These stresses are also tabulated and drawn on the lower profile of figure F-11. All stresses are shown in pounds per square inch.

It can be seen that the maximum principal stress at the downstream face occurs at elevation 250 on the right side and amounts to 566 pounds per square inch. At the upstream face, the maximum principal stress occurs at elevation 300 at the base of cantilever *C*, and amounts to 297 pounds per square inch. These principal stresses act in directions parallel to the face of the dam at the point shown. The third principal stress is equal to reservoir pressure at the upstream face and to zero pressure at the downstream face.



PRINCIPAL STRESSES — UPSTREAM FACE (left abutment)

| PRINCIPAL STRESSES | | | Δ Max. Shear | Elev. |
|---|---|---|---|---|
| $\sigma P_1$ | $\sigma P_2$ | α | | |
| +156 | 0 | 90° | | 456 |
| +40 | +95 | 32° 36' | Maximum shear occurs at the downstream face | 400 |
| +44 | +265 | 27° 54' | | 350 |
| +58 | +275 | 16° 53' | | 300 |
| -16 | +292 | 18° 25' | | 250 |
| -43 | +207 | 17° 18' | | 200 |
| | | | | 165 |

PRINCIPAL STRESSES — UPSTREAM FACE (right abutment)

| Elev. | PRINCIPAL STRESSES | | | Δ Max. Shear |
|---|---|---|---|---|
| | $\sigma P_1$ | $\sigma P_2$ | α | |
| 456 | +106 | 0 | 90° | |
| 400 | +44 | +54 | 86° 48' | Maximum shear occurs at the downstream face |
| 350 | +53 | +238 | 22° 53' | |
| 300 | +48 | +297 | 18° 02' | |
| 250 | -5 | +241 | 22° 30' | |
| 200 | -27 | +194 | 24° 26' | |
| 165 | -1 | +8 | 73° 17' | |

PRINCIPAL STRESSES — DOWNSTREAM FACE (left abutment)

| PRINCIPAL STRESSES | | | Δ Max. Shear | Elev. |
|---|---|---|---|---|
| $\sigma P_1$ | $\sigma P_2$ | α | | |
| +57 | 0 | 90° | +36 | 456 |
| +209 | 0 | 71° 13' | +57 | 400 |
| +335 | -83 | 56° 55' | +133 | 350 |
| +516 | +56 | 65° 14' | +188 | 300 |
| +548 | +155 | 53° 28' | +224 | 250 |
| +528 | +185 | 37° 45' | +207 | 200 |
| | | | | 165 |

PRINCIPAL STRESSES — DOWNSTREAM FACE (right abutment)

| Elev. | PRINCIPAL STRESSES | | | Δ Max. Shear |
|---|---|---|---|---|
| | $\sigma P_1$ | $\sigma P_2$ | α | |
| 456 | +28 | 0 | 90° | +39 |
| 400 | +187 | -5 | 67° 20' | +63 |
| 350 | +334 | -64 | 56° 46' | +137 |
| 300 | +544 | +55 | 65° 03' | +200 |
| 250 | +566 | +155 | 59° 09' | +215 |
| 200 | +472 | +149 | 37° 57' | +171 |
| 165 | +395 | -4 | 5° 24' | +117 |

α = Angle first principal stress ($\sigma P_1$) makes with the vertical. Positive angle measured in a clockwise direction on the left side of the dam, and in a counter-clockwise direction on the right side of the dam.

$\sigma P_1$ = First principal stress. $\sigma P_2$ = Second principal stress.

Δ = Maximum horizontal shear stress at rock abutment planes.

►——◄ Compression. ◄——► Tension.

See Drawing No. 413-D-110 for constants, assumptions, and loading conditions.

*Figure F-11.* **Monticello Dam study—principal stresses at upstream and downstream faces.**

## CANTILEVER STRESS ANALYSIS

MONTICELLO DAM. SECTION. STUDY NO. A-11
RADIAL-SIDE CANTILEVER STRESS ANALYSIS- TRIAL LOAD
PRINCIPAL STRESSES PARALLEL TO DOWNSTREAM FACE
(LEFT SIDE) By Date

| ELEV. | $\sigma'_{zD}$ | $\sigma'_{xD}$ | $\tau'_{xzD}$ | $\frac{\sigma'_{zD}+\sigma'_{xD}}{2}$ | $\frac{\sigma'_{zD}-\sigma'_{xD}}{2}$ | tan 2ξ | 2ξ | ξ | $\sqrt{\left(\frac{\sigma'_{zD}-\sigma'_{xD}}{2}\right)^2+(\tau'_{xzD})^2}$ | PRINCIPAL STRESSES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | POUNDS PER FT.$^2$ | | POUNDS PER IN.$^2$ | |
| | | | | | | | | | | $\sigma_{P_1}$ | $\sigma_{P_2}$ | $\sigma_{P_1}$ | $\sigma_{P_2}$ |
| 456 | 0 | +3966 | 0 | +1,983 | -1,983 | 0 | 0 | 0 | 1,983 | 0 | +3,966 | 0 | +28 |
| 400 | +3,330 | +22,758 | -9,830 | +13,044 | -9,714 | -1.011,94 | -45° 20' | -22° 40' | 13,820 | -776 | +26,864 | -5 | +187 |
| 350 | +7,999 | +30,872 | -26,272 | +19,436 | -11,436 | -2.297,31 | -66° 28' | -33° 14' | 28,653 | -9,217 | +48,089 | -64 | +334 |
| 300 | +20,470 | +65,789 | -26,901 | +43,130 | -22,660 | -1,187,16 | -49° 54' | -24° 57' | 35,173 | +7,957 | +78,303 | +55 | +544 |
| 250 | +37,882 | +65,995 | -26,089 | +51,938 | -14,056 | -1.856,08 | -61° 42' | -30° 51' | 29,635 | +22,303 | +81,573 | +155 | +566 |
| 200 | +50,343 | +39,031 | -22,524 | +44,687 | +5,656 | +3.982,32 | +75° 54' | +37° 57' | 23,223 | +67,910 | +21,464 | +472 | +149 |
| 165 | +56,413 | 0 | -5,372 | +28,206 | +28,206 | +.190,46 | +10° 48' | +5° 24' | 28,713 | +56,919 | -507 | +395 | -4 |

*Figure F-12.* **Monticello Dam study—principal stresses parallel to downstream face. –DS2-1(223)**

# Arch Dam Stress Analysis System (ADSAS)

G-1. *General.*—The Arch Dam Stress Analysis System (ADSAS) used in the design or analysis of arch dams is explained in subchapter C of chapter IV. The example shown in this appendix is a nonsymmetrical analysis of Morrow Point Dam. The illustrations show representative input data and output prints from the ADSAS. A plan, profile, and section of the dam are shown on figure G-1.

G-2. ***Input.***—Samples of the writeup of data for input cards are shown on figures G-2 through G-7. Data required for input to ADSAS are: a geometrical description of the maximum section through the dam at the reference plane; the loci of centers in the reference plane described with respect to the axis of the dam; and angles measured at the extrados centers from the reference plane to the arch abutments. These data are included on the forms shown on figures G-5, G-6, and G-7.

Physical properties of concrete and foundation rock such as the modulus of elasticity, Poisson's ratio, thermal coefficient of expansion, and weight are included on the form shown on figure G-3. The loading conditions used in the analysis are described on figures G-3 and G-4. Temperature loads are included on figure G-7.

G-3. ***Output.***—Representative examples of computer printout from ADSAS for an analysis of Morrow Point Dam are shown on figures G-8 through G-26. These data include design criteria printout, geometrical statistics page, arch and cantilever displacements for the adjustments, distribution of loads, arch forces, cantilever forces, and stresses. Stress printouts shown include dead load stresses during construction, shear stresses, principal stresses along the abutments, arch stresses, and cantilever stresses. Additional output may be obtained if desired.

G-4. ***Summary.***—ADSAS offers a convenient and economical means of analyzing an arch dam. As shown in the example, the output prints are versatile and easily read.

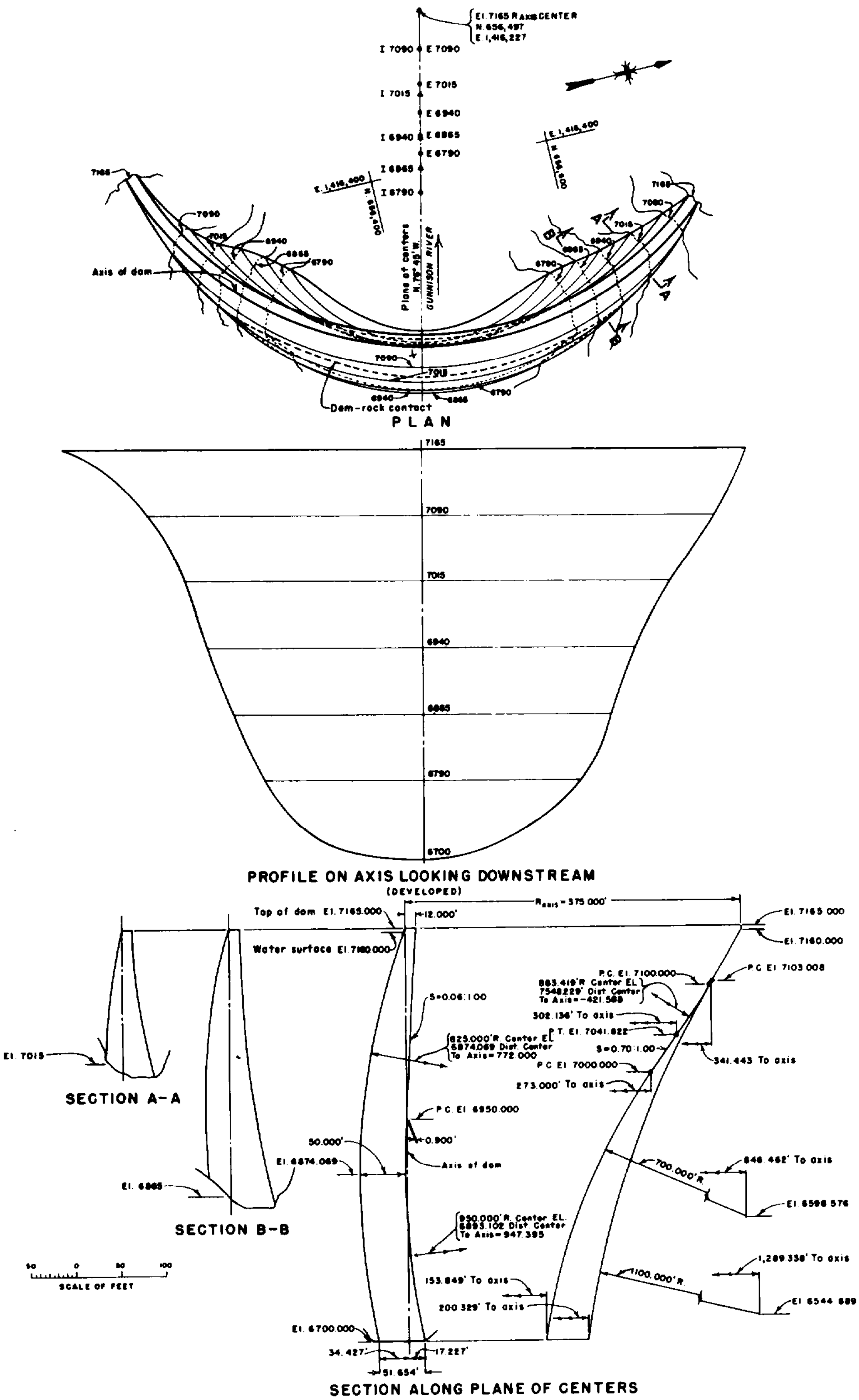


*Figure G-1.* **Morrow Point Dam as constructed–plan, profile, and sections.–288-D-3118**

# IDENTIFICATION AND CONTROLS

USE ------- COLORED CARDS
PRINT

DAM M. P. DAM
STUDY

JOB NUMBER
SUBMITTED BY

## CARD 1 IDENTIFICATION OF STUDY

| | | | | | | |
|---|---|---|---|---|---|---|
| Card Col | 1 | 5 | 11–40 | 41–44 | 51 52 | 77–80 |
| | 1 | 0 | M P DAM (6-0 NON-SYM TEST PRB) | | 0 | 9701 |
| | | | Name of Dam | Study Number | Case Number | Problem Number |

Col's. 53-76 Not Used

### NOTES

1. Name of dam should start in col. 11 and include the word "Dam".
2 The study number may include any combination of alphabetic and numeric characters necessary. (Do not inclue the "A" in A-1, left justify the data).
3. Case number must be only numeric (need not be punched if not necessary).
4. Problem number must be only numeric and must be punched.

## CARD 2 INPUT - OUTPUT CONTROL AND OPERATION CONTROL CARD

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Card Col. | 1 | 5 | 11 | | | | 15 | | | | 20 | 77–80 |
| | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 9701 |
| | | Control number | 1 | | | | 5 | | | | 10 | |

Card Col. 11–30 = Control number 1–20

Col's. 31-76 Not Used

Normal Set-up 0 1 1 0 1 1 1 0 1

### NOTES

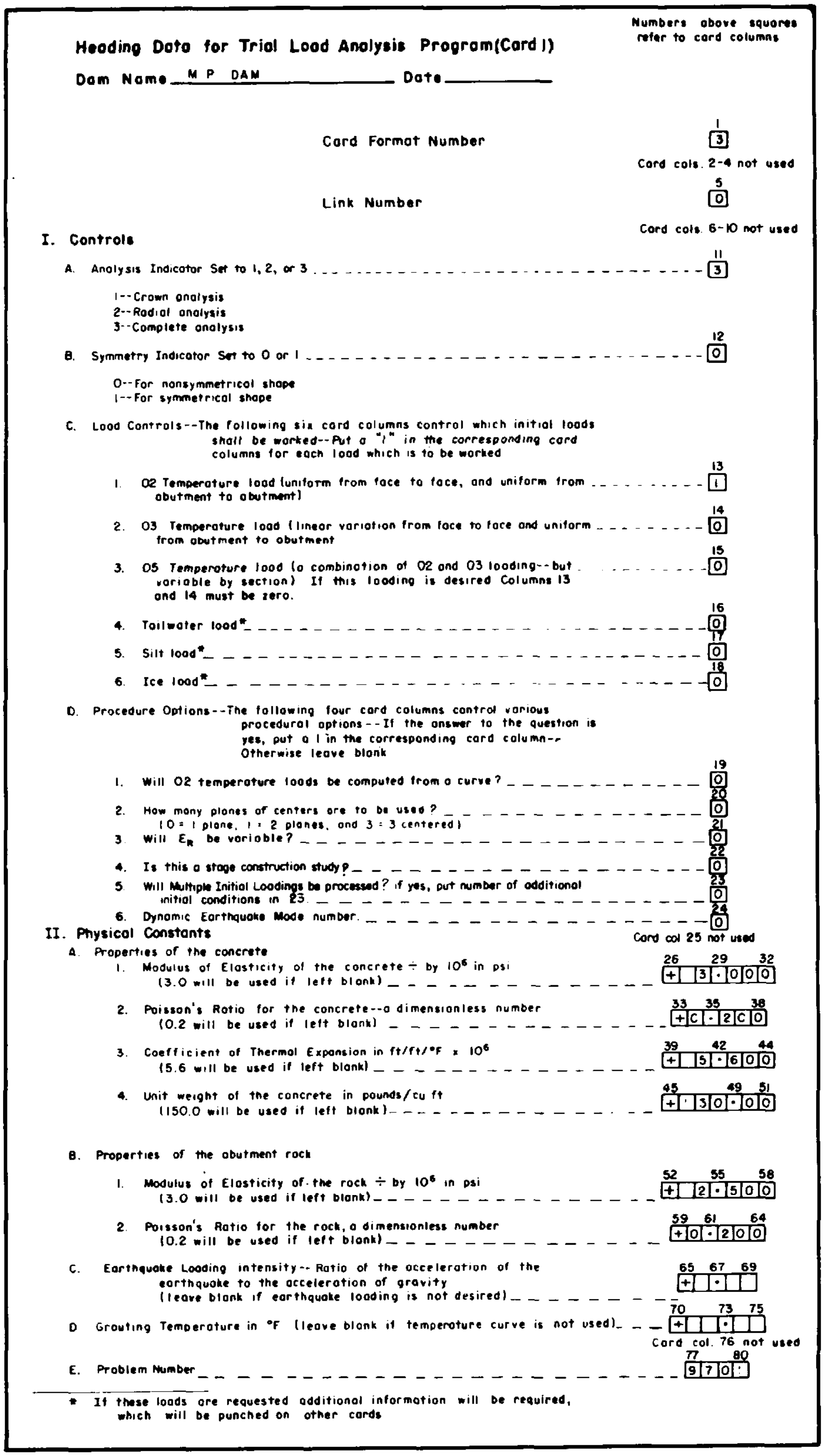
Controls are set up as follows:
1. Will an IOC card be read for DC0100? (0 = No, use the standard set-up; 1 = Yes, override the standard set-up)
2. Same as 1 for DC0200
3. Should an output tape be created? (0 = No, 1 = Yes)
4. Should the following message be printed: DATA ERROR PULL THIS RUN, if the run is aborted? (0 = No, 1 = Yes).
5. Inactive

6. Will Geometric properties be computed? (0 = No, 1 = Yes)
7. Will abutment constants be worked? (0 = No, 1 = Yes)
8. Shall the debugging print from the statistical routine be used? (0 = No, 1 = Yes)
9. Is this an intergrated job? (0 = No, 1 = Yes).

Sheet 1 of 6

ADSAS

*Figure G-2.* **Identification and input-output cards used with ASDAS.–288-D-3119**



Numbers above squares refer to card columns

**Heading Data for Trial Load Analysis Program(Card I)**

**Dam Name** M P DAM **Date**

Card Format Number — 1: 3

Card cols. 2-4 not used

Link Number — 5: 0

Card cols. 6-10 not used

**I. Controls**

A. Analysis Indicator Set to 1, 2, or 3 — 11: 3

1--Crown analysis
2--Radial analysis
3--Complete analysis

B. Symmetry Indicator Set to 0 or 1 — 12: 0

0--For nonsymmetrical shape
1--For symmetrical shape

C. Load Controls--The following six card columns control which initial loads shall be worked--Put a "1" in the corresponding card columns for each load which is to be worked

1. 02 Temperature load (uniform from face to face, and uniform from abutment to abutment) — 13: 1
2. 03 Temperature load (linear variation from face to face and uniform from abutment to abutment — 14: 0
3. 05 Temperature load (a combination of 02 and 03 loading--but variable by section) If this loading is desired Columns 13 and 14 must be zero. — 15: 0
4. Tailwater load* — 16: 0
5. Silt load* — 17: 0
6. Ice load* — 18: 0

D. Procedure Options--The following four card columns control various procedural options--If the answer to the question is yes, put a 1 in the corresponding card column-- Otherwise leave blank

1. Will 02 temperature loads be computed from a curve? — 19: 0
2. How many planes of centers are to be used? (0 = 1 plane, 1 = 2 planes, and 3 = 3 centered) — 20: 0
3. Will $E_R$ be variable? — 21: 0
4. Is this a stage construction study? — 22: 0
5. Will Multiple Initial Loadings be processed? if yes, put number of additional initial conditions in 23. — 23: 0
6. Dynamic Earthquake Mode number. — 24: 0

**II. Physical Constants**

Card col 25 not used

A. Properties of the concrete

1. Modulus of Elasticity of the concrete ÷ by $10^6$ in psi (3.0 will be used if left blank) — 26 29 32: + 3.000
2. Poisson's Ratio for the concrete--a dimensionless number (0.2 will be used if left blank) — 33 35 38: +C.2C0
3. Coefficient of Thermal Expansion in ft/ft/°F x $10^6$ (5.6 will be used if left blank) — 39 42 44: + 5.600
4. Unit weight of the concrete in pounds/cu ft (150.0 will be used if left blank) — 45 49 51: +'30.00

B. Properties of the abutment rock

1. Modulus of Elasticity of the rock ÷ by $10^6$ in psi (3.0 will be used if left blank) — 52 55 58: + 2.500
2. Poisson's Ratio for the rock, a dimensionless number (0.2 will be used if left blank) — 59 61 64: +0.200

C. Earthquake Loading intensity--Ratio of the acceleration of the earthquake to the acceleration of gravity (leave blank if earthquake loading is not desired) — 65 67 69: + .

D. Grouting Temperature in °F (leave blank if temperature curve is not used) — 70 73 75: + .

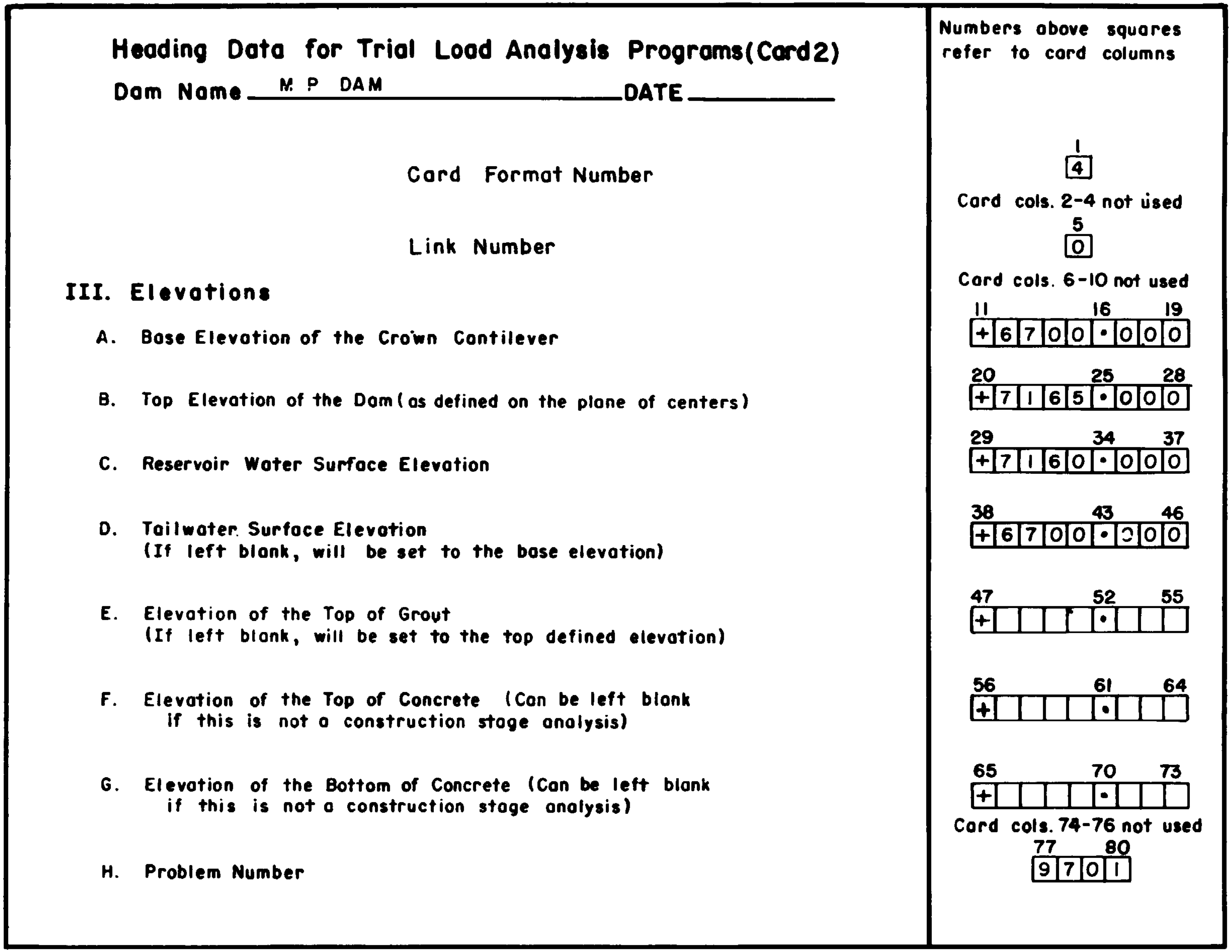
Card col. 76 not used

E. Problem Number — 77 80: 970:

---

* If these loads are requested additional information will be required, which will be punched on other cards

*Figure G-3.* Controls and physical property constants.–288-D-3120



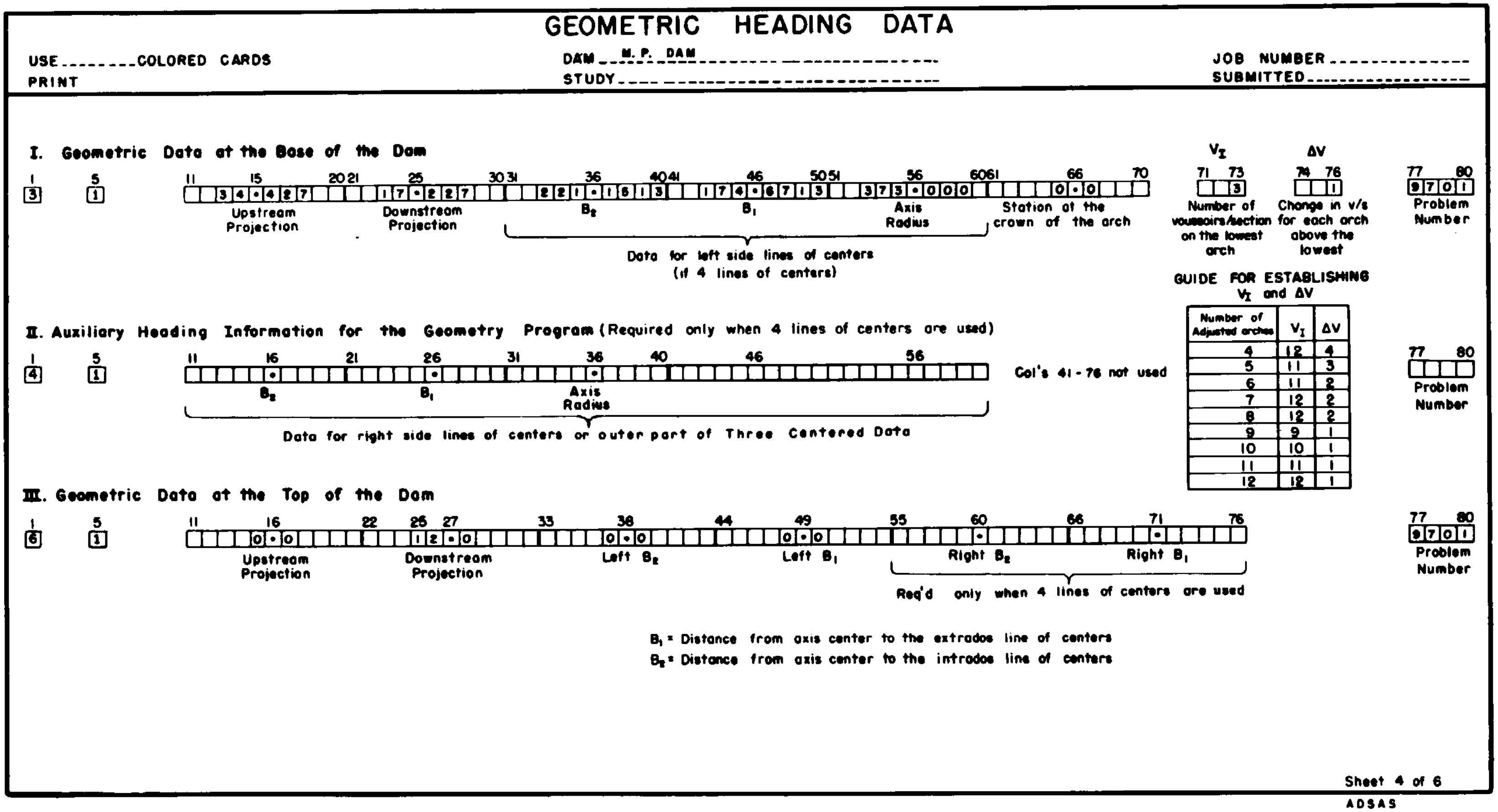
## Heading Data for Trial Load Analysis Programs(Card2)

Dam Name M P DAM DATE ____________

Numbers above squares refer to card columns

| | | |
|---|---|---|
| Card Format Number | 1 | 4 |
| | Card cols. 2-4 not used | |
| Link Number | 5 | 0 |
| | Card cols. 6-10 not used | |
| **III. Elevations** | | |
| A. Base Elevation of the Crown Cantilever | 11 16 19 | +6700.000 |
| B. Top Elevation of the Dam (as defined on the plane of centers) | 20 25 28 | +7165.000 |
| C. Reservoir Water Surface Elevation | 29 34 37 | +7160.000 |
| D. Tailwater Surface Elevation<br>(If left blank, will be set to the base elevation) | 38 43 46 | +6700.000 |
| E. Elevation of the Top of Grout<br>(If left blank, will be set to the top defined elevation) | 47 52 55 | + . |
| F. Elevation of the Top of Concrete (Can be left blank if this is not a construction stage analysis) | 56 61 64 | + . |
| G. Elevation of the Bottom of Concrete (Can be left blank if this is not a construction stage analysis) | 65 70 73 | + . |
| | Card cols. 74-76 not used | |
| H. Problem Number | 77 80 | 9701 |

*Figure G-4.* Elevation information.–288-D-3121



GEOMETRIC HEADING DATA

USE ________ COLORED CARDS
PRINT
DAM M. P. DAM
STUDY
JOB NUMBER
SUBMITTED

I. Geometric Data at the Base of the Dam

1: 3
5: 1
11–20: 34.427 (Upstream Projection)
21–30: 17.227 (Downstream Projection)
31–40: 221.1613 ($B_2$)
41–50: 174.6713 ($B_1$)
51–60: 373.000 (Axis Radius)
Data for left side lines of centers (if 4 lines of centers)
61–70: 0.0 (Station at the crown of the arch)
$V_I$ 71–73: 3 (Number of voussoirs/section on the lowest arch)
$\Delta V$ 74–76: 1 (Change in v/s for each arch above the lowest)
77–80: 9701 (Problem Number)

GUIDE FOR ESTABLISHING $V_I$ and $\Delta V$

| Number of Adjusted arches | $V_I$ | $\Delta V$ |
|---|---|---|
| 4 | 12 | 4 |
| 5 | 11 | 3 |
| 6 | 11 | 2 |
| 7 | 12 | 2 |
| 8 | 12 | 2 |
| 9 | 9 | 1 |
| 10 | 10 | 1 |
| 11 | 11 | 1 |
| 12 | 12 | 1 |

II. Auxiliary Heading Information for the Geometry Program (Required only when 4 lines of centers are used)

1: 4
5: 1
11–20: . ($B_2$)
21–30: . ($B_1$)
31–40: . (Axis Radius)
Data for right side lines of centers or outer part of Three Centered Data
Col's 41-76 not used
77–80: (Problem Number)

III. Geometric Data at the Top of the Dam

1: 6
5: 1
11–21: 0.0 (Upstream Projection)
22–32: 12.0 (Downstream Projection)
33–43: 0.0 (Left $B_2$)
44–54: 0.0 (Left $B_1$)
55–65: . (Right $B_2$)
66–76: . (Right $B_1$)
Req'd only when 4 lines of centers are used
77–80: 9701 (Problem Number)

$B_1$ = Distance from axis center to the extrados line of centers
$B_2$ = Distance from axis center to the introdos line of centers

Sheet 4 of 6
ADSAS

*Figure G-5.* Geometric data at top and base of dam.–288-D-3122

# PLANE OF CENTERS DESCRIPTION

Use_ _ _ _ _ _ _ stripe cards

DAM_ M. P. DAM _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

STUDY_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Job No._ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Return to_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
(ENGR.,ROOM NO, PHONE, ETC.)

| 5 | I | LN | SN | Maximum Elevation | Straight Line Segment | Circular Line Segment | | | Problem Number | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Slope (run to unit rise) | X Coord. of Center | Y Coord. of Center | Radius | | |
| 1 | 5 | 7 | 9 | 13–22 | 28–34 | 38–46 | 48–58 | 62–70 | 77–80 | COMMENTS |
| 5 | 1 | 1 | 1 | 7165.0 | 0.0 | 772.0 | 6874.06870 | 825.0 | 9701 | |
| | | 2 | 1 | 6950.0 | 0.0 | 947.39460 | 6893.10230 | 950.0 | 9701 | |
| | | 2 | 2 | 7165.0 | 0.06 | 0.0 | 0.0 | 0.0 | 9701 | |
| | | 3 | 1 | 7000.0 | 0.0 | 846.46210 | 6598.57650 | 700.0 | 9701 | |
| | | 3 | 2 | 7041.62220 | 0.70 | 0.0 | 0.0 | 0.0 | 9701 | |
| | | 3 | 3 | 7100.0 | 0.0 | -421.58820 | 7548.22880 | -883.41740 | 9701 | |
| | | 3 | 4 | 7160.0 | 0.58880 | 0.0 | 0.0 | 0.0 | 9701 | |
| | | 3 | 5 | 7165.0 | 0.0 | 0.0 | 0.0 | 0.0 | 9701 | |
| | | 4 | 1 | 7103.00800 | 0.0 | 1289.33770 | 6544.88930 | 1100.0 | 9701 | |
| | | 4 | 2 | 7160.0 | 0.58880 | 0.0 | 0.0 | 0.0 | 9701 | |
| | | 4 | 3 | 7165.0 | 0.0 | 0.0 | 0.0 | 0.0 | 9701 | |

## NOTES

LN—Line Number, 1 = upstream face; 2 = downstream face; 3 = intrados line of centers on the left; 4 = extrados line of center on the left, 5 = intrados line of centers on the right, 6 = extrados line of centers on the right. Data for line 5 & 6 need not be supplied if same as for line 3 & 4.

SN—Segment Number, starts with 1 for the lowest segment in the line.

Maximum Elevation— of each line segment.

Slope— filled in only for a straight line segment, in this case nothing (excepting zero and/or a decimal point) may be punched in columns 60 — 70.

X & Y coordinates of Center— filled in only for a circular line segment X coordinates have their origin at the axis of the dam, the positive direction is downstream. Y coordinates are sea level elevations; or any other elevations, postive direction is upward.

Radius— filled in only for a circular line segment. The sign given to the radius indicates which side of the circle to compute the coordinate on;
+ = upstream, — = downstream

| | BY: | DATE: | CHECKED: | Sheet 5 of 6 |
|---|---|---|---|---|

*Figure G-6.* Description of plane of centers.—288-D-3123

# ANGLES AND CONTROLS FOR ARCH DAM DESCRIPTION

Use________ stripe cards

DAM M. P. DAM

STUDY________

Job No.________

Return to________ (ENGR,ROOM NO, PHONE, ETC.)

| 7 | | I | P | SP | VA | W | A | ELEVATION | $\phi_L$ | $\phi_R$ | $\phi_{WL}$ | $\phi_{WR}$ | 02 LOAD | 03 LOAD | $E_R$ LEFT | $E_R$ RIGHT | | Problem Number | COMMENTS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 3 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11–21 | 22–28 | 29–35 | 36–42 | 43–49 | 50–55 | 56–61 | 62–66 | 67–71 | 72–76 | 77–80 | |
| 7 | | 1 | 1 | 0 | 0 | 0 | 0 | 6700.0 | 0.0 | 0.0 | . | . | . | . | . | . | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 6790.0 | 44.35 | 44.35 | | | -1.5 | | | | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 6865.0 | 48.125 | 48.125 | | | -1.5 | | | | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 6940.0 | 49.60 | 49.60 | | | -1.5 | | | | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 7015.0 | 50.25 | 50.25 | | | -2.0 | | | | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 7090.0 | 51.625 | 51.625 | | | -4.5 | | | | | 9701 | |
| | | | 0 | 0 | 0 | 0 | 0 | 7165.0 | 54.875 | 54.875 | | | -15.0 | | | | | 9701 | |

## P = PROCEDURE CONTROL

- 0 - An arch element will be placed at this elevation with cantilevers based at both abutments
- 1 - Base elevation of the Crown Cantilever (no arch at this elevation)
- 2 - Base elevation of a free cantilever on the left side (no arch at this elevation)
- 3 - No function at the present time
- 4 - Base elevation of a free cantilever) on the right side (no arch at this elevation )
- 5 - Base elevation of free cantilever on both sides (no arch at this elevation)
- 6* Angles provided at this elevation only to aid in the computation of the volume of concrete
- 7 - Base elevation of a thrust block-or other break point in the abutment (used to correctly define angle Ψ, see Supplementary Procedure Control)
- * - Not to be used for ungrouted elevations

## SP = SUPPLEMENTARY PROCEDURE CONTROL

- 0 - Not used
- 1 - Break point is on the left abutment, at this elevation
- 2 - Break point is on the right abutment, at this elevation
- 3 - Break point is on both abutments, at this elevation

## A = ADUSTMENT CONTROL

- 0 - Cantilever based at this elevation will be adjusted
- 1 - Cantilevr based at this elevation will not be adjusted

## VA = VERTICAL ABUTMENT CONTROL

- 0 - Sloping abutment on both sides
- 1 - Vertical abutment on left side
- 2 - Vertical abutment on right side
- 3 - Vertical abutment on both sides
  Note: if a thrust block starts at an arch elevation arch abutment is not vertical

## W = WEDGE CONTROL

- 0 - Radial abutments on both sides
- 1 - Non-radial abutment on the left side
- 2 - Non-radial abutment on the right side
- 3 - Non-radial abutment on both sides

## NOTES

1. Wedge Angles ($\phi_{WL}$ and $\phi_{WR}$) are measured about the extrados center from the reference plane the downstream contact of the abutment and wedge.
2. O2 LOAD = Uniform temperature load.
   O3 LOAD = Linear temperature load.
3. Uniform temperature loads are differences between grouting temperature and expected temperature in arches, in °F. The linear temperature load is the change in temperature from the U.S. face to the D.S. face in °F. (A rise, from U.S. to D.S. indicates a + load).
4. $E_{RL}$ and $E_{RR}$ are the moduli of elasticity of the rock in psi (÷ $10^6$) filled in only when a variable modulus is desired.

BY:

DATE:

CHECKED

Sheet 6 of 6

ADSAS

*Figure G-7.* Geometry of arches.–288-D-3124

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 0 - ARCH DAM STRESS ANALYSIS SYSTEM      PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                        DATE  02/16/72
ON  02/11/72                          STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)           PAGE 1

00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000

                                                  DESIGN CRITERIA

 1.  DATA WILL BE PREPARED FOR A COMPLETE ADJUSTMENT.

 2.  THE SHAPE WILL BE CONSIDERED NONSYMMETRIC.

 3.  TEMPERATURE LOADING IS UNIFORM ACROSS THE ARCH.

 4.  TEMPERATURE LOADING IS PRESENTED IN TABULAR FORM (SEE TABLE ON PAGE 1, LINK 1 OUTPUT).

 5.  THE GROUTING TEMPERATURE = 38.00 DEGREES FAHRENHEIT.

 6.  THE EFFECTS OF TAILWATER, UPLIFT, SILT AND ICE LOADING, IF ANY, ARE NOT INCLUDED.

 7.  NO EARTHQUAKE LOADING WILL BE APPLIED.

 8.  TWO LINES OF CENTERS WILL BE USED TO DESCRIBE THE DAM.

 9.  MODULUS OF ELASTICITY OF THE ROCK IS CONSTANT FROM TOP TO BOTTOM OF THE DAM.

10.  EFFECTS OF THE CONSTRUCTION PROGRAM ARE NOT INCLUDED IN THIS STUDY.

11.  PHYSICAL PROPERTIES OF THE CONCRETE

     A.  YOUNGS MODULUS OF ELASTICITY = 3.000 MILLION POUNDS/SQUARE INCH.
     B.  POISSONS RATIO = 0.200.
     C.  COEFFICIENT OF THERMAL EXPANSION = 0.000005600 FOOT/FOOT/DEGREE FAHRENHEIT.
     D.  UNIT WEIGHT OF CONCRETE = 150.00 POUNDS/CUBIC FOOT.

12.  PHYSICAL PROPERTIES OF THE ABUTMENT ROCK

     A.  YOUNGS MODULUS OF ELASTICITY = 2.500 MILLION POUNDS/SQUARE INCH.
     B.  POISSONS RATIO = 0.200.

13.  THE BASE ELEVATION OF THE DAM IS 6700.000.

14.  THE TOP ELEVATION OF THE DAM (AS DEFINED ON THE PLANE OF CENTERS) IS 7165.000.

15.  THE RESERVOIR WATER SURFACE ELEVATION IS 7160.000.

16.  THE TAILWATER SURFACE ELEVATION IS 6700.000.

17.  THE DAM IS GROUTED TO ELEVATION 7165.000.

18.  THE ELEVATION OF TOP OF CONCRETE IS 7165.000.

19.  THE ELEVATION OF BOTTOM OF CONCRETE IS 6700.000.
```

*Figure G-8.* **ADSAS output–design criteria.–288-D-3125**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 1 - GEOMETRY PROGRAM (DOUBLE CNTR)     PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                               STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)    PAGE 2

                   STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                                         GEOMETRICAL STATISTICS


1.   HEIGHT OF DAM = 465.0 FEET (141.7M.).                                            THE CANYON SHAPE IS A
                                                                                      NARROW U
2.   A.  CHORD LENGTH AT THE CREST = 613.4 FEET (187.0M.).
     B.  LENGTH ALONG AXIS AT THE CREST = 718.3 FEET (218.9M.).

3.   A.  SUBTENDED ANGLE AT THE CREST = 109.8 DEGREES.                                6 ARCHES AND 0 FREE CANTILEVERS
     B.  AXIS RADIUS = 375.0 FEET (114.3M.).                                          WILL BE ADJUSTED

4.                                         UPSTREAM          DOWNSTREAM              AT 36 POINTS
                         THICKNESS        PROJECTION         PROJECTION

     A.  AT CREST     12.00 (3.66M.)     -0.00 (-0.00M.)    112.00 (3.66M.)
     B.  AT BASE      51.65 (15.74M.)    34.43 (10.49M.)     17.23 (5.25M.)

5.   THE SLENDERNESS RATIO (TB/H) = 0.111.

6.   THE LENGTH (ALONG AXIS) TO HEIGHT RATIO = 1.54.

7.   THE VOLUME OF CONCRETE IN THE DAM = 351439.7 CUBIC YARDS (268851.3 CUBIC METERS).

8.   LENGTH TO WIDTH RATIO OF THE IDEALIZED ABUTMENT CONTACT AREA (B/A) = 18.9.

  COMPARISON OF THE ACTUAL PROPERTIES OF THIS DAM WITH THE PROPERTIES PREDICTED BY THE PRELIMINARY DESIGN CRITERIA

                                                            PERCENT        THESE PREDICTIONS ARE
                PROPERTY           ACTUAL       PREDICTED   DIFFERENCE    BASED ON CHORD LENGTHS OF

             BASE THICKNESS          51.65         47.93         7.2
             CREST THICKNESS         12.00         12.01        -0.1        613.4 FEET AT ELEV 7165
             VOLUME                351440        440489      -20.2          269.9 FEET AT ELEV 6770

                                       GEOMETRIC PROPERTIES OF THE ARCHES

               PHI     PHI      CROWN      LEFT ABUT    RIGHT ABUT                          AVERAGE                              RISE
ELEVATION     LEFT    RIGHT   THICKNESS    THICKNESS    THICKNESS     TAL/TO     TAR/TO    THICKNESS      RE       TAVE/RO      CHORD

  7165.0      54.9    54.9      12.00        12.00        12.00       1.0000     1.0000      12.00      375.00     0.0325      0.2596
  7090.0      51.6    51.6      31.74        31.82        31.82       1.0025     1.0025      31.77      358.14     0.0928      0.2418
  7015.0      50.3    50.3      43.87        48.10        48.10       1.0964     1.0964      45.55      335.73     0.1452      0.2345
  6940.0      49.6    49.6      48.91        59.47        59.47       1.2157     1.2157      53.33      313.11     0.1847      0.2310
  6865.0      48.1    48.1      50.76        65.31        65.31       1.2865     1.2865      57.27      289.90     0.2165      0.2233
  6790.0      44.3    44.3      51.71        66.99        66.99       1.2954     1.2954      59.35      265.70     0.2475      0.2038
```

*Figure G-9.* Geometrical statistics.–288-D-3126

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 3 - *    CANTILEVER PROGRAM     *      PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                    DATE  02/16/72
ON  02/11/72                              STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)      PAGE  22

                  STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                               * MINIMUM DEAD LOAD STRESSES (IN PSI) BY CANTILEVER

                    STRESS                                      STRESS
        CANTILEVER  BASED ON     ELEVATION      MINIMUM       BASED ON     ELEVATION      MINIMUM
         NUMBER     CONCRETE     OF MINIMUM    DOWNSTREAM     CONCRETE     OF MINIMUM     UPSTREAM
                    PLACED TO      STRESS        STRESS       PLACED TO      STRESS        STRESS

           20        7015.00      6700.00        -233          7165.00      6940.00          -40
           21        7165.00      6790.00         -87          7165.00      7090.00           19
           22        7090.00      6865.00          25          7165.00      7090.00           29
           23        7015.00      6940.00          46          7165.00      7090.00           38
           24        7090.00      7015.00          45          7165.00      7090.00           48

           25        7165.00      7090.00          48          7165.00      7090.00           63
           19        7165.00      6790.00         -87          7165.00      7090.00           19
           18        7090.00      6865.00          25          7165.00      7090.00           29
           17        7015.00      6940.00          46          7165.00      7090.00           38
           16        7090.00      7015.00          45          7165.00      7090.00           48
           15        7165.00      7090.00          48          7165.00      7090.00           63
      ______________________________

      * MINIMUM COMPRESSIVE OR MAXIMUM TENSILE STRESS.
```

*Figure G-10.* **Minimum dead load stresses.–288-D-3127**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 7 - *     ADJUSTMENT PROGRAM     *     PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                          STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)         PAGE 47

                 STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE


                        CANTILEVER NO 20                        TANGEN ADJUSTMENT NO 2
                          CANTILEVER                                 DEFLECTIONS
           ELEVATION         LOAD           CANTILEVER                 ARCH            DIFFERENCE

            7165.00         0.0000         -0.000000000        -0.000000000        -0.000000000
            7090.00        -0.0000         -0.000000000        -0.000000000         0.000000000
            7015.00         0.0000         -0.000000000        -0.000000000        -0.000000000
            6940.00        -0.0000          0.000000000         0.000000000        -0.000000000

            6865.00        -0.0000         -0.000000000         0.000000000        -0.000000000
            6790.00         0.0000          0.000000000         0.000000000        -0.000000000
            6700.00         0.0000          0.000000000

                        CANTILEVER NO 21                        TANGEN ADJUSTMENT NO 2
                          CANTILEVER                                 DEFLECTIONS
           ELEVATION         LOAD           CANTILEVER                 ARCH            DIFFERENCE

            7165.00         0.2564         -0.018598326        -0.018598326        -0.000000000
            7090.00         1.8490         -0.019114499        -0.019114499         0.000000000
            7015.00        -1.0695         -0.020111714        -0.020111714         0.000000000
            6940.00        -5.7322         -0.020021728        -0.020021728         0.000000000
            6865.00       -10.8966         -0.016767223        -0.016767223        -0.000000000
            6790.00       -16.0610         -0.008180797

                        CANTILEVER NO 22                        TANGEN ADJUSTMENT NO 2
                          CANTILEVER                                 DEFLECTIONS
           ELEVATION         LOAD           CANTILEVER                 ARCH            DIFFERENCE

            7165.00         0.7928         -0.017163554        -0.017163554        -0.000000000
            7090.00         1.3152         -0.017677943        -0.017677943         0.000000000
            7015.00        -1.0730         -0.018580673        -0.018580673         0.000000000
            6940.00        -4.9329         -0.018459367        -0.018459367         0.000000000
            6865.00       -12.7049         -0.015301208

                        CANTILEVER NO 23                        TANGEN ADJUSTMENT NO 2
                          CANTILEVER                                 DEFLECTIONS
           ELEVATION         LOAD           CANTILEVER                 ARCH            DIFFERENCE

            7165.00         1.0806         -0.014854478        -0.014854478        -0.000000000
            7090.00         0.9309         -0.015344986        -0.015344986         0.000000000
            7015.00        -0.7984         -0.016188499        -0.016188499         0.000000000
            6940.00        -4.2773         -0.016148989

                        CANTILEVER NO 24                        TANGEN ADJUSTMENT NO 2
                          CANTILEVER                                 DEFLECTIONS
           ELEVATION         LOAD           CANTILEVER                 ARCH            DIFFERENCE

            7165.00         0.8058         -0.011663094        -0.011663094        -0.000000000
            7090.00         0.9855         -0.012101511        -0.012101511        -0.000000000
            7015.00        -0.5178         -0.012911661
```

*Figure G-11.* **Tangential adjustment.—288-D-3128**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 7 - *     ADJUSTMENT PROGRAM     *      PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                        DATE  02/16/72
ON  02/11/72                              STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)       PAGE 54

                   STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                         CANTILEVER NO 20                       TWIST ADJUSTMENT NO 2
                            CANTILEVER                             DEFLECTIONS
                  ELEVATION    LOAD          CANTILEVER            ARCH            DIFFERENCE

                   7165.00       0.0000      0.000000000        0.000000000       -0.000000000
                   7090.00      -0.0000      0.000000000        0.000000000       -0.000000000
                   7015.00       0.0000     -0.000000000       -0.000000000       -0.000000000
                   6940.00       0.0000     -0.000000000       -0.000000000       -0.000000000

                   6865.00      -0.0000      0.000000000        0.000000000       -0.000000000
                   6790.00       0.0000     -0.000000000        0.000000000       -0.000000000
                   6700.00       0.0000     -0.000000000

                         CANTILEVER NO 21                       TWIST ADJUSTMENT NO 2
                            CANTILEVER                             DEFLECTIONS
                  ELEVATION    LOAD          CANTILEVER            ARCH            DIFFERENCE

                   7165.00      -4.7404     -0.000430900       -0.000430900        0.000000000
                   7090.00       9.6379     -0.000427667       -0.000427667       -0.000000000
                   7015.00      17.4814     -0.000418016       -0.000418016       -0.000000000
                   6940.00      89.6323     -0.000381081       -0.000381081       -0.000000000
                   6865.00     239.2127     -0.000285210       -0.000285210       -0.000000000
                   6790.00     388.7930     -0.000149164

                         CANTILEVER NO 22                       TWIST ADJUSTMENT NO 2
                            CANTILEVER                             DEFLECTIONS
                  ELEVATION    LOAD          CANTILEVER            ARCH            DIFFERENCE

                   7165.00      -3.5693     -0.000361525       -0.000361525        0.000000000
                   7090.00       6.9793     -0.000365464       -0.000365464       -0.000000000
                   7015.00      39.4483     -0.000351628       -0.000351628       -0.000000000
                   6940.00     101.1084     -0.000291913       -0.000291913        0.000000000
                   6865.00     278.9104     -0.000183295

                         CANTILEVER NO 23                       TWIST ADJUSTMENT NO 2
                            CANTILEVER                             DEFLECTIONS
                  ELEVATION    LOAD          CANTILEVER            ARCH            DIFFERENCE

                   7165.00      -7.0737     -0.000274718       -0.000274718        0.000000000
                   7090.00       7.9266     -0.000282477       -0.000282477       -0.000000000
                   7015.00      59.6905     -0.000261463       -0.000261463        0.000000000
                   6940.00     110.5211     -0.000181190

                         CANTILEVER NO 24                       TWIST ADJUSTMENT NO 2
                            CANTILEVER                             DEFLECTIONS
                  ELEVATION    LOAD          CANTILEVER            ARCH            DIFFERENCE

                   7165.00     -13.9158     -0.000173883       -0.000173883        0.000000000
                   7090.00      13.7201     -0.000178453       -0.000178453       -0.000000000
                   7015.00      80.3851     -0.000140118
```

*Figure G-12.* Twist adjustment.—288-D-3129

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 7 - *     ADJUSTMENT PROGRAM      *     PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                              STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)     PAGE 61

                    STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                       CANTILEVER NO 20                        RADIAL ADJUSTMENT NO 3
                          CANTILEVER                                DEFLECTIONS
            ELEVATION        LOAD          CANTILEVER              ARCH           DIFFERENCE

             7165.00       -0.2363        -0.092581339      -0.092581343        0.000000004
             7090.00       -4.1378        -0.102033856      -0.102033859        0.000000003
             7015.00       -9.6361        -0.107171450      -0.107171452        0.000000002
             6940.00      -14.0913        -0.105137200      -0.105137202        0.000000001

             6865.00      -17.9318        -0.091076481      -0.091076482        0.000000001
             6790.00      -16.9147        -0.058104120      -0.058104120        0.000000000
             6700.00      -15.6943        -0.010778398

                       CANTILEVER NO 21                        RADIAL ADJUSTMENT NO 3
                          CANTILEVER                                DEFLECTIONS
            ELEVATION        LOAD          CANTILEVER              ARCH           DIFFERENCE

             7165.00       -0.2807        -0.036615285      -0.036615281       -0.000000005
             7090.00       -4.0403        -0.038865660      -0.038865658       -0.000000002
             7015.00       -8.6828        -0.037630500      -0.037630499       -0.000000001
             6940.00      -14.0290        -0.031860744      -0.031860744       -0.000000001
             6865.00      -15.6044        -0.021632400      -0.021632400       -0.000000000
             6790.00      -16.9147        -0.009464039

                       CANTILEVER NO 22                        RADIAL ADJUSTMENT NO 3
                          CANTILEVER                                DEFLECTIONS
            ELEVATION        LOAD          CANTILEVER              ARCH           DIFFERENCE

             7165.00       -0.3170        -0.022689681      -0.022689677       -0.000000004
             7090.00       -3.8184        -0.023581336      -0.023581335       -0.000000001
             7015.00       -8.9389        -0.021662129      -0.021662129       -0.000000000
             6940.00      -12.2908        -0.016306709      -0.016306709       -0.000000000
             6865.00      -15.2181        -0.009089142

                       CANTILEVER NO 23                        RADIAL ADJUSTMENT NO 3
                          CANTILEVER                                DEFLECTIONS
            ELEVATION        LOAD          CANTILEVER              ARCH           DIFFERENCE

             7165.00       -0.1245        -0.013044613      -0.013044611       -0.000000002
             7090.00       -3.8854        -0.013037057      -0.013037057       -0.000000000
             7015.00       -8.3576        -0.010741883      -0.010741883        0.000000000
             6940.00      -10.8651        -0.006296842

                       CANTILEVER NO 24                        RADIAL ADJUSTMENT NO 3
                          CANTILEVER                                DEFLECTIONS
            ELEVATION        LOAD          CANTILEVER              ARCH           DIFFERENCE

             7165.00       -0.2877        -0.005722232      -0.005722230       -0.000000002
             7090.00       -3.4475        -0.005159835      -0.005159835       -0.000000000
             7015.00       -7.7633        -0.002767983
```

*Figure G-13.* Radial adjustment by cantilevers.—288-D-3130

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS    LINK 7 - *    ADJUSTMENT PROGRAM    *     PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                   DATE  02/16/72
ON  02/11/72                              STUDY A-16 OF M.P. DAM (6-O NON-SYM TEST PRB)  PAGE 63

                   STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                                         RADIAL ADJUSTMENT NO 3
                                        ARCH AT ELEVATION 7165.00
                                    ARCH LOADS, DEFLECTIONS, AND LENGTHS
            LEFT ABUTMENT TO CROWN                                        RIGHT ABUTMENT TO CROWN
  LENGTH      STATION      LOAD      DEFLECTION      LENGTH      STATION      LOAD      DEFLECTION

    0.00      1359.16      0.231      0.000200         0.00       640.84      0.231      0.000200
   52.67      1305.63      0.264     -0.000416        52.67       694.37      0.264     -0.000416
   90.70      1266.99      0.288     -0.005722        90.70       733.01      0.288     -0.005722
  118.84      1238.39      0.125     -0.013045       118.84       761.61      0.125     -0.013045

  145.93      1210.85      0.317     -0.022690       145.93       789.15      0.317     -0.022690
  178.11      1178.15      0.281     -0.036615       178.11       821.85      0.281     -0.036615
  353.41      1000.00      0.236     -0.092581       353.41      1000.00      0.236     -0.092581

                                         RADIAL ADJUSTMENT NO 3
                                        ARCH AT ELEVATION 7090.00
                                    ARCH LOADS, DEFLECTIONS, AND LENGTHS
            LEFT ABUTMENT TO CROWN                                        RIGHT ABUTMENT TO CROWN
  LENGTH      STATION      LOAD      DEFLECTION      LENGTH      STATION      LOAD      DEFLECTION

    0.00      1305.63      2.861     -0.000025         0.00       694.37      2.861     -0.000025
   38.04      1266.99      3.448     -0.005160        38.04       733.01      3.448     -0.005160
   66.43      1238.39      3.885     -0.013037        66.43       761.61      3.885     -0.013037
   93.94      1210.85      3.818     -0.023581        93.94       789.15      3.818     -0.023581
  126.80      1178.15      4.040     -0.038866       126.80       821.85      4.040     -0.038866
  308.40      1000.00      4.138     -0.102034       308.40      1000.00      4.138     -0.102034

                                         RADIAL ADJUSTMENT NO 3
                                        ARCH AT ELEVATION 7015.00
                                    ARCH LOADS, DEFLECTIONS, AND LENGTHS
            LEFT ABUTMENT TO CROWN                                        RIGHT ABUTMENT TO CROWN
  LENGTH      STATION      LOAD      DEFLECTION      LENGTH      STATION      LOAD      DEFLECTION

    0.00      1266.99      7.763     -0.002768         0.00       733.01      7.763     -0.002768
   28.01      1238.39      8.358     -0.010742        28.01       761.61      8.358     -0.010742
   55.41      1210.85      8.939     -0.021662        55.41       789.15      8.939     -0.021662
   88.44      1178.15      8.683     -0.037630        88.44       821.85      8.683     -0.037630
  275.20      1000.00      9.636     -0.107171       275.20      1000.00      9.636     -0.107171

                                         RADIAL ADJUSTMENT NO 3
                                        ARCH AT ELEVATION 6940.00
                                    ARCH LOADS, DEFLECTIONS, AND LENGTHS
            LEFT ABUTMENT TO CROWN                                        RIGHT ABUTMENT TO CROWN
  LENGTH      STATION      LOAD      DEFLECTION      LENGTH      STATION      LOAD      DEFLECTION

    0.00      1238.39     10.865     -0.006297         0.00       761.61     10.865     -0.006297
   26.81      1210.85     12.291     -0.016307        26.81       789.15     12.291     -0.016307
   59.50      1178.15     14.029     -0.031861        59.50       821.85     14.029     -0.031861
  249.88      1000.00     14.091     -0.105137       249.88      1000.00     14.091     -0.105137
```

*Figure G-14.* **Radial adjustment by arches.—288-D-3131**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM      *        PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                                 STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)  PAGE 66

                        STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                                 MOMENTS, THRUSTS, AND SHEARS AT ARCH POINTS DUE TOTAL LOADS
                               (PRINTED FROM LEFT ABUTMENT TO RIGHT ABUTMENT LOOKING UPSTREAM)


                                       ARCH NUMBER 1 AT ELEVATION 7165.00
                            STATION            MOMENT              THRUST             SHEAR

                            1359.16             10500              119402             32202
                            1305.63           -170769              188073             22444
                            1266.99           -203113              182122             13628
                            1238.39           -198216              156368              6563
                            1210.85           -155617              131259              2127

                            1178.15            -76459              114529              1308
                            1000.00            263386               92260                -0
                             821.85            -76459              114529              1308
                             789.15           -155617              131259              2127

                             761.61           -198216              156368              6563
                             733.01           -203113              182122             13628
                             694.37           -170769              188073             22444
                             640.84             10500              119402             32202

                                       ARCH NUMBER 2 AT ELEVATION 7090.00
                            STATION            MOMENT              THRUST             SHEAR

                            1305.63          -3328858             1634856             39085
                            1266.99          -4384640             1597060            -14997
                            1238.39          -3922832             1567583            -37325
                            1210.85          -2918002             1533127            -51133

                            1178.15          -1328410             1475670            -60604
                            1000.00           5086524             1286420                 0
                             821.85          -1328410             1475670            -60604
                             789.15          -2918002             1533127            -51133

                             761.61          -3922832             1567583            -37325
                             733.01          -4384640             1597060            -14997
                             694.37          -3328858             1634856             39085

                                       ARCH NUMBER 3 AT ELEVATION 7015.00
                            STATION            MOMENT              THRUST             SHEAR

                            1266.99         -19032393             2689442           -259432
                            1238.39         -14753131             2684660           -257671
                            1210.85         -10196602             2688506           -238669
                            1178.15          -4728840             2700320           -210892
                            1000.00          13315925             2728774                -0

                             821.85          -4728840             2700320           -210892
                             789.15         -10196602             2688506           -238669
                             761.61         -14753131             2684660           -257671
                             733.01         -19032392             2689442           -259432
```

*Figure G-15.* **Forces at arch points.–288-D-3132**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM     *         PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                               STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)    PAGE 68

                  STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE


                       * TOTAL HORIZONTAL SHEAR FORCE (VC) ON A HORIZONTAL SECTION
                                     ONE FOOT WIDE AT THE CENTERLINE

          DEFINITIONS
             A = SUMMATION OF RADIAL SHEARS IN CANTILEVER DUE UNIT RADIAL LOADS TIMES (RAXIS/RA) TAN SI
                 TIMES RADIAL CANTILEVER LOAD
             B = TOTAL RADIAL SHEAR DUE INITIAL LOAD

                    CANTILEVER                                        I.O OVER
                        NO              A               B              TAN SI            VC

                        15             63780          -83546          1.9180            -37912
                        16            233715         -274162          2.5634           -103680
                        17            516236         -587223          2.8287           -200804
                        18           1189909        -1315280          2.3354           -292791
                        19           5358039        -5995526          0.8311           -529795

                        20         633255791      -756078651          0.0100          -1228479
                        21           5358039        -5995526          0.8311           -529795
                        22           1189909        -1315280          2.3354           -292791
                        23            516236         -587223          2.8287           -200804
                        24            233715         -274162          2.5634           -103680
                        25             63780          -83546          1.9180            -37912
```

*Figure G-16.* **Total horizontal shear.–288-D-3133**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM     *        PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                     DATE  02/16/72
ON  02/11/72                             STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)     PAGE 69

              STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                    * TOTAL MOMENT OF CANTILEVER (M), ONE FOOT WIDE AT THE AXIS,
                          ABOUT CENTER OF GRAVITY OF HORIZONTAL SECTION


       DEFINITIONS
         A = SUMMATION OF RADIAL BENDING MOMENTS DUE TO RADIAL CANTILEVER LOADS
         B = RADIAL BENDING MOMENTS DUE TO TWIST EFFECTS
         C = RADIAL BENDING MOMENTS DUE TO CONCRETE AND INITIAL LOAD

                                      CANTILEVER NO. 15
          ELEVATION            A                B               C               MOMENT

           7165.00             0                0               0                    0
           7090.00       3254185           187048        -3252200               189033

                                      CANTILEVER NO. 16
          ELEVATION            A                B               C              'MOMENT

           7165.00             0                0               0                    0
           7090.00       3895268           130082        -3587064               438286
           7015.00      29151731          2227894       -30212453              1167172

                                      CANTILEVER NO. 17
          ELEVATION            A                B               C               MOMENT

           7165.00             0                0               0                    0
           7090.00       4039122           -13931        -3822844               202348
           7015.00      31791233          1509706       -31718610              1582330
           6940.00     107752544          6145417      -109028739              4869223

                                      CANTILEVER NO. 18
          ELEVATION            A                B               C               MOMENT

           7165.00             0                0               0                    0
           7090.00       4354770          -168609        -4036176               149985
           7015.00      33119869           874209       -33132078               862000
           6940.00     114530003          4765744      -113445353              5850394
           6865.00     270813553          9788106      -270290280             10311379

                                      CANTILEVER NO. 19
          ELEVATION            A                B               C               MOMENT

           7165.00             0                0               0                    0
           7090.00       4528318          -176248        -4267192                84878
           7015.00      34260497           573571       -34716853               117215
           6940.00     118437274          3690338      -118529486              3598125
           6865.00     286701620          8236813      -281510005             13428429
           6790.00     552088220         13819761      -548250185             17657795
```

*Figure G-17.* Cantilever moments.–288-D-3134

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM     *         PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                               STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)    PAGE 71

                  STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                * TOTAL HORIZONTAL TANGENTIAL SHEAR FORCE (VTA), ON HORIZONTAL CANTILEVER SECTION
                                          ONE FOOT WIDE AT THE AXIS

          DEFINITIONS
             A = SUMMATION OF TANGENTIAL SHEARS IN CANTILEVERS DUE UNIT LOADS TIMES TANGENTIAL CANTILEVER LOAD
             B = TANGENTIAL COMPONENT OF EARTHQUAKE INERTIA

                                         CANTILEVER NO 15
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00         -24432            0              -24432

                                         CANTILEVER NO 16
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00         -66403            0              -66403
                          7015.00         -83952            0              -83952

                                         CANTILEVER NO 17
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00         -74728            0              -74728
                          7015.00         -79674            0              -79674
                          6940.00         108590            0              108590

                                         CANTILEVER NO 18
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00         -78770            0              -78770
                          7015.00         -87576            0              -87576
                          6940.00         139601            0              139601
                          6865.00         795464            0              795464

                                         CANTILEVER NO 19
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00         -79474            0              -79474
                          7015.00        -108351            0             -108351
                          6940.00         154442            0              154442
                          6865.00         791789            0              791789
                          6790.00        1796185            0             1796185

                                         CANTILEVER NO 20
                         ELEVATION         A                B               VTA

                          7165.00              0            0                   0
                          7090.00              0            0                   0
                          7015.00              0            0                   0
                          6940.00             -0            0                  -0

                          6865.00              0            0                   0
                          6790.00              0            0                   0
                          6700.00             -0            0                  -0
```

*Figure G-18.* **Cantilever shears.–288-D-3135**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM     *         PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                                STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)   PAGE 73

                   STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                       * SHEAR STRESSES (TXZ) IN HORIZONTAL PLANES OF CANTILEVER ELEMENTS
                                       (ACTING IN TANGENTIAL DIRECTION)

          DEFINITIONS
             AC = AREA OF HORIZONTAL CANTILEVER SECTION
             LG = DISTANCE FROM UPSTREAM FACE TO CENTER OF GRAVITY OF HORIZONTAL CANTILEVER SECTION
             IC = MOMENT OF INERTIA OF HORIZONTAL CANTILEVER SECTION
             T = THICKNESS OF DAM IN RADIAL DIRECTION
             VTA = TOTAL HORIZONTAL TANGENTIAL SHEAR FORCE
             MTW = TOTAL TWISTING MOMENT ON HORIZONTAL CANTILEVER SECTION

                                                CANTILEVER NO 15
                                                                                  SHEAR STRESS
          ELEVATION      1/AC          LG/IC        (T-LG)/IC       VTA        MTW         POUNDS/SQ.FT.
                                                                                     UPSTREAM   DOWNSTREAM

           7165.00    0.08468792    0.04211796    0.04257701         0           0         -0          -0
           7090.00    0.03196599    0.00593850    0.00612544    -24432      -44281       1044         510

                                                CANTILEVER NO 16
                                                                                  SHEAR STRESS
          ELEVATION      1/AC          LG/IC        (T-LG)/IC       VTA        MTW         POUNDS/SQ.FT.
                                                                                     UPSTREAM   DOWNSTREAM

           7165.00    0.08468804    0.04211808    0.04257713         0           0         -0          -0
           7090.00    0.03169161    0.00589079    0.00607612    -66403      116578       1418        2813
           7015.00    0.02125387    0.00258795    0.00272461    -83952    -3729473      11436       -8377

                                                CANTILEVER NO 17
                                                                                  SHEAR STRESS
          ELEVATION      1/AC          LG/IC        (T-LG)/IC       VTA        MTW         POUNDS/SQ.FT.
                                                                                     UPSTREAM   DOWNSTREAM

           7165.00    0.08468812    0.04211816    0.04257721         0           0         -0          -0
           7090.00    0.03150684    0.00585864    0.00604289    -74728      198960       1189        3557
           7015.00    0.02126692    0.00263267    0.00276923    -79674    -2420413       8067       -5008
           6940.00    0.01735027    0.00169560    0.00181851    108590    -8366048      12301      -17098

                                                CANTILEVER NO 18
                                                                                  SHEAR STRESS
          ELEVATION      1/AC          LG/IC        (T-LG)/IC       VTA        MTW         POUNDS/SQ.FT.
                                                                                     UPSTREAM   DOWNSTREAM

           7165.00    0.08468819    0.04211823    0.04257728         0           0         -0          -0
           7090.00    0.03134508    0.00583050    0.00601380    -78770      101466       1877        3079
           7015.00    0.02128470    0.00267485    0.00281136    -87576    -1450801       5745       -2215
           6940.00    0.01752359    0.00177417    0.00189784    139601    -6300365       8732      -14403
           6865.00    0.01598168    0.00141379    0.00153872    795464   -14158630       7305      -34499
```

*Figure G-19.* Cantilever shear stresses.–288-D-3136

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM      *          PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                        DATE  02/16/72
ON  02/11/72                                  STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)   PAGE 76

                   STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                                         ARCH STRESS PARALLEL TO FACES
                    STRESSES PRINTED FROM LEFT ABUTMENT TO RIGHT ABUTMENT LOOKING UPSTREAM

                                       ARCH NUMBER 1 AT ELEVATION 7165.00
                                         ARCH FORCES                           TAN                ARCH STRESSES (PSF)
STATION      1/AREA       C/I       MOMENT      THRUST      SHEAR       BETA        PHID        TXZD   EXTRADOS    INTRADOS

1359.16    0.083333    0.041666       10500     119402      32202    0.000027    0.000000       -0      10388        9513
1305.63    0.083333    0.041666     -170769     188073      22444    0.000027    0.000000       -0       8557       22788
1266.99    0.083333    0.041666     -203113     182122      13628    0.000024    0.000000       -0       6714       23640
1238.39    0.083333    0.041666     -198216     156368       6563    0.000022    0.000000       -0       4772       21290
1210.85    0.083333    0.041667     -155617     131259       2127    0.000020    0.000000       -0       4454       17422

1178.15    0.083333    0.041667      -76459     114529       1308    0.000017    0.000000       -0       6358       12730
1000.00    0.083333    0.041667      263386      92260         -0    0.000000    0.000000       -0      18663       -3286
 821.85    0.083333    0.041667      -76459     114529       1308    0.000017    0.000000       -0       6358       12730
 789.15    0.083333    0.041667     -155617     131259       2127    0.000020    0.000000       -0       4454       17422
 761.61    0.083333    0.041666     -198216     156368       6563    0.000022    0.000000       -0       4772       21290

 733.01    0.083333    0.041666     -203113     182122      13628    0.000024    0.000000       -0       6714       23640
 694.37    0.083333    0.041666     -170769     188073      22444    0.000027    0.000000       -0       8557       22788
 640.84    0.083333    0.041666       10500     119402      32202    0.000027    0.000000       -0      10388        9513

                                       ARCH NUMBER 2 AT ELEVATION 7090.00
                                         ARCH FORCES                           TAN                ARCH STRESSES (PSF)
STATION      1/AREA       C/I       MOMENT      THRUST      SHEAR       BETA        PHID        TXZD   EXTRADOS    INTRADOS

1305.63    0.031427    0.005926    -3328858    1634856      39085    0.028728    0.176158      510      31652       71106
1266.99    0.031444    0.005933    -4384640    1597060     -14997    0.026028    0.124070     2813      24207       76230
1238.39    0.031456    0.005937    -3922832    1567583     -37325    0.023801    0.089079     3557      26021       72599
1210.85    0.031466    0.005941    -2918002    1533127     -51133    0.021487    0.058228     3079      30907       65577
1178.15    0.031477    0.005945    -1328410    1475670     -60604    0.018543    0.025616     1977      38553       54347

1000.00    0.031506    0.005956     5086524    1286420          0    0.000000   -0.060000        0      70823       10236
 821.85    0.031477    0.005945    -1328410    1475670     -60604    0.018543    0.025616     1977      38553       54347
 789.15    0.031466    0.005941    -2918002    1533127     -51133    0.021487    0.058228     3079      30907       65577
 761.61    0.031456    0.005937    -3922832    1567583     -37325    0.023801    0.089079     3557      26021       72599
 733.01    0.031444    0.005933    -4384640    1597060     -14997    0.026028    0.124070     2813      24207       76230
 694.37    0.031427    0.005926    -3328858    1634856      39085    0.028728    0.176158      510      31652       71106

                                       ARCH NUMBER 3 AT ELEVATION 7015.00
                                         ARCH FORCES                           TAN                ARCH STRESSES (PSF)
STATION      1/AREA       C/I       MOMENT      THRUST      SHEAR       BETA        PHID        TXZD   EXTRADOS    INTRADOS

1266.99    0.020788    0.002593   -19032393    2689442    -259432    1.783594    0.213336    -8377       6559      105473
1238.39    0.021126    0.002678   -14753131    2684660    -257671    1.644222    0.157436    -5008      17209       96350
1210.85    0.021440    0.002758   -10196602    2688506    -238669    1.495230    0.114376    -2215      29519       85834
1178.15    0.021787    0.002848    -4728840    2700320    -210892    1.300570    0.067821      860      45364       72335
1000.00    0.022793    0.003117    13315925    2728774         -0    0.000000   -0.060000       -0     103703       20690

 821.85    0.021787    0.002848    -4728840    2700320    -210892    1.300570    0.067821      860      45364       72335
 789.15    0.021440    0.002758   -10196602    2688506    -238669    1.495230    0.114376    -2215      29519       85834
 761.61    0.021126    0.002678   -14753131    2684660    -257671    1.644222    0.157436    -5008      17209       96350
 733.01    0.020788    0.002593   -19032392    2689442    -259432    1.783594    0.213336    -8377       6559      105473
```

*Figure G-20.* Arch stresses.—288-D-3137

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM      *          PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                        DATE  02/16/72
ON  02/11/72                                   STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)  PAGE 78

                          STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE

                                            * CANTILEVER STRESSES PARALLEL TO FACE

           DEFINITIONS

              PHIU = SLOPE UPSTREAM EDGE OF CANTILEVER MAKES WITH VERTICAL
              PHID = SLOPE DOWNSTREAM EDGE OF CANTILEVER MAKES WITH VERTICAL
              BETAD = ANGLE A LINE TANGENT TO THE INTRADOS MAKES WITH NORMAL TO RADIAL ARCH SECTION
              PU = NORMAL PRESSURE AT UPSTREAM FACE OF DAM
              PD = NORMAL PRESSURE AT DOWNSTREAM FACE OF DAM
              CW = TOTAL WEIGHT OF CANTILEVER, INCLUDING VERTICAL WATER LOAD
              CM = TOTAL MOMENT OF CANTILEVER

                                                   CANTILEVER NO 15
                                                                                                CANTILEVER STRESSES
ELEVATION    1/AC     LG/IC    (T-LG)/IC    BETAD     PHIU      PHID       PU    PD     CW         CM          POUNDS/SQ.FT.
                                                                                                  UPSTREAM  DOWNSTREAM

 7165.00   0.08469  0.04212    0.04258    0.0000   0.00000  0.00000       0    0          0          0         0          0
 7090.00   0.03197  0.00594    0.00613    0.0287   0.03742  0.17616    4375    0     268473     189033      9712       7654

                                                   CANTILEVER NO 16
                                                                                                CANTILEVER STRESSES
ELEVATION    1/AC     LG/IC    (T-LG)/IC    BETAD     PHIU      PHID       PU    PD     CW         CM          POUNDS/SQ.FT.
                                                                                                  UPSTREAM  DOWNSTREAM

 7165.00   0.08469  0.04212    0.04258    0.0000   0.00000  0.00000       0    0          0          0         0          0
 7090.00   0.03169  0.00589    0.00608    0.0260   0.10438  0.12407    4375    0     277007     438286     11437       6210
 7015.00   0.02125  0.00259    0.00272    1.7836   0.00185  0.21334    9063    0     748000    1167172     18919      13412

                                                   CANTILEVER NO 17
                                                                                                CANTILEVER STRESSES
ELEVATION    1/AC     LG/IC    (T-LG)/IC    BETAD     PHIU      PHID       PU    PD     CW         CM          POUNDS/SQ.FT.
                                                                                                  UPSTREAM  DOWNSTREAM

 7165.00   0.08469  0.04212    0.04258    0.0000   0.00000  0.00000       0    0          0          0         0          0
 7090.00   0.03151  0.00586    0.00604    0.0238   0.13621  0.08908    4375    0     282835     202348     10203       7749
 7015.00   0.02127  0.00263    0.00277    1.6442   0.03458  0.15744    9063    0     772805    1582330     20615      12399
 6940.00   0.01735  0.00170    0.00182    5.0502  -0.05611  0.17746   13750    0    1357940    4869223     31874      15733

                                                   CANTILEVER NO 18
                                                                                                CANTILEVER STRESSES
ELEVATION    1/AC     LG/IC    (T-LG)/IC    BETAD     PHIU      PHID       PU    PD     CW         CM          POUNDS/SQ.FT.
                                                                                                  UPSTREAM  DOWNSTREAM

 7165.00   0.08469  0.04212    0.04258    0.0000   0.00000  0.00000       0    0          0          0         0          0
 7090.00   0.03135  0.00583    0.00601    0.0215   0.16425  0.05823    4375    0     287991     149985     10051       8153
 7015.00   0.02128  0.00267    0.00281    1.4952   0.06247  0.11438    9063    0     795121     862000     19269      14704
 6940.00   0.01752  0.00177    0.00190    4.6288  -0.02880  0.14491   13750    0    1401748    5850394     34961      14095
 6865.00   0.01598  0.00141    0.00154    8.0361  -0.11274  0.21020   18438    0    1984238   10311379     46644      18678
```

*Figure G-21.* Cantilever stresses.—288-D-3138

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM      *      PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                    DATE  02/16/72
ON  02/11/72                            STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)     PAGE 83

                STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE


                           STRESSES (PSI) FOR ARCH NO 3 AT ELEVATION 7015.00
                 STRESSES PRINTED FROM LEFT ABUTMENT TO RIGHT ABUTMENT LOOKING UPSTREAM

                                  NORMAL TO RE                  PARALLEL TO FACES          TXYM
                 STATION       EXTRADOS     INTRADOS         EXTRADOS     INTRADOS         MAX.

                 1266.99           46          731               46          732             50
                 1238.39          120          668              120          669
                 1210.85          205          596              205          596
                 1178.15          315          502              315          502
                 1000.00          720          144              720          144

                  821.85          315          502              315          502
                  789.15          205          596              205          596
                  761.61          120          668              120          669
                  733.01           46          731               46          732             50

                           STRESSES (PSI) FOR ARCH NO 4 AT ELEVATION 6940.00
                 STRESSES PRINTED FROM LEFT ABUTMENT TO RIGHT ABUTMENT LOOKING UPSTREAM

                                  NORMAL TO RE                  PARALLEL TO FACES          TXYM
                 STATION       EXTRADOS     INTRADOS         EXTRADOS     INTRADOS         MAX.

                 1238.39          -54          794              -54          803            107
                 1210.85           85          695               85          702
                 1178.15          265          574              265          578
                 1000.00          820          235              820          235

                  821.85          265          574              265          578
                  789.15           85          695               85          702
                  761.61          -54          794              -54          803            107

                           STRESSES (PSI) FOR ARCH NO 5 AT ELEVATION 6865.00
                 STRESSES PRINTED FROM LEFT ABUTMENT TO RIGHT ABUTMENT LOOKING UPSTREAM

                                  NORMAL TO RE                  PARALLEL TO FACES          TXYM
                 STATION       EXTRADOS     INTRADOS         EXTRADOS     INTRADOS         MAX.

                 1210.85          -83          702              -83          730            156
                 1178.15          115          599              115          617
                 1000.00          854          212              854          212
                  821.85          115          599              115          617
                  789.15          -83          702              -83          730            156
```

*Figure G-22.* **Arch stresses in pounds per square inch.–288-D-3139**

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAMS     LINK 8 - *     STRESS PROGRAM     *         PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                      DATE  02/16/72
ON  02/11/72                             STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)      PAGE  86

                STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE


                                STRESSES (PSI) FOR CANTILEVER NO 19

                                  VERTICAL STRESSES          PARALLEL STRESSES
                 ELEVATION      US FACE      DS FACE      US FACE      DS FACE

                  7165.00           0            0            0            0
                  7090.00          67           60           68           60
                  7015.00         123          119          124          119

                  6940.00         225          129          225          131
                  6865.00         375           78          376           90
                  6790.00         445           96          453          132
                 MAXIMUM ROCK PLANE SHEAR STRESS = 183


                                STRESSES (PSI) FOR CANTILEVER NO 20

                                  VERTICAL STRESSES          PARALLEL STRESSES
                 ELEVATION      US FACE      DS FACE      US FACE      DS FACE

                  7165.00           0            0            0            0
                  7090.00          55           77           56           77
                  7015.00          75          191           75          192
                  6940.00         143          270          143          270

                  6865.00         312          239          312          239
                  6790.00         522          143          526          144
                  6700.00         250          563          252          587
                 MAXIMUM ROCK PLANE SHEAR STRESS = 117


                                STRESSES (PSI) FOR CANTILEVER NO 21

                                  VERTICAL STRESSES          PARALLEL STRESSES
                 ELEVATION      US FACE      DS FACE      US FACE      DS FACE

                  7165.00           0            0            0            0
                  7090.00          67           60           68           60
                  7015.00         123          119          124          119

                  6940.00         225          129          225          131
                  6865.00         375           78          376           90
                  6790.00         445           96          453          132
                 MAXIMUM ROCK PLANE SHEAR STRESS = 183
```

*Figure G-23.* Cantilever stresses in pounds per square inch.–288-D-3140

```
OUTPUT OF ARCH DAM STRESS ANALYSIS PROGRAM     LINK 8 - *     STRESS PROGRAM     *      PROBLEM NUMBER 9701
VERSION 44.0 COLLECTED                                                                DATE  02/16/72
ON  02/11/72                           STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)  PAGE  91

                STANDARD TEST PROBLEM FOR THE ARCH DAM STRESS ANALYSIS SYSTEM OVERLAY TAPE


                     PRINCIPAL STRESSES (PSI) PARALLEL TO THE FACES AT THE ABUTMENTS

                                   LEFT ABUTMENT (LOOKING UPSTREAM)

               AT THE UPSTREAM FACE                                   AT THE DOWNSTREAM FACE
                                 ALPHA           CANT                               ALPHA
      ELEV        P1       P2     DEG     MIN     NO        P1       P2     DEG     MIN       ELEV

     7165.0        0       72       0       0     26         0       66       0       0     7165.0
     7090.0       67      220       2      43     25        53      494       0      28     7090.0
     7015.0      179       -2     -30      48     24        87      738      -5      19     7015.0
     6940.0      246      -78     -15      55     23        84      824      -9      40     6940.0

     6865.0      330      -89      -7       2     22        26      819     -19      34     6865.0
     6790.0      457      -78       5       4     21      -110      791     -28      56     6790.0
     6700.0      252        0      -0       0     20       587        0       0       0     6700.0

                                   RIGHT ABUTMENT (LOOKING UPSTREAM)

               AT THE UPSTREAM FACE                                   AT THE DOWNSTREAM FACE
                                 ALPHA           CANT                               ALPHA
      ELEV        P1       P2     DEG     MIN     NO        P1       P2     DEG     MIN       ELEV

     7165.0        0       72       0       0     14         0       66       0       0     7165.0
     7090.0       67      220       2      43     15        53      494       0      28     7090.0
     7015.0      179       -2     -30      48     16        87      738      -5      19     7015.0

     6940.0      246      -78     -15      55     17        84      824      -9      40     6940.0
     6865.0      330      -89      -7       2     18        26      819     -19      34     6865.0
     6790.0      457      -78       5       4     19      -110      791     -28      56     6790.0
```

*Figure G-24.* **Principal stresses at abutments, in pounds per square inch.—288-D-3141**

```
                    STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)    (9701)         RUN 02/16/72

                                  ARCH STRESSES PARALLEL TO THE FACE OF THE DAM
                                                LOOKING UPSTREAM

ELEV  STA  1359.16  1305.63  1266.99  1238.39  1210.85  1178.15  1000.00  821.85  789.15  761.61  733.01  694.37  640.84

      TAU    -28                                                                                                   -28   TAU
7165  E       72       59       47       33       31       44      130      44      31      33      47      59      72
      I       66      158      164      148      121       88      -23      88     121     148     164     158      66

             TAU      -13                                                                                  -13     TAU
7090         E        220      168      181      215      268      492     268     215     181     168     220
             I        494      529      504      455      377       71     377     455     504     529     494

                      TAU       50                                                                  50     TAU
7015                  E         46      120      205      315      720     315     205     120      46
                      I        732      669      596      502      144     502     596     669     732

                               TAU      107                                                 107     TAU
6940                           E        -54       85      265      820     265      85     -54
                               I        803      702      578      235     578     702     803

                                        TAU      156                                156     TAU
6865                                    E        -83      115      854     115     -83
                                        I        730      617      212     617     730

                                                 TAU      181              181     TAU
6790                                             E        -74      674     -74
                                                 I        580      207     580
```

*Figure G-25.* Arch stresses parallel to faces, in pounds per square inch.–288-D-3142

```
              STUDY A-16 OF M.P. DAM (6-0 NON-SYM TEST PRB)     (9701)          RUN 02/16/72

                                       CANTILEVER STRESSES PARALLEL TO THE FACE OF THE DAM
                                                      LOOKING UPSTREAM

ELEV  STA     1305.63   1266.99   1238.39   1210.85   1178.15   1000.00    821.85    789.15    761.61    733.01    694.37

      TAU           0                                                                                              0   TAU
7165  U             0         0         0         0         0         0         0         0         0         0         0
      D             0         0         0         0         0         0         0         0         0         0         0

      TAU         -12                                                                                            -12   TAU
7090  U            67        79        71        70        68        56        68        70        71        79        67
      D            53        43        54        57        60        77        60        57        54        43        53

                  TAU        47                                                                        47       TAU
7015                U       131       143       134       124        75       124       134       143       131
                    D        93        86       102       119       192       119       102        86        93

                            TAU       102                                                   102       TAU
6940                          U       221       243       225       143       225       243       221
                              D       109        98       131       270       131        98       109

                                      TAU       160                                 160       TAU
6865                                    U       324       376       312       376       324
                                        D       130        90       239        90       130

                                                TAU       183                183       TAU
6790                                              U       453       526       453
                                                  D       132       144       132

                                                          TAU       117
6700                                                        U       252
                                                            D       587
```

*Figure G-26.* **Cantilever stresses parallel to faces, in pounds per square inch.—288-D-3143**

# Tables for Arch Analyses

H-1. *Introduction.*—The tables shown on the following pages have been prepared to simplify and expedite calculations for circular arches. They can also be used for noncircular arches, as described in appendix C, sections C-12 and C-16. Formulas used for quantities in the tables are derived in section 4-34. The use of tabular values has already been described in the various sections on calculation of arch data. Methods used in calculating tabular quantities for circular arches, together with discussions of the tables, are included in the following sections. These tables represent a large expenditure of time and effort, which has been justified by savings in time required for conducting the complicated trial-load arch analyses. The saving has resulted because the major expenditure of time in calculation is spent on the preparation of arch data. Development of electronic computer programs has further reduced the time required. Programs are discussed on pages following the tables.

H-2. *Arrangement of Tables.*—There are 13 tables, numbered H-1 through H-13, which follow section H-11. These tables are for evaulating arch constants and load constants, including necessary trigonometric functions. All tabulations are given to seven significant figures except some values in tables H-2 through H-6. The latter values, which are trigonometric functions and integrals needed for calculating load constants, are given to 12 places because several significant figures are lost when the values are used in computing subsequent functions and integrals. Computations must be made with *extreme* care and *carefully checked.*

Some tabular values have several zeros between the decimal point and the first significant figure. For purposes of condensation, these zeros are designated in the tables by one zero with an appropriate superior number. For example, .000,012,984,18 is shown as $.0^4$ 12,984,18. The superior number after the zero gives the number of zeros between the decimal point and the first significant figure.

H-3. *Tables of Trigonometric Data for Arch Constants.*—Trigonometric parts used in determining arch constants have been evaluated and are tabulated in table H-1. An arch constant is the product, or sum of products, of a multiplier and a trigonometric function. These constants are used in determining arch movements due to moment, thrust, and radial shear at a section.

H-4. *Constants for Circular Arches.*—Tabular quantities in table H-1 represent the following trigonometric parts for arch constants:

$$A_1 = \Phi$$

$$B_1 = \Phi - \sin \Phi$$

$$B_3,\ 1^{st}\ \text{term} = \Phi - 2 \sin \Phi + \frac{\Phi + \sin \Phi \cos \Phi}{2}$$

$$B_3,\ 2^{nd}\ \text{term} = \frac{\Phi + \sin \Phi \cos \Phi}{2} + 3\,\frac{\Phi - \sin \Phi \cos \Phi}{2}$$

$$C_1 = \text{vers } \Phi$$

$$B_2,\ 1^{st} \text{ term} = \text{vers } \Phi - \frac{\sin^2 \Phi}{2}$$

$$B_2,\ 2^{nd} \text{ term} = \sin^2 \Phi$$

$$C_2,\ 1^{st} \text{ term} = \frac{\Phi - \sin \Phi \cos \Phi}{2}$$

$$C_2,\ 2^{nd} \text{ term} = \frac{\Phi - \sin \Phi \cos \Phi}{2} + 3\,\frac{\Phi + \sin \Phi \cos \Phi}{2}$$

**H-5. *Explanation and Arrangement of Tables.***—Trigonometric functions and integrals needed to evaluate load constants for circular arches are given in tables H-2 to H-6, inclusive. Quantities are expressed in terms of $\Phi$, $\Phi_1$, and $\Phi_o$, in which $\Phi$ is the angle from the arch point considered to the abutment, $\Phi_1$, is the angle under the loaded portion of the arch, and $\Phi_o$ is the angle between the point considered and the point where the loading begins.

Integrals in tables H-4, H-5, and H-6 are shown in trigonometric form. In many cases, however, the value obtained from the formulas is given as a small difference between two relatively large numbers. This required the use of trigonometric functions expressed to 12 or more places in order to obtain sufficient accuracy in results. Terms that required more than 12 places were evaluated by using series in which enough terms were used to give the required accuracy. All series used were alternating, so that the error involved by stopping at any given term was less than the magnitude of the succeeding term.

The arrangement in tables H-4, H-5, and H-6 is self-explanatory. Tabular quantities are values of the functions $\Phi_1$ shown in column headings. In table H-4 tabular quantities in five of the columns are also equal to integrals of the functions of $\Phi_1$ given at the bottom of the table.

Tables H-5 and H-6 give quantities needed in evaluating the integrals of functions of $\Phi_1$, and $\Phi$, shown at the tops and bottoms of the tables. In each case the evaluation of the integral, after substituting $\Phi_1 + \Phi_o$ for $\Phi$, is a trigonometric function of $\Phi_1$ multiplied by $\pm \sin \Phi_o$, plus a trigonometric function of $\Phi_1$ multiplied by + $\cos \Phi_o$. Values of trigonometric functions of $\Phi_1$ are given in the tables and the function of $\Phi_o$ by which each value is to be multiplied is given at the top or bottom of the column. For instance, to obtain the integral of $(\Phi_1 - \sin \Phi_1) \sin \Phi \, d\Phi_1$, for given values of $\Phi_1$ and $\Phi_o$, take the quantity opposite the given value of $\Phi_1$ in the second column of table H-5, multiply by $\sin \Phi_o$ and add the quantity in the third column multiplied by $\cos \Phi_o$. If the integral of $(\Phi_1 - \sin \Phi_1) \cos \Phi d\Phi_1$ is desired, multiply the quantity in the second column of table H-5 by $\cos \Phi_o$ and subtract the quantity in the third column multiplied by $\sin \Phi_o$. The same procedure is followed in obtaining values of other integrals in tables H-5 and H-6.

**H-6. *Formulas in Trigonometric Form.***—Integrals expressed in trigonometric form in tables H-4, H-5, and H-6 are as follows:

$$\int_0^{\Phi_1} \Phi_1 \, d\Phi_1 = \frac{\Phi_1{}^2}{2}$$

$$\int_0^{\Phi_1} \frac{\Phi_1{}^2}{2} \, d\Phi_1 = \frac{\Phi_1{}^3}{6}$$

$$\int_0^{\Phi_1} \text{vers}\,\Phi_1\, d\Phi_1 = \Phi_1 - \sin\Phi_1$$

$$\int_0^{\Phi_1} [\Phi_1 - \sin\Phi_1]\, d\Phi_1 = \frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1$$

$$\int_0^{\Phi_1} \left[\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right] d\Phi_1 = \sin\Phi_1 - \Phi_1 + \frac{\Phi_1{}^3}{6}$$

$$\int_0^{\Phi_1} \left[\Phi_1 - \sin\Phi_1\right] \sin\Phi\, d\Phi_1 = \sin\Phi_o \left[\Phi_1 \sin\Phi_1 - \text{vers}\,\Phi_1 - \frac{\sin^2\Phi_1}{2}\right]$$

$$+ \cos\Phi_o \left[\sin\Phi_1 - \frac{\Phi_1 - \sin\Phi_1 \cos\Phi_1}{2} - \Phi_1 \cos\Phi_1\right]$$

$$\int_0^{\Phi_1} \text{vers}\,\Phi_1 \sin\Phi\, d\Phi_1 = \sin\Phi_o \left[\sin\Phi_1 - \frac{\Phi_1 + \sin\Phi_1 \cos\Phi_1}{2}\right]$$

$$+ \cos\Phi_o \left[\text{vers}\,\Phi_1 - \frac{\sin^2\Phi_1}{2}\right]$$

$$\int_0^{\Phi_1} \sin\Phi_1 \sin\Phi\, d\Phi_1 = \sin\Phi_o \left[\frac{\sin^2\Phi_1}{2}\right] + \cos\Phi_o \left[\frac{\Phi_1 - \sin\Phi_1 \cos\Phi_1}{2}\right]$$

$$\int_0^{\Phi_1} \frac{\Phi_1{}^2}{2} \sin\Phi\, d\Phi_1 = \sin\Phi_o \left[\frac{\Phi_1{}^2}{2} \sin\Phi_1 + \Phi_1 \cos\Phi_1 - \sin\Phi_1\right]$$

$$+ \cos\Phi_o \left[\Phi_1 \sin\Phi_1 - \frac{\Phi_1{}^2}{2} \cos\Phi_1 - \text{vers}\,\Phi_1\right]$$

$$\int_0^{\Phi_1} \Phi_1 \sin\Phi\, d\Phi_1 = \sin\Phi_o\,[\Phi_1 \sin\Phi_1 - \text{vers}\,\Phi_1] + \cos\Phi_o\,[\sin\Phi_1 - \Phi_1 \cos\Phi_1]$$

$$\int_0^{\Phi_1} \left[\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right] \sin\Phi\, d\Phi_1$$

$$= \sin\Phi_o \left[\frac{\Phi_1{}^2}{2} \sin\Phi_1 + \Phi_1 \cos\Phi_1 - 2\sin\Phi_1 + \frac{\Phi_1 + \sin\Phi_1 \cos\Phi_1}{2}\right]$$

$$+ \cos\Phi_o \left[\Phi_1 \sin\Phi_1 - \frac{\Phi_1{}^2}{2} \cos\Phi_1 - 2\,\text{vers}\,\Phi_1 + \frac{\sin^2\Phi_1}{2}\right]$$

H-7. ***Formulas in Power Series.***—The integrals expressed in terms of power series are given below. Developments for vers $\Phi_1$, sin $\Phi_1$,

$\int_0^{\Phi_1} \Phi_1\, d\Phi_1$ and $\int_0^{\Phi_1} \frac{\Phi_1{}^2}{2} d\Phi_1$ are added for completeness.

$$\int_0^{\Phi_1} \Phi_1\, d\Phi_1 = \frac{\Phi_1{}^2}{2} = 0.500{,}000{,}000{,}000\ \Phi_1{}^2$$

$$\int_0^{\Phi_1} \frac{\Phi_1{}^2}{2}\, d\Phi_1 = \frac{\Phi_1{}^3}{6} = 0.166{,}666{,}666{,}666\ \Phi_1{}^3$$

$$\begin{aligned} \text{vers}\,\Phi_1 = &+ 0.500{,}000{,}000{,}000\ \Phi_1{}^2 - 0.041{,}666{,}666{,}666\ \Phi_1{}^4 \\ &+ 0.001{,}388{,}888{,}888\ \Phi_1{}^6 - 0.000{,}024{,}801{,}587\ \Phi_1{}^8 \\ &+ 0.000{,}000{,}275{,}573\ \Phi_1{}^{10} - 0.000{,}000{,}002{,}087\ \Phi_1{}^{12} \\ &+ 0.000{,}000{,}000{,}011\ \Phi_1{}^{14} - \ldots\ldots \end{aligned}$$

$$\begin{aligned} \sin\Phi_1 = &+ 1.000{,}000{,}000{,}000\ \Phi_1 - 0.166{,}666{,}666{,}666\ \Phi_1{}^3 \\ &+ 0.008{,}333{,}333{,}333\ \Phi_1{}^5 - 0.000{,}198{,}412{,}698\ \Phi_1{}^7 \\ &+ 0.000{,}002{,}755{,}731\ \Phi_1{}^9 - 0.000{,}000{,}025{,}052\ \Phi_1{}^{11} \\ &+ 0.000{,}000{,}000{,}160\ \Phi_1{}^{13} - 0.000{,}000{,}000{,}007\ \Phi_1{}^{15} + \ldots \end{aligned}$$

$$\begin{aligned} \int_0^{\Phi_1} \text{vers}\,\Phi_1\, d\Phi_1 = &+ 0.166{,}666{,}666{,}666\ \Phi_1{}^3 - 0.008{,}333{,}333{,}333\ \Phi_1{}^5 \\ &+ 0.000{,}198{,}412{,}698\ \Phi_1{}^7 - 0.000{,}002{,}755{,}731\ \Phi_1{}^9 \\ &+ 0.000{,}000{,}025{,}052\ \Phi_1{}^{11} - 0.000{,}000{,}000{,}160\ \Phi_1{}^{13} \\ &+ 0.000{,}000{,}000{,}000{,}7\ \Phi_1{}^{15} - \ldots . \end{aligned}$$

$$\begin{aligned} \int_0^{\Phi_1} [\Phi_1 - \sin\Phi_1]\, d\Phi_1 = &+ 0.041{,}666{,}666{,}666\ \Phi_1{}^4 - 0.001{,}388{,}888{,}888\ \Phi_1{}^6 \\ &+ 0.000{,}024{,}801{,}587\ \Phi_1{}^8 - 0.000{,}000{,}275{,}573\ \Phi_1{}^{10} \\ &+ 0.000{,}000{,}002{,}087\ \Phi_1{}^{12} - 0.000{,}000{,}000{,}011\ \Phi_1{}^{14} + \ldots . \end{aligned}$$

$$\int_0^{\Phi_1} \left[\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right] d\Phi_1 = +0.008{,}333{,}333{,}333\ \Phi_1{}^5 - 0.000{,}198{,}412{,}698\ \Phi_1{}^7$$

$$+0.000{,}002{,}755{,}731\ \Phi_1{}^9 - 0.000{,}000{,}025{,}052\ \Phi_1{}^{11}$$

$$+0.000{,}000{,}000{,}160\ \Phi_1{}^{13} - 0.000{,}000{,}000{,}000{,}7\ \Phi_1{}^{15} + \ldots.$$

$$\int_0^{\Phi_1} [\Phi_1 - \sin\Phi_1]\sin\Phi\, d\Phi_1 = \sin\Phi_o[+0.041{,}666{,}666{,}66\ \Phi_1{}^4$$

$$-0.015{,}277{,}777{,}77\ \Phi_1{}^6 + 0.001{,}413{,}690{,}47\ \Phi_1{}^8$$

$$-0.000{,}068{,}066{,}57\ \Phi_1{}^{10} + 0.000{,}002{,}114{,}81\ \Phi_1{}^{12}$$

$$-0.000{,}000{,}046{,}80\ \Phi_1{}^{14} + \ldots.]$$

$$+\cos\Phi_o[+0.033{,}333{,}333{,}33\ \Phi_1{}^5 - 0.005{,}158{,}730{,}15\ \Phi_1{}^7$$

$$+0.000{,}330{,}687{,}83\ \Phi_1{}^9 - 0.000{,}012{,}576{,}11\ \Phi_1{}^{11}$$

$$+0.000{,}000{,}326{,}96\ \Phi_1{}^{13} - 0.000{,}000{,}006{,}2\ \Phi_1{}^{15} + \ldots.]$$

$$\int_0^{\Phi_1} \text{vers}\,\Phi_1\ \sin\Phi\, d\Phi_1 = \sin\Phi_o[0.166{,}666{,}666{,}66\ \Phi_1{}^3$$

$$-0.058{,}333{,}333{,}33\ \Phi_1{}^5 + 0.006{,}150{,}793{,}65\ \Phi_1{}^7$$

$$-0.000{,}349{,}977{,}95\ \Phi_1{}^9 + 0.000{,}012{,}801{,}62\ \Phi_1{}^{11}$$

$$-0.000{,}000{,}328{,}72\ \Phi_1{}^{13} + 0.000{,}000{,}006{,}2\ \Phi_1{}^{15} - \ldots.]$$

$$+\cos\Phi_o[+0.125{,}000{,}000{,}00\ \Phi_1{}^4 - 0.020{,}833{,}333{,}33\ \Phi_1{}^6$$

$$+0.001{,}562{,}500{,}00\ \Phi_1{}^8 - 0.000{,}070{,}271{,}16\ \Phi_1{}^{10}$$

$$+0.000{,}002{,}135{,}69\ \Phi_1{}^{12} - 0.000{,}000{,}046{,}97\ \Phi_1{}^{14} + \ldots.]$$

$$\int_0^{\Phi_1} \sin\Phi_1\ \sin\Phi\, d\Phi_1 = \sin\Phi_o[+0.500{,}000{,}000{,}000\ \Phi_1{}^2$$

$$-0.166{,}666{,}666{,}666\ \Phi_1{}^4 + 0.022{,}222{,}222{,}222\ \Phi_1{}^6$$

$$-0.001{,}587{,}301{,}587\ \Phi_1{}^8 + 0.000{,}070{,}546{,}737\ \Phi_1{}^{10}$$

$$-0.000{,}002{,}137{,}779\ \Phi_1{}^{12} + 0.000{,}000{,}046{,}984\ \Phi_1{}^{14}$$

$$-0.000{,}000{,}000{,}783\ \Phi_1{}^{16} + \ldots.]$$

$$+ \cos \Phi_o [+ 0.333{,}333{,}333{,}333 \ \Phi_1{}^3 - 0.066{,}666{,}666{,}666 \ \Phi_1{}^5$$

$$+ 0.006{,}349{,}206{,}336 \ \Phi_1{}^7 - 0.000{,}352{,}733{,}686 \ \Phi_1{}^9$$

$$+ 0.000{,}012{,}826{,}679 \ \Phi_1{}^{11} - 0.000{,}000{,}328{,}889 \ \Phi_1{}^{13}$$

$$+ 0.000{,}000{,}006{,}264 \ \Phi_1{}^{15} - 0.000{,}000{,}000{,}092 \ \Phi_1{}^{17} + \ldots .]$$

$$\int_0^{\Phi_1} \frac{\Phi_1{}^2}{2} \sin \Phi \ d\Phi_1 = \sin \Phi_o [+ 0.166{,}666{,}666{,}666 \ \Phi_1{}^3$$

$$- 0.050{,}000{,}000{,}000 \ \Phi_1{}^5 + 0.002{,}976{,}190{,}470 \ \Phi_1{}^7$$

$$- 0.000{,}077{,}160{,}493 \ \Phi_1{}^9 + 0.000{,}001{,}127{,}344 \ \Phi_1{}^{11}$$

$$- 0.000{,}000{,}010{,}598 \ \Phi_1{}^{13} + 0.000{,}000{,}000{,}069 \ \Phi_1{}^{15} - \ldots .]$$

$$+ \cos \Phi_o [+ 0.125{,}000{,}000{,}000 \ \Phi_1{}^4 - 0.013{,}888{,}888{,}888 \ \Phi_1{}^6$$

$$+ 0.000{,}520{,}833{,}333 \ \Phi_1{}^8 - 0.000{,}009{,}920{,}634 \ \Phi_1{}^{10}$$

$$+ 0.000{,}000{,}114{,}822 \ \Phi_1{}^{12} - 0.000{,}000{,}000{,}894 \ \Phi_1{}^{14}$$

$$+ 0.000{,}000{,}000{,}005 \ \Phi_1{}^{16} - \ldots .]$$

$$\int_0^{\Phi_1} \Phi_1 \sin \Phi \ d\Phi_1 = \sin \Phi_o [+ 0.500{,}000{,}000{,}000 \ \Phi_1{}^2$$

$$- 0.125{,}000{,}000{,}000 \ \Phi_1{}^4 + 0.006{,}944{,}444{,}444 \ \Phi_1{}^6$$

$$- 0.000{,}173{,}611{,}111 \ \Phi_1{}^8 + 0.000{,}002{,}480{,}158 \ \Phi_1{}^{10}$$

$$- 0.000{,}000{,}022{,}964 \ \Phi_1{}^{12} + 0.000{,}000{,}000{,}149 \ \Phi_1{}^{14} - \ldots .]$$

$$+ \cos \Phi_o [+ 0.333{,}333{,}333{,}333 \ \Phi_1{}^3 - 0.033{,}333{,}333{,}333 \ \Phi_1{}^5$$

$$+ 0.001{,}190{,}476{,}188 \ \Phi_1{}^7 - 0.000{,}022{,}045{,}855 \ \Phi_1{}^9$$

$$+ 0.000{,}000{,}250{,}521 \ \Phi_1{}^{11} - 0.000{,}000{,}001{,}927 \ \Phi_1{}^{13}$$

$$+ 0.000{,}000{,}000{,}010 \ \Phi_1{}^{15} - \ldots .]$$

$$\int_0^{\Phi_1} \left[\frac{\Phi_1{}^2}{2} - \text{vers}\,\Phi_1\right] \sin\Phi\, d\Phi_1 = \sin\Phi_o\,[+\,0.008{,}333{,}333{,}333\ \Phi_1{}^5$$

$$-\,0.003{,}174{,}603{,}168\ \Phi_1{}^7 + 0.000{,}272{,}817{,}460\ \Phi_1{}^9$$

$$-\,0.000{,}011{,}674{,}282\ \Phi_1{}^{11} + 0.000{,}000{,}318{,}129\ \Phi_1{}^{13}$$

$$-\,0.000{,}000{,}006{,}194\ \Phi_1{}^{15} + 0.000{,}000{,}000{,}091\ \Phi_1{}^{17} - \ldots.]$$

$$+\cos\Phi_o\,[+\,0.006{,}944{,}444{,}444\ \Phi_1{}^6 - 0.001{,}041{,}666{,}666\ \Phi_1{}^8$$

$$+\,0.000{,}060{,}350{,}529\ \Phi_1{}^{10} - 0.000{,}002{,}020{,}870\ \Phi_1{}^{12}$$

$$+\,0.000{,}000{,}046{,}077\ \Phi_1{}^{14} - 0.000{,}000{,}000{,}778\ \Phi_1{}^{16}$$

$$+\,0.000{,}000{,}000{,}010\ \Phi_1{}^{18} - \ldots.]$$

H-8. ***Tables of D-Terms or Load Constants.***—Load constants, which are the trigonometric parts of the expressions for deflections due to applied loads, are tabulated in tables I-8 to I-12, inclusive. These quantities are designated *D*-terms, $D_1$ being used for twist deflections, $D_2$ for radial deflections, and $D_3$ for tangential deflections. These *D*-terms, multiplied by proper factors, give deflections at an arch point due to applied load between the point and the abutment.

H-9. ***Evaluation of Tables of D-terms or Load Constants.***—The formulas in section 4-34(h), together with tabular values in figure C-31 and tables H-1 to H-3, inclusive, give all quantities needed to evaluate the trigonometric parts of load formulas, or *D*-terms, which are tabulated in tables H-8 to H-12, inclusive. These tabular load constants are the trigonometric parts of the *D*-terms with $P$ = 1,000 included in the uniform loads and $P/\Phi_1$ = 1,000/$\Phi_1$ included in the triangular loads. Forms and sample computations for tabular *D*-terms of several unit loads are shown in table H-7.

Sheet 1 of table H-7 shows values of trigonometric functions and integrals needed in evaluating load constants for an arch with a 48° half-central angle, proper sine and cosine multipliers being shown beneath the quantities where needed. These values were taken from tables H-2 to H-6, inclusive, and are tabulated on sheet 1 for convenience. Since *D*-terms are determined for the quarter-points of the half-arch, the quarter values of Φ are listed in descending order, and the quantities for these angles are tabulated.

Sample computations for uniform and triangular radial, tangential, and twist loads are given on sheets 2, 3, and 4 of table H-7. In computing values for uniform loads, columns having integrals underneath are 1,000 times the values for the integrals on sheet 1, except that some values are multiplied by three as indicated. Formulas for calculating the remaining columns are given on the computation sheets.

Computations for triangular loads are complicated, since some points are outside the loaded portion of the arch while other points are under the load. For points outside the load, values are obtained from sheet 1, using $\Phi_1$ as the angle between the abutment and the end of the load, and $\Phi_o$ as the angle between the end of the load and the point considered. As explained previously, these values are multiplied by the factor 1,000/$\Phi_1$. In calculating *D*-terms for points under the load, only deflections produced by the load between the point and the abutment are considered. This load is divided into a triangular load and a uniform load. Values of the triangular part of the load are equal to the indicated integrals for the angle between the abutment and the point considered. These values are multiplied by 1,000/Φ, which automatically corrects for the fact that the ordinate of the triangular load at the abutment is a fraction of *P*. The value of the uniform load to be included with the triangular part is equal to a fractional part of the 1,000-pound uniform load values, the fraction being indicated on sheet 2 of table H-7.

**H-10.** ***Tabulation of D-Terms.***—Values of load constants for circular arches of uniform thickness, for all angles between 10° and 90°, are tabulated in tables H-8 to H-12, inclusive. Tables H-8 and H-9 show load constants for radial loads, tables H-10 and H-11 show load constants for tangential loads, and table H-12 shows load constants for twist loads.

All *D*-terms for the crown point are shown on the first sheets for radial and tangential loads. This is because all *D*-terms are needed in determining the moment, thrust, and shear at the crown. The $D_2$ and $D_3$ terms for radial and tangential loads are divided into two parts, since each part has a different multiplier. The first part is used in calculating the deflection due to bending, and the second part in calculating the deflection due to rib-shortening and shear.

**H-11.** ***Trigonometric Tables.***—A convenient table of trigonometric functions is shown in table H-13. These are used in calculating arch data. Some are necessary in evaluating moments, thrusts, and shears due to applied loads. Others are needed at various places in arch computation forms. Values may be taken quickly from these tables.

Table H-1.–*Constants for circular arches* (sheet 1).

| $\Phi$ | SIN $\Phi$ | COS $\Phi$ | A$_1$ | B$_1$ | B$_3$ | | C$_1$ | B$_2$ | | C$_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° ′ | | | (Radians) | ($\Phi$-SIN $\Phi$) | 1ST TERM | 2ND TERM | (VERS $\Phi$) | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| 0 15 | .004,363,309 | .999,990,5 | .004,363,323 | .0$^{7}$13,845,24 | .0$^{13}$79,078,03 | .004,363,379 | .0$^{5}$9,519,279 | .0$^{9}$45,308,34 | .0$^{4}$19,038,47 | .0$^{7}$27,690,40 | .013,089,91 |
| 0 30 | .008,726,535 | .999,961,9 | .008,726,646 | .0$^{6}$110,761,6 | .0$^{11}$2,530,480 | .008,727,089 | .0$^{4}$38,076,94 | .0$^{9}$724,926,5 | .0$^{4}$76,152,42 | .0$^{6}$221,520,7 | .026,179,50 |
| 0 45 | .013,089,60 | .999,914,3 | .013,089,97 | .0$^{6}$373,818,6 | .0$^{10}$19,215,61 | .013,091,46 | .0$^{4}$85,672,43 | .0$^{8}$3,669,882 | .0$^{3}$171,337,5 | .0$^{6}$747,618,0 | .039,268,41 |
| 1 00 | .017,452,41 | .999,847,7 | .017,453,29 | .0$^{6}$886,082,6 | .0$^{10}$80,973,15 | .017,456,84 | .0$^{3}$152,304,8 | .0$^{7}$11,598,38 | .0$^{3}$304,586,5 | .0$^{5}$1,772,084 | .052,356,33 |
| 1 15 | .021,814,89 | .999,762,0 | .021,816,62 | .0$^{5}$1,730,615 | .0$^{9}$247,105,4 | .021,823,54 | .0$^{3}$237,972,9 | .0$^{7}$28,315,55 | .0$^{3}$475,889,2 | .0$^{5}$3,460,984 | .065,442,93 |
| 1 30 | .026,176,95 | .999,657,3 | .026,179,94 | .0$^{5}$2,990,472 | .0$^{9}$614,862,1 | .026,191,90 | .0$^{3}$342,675,0 | .0$^{7}$58,713,09 | .0$^{3}$685,232,6 | .0$^{5}$5,980,329 | .078,527,85 |
| 1 45 | .030,538,51 | .999,533,6 | .030,543,26 | .0$^{5}$4,748,700 | .0$^{8}$1,328,920 | .030,562,25 | .0$^{3}$466,409,2 | .0$^{6}$108,768,8 | .0$^{3}$932,600,8 | .0$^{5}$9,496,071 | .091,610,79 |
| 2 00 | .034,899,50 | .999,390,8 | .034,906,59 | .0$^{5}$7,088,337 | .0$^{8}$2,590,859 | .034,934,93 | .0$^{3}$609,173,0 | .0$^{6}$185,545,9 | .001,217,975 | .0$^{4}$14,174,08 | .104,691,4 |
| 2 15 | .039,259,82 | .999,229,0 | .039,269,91 | .0$^{4}$10,092,41 | .0$^{8}$4,668,833 | .039,310,27 | .0$^{3}$770,963,8 | .0$^{6}$297,192,6 | .001,541,333 | .0$^{4}$20,180,15 | .117,769,4 |
| 2 30 | .043,619,39 | .999,048,2 | .043,633,23 | .0$^{4}$13,843,93 | .0$^{8}$7,906,030 | .043,688,59 | .0$^{3}$951,776,4 | .0$^{6}$452,941,1 | .001,902,651 | .0$^{4}$27,679,96 | .130,844,3 |
| 2 45 | .047,978,13 | .998,848,4 | .047,996,55 | .0$^{4}$18,425,91 | .0$^{7}$12,732,13 | .048,070,23 | .001,151,614 | .0$^{6}$663,106,9 | .002,301,901 | .0$^{4}$36,839,08 | .143,916,0 |
| 3 00 | .052,335,96 | .998,629,5 | .052,359,88 | .0$^{4}$23,921,32 | .0$^{7}$19,670,77 | .052,455,52 | .001,370,465 | .0$^{6}$939,087,5 | .002,739,052 | .0$^{4}$47,822,96 | .156,984,0 |
| 3 15 | .056,692,79 | .998,391,7 | .056,723,20 | .0$^{4}$30,413,13 | .0$^{7}$29,349,95 | .056,844,79 | .001,608,329 | .0$^{5}$1,293,362 | .003,214,072 | .0$^{4}$60,796,90 | .170,048,0 |
| 3 30 | .061,048,54 | .998,134,8 | .061,088,52 | .0$^{4}$37,984,28 | .0$^{7}$42,511,28 | .061,238,38 | .001,865,202 | .0$^{5}$1,739,489 | .003,726,924 | .0$^{4}$75,926,06 | .183,107,7 |
| 3 45 | .065,403,13 | .997,858,9 | .065,449,85 | .0$^{4}$46,717,72 | .0$^{7}$60,019,41 | .065,636,60 | .002,141,077 | .0$^{5}$2,292,105 | .004,277,589 | .0$^{4}$93,375,42 | .196,162,8 |
| 4 00 | .069,756,47 | .997,564,1 | .069,813,17 | .0$^{4}$56,696,34 | .0$^{7}$82,871,43 | .070,039,79 | .002,435,950 | .0$^{5}$2,966,925 | .004,885,966 | .0$^{3}$113,309,8 | .209,212,9 |
| 4 15 | .074,108,49 | .997,250,2 | .074,178,49 | .0$^{4}$68,003,01 | .0$^{6}$112,206,2 | .074,448,28 | .002,749,815 | .0$^{5}$3,780,741 | .005,492,068 | .0$^{3}$135,893,8 | .222,257,7 |
| 4 30 | .078,459,10 | .996,917,3 | .078,539,82 | .0$^{4}$80,720,61 | .0$^{6}$149,314,0 | .078,862,40 | .003,082,666 | .0$^{5}$4,751,416 | .006,155,830 | .0$^{3}$161,291,9 | .235,296,9 |
| 4 45 | .082,808,21 | .996,565,5 | .082,903,14 | .0$^{4}$94,931,96 | .0$^{6}$195,645,4 | .083,282,48 | .003,434,498 | .0$^{5}$5,897,887 | .006,857,199 | .0$^{3}$189,668,3 | .248,330,1 |
| 5 00 | .087,155,74 | .996,194,7 | .087,266,46 | .0$^{3}$110,719,9 | .0$^{6}$252,821,0 | .087,708,84 | .003,805,302 | .0$^{5}$7,240,160 | .007,596,124 | .0$^{3}$221,186,9 | .261,357,0 |
| 5 15 | .091,501,62 | .995,804,9 | .091,629,79 | .0$^{3}$128,167,1 | .0$^{6}$322,640,8 | .092,141,81 | .004,195,072 | .0$^{5}$8,799,316 | .008,372,546 | .0$^{3}$256,011,5 | .274,377,3 |
| 5 30 | .095,845,75 | .995,396,2 | .095,993,11 | .0$^{3}$147,356,3 | .0$^{6}$407,093,6 | .096,581,72 | .004,603,802 | .0$^{4}$10,597,50 | .009,186,408 | .0$^{3}$294,305,6 | .287,390,7 |
| 5 45 | .100,188,1 | .994,968,5 | .100,356,4 | .0$^{3}$168,370,4 | .0$^{6}$508,364,6 | .101,028,9 | .005,031,482 | .0$^{4}$12,657,90 | .010,037,65 | .0$^{3}$336,232,4 | .300,396,8 |
| 6 00 | .104,528,5 | .994,521,9 | .104,719,8 | .0$^{3}$191,291,9 | .0$^{6}$628,848,8 | .105,483,7 | .005,478,105 | .0$^{4}$15,004,82 | .010,926,20 | .0$^{3}$381,954,9 | .313,395,4 |
| 6 15 | .108,866,9 | .994,056,3 | .109,083,1 | .0$^{3}$216,203,4 | .0$^{6}$771,155,3 | .109,946,4 | .005,943,682 | .0$^{4}$17,683,56 | .011,852,00 | .0$^{3}$431,635,6 | .326,386,0 |
| 6 30 | .113,203,2 | .993,571,9 | .113,446,4 | .0$^{3}$243,187,6 | .0$^{6}$938,119,6 | .114,417,3 | .006,428,144 | .0$^{4}$20,660,52 | .012,814,97 | .0$^{3}$485,437,1 | .339,368,3 |
| 6 45 | .117,537,4 | .993,068,5 | .117,809,7 | .0$^{3}$272,327,1 | .0$^{5}$1,132,813 | .118,896,8 | .006,931,543 | .0$^{4}$24,023,14 | .013,815,04 | .0$^{3}$543,521,3 | .352,342,2 |
| 7 00 | .121,869,3 | .992,546,2 | .122,173,0 | .0$^{3}$303,704,2 | .0$^{5}$1,358,549 | .123,385,1 | .007,453,848 | .0$^{4}$27,779,93 | .014,852,14 | .0$^{3}$606,049,9 | .365,307,0 |
| 7 15 | .126,199,0 | .992,004,9 | .126,536,4 | .0$^{3}$337,401,6 | .0$^{5}$1,618,896 | .127,882,7 | .007,995,050 | .0$^{4}$31,960,42 | .015,926,18 | .0$^{3}$673,184,4 | .378,262,8 |
| 7 30 | .130,526,2 | .991,444,9 | .130,899,7 | .0$^{3}$373,501,7 | .0$^{5}$1,917,685 | .132,389,9 | .008,555,139 | .0$^{4}$36,595,20 | .017,037,09 | .0$^{3}$745,085,7 | .391,208,9 |
| 7 45 | .134,850,9 | .990,865,9 | .135,263,0 | .0$^{3}$412,086,8 | .0$^{5}$2,259,017 | .136,906,8 | .009,134,103 | .0$^{4}$41,715,92 | .018,184,77 | .0$^{3}$821,914,5 | .404,145,2 |
| 8 00 | .139,173,1 | .990,268,1 | .139,626,3 | .0$^{3}$453,239,2 | .0$^{5}$2,647,273 | .141,434,0 | .009,731,931 | .0$^{4}$47,355,24 | .019,369,15 | .0$^{3}$903,831,1 | .417,071,4 |
| 8 15 | .143,492,6 | .989,651,4 | .143,989,7 | .0$^{3}$497,041,3 | .0$^{5}$3,087,128 | .145,971,7 | .010,348,61 | .0$^{4}$53,546,90 | .020,590,13 | .0$^{3}$990,995,5 | .429,987,0 |
| 8 30 | .147,809,4 | .989,015,9 | .148,353,0 | .0$^{3}$543,575,3 | .0$^{5}$3,583,551 | .150,520,1 | .010,984,14 | .0$^{4}$60,325,63 | .021,847,62 | .001,083,567 | .442,891,8 |
| 8 45 | .152,123,4 | .988,361,5 | .152,716,3 | .0$^{3}$592,923,4 | .0$^{5}$4,141,821 | .155,079,7 | .011,638,49 | .0$^{4}$67,727,22 | .023,141,52 | .001,181,705 | .455,785,5 |
| 9 00 | .156,434,5 | .987,688,3 | .157,079,6 | .0$^{3}$645,167,6 | .0$^{5}$4,767,532 | .159,650,8 | .012,311,66 | .0$^{4}$75,788,48 | .024,471,74 | .001,285,568 | .468,867,8 |
| 9 15 | .160,742,6 | .986,996,4 | .161,443,0 | .0$^{3}$700,390,2 | .0$^{5}$5,466,608 | .164,233,6 | .013,003,63 | .0$^{4}$84,547,24 | .025,838,17 | .001,395,314 | .481,538,2 |
| 9 30 | .165,047,6 | .986,285,6 | .165,806,3 | .0$^{3}$758,673,1 | .0$^{5}$6,245,302 | .168,828,5 | .013,714,40 | .0$^{4}$94,042,36 | .027,240,71 | .001,511,101 | .494,396,6 |
| 9 45 | .169,349,5 | .985,556,1 | .170,169,6 | .0$^{3}$820,098,2 | .0$^{5}$7,110,215 | .173,435,8 | .014,443,94 | .0$^{3}$104,313,7 | .028,679,25 | .001,633,086 | .507,242,6 |
| 10 00 | .173,648,2 | .984,807,8 | .174,532,9 | .0$^{3}$884,747,5 | .0$^{5}$8,068,297 | .178,055,8 | .015,192,25 | .0$^{3}$115,402,2 | .030,153,69 | .001,761,427 | .520,075,9 |
| 10 15 | .177,943,5 | .984,040,7 | .178,896,2 | .0$^{3}$952,702,9 | .0$^{5}$9,126,867 | .182,688,8 | .015,959,30 | .0$^{3}$127,349,7 | .031,663,91 | .001,896,279 | .532,896,2 |
| 10 30 | .182,235,5 | .983,254,9 | .183,259,6 | .001,024,046 | .0$^{4}$10,293,59 | .187,335,2 | .016,745,09 | .0$^{3}$140,199,1 | .033,209,79 | .002,037,798 | .545,703,1 |
| 10 45 | .186,524,0 | .982,450,4 | .187,622,9 | .001,098,859 | .0$^{4}$11,576,55 | .191,995,2 | .017,549,60 | .0$^{3}$153,994,3 | .034,791,22 | .002,186,141 | .558,496,4 |
| 11 00 | .190,809,0 | .981,627,2 | .191,986,2 | .001,177,222 | .0$^{4}$12,984,18 | .196,669,1 | .018,372,82 | .0$^{3}$168,780,2 | .036,408,07 | .002,341,461 | .571,275,7 |
| 11 15 | .195,090,3 | .980,785,3 | .196,349,5 | .001,259,219 | .0$^{4}$14,525,33 | .201,357,4 | .019,214,72 | .0$^{3}$184,602,7 | .038,060,23 | .002,503,912 | .584,040,8 |
| 11 30 | .199,367,9 | .979,924,7 | .200,712,9 | .001,344,930 | .0$^{4}$16,209,26 | .206,060,2 | .020,075,30 | .0$^{3}$201,508,7 | .039,747,57 | .002,673,650 | .596,791,3 |
| 11 45 | .203,641,8 | .979,045,5 | .205,076,2 | .001,434,436 | .0$^{4}$18,045,57 | .210,777,8 | .020,954,53 | .0$^{3}$219,546,1 | .041,469,96 | .002,850,826 | .609,526,9 |
| 12 00 | .207,911,7 | .978,147,6 | .209,439,5 | .001,527,819 | .0$^{4}$20,044,49 | .215,510,7 | .021,852,40 | .0$^{3}$238,763,7 | .043,227,27 | .003,035,594 | .622,247,3 |
| 12 15 | .212,177,7 | .977,231,1 | .213,802,8 | .001,625,161 | .0$^{4}$22,216,41 | .220,259,1 | .022,768,89 | .0$^{3}$259,211,3 | .045,019,36 | .003,228,106 | .634,952,3 |
| 12 30 | .216,439,6 | .976,298,0 | .218,166,2 | .001,726,543 | .0$^{4}$24,572,31 | .225,023,2 | .023,703,99 | .0$^{3}$280,939,6 | .046,846,11 | .003,428,513 | .647,641,4 |
| 12 45 | .220,697,4 | .975,342,3 | .222,529,5 | .001,832,045 | .0$^{4}$27,123,60 | .229,803,4 | .024,657,68 | .0$^{3}$304,000,6 | .048,707,36 | .003,636,966 | .660,314,5 |
| 13 00 | .224,951,1 | .974,370,1 | .226,892,8 | .001,941,748 | .0$^{4}$29,882,15 | .234,600,0 | .025,629,94 | .0$^{3}$328,446,8 | .050,602,98 | .003,853,615 | .672,971,2 |
| 13 15 | .229,200,4 | .973,379,3 | .231,256,1 | .002,055,735 | .0$^{4}$32,860,27 | .239,413,4 | .026,620,74 | .0$^{3}$354,331,9 | .052,532,82 | .004,078,610 | .685,611,2 |
| 13 30 | .233,445,4 | .972,369,9 | .235,619,4 | .002,174,085 | .0$^{4}$36,070,75 | .244,243,7 | .027,630,08 | .0$^{3}$381,710,7 | .054,496,74 | .004,312,100 | .698,234,2 |
| 13 45 | .237,685,9 | .971,342,1 | .239,982,8 | .002,296,880 | .0$^{4}$39,526,88 | .249,091,2 | .028,657,93 | .0$^{3}$410,638,4 | .056,494,58 | .004,554,233 | .710,839,9 |
| 14 00 | .241,921,9 | .970,295,7 | .244,346,1 | .002,424,200 | .0$^{4}$43,242,42 | .253,956,4 | .029,704,27 | .0$^{3}$441,171,9 | .058,526,20 | .004,805,157 | .723,428,0 |
| 14 15 | .246,153,3 | .969,230,9 | .248,709,4 | .002,556,125 | .0$^{4}$47,231,62 | .258,839,5 | .030,769,09 | .0$^{3}$473,368,5 | .060,591,44 | .005,065,019 | .735,998,2 |
| 14 30 | .250,380,0 | .968,147,6 | .253,072,7 | .002,692,737 | .0$^{4}$51,509,26 | .263,740,7 | .031,852,36 | .0$^{3}$507,286,4 | .062,690,15 | .005,333,966 | .748,550,3 |
| 14 45 | .254,601,9 | .967,045,9 | .257,436,1 | .002,834,117 | .0$^{4}$56,090,62 | .268,660,4 | .032,954,06 | .0$^{3}$542,985,1 | .064,822,15 | .005,612,142 | .761,083,9 |
| 15 00 | .258,819,0 | .965,925,8 | .261,799,4 | .002,980,343 | .0$^{4}$60,991,49 | .273,598,8 | .034,074,17 | .0$^{3}$580,524,7 | .066,987,30 | .005,899,694 | .773,598,8 |
| 15 15 | .263,031,2 | .964,787,3 | .266,162,7 | .003,131,496 | .0$^{4}$66,228,22 | .278,556,2 | .035,212,68 | .0$^{3}$619,986,3 | .069,185,42 | .006,196,765 | .786,094,6 |
| 15 30 | .267,238,4 | .963,630,5 | .270,526,0 | .003,287,658 | .0$^{4}$71,817,66 | .283,533,0 | .036,369,55 | .0$^{3}$661,372,0 | .071,416,35 | .006,503,498 | .798,571,1 |
| 15 45 | .271,440,4 | .962,455,2 | .274,889,4 | .003,448,907 | .0$^{4}$77,777,23 | .288,529,4 | .037,544,76 | .0$^{3}$704,804,6 | .073,679,92 | .006,820,037 | .811,028,0 |
| 16 00 | .275,637,4 | .961,261,7 | .279,252,7 | .003,615,325 | .0$^{4}$84,124,90 | .293,545,7 | .038,738,30 | .0$^{3}$750,328,1 | .075,975,95 | .007,146,524 | .823,465,0 |
| 16 15 | .279,829,0 | .960,049,9 | .283,616,0 | .003,786,989 | .0$^{4}$90,879,20 | .298,582,2 | .039,950,15 | .0$^{3}$798,007,1 | .078,304,28 | .007,483,100 | .835,881,8 |
| 16 30 | .284,015,3 | .958,819,7 | .287,979,3 | .003,963,982 | .0$^{4}$98,059,22 | .303,639,1 | .041,180,27 | .0$^{3}$847,907,1 | .080,664,72 | .007,829,905 | .848,278,2 |
| 16 45 | .288,196,3 | .957,571,4 | .292,342,6 | .004,146,382 | .0$^{3}$105,684,6 | .308,716,8 | .042,428,64 | .0$^{3}$900,094,7 | .083,057,09 | .008,187,079 | .860,653,8 |
| 17 00 | .292,371,7 | .956,304,8 | .296,706,0 | .004,334,268 | .0$^{3}$113,775,7 | .313,815,5 | .043,695,24 | .0$^{3}$954,637,2 | .085,481,21 | .008,554,761 | .873,008,4 |
| 17 15 | .296,541,6 | .955,019,9 | .301,069,3 | .004,527,721 | .0$^{3}$122,353,2 | .318,935,5 | .044,980,08 | .001,011,603 | .087,936,91 | .008,933,089 | .885,341,7 |
| 17 30 | .300,705,8 | .953,717,0 | .305,432,6 | .004,726,820 | .0$^{3}$131,438,7 | .324,077,0 | .046,283,05 | .001,071,060 | .090,423,98 | .009,322,200 | .897,653,5 |
| 17 45 | .304,864,3 | .952,395,8 | .309,795,9 | .004,931,643 | .0$^{3}$141,054,2 | .329,240,4 | .047,604,20 | .001,133,080 | .092,942,24 | .009,722,232 | .909,943,4 |
| 18 00 | .309,017,0 | .951,056,5 | .314,159,3 | .005,142,271 | .0$^{3}$151,222,4 | .334,425,9 | .048,943,48 | .001,197,732 | .095,491,50 | .010,133,32 | .922,211,2 |
| 18 15 | .313,163,8 | .949,699,1 | .318,522,6 | .005,358,782 | .0$^{3}$161,966,5 | .339,633,8 | .050,300,87 | .001,265,089 | .098,071,57 | .010,555,60 | .934,456,6 |
| 18 30 | .317,304,7 | .948,323,7 | .322,885,9 | .005,581,255 | .0$^{3}$173,310,4 | .344,864,3 | .051,676,34 | .001,335,222 | .100,682,2 | .010,989,20 | .946,679,3 |
| 18 45 | .321,439,5 | .946,930,1 | .327,249,2 | .005,809,769 | .0$^{3}$185,278,8 | .350,117,8 | .053,069,87 | .001,408,206 | .103,323,3 | .011,434,26 | .958,879,2 |
| 19 00 | .325,568,2 | .945,518,6 | .331,612,6 | .006,044,403 | .0$^{3}$197,896,7 | .355,394,4 | .054,481,42 | .001,484,113 | .105,994,6 | .011,890,91 | .971,055,9 |
| 19 15 | .329,690,6 | .944,089,0 | .335,975,9 | .006,285,236 | .0$^{3}$211,190,2 | .360,694,4 | .055,910,98 | .001,563,019 | .108,695,9 | .012,359,28 | .983,209,1 |
| 19 30 | .333,806,9 | .942,641,5 | .340,339,2 | .006,532,345 | .0$^{3}$225,185,5 | .366,018,2 | .057,358,51 | .001,644,999 | .111,427,0 | .012,839,50 | .995,338,6 |
| 19 45 | .337,916,7 | .941,176,0 | .344,702,5 | .006,785,809 | .0$^{3}$239,910,0 | .371,365,9 | .058,823,98 | .001,730,131 | .114,187,7 | .013,331,71 | 1.007,444,2 |
| 20 00 | .342,020,1 | .939,692,6 | .349,065,9 | .007,045,707 | .0$^{3}$255,391,4 | .376,737,9 | .060,307,38 | .001,818,490 | .116,977,8 | .013,836,02 | 1.019,525,5 |
| 20 15 | .346,117,1 | .938,191,3 | .353,429,2 | .007,312,116 | .0$^{3}$271,658,2 | .382,134,3 | .061,808,66 | .001,910,156 | .119,797,0 | .014,352,57 | 1.031,582,4 |
| 20 30 | .350,207,4 | .936,672,2 | .357,792,5 | .007,585,115 | .0$^{3}$288,739,7 | .387,555,5 | .063,327,81 | .002,005,206 | .122,645,2 | .014,881,49 | 1.043,614,5 |
| 20 45 | .354,291,0 | .935,135,2 | .362,155,8 | .007,864,782 | .0$^{3}$306,665,7 | .393,001,6 | .064,864,79 | .002,103,721 | .125,522,1 | .015,422,90 | 1.055,621,7 |
| 21 00 | .358,367,9 | .933,580,4 | .366,519,1 | .008,151,193 | .0$^{3}$325,466,9 | .398,473,0 | .066,419,57 | .002,205,780 | .128,427,6 | .015,976,92 | 1.067,603,6 |
| 21 15 | .362,438,0 | .932,007,9 | .370,882,5 | .008,444,428 | .0$^{3}$345,174,4 | .403,969,8 | .067,992,13 | .002,311,465 | .131,361,3 | .016,543,68 | 1.079,560,0 |
| 21 30 | .366,501,2 | .930,417,6 | .375,245,8 | .008,744,562 | .0$^{3}$365,820,3 | .409,492,4 | .069,582,43 | .002,420,857 | .134,323,1 | .017,123,30 | 1.091,490,8 |
| 21 45 | .370,557,4 | .928,809,6 | .379,609,1 | .009,051,675 | .0$^{3}$387,437,4 | .415,040,9 | .071,190,45 | .002,534,040 | .137,312,8 | .017,715,91 | 1.103,395,5 |
| 22 00 | .374,606,6 | .927,183,9 | .383,972,4 | .009,365,842 | .0$^{3}$410,058,9 | .420,615,7 | .072,816,15 | .002,651,096 | .140,330,1 | .018,321,63 | 1.115,274,1 |
| 22 15 | .378,648,6 | .925,540,5 | .388,335,8 | .009,687,141 | .0$^{3}$433,719,2 | .426,216,9 | .074,459,50 | .002,772,108 | .143,374,8 | .018,940,56 | 1.127,126,1 |
| 22 30 | .382,683,4 | .923,879,5 | .392,699,1 | .010,015,65 | .0$^{3}$458,453,1 | .431,844,8 | .076,120,47 | .002,897,163 | .146,446,6 | .019,572,85 | 1.138,951,6 |

SHEET 1 OF 4

205-D-230

Table H-1.–*Constants for circular arches* (sheet 2).

| $\Phi$ | SIN $\Phi$ | COS $\Phi$ | $A_1$ | $B_1$ | $B_3$ | | $C_1$ | $B_2$ | | $C_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° ′ | | | (Radians) | ($\Phi$-SIN $\Phi$) | 1ST TERM | 2ND TERM | (VERS $\Phi$) | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| 22 45 | .386,711,0 | .922,201,0 | .397,062,4 | .010,351,44 | .0$^3$484,296,3 | .437,499,6 | .077,799,03 | .003,026,344 | .149,545,4 | .020,218,59 | 1,150,750,0 |
| 23 00 | .390,731,1 | .920,504,9 | .401,425,7 | .010,694,60 | .0$^3$511,285,1 | .443,181,6 | .079,495,15 | .003,159,739 | .152,670,8 | .020,877,91 | 1,162,521,4 |
| 23 15 | .394,743,9 | .918,791,2 | .405,789,1 | .011,045,19 | .0$^3$539,456,6 | .448,890,9 | .081,206,79 | .003,297,434 | .155,822,7 | .021,550,93 | 1,174,265,3 |
| 23 30 | .398,749,1 | .917,060,1 | .410,152,4 | .011,403,31 | .0$^3$568,848,9 | .454,627,9 | .082,939,93 | .003,439,516 | .159,000,8 | .022,237,76 | 1,185,981,6 |
| 23 45 | .402,746,7 | .915,311,5 | .414,515,7 | .011,769,01 | .0$^3$599,500,5 | .460,392,7 | .084,688,52 | .003,586,073 | .162,204,9 | .022,938,51 | 1,197,670,1 |
| 24 00 | .406,736,6 | .913,545,5 | .418,879,0 | .012,142,38 | .0$^3$631,450,9 | .466,185,6 | .086,454,54 | .003,737,194 | .165,434,7 | .023,653,30 | 1,209,330,5 |
| 24 15 | .410,718,9 | .911,762,0 | .423,242,3 | .012,523,49 | .0$^3$664,740,4 | .472,006,8 | .088,237,96 | .003,892,968 | .168,690,0 | .024,382,24 | 1,220,962,5 |
| 24 30 | .414,693,2 | .909,961,3 | .427,605,7 | .012,912,42 | .0$^3$699,409,9 | .477,856,5 | .090,038,73 | .004,053,486 | .171,970,5 | .025,125,44 | 1,232,566,1 |
| 24 45 | .418,659,7 | .908,143,2 | .431,969,0 | .013,309,25 | .0$^3$735,501,1 | .483,735,0 | .091,856,83 | .004,218,838 | .175,276,0 | .025,883,00 | 1,244,141,0 |
| 25 00 | .422,618,3 | .906,307,8 | .436,332,3 | .013,714,05 | .0$^3$773,056,8 | .489,642,4 | .093,692,21 | .004,389,115 | .178,606,2 | .026,655,05 | 1,255,686,8 |
| 25 15 | .426,568,7 | .904,455,1 | .440,695,6 | .014,126,90 | .0$^3$812,120,2 | .495,579,0 | .095,544,85 | .004,564,410 | .181,960,9 | .027,441,67 | 1,267,203,6 |
| 25 30 | .430,511,1 | .902,585,3 | .445,059,0 | .014,547,86 | .0$^3$852,735,6 | .501,544,9 | .097,414,72 | .004,744,813 | .185,339,8 | .028,242,99 | 1,278,690,9 |
| 25 45 | .434,445,3 | .900,698,2 | .449,422,3 | .014,977,02 | .0$^3$894,948,0 | .507,540,5 | .099,301,76 | .004,930,420 | .188,742,7 | .029,059,10 | 1,290,148,6 |
| 26 00 | .438,371,1 | .898,794,0 | .453,785,6 | .015,414,46 | .0$^3$938,803,1 | .513,565,8 | .101,206,0 | .005,121,323 | .192,169,3 | .029,890,11 | 1,301,576,6 |
| 26 15 | .442,288,7 | .896,872,7 | .458,148,9 | .015,860,24 | .0$^3$984,347,6 | .519,621,2 | .103,127,3 | .005,317,616 | .195,619,3 | .030,736,13 | 1,312,974,5 |
| 26 30 | .446,197,8 | .894,934,4 | .462,512,3 | .016,314,44 | .001,031,629 | .525,706,8 | .105,065,6 | .005,519,394 | .199,092,5 | .031,597,25 | 1,324,342,3 |
| 26 45 | .450,098,4 | .892,978,9 | .466,875,6 | .016,777,13 | .001,080,695 | .531,822,7 | .107,021,1 | .005,726,753 | .202,588,6 | .032,473,57 | 1,335,679,6 |
| 27 00 | .453,990,5 | .891,006,5 | .471,238,9 | .017,248,40 | .001,131,596 | .537,969,3 | .108,993,5 | .005,939,789 | .206,107,4 | .033,365,20 | 1,346,986,3 |
| 27 15 | .457,873,9 | .889,017,1 | .475,602,2 | .017,728,31 | .001,184,381 | .544,146,7 | .110,982,9 | .006,158,597 | .209,648,5 | .034,272,23 | 1,358,262,2 |
| 27 30 | .461,748,6 | .887,010,8 | .479,965,5 | .018,216,93 | .001,239,101 | .550,355,1 | .112,989,2 | .006,383,278 | .213,211,8 | .035,194,76 | 1,369,507,1 |
| 27 45 | .465,614,5 | .884,987,6 | .484,328,9 | .018,714,35 | .001,295,808 | .556,594,6 | .115,012,4 | .006,613,922 | .216,796,9 | .036,132,89 | 1,380,720,8 |
| 28 00 | .469,471,6 | .882,947,6 | .488,692,2 | .019,220,63 | .001,354,553 | .562,865,6 | .117,052,4 | .006,850,833 | .220,403,5 | .037,086,70 | 1,391,903,2 |
| 28 15 | .473,319,7 | .880,890,7 | .493,055,5 | .019,735,85 | .001,415,392 | .569,168,1 | .119,109,3 | .007,093,508 | .224,031,5 | .038,056,30 | 1,403,053,9 |
| 28 30 | .477,158,8 | .878,817,1 | .497,418,8 | .020,260,08 | .001,478,377 | .575,502,4 | .121,182,9 | .007,342,646 | .227,680,5 | .039,041,78 | 1,414,173,0 |
| 28 45 | .480,988,8 | .876,726,8 | .501,782,2 | .020,793,39 | .001,543,564 | .581,868,6 | .123,273,2 | .007,598,146 | .231,350,2 | .040,043,22 | 1,425,260,0 |
| 29 00 | .484,809,6 | .874,619,7 | .506,145,5 | .021,335,88 | .001,611,008 | .588,266,9 | .125,380,3 | .007,860,109 | .235,040,4 | .041,060,72 | 1,436,315,0 |
| 29 15 | .488,621,2 | .872,496,0 | .510,508,8 | .021,887,56 | .001,680,767 | .594,697,5 | .127,504,0 | .008,128,634 | .238,750,7 | .042,094,36 | 1,447,337,7 |
| 29 30 | .492,423,6 | .870,355,7 | .514,872,1 | .022,448,57 | .001,752,899 | .601,160,6 | .129,644,3 | .008,403,823 | .242,481,0 | .043,144,24 | 1,458,327,9 |
| 29 45 | .496,216,5 | .868,198,8 | .519,235,5 | .023,018,95 | .001,827,461 | .607,656,3 | .131,801,2 | .008,685,776 | .246,230,8 | .044,210,44 | 1,469,285,5 |
| 30 00 | .500,000,0 | .866,025,4 | .523,598,8 | .023,598,78 | .001,904,514 | .614,184,9 | .133,974,6 | .008,974,596 | .250,000,0 | .045,293,04 | 1,480,210,3 |
| 30 15 | .503,774,0 | .863,835,5 | .527,962,1 | .024,188,12 | .001,984,118 | .620,746,3 | .136,164,5 | .009,270,385 | .253,788,2 | .046,392,13 | 1,491,102,0 |
| 30 30 | .507,538,4 | .861,629,2 | .532,325,4 | .024,787,06 | .002,066,334 | .627,341,0 | .138,370,8 | .009,573,245 | .257,595,2 | .047,507,78 | 1,501,960,7 |
| 30 45 | .511,293,1 | .859,406,4 | .536,686,7 | .025,395,66 | .002,151,224 | .633,968,9 | .140,593,6 | .009,883,279 | .261,420,6 | .048,640,09 | 1,512,786,0 |
| 31 00 | .515,038,1 | .857,167,3 | .541,052,1 | .026,013,99 | .002,238,851 | .640,630,3 | .142,832,7 | .010,200,59 | .265,264,2 | .049,789,14 | 1,523,577,9 |
| 31 15 | .518,773,3 | .854,911,9 | .545,415,4 | .026,642,13 | .002,329,279 | .647,325,4 | .145,088,1 | .010,525,28 | .269,125,7 | .050,954,99 | 1,534,336,2 |
| 31 30 | .522,498,6 | .852,640,2 | .549,778,7 | .027,280,15 | .002,422,573 | .654,054,2 | .147,359,8 | .010,857,46 | .273,004,8 | .052,137,73 | 1,545,060,7 |
| 31 45 | .526,213,9 | .850,352,2 | .554,142,0 | .027,928,11 | .002,518,799 | .660,816,9 | .149,647,8 | .011,197,23 | .276,901,1 | .053,337,43 | 1,555,751,3 |
| 32 00 | .529,919,3 | .848,048,1 | .558,505,4 | .028,586,10 | .002,618,024 | .667,613,7 | .151,951,9 | .011,544,69 | .280,814,4 | .054,554,17 | 1,566,407,7 |
| 32 15 | .533,614,5 | .845,727,8 | .562,868,7 | .029,254,17 | .002,720,315 | .674,444,7 | .154,272,2 | .011,899,95 | .284,744,5 | .055,788,02 | 1,577,030,0 |
| 32 30 | .537,299,6 | .843,391,4 | .567,232,0 | .029,932,40 | .002,825,740 | .681,310,1 | .156,608,6 | .012,263,12 | .288,690,9 | .057,039,06 | 1,587,617,9 |
| 32 45 | .540,974,5 | .841,039,0 | .571,595,3 | .030,620,86 | .002,934,370 | .688,210,0 | .158,961,0 | .012,634,30 | .292,653,4 | .058,307,35 | 1,598,171,3 |
| 33 00 | .544,639,0 | .838,670,6 | .575,958,7 | .031,319,62 | .003,046,274 | .695,144,6 | .161,329,4 | .013,013,59 | .296,631,7 | .059,592,98 | 1,608,690,0 |
| 33 15 | .548,293,2 | .836,286,2 | .580,322,0 | .032,028,75 | .003,161,524 | .702,113,9 | .163,713,8 | .013,401,11 | .300,625,5 | .060,895,97 | 1,619,174,0 |
| 33 30 | .551,937,0 | .833,885,8 | .584,685,3 | .032,748,31 | .003,280,192 | .709,118,2 | .166,114,2 | .013,796,96 | .304,634,4 | .062,216,44 | 1,629,623,0 |
| 33 45 | .555,570,2 | .831,469,6 | .589,048,6 | .033,478,39 | .003,402,351 | .716,157,5 | .168,530,4 | .014,201,25 | .308,658,3 | .063,554,43 | 1,640,037,0 |
| 34 00 | .559,192,9 | .829,037,6 | .593,411,9 | .034,219,04 | .003,528,075 | .723,232,0 | .170,962,4 | .014,614,08 | .312,696,7 | .064,910,01 | 1,650,415,8 |
| 34 15 | .562,804,9 | .826,589,7 | .597,775,3 | .034,970,34 | .003,657,440 | .730,341,8 | .173,410,3 | .015,035,56 | .316,749,4 | .066,283,24 | 1,660,759,3 |
| 34 30 | .566,406,2 | .824,126,2 | .602,138,6 | .035,732,36 | .003,790,521 | .737,487,0 | .175,873,8 | .015,465,80 | .320,816,0 | .067,674,19 | 1,671,067,4 |
| 34 45 | .569,996,8 | .821,646,9 | .606,501,9 | .036,505,15 | .003,927,395 | .744,667,7 | .178,353,1 | .015,904,91 | .324,896,3 | .069,082,91 | 1,681,339,9 |
| 35 00 | .573,576,4 | .819,152,0 | .610,865,2 | .037,288,80 | .004,068,140 | .751,884,2 | .180,848,0 | .016,352,99 | .328,989,9 | .070,509,46 | 1,691,576,8 |
| 35 15 | .577,145,2 | .816,641,6 | .615,228,6 | .038,083,37 | .004,212,835 | .759,136,4 | .183,358,4 | .016,810,16 | .333,096,6 | .071,953,91 | 1,701,777,9 |
| 35 30 | .580,703,0 | .814,115,5 | .619,591,9 | .038,888,93 | .004,361,559 | .766,424,5 | .185,884,5 | .017,276,52 | .337,215,9 | .073,416,30 | 1,711,943,1 |
| 35 45 | .584,249,7 | .811,574,0 | .623,955,2 | .039,705,54 | .004,514,394 | .773,748,6 | .188,426,0 | .017,752,18 | .341,347,7 | .074,896,69 | 1,722,072,2 |
| 36 00 | .587,785,3 | .809,017,0 | .628,318,5 | .040,533,28 | .004,671,421 | .781,108,8 | .190,983,0 | .018,237,25 | .345,491,5 | .076,395,14 | 1,732,165,3 |
| 36 15 | .591,309,6 | .806,444,6 | .632,681,9 | .041,372,21 | .004,832,722 | .788,505,2 | .193,555,4 | .018,731,85 | .349,647,1 | .077,911,69 | 1,742,222,2 |
| 36 30 | .594,822,8 | .803,856,9 | .637,045,2 | .042,222,39 | .004,998,381 | .795,938,0 | .196,143,1 | .019,236,07 | .353,814,1 | .079,446,40 | 1,752,242,7 |
| 36 45 | .598,324,6 | .801,253,8 | .641,408,5 | .043,083,90 | .005,168,483 | .803,407,1 | .198,746,2 | .019,750,02 | .357,992,3 | .080,999,32 | 1,762,226,9 |
| 37 00 | .601,815,0 | .798,635,5 | .645,771,8 | .043,956,80 | .005,343,113 | .810,912,8 | .201,364,5 | .020,273,83 | .362,181,3 | .082,570,49 | 1,772,174,5 |
| 37 15 | .605,294,0 | .796,002,0 | .650,135,1 | .044,841,16 | .005,522,357 | .818,455,1 | .203,998,0 | .020,807,59 | .366,380,8 | .084,159,96 | 1,782,085,5 |
| 37 30 | .608,761,4 | .793,353,3 | .654,498,5 | .045,737,04 | .005,706,303 | .826,034,0 | .206,646,7 | .021,351,42 | .370,590,5 | .085,767,78 | 1,791,959,9 |
| 37 45 | .612,217,3 | .790,689,6 | .658,861,8 | .046,644,51 | .005,895,039 | .833,649,8 | .209,310,4 | .021,905,43 | .374,810,0 | .087,393,99 | 1,801,797,4 |
| 38 00 | .615,661,5 | .788,010,8 | .663,225,1 | .047,563,64 | .006,088,655 | .841,302,4 | .211,989,2 | .022,469,72 | .379,039,1 | .089,038,63 | 1,811,598,1 |
| 38 15 | .619,093,9 | .785,316,9 | .667,588,4 | .048,494,49 | .006,287,240 | .848,991,9 | .214,683,1 | .023,044,41 | .383,277,3 | .090,701,74 | 1,821,361,8 |
| 38 30 | .622,514,6 | .782,608,2 | .671,951,8 | .049,437,13 | .006,490,886 | .856,718,5 | .217,391,8 | .023,629,61 | .387,524,5 | .092,383,36 | 1,831,088,6 |
| 38 45 | .625,923,5 | .779,884,5 | .676,315,1 | .050,391,81 | .006,699,685 | .864,482,2 | .220,115,5 | .024,225,42 | .391,780,2 | .094,083,54 | 1,840,778,2 |
| 39 00 | .629,320,4 | .777,146,0 | .680,678,4 | .051,356,02 | .006,913,731 | .872,283,0 | .222,854,0 | .024,831,96 | .396,044,2 | .095,802,30 | 1,850,430,6 |
| 39 15 | .632,705,3 | .774,392,6 | .685,041,7 | .052,336,40 | .007,133,116 | .880,121,1 | .225,607,4 | .025,449,34 | .400,316,0 | .097,539,69 | 1,860,045,8 |
| 39 30 | .636,078,2 | .771,624,6 | .689,405,1 | .053,326,83 | .007,357,937 | .887,996,5 | .228,375,4 | .026,077,67 | .404,595,5 | .099,295,73 | 1,869,623,7 |
| 39 45 | .639,439,0 | .768,841,8 | .693,768,4 | .054,329,38 | .007,588,289 | .895,909,3 | .231,158,2 | .026,717,05 | .408,882,2 | .101,070,5 | 1,879,164,2 |
| 40 00 | .642,787,6 | .766,044,4 | .698,131,7 | .055,344,09 | .007,824,270 | .903,859,5 | .233,955,6 | .027,367,60 | .413,175,9 | .102,863,9 | 1,888,667,3 |
| 40 15 | .646,124,0 | .763,232,5 | .702,495,0 | .056,371,04 | .008,065,977 | .911,847,2 | .236,767,5 | .028,029,43 | .417,476,2 | .104,676,1 | 1,898,132,8 |
| 40 30 | .649,448,0 | .760,406,0 | .706,858,3 | .057,410,30 | .008,313,509 | .919,872,5 | .239,594,0 | .028,702,65 | .421,782,8 | .106,507,1 | 1,907,560,9 |
| 40 45 | .652,759,8 | .757,565,0 | .711,221,7 | .058,461,92 | .008,566,966 | .927,935,4 | .242,435,0 | .029,387,37 | .426,095,3 | .108,356,9 | 1,916,951,3 |
| 41 00 | .656,059,0 | .754,709,6 | .715,585,0 | .059,525,96 | .008,826,449 | .936,036,0 | .245,290,4 | .030,083,70 | .430,413,4 | .110,225,5 | 1,926,304,0 |
| 41 15 | .659,345,8 | .751,839,8 | .719,948,3 | .060,602,50 | .009,092,060 | .944,174,2 | .248,160,2 | .030,791,74 | .434,736,9 | .112,112,9 | 1,935,619,1 |
| 41 30 | .662,620,0 | .748,955,7 | .724,311,6 | .061,691,59 | .009,363,901 | .952,350,2 | .251,044,3 | .031,511,62 | .439,065,3 | .114,019,3 | 1,944,896,4 |
| 41 45 | .665,881,7 | .746,057,4 | .728,675,0 | .062,793,30 | .009,642,076 | .960,564,0 | .253,942,6 | .032,243,43 | .443,398,4 | .115,944,5 | 1,954,135,9 |
| 42 00 | .669,130,6 | .743,144,8 | .733,038,3 | .063,907,68 | .009,926,690 | .968,815,6 | .256,855,2 | .032,987,29 | .447,735,8 | .117,888,7 | 1,963,337,5 |
| 42 15 | .672,366,8 | .740,218,1 | .737,401,6 | .065,034,80 | .010,217,85 | .977,105,1 | .259,781,9 | .033,743,31 | .452,077,1 | .119,851,8 | 1,972,501,3 |
| 42 30 | .675,590,2 | .737,277,3 | .741,764,9 | .066,174,72 | .010,515,66 | .985,432,5 | .262,722,7 | .034,511,60 | .456,422,1 | .121,833,8 | 1,981,627,2 |
| 42 45 | .678,800,7 | .734,322,5 | .746,128,3 | .067,327,51 | .010,820,23 | .993,797,8 | .265,677,5 | .035,292,26 | .460,770,5 | .123,834,8 | 1,990,715,2 |
| 43 00 | .681,998,4 | .731,353,7 | .750,491,6 | .068,493,22 | .011,131,66 | 1,002,201,1 | .268,646,3 | .036,085,42 | .465,121,8 | .125,854,8 | 1,999,765,2 |
| 43 15 | .685,183,0 | .728,371,0 | .754,854,9 | .069,671,91 | .011,450,07 | 1,010,642,4 | .271,629,0 | .036,891,17 | .469,475,7 | .127,893,8 | 2,008,777,2 |
| 43 30 | .688,354,6 | .725,374,4 | .759,218,2 | .070,863,65 | .011,775,57 | 1,019,121,7 | .274,625,6 | .037,709,62 | .473,832,0 | .129,951,7 | 2,017,751,2 |
| 43 45 | .691,513,1 | .722,364,0 | .763,581,5 | .072,068,49 | .012,108,27 | 1,027,639,0 | .277,636,0 | .038,540,88 | .478,190,3 | .132,028,7 | 2,026,687,2 |
| 44 00 | .694,658,4 | .719,339,8 | .767,944,9 | .073,286,50 | .012,448,27 | 1,036,194,3 | .280,660,2 | .039,385,07 | .482,550,3 | .134,124,7 | 2,035,585,2 |
| 44 15 | .697,790,5 | .716,301,9 | .772,308,2 | .074,517,73 | .012,795,70 | 1,044,787,7 | .283,698,1 | .040,242,29 | .486,911,5 | .136,239,8 | 2,044,445,1 |
| 44 30 | .700,909,3 | .713,250,4 | .776,671,5 | .075,762,25 | .013,150,67 | 1,053,419,2 | .286,749,6 | .041,112,65 | .491,273,8 | .138,373,8 | 2,053,266,9 |
| 44 45 | .704,014,7 | .710,185,4 | .781,034,8 | .077,020,12 | .013,513,29 | 1,062,088,7 | .289,814,6 | .041,996,26 | .495,636,7 | .140,528,9 | 2,062,050,6 |
| 45 00 | .707,106,8 | .707,106,8 | .785,398,2 | .078,291,38 | .013,883,68 | 1,070,796,3 | .292,893,2 | .042,893,22 | .500,000,0 | .142,899,1 | 2,070,796,3 |

SHEET 2 OF 4

205-D-231

Table H-1.–*Constants for circular arches* (sheet 3).

| Φ | SIN Φ | COS Φ | A₁ | B₁ | B₃ | | C₁ | B₂ | | C₂ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° ′ | | | (Radians) | (Φ–SIN Φ) | 1ST TERM | 2ND TERM | (VERS Φ) | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| 45 15 | .710,185,4 | .704,014,7 | .789,781,5 | .079,576,11 | .014,261,96 | 1.079,542,0 | .295,985,3 | .043,803,64 | .504,363,3 | .144,890,3 | 2.079,503,9 |
| 45 30 | .713,250,4 | .700,909,3 | .794,124,8 | .080,874,36 | .014,648,24 | 1.088,325,8 | .299,090,7 | .044,727,63 | .508,726,2 | .147,100,5 | 2.088,173,5 |
| 45 45 | .716,301,9 | .697,790,5 | .798,488,1 | .082,186,19 | .015,042,64 | 1.097,147,6 | .302,209,5 | .045,665,30 | .513,088,5 | .149,329,7 | 2.096,804,9 |
| 46 00 | .719,339,8 | .694,658,4 | .802,851,5 | .083,511,66 | .015,445,29 | 1.106,007,5 | .305,341,6 | .046,616,76 | .517,449,7 | .151,578,0 | 2.105,398,3 |
| 46 15 | .722,364,0 | .691,513,1 | .807,214,8 | .084,850,82 | .015,856,30 | 1.114,905,4 | .308,486,9 | .047,582,10 | .521,809,7 | .153,845,3 | 2.113,953,7 |
| 46 30 | .725,374,4 | .688,354,6 | .811,578,1 | .086,203,73 | .016,275,80 | 1.123,841,4 | .311,645,4 | .048,561,44 | .526,168,0 | .156,131,7 | 2.122,471,0 |
| 46 45 | .728,371,0 | .685,183,0 | .815,941,4 | .087,570,46 | .016,703,90 | 1.132,815,5 | .314,817,0 | .049,554,87 | .530,524,3 | .158,437,0 | 2.130,950,2 |
| 47 00 | .731,353,7 | .681,998,4 | .820,304,7 | .088,951,05 | .017,140,73 | 1.141,827,5 | .318,001,6 | .050,562,52 | .534,878,2 | .160,761,4 | 2.139,391,5 |
| 47 15 | .734,322,5 | .678,800,7 | .824,668,1 | .090,345,56 | .017,588,42 | 1.150,877,5 | .321,199,3 | .051,584,48 | .539,229,5 | .163,104,7 | 2.147,794,8 |
| 47 30 | .737,277,3 | .675,590,2 | .829,031,4 | .091,754,06 | .018,041,09 | 1.159,965,4 | .324,409,8 | .052,620,86 | .543,577,9 | .165,467,0 | 2.156,160,1 |
| 47 45 | .740,218,1 | .672,366,8 | .833,394,7 | .093,176,59 | .018,504,87 | 1.169,091,3 | .327,633,2 | .053,671,75 | .547,922,9 | .167,848,3 | 2.164,487,5 |
| 48 00 | .743,144,8 | .669,130,6 | .837,758,0 | .094,613,22 | .018,977,88 | 1.178,255,1 | .330,869,4 | .054,737,28 | .552,264,2 | .170,248,5 | 2.172,777,0 |
| 48 15 | .746,057,4 | .665,881,7 | .842,121,4 | .096,063,99 | .019,460,26 | 1.187,456,8 | .334,118,3 | .055,817,53 | .556,601,6 | .172,667,7 | 2.181,028,7 |
| 48 30 | .748,955,7 | .662,620,0 | .846,484,7 | .097,528,97 | .019,952,13 | 1.196,696,3 | .337,380,0 | .056,912,62 | .560,934,7 | .175,105,8 | 2.189,242,5 |
| 48 45 | .751,839,8 | .659,345,8 | .850,848,0 | .099,008,20 | .020,453,62 | 1.205,973,6 | .340,654,2 | .058,022,64 | .565,263,1 | .177,562,8 | 2.197,418,5 |
| 49 00 | .754,709,6 | .656,059,0 | .855,211,3 | .100,501,8 | .020,964,86 | 1.215,288,6 | .343,941,0 | .059,147,70 | .569,586,6 | .180,038,6 | 2.205,556,7 |
| 49 15 | .757,565,0 | .652,759,8 | .859,574,7 | .102,009,7 | .021,485,98 | 1.224,641,4 | .347,240,2 | .060,287,89 | .573,904,7 | .182,533,4 | 2.213,657,2 |
| 49 30 | .760,406,0 | .649,448,0 | .863,938,0 | .103,532,0 | .022,017,12 | 1.234,031,6 | .350,552,0 | .061,443,34 | .578,217,2 | .185,046,9 | 2.221,720,1 |
| 49 45 | .763,232,5 | .646,124,0 | .868,301,3 | .105,068,8 | .022,558,42 | 1.243,459,8 | .353,876,0 | .062,614,12 | .582,523,6 | .187,579,3 | 2.229,745,4 |
| 50 00 | .766,044,4 | .642,787,6 | .872,664,6 | .106,620,2 | .023,109,99 | 1.252,925,4 | .357,212,4 | .063,800,35 | .586,824,1 | .190,130,4 | 2.237,733,1 |
| 50 15 | .768,841,8 | .639,439,0 | .877,027,9 | .108,186,1 | .023,671,99 | 1.262,428,4 | .360,561,0 | .065,002,12 | .591,117,8 | .192,700,2 | 2.245,683,4 |
| 50 30 | .771,624,6 | .636,078,2 | .881,391,3 | .109,766,7 | .024,244,54 | 1.271,969,0 | .363,921,8 | .066,219,53 | .595,404,5 | .195,288,8 | 2.253,596,1 |
| 50 45 | .774,392,6 | .632,705,3 | .885,754,6 | .111,362,0 | .024,827,78 | 1.281,546,8 | .367,294,7 | .067,452,89 | .599,684,0 | .197,896,1 | 2.261,471,5 |
| 51 00 | .777,146,0 | .629,320,4 | .890,117,9 | .112,972,0 | .025,421,86 | 1.291,162,0 | .370,679,6 | .068,701,69 | .603,955,6 | .200,522,1 | 2.269,309,6 |
| 51 15 | .779,884,5 | .625,923,5 | .894,481,2 | .114,596,8 | .026,026,90 | 1.300,814,5 | .374,076,5 | .069,966,62 | .608,219,6 | .203,166,6 | 2.277,110,5 |
| 51 30 | .782,606,2 | .622,514,6 | .898,844,6 | .116,236,4 | .026,643,05 | 1.310,504,1 | .377,485,4 | .071,247,60 | .612,475,5 | .205,829,8 | 2.284,874,2 |
| 51 45 | .785,316,9 | .619,093,9 | .903,207,9 | .117,891,0 | .027,270,45 | 1.320,230,8 | .380,906,1 | .072,544,71 | .616,722,7 | .208,511,5 | 2.292,600,7 |
| 52 00 | .788,010,8 | .615,661,5 | .907,571,2 | .119,560,5 | .027,909,24 | 1.329,994,6 | .384,338,5 | .073,858,05 | .620,960,9 | .211,211,7 | 2.300,290,3 |
| 52 15 | .790,689,6 | .612,217,3 | .911,934,5 | .121,245,0 | .028,559,56 | 1.339,795,2 | .387,782,7 | .075,187,72 | .625,190,0 | .213,930,4 | 2.307,942,9 |
| 52 30 | .793,353,3 | .608,761,4 | .916,297,9 | .122,944,5 | .029,221,56 | 1.349,632,8 | .391,238,6 | .076,533,81 | .629,409,5 | .216,667,5 | 2.315,558,6 |
| 52 45 | .796,002,0 | .605,294,0 | .920,661,2 | .124,659,2 | .029,895,38 | 1.359,507,1 | .394,706,0 | .077,896,42 | .633,619,2 | .219,423,0 | 2.323,137,8 |
| 53 00 | .798,635,5 | .601,815,0 | .925,024,5 | .126,389,0 | .030,581,16 | 1.369,418,2 | .398,185,0 | .079,275,64 | .637,818,7 | .222,196,8 | 2.330,679,9 |
| 53 15 | .801,253,8 | .598,324,6 | .929,387,8 | .128,134,0 | .031,279,05 | 1.379,365,8 | .401,675,4 | .080,671,56 | .642,007,7 | .224,989,0 | 2.338,185,5 |
| 53 30 | .803,856,9 | .594,822,8 | .933,751,1 | .129,894,3 | .031,989,19 | 1.389,349,9 | .405,177,2 | .082,084,29 | .646,185,9 | .227,799,4 | 2.345,654,7 |
| 53 45 | .806,444,6 | .591,309,6 | .938,114,5 | .131,669,9 | .032,711,74 | 1.399,370,5 | .408,690,4 | .083,513,90 | .650,352,9 | .230,628,0 | 2.353,087,4 |
| 54 00 | .809,017,0 | .587,785,3 | .942,477,8 | .133,460,8 | .033,446,83 | 1.409,427,3 | .412,214,7 | .084,960,50 | .654,508,5 | .233,474,8 | 2.360,483,9 |
| 54 15 | .811,574,0 | .584,249,7 | .946,841,1 | .135,267,1 | .034,194,63 | 1.419,520,4 | .415,750,3 | .086,424,17 | .658,652,3 | .236,339,6 | 2.367,844,1 |
| 54 30 | .814,115,5 | .580,703,0 | .951,204,4 | .137,088,9 | .034,955,27 | 1.429,649,6 | .419,297,0 | .087,905,01 | .662,784,1 | .239,222,6 | 2.375,168,2 |
| 54 45 | .816,641,6 | .577,145,2 | .955,567,8 | .138,926,2 | .035,728,91 | 1.439,814,8 | .422,854,8 | .089,403,10 | .666,903,4 | .242,123,5 | 2.382,456,3 |
| 55 00 | .819,152,0 | .573,576,4 | .959,931,1 | .140,779,0 | .036,515,70 | 1.450,015,9 | .426,423,6 | .090,918,53 | .671,010,1 | .245,042,4 | 2.389,708,5 |
| 55 15 | .821,646,9 | .569,996,6 | .964,294,4 | .142,647,5 | .037,315,79 | 1.460,252,7 | .430,003,2 | .092,451,39 | .675,103,7 | .247,979,2 | 2.396,924,9 |
| 55 30 | .824,126,2 | .566,406,2 | .968,657,7 | .144,531,5 | .038,129,33 | 1.470,525,3 | .433,593,8 | .094,001,76 | .679,184,0 | .250,933,8 | 2.404,105,7 |
| 55 45 | .826,589,7 | .562,804,9 | .973,021,1 | .146,431,3 | .038,956,48 | 1.480,833,3 | .437,195,1 | .095,569,77 | .683,250,6 | .253,906,1 | 2.411,250,9 |
| 56 00 | .829,037,6 | .559,192,9 | .977,384,4 | .148,346,8 | .039,797,39 | 1.491,176,8 | .440,807,1 | .097,155,45 | .687,303,3 | .256,896,2 | 2.418,360,7 |
| 56 15 | .831,469,6 | .555,570,2 | .981,747,7 | .150,278,1 | .040,652,22 | 1.501,555,6 | .444,429,8 | .098,758,91 | .691,341,7 | .259,904,0 | 2.425,435,2 |
| 56 30 | .833,885,8 | .551,937,0 | .986,111,0 | .152,225,2 | .041,521,11 | 1.511,969,6 | .448,063,0 | .100,380,2 | .695,365,6 | .262,929,3 | 2.432,474,5 |
| 56 45 | .836,286,2 | .548,293,2 | .990,474,4 | .154,188,2 | .042,404,23 | 1.522,418,7 | .451,706,8 | .102,019,5 | .699,374,5 | .265,972,2 | 2.439,478,7 |
| 57 00 | .838,670,6 | .544,639,0 | .994,837,7 | .156,167,1 | .043,301,74 | 1.532,902,8 | .455,361,0 | .103,676,8 | .703,368,3 | .269,032,5 | 2.446,448,1 |
| 57 15 | .841,039,0 | .540,974,5 | .999,201,0 | .158,162,0 | .044,213,79 | 1.543,421,4 | .459,025,5 | .105,352,2 | .707,346,6 | .272,110,2 | 2.453,382,6 |
| 57 30 | .843,391,4 | .537,299,6 | 1.003,564,3 | .160,172,9 | .045,140,54 | 1.553,974,7 | .462,700,4 | .107,045,8 | .711,309,1 | .275,205,2 | 2.460,282,5 |
| 57 45 | .845,727,8 | .533,614,5 | 1.007,927,6 | .162,199,6 | .046,082,14 | 1.564,562,6 | .466,385,5 | .108,757,7 | .715,255,5 | .278,317,5 | 2.467,147,9 |
| 58 00 | .848,048,1 | .529,919,3 | 1.012,291,0 | .164,242,9 | .047,038,77 | 1.575,184,9 | .470,080,7 | .110,488,0 | .719,185,6 | .281,447,0 | 2.473,979,0 |
| 58 15 | .850,352,2 | .526,213,9 | 1.016,654,3 | .166,302,1 | .048,010,58 | 1.585,841,4 | .473,786,1 | .112,236,6 | .723,098,9 | .284,593,6 | 2.480,775,8 |
| 58 30 | .852,640,2 | .522,498,6 | 1.021,017,6 | .168,377,4 | .048,997,72 | 1.596,532,0 | .477,501,4 | .114,003,8 | .726,995,2 | .287,757,2 | 2.487,538,5 |
| 58 45 | .854,911,9 | .518,773,3 | 1.025,380,9 | .170,469,1 | .050,000,37 | 1.607,256,5 | .481,226,7 | .115,789,6 | .730,874,3 | .290,937,8 | 2.494,267,3 |
| 59 00 | .857,167,3 | .515,038,1 | 1.029,744,3 | .172,577,0 | .051,018,69 | 1.618,014,7 | .484,961,9 | .117,594,0 | .734,735,8 | .294,135,2 | 2.500,962,3 |
| 59 15 | .859,406,4 | .511,293,1 | 1.034,107,6 | .174,701,2 | .052,052,83 | 1.628,806,6 | .488,706,9 | .119,417,2 | .738,579,4 | .297,349,5 | 2.507,623,7 |
| 59 30 | .861,629,2 | .507,538,4 | 1.038,470,9 | .176,841,7 | .053,102,96 | 1.639,632,0 | .492,461,6 | .121,259,2 | .742,404,8 | .300,580,5 | 2.514,251,7 |
| 59 45 | .863,835,5 | .503,774,0 | 1.042,834,2 | .178,998,7 | .054,169,26 | 1.650,490,6 | .496,226,0 | .123,120,1 | .746,211,6 | .303,828,2 | 2.520,846,3 |
| 60 00 | .866,025,4 | .500,000,0 | 1.047,197,6 | .181,172,1 | .055,251,87 | 1.661,382,4 | .500,000,0 | .125,000,0 | .750,000,0 | .307,092,4 | 2.527,407,8 |
| 60 15 | .868,198,8 | .496,216,5 | 1.051,560,9 | .183,362,1 | .056,350,97 | 1.672,307,2 | .503,783,5 | .126,898,9 | .753,769,2 | .310,373,1 | 2.533,936,3 |
| 60 30 | .870,355,7 | .492,423,6 | 1.055,924,2 | .185,568,5 | .057,466,73 | 1.683,264,7 | .507,576,4 | .128,816,9 | .757,519,0 | .313,670,3 | 2.540,432,0 |
| 60 45 | .872,496,0 | .488,621,2 | 1.060,287,5 | .187,791,5 | .058,599,31 | 1.694,255,0 | .511,378,8 | .130,754,1 | .761,249,3 | .316,983,7 | 2.546,895,1 |
| 61 00 | .874,619,7 | .484,809,6 | 1.064,650,8 | .190,031,1 | .059,748,88 | 1.705,277,6 | .515,190,4 | .132,710,6 | .764,959,6 | .320,313,4 | 2.553,325,7 |
| 61 15 | .876,726,8 | .480,988,8 | 1.069,014,2 | .192,287,4 | .060,915,60 | 1.716,332,6 | .519,011,2 | .134,686,3 | .768,649,8 | .323,659,2 | 2.559,724,1 |
| 61 30 | .878,817,1 | .477,158,8 | 1.073,377,5 | .194,560,4 | .062,099,65 | 1.727,419,7 | .522,841,2 | .136,681,5 | .772,319,5 | .327,021,1 | 2.566,090,3 |
| 61 45 | .880,890,7 | .473,319,7 | 1.077,740,8 | .196,850,1 | .063,301,20 | 1.738,538,7 | .526,680,3 | .138,698,1 | .775,968,5 | .330,399,0 | 2.572,424,5 |
| 62 00 | .882,947,6 | .469,471,6 | 1.082,104,1 | .199,156,5 | .064,520,41 | 1.749,689,5 | .530,528,4 | .140,730,2 | .779,596,5 | .333,792,7 | 2.578,727,1 |
| 62 15 | .884,987,6 | .465,614,5 | 1.086,467,5 | .201,479,6 | .065,757,46 | 1.760,871,6 | .534,385,5 | .142,783,9 | .783,203,1 | .337,202,2 | 2.584,998,0 |
| 62 30 | .887,010,8 | .461,748,6 | 1.090,830,8 | .203,819,9 | .067,012,52 | 1.772,085,5 | .538,251,4 | .144,857,3 | .786,788,2 | .340,627,4 | 2.591,237,6 |
| 62 45 | .889,017,1 | .457,873,9 | 1.095,194,1 | .206,177,0 | .068,285,76 | 1.783,330,5 | .542,126,1 | .146,950,3 | .790,351,5 | .344,068,2 | 2.597,446,0 |
| 63 00 | .891,006,5 | .453,990,5 | 1.099,557,4 | .208,550,9 | .069,577,34 | 1.794,606,4 | .546,009,5 | .149,063,2 | .793,892,6 | .347,524,5 | 2.603,623,4 |
| 63 15 | .892,978,9 | .450,098,4 | 1.103,920,8 | .210,941,8 | .070,887,46 | 1.805,913,1 | .549,901,6 | .151,195,9 | .797,411,4 | .350,996,2 | 2.609,769,9 |
| 63 30 | .894,934,4 | .446,197,8 | 1.108,284,1 | .213,349,7 | .072,216,27 | 1.817,250,4 | .553,802,2 | .153,348,4 | .800,907,5 | .354,483,2 | 2.615,885,9 |
| 63 45 | .896,872,7 | .442,288,7 | 1.112,647,4 | .215,774,7 | .073,563,95 | 1.828,618,1 | .557,711,3 | .155,521,0 | .804,380,7 | .357,985,4 | 2.621,971,5 |
| 64 00 | .898,794,0 | .438,371,1 | 1.117,010,7 | .218,216,7 | .074,930,68 | 1.840,016,1 | .561,628,9 | .157,713,5 | .807,830,7 | .361,502,7 | 2.628,026,8 |
| 64 15 | .900,698,2 | .434,445,3 | 1.121,374,0 | .220,675,8 | .076,316,63 | 1.851,444,0 | .565,554,7 | .159,926,1 | .811,257,3 | .365,035,0 | 2.634,052,2 |
| 64 30 | .902,585,3 | .430,511,1 | 1.125,737,4 | .223,152,1 | .077,721,97 | 1.862,901,8 | .569,488,9 | .162,158,8 | .814,660,2 | .368,582,2 | 2.640,047,7 |
| 64 45 | .904,455,1 | .426,568,7 | 1.130,100,7 | .225,645,5 | .079,146,89 | 1.874,389,1 | .573,431,3 | .164,411,7 | .818,039,1 | .372,144,2 | 2.646,013,7 |
| 65 00 | .906,307,8 | .422,618,3 | 1.134,464,0 | .228,156,2 | .080,591,56 | 1.885,905,8 | .577,381,7 | .166,684,6 | .821,393,8 | .375,720,9 | 2.651,950,2 |
| 65 15 | .908,143,2 | .418,659,7 | 1.138,827,3 | .230,684,2 | .082,056,15 | 1.897,451,7 | .581,340,3 | .168,978,3 | .824,724,0 | .379,312,2 | 2.657,857,7 |
| 65 30 | .909,961,3 | .414,693,2 | 1.143,190,7 | .233,229,4 | .083,540,85 | 1.909,026,5 | .585,306,8 | .171,292,0 | .828,029,5 | .382,917,9 | 2.663,736,1 |
| 65 45 | .911,762,0 | .410,718,9 | 1.147,554,0 | .235,791,9 | .085,045,82 | 1.920,630,1 | .589,281,1 | .173,626,1 | .831,310,0 | .386,538,1 | 2.669,585,8 |
| 66 00 | .913,545,5 | .406,736,6 | 1.151,917,3 | .238,371,8 | .086,571,25 | 1.932,262,2 | .593,263,4 | .175,980,7 | .834,565,3 | .390,172,4 | 2.675,407,0 |
| 66 15 | .915,311,5 | .402,746,7 | 1.156,280,6 | .240,969,2 | .088,117,32 | 1.943,922,6 | .597,253,3 | .178,355,8 | .837,795,1 | .393,821,0 | 2.681,199,9 |
| 66 30 | .917,060,1 | .398,749,1 | 1.160,644,0 | .243,583,9 | .089,684,21 | 1.955,611,1 | .601,250,9 | .180,751,3 | .840,999,2 | .397,483,6 | 2.686,964,6 |
| 66 45 | .918,791,2 | .394,743,9 | 1.165,007,3 | .246,216,1 | .091,272,09 | 1.967,327,4 | .605,256,1 | .183,167,5 | .844,177,3 | .401,160,0 | 2.692,701,7 |
| 67 00 | .920,504,9 | .390,731,1 | 1.169,370,6 | .248,865,7 | .092,881,14 | 1.979,071,3 | .609,268,9 | .185,604,3 | .847,329,2 | .404,850,3 | 2.698,411,1 |
| 67 15 | .922,201,0 | .386,711,0 | 1.173,733,9 | .251,533,0 | .094,511,55 | 1.990,842,6 | .613,289,0 | .188,061,7 | .850,454,6 | .408,554,3 | 2.704,093,1 |
| 67 30 | .923,879,5 | .382,683,4 | 1.178,097,2 | .254,217,7 | .096,163,50 | 2.002,841,1 | .617,316,6 | .190,539,9 | .853,553,4 | .412,271,9 | 2.709,747,9 |

SHEET 3 OF 4

205-D-232

Table H-1.—*Constants for circular arches* (sheet 4).

| Φ | SIN Φ | COS Φ | $A_1$ | $B_1$ | $B_3$ | | $C_1$ | $B_2$ | | $C_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° ′ | | | (Radians) | (Φ-SIN Φ) | 1ST TERM | 2ND TERM | (VERS Φ) | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| 67 45 | .925,540,5 | .378,648,6 | 1.182,460,6 | .256,920,1 | .097,837,16 | 2.014,466,5 | .621,351,4 | .193,038,8 | .856,625,2 | .416,003,0 | 2.715,375,8 |
| 68 00 | .927,183,9 | .374,606,6 | 1.186,823,9 | .259,640,0 | .099,532,72 | 2.026,318,6 | .625,393,4 | .195,558,5 | .859,669,9 | .419,747,4 | 2.720,977,0 |
| 68 15 | .928,809,6 | .370,557,4 | 1.191,187,2 | .262,377,7 | .101,250,4 | 2.038,197,1 | .629,442,6 | .198,099,0 | .862,687,2 | .423,505,0 | 2.726,551,7 |
| 68 30 | .930,417,6 | .366,501,2 | 1.195,550,5 | .265,133,0 | .102,990,3 | 2.050,101,9 | .633,498,8 | .200,660,3 | .865,676,9 | .427,275,7 | 2.732,100,3 |
| 68 45 | .932,007,9 | .362,438,0 | 1.199,913,9 | .267,906,0 | .104,752,6 | 2.062,032,6 | .637,562,0 | .203,242,6 | .868,638,7 | .431,059,4 | 2.737,622,8 |
| 69 00 | .933,580,4 | .358,367,9 | 1.204,277,2 | .270,696,8 | .106,537,6 | 2.073,989,1 | .641,632,1 | .205,845,8 | .871,572,4 | .434,855,9 | 2.743,119,7 |
| 69 15 | .935,135,2 | .354,291,0 | 1.208,640,5 | .273,505,3 | .108,345,4 | 2.085,970,9 | .645,709,0 | .208,470,0 | .874,477,9 | .438,665,2 | 2.748,591,0 |
| 69 30 | .936,672,2 | .350,207,4 | 1.213,003,8 | .276,331,6 | .110,176,1 | 2.097,978,1 | .649,792,6 | .211,115,2 | .877,354,8 | .442,487,2 | 2.754,037,2 |
| 69 45 | .938,191,3 | .346,117,1 | 1.217,367,2 | .279,175,8 | .112,030,1 | 2.110,010,3 | .653,882,9 | .213,781,5 | .880,203,0 | .446,321,6 | 2.759,458,3 |
| 70 00 | .939,692,6 | .342,020,1 | 1.221,730,5 | .282,037,9 | .113,907,4 | 2.122,067,1 | .657,979,9 | .216,468,8 | .883,022,2 | .450,168,3 | 2.764,854,8 |
| 70 15 | .941,176,0 | .337,916,7 | 1.226,093,8 | .284,917,8 | .115,808,2 | 2.134,148,5 | .662,083,3 | .219,177,1 | .885,812,3 | .454,027,3 | 2.770,226,7 |
| 70 30 | .942,641,5 | .333,806,9 | 1.230,457,1 | .287,815,6 | .117,732,8 | 2.146,254,0 | .666,193,1 | .221,906,7 | .888,573,0 | .457,898,5 | 2.775,574,4 |
| 70 45 | .944,089,0 | .329,690,6 | 1.234,820,4 | .290,731,4 | .119,681,3 | 2.158,383,6 | .670,309,4 | .224,657,3 | .891,304,1 | .461,781,6 | 2.780,898,2 |
| 71 00 | .945,518,6 | .325,568,2 | 1.239,183,8 | .293,665,2 | .121,653,9 | 2.170,536,8 | .674,431,8 | .227,429,2 | .894,005,4 | .465,676,5 | 2.786,198,3 |
| 71 15 | .946,930,1 | .321,439,5 | 1.243,547,1 | .296,617,0 | .123,650,7 | 2.182,713,5 | .678,560,5 | .230,222,2 | .896,676,7 | .469,583,2 | 2.791,474,9 |
| 71 30 | .948,323,7 | .317,304,7 | 1.247,910,4 | .299,586,8 | .125,672,1 | 2.194,913,3 | .682,695,3 | .233,036,5 | .899,317,8 | .473,501,5 | 2.796,728,3 |
| 71 45 | .949,699,1 | .313,163,8 | 1.252,273,7 | .302,574,6 | .127,718,1 | 2.207,136,1 | .686,836,2 | .235,872,0 | .901,928,4 | .477,431,2 | 2.801,958,9 |
| 72 00 | .951,056,5 | .309,017,0 | 1.256,637,1 | .305,580,5 | .129,788,9 | 2.219,381,5 | .690,983,0 | .238,728,8 | .904,508,5 | .481,372,2 | 2.807,166,7 |
| 72 15 | .952,395,8 | .304,864,3 | 1.261,000,4 | .308,604,6 | .131,884,7 | 2.231,649,3 | .695,135,7 | .241,606,8 | .907,057,8 | .485,324,5 | 2.812,352,2 |
| 72 30 | .953,717,0 | .300,705,8 | 1.265,363,7 | .311,646,8 | .134,005,8 | 2.243,939,2 | .699,294,2 | .244,506,2 | .909,576,0 | .489,287,7 | 2.817,515,6 |
| 72 45 | .955,019,9 | .296,541,6 | 1.269,727,0 | .314,707,1 | .136,152,2 | 2.256,250,9 | .703,458,4 | .247,426,9 | .912,063,1 | .493,262,0 | 2.822,657,2 |
| 73 00 | .956,304,8 | .292,371,7 | 1.274,090,4 | .317,785,6 | .138,324,2 | 2.268,584,3 | .707,628,3 | .250,368,9 | .914,518,8 | .497,247,0 | 2.827,777,2 |
| 73 15 | .957,571,4 | .288,196,3 | 1.278,453,7 | .320,882,3 | .140,522,0 | 2.280,938,9 | .711,803,7 | .253,332,3 | .916,942,9 | .501,242,6 | 2.832,875,8 |
| 73 30 | .958,819,7 | .284,015,3 | 1.282,817,0 | .323,997,3 | .142,745,8 | 2.293,314,5 | .715,984,7 | .256,317,0 | .919,335,3 | .505,248,7 | 2.837,953,5 |
| 73 45 | .960,049,9 | .279,829,0 | 1.287,180,3 | .327,130,5 | .144,995,7 | 2.305,710,8 | .720,171,0 | .259,323,1 | .921,695,7 | .509,265,3 | 2.843,010,5 |
| 74 00 | .961,261,7 | .275,637,4 | 1.291,543,6 | .330,282,0 | .147,271,9 | 2.318,127,7 | .724,362,6 | .262,350,6 | .924,024,1 | .513,292,0 | 2.848,046,9 |
| 74 15 | .962,455,2 | .271,440,4 | 1.295,907,0 | .333,451,7 | .149,574,6 | 2.330,564,7 | .728,559,6 | .265,399,5 | .926,320,1 | .517,328,8 | 2.853,063,2 |
| 74 30 | .963,630,5 | .267,238,4 | 1.300,270,3 | .336,639,8 | .151,904,1 | 2.343,021,5 | .732,761,6 | .268,469,8 | .928,583,7 | .521,375,6 | 2.858,059,6 |
| 74 45 | .964,787,3 | .263,031,2 | 1.304,633,6 | .339,846,3 | .154,260,4 | 2.355,498,1 | .736,968,8 | .271,561,5 | .930,814,6 | .525,432,2 | 2.863,036,4 |
| 75 00 | .965,925,8 | .258,819,0 | 1.308,996,9 | .343,071,1 | .156,643,8 | 2.367,993,9 | .741,181,0 | .274,674,6 | .933,012,7 | .529,498,5 | 2.867,993,9 |
| 75 15 | .967,045,9 | .254,601,9 | 1.313,360,3 | .346,314,3 | .159,054,4 | 2.380,508,7 | .745,398,1 | .277,809,1 | .935,177,8 | .533,574,2 | 2.872,932,3 |
| 75 30 | .968,147,6 | .250,380,0 | 1.317,723,6 | .349,575,9 | .161,492,5 | 2.393,042,4 | .749,620,0 | .280,965,1 | .937,309,9 | .537,659,4 | 2.877,852,0 |
| 75 45 | .969,230,9 | .246,153,3 | 1.322,086,9 | .352,856,0 | .163,958,2 | 2.405,594,4 | .753,846,7 | .284,142,4 | .939,408,6 | .541,753,8 | 2.882,753,2 |
| 76 00 | .970,295,7 | .241,921,9 | 1.326,450,2 | .356,154,5 | .166,451,8 | 2.418,164,7 | .758,078,1 | .287,341,2 | .941,473,8 | .545,857,2 | 2.887,636,2 |
| 76 15 | .971,342,1 | .237,685,9 | 1.330,813,6 | .359,471,5 | .168,973,3 | 2.430,752,8 | .762,314,1 | .290,561,4 | .943,505,4 | .549,969,6 | 2.892,501,4 |
| 76 30 | .972,369,9 | .233,445,4 | 1.335,176,9 | .362,807,0 | .171,523,1 | 2.443,358,5 | .766,554,6 | .293,803,0 | .945,503,3 | .554,090,8 | 2.897,349,0 |
| 76 45 | .973,379,3 | .229,200,4 | 1.339,540,2 | .366,160,9 | .174,101,2 | 2.455,981,5 | .770,799,6 | .297,066,0 | .947,467,2 | .558,220,6 | 2.902,179,3 |
| 77 00 | .974,370,1 | .224,951,1 | 1.343,903,5 | .369,533,5 | .176,707,9 | 2.468,621,5 | .775,048,9 | .300,350,4 | .949,397,0 | .562,359,0 | 2.906,992,6 |
| 77 15 | .975,342,3 | .220,697,4 | 1.348,266,8 | .372,924,5 | .179,343,4 | 2.481,278,1 | .779,302,6 | .303,656,2 | .951,292,6 | .566,505,6 | 2.911,789,2 |
| 77 30 | .976,296,0 | .216,439,6 | 1.352,630,2 | .376,334,2 | .182,007,8 | 2.493,951,2 | .783,560,4 | .306,983,4 | .953,153,9 | .570,660,5 | 2.916,569,5 |
| 77 45 | .977,231,1 | .212,177,7 | 1.356,993,5 | .379,762,4 | .184,701,3 | 2.506,640,4 | .787,822,3 | .310,332,0 | .954,980,6 | .574,823,4 | 2.921,333,6 |
| 78 00 | .978,147,6 | .207,911,7 | 1.361,356,8 | .383,209,2 | .187,424,2 | 2.519,345,3 | .792,088,3 | .313,701,9 | .956,772,7 | .578,994,2 | 2.926,082,0 |
| 78 15 | .979,045,5 | .203,641,8 | 1.365,720,1 | .386,674,7 | .190,176,5 | 2.532,065,7 | .796,358,2 | .317,093,2 | .958,530,0 | .583,172,8 | 2.930,814,8 |
| 78 30 | .979,924,7 | .199,367,9 | 1.370,083,5 | .390,158,8 | .192,958,6 | 2.544,801,4 | .800,632,1 | .320,505,9 | .960,252,4 | .587,358,9 | 2.935,532,5 |
| 78 45 | .980,785,3 | .195,090,3 | 1.374,446,8 | .393,661,5 | .195,770,5 | 2.557,551,9 | .804,909,7 | .323,939,6 | .961,939,8 | .591,552,5 | 2.940,235,3 |
| 79 00 | .981,627,2 | .190,809,0 | 1.378,810,1 | .397,182,9 | .198,612,5 | 2.570,316,9 | .809,191,0 | .327,395,0 | .963,591,9 | .595,753,4 | 2.944,923,5 |
| 79 15 | .982,450,4 | .186,524,0 | 1.383,173,4 | .400,723,0 | .201,484,7 | 2.583,096,3 | .813,476,0 | .330,871,6 | .965,208,6 | .599,961,4 | 2.949,597,5 |
| 79 30 | .983,254,9 | .182,235,5 | 1.387,536,8 | .404,281,8 | .204,387,3 | 2.595,889,5 | .817,764,5 | .334,369,4 | .966,790,2 | .604,176,4 | 2.954,257,5 |
| 79 45 | .984,040,7 | .177,943,5 | 1.391,900,1 | .407,859,4 | .207,320,6 | 2.608,696,5 | .822,056,5 | .337,888,4 | .968,336,1 | .608,398,2 | 2.958,903,8 |
| 80 00 | .984,807,8 | .173,648,2 | 1.396,263,4 | .411,455,6 | .210,284,6 | 2.621,516,7 | .826,351,8 | .341,428,7 | .969,846,3 | .612,626,7 | 2.963,536,9 |
| 80 15 | .985,556,1 | .169,349,5 | 1.400,626,7 | .415,070,7 | .213,279,7 | 2.634,350,0 | .830,650,5 | .344,990,1 | .971,320,7 | .616,861,6 | 2.968,156,9 |
| 80 30 | .986,285,6 | .165,047,6 | 1.404,990,0 | .418,704,4 | .216,305,9 | 2.647,196,0 | .834,952,4 | .348,572,8 | .972,759,3 | .621,103,0 | 2.972,764,2 |
| 80 45 | .986,996,4 | .160,742,6 | 1.409,353,4 | .422,357,0 | .219,363,5 | 2.660,054,4 | .839,257,4 | .352,176,5 | .974,161,8 | .625,350,5 | 2.977,359,1 |
| 81 00 | .987,688,3 | .156,434,5 | 1.413,716,7 | .426,028,4 | .222,452,6 | 2.672,924,9 | .843,565,5 | .355,801,4 | .975,528,3 | .629,604,1 | 2.981,941,9 |
| 81 15 | .988,361,5 | .152,123,4 | 1.418,080,0 | .429,718,5 | .225,573,5 | 2.685,807,1 | .847,876,6 | .359,447,4 | .976,858,5 | .633,863,6 | 2.986,512,9 |
| 81 30 | .989,015,9 | .147,809,4 | 1.422,443,3 | .433,427,5 | .228,726,2 | 2.698,700,8 | .852,190,6 | .363,114,4 | .978,152,4 | .638,128,7 | 2.991,072,5 |
| 81 45 | .989,651,4 | .143,492,6 | 1.426,806,7 | .437,155,3 | .231,911,1 | 2.711,605,7 | .856,507,4 | .366,802,4 | .979,409,9 | .642,399,5 | 2.995,621,0 |
| 82 00 | .990,268,1 | .139,173,1 | 1.431,170,0 | .440,901,9 | .235,128,2 | 2.724,521,3 | .860,826,9 | .370,511,5 | .980,630,9 | .646,675,7 | 3.000,156,7 |
| 82 15 | .990,865,9 | .134,850,9 | 1.435,533,3 | .444,667,4 | .238,377,8 | 2.737,447,4 | .865,149,1 | .374,241,5 | .981,815,2 | .650,957,1 | 3.004,685,6 |
| 82 30 | .991,444,9 | .130,526,2 | 1.439,896,6 | .448,451,8 | .241,660,0 | 2.750,383,7 | .869,473,8 | .377,992,4 | .982,962,9 | .655,243,6 | 3.009,202,8 |
| 82 45 | .992,004,9 | .126,199,0 | 1.444,260,0 | .452,255,0 | .244,975,0 | 2.763,329,9 | .873,801,0 | .381,764,1 | .984,073,8 | .659,535,0 | 3.013,709,9 |
| 83 00 | .992,546,2 | .121,869,3 | 1.448,623,3 | .456,077,1 | .248,323,1 | 2.776,285,6 | .878,130,7 | .385,556,7 | .985,147,9 | .663,831,2 | 3.018,207,5 |
| 83 15 | .993,068,5 | .117,537,4 | 1.452,986,6 | .459,918,1 | .251,704,3 | 2.789,250,5 | .882,462,6 | .389,370,1 | .986,185,0 | .668,132,0 | 3.022,695,9 |
| 83 30 | .993,571,9 | .113,203,2 | 1.457,349,9 | .463,778,1 | .255,118,9 | 2.802,224,3 | .886,796,8 | .393,204,3 | .987,185,0 | .672,437,2 | 3.027,175,4 |
| 83 45 | .994,056,3 | .108,866,9 | 1.461,713,2 | .467,656,9 | .258,567,1 | 2.815,206,7 | .891,133,1 | .397,059,1 | .988,148,0 | .676,746,7 | 3.031,646,3 |
| 84 00 | .994,521,9 | .104,528,5 | 1.466,076,6 | .471,554,7 | .262,049,0 | 2.828,197,3 | .895,471,5 | .400,934,6 | .989,073,8 | .681,060,4 | 3.036,109,0 |
| 84 15 | .994,968,5 | .100,188,1 | 1.470,439,9 | .475,471,4 | .265,564,8 | 2.841,195,8 | .899,811,9 | .404,830,8 | .989,962,4 | .685,378,0 | 3.040,563,8 |
| 84 30 | .995,396,2 | .095,845,75 | 1.474,803,2 | .479,407,0 | .269,114,7 | 2.854,201,9 | .904,154,2 | .408,747,5 | .990,813,6 | .689,699,4 | 3.045,010,9 |
| 84 45 | .995,804,9 | .091,501,62 | 1.479,166,5 | .483,361,6 | .272,698,8 | 2.867,215,3 | .908,498,4 | .412,684,7 | .991,627,5 | .694,024,4 | 3.049,450,8 |
| 85 00 | .996,194,7 | .087,155,74 | 1.483,529,9 | .487,335,2 | .276,317,4 | 2.880,235,6 | .912,844,3 | .416,642,3 | .992,403,9 | .698,352,9 | 3.053,883,8 |
| 85 15 | .996,565,5 | .082,808,21 | 1.487,893,2 | .491,327,7 | .279,970,7 | 2.893,262,6 | .917,191,8 | .420,620,4 | .993,142,8 | .702,684,7 | 3.058,310,2 |
| 85 30 | .996,917,3 | .078,459,10 | 1.492,256,5 | .495,339,2 | .283,658,7 | 2.906,295,8 | .921,540,9 | .424,618,6 | .993,844,2 | .707,019,6 | 3.062,730,3 |
| 85 45 | .997,250,2 | .074,108,49 | 1.496,619,8 | .499,369,6 | .287,381,7 | 2.919,335,0 | .925,891,5 | .428,637,5 | .994,507,9 | .711,357,6 | 3.067,144,4 |
| 86 00 | .997,564,1 | .069,756,47 | 1.500,983,2 | .503,419,1 | .291,139,9 | 2.932,379,8 | .930,243,5 | .432,676,5 | .995,134,0 | .715,698,3 | 3.071,552,9 |
| 86 15 | .997,858,9 | .065,403,13 | 1.505,346,5 | .507,487,6 | .294,933,4 | 2.945,429,9 | .934,596,9 | .436,735,7 | .995,722,4 | .720,041,7 | 3.075,956,1 |
| 86 30 | .998,134,8 | .061,048,54 | 1.509,709,8 | .511,575,0 | .298,762,4 | 2.958,484,9 | .938,951,5 | .440,814,9 | .996,273,1 | .724,387,6 | 3.080,354,3 |
| 86 45 | .998,391,7 | .056,692,79 | 1.514,073,1 | .515,681,5 | .302,627,2 | 2.971,544,6 | .943,307,2 | .444,914,2 | .996,785,9 | .728,735,8 | 3.084,747,9 |
| 87 00 | .998,629,5 | .052,335,96 | 1.518,436,4 | .519,806,9 | .306,527,7 | 2.984,608,7 | .947,664,0 | .449,033,6 | .997,260,9 | .733,086,1 | 3.089,137,1 |
| 87 15 | .998,848,4 | .047,978,13 | 1.522,799,8 | .523,951,4 | .310,464,3 | 2.997,676,7 | .952,021,9 | .453,172,8 | .997,698,1 | .737,438,4 | 3.093,522,4 |
| 87 30 | .999,048,2 | .043,619,39 | 1.527,163,1 | .528,114,9 | .314,437,1 | 3.010,748,3 | .956,380,6 | .457,331,9 | .998,097,3 | .741,792,6 | 3.097,904,1 |
| 87 45 | .999,229,0 | .039,259,82 | 1.531,526,4 | .532,297,4 | .318,446,3 | 3.023,823,3 | .960,740,2 | .461,510,9 | .998,458,7 | .746,148,4 | 3.102,282,4 |
| 88 00 | .999,390,8 | .034,899,50 | 1.535,889,7 | .536,498,9 | .322,492,1 | 3.036,901,2 | .965,100,5 | .465,709,5 | .998,782,0 | .750,505,8 | 3.106,657,7 |
| 88 15 | .999,533,6 | .030,538,51 | 1.540,253,1 | .540,719,5 | .326,574,6 | 3.049,981,9 | .969,461,5 | .469,927,8 | .999,067,4 | .754,864,4 | 3.111,030,4 |
| 88 30 | .999,657,3 | .026,176,95 | 1.544,616,4 | .544,959,1 | .330,693,9 | 3.063,064,8 | .973,823,1 | .474,165,7 | .999,314,8 | .759,224,2 | 3.115,400,8 |
| 88 45 | .999,762,0 | .021,814,89 | 1.548,979,7 | .549,217,7 | .334,850,4 | 3.076,149,7 | .978,185,1 | .478,423,1 | .999,524,1 | .763,585,0 | 3.119,769,1 |
| 89 00 | .999,847,7 | .017,452,41 | 1.553,343,0 | .553,495,3 | .339,044,0 | 3.089,236,3 | .982,547,6 | .482,699,9 | .999,695,4 | .767,946,6 | 3.124,135,8 |
| 89 15 | .999,914,3 | .013,089,60 | 1.557,706,4 | .557,792,0 | .343,275,1 | 3.102,324,2 | .986,910,4 | .486,996,1 | .999,828,7 | .772,308,9 | 3.128,501,2 |
| 89 30 | .999,961,9 | .008,726,535 | 1.562,069,7 | .562,107,8 | .347,543,8 | 3.115,413,2 | .991,273,5 | .491,311,5 | .999,923,8 | .776,671,7 | 3.132,865,6 |
| 89 45 | .999,990,5 | .004,363,309 | 1.566,433,0 | .566,442,5 | .351,850,2 | 3.128,502,7 | .995,636,7 | .495,646,2 | .999,981,0 | .781,034,9 | 3.137,229,3 |
| 90 00 | 1.000,000,0 | .000,000,000 | 1.570,796,3 | .570,796,3 | .356,194,5 | 3.141,592,6 | 1.000,000,0 | .500,000,0 | 1.000,000,0 | .785,398,2 | 3.141,592,6 |

SHEET 4 OF 4

205-D-233

Table H-2.–*Trigonometric functions for arch load constants* (sheet 1).

| $\Phi_1$ ° ′ | $\Phi_1$ RADIANS | SIN $\Phi_1$ | COS $\Phi_1$ | VERS $\Phi_1$ | $\Phi_1$ SIN $\Phi_1$ | $\Phi_1$ COS $\Phi_1$ |
|---|---|---|---|---|---|---|
| 0 15 | .004,363,323,130 | .004,363,309,285 | .999,990,480,721 | .0$^{3}$,009,519,279 | .0$^{3}$,019,038,528 | .004,363,281,594 |
| 0 30 | .008,726,646,260 | .008,726,535,498 | .999,961,923,064 | .0$^{3}$,038,076,936 | .0$^{3}$,076,153,388 | .008,726,313,976 |
| 0 45 | .013,089,969,390 | .013,089,595,571 | .999,914,327,574 | .0$^{3}$,085,672,426 | .0$^{3}$,171,342,405 | .013,088,847,941 |
| 1 00 | .017,453,292,520 | .017,452,406,437 | .999,847,695,156 | .0$^{3}$,152,304,844 | .0$^{3}$,304,601,955 | .017,450,634,299 |
| 1 15 | .021,816,615,650 | .021,814,885,035 | .999,762,027,080 | .0$^{3}$,237,972,920 | .0$^{3}$,475,926,962 | .021,811,423,886 |
| 1 30 | .026,179,938,780 | .026,176,948,308 | .999,657,324,976 | .0$^{3}$,342,675,024 | .0$^{3}$,685,310,904 | .026,170,967,569 |
| 1 45 | .030,543,261,910 | .030,538,513,210 | .999,533,590,837 | .0$^{3}$,466,409,163 | .0$^{3}$,932,745,807 | .030,529,016,253 |
| 2 00 | .034,906,585,040 | .034,899,496,703 | .999,390,827,019 | .0$^{3}$,609,172,981 | .001,218,222,250 | .034,885,320,892 |
| 2 15 | .039,269,908,170 | .039,259,815,759 | .999,229,036,241 | .0$^{3}$,770,963,759 | .001,541,729,360 | .039,239,632,494 |
| 2 30 | .043,633,231,300 | .043,619,387,365 | .999,048,221,582 | .0$^{3}$,951,778,418 | .001,903,254,818 | .043,591,702,132 |
| 2 45 | .047,996,554,430 | .047,978,128,521 | .998,848,386,485 | .001,151,613,515 | .002,302,784,857 | .047,941,280,949 |
| 3 00 | .052,359,877,560 | .052,335,956,243 | .998,629,534,755 | .001,370,465,245 | .002,740,304,261 | .052,288,120,167 |
| 3 15 | .056,723,200,690 | .056,692,787,563 | .998,391,670,557 | .001,608,329,443 | .003,215,796,367 | .056,631,971,096 |
| 3 30 | .061,086,523,820 | .061,048,539,535 | .998,134,798,422 | .001,865,201,578 | .003,729,243,064 | .060,972,585,139 |
| 3 45 | .065,449,846,950 | .065,403,129,230 | .997,858,923,239 | .002,141,076,761 | .004,280,624,798 | .065,309,713,803 |
| 4 00 | .069,813,170,080 | .069,756,473,744 | .997,564,050,260 | .002,435,949,740 | .004,869,920,566 | .069,643,108,706 |
| 4 15 | .074,176,493,210 | .074,108,490,195 | .997,250,185,099 | .002,749,814,901 | .005,497,107,920 | .073,972,521,583 |
| 4 30 | .078,539,816,340 | .078,459,095,728 | .996,917,333,733 | .003,082,666,267 | .006,162,162,969 | .078,297,704,297 |
| 4 45 | .082,903,139,470 | .082,808,207,512 | .996,565,502,498 | .003,434,497,502 | .006,865,060,377 | .082,618,408,844 |
| 5 00 | .087,266,462,600 | .087,155,742,748 | .996,194,698,092 | .003,805,301,908 | .007,605,773,365 | .086,934,387,363 |
| 5 15 | .091,629,785,730 | .091,501,618,663 | .995,804,927,575 | .004,195,072,425 | .008,384,273,712 | .091,245,392,142 |
| 5 30 | .095,993,108,860 | .095,845,752,520 | .995,396,198,367 | .004,603,801,633 | .009,200,531,755 | .095,551,175,628 |
| 5 45 | .100,356,431,990 | .100,188,061,612 | .994,968,518,251 | .005,031,481,749 | .010,054,516,391 | .099,851,490,434 |
| 6 00 | .104,719,755,120 | .104,528,463,268 | .994,521,895,368 | .005,478,104,632 | .010,946,195,076 | .104,146,089,344 |
| 6 15 | .109,083,078,250 | .108,866,874,852 | .994,056,338,222 | .005,943,661,778 | .011,875,533,828 | .108,434,725,327 |
| 6 30 | .113,446,401,380 | .113,203,213,768 | .993,571,855,677 | .006,428,144,323 | .012,842,497,227 | .112,717,151,539 |
| 6 45 | .117,809,724,510 | .117,537,397,458 | .993,068,456,955 | .006,931,543,045 | .013,847,048,414 | .116,993,121,333 |
| 7 00 | .122,173,047,640 | .121,869,343,405 | .992,546,151,641 | .007,453,848,359 | .014,889,149,098 | .121,262,388,269 |
| 7 15 | .126,536,370,770 | .126,198,969,136 | .992,004,949,680 | .007,995,050,320 | .015,968,759,549 | .125,524,706,118 |
| 7 30 | .130,899,693,900 | .130,526,192,220 | .991,444,861,374 | .008,555,138,626 | .017,085 838 608 | .129,779,828,873 |
| 7 45 | .135,263,017,030 | .134,850,930,274 | .990,865,897,387 | .009,134,102,613 | .018,240,343,678 | .134,027,510,753 |
| 8 00 | .139,626,340,160 | .139,173,100,960 | .990,268,068,742 | .009,731,931,258 | .019,432,230,736 | .138,267,506,216 |
| 8 15 | .143,989,663,290 | .143,492,621,991 | .989,651,386,820 | .010,348,613,180 | .020,661,454,325 | .142,499,569,963 |
| 8 30 | .148,352,986,419 | .147,809,411,130 | .989,015,863,362 | .010,984,136,638 | .021,927,967,562 | .146,723,456,946 |
| 8 45 | .152,716,309,549 | .152,123,386,190 | .988,361,510,468 | .011,638,489,532 | .023,231,722,135 | .150,938,922,379 |
| 9 00 | .157,079,632,679 | .156,434,465,040 | .987,688,340,595 | .012,311,659,405 | .024,572,668,307 | .155,145,721,742 |
| 9 15 | .161,442,955,809 | .160,742,565,604 | .986,996,366,560 | .013,003,633,440 | .025,950,754,915 | .159,343,610,790 |
| 9 30 | .165,806,278,939 | .165,047,605,861 | .986,285,601,537 | .013,714,398,463 | .027,365,929,376 | .163,532,345,562 |
| 9 45 | .170,169,602,069 | .169,349,503,849 | .985,556,059,058 | .014,443,940,942 | .028,818,137,681 | .167,711,682,387 |
| 10 00 | .174,532,925,199 | .173,648,177,667 | .984,807,753,012 | .015,192,246,988 | .030,307,324,404 | .171,881,377,892 |
| 10 15 | .178,896,248,329 | .177,943,545,474 | .984,040,697,646 | .015,959,302,354 | .031,833,432,700 | .176,041,189,012 |
| 10 30 | .183,259,571,459 | .182,235,525,492 | .983,254,907,564 | .016,745,092,436 | .033,396,404,306 | .180,190,872,995 |
| 10 45 | .187,622,894,589 | .186,524,036,009 | .982,450,397,726 | .017,549,602,274 | .034,996,179,546 | .184,330,187,411 |
| 11 00 | .191,986,217,719 | .190,808,995,377 | .981,627,183,448 | .018,372,816,552 | .036,632,697,329 | .188,458,890,160 |
| 11 15 | .196,349,540,849 | .195,090,322,016 | .980,785,280,403 | .019,214,719,597 | .038,305,895,152 | .192,576,739,479 |
| 11 30 | .200,712,863,979 | .199,367,934,417 | .979,924,704,621 | .020,075,295,379 | .040,015,709,102 | .196,683,493,948 |
| 11 45 | .205,076,187,109 | .203,641,751,140 | .979,045,472,485 | .020,954,527,515 | .041,762,073,860 | .200,778,912,504 |
| 12 00 | .209,439,510,239 | .207,911,690,818 | .978,147,600,734 | .021,852,399,266 | .043,544,922,698 | .204,862,754,439 |
| 12 15 | .213,802,833,369 | .212,177,672,156 | .977,231,106,463 | .022,768,893,537 | .045,364,187,485 | .208,934,779,418 |
| 12 30 | .218,166,156,499 | .216,439,613,938 | .976,296,007,120 | .023,703,992,880 | .047,219,798,687 | .212,994,747,479 |
| 12 45 | .222,529,479,629 | .220,697,435,022 | .975,342,320,509 | .024,657,679,491 | .049,111,685,371 | .217,042,419,043 |
| 13 00 | .226,892,802,759 | .224,951,054,344 | .974,370,064,785 | .025,629,935,215 | .051,039,775,204 | .221,077,554,924 |
| 13 15 | .231,256,125,889 | .229,200,390,922 | .973,379,258,460 | .026,620,741,540 | .053,003,994,457 | .225,099,916,332 |
| 13 30 | .235,619,449,019 | .233,445,363,856 | .972,369,920,398 | .027,630,079,602 | .055,004,268,008 | .229,109,264,887 |
| 13 45 | .239,982,772,149 | .237,685,892,326 | .971,342,069,813 | .028,657,930,187 | .057,040,519,341 | .233,105,362,619 |
| 14 00 | .244,346,095,279 | .241,921,895,600 | .970,295,726,276 | .029,704,273,724 | .059,112,670,552 | .237,087,971,981 |
| 14 15 | .248,709,418,409 | .246,153,293,029 | .969,230,909,707 | .030,769,090,293 | .061,220,642,349 | .241,056,855,857 |
| 14 30 | .253,072,741,539 | .250,380,004,054 | .968,147,640,378 | .031,852,359,622 | .063,364,354,052 | .245,011,777,565 |
| 14 45 | .257,436,064,669 | .254,601,948,206 | .967,045,938,914 | .032,954,061,086 | .065,543,723,603 | .248,952,500,868 |
| 15 00 | .261,799,387,799 | .258,819,045,103 | .965,925,826,289 | .034,074,173,711 | .067,758,667,559 | .252,878,789,982 |
| 15 15 | .266,162,710,929 | .263,031,214,458 | .964,787,323,829 | .035,212,676,171 | .070,009,101,099 | .256,790,409,580 |
| 15 30 | .270,526,034,059 | .267,238,376,078 | .963,630,453,209 | .036,369,546,791 | .072,294,938,029 | .260,687,124,805 |
| 15 45 | .274,889,357,189 | .271,440,449,865 | .962,455,236,454 | .037,544,763,546 | .074,616,090,778 | .264,568,701,272 |
| 16 00 | .279,252,680,319 | .275,637,355,817 | .961,261,695,938 | .038,738,304,062 | .076,972,470,408 | .268,434,905,079 |
| 16 15 | .283,616,003,449 | .279,829,014,031 | .960,049,854,386 | .039,950,145,614 | .079,363,986,609 | .272,285,502,813 |
| 16 30 | .287,979,326,579 | .284,015,344,704 | .958,819,734,868 | .041,180,265,132 | .081,790,547,706 | .276,120,261,558 |
| 16 45 | .292,342,649,709 | .288,196,268,134 | .957,571,360,805 | .042,428,639,195 | .084,252,060,663 | .279,938,948,903 |
| 17 00 | .296,705,972,839 | .292,371,704,723 | .956,304,755,963 | .043,695,244,037 | .086,748,431,080 | .283,741,332,949 |
| 17 15 | .301,069,295,969 | .296,541,574,976 | .955,019,944,457 | .044,980,055,543 | .089,279,563,204 | .287,527,182,314 |
| 17 30 | .305,432,619,099 | .300,705,799,504 | .953,716,950,748 | .046,283,049,252 | .091,845,359,921 | .291,296,266,146 |
| 17 45 | .309,795,942,229 | .304,864,299,028 | .952,395,799,643 | .047,604,200,357 | .094,445,722,769 | .295,048,354,125 |
| 18 00 | .314,159,265,359 | .309,016,994,375 | .951,056,516,295 | .048,943,483,705 | .097,080,551,936 | .298,783,216,474 |
| 18 15 | .318,522,588,489 | .313,163,806,484 | .949,699,126,202 | .050,300,873,798 | .099,749,746,262 | .302,500,623,964 |
| 18 30 | .322,885,911,619 | .317,304,656,405 | .948,323,655,206 | .051,676,344,794 | .102,453,203,244 | .306,200,347,921 |
| 18 45 | .327,249,234,749 | .321,439,465,303 | .946,930,129,495 | .053,069,870,505 | .105,190,819,039 | .309,882,160,238 |
| 19 00 | .331,612,557,879 | .325,568,154,457 | .945,518,575,599 | .054,481,424,401 | .107,962,488,463 | .313,545,833,376 |
| 19 15 | .335,975,881,009 | .329,690,645,263 | .944,089,020,393 | .055,910,979,607 | .110,768,105,003 | .317,191,140,377 |
| 19 30 | .340,339,204,139 | .333,806,859,234 | .942,641,491,092 | .057,358,508,908 | .113,607,560,808 | .320,817,854,867 |
| 19 45 | .344,702,527,269 | .337,916,718,003 | .941,176,015,256 | .058,823,984,744 | .116,480,746,702 | .324,425,751,064 |
| 20 00 | .349,065,850,399 | .342,020,143,326 | .939,692,620,786 | .060,307,379,214 | .119,387,552,184 | .328,014,603,788 |
| 20 15 | .353,429,173,529 | .346,117,057,077 | .938,191,335,922 | .061,808,664,078 | .122,327,865,427 | .331,584,188,467 |
| 20 30 | .357,792,496,659 | .350,207,381,259 | .936,672,189,248 | .063,327,810,752 | .125,301,573,289 | .335,134,281,142 |
| 20 45 | .362,155,819,789 | .354,291,037,998 | .935,135,209,686 | .064,864,790,314 | .128,308,561,310 | .338,664,658,477 |
| 21 00 | .366,519,142,919 | .358,367,949,545 | .933,580,426,497 | .066,419,573,503 | .131,348,713,717 | .342,175,097,766 |
| 21 15 | .370,882,466,049 | .362,438,038,284 | .932,007,869,283 | .067,992,130,717 | .134,421,913,429 | .345,665,376,937 |
| 21 30 | .375,245,789,179 | .366,501,226,724 | .930,417,567,982 | .069,582,432,018 | .137,528,042,057 | .349,135,274,563 |
| 2 45 | .379,609,112,309 | .370,557,437,510 | .928,809,552,872 | .071,190,447,128 | .140,666,979,913 | .352,584,569,870 |
| 22 00 | .383,972,435,439 | .374,606,593,416 | .927 183 854 567 | .072 816 145 433 | .143 838 606 005 | .356 013 042 738 |
| 22 15 | .388,335,758,569 | .378,648,617,352 | .925,540,504,018 | .074,459,495,982 | .147,042,798,050 | .359,420,473,714 |
| 22 30 | .392,699,081,699 | .382,683,432,365 | .923,879,532,511 | .076,120,467,489 | .150,279,432,471 | .362,806,644,018 |

SHEET 1 OF 4

205-D-1857

Table H-2.–*Trigonometric functions for arch load constants* (sheet 2).

| $\Phi_1$ ° ' | $\Phi_1$ RADIANS | SIN $\Phi_1$ | COS $\Phi_1$ | VERS $\Phi_1$ | $\Phi_1$ SIN $\Phi_1$ | $\Phi_1$ COS $\Phi_1$ |
|---|---|---|---|---|---|---|
| 22 45 | .397,062,404,829 | .386,710,961,637 | .922,200,371,670 | .077,799,028,330 | .153,548,384,401 | .366,171,335,547 |
| 23 00 | .401,425,727,959 | .390,731,128,489 | .920,504,853,452 | .079,495,146,548 | .156,849,527,690 | .369,514,330,887 |
| 23 15 | .405,789,051,089 | .394,743,856,384 | .918,791,210,149 | .081,208,789,851 | .160,182,734,905 | .372,835,413,315 |
| 23 30 | .410,152,374,219 | .398,749,068,925 | .917,060,074,385 | .082,939,925,615 | .163,547,877,337 | .376,134,366,810 |
| 23 45 | .414,515,697,349 | .402,746,689,859 | .915,311,479,119 | .084,688,520,881 | .166,944,825,002 | .379,410,976,059 |
| 24 00 | .418,879,020,479 | .406,736,643,076 | .913,545,457,643 | .086,454,542,357 | .170,373,446,645 | .382,665,026,461 |
| 24 15 | .423,242,343,609 | .410,718,852,613 | .911,762,043,577 | .088,237,956,423 | .173,833,609,744 | .385,896,304,137 |
| 24 30 | .427,605,666,739 | .414,693,242,656 | .909,961,270,877 | .090,038,729,123 | .177,325,180,518 | .389,104,595,940 |
| 24 45 | .431,968,989,869 | .418,659,737,537 | .908,143,173,825 | .091,856,826,175 | .180,848,023,923 | .392,289,689,454 |
| 25 00 | .436,332,312,999 | .422,618,261,741 | .906,307,787,037 | .093,692,212,963 | .184,402,003,661 | .395,451,373,007 |
| 25 15 | .440,695,636,128 | .426,568,739,901 | .904,455,145,454 | .095,544,854,546 | .187,986,982,183 | .398,589,435,675 |
| 25 30 | .445,058,959,258 | .430,511,096,808 | .902,585,284,350 | .097,414,715,650 | .191,602,820,694 | .401,703,667,294 |
| 25 45 | .449,422,282,388 | .434,445,257,404 | .900,698,239,323 | .099,301,760,677 | .195,249,379,155 | .404,793,858,459 |
| 26 00 | .453,785,605,518 | .438,371,146,789 | .898,794,046,299 | .101,205,953,701 | .198,926,516,287 | .407,859,800,536 |
| 26 15 | .458,148,928,648 | .442,288,690,219 | .896,872,741,533 | .103,127,258,467 | .202,634,089,577 | .410,901,285,667 |
| 26 30 | .462,512,251,778 | .446,197,813,110 | .894,934,361,602 | .105,065,638,398 | .206,371,955,280 | .413,918,106,778 |
| 26 45 | .466,875,574,908 | .450,098,441,037 | .892,978,943,411 | .107,021,056,589 | .210,139,968,424 | .416,910,057,586 |
| 27 00 | .471,238,898,038 | .453,990,499,740 | .891,006,524,188 | .108,993,475,812 | .213,937,982,817 | .419,876,932,603 |
| 27 15 | .475,602,221,168 | .457,873,915,117 | .889,017,141,486 | .110,982,858,514 | .217,765,851,045 | .422,818,527,147 |
| 27 30 | .479,965,544,298 | .461,748,613,235 | .887,010,833,178 | .112,989,166,822 | .221,623,424,480 | .425,734,637,345 |
| 27 45 | .484,328,867,428 | .465,614,520,325 | .884,987,637,463 | .115,012,362,537 | .225,510,553,287 | .428,625,060,140 |
| 28 00 | .488,692,190,558 | .469,471,562,786 | .882,947,592,859 | .117,052,407,141 | .229,427,086,423 | .431,489,593,302 |
| 28 15 | .493,055,513,688 | .473,319,667,185 | .880,890,738,205 | .119,109,261,795 | .233,372,871,643 | .434,328,035,429 |
| 28 30 | .497,418,836,818 | .477,158,760,260 | .878,817,112,662 | .121,182,887,338 | .237,347,755,506 | .437,140,185,956 |
| 28 45 | .501,782,159,948 | .480,988,768,919 | .876,726,755,708 | .123,273,244,292 | .241,351,583,379 | .439,925,845,163 |
| 29 00 | .506,145,483,078 | .484,809,620,246 | .874,619,707,139 | .125,380,292,861 | .245,384,199,440 | .442,684,814,179 |
| 29 15 | .510,508,806,208 | .488,621,241,497 | .872,496,007,073 | .127,503,992,927 | .249,445,446,685 | .445,416,894,992 |
| 29 30 | .514,872,129,338 | .492,423,560,103 | .870,355,695,940 | .129,644,304,060 | .253,535,166,926 | .448,121,890,450 |
| 29 45 | .519,235,452,468 | .496,216,503,675 | .868,198,814,489 | .131,801,185,511 | .257,653,200,808 | .450,799,604,273 |
| 30 00 | .523,598,775,598 | .500,000,000,000 | .866,025,403,784 | .133,974,596,216 | .261,799,387,799 | .453,449,841,058 |
| 30 15 | .527,962,098,728 | .503,773,977,040 | 863,835,505,204 | .136,164,494,796 | .265,973,566,206 | .456,072,406,283 |
| 30 30 | .532,325,421,858 | .507,538,362,961 | .861,629,150,442 | .138,370,839,558 | .270,175,573,172 | .458,667,106,317 |
| 30 45 | .536,688,744,988 | .511,293,086,077 | .859,406,411,501 | .140,593,588,499 | .274,405,244,688 | .461,233,748,423 |
| 31 00 | .541,052,068,118 | .515,038,074,910 | .857,167,300,702 | .142,832,699,298 | .278,662,415,590 | .463,772,140,768 |
| 31 15 | .545,415,391,248 | .518,773,258,161 | .854,911,870,673 | .145,088,129,327 | .282,946,919,569 | .466,282,092,426 |
| 31 30 | .549,778,714,378 | .522,498,564,716 | .852,640,164,354 | .147,359,835,646 | .287,258,589,174 | .468,763,413,386 |
| 31 45 | .554,142,037,508 | .526,213,923,652 | .850,352,224,996 | .149,647,775,004 | .291,597,255,818 | .471,215,914,559 |
| 32 00 | .558,505,360,638 | .529,919,264,233 | .848,048,096,156 | .151,951,903,844 | .295,962,749,779 | .473,639,407,782 |
| 32 15 | .562,868,683,768 | .533,614,515,916 | .845,727,821,704 | .154,272,178,296 | .300,354,900,213 | .476,033,705,829 |
| 32 30 | .567,232,006,898 | .537,299,608,347 | .843,391,445,813 | .156,608,554,187 | .304,773,535,148 | .478,398,622,409 |
| 32 45 | .571,595,330,028 | .540,974,471,368 | .841,039,012,964 | .158,960,987,036 | .309,218,481,498 | .480,733,972,182 |
| 33 00 | .575,958,653,158 | .544,639,035 015 | .838,670,567,945 | .161,329,432,055 | .313,689,565,065 | .483,039,570,757 |
| 33 15 | .580,321,976,288 | .548,293,229,520 | .836,286,155,848 | .163,713,844,152 | .318,186,610,540 | .485,315,234,704 |
| 33 30 | .584,685,299,418 | .551,936,985,312 | .833,885,822,067 | .166,114,177,933 | .322,709,441,517 | .487,560,781,556 |
| 33 45 | .589,048,622,548 | .555,570,233,020 | .831,469,612,303 | .168,530,387,697 | .327,257,880,489 | .489,776,029,818 |
| 34 00 | .593,411,945,678 | .559,192,903,471 | .829,037,572,555 | .170,962,427,445 | .331,831,748,858 | .491,960,798,970 |
| 34 15 | .597,775,268,808 | .562,804,927,695 | .826,589,749,127 | .173,410,250,873 | .336,430,866,939 | .494,114,909,478 |
| 34 30 | .602,138,591,938 | .566,406,236,925 | .824,126,188,622 | .175,873,811,378 | .341,055,053,967 | .496,238,182,796 |
| 34 45 | .606,501,915,068 | .569,996,762,596 | .821,646,937,942 | .178,353,062,058 | .345,704,128,097 | .498,330,441,372 |
| 35 00 | .610,865,238,198 | .573,576,436,351 | .819,152,044,289 | .180,847,955,711 | .350,377,906,416 | .500,391,508,655 |
| 35 15 | .615,228,561,328 | .577,145,190,037 | .816,641,555,162 | .183,358,444,838 | .355,076,204,944 | .502,421,209,103 |
| 35 30 | .619,591,884,458 | .580,702,955,711 | .814,115,518,356 | .185,884,481,644 | .359,798,838,639 | .504,419,368,185 |
| 35 45 | .623,955,207,588 | .584,249,665,637 | .811,573,981,965 | .188,426,018,035 | .364,545,621,406 | .506,385,812,390 |
| 36 00 | .628,318,530,718 | .587,785,252,292 | .809,016,994,375 | .190,983,005,625 | .369,316,366,098 | .508,320,369,232 |
| 36 15 | .632,681,853,848 | .591,309,648,364 | .806,444,604,267 | .193,555,395,733 | .374,110,884,525 | .510,222,867,253 |
| 36 30 | .637,045,176,978 | .594,822,786,751 | .803,856,860,617 | .196,143,139,383 | .378,918,987,456 | .512,093,136,037 |
| 36 45 | .641,408,500,108 | .598,324,600,571 | .801,253,812,691 | .198,746,187,309 | .383,770,484,630 | .513,931,006,204 |
| 37 00 | .645,771,823,238 | .601,815,023,152 | .798,635,510,047 | .201,364,489,953 | .388,635,184,753 | .515,736,309,426 |
| 37 15 | .650,135,146,368 | .605,293,988,043 | .796,002,002,535 | .203,997,997,465 | .393,522,895,512 | .517,508,878,427 |
| 37 30 | .654,498,469,498 | .608,761,429,009 | .793,353,340,291 | .206,646,659,709 | .398,433,423,576 | .519,248,546,992 |
| 37 45 | .658,861,792,628 | .612,217,280,034 | .790,689,573,744 | .209,310,426,256 | .403,366,574,601 | .520,955,149,969 |
| 38 00 | .663,225,115,758 | .615,661,475,326 | .788,010,753,607 | .211,989,246,393 | .408,322,153,241 | .522,628,523,280 |
| 38 15 | .667,588,438,888 | .619,093,949,310 | .785,316,930,881 | .214,683,069,119 | .413,299,963,145 | .524,268,503,919 |
| 38 30 | .671,951,762,018 | .622,514,636,638 | .782,608,156,852 | .217,391,843,148 | .418,299,806,971 | .525,874,929,966 |
| 38 45 | .676,315,085,148 | .625,923,472,184 | .779,884,483,093 | .220,115,516,907 | 423,321,486,386 | .527,447,640,589 |
| 39 00 | .680,678,408,278 | .629,320,391,050 | .777,145,961,457 | .222,854,038,543 | 428,364,802,077 | .528,986,476,044 |
| 39 15 | .685,041,731,408 | .632,705,328,563 | .774,392,644,082 | .225,607,355,918 | .433,429,553,750 | .530,491,277,692 |
| 39 30 | .689,405,054,538 | .636,078,220,278 | .771,624,583,388 | .228,375,416,612 | .438,515,540,141 | .531,961,887,993 |
| 39 45 | .693,768,377,668 | .639,439,001,981 | .768,841,832,073 | .231,158,167,927 | .443,622,559,022 | .533,398,150,521 |
| 40 00 | .698,131,700,798 | .642,787,609,687 | .766,044,443,119 | .233,955,556,881 | .448,750,407,203 | .534,799,909,962 |
| 40 15 | .702,495,023,928 | .646,123,979,643 | .763,232,469,783 | .236,767,530,217 | .453,896,880,540 | .536,167,012,123 |
| 40 30 | .706,858,347,058 | .649,448,048,330 | .760,405,965,600 | .239,594,034,400 | .459,067,773,943 | .537,499,303,937 |
| 40 45 | .711,221,670,188 | .652,759,752,463 | .757,564,984,384 | .242,435,015,616 | .464,256,881,378 | .538,796,633,470 |
| 41 00 | .715,584,993,318 | .656,059,028,991 | .754,709,580,223 | .245,290,419,777 | .469,465,995,877 | .540,058,849,921 |
| 41 15 | .719,948,316,448 | .659,345,815,100 | .751,839,807,479 | .248,160,192,251 | .474,694,909,538 | .541,285,803,633 |
| 41 30 | .724,311,639,578 | .662,620,048,216 | .748,955,720,789 | .251,044,279,211 | .479,943,413,541 | .542,477,346,096 |
| 41 45 | .728,674,962,707 | .665,881,666,001 | .746,057,375,062 | .253,942,624,938 | .485,211,298,141 | .543,633,329,951 |
| 42 00 | .733,038,285,837 | .669,130,606,359 | .743,144,825,477 | .256,855,174,523 | .490,498,352,686 | .544,753,608,996 |
| 42 15 | .737,401,608,967 | .672,366,807,435 | .740,218,127,487 | .259,781,872,513 | .495,804,365,619 | .545,838,038,195 |
| 42 30 | .741,764,932,097 | .675,590,207,616 | .737,277,336,810 | .262,722,663,190 | .501,129,124,478 | .546,886,473,676 |
| 42 45 | .746,128,255,227 | .678,800,745,533 | .734,322,509,436 | .265,677,490,564 | .506,472,415,911 | .547,898,772,739 |
| 43 00 | .750,491,578,357 | .681,998,360,062 | .731,353,701,619 | .268,646,298,38 | .511,834,025,680 | .548,874,793,865 |
| 43 15 | .754,854,901,487 | .685,182,990,326 | .728,370,969,882 | .271,629,030,118 | .517,213,738,663 | .549,814,396,716 |
| 43 30 | .759,218,224,617 | .688,354,575,694 | .725,374,371,012 | .274,625,628,988 | .522,611,338,865 | .550,717,442,142 |
| 43 45 | .763,581,547,747 | .691,513,055,782 | .722,363,962,060 | .277,636,037,940 | .528,026,609,421 | .551,583,792,186 |
| 44 00 | .767,944,870,877 | .694,658,370,459 | .719,339,800,339 | .280,660,199,66 | .533,459,332,606 | .552,413,310,088 |
| 44 15 | .772,308,194,007 | .697,790,459,842 | .716,301,943,425 | .283,698,056,575 | .538,909,289,836 | .553,205,860,290 |
| 44 30 | .776,671,517,137 | .700,909,264,300 | .713,250,449,154 | .286,749,550,846 | .544,376,261,679 | .553,961,308,443 |
| 44 45 | .781,034,840,267 | .704,014,724,456 | .710,185,375,623 | .289,814,624,377 | .549,860,027,861 | .554,679,521,410 |
| 45 00 | .785,398,163,397 | .707,106,781,187 | .707,106,781,187 | .292,893,218,813 | .555,360,367,270 | .555,360,367,270 |

SHEET 2 OF 4

205-D-1858

Table H-2.–*Trigonometric functions for arch load constants* (sheet 3).

| $\Phi_1$ ° ′ | $\Phi_1$ RADIANS | SIN $\Phi_1$ | COS $\Phi_1$ | VERS $\Phi_1$ | $\Phi_1$ SIN $\Phi_1$ | $\Phi_1$ COS $\Phi_1$ |
|---|---|---|---|---|---|---|
| 45 15 | .789,761,486,527 | .710,185,375,623 | .704,014,724,456 | .295,985,275,544 | .560,877,057,962 | .556,003,715,323 |
| 45 30 | .794,124,809,657 | .713,250,449,154 | .700,909,264,300 | .299,090,735,700 | .566,409,877,172 | .556,609,436,099 |
| 45 45 | .798,488,132,787 | .716,301,943,425 | .697,790,459,842 | .302,209,540,158 | .571,958,601,317 | .557,177,401,356 |
| 46 00 | .802,851,455,917 | .719,339,800,339 | .694,658,370,459 | .305,341,629,541 | .577,523,006,001 | .557,707,484,088 |
| 46 15 | .807,214,779,047 | .722,363,962,060 | .691,513,055,782 | .308,486,944,218 | .583,102,866,026 | .558,199,558,531 |
| 46 30 | .811,578,102,177 | .725,374,371,012 | .688,354,575,694 | .311,645,424,306 | .588,697,955,394 | .558,653,500,167 |
| 46 45 | .815,941,425,307 | .728,370,969,882 | .685,182,990,326 | .314,817,009,674 | .594,308,047,318 | .559,069,185,723 |
| 47 00 | .820,304,748,437 | .731,353,701,619 | .681,998,360,062 | .318,001,639,938 | .599,932,914,225 | .559,446,493,185 |
| 47 15 | .824,668,071,567 | .734,322,509,436 | .678,800,745,533 | .321,199,254,467 | .605,572,327,765 | .559,785,301,797 |
| 47 30 | .829,031,394,697 | .737,277,336,810 | .675,590,207,616 | .324,409,792,384 | .611,226,058,814 | .560,085,492,064, |
| 47 45 | .833,394,717,827 | .740,218,127,487 | .672,366,807,435 | .327,633,192,565 | .616,893,877,487 | .560,346,945,759 |
| 48 00 | .837,758,040,957 | .743,144,825,477 | .669,130,606,359 | .330,869,393,641 | .622,575,553,139 | .560,569,545,928 |
| 48 15 | .842,121,364,087 | .746,057,375,062 | .665,881,666,001 | .334,118,333,999 | .628,270,854,374 | .560,753,176,893 |
| 48 30 | .846,484,687,217 | .748,955,720,789 | .662,620,048,216 | .337,379,951,784 | .633,979,549,051 | .560,897,724,258 |
| 48 45 | .850,848,010,347 | .751,839,807,479 | .659,345,815,100 | .340,654,184,900 | .639,701,404,293 | .561,003,074,908 |
| 49 00 | .855,211,333,477 | .754,709,580,223 | .656,059,028,991 | .343,940,971,009 | .645,436,186,490 | .561,069,117,023 |
| 49 15 | .859,574,656,607 | .757,564,984,384 | .652,759,752,463 | .347,240,247,537 | .651,183,661,309 | .561,095,740,070 |
| 49 30 | .863,937,979,737 | .760,405,965,600 | .649,448,048,330 | .350,551,951,670 | .656,943,593,700 | .561,082,834,818 |
| 49 45 | .868,301,302,867 | .763,232,469,783 | .646,123,979,643 | .353,876,020,357 | .662,715,747,903 | .561,030,293,338 |
| 50 00 | .872,664,625,997 | .766,044,443,119 | .642,787,609,687 | .357,212,390,313 | .668,499,887,452 | .560,938,009,003 |
| 50 15 | .877,027,949,127 | .768,841,832,073 | .639,439,001,981 | .360,560,998,019 | .674,295,775,186 | .560,805,876,499 |
| 50 30 | .881,391,272,257 | .771,624,583,388 | .636,078,220,278 | .363,921,779,722 | .680,103,173,257 | .560,633,791,826 |
| 50 45 | .885,754,595,387 | .774,392,644,082 | .632,705,328,563 | .367,294,671,437 | .685,921,843,130 | .560,421,652,301 |
| 51 00 | .890,117,918,517 | .777,145,961,457 | .629,320,391,050 | .370,679,608,950 | .691,751,545,596 | .560,169,356,562 |
| 51 15 | .894,481,241,647 | .779,884,483,093 | .625,923,472,184 | .374,076,527,816 | .697,592,040,778 | .559,876,804,575 |
| 51 30 | .898,844,564,777 | .782,608,156,852 | .622,514,636,638 | .377,485,363,362 | .703,443,088,137 | .559,543,897,636 |
| 51 45 | .903,207,887,907 | .785,316,930,881 | .619,093,949,310 | .380,906,050,690 | .709,304,446,479 | .559,170,538,372 |
| 52 00 | .907,571,211,037 | .788,010,753,607 | .615,661,475,326 | .384,338,524,674 | .715,175,873,961 | .558,756,630,750 |
| 52 15 | .911,934,534,167 | .790,689,573,744 | .612,217,280,034 | .387,782,719,966 | .721,057,128,103 | .558,302,080,077 |
| 52 30 | .916,297,857,297 | .793,353,340,291 | .608,761,429,009 | .391,238,570,991 | .726,947,965,788 | .557,806,793,006 |
| 52 45 | .920,661,180,427 | .796,002,002,535 | .605,293,988,043 | .394,706,011,957 | .732,848,143,276 | .557,270,677,537 |
| 53 00 | .925,024,503,557 | .798,635,510,047 | .601,815,023,152 | .398,184,976,848 | .738,757,416,204 | .556,693,643,024 |
| 53 15 | .929,387,826,687 | .801,253,812,691 | .598,324,600,571 | .401,675,399,429 | .744,675,539,601 | .556,075,600,178 |
| 53 30 | .933,751,149,817 | .803,856,860,617 | .594,822,786,751 | .405,177,213,249 | .750,602,267,889 | .555,416,461,066 |
| 53 45 | .938,114,472,947 | .806,444,604,267 | .591,309,648,364 | .408,690,351,636 | .756,537,354,893 | .554,716,139,123 |
| 54 00 | .942,477,796,077 | .809,016,994,375 | .587,785,252,292 | .412,214,747,708 | .762,480,553,847 | .553,974,549,147 |
| 54 15 | .946,841,119,207 | .811,573,981,965 | .584,249,665,637 | .415,750,334,363 | .768,431,617,403 | .553,191,607,308 |
| 54 30 | .951,204,442,337 | .814,115,518,356 | .580,702,955,711 | .419,297,044,289 | .774,390,297,636 | .552,367,231,151 |
| 54 45 | .955,567,765,467 | .816,641,555,162 | .577,145,190,037 | .422,854,809,963 | .780,356,346,054 | .551,501,339,594 |
| 55 00 | .959,931,088,597 | .819,152,044,289 | .573,576,436,351 | .426,423,563,649 | .786,329,513,601 | .550,593,852,940 |
| 55 15 | .964,294,411,727 | .821,646,937,942 | .569,996,762,596 | .430,003,237,404 | .792,309,550,670 | .549,644,692,874 |
| 55 30 | .968,657,734,857 | .824,126,188,622 | .566,406,236,925 | .433,593,763,075 | .798,296,207,107 | .548,653,782,469 |
| 55 45 | .973,021,057,987 | .826,589,749,127 | .562,804,927,695 | .437,195,072,305 | .804,289,232,217 | .547,621,046,186 |
| 56 00 | .977,384,381,117 | .829,037,572,555 | .559,192,903,471 | .440,807,096,529 | .810,288,374,774 | .546,546,409,884 |
| 56 15 | .981,747,704,247 | .831,469,612,303 | .555,570,233,020 | .444,429,766,980 | .816,293,383,030 | .545,429,800,815 |
| 56 30 | .986,111,027,377 | .833,885,822,067 | .551,936,985,312 | .448,063,014,688 | .822,304,004,714 | .544,271,147,633 |
| 56 45 | .990,474,350,507 | .836,286,155,848 | .548,293,229,520 | .451,706,770,480 | .828,319,987,052 | .543,070,380,396 |
| 57 00 | .994,837,673,637 | .838,670,567,945 | .544,639,035,015 | .455,360,964,985 | .834,341,076,762 | .541,827,430,566 |
| 57 15 | .999,200,996,767 | .841,039,012,964 | .540,974,471,368 | .459,025,528,632 | .840,367,020,074 | .540,542,231,016 |
| 57 30 | 1.003,564,319,897 | .843,391,445,813 | .537,299,608,347 | .462,700,391,653 | .846,397,562,724 | .539,214,716,032 |
| 57 45 | 1.007,927,643,027 | .845,727,821,704 | .533,614,515,916 | .466,385,484,084 | .852,432,449,972 | .537,844,821,312 |
| 58 00 | 1.012,290,966,157 | .848,048,096,156 | .529,919,264,233 | .470,080,735,767 | .858,471,426,605 | .536,432,483,976 |
| 58 15 | 1.016,654,289,287 | .850,352,224,996 | .526,213,923,652 | .473,786,076,348 | .864,514,236,947 | .534,977,642,563 |
| 58 30 | 1.021,017,612,416 | .852,640,164,354 | .522,498,564,716 | .477,501,435,284 | .870,560,624,859 | .533,480,237,037 |
| 58 45 | 1.025,380,935,546 | .854,911,870,673 | .518,773,258,161 | .481,226,741,839 | .876,610,333,760 | .531,940,208,789 |
| 59 00 | 1.029,744,258,676 | .857,167,300,702 | .515,038,074,910 | .484,961,925,090 | .882,663,106,623 | .530,357,500,638 |
| 59 15 | 1.034,107,581,806 | .859,406,411,501 | .511,293,086,077 | .488,706,913,923 | .888,718,685,986 | .528,732,056,837 |
| 59 30 | 1.038,470,904,936 | .861,629,160,442 | .507,538,362,961 | .492,461,637,039 | .894,776,813,963 | .527,063,823,074 |
| 59 45 | 1.042,834,228,066 | .863,835,505,204 | .503,773,977,046 | .496,226,022,954 | .900,837,232,245 | .525,352,746,473 |
| 60 00 | 1.047,197,551,196 | .866,025,403,784 | .500,000,000,000 | .500,000,000,000 | .906,899,682,116 | .523,598,775,598 |
| 60 15 | 1.051,560,874,326 | .868,198,814,489 | .496,216,503,675 | .503,783,496,325 | .912,963,904,453 | .521,801,860,459 |
| 60 30 | 1.055,924,197,456 | .870,355,695,940 | .492,423,560,103 | .507,576,439,897 | .919,029,639,737 | .519,961,952,510 |
| 60 45 | 1.060,287,520,586 | .872,496,007,073 | .488,621,241,497 | .511,378,758,503 | .925,096,628,061 | .518,079,004,653 |
| 61 00 | 1.064,650,843,716 | .874,619,707,139 | .484,809,620,246 | .515,190,379,754 | .331,164,609,136 | .516,152,971,237 |
| 61 15 | 1.069,014,166,846 | .876,726,755,708 | .480,988,768,919 | .519,011,231,081 | .937,233,322,305 | .514,183,808,068 |
| 61 30 | 1.073,377,489,976 | .878,817,112,662 | .477,158,760,260 | .522,841,239,740 | .943,302,506,537 | .512,171,472,408 |
| 61 45 | 1.077,740,813,106 | .880,890,738,205 | .473,319,667,185 | .526,680,332,815 | .949,371,900,451 | .510,115,922,971 |
| 62 00 | 1.082,104,136,236 | .882,947,592,859 | .469,471,562,786 | .530,528,437,214 | .955,441,242,312 | .508,017,119,936 |
| 62 15 | 1.086,467,459,366 | .884,987,637,463 | .465,614,520,325 | .534,385,479,675 | .961,510,270,045 | .505,875,024,941 |
| 62 30 | 1.090,830,782,496 | .887,010,833,178 | .461,748,613,235 | .538,251,386,765 | .967,578,721,238 | .503,689,601,092 |
| 62 45 | 1.095,194,105,626 | .889,017,141,486 | .457,873,915,117 | .542,126,084,883 | .973,646,333,156 | .501,460,812,956 |
| 63 00 | 1.099,557,428,756 | .891,006,524,188 | .453,990,499,740 | .546,009,500,260 | .979,712,842,741 | .499,188,626,574 |
| 63 15 | 1.103,920,751,886 | .892,978,943,411 | .450,098,441,037 | .549,901,558,963 | .985,777,986,629 | .496,873,009,452 |
| 63 30 | 1.108,284,075,016 | .894,934,361,602 | .446,197,813,110 | .553,802,186,890 | .991,841,501,148 | .494,513,930,577 |
| 63 45 | 1.112,647,398,146 | .896,872,741,533 | .442,288,690,219 | .557,711,309,781 | .997,903,122,335 | .492,111,360,402 |
| 64 00 | 1.117,010,721,276 | .898,794,046,299 | .438,371,146,789 | .561,628,853,211 | 1.003,962,585,935 | .489,665,270,861 |
| 64 15 | 1.121,374,044,406 | .900,698,239,323 | .434,445,257,404 | .565,554,742,596 | 1.010,019,627,419 | .487,175,635,368 |
| 64 30 | 1.125,737,367,536 | .902,585,284,350 | .430,511,096,808 | .569,488,903,192 | 1.016,073,981,981 | .484,642,428,816 |
| 64 45 | 1.130,100,690,666 | .904,455,145,454 | .426,568,739,901 | .573,431,260,099 | 1.022,125,384,554 | .482,065,627,579 |
| 65 00 | 1.134,464,013,796 | .906,307,787,037 | .422,618,261,741 | .577,381,738,259 | 1.028,173,569,817 | .479,445,209,518 |
| 65 15 | 1.138,827,336,926 | .908,143,173,825 | .418,659,737,537 | .581,340,262,463 | 1.034,218,272,195 | .476,781,153,977 |
| 65 30 | 1.143,190,660,056 | .909,961,270,877 | .414,693,242,656 | .585,306,757,344 | 1.040,259,225,879 | .474,073,441,793 |
| 65 45 | 1.147,553,983,186 | .911,762,043,577 | .410,718,852,613 | .589,281,147,387 | 1.046,296,164,825 | .471,322,055,286 |
| 66 00 | 1.151,917,306,316 | .913,545,457,643 | .406,736,643,076 | .593,263,356,924 | 1.052,328,822,765 | .468,526,978,272 |
| 66 15 | 1.156,280,629,446 | .915,311,479,119 | .402,746,689,859 | .597,253,310,141 | 1.058,356,933,215 | .465,688,196,057 |
| 66 30 | 1.160,643,952,576 | .917,060,074,385 | .398,749,068,925 | .601,250,931,075 | 1.064,380,229,484 | .462,805,695,443 |
| 66 45 | 1.165,007,275,706 | .918,791,210,149 | .394,743,856,384 | .605,256,143,616 | 1.070,398,444,678 | .459,879,464,728 |
| 67 00 | 1.169,370,598,836 | .920,504,853,452 | .390,731,128,489 | .609,268,871,511 | 1.076,411,311,713 | .456,909,493,705 |
| 67 15 | 1.173,733,921,966 | .922,200,971,670 | .386,710,961,637 | .613,289,038,363 | 1.082,418,563,319 | .453,895,773,669 |
| 67 30 | 1.178,097,245,096 | .923,879,532,511 | .382,683,432,365 | .617,316,567,635 | 1.088,419,932,052 | .450,838,297,413 |

SHEET 3 OF 4

205-D-1859

Table H-2.–*Trigonometric functions for arch load constants* (sheet 4).

| $\Phi_1$ ° ′ | $\Phi_1$ RADIANS | SIN $\Phi_1$ | COS $\Phi_1$ | VERS $\Phi_1$ | $\Phi_1$ SIN $\Phi_1$ | $\Phi_1$ COS $\Phi_1$ |
|---|---|---|---|---|---|---|
| 67 45 | 1.182,460,568,226 | .925,540,504,018 | .378,648,617,352 | .621,351,382,648 | 1.094,415,150,297 | .447,737,059,232 |
| 68 00 | 1.186,823,891,356 | .927,183,854,567 | .374,606,593,416 | .625,393,406,584 | 1.100,403,950,280 | .444,592,054,926 |
| 68 15 | 1.191,187,214,486 | .928,809,552,872 | .370,557,437,510 | .629,442,562,490 | 1.106,386,064,074 | .441,403,281,795 |
| 68 30 | 1.195,550,537,616 | .930,417,567,982 | .366,501,226,724 | .633,498,773,276 | 1.112,361,223,608 | .438,170,738,647 |
| 68 45 | 1.199,913,860,746 | .932,007,869,283 | .362,438,038,284 | .637,561,961,716 | 1.118,329,160,677 | .434,894,425,799 |
| 69 00 | 1.204,277,183,876 | .933,580,426,497 | .358,367,949,545 | .641,632,050,455 | 1.124,289,606,944 | .431,574,345,069 |
| 69 15 | 1.208,640,507,006 | .935,135,209,686 | .354,291,037,998 | .645,708,962,002 | 1.130,242,293,954 | .428,210,499,794 |
| 69 30 | 1.213,003,830,136 | .936,672,189,248 | .350,207,381,259 | .649,792,618,741 | 1.136,186,953,140 | .424,802,894,809 |
| 69 45 | 1.217,367,153,266 | .938,191,335,922 | .346,117,057,077 | .653,882,942,923 | 1.142,123,315,830 | .421,351,536,471 |
| 70 00 | 1.221,730,476,376 | .939,692,620,786 | .342,020,143,326 | .657,979,856,674 | 1.148,051,113,259 | .417,856,432,643 |
| 70 15 | 1.226,093,799,526 | .941,176,015,256 | .337,916,718,003 | .662,083,281,997 | 1.153,970,076,568 | .414,317,592,700 |
| 70 30 | 1.230,457,122,656 | .942,641,491,092 | .333,806,859,234 | .666,193,140,766 | 1.159,879,936,825 | .410,735,027,536 |
| 70 45 | 1.234,820,445,786 | .944,089,020,393 | .329,690,645,263 | .670,309,354,737 | 1.165,780,425,023 | .407,108,749,555 |
| 71 00 | 1.239,183,768,916 | .945,518,575,599 | .325,568,154,457 | .674,431,845,543 | 1.171,671,272,091 | .403,438,772,679 |
| 71 15 | 1.243,547,092,046 | .946,930,129,495 | .321,439,465,303 | .678,560,534,697 | 1.177,552,208,904 | .399,725,112,346 |
| 71 30 | 1.247,910,415,176 | .948,323,655,206 | .317,304,656,405 | .682,695,343,595 | 1.183,422,966,289 | .395,967,785,512 |
| 71 45 | 1.252,273,738,306 | .949,699,126,202 | .313,163,806,484 | .686,836,193,516 | 1.189,283,275,035 | .392,166,810,648 |
| 72 00 | 1.256,637,061,436 | .951,056,516,295 | .309,016,994,375 | .690,983,005,625 | 1.195,132,865,897 | .388,322,207,745 |
| 72 15 | 1.261,000,384,566 | .952,395,799,643 | .304,864,299,028 | .695,135,700,972 | 1.200,971,469,609 | .384,433,998,315 |
| 72 30 | 1.265,363,707,696 | .953,716,950,748 | .300,705,799,504 | .699,294,200,496 | 1.206,798,816,891 | .380,502,205,386 |
| 72 45 | 1.269,727,030,826 | .955,019,944,457 | .296,541,574,976 | .703,458,425,024 | 1.212,614,638,455 | .376,526,853,511 |
| 73 00 | 1.274,090,353,956 | .956,304,755,963 | .292,371,704,723 | .707,628,295,277 | 1.218,418,665,015 | .372,507,968,757 |
| 73 15 | 1.278,453,677,086 | .957,571,360,805 | .288,196,268,134 | .711,803,731,866 | 1.224,210,627,293 | .368,445,578,718 |
| 73 30 | 1.282,817,000,216 | .958,819,734,868 | .284,015,344,704 | .715,984,655,296 | 1.229,990,256,031 | .364,339,712,508 |
| 73 45 | 1.287,180,323,346 | .960,049,854,386 | .279,829,014,031 | .720,170,985,969 | 1.235,757,281,997 | .360,190,400,762 |
| 74 00 | 1.291,543,646,476 | .961,261,695,938 | .275,637,355,817 | .724,362,644,183 | 1.241,511,435,989 | .355,997,675,637 |
| 74 15 | 1.295,906,969,606 | .962,455,236,454 | .271,440,449,865 | .728,559,550,135 | 1.247,252,448,855 | .351,761,570,813 |
| 74 30 | 1.300,270,292,736 | .963,630,453,209 | .267,238,376,078 | .732,761,623,922 | 1.252,980,051,483 | .347,482,121,493 |
| 74 45 | 1.304,633,615,866 | .964,787,323,829 | .263,031,214,458 | .736,968,785,542 | 1.258,693,974,829 | .343,159,364,404 |
| 75 00 | 1.308,996,938,996 | .965,925,826,289 | .258,819,045,103 | .741,180,954,897 | 1.264,393,949,909 | .338,793,337,794 |
| 75 15 | 1.313,360,262,125 | .967,045,938,914 | .254,601,948,206 | .745,398,051,794 | 1.270,079,707,819 | .334,384,081,433 |
| 75 30 | 1.317,723,585,255 | .968,147,640,378 | .250,380,004,054 | .749,619,995,946 | 1.275,750,979,735 | .329,931,636,618 |
| 75 45 | 1.322,086,908,385 | .969,230,909,707 | .246,153,293,029 | .753,846,706,971 | 1.281,407,496,926 | .325,436,046,169 |
| 76 00 | 1.326,450,231,515 | .970,295,726,276 | .241,921,895,600 | .758,078,104,400 | 1.287,048,990,757 | .320,897,354,427 |
| 76 15 | 1.330,813,554,645 | .971,342,069,813 | .237,685,892,326 | .762,314,107,674 | 1.292,675,192,704 | .316,315,607,255 |
| 76 30 | 1.335,176,877,775 | .972,369,920,398 | .233,445,363,856 | .766,554,636,144 | 1.298,285,834,359 | .311,690,852,044 |
| 76 45 | 1.339,540,200,905 | .973,379,258,460 | .229,200,390,922 | .770,799,609,078 | 1.303,880,647,434 | .307,023,137,703 |
| 77 00 | 1.343,903,524,035 | .974,370,064,785 | .224,951,054,344 | .775,048,945,656 | 1.309,459,363,779 | .302,312,514,668 |
| 77 15 | 1.348,266,847,165 | .975,342,320,509 | .220,697,435,022 | .779,302,564,978 | 1.315,021,715,379 | .297,559,034,895 |
| 77 30 | 1.352,630,170,295 | .976,296,007,120 | .216,439,613,938 | .783,560,386,062 | 1.320,567,434,369 | .292,762,751,860 |
| 77 45 | 1.356,993,493,425 | .977,231,106,463 | .212,177,672,156 | .787,822,327,844 | 1.326,096,253,043 | .287,923,720,566 |
| 78 00 | 1.361,356,816,555 | .978,147,600,734 | .207,911,690,818 | .792,088,309,182 | 1.331,607,903,856 | .283,041,997,537 |
| 78 15 | 1.365,720,139,685 | .979,045,472,485 | .203,641,751,140 | .796,358,248,860 | 1.337,102,119,440 | .278,117,640,813 |
| 78 30 | 1.370,083,462,815 | .979,924,704,621 | .199,367,934,417 | .800,632,065,583 | 1.342,578,632,605 | .273,150,709,960 |
| 78 45 | 1.374,446,785,945 | .980,785,280,403 | .195,090,322,016 | .804,909,677,984 | 1.348,037,176,352 | .268,141,266,064 |
| 79 00 | 1.378,810,109,075 | .981,627,183,448 | .190,808,995,377 | .809,191,004,623 | 1.353,477,483,881 | .263,089,371,728 |
| 79 15 | 1.383,173,432,205 | .982,450,397,726 | .186,524,036,009 | .813,475,963,991 | 1.358,899,288,594 | .257,995,091,075 |
| 79 30 | 1.387,536,755,335 | .983,254,907,564 | .182,235,525,492 | .817,764,474,508 | 1.364,302,324,109 | .252,858,489,748 |
| 79 45 | 1.391,900,078,465 | .984,040,697,646 | .177,943,545,474 | .822,056,454,526 | 1.369,686,324,266 | .247,679,634,908 |
| 80 00 | 1.396,263,401,595 | .984,807,753,012 | .173,648,177,667 | .826,351,822,333 | 1.375,051,023,138 | .242,458,595,230 |
| 80 15 | 1.400,626,724,725 | .985,556,059,058 | .169,349,503,849 | .830,650,496,151 | 1.380,396,155,031 | .237,195,440,910 |
| 80 30 | 1.404,990,047,855 | .986,285,601,537 | .165,047,605,861 | .834,952,394,139 | 1.385,721,454,502 | .231,890,243,657 |
| 80 45 | 1.409,353,370,985 | .986,996,366,560 | .160,742,565,604 | .839,257,434,396 | 1.391,026,656,361 | .226,543,076,695 |
| 81 00 | 1.413,716,694,115 | .987,688,340,595 | .156,434,465,040 | .843,565,534,960 | 1.396,311,495,682 | .221,154,014,762 |
| 81 15 | 1.418,080,017,245 | .888,361,510,468 | .152,123,386,190 | .847,876,613,810 | 1.401,575,707,809 | .215,723,134,112 |
| 81 30 | 1.422,443,340,375 | .989,015,863,362 | .147,809,411,130 | .852,190,588,870 | 1.406,819,028,365 | .210,250,512,507 |
| 81 45 | 1.426,806,663,505 | .989,651,386,820 | .143,492,621,991 | .856,507,378,009 | 1.412,041,193,262 | .204,736,229,221 |
| 82 00 | 1.431,169,986,635 | .990,268,068,742 | .139,173,100,960 | .860,826,899,040 | 1.417,241,938,707 | .199,180,365,041 |
| 82 15 | 1.435,533,309,765 | .990,865,897,387 | .134,850,930,274 | .865,149,069,726 | 1.422,421,001,209 | .193,583,002,261 |
| 82 30 | 1.439,896,632,895 | .991,444,861,374 | .130,526,192,220 | .869,473,807,780 | 1.427,578,117,593 | .187,944,224,682 |
| 82 45 | 1.444,259,956,025 | .992,004,949,680 | .126,198,969,136 | .873,801,030,864 | 1.432,713,025,001 | .182,264,117,615 |
| 83 00 | 1.448,623,279,155 | .992,546,151,641 | .121,869,343,405 | .878,130,656,595 | 1.437,825,460,903 | .176,542,767,872 |
| 83 15 | 1.452,986,602,285 | .993,068,456,955 | .117,537,397,458 | .882,462,602,542 | 1.442,915,163,107 | .170,780,263,774 |
| 83 30 | 1.457,349,925,415 | .993,571,855,677 | .113,203,213,768 | .886,796,786,232 | 1.447,981,869,765 | .164,976,695,142 |
| 83 45 | 1.461,713,248,545 | .994,056,338,222 | .108,866,874,852 | .891,133,125,148 | 1.453,025,319,379 | .159,132,153,299 |
| 84 00 | 1.466,076,571,675 | .994,521,895,368 | .104,528,463,268 | .895,471,536,732 | 1.458,045,250,817 | .153,246,731,070 |
| 84 15 | 1.470,439,894,805 | .994,968,518,251 | .100,188,061,612 | .899,811,938,388 | 1.463,041,403,311 | .147,320,522,777 |
| 84 30 | 1.474,803,217,935 | .995,396,198,367 | .095,845,752,520 | .904,154,247,480 | 1.468,013,516,472 | .141,353,624,242 |
| 84 45 | 1.479,166,541,065 | .995,804,927,575 | .091,501,618,663 | .908,498,381,337 | 1.472,961,330,297 | .135,346,132,780 |
| 85 00 | 1.483,529,864,195 | .996,194,698,092 | .087,155,742,748 | .912,844,257,252 | 1.477,884,585,172 | .129,298,147,203 |
| 85 15 | 1.487,893,187,325 | .996,565,502,498 | .082,808,207,512 | .917,191,792,488 | 1.482,783,021,890 | .123,209,767,812 |
| 85 30 | 1.492,256,510,455 | .996,917,333,733 | .078,459,095,728 | .921,540,904,272 | 1.487,656,381,649 | .117,081,096,405 |
| 85 45 | 1.496,619,833,585 | .997,250,185,099 | .074,108,490,195 | .925,891,509,805 | 1.492,504,406,065 | .110,912,236,263 |
| 86 00 | 1.500,983,156,715 | .997,564,050,260 | .069,756,473,744 | .930,243,526,256 | 1.497,326,837,185 | .104,703,292,162 |
| 86 15 | 1.505,346,479,845 | .997,858,923,239 | .065,403,129,230 | .934,596,870,770 | 1.502,123,417,480 | .098,454,370,357 |
| 86 30 | 1.509,709,802,975 | .998,134,798,422 | .061,048,539,535 | .938,951,460,465 | 1.506,893,889,868 | .092,165,578,593 |
| 86 45 | 1.514,073,126,105 | .998,391,670,557 | .056,692,787,563 | .943,307,212,437 | 1.511,637,997,717 | .085,837,026,093 |
| 87 00 | 1.518,436,449,235 | .998,629,534,755 | .052,335,956,243 | .947,664,043,757 | 1.516,355,484,855 | .079,468,823,565 |
| 87 15 | 1.522,799,772,365 | .998,848,386,485 | .047,978,128,521 | .952,021,871,479 | 1.521,046,095,567 | .073,061,083,190 |
| 87 30 | 1.527,163,095,495 | .999,048,221,582 | .043,619,387,365 | .956,380,612,635 | 1.525,709,574,620 | .066,613,918,632 |
| 87 45 | 1.531,526,418,625 | .999,229,036,241 | .039,259,815,759 | .960,740,184,241 | 1.530,345,667,260 | .060,127,445,025 |
| 88 00 | 1.535,889,741,755 | .999,390,827,019 | .034,899,496,703 | .965,100,503,297 | 1.534,954,119,223 | .053,601,778,979 |
| 88 15 | 1.540,253,064,885 | .999,533,590,837 | .030,538,513,210 | .969,461,486,790 | 1.539,534,676,742 | .047,037,038,569 |
| 88 30 | 1.544,616,388,015 | .999,657,324,976 | .026,176,948,308 | .973,823,051,692 | 1.544,087,086,557 | .040,433,343,345 |
| 88 45 | 1.548,979,711,145 | .999,762,027,080 | .021,814,885,035 | .978,185,114,965 | 1.548,611,095,920 | .033,790,814,320 |
| 89 00 | 1.553,343,034,275 | .999,847,695,156 | .017,452,406,437 | .982,547,593,563 | 1.553,106,452,606 | .027,109,573,970 |
| 89 15 | 1.557,706,357,405 | .999,914,327,574 | .013,089,595,571 | .986,910,404,429 | 1.557,572,904,922 | .020,389,746,237 |
| 89 30 | 1.562,069,680,535 | .999,961,923,064 | .008,726,535,498 | .991,273,464,502 | 1.562,010,201,708 | .013,631,456,518 |
| 89 45 | 1.566,433,003,665 | .999,990,480,721 | .004,363,309,285 | .995,636,690,715 | 1.566,418,092,352 | .006,834,831,669 |
| 90 00 | 1.570,796,326,795 | 1.0 | 0 | 1.0 | 1.570,796,326,795 | 0 |

SHEET 4 OF 4

205-D-1860

Table H-3.—*Additional trigonometric functions for arch load constants* (sheet 1).

| $\Phi_1$ ° ′ | $\frac{\sin\Phi_1}{\Phi_1}$ | $\frac{\sin^2\Phi_1}{\Phi_1}$ | $\frac{\Phi_1^2}{2}\sin\Phi_1$ | $\frac{\Phi_1^2}{2}\cos\Phi_1$ | $\frac{\Phi_1+\sin\Phi_1\cos\Phi_1}{2}$ | $\frac{\Phi_1-\sin\Phi_1\cos\Phi_1}{2}$ |
|---|---|---|---|---|---|---|
| 0 15 | .999,996,826,906 | .004,363,295,440 | .0$^{6}$ ,041,535,626 | .0$^{5}$9,519,203,752 | .004,363,295,440 | .0$^{6}$ ,027,690 |
| 0 30 | .999,987,307,656 | .008,726,424,738 | .0$^{6}$ ,332,281,841 | .0$^{3}$ ,038,075,728 | .008,726,424,739 | .0$^{6}$ ,221,521 |
| 0 45 | .999,971,442,361 | .013,089,221,763 | .0$^{5}$1,121,433,421 | .0$^{3}$ ,085,666,309 | .013,089,221,772 | .0$^{6}$ ,747,618 |
| 1 00 | .999,949,231,203 | .017,451,520,400 | .0$^{5}$2,658,153,509 | .0$^{3}$ ,152,285,513 | .017,451,520,436 | .0$^{3}$ ,001,772,084 |
| 1 15 | .999,920,674,435 | .021,813,154,556 | .0$^{5}$5,191,557,806 | .0$^{3}$ ,237,925,726 | .021,813,154,666 | .0$^{3}$ ,003,460,984 |
| 1 30 | .999,885,772,383 | .026,173,958,177 | .0$^{5}$8,970,698,758 | .0$^{3}$ ,342,577,164 | .026,173,958,451 | .0$^{3}$ ,005,980,329 |
| 1 45 | .999,844,525,444 | .030,533,765,248 | .0$^{3}$ ,014,244,550 | .0$^{3}$ ,466,227,870 | .030,533,765,839 | .0$^{3}$ ,009,496,071 |
| 2 00 | .999,796,934,092 | .034,892,409,804 | .0$^{3}$ ,021,261,989 | .0$^{3}$ ,608,863,710 | .034,892,410,956 | .0$^{3}$ ,014,174,084 |
| 2 15 | .999,742,998,869 | .039,249,725,942 | .0$^{3}$ ,030,271,785 | .0$^{3}$ ,770,468,382 | .039,249,728,017 | .0$^{3}$ ,020,180,153 |
| 2 30 | .999,682,720,393 | .043,605,547,823 | .0$^{3}$ ,041,522,579 | .0$^{3}$ ,951,023,411 | .043,605,551,337 | .0$^{3}$ ,027,679,963 |
| 2 45 | .999,616,099,350 | .047,959,709,686 | .0$^{3}$ ,055,262,869 | .001,150,508,150 | .047,959,715,345 | .0$^{3}$ ,036,839,085 |
| 3 00 | .999,543,136,501 | .052,312,045,855 | .0$^{3}$ ,071,740,998 | .001,368,899,785 | .052,312,054,597 | .0$^{3}$ ,047,822,963 |
| 3 15 | .999,463,832,681 | .056,662,390,743 | .0$^{3}$ ,091,205,131 | .001,606,173,331 | .056,662,403,787 | .0$^{3}$ ,060,796,903 |
| 3 30 | .999,378,188,796 | .061,010,578,869 | .0$^{3}$ ,113,903,248 | .001,862,301,637 | .061,010,597,761 | .0$^{3}$ ,075,926,059 |
| 3 45 | .999,286,205,823 | .065,356,444,857 | .0$^{3}$ ,140,083,119 | .002,137,255,386 | .065,356,471,530 | .0$^{3}$ ,093,375,420 |
| 4 00 | .999,187,884,813 | .069,699,823,452 | .0$^{3}$ ,169,992,296 | .002,431,003,096 | .069,699,860,280 | .0$^{3}$ ,113,309,800 |
| 4 15 | .999,083,226,891 | .074,040,549,524 | .0$^{3}$ ,203,878,094 | .002,743,511,122 | .074,040,599,387 | .0$^{3}$ ,135,893,822 |
| 4 30 | .998,972,233,249 | .078,378,458,078 | .0$^{3}$ ,241,987,574 | .003,074,743,658 | .078,378,524,430 | .0$^{3}$ ,161,291,910 |
| 4 45 | .998,854,905,157 | .082,713,384,261 | .0$^{3}$ ,284,567,529 | .003,424,662,736 | .082,713,471,200 | .0$^{3}$ ,189,668,270 |
| 5 00 | .998,731,243,954 | .087,045,163,372 | .0$^{3}$ ,331,864,468 | .003,793,228,232 | .087,045,275,717 | .0$^{3}$ ,221,186,883 |
| 5 15 | .998,601,251,053 | .091,373,630,871 | .0$^{3}$ ,384,124,602 | .004,180,397,865 | .091,373,774,238 | .0$^{3}$ ,256,011,492 |
| 5 30 | .998,464,927,939 | .095,698,622,383 | .0$^{3}$ ,441,593,823 | .004,586,127,202 | .095,698,803,274 | .0$^{3}$ ,294,305,586 |
| 5 45 | .998,322,276,165 | .100,019,973,713 | .0$^{3}$ ,504,517,695 | .005,010,369,655 | .100,020,199,599 | .0$^{3}$ ,336,232,391 |
| 6 00 | .998,173,297,371 | .104,337,520,849 | .0$^{3}$ ,573,141,434 | .005,453,076,486 | .104,337,800,265 | .0$^{3}$ ,381,954,855 |
| 6 15 | .998,017,993,245 | .108,651,099,971 | .0$^{3}$ ,647,709,893 | .005,914,196,814 | .108,651,442,610 | .0$^{3}$ ,431,635,640 |
| 6 30 | .997,856,365,570 | .112,960,547,461 | .0$^{3}$ ,728,467,548 | .006,393,677,608 | .112,960,964,276 | .0$^{3}$ ,485,437,104 |
| 6 45 | .997,688,416,189 | .117,265,699,913 | .0$^{3}$ ,815,658,480 | .006,891,463,697 | .117,266,203 219 | .0$^{3}$ ,543 521 291 |
| 7 00 | .997,514,147,016 | .121,566,394,134 | .0$^{3}$ ,909,526,361 | .007,407,497,770 | .121,566,997,720 | .0$^{3}$ ,606,049,920 |
| 7 15 | .997,333,560,051 | .125,862,467,163 | .001,010,314,440 | .007,941,720,377 | .125,863,186,399 | .0$^{3}$ ,673,184,371 |
| 7 30 | .997,146,657,346 | .130,153,756,268 | .001,118,265,522 | .008,494,069,937 | .130,154,608,226 | .0$^{3}$ ,745,085,674 |
| 7 45 | .996,953,441,044 | .134,440,098,965 | .001,233,621,959 | .009,064,482,735 | .134,441,102,535 | .0$^{3}$ ,821,914,495 |
| 8 00 | .996,753,913,341 | .138,721,333,014 | .001,356,625,629 | .009,652,892,928 | .138,722,509,034 | .0$^{3}$ ,903,831,126 |
| 8 15 | .996,548,076,524 | .142,997,296,441 | .001,487,517,926 | .010,259,232,549 | .142,998,667,821 | .0$^{3}$ ,990,995,469 |
| 8 30 | .996,335,932,952 | .147,267,827,537 | .001,626,539,737 | .010,883,431,508 | .147,269,419,390 | .001,083,567,029 |
| 8 45 | .996,117,485,023 | .151,532,764,865 | .001,773,931,434 | .011,525,417,596 | .151,534,604,651 | .001,181,704,898 |
| 9 00 | .995,892,735,245 | .155,791,947,275 | .001,929,932,856 | .012,185,116,491 | .155,794,064,933 | .001,285,567,746 |
| 9 15 | .995,661,686,188 | .160,045,213,911 | .002,094,783,289 | .012,862,451,757 | .160,047,642,006 | .001,395,313,803 |
| 9 30 | .995,424,340,484 | .164,292,404,213 | .002,268,721,460 | .013,557,344,852 | .164,295,178,084 | .001,511,100,855 |
| 9 45 | .995,180,700,842 | .168,533,357,928 | .002,451,985,511 | .014,269,715,127 | .168,536,515,843 | .001,633,086,226 |
| 10 00 | .994,930,770,048 | .172,767,915,124 | .002,644,812,992 | .014,999,479,835 | .172,771,498,431 | .001,761,426,768 |
| 10 15 | .994,674,550,954 | .176,995,916,190 | .002,847,440,841 | .015,746,554,133 | .176,999,969,479 | .001,896,278,850 |
| 10 30 | .994,412,046,482 | .181,217,201,846 | .003,080,105,371 | .016,510,851,083 | .181,221,773,116 | .002,037,798,343 |
| 10 45 | .994,143,259,636 | .185,431,613,158 | .003,283,042,253 | .017,292,281,661 | .185,436,753,976 | .002,186,140,613 |
| 11 00 | .993,868,193,478 | .189,638,991,535 | .003,516,486,503 | .018,090,754,759 | .189,644,757,214 | .002,341,460,505 |
| 11 15 | .993,586,851,145 | .193,839,178,741 | .003,760,672,462 | .018,906,177,188 | .193,845,628,516 | .002,503,912,333 |
| 11 30 | .993,299,235,857 | .198,032,016,911 | .004,015,833,789 | .019,738,453,684 | .198,039,214,112 | .002,673,649,867 |
| 11 45 | .993,005,350,893 | .202,217,348,547 | .004,282,203,437 | .020,587,486,914 | .202,225,360,786 | .002,850,826,323 |
| 12 00 | .992,705,199,610 | .206,395,016,535 | .004,560,013,642 | .021,453,177,478 | .206,403,915,889 | .003,035,594,350 |
| 12 15 | .992,398,785,426 | .210,564,864,142 | .004,849,495,909 | .022,335,423,914 | .210,574,727,328 | .003,228,106,021 |
| 12 30 | .992,086,111,848 | .214,726,735,042 | .005,150,880,995 | .023,234,122,706 | .214,737,643,685 | .003,428,512,814 |
| 12 45 | .991,767,182,442 | .218,880,473,304 | .005,464,398,895 | .024,149,168,284 | .218,892,514,017 | .003,636,965,612 |
| 13 00 | .991,442,000,842 | .223,025,923,410 | .005,790,278,824 | .025,080,453,032 | .223,039,188,077 | .003,853,614,682 |
| 13 15 | .991,110,570,762 | .227,162,930,266 | .006,128,749,207 | .026,027,867,295 | .227,177,516,222 | .004,078,609,667 |
| 13 30 | .990,772,895,989 | .231,291,339,203 | .006,480,037,661 | .026,991,299,379 | .231,307,349,444 | .004,312,099,575 |
| 13 45 | .990,428,980,370 | .235,410,995,985 | .006,844,370,978 | .027,970,635,562 | .235,428,539,383 | .004,554,232,766 |
| 14 00 | .990,078,827,835 | .239,521,746,823 | .007,221,975,115 | .028,965,760,096 | .239,540,938,336 | .004,805,156,943 |
| 14 15 | .989,722,442,373 | .243,623,438,375 | .007,613,075,177 | .029,976,555,212 | .243,644,399,269 | .005,065,019,140 |
| 14 30 | .989,359,828,053 | .247,715,917,759 | .008,017,895,398 | .031,002,901,129 | .247,738,775,831 | .005,333,965,708 |
| 14 45 | .988,990,989,018 | .251,799,032,562 | .008,436,659,134 | .032,044,676,056 | .251,823,922,361 | .005,612,142,308 |
| 15 00 | .988,615,929,468 | .255,872,630,839 | .008,869,588,842 | .033,101,756,202 | .255,899,693,900 | .005,899,693,900 |
| 15 15 | .988,234,653,682 | .259,936,561,127 | .009,316,906,069 | .034,174,015,778 | .259,965,946,205 | .006,196,764,724 |
| 15 30 | .987,847,166,013 | .263,990,672,459 | .009,778,831,434 | .035,261,327,002 | .264,022,535,757 | .006,503,498,302 |
| 15 45 | .987,453,470,882 | .268,034,814,357 | .010,255,584,615 | .036,363,560,113 | .268,069,319,773 | .006,820,037,416 |
| 16 00 | .987,053,572,779 | .272,068,836,851 | .010,747,384,336 | .037,480,583,368 | .272,106,156,218 | .007,146,524,101 |
| 16 15 | .986,647,476,264 | .276,092,590,479 | .011,254,448,350 | .038,612,263,052 | .276,132,903,811 | .007,483,099,638 |
| 16 30 | .986,235,185,969 | .280,105,926,302 | .011,776,993,424 | .039,758,463,489 | .280,149,422,043 | .007,829,904,536 |
| 16 45 | .985,816,706,597 | .284,108,695,905 | .012,315,235,329 | .040,919,047,039 | .284,155,571,182 | .008,187,078,527 |
| 17 00 | .985,392,042,922 | .288,100,751,410 | .012,869,388,818 | .042,093,874,113 | .288,151,212,287 | .008,554,760,552 |
| 17 15 | .984,961,199,785 | .292,081,945,474 | .013,439,667,619 | .043,282,803,176 | .292,136,207,216 | .008,933,088,753 |
| 17 30 | .984,524,182,096 | .296,052,131,308 | .014,026,284,416 | .044,485,690,752 | .296,110,418,637 | .009,322,200,462 |
| 17 45 | .984,080,994,846 | .300,011,162,681 | .014,629,450,837 | .045,702,391,435 | .300,073,710,042 | .009,722,232,187 |
| 18 00 | .983,631,643,084 | .303,958,893,913 | .015,249,377,438 | .046,932,757,894 | .304,025,945,783 | .010,133,319,606 |
| 18 15 | .983,176,131,933 | .307,895,179,920 | .015,886,273,690 | .048,176,640,882 | .307,966,990,932 | .010,555,597,557 |
| 18 30 | .982,714,466,587 | .311,819,876,165 | .016,540,347,964 | .049,433,889,238 | .311,896,711,597 | .010,989,200,022 |
| 18 45 | .982,246,652,309 | .315,732,838,714 | .017,211,807,517 | .050,704,349,900 | .315,814,974,627 | .011,434,260,122 |
| 19 00 | .981,772,694,434 | .319,633,924,223 | .017,900,858,477 | .051,987,867,910 | .319,721,647,771 | .011,890,910,108 |
| 19 15 | .981,292,598,364 | .323,522,989,946 | .018,607,705,833 | .053,284,286,418 | .323,616,599,664 | .012,359,281,345 |
| 19 30 | .980,806,369,570 | .327,399,893,743 | .019,332,553,415 | .054,593,446,699 | .327,499,699,832 | .012,839,504,307 |
| 19 45 | .980,314,013,594 | .331,264,494,086 | .020,075,603,883 | .055,915,188,152 | .331,370,818,704 | .013,331,708,565 |
| 20 00 | .979,815,536,052 | .335,116,650,074 | .020,837,058,715 | .057,249,348,307 | .335,229,827,621 | .013,836,022,778 |
| 20 15 | .979,310,942,617 | .338,956,221,422 | .021,617,118,189 | .058,595,762,843 | .339,076,598,847 | .014,352,574,682 |
| 20 30 | .978,800,239,047 | .342,783,068,492 | .022,415,981,371 | .059,954,265,583 | .342,911,005,577 | .014,881,491,082 |
| 20 45 | .978,283,431,161 | .346,597,052,282 | .023,233,846,104 | .061,324,688,513 | .346,732,921,949 | .015,422,897,840 |
| 21 00 | .977,760,524,842 | .350,398,034,434 | .024,070,908,987 | .062,706,861,781 | .350,542,223,049 | .015,976,919,870 |
| 21 15 | .977,231,526,055 | .354,185,877,253 | .024,927,365,372 | .064,100,613,713 | .354,338,784,929 | .016,543,681,120 |
| 21 30 | .976,696,440,820 | .357,960,443,697 | .025,803,409,338 | .065,505,770,817 | .358,122,484,605 | .017,123,304,574 |
| 21 45 | .976,155,275,241 | .361,721,597,405 | .026,699,233,688 | .066,922,157,791 | .361,893,200,078 | .017,715,912,231 |
| 22 00 | .975,608,035,477 | .365,469,202,679 | .027,615,029,929 | .068,349,597,534 | .365,650,810,334 | .018,321,625,105 |
| 22 15 | .975,054,727,763 | .369,203,124,510 | .028,550,988,262 | .069,787,911,153 | .369,395,195,359 | .018,940,563,210 |
| 22 30 | .974,495,358,404 | .372,923,228,578 | .029,507,297,565 | .071,236,917,970 | .373,126,236,146 | .019,572,845,553 |

SHEET 1 OF 4

205-D-1861

Table H-3.–*Additional trigonometric functions for arch load constants* (sheet 2).

| $\Phi_1$ ° ′ | $\frac{\sin\Phi_1}{\Phi_1}$ | $\frac{\sin^2\Phi_1}{\Phi_1}$ | $\frac{\Phi_1^2}{2}\sin\Phi_1$ | $\frac{\Phi_1^2}{2}\cos\Phi_1$ | $\frac{\Phi_1+\sin\Phi_1\cos\Phi_1}{2}$ | $\frac{\Phi_1-\sin\Phi_1\cos\Phi_1}{2}$ |
|---|---|---|---|---|---|---|
| 22 45 | .973,929,933,768 | .376,629,381,254 | .030,484,145,384 | .072,696,435,536 | .376,843,814,703 | .020,218,590,126 |
| 23 00 | .973,358,460,295 | .380,321,449,615 | .031,481,717,917 | .074,166,279,634 | .380,547,814,064 | .020,877,913,895 |
| 23 15 | .972,780,944,495 | .383,999,301,447 | .032,500,199,999 | .075,646,264,291 | .384,238,118,297 | .021,550,932,792 |
| 23 30 | .972,197,392,943 | .387,662,805,247 | .033,539,775,094 | .077,136,201,787 | .387,914,612,514 | .022,237,761,705 |
| 23 45 | .971,607,812,285 | .391,311,830,239 | .034,600,625,277 | .078,635,902,661 | .391,577,182,877 | .022,938,514,472 |
| 24 00 | .971,012,209,231 | .394,946,246,368 | .035,682,931,223 | .080,145,175,728 | .395,225,716,609 | .023,653,303,870 |
| 24 15 | .970,410,590,563 | .398,565,924,320 | .036,786,872,193 | .081,663,828,077 | .398,860,102,002 | .024,382,241,607 |
| 24 30 | .969,802,963,133 | .402,170,735,519 | .037,912,626,023 | .083,191,665,089 | .402,480,228,425 | .025,125,438,314 |
| 24 45 | .969,189,333,855 | .405,760,552,135 | .039,060,369,107 | .084,728,490,444 | .406,085,986,334 | .025,883,003,535 |
| 25 00 | .968,569,709,716 | .409,335,247,095 | .040,230,276,390 | .086,274,106,132 | .409,677,267,280 | .026,655,045,719 |
| 25 15 | .967,944,097,765 | .412,894,694,078 | .041,422,521,348 | .087,828,312,454 | .413,253,963,911 | .027,441,672,217 |
| 25 30 | .967,312,505,125 | .416,438,767,537 | .042,637,275,985 | .089,390,908,048 | .416,815,969,993 | .028,242,989,265 |
| 25 45 | .966,674,938,981 | .419,967,342,692 | .043,874,710,807 | .090,961,689,882 | .420,363,180,407 | .029,059,101,981 |
| 26 00 | .966,031,406,590 | .423,480,295,541 | .045,134,994,824 | .092,540,453,277 | .423,895,491,161 | .029,890,114,357 |
| 26 15 | .965,381,915,274 | .426,977,502,868 | .046,418,295,524 | .094,126,991,905 | .427,412,799,397 | .030,736,129,251 |
| 26 30 | .964,726,472,423 | .430,458,842,244 | .047,724,778,870 | .095,721,097,808 | .430,915,003,401 | .031,597,248,377 |
| 26 45 | .964,065,085,490 | .433,924,192,037 | .049,054,609,285 | .097,322,561,410 | .434,402,002,608 | .032,473,572,300 |
| 27 00 | .963,397,762,006 | .437,373,431,422 | .050,407,949,636 | .098,931,171,516 | .437,873,697,613 | .033,365,200,425 |
| 27 15 | .962,724,509,555 | .440,806,440,369 | .051,784,961,226 | .100,546,715,331 | .441,329,990,173 | .034,272,230,995 |
| 27 30 | .962,045,335,797 | .444,223,099,673 | .053,185,803,780 | .102,168,978,470 | .444,770,783,221 | .035,194,761,077 |
| 27 45 | .961,360,248,456 | .447,623,290,944 | .054,610,635,433 | .103,797,744,964 | .448,195,980,869 | .036,132,886,559 |
| 28 00 | .960,669,255,324 | .451,006,896,617 | .056,059,612,719 | .105,432,797,277 | .451,605,488,418 | .037,086,702,140 |
| 28 15 | .959,972,364,257 | .454,373,799,957 | .057,532,890,554 | .107,073,916,309 | .454,999,212,361 | .038,056,301,327 |
| 28 30 | .959,269,583.179 | .457,723,885,065 | .059,030,622,233 | .108,720,881,413 | .458,377,060,396 | .039,041,776,422 |
| 28 45 | .958,560,920,079 | .461,057,036,883 | .060,552,959,407 | .110,373,470,401 | .461,738,941,427 | .040,043,218,521 |
| 29 00 | .957,846,383,016 | .464,373,141,204 | .062,100,052,083 | .112,031,459,562 | .465,084,765,578 | .041,060,717,500 |
| 29 15 | .957,125,980,111 | .467,672,084,671 | .063,672,048,600 | .113,694,623,664 | .468,414,444,193 | .042,094,362,015 |
| 29 30 | .956,399,719,550 | .470,953,754,782 | .065,269,095,629 | .115,362,735,970 | .471,727,889,844 | .043,144,239,494 |
| 29 45 | .955,667,609,591 | .474,218,039,907 | .066,891,338,151 | .117,035,568,249 | .475,025,016,344 | .044,210,436,124 |
| 30 00 | .954,929,658,552 | .477,464,829,276 | .068,538,919,452 | .118,712,890,787 | .478,305,738,745 | .045,293,036,853 |
| 30 15 | .954,185,874,819 | .480,694,012,998 | .070,211,981,110 | .120,394,472,397 | .481,569,973,349 | .046,392,125,379 |
| 30 30 | .953,436,266,841 | .483,905,482,060 | .071,910,662,982 | .122,080,080,431 | .484,817,637,714 | .047,507,784,144 |
| 30 45 | .952,680,843,136 | .487,099,128,333 | .073,635,103,195 | .123,769,480,793 | .488,048,650,659 | .048,640,094,329 |
| 31 00 | .951,919,612,287 | .490,274,844,581 | .075,385,438,131 | .125,462,437,949 | .491,262,932,274 | .049,789,135,844 |
| 31 15 | .951,152,582,940 | .493,432,524,460 | .077,161,802,419 | .127,158,714,936 | .494,460,403,919 | .050,954,987,329 |
| 31 30 | .950,379,763,806 | .496,572,062,524 | .078,964,328,925 | .128,858,073,380 | .497,640,988,236 | .052,137,726,142 |
| 31 45 | .949,601,163,663 | .499,693,354,236 | .080,793,148,735 | .130,560,273,500 | .500,804,609,155 | .053,337,428,353 |
| 32 00 | .948,816,791,351 | .502,796,295,965 | .082,648,391,151 | .132,265,074,128 | .503,951,191,894 | .054,554,168,744 |
| 32 15 | .948,026,655,780 | .505,880,785,000 | .084,530,183,673 | .133,972,232,714 | .507,080,662,972 | .055,788,020,796 |
| 32 30 | .947,230,765,918 | .508,946,719,542 | .086,438,651,996 | .135,681,505,343 | .510,192,950,208 | .057,039,056,690 |
| 32 45 | .946,429,130,800 | .511,993,998,722 | .088,373,919,991 | .137,392,646,742 | .513,287,982,733 | .058,307,347,295 |
| 33 00 | .945,621,759,529 | .515,022,522,599 | .090,336,109,702 | .139,105,410,298 | .516,365,690,990 | .059,592,962,168 |
| 33 15 | .944,808,661,266 | .518,032,192,164 | .092,325,341,328 | .140,819,548,063 | .519,426,006,740 | .060,895,969,548 |
| 33 30 | .943,989,845,241 | .521,022,909,347 | .094,341,733,219 | .142,534,810,774 | .522,468,863,072 | .062,216,436,346 |
| 33 45 | .943,165,320,745 | .523,994,577,023 | .096,385,401,860 | .144,250,947,861 | .525,494,194,402 | .063,554,428,146 |
| 34 00 | .942,335,097,134 | .526,947,099,009 | .098,456,461,864 | .145,967,707,457 | .528,501,936,481 | .064,910,009,197 |
| 34 15 | .941,499,183,828 | .529,880,380,079 | .100,555,025,960 | .147,684,836,418 | .531,492,026,399 | .066,283,242,409 |
| 34 30 | .940,657,590,310 | .532,794,325,962 | .102,681,204,985 | .149,402,080,328 | .534,464,402,593 | .067,674,189,345 |
| 34 45 | .939,810,326,126 | .535,688,843,346 | .104,835,107,869 | .151,119,183,515 | .537,419,004,846 | .069,082,910,222 |
| 35 00 | .938,957,400,888 | .538,563,839,887 | .107,016,841,631 | .152,835,889,063 | .540,355,774,295 | .070,509,463,903 |
| 35 15 | .938,098,824,267 | .541,419,224,205 | .109,226,511,365 | .154,551,938,829 | .543,274,653,437 | .071,953,907,891 |
| 35 30 | .937,234,606,000 | .544,254,905,899 | .111,464,220,229 | .156,267,073,445 | .546,175,586,129 | .073,416,298,329 |
| 35 45 | .936,364,755,886 | .547,070,795,541 | .113,730,069,440 | .157,981,032,345 | .549,058,517,595 | .074,896,689,993 |
| 36 00 | .935,489,283,788 | .549,866,804,688 | .116,024,158,258 | .159,693,553,765 | .551,923,394,433 | .076,395,136,285 |
| 36 15 | .934,608,199,631 | .552,642,845,882 | .118,346,583,983 | .161,404,374,765 | .554,770,164,611 | .077,911,689,237 |
| 36 30 | .933,721,513,398 | .555,398,832,649 | .120,697,441,938 | .163,113,231,237 | .557,598,777,480 | .079,446,399,498 |
| 36 45 | .932,829,235,145 | .558,134,679,519 | .123,076,825,466 | .164,819,857,924 | .560,409,183,771 | .080,999,316,337 |
| 37 00 | .931,931,374,978 | .560,850,302,008 | .125,484,825,916 | .166,523,988,424 | .563,201,335,603 | .082,570,487,635 |
| 37 15 | .931,027,943,074 | .563,545,616,643 | .127,921,532,636 | .168,225,355,211 | .565,975,186,486 | .084,159,959,882 |
| 37 30 | .930,118,949,668 | .566,220,540,948 | .130,387,032,964 | .169,923,689,648 | .568,730,691,321 | .085,767,778,177 |
| 37 45 | .929,204,405,057 | .568,874,993,460 | .132,881,412,214 | .171,618,721,993 | .571,467,806,408 | .087,393,986,220 |
| 38 00 | .928,284,319,604 | .571,508,893,729 | .135,404,753,675 | .173,310,181,425 | .574,186,489,448 | .089,038,626,310 |
| 38 15 | .927,358,703,727 | .574,122,162,317 | .137,957,138,594 | .174,997,796,044 | .576,886,699,544 | .090,701,739,344 |
| 38 30 | .926,427,567,908 | .576,714,720,808 | .140,538,646,173 | .176,681,292,896 | .579,568,397,205 | .092,383,364,813 |
| 38 45 | .925,490,922,692 | .579,286,491,806 | .143,149,353,555 | .178,360,397,978 | .582,231,544,354 | .094,083,540,794 |
| 39 00 | .924,548,778,684 | .581,837,398,946 | .145,789,335,820 | .180,034,836,257 | .584,876,104,323 | .095,802,303,955 |
| 39 15 | .923,601,146,550 | .584,367,366,889 | .148,458,665,972 | .181,704,331,683 | .587,502,041,859 | .097,539,689,549 |
| 39 30 | .922,648,037,015 | .586,876,321,327 | .151,157,414,933 | .183,368,607,202 | .590,109,323,131 | .099,295,731,407 |
| 39 45 | .921,689,460,869 | .589,364,188,994 | .153,885,651,535 | .185,027,384,769 | .592,697,915,725 | .101,070,461,943 |
| 40 00 | .920,725,428,959 | .591,830,897,659 | .156,643,442,507 | .186,680,385,364 | .595,267,788,652 | .102,863,912,146 |
| 40 15 | .919,755,952,192 | .594,276,376,131 | .159,430,852,473 | .188,327,329,005 | .597,818,912,348 | .104,676,111,580 |
| 40 30 | .918,781,041,538 | .596,700,554,269 | .162,247,943,938 | .189,967,934,763 | .600,351,258,678 | .106,507,088,380 |
| 40 45 | .917,800,708,027 | .599,103,362,982 | .165,094,777,285 | .191,601,920,774 | .602,864,800,935 | .108,356,869,253 |
| 41 00 | .916,814,962,747 | .601,484,734,224 | .167,971,410,761 | .193,229,004,256 | .605,359,513,845 | .110,225,479,473 |
| 41 15 | .915,823,816,844 | .603,844,601,005 | .170,877,900,474 | .194,848,901,521 | .607,835,373,567 | .112,112,942,881 |
| 41 30 | .914,827,281,530 | .606,182,897,397 | .173,814,300,383 | .196,461,327,992 | .610,292,357,699 | .114,019,281,879 |
| 41 45 | .913,825,368,073 | .608,499,558,526 | .176,780,662,289 | .198,065,998,214 | .612,730,445,273 | .115,944,517,434 |
| 42 00 | .912,818,087,796 | .610,794,520,582 | .179,777,035,829 | .199,662,625,871 | .615,149,616,760 | .117,888,669,077 |
| 42 15 | .911,805,452,089 | .613,067,720,823 | .182,803,468,470 | .201,250,923,801 | .617,549,854,075 | .119,851,754,892 |
| 42 30 | .910,787,472,395 | .615,319,097,569 | .185,860,005,495 | .202,830,604,005 | .619,931,140,572 | .121,833,791,525 |
| 42 45 | .909,764,160,220 | .617,548,590,217 | .188,946,690,002 | .204,401,377,672 | .622,293,461,047 | .123,834,794,180 |
| 43 00 | .908,735,527,126 | .619,756,139,230 | .192,063,562,894 | .205,962,955,184 | .624,636,801,743 | .125,854,776,614 |
| 43 15 | .907,701,584,737 | .621 941,686,154 | .195,210,662,873 | .207,515,046,134 | .626,961,150,349 | .127,893,751,138 |
| 43 30 | .906,662,344,731 | .624,105,173,605 | .198,388,026,429 | .209,057,359,344 | .629,266,495,997 | .129,951,728,620 |
| 43 45 | .905,617,818,846 | .626,246,545,281 | .201,595,687,837 | .210,589,602,875 | .631,552,829,269 | .132,028,718,478 |
| 44 00 | .904,568,018,881 | .628,365,745,965 | .204,833,679,148 | .212,111,484,043 | .633,820,142,193 | .134,124,728,684 |
| 44 15 | .903,512,956,688 | .630,462,721,521 | .208,102,030,183 | .213,622,709,437 | .636,068,428,248 | .136,239,765,759 |
| 44 30 | .902,452,644,181 | .632,537,418,898 | .211,400,768,526 | .215,122,984,932 | .638,297,682,358 | .138,373,834,779 |
| 44 45 | .901,387,093,328 | .634,589,786,138 | .214,729,919,515 | .216,612,015 701 | .640,507,900,899 | .140,526,939,368 |
| 45 00 | .900,316,316,158 | .636,619,772,369 | .218,089,506,239 | .218,089,506,239 | .642,699,081,699 | .142,699,081,699 |

SHEET 2 OF 4

205-D-1862

Table H-3.–*Additional trigonometric functions for arch load constants* (sheet 3).

| $\Phi_1$<br>° ′ | $\frac{\text{SIN}\,\Phi_1}{\Phi_1}$ | $\frac{\text{SIN}^2\,\Phi_1}{\Phi_1}$ | $\frac{\Phi_1^2}{2}\text{SIN}\,\Phi_1$ | $\frac{\Phi_1^2}{2}\text{COS}\,\Phi_1$ | $\frac{\Phi_1+\text{SIN}\,\Phi_1\,\text{COS}\,\Phi_1}{2}$ | $\frac{\Phi_1-\text{SIN}\,\Phi_1\,\text{COS}\,\Phi_1}{2}$ |
|---|---|---|---|---|---|---|
| 45 15 | .899,240,324,754 | .638,627,327,811 | .221,479,549,528 | .219,555,160,364 | .644,871,224,029 | .144,890,262,498 |
| 45 30 | .898,159,131,261 | .640,612,403,784 | .224,900,067,948 | .221,008,681,247 | .647,024,328,618 | .147,100,481,039 |
| 45 45 | .897,072,747,875 | .642,574,952,696 | .228,351,077,799 | .222,449,771,420 | .649,158,397,638 | .149,329,735,149 |
| 46 00 | .895,981,186,853 | .644,514,928,058 | .231,832,593,097 | .223,878,132,788 | .651,273,434,713 | .151,578,021,204 |
| 46 15 | .894,884,460,506 | .646,432,284,477 | .235,344,625,580 | .225,293,466,652 | .653,369,444,919 | .153,845,334,128 |
| 46 30 | .893,782,581,204 | .648,326,977,662 | .238,887,184,697 | .226,695,473,720 | .655,446,434,777 | .156,131,667,400 |
| 46 45 | .892,675,561,371 | .650,198,964,426 | .242,460,277,600 | .228,083,854,122 | .657,504,412,259 | .158,437,013,048 |
| 47 00 | .891,563,413,491 | .652,048,202,685 | .246,063,909,141 | .229,458,307,428 | .659,543,386,783 | .160,761,361,654 |
| 47 15 | .890,446,150,099 | .653,874,651,458 | .249,698,081,866 | .230,818,532,662 | .661,563,369,217 | .163,104,702,350 |
| 47 30 | .889,323,783,787 | .655,678,270,872 | .253,362,796,007 | .232,164,228,318 | .663,564,371,872 | .165,467,022,825 |
| 47 45 | .888,196,327,206 | .657,459,022,165 | .257,058,049,479 | .233,495,092,373 | .665,546,408,505 | .167,848,309,322 |
| 48 00 | .887,063,793,059 | .659,216,867,680 | .260,783,837,873 | .234,810,822,308 | .667,509,494,320 | .170,248,546,637 |
| 48 15 | .885,926,194,107 | .660,951,770,874 | .264,540,154,451 | .236,111,115,121 | .669,453,645,963 | .172,667,718,124 |
| 48 30 | .884,783,543,163 | .662,663,696,311 | .268,326,990,140 | .237,395,667,339 | .671,378,881,519 | .175,105,805,698 |
| 48 45 | .883,635,853,097 | .664,352,609,674 | .272,144,333,530 | .238,664,175,042 | .673,285,220,517 | .177,562,789,830 |
| 49 00 | .882,483,136,834 | .666,018,477,754 | .275,992,170,862 | .239,916,333,871 | .675,172,683,924 | .180,038,649,553 |
| 49 15 | .881,325,407,352 | .667,661,268,458 | .279,870,486,029 | .241,151,839,048 | .677,041,294,144 | .182,533,362,463 |
| 49 30 | .880,162,677,686 | .669,280,950,811 | .283,779,260,571 | .242,370,385,389 | .678,891,075,017 | .185,046,904,720 |
| 49 45 | .878,994,960,923 | .670,877,494,952 | .287,718,473,667 | .243,571,667,326 | .680,722,051,818 | .187,579,251,049 |
| 50 00 | .877,822,270,204 | .672,450,872,136 | .291,688,102,131 | .244,755,378,917 | .682,534,251,252 | .190,130,374,745 |
| 50 15 | .876,644,618,724 | .674,001,054,737 | .295,688,120,408 | .245,921,213,862 | .684,327,701,455 | .192,700,247,672 |
| 50 30 | .875,462,019,736 | .675,528,016,251 | .299,718,500,571 | .247,068,865,524 | .686,102,431,991 | .195,288,840,266 |
| 50 45 | .874,274,486,540 | .677,031,731,285 | .303,779,212,314 | .248,198,026,940 | .687,858,473,849 | .197,896,121,538 |
| 51 00 | .873,082,032,493 | .678,512,175,573 | .307,870,222,949 | .249,308,390,840 | .689,595,859,442 | .200,522,059,075 |
| 51 15 | .871,884,671,004 | .679,969,325,963 | .311,991,497,399 | .250,399,649,663 | .691,314,622,603 | .203,166,619,044 |
| 51 30 | .870,682,415,537 | .681,403,160,427 | .316,142,998,201 | .251,471,495,572 | .693,014,798,585 | .205,829,766,192 |
| 51 45 | .869,475,279,607 | .682,813,658,058 | .320,324,685,494 | .252,523,620,472 | .694,696,424,053 | .208,511,463,854 |
| 52 00 | .868,263,276,781 | .684,200,799,065 | .324,536,517,018 | .253,555,716,023 | .696,359,537,088 | .211,211,673,949 |
| 52 15 | .867,046,420,680 | .685,564,564,784 | .328,778,448,112 | .254,567,473,660 | .698,004,177,178 | .213,930,356,989 |
| 52 30 | .865,824,724,977 | .686,904,937,667 | .333,050,431,709 | .255,558,584,609 | .699,630,385,221 | .216,667,472,076 |
| 52 45 | .864,598,203,397 | .688,221,901,292 | .337,352,418,331 | .256,528,739,900 | .701,238,203,516 | .219,422,976,911 |
| 53 00 | .863,366,869,716 | .689,515,440,353 | .341,684,356,086 | .257,477,630,386 | .702,827,675,763 | .222,196,827,794 |
| 53 15 | .862,130,737,764 | .690,785,540,672 | .346,046,190,669 | .258,404,946,762 | .704,398,847,061 | .224,988,979,626 |
| 53 30 | .860,889,821,420 | .692,032,189,184 | .350,437,865,348 | .259,310,379,574 | .705,951,763,899 | .227,799,385,918 |
| 53 45 | .859,644,134,616 | .693,255,373,951 | .354,859,320,975 | .260,193,619,244 | .707,486,474,161 | .230,627,998,786 |
| 54 00 | .858,393,691,334 | .694,455,084,153 | .359,310,495,971 | .261,054,356,081 | .709,003,027,112 | .233,474,768,965 |
| 54 15 | .857,138,505,608 | .695,631,310,092 | .363,791,326,328 | .261,892,280,300 | .710,501,473,405 | .236,339,645,802 |
| 54 30 | .855,878,591,521 | .696,784,043,186 | .368,301,745,607 | .262,707,082,036 | .711,981,865,068 | .239,222,577,269 |
| 54 45 | .854,613,963,211 | .697,913,275,980 | .372,841,684,933 | .263,498,451,364 | .713,444,255,507 | .242,123,509,960 |
| 55 00 | .853,344,634,859 | .699,019,002,128 | .377,411,072,994 | .264,266,078,314 | .714,888,699,495 | .245,042,389,102 |
| 55 15 | .852,070,620,704 | .700,101,216,412 | .382,009,836,035 | .265,009,652,887 | .716,315,253,175 | .247,979,158,552 |
| 55 30 | .850,791,935,031 | .701,159,914,727 | .386,637,897,860 | .265,728,865,073 | .717,723,974,053 | .250,933,760,804 |
| 55 45 | .849,508,592,175 | .702,195,094,087 | .391,295,179,829 | .266,423,404,868 | .719,114,920,989 | .253,906,136,998 |
| 56 00 | .848,220,606,521 | .703,206,752,621 | .395,981,600,853 | .267,092,962,288 | .720,488,154,200 | .256,896,226,917 |
| 56 15 | .846,927,992,504 | .704,194,889,576 | .400,697,077,391 | .267,737,227,389 | .721,843,735,252 | .259,903,968,995 |
| 56 30 | .845,630,764,606 | .705,159,505,309 | .405,441,523,452 | .268,355,890,282 | .723,181,727,052 | .262,929,300,325 |
| 56 45 | .844,328,937,362 | .706,100,601,298 | .410,214,850,593 | .268,948,641,151 | .724,502,193,850 | .265,972,156,657 |
| 57 00 | .843,022,525,352 | .707,018,180,127 | .415,016,967,913 | .269,515,170,269 | .725,805,201,229 | .269,032,472,408 |
| 57 15 | .841,711,543,208 | .707,912,245,500 | .419,847,782,054 | .270,055,168,013 | .727,090,816,103 | .272,110,180,664 |
| 57 30 | .840,396,005,609 | .708,782,802,226 | .424,707,197,199 | .270,568,324,886 | .728,359,106,708 | .275,205,213,189 |
| 57 45 | .839,075,927,280 | .709,629,856,223 | .429,595,115,070 | .271,054,331,530 | .729,610,142,601 | .278,317,500,426 |
| 58 00 | .837,751,322,997 | .710,453,414,520 | .434,511,434,929 | .271,512,878,741 | .730,843,994,653 | .281,446,971,504 |
| 58 15 | .836,422,207,585 | .711,253,485,256 | .439,456,053,571 | .271,943,657,492 | .732,060,735,044 | .284,593,554,243 |
| 58 30 | .835,088,595,912 | .712,030,077,669 | .444,428,865,328 | .272,346,358,945 | .733,260,437,255 | .287,757,175,161 |
| 58 45 | .833,750,502,898 | .712,783,202,107 | .449,429,762,070 | .272,720,674,472 | .734,443,176,068 | .290,937,759,478 |
| 59 00 | .832,407,943,506 | .713,512,870 018 | .454,458,633,195 | .273,066,295,664 | .735,609,027,553 | .294,135,231,123 |
| 59 15 | .831,060,932,751 | .714,219,093,954 | .459,515,365,635 | .273,382,914,359 | .736,758,069,068 | .297,349,512,738 |
| 59 30 | .829,709,485,693 | .714,901,887,568 | .464,599,843,856 | .273,670,222,653 | .737,890,379,253 | .300,580,525,683 |
| 59 45 | .828,353,617,435 | .715,561,265,605 | .469,711,949,851 | .273,927,912,915 | .739,006,038,018 | .303,828,190,048 |
| 60 00 | .826,993,343,133 | .716,197,243,913 | .474,851,563,146 | .274,155,677,808 | .740,105,126,544 | .307,092,424,652 |
| 60 15 | .825,628,677,983 | .716,809,839,433 | .480,018,560,798 | .274,353,210,305 | .741,187,727,273 | .310,373,147,053 |
| 60 30 | .824,259,637,232 | .717,399,070,198 | .485,212,817,389 | .274,520,203,706 | .742,253,923,903 | .313,670,273,553 |
| 60 45 | .822,886,236,170 | .717,964,955,334 | .490,434,205,034 | .274,656,351,655 | .743,303,801,382 | .316,983,719,204 |
| 61 00 | .821,508,490,132 | .718,507,515,051 | .495,682,593,378 | .274,761,348,157 | .744,337,445,897 | .320,313,397,819 |
| 61 15 | .820,126,414,502 | .719,026,770,657 | .500,957,849,592 | .274,834,887,594 | .745,354,944,876 | .323,659,221,970 |
| 61 30 | .818,740,024,706 | .719,522,744,533 | .506,259,838,378 | .274,876,664,745 | .746,356,386,975 | .327,021,103,002 |
| 61 45 | .817,349,336,215 | .719,995,460,150 | .511,588,421,966 | .274,886,374,800 | .747,341,862,070 | .330,398,951,036 |
| 62 00 | .815,954,364,549 | .720,444,942,061 | .516,943,460,119 | .274,863,713,381 | .748,311,461,257 | .333,792,674,979 |
| 62 15 | .814,555,125,268 | .720,871,215,894 | .522,324,810,125 | .274,808,376,553 | .749,265,276,838 | .337,202,182,528 |
| 62 30 | .813,151,633,976 | .721,274,308,353 | .527,732,326,807 | .274,720,060,847 | .750,203,402,320 | .340,627,380,176 |
| 62 45 | .811,743,906,326 | .721,654,247,221 | .533,165,862,518 | .274,598,463,276 | .751,125,932,402 | .344,068,173,224 |
| 63 00 | .810,331,958,009 | .722,011,061,344 | .538,625,267,142 | .274,443,281,350 | .752,032,962,972 | .347,524,465,784 |
| 63 15 | .808,915,804,767 | .722,344,780,649 | .544,110,388,096 | .274,254,213,093 | .752,924,591,097 | .350,996,160,789 |
| 63 30 | .807,495,462,378 | .722,655,436,120 | .549,621,070,331 | .274,030,957,066 | .753,800,915,020 | .354,483,159,996 |
| 63 45 | .806,070,946,669 | .722,943,059,809 | .555,157,156,334 | .273,773,212,375 | .754,662,034,146 | .357,985,364,000 |
| 64 00 | .804,642,273,507 | .723,207,684,829 | .560,718,486,125 | .273,480,678,694 | .755,508,049,040 | .361,502,672,236 |
| 64 15 | .803,209,458,803 | .723,449,345,351 | .566,304,897,264 | .273,153,056,285 | .756,339,061,416 | .365,034,982,990 |
| 64 30 | .801,772,518,510 | .723,668,076,603 | .571,916,224,848 | .272,790,046,005 | .757,155,174,132 | .368,582,193,404 |
| 64 45 | .800,331,468,624 | .723,863,914,866 | .577,552,301,516 | .272,391,349,337 | .757,956,491,180 | .372,144,199,486 |
| 65 00 | .798,886,325,186 | .724,036,897,473 | .583,212,957,446 | .271,956,668,393 | .758,743,117,678 | .375,720,896,118 |
| 65 15 | .797,437,104,273 | .724,187,062,800 | .588,898,020,362 | .271,485,705,940 | .759,515,159,863 | .379,312,177,063 |
| 65 30 | .795,983,822,010 | .724,314,450,274 | .594,607,315,531 | .270,978,165,419 | .760,272,725,084 | .382,917,934,972 |
| 65 45 | .794,526,494,558 | .724,419,100,354 | .600,340,665,768 | .270,433,750,953 | .761,015,921,790 | .386,538,061,396 |
| 66 00 | .793,065,138,126 | .724,501,054,550 | .606,097,891,439 | .269,852,167,374 | .761,744,859,527 | .390,172,446,789 |
| 66 15 | .791,599,768,957 | .724,560,355,394 | .611,878,810,458 | .269,233,120,231 | .762,459,648,926 | .393,820,980,520 |
| 66 30 | .790,130,403,342 | .724,597,046,463 | .617,683,238,296 | .268,576,315,817 | .763,160,401,693 | .397,483,550,883 |
| 66 45 | .788,657,057,607 | .724,611,172,351 | .623,510,987,977 | .267,881,461,178 | .763,847,230,606 | .401,160,045,100 |
| 67 00 | .787,179,748,121 | .724,602,778,685 | .629,361,870,086 | .267,148,264,134 | .764,520,249,502 | .404,850,349,334 |
| 67 15 | .785,698,491,295 | .724,571,912,112 | .635,235,692,767 | .266,376,433,296 | .765,179,573,272 | .408,554,348,694 |
| 67 30 | .784,213,303,576 | .724,518,620,297 | .641,132,261,729 | .265,565,678,083 | .765,825,317,845 | .412,271,927,251 |

SHEET 3 OF 4

205-D-1863

Table H-3.—*Additional trigonometric functions for arch load constants* (sheet 4).

| $\Phi_1$<br>° ' | $\frac{\sin\Phi_1}{\Phi_1}$ | $\frac{\sin^2\Phi_1}{\Phi_1}$ | $\frac{\Phi_1^2}{2}\sin\Phi_1$ | $\frac{\Phi_1^2}{2}\cos\Phi_1$ | $\frac{\Phi_1+\sin\Phi_1\cos\Phi_1}{2}$ | $\frac{\Phi_1-\sin\Phi_1\cos\Phi_1}{2}$ |
|---|---|---|---|---|---|---|
| 67 45 | .782,724,201,456 | .724,442,951,923 | .647,051,380,248 | .264,715,708,738 | .766,457,600,188 | .416,002,968,038 |
| 68 00 | .781,231,201,461 | .724,344,956,679 | .652,992,849,167 | .263,826,236,346 | .767,076,538,293 | .419,747,353,063 |
| 68 15 | .779,734,320,161 | .724,224,685,268 | .658,956,466,905 | .262,896,972,853 | .767,682,251,167 | .423,504,963,319 |
| 68 30 | .778,233,574,164 | .724,082,189,395 | .664,942,029,454 | .261,927,631,078 | .768,274,858,823 | .427,275,678,793 |
| 68 45 | .776,728,980,115 | .723,917,521,767 | .670,949,330,386 | .260,917,924,738 | .768,854,482,277 | .431,059,378,469 |
| 69 00 | .775,220,554,700 | .723,730,736,086 | .676,978,160,855 | .259,867,568,457 | .769,421,243,528 | .434,855,940,348 |
| 69 15 | .773,708,314,644 | .723,521,887,050 | .683,028,309,602 | .258,776,277,788 | .769,975,265,557 | .438,665,241,449 |
| 69 30 | .772,192,276,708 | .723,291,030,344 | .689,099,562,955 | .257,643,769,228 | .770,516,672,315 | .442,487,157,821 |
| 69 45 | .770,672,457,693 | .723,038,222,641 | .695,191,704,836 | .256,469,760,239 | .771,045,588,715 | .446,321,564,551 |
| 70 00 | .769,148,874,438 | .722,763,521,595 | .701,304,516,764 | .255,253,969,259 | .771,562,140,620 | .450,168,335,776 |
| 70 15 | .767,621,543,817 | .722,466,985,834 | .707,437,777,859 | .253,996,115,722 | .772,066,454,832 | .454,027,344,694 |
| 70 30 | .766,090,482,745 | .722,148,674,966 | .713,591,264,847 | .252,695,920,078 | .772,558,659,091 | .457,898,463,565 |
| 70 45 | .764,555,708,172 | .721,808,649,564 | .719,764,752,058 | .251,353,103,805 | .773,038,882,053 | .461,781,563,733 |
| 71 00 | .763,017,237,085 | .721,446,971,166 | .725,958,011,440 | .249,967,389,427 | .773,507,253,290 | .465,676,515,626 |
| 71 15 | .761,475,086,510 | .721,063,702,276 | .732,170,812,558 | .248,538,500,538 | .773,963,903,275 | .469,583,188,771 |
| 71 30 | .759,929,273,507 | .720,658,906,350 | .738,402,922,596 | .247,066,161,807 | .774,408,963,376 | .473,501,451,800 |
| 71 45 | .758,379,815,173 | .720,232,647,799 | .744,654,106,367 | .245,550,099,005 | .774,842,565,841 | .477,431,172,465 |
| 72 00 | .756,826,728,640 | .719,784,991,979 | .750,924,126,313 | .243,990,039,016 | .775,264,843,791 | .481,372,217,645 |
| 72 15 | .755,270,031,080 | .719,316,005,197 | .757,212,742,515 | .242,385,709,858 | .775,675,931,211 | .485,324,453,355 |
| 72 30 | .753,709,739,696 | .718,825,754,692 | .763,519,712,692 | .240,736,840,697 | .776,075,962,936 | .489,287,744,760 |
| 72 45 | .752,145,871,728 | .718,314,308,641 | .769,844,792,211 | .239,043,161,867 | .776,465,074,644 | .493,261,956,182 |
| 73 00 | .750,578,444,452 | .717,781,736,152 | .776,187,734,088 | .237,304,404,883 | .776,843,402,846 | .497,246,951,110 |
| 73 15 | .749,007,475,177 | .717,228,107,258 | .782,548,288,996 | .235,520,302,459 | .777,211,084,871 | .501,242,592,215 |
| 73 30 | .747,432,981,249 | .716,653,492,913 | .788,926,205,269 | .233,690,588,530 | .777,568,258,862 | .505,248,741,354 |
| 73 45 | .745,854,980,047 | .716,057,964,987 | .795,321,228,909 | .231,814,998,260 | .777,915,063,760 | .509,265,259,586 |
| 74 00 | .744,273,488,984 | .715,441,596,262 | .801,733,103,589 | .229,893,268,064 | .778,251,639,297 | .513,292,007,179 |
| 74 15 | .742,688,525,509 | .714,804,460,430 | .808,161,570,665 | .227,925,135,628 | .778,578,125,982 | .517,328,843,624 |
| 74 30 | .741,100,107,103 | .714,146,632,081 | .814,606,369,167 | .225,910,339,917 | .778,894,665,095 | .521,375,627,641 |
| 74 45 | .739,508,251,279 | .713,468,186,701 | .821,067,235,825 | .223,848,621,200 | .779,201,398,673 | .525,432,217,193 |
| 75 00 | .737,912,975,587 | .712,769,200,673 | .827,543,905,058 | .221,739,721,062 | .779,498,469,498 | .529,498,469,498 |
| 75 15 | .736,314,297,609 | .712,049,751,267 | .834,036,108,991 | .219,583,382,421 | .779,786,021,089 | .533,574,241,036 |
| 75 30 | .734,712,234,957 | .711,309,916,630 | .840,543,577,455 | .217,379,349,547 | .780,064,197,689 | .537,659,387,566 |
| 75 45 | .733,106,805,279 | .710,549,775,793 | .847,066,037,996 | .215,127,368,079 | .780,333,144,257 | .541,753,764,128 |
| 76 00 | .731,498,026,253 | .709,769,408,653 | .853,603,215,880 | .212,827,185,036 | .780,593,006,454 | .545,857,225,061 |
| 76 15 | .729,885,915,591 | .708,968,895,978 | .860,154,834,103 | .210,478,548,841 | .780,843,930,631 | .549,969,624,014 |
| 76 30 | .728,270,491,037 | .708,148,319,398 | .866,720,613,390 | .208,081,209,332 | .781,086,063,822 | .554,090,813,953 |
| 76 45 | .726,651,770,363 | .707,307,761,394 | .873,300,272,211 | .205,634,917,781 | .781,319,553,730 | .558,220,647,175 |
| 77 00 | .725,029,771,378 | .706,447,305,309 | .879,893,526,782 | .203,139,426,911 | .781,544,548,715 | .562,358,975,320 |
| 77 15 | .723,404,511,918 | .705,567,035,321 | .886,500,091,074 | .200,594,490,911 | .781,761,197,785 | .566,505,649,380 |
| 77 30 | .721,776,009,851 | .704,667,036,453 | .893,119,676,818 | .197,999,865,452 | .781,969,650,583 | .570,660,519,712 |
| 77 45 | .720,144,283,077 | .703,747,394,564 | .899,751,993,517 | .195,355,307,705 | .782,170,057,376 | .574,823,436,049 |
| 78 00 | .718,509,349,525 | .702,808,196,343 | .906,396,748,446 | .192,660,576,359 | .782,362,569,046 | .578,994,247,509 |
| 78 15 | .716,871,227,154 | .701,849,529,300 | .913,053,646,668 | .189,915,431,630 | .782,547,337,074 | .583,172,802,611 |
| 78 30 | .715,229,933,954 | .700,871,481,766 | .919,722,391,031 | .187,119,635,286 | .782,724,513,530 | .587,358,949,285 |
| 78 45 | .713,585,487,945 | .699,874,142,886 | .926,402,682,186 | .184,272,950,660 | .782,894,251,064 | .591,552,534,881 |
| 79 00 | .711,937,907,176 | .698,857,602,611 | .933,094,218,590 | .181,375,142,665 | .783,056,702,892 | .595,753,406,183 |
| 79 15 | .710,287,209,725 | .697,821,951,694 | .939,796,696,513 | .178,425,977,807 | .783,212,022,784 | .599,961,409,421 |
| 79 30 | .708,633,413,698 | .696,767,281,682 | .946,509,810,045 | .175,425,224,212 | .783,360,365,054 | .604,176,390,281 |
| 79 45 | .706,976,537,232 | .695,693,684,917 | .953,233,251,109 | .172,372,651,631 | .783,501,884,547 | .608,398,193,918 |
| 80 00 | .705,316,598,492 | .694,601,254,523 | .959,966,709,467 | .169,268,031,461 | .783,636,736,629 | .612,626,664,966 |
| 80 15 | .703,653,615,671 | .693,490,084,403 | .966,709,872,722 | .166,111,136,761 | .783,765,077,171 | .616,861,647,554 |
| 80 30 | .701,987,606,989 | .692,360,269,231 | .973,462,426,338 | .162,901,742,266 | .783,887,062,542 | .621,102,985,313 |
| 80 45 | .700,318,590,695 | .691,211,904,450 | .980,224,053,636 | .159,639,624,406 | .784,002,849,594 | .625,350,521,391 |
| 81 00 | .698,646,585,067 | .690,045,086,267 | .986,994,435,815 | .156,324,561,320 | .784,112,595,651 | .629,604,098,464 |
| 81 15 | .696,971,608,406 | .688,859,911,637 | .993,773,251,950 | .152,956,332,871 | .784,216,458,499 | .633,863,558,746 |
| 81 30 | .695,293,679,045 | .687,656,478,271 | 1.000,560,179,005 | .149,534,720,663 | .784,314,596,368 | .638,128,744,007 |
| 81 45 | .693,612,815,340 | .686,434,884,617 | 1.007,354,891,845 | .146,059,508,056 | .784,407,167,928 | .642,399,495,577 |
| 82 00 | .691,929,035,677 | .685,195,229,866 | 1.014,157,063,238 | .142,530,480,187 | .784,494,332,271 | .646,675,654,364 |
| 82 15 | .690,242,358,465 | .683,937,613,935 | 1.020,966,363,872 | .138,947,423,975 | .784,576,248,902 | .650,957,060,863 |
| 82 30 | .688,552,802,142 | .682,662,137,468 | 1.027,782,462,359 | .135,310,128,146 | .784,653,077,723 | .655,243,555,172 |
| 82 45 | .686,860,385,169 | .681,368,901,827 | 1.034,605,025,242 | .131,618,383,246 | .784,724,979,026 | .659,534,976,999 |
| 83 00 | .685,165,126,036 | .680,058,009,086 | 1.041,433,717,013 | .127,87[illegible],981,653 | .784,792,113,477 | .663,831,165,678 |
| 83 15 | .683,467,043,256 | .678,729,562,026 | 1.048,268,200,115 | .124,070,717,599 | .784,354,642,107 | .668,131,960,178 |
| 83 30 | .681,766,155,369 | .677,383,664,127 | 1.055,108,134,953 | .120,214,387,180 | .784,912,726,294 | .672,437,199,121 |
| 83 45 | .680,062,480,936 | .676,020,419,561 | 1.061,953,179,904 | .116,302,788,373 | .784,966,527,757 | .676,746,720,788 |
| 84 00 | .678,356,038,547 | .674,639,933,190 | 1.068,802,991,332 | .112,335,721,054 | .785,016,208,542 | .681,060,363,133 |
| 84 15 | .676,646,846,815 | .673,242,310,555 | 1.075,657,223,590 | .108,312,987,008 | .785,061,931,007 | .685,377,963,798 |
| 84 30 | .674,934,924,376 | .671,827,657,869 | 1.082,515,529,033 | .104,234,389,949 | .785,103,857,812 | .689,699,360,123 |
| 84 45 | .673,220,289,892 | .670,396,082,018 | 1.089,377,558,029 | .100,099,735,535 | .785,142,151,905 | .694,024,389,160 |
| 85 00 | .671,502,962,047 | .668,947,690,544 | 1.096,242,958,968 | .095,908,831,380 | .785,176,976,515 | .698,352,887,680 |
| 85 15 | .669,782,959,548 | .667,482,591,647 | 1.103,111,378,275 | .091,661,487,069 | .785,208,495,128 | .702,684,692,197 |
| 85 30 | .668,060,301,127 | .666,000,894,172 | 1.109,982,460,418 | .087,357,514,180 | .785,236,871,488 | .707,019,638,967 |
| 85 45 | .666,335,005,537 | .664,502,707,610 | 1.116,855,847,915 | .082,996,726,289 | .785,262,269,575 | .711,357,564,010 |
| 86 00 | .664,607,091,557 | .662,988,142,085 | 1.123,731,181,356 | .078,578,938,994 | .785,284,853,597 | .715,698,303,118 |
| 86 15 | .662,876,577,983 | .661,457,308,347 | 1.130,608,099,398 | .074,103,969,921 | .785,304,787,977 | .720,041,691,868 |
| 86 30 | .661,143,483,638 | .659,910,317,769 | 1.137,486,238,788 | .069,571,638,750 | .785,322,237,339 | .724,387,565,636 |
| 86 45 | .659,407,827,365 | .658,347,282,341 | 1.144,365,234,372 | .064,981,767,216 | .785,337,366,494 | .728,735,759,611 |
| 87 00 | .657,669,628,029 | .656,768,314,661 | 1.151,244,719,101 | .060,334,179,139 | .785,350,340,435 | .733,086,108,800 |
| 87 15 | .655,928,904,516 | .655,173,527,925 | 1.158,124,324,042 | .055,628,700,425 | .785,361,324,312 | .737,438,448,053 |
| 87 30 | .654,185,675,734 | .653,563,035,926 | 1.165,003,678,402 | .050,865,159,090 | .785,370,483,434 | .741,792,612,061 |
| 87 45 | .652,439,960,610 | .651,936,953,045 | 1.171,882,409,519 | .046,043,385,270 | .785,377,983,244 | .746,148,435,381 |
| 88 00 | .650,691,778,094 | .650,295,394,244 | 1.178,760,142,889 | .041,163,211,236 | .785,383,989,314 | .750,505,752,441 |
| 88 15 | .648,941,147,156 | .648,638,475,059 | 1.185,636,502,175 | .036,224,471,409 | .785,388,667,326 | .754,864,397,559 |
| 88 30 | .647,188,086,785 | .646,966,311,592 | 1.192,511,109,209 | .031,227,002,376 | .785,392,183,068 | .759,224,204,947 |
| 88 45 | .645,432,615,990 | .645,279,020,506 | 1.199,383,584,017 | .026,170,642,902 | .785,394,702,414 | .763,585,008,731 |
| 89 00 | .643,674,753,801 | .643,576,719,018 | 1.206,253,544,822 | .021,055,233,944 | .785,396,391,313 | .767,946,642,962 |
| 89 15 | .641,914,519,268 | .641,859,524,894 | 1.213,120,608,060 | .015,880,6[illegible]8,669 | .785,397,415,779 | .772,308,941,626 |
| 89 30 | .640,151,531,456 | .640,127,556,432 | 1.219,984,388,387 | .010,646,642,464 | .785,397,941,877 | .776,671,738,658 |
| 89 45 | .638,387,009,455 | .638,380,932,471 | 1.226,844,498,699 | .005,353,152,951 | .785,398,135,707 | .781,034,867,958 |
| 90 00 | .636,619,772,368 | .636,619,772,368 | 1.233,700,550,136 | 0 | .785,398,163,397 | .785,398,163,397 |

SHEET 4 OF 4

205-D-1864

Table H-4.—*Trigonometric functions and integrals for arch load constants* (sheet 1).

| $\Phi_1$<br>° ′ | $\frac{\Phi_1^2}{2}$ | $\frac{\Phi_1^3}{6}$ | $\frac{\sin^2 \Phi_1}{2}$ | $\Phi_1 - \sin \Phi_1$ | $\frac{\Phi_1^2}{2} - \text{vers}\,\Phi_1$ | $\sin \Phi_1 - \Phi_1 + \frac{\Phi_1^3}{6}$ |
|---|---|---|---|---|---|---|
| 0 15 | .0$^{5}$9,519,294,368 | .0$^{7}$13,845,252,433 | .0$^{5}$9,519,233,958 | .0$^{7}$13,845,239 | .0$^{10}$15,102,818 | .0$^{13}$13,179,697 |
| 0 30 | .0$^{4}$38,077,177,473 | .0$^{6}$110,762,019,463 | .0$^{4}$38,076,210,90 | .0$^{6}$110,761,598 | .0$^{9}$241,644,627 | .0$^{12}$421,749,742 |
| 0 45 | .0$^{4}$85,673,649,315 | .0$^{6}$373,821,815,69 | .0$^{4}$85,668,756,11 | .0$^{6}$373,818,613 | .0$^{8}$1,223,322 | .0$^{11}$3,202,654,8 |
| 1 00 | .0$^{3}$152,308,709,89 | .0$^{6}$886,096,155,70 | .0$^{3}$152,293,245,22 | .0$^{6}$886,082,660 | .0$^{8}$3,866,284 | .0$^{10}$13,495,918 |
| 1 15 | .0$^{3}$237,982,359,21 | .0$^{5}$1,730,656,554 | .0$^{3}$237,944,604,54 | .0$^{5}$1,730,615 | .0$^{8}$9,439,117 | .0$^{10}$41,186,106 |
| 1 30 | .0$^{3}$342,694,597,26 | .0$^{5}$2,990,574,525 | .0$^{3}$342,616,311,36 | .0$^{5}$2,990,472 | .0$^{7}$19,572,817 | .0$^{9}$102,483,701 |
| 1 45 | .0$^{3}$466,445,424,05 | .0$^{5}$4,748,921,584 | .0$^{3}$466,300,394,53 | .0$^{5}$4,748,700 | .0$^{7}$36,260,761 | .0$^{9}$221,506,354 |
| 2 00 | .0$^{3}$609,234,839,57 | .0$^{5}$7,088,769,246 | .0$^{3}$608,987,435,04 | .0$^{5}$7,088,337 | .0$^{7}$61,858,669 | .0$^{9}$431,859,990 |
| 2 15 | .0$^{3}$771,062,843,84 | .0$^{4}$10,093,189,024 | .0$^{3}$770,666,566,72 | .0$^{4}$10,092,411 | .0$^{7}$99,084,559 | .0$^{9}$778,219,728 |
| 2 30 | .0$^{3}$951,929,436,84 | .0$^{4}$13,845,252,433 | .0$^{3}$951,325,477,05 | .0$^{4}$13,843,935 | .0$^{6}$151,018,691 | .0$^{8}$1,317,910,6 |
| 2 45 | .001,151,834,619 | .0$^{4}$18,428,030,988 | .001,150,950,408 | .0$^{4}$18,425,909 | .0$^{6}$221,103,519 | .0$^{8}$2,122,488 |
| 3 00 | .001,370,778,389 | .0$^{4}$23,924,596,204 | .001,369,526,158 | .0$^{4}$23,921,317 | .0$^{6}$313,143,614 | .0$^{8}$3,279,317,9 |
| 3 15 | .001,608,760,748 | .0$^{4}$30,418,019,595 | .001,607,036,081 | .0$^{4}$30,413,127 | .0$^{6}$431,305,597 | .0$^{8}$4,893,157 |
| 3 30 | .001,865,781,696 | .0$^{4}$37,991,372,676 | .001,863,462,090 | .0$^{4}$37,984,285 | .0$^{6}$580,118,061 | .0$^{8}$7,087,731 |
| 3 45 | .002,141,841,233 | .0$^{4}$46,727,726,961 | .002,138,784,657 | .0$^{4}$46,717,720 | .0$^{6}$764,471,479 | .0$^{7}$10,007,317 |
| 4 00 | .002,436,939,358 | .0$^{4}$56,710,153,965 | .002,432,982,815 | .0$^{4}$56,696,336 | .0$^{6}$989,618,118 | .0$^{7}$13,818,317 |
| 4 15 | .002,751,076,072 | .0$^{4}$68,021,725,202 | .002,746,034,159 | .0$^{4}$68,003,014 | .0$^{5}$1,261,172 | .0$^{7}$18,710,843 |
| 4 30 | .003,084,251,375 | .0$^{4}$80,745,512,188 | .003,077,914,851 | .0$^{4}$80,720,612 | .0$^{5}$1,585,109 | .0$^{7}$24,900,288 |
| 4 45 | .003,436,465,267 | .0$^{4}$94,964,586,437 | .003,428,599,616 | .0$^{4}$94,931,958 | .0$^{5}$1,967,765 | .0$^{7}$32,628,910 |
| 5 00 | .003,807,717,747 | .0$^{3}$110,762,019,46 | .003,798,061,747 | .0$^{3}$110,719,852 | .0$^{5}$2,415,839 | .0$^{7}$42,167,404 |
| 5 15 | .004,198,008,816 | .0$^{3}$128,220,882,78 | .004,186,273,109 | .0$^{3}$128,167,067 | .0$^{5}$2,936,391 | .0$^{7}$53,816,480 |
| 5 30 | .004,607,338,474 | .0$^{3}$147,424,247,90 | .004,593,204,138 | .0$^{3}$147,356,340 | .0$^{5}$3,536,841 | .0$^{7}$67,908,441 |
| 5 45 | .005,035,706,721 | .0$^{3}$168,455,186,35 | .005,018,823,845 | .0$^{3}$168,370,378 | .0$^{5}$4,224,972 | .0$^{7}$84,808,753 |
| 6 00 | .005,483,113,556 | .0$^{3}$191,396,769,63 | .005,463,099,817 | .0$^{3}$191,291,852 | .0$^{5}$5,008,924 | .0$^{6}$104,917,625 |
| 6 15 | .005,949,558,980 | .0$^{3}$216,332,069,26 | .005,925,998,220 | .0$^{3}$216,203,398 | .0$^{5}$5,897,202 | .0$^{6}$128,671,582 |
| 6 30 | .006,435,042,993 | .0$^{3}$243,344,156,76 | .006,407,483,804 | .0$^{3}$243,187,612 | .0$^{5}$6,898,670 | .0$^{6}$156,545,035 |
| 6 45 | .006,939,565,595 | .0$^{3}$272,516,103,66 | .006,907,519,901 | .0$^{3}$272,327,052 | .0$^{5}$8,022,550 | .0$^{6}$189,051,856 |
| 7 00 | .007,463,126,785 | .0$^{3}$303,930,981,42 | .007,426,068,431 | .0$^{3}$303,704,235 | .0$^{5}$9,278,426 | .0$^{6}$226,746,950 |
| 7 15 | .008,005,726,564 | .0$^{3}$337,671,861,59 | .007,963,089,905 | .0$^{3}$337,401,634 | .0$^{4}$10,676,244 | .0$^{6}$270,227,825 |
| 7 30 | .008,567,364,932 | .0$^{3}$373,821,815,71 | .008,518,543,428 | .0$^{3}$373,501,680 | .0$^{4}$12,226,306 | .0$^{6}$320,136,163 |
| 7 45 | .009,148,041,888 | .0$^{3}$412,463,915,23 | .009,092,386,698 | .0$^{3}$412,086,756 | .0$^{4}$13,939,275 | .0$^{6}$377,159,389 |
| 8 00 | .009,747,757,433 | .0$^{3}$453,681,231,71 | .009,684,576,015 | .0$^{3}$453,239,200 | .0$^{4}$15,826,175 | .0$^{6}$442,032,238 |
| 8 15 | .010,366,511,567 | .0$^{3}$497,556,836,67 | .010,295,066,283 | .0$^{3}$497,041,299 | .0$^{4}$17,898,387 | .0$^{6}$515,538,326 |
| 8 30 | .011,004,304,290 | .0$^{3}$544,173,801,63 | .010,923,811,009 | .0$^{3}$543,575,289 | .0$^{4}$20,167,652 | .0$^{6}$598,511,713 |
| 8 45 | .011,661,135,601 | .0$^{3}$593,615,198,05 | .011,570,762,313 | .0$^{3}$592,923,359 | .0$^{4}$22,646,069 | .0$^{6}$691,838,471 |
| 9 00 | .012,337,005,501 | .0$^{3}$645,964,097,49 | .012,235,870,926 | .0$^{3}$645,167,639 | .0$^{4}$25,346,096 | .0$^{6}$796,458,247 |
| 9 15 | .013,031,913,990 | .0$^{3}$701,303,571,46 | .012,919,086,198 | .0$^{3}$700,390,205 | .0$^{4}$28,280,550 | .0$^{6}$913,365,831 |
| 9 30 | .013,745,861,068 | .0$^{3}$759,716,691,50 | .013,620,356,100 | .0$^{3}$758,673,078 | .0$^{4}$31,462,605 | .0$^{5}$1,043,613 |
| 9 45 | .014,478,846,734 | .0$^{3}$821,286,529,05 | .014,339,627,227 | .0$^{3}$820,098,220 | .0$^{4}$34,905,792 | .0$^{5}$1,188,309 |
| 10 00 | .015,230,870,989 | .0$^{3}$886,096,155,68 | .015,076,844,804 | .0$^{3}$884,747,532 | .0$^{4}$38,624,001 | .0$^{5}$1,348,623 |
| 10 15 | .015,001,933,833 | .0$^{3}$954,228,642,92 | .015,831,952,688 | .0$^{3}$952,702,855 | .0$^{4}$42,631,479 | .0$^{5}$1,525,787 |
| 10 30 | .016,792,035,266 | .001,025,767,062 | .016,604,893,376 | .001,024,045,967 | .0$^{4}$46,942,830 | .0$^{5}$1,721,095 |
| 10 45 | .017,601,175,287 | .001,100,794,485 | .017,395,608,005 | .001,098,858,580 | .0$^{4}$51,573,013 | .0$^{5}$1,935,905 |
| 11 00 | .018,429,353,897 | .001,179,393,983 | .018,204,036,358 | .001,177,222,342 | .0$^{4}$56,537,345 | .0$^{5}$2,171,640 |
| 11 15 | .019,276,571,096 | .001,261,648,628 | .019,030,116,872 | .001,259,218,833 | .0$^{4}$61,851,499 | .0$^{5}$2,429,795 |
| 11 30 | .020,142,826,883 | .001,347,641,491 | .019,873,786,637 | .001,344,929,562 | .0$^{4}$67,531,504 | .0$^{5}$2,711,929 |
| 11 45 | .021,028,121,260 | .001,437,455,643 | .020,734,981,404 | .001,434,435,969 | .0$^{4}$73,593,745 | .0$^{5}$3,019,674 |
| 12 00 | .021,932,454,225 | .001,531,174,157 | .021,613,635,589 | .001,527,819,421 | .0$^{4}$80,054,959 | .0$^{5}$3,354,735 |
| 12 15 | .022,855,825,778 | .001,628,880,103 | .022,509,682,281 | .001,625,161,213 | .0$^{4}$86,932,241 | .0$^{5}$3,718,891 |
| 12 30 | .023,798,235,921 | .001,730,656,554 | .023,423,053,241 | .001,726,542,561 | .0$^{4}$94,243,041 | .0$^{5}$4,113,993 |
| 12 45 | .024,759,684,652 | .001,836,586,580 | .024,353,678,913 | .001,832,044,607 | .0$^{3}$102,005,161 | .0$^{5}$4,541,973 |
| 13 00 | .025,740,171,972 | .001,946,753,254 | .025,301,488,425 | .001,941,748,415 | .0$^{3}$110,236,757 | .0$^{5}$5,004,839 |
| 13 15 | .026,739,697,881 | .002,061,239,646 | .026,266,409,599 | .002,055,734,967 | .0$^{3}$118,956,341 | .0$^{5}$5,504 680 |
| 13 30 | .027,758,262,378 | .002,180,128,829 | .027,248,368,953 | .002,174,085,163 | .0$^{3}$128,182,776 | .0$^{5}$6,043,666 |
| 13 45 | .028,795,865,464 | .002,303,503,873 | .028,247,291,705 | .002,296,879,823 | .0$^{3}$137,935,277 | .0$^{5}$6,624,050 |
| 14 00 | .029,852,507,139 | .002,431,447,851 | .029,263,101,785 | .002,424,199,679 | .0$^{3}$148,233,415 | .0$^{5}$7,248,172 |
| 14 15 | .030,928,187,403 | .002,564,043,834 | .030,295,721,835 | .002,556,125,380 | .0$^{3}$159,097,110 | .0$^{5}$7,918,454 |
| 14 30 | .032,022,906,255 | .002,701,374,893 | .031,345,073,215 | .002,692,737,485 | .0$^{3}$170,546,633 | .0$^{5}$8,637,408 |
| 14 45 | .033,136,663,696 | .002,843,524,099 | .032,411,076,015 | .002,834,116,463 | .0$^{3}$182,602,610 | .0$^{5}$9,407,636 |
| 15 00 | .034,269,459,726 | .002,990,574,525 | .033,493,649,054 | .002,980,342,696 | .0$^{3}$195,286,015 | .0$^{4}$10,231,829 |
| 15 15 | .035,421,294,345 | .003,142,609,242 | .034,592,709,890 | .003,131,496,471 | .0$^{3}$208,618,174 | .0$^{4}$11,112,771 |
| 15 30 | .036,592,167,552 | .003,299,711,322 | .035,708,174,824 | .003,287,657,981 | .0$^{3}$222,620,761 | .0$^{4}$12,053,341 |
| 15 45 | .037,782,079,348 | .003,461,963,835 | .036,839,958,911 | .003,448,907,324 | .0$^{3}$237,315,802 | .0$^{4}$13,056,511 |
| 16 00 | .038,991,029,733 | .003,629,449,854 | .037,987,975,961 | .003,615,324,502 | .0$^{3}$252,725,671 | .0$^{4}$14,125,352 |
| 16 15 | .040,219,018,706 | .003,802,252,449 | .039,152,138,547 | .003,786,989,418 | .0$^{3}$268,873,092 | .0$^{4}$15,263,031 |
| 16 30 | .041,466,046,268 | .003,980,454,693 | .040,332,358,014 | .003,963,981,875 | .0$^{3}$285,781,136 | .0$^{4}$16,472,818 |
| 16 45 | .042,732,112,419 | .004,164,139,657 | .041,528,544,483 | .004,146,381,575 | .0$^{3}$303,473,224 | .0$^{4}$17,758,082 |
| 17 00 | .044,017,217,159 | .004,353,390,413 | .042,740,606,861 | .004,334,268,116 | .0$^{3}$321,973,122 | .0$^{4}$19,122,297 |
| 17 15 | .045,321,360,488 | .004,548,290,031 | .043,968,452,845 | .004,527,720,993 | .0$^{3}$341,304,945 | .0$^{4}$20,569,038 |
| 17 30 | .046,644,542,405 | .004,748,921,584 | .045,211,988,928 | .004,726,819,595 | .0$^{3}$361,493,153 | .0$^{4}$22,101,989 |
| 17 45 | .047,986,762,911 | .004,955,368,144 | .046,471,120,411 | .004,931,643,201 | .0$^{3}$382,562,554 | .0$^{4}$23,724,943 |
| 18 00 | .049,348,022,005 | .005,167,712,780 | .047,745,751,406 | .005,142,270,984 | .0$^{3}$404,538,300 | .0$^{4}$25,441,796 |
| 18 15 | .050,728,319,689 | .005,386,038,566 | .049,035,784,846 | .005,358,782,005 | .0$^{3}$427,445,891 | .0$^{4}$27,256,561 |
| 18 30 | .052,127,655,961 | .005,610,428,572 | .050,341,122,488 | .005,581,255,214 | .0$^{3}$451,311,167 | .0$^{4}$29,173,358 |
| 18 45 | .053,546,030,822 | .005,840,965,870 | .051,661,664,927 | .005,809,769,446 | .0$^{3}$476,160,317 | .0$^{4}$31,196,424 |
| 19 00 | .054,983,444,272 | .006,077,733,532 | .052,997,311,598 | .006,044,403,422 | .0$^{3}$502,019,871 | .0$^{4}$33,330,110 |
| 19 15 | .056,439,896,310 | .006,320,814,629 | .054,347,960,787 | .006,285,235,746 | .0$^{3}$528,916,703 | .0$^{4}$35,578,883 |
| 19 30 | .057,915,386,937 | .006,570,292,233 | .055,713,509,636 | .006,532,344,905 | .0$^{3}$556,878,029 | .0$^{4}$37,947,328 |
| 19 45 | .059,409,916,153 | .006,826,249,414 | .057,093,854,153 | .006,785,809,266 | .0$^{3}$585,931,409 | .0$^{4}$40,440,148 |
| 20 00 | .060,923,483,957 | .007,088,769,246 | .058,488,889,220 | .007,045,707,073 | .0$^{3}$616,104,743 | .0$^{4}$43,062,173 |
| 20 15 | .062,456,090,351 | .007,357,934,798 | .059,898,508,600 | .007,312,116,452 | .0$^{3}$647,426,273 | .0$^{4}$45,818,347 |
| 20 30 | .064,007,735,333 | .007,633,829,143 | .061,322,604,944 | .007,585,115,400 | .0$^{3}$679,924,581 | .0$^{4}$48,713,744 |
| 20 45 | .065,578,418,904 | .007,916,535,353 | .062,761,069,803 | .007,864,781,791 | .0$^{3}$713,628,590 | .0$^{4}$51,753,563 |
| 21 00 | .067,168,141,063 | .008,206,136,498 | .064,213,793,631 | .008,151,193,374 | .0$^{3}$748,567,560 | .0$^{4}$54,943,125 |
| 21 15 | .068,776,901,811 | .008,502,715,650 | .065,680,665,798 | .008,444,427,765 | .0$^{3}$784,771,094 | .0$^{4}$58,287,886 |
| 21 30 | .070,404,701,148 | .008,806,355,881 | .067,161,574,595 | .008,744,562,455 | .0$^{3}$822,269,130 | .0$^{4}$61,793,427 |
| 21 45 | .072,051,539,074 | .009,117,140,263 | .068,656,407,247 | .009,051,674,799 | .0$^{3}$861,091,946 | .0$^{4}$65,465,465 |
| 22 00 | .073,717,415,588 | .009,435,151,866 | .070,165,049,915 | .009,365,842,023 | .0$^{3}$901,270,155 | .0$^{4}$69,309,844 |
| 22 15 | .075,402,330,692 | .009,760,473,762 | .071,687,387,711 | .009,687,141,217 | .0$^{3}$942,834,710 | .0$^{4}$73,332,545 |
| 22 30 | .077,106,284,384 | .010,093,189,024 | .073,223,304,703 | .010,015,649,334 | .0$^{3}$985,816,895 | .0$^{4}$77,539,690 |
| ° ′<br>$\Phi_1$ | $\int \Phi_1 d\Phi_1$ | $\int \frac{\Phi_1^2}{2} d\Phi_1$ |  | $\int \text{vers}\,\Phi_1 d\Phi_1$ | $\int(\Phi_1 - \sin \Phi_1)d\Phi_1$ | $\int\left(\frac{\Phi_1^2}{2} - \text{vers}\,\Phi_1\right)d\Phi_1$ |

SHEET 1 OF 4

205-D-1865

Table H-4.–*Trigonometric functions and integrals for arch load constants* (sheet 2).

| $\Phi_1$ ° ′ | $\frac{\Phi_1^2}{2}$ | $\frac{\Phi_1^3}{6}$ | $\frac{\text{SIN}^2\Phi_1}{2}$ | $\Phi_1-\text{SIN}\Phi_1$ | $\frac{\Phi_1^2}{2}-\text{VERS }\Phi_1$ | $\text{SIN }\Phi_1-\Phi_1+\frac{\Phi_1^3}{6}$ |
|---|---|---|---|---|---|---|
| 22 45 | .078,829,276,664 | .010,433,380,721 | .074,772,683,925 | .010,351,443,192 | .001,030,248,334 | .0$^{4}$61,937,529 |
| 23 00 | .080,571,307,534 | .010,781,131,926 | .076,335,407,385 | .010,694,599,470 | .001,076,160,986 | .0$^{4}$86,532,456 |
| 23 15 | .082,332,376,992 | .011,136,525,711 | .077,911,356,076 | .011,045,194,705 | .001,123,587,141 | .0$^{4}$91,331,006 |
| 23 30 | .084,112,485,039 | .011,499,645,147 | .079,500,409,984 | .011,403,305,294 | .001,172,559,424 | .0$^{4}$96,339,853 |
| 23 45 | .085,911,631,674 | .011,870,573,305 | .081,102,448,096 | .011,769,007,490 | .001,223,110,793 | .0$^{3}$101,565,815 |
| 24 00 | .087,729,816,899 | .012,249,393,256 | .082,717,348,410 | .012,142,377,403 | .001,275,274,542 | .0$^{3}$107,015,853 |
| 24 15 | .089,567,040,712 | .012,636,188,074 | .084,344,987,946 | .012,523,490,996 | .001,329,084,289 | .0$^{3}$112,697,078 |
| 24 30 | .091,423,303,114 | .013,031,040,828 | .085,985,242,752 | .012,912,424,083 | .001,384,573,991 | .0$^{3}$118,616,745 |
| 24 45 | .093,298,604,104 | .013,434,034,590 | .087,637,987,917 | .013,309,252,332 | .001,441,777,929 | .0$^{3}$124,782,258 |
| 25 00 | .095,192,943,684 | .013,845,252,433 | .089,303,097,578 | .013,714,051,258 | .001,500,730,721 | .0$^{3}$131,201,175 |
| 25 15 | .097,106,321,851 | .014,264,777,427 | .090,980,444,930 | .014,126,896,227 | .001,561,467,305 | .0$^{3}$137,881,200 |
| 25 30 | .099,038,738,608 | .014,692,692,644 | .092,669,902,237 | .014,547,862,450 | .001,624,022,958 | .0$^{3}$144,830,194 |
| 25 45 | .100,990,193,953 | .015,129,081,155 | .094,371,340,840 | .014,977,024,984 | .001,688,433,276 | .0$^{3}$152,056,171 |
| 26 00 | .102,960,687,888 | .015,574,026,033 | .096,084,631,169 | .015,414,458,729 | .001,754,734,187 | .0$^{3}$159,567,304 |
| 26 15 | .104,950,220,411 | .016,027,610,348 | .097,809,642,748 | .015,860,238,429 | .001,822,961,944 | .0$^{3}$167,371,919 |
| 26 30 | .106,958,791,522 | .016,489,917,171 | .099,546,244,212 | .016,314,438,668 | .001,893,153,124 | .0$^{3}$175,478,503 |
| 26 45 | .108,986,401,223 | .016,961,029,576 | .101,294,303,312 | .016,777,133,871 | .001,965,344,634 | .0$^{3}$183,895,705 |
| 27 00 | .111,033,049,512 | .017,441,030,633 | .103,053,686,927 | .017,248,398,298 | .002,039,573,700 | .0$^{3}$192,632,335 |
| 27 15 | .113,098,736,390 | .017,930,003,413 | .104,824,261,072 | .017,728,306,051 | .002,115,877,876 | .0$^{3}$201,697,362 |
| 27 30 | .115,183,461,857 | .018,428,030,988 | .106,605,890,912 | .018,216,931,063 | .002,194,295,035 | .0$^{3}$211,099,925 |
| 27 45 | .117,287,225,912 | .018,935,196,430 | .108,398,440,769 | .018,714,347,103 | .002,274,863,375 | .0$^{3}$220,849,327 |
| 28 00 | .119,410,028,556 | .019,451,582,810 | .110,201,774,132 | .019,220,627,772 | .002,357,621,415 | .0$^{3}$230,955,038 |
| 28 15 | .121,551,869,789 | .019,977,273,200 | .112,015,753,672 | .019,735,846,503 | .002,442,607,994 | .0$^{3}$241,426,697 |
| 28 30 | .123,712,749,611 | .020,512,350,670 | .113,840,241,246 | .020,260,076,558 | .002,529,862,273 | .0$^{3}$252,274,1 2 |
| 28 45 | .125,892,668,021 | .021,056,898,294 | .115,675,097,913 | .020,793,391,029 | .002,619,423,729 | .0$^{3}$263,507,265 |
| 29 00 | .128,091,625,020 | .021,610,999,141 | .117,520,183,942 | .021,335,862,832 | .002,711,332,159 | .0$^{3}$275,136,309 |
| 29 15 | .130,309,620,608 | .022,174,736,285 | .119,375,358,821 | .021,887,564,711 | .002,805,627,681 | .0$^{3}$287,171,574 |
| 29 30 | .132,546,654,785 | .022,748,192,795 | .121,240,481,272 | .022,448,569,235 | .002,902,350,725 | .0$^{3}$299,623,560 |
| 29 45 | .134,802,727,550 | .023,331,451,744 | .123,115,409,260 | .023,018,948,793 | .003,001,542,039 | .0$^{3}$312,502,951 |
| 30 00 | .137,077,838,904 | .023,924,596,204 | .125,000,000,000 | .023,598,775,598 | .003,103,242,688 | .0$^{3}$325,820,606 |
| 30 15 | .139,371,988,847 | .024,527,709,245 | .126,894,109,974 | .024,188,121,682 | .003,207,494,051 | .0$^{3}$339,587,563 |
| 30 30 | .141,685,177,378 | .025,140,873,940 | .128,797,594,939 | .024,787,058,897 | .003,314,337,820 | .0$^{3}$353,815,043 |
| 30 45 | .144,017,404,498 | .025,764,173,359 | .130,710,309,935 | .025,395,658,911 | .003,423,815,999 | .0$^{3}$368,514,448 |
| 31 00 | .146,368,670,207 | .026,397,690,574 | .132,632,109,303 | .026,013,993,208 | .003,535,970,909 | .0$^{3}$383,697,366 |
| 31 15 | .148,738,974,505 | .027,041,508,658 | .134,562,846,691 | .026,642,133,087 | .003,650,845,178 | .03399,375,571 |
| 31 30 | .151,128,317,392 | .027,695,710,681 | .136,502,375,065 | .027,280,149,662 | .003,768,481,746 | .0$^{3}$415,561,019 |
| 31 45 | .153,536,698,867 | .028,360,379,714 | .138,450,546,723 | .027,928,113,856 | .003,888,923,863 | .0$^{3}$432,265,858 |
| 32 00 | .155,964,118,931 | .029,035,598,830 | .140,407,213,303 | .028,586,096,405 | .004,012,215,087 | .0$^{3}$449,502,425 |
| 32 15 | .158,410,577,583 | .029,721,451,100 | .142,372,225,798 | .029,254,167,852 | .004,138,399,287 | .0$^{3}$467,283,248 |
| 32 30 | .160,876,074,825 | .030,418,019,595 | .144,345,434,565 | .029,932,398,551 | .004,267,520,638 | .0$^{3}$485,621,044 |
| 32 45 | .163,360,610,655 | .031,125,387,387 | .146,326,689,336 | .030,620,858,660 | .004,399,623,619 | .0$^{3}$504,528,727 |
| 33 00 | .165,864,185,074 | .031,843,637,547 | .148,315,839,231 | .031,319,618,143 | .004,534,753,019 | .0$^{3}$524,019,404 |
| 33 15 | .168,386,798,081 | .032,572,853,148 | .150,312,732,769 | .032,028,746,768 | .004,672,953,929 | .0$^{3}$544,106,380 |
| 33 30 | .170,928,449,678 | .033,313,117,260 | .152,317,217,878 | .032,748,314,106 | .004,814,271,745 | .0$^{3}$564,803,154 |
| 33 45 | .173,489,139,863 | .034,064,512,954 | .154,329,141,909 | .033,478,389,528 | .004,958,752,166 | .0$^{3}$586,123,426 |
| 34 00 | .176,068,868,637 | .034,827,123,304 | .156,348,351,646 | .034,219,042,207 | .005,106,441,192 | .0$^{3}$608,081,097 |
| 34 15 | .178,667,635,999 | .035,601,031,379 | .158,374,693,319 | .034,970,341, 13 | .005,257,385,126 | .0$^{3}$630,690,266 |
| 34 30 | .181,285,441,951 | .036,386,320,252 | .160,408,012,614 | .035,732,355,013 | .005,411,630,573 | .0$^{3}$653,965,239 |
| 34 45 | .183,922,286,491 | .037,183,072,993 | .162,448,154,685 | .036,505,152,472 | .005,569,224,433 | .0$^{3}$677,920,521 |
| 35 00 | .186,578,169,619 | .037,991,372,676 | . 64,494,964,169 | .037,288,801,847 | .005,730,213,908 | .0$^{3}$702,570,829 |
| 35 15 | .189,253,091,337 | .038,811,302,370 | .166,548,285,191 | .038,083,371,291 | .005,894,646,499 | .0$^{3}$727,931,079 |
| 35 30 | .191,947,051,643 | .039,642,945,148 | .168,607,961,386 | .038,888,928,747 | .006,062,569,999 | .0$^{3}$754,016,401 |
| 35 45 | .194,660,050,538 | .040,486,384,081 | .170,673,835,898 | .039,705,541,951 | .006,234,032,503 | .0$^{3}$780,842,130 |
| 36 00 | .197,392,088,022 | .041,341,702,240 | .172,745,751,406 | .040,533,278,426 | .006,409,082,397 | .0$^{3}$808,423,814 |
| 36 15 | .200,143,164,094 | .042,209,982,698 | .174,823,550,124 | .041,372,205,484 | .006,587,768,361 | .0$^{3}$836,777,214 |
| 36 30 | .202,913,278,755 | .043,088,308,525 | .176,907,073,819 | .042,222,390,227 | .006,770,139,372 | .0$^{3}$865,918,298 |
| 36 45 | .205,702,432,005 | .043,979,762,794 | .178,996,163,824 | .043,083,899,537 | .006,956,244,696 | .0$^{3}$895,863,257 |
| 37 00 | .208,510,623,844 | .044,883,428,575 | .181,090,661,046 | .043,956,800,086 | .007,146,133,891 | .0$^{3}$925,628,489 |
| 37 15 | .211,337,854,271 | .045,799,388,940 | .183,190,405,980 | .044,841,158,325 | .007,339,856,806 | .0$^{3}$958,230,615 |
| 37 30 | .214,184,123,288 | .046,727,726,961 | .185,295,238,725 | .045,737,040,499 | .007,537,463,579 | .0$^{3}$990,686,472 |
| 37 45 | .217,049,430,892 | .047,668,525,709 | .187,404,998,986 | .046,644,512,594 | .007,739,004,636 | .001,024,013,115 |
| 38 00 | .219,933,777,086 | .048,621,868,256 | .189,519,526,100 | .047,563,640,432 | .007,944,530,693 | .001,058,227,824 |
| 38 15 | .222,837,161,868 | .049,587,837,673 | .191,638,659,036 | .048,494,489,578 | .008,154,092,749 | .001,093,348,095 |
| 38 30 | .225,759,585,240 | .050,566,517,031 | .193,762,236,414 | .049,437,125,380 | .008,367,742,092 | .001,129,391,651 |
| 38 45 | .228,701,047,199 | .051,557,989,403 | .195,890,096,515 | .050,391,612,964 | .008,585,530,292 | .001,166,376,439 |
| 39 00 | .231,661,547,748 | .052,562,337,860 | .198,022,077,296 | .051,358,017,228 | .008,807,509,205 | .001,204,320,632 |
| 39 15 | .234,641,086,885 | .053,579,645,473 | .200,158,016,396 | .052,336,402,845 | .009,033,730,967 | .001,243,242,628 |
| 39 30 | .237,639,664,611 | .054,609,995,314 | .202,297,751,156 | .053,326,834,260 | .009,264,247,999 | .001,283,161,054 |
| 39 45 | .240,657,280,926 | .055,653,470,454 | .204,441,118,627 | .054,329,375,687 | .009,499,112,999 | .001,324,094,767 |
| 40 00 | .243,693,935,830 | .056,710,153,965 | .206,587,955,584 | .055,344,09 | .009,738,378,949 | .001,366,062,854 |
| 40 15 | .246,749,629,322 | .057,780,128,918 | .208,738,098,535 | .056,371,044,285 | .009,982,099, 05 | .001,409,084,633 |
| 40 30 | .249,824,36 ,403 | .058,863,478,385 | .210,891,383,740 | .057,410,298,728 | .010,230,327,003 | .001,453,179,657 |
| 40 45 | .252,918,132,073 | .059,960,285,438 | .213,047,647,218 | .058,461,9 7,725 | .010,483,116,457 | .001,498,367,713 |
| 41 00 | .256,030,941,331 | .061,070,633,147 | .215,206,724,760 | .059,525,964,327 | .010,740,521,554 | .001,544,668,820 |
| 41 15 | .259,162,789,178 | .062,194,604,585 | .217,368,451,945 | .060,602,501,348 | .011,002,596,657 | .001,592, 03,237 |
| 41 30 | .262,313,675,614 | .063,332,282,823 | .219,532,664,149 | .061,691,591,362 | .011,269,396,403 | .001,640,691,46 |
| 41 45 | .265,483,600,638 | .064,483,750,931 | .221,699,196,558 | .062,793,296,706 | .011,540,975,700 | .001,690,454,225 |
| 42 00 | .268,672,564,251 | .065,649,091,983 | .223,867,884,183 | .063,907,679,478 | .011,817,389,728 | .001,741,412,005 |
| 42 15 | .271,880,566,454 | .066,828,389,050 | .226,038,561,870 | .065,034,801,532 | .012,098,693,941 | .001,793,587,518 |
| 42 30 | 275,107,607,244 | .068,021,725,202 | .228,211,064,313 | .066,174,724,48 | .012,384,944,054 | .001,847,000,721 |
| 42 45 | .278,353,686,624 | .069,229,183,512 | .230,385,226,068 | .067,327,5 3,694 | .012,676,196,060 | .001,901,673,8 8 |
| 43 00 | .281,6 9,804,592 | .070,450,847,05 | .232,560,881,564 | .068,493,219,295 | .012,972,506,21 | .001,957,628,756 |
| 43 15 | .284,902,961,149 | .071,686,798,890 | .234,737,865, 6 | .069,671,911,161 | .013,273,93 ,031 | .002,014,887,729 |
| 43 30 | .288,206,156,295 | .072,937,122,102 | .236,916,000,339 | .070,863,649,923 | .013,580,527,307 | .002,073,473,179 |
| 43 45 | .291,528,390,030 | .074,20 ,899,757 | .239,095, 53,158 | .072,066,49 ,965 | .013,892,352,090 | .002, 33,407,792 |
| 44 00 | .294,869,662,353 | .075,481,214,927 | .241,275, 25,824 | .073,266,500,4 8 | .014,209,462,692 | .002,194,714,509 |
| 44 15 | 298,229,973,265 | .076,775,150,684 | .243,455,762,923 | .074,51 ,734,165 | .014,531,916,690 | .002,257,4 6,519 |
| 44 30 | .301,609,322,766 | .078,083,790,098 | .245,636,6 | .075,762,252,64 | .014,859,77 ,920 | .002,32 ,537,26 |
| 44 45 | .305,007,710,855 | .079,407,216,243 | .247,818,366,125 | .077,020,115,811 | .015, 93,086,478 | .002,387,100,432 |
| 45 00 | .308,425,137,534 | .080,745,512,168 | .250,000,000,000 | .078,291,382,2 0 | .015,531,9 8,721 | .002,454, 29,976 |
| ° ′ $\Phi_1$ | $\int \Phi_1 d\Phi_1$ | $\int \frac{\Phi_1^2}{2} d\Phi_1$ |  | $\int \text{VERS}\,\Phi_1\, d\Phi_1$ | $\int(\Phi_1-\text{SIN }\Phi_1)d\Phi_1$ | $\int(\frac{\Phi_1^2}{2}-\text{VERS}\Phi_1)d\Phi_1$ |

SHEET 2 OF 4

205-D-1866

Table H-4.—*Trigonometric functions and integrals for arch load constants* (sheet 3).

| $\Phi_1$ ° ' | $\frac{\Phi_1^2}{2}$ | $\frac{\Phi_1^3}{6}$ | $\frac{\sin^2\Phi_1}{2}$ | $\Phi_1 - \sin\Phi_1$ | $\frac{\Phi_1^2}{2} - \text{vers}\,\Phi_1$ | $\sin\Phi_1 - \Phi_1 + \frac{\Phi_1^3}{6}$ |
|---|---|---|---|---|---|---|
| 45 15 | .311,861,602,801 | .082,098,761,006 | .252,181,633,874 | .079,576,110,904 | .015,876,327,257 | .002,522,650,102 |
| 45 30 | .315,317,106,656 | .083,467,045,768 | .254,363,101,609 | .080,874,360,503 | .016,226,370,956 | .002,592,685,265 |
| 45 45 | .318,791,649,101 | .084,850,449,546 | .256,544,237,077 | .082,186,189,362 | .016,582,108,943 | .002,664,260,184 |
| 46 00 | .322,285,230,134 | .086,249,055,411 | .258,724,874,176 | .083,511,655,578 | .016,943,600,593 | .002,737,399,833 |
| 46 15 | .325,797,849,756 | .087,662,946,435 | .260,904,846,842 | .084,850,816,987 | .017,310,905,538 | .002,812,129,448 |
| 46 30 | .329,329,507,967 | .089,092,205,689 | .263,083,989,061 | .086,203,731,165 | .017,684,083,661 | .002,888,474,524 |
| 46 45 | .332,880,204,766 | .090,536,916,244 | .265,262,134,883 | .087,570,455,425 | .018,063,195,092 | .002,966,460,819 |
| 47 00 | .336,449,940,154 | .091,997,161,173 | .267,439,118,436 | .088,951,046,818 | .018,448,300,216 | .003,046,114,355 |
| 47 15 | .340,038,714,131 | .093,473,023,547 | .269,614,773,932 | .090,345,562,131 | .018,839,459,664 | .003,127,461,416 |
| 47 30 | .343,646,526,697 | .094,964,586,437 | .271,788,935,687 | .091,754,057,887 | .019,236,734,313 | .003,210,528,550 |
| 47 45 | .347,273,377,851 | .096,471,932,914 | .273,961,438,130 | .093,176,590,340 | .019,640,185,286 | .003,295,342,574 |
| 48 00 | .350,919,267,594 | .097,995,146,051 | .276,132,115,817 | .094,613,215,480 | .020,049,873,953 | .003,381,930,571 |
| 48 15 | .354,584,195,926 | .099,534,308,919 | .278,300,803,442 | .096,063,989,025 | .020,465,861,927 | .003,470,319,894 |
| 48 30 | .358,268,162,846 | .101,089,504,589 | .280,467,335,851 | .097,528,966,428 | .020,888,211,062 | .003,560,538,161 |
| 48 45 | .361,971,168,356 | .102,660,816,133 | .282,631,548,055 | .099,008,202,868 | .021,316,983,456 | .003,652,613,265 |
| 49 00 | .365,693,212,454 | .104,248,326,622 | .284,793,275,240 | .100,501,753,254 | .021,752,241,445 | .003,746,573,368 |
| 49 15 | .369,434,295,141 | .105,852,119,128 | .286,952,352,782 | .102,009,672,223 | .022,194,047,604 | .003,842,446,905 |
| 49 30 | .373,194,416,416 | .107,472,276,723 | .289,108,616,260 | .103,532,014,137 | .022,642,464,746 | .003,940,262,586 |
| 49 45 | .376,973,576,280 | .109,108,882,477 | .291,261,901,466 | .105,068,833,084 | .023,097,555,923 | .004,040,049,393 |
| 50 00 | .380,771,774,733 | .110,762,019,463 | .293,412,044,417 | .106,620,182,878 | .023,559,384,420 | .004,141,836,585 |
| 50 15 | .384,589,011,775 | .112,431,770,751 | .295,558,881,373 | .108,186,117,054 | .024,028,013,756 | .004,245,653,697 |
| 50 30 | .388,425,287,405 | .114,118,219,414 | .297,702,248,844 | .109,766,688,869 | .024,503,507,683 | .004,351,530,545 |
| 50 45 | .392,280,601,625 | .115,821,448,524 | .299,841,983,604 | .111,361,951,305 | .024,985,930,188 | .004,459,497,219 |
| 51 00 | .396,154,954,433 | .117,541,541,150 | .301,977,922,704 | .112,971,957,060 | .025,475,345,483 | .004,569,584,090 |
| 51 15 | .400,048,345,829 | .119,278,580,365 | .304,109,903,485 | .114,596,758,554 | .025,971,818,013 | .004,681,821,811 |
| 51 30 | .403,960,775,815 | .121,032,649,241 | .306,237,763,586 | .116,236,407,925 | .026,475,412,453 | .004,796,241,316 |
| 51 45 | .407,892,244,389 | .122,803,830,849 | .308,361,340,964 | .117,890,957,026 | .026,986,193,699 | .004,912,873,823 |
| 52 00 | .411,842,751,552 | .124,592,208,261 | .310,480,473,900 | .119,560,457,430 | .027,504,226,878 | .005,031,750,831 |
| 52 15 | .415,812,297,303 | .126,397,864,547 | .312,595,001,014 | .121,244,960,423 | .028,029,577,337 | .005,152,904,124 |
| 52 30 | .419,800,881,644 | .128,220,882,781 | .314,704,761,275 | .122,944,517,006 | .028,562,310,653 | .005,276,365,775 |
| 52 45 | .423,808,504,573 | .130,061,346,032 | .316,809,594,020 | .124,659,177,892 | .029,102,492,616 | .005,402,168,140 |
| 53 00 | .427,835,166,090 | .131,919,337,372 | .318,909,338,954 | .126,388,993,510 | .029,650,189,242 | .005,530,343,862 |
| 53 15 | .431,880,866,197 | .133,794,939,874 | .321,003,836,176 | .128,134,013,996 | .030,205,466,768 | .005,660,925,878 |
| 53 30 | .435,945,604,892 | .135,688,236,609 | .323,092,926,181 | .129,894,289,200 | .030,768,391,643 | .005,793,947,409 |
| 53 45 | .440,029,382,176 | .137,599,310,647 | .325,176,449,876 | .131,669,868,680 | .031,339,030,540 | .005,929,441,967 |
| 54 00 | .444,132,198,049 | .139,528,245,061 | .327 254,248,594 | .133,460,801,702 | .031,917,450,341 | .006,067,443,359 |
| 54 15 | .448,254,052,511 | .141,475,122,923 | .329,326,164,101 | .135,267,137,242 | .032,503,718,148 | .006,207,985,681 |
| 54 30 | .452,394,945,561 | .143,440,027,303 | .331,392,038,614 | .137,088,923,981 | .033,097,901,272 | .006,351,103,322 |
| 54 45 | .456,554,877,200 | .145,423,041,273 | .333,451,714,809 | .138,926,210,305 | .033,700,067,237 | .006,496,830,968 |
| 55 00 | .460,733,847,428 | .147,424,247,905 | .335,505,035,831 | .140,779,044,308 | .034,310,283,779 | .006,645,203,597 |
| 55 15 | .464,931,856,244 | .149,443,730,270 | .337,551,845,315 | .142,647,473,785 | .034,928,618,840 | .006,796,256,485 |
| 55 30 | .469,148,903,649 | .151,481,571,440 | .339,591,987,386 | .144,531,546,235 | .035,555,140,574 | .006,950,025,205 |
| 55 45 | .473,384,989,643 | .153,537,854,486 | .341,625,306,681 | .146,431,308,860 | .036,189,917,338 | .007,106,545,626 |
| 56 00 | .477,640,114,226 | .155,612,662,480 | .343,651,648,354 | .148,346,808,562 | .036,833,017,697 | .007,265,853,918 |
| 56 15 | .481,914,277,397 | .157,706,078,493 | .345,670,858,092 | .150,278,091,944 | .037,484,510,417 | .007,427,986,549 |
| 56 30 | .486,207,479,157 | .159,818,185,597 | .347,682,782,122 | .152,225,205,310 | .038,144,464,469 | .007,592,980,287 |
| 56 45 | .490,519,719,506 | .161,949,066,863 | .349,687,267,232 | .154,188,194,659 | .038,812,949,026 | .007,760,872,204 |
| 57 00 | .494,850,998,444 | .164,098,805,363 | .351,684,160,769 | .156,167,105,692 | .039,490,033,459 | .007,931,699,671 |
| 57 15 | .499,201,315,970 | .166,267,484,168 | .353,673,310,664 | .158,161,983,803 | .040,175,787,338 | .008,105,500,365 |
| 57 30 | .503,570,672,085 | .168,455,186,350 | .355,654,565,435 | .160,172,874,084 | .040,870,280,432 | .008,282,312,266 |
| 57 45 | .507,959,066,789 | .170,661,994,981 | .357,627,774,202 | .162,199,821,323 | .041,573,582,705 | .008,462,173,658 |
| 58 00 | .512,366,500,082 | .172,887,993,131 | .359,592,786,697 | .164,242,870,001 | .042,285,764,315 | .008,645,123,130 |
| 58 15 | .516,792,971,963 | .175,133,263,873 | .361,549,453,278 | .166,302,064,291 | .043,006,895,615 | .008,831,199,582 |
| 58 30 | .521,238,482,432 | .177,397,890,277 | .363,497,624,935 | .168,377,448,062 | .043,737,047,148 | .009,020,442,215 |
| 58 45 | .525,703,031,491 | .179,681,955,417 | .365,437,153,309 | .170,469,064.873 | .044,476,289,652 | .009,212,890,544 |
| 59 00 | .530,186,619,138 | .181,985,542,361 | .367,367,890,696 | .172,576,957,974 | .045,224,694,048 | .009,408,584,387 |
| 59 15 | .534,689,245,374 | .184,308,734,184 | .369,289,690,065 | .174,701,170,305 | .045,982,331,451 | .009,607,563,879 |
| 59 30 | .539,210,910,199 | .186,651,613,955 | .371,202,405,062 | .176,841,744,494 | .046,749,273,160 | .009,809,869,461 |
| 59 45 | .543,751,613,613 | .189,014,264,747 | .373,105,890,026 | .178,998,722,862 | .047,525,590,659 | .010,015,541,885 |
| 60 00 | .548,311,355,615 | .191,396,769,631 | .375,000,000,000 | .181,172,147,412 | .048,311,355,615 | .010,224,622,219 |
| 60 15 | .552,890,136,207 | .193,799,211,678 | .376,884,590,740 | .183,362,059,837 | .049,106,639,882 | .010,437,151,841 |
| 60 30 | .557,487,955,387 | .196,221,673,961 | .378,759,518,728 | 185,568,501,516 | .049,911,515,490 | .010,653,172,445 |
| 60 45 | .562,104,813,155 | .198,664,239,550 | .380,624,641,179 | .187,791,513,513 | .050,726,054,652 | .010,872,726,037 |
| 61 00 | .566,740,709,513 | .201,126,991,517 | .382,479,816,058 | .190,031,136,577 | .051,550,329,759 | .011,095,854,940 |
| 61 15 | .571,395,644,459 | .203,610,012,934 | .384,324,902,087 | .192,287,411,138 | .052,384,413,378 | .011,322,601,796 |
| 61 30 | .576,069,617,994 | .206,113,386,871 | .386,159,758,754 | .194,560,377,314 | .053,228,378,254 | .011,553,009,557 |
| 61 45 | .580,762,630,117 | .208,637,196,401 | .387,984,246,328 | .196,850,074,901 | .054,082,297,302 | .011,787,121,500 |
| 62 00 | .585,474,680,830 | .211,181,524,596 | .389,798,225,868 | .199,156,543,377 | .054,946,243,616 | .012,024,981,219 |
| 62 15 | .590,205,770,131 | .213,746,454,526 | .391,601,559,231 | .201,479,821,903 | .055,820,290,456 | .012,266,632,623 |
| 62 30 | .594,955,898,020 | .216,332,069,263 | .393,394,109,088 | .203,819,949,318 | .056,704,511,255 | .012,512,119,945 |
| 62 45 | .599,725,064,499 | .218,938,451,878 | .395,175,738,928 | .206,176,964,140 | .057,598,979,616 | .012,761,487,738 |
| 63 00 | .604,513,269,566 | .221,565,685,444 | .396,946,313,073 | .208,550,904,568 | .058,503,769,306 | .013,014,780,876 |
| 63 15 | .609,320,513,222 | .224,213,853,032 | .398,705,696,688 | .210,941,808,475 | .059,418,954,259 | .013,272,044,557 |
| 63 30 | .614,146,795,467 | .226,883,037,713 | .400,453,755,788 | .213,349,713,414 | .060,344,608,577 | .013,533,324,299 |
| 63 45 | .618,992,116,301 | .229,573,322,558 | .402,190,357,252 | .215,774,656,613 | .061,280,806,520 | .013,798,665,945 |
| 64 00 | .623,856,475,723 | .232,284,790,640 | .403,915,368,831 | .218,216,674,977 | .062,227,622,512 | .014,068,115,663 |
| 64 15 | .628,739,873,734 | .235,017,525,029 | .405,628,659,160 | .220,675,805,083 | .063,185,131,138 | .014,341,719,946 |
| 64 30 | .633,642,310,333 | .237,771,608,798 | .407,330,097,763 | .223,152,083,185 | .064,153,407,141 | .014,619,525,612 |
| 64 45 | .638,563,785,522 | .240,547,125,018 | .409,019,555,069 | .225,645,545,212 | .065,132,525,423 | .014,901,579,806 |
| 65 00 | .643,504,299,299 | .243,344,156,759 | .410,696,902,422 | .228,156,226,759 | .066,122,561,040 | .015,187,930,000 |
| 65 15 | .648,463,851,665 | .246,162,787,095 | .412,362,012,082 | .230,684,163,101 | .067,123,589,202 | .015,478,623,994 |
| 65 30 | .653,442,442,620 | .249,003,099,096 | .414,014,757,248 | .233,229,389,179 | .068,135,685,276 | .015,773,709,917 |
| 65 45 | .658,440,072,163 | .251,865,175,833 | .415,655,012,054 | .235,791,939,609 | .069,158,924,776 | .016,073,236,224 |
| 66 00 | .663,456,740,295 | .254,749,100,379 | .417,282,651,590 | .238,371,848,673 | .070,193,383,371 | .016,377,251,706 |
| 66 15 | .668,492,447,016 | .257,654,955,805 | .418,997,551,904 | .240,969,150,327 | .071,239,136,875 | .016,685,805,478 |
| 66 30 | .673,547,192,326 | .260,582,825,183 | 420,499,590,016 | .243,583,878,191 | .072,296,261,251 | .016,998,946,992 |
| 66 45 | .678,620,976,224 | .263,532,791,583 | .422,088,643,924 | .246,216,065,557 | .073,364,832,608 | .017,316,726,026 |
| 67 00 | .683,713,798,711 | .266,504,938,077 | .423,664,592,614 | .248,865,745,384 | .074,444,927,200 | .017,639,192,693 |
| 67 15 | .688,825,659,787 | .269,499,347,738 | .425,227,316,075 | .251,532,950,296 | .075,536,621,424 | .017,966,397,442 |
| 67 30 | .693,956,559,451 | .272,516,103,635 | .426,776,695,296 | .254,217,712,585 | .076,639,991,816 | .018,298,391,050 |
| ° ' $\Phi_1$ | $\int \Phi_1 d\Phi_1$ | $\int \frac{\Phi_1^2}{2} d\Phi_1$ |  | $\int \text{vers}\,\Phi_1 d\Phi_1$ | $\int (\Phi_1 - \sin\Phi_1) d\Phi_1$ | $\int (\frac{\Phi_1^2}{2} \cdot \text{vers}\,\Phi_1) d\Phi_1$ |

**Table H-4.**—*Trigonometric functions and integrals for arch load constants* (sheet 4).

| $\Phi_1$ ° ′ | $\frac{\Phi_1^2}{2}$ | $\frac{\Phi_1^3}{6}$ | $\frac{\sin^2\Phi_1}{2}$ | $\Phi_1 - \sin\Phi_1$ | $\frac{\Phi_1^2}{2} - \text{vers}\,\Phi_1$ | $\sin\Phi_1 - \Phi_1 + \frac{\Phi_1^3}{6}$ |
|---|---|---|---|---|---|---|
| 67 45 | .699,106,497,705 | .275,555,288,842 | .428,312,612,289 | .256,920,064,208 | .077,755,115,057 | .018,635,224,634 |
| 68 00 | .704,275,474,547 | .278,616,986,429 | .429,834,950,085 | .259,640,036,789 | .078,882,067,963 | .018,976,949,640 |
| 68 15 | .709,463,489,977 | .281,701,279,468 | .431,343,592,753 | .262,377,661,614 | .080,020,927,487 | .019,323,617,854 |
| 68 30 | .714,670,543,997 | .284,808,251,031 | .432,838,425,405 | .265,132,969,634 | .081,171,770,721 | .019,675,281,397 |
| 68 45 | .719,896,636,605 | .287,937,984,189 | .434,319,334,203 | .267,905,991,463 | .082,334,674,889 | .020,031,992,726 |
| 69 00 | .725,141,767,802 | .291,090,562,013 | .435,786,206,369 | .270,696,757,379 | .083,509,717,347 | .020,393,804,634 |
| 69 15 | .730,405,937,588 | .294,266,067,576 | .437,238,930,197 | .273,505,297,320 | .084,696,975,586 | .020,760,770,256 |
| 69 30 | .735,689,145,962 | .297,464,583,947 | .438,677,395,055 | .276,331,640,888 | .085,896,527,221 | .021,132,943,059 |
| 69 45 | .740,991,392,926 | .300,686,194,200 | .440,101,491,400 | .279,175,817,344 | .087,108,450,003 | .021,510,376,856 |
| 70 00 | .746,312,678,477 | .303,930,981,405 | .441,511,110,780 | .282,037,855,610 | .088,332,821,803 | .021,893,125,795 |
| 70 15 | .751,653,002,618 | .307,199,028,635 | .442,906,145,847 | .284,917,784,270 | .089,569,720,621 | .022,281,244,365 |
| 70 30 | .757,012,365,348 | .310,490,418,960 | .444,286,490,364 | .287,815,631,564 | .090,819,224,582 | .022,674,787,396 |
| 70 45 | .762,390,766,666 | .313,805,235,452 | .445,652,039,213 | .290,731,425,393 | .092,081,411,929 | .023,073,810,059 |
| 71 00 | .767,788,206,572 | .317,143,561,183 | .447,002,688,401 | .293,665,193,317 | .093,356,361,029 | .023,478,367,866 |
| 71 15 | .773,204,685,068 | .320,505,479,224 | .448,338,335,073 | .296,616,962,551 | .094,644,150,371 | .023,888,516,673 |
| 71 30 | .778,640,202,153 | .323,891,072,647 | .449,658,877,512 | .299,586,759,970 | .095,944,858,558 | .024,304,312,677 |
| 71 45 | .784,094,757,825 | .327,300,424,523 | .450,964,215,154 | .302,574,612,104 | .097,258,564,309 | .024,725,812,419 |
| 72 00 | .789,568,352,087 | .330,733,617,923 | .452,254,248,594 | .305,580,545,141 | .098,585,346,462 | .025,153,072,782 |
| 72 15 | .795,060,984,938 | .334,190,735,920 | .453,528,879,589 | .308,604,584,923 | .099,925,283,966 | .025,586,150,997 |
| 72 30 | .800,572,656,377 | .337,671,861,584 | .454,788,011,072 | .311,646,756,948 | .101,278,455,881 | .026,025,104,636 |
| 72 45 | .806,103,366,405 | .341,177,077,988 | .456,031,547,155 | .314,707,086,369 | .102,644,941,381 | .026,469,991,619 |
| 73 00 | .811,653,115,022 | .344,706,468,203 | .457,259,393,139 | .317,785,597,993 | .104,024,819,745 | .026,920,870,210 |
| 73 15 | .817,221,902,228 | .348,260,115,299 | .458,471,455,517 | .320,882,316,281 | .105,418,170,362 | .027,377,799,018 |
| 73 30 | .822,809,728,022 | .351,838,102,350 | .459,667,641,986 | .323,997,265,348 | .106,825,072,726 | .027,840,837,002 |
| 73 45 | .828,416,592,405 | .355,440,512,425 | .460,847,861,453 | .327,130,468,960 | .108,245,606,436 | .028,310,043,465 |
| 74 00 | .834,042,495,376 | .359,067,428,598 | .462,012,024,039 | .330,281,950,538 | .109,679,851,193 | .028,785,478,060 |
| 74 15 | .839,687,436,937 | .362,718,933,939 | .463,160,041,089 | .333,451,733,152 | .111,127,886,802 | .029,267,200,787 |
| 74 30 | .845,351,417,086 | .366,395,111,520 | .464,291,825,176 | .336,639,839,527 | .112,589,793,164 | .029,755,271,993 |
| 74 45 | .851,034,435,824 | .370,096,044,412 | 465,407,290,110 | .339,846,292,037 | .114,065,650,282 | .030,249,752,375 |
| 75 00 | .856,736,493,150 | .373,821,815,687 | .466,506,350,946 | .343,071,112,707 | .115,555,538,253 | .030,750,702,980 |
| 75 15 | .862,457,589,065 | .377,572,508,415 | .467,588,923,985 | .346,314,323,211 | .117,059,537,271 | .031,258,185,204 |
| 75 30 | .868,197,723,569 | .381,348,205,670 | .468,654,926,785 | .349,575,944,877 | .118,577,727,623 | .031,772,260,793 |
| 75 45 | .873,956,896,662 | .385,148,990,523 | .469,704,278,166 | .352,855,998,678 | .120,110,189,691 | .032,292,991,845 |
| 76 00 | .879,735,108,343 | .388,974,946,044 | .470,736,898,215 | .356,154,505,239 | .121,657,003,943 | .032,820,440,805 |
| 76 15 | .885,532,358,613 | .392,826,155,306 | .471,752 708,294 | .359,471,484,832 | .123,218,250,939 | .033,354,670,474 |
| 76 30 | .891,348,647,473 | .396,702,701,380 | .472,751,631,047 | .362,806,957,377 | .124,794,011,329 | .033,895,744,003 |
| 76 45 | .897,183,974,920 | .400,604,667,338 | .473,733,590,400 | .363,160,942,445 | .126,384,365,842 | .034,443,724,893 |
| 77 00 | .903,038,340,957 | .404,532,136,250 | .474,698,511,575 | .369,533,459,250 | .127,989,395,301 | .034,998,677,000 |
| 77 15 | .908,911,745,582 | .408,485,191,189 | .475,646,321,088 | .372,924,526,656 | .129,609,180,604 | .035,560,664,533 |
| 77 30 | .914,804,188,796 | .412,463,915,226 | .476,576,946,759 | .376,334,163,175 | .131,243,802,734 | .036,129,752,051 |
| 77 45 | .920,715,670,599 | .416,468,391,432 | .477,490,317,719 | .379,762,386,962 | .132,893,342,755 | .036,706,004,470 |
| 78 00 | .926,646,190,990 | .420,498,702,880 | .478,386,364,411 | .383,209,215,821 | .134,557,881,808 | .037,289,487,059 |
| 78 15 | .932,595,749,971 | .424,554,932,640 | .479,265,018,597 | .386,674,667,200 | .136,237,501,111 | .037,880,265,440 |
| 78 30 | .938,564,347,540 | .428,637,163,784 | .480,126,213,363 | .390,158,758,194 | .137,932,281,957 | .038,478,405,590 |
| 78 45 | .944,551,983,697 | .432,745,479,383 | .480,969,883,128 | .393,661,505,542 | .139,642,305,713 | .039,083,973,841 |
| 79 00 | .950,558,658,444 | .436,879,962,510 | .481,795,963,642 | .397,182,925,627 | .141,367,653,821 | .039,697,036,883 |
| 79 15 | .956,584,371,779 | .441,040,696,236 | .482,604,391,996 | .400,723,034,479 | .143,108,407,788 | .040,317,661,757 |
| 79 30 | .962,629,123,703 | .445,227,763,631 | .483,395,106,624 | .404,281,847,771 | .144,864,649,195 | .040,945,915,860 |
| 79 45 | .968,692,914,215 | .449,441,247,768 | .484,168,047,312 | .407,859,380,819 | .146,636,459,689 | .041,581,866,949 |
| 80 00 | .974,775,743,317 | .453,681,231,719 | .484,923,155,196 | .411,455,648,583 | .148,423,920,984 | .042,225,583,136 |
| 80 15 | .980,877,611,007 | .457,947,798,554 | .485,660,372,773 | .415,070,665,667 | .150,227,114,856 | .042,877,132,887 |
| 80 30 | .986,998,517,286 | .462,241,031,345 | .486,379,643,900 | .418,704,446,318 | .152,046,123,147 | .043,536,585,027 |
| 80 45 | .993,138,462,153 | .466,561,013,164 | .487,080,913,801 | .422,357,004,425 | .153,881,027,757 | .044,204,008,739 |
| 81 00 | .999,297,445,610 | .470,907,827,082 | .487,764,129,074 | .426,028,353,520 | .155,731,910,650 | .044,879,473,562 |
| 81 15 | 1.005,475,467,655 | .475,281,556,170 | .488,429,237,687 | .429,718,506,777 | .157,598,853,845 | .045,563,049,393 |
| 81 30 | 1.011,672,528,289 | .479,682,283,502 | .489,076,188,991 | .433,427,477,013 | .159,481,939,419 | .046,254,806,489 |
| 81 45 | 1.017,888,627,511 | .484,110,092,146 | .489,704,933,717 | .437,155,276,685 | .161,381,249,502 | .046,954,815,461 |
| 82 00 | 1.024,123,765,322 | .488,565,065,176 | .490,315,423,985 | .440,901,917,893 | .163,296,866,282 | .047,663,147,283 |
| 82 15 | 1.030,377,941,722 | .493,047,285,663 | .490,907,613,302 | .444,667,412,378 | .165,228,871,996 | .048,379,873,285 |
| 82 30 | 1.036,651,156,711 | .497,556,836,678 | .491,481,456,573 | .448,451,771,521 | .167,177,348,931 | .049,105,065,157 |
| 82 45 | 1.042,943,410,289 | .502,093,801,294 | .492,036,910,095 | .452,255,006,345 | .169,142,379,425 | .049,838,794,949 |
| 83 00 | 1.049,254,702,455 | .506,658,262,580 | .492,573,931,569 | .456,077,127,514 | .171,124,045,860 | .050,581,135,066 |
| 83 15 | 1.055,585,033,210 | .511,250,303,609 | .493,092,480,100 | .459,918,145,330 | .173,122,430,668 | .051,332,158,279 |
| 83 30 | 1.061,934,402,554 | .515,870,007,453 | .493,592,516,197 | .463,778,069,738 | .175,137,616,322 | .052,091,937,715 |
| 83 45 | 1.068,302,810,486 | .520,517,457,182 | .494,074,001,780 | .467,656,910,323 | .177,169,685,338 | .052,860,546,859 |
| 84 00 | 1.074,690,257,007 | .525,192,735,868 | .494,536,900,183 | .471,554,676,307 | .179,218,720,275 | .053,638,059,561 |
| 84 15 | 1.081,096,742,117 | .529,895,926,584 | 494,981,176,156 | .475,471,376,554 | .181,284,803,729 | .054,424,550,030 |
| 84 30 | 1.087,522,265,816 | .534,627,112,400 | .495,406,795,862 | .479,407,019,568 | .183,368,018,336 | .055,220,092,832 |
| 84 45 | 1.093,966,828,103 | .539,386,376,388 | .495,813,726,832 | .483,361,613,490 | .185,468,446,766 | .056,024,762,898 |
| 85 00 | 1.100,430,428,979 | .544,173,801,620 | .496,201,938,253 | .487,335,166,103 | .187,586,171,727 | .056,838,635,517 |
| 85 15 | 1.106,913,068,444 | .548,989,471,166 | .496,571,400,385 | .491,327,684,827 | .189,721,275,956 | .057,661,786,339 |
| 85 30 | 1.113,414,746,498 | .553,833,468,099 | .496,922,085,149 | .495,339,175,722 | .191,873,842,226 | .058,494,291,377 |
| 85 45 | 1.119,935,463,140 | .558,705,875,490 | .497,253,965,840 | .499,369,648,486 | .194,043,953,335 | .059,336,227,004 |
| 86 00 | 1.126,475,218,371 | .563,606,776,411 | .497,567,017,186 | .503,419,106,455 | .196,231,692,115 | .060,187,669,956 |
| 86 15 | 1.133,034,012,191 | .568,536,253,932 | .497,861,215,344 | .507,487,556,606 | .198,437,141,421 | .061,048,697,326 |
| 86 30 | 1.139,611,844,599 | .573,494,391,126 | .498,136,537,910 | .511,575,004,553 | .200,660,384,134 | .061,919,386,573 |
| 86 45 | 1.146,208,715,597 | .578,481,271,064 | .498,392,963,919 | .515,681,455,548 | .202,901,503,160 | .062,799,815,516 |
| 87 00 | 1.152,824,625,183 | .583,496,976,818 | .498,630,473,842 | .519,806,914,480 | .205,160,581,426 | .063,690,062,338 |
| 87 15 | 1.159,459,573,357 | .588,541,591,458 | .498,849,049,592 | .523,951,385,880 | .207,437,701,878 | .064,590,205,578 |
| 87 30 | 1.166,113,560,121 | .593,615,198,058 | .499,048,674,523 | .528,114,873,913 | .209,732,947,486 | .065,500,324,145 |
| 87 45 | 1.172,786,585,473 | .598,717,879,687 | .499,229,333,434 | .532,297,382,384 | .212,046,401,232 | .066,420,497,303 |
| 88 00 | 1.179,478,649,414 | .603,849,719,418 | .499,391,012,565 | .536,498,914,736 | .214,378,146,117 | .067,350,804,682 |
| 88 15 | 1.186,189,751,944 | .609,010,800,322 | .499,533,699,606 | .540,719,474,048 | .216,728,265,154 | .068,291,326,274 |
| 88 30 | 1.192,919,893,062 | .614,201,205,471 | .499,657,383,699 | .544,959,063,039 | .219,096,841,370 | .069,242,142,432 |
| 88 45 | 1.199,669,072,769 | .619,421,017,936 | 499,762,055,396 | .549,217,684,065 | .221,483,957,804 | .070,203,333,871 |
| 89 00 | 1.206,437,291,065 | .624,670,320,786 | .499,847,706,754 | .553,495,339,119 | .223,889,697,502 | .071,174,981,669 |
| 89 15 | 1.213,224,547,950 | .629,949,197,100 | .499,914,331,244 | .557,792,029,831 | .226,314,143,521 | .072,157,167,269 |
| 89 30 | 1.220,030,843,423 | .635,257,729,943 | .499,961,923,789 | .562,107,757,471 | .228,757,378,921 | .073,149,972,472 |
| 89 45 | 1.226,856,177,485 | .640,590,002,388 | .499,990,480,766 | .566,442,522,944 | .231,219,486,770 | .074,153,479,444 |
| 90 00 | 1.233,700,550,136 | .645,964,097,506 | .500,000,000,000 | .570,796,326,795 | .233,700,550,136 | .075,167,770,711 |
| $\Phi_1$ | $\int \Phi_1 \, d\Phi_1$ | $\int \frac{\Phi_1^2}{2} \, d\Phi_1$ |  | $\int \text{vers}\,\Phi_1 \, d\Phi_1$ | $\int (\Phi_1 - \sin\Phi_1) \, d\Phi_1$ | $\int \left(\frac{\Phi_1^2}{2} - \text{vers}\,\Phi_1\right) d\Phi_1$ |

SHEET 4 OF 4

205-D-1868

**Table H-5.**—*Integrals for arch load constants* (sheet 1).

| $\Phi_1$ | $\int(\Phi_1-\text{SIN }\Phi_1)\text{ SIN }\Phi\, d\Phi_1$ | | $\int\text{VERS }\Phi_1\text{ SIN }\Phi\, d\Phi_1$ | | $\int\text{SIN }\Phi_1\text{ SIN }\Phi\, d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ |
| 0 15 | .0$^{10}$15,102,722 | .0$^{13}$52,718,658 | .0$^{7}$13,845,160 | .0$^{10}$45,308,339 | .0$^{5}$9,519,233,958 | .0$^{7}$27,690,399 |
| 0 30 | .0$^{9}$,241,638,493 | .0$^{11}$1,686,982 | .0$^{6}$,110,759,067 | .0$^{9}$,724,926,521 | .0$^{4}$38,076,210,902 | .0$^{6}$,221,520,664 |
| 0 45 | .0$^{8}$1,223,252 | .0$^{10}$12,810,332 | .0$^{6}$,373,799,397 | .0$^{8}$3,669,882 | .0$^{4}$85,668,756,111 | .0$^{6}$,747,618,010 |
| 1 00 | .0$^{8}$3,865,892 | .0$^{9}$,053,981,520 | .0$^{6}$,886,001,687 | .0$^{7}$11,598,383 | .0$^{3}$,152,293,245 | .0$^{5}$1,772,084 |
| 1 15 | .0$^{8}$9,437,620 | .0$^{9}$,164,734 157 | .0$^{5}$1,730,368 | .0$^{7}$28,315,555 | .0$^{3}$,237,944,605 | .0$^{5}$3,460,984 |
| 1 30 | .0$^{7}$19,568,346 | .0$^{9}$,409,898,012 | .0$^{5}$2,989,857 | .0$^{7}$58,713,086 | .0$^{3}$,342,616,311 | .0$^{5}$5,980,329 |
| 1 45 | .0$^{7}$36,249,487 | .0$^{9}$,885,917,181 | .0$^{5}$4,747,371 | .0$^{6}$,108,768,754 | .0$^{3}$,466,300,395 | .0$^{5}$9,496,071 |
| 2 00 | .0$^{7}$61,833,547 | .0$^{8}$1,727,164 | .0$^{5}$7,085,747 | .0$^{6}$,185,545,860 | .0$^{3}$,608,987,435 | .0$^{4}$14,174,084 |
| 2 15 | .0$^{7}$99,033,630 | .0$^{8}$3,112,250 | .0$^{4}$10,087,742 | .0$^{6}$,297,192,559 | .0$^{3}$,770,666,567 | .0$^{4}$20,180,153 |
| 2 30 | .0$^{6}$,150,922,864 | .0$^{8}$5,270,328 | .0$^{4}$13,836,028 | .0$^{6}$,452,941,079 | .0$^{3}$,951,325,477 | .0$^{4}$27,679,963 |
| 2 45 | .0$^{6}$,220,933,762 | .0$^{8}$8,487,391 | .0$^{4}$18,413,176 | .0$^{6}$,663,106 844 | .0$^{2}$1,150,950,408 | .0$^{4}$36,839,085 |
| 3 00 | .0$^{6}$,312,857,499 | .0$^{7}$13,112,563 | .0$^{4}$23,901,646 | .0$^{6}$,939,087,494 | .0$^{2}$1,369,526,158 | .0$^{4}$47,822,963 |
| 3 15 | .0$^{6}$,430,843,118 | .0$^{7}$19,564,382 | .0$^{4}$30,383,776 | .0$^{5}$1,293,362 | .0$^{2}$1,607,036,081 | .0$^{4}$60,796,903 |
| 3 30 | .0$^{6}$,579,396,658 | .0$^{7}$28,337,073 | .0$^{4}$37,941,774 | .0$^{5}$1,739,488 | .0$^{2}$1,863,462,090 | .0$^{4}$75,926,059 |
| 3 45 | .0$^{6}$,763,380,206 | .0$^{7}$40,006,816 | .0$^{4}$46,657,700 | .0$^{5}$2,292,105 | .0$^{2}$2,138,784,657 | .0$^{4}$93,375,420 |
| 4 00 | .0$^{6}$,988,010,881 | .0$^{7}$55,237,999 | .0$^{4}$56,613,464 | .0$^{5}$2,966,925 | .0$^{2}$2,432,982,815 | .0$^{3}$,113,309,800 |
| 4 15 | .0$^{5}$1,258,860 | .0$^{7}$74,789,459 | .0$^{4}$67,890,808 | .0$^{5}$3,780,741 | .0$^{2}$2,746,034,159 | .0$^{3}$,135,893,822 |
| 4 30 | .0$^{5}$1,581,851 | .0$^{7}$99,520,722 | .0$^{4}$80,571,298 | .0$^{5}$4,751,416 | .0$^{2}$3,077,914 851 | .0$^{3}$,161,291,910 |
| 4 45 | .0$^{5}$1,963,259 | .0$^{6}$,130,398,214 | .0$^{4}$94,736,312 | .0$^{5}$5,897,886 | .0$^{2}$3,428,599,616 | .0$^{3}$,189,668,270 |
| 5 00 | .0$^{5}$2,409,710 | .0$^{6}$,168,501,473 | .0$^{3}$,110,467,031 | .0$^{5}$7,240,161 | .0$^{2}$3,798,061,747 | .0$^{3}$,221,186,883 |
| 5 15 | .0$^{5}$2,928,178 | .0$^{6}$,215,029,340 | .0$^{3}$,127,844,425 | .0$^{5}$8,799,316 | .0$^{2}$4,186,273,109 | .0$^{3}$,256,011,492 |
| 5 30 | .0$^{5}$3,525,984 | .0$^{6}$,271,306,136 | .0$^{3}$,146,949,246 | .0$^{4}$10,597,495 | .0$^{2}$4,593,204,138 | .0$^{3}$,294,305,586 |
| 5 45 | .0$^{5}$4,210,797 | .0$^{6}$,338,787,824 | .0$^{3}$,167,862,013 | .0$^{4}$12,657,904 | .0$^{2}$5,018,823,845 | .0$^{3}$,336,232,391 |
| 6 00 | .0$^{5}$4,990,627 | .0$^{6}$,419,068,158 | .0$^{3}$,190,663,003 | .0$^{4}$15,004,815 | .0$^{2}$5,463,099,817 | .0$^{3}$,381,954,855 |
| 6 15 | .0$^{5}$5,873,830 | .0$^{6}$,513,884,806 | .0$^{3}$,215,432,242 | .0$^{4}$17,663,558 | .0$^{2}$5,925,998,220 | .0$^{3}$,431,635,640 |
| 6 30 | .0$^{5}$6,869,100 | .0$^{6}$,625,125,466 | .0$^{3}$,242,249,492 | .0$^{4}$20,660,520 | .0$^{2}$6,407,483,804 | .0$^{3}$,485,437,104 |
| 6 45 | .0$^{5}$7,985,468 | .0$^{6}$,754,833,956 | .0$^{3}$,271,194,239 | .0$^{4}$24,023,144 | .0$^{2}$6,907,519,901 | .0$^{3}$,543,521,291 |
| 7 00 | .0$^{5}$9,232,308 | .0$^{6}$,905,216,284 | .0$^{3}$,302,345,685 | .0$^{4}$27,779,928 | .0$^{2}$7,426,068,431 | .0$^{3}$,606,049,920 |
| 7 15 | .0$^{4}$10,619,324 | .0$^{5}$1,078,647 | .0$^{3}$,335,782,737 | .0$^{4}$31,960,415 | .0$^{2}$7,963,089,905 | .0$^{3}$,673,184,371 |
| 7 30 | .0$^{4}$12,156,554 | .0$^{5}$1,277,674 | .0$^{3}$,371,583,994 | .0$^{4}$36,595,199 | .0$^{2}$8,518,543,428 | .0$^{3}$,745,085,674 |
| 7 45 | .0$^{4}$13,854,367 | .0$^{5}$1,505,026 | .0$^{3}$,409,827,739 | .0$^{4}$41,715,915 | .0$^{2}$9,092,386,698 | .0$^{3}$,821,914,495 |
| 8 00 | .0$^{4}$15,723,463 | .0$^{5}$1,763,619 | .0$^{3}$,450,591,926 | .0$^{4}$47,355,243 | .0$^{2}$9,684,576,015 | .0$^{3}$,903,831,126 |
| 8 15 | .0$^{4}$17,774,862 | .0$^{5}$2,056,560 | .0$^{3}$,493,954,170 | .0$^{4}$53,546,898 | .010,295,066,283 | .0$^{3}$,990,995,469 |
| 8 30 | .0$^{4}$20,019,915 | .0$^{5}$2,387,155 | .0$^{3}$,539,991,740 | .0$^{4}$60,325,629 | .010,923,811,009 | .0$^{2}$1,083,567,029 |
| 8 45 | .0$^{4}$22,470,290 | .0$^{5}$2,758,912 | .0$^{3}$,588,781,539 | .0$^{4}$67,727,219 | .011,570,762,313 | .0$^{2}$1,181,704,898 |
| 9 00 | .0$^{4}$25,137,976 | .0$^{5}$3,175,552 | .0$^{3}$,640,400,107 | .0$^{4}$75,788,479 | .012,235,870,926 | .0$^{2}$1,285,567,746 |
| 9 15 | .0$^{4}$28,035,277 | .0$^{5}$3,641,010 | .0$^{3}$,694,923,598 | .0$^{4}$84,547,242 | .012,919,086,198 | .0$^{2}$1,395,313,803 |
| 9 30 | .0$^{4}$31,174,813 | .0$^{5}$4,159,443 | .0$^{3}$,752,427,777 | .0$^{4}$94,042,363 | .013,620,356,100 | .0$^{2}$1,511,100,855 |
| 9 45 | .0$^{4}$34,569,512 | .0$^{5}$4,735,236 | .0$^{3}$,812,988,006 | .0$^{3}$,104,313,715 | .014,339,627,227 | .0$^{2}$1,633,086,226 |
| 10 00 | .0$^{4}$38,232,612 | .0$^{5}$5,373,006 | .0$^{3}$,876,679,236 | .0$^{3}$,115,402,184 | .015,076,844,804 | .0$^{2}$1,761,426,768 |
| 10 15 | .0$^{4}$42,177,658 | .0$^{5}$6,077,612 | .0$^{3}$,943,575,995 | .0$^{3}$,127,349,666 | .015,831,952,688 | .0$^{2}$1,896,278,850 |
| 10 30 | .0$^{4}$46,418,494 | .0$^{5}$6,854,154 | .0$^{2}$1,013,752,376 | .0$^{3}$,140,199,060 | .016,604,893,376 | .0$^{2}$2,037,798,343 |
| 10 45 | .0$^{4}$50,969,267 | .0$^{5}$7,707,985 | .0$^{2}$1,087,282,033 | .0$^{3}$,153,994,269 | .017,395,608,005 | .0$^{2}$2,186,140,613 |
| 11 00 | .0$^{4}$55,844,419 | .0$^{5}$8,644,712 | .0$^{2}$1,164,238,163 | .0$^{3}$,168,780,194 | .018,204,036,358 | .0$^{2}$2,341,460,505 |
| 11 15 | .0$^{4}$61,058,683 | .0$^{5}$9,670,204 | .0$^{2}$1,244,693,500 | .0$^{3}$,184,602,725 | .019,030,116,872 | .0$^{2}$2,503,912,333 |
| 11 30 | .0$^{4}$66,627,086 | .0$^{4}$10,790,602 | .0$^{2}$1,328,720,305 | .0$^{3}$,201,508,742 | .019,873,786,637 | .0$^{2}$2,673,649,867 |
| 11 45 | .0$^{4}$72,564,941 | .0$^{4}$12,012,313 | .0$^{2}$1,416,390,354 | .0$^{3}$,219,546,111 | .020,734,981,404 | .0$^{2}$2,850,826,323 |
| 12 00 | .0$^{4}$78,887,843 | .0$^{4}$13,342,029 | .0$^{2}$1,507,774,929 | .0$^{3}$,238,763,677 | .021,613,635,589 | .0$^{2}$3,035,594,350 |
| 12 15 | .0$^{4}$85,611,667 | .0$^{4}$14,786,717 | .0$^{2}$1,602,944,808 | .0$^{3}$,259,211,256 | .022,509,682,281 | .0$^{2}$3,228,106,021 |
| 12 30 | .0$^{4}$92,752,566 | .0$^{4}$16,353,645 | .0$^{2}$1,701,970,253 | .0$^{3}$,280,939,639 | .023,423,053,241 | .0$^{2}$3,428,512,814 |
| 12 45 | .0$^{3}$,100,326,967 | .0$^{4}$18,050,367 | .0$^{2}$1,804,921,005 | .0$^{3}$,304,000,578 | .024,353,678,913 | .0$^{2}$3,636,965,612 |
| 13 00 | .0$^{3}$,108,351,564 | .0$^{4}$19,884,738 | .0$^{2}$1,911,866,267 | .0$^{3}$,328,446,790 | .025,301,488,425 | .0$^{2}$3,853,614,682 |
| 13 15 | .0$^{3}$,116,843,318 | .0$^{4}$21,864,923 | .0$^{2}$2,022,874,700 | .0$^{3}$,354,331,941 | .026,266,409,599 | .0$^{2}$4,078,609,667 |
| 13 30 | .0$^{3}$,125,819,453 | .0$^{4}$23,999,394 | .0$^{2}$2,138,014,412 | .0$^{3}$,381,710,649 | .027,248,368,953 | .0$^{2}$4,312,099,575 |
| 13 45 | .0$^{3}$,135,297,49 | .0$^{4}$26,296,941 | .0$^{2}$2,257,352,943 | .0$^{3}$,410,638,482 | .028,247,291,705 | .0$^{2}$4,554,232,766 |
| 14 00 | .0$^{3}$,145,295,043 | .0$^{4}$28,766,676 | .0$^{2}$2,380,957,264 | .0$^{3}$,441,171,939 | .029,263,101,785 | .0$^{2}$4,805,156,943 |
| 14 15 | .0$^{3}$,155,830,221 | .0$^{4}$31,418,032 | .0$^{2}$2,508,893,760 | .0$^{3}$,473,368,458 | .030,295,721,835 | .0$^{2}$5,065,019,140 |
| 14 30 | .0$^{3}$,166,921,215 | .0$^{4}$34,260,781 | .0$^{2}$2,641,228,223 | .0$^{3}$,507,286,407 | .031,345,073,215 | .0$^{2}$5,333,965,708 |
| 14 45 | .0$^{3}$,178,586,502 | .0$^{4}$37,305,030 | .0$^{2}$2,778,025,845 | .0$^{3}$,542,985,071 | .032,411,076,015 | .0$^{2}$5,612,142,308 |
| 15 00 | .0$^{3}$,190,844,794 | .0$^{4}$40,561,221 | .0$^{2}$2,919,351,203 | .0$^{3}$,580,524,657 | .033,493,649,054 | .0$^{2}$5,899,693,900 |
| 15 15 | .0$^{3}$,203,715,038 | .0$^{4}$44,040,154 | .0$^{2}$3,065,268,253 | .0$^{3}$,619,966,281 | .034,592,709,890 | .0$^{2}$6,196,764,724 |
| 15 30 | .0$^{3}$,217,216,414 | .0$^{4}$47,752,971 | .0$^{2}$3,215,840,321 | .0$^{3}$,661,371,967 | .035,708,174,824 | .0$^{2}$6,503,498,302 |
| 15 45 | .0$^{3}$,231,368,321 | .0$^{4}$51,711,177 | .0$^{2}$3,371,130,092 | .0$^{3}$,704,804,635 | .036,839,958,911 | .0$^{2}$6,820,037,416 |
| 16 00 | .0$^{3}$,246,190,385 | .0$^{4}$55,926,637 | .0$^{2}$3,531,199,599 | .0$^{3}$,750,328,101 | .037,987,975,961 | .0$^{2}$7,146,524,101 |
| 16 15 | .0$^{3}$,261,702,448 | .0$^{4}$60,411,580 | .0$^{2}$3,696,110,220 | .0$^{3}$,798,007,067 | .039,152,138,547 | .0$^{2}$7,483,099,638 |
| 16 30 | .0$^{3}$,277,924,560 | .0$^{4}$65,178,610 | .0$^{2}$3,865,922,661 | .0$^{3}$,847,907,118 | .040,332,358,014 | .0$^{2}$7,829,904,536 |
| 16 45 | .0$^{3}$,294,876,985 | .0$^{4}$70,240,704 | .0$^{2}$4,040,696,952 | .0$^{3}$,900,094,712 | .041,528,544,483 | .0$^{2}$8,187,078,527 |
| 17 00 | .0$^{3}$,312,580,182 | .0$^{4}$75,611,222 | .0$^{2}$4,220,492,436 | .0$^{3}$,954,637,176 | .042,740,606,861 | .0$^{2}$8,554,760,552 |
| 17 15 | .0$^{3}$,331,054,816 | .0$^{4}$81,303,909 | .0$^{2}$4,405,367,760 | .0$^{2}$1,011,602,698 | .043,968,452,845 | .0$^{2}$8,933,088,753 |
| 17 30 | .0$^{3}$,350,321,741 | .0$^{4}$87,332,896 | .0$^{2}$4,595,380,867 | .0$^{2}$1,071,060,324 | .045,211,988,928 | .0$^{2}$9,322,200,462 |
| 17 45 | .0$^{3}$,370,402,001 | .0$^{4}$93,712,716 | .0$^{2}$4,790,588,986 | .0$^{2}$1,133,079,946 | .046,471,120,411 | .0$^{2}$9,722,232,187 |
| 18 00 | .0$^{3}$,391,316,825 | .0$^{3}$,100,458,295 | .0$^{2}$4,991,048,622 | .0$^{2}$1,197,732,299 | .047,745,751,406 | .010,133,319,606 |
| 18 15 | .0$^{3}$,413,087,618 | .0$^{3}$,107,584,963 | .0$^{2}$5,196,815,552 | .0$^{2}$1,265,088,952 | .049,035,784,846 | .010,555,597,557 |
| 18 30 | .0$^{3}$,435,735,962 | .0$^{3}$,115,108,462 | .0$^{2}$5,407,944,808 | .0$^{2}$1,335,222,306 | .050,341,122,488 | .010,989,200,022 |
| 18 45 | .0$^{3}$,459,283,607 | .0$^{3}$,123,044,943 | .0$^{2}$5,624,490,676 | .0$^{2}$1,408,205,578 | .051,661,664,927 | .011,434,260,122 |
| 19 00 | .0$^{3}$,483,752,464 | .0$^{3}$,131,410,973 | .0$^{2}$5,846,506,686 | .0$^{2}$1,484,112,803 | .052,997,311,598 | .011,890,910,108 |
| 19 15 | .0$^{3}$,509,164,609 | .0$^{3}$,140,223,541 | .0$^{2}$6,074,045,599 | .0$^{2}$1,563,018,820 | .054,347,960,787 | .012,359,281,345 |
| 19 30 | .0$^{3}$,535,542,264 | .0$^{3}$,149,500,060 | .0$^{2}$6,307,159,402 | .0$^{2}$1,644,999,272 | .055,713,509,636 | .012,839,504,307 |
| 19 45 | .0$^{3}$,562,907,805 | .0$^{3}$,159,258,374 | .0$^{2}$6,545,899,299 | .0$^{2}$1,730,130,591 | .057,093,854,153 | .013,331,708,565 |
| 20 00 | .0$^{3}$,591,283,750 | .0$^{3}$,169,516,760 | .0$^{2}$6,790,315,705 | .0$^{2}$1,818,489,994 | .058,488,889,220 | .013,836,022,778 |
| 20 15 | .0$^{3}$,620,692,749 | .0$^{3}$,180,293,928 | .0$^{2}$7,040,458,230 | .0$^{2}$1,910,155,478 | .059,898,508,600 | .014,352,574,682 |
| 20 30 | .0$^{3}$,651,157,593 | .0$^{3}$,191,609,035 | .0$^{2}$7,296,375,682 | .0$^{2}$2,005,205,808 | .061,322,604,944 | .014,881,491,082 |
| 20 45 | .0$^{3}$,682,701,193 | .0$^{3}$,203,481,681 | .0$^{2}$7,558,116,049 | .0$^{2}$2,103,720,511 | .062,761,069,803 | .015,422,897,840 |
| 21 00 | .0$^{3}$,715,346,583 | .0$^{3}$,215,931,909 | .0$^{2}$7,825,726,496 | .0$^{2}$2,205,779,872 | .064,213,793,631 | .015,976,919,870 |
| 21 15 | .0$^{3}$,749,116,914 | .0$^{3}$,228,980,227 | .0$^{2}$8,099,253,355 | .0$^{2}$2,311,464,919 | .065,680,665,798 | .016,543,681,120 |
| 21 30 | .0$^{3}$,784,035,444 | .0$^{3}$,242,647,587 | .0$^{2}$8,378,742,119 | .0$^{2}$2,420,857,423 | .067,161,574,595 | .017,123,304,574 |
| 21 45 | .0$^{3}$,820,125,538 | .0$^{3}$,256,955,409 | .0$^{2}$8,664,237,432 | .0$^{2}$2,534,039,881 | .068,656,407,247 | .017,715,912,231 |
| 22 00 | .0$^{3}$,857,410,657 | .0$^{3}$,271,925,573 | .0$^{2}$8,955,783,082 | .0$^{2}$2,651,095,518 | .070,165,049,915 | .018,321,625,105 |
| 22 15 | .0$^{3}$,895,914,357 | .0$^{3}$,287,580,428 | .0$^{2}$9,253,421,993 | .0$^{2}$2,772,108,271 | .071,687,387,711 | .018,940,563,210 |
| 22 30 | .0$^{3}$,935,660,279 | .0$^{3}$,303,942,794 | .0$^{2}$9,557,196,219 | .0$^{2}$2,897,162,786 | .073,223,304,703 | .019,572,845,553 |
| ° ′ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ |
| $\Phi_1$ | $\int(\Phi_1-\text{SIN }\Phi_1)\text{COS }\Phi\, d\Phi_1$ | | $\int\text{VERS }\Phi_1\text{ COS }\Phi\, d\Phi_1$ | | $\int\text{SIN }\Phi_1\text{ COS }\Phi\, d\Phi_1$ | |

SHEET 1 OF 4

205-D-1869

Table H-5.–*Integrals for arch load constants* (sheet 2).

| $\Phi_1$ | $\int(\Phi_1-\sin\Phi_1)\sin\Phi\,d\Phi_1$ | | $\int \text{vers}\,\Phi_1\sin\Phi\,d\Phi_1$ | | $\int\sin\Phi_1\sin\Phi\,d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ |
| 22 45 | $.0^3$, 976,672,146 | $.0^3$, 321,035,964 | .009,867,146,934 | .003,026,344,405 | .074,772,683,925 | .020,218,590,126 |
| 23 00 | .001,018,973,757 | $.0^3$, 338,883,707 | .010,183,314,425 | .003,159,739,163 | .076,335,407,385 | .020,877,913,895 |
| 23 15 | .001,062,588,978 | $.0^3$, 357,510,277 | .010,505,738,087 | .003,297,433,775 | .077,911,356,076 | .021,550,932,792 |
| 23 30 | .001,107,541,738 | $.0^3$, 376,940,410 | .010,834,456,411 | .003,439,515,631 | .079,500,409,984 | .022,237,761,705 |
| 23 45 | .001,153,856,025 | $.0^3$, 397,199,328 | .011,169,506,982 | .003,586,072,785 | .081,102,448,096 | .022,938,514,472 |
| 24 00 | .001,201,555,878 | $.0^3$, 418,312,745 | .011,510,926,467 | .003,737,193,947 | .082,717,348,410 | .023,653,303,870 |
| 24 15 | .001,250,665,375 | $.0^3$, 440,306,869 | .011,858,850,611 | .003,892,968,477 | .084,344,987,946 | .024,382,241,607 |
| 24 30 | .001,301,208,643 | $.0^3$, 463,208,402 | .012,213,014,231 | .004,053,486,371 | .085,985,242,752 | .025,125,438,314 |
| 24 45 | .001,353,209,831 | $.0^3$, 487,044,548 | .012,573,751,203 | .004,218,838,258 | .087,637,987,917 | .025,883,003,535 |
| 25 00 | .001,406,693,120 | $.0^3$, 511,843,015 | .012,940,994,461 | .004,389,115,385 | .089,303,097,578 | .026,655,045,719 |
| 25 15 | .001,461,682,707 | $.0^3$, 537,632,009 | .013,314,775,990 | .004,564,409,616 | .090,980,444,930 | .027,441,672,217 |
| 25 30 | .001,518,202,807 | $.0^3$, 564,440,249 | .013,695,126,815 | .004,744,813,413 | .092,669,902,237 | .028,242,989,265 |
| 25 45 | .001,576,277,638 | $.0^3$, 592,296,964 | .014,082,076,997 | .004,930,419,837 | .094,371,340,840 | .029,059,101,981 |
| 26 00 | .001,635,931,417 | $.0^3$, 621,231,896 | .014,475,655,628 | .005,121,322,532 | .096,084,631,169 | .029,890,114,357 |
| 26 15 | .001,697,188,362 | $.0^3$, 651,275,301 | .014,875,890,822 | .005,317,615,719 | .097,809,642,748 | .030,736,129,251 |
| 26 30 | .001,760,072,670 | $.0^3$, 682,457,955 | .015,282,809,709 | .005,519,394,186 | .099,546,244,212 | .031,597,248,377 |
| 26 45 | .001,824,608,523 | $.0^3$, 714,811,151 | .015,696,438,429 | .005,726,753,277 | .101,294,303,312 | .032,473,572,300 |
| 27 00 | .001,890,820,078 | $.0^3$, 748,366,712 | .016,116,802,127 | .005,939,788,885 | .103,053,686,927 | .033,365,200,425 |
| 27 15 | .001,958,731,459 | $.0^3$, 783,156,975 | .016,543,924,944 | .006,158,597,442 | .104,824,261,072 | .034,272,230,995 |
| 27 30 | .002,028,366,746 | $.0^3$, 819,214,813 | .016,977,830,014 | .006,383,275,910 | .106,605,890,912 | .035,194,761,077 |
| 27 45 | .002,099,749,981 | $.0^3$, 856,573,626 | .017,418,539,456 | .006,613,921,768 | .108,398,440,769 | .036,132,886,559 |
| 28 00 | .002,172,905,150 | $.0^3$, 895,267,344 | .017,866,074,368 | .006,850,633,009 | .110,201,774,132 | .037,086,702,140 |
| 28 15 | .002,247,856,176 | $.0^3$, 935,330,429 | .018,320,454,824 | .007,093,508,123 | .112,015,753,672 | .038,056,301,327 |
| 28 30 | .002,324,626,922 | $.0^3$, 976,797,882 | .018,781,699,864 | .007,342,646,092 | .113,840,241,246 | .039,041,776,422 |
| 28 45 | .002,403,241,174 | .001,019,705,235 | .019,249,827,492 | .007,598,146,379 | .115,675,097,913 | .040,043,218,521 |
| 29 00 | .002,483,722,637 | .001,064,088,567 | .019,724,854,668 | .007,860,108,919 | .117,520,183,942 | .041,060,717,500 |
| 29 15 | .002,566,094,937 | .001,109,994,490 | .020,206,797,304 | .008,128,634,106 | .119,375,358,821 | .042,094,362,015 |
| 29 30 | .002,650,381,594 | .001,157,430,159 | .020,695,670,259 | .008,403,822,788 | .121,240,481,272 | .043,144,239,494 |
| 29 45 | .002,736,606,037 | .001,206,463,278 | .021,191,487,331 | .008,685,776,251 | .123,115,409,260 | .044,210,436,124 |
| 30 00 | .002,824,791,583 | .001,257,122,089 | .021,694,261,255 | .008,974,596,216 | .125,000,000,000 | .045,293,036,853 |
| 30 15 | .002,914,961,436 | .001,309,445,384 | .022,204,003,697 | .009,270,384,822 | .126,894,109,974 | .046,392,125,379 |
| 30 30 | .003,007,138,675 | .001,363,472,500 | .022,720,725,247 | .009,573,244,619 | .128,797,594,939 | .047,507,784,144 |
| 30 45 | .003,101,346,254 | .001,419,243,325 | .023,244,435,418 | .009,883,278,564 | .130,710,309,935 | .048,640,094,329 |
| 31 00 | .003,197,606,989 | .001,476,798,298 | .023,775,142,636 | .010,200,589,995 | .132,632,109,303 | .049,789,135,844 |
| 31 15 | .003,295,943,551 | .001,536,178,406 | .024,312,854,242 | .010,525,282,636 | .134,562,846,691 | .050,954,987,329 |
| 31 30 | .003,396,378,463 | .001,597,425,188 | .024,857,576,480 | .010,857,460,581 | .136,502,375,065 | .052,137,726,142 |
| 31 45 | .003,498,934,091 | .001,660,580,740 | .025,409,314,497 | .011,197,228,281 | .138,450,546,723 | .053,337,428,353 |
| 32 00 | .003,603,632,632 | .001,725,687,707 | .025,968,072,339 | .011,544,690,541 | .140,407,213,303 | .054,554,168,744 |
| 32 15 | .003,710,496,119 | .001,792,789,291 | .026,533,852,944 | .011,899,952,498 | .142,372,225,798 | .055,788,020,796 |
| 32 30 | .003,819,546,396 | .001,861,929,248 | .027,106,658,139 | .012,263,119,622 | .144,345,434,565 | .057,039,056,690 |
| 32 45 | .003,930,805,126 | .001,933,151,891 | .027,686,488,635 | .012,634,297,700 | .146,326,689,336 | .058,307,347,295 |
| 33 00 | .004,044,293,779 | .002,006,502,090 | .028,273,344,025 | .013,013,592,824 | .148,315,839,231 | .059,592,962,168 |
| 33 15 | .004,160,033,619 | .002,082,025,268 | .028,867,222,780 | .013,401,111,383 | .150,312,732,769 | .060,895,969,548 |
| 33 30 | .004,278,045,706 | .002,159,767,410 | .029,468,122,240 | .013,796,960,055 | .152,317,217,878 | .062,216,436,346 |
| 33 45 | .004,398,350,883 | .002,239,775,056 | .030,076,038,618 | .014,201,245,788 | .154,329,141,909 | .063,554,428,146 |
| 34 00 | .004,520,969,767 | .002,322,095,304 | .030,690,966,990 | .014,614,075,799 | .156,348,351,646 | .064,910,009,197 |
| 34 15 | .004,645,922,747 | .002,406,775,808 | .031,312,901,296 | .015,035,557,554 | .158,374,693,319 | .066,283,242,409 |
| 34 30 | .004,773,229,975 | .002,493,864,784 | .031,941,834,332 | .015,465,798,764 | .160,408,012,614 | .067,674,189,345 |
| 34 45 | .004,902,911,354 | .002,583,411,002 | .032,577,757,750 | .015,904,907,373 | .162,448,154,685 | .069,082,910,222 |
| 35 00 | .005,034,986,536 | .002,675,463,793 | .033,220,662,056 | .016,352,991,542 | .164,494,964,169 | .070,509,463,903 |
| 35 15 | .005,169,474,915 | .002,770,073,043 | .033,870,536,600 | .016,810,159,647 | .166,548,285,191 | .071,953,907,891 |
| 35 30 | .005,306,395,609 | .002,867,289,197 | .034,527,369,582 | .017,276,520,258 | .168,607,961,386 | .073,416,298,329 |
| 35 45 | .005,445,767,473 | .002,367,163,254 | .035,191,148 042 | .017,752,182,137 | .170,673,835,898 | .074,896,689,993 |
| 36 00 | .005,587,609,067 | .003,069,746,775 | .035,861,857,859 | .018,237,254,219 | .172,745,751,406 | .076,395,136,285 |
| 36 15 | .005,731,938,668 | .003,175,091,874 | .036,539,483,753 | .018,731,845,609 | .174,823,550,124 | .077,911,689,237 |
| 36 30 | .005,878,774,254 | .003,283,251,216 | .037,224,009,271 | .019,236,065,564 | .176,907,073,819 | .079,446,399,498 |
| 36 45 | .006,028,133,497 | .003,394,278,030 | .037,915,416,800 | .019,750,023,485 | .178,996,163,824 | .080,999,316,337 |
| 37 00 | .006,180,033,754 | .003,508,226,091 | .038,613,687,549 | .020,273,828,907 | .181,090,661,046 | .082,570,487,635 |
| 37 15 | .006,334,492,067 | .003,625,149,734 | .039,318,801,557 | .020,807,591,485 | .183,190,405,980 | .084,159,959,882 |
| 37 30 | .006,491,525,142 | .003,745,103,840 | .040,030,737,688 | .021,351,420,984 | .185,295,238,725 | .085,767,778,177 |
| 37 45 | .006,651,149,359 | .003,868,143,845 | .040,749,473,626 | .021,905,427,270 | .187,404,998,986 | .087,393,988,220 |
| 38 00 | .006,813,380,748 | .003,994,325,736 | .041,474,985,878 | .022,469,720,293 | .189,519,526,100 | .089,038,626,310 |
| 38 15 | .006,978,234,990 | .004,123,706,047 | .042,207,249,766 | .023,044,410,083 | .191,638,659,036 | .090,701,739,344 |
| 38 30 | .007,145,727,409 | .004,256,341,859 | .042,946,239,433 | .023,629,606,734 | .193,762,236,414 | .092,383,364,813 |
| 38 45 | .007,315,872,964 | .004,392,290,801 | .043,691,927,830 | .024,225,420,392 | .195,890,096,515 | .094,083,540,794 |
| 39 00 | .007,488,686,238 | .004,531,611,051 | .044,444,286,727 | .024,831,961,247 | .198,022,077,296 | .095,802,303,955 |
| 39 15 | .007,664,181,436 | .004,674,361,322 | .045,203,286,704 | .025,449,339,522 | .200,158,016,396 | .097,539,689,549 |
| 39 30 | .007,842,372,373 | .004,820,600,878 | .045,968,897,147 | .026,077,665,456 | .202,297,751,156 | .099,295,731,407 |
| 39 45 | .008,023,272,468 | .004,970,389,517 | .046,741,086,256 | .026,717,049,300 | .204,441,118,627 | .101,070,461,943 |
| 40 00 | .008,206,894,738 | .005,123,787,579 | .047,519,821,035 | .027,367,601,297 | .206,587,955,584 | .102,863,912,146 |
| 40 15 | .008,393,251,788 | .005,280,855,940 | .048,305,067,295 | .028,029,431,682 | .208,738,098,535 | .104,676,111,580 |
| 40 30 | .008,582,355,803 | .005,441,656,013 | .049,096,789,652 | .028,702,650,660 | .210,891,383,740 | .106,507,088,380 |
| 40 45 | .008,774,218,544 | .005,606,249,740 | .049,894,951,528 | .029,387,368,398 | .213,047,647,218 | .108,356,869,253 |
| 41 00 | .008,968,851,340 | .005,774,699,597 | .050,699,515,146 | .030,083,695,017 | .215,206,724,760 | .110,225,479,473 |
| 41 15 | .009,166,265,072 | .005,947,068,586 | .051,510,441,533 | .030,791,740,576 | .217,368,451,945 | .112,112,942,881 |
| 41 30 | .009,366,470,181 | .006,123,420,241 | .052,327,690,517 | .031,511,615,062 | .219,532,664,149 | .114,019,281,879 |
| 41 45 | .009,569,476,645 | .006,303,818,616 | .053,151,220,728 | .032,243,428,380 | .221,699,196,558 | .115,944,517,434 |
| 42 00 | .009,775,293,980 | .006,488,328,286 | .053,980,989,599 | .032,987,290,340 | .223,867,884,183 | .187,888,669,077 |
| 42 15 | .009,983,931,236 | .006,677,014,348 | .054,816,953,360 | .033,743,310,643 | .226,038,561,870 | .119,851,754,892 |
| 42 30 | .010,195,396,975 | .006,869,942,415 | .055,659,067,044 | .034,511,598,877 | .228,211,064,313 | .121,833,791,525 |
| 42 45 | .010,409,699,279 | .007,067,178,614 | .056,507,284,486 | .035,292,264,496 | .230,385,226,068 | .123,834,794,180 |
| 43 00 | .010,626,845,735 | .007,268,789,583 | .057,361,558,319 | .036,085,416,817 | .232,560,881,564 | .125,854,776,614 |
| 43 15 | .010,846,843,429 | .007,474,842,472 | .058,221,839,977 | .036,891,165,002 | .234,737,865,116 | .127,893,751,138 |
| 43 30 | .011,069,698,938 | .007,685,404,932 | .059,088,079,697 | .037,709,618,049 | .236,916,010,939 | .129,951,728,620 |
| 43 45 | .011,295,418,323 | .007,900,545,118 | .059,960,226,513 | .038,540,884,782 | .239,095,153,158 | .132,028,718,478 |
| 44 00 | .011,524,007,121 | .008,120,331,687 | .060,838,228,266 | .039,385,073,837 | .241,275,125,824 | .134,124,728,684 |
| 44 15 | .011,755,470,338 | .008,344,833,793 | .061,722,031,594 | .040,242,293,652 | .243,455,762,923 | .136,239,765,759 |
| 44 30 | .011,989,812,442 | .008,574,121,078 | .062,611,581,942 | .041,112,652,455 | .245,636,898,391 | .138,373,834,779 |
| 44 45 | .012,227,037,359 | .008,808,263,678 | .063,506,823,557 | .041,996,258,252 | .247,818,366,125 | .140,526,939,368 |
| 45 00 | .012,467,148,457 | .009,047,332,218 | .064,407,699,488 | .042,893,218,813 | .250,000,000,000 | .142,699,081,699 |
| ° ′ | + COS $\Phi_0$ | - SIN $\Phi_0$ | +COS $\Phi_0$ | -SIN $\Phi_0$ | +COS $\Phi_0$ | -SIN $\Phi_0$ |
| $\Phi_1$ | $\int(\Phi_1-\sin\Phi_1)\cos\Phi\,d\Phi_1$ | | $\int \text{vers}\,\Phi_1\cos\Phi\,d\Phi_1$ | | $\int\sin\Phi_1\cos\Phi\,d\Phi_1$ | |

SHEET 2 OF 4

205-D-1870

Table H-5.–*Integrals for arch load constants* (sheet 3).

| $\Phi_1$ | $\int(\Phi_1-\sin\Phi_1)\sin\Phi\,d\Phi_1$ | | $\int\text{vers}\,\Phi_1\sin\Phi\,d\Phi_1$ | | $\int\sin\Phi_1\sin\Phi\,d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | $+\sin\Phi_0$ | $+\cos\Phi_0$ | $+\sin\Phi_0$ | $+\cos\Phi_0$ | $+\sin\Phi_0$ | $+\cos\Phi_0$ |
| 45 15 | .012,710,148,544 | .009,291,397,802 | .065,314,151,594 | .043,803,641,670 | .252,181,633,874 | .144,890,262,498 |
| 45 30 | .012,956,039,863 | .009,540,532,016 | .066,226,120,536 | .044,727,634,091 | .254,363,101,609 | .147,100,481,039 |
| 45 45 | .013,204,824,082 | .009,794,806,920 | .067,143,545,787 | .045,665,303,081 | .256,544,237,077 | .149,329,735,149 |
| 46 00 | .013,456,502,284 | .010,054,295,047 | .068,066,365,626 | .046,616,755,365 | .258,724,874,176 | .151,578,021,204 |
| 46 15 | .013,711,074,966 | .010,319,069,401 | .068,994,517,141 | .047,582,097,376 | .260,904,846,842 | .153,845,334,128 |
| 46 30 | .013,968,542,027 | .010,589,203,445 | .069,927,936,235 | .048,561,435,245 | .263,083,989,061 | .156,131,667,400 |
| 46 45 | .014,228,902,761 | .010,864,771,111 | .070,866,557,623 | .049,554,874,791 | .265,262,134,883 | .158,437,013,048 |
| 47 00 | .014,492,155,851 | .011,145,846,780 | .071,810,314,836 | .050,562,521,502 | .267,439,118,436 | .160,761,361,654 |
| 47 15 | .014,758,299,366 | .011,432,505,289 | .072,759,140,219 | .051,584,480,535 | .269,614,773,932 | .163,104,702,350 |
| 47 30 | .015,027,330,743 | .011,724,821,921 | .073,712,964,938 | .052,620,856,697 | .271,788,935,687 | .165,467,022,825 |
| 47 45 | .015,299,246,792 | .012,022,872,406 | .074,671,718,982 | .053,671,754,435 | .273,961,438,130 | .167,848,309,322 |
| 48 00 | .015,574,043,681 | .012,326,732,912 | .075,635,331,157 | .054,737,277,824 | .276,132,115,817 | .170,248,546,637 |
| 48 15 | .015,851,716,933 | .012,636,480,045 | .076,603,729,099 | .055,817,530,557 | .278,300,803,442 | .172,667,718,124 |
| 48 30 | .016,132,261,416 | .012,952,190,833 | .077,576,839,270 | .056,912,615,933 | .280,467,335,851 | .175,105,805,698 |
| 48 45 | .016,415,671,338 | .013,273,942,741 | .078,554,586,962 | .058,022,636,845 | .282,631,548,055 | .177,562,789,830 |
| 49 00 | .016,701,940,241 | .013,601,813,647 | .079,536,896,299 | .059,147,695,769 | .284,793,275,240 | .180,038,649,553 |
| 49 15 | .016,991,060,990 | .013,935,881,851 | .080,523,690,240 | .060,287,894,755 | .286,952,352,782 | .182,533,362,463 |
| 49 30 | .017,283,025,770 | .014,276,226,062 | .081,514,890,583 | .061,443,335,410 | .289,108,616,260 | .185,046,904,720 |
| 49 45 | .017,577,826,080 | .014,622,925,396 | .082,510,417,965 | .062,614,118,891 | .291,261,901,466 | .187,579,251,049 |
| 50 00 | .017,875,452,722 | .014,976,059,371 | .083,510,191,867 | .063,800,345,896 | .293,412,044,417 | .190,130,374,745 |
| 50 15 | .018,175,895,794 | .015,335,707,902 | .084,514,130,618 | .065,002,116,646 | .295,558,881,373 | .192,700,247,672 |
| 50 30 | .018,479,144,691 | .015,701,951,296 | .085,504,151,397 | .066,219,530,878 | .297,702,248,844 | .195,288,840,266 |
| 50 45 | .018,785,188,089 | .016,074,870,243 | .086,534,170,233 | .067,452,687,833 | .299,841,983,604 | .197,896,121,538 |
| 51 00 | .019,094,013,942 | .016,454,545,820 | .087,550,102,015 | .068,701,686,246 | .301,977,922,704 | .200,522,059,075 |
| 51 15 | .019,405,609,477 | .016,841,059,474 | .088,569,860,490 | .069,966,624,331 | .304,109,903,485 | .203,166,619,044 |
| 51 30 | .019,719,961,189 | .017,234,493,024 | .089,593,358,267 | .071,247,599,776 | .306,237,763,586 | .205,829,766,192 |
| 51 45 | .020,037,054,825 | .017,634,928,655 | .090,620,506,828 | .072,544,709,726 | .308,361,340,964 | .208,511,463,854 |
| 52 00 | .020,356,875,387 | .018,042,448,908 | .091,651,216,519 | .073,858,050,774 | .310,480,473,900 | .211,211,673,949 |
| 52 15 | .020,679,407,123 | .018,457,136,678 | .092,685,396,566 | .075,187,718,952 | .312,595,001,014 | .213,930,356,989 |
| 52 30 | .021,004,633,522 | .018,879,075,209 | .093,722,955,070 | .076,533,809,716 | .314,704,761,275 | .216,667,472,076 |
| 52 45 | .021,332,537,299 | .019,308,348,087 | .094,763,799,019 | .077,896,417,937 | .316,809,594,020 | .219,422,976,911 |
| 53 00 | .021,663,100,402 | .019,745,039,229 | .095,807,834,284 | .079,275,637,894 | .318,909,338,954 | .222,196,827,794 |
| 53 15 | .021,996,303,996 | .020,189,232,887 | .096,854,965,630 | .080,671,563,253 | .321,003,836,176 | .224,988,979,626 |
| 53 30 | .022,332,128,459 | .020,641,013,633 | .097,905,096,718 | .082,084,287,068 | .323,092,926,181 | .227,799,385,918 |
| 53 45 | .022,670,553,381 | .021,100,466,358 | .098,958,130,106 | .083,513,901,760 | .325,176,449,876 | .230,627,998,786 |
| 54 00 | .023,011,557,545 | .021,567,676,263 | .100,013,967,263 | .084,960,499,114 | .327,254,248,594 | .233,474,768,965 |
| 54 15 | .023,355,118,939 | .022,042,728,855 | .101,036,508,560 | .086,424,170,262 | .329,326,164,101 | .236,339,645,802 |
| 54 30 | .023,701,214,733 | .022,525,709,936 | .102,133,653,288 | .087,905,005,675 | .331,392,038,614 | .239,222,577,269 |
| 54 45 | .024,049,821,282 | .023,016,705,608 | .103,197,299,655 | .089,403,095,154 | .333,451,714,809 | .242,123,509,960 |
| 55 00 | .024,400,914,121 | .023,515,802,247 | .104,263,344,794 | .090,918,527,818 | .335,505,035,831 | .245,042,389,102 |
| 55 15 | .024,754,467,951 | .024,023,086,516 | .105,331,684,767 | .092,451,392,089 | .337,551,845,315 | .247,979,158,552 |
| 55 30 | .025,110,456,646 | .024,538,645,349 | .106,402,214,569 | .094,001,775,689 | .339,591,987,386 | .250,933,760,804 |
| 55 45 | .025,468,853,231 | .025,062,565,943 | .107,474,828,138 | .095,569,765,624 | .341,625,306,681 | .253,906,136,998 |
| 56 00 | .025,829,629,891 | .025,594,935,754 | .108,549,418,355 | .097,155,448,175 | .343,651,648,354 | .256,896,226,917 |
| 56 15 | .026,192,757,958 | .026,135,842,493 | .109,625,877,051 | .098,758,908,888 | .345,670,858,092 | .259,903,968,995 |
| 56 30 | .026,558,207,904 | .026,685,374,109 | .110,704,095,015 | .100,380,232,566 | .347,682,782,122 | .262,929,300,325 |
| 56 45 | .026,925,949,340 | .027,243,618,795 | .111,783,961,998 | .102,019,503,248 | .349,687,267,232 | .265,972,156,657 |
| 57 00 | .027,295,951,008 | .027,810,664,971 | .112,865,366,716 | .103,676,804,216 | .351,684,160,769 | .269,032,472,408 |
| 57 15 | .027,668,180,778 | .028,386,601,284 | .113,948,196,861 | .105,352,217,968 | .353,673,310,664 | .272,110,180,664 |
| 57 30 | .028,042,605,636 | .028,971,516,592 | .115,034,339,105 | .107,045,826,218 | .355,654,565,435 | .275,205,213,189 |
| 57 45 | .028,419,191,686 | .029,565,499,966 | .116,117,679,103 | .108,757,709,882 | .357,627,774,202 | .278,317,500,426 |
| 58 00 | .028,797,904,141 | .030,168,640,676 | .117,204,101,503 | .110,487,949,070 | .359,592,786,697 | .281,446,971,504 |
| 58 15 | .029,178,707,321 | .030,781,028,190 | .118,291,489,952 | .112,236,623,070 | .361,549,453,278 | .284,593,554,243 |
| 58 30 | .029,561,564,640 | .031,402,752,156 | .119,379,727,099 | .114,003,810,349 | .363,497,624,935 | .287,757,175,161 |
| 58 45 | .029,946,438,612 | .032,033,902,406 | .120,468,694,605 | .115,789,588,530 | .365,437,153,309 | .290,937,759,478 |
| 59 00 | .030,333,290,837 | .032,674,568,941 | .121,558,273,149 | .117,594,034,394 | .367,367,890,696 | .294,135,231,123 |
| 59 15 | .030,722,081,998 | .033,324,841,926 | .122,648,342,433 | .119,417,223,858 | .369,289,690,065 | .297,349,512,738 |
| 59 30 | .031,112,771,862 | .033,984,811,685 | .123,801,781,189 | .121,259,231,977 | .371,202,405,062 | .300,580,525,683 |
| 59 45 | .031,505,319,265 | .034,654,568,683 | .124,829,467,186 | .123,120,132,928 | .373,105,890,026 | .303,828,190,048 |
| 60 00 | .031,899,682,116 | .035,334,203,534 | .125,920,277,240 | .125,000,000,000 | .375,000,000,000 | .307,092,424,652 |
| 60 15 | .032,295,817,388 | .036,023,806,977 | .127,011,087,216 | .126,898,905,585 | .376,884,590,740 | .310,373,147,053 |
| 60 30 | .032,693,681,112 | .036,723,469,877 | .128,101,772,037 | .128,816,921,169 | .378,759,518,728 | .313,670,273,553 |
| 60 45 | .033,093,228,379 | .037,433,283,216 | .129,192,205,691 | .130,754,117,324 | .380,624,641,179 | .316,983,719,204 |
| 61 00 | .033,494,413,324 | .038,153,338,083 | .130,282,261,242 | .132,710,563,696 | .382,479,816,058 | .320,313,397,819 |
| 61 15 | .033,897,189,137 | .038,883,725,670 | .131,371,810,832 | .134,686,328,994 | .384,324,902,087 | .323,659,221,970 |
| 61 30 | .034,301,508,043 | .039,624,537,252 | .132,460,725,687 | .136,681,480,986 | .386,159,758,754 | .327,021,103,002 |
| 61 45 | .034,707,321,308 | .040,375,864,198 | .133,548,876,135 | .138,696,086,487 | .387,984,246,328 | .330,398,951,036 |
| 62 00 | .035,114,579,230 | .041,137,797,944 | .134,636,131,602 | .140,730,211,346 | .389,798,225,868 | .333,792,674,979 |
| 62 15 | .035,523,231,139 | .041,910,429,994 | .135,722,360,625 | .142,783,920,444 | .391,601,559,231 | .337,202,182,528 |
| 62 30 | .035,933,225,385 | .042,693,851,910 | .136,807,430,858 | .144,857,277,677 | .393,394,109,088 | .340,627,380,176 |
| 62 45 | .036,344,509,345 | .043,388,155,306 | .137,891,209,084 | .146,950,345,955 | .395,175,738,928 | .344,068,173,224 |
| 63 00 | .036,757,029,408 | .044,293,431,830 | .138,973,561,216 | .149,063,187,187 | .396,946,313,073 | .347,524,465,784 |
| 63 15 | .037,170,730,978 | .045,109,773,170 | .140,054,352,314 | .151,195,862,275 | .398,705,696,688 | .350,996,160,789 |
| 63 30 | .037,585,558,470 | .045,937,271,029 | .141,133,446,582 | .153,348,431,102 | .400,453,755,788 | .354,483,159,996 |
| 63 45 | .038,001,455,302 | .046,776,017,131 | .142,210,707,387 | .155,520,952,529 | .402,190,357,252 | .357,985,364,000 |
| 64 00 | .038,418,363,893 | .047,626,103,202 | .143,285,997,259 | .157,713,484,380 | .403,915,368,831 | .361,502,672,236 |
| 64 15 | .038,836,225,663 | .048,487,620,965 | .144,359,177,907 | .159,926,083,436 | .405,628,659,160 | .365,034,982,990 |
| 64 30 | .039,254,981,026 | .049,360,662,130 | .145,430,110,218 | .162,158,805,429 | .407,330,097,763 | .368,582,193,404 |
| 64 45 | .039,674,569,386 | .050,245,318,389 | .146,498,654,274 | .164,411,705,030 | .409,019,555,069 | .372,144,199,486 |
| 65 00 | .040,094,929,136 | .051,141,681,401 | .147,564,669,359 | .166,684,835,837 | .410,696,902,422 | .375,720,896,118 |
| 65 15 | .040,515,997,650 | .052,049,842,785 | .148,628,013,962 | .168,978,250,381 | .412,362,012,082 | .379,312,177,063 |
| 65 30 | .040,937,711,287 | .052,969,894,112 | .149,688,545,793 | .171,292,000,096 | .414,014,757,248 | .382,917,934,972 |
| 65 45 | .041,360,005,384 | .053,901,926,895 | .150,746,121,787 | .173,626,135,333 | .415,655,012,054 | .386,538,061,396 |
| 66 00 | .041,782,814,251 | .054,846,032,582 | .151,800,598,116 | .175,980,705,334 | .417,282,651,590 | .390,172,446,789 |
| 66 15 | .042,206,071,170 | .055,802,302,542 | .152,851,830,193 | .178,355,758,237 | .418,897,551,904 | .393,820,980,520 |
| 66 30 | .042,629,708,393 | .056,770,828,059 | .153,899,672,692 | .180,751,341,059 | .420,499,590,016 | .397,483,550,883 |
| 66 45 | .043,053,657,138 | .057,751,700,321 | .154,943,979,543 | .183,167,499,692 | .422,088,643,924 | .401,160,045,100 |
| 67 00 | .043,477,847,588 | .058,745,010,413 | .155,984,603,950 | .185,604,278,897 | .423,664,592,614 | .404,850,349,334 |
| 67 15 | .043,902,208,881 | .059,750,849,307 | .157,021,398,398 | .188,061,722,288 | .425,227,316,075 | .408,554,348,694 |
| 67 30 | .044,326,669,121 | .060,769,307,847 | .158,054,214,666 | .190,539,872,339 | .426,776,695,296 | .412,271,927,251 |
| ° ′ | $+\cos\Phi_0$ | $-\sin\Phi_0$ | $+\cos\Phi_0$ | $-\sin\Phi_0$ | $+\cos\Phi_0$ | $-\sin\Phi_0$ |
| $\Phi_1$ | $\int(\Phi_1-\sin\Phi_1)\cos\Phi\,d\Phi_1$ | | $\int\text{vers}\,\Phi_1\cos\Phi\,d\Phi_1$ | | $\int\sin\Phi_1\cos\Phi\,d\Phi_1$ | |

Table H-5.–*Integrals for arch load constants* (sheet 4).

| $\Phi_1$ | $\int(\Phi_1-\text{SIN }\Phi_1)\text{ SIN }\Phi\, d\Phi_1$ | | $\int\text{VERS }\Phi_1\text{ SIN }\Phi\, d\Phi_1$ | | $\int\text{SIN }\Phi_1\text{ SIN }\Phi\, d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ |
| 67 45 | .044,751,155,360 | .061,800,476,748 | .159,082,903,830 | .193,038,770,359 | .428,312,612,289 | .416,002,968,038 |
| 68 00 | .045,175,593,611 | .062,844,446,578 | .160,107,316,274 | .195,558,456,499 | .429,834,950,085 | .419,747,353,063 |
| 68 15 | .045,599,908,831 | .063,901,307,758 | .161,127,301,705 | .198,098,969,737 | .431,343,592,753 | .423,504,963,319 |
| 68 30 | .046,024,024,927 | .064,971,150,542 | .162,142,709,159 | .200,660,347,871 | .432,838,425,405 | .427,275,678,793 |
| 68 45 | .046,447,864,758 | .066,054,065,015 | .163,153,387,006 | .203,242,627,513 | .434,319,334,203 | .431,059,378,469 |
| 69 00 | .046,871,350,120 | .067,150,141,080 | .164,159,182,969 | .205,845,844,086 | .435,786,206,369 | .434,855,940,348 |
| 69 15 | .047,294,401,755 | .068,259,468,443 | .165,159,944,129 | .208,470,031,805 | .437,238,930,197 | .438,665,241,449 |
| 69 30 | .047,716,939,344 | .069,382,136,618 | .166,155,516,933 | .211,115,223,686 | .438,677,395,055 | .442,487,157,821 |
| 69 45 | .048,138,881,507 | .070,518,234,900 | .167,145,747,207 | .213,781,451,523 | .440,101,491,400 | .446,321,564,551 |
| 70 00 | .048,560,145,805 | .071,667,852,367 | .168,130,480,166 | .216,468,745,894 | .441,511,110,780 | .450,168,335,776 |
| 70 15 | .048,980,648,724 | .072,831,077,862 | .169,109,560,424 | .219,177,136,150 | .442,906,145,847 | .454,027,344,694 |
| 70 30 | .049,400,305,695 | .074,007,999,991 | .170,082,832,001 | .221,906,650,402 | .444,286,490,364 | .457,898,463,565 |
| 70 45 | .049,819,031,073 | .075,198,707,105 | .171,050,138,340 | .224,657,315,524 | .445,652,039,213 | .461,781,563,733 |
| 71 00 | .050,236,738,147 | .076,403,287,294 | .172,011,322,309 | .227,429,157,142 | .447,002,688,401 | .465,676,515,626 |
| 71 15 | .050,653,339,134 | .077,621,828,378 | .172,966,226,220 | .230,222,199,624 | .448,338,335,073 | .469,583,188,771 |
| 71 30 | .051,068,745,182 | .078,854,417,894 | .173,914,691,830 | .233,036,466,083 | .449,658,877,512 | .473,501,451,800 |
| 71 45 | .051,482,866,365 | .080,101,143,089 | .174,856,560,361 | .235,871,978,362 | .450,964,215,154 | .477,431,172,465 |
| 72 00 | .051,895,611,678 | .081,362,090,905 | .175,791,672,504 | .238,728,757,031 | .452,254,248,594 | .481,372,217,645 |
| 72 15 | .052,306,889,048 | .082,637,347,973 | .176,719,868,432 | .241,606,821,383 | .453,528,879,589 | .485,324,453,355 |
| 72 30 | .052,716,605,323 | .083,927,000,602 | .177,640,987,812 | .244,506,189,424 | .454,788,011,072 | .489,287,744,760 |
| 72 45 | .053,124,666,276 | .085,231,134,764 | .178,554,869,813 | .247,426,877,869 | .456,031,547,155 | .493,261,956,182 |
| 73 00 | .053,530,976,599 | .086,549,836,096 | .179,461,353,117 | .250,368,902,138 | .457,259,393,139 | .497,246,951,110 |
| 73 15 | .053,935,439,910 | .087,883,189,872 | .180,360,275,934 | .253,332,276,349 | .458,471,455,517 | .501,242,592,215 |
| 73 30 | .054,337,958,749 | .089,231,281,006 | .181,251,476,006 | .256,317,013,310 | .459,667,641,986 | .505,248,741,354 |
| 73 45 | .054,738,434,575 | .090,594,194,038 | .182,134,790,626 | .259,323,124,516 | .460,847,861,453 | .509,265,259,586 |
| 74 00 | .055,136,767,767 | .091,972,013,122 | .183,010,056,641 | .262,350,620,144 | .462,012,024,039 | .513,292,007,179 |
| 74 15 | .055,532,857,631 | .093,364,822,017 | .183,877,110,472 | .265,399,509,046 | .463,160,041,089 | .517,328,843,624 |
| 74 30 | .055,926,602,385 | .094,772,704,075 | .184,735,788,114 | .268,469,798,746 | .464,291,825,176 | .521,375,627,641 |
| 74 45 | .056,317,899,177 | .096,195,742,232 | .185,585,925,156 | .271,561,495,432 | .465,407,290,110 | .525,432,217,193 |
| 75 00 | .056,706,644,066 | .097,634,018,997 | .186,427,356,791 | .274,674,603,951 | .466,506,350,946 | .529,498,469,498 |
| 75 15 | .057,092,732,040 | .099,087,616,445 | .187,259,917,825 | .277,809,127,809 | .467,588,923,985 | .533,574,241,036 |
| 75 30 | .057,476,057,004 | .100,556,616,194 | .188,083,442,689 | .280,965,069,161 | .468,654,926,785 | .537,659,387,566 |
| 75 45 | .057,856,511,789 | .102,041,099,410 | .188,897,765,450 | .284,142,428,805 | .469,704,278,166 | .541,753,764,128 |
| 76 00 | .058,233,988,142 | .103,541,146,788 | .189,702,719,822 | .287,341,206,185 | .470,736,898,215 | .545,857,225,061 |
| 76 15 | .058,608,376,736 | .105,056,838,544 | .190,498,139,182 | .290,561,399,380 | .471,752,708,294 | .549,969,624,014 |
| 76 30 | .058,979,567,168 | .106,588,254,401 | .191,283,856,576 | .293,803,005,097 | .472,751,631,047 | .554,090,813,953 |
| 76 45 | .059,347,447,956 | .108,135,473,582 | .192,059,704,730 | .297,066,018,678 | .473,733,590,400 | .558,220,647,175 |
| 77 00 | .059,711,906,548 | .109,698,574,797 | .192,825,516,070 | .300,350,434,081 | .474,698,511,575 | .562,358,975,320 |
| 77 15 | .060,072,829,313 | .111,277,636,234 | .193,581,122,724 | .303,656,243,890 | .475,646,321,088 | .566,505,649,380 |
| 77 30 | .060,430,101,548 | .112,872,735,548 | .194,326,356,537 | .306,983,439,303 | .476,576,946,759 | .570,660,519,712 |
| 77 45 | .060,783,607,480 | .114,483,949,848 | .195,061,049,087 | .310,332,010,125 | .477,490,317,719 | .574,823,436,049 |
| 78 00 | .061,133,230,263 | .116,111,355,688 | .195,785,031,688 | .313,701,944,771 | .478,386,364,411 | .578,994,247,509 |
| 78 15 | .061,478,851,983 | .117,755,029,061 | .196,498,135,411 | .317,093,230,263 | .479,265,018,597 | .583,172,802,611 |
| 78 30 | .061,820,353,659 | .119,415,045,376 | .197,200,191,091 | .320,505,852,220 | .480,126,213,363 | .587,358,949,285 |
| 78 45 | .062,157,615,240 | .121,091,479,458 | .197,891,029,339 | .323,939,794,856 | .480,969,883,128 | .591,552,534,881 |
| 79 00 | .062,490,515,616 | .122,784,405,537 | .198,570,480,556 | .327,395,040,981 | .481,795,963,642 | .595,753,406,183 |
| 79 15 | .062,818,932,607 | .124,493,897,230 | .199,238,374,942 | .330,871,571,995 | .482,604,391,996 | .599,961,409,421 |
| 79 30 | .063,142,742,977 | .126,220,027,535 | .199,894,542,510 | .334,369,367,884 | .483,395,106,624 | .604,176,390,281 |
| 79 45 | .063,461,822,428 | .127,962,868,820 | .200,538,813,099 | .337,888,407,214 | .484,168,047,312 | .608,398,193,918 |
| 80 00 | .063,776,045,609 | .129,722,492,816 | .201,171,016,383 | .341,428,667,137 | .484,923,155,196 | .612,626,664,966 |
| 80 15 | .064,085,286,107 | .131,498,970,594 | .201,790,981,887 | .344,990,123,378 | .485,660,372,773 | .616,861,647,554 |
| 80 30 | .064,389,416,463 | .133,292,372,567 | .202,398,538,995 | .348,572,750,239 | .486,379,643,900 | .621,102,985,313 |
| 80 45 | .064,688,308,164 | .135,102,768,474 | .202,993,516,966 | .352,176,520,595 | .487,080,913,801 | .625,350,521,391 |
| 81 00 | .064,981,831,648 | .136,930,227,369 | .203,575,744,944 | .355,801,405,886 | .487,764,129,074 | .629,604,098,464 |
| 81 15 | .065,269,856,312 | .138,774,817,610 | .204,145,051,969 | .359,447,376,123 | .488,429,237,687 | .633,863,558,746 |
| 81 30 | .065,552,250,504 | .140,636,606,848 | .204,701,266,994 | .363,114,399,879 | .489,076,188,991 | .638,128,744,007 |
| 81 45 | .065,828,881,536 | .142,515,662,022 | .205,244,218,892 | .366,802,444,292 | .489,704,933,717 | .642,399,495,577 |
| 82 00 | .066,099,615,682 | .144,412,049,337 | .205,773,736,471 | .370,511,475,055 | .490,315,423,985 | .646,675,654,364 |
| 82 15 | .066,364,318,181 | .146,325,834,263 | .206,289,648,485 | .374,241,456,424 | .490,907,613,302 | .650,957,060,863 |
| 82 30 | .066,622,853,240 | .148,257,081,520 | .206,791,783,651 | .377,992,351,207 | .491,481,456,573 | .655,243,555,172 |
| 82 45 | .066,875,084,042 | .150,205,855,066 | .207,279,970,654 | .381,764,120,769 | .492,036,910,095 | .659,534,976,999 |
| 83 00 | .067,120,872,739 | .152,172,218,091 | .207,754,038,164 | .385,556,725,026 | .492,573,931,569 | .663,831,165,678 |
| 83 15 | .067,360,080,465 | .154,156,233,003 | .208,213,814,848 | .389,370,122,442 | .493,092,480,100 | .668,131,960,176 |
| 83 30 | .067,592,567,336 | .156,157,961,414 | .208,659,129,383 | .393,204,270,035 | .493,592,516,197 | .672,437,199,121 |
| 83 45 | .067,818,192,451 | .158,177,464,135 | .209,089,810,465 | .397,059,123,368 | .494,074,001,780 | .676,746,720,788 |
| 84 00 | .068,036,813,902 | .160,214,801,165 | .209,505,686,826 | .400,934,636,549 | .494,536,900,183 | .681,060,363,133 |
| 84 15 | .068,248,288,767 | .162,270,031,676 | .209,906,587,244 | .404,830,762,232 | .494,981,176,156 | .685,377,963,798 |
| 84 30 | .068,452,473,130 | .164,343,214,002 | .210,292,340,555 | .408,747,451,618 | .495,406,795,862 | .689,699,360,123 |
| 84 45 | .068,649,222,068 | .166,434,405,635 | .210,662,775,670 | .412,684,654,445 | .495,813,726,892 | .694,024,389,160 |
| 85 00 | .068,838,389,667 | .168,543,663,209 | .211,017,721,577 | .416,642,318,999 | .496,201,938,253 | .698,352,887,680 |
| 85 15 | .069,019,829,017 | .170,671,042,489 | .211,357,007,370 | .420,620,392,103 | .496,571,400,385 | .702,684,692,197 |
| 85 30 | .069,193,392,228 | .172,816,598,361 | .211,680,462,245 | .424,618,819,123 | .496,922,085,149 | .707,019,638,967 |
| 85 45 | .069,358,930,420 | .174,980,384,826 | .211,987,915,524 | .428,637,543,965 | .497,253,965,840 | .711,357,564,010 |
| 86 00 | .069,516,293,743 | .177,162,454,980 | .212,279,196,663 | .432,676,509,070 | .497,567,017,186 | .715,698,303,118 |
| 86 15 | .069,665,331,366 | .179,362,861,014 | .212,554,135,262 | .436,735,655,426 | .497,861,215,344 | .720,041,691,868 |
| 86 30 | .069,805,891,493 | .181,581,654,193 | .212,812,561,083 | .440,814,922,555 | .498,136,537,910 | .724,387,565,636 |
| 86 45 | .069,937,821,361 | .183,818,884,853 | .213,054,304,063 | .444,914,248,518 | .498,392,963,919 | .728,735,759,611 |
| 87 00 | .070,060,967,256 | .186,074,602,390 | .213,279,194,320 | .449,033,569,915 | .498,630,473,842 | .733,086,108,800 |
| 87 15 | .070,175,174,496 | .188,348,855,242 | .213,487,062,173 | .453,172,821,887 | .498,849,049,592 | .737,438,448,053 |
| 87 30 | .070,280,287,462 | .190,641,690,889 | .213,677,738,148 | .457,331,938,112 | .499,048,674,523 | .741,792,612,061 |
| 87 45 | .070,376,149,585 | .192,953,155,835 | .213,851,052,997 | .461,510,850,807 | .499,229,333,434 | .746,148,435,381 |
| 88 00 | .070,462,603,361 | .195,283,295,599 | .214,006,837,705 | .465,709,490,732 | .499,391,012,565 | .750,505,752,441 |
| 88 15 | .070,539,490,346 | .197,632,154,709 | .214,144,923,511 | .469,927,787,184 | .499,533,699,606 | .754,864,397,559 |
| 88 30 | .070,606,651,176 | .199,999,776,684 | .214,265,141,908 | .474,165,668,003 | .499,657,383,689 | .759,224,204,947 |
| 88 45 | .070,663,925,559 | .202,386,204,029 | .214,367,324,666 | .478,423,059,569 | .499,762,055,396 | .763,585,008,731 |
| 89 00 | .070,711,152,289 | .204,791,478,224 | .214,451,303,843 | .482,699,886,809 | .499,847,706,754 | .767,946,642,962 |
| 89 15 | .070,748,169,249 | .207,215,639,711 | .214,516,911,795 | .486,996,073,185 | .499,914,331,244 | .772,308,941,626 |
| 89 30 | .070,774,813,417 | .209,658,727,888 | .214,563,981,187 | .491,311,540,713 | .499,961,923,789 | .776,671,738,658 |
| 89 45 | .070,790,920,871 | .212,120,781,094 | .214,592,345,014 | .495,646,209,949 | .499,990,480,766 | .781,034,867,958 |
| 90 00 | .070,796,326,795 | .214,601,836,603 | .214,601,836,603 | .500,000,000,000 | .500,000,000,000 | .785,398,163,397 |
| ° ′ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ |
| $\Phi_1$ | $\int(\Phi_1-\text{SIN }\Phi_1)\text{ COS }\Phi\, d\Phi_1$ | | $\int\text{VERS }\Phi_1\text{ COS }\Phi\, d\Phi_1$ | | $\int\text{SIN }\Phi_1\text{ COS }\Phi\, d\Phi_1$ | |

SHEET 4 OF 4

205-D-1872

Table H-6.—*Additional integrals for arch load constants* (sheet 1).

| $\Phi_1$ ° ′ | $\int \frac{\Phi_1^2}{2} \sin \Phi \, d\Phi_1$ | | $\int \Phi_1 \sin \Phi \, d\Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{vers } \Phi_1 \right) \sin \Phi \, d\Phi_1$ | |
|---|---|---|---|---|---|---|
| | + SIN $\Phi_0$ | + COS $\Phi_0$ | + SIN $\Phi_0$ | + COS $\Phi_0$ | + SIN $\Phi_0$ | + COS $\Phi_0$ |
| 0 15 | $.0^{7}$13,845,173 | $.0^{10}$45,308,387 | $.0^{5}$9,519,249 | $.0^{7}$27,690,452 | $.0^{13}$13,179,608 | $.0^{16}$47,922,617 |
| 0 30 | $.0^{6}$,110,759,489 | $.0^{9}$,724,929,588 | $.0^{4}$38,076,452 | $.0^{6}$,221,522,352 | $.0^{12}$,421,738,272 | $.0^{14}$3,067,021 |
| 0 45 | $.0^{6}$,373,802,600 | $.0^{8}$3,669,917 | $.0^{4}$85,669,979 | $.0^{6}$,747,630,820 | $.0^{11}$3,202,459 | $.0^{13}$34,934,789 |
| 1 00 | $.0^{6}$,886,015,181 | $.0^{7}$11,598,579 | $.0^{3}$,152,297,111 | $.0^{5}$1,772,138 | $.0^{10}$13,494,450 | $.0^{12}$,196,282,630 |
| 1 15 | $.0^{5}$1,730,409,4 | $.0^{7}$28,316,304 | $.0^{3}$,237,954,042 | $.0^{5}$3,461,148 | $.0^{10}$41,179,105 | $.0^{12}$,748,739,569 |
| 1 30 | $.0^{5}$2,989,959,6 | $.0^{7}$58,715,322 | $.0^{3}$,342,635,880 | $.0^{5}$5,980,739 | $.0^{9}$,102,458,616 | $.0^{11}$2,235,654 |
| 1 45 | $.0^{5}$4,747,592,6 | $.0^{6}$,108,774,391 | $.0^{3}$,466,336,644 | $.0^{5}$9,496,957 | $.0^{9}$,221,432,558 | $.0^{11}$5,637,275 |
| 2 00 | $.0^{5}$7,086,178 | $.0^{6}$,185,558,421 | $.0^{3}$,609,049,269 | $.0^{4}$14,175,811 | $.0^{9}$,431,672,074 | $.0^{10}$12,560,366 |
| 2 15 | $.0^{4}$10,088,520 | $.0^{6}$,297,218,021 | $.0^{3}$,770,765,601 | $.0^{4}$20,183,265 | $.0^{9}$,777,791,161 | $.0^{10}$25,462,225 |
| 2 30 | $.0^{4}$13,837,346 | $.0^{6}$,452,988,988 | $.0^{3}$,951,476,400 | $.0^{4}$27,685,233 | $.0^{8}$1,317,014,595 | $.0^{10}$47,909,069 |
| 2 45 | $.0^{4}$18,115,297 | $.0^{6}$,663,191,713 | $.0^{2}$1,151,171,342 | $.0^{4}$36,847,572 | $.0^{8}$2,120,742,000 | $.0^{10}$84,868,749 |
| 3 00 | $.0^{4}$23,904,922 | $.0^{6}$,939,230,532 | $.0^{2}$1,369,839,016 | $.0^{4}$47,836,076 | $.0^{8}$3,276,107,600 | $.0^{9}$,143,037,739 |
| 3 15 | $.0^{4}$30,388,664 | $.0^{5}$1,293,593 | $.0^{2}$1,607,466,924 | $.0^{4}$60,816,467 | $.0^{8}$4,887,535,143 | $.0^{9}$,231,202,361 |
| 3 30 | $.0^{4}$37,948,852 | $.0^{5}$1,739,849 | $.0^{2}$1,864,041,486 | $.0^{4}$75,954,396 | $.0^{8}$7,078,287,561 | $.0^{9}$,360,634,170 |
| 3 45 | $.0^{4}$46,667,692 | $.0^{5}$2,292,650 | $.0^{2}$2,139,548,037 | $.0^{4}$93,415,427 | $.0^{8}$9,992,010,849 | $.0^{9}$,545,519,454 |
| 4 00 | $.0^{4}$56,627,258 | $.0^{5}$2,967,729 | $.0^{2}$2,433,970,826 | $.0^{3}$,113,365,038 | $.0^{7}$13,794,272 | $.0^{9}$,803,422,780 |
| 4 15 | $.0^{4}$67,909,482 | $.0^{5}$3,781,897 | $.0^{2}$2,747,293,019 | $.0^{3}$,135,968,612 | $.0^{7}$18,674,088 | $.0^{8}$1,155,788 |
| 4 30 | $.0^{4}$80,596,143 | $.0^{5}$4,753,044 | $.0^{2}$3,079,496,702 | $.0^{3}$,161,391,431 | $.0^{7}$24,845,455 | $.0^{8}$1,628,452 |
| 4 45 | $.0^{4}$94,768,861 | $.0^{5}$5,900,139 | $.0^{2}$3,430,562,875 | $.0^{3}$,189,798,668 | $.0^{7}$32,548,856 | $.0^{8}$2,252,245 |
| 5 00 | $.0^{3}$,110,509,083 | $.0^{5}$7,243,225 | $.0^{2}$3,800,471,457 | $.0^{3}$,221,355,385 | $.0^{7}$42,052,776 | $.0^{8}$3,063,554 |
| 5 15 | $.0^{3}$,127,898,081 | $.0^{5}$8,803,421 | $.0^{2}$4,189,201,287 | $.0^{3}$,256,226,521 | $.0^{7}$53,655,198 | $.0^{8}$4,104,975 |
| 5 30 | $.0^{3}$,147,016,931 | $.0^{4}$10,602,921 | $.0^{2}$4,596,730,122 | $.0^{3}$,294,576,892 | $.0^{7}$67,685,095 | $.0^{8}$5,425,971 |
| 5 45 | $.0^{3}$,167,946,517 | $.0^{4}$12,664,988 | $.0^{2}$5,023,034,642 | $.0^{3}$,336,571,178 | $.0^{7}$84,503,907 | $.0^{8}$7,083,576 |
| 6 00 | $.0^{3}$,190,767,510 | $.0^{4}$15,013,958 | $.0^{2}$5,468,090,444 | $.0^{3}$,382,373,924 | $.0^{6}$,104,507,016 | $.0^{8}$9,143,126 |
| 6 15 | $.0^{3}$,215,560,368 | $.0^{4}$17,675,237 | $.0^{2}$5,931,872,050 | $.0^{3}$,432,149,525 | $.0^{6}$,128,125,204 | $.0^{7}$11,679,023 |
| 6 30 | $.0^{3}$,242,405,319 | $.0^{4}$20,675,295 | $.0^{2}$6,414,352,904 | $.0^{3}$,486,062,229 | $.0^{6}$,155,826,102 | $.0^{7}$14,775,536 |
| 6 45 | $.0^{3}$,271,382,355 | $.0^{4}$24,041,672 | $.0^{2}$6,915,505,369 | $.0^{3}$,544,276,125 | $.0^{6}$,188,115.628 | $.0^{7}$18,527,635 |
| 7 00 | $.0^{3}$,302,571,225 | $.0^{4}$27,802,970 | $.0^{2}$7,435,300,739 | $.0^{3}$,606,955,136 | $.0^{6}$,225,539,414 | $.0^{7}$23,041,850 |
| 7 15 | $.0^{3}$,336,051,422 | $.0^{4}$31,988,852 | $.0^{2}$7,973,709,229 | $.0^{3}$.674,263,018 | $.0^{6}$,268,684,218 | $.0^{7}$28,437,173 |
| 7 30 | $.0^{3}$,371,902,175 | $.0^{4}$36,630,045 | $.0^{2}$8,530,699,982 | $.0^{3}$,746,363,347 | $.0^{6}$,318,179,323 | $.0^{7}$34,845,985 |
| 7 45 | $.0^{3}$,410,202,438 | $.0^{4}$41,758,330 | $.0^{2}$9,106,241,065 | $.0^{3}$,823,419,521 | $.0^{6}$,374,697,930 | $.0^{7}$42,415,020 |
| 8 00 | $.0^{3}$,451,030,885 | $.0^{4}$47,406,549 | $.0^{2}$9,700,299,478 | $.0^{3}$,905,594,744 | $.0^{6}$,438,958,526 | $.0^{7}$51,306,359 |
| 8 15 | $.0^{3}$,494,465,898 | $.0^{4}$53,608,596 | .010,312,841,145 | $.0^{3}$,993,052,028 | $.0^{6}$,511,726,245 | $.0^{7}$61,698,456,5 |
| 8 30 | $.0^{3}$,540,585,553 | $.0^{4}$60,399,416 | .010,943,830,924 | .001,085,954,184 | $.0^{6}$,593,814,215 | $.0^{7}$73,787,194 |
| 8 45 | $.0^{3}$,589,467,623 | $.0^{4}$67,815,006 | .011,593,232,603 | .001,184,463,811 | $.0^{6}$,686,084,885 | $.0^{7}$87,786,982 |
| 9 00 | $.0^{3}$,641,189,558 | $.0^{4}$75,892,411 | .012,261,008,902 | .001,288,743,298 | $.0^{6}$,789,451,340 | $.0^{6}$,103,931,866 |
| 9 15 | $.0^{3}$,695,828,475 | $.0^{4}$84,669,717 | .012,947,121,475 | .001,398,954,814 | $.0^{6}$,904,878,599 | $.0^{6}$,122,476,692 |
| 9 30 | $.0^{3}$,753,461,161 | $.0^{4}$94,186,061 | .013,651,530,913 | .001,515,260,299 | $.0^{5}$1,033,385 | $.0^{6}$,143,698,283 |
| 9 45 | $.0^{3}$,814,164,049 | $.0^{3}$,104,481,612 | .014,374,196,739 | .001,637,821,462 | $.0^{5}$1,176,043 | $.0^{6}$,167,896,657 |
| 10 00 | $.0^{3}$,878,013,217 | $.0^{3}$,115,597,581 | .015,115,077,416 | .001,766,799,775 | $.0^{5}$1,333,981 | $.0^{6}$,195,396 273 |
| 10 15 | $.0^{3}$,945,084,379 | $.0^{3}$,127,578,213 | .015,874,130,346 | .001,902,356,462 | $.0^{5}$1,508,384 | $.0^{6}$,226,547,305 |
| 10 30 | $.0^{2}$1,015,452,874 | $.0^{3}$,140,460,787 | .016,651,311,870 | .002,044,652,497 | $.0^{5}$1,700,498 | $.0^{6}$,261,726,951 |
| 10 45 | $.0^{2}$1,089,193,655 | $.0^{3}$,154,295,611 | .017,446,577,272 | .002,193,848,598 | $.0^{5}$1,911,623 | $.0^{6}$,301,340,768 |
| 11 00 | $.0^{2}$1,166,381,286 | $.0^{3}$,169,126,018 | .018,259,880,777 | .002,350,105,217 | $.0^{5}$2,143,124 | $.0^{6}$,345,824,041 |
| 11 15 | $.0^{2}$1,247,089,925 | $.0^{3}$,184,998,367 | .019,091,175,555 | .002,513,582,537 | $.0^{5}$2,396,425 | $.0^{6}$,395,643,177 |
| 11 30 | $.0^{2}$1,331,393,320 | $.0^{3}$,201,960,039 | .019,940,413,723 | .002,684,440,469 | $.0^{5}$2,673,015 | $.0^{6}$,451,297,131 |
| 11 45 | $.0^{2}$1,419,364,801 | $.0^{3}$,220,059,431 | .020,807,546,345 | .002,862,838,636 | $.0^{5}$2,974,447 | $.0^{6}$,513,318,864 |
| 12 00 | $.0^{2}$1,511,077,263 | $.0^{3}$,239,345,954 | .021,692,523,432 | .003,048,936,379 | $.0^{5}$3,302,334 | $.0^{6}$,582,278,823 |
| 12 15 | $.0^{2}$1,606,603,171 | $.0^{3}$,259,870,034 | .022,595,293,948 | .003,242,892,738 | $.0^{5}$3,658,363 | $.0^{6}$,658,776,457 |
| 12 30 | $.0^{2}$1,706,014,536 | $.0^{3}$,281,683,101 | .023,515,805,807 | .003,444,866,459 | $.0^{5}$4,044,283 | $.0^{6}$,743,461,760 |
| 12 45 | $.0^{2}$1,809,382,916 | $.0^{3}$,304,837,596 | .024,454,005,880 | .003,655,015,979 | $.0^{5}$4,461,911 | $.0^{6}$,837,016,833 |
| 13 00 | $.0^{2}$1,916,779,404 | $.0^{3}$,329,386,957 | .025,409,839,989 | .003,873,499,420 | $.0^{5}$4,913,137 | $.0^{6}$,940,167,491 |
| 13 15 | $.0^{2}$2,028,274,617 | $.0^{3}$,355,385,622 | .026,383,252,917 | .004,100,474,590 | $.0^{5}$5,399,917 | $.0^{5}$1,053,683 |
| 13 30 | $.0^{2}$2,143,938,692 | $.0^{3}$,382,889,027 | .027,374,188,406 | .004,336,098,969 | $.0^{5}$5,924,280 | $.0^{5}$1,178,377 |
| 13 45 | $.0^{2}$2,263,841,271 | $.0^{3}$,411,953,592 | .028,382,589,154 | .004,580,529,707 | $.0^{5}$6,488,328 | $.0^{5}$1,315,111 |
| 14 00 | $.0^{2}$2,388,051,496 | $.0^{3}$,442,636,732 | .029,408,396,828 | .004,833,923,619 | $.0^{5}$7,094,232 | $.0^{5}$1,464,793 |
| 14 15 | $.0^{2}$2,516,638,005 | $.0^{3}$,474,996,844 | .030,451,552,056 | .005,096,437,172 | $.0^{5}$7,744,245 | $.0^{5}$1,628,386 |
| 14 30 | $.0^{2}$2,649,668,909 | $.0^{3}$,509,093,301 | .031,511,994,430 | .005,368,226,489 | $.0^{5}$8,440,686 | $.0^{5}$1,806,894 |
| 14 45 | $.0^{2}$2,787,211,796 | $.0^{3}$,544,986,461 | .032,589,662,517 | .005,649,447,338 | $.0^{5}$9,185,951 | $.0^{5}$2,001,390 |
| 15 00 | $.0^{2}$2,929,333,721 | $.0^{3}$,582,737,646 | .033,684,493,848 | .005,940,255,121 | $.0^{5}$9,982,518 | $.0^{5}$2,212,989 |
| 15 15 | $.0^{2}$3,076,101,191 | $.0^{3}$,622,409,150 | .034,796,424,928 | .006,240,804,878 | $.0^{4}$10,832,938 | $.0^{5}$2,442,869 |
| 15 30 | $.0^{2}$3,227,580,161 | $.0^{3}$,664,064,236 | .035,925,391,238 | .006,551,251,273 | $.0^{4}$11,739,840 | $.0^{5}$2,692,269 |
| 15 45 | $.0^{2}$3,383,836,022 | $.0^{3}$,707,767,119 | .037,071,327,232 | .006,871,748,593 | $.0^{4}$12,705,930 | $.0^{5}$2,962,484 |
| 16 00 | $.0^{2}$3,544,933,598 | $.0^{3}$,753,582,978 | .038,234,166,346 | .007,202,450,738 | $.0^{4}$13,733,999 | $.0^{5}$3,254,877 |
| 16 15 | $.0^{2}$3,710,937,132 | $.0^{3}$,801,577,943 | .039,413,840,995 | .007,543,511,218 | $.0^{4}$14,826,912 | $.0^{5}$3,570,876 |
| 16 30 | $.0^{2}$3,881,910,278 | $.0^{3}$,851,819,085 | .040,610,282,574 | .007,895,083,146 | $.0^{4}$15,987,617 | $.0^{5}$3,911,967 |
| 16 45 | $.0^{2}$4,057,916,098 | $.0^{3}$,904,374,429 | .041,823,421,468 | .008,257,319,231 | $.0^{4}$17,219,146 | $.0^{5}$4,279,717 |
| 17 00 | $.0^{2}$4,239,017,044 | $.0^{3}$,959,312,930 | .043,053,187,043 | .008,630,371,774 | $.0^{4}$18,524,608 | $.0^{5}$4,675,754 |
| 17 15 | $.0^{2}$4,425,274,957 | $.0^{2}$1,016,704,485 | .044,299,507,661 | .009,014,392,662 | $.0^{4}$19,907,197 | $.0^{5}$5,101,787 |
| 17 30 | $.0^{2}$4,616,751,058 | $.0^{2}$1,076,619,917 | .045,562,310,669 | .009,409,533,358 | $.0^{4}$21,370,191 | $.0^{5}$5,559,593 |
| 17 45 | $.0^{2}$4,813,505,934 | $.0^{2}$1,139,130,977 | .046,841,522,412 | .009,815,944,903 | $.0^{4}$22,916,948 | $.0^{5}$6,051,031 |
| 18 00 | $.0^{2}$5,015,599,537 | $.0^{2}$1,204,310,337 | .048,137,068,231 | .010,233,777,901 | $.0^{4}$24,550,915 | $.0^{5}$6,578,038 |
| 18 15 | $.0^{2}$5,223,091,170 | $.0^{2}$1,272,231,582 | .049,448,872,464 | .010,663,182,520 | $.0^{4}$26,275,618 | $.0^{5}$7,142,630 |
| 18 30 | $.0^{2}$5,436,039,480 | $.0^{2}$1,342,969,212 | .050,776,858,450 | .011,104,308,484 | $.0^{4}$28,094,672 | $.0^{5}$7,746,906 |
| 18 45 | $.0^{2}$5,654,502,452 | $.0^{7}$1,416,598,634 | .052,120,948,534 | .011,557,305,065 | $.0^{4}$30,011,776 | $.0^{5}$8,393,056 |
| 19 00 | $.0^{2}$5,878,537,396 | $.0^{2}$1,493,196,152 | .053,481,064,062 | .012,022,321,081 | $.0^{4}$32,030,710 | $.0^{5}$9,083,349 |
| 19 15 | $.0^{2}$6,108,200,947 | $.0^{2}$1,572,838,978 | .054,857,125,396 | .012,499,504,886 | $.0^{4}$34,155,348 | $.0^{5}$9,820,158 |
| 19 30 | $.0^{2}$6,343,549,048 | $.0^{2}$1,655,605,201 | .056,249,051,900 | .012,989,004,367 | $.0^{4}$36,389,646 | $.0^{4}$10,605,929 |
| 19 45 | $.0^{2}$6,584,636,944 | $.0^{2}$1,741,573,806 | .057,656,761,958 | .013,490,966,939 | $.0^{4}$38,737,645 | $.0^{4}$11,443,215 |
| 20 00 | $.0^{2}$6,831,519,177 | $.0^{2}$1,830,824,663 | .059,080,172,970 | .014,005,539,538 | $.0^{4}$41,203,472 | $.0^{4}$12,334,669 |
| 20 15 | $.0^{2}$7,084,249,579 | $.0^{2}$1,923,438,506 | .060,519,201 349 | .014,532,868,610 | $.0^{4}$43,791,349 | $.0^{4}$13,283,028 |
| 20 30 | $.0^{2}$7,342,881,254 | $.0^{2}$2,019,496,954 | .061,973,762,537 | .015,073,100,117 | $.0^{4}$46,505,572 | $.0^{4}$14,291,146 |
| 20 45 | $.0^{2}$7,607,466,583 | $.0^{2}$2,119,082,483 | .063,443,770,996 | .015,626,379,521 | $.0^{4}$49,350,534 | $.0^{4}$15,361,972 |
| 21 00 | $.0^{2}$7,878,057,208 | $.0^{2}$2,222,278,433 | .064,929,140,214 | .016,192,851,779 | $.0^{4}$52,330,712 | $.0^{4}$16,498,561 |
| 21 15 | $.0^{2}$8,154,704,025 | $.0^{2}$2,329,168,999 | .066,429,782,712 | .016,772,661,347 | $.0^{4}$55,450,670 | $.0^{4}$17,704,080 |
| 21 30 | $.0^{2}$8,437,457,177 | $.0^{2}$2,439,839,222 | .067,945,610,039 | .017,365,952,161 | $.0^{4}$58,715,058 | $.0^{4}$18,981,799 |
| 21 45 | $.0^{2}$8,726,366,048 | $.0^{2}$2,554,374,994 | .069,476,532,785 | .017,972,867,640 | $.0^{4}$62,128,616 | $.0^{4}$20,335,113 |
| 22 00 | $.0^{2}$9,021,479,251 | $.0^{2}$2,672,863,038 | .071,022,460,572 | .018,593,550,678 | $.0^{4}$65,696,169 | $.0^{4}$21,767,520 |
| 22 15 | $.0^{2}$9,322,844,624 | $.0^{2}$2,795,390,915 | .072,583,302,068 | .019,228,143,638 | $.0^{4}$69,422,631 | $.0^{4}$23,282,644 |
| 22 30 | $.0^{2}$9,630,509,218 | $.0^{2}$2,922,047,012 | .074,158,964,982 | .019,876,788,347 | $.0^{4}$73,312,999 | $.0^{4}$24,884,226 |
| ° ′ $\Phi_1$ | + COS $\Phi_0$ | - SIN $\Phi_0$ | + COS $\Phi_0$ | - SIN $\Phi_0$ | + COS $\Phi_0$ | - SIN $\Phi_0$ |
| | $\int \frac{\Phi_1^2}{2} \cos \Phi \, d\Phi_1$ | | $\int \Phi_1 \cos \Phi \, d\Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{vers } \Phi_1 \right) \cos \Phi \, d\Phi_1$ | |

SHEET 1 OF 4

205-D-1873

Table H-6.–*Additional integrals for arch load constants* (sheet 2).

| $\Phi_1$ | | $\int \frac{\Phi_1^2}{2}\ \mathrm{SIN}\ \Phi\, d\Phi_1$ | | $\int \Phi_1\ \mathrm{SIN}\ \Phi\, d\Phi_1$ | | $\int \left(\frac{\Phi_1^2}{2} - \mathrm{VERS}\ \Phi_1\right) \mathrm{SIN}\ \Phi\, d\Phi_1$ | |
|---|---|---|---|---|---|---|---|
| ° | ′ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ |
| 22 | 45 | .009,944,519,294 | .003,052,920,535 | .075,749,356,071 | .020,539,626,090 | .0$^4$77,372,360 | .0$^4$26,576,130 |
| 23 | 00 | .010,264,920,315 | .003,188,101,508 | .077,354,381,142 | .021,216,797,602 | .0$^4$81,605,890 | .0$^4$28,362,345 |
| 23 | 15 | .010,591,756,930 | .003,327,680,763 | .078,973,945,054 | .021,908,443,069 | .0$^4$86,018,843 | .0$^4$30,246,988 |
| 23 | 30 | .010,925,072,979 | .003,471,749,935 | .080,607,951,722 | .022,614,702,115 | .0$^4$90,616,568 | .0$^4$32,234,304 |
| 23 | 45 | .011,264,911,477 | .003,620,401,460 | .082,256,304,121 | .023,335,713,800 | .0$^4$95,404,495 | .0$^4$34,328,675 |
| 24 | 00 | .011,611,314,608 | .003,773,728,560 | .083,918,904,288 | .024,071,616,615 | .0$^3$,100,388,141 | .0$^4$36,534,613 |
| 24 | 15 | .011,964,323,717 | .003,931,825,244 | .085,595,653,321 | .024,822,548,476 | .0$^3$,105,573,106 | .0$^4$38,856,767 |
| 24 | 30 | .012,323,979,307 | .004,094,786,306 | .087,286,451,395 | .025,588,646,716 | .0$^3$,110,965,076 | .0$^4$41,299,935 |
| 24 | 45 | .012,690,321,024 | .004,262,707,304 | .088,991,197,748 | .026,370,048,083 | .0$^3$,116,569,821 | .0$^4$43,869,046 |
| 25 | 00 | .013,063,387,656 | .004,435,684,566 | .090,709,790,698 | .027,166,888,734 | .0$^3$,122,393,195 | .0$^4$46,569,181 |
| 25 | 15 | .013,443,217,122 | .004,613,815,183 | .092,442,127,637 | .027,979,304,226 | .0$^3$,128,441,132 | .0$^4$49,405,567 |
| 25 | 30 | .013,829,846,471 | .004,797,196,996 | .094,188,105,044 | .028,807,429,514 | .0$^3$,134,719,656 | .0$^4$52,383,583 |
| 25 | 45 | .014,223,311,862 | .004,985,928,596 | .095,947,618,478 | .029,651,398,945 | .0$^3$,141,234,865 | .0$^4$55,508,759 |
| 26 | 00 | .014,623,648,571 | .005,180,109,309 | .097,720,562,586 | .030,511,346,253 | .0$^3$,147,992,943 | .0$^4$58,786,777 |
| 26 | 15 | .015,030,890,972 | .005,379,839,205 | .099,506,831,110 | .031,387,404,552 | .0$^3$,155,000,150 | .0$^4$62,223,486 |
| 26 | 30 | .015,445,072,538 | .005,585,219,074 | .101,306,316,882 | .032,279,706,332 | .0$^3$,162,262,829 | .0$^4$65,824,888 |
| 26 | 45 | .015,866,225,834 | .005,796,350,425 | .103,118,911,835 | .033,188,383,451 | .0$^3$,169,787,405 | .0$^4$69,597,148 |
| 27 | 00 | .016,294,382,499 | .006,013,335,489 | .104,944,507,005 | .034,113,567,137 | .0$^3$,177,580,372 | .0$^4$73,546,604 |
| 27 | 15 | .016,729,573,256 | .006,236,277,200 | .106,782,992,531 | .035,055,387,970 | .0$^3$,185,648,312 | .0$^4$77,679,758 |
| 27 | 30 | .017,171,827,890 | .006,465,279,188 | .108,634,257,658 | .036,013,975,890 | .0$^3$,193,997,876 | .0$^4$82,003,278 |
| 27 | 45 | .017,621,175,248 | .006,700,445,786 | .110,498,190,750 | .036,989,460,185 | .0$^3$,202,635,792 | .0$^4$86,524,018 |
| 28 | 00 | .018,077,643,235 | .006,941,882,005 | .112,374,679,282 | .037,981,969,484 | .0$^3$,211,568,867 | .0$^4$91,248,996 |
| 28 | 15 | .018,541,258,798 | .007,189,693,539 | .114,263,609,848 | .038,991,631,756 | .0$^3$,220,803,974 | .0$^4$96,185,416 |
| 28 | 30 | .019,012,047,929 | .007,443,986,755 | .116,164,868,168 | .040,018,574,304 | .0$^3$,230,348,065 | .0$^3$,101,340,663 |
| 26 | 45 | .019,490,035,651 | .007,704,868,686 | .118,078,339,087 | .041,062,923,756 | .0$^3$,240,208,159 | .0$^3$,106,722,307 |
| 29 | 00 | .019,975,246,016 | .007,972,447,017 | .120,003,906,579 | .042,124,806,067 | .0$^3$,250,391,348 | .0$^3$,112,338,098 |
| 29 | 15 | .020,467,702,095 | .008,246,830,094 | .121,941,453,758 | .043,204,346,505 | .0$^3$,260,904,791 | .0$^3$,118,195,988 |
| 29 | 30 | .020,967,425,976 | .008,528,126,896 | .123,890,862,866 | .044,301,669,653 | .0$^3$,271,755,717 | .0$^3$,124,304,108 |
| 29 | 45 | .021,474,438,749 | .008,816,447,048 | .125,852,015,297 | .045,416,899,402 | .0$^3$,282,951,418 | .0$^3$,130,670,797 |
| 30 | 00 | .021,988,760,510 | .009,111,900,796 | .127,824,791,583 | .046,550,158,942 | .0$^3$,294,499,255 | .0$^3$,137,304,580 |
| 30 | 15 | .022,510,410,347 | .009,414,599,013 | .129,809,071,410 | .047,701,570,763 | .0$^3$,306,406,650 | .0$^3$,144,214,191 |
| 30 | 30 | .023,039,406,338 | .009,724,653,183 | .131,804,733,614 | .048,871,256,644 | .0$^3$,318,681,091 | .0$^3$,151,408,564 |
| 30 | 45 | .023,575,765,541 | .010,042,175,396 | .133,811,656,189 | .050,059,337,654 | .0$^3$,331,330,123 | .0$^3$,158,896,832 |
| 31 | 00 | .024,119,503,989 | .010,367,278,343 | .135,829,716,292 | .051,265,934,142 | .0$^3$,344,361,353 | .0$^3$,166,688,348 |
| 31 | 15 | .024,670,636,684 | .010,700,075,306 | .137,858,790,242 | .052,491,165,735 | .0$^3$,357,782,442 | .0$^3$,174,792,670 |
| 31 | 30 | .025,229,177,595 | .011,040,680,148 | .139,898,753,528 | .053,735,151,330 | .0$^3$,371,601,115 | .0$^3$,183,219,567 |
| 31 | 45 | .025,795,139,642 | .011,389,207,314 | .141,949,480,814 | .054,998,009,093 | .0$^3$,385,825,145 | .0$^3$,191,979,033 |
| 32 | 00 | .026,368,534,700 | .011,745,771,807 | .144,010,845,935 | .056,279,856,451 | .0$^3$,400,462,361 | .0$^3$,201,081,266 |
| 32 | 15 | .026,949,373,586 | .012,110,489,203 | .146,082,721,917 | .057,580,810,087 | .0$^3$,415,520,642 | .0$^3$,210,536,705 |
| 32 | 30 | .027,537,666,058 | .012,483,475,618 | .148,164,980,961 | .058,900,985,938 | .0$^3$,431,007,919 | .0$^3$,220,355,996 |
| 32 | 45 | .028,133,420,805 | .012,864,847,720 | .150,257,494,462 | .060,240,499,186 | .0$^3$,446,932,170 | .0$^3$,230,550,020 |
| 33 | 00 | .028,736,645,444 | .013,254,722,712 | .152,360,133,010 | .061,599,464,258 | .0$^3$,463,301,419 | .0$^3$,241,129,888 |
| 33 | 15 | .029,347,346,512 | .013,653,218,325 | .154,472,766,388 | .062,977,994,816 | .0$^3$,480,123,732 | .0$^3$,252,106,942 |
| 33 | 30 | .029,965,529,463 | .014,060,452,810 | .156,595,263,584 | .064,376,203,756 | .0$^3$,497,407,223 | .0$^3$,263,492,755 |
| 33 | 45 | .030,591,198,658 | .014,476,544,931 | .158,727,492,792 | .065,794,203,202 | .0$^3$,515,160,040 | .0$^3$,275,299,143 |
| 34 | 00 | .031,224,357,363 | .014,901,613,956 | .160,869,321,413 | .067,232,104,501 | .0$^3$,533,390,373 | .0$^3$,287,538,157 |
| 34 | 15 | .031,865,007,743 | .015,335,779,648 | .163,020,616,066 | .068,690,018,217 | .0$^3$,552,106,447 | .0$^3$,300,222,094 |
| 34 | 30 | .032,513 150,856 | .015,779,162,261 | .165,181,242,589 | .070,168,054,129 | .0$^3$,571,316,524 | .0$^3$,313,363,497 |
| 34 | 45 | .033,168,786,645 | .016,231,882,524 | .167,351,066,039 | .071,666,321,224 | .0$^3$,591,028,895 | .0$^3$,326,975,151 |
| 35 | 00 | .033,831,913,935 | .016,694,061,642 | .169,529,950,705 | .073,184,927,696 | .0$^3$,611,251,879 | .0$^3$,341,070,100 |
| 35 | 15 | .034,502,530,431 | .017,165,821,277 | .171,717,760,106 | .074,723,980,934 | .0$^3$,631,993,831 | .0$^3$,355,661,630 |
| 35 | 30 | .035,180,632,703 | .017,647,283,550 | .173,914,356,995 | .076,283,587,526 | .0$^3$,653,263,121 | .0$^3$,370,763,292 |
| 35 | 45 | .035,866,216,193 | .018,138,571,026 | .176,119,603,371 | .077,863,853,247 | .0$^3$,675,068,151 | .0$^3$,386,388,889 |
| 36 | 00 | .036,559,275,198 | .018,639,806,708 | .178,333,360,473 | .079,464,883,060 | .0$^3$,697,417,339 | .0$^3$,402,552,489 |
| 36 | 15 | .037,259,602,872 | .019,151,114,027 | .180,555,488,792 | .081,086,781,111 | .0$^3$,720,319,119 | .0$^3$,419,268,418 |
| 36 | 30 | .037,967,791,224 | .019,672,616,836 | .182,785,848,073 | .082,729,650,714 | .0$^3$,743,781,953 | .0$^3$,436,551,272 |
| 36 | 45 | .038,683,231,099 | .020,204,439,397 | .185,024,297,321 | .084,393,594,367 | .0$^3$,767,814,299 | .0$^3$,454,415,912 |
| 37 | 00 | .039,406,112,190 | .020,746,706,376 | .187,270,694,800 | .086,078,713,726 | .0$^3$,792,424,641 | .0$^3$,472,877,469 |
| 37 | 15 | .040,136,423,020 | .021,299,542,836 | .189,524,898,047 | .087,785,109,616 | .0$^3$,817,621,463 | .0$^3$,491,951,351 |
| 37 | 30 | .040,874,150,947 | .021,863,074,219 | .191,786,763,867 | .089,512,882,017 | .0$^3$,843,413,259 | .0$^3$,511,653,235 |
| 37 | 45 | .041,619,282,149 | .022,437,428,352 | .194,056,148,345 | .091,262,130,065 | .0$^3$,869,808,523 | .0$^3$,531,999,082 |
| 38 | 00 | .042,371,801,629 | .023,022,725,423 | .196,332,906,848 | .093,032,952,046 | .0$^3$,896,815,751 | .0$^3$,553,005,130 |
| 38 | 15 | .043,131,693,203 | .023,619,097,982 | .198,616,894,026 | .094,825,445,391 | .0$^3$,924,443,437 | .0$^3$,574,687,899 |
| 38 | 30 | .043,898,939,501 | .024,226,670,927 | .200,907,963,823 | .096,639,706,672 | .0$^3$,952,700,068 | .0$^3$,597,064,193 |
| 38 | 45 | .044,673,521,960 | .024,845,571,501 | .203,205,969,479 | .098,475,831,595 | .0$^3$,981,594,130 | .0$^3$,620,151,109 |
| 39 | 00 | .045,455,420,814 | .025,475,927,277 | .205,510,763,534 | .100,333,915,006 | .001,011,134,087 | .0$^3$,643,966,030 |
| 39 | 15 | .046,244,615,101 | .026,117,866,149 | .207,822,197,832 | .102,214,050,871 | .001,041,328,397 | .0$^3$,668,526,627 |
| 39 | 30 | .047,041,082,648 | .026,771,516,327 | .210,140,123,529 | .104,116,332,285 | .001,072,185,501 | .0$^3$,693,850,871 |
| 39 | 45 | .047,844,800,075 | .027,437,006,326 | .212,464,391,095 | .106,040,851,460 | .001,103,713,819 | .0$^3$,719,957,026 |
| 40 | 00 | .048,655,742,782 | .028,114,464,958 | .214,794,850,322 | .107,987,699,725 | .001,135,921,747 | .0$^3$,746,863,661 |
| 40 | 15 | .049,473,884,953 | .028,804,021,318 | .217,131,350,323 | .109,956,967,520 | .001,168,817,658 | .0$^3$,774,589,636 |
| 40 | 30 | .050,299,199,545 | .029,505,804,780 | .219,473,739,543 | .111,948,744,393 | .001,202,409,893 | .0$^3$,803,154,120 |
| 40 | 45 | .051,131,658,292 | .030,219,944,988 | .221,821,865,762 | .113,963,118,993 | .001,236,706,764 | .0$^3$,832,576,590 |
| 41 | 00 | .051,971,231,691 | .030,946,571,844 | .224,175,576,100 | .116,000,179,070 | .001,271,716,545 | .0$^3$,862,876,827 |
| 41 | 15 | .052,817,889,007 | .031,685,815,496 | .226,534,717,017 | .118,060,011,467 | .001,307,447,474 | .0$^3$,894,074,920 |
| 41 | 30 | .053,671,598,263 | .032,437,806,338 | .228,899,134,330 | .120,142,702,120 | .001,343,907,746 | .0$^3$,926,191,276 |
| 41 | 45 | .054,532,326,239 | .033,202,674,989 | .231,268,673,203 | .122,248,336,050 | .001,381,105,511 | .0$^3$,959,246,609 |
| 42 | 00 | .055,400,038,466 | .033,980,552,292 | .233,643,178,163 | .124,376,997,363 | .001,419,048,867 | .0$^3$,993,261,952 |
| 42 | 15 | .056,274,699,230 | .034,771,569,305 | .236,022,493,106 | .126,528,769,240 | .001,457,745,870 | .001,028,258,662 |
| 42 | 30 | .057,156,271,555 | .035,575,857,283 | .238,406,461,288 | .128,703,733,940 | .001,497,204,511 | .001,064,258,406 |
| 42 | 45 | .058,044,717,208 | .036,393,547,675 | .240,794,925,347 | .130,901,972,794 | .001,537,432,722 | .001,101,283,179 |
| 43 | 00 | .058,939,996,697 | .037,224,772,115 | .243,187,727,299 | .133,123,566,197 | .001,578,438,378 | .001,139,355,298 |
| 43 | 15 | .059,842,069,263 | .038,069,662,411 | .245,584,708,545 | .135,368,593,610 | .001,620,229,286 | .001,178,497,409 |
| 43 | 30 | .060,750,892,877 | .038,928,350,533 | .247,985,709,877 | .137,637,133,552 | .001,662,813,180 | .001,218,732,484 |
| 43 | 45 | .061,666,424,241 | .039,800,968,606 | .250,390,571,481 | .139,929,263,596 | .001,706,197,728 | .001,260,083,824 |
| 44 | 00 | .062,588,618,777 | .040,687,648,902 | .252,799,132,945 | .142,245,060,371 | .001,750,390,511 | .001,302,575,065 |
| 44 | 15 | .063,517,430,631 | .041,588,523,824 | .255,211,233,261 | .144,584,599,552 | .001,795,399,037 | .001,346,230,172 |
| 44 | 30 | .064,452,812,669 | .042,503,725,901 | .257,626,710,833 | .146,947,955,857 | .001,841,230,727 | .001,391,073,446 |
| 44 | 45 | .065,394,716,469 | .043,433,387,783 | .260,045,403,484 | .149,335,203,046 | .001,887,892,912 | .001,437,129,531 |
| 45 | 00 | .066,343,092,322 | .044,377,642,218 | .262,467,148,457 | .151,746,413,917 | .001,935,392,834 | .001,484,423,405 |
| ° | ′ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ |
| $\Phi_1$ | | $\int \frac{\Phi_1^2}{2}\ \mathrm{COS}\ \Phi\, d\Phi_1$ | | $\int \Phi_1\ \mathrm{COS}\ \Phi\, d\Phi_1$ | | $\int \left(\frac{\Phi_1^2}{2} - \mathrm{VERS}\ \Phi_1\right) \mathrm{COS}\ \Phi\, d\Phi_1$ | |

SHEET 2 OF 4

205-D-1874

Table H-6.—*Additional integrals for arch load constants* (sheet 3).

| $\Phi_1$ | $\int \frac{\Phi_1^2}{2}$ SIN $\Phi \, d \Phi_1$ | | $\int \Phi_1$ SIN $\Phi \, d \Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{VERS } \Phi_1 \right)$ SIN $\Phi \, d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ | +SIN $\Phi_0$ | +COS $\Phi_0$ |
| 45 15 | .067,297,889,228 | .045,336,622,054 | .264,891,782,418 | .154,181,660,300 | .001,983,737,634 | .001,532,980,384 |
| 45 30 | .068,259,054,893 | .046,310,460,225 | .267,319,141,472 | .156,641,013,055 | .002,032,934,357 | .001,582,826,134 |
| 45 45 | .069,226,535,730 | .047,299,289,739 | .269,749,061,159 | .159,124,542,069 | .002,082,989,943 | .001,633,986,658 |
| 46 00 | .070,200,276,846 | .048,303,243,672 | .272,181,376,460 | .161,632,316,251 | .002,133,911,220 | .001,686,488,307 |
| 46 15 | .071,180,222,051 | .049,322,455,156 | .274,615,921,808 | .164,164,403,529 | .002,185,704,910 | .001,740,357,780 |
| 46 30 | .072,166,313,852 | .050,357,057,368 | .277,052,531,088 | .166,720,870,845 | .002,238,377,617 | .001,795,622,123 |
| 46 45 | .073,158,493,441 | .051,407,183,522 | .279,491,037,644 | .169,301,784,159 | .002,291,935,818 | .001,852,308,731 |
| 47 00 | .074,156,700,707 | .052,472,966,859 | .281,931,274,287 | .171,907,208,434 | .002,346,385,871 | .001,910,445,357 |
| 47 15 | .075,160,874,227 | .053,554,540,636 | .284,373,073,298 | .174,537,207,639 | .002,401,734,008 | .001,970,060,101 |
| 47 30 | .076,170,951,261 | .054,652,038,112 | .286,816,266,430 | .177,191,844,746 | .002,457,986,323 | .002,031,181,415 |
| 47 45 | .077,186,867,751 | .055,765 592 549 | .289,260,684,922 | .179,871,181,728 | .002,515,148,769 | .002,093,838,114 |
| 48 00 | .078,208,558,324 | .056,895,337,190 | .291,706,159,498 | .182,575,279,549 | .002,573,227,167 | .002,158,059,366 |
| 48 15 | .079,235,956,282 | .058,041,405,254 | .294,152,520,375 | .185,304,198,169 | .002,632,227,183 | .002,223,874,697 |
| 48 30 | .080,268,993,609 | .059,203,929,928 | .296,599,597,267 | .188,057,996,531 | .002,692,154,339 | .002,291,313,995 |
| 48 45 | .081,307,600,959 | .060,383,044,351 | .299,047,219,393 | .190,836,732,571 | .002,753,013,997 | .002,360,407,506 |
| 49 00 | .082,351,707,662 | .061,578,881,610 | .301,495,215,481 | .193,640,463,200 | .002,814,811,363 | .002,431,185,841 |
| 49 15 | .083,401,241,715 | .062,791,574,724 | .303,943,413,772 | .196,469,244,314 | .002,877,551,475 | .002,503,679,969 |
| 49 30 | .084,456,129,789 | .064,021,256,641 | .306,391,642,030 | .199,323,130,782 | .002,941,239,206 | .002,577,921,231 |
| 49 45 | .085,516,297,222 | .065,268,060,220 | .308,839,727,546 | .202,202,176,445 | .003,005,879,257 | .002,653,941,329 |
| 50 00 | .086,581,668,015 | .066,532,118,222 | .311,287,497,139 | .205,106,434,116 | .003,071,476,148 | .002,731,772,326 |
| 50 15 | .087,652,164,834 | .067,813,563,305 | .313,734,777,167 | .208,035,955,574 | .003,138,034,216 | .002,811,446,659 |
| 50 30 | .088,727,709,009 | .069,112,528,011 | .316,181,393,535 | .210,990,791,562 | .003,205,557,612 | .002,892,997,133 |
| 50 45 | .089,808,220,533 | .070,429,144,753 | .318,627,171,693 | .213,970,991,781 | .003,274,050,300 | .002,976,456,920 |
| 51 00 | .090,893,618,054 | .071,763,545,806 | .321,071,936,646 | .216,976,604,895 | .003,343,516,039 | .003,061,859,560 |
| 51 15 | .091,983,818,881 | .073,115,863,299 | .323,515,512,962 | .220,007,678,518 | .003,413,958,391 | .003,149,238,968 |
| 51 30 | .093,078,738,985 | .074,486,229,203 | .325,957,724,775 | .223,064,259,216 | .003,485,380,718 | .003,238,629,427 |
| 51 45 | .094,178,292,985 | .075,874,775,317 | .328,398,395,789 | .226,146,392,509 | .003,557,786,157 | .003,330,065,591 |
| 52 00 | .095,282,394,161 | .077,281,633,264 | .330,837,349,287 | .229,254,122,857 | .003,631,177,642 | .003,423,582,490 |
| 52 15 | .096,390,954,445 | .078,706,934,477 | .333,274,408,137 | .232,387,493,667 | .003,705,557,879 | .003,519,215,525 |
| 52 30 | .097,503,884,424 | .080,150,810,188 | .335,709,394,797 | .235,546,547,285 | .003,780,929,354 | .003,617,000,472 |
| 52 45 | .098,621,093,333 | .081,613,391,419 | .338,142,131,319 | .238,731,324,998 | .003,857,294,314 | .003,716,973,482 |
| 53 00 | .099,742,489,063 | .083,094,808,970 | .340,572,439,356 | .241,941,867,023 | .003,934,654,779 | .003,819,171,076 |
| 53 15 | .100,867,978,156 | .084,595,193,410 | .343,000,140,172 | .245,178,212,513 | .004,013,012,523 | .003,923,630,157 |
| 53 30 | .101,997,465,797 | .086,114,675,066 | .345,425,054,640 | .248,440,399,551 | .004,092,369,079 | .004,030,387,998 |
| 53 45 | .103,130,855,831 | .087,653,384,013 | .347,847,003,257 | .251,728,465,144 | .004,172,725,725 | .004,139,482,253 |
| 54 00 | .104,268,050,743 | .089,211,450,058 | .350,265,806,139 | .255,042,445,228 | .004,254,083,480 | .004,250,950,944 |
| 54 15 | .105,408,951,671 | .090,789,002,740 | .352,681,283,040 | .258,382,374,657 | .004,336,443,111 | .004,364,832,478 |
| 54 30 | .106,553,458,402 | .092,386,171,311 | .355,093,253,347 | .261,748,287,205 | .004,419,805,114 | .004,481,165,636 |
| 54 45 | .107,701,469,365 | .094,003,084,727 | .357,501,536,091 | .265,140,215,568 | .004,504,169,710 | .004,599,989,573 |
| 55 00 | .108,852,881,645 | .095,639,871,638 | .359,905,949,952 | .268,558,191,349 | .004,589,536,851 | .004,721,343,820 |
| 55 15 | .110,007,590,967 | .097,296,660,379 | .362,306,313,266 | .272,002,245,068 | .004,675,906,200 | .004,845,268,290 |
| 55 30 | .111,165,491,707 | .098,973,578,959 | .364,702,444,032 | .275,472,406,153 | .004,763,277,138 | .004,971,803,270 |
| 55 45 | .112,326,476,888 | .100,670,755,044 | .367,094,159,912 | .278,968,702,941 | .004,851,648,750 | .005,100,989,420 |
| 56 00 | .113,490,438,182 | .102,388,315,957 | .369,481,278,245 | .282,491,162,671 | .004,941,019,827 | .005,232,867,782 |
| 56 15 | .114,657,265,903 | .104,126,388,661 | .371,863,616,050 | .286,039,811,488 | .005,031,388,852 | .005,367,479,773 |
| 56 30 | .115,826,849,018 | .105,885,099,744 | .374,240,990,026 | .289,614,674,434 | .005,122,754,003 | .005,504,867,178 |
| 56 45 | .116,999,075,141 | .107,664,575,421 | .376,613,216,572 | .293,215,775,452 | .005,215,113,143 | .005,645,072,173 |
| 57 00 | .118,173,830,534 | .109,464,941,508 | .378,980,111,777 | .296,843,137,379 | .005,308,463,818 | .005,788,137,292 |
| 57 15 | .119,351,000,106 | .111,286,323,429 | .381,341,491,442 | .300,496,781,948 | .005,402,803,245 | .005,934,105,461 |
| 57 30 | .120,530,467,418 | .113,128,846,185 | .383,697,171,071 | .304,176,729,781 | .005,498,128,313 | .006,083,019,967 |
| 57 45 | .121,712,114,678 | .114,992,634,358 | .386,046,965,888 | .307,883,000,392 | .005,594,435,575 | .006,234,924,476 |
| 58 00 | .122,895,822,749 | .116,877,812,097 | .388,390,690,838 | .311,615,612,180 | .005,691,721,246 | .006,389,863,027 |
| 58 15 | .124,081,471,138 | .118,784,503,107 | .390,728,160,599 | .315,374,582,433 | .005,789,981,186 | .006,547,880,037 |
| 58 30 | .125,268,938,011 | .120,712,830,630 | .393,059,189,575 | .319,159,927,317 | .005,889,210,912 | .006,709,020,281 |
| 58 45 | .126,458,100,186 | .122,662,917,449 | .395,383,591,921 | .322,971,661,884 | .005,989,405,581 | .006,873,328,919 |
| 59 00 | .127,648,833,131 | .124,634,885,869 | .397,701,181,533 | .326,809,800,064 | .006,090,559,982 | .007,040,851,475 |
| 59 15 | .128,841,010,971 | .126,628,857,704 | .400,011,772,063 | .330,674,354,664 | .006,192,668,538 | .007,211,633,846 |
| 59 30 | .130,034,506,488 | .128,644,954,271 | .402,315,176,924 | .334,565,337,368 | .006,295,725,299 | .007,385,722,294 |
| 59 45 | .131,229,191,120 | .130,683,296,376 | .404,611,209,291 | .338,482,758,731 | .006,399,723,934 | .007,563,163,448 |
| 60 00 | .132,424,934,960 | .132,744,004,308 | .406,899,682,116 | .342,426,628,186 | .006,504,657,720 | .007,744,004,308 |
| 60 15 | .133,621,606,768 | .134,827,197,823 | .409,180,408,128 | .346,396,954,030 | .006,610,519,552 | .007,928,292,238 |
| 60 30 | .134,819,073,959 | .136,932,996,134 | .411,453,199,840 | .350,393,743,430 | .006,717,301,922 | .008,116,074,965 |
| 60 45 | .136,017,202,614 | .139,061,517,903 | .413,717,869,558 | .354,417,002,420 | .006,824,996,923 | .008,307,400,579 |
| 61 00 | .137,215,857,476 | .141,212,881,225 | .415,974,229,382 | .358,446,735,902 | .006,933,596,234 | .008,502,317,529 |
| 61 15 | .138,414,901,952 | .143,387,203,630 | .418,222,091,224 | .362,542,947,640 | .007,043,091,120 | .008,700,874,636 |
| 61 30 | .139,614,198,124 | .145,584,602,052 | .420,461,266,797 | .366,645,640,254 | .007,153,472,437 | .008,903,121,066 |
| 61 45 | .140,813,606,732 | .147,805,192,836 | .422,691,567,636 | .370,774,815,234 | .007,264,730,597 | .009,109,106,349 |
| 62 00 | .142,012,987,196 | .150,049,091,717 | .424,912,805,098 | .374,930,472,923 | .007,376,855,594 | .009,318,880,371 |
| 62 15 | .143,212,197,603 | .152,316,413,817 | .427,124,790,370 | .379,112,612,522 | .007,489,836,978 | .009,532,493,373 |
| 62 30 | .144,411,094,721 | .154,607,273,626 | .429,327,334,473 | .383,321,232,086 | .007,603,663,863 | .009,749,995,949 |
| 62 45 | .145,609,533,988 | .156,921,784,997 | .431,520,248,273 | .387,556,328,530 | .007,718,324,904 | .009,971,439,042 |
| 63 00 | .146,807,369,528 | .159,260,061,131 | .433,703,342,481 | .391,817,897,614 | .007,833,808,312 | .010,196,873,944 |
| 63 15 | .148,004,454,137 | .161,622,214,573 | .435,876,427,666 | .396,105,933,959 | .007,950,101,823 | .010,426,352,298 |
| 63 30 | .149,200,639,306 | .164,008,357,192 | .438,039,314,258 | .400,420,431,025 | .008,067,192,724 | .010,659,926,090 |
| 63 45 | .150,395,775,203 | .166,418,600,179 | .440,191,812,554 | .404,761,381,131 | .008,185,067,816 | .010,897,647,650 |
| 64 00 | .151,589,710,687 | .168,853,054,030 | .442,333,732,724 | .409,128,775,438 | .008,303,713,428 | .011,139,569,650 |
| 64 15 | .152,782,293,309 | .171,311,828,538 | .444,464,884,823 | .413,522,603,955 | .008,423,115,402 | .011,385,745,102 |
| 64 30 | .153,973,369,314 | .173,795,032,784 | .446,585,078,789 | .417,942,855,534 | .008,543,259,096 | .011,636,227,355 |
| 64 45 | .155,162,783,641 | .176,302,775,118 | .448,694,124,455 | .422,389,517,875 | .008,664,129,367 | .011,891,070,088 |
| 65 00 | .156,350,379,927 | .178,835,163,165 | .450,791,831,558 | .426,862,577,519 | .008,785,710,568 | .012,150,327,328 |
| 65 15 | .157,536,000,514 | .181,392,303,792 | .452,878,009,732 | .431,362,019,848 | .008,907,986,552 | .012,414,053,411 |
| 65 30 | .158,719,486,447 | .183,974,303,116 | .454,952,468,535 | .435,887,829,084 | .009,030,940,654 | .012,682,303,020 |
| 65 45 | .159,900,677,477 | .186,581,266,485 | .457,015,017,438 | .440,439,988,291 | .009,154,555,690 | .012,955,131,152 |
| 66 00 | .161,079,412,068 | .189,213,298,467 | .459,065,465,841 | .445,018,479,371 | .009,278,813,952 | .013,232,593,133 |
| 66 15 | .162,255,527,396 | .191,870,502,843 | .461,103,623,074 | .449,623,283,062 | .009,403,697,203 | .013,514,744,606 |
| 66 30 | .163,428,859,354 | .194,552,982,592 | .463,129,298,409 | .454,254,378,942 | .009,529,186,662 | .013,801,641,533 |
| 66 45 | .164,599,242,556 | .197,260,839,884 | .465,142,301,062 | .458,911,745,421 | .009,655,263,013 | .014,093,340,192 |
| 67 00 | .165,766,510,339 | .199,994,176,068 | .467,142,440,202 | .463,595,359,747 | .009,781,906,389 | .014,389,897,171 |
| 67 15 | .166,930,494,766 | .202,753,091,660 | .469,129,524,956 | .468,305,198,001 | .009,909,096,368 | .014,691,369,372 |
| 67 30 | .168,091,026,631 | .205,537,686,334 | .471,103,364,417 | .473,041,235,098 | .010,036,811,965 | .014,997,813,995 |
| ° ′ $\Phi_1$ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ | +COS $\Phi_0$ | −SIN $\Phi_0$ |
| | $\int \frac{\Phi_1^2}{2}$ COS $\Phi \, d \Phi_1$ | | $\int \Phi_1$ COS $\Phi \, d \Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{VERS } \Phi_1 \right)$ COS $\Phi \, d\Phi_1$ | |

SHEET 3 OF 4

205-D-1875

Table H-6.–*Additional integrals for arch load constants* (sheet 4).

| $\Phi_1$ | $\int \frac{\Phi_1^2}{2} \sin \Phi \, d\Phi_1$ | | $\int \Phi_1 \sin \Phi \, d\Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{vers} \ \Phi_1 \right) \sin \Phi \, d\Phi_1$ | |
|---|---|---|---|---|---|---|
| ° ′ | + SIN $\Phi_0$ | + COS $\Phi_0$ | + SIN $\Phi_0$ | + COS $\Phi_0$ | + SIN $\Phi_0$ | + COS $\Phi_0$ |
| 67 45 | .169,247,935,462 | .208,348,058,911 | .473,063,767,649 | .477,803,444,786 | .010,165,031,632 | .015,309,288,552 |
| 68 00 | .170,401,049,526 | .211,184,307,350 | .475,010,543,696 | .482,591,799,641 | .010,293,733,252 | .015,625,850,851 |
| 68 15 | .171,550,195,828 | .214,046,528,731 | .476,943,501,584 | .487,406,271,077 | .010,422,894,123 | .015,947,558,994 |
| 68 30 | .172,695,200,119 | .216,934,819,254 | .478,862,450,332 | .492,246,829,335 | .010,552,490,960 | .016,274,471,383 |
| 68 45 | .173,835,886,902 | .219,849,274,223 | .480,767,198,961 | .497,113,443,484 | .010,682,499,896 | .016,606,646,710 |
| 69 00 | .174,972,079,427 | .222,789,988,032 | .482,657,556,489 | .502,006,081,428 | .010,812,896,458 | .016,944,143,946 |
| 69 15 | .176,103,599,710 | .225,757,054,164 | .484,533,331,952 | .506,924,709,892 | .010,943,655,581 | .017,287,022,359 |
| 69 30 | .177,230,268,516 | .228,750,565,171 | .486,394,334,399 | .511,869,294,439 | .011,074,751,583 | .017,635,341,485 |
| 69 45 | .178,351,905,385 | .231,770,612,668 | .488,240,372,907 | .516,839,799,451 | .011,206,158,178 | .017,989,161,145 |
| 70 00 | .179,468,328,621 | .234,817,287,326 | .490,071,256,585 | .521,836,188,143 | .011,337,848,455 | .018,348,541,432 |
| 70 15 | .180,579,355,303 | .237,890,678,849 | .491,886,794,571 | .526,858,422,556 | .011,469,794,879 | .018,713,542,699 |
| 70 30 | .181,684,801,291 | .240,990,875,981 | .493,686,796,059 | .531,906,463,556 | .011,601,969,290 | .019,084,225,579 |
| 70 45 | .182,784,481,220 | .244,117,966,481 | .495,471,070,286 | .536,980,270,838 | .011,734,342,880 | .019,460,650,957 |
| 71 00 | .183,878,208,520 | .247,272,037,121 | .497,239,426,548 | .542,079,802,920 | .011,866,886,211 | .019,842,879,979 |
| 71 15 | .184,965,795,409 | .250,453,173,669 | .498,991,674,207 | .547,205,017,149 | .011,999,569,189 | .020,230,974,045 |
| 71 30 | .186,047,052,902 | .253,661,460,887 | .500,727,622,694 | .552,355,869,694 | .012,132,361,072 | .020,624,994,804 |
| 71 45 | .187,121,790,813 | .256,896,982,514 | .502,447,081,519 | .557,532,315,554 | .012,265,230,452 | .021,025,004,152 |
| 72 00 | .188,189,817,763 | .260,159,821,256 | .504,149,860,272 | .562,734,308,550 | .012,398,145,259 | .021,431,064,225 |
| 72 15 | .189,250,941,187 | .263,450,058,779 | .505,835,768,637 | .567,961,801,328 | .012,531,072,755 | .021,843,237,396 |
| 72 30 | .190,304,967,330 | .266,767,775,698 | .507,504,616,395 | .573,214,745,362 | .012,663,979,518 | .022,261,586,274 |
| 72 45 | .191,351,701,265 | .270,113,051,564 | .509,156,213,431 | .578,493,090,946 | .012,796,831,452 | .022,686,173,695 |
| 73 00 | .192,390,946,882 | .273,485,964,855 | .510,790,369,738 | .583,796,787,206 | .012,929,593,765 | .023,117,062,717 |
| 73 15 | .193,422,506,909 | .276,886,592,968 | .512,406,835,427 | .589,125,782,087 | .013,062,230,975 | .023,554,316,619 |
| 73 30 | .194,446,182,909 | .280,315,012,205 | .514,005,600,735 | .594,480,022,360 | .013,194,706,903 | .023,997,998 895 |
| 73 45 | .195,461,775,285 | .283,771,297,768 | .515,586,296,028 | .599,859,453,624 | .013,326,984,659 | .024,448,173,252 |
| 74 00 | .196,469,083,288 | .287,255,523,742 | .517,148,791,806 | .605,264,020,301 | .013,459,026,647 | .024,904,903,598 |
| 74 15 | .197,467,905,024 | .290,767,763,092 | .518,692,898,720 | .610,693,665,641 | .013,590,794,552 | .025,368,254,046 |
| 74 30 | .198,458,037,451 | .294,308,087,644 | .520,218,427,561 | .616,148,331,716 | .013,722,249,337 | .025,838,288,898 |
| 74 45 | .199,439,276,400 | .297,876,568,087 | .521,725,189,287 | .621,627,959,425 | .013,853,351,244 | .026,315,072,655 |
| 75 00 | .200,411,416,563 | .301,473,273,950 | .523,212,995,012 | .627,132,488,495 | .013,984,059,772 | .026,798,669,999 |
| 75 15 | .201,374,251,510 | .305,098,273,604 | .524,681,656,025 | .632,661,857,481 | .014,114,333,685 | .027,289,145,795 |
| 75 30 | .202,327,573,695 | .308,751,634,242 | .526,130,983,789 | .638,216,003,760 | .014,244,131,006 | .027,786,565,081 |
| 75 45 | .203,271,174,458 | .312,433,421,876 | .527,560,789,955 | .643,794,863,538 | .014,373,409,008 | .028,290,993,071 |
| 76 00 | .204,204,844,031 | .316,143,701,321 | .528,970,886,357 | .649,398,371,849 | .014,502,124,209 | .028,802,495,136 |
| 76 15 | .205,128,371,545 | .319,882,536,189 | .530,361,085,030 | .655,026,462,558 | .014,630,232,363 | .029,321,136,809 |
| 76 30 | .206,041,545,036 | .323,649,988,883 | .531,731,198,215 | .660,679,068,354 | .014,757,688,460 | .029,846,983,786 |
| 76 45 | .206,944,151,454 | .327,446,120,575 | .533,081,038,356 | .666,356,120,757 | .014,884,446,724 | .030,380,101,897 |
| 77 00 | .207,835,976,665 | .331,270,991,212 | .534,410,418,123 | .672,057,550,117 | .015,010,460,595 | .030,920,557,131 |
| 77 15 | .208,716,805,460 | .335,124,659,490 | .535,719,150,401 | .677,783,285,614 | .015,135,682,736 | .031,468,415,600 |
| 77 30 | .209,586,421,558 | .339,007,182,855 | .537,007,048,307 | .683,533,255,260 | .015,260,065,021 | .032,023,743,552 |
| 77 45 | .210,444,607,620 | .342,918,617,494 | .538,273,925,199 | .689,307,385,897 | .015,383,558,533 | .032,586,607,369 |
| 78 00 | .211,291,145,249 | .346,859,018,315 | .539,519,594,674 | .695,105,603,197 | .015,506,113,56 | .033,157,073,544 |
| 78 15 | .212,125,814,996 | .350,828,438,950 | .540,743,870,580 | .700,927,831,672 | .015,627,679,585 | .033,735,208,687 |
| 78 30 | .212,948,396,370 | .354,826,931,736 | .541,946,567,022 | .706,773,994,661 | .015,748,205,279 | .034,321,079,516 |
| 78 45 | .213,758,667,847 | .358,854,547,708 | .543,127,498,368 | .712,644,014,339 | .015,867,638,508 | .034,914,752,852 |
| 79 00 | .214,556,406,870 | .362,911,336,593 | .544,286,479,258 | .718,537,811,720 | .015,985,926,314 | .035,516,295,612 |
| 79 15 | .215,341,389,862 | .366,997,346,796 | .545,423,324,603 | .724,455,306,651 | .016,103,014,920 | .036,125,774,601 |
| 79 30 | .216,113,392,229 | .371,112,625,389 | .546,537,849,601 | .730,396,417,816 | .016,218,849,719 | .036,743,257,505 |
| 79 45 | .216,872,188,371 | .375,257,218,109 | .547,629,869,740 | .736,361,062,738 | .016,333,375,272 | .037,368,810,895 |
| 80 00 | .217,617,551,685 | .379,431,169,344 | .548,699,200,805 | .742,349,157,782 | .016,446,535,302 | .038,002,502,207 |
| 80 15 | .218,349,254,574 | .383,634,522,119 | .549,745,658,880 | .748,360,618,148 | .016,558,272,687 | .038,644,398,741 |
| 80 30 | .219,067,068,458 | .387,867,318,097 | .550,769,060,363 | .754,395,357,880 | .016,668,529,463 | .039,294,567,858 |
| 80 45 | .219,770,763,771 | .392,129,597,559 | .551,769,221,965 | .760,453,289,865 | .016,777,246,805 | .039,953,076,964 |
| 81 00 | .220,460,109,982 | .336,421,399,402 | .552,745,960,722 | .766,534,325,833 | .016,884,365,038 | .040,619,993,516 |
| 81 15 | .221,134,875,594 | .400,742,761,128 | .553,699,093,999 | .772,638,376,356 | .016,989,823,625 | .041,295,385,005 |
| 81 30 | .221,794,828,150 | .405,093,718,832 | .554,628,439,495 | .778,765,350,855 | .017,093,561,156 | .041,979,318,953 |
| 81 45 | .222,439,734,246 | .409,474,307,197 | .555,533,815,253 | .784,915,157,599 | .017,195,515,354 | .042,671,862,905 |
| 82 00 | .223,069,359,537 | .413,884,559,480 | .556,415,039,667 | .791,087,703,701 | .017,295,623,066 | .043,373,084,425 |
| 82 15 | .223,683,468,746 | .418,324,507,508 | .557,271,931,483 | .797,282,895,126 | .017,393,820,261 | .044,083,051,084 |
| 82 30 | .224,281,825,667 | .422,794,181,667 | .558,104,309,813 | .803,500,636,692 | .017,490,042,016 | .044,801,830,460 |
| 82 45 | .224,864,193,177 | .427,293,610,891 | .558,911,994,137 | .809,740,832,065 | .017,584,222,523 | .045,529,490,122 |
| 83 00 | .225,430,333,244 | .431,822,822,655 | .559,694,804,308 | .816,003,383,769 | .017,676,295,080 | .046,266,097,629 |
| 83 15 | .225,980,006,934 | .436,381,842,966 | .560,452,560,565 | .822,288,193,181 | .017,766,192,086 | .047,011,720,524 |
| 83 30 | .226,512,974,416 | .440,970,696,353 | .561,185,083,533 | .828,595,160,535 | .017,853,845,035 | .047,766,426,318 |
| 83 45 | .227,028,994,981 | .445,589,405,858 | .561,892,194,231 | .834,924,184,923 | .017,939,184,516 | .048,530,282,490 |
| 84 00 | .227,527,827,034 | .450,237,993,031 | .562,573,714,085 | .841,275,164,298 | .018,022,140,208 | .049,303,356,482 |
| 84 15 | .228,009,228,116 | .454,916,477,915 | .563,229,464,923 | .847,647,995,474 | .018,102,640,872 | .050,085,715,683 |
| 84 30 | .228,472,954,908 | .459,624,879,043 | .563,859,268,992 | .854,042,574,125 | .018,180,614,353 | .050,877,427,425 |
| 84 45 | .228,918,763,234 | .464,363,213,425 | .564,462,948,960 | .860,458,794,795 | .018,255,987,564 | .051,678,558,980 |
| 85 00 | .229,346,408,079 | .469,131,496,540 | .565,040,327,920 | .866,896,550,889 | .018,328,686,502 | .052,489,177,541 |
| 85 15 | .229,755,643,589 | .473,929,742,333 | .565,591,229,402 | .873,355,734,686 | .018,398,636,219 | .053,309,350,230 |
| 85 30 | .230,146,223,090 | .478,757,963,197 | .566,115,477,377 | .879,836,237,328 | .018,465,760,845 | .054,139,144,074 |
| 85 45 | .230,517,899,079 | .483,616,169,971 | .566,612,896,260 | .886,337,948,836 | .018,529,983,555 | .054,978,626,006 |
| 86 00 | .230,870,423,258 | .488,504,371,935 | .567,083,310,929 | .892,860,758,098 | .018,591,226,595 | .055,827,862,865 |
| 86 15 | .231,203,546,516 | .493,422,576,789 | .567,526,546,710 | .899,404,552,882 | .018,649,411,254 | .056,686,921,363 |
| 86 30 | .231,517,018,959 | .498,372,790,653 | .567,942,429,403 | .905,969,219,829 | .018,704,457,876 | .057,555,868,098 |
| 86 45 | .231,810,589,908 | .503,349,018,064 | .568,330,785,280 | .912,554,644,464 | .018,756,285,845 | .058,434,769,546 |
| 87 00 | .232,084,007,911 | .508,357,261,959 | .568,691,441,098 | .919,160,711,190 | .018,804,813,591 | .059,323,692,044 |
| 87 15 | .232,337,020,747 | .513,395,523,663 | .569,024,224,088 | .925,787,303,295 | .018,849,958,574 | .060,222,701,776 |
| 87 30 | .232,569,375,452 | .518,463,802,895 | .569,328,961,985 | .932,434,302,950 | .018,891,637,304 | .061,131,864,783 |
| 87 45 | .232,780,818,303 | .523,562,097,749 | .569,605,483,019 | .939,101,591,216 | .018,929,765,306 | .062,051,246,942 |
| 88 00 | .232,971,094,849 | .528,690,404,690 | .569,853,615,926 | .945,789,048,040 | .018,964,257,144 | .062,980,913,958 |
| 88 15 | .233,139,949,907 | .533,848,718,543 | .570,073,189,552 | .952,496,552,268 | .018,995,026,396 | .063,920,931,359 |
| 88 30 | .233,287,127,578 | .539,037,032,489 | .570,264,034,865 | .959,223,981,631 | .019,021,985,670 | .064,871,364,486 |
| 88 45 | .233,412,371,257 | .544,255,358,052 | .570,425,980,955 | .965,971,212,760 | .019,045,046,591 | .065,832,278,484 |
| 89 00 | .233,515,423,636 | .549,503,625,099 | .570,558,859,043 | .972,738,121,186 | .019,064,119,793 | .066,803,738,290 |
| 89 15 | .233,596,026,723 | .554,781,881,824 | .570,662,500,493 | .979,524,581,337 | .019,079,114,928 | .067,785,808,639 |
| 89 30 | .233,653,921,841 | .560,090,094,742 | .570,736,737,206 | .986,330,466,546 | .019,089,940,654 | .068,778,554,029 |
| 89 45 | .233,688,849,647 | .565,428,248,686 | .570,781,101,637 | .993,155,649,052 | .019,096,504,633 | .069,782,038,737 |
| 90 00 | .233,700,550,136 | .570,796,326,795 | .570,796,326,795 | 1.000,000,000,000 | .019,098,713,533 | .070,796,326,795 |
| ° ′ | + COS $\Phi_0$ | − SIN $\Phi_0$ | + COS $\Phi_0$ | − SIN $\Phi_0$ | + COS $\Phi_0$ | − SIN $\Phi_0$ |
| $\Phi_1$ | $\int \frac{\Phi_1^2}{2} \cos \Phi \, d\Phi_1$ | | $\int \Phi_1 \cos \Phi \, d\Phi_1$ | | $\int \left( \frac{\Phi_1^2}{2} - \text{vers} \ \Phi_1 \right) \cos \Phi \, d\Phi_1$ | |

SHEET 4 OF 4

205-D-1876

Table H-7.—*Computations for load constants* (sheet 1).

$\Phi = 48°$

INTEGRALS FOR LOAD FORMULAS, SHEET 1 OF 4

| $\Phi$ DEG. | $\sin \Phi_1$ | $\cos \Phi_1$ | $\text{vers } \Phi_1$ | $\Phi_1 - \sin \Phi_1$ | $\frac{\Phi_1^2}{2} - \text{vers } \Phi_1$ | $\frac{\Phi_1^2}{2}$ | $\frac{\Phi_1^3}{6}$ | $\Phi_1$ |
|---|---|---|---|---|---|---|---|---|
| 48 | .743,144,83 | .669,130,61 | .330,869,39 | .094,613,215 | .020,049,874 | .350,919,26 | .097,995,146 | .837,758,04 |
| 36 | .587,785,25 | .809,016,99 | .190,983,00 | .040,533,278 | .006,409,082,3 | .197,392,08 | .041,341,702 | .628,318,53 |
| 24 | .406,736,64 | .913,545,46 | .086,454,542 | .012,142,377 | .001,275,274,5 | .087,729,817 | .012,249,393 | .418,879,02 |
| 12 | .207,911,69 | .978,147,60 | .021,852,399 | .001,527,819,4 | .000,080,054,958 | .021,932,454 | .001,531,174,2 | .209,439,51 |
|  |  |  |  | $\int_0^{\Phi_1} \text{vers } \Phi_1 \, d\Phi_1$ | $\int_0^{\Phi_1} (\Phi_1 - \sin \Phi_1) d\Phi_1$ | $\int_0^{\Phi_1} \Phi_1 \, d\Phi$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \, d\Phi_1$ |  |

| $\Phi$ DEG. | $\int_0^{\Phi_1} \left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right) d\Phi_1$ | $\int_0^{\Phi_1} (\Phi_1 - \sin \Phi_1) \sin \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \text{vers } \Phi_1 \sin \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \sin \Phi_1 \sin \Phi \, d\Phi_1$ |  |
|---|---|---|---|---|---|---|---|
|  |  | $+\sin \Phi_0$ | $+\cos \Phi_0$ | $+\sin \Phi_0$ | $+\cos \Phi_0$ | $+\sin \Phi_0$ | $+\cos \Phi_0$ |
| 48 | .003,381,930,6 | .015,574,044 | .012,326,733 | .075,635,331 | .054,737,278 | .276,132,11 | .170,248,55 |
| 36 | .000,808,423,81 | .005,587,609,1 | .003,069,746,8 | .035,861,858 | .018,237,254 | .172,745,75 | .076,395,136 |
| 24 | .000,107,015,85 | .001,201,555,9 | .000,418,312,75 | .011,501,926 | .003,737,194,0 | .082,717,348 | .023,653,303 |
| 12 | .000,003,354,735,5 | .000,078,887,841 | .000,013,342,029 | .001,507,774,9 | .000,238,763,68 | .021,613,636 | .003,035,594,4 |
|  |  | $+\cos \Phi_0$ | $-\sin \Phi_0$ | $+\cos \Phi_0$ | $-\sin \Phi_0$ | $+\cos \Phi_0$ | $-\sin \Phi_0$ |
|  |  | $\int_0^{\Phi_1} (\Phi_1 - \sin \Phi_1) \cos \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \text{vers } \Phi_1 \cos \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \sin \Phi_1 \cos \Phi \, d\Phi_1$ |  |

| $\Phi$ DEG. | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \Phi_1 \sin \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right) \sin \Phi \, d\Phi_1$ |  |
|---|---|---|---|---|---|---|
|  | $+\sin \Phi_0$ | $+\cos \Phi_0$ | $+\sin \Phi_0$ | $+\cos \Phi_0$ | $+\sin \Phi_0$ | $+\cos \Phi_0$ |
| 48 | .078,208,558 | .056,895,337 | .291,706,16 | .182,575,27 | .002,573,227,2 | .002,158,059,4 |
| 36 | .036,559,275 | .018,639,806 | .178,333,36 | .079,464,883 | .000,697,417,34 | .000,402,552,49 |
| 24 | .011,611,315 | .003,773,728,6 | .083,918,904 | .024,071,617 | .000,100,388,14 | .000,036,534,612 |
| 12 | .001,511,077,3 | .000,239,345,95 | .021,692,523 | .003,048,936,4 | .000,003,302,334,2 | .000,000,582,276,82 |
|  | $+\cos \Phi_0$ | $-\sin \Phi_0$ | $+\cos \Phi_0$ | $-\sin \Phi_0$ | $+\cos \Phi_0$ | $-\sin \Phi_0$ |
|  | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \Phi_1 \cos \Phi \, d\Phi_1$ |  | $\int_0^{\Phi_1} \left(\frac{\Phi_1^2}{2} - \text{vers } \Phi_1\right) \cos \Phi \, d\Phi_1$ |  |

288-D-452

Table H-7.–*Computations for load constants* (sheet 2).

$\Phi = 48°$

## UNIFORM RADIAL LOAD, SHEET 2 OF 4

| POINT | $D_2$ 2ND TERM | $D_3$ 2ND TERM | | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|
| | 2ND PART | 1ST PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 828.396,33 | 75.635,331 | 510.745,62 | 94.613,215 | 54.737,277 | 883.133,61 | 18.977,884 | 435.110,29 |
| 1/4 | 518.237,25 | 35.861,857 | 229.185,41 | 40.533,278 | 18.237,254 | 536.474,50 | 4.671,420 | 193.323,55 |
| 1/2 | 248.152,04 | 11.510,926 | 70.959,912 | 12.142,377 | 3.737,193,9 | 251.889,23 | .631,451 | 59.448,986 |
| 3/4 | 64.840,905 | 1.507,774,9 | 9.106,782,9 | 1.527,819,4 | .238,763,67 | 65.079,669 | .020,044,5 | 7.599,008 |
| | $\int_0^{\Phi}\sin\Phi_1\cos\Phi\,d\Phi_1$ | $\int_0^{\Phi}\text{vers}\,\Phi_1\cos\Phi\,d\Phi_1$ | $3\int_0^{\Phi}\sin\Phi_1\sin\Phi\,d\Phi_1$ | $\int_0^{\Phi}\text{vers}\,\Phi_1\,d\Phi_1$ | $\int_0^{\Phi}\text{vers}\,\Phi_1\sin\Phi\,d\Phi_1$ | | | |

UNIFORM LOAD INCLUDES FACTOR, P = 1000

$D_2$ 2ND TERM = ($D_2$ 1ST TERM) + ($D_2$ 2ND TERM, 2ND PART)
$D_3$ 1ST TERM = $D_1$ − ($D_3$ 2ND TERM, 1ST PART)
$D_3$ 2ND TERM = ($D_3$ 2ND TERM, 2ND PART) − ($D_3$ 2ND TERM, 1ST PART)

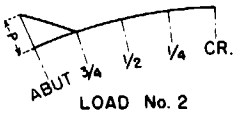
TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\Phi_1} = \frac{1000}{\Phi_1}$

## TRIANGULAR RADIAL LOAD No. 2

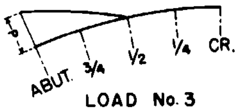
| POINT | $D_2$ 2ND TERM | $D_3$ 2ND TERM | | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|
| | 2ND PART | 1ST PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 15.462,322 | .267,281,74 | 15.461,434 | .382,234,27 | .272,933,41 | 15.735,256 | .114,952,53 | 15.194,152 |
| 1/4 | 18.339,046 | .318,187,05 | 11.908,767 | .382,234,27 | .211,398,16 | 18.550,445 | .064,047,23 | 11.590,580 |
| 1/2 | 20.414,266 | .355,186,04 | 7.835,629,6 | .382,234,27 | .140,623,79 | 20.554,890 | .027,048,237 | 7.480,443,5 |
| 3/4 | 21.597,285 | .376,661,71 | 3.420,037,7 | .382,234,27 | .063,703,496 | 21.660,988 | .005,572,569 | 3.043,376,0 |
| | $3\int_0^{\Phi}\text{vers}\,\Phi_1\cos\Phi\,d\Phi_1$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\cos\Phi\,d\Phi_1$ | $3\int_0^{\Phi}\text{vers}\,\Phi_1\sin\Phi\,d\Phi_1$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\,d\Phi$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\sin\Phi\,d\Phi_1$ | | | |

FRACTION OF UNIFORM LOAD TO INCLUDE IN TRIANGULAR LOAD

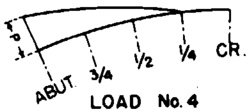
| POINT \ LOAD | No. 2 | No. 3 | No. 4 | No. 5 |
|---|---|---|---|---|
| CROWN | 0 | 0 | 0 | 0 |
| 1/4 | 0 | 0 | 0 | 1/4 |
| 1/2 | 0 | 0 | 1/3 | 1/2 |
| 3/4 | 0 | 1/2 | 2/3 | 3/4 |



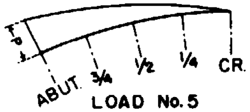
LOAD No. 2



LOAD No. 3



LOAD No. 4



LOAD No. 5

## TRIANGULAR RADIAL LOAD No. 4

| POINT | $D_2$ 2ND TERM | $D_3$ 2ND TERM | | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|
| | 2ND PART | 1ST PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 149.381,80 | 7.682,838,9 | 120.773,90 | 10.200,371 | 6.627,840,2 | 156.009,64 | 2.517,532,1 | 113.091,06 |
| 1/4 | 171.227,75 | 8.892,955,9 | 87.076,474 | 10.200,371 | 4.885,653,9 | 176.113,40 | 1.307,415,1 | 78.183,518 |
| 1/2 | 137.677,97 | 5.749,310,8 | 41.497,089 | 6.077,121,4 | 1.911,496,8 | 139.589,47 | .327,810,6 | 35.747,778 |
| 3/4 | 50.426,365 | 1.130,737,2 | 7.211,201,2 | 1.145,957,7 | .180,410,29 | 50.606,775 | .015,220,5 | 6.080,464,0 |
| | $3\int_0^{\Phi}\text{vers}\,\Phi_1\cos\Phi\,d\Phi_1$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\cos\Phi\,d\Phi_1$ | $3\int_0^{\Phi}\text{vers}\,\Phi_1\sin\Phi\,d\Phi_1$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\,d\Phi_1$ | $\int_0^{\Phi}(\Phi_1-\sin\Phi_1)\sin\Phi\,d\Phi_1$ | | | |

288-D-452

Table H-7.—*Computations for load constants* (sheet 3).

$\Phi = 48^\circ$

UNIFORM TANGENTIAL LOAD, SHEET 3 OF 4

| POINT | $D_2$ 2ND TERM | | $D_3$ 2ND TERM | | $D_3$ 1ST TERM | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1ST PART | 2ND PART | 1ST PART | 2ND PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 170.248,55 | 226.905,99 | 276.132,12 | 164.211,83 | 15.574,044 | 20.049,874 | 12.326,733 | 56.657,44 | 4.475,830 | 440.343,95 |
| 1/4 | 76.395,136 | 107.585,57 | 172.745,75 | 54.711,762 | 5.587,609,1 | 6.409,082,3 | 3.069,746,8 | 31.190,438 | .821,473,2 | 227.457,51 |
| 1/2 | 23.653,303 | 34.532,778 | 82.717,348 | 11.211,582 | 1.201,555,9 | 1.275,274,5 | .418,312,75 | 10.879,475 | .073,718,6 | 93.928,93 |
| 3/4 | 3.035,594,4 | 4.523,324,7 | 21.613,636 | .716,291,04 | .078,887,842 | .080,054,958 | .013,342,029 | 1.487,730 | .001,167,116 | 22.329,93 |
| | $\int_0^{\phi_1} \sin\phi_1 \sin\phi \, d\phi_1$ | $3\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} \sin\phi_1 \cos\phi \, d\phi_1$ | $3\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$ | $\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \, d\phi_1$ | $\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$ | | | |

UNIFORM LOAD INCLUDES FACTOR, P = 1000

$D_2$ 2ND TERM = ($D_2$ 2ND TERM, 2ND PART) − ($D_2$ 2ND TERM, 1ST PART)
$D_3$ 1ST TERM = $D_1$ − ($D_3$ 1ST TERM, 2ND PART)
$D_3$ 2ND TERM = ($D_3$ 2ND TERM, 1ST PART) + ($D_3$ 2ND TERM, 2ND PART)

TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\Phi_1} = \frac{1000}{\Phi_1}$

TRIANGULAR TANGENTIAL LOAD No. 3

| POINT | $D_2$ 2ND TERM | | $D_3$ 2ND TERM | | $D_3$ 1ST TERM | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1ST PART | 2ND PART | 1ST PART | 2ND PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 19.327,805 | 6.642,964,5 | 21.475,653 | 6.237,107,7 | .183,463,87 | .255,481,52 | .177,157,50 | -12.684,840 | .072,017,65 | 27.712,761 |
| 1/4 | 14.440,406 | 7.794,567,3 | 25.024,836 | 4.719,662,0 | .216,287,86 | .255,481,52 | .135,141,91 | −6.645,839,2 | .039,193,66 | 29.744,498 |
| 1/2 | 8.921,893,2 | 8.605,510,2 | 27.480,312 | 2.995,944,4 | .239,659,03 | .255,481,52 | .087,219,962 | −.316,383,0 | .015,822,49 | 30.476,256 |
| 3/4 | 2.087,803,5 | 2.826,654,9 | 14.406,365 | .453,700,7 | .047,327,663 | .048,036,321 | .008,061,098 | +.738,851,45 | .000,708,658 | 14.860,066 |
| | $\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$ | $3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$ | $3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) \sin\phi \, d\phi_1$ | | | |

TRIANGULAR TANGENTIAL LOAD No. 5

| POINT | $D_2$ 2ND TERM | | $D_3$ 2ND TERM | | $D_3$ 1ST TERM | $D_1$ | $D_2$ | | $D_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1ST PART | 2ND PART | 1ST PART | 2ND PART | 2ND PART | | 1ST TERM | 2ND TERM | 1ST TERM | 2ND TERM |
| CROWN | 65.337,813 | 55.770,437 | 90.283,026 | 44.141,861 | 3.071,563,7 | 4.036,882,3 | 2.575,993,7 | − 9.567,376 | .965,318,58 | 134.424,89 |
| 1/4 | 40.867,902 | 46.905,543 | 85.993,377 | 24.670,661 | 2.229,382,7 | 2.567,255,4 | 1.247,948,3 | + 6.037,641 | .337,872,7 | 110.664,04 |
| 1/2 | 16.287,598 | 21.569,144 | 55.098,830 | 7.103,763,2 | .720,607,46 | .765,378,01 | .252,766,36 | + 5.281,546 | .044,770,55 | 62.202,593 |
| 3/4 | 2.561,698,9 | 3.674,989,9 | 18.010,000 | .584,995,91 | .063,107,752 | .064,045,639 | .010,701,564 | + 1.113,291,0 | .000,937,887 | 18.594,996 |
| | $\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$ | $3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$ | $3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) \cos\phi \, d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) d\phi_1$ | $\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) \sin\phi \, d\phi_1$ | | | |

288-D-452

Table H-7.–*Computations for load constants* (sheet 4).

$\Phi = 48^\circ$

### UNIFORM TWIST LOAD, SHEET 4 OF 4

UNIFORM LOAD INCLUDES FACTOR, P = 1000

| POINT | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| | 2ND PART | | | |
| CROWN | 291.706,16 | 350.919,26 | 182.575,27 | 59.213,10 |
| $1/4$ | 178.333,36 | 197.392,08 | 79.464,883 | 19.058,72 |
| $1/2$ | 83.918,904 | 87.729,817 | 24.071,617 | 3.810,913 |
| $3/4$ | 21.692,523 | 21.932,454 | 3.048,936 4 | .239,931 |
| | $\int_0^{\Phi_1} \Phi_1 \cos\Phi \, d\Phi_1$ | $\int_0^{\Phi_1} \Phi_1 \, d\Phi_1$ | $\int_0^{\Phi_1} \Phi_1 \sin\Phi \, d\Phi_1$ | |

$D_3 = D_1 - D_3$ 2ND PART

### TRIANGULAR TWIST LOAD No. 2

| POINT | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| | 2ND PART | | | |
| CROWN | 5.165,229,8 | 7.310,818,3 | 5.165,328,6 | 2.145,588,5 |
| $1/4$ | 6.126,289,4 | 7.310,818,3 | 3.978,542,1 | 1.184,528,9 |
| $1/2$ | 6.819,600,6 | 7.310,818,3 | 2.617,874,2 | .491,217,7 |
| $3/4$ | 7.214,862,7 | 7.310,818,3 | 1.142,792,7 | .095,955,6 |
| | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos\Phi \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin\Phi \, d\Phi_1$ | |

### TRIANGULAR TWIST LOAD No. 3

| POINT | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| | 2ND PART | | | |
| CROWN | 21.659,119 | 29.243,272 | 19.504,962 | 7.584,153 |
| $1/4$ | 25.241,125 | 29.243,272 | 14.575,548 | 4.002,147 |
| $1/2$ | 27.719,973 | 29.243,272 | 9.009,113,2 | 1.523,299 |
| $3/4$ | 14.453,693 | 14.621,636 | 2.095,864,5 | 0.167,943 |
| | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos\Phi \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin\Phi \, d\Phi_1$ | |

TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\Phi_1} = \frac{1000}{\Phi_1}$

### TRIANGULAR TWIST LOAD No. 4

| POINT | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| | 2ND PART | | | |
| CROWN | 50.746,448 | 65.797,362 | 41.115,423 | 15.050,915 |
| $1/4$ | 58.185,893 | 65.797,362 | 29.666,173 | 7.611,469,0 |
| $1/2$ | 46.452,950 | 48.738,787 | 14.029,948 | 2.285,837,0 |
| $3/4$ | 16.866,636 | 17.058,575 | 2.413,555,2 | .191,939,2 |
| | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos\Phi \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin\Phi \, d\Phi_1$ | |

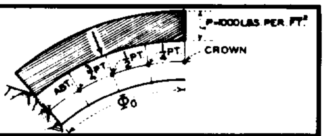
### TRIANGULAR TWIST LOAD No. 5

| POINT | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| | 2ND PART | | | |
| CROWN | 93.354,589 | 116.973,09 | 67.913,806 | 23.618,500 |
| $1/4$ | 88.222,760 | 98.696,042 | 42.115,850 | 10.473,282 |
| $1/2$ | 55.819,438 | 58.486,544 | 16.540,365 | 2.667,106 |
| $3/4$ | 18.073,108 | 18.277,045 | 2.572,400,5 | 0.203,937 |
| | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \cos\Phi \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \, d\Phi_1$ | $\int_0^{\Phi_1} \frac{\Phi_1^2}{2} \sin\Phi \, d\Phi_1$ | |

288-D-452

Table H-8.—*Load constants for circular arches—radial deflections* (sheet 1).

UNIT RADIAL LOAD NO. 1
RADIAL DEFLECTIONS

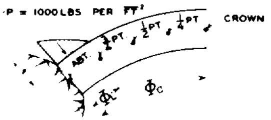


| $\Phi$ | Crown | | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_2$ | | $D_2$ | | $D_2$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .884.747.5 | .115.402.2 | 45.345.94 | .008.068.29 | 4.407.601 | .036,595,20 | 25.592,23 | .007,240,160 | 11.401,42 | .0$^{3}$452,941,1 | 2.854,429 |
| 11 | 1.177,222 | .168.780.2 | 54.780,89 | .012,984,1 | 5.860,143 | .053,546,90 | 30.938,74 | .010,597,50 | 13.790,21 | .0$^{3}$663,106,9 | 3453,514 |
| 12 | 1.527,819 | .238.763.7 | 65.079,67 | .020,044,5 | 7.599,008 | .075,788,48 | 36.783,40 | .015,004,82 | 16.404,30 | .0$^{3}$939,087,5 | 4.109,518 |
| 13 | 1.941.748 | .328,446,8 | 76.232.91 | .029,882,1 | 9.648.978 | .104,313,7 | 43.123,20 | .020,660,52 | 19.243,11 | .001,293,362 | 4.822,402 |
| 14 | 2.424,200 | .441,171,9 | 88.230,48 | .043,242,4 | 12.034,51 | .140,199,1 | 49.954,88 | .027,779,93 | 22.305,99 | .001,739,488 | 5.592,126 |
| 15 | 2.980.343 | .580,524,7 | 101.061,5 | .060,991,5 | 14.779,73 | .184,602,7 | 57.274,95 | .036,595,20 | 25.592,23 | .002,292,105 | 6.418,646 |
| 16 | 3.615.325 | .750,328,1 | 114.714,3 | .084,124,9 | 17.908,37 | .238,763,7 | 65.079,67 | .047,355,24 | 29.101,08 | .002,966,925 | 7.301,915 |
| 17 | 4.334,268 | .954,637,2 | 129.176,5 | .113,775,6 | 21.443,79 | .304,000,6 | 73.365,04 | .060,325,63 | 32.831,76 | .003,780,741 | 8.241,883 |
| 18 | 5.142.271 | 1.197,732 | 144.435,0 | .151,222,4 | 25.408,91 | .381,710,7 | 82.126,82 | .075,788,48 | 36.783,40 | .004,751,416 | 9.238,496 |
| 19 | 6.044,403 | 1.484,113 | 160.476,1 | .197,896,7 | 29.826,22 | .473,368,5 | 91.360,53 | .094,042,36 | 40.955,11 | .005,897,886 | 10.291,70 |
| 20 | 7.045,707 | 1.818,490 | 177.285,2 | .255,391,4 | 34.717,75 | .580,524,7 | 101.061,5 | .115,402,2 | 45.345,94 | .007,240,160 | 11.401,42 |
| 21 | 8.151.193 | 2.205,780 | 194.847,2 | .325,466,9 | 40.105,03 | .704,804,6 | 111.224,7 | .140,199,1 | 49.954,88 | .008,799,316 | 12.567,62 |
| 22 | 9.365,842 | 2.651,096 | 213.146,2 | .410,058,9 | 46.009,09 | .847,907,1 | 121.845,0 | .168.780,2 | 54.780,89 | .010,597,50 | 13.790,21 |
| 23 | 10.694.60 | 3.159,739 | 232.166,0 | .511,285 | 52.450,43 | 1.011,603 | 132.917,0 | .201,508,7 | 59.822,87 | .012,657,90 | 15.069,13 |
| 24 | 12.142.38 | 3.737,194 | 251,889,2 | .631,451 | 59.448,99 | 1.197,732 | 144.435,0 | .238,763,7 | 65.079,67 | .015,004,82 | 16.404,30 |
| 25 | 13.714.05 | 4.389,115 | 272.298,4 | .773,057 | 67.024,14 | 1.408,206 | 156.393,2 | .280,939,6 | 70.550,10 | .017,663,56 | 17.795,86 |
| 26 | 15.414.46 | 5.121,323 | 293.375,2 | .938,803 | 75.194,69 | 1.644,999 | 168.785,5 | .328,446,8 | 76.232,91 | .020,660,52 | 19.243,11 |
| 27 | 17.248.40 | 5.939,789 | 315.100,9 | 1.131,596 | 83.978,80 | 1.910,156 | 181.605,7 | .381,710,7 | 82.126,82 | .024,023,14 | 20.746,58 |
| 28 | 19.220.63 | 6.850,633 | 337.455,9 | 1.354,554 | 93.394,03 | 2.205,780 | 194.847,2 | .441,171,9 | 88.230,48 | .027,779,93 | 22.305,99 |
| 29 | 21.335,86 | 7.860,109 | 360.420,7 | 1.611,008 | 103.457,3 | 2.534,040 | 208.503,3 | .507,286,4 | 94.542,51 | .031,960,42 | 23.921,23 |
| 30 | 23.598,78 | 8.974,596 | 383.974,6 | 1.904,515 | 114.184,9 | 2.897,163 | 222.567,1 | .580,524,7 | 101.061,5 | .036,595,20 | 25.592,23 |
| 31 | 26.013,99 | 10.200,59 | 408.096,9 | 2.238,850 | 125.592,3 | 3.297,434 | 237.031,5 | .661,372,0 | 107.785,9 | .041,715,92 | 27.318,88 |
| 32 | 28.586.10 | 11.544,69 | 432.766,3 | 2.618.024 | 137.694,4 | 3.737,194 | 251.889,2 | .750,328,1 | 114.714,3 | .047,355,24 | 29.101,08 |
| 33 | 31.319,62 | 13.013,59 | 457.961,1 | 3.046,274 | 150.505,5 | 4.218,838 | 267.132.8 | .847,907,1 | 121.845,0 | .053,546,90 | 30.938,74 |
| 34 | 34.219.04 | 14.614,08 | 483.659.1 | 3.528,075 | 164.039,1 | 4.744,813 | 282.754,5 | .954,637,2 | 129.176,5 | .060,325,63 | 32.831,76 |
| 35 | 37.288.80 | 16.352,99 | 509.837,9 | 4.068,140 | 178.307,7 | 5.317,616 | 298.746,5 | 1.071,060 | 136.707,0 | .067,727,22 | 34.780,01 |
| 36 | 40.533.28 | 18.237,25 | 536.474,5 | 4.671,420 | 193.323,6 | 5.939,789 | 315.100,9 | 1.197,732 | 144.435,0 | .075,788,48 | 36.783,40 |
| 37 | 43.956,80 | 20.273,83 | 563.545,8 | 5.343,112 | 209.097,8 | 6.613,922 | 331.809,2 | 1.335,222 | 152.358,6 | .084,547,24 | 38.841,81 |
| 38 | 47.563,64 | 22.469,72 | 591.028,3 | 6.088,654 | 225.640,9 | 7.342,646 | 348.863,4 | 1.484,113 | 160.476,1 | .094,042,36 | 40.955,11 |
| 39 | 51.358,02 | 24.831,96 | 618.898,2 | 6.913,730 | 242.962,6 | 8.128,634 | 366.254,7 | 1.644,999 | 168.785,5 | .104,313,7 | 43.123,20 |
| 40 | 55.344,09 | 27.367,60 | 647.131,5 | 7.824,269 | 261.071,9 | 8.974,596 | 383.974,6 | 1.818,490 | 177.285,2 | .115,402,2 | 45.345,94 |
| 41 | 59.525,96 | 30.083,70 | 675.703,9 | 8.826,449 | 279.976,9 | 9.883,279 | 402.014,2 | 2.005,206 | 185.973,0 | .127,349,7 | 47.623,21 |
| 42 | 63.907,68 | 32.987,29 | 704.590,9 | 9.926,690 | 299.685,0 | 10.857,46 | 420.364,6 | 2.205,780 | 194.847,2 | .140,199,1 | 49.954,88 |
| 43 | 68.493,22 | 36.085,42 | 733.768,1 | 11.131,66 | 320.202,8 | 11.899,95 | 439.016,6 | 2.420,857 | 203.905,6 | .153,994,3 | 52.340,82 |
| 44 | 73.286,50 | 39.385,07 | 763.210,5 | 12.448,27 | 341.536,0 | 13.013,59 | 457.961,1 | 2.651,096 | 213.146,2 | .168,780,2 | 54.780,89 |
| 45 | 78.291,38 | 42.893,22 | 792.893,2 | 13.863,68 | 363.689,5 | 14.201,25 | 477.188,7 | 2.897,163 | 222.567,1 | .184,602,7 | 57.274,95 |
| 46 | 83.511,66 | 46.616,76 | 822.791,5 | 15.445,29 | 386.667,7 | 15.465,80 | 496.689,8 | 3.159,739 | 232.166,0 | .201,508,7 | 59.822,87 |
| 47 | 88.951,05 | 50.562,52 | 852.879,9 | 17.140,73 | 410.473,8 | 16.810,16 | 516.455,0 | 3.439,516 | 241.940,8 | .219,546,1 | 62.424,49 |
| 48 | 94.613,22 | 54.737,28 | 883.133,6 | 18.977,88 | 435.110,3 | 18.237,25 | 536.474,5 | 3.737,194 | 251.889,2 | .238,763,7 | 65.079,67 |
| 49 | 100.501,8 | 59.147,70 | 913.527,5 | 20.964,86 | 460.579,0 | 19.750,02 | 556.738,5 | 4.053,486 | 262.009,2 | .259,211,3 | 67.788,26 |
| 50 | 106.620,2 | 63.800,35 | 944.036,5 | 23.110,00 | 486.880,9 | 21.351,42 | 577.237,1 | 4.389,115 | 272.298,4 | .280,939,6 | 70.550,10 |
| 51 | 112.972,0 | 68.701,69 | 974.635,4 | 25.421,86 | 514.016,1 | 23.044,41 | 597.960,4 | 4.744,813 | 282.754,5 | .304,000,6 | 73.365,04 |
| 52 | 119.560,5 | 73.858,05 | 1,005.299 | 27.909,24 | 541.983,8 | 24.831,96 | 618.898,2 | 5.121,323 | 293.375,2 | .328,446,8 | 76.232,91 |
| 53 | 126.389,0 | 79.275,64 | 1,036.004 | 30.581,16 | 570.782,7 | 26.717,05 | 640.040,4 | 5.519,394 | 304.158,1 | .354,331,9 | 79.153,56 |
| 54 | 133.460,8 | 84.960,50 | 1,066.723 | 33.446,84 | 600.410,3 | 28.702,65 | 661.376,8 | 5.939,789 | 315.100,9 | .381,710,7 | 82.126,82 |
| 55 | 140.779,1 | 90.918,53 | 1,097.434 | 36.515,71 | 630.863,8 | 30.791,74 | 682.897,1 | 6.383,276 | 326.200,9 | .410,638,4 | 85.152,51 |
| 56 | 148.346,8 | 97.155,45 | 1,128.110 | 39.797,39 | 662.139,3 | 32.987,29 | 704.590,9 | 6.850,633 | 337.455,9 | .441,171,9 | 88.230,48 |
| 57 | 156.167,1 | 103.676,8 | 1,158.729 | 43.301,73 | 694.232,0 | 35.292,26 | 726.448,0 | 7.342,646 | 348.863,4 | .473,368,5 | 91.360,53 |
| 58 | 164.242,9 | 110.488,0 | 1,189.266 | 47.038,77 | 727.136,8 | 37.709,62 | 748.457,7 | 7.860,109 | 360.420,7 | .507,286,4 | 94.542,51 |
| 59 | 172.577,0 | 117.594,0 | 1,219.698 | 51.018,69 | 760.847,4 | 40.242,29 | 770.609,6 | 8.403,823 | 372.125,3 | .542,985,1 | 97.776,21 |
| 60 | 181.172,2 | 125.000,0 | 1,250.000 | 55.251,87 | 795.357,0 | 42.893,22 | 792.893,2 | 8.974,596 | 383.974,6 | .580,524,7 | 101.061,5 |
| 61 | 190.031,1 | 132.710,6 | 1,280.150 | 59.748,87 | 830.657,9 | 45.665,30 | 815.298,0 | 9.573,245 | 395.966,0 | .619,966,3 | 104.398,1 |
| 62 | 199.156,6 | 140.730,2 | 1,310.125 | 64.520,41 | 866.741,9 | 48.561,44 | 837.813,4 | 10.200,59 | 408.096,9 | .661,372,0 | 107.785,9 |
| 63 | 208.550,9 | 149.063,2 | 1,339.902 | 69.577,35 | 903.599,8 | 51.584,48 | 860.428,8 | 10.857,46 | 420.364,6 | .704,804,6 | 111.224,7 |
| 64 | 218.216,7 | 157.713,5 | 1,369.460 | 74.930,67 | 941.222,0 | 54.737,28 | 883.133,6 | 11.544,69 | 432.766,3 | .750,328,1 | 114.714,3 |
| 65 | 228.156,2 | 166.684,8 | 1,398.776 | 80.591,55 | 979.598,0 | 58.022,64 | 905.917,3 | 12.263,12 | 445.299,4 | .798,007,1 | 118.254,4 |
| 66 | 238.371,9 | 175.980,7 | 1,427.829 | 86.571,25 | 1,018.717 | 61.443,34 | 928.769,2 | 13.013,59 | 457.961,1 | .847,907,1 | 121.845,0 |
| 67 | 248.865,8 | 185.604,3 | 1,456.598 | 92.881,15 | 1,058.567 | 65.002,12 | 951.678,8 | 13.796,96 | 470.748,6 | .900,094,7 | 125.485,7 |
| 68 | 259.640,0 | 195.558,5 | 1,485.063 | 99.532,72 | 1,099.135 | 68.701,69 | 974.635,4 | 14.614,08 | 483.659,1 | .954,637,2 | 129.176,5 |
| 69 | 270.696,8 | 205.845,8 | 1,513.204 | 106.537,6 | 1,140.409 | 72.544,71 | 997.628,7 | 15.465,80 | 496.689,8 | 1.011,603 | 132.917,0 |
| 70 | 282.037,9 | 216.468,7 | 1,541.002 | 113.907,4 | 1,182.375 | 76.533,81 | 1,020.648 | 16.352,99 | 509.837,9 | 1.071,060 | 136.707,0 |
| 71 | 293.665,2 | 227.429,2 | 1,568.437 | 121.653,9 | 1,225.018 | 80.671,56 | 1,043.683 | 17.276,52 | 523.100,4 | 1.133,080 | 140.546,4 |
| 72 | 305.580,5 | 238.728,8 | 1,595.492 | 129.788,9 | 1,268.325 | 84.960,50 | 1,066.723 | 18.237,25 | 536.474,5 | 1.197,732 | 144.435,0 |
| 73 | 317.785,6 | 250.368,9 | 1,622.147 | 138.324,2 | 1,312.280 | 89.403,10 | 1,089.758 | 19.236,07 | 549.957,3 | 1.265,089 | 148.372,5 |
| 74 | 330.282,0 | 262.350,6 | 1,648.387 | 147.271,9 | 1,356.866 | 94.001,78 | 1,112.778 | 20.273,83 | 563.545,8 | 1.335,222 | 152.358,6 |
| 75 | 343.071,1 | 274.674,6 | 1,674.194 | 156.643,8 | 1,402.068 | 98.758,91 | 1,135.772 | 21.351,42 | 577.237,1 | 1.408,206 | 156.393,2 |
| 76 | 356.154,5 | 287.341,2 | 1,699.552 | 166.451,8 | 1,447.869 | 103.676,8 | 1,158.729 | 22.469,72 | 591.028,3 | 1.484,113 | 160.476,1 |
| 77 | 369.533,5 | 300.350,4 | 1,724.446 | 176.707,9 | 1,494.251 | 108.757,7 | 1,181.641 | 23.629,61 | 604.916,3 | 1.563,019 | 164.606,9 |
| 78 | 383.209,2 | 313.701,9 | 1,748.861 | 187.424,2 | 1,541.198 | 114.003,8 | 1,204.497 | 24.831,96 | 618.898,2 | 1.644,999 | 168.785,5 |
| 79 | 397.182,9 | 327.395,0 | 1,772.783 | 198.612,5 | 1,588.690 | 119.417,2 | 1,227.286 | 26.077,67 | 632.970,9 | 1.730,131 | 173.011,7 |
| 80 | 411.455,7 | 341.428,7 | 1,796.198 | 210.284,6 | 1,636.709 | 125.000,0 | 1,250.000 | 27.367,60 | 647.131,5 | 1.818,490 | 177.285,2 |
| 81 | 426.028,4 | 355.801,4 | 1,819.094 | 222.452,6 | 1,685.237 | 130.754,2 | 1,272.627 | 28.702,65 | 661.376,8 | 1.910,156 | 181.605,7 |
| 82 | 440.901,9 | 370.511,5 | 1,841.458 | 235.128,2 | 1,734.253 | 136.681,5 | 1,295.161 | 30.083,70 | 675.703,9 | 2.005,206 | 185.973,0 |
| 83 | 456.077,2 | 385.556,7 | 1,863.279 | 248.323,1 | 1,783.740 | 142.783,9 | 1,317.589 | 31.511,62 | 690.109,6 | 2.103,721 | 190.386,9 |
| 84 | 471.554,7 | 400.934,6 | 1,884.545 | 262.049,0 | 1,833.675 | 149.063,2 | 1,339.902 | 32.987,29 | 704.590,9 | 2.205,780 | 194.847,2 |
| 85 | 487.335,2 | 416.642,3 | 1,905.248 | 276.317,4 | 1,884.041 | 155.521,0 | 1,362.092 | 34.511,60 | 719.144,8 | 2.311,465 | 199.353,5 |
| 86 | 503.419,1 | 432.676,5 | 1,925.378 | 291.139,9 | 1,934.816 | 162.158,8 | 1,384.149 | 36.085,42 | 733.768,1 | 2.420,857 | 203.905,6 |
| 87 | 519.806,9 | 449.033,6 | 1,944.925 | 306.527,7 | 1,985.979 | 168.978,3 | 1,406.064 | 37.709,62 | 748.457,7 | 2.534,040 | 208.503,3 |
| 88 | 536.498,9 | 465.709,5 | 1,963.883 | 322.492,1 | 2,037.510 | 175.980,7 | 1,427.829 | 39.385,07 | 763.210,5 | 2.651,096 | 213.146,2 |
| 89 | 553.495,3 | 482.699,9 | 1,982.243 | 339.044,0 | 2,089.389 | 183.167,5 | 1,449.433 | 41.112,65 | 778.023,4 | 2.772,108 | 217.834,3 |
| 90 | 570.796,3 | 500.000,0 | 2,000.000 | 356.194,5 | 2,141.593 | 190.539,9 | 1,470.870 | 42.893,22 | 792.893,2 | 2.897,163 | 222.567,1 |

SHEET 1 OF 5 205-D-394

Table H-8.—*Load constants for circular arches–radial deflections* (sheet 2).

UNIT RADIAL LOAD NO. 2
RADIAL DEFLECTIONS

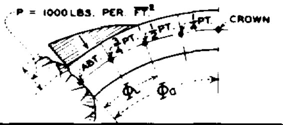


| $\phi$ | Crown | | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_2$ | | $D_2$ | | $D_2$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .003.461,094 | .0$^{3}$571,230.3 | 939.663,1 | .0$^{4}$47.553.5 | 151.630.9 | .0$^{3}$421,790.1 | .945.382.8 | 0$^{3}$271 5470 | .949.302.9 | .0$^{3}$.120.787.0 | .951 416.0 |
| 11 | .004.606.654 | .0$^{3}$835,516.7 | 1.133884 | .0$^{4}$76.546.9 | .201.634.6 | .0$^{3}$.617.208.5 | 1.142.252 | 0$^{3}$.397,478.6 | 1.147,990 | .0$^{3}$.176,833.3 | 1.151,083 |
| 12 | .005.980.602 | .001.182,066 | 1.345,368 | .0$^{3}$.118.204.3 | .261,512.2 | .0$^{3}$873.631,6 | 1.357211 | 0$^{3}$.562,802,9 | 1.365,333 | .0$^{3}$250,431 5 | 1.369.714 |
| 13 | .007.603,689 | .001,626,228 | 1.573.781 | .0$^{3}$.176.272.8 | .332,124.5 | .001,202.532 | 1.590,079 | 0$^{3}$.774,967.0 | 1.601,262 | .0$^{3}$.344.909.7 | 1.607.295 |
| 14 | .009.496.661 | .002.184,593 | 1.818.761 | .0$^{3}$.255,170.3 | .414.323.3 | .001,616,339 | 1.840.662 | .001,042,055 | 1.855.697 | .0$^{3}$.463.884,2 | 1.863.810 |
| 15 | .011,680.26 | .002,874,968 | 2.079.917 | .0$^{3}$.360.036 | .508,949.7 | .002.128434 | 2.108.750 | .001,372,786 | 2.128,554 | .0$^{3}$.611,259.1 | 2.139,243 |
| 16 | .014,175.24 | .003.716,346 | 2.356,831 | .0$^{3}$.496.787 | .616.834.0 | .002.753.133 | 2 394.118 | .001.776,507 | 2.419,741 | .0$^{3}$,791,225.8 | 2.433.576 |
| 17 | .017,002.31 | .004.728889 | 2.649.059 | .0$^{3}$,672,161 | .738.795.0 | .003.505,683 | 2.696 526 | 002.263,197 | 2.729,162 | .001,008,264 | 2.746.790 |
| 18 | .020,182.23 | .005.933,890 | 2.956,128 | .0$^{3}$,893,778 | .875,638.4 | .004.402,244 | 3.015.717 | 002,843,456 | 3.056,714 | .001,267,137 | 3.078,864 |
| 19 | .023.735.71 | .007.353,745 | 3.277.540 | .001,170,180 | 1.028,157 | .005.459,878 | 3.351,424 | 003,528,507 | 3.402.286 | .001.572,898 | 3.429,778 |
| 20 | .027,683.48 | .009,011,923 | 3.612,770 | .001,510,887 | 1.197,130 | .006,696,539 | 3.703.359 | .004.330,189 | 3.765,763 | .001,930,885 | 3.799,508 |
| 21 | .032,046.25 | .010.932.93 | 3.961,265 | .001,926434 | 1.383.321 | .008.131,053 | 4.071.224 | 005,260,956 | 4.147,024 | .002,346.719 | 4.188.030 |
| 22 | .036,844,74 | .013.142.27 | 4.322,451 | .002428435 | 1.587,479 | .009.783,111 | 4.454.705 | .006,333,871 | 4,545,941 | .002.826.311 | 4.595.320 |
| 23 | .042 099.66 | .015.666.42 | 4.695.724 | .003,029,610 | 1.810.337 | .011,673,24 | 4.853.472 | .007,562,593 | 4.962,379 | .003 375,845 | 5.021.350 |
| 24 | .047,831,71 | .018.532.76 | 5.080,459 | .003,743841 | 2.052,609 | .013,822,80 | 5.267.184 | .008,961,396 | 5.396.200 | .004.001.806 | 5.466,095 |
| 25 | .054.061.57 | .021.769,59 | 5.476,003 | .004.586,214 | 2.314.995 | .016.253,97 | 5.695.482 | .010,545,14 | 5.847.256 | .004,710,950 | 5.929.523 |
| 26 | .060.809.95 | .025.406.02 | 5.881,682 | .005.573.052 | 2.598,175 | .018,989,71 | 6.137.995 | .012,329,27 | 6.315.397 | .005,510.315 | 6.411,606 |
| 27 | .068.097,52 | .029.471,98 | 6.296.797 | .006.721.967 | 2.902,809 | .022,053,77 | 6.594.339 | .014.329,83 | 6.800.463 | .006.407.230 | 6.912,312 |
| 28 | .075,944.95 | .033.998.13 | 6.720,626 | .008.051,894 | 3.229,540 | .025,470,63 | 7.064 114 | .016,563,43 | 7.302,291 | .007.409,296 | 7.431,609 |
| 29 | .084.372,92 | .039.015,87 | 7.152,426 | 009.583.134 | 3.578.988 | .029,265,54 | 7.546,907 | .019,047,26 | 7.820,712 | .008.524,401 | 7.969.462 |
| 30 | .093,402,09 | .044,557,24 | 7.591,431 | .011,337,37 | 3.951,755 | .033,464,45 | 8.042,293 | .021,799,07 | 8.355.548 | .009,760.710 | 8.525.839 |
| 31 | .103.053.1 | .050,654,88 | 8.036,851 | .013,337.75 | 4.348,420 | .038,094,17 | 8.549.831 | .024,837,19 | 8.906,620 | .011,126.67 | 9.100,701 |
| 32 | .113.3466 | .057,341.95 | 8487,882 | .015,608.84 | 4.769,537 | .043,181,48 | 9 069 069 | .028.180.48 | 9.473,738 | .012,630,99 | 9.694,012 |
| 33 | .124.303.3 | .064.652.34 | 8.943,689 | .018.176.77 | 5.215,651 | .048,754,91 | 9.599.542 | .031,848.39 | 10,056.71 | .014.282,69 | 10.305,73 |
| 34 | .135.943.7 | .072.620.06 | 9.403,426 | .021,069.12 | 5.687,261 | .054,842,85 | 10.140.77 | .035,860.86 | 10.655.33 | .016,091,04 | 10.935.82 |
| 35 | .148,288.5 | .081,279,78 | 9.866,229 | .024.315.11 | 6.184,864 | .061,474,56 | 10.692.26 | 040.238,39 | 11,269,41 | .018,065,60 | 11.584.25 |
| 36 | .161,358.3 | .090.666.37 | 10.331,21 | .027,945,47 | 6.708.918 | .068,679.76 | 11.253.52 | 045,002.02 | 11.898.73 | .020,216,19 | 12.250.96 |
| 37 | .175.173,6 | .100.815.1 | 10.797,45 | .031.992.62 | 7.259,863 | .076,488,81 | 11 824.02 | .050.173,30 | 12.543,07 | .022,552.92 | 12.935.91 |
| 38 | .189.755.2 | .111,761,3 | 11.264,05 | .036,490.54 | 7.838,113 | .084.932.59 | 12403.23 | 055,774,29 | 13.202.21 | 025.086,16 | 13.639.07 |
| 39 | .205.123,6 | .123.540.7 | 11.730,06 | .041.474.91 | 8444,055 | .094,042,47 | 12990,63 | .061,827,55 | 13.875,92 | .027.826,57 | 14.360.37 |
| 40 | .221.299.2 | .136,189.0 | 12.194,52 | .046,983.06 | 9.078,050 | .103,850.3 | 13.585.65 | .068,356,16 | 14.563,98 | .030,785,06 | 15.099,79 |
| 41 | 238.302.8 | .149,742.0 | 12.656,47 | .053,054,05 | 9.740,432 | .114,388,4 | 14.187.73 | 075,383,69 | 15.266.13 | .033,972,83 | 15.857.27 |
| 42 | .256,154.9 | .164.235.5 | 13.114,92 | .059,728,64 | 10431,51 | .125,689.5 | 14.796.30 | .082,934.16 | 15.982.15 | .037401,34 | 16.632.75 |
| 43 | .274,875.9 | .179,705.1 | 13.568,87 | .067,049,26 | 11.151,56 | .137,786.7 | 15410.77 | .091,032,10 | 16.711.77 | .041,082,32 | 17.426 20 |
| 44 | .294.486.5 | .196.186.6 | 14.017,30 | .075,060,18 | 11.900,83 | .150,713.5 | 16030.55 | .099,702.47 | 17.454.75 | .045,027,76 | 18.237.56 |
| 45 | .315,007.1 | .213.715.1 | 14.459,20 | .083,807,36 | 12.679,54 | .164,503,8 | 16.655 03 | .108,970,7 | 18.210.81 | .049.249,94 | 19.066.77 |
| 46 | .336.458.3 | .232.326.0 | 14.893,53 | .093,338,54 | 13.487,90 | .179,191,7 | 17.283.59 | .118.862,7 | 18.979.71 | .053,761,38 | 19.913.78 |
| 47 | .358,860.5 | .252,054.1 | 15.319,23 | .103,703,3 | 14.326,06 | .194.811.8 | 17.915,61 | .129,405,1 | 19.761.16 | .058.575,10 | 20.778,54 |
| 48 | .382,234.3 | .272.933.4 | 15.735,26 | .114,952,6 | 15.194,15 | .211,398,2 | 18.550.44 | .140,623.8 | 20.554.89 | .063,703,50 | 21.660.99 |
| 49 | .406.600.0 | .294.998.3 | 16.140,53 | .127,139.8 | 16.092,29 | .228.986.2 | 19.187.45 | .152,546.7 | 21.360.62 | .069.160,53 | 22.561.08 |
| 50 | .431,978.3 | .318,282.2 | 16.533,98 | .140319.4 | 17.020,54 | .247,611,1 | 19.825.97 | .165,201,3 | 22.178,06 | .074,959,59 | 23.478,73 |
| 51 | .458.389.4 | .342,817.9 | 16.914,51 | .154,548.4 | 17.978,94 | .267,307.8 | 20465.34 | .178,615.4 | 23.006,92 | .081,114,50 | 24.413,90 |
| 52 | .485,853.9 | .368,637.5 | 17.281,04 | .169,884.9 | 18.967,51 | .288,111.7 | 21.104,89 | .192,817.4 | 23.846.91 | .087.639,34 | 25.366,53 |
| 53 | .514,392.2 | .395,772.6 | 17.632,47 | .186,389,0 | 19.986.23 | .310,058.4 | 21.743,93 | .207,836.2 | 24.697,71 | .094.548.51 | 26.336.55 |
| 54 | .544,024.6 | .424.253.9 | 17.967,68 | .204,122,7 | 21.035,04 | .333,183,2 | 22.381.77 | .223,700.8 | 25.559,03 | .101,856.6 | 27.323,90 |
| 55 | .574,771,6 | .454,111.2 | 18.285.58 | .223,149,7 | 22.113,87 | .357,521.7 | 23.017 71 | .240440.6 | 26.430.56 | .109,578.5 | 28.328.52 |
| 56 | .606,653.5 | .485,373.7 | 18.585.04 | .243,535.0 | 23.222.60 | 383,109.7 | 23.651.04 | .258,085.7 | 27.311,97 | .117,729,2 | 29.350,33 |
| 57 | .639,690.7 | .518,069.0 | 18.864.95 | .265,346,0 | 24.361,08 | .409,982.3 | 24.281.06 | 276,666,0 | 28.202,95 | .126,324.3 | 30.389,28 |
| 58 | .673,903.6 | .552,224.2 | 19.124,19 | .288,651.5 | 25.529,12 | .438,175,1 | 24.907.02 | .296,212,2 | 29.103,16 | .135,379.2 | 31.445.29 |
| 59 | .709,312,5 | .587,864.9 | 19.361.64 | .313,521.9 | 26.726,53 | .467,723,3 | 25.528.22 | .316,755,0 | 30.012,29 | .144,909,9 | 32.518,30 |
| 60 | .745,937,6 | .625,015.9 | 19.576,17 | .340,029.5 | 27.953.05 | .498,662,2 | 26.143.91 | .338.325.5 | 30.929,99 | .154,932,4 | 33.608,23 |
| 61 | .783,799.4 | .663,700,3 | 19.766.66 | .368.247,7 | 29.208.41 | .531,026,6 | 26753.35 | .360,955,1 | 31.855.92 | .165,463.3 | 34.715,03 |
| 62 | .822,918.1 | .703.940.3 | 19.931,99 | .398.252.4 | 30492,28 | .564851.4 | 27.355.78 | .384,675.6 | 32.789,74 | .176,518.9 | 35.838.60 |
| 63 | .863,313.9 | .745.756.6 | 20.071.04 | .430,120.3 | 31.804,34 | .600.171.1 | 27950.46 | .409.519,0 | 33.731,10 | .188,116.3 | 36.978,88 |
| 64 | .905.007.1 | .789,168.2 | 20.182,69 | .463930.1 | 33.144,21 | .637,019.9 | 28.536,63 | .435,517,4 | 34679.65 | .200.272.5 | 38.135,80 |
| 65 | .948.018,1 | .834,192.8 | 20.265,83 | .499,762.0 | 34.511,47 | .675431,7 | 29113.52 | 462.703,3 | 35.635,03 | .213,004.8 | 39.309,28 |
| 66 | .992.366.9 | .880,846,7 | 20.319,35 | .537,697,5 | 35.905.67 | .715440.2 | 29680.36 | .491.109.5 | 36596.87 | .226.330,9 | 40.499.25 |
| 67 | 1.038,074 | .929.144.0 | 20.342,14 | .577,819.8 | 37.326.36 | .757,078.2 | 30.236.37 | .520.768.8 | 37.564.82 | .240,268.4 | 41.705,62 |
| 68 | 1.085.159 | .979,097.7 | 20.333.10 | .620,213.4 | 38.773.00 | .800,378,9 | 30 780.77 | .551,714.6 | 38.538.49 | .254.835,7 | 42.928,32 |
| 69 | 1.133.643 | 1.030.719 | 20.291,15 | .664,963.9 | 40.245.07 | .845,374,2 | 31.312.81 | .583,979.8 | 39.517.53 | 270,050.5 | 44.167,27 |
| 70 | 1.183.545 | 1.084.016 | 20.215,19 | .712.159,3 | 41.741,99 | .892.096,1 | 31.831.63 | .617,598,2 | 40.501,53 | .285,931,8 | 45.422.38 |
| 71 | 1.234.886 | 1.138,997 | 20.104,15 | .761,887,3 | 43.263.15 | 940,575.6 | 32.336.50 | .652.603.6 | 41,490.14 | .302.498,2 | 46.693.57 |
| 72 | 1.287,685 | 1.195.667 | 19.956,97 | .814,238,3 | 44.807,91 | .990.843.7 | 32826.61 | .689,029.7 | 42482,96 | .319.768.7 | 47,980,77 |
| 73 | 1.341,964 | 1.254,029 | 19.772,59 | .869.303,2 | 46.375.59 | 1.042.930 | 33.301.16 | .726,910.6 | 43479,59 | .337,762,4 | 49.283.89 |
| 74 | 1.397.742 | 1.314,085 | 19.549.96 | .927174,2 | 47.965.49 | 1.096,865 | 33.759.36 | .766.280.4 | 44.479.64 | .356.498.9 | 50.602,84 |
| 75 | 1.455,039 | 1.375,834 | 19.288,05 | .987,944.7 | 49.576.87 | 1.152 676 | 34.200 40 | .807,173.3 | 45.482,73 | .375 997.6 | 51.937.53 |
| 76 | 1.513,875 | 1.439.272 | 18.985,84 | 1.051,709 | 51.208,97 | 1.210,391 | 34623.47 | .849,623.8 | 46.488.43 | .396,278.6 | 53.287,88 |
| 77 | 1.574.270 | 1.504.394 | 18.642,33 | 1.118,563 | 52.860.97 | 1.270.039 | 35027.78 | .893,666,3 | 47.496,36 | .417,361,9 | 54.653,80 |
| 78 | 1.636,244 | 1.571.193 | 18.256,51 | 1.188,602 | 54.532,04 | 1.331.645 | 35.412.51 | .939,335.3 | 48.506.10 | .439.267.8 | 56.035.21 |
| 79 | 1.699,818 | 1.639,658 | 17.827,42 | 1.261,923 | 56.221,32 | 1.395.235 | 35.776.87 | .986.665.6 | 49.517,24 | .462,016.8 | 57.432.00 |
| 80 | 1.765,010 | 1.709.778 | 17.354,10 | 1.338,626 | 57.927.91 | 1.460.834 | 36.120,03 | 1.035,692 | 50.529,36 | .485.629.8 | 58.844.09 |
| 81 | 1.831,842 | 1.781.538 | 16.835,60 | 1.418,809 | 59.650.89 | 1.528.465 | 36441 20 | 1.086,448 | 51.542,04 | .510.127.5 | 60.271 39 |
| 82 | 1.900.332 | 1.854920 | 16.270,99 | 1.502,570 | 61.389,29 | 1.598.153 | 36.739,57 | 1.138,970 | 52.554,87 | .535,531,2 | 61.713,80 |
| 83 | 1.970,502 | 1.929.905 | 15.659,37 | 1.590,011 | 63.142,13 | 1.669,917 | 37014.32 | 1.193 292 | 53.567.42 | 561,862,2 | 63.171,23 |
| 84 | 2.042,370 | 2.006470 | 14.999.85 | 1.681,232 | 64.908.38 | .743.780 | 37.264,65 | 1.249,449 | 54.579,25 | .589.142.3 | 64.643.59 |
| 85 | 2.115.956 | 2.084.590 | 14.291,57 | 1.776,334 | 66.687.01 | 1.819.763 | 37.489,76 | 1.307.476 | 55.589,94 | .617.393.0 | 66.130.76 |
| 86 | 2.191,281 | 2.164.238 | 13.533,68 | 1.875,420 | 68.476,93 | 1.897.882 | 37.688.85 | 1.367407 | 56.599,06 | .646.636.4 | 67.632,67 |
| 87 | 2.268.365 | 2.245.384 | 12.725,35 | 1.978,590 | 70.277.04 | 1.978,156 | 37.861,10 | 1.429.276 | 57.606.16 | .676,894.7 | 69.149,20 |
| 88 | 2.347,226 | 2.327.994 | 11.865,78 | 2085947 | 72086,20 | 2.060,602 | 38.005.73 | 1.493,119 | 58.610.81 | .708,190.4 | 70.680,27 |
| 89 | 2.427,885 | 2.412.033 | 10.954,20 | 2.197,594 | 73.903,25 | 2.145.235 | 38.121,92 | 1.558,971 | 59.612.56 | .740.545.8 | 72.225,76 |
| 90 | 2.510,362 | 2.497463 | 9.989.850 | 2.313.633 | 75.727,01 | 2.232,069 | 38.208.86 | 1.626.865 | 60.610,97 | .773.984.0 | 73.795,58 |

SHEET 2 OF 5 205-D-395

Table H-8.–*Load constants for circular arches–radial deflections* (sheet 3).

UNIT RADIAL LOAD NO. 3
RADIAL DEFLECTIONS



| $\Phi$ | $D_1$ | Crown | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $D_2$ | | $D_3$ | | $D_2$ | | $D_2$ | | $D_2$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .027,683,48 | .004,330,189 | 3.765,763 | .0$^3$,343,603 | .551,592,0 | .003,133,519 | 3.786,239 | .001,930,885 | 3.799,508 | .0$^3$,286,864,0 | 1.902,923 |
| 11 | .036,844,74 | .006,333,871 | 4.545,941 | .0$^3$,553,096 | .733,550,1 | .004,585,372 | 4.575,900 | .002,826,311 | 4.595,320 | .0$^3$,419,970,1 | 2.302,299 |
| 12 | .047,831,71 | .008,961,396 | 5.396,200 | .0$^3$,854,090 | .951,468,0 | .006,490,496 | 5.438,601 | .004,001,806 | 5.466,095 | .0$^3$,594,759,5 | 2.739,616 |
| 13 | .060,809,95 | .012,329,27 | 6.315,397 | .001,273,658 | 1.208,493 | .008,934,161 | 6.373,752 | .005,510,315 | 6.411,606 | .0$^3$,819,135,8 | 3.214,848 |
| 14 | .075,944,95 | .016,563,43 | 7.302,291 | .001,843,716 | 1.507,742 | .012,008,76 | 7.380,717 | .007,409,296 | 7.431,609 | .001,101,686 | 3.727,968 |
| 15 | .093,402,09 | .021,799,07 | 8.355,548 | .002,601,396 | 1.852,296 | .015,813,75 | 8.458,805 | .009,760,710 | 8.525,839 | .001,451,682 | 4.278,944 |
| 16 | .113,346,6 | .028,180,48 | 9.473,738 | .003,589,45 | 2.245,200 | .020,455,57 | 9.607,278 | .012,630,99 | 9.694,012 | .001,879,075 | 4.867,746 |
| 17 | .135,943,7 | .035,860,86 | 10.655,33 | .004,856,55 | 2.689,460 | .026,047,58 | 10.825,35 | .016,091,04 | 10.935,82 | .002,394,502 | 5.494,336 |
| 18 | .161,358,3 | .045,002,02 | 11.898,73 | .006,457,71 | 3.188,042 | .032,709,94 | 12.112,18 | .020,216,19 | 12.250,96 | .003,009,276 | 6.158,680 |
| 19 | .189,755,2 | .055,774,29 | 13.202,21 | .008,454,70 | 3.743,865 | .040,569,56 | 13.466,89 | .025,086,16 | 13.639,07 | .003,735,392 | 6.860,737 |
| 20 | .221,299,2 | .068,356,16 | 14.563,98 | .010,916,23 | 4.359,804 | .049,759,96 | 14.888,54 | .030,785,06 | 15.099,79 | .004,585,523 | 7.600,466 |
| 21 | .256,154,9 | .082,934,16 | 15.982,15 | .013,918,44 | 5.038,685 | .060,421,22 | 16.376,15 | .037,401,34 | 16.632,75 | .005,573,018 | 8.377,824 |
| 22 | .294,486,5 | .099,702,47 | 17.454,75 | .017,545,19 | 5.783,281 | .072,699,81 | 17.928,69 | .045,027,76 | 18.237,56 | .006,711,903 | 9.192,765 |
| 23 | .336,458,3 | .118,862,7 | 18.979,71 | .021,888,37 | 6.596,312 | .086,748,53 | 19.545,08 | .053,761,38 | 19.913,78 | .008,016,875 | 10.045,24 |
| 24 | .382,234,3 | .140,623,8 | 20.554,89 | .027,048,23 | 7.480,444 | .102,726,4 | 21.224,21 | .063,703,50 | 21.660,99 | .009,503,311 | 10.935,20 |
| 25 | .431,978,3 | .165,201,3 | 22.178,06 | .033,133,70 | 8.438,282 | .120,798,4 | 22.964,89 | .074,959,59 | 23.478,73 | .011,187,25 | 11.862,59 |
| 26 | .485,853,9 | .192,817,4 | 23.846,91 | .040,262,80 | 9.472,375 | .141,135,6 | 24.765,92 | .087,639,34 | 25.366,53 | .013,085,42 | 12.827,36 |
| 27 | .544,024,6 | .223,700,8 | 25.559,03 | .048,562,54 | 10.585,20 | .163,914,7 | 26.626,02 | .101,856,6 | 27.323,90 | .015,215,19 | 13.829,45 |
| 28 | .606,653,5 | .258,085,7 | 27.311,97 | .058,169,75 | 11.779,19 | .189,318,6 | 28.543,91 | .117,729,2 | 29.350,33 | .017,594,61 | 14.868,80 |
| 29 | .673,903,6 | .296,212,2 | 29.103,16 | .069,230,92 | 13.056,68 | .217,534,9 | 30.518,22 | .135,379,2 | 31.445,29 | .020,242,41 | 15.945,35 |
| 30 | .745,937,6 | .338,325,5 | 30.929,99 | .081,902,86 | 14.419,97 | .248,757,1 | 32.547,56 | .154,932,4 | 33.608,23 | .023,177,95 | 17.059,03 |
| 31 | .822,918,1 | .384,675,6 | 32.789,74 | .096,352,42 | 15.871,25 | .283,183,9 | 34.630,49 | .176,518,9 | 35.838,60 | .026,421,29 | 18.209,79 |
| 32 | .905,007,1 | .435,517,4 | 34.679,65 | .112,757,0 | 17.412,68 | .321,019,1 | 36.765,53 | .200,272,5 | 38.135,80 | .029,993,12 | 19.397,55 |
| 33 | .992,366,9 | .491,109,5 | 36.596,87 | .131,305,7 | 19.046,32 | .362,471,2 | 38.951,35 | .226,330,9 | 40.499,25 | .033,914,79 | 20.622,24 |
| 34 | 1.085,159 | .551,714,6 | 38.538,49 | .152,197,2 | 20.774,15 | .407,753,9 | 41.185,80 | .254,835,7 | 42.928,32 | .038,208,33 | 21.883,79 |
| 35 | 1.183,545 | .617,598,2 | 40.501,53 | .175,642,3 | 22.598,08 | .457,084,8 | 43.467,86 | .285,931,8 | 45.422,38 | .042,896,41 | 23.182,13 |
| 36 | 1.287,685 | .689,029,7 | 42.482,96 | .201,863,2 | 24.519,95 | .510,686,6 | 45.795,69 | .319,768,7 | 47.980,77 | .048,002,34 | 24.517,18 |
| 37 | 1.397,742 | .766,280,4 | 44.479,64 | .231,093,8 | 26.541,49 | .568,785,9 | 48.167,59 | .356,498,9 | 50.602,84 | .053,550,08 | 25.888,86 |
| 38 | 1.513,875 | .849,823,8 | 46.488,43 | .263,579,2 | 28.664,36 | .631,613,4 | 50.581,86 | .396,278,6 | 53.287,88 | .059,564,26 | 27.297,09 |
| 39 | 1.636,244 | .939,335,3 | 48.506,10 | .299,577,1 | 30.890,16 | .699,403,7 | 53.036,71 | .439,267,8 | 56.035,21 | .066,070,14 | 28.741,78 |
| 40 | 1.765,010 | 1.035,692 | 50.529,36 | .339,357,0 | 33.220,36 | .772,395,3 | 55.530,36 | .485,629,8 | 58.844,09 | .073,093,62 | 30.222,87 |
| 41 | 1.900,332 | 1.138,970 | 52.554,87 | .383,200,5 | 35.656,36 | .850,829,4 | 58.060,95 | .535,531,2 | 61.713,80 | .080,661,25 | 31.740,24 |
| 42 | 2.042,370 | 1.249,449 | 54.579,25 | .431,402,0 | 38.199,47 | .934,951,8 | 60.626,62 | .589,142,3 | 64.643,59 | .088,800,21 | 33.293,82 |
| 43 | 2.191,281 | 1.367,407 | 56.599,06 | .484,267,7 | 40.850,91 | 1.025,010 | 63.225,44 | .646,636,4 | 67.632,67 | .097,538,30 | 34.883,51 |
| 44 | 2.347,226 | 1.493,119 | 58.610,81 | .542,117,0 | 43.611,81 | 1.121,256 | 65.855,49 | .708,190,4 | 70.680,27 | .106,904,0 | 36.509,22 |
| 45 | 2.510,362 | 1.626,865 | 60.610,97 | .605,281,3 | 46.483,19 | 1.223,942 | 68.514,77 | .773,984,0 | 73.785,58 | .116,926,3 | 38.170,86 |
| 46 | 2.680,847 | 1.768,917 | 62.595,97 | .674,105,1 | 49.465,99 | 1.333,325 | 71.201,27 | .844,200,3 | 76.947,80 | .127,635,1 | 39.868,32 |
| 47 | 2.858,839 | 1.919,551 | 64.562,20 | .748,945,5 | 52.561,11 | 1.449,665 | 73.912,95 | .919,025,3 | 80.166,08 | .139,060,6 | 41.601,51 |
| 48 | 3.044,494 | 2.079,036 | 66.505,99 | .830,172,4 | 55.769,09 | 1.573,221 | 76.647,73 | .998,648,1 | 83.439,58 | .151,233,6 | 43.370,33 |
| 49 | 3.237,969 | 2.247,641 | 68.423,68 | .918,168,1 | 59.090,77 | 1.704,255 | 79.403,49 | 1.083,261 | 86.767,44 | .164,185,9 | 45.174,66 |
| 50 | 3.439,421 | 2.425,632 | 70.311,53 | 1.013,329 | 62.526,62 | 1.843,032 | 82.178,10 | 1.173,058 | 90.148,78 | .177,949,6 | 47.014,41 |
| 51 | 3.649,006 | 2.613,269 | 72.165,80 | 1.116,027 | 66.077,05 | 1.989,817 | 84.969,40 | 1.268,237 | 93.582,70 | .192,557,5 | 48.889,47 |
| 52 | 3.866,879 | 2.810,809 | 73.982,70 | 1.226,755 | 69.742,49 | 2.144,877 | 87.775,18 | 1.368,999 | 97.068,31 | .208,043,1 | 50.799,72 |
| 53 | 4.093,196 | 3.018,505 | 75.758,44 | 1.345,942 | 73.523,06 | 2.308,479 | 90.593,21 | 1.475,546 | 100.604,6 | .224,440,2 | 52.745,06 |
| 54 | 4.328,110 | 3.236,605 | 77.489,18 | 1.473,970 | 77.418,94 | 2.480,891 | 93.421,24 | 1.588,083 | 104.190,8 | .241,783,6 | 54.725,36 |
| 55 | 4.571,776 | 3.465,349 | 79.171,10 | 1.611,330 | 81.430,14 | 2.662,383 | 96.257,01 | 1.706,820 | 107.825,9 | .260,108,4 | 56.740,51 |
| 56 | 4.824,348 | 3.704,973 | 80.800,31 | 1.758,495 | 85.556,58 | 2.853,222 | 99.098,21 | 1.831,966 | 111.508,8 | .279,450,6 | 58.790,40 |
| 57 | 5.085,980 | 3.955,706 | 82.372,95 | 1.915,947 | 89.798,07 | 3.053,679 | 101.942,5 | 1.963,733 | 115.238,7 | .299,846,4 | 60.874,91 |
| 58 | 5.356,824 | 4.217,771 | 83.885,13 | 2.084,183 | 94.154,32 | 3.264,021 | 104.787,5 | 2.102,337 | 119.014,5 | .321,332,8 | 62.993,90 |
| 59 | 5.637,032 | 4.491,380 | 85.332,93 | 2.263,711 | 98.624,92 | 3.484,517 | 107.631,0 | 2.247,995 | 122.835,2 | .343,947,5 | 65.147,26 |
| 60 | 5.926,757 | 4.776,741 | 86.712,47 | 2.455,053 | 103.209,4 | 3.715,434 | 110.470,4 | 2.400,927 | 126.699,9 | .367,728,6 | 67.334,85 |
| 61 | 6.226,150 | 5.074,050 | 88.019,84 | 2.658,738 | 107.907,0 | 3.957,039 | 113.303,3 | 2.561,352 | 130.607,4 | .392,714,8 | 69.556,56 |
| 62 | 6.535,362 | 5.383,498 | 89.251,12 | 2.875,313 | 112.717,2 | 4.209,597 | 116.127,5 | 2.729,494 | 134.556,8 | .418,945,5 | 71.812,25 |
| 63 | 6.854,543 | 5.705,262 | 90.402,39 | 3.105,332 | 117.639,0 | 4.473,372 | 118.940,2 | 2.905,578 | 138.546,9 | .446,460,5 | 74.101,78 |
| 64 | 7.183,843 | 6.039,513 | 91.469,78 | 3.349,362 | 122.671,6 | 4.748,626 | 121.739,3 | 3.089,832 | 142.576,8 | .475,300,3 | 76.425,03 |
| 65 | 7.523,413 | 6.386,410 | 92.449,36 | 3.607,980 | 127.813,8 | 5.035,620 | 124.522,0 | 3.282,483 | 146.645,3 | .505,506,0 | 78.781,86 |
| 66 | 7.873,400 | 6.746,100 | 93.337,27 | 3.881,776 | 133.064,5 | 5.334,611 | 127.286,0 | 3.483,761 | 150.751,3 | .537,119,0 | 81.172,12 |
| 67 | 8.233,954 | 7.118,720 | 94.129,61 | 4.171,348 | 138.422,4 | 5.645,855 | 130.028,6 | 3.693,897 | 154.893,8 | .570,181,6 | 83.595,67 |
| 68 | 8.605,221 | 7.504,397 | 94.822,55 | 4.477,306 | 143.886,2 | 5.969,604 | 132.747,5 | 3.913,125 | 159.071,6 | .604,736,4 | 86.052,39 |
| 69 | 8.987,350 | 7.903,241 | 95.412,22 | 4.800,269 | 149.454,4 | 6.306,109 | 135.440,0 | 4.141,679 | 163.283,6 | .640,826,6 | 88.542,11 |
| 70 | 9.380,488 | 8.315,355 | 95.894,81 | 5.140,868 | 155.125,4 | 6.655,617 | 138.103,6 | 4.379,794 | 167.528,7 | .678,496,0 | 91.064,70 |
| 71 | 9.784,780 | 8.740,825 | 96.266,54 | 5.499,740 | 160.897,4 | 7.018,370 | 140.735,7 | 4.627,706 | 171.805,7 | .717,789,0 | 93.620,01 |
| 72 | 10.200,37 | 9.179,725 | 96.523,62 | 5.877,534 | 166.768,7 | 7.394,607 | 143.333,8 | 4.885,654 | 176.113,4 | .758,750,5 | 96.207,88 |
| 73 | 10.627,41 | 9.632,116 | 96.662,32 | 6.274,907 | 172.737,3 | 7.784,566 | 145.895,2 | 5.153,875 | 180.450,7 | .801,425,7 | 98.828,17 |
| 74 | 11.066,04 | 10.098,04 | 96.678,94 | 6.692,523 | 178.801,2 | 8.188,477 | 148.417,4 | 5.432,609 | 184.816,5 | .845,860,6 | 101.480,7 |
| 75 | 11.516,40 | 10.577,54 | 96.569,78 | 7.131,056 | 184.958,6 | 8.606,566 | 150.897,8 | 5.722,097 | 189.209,5 | .892,101,6 | 104.165,4 |
| 76 | 11.978,63 | 11.070,61 | 96.331,23 | 7.591,187 | 191.206,5 | 9.039,056 | 153.333,7 | 6.022,579 | 193.628,5 | .940,195,7 | 106.882,0 |
| 77 | 12.452,89 | 11.577,27 | 95.959,67 | 8.073,604 | 197.543,2 | 9.486,165 | 155.722,6 | 6.334,297 | 198.072,3 | .990,190,4 | 109.630,4 |
| 78 | 12.939,31 | 12.097,50 | 95.451,57 | 8.579,004 | 203.966,1 | 9.948,104 | 158.061,9 | 6.657,492 | 202.539,8 | 1.042,134 | 112.410,4 |
| 79 | 13.438,03 | 12.631,26 | 94.803,40 | 9.108,085 | 210.472,6 | 10.425,08 | 160.348,9 | 6.992,407 | 207.029,7 | 1.096,074 | 115.221,8 |
| 80 | 13.949,20 | 13.178,51 | 94.011,72 | 9.661,558 | 217.060,1 | 10.917,29 | 162.581,0 | 7.339,285 | 211.540,7 | 1.152,060 | 118.064,6 |
| 81 | 14.472,95 | 13.739,19 | 93.073,10 | 10.240,14 | 223.725,8 | 11.424,94 | 164.755,7 | 7.698,368 | 216.071,6 | 1.210,142 | 120.938,5 |
| 82 | 15.009,43 | 14.313,21 | 91.984,19 | 10.844,54 | 230.466,9 | 11.948,21 | 166.870,2 | 8.069,900 | 220.621,2 | 1.270,368 | 123.843,4 |
| 83 | 15.558,77 | 14.900,47 | 90.741,70 | 11.475,48 | 237.280,4 | 12.487,28 | 168.922,1 | 8.454,124 | 225.188,3 | 1.332,791 | 126.779,1 |
| 84 | 16.121,11 | 15.500,85 | 89.342,37 | 12.133,70 | 244.163,4 | 13.042,33 | 170.908,6 | 8.851,282 | 229.771,5 | 1.397,461 | 129.745,4 |
| 85 | 16.696,59 | 16.114,22 | 87.783,03 | 12.819,93 | 251.112,5 | 13.613,53 | 172.827,2 | 9.261,617 | 234.369,6 | 1.464,429 | 132.742,1 |
| 86 | 17.285,35 | 16.740,43 | 86.060,56 | 13.534,90 | 258.124,7 | 14.201,04 | 174.675,3 | 9.685,371 | 238.981,3 | 1.533,747 | 135.769,1 |
| 87 | 17.887,52 | 17.379,29 | 84.171,91 | 14.279,34 | 265.196,5 | 14.805,02 | 176.450,2 | 10.122,79 | 243.605,4 | 1.605,467 | 138.826,2 |
| 88 | 18.503,23 | 18.030,62 | 82.114,08 | 15.054,00 | 272.324,6 | 15.425,60 | 178.149,4 | 10.574,11 | 248.240,5 | 1.679,643 | 141.913,3 |
| 89 | 19.132,63 | 18.694,22 | 79.884,18 | 15.859,62 | 279.505,4 | 16.062,93 | 179.770,4 | 11.039,57 | 252.885,4 | 1.756,327 | 145.030,0 |
| 90 | 19.775,85 | 19.369,84 | 77.479,35 | 16.696,94 | 286.735,4 | 16.717,15 | 181.310,5 | 11.519,42 | 257.538,7 | 1.835,573 | 148.176,3 |

SHEET 3 OF 5

205-D-396

Table H-8.–*Load constants for circular arches–radial deflections* (sheet 4).

UNIT RADIAL LOAD NO. 4
RADIAL DEFLECTIONS

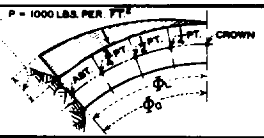
| $\phi$ | Crown | | | | | ¼ Point | | ½ Point | | ¾ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_2$ | | $D_2$ | | $D_2$ | |
| ° | | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term |
| 10 | .093,402,09 | .013,802,32 | 8.485,191 | .001,047,005 | 1.117,013 | .009,760,710 | 8.525,839 | .003,700,643 | 6.333,480 | .0$^{3}$,342,223,1 | 2.220,092 |
| 11 | .124,303,3 | .020,188,92 | 10.246,26 | .001,685,29 | 1.485,502 | .014,282,69 | 10.305,73 | .005,416,706 | 7.660,283 | .0$^{3}$,501,015,7 | 2.686,037 |
| 12 | .161,358,3 | .028,563,98 | 12.166,79 | .002,602,28 | 1.926,801 | .020,216,19 | 12.250,96 | .007,669,476 | 9.112,164 | .0$^{3}$,709,535,5 | 3.196,250 |
| 13 | .205,123,6 | .039,298,80 | 14.244,54 | .003,880,45 | 2.447,347 | .027,826,57 | 14.360,37 | .010,560,38 | 10.688,77 | .0$^{3}$977,211,7 | 3.750,699 |
| 14 | .256,154,9 | .052,794,79 | 16.477,08 | .005,616,91 | 3.053,393 | .037,401,34 | 16.632,75 | .014,199,51 | 12.389,73 | .001,314,287 | 4.349,354 |
| 15 | .315,007,1 | .069,482,86 | 18.861,81 | .007,924,69 | 3.751,206 | .049,249,94 | 19.066,77 | .018,705,54 | 14.214,64 | .001,731,823 | 4.992,178 |
| 16 | .382,234,3 | .089,822,91 | 21.395,93 | .010,933,83 | 4.546,957 | .063,703,50 | 21.660,99 | .024,205,74 | 16.163,04 | .002,241,692 | 5.679,135 |
| 17 | .458,389,4 | .114,303,1 | 24.076,46 | .014,792,45 | 5.446,741 | .081,114,50 | 24.413,90 | .030,835,91 | 18.234,47 | .002,856,582 | 6.410,185 |
| 18 | .544,024,6 | .143,439,3 | 26.900,25 | .019,667,98 | 6.456,569 | .101,856,6 | 27.323,90 | .038,740,29 | 20.428,44 | .003,589,990 | 7.185,285 |
| 19 | .639,690,7 | .177,774,3 | 29.863,96 | .025,747,95 | 7.582,365 | .126,324,3 | 30.389,28 | .048,071,56 | 22.744,41 | .004,456,224 | 8.004,390 |
| 20 | .745,937,6 | .217,877,1 | 32.964,09 | .033,241,50 | 8.829,960 | .154,932,4 | 33.608,23 | .058,990,77 | 25.181,84 | .005,470,402 | 8.887,452 |
| 21 | .863,313,9 | .264,342,1 | 36.196,95 | .042,379,78 | 10.205,09 | .188,116,3 | 36.978,88 | .071,667,25 | 27.740,13 | .006,648,451 | 9.774,422 |
| 22 | .992,366,9 | .317,788,4 | 39.558,69 | .053,417,65 | 11.713,37 | .226,330,9 | 40.499,25 | .086,278,57 | 30.418,67 | .008,007,100 | 10.725,24 |
| 23 | 1.133,643 | .378,858,2 | 43.045,30 | .066,635,2 | 13.360,35 | .270,050,5 | 44.167,27 | .103,010,5 | 33.216,81 | .009,563,885 | 11.719,87 |
| 24 | 1.287,685 | .448,217,6 | 46.652,59 | .082,333,7 | 15.151,43 | .319,768,7 | 47.980,77 | .122,058,9 | 36.133,88 | .011,337,15 | 12.758,23 |
| 25 | 1.455,039 | .526,553,9 | 50.376,21 | .100,846,9 | 17.091,90 | .375,997,6 | 51.937,53 | .143,619,6 | 39.169,19 | .013,346,02 | 13.840,28 |
| 26 | 1.636,244 | .614,575,8 | 54.211,67 | .122,531,2 | 19.186,95 | .439,267,8 | 56.035,21 | .167,908,5 | 42.321,99 | .015,610,45 | 14.965,94 |
| 27 | 1.831,842 | .713,010,8 | 58.154,30 | .147,772,6 | 21.441,63 | .510,127,5 | 60.271,39 | .195,141,3 | 45.591,54 | .018,151,17 | 16.135,16 |
| 28 | 2.042,370 | .822,607,0 | 62.199,30 | .176,985,5 | 23.860,87 | .589,142,3 | 64.643,58 | .225,543,5 | 48.977,05 | .020,989,72 | 17.347,86 |
| 29 | 2.268,365 | .944,129,2 | 66.341,72 | .210,613,6 | 26.449,45 | .676,894,7 | 69.149,20 | .259,348,3 | 52.477,69 | .024,148,41 | 18.603,97 |
| 30 | 2.510,362 | 1.078,359 | 70.576,44 | .249,131,7 | 29.212,04 | .773,984,0 | 73.785,58 | .296,796,5 | 56.092,65 | .027,650,37 | 19.903,43 |
| 31 | 2.768,895 | 1.226,095 | 74.898,23 | .293,045,1 | 32.153,15 | .881,024,9 | 78.549,99 | .338,136,6 | 59.821,03 | .031,519,50 | 21.248,15 |
| 32 | 3.044,494 | 1.388,148 | 79.301,71 | .342,891,2 | 35.277,15 | .998,648,1 | 83.439,58 | .383,624,4 | 63.661,95 | .035,780,49 | 22.632,06 |
| 33 | 3.337,688 | 1.565,344 | 83.781,37 | .399,239,9 | 38.588,28 | 1.127,499 | 88.451,48 | .433,522,9 | 67.614,49 | .040,458,83 | 24.061,07 |
| 34 | 3.649,006 | 1.758,520 | 88.331,56 | .462,693,6 | 42.090,59 | 1.268,237 | 93.582,70 | .488,102,8 | 71.677,70 | .045,580,76 | 25.533,11 |
| 35 | 3.978,972 | 1.968,525 | 92.946,52 | .533,888,4 | 45.788,02 | 1.421,536 | 98.830,20 | .547,641,3 | 75.850,59 | .051,173,35 | 27.048,09 |
| 36 | 4.328,110 | 2.196,216 | 97.820,36 | .613,495,8 | 49.684,31 | 1.588,083 | 104.190,8 | .612,423,2 | 80.132,18 | .057,264,38 | 28.605,92 |
| 37 | 4.696,940 | 2.442,461 | 102.347,1 | .702,220,4 | 53.783,07 | 1.768,579 | 109.661,4 | .682,740,0 | 84.521,42 | .063,882,47 | 30.206,51 |
| 38 | 5.085,980 | 2.708,132 | 107.120,5 | .800,802,8 | 58.087,74 | 1.963,733 | 115.238,7 | .758,890,0 | 89.017,27 | .071,056,96 | 31.849,76 |
| 39 | 5.495,748 | 2.994,109 | 111.934,5 | .910,018,9 | 62.601,57 | 2.174,271 | 120.919,3 | .841,178,3 | 93.618,65 | .078,818,00 | 33.535,59 |
| 40 | 5.926,757 | 3.301,275 | 116.782,8 | 1.030,680 | 67.327,66 | 2.400,927 | 126.699,9 | .929,916,6 | 98.324,45 | .087,196,47 | 35.263,89 |
| 41 | 6.379,519 | 3.630,517 | 121.658,7 | 1.163,636 | 72.268,95 | 2.644,444 | 132.576,9 | 1.025,423 | 103.133,5 | .096,224,06 | 37.034,56 |
| 42 | 6.854,543 | 3.982,724 | 126.556,0 | 1.309,770 | 77.428,16 | 2.905,578 | 138.546,9 | 1.128,022 | 108.044,8 | .105,933,2 | 38.847,50 |
| 43 | 7.352,335 | 4.358,784 | 131.467,8 | 1.470,004 | 82.807,88 | 3.185,093 | 144.606,3 | 1.238,043 | 113.057,0 | .116,357,0 | 40.702,61 |
| 44 | 7873.400 | 4.759,586 | 136.387,6 | 1.645,297 | 88.410,49 | 3.483,761 | 150.751,3 | 1.355,825 | 118.168,9 | .127,529,4 | 42.599,78 |
| 45 | 8418.239 | 5.186,014 | 141.308,6 | 1.836,644 | 94.238,18 | 3.802,360 | 156.978,4 | 1.481,710 | 123.379,4 | .139,485,1 | 44.538,89 |
| 46 | 8.987,350 | 5.638,949 | 146.223,9 | 2.045,079 | 100.293,0 | 4.141,679 | 163.283,6 | 1.616,047 | 128.687,2 | .152,259,6 | 46.519,84 |
| 47 | 9.581,230 | 6.119,268 | 151.126,6 | 2.271,673 | 106.576,7 | 4.502,510 | 169.663,3 | 1.759,189 | 134.091,0 | .165,889,1 | 48.542,51 |
| 48 | 10.200,37 | 6.627,840 | 156.009,6 | 2.517,532 | 113.091,1 | 4.885,654 | 176.113,4 | 1.911,497 | 139.589,5 | .180,410,3 | 50.606,78 |
| 49 | 10.845,26 | 7.165,526 | 160.866,1 | 2.783,803 | 119.837,4 | 5.291,913 | 182.630,1 | 2.073,336 | 145.181,4 | .195,860,2 | 52.712,53 |
| 50 | 11.516,40 | 7.733,178 | 165.688,7 | 3.071,668 | 126.817,1 | 5.722,097 | 189.209,5 | 2.245,077 | 150.865,3 | .212,279,6 | 54.859,64 |
| 51 | 12214,25 | 8.331,636 | 170470,5 | 3.382,348 | 134.031,1 | 6.177,018 | 195.847,4 | 2.427,096 | 156.640,0 | .229,705,2 | 57.047,99 |
| 52 | 12.939,31 | 8.961,727 | 175.204,1 | 3.717,098 | 141.480,3 | 6.657,492 | 202.539,8 | 2.619,773 | 162.503,9 | .248,177,6 | 59.277,45 |
| 53 | 13.692,05 | 9.624,266 | 179.882,4 | 4.077,212 | 149.165,4 | 7.164,336 | 209.282,6 | 2.823,495 | 168.455,8 | .267,737,5 | 61.547,89 |
| 54 | 14.472,95 | 10.320,05 | 184.498,1 | 4.464,022 | 157.086,8 | 7.698,368 | 216.071,6 | 3.038,652 | 174.494,2 | .288,426,0 | 63.859,18 |
| 55 | 15.282,48 | 11.049,86 | 189.043,9 | 4.878,895 | 165.244,9 | 8.260,410 | 222.902,7 | 3.265,639 | 180.617,6 | .310,285,1 | 66.211,18 |
| 56 | 16.121,11 | 11.814,47 | 193.512,4 | 5.323,233 | 173.639,7 | 8.851,282 | 229.771,5 | 3.504,855 | 186.824,5 | .333,357,7 | 68.603,76 |
| 57 | 16.989,30 | 12.614,60 | 197.896,3 | 5.798,470 | 182.271,1 | 9.471,801 | 236.673,8 | 3.756,704 | 193.113,6 | .357,687,1 | 71.036,78 |
| 58 | 17.887,52 | 13.450,99 | 202.188,2 | 6.306,088 | 191.138,8 | 10.122,79 | 243.605,4 | 4.021,595 | 199.483,2 | .383,317,3 | 73.510,10 |
| 59 | 18.816,21 | 14.324,34 | 206.380,9 | 6.847,587 | 200.242,1 | 10.805,06 | 250.561,8 | 4.299,938 | 205.931,9 | .410,293,3 | 76.023,57 |
| 60 | 19.775,85 | 15.235,31 | 210.466,8 | 7.424,515 | 209.579,4 | 11.519,42 | 257.538,7 | 4.592,150 | 212.458,1 | .438,660,6 | 78.577,06 |
| 61 | 20.766,90 | 16.184,57 | 214.438,6 | 8.038,460 | 219.152,8 | 12.266,69 | 264.531,7 | 4.898,649 | 219.060,3 | .468,465,3 | 81.170,40 |
| 62 | 21.789,75 | 17.172,73 | 218.289,1 | 8.690,985 | 228.958,2 | 13.047,67 | 271.536,4 | 5.219,859 | 225.736,8 | .499,754,3 | 83.803,46 |
| 63 | 22.844,90 | 18.200,38 | 222.010,8 | 9.383,777 | 238.995,1 | 13.863,16 | 278.548,3 | 5.556,208 | 232.486,2 | .532,575,2 | 86.476,08 |
| 64 | 23.932,77 | 19.268,10 | 225.596,3 | 10.118,49 | 249.262,1 | 14.713,95 | 285.563,0 | 5.908,118 | 239.306,6 | .566,976,2 | 89.188,10 |
| 65 | 25.053,81 | 20.376,41 | 229.038,5 | 10.896,84 | 259.757,3 | 15.600,84 | 292.576,0 | 6.276,028 | 246.196,7 | .603,006,3 | 91.939,38 |
| 66 | 26.208,44 | 21.525,82 | 232.330,0 | 11.720,55 | 270.479,0 | 16.524,60 | 299.582,7 | 6.660,371 | 253.154,6 | .640,715,1 | 94.729,73 |
| 67 | 27.397,09 | 22.716,79 | 235.463,5 | 12.591,38 | 281.424,8 | 17.486,00 | 306.578,7 | 7.061,585 | 260.178,7 | .680,152,6 | 97.559,02 |
| 68 | 28.620,19 | 23.949,76 | 238.432,0 | 13.511,12 | 292.592,6 | 18.485,80 | 313.559,4 | 7.480,109 | 267.267,4 | .721,370,0 | 100.427,1 |
| 69 | 29.878,16 | 25.225,12 | 241.228,1 | 14.481,60 | 303.979,7 | 19.524,77 | 320.520,3 | 7.916,385 | 274.419,0 | .764,418,6 | 103.333,7 |
| 70 | 31.171,42 | 26.543,23 | 243.844,9 | 15.504,66 | 315.583,5 | 20.603,64 | 327.456,8 | 8.370,860 | 281.631,8 | .809,350,8 | 106.278,8 |
| 71 | 32.500,39 | 27.904,42 | 246.275,2 | 16.582,16 | 327.400,9 | 21.723,15 | 334.364,3 | 8.843,978 | 288.903,9 | .856,219,3 | 109.262,2 |
| 72 | 33.865,47 | 29.308,96 | 248.512,1 | 17.716,00 | 339.428,9 | 22.884,02 | 341.238,4 | 9.336,187 | 296.233,8 | .905,077,8 | 112.283,6 |
| 73 | 35.267,06 | 30.757,09 | 250.548,6 | 18.908,10 | 351.664,2 | 24.086,94 | 348.074,3 | 9.847,939 | 303.619,6 | .955,980,1 | 115.342,9 |
| 74 | 36.705,58 | 32.249,00 | 252.377,9 | 20.160,40 | 364.103,3 | 25.332,63 | 354.867,6 | 10.379,68 | 311.059,6 | 1.008,981 | 118.440,0 |
| 75 | 38.181,41 | 33.784,85 | 253.993,3 | 21.474,86 | 376.742,4 | 26.621,75 | 361.613,8 | 10.931,87 | 318.552,0 | 1.064,136 | 121.574,7 |
| 76 | 39.694,95 | 35.364,76 | 255.387,9 | 22.853,45 | 389.577,7 | 27.954,98 | 368.308,1 | 11.504,96 | 326.095,1 | 1.121,501 | 124.746,7 |
| 77 | 41.246,60 | 36.988,77 | 256.555,2 | 24.298,18 | 402.605,1 | 29.332,96 | 374.946,1 | 12.099,40 | 333.687,0 | 1.181,133 | 127.955,9 |
| 78 | 42.836,72 | 38.656,91 | 257.488,7 | 25.811,06 | 415.820,4 | 30.756,33 | 381.523,2 | 12.715,65 | 341.325,9 | 1.243,089 | 131.202,1 |
| 79 | 44.465,71 | 40.369,15 | 258.181,9 | 27.394,12 | 429.219,2 | 32.225,70 | 388.034,9 | 13.354,16 | 349.010,1 | 1.307,426 | 134.485,1 |
| 80 | 46.133,94 | 42.125,41 | 258.628,5 | 29.049,41 | 442.796,8 | 33.741,68 | 394.476,7 | 14.015,39 | 356.737,6 | 1.374,203 | 137.804,8 |
| 81 | 47.841,79 | 43.925,55 | 258.822,4 | 30.778,98 | 456.548,5 | 35.304,84 | 400.844,0 | 14.699,80 | 364.506,7 | 1.443,480 | 141.160,9 |
| 82 | 49.589,62 | 45.769,40 | 258.757,2 | 32.584,92 | 470.469,1 | 36.915,75 | 407.132,4 | 15.407,83 | 372.315,4 | 1.515,314 | 144.553,3 |
| 83 | 51.377,78 | 47.656,73 | 258.427,0 | 34.469,29 | 484.553,8 | 38.574,95 | 413.337,3 | 16.139,95 | 380.162,1 | 1.589,768 | 147.981,7 |
| 84 | 53.206,65 | 49.587,24 | 257.826,[illegible] | 36.434,18 | 498.797,2 | 40.282,96 | 419.454,3 | 16.896,62 | 388.044,6 | 1.666,901 | 151.446,0 |
| 85 | 55.076,57 | 51.560,62 | 256.948,6 | 38.481,70 | 513.193,8 | 42.040,29 | 425.479,1 | 17.678,28 | 395.961,3 | 1.748,774 | 154.945,9 |
| 86 | 56.987,90 | 53.576,46 | 255.788,8 | 40.613,94 | 527.738,0 | 43.847,41 | 431.407,0 | 18.485,39 | 403.910,2 | 1.829,450 | 158.481,3 |
| 87 | 58.940,97 | 55.[illegible]4,33 | 254.341,3 | 42.833,01 | 542.424,0 | 45.704,77 | 437.233,9 | 19.318,40 | 411.889,5 | 1.914,992 | 162.051,9 |
| 88 | 60.936,13 | 57.733,72 | 252.600,7 | 45.141,02 | 557.246,0 | 47.612,82 | 442.955,3 | 20.177,76 | 419.897,2 | 2.003,460 | 165.657,6 |
| 89 | 62.973,71 | 59.874,09 | 250.561,7 | 47.540,07 | 572.197,7 | 49.571,97 | 448.566,8 | 21.063,93 | 427.931,4 | 2.094,921 | 169.298,1 |
| 90 | 65.054,05 | 62.054,81 | 248.219,2 | 50.032,28 | 587.273,0 | 51.582,59 | 454.[illegible] | 21.9[illegible]7,35 | 435.990,2 | 2.189,437 | 172.973,2 |

SHEET 4 OF 5

205-D-397

Table H-8.–*Load constants for circular arches–radial deflections* (sheet 5).

UNIT RADIAL LOAD NO. 5
RADIAL DEFLECTIONS



| $\Phi$ | Crown | | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_2$ | | $D_2$ | | $D_2$ | |
| ° | | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term |
| 10 | .221,299,2 | .030,785,06 | 15,099,79 | .002,242,50 | 1.764,561 | .016,469,33 | 12.792,44 | .004,585,523 | 7.600,466 | .0$^3$369,902,6 | 2.378,676 |
| 11 | .294,486,5 | .045,027,76 | 18.237,56 | .003,609,25 | 2.346,503 | .024,098,74 | 15.463,99 | .006,711,903 | 9.192,765 | .0$^3$541,538,5 | 2.877,906 |
| 12 | .382,234,3 | .063,703,50 | 21.660,99 | .005,572,56 | 3.043,376 | .034,109,26 | 18.384,07 | .009,503,311 | 10.935,20 | .0$^3$766,923,5 | 3.424,567 |
| 13 | .485,853,9 | .087,639,34 | 25.366,53 | .008,308,77 | 3.865,212 | .046,948,35 | 21.551,08 | .013,085,42 | 12.827,36 | .001,056,249 | 4.018,625 |
| 14 | .606,653,5 | .117,729,2 | 29.350,33 | .012,025,49 | 4.821,934 | .063,100,77 | 24.963,29 | .017,594,61 | 14.868,80 | .001,420,587 | 4.660,047 |
| 15 | .745,937,6 | .154,932,4 | 33.608,23 | .016,964,25 | 5.923,349 | .083,088,14 | 28.618,81 | .023,177,95 | 17.059,03 | .001,871,894 | 5.348,795 |
| 16 | .905,007,1 | .200,272,5 | 38.135,80 | .023,402,75 | 7.179,140 | .107,468,5 | 32.515,66 | .029,993,12 | 19.397,55 | .002,423,000 | 6.064,830 |
| 17 | 1.065,159 | .254,835,7 | 42.928,32 | .031,657,4 | 8.598,854 | .136,836,0 | 36.651,69 | .038,208,33 | 21.883,79 | .003,087,622 | 6.868,110 |
| 18 | 1.287,685 | .319,768,7 | 47.980,77 | .042,085,2 | 10.191,90 | .171,820,1 | 41.024,63 | .048,002,34 | 24.517,18 | .003,880,346 | 7.698,586 |
| 19 | 1.513,875 | .396,278,6 | 53.287,88 | .055,086,6 | 11.967,54 | .213,085,3 | 45.632,09 | .059,564,26 | 27.297,09 | .004,816,640 | 8.576,217 |
| 20 | 1.765,010 | .485,629,8 | 58.844,09 | .071,106,9 | 13.934,87 | .261,330,5 | 50.471,54 | .073,093,62 | 30.222,87 | .005,912,842 | 9.500,945 |
| 21 | 2.042,370 | .589,142,3 | 64.643,59 | .090,639,2 | 16.102,82 | .317,288,4 | 55.540,33 | .088,800,21 | 33.293,82 | .007,186,167 | 10.472,72 |
| 22 | 2.347,226 | .708,190,4 | 70.680,27 | .114,225,7 | 18.480,17 | .381,724,9 | 60.835,88 | .106,904,0 | 36.509,22 | .008,654,899 | 11.491,49 |
| 23 | 2.680,847 | .844,200,3 | 76.947,80 | .142,460,3 | 21.075,49 | .455,438,5 | 66.354,69 | .127,635,1 | 39.868,32 | .010,337,39 | 12.557,19 |
| 24 | 3.044,494 | .998,648,1 | 83.439,58 | .175,990,2 | 23.897,18 | .539,259,6 | 72.094,32 | .151,233,6 | 43.370,33 | .012,254,06 | 13.669,75 |
| 25 | 3.439,421 | 1.173,058 | 90.148,78 | .215,518,3 | 26.953,43 | .634,049,6 | 78.051,45 | .177,949,6 | 47.014,41 | .014,425,41 | 14.829,12 |
| 26 | 3.866,879 | 1.368,999 | 97.068,31 | .261,803,8 | 30.252,25 | .740,700,7 | 84.222,79 | .208,043,1 | 50.799,72 | .016,872,97 | 16.035,24 |
| 27 | 4.328,110 | 1.588,083 | 104.190,8 | .315,664,9 | 33.801,43 | .860,134,5 | 90.604,97 | .241,783,6 | 54.725,36 | .019,619,17 | 17.288,02 |
| 28 | 4.824,348 | 1.831,966 | 111.508,8 | .377,980,6 | 37.608,53 | .993,301,7 | 97.194,48 | .279,450,6 | 58.790,40 | .022,687,27 | 18.587,40 |
| 29 | 5.356,824 | 2.102,337 | 119.014,5 | .449,692,1 | 41.680,91 | 1.141,181 | 103.987,7 | .321,332,8 | 62.993,90 | .026,101,41 | 19.933,30 |
| 30 | 5.926,757 | 2.400,927 | 126.699,9 | .531,802,4 | 46.025,69 | 1.304,779 | 110.980,9 | .367,728,6 | 67.334,85 | .029,886,58 | 21.325,63 |
| 31 | 6.535,362 | 2.729,494 | 134.556,8 | .625,381,4 | 50.649,77 | 1.485,127 | 118.170,4 | .418,945,5 | 71.812,25 | .034,068,60 | 22.764,33 |
| 32 | 7.183,843 | 3.089,832 | 142.576,8 | .731,564,2 | 55.559,79 | 1.683,285 | 125.552,0 | .475,300,3 | 76.425,03 | .038,674,18 | 24.249,32 |
| 33 | 7.873,400 | 3.483,761 | 150.751,3 | .851,552,8 | 60.762,15 | 1.900,334 | 133.121,8 | .537,119,0 | 81.172,12 | .043,730,85 | 25.780,49 |
| 34 | 8.605,221 | 3.913,125 | 159.071,6 | .986,618,8 | 66.263,00 | 2.137,381 | 140.875,7 | .604,736,4 | 86.052,39 | .049,266,97 | 27.357,77 |
| 35 | 9.380,488 | 4.379,794 | 167.528,7 | 1.138,103 | 72.068,25 | 2.395,556 | 148.809,3 | .678,496,0 | 91.064,70 | .055,311,81 | 28.981,07 |
| 36 | 10.200,37 | 4.885,654 | 176.113,4 | 1.307,415 | 78.183,52 | 2.676,010 | 156.918,3 | .758,750,5 | 96.207,88 | .061,895,41 | 30.650,29 |
| 37 | 11.066,04 | 5.432,609 | 184.816,5 | 1.496,039 | 84.614,18 | 2.979,914 | 165.198,4 | .845,860,8 | 101.480,7 | .069,048,66 | 32.365,33 |
| 38 | 11.978,63 | 6.022,579 | 193.628,5 | 1.705,530 | 91.365,33 | 3.308,461 | 173.644,9 | .940,195,7 | 106.882,0 | .076,803,07 | 34.126,10 |
| 39 | 12.939,31 | 6.657,492 | 202.539,8 | 1.937,513 | 98.441,78 | 3.662,862 | 182.253,2 | 1.042,134 | 112.410,4 | .085,191,93 | 35.932,49 |
| 40 | 13.949,20 | 7.339,285 | 211.540,7 | 2.193,688 | 105.848,1 | 4.044,344 | 191.018,8 | 1.152,060 | 118.064,8 | .094,247,90 | 37.784,40 |
| 41 | 15.009,43 | 8.069,900 | 220.621,2 | 2.475,834 | 113.588,5 | 4.454,153 | 199.936,3 | 1.270,368 | 123.843,4 | .104,005,5 | 39.681,72 |
| 42 | 16.121,11 | 8.851,282 | 229.771,5 | 2.785,797 | 121.667,0 | 4.893,549 | 209.001,3 | 1.397,461 | 129.745,4 | .114,499,6 | 41.624,35 |
| 43 | 17.285,35 | 9.685,371 | 238.981,3 | 3.125,498 | 130.087,3 | 5.363,808 | 218.208,9 | 1.533,747 | 135.769,1 | .125,766,3 | 43.612,16 |
| 44 | 18.503,23 | 10.574,11 | 248.240,5 | 3.498,937 | 138.852,7 | 5.866,219 | 227.553,8 | 1.679,643 | 141.913,3 | .137,842,1 | 45.645,06 |
| 45 | 19.775,85 | 11.519,42 | 257.538,7 | 3.902,186 | 147.966,4 | 6.402,082 | 237.030,9 | 1.835,573 | 148.176,3 | .150,764,5 | 47.722,91 |
| 46 | 21.104,28 | 12.523,23 | 266.865,6 | 4.343,392 | 157.431,1 | 6.972,709 | 246.635,2 | 2.001,970 | 154.556,9 | .164,571,9 | 49.845,59 |
| 47 | 22.489,57 | 13.587,45 | 276.210,5 | 4.822.773 | 167.249,3 | 7.579,423 | 256.361,2 | 2.179,270 | 161.053,4 | .179,303,4 | 52.014,00 |
| 48 | 23.932,77 | 14.713,95 | 285.563,0 | 5.342,629 | 177.423,3 | 8.223,554 | 266.203,7 | 2.367,921 | 167.664,4 | .194,998,6 | 54.225,00 |
| 49 | 25.434,93 | 15.904,62 | 294.912,5 | 5.905,325 | 187.954,9 | 8.906,441 | 276.157,2 | 2.568,374 | 174.388,3 | .211,698,6 | 56.481,46 |
| 50 | 26.997,07 | 17.161,30 | 304.248,2 | 6.513,305 | 198.845,7 | 9.629,428 | 286.216,4 | 2.781,087 | 181.223,6 | .229,444,6 | 58.782,26 |
| 51 | 28.620,19 | 18.485,80 | 313.559,4 | 7.169,085 | 210.096,9 | 10.393,87 | 296.375,6 | 3.006,525 | 188.168,6 | .248,279,1 | 61.127,25 |
| 52 | 30.305,31 | 19.879,93 | 322.835,4 | 7.875,252 | 221.709,6 | 11.201,11 | 306.629,4 | 3.245,161 | 195.221,8 | .268,244,9 | 63.516,32 |
| 53 | 32.053,41 | 21.345,42 | 332.065,3 | 8.634,462 | 233.684,4 | 12.052,51 | 316.972,1 | 3.497,470 | 202.381,4 | .289,386,1 | 65.949,31 |
| 54 | 33.865,47 | 22.884,02 | 341.238,4 | 9.449,447 | 246.021,7 | 12.949,44 | 327.397,9 | 3.763,937 | 209.645,8 | .311,747,2 | 68.426,09 |
| 55 | 35.742,44 | 24.497,39 | 350.343,7 | 10.323,00 | 258.721,4 | 13.893,24 | 337.901,3 | 4.045,048 | 217.013,4 | .335,373,4 | 70.946,51 |
| 56 | 37.685,29 | 26.187,18 | 359.370,6 | 11.257,99 | 271.783,3 | 14.885,28 | 348.476,4 | 4.341,299 | 224.482,4 | .360,311,3 | 73.510,44 |
| 57 | 39.694,95 | 27.954,98 | 368.308,1 | 12.257,36 | 285.206,8 | 15.926,92 | 359.117,4 | 4.653,190 | 232.051,0 | .386,607,4 | 76.117,72 |
| 58 | 41.772,34 | 29.802,34 | 377.145,5 | 13.324,09 | 298.991,1 | 17.019,50 | 369.816,5 | 4.981,223 | 239.717,6 | .414,309,6 | 78.768,20 |
| 59 | 43.918,37 | 31.730,76 | 385.871,9 | 14.461,26 | 313.134,8 | 18.164,37 | 380.573,7 | 5.325,909 | 247.480,2 | .443,466,3 | 81.461,73 |
| 60 | 46.133,94 | 33.741,66 | 394.476,7 | 15.671,99 | 327.636,7 | 19.362,87 | 391.377,3 | 5.687,761 | 255.337,2 | .474,126,6 | 84.198,16 |
| 61 | 48.419,94 | 35.836,48 | 402.949,1 | 16.959,47 | 342.494,7 | 20.616,34 | 402.223,3 | 6.067,298 | 263.286,7 | .506,340,5 | 86.977,33 |
| 62 | 50.777,22 | 38.016,49 | 411.278,5 | 18.326,95 | 357.706,8 | 21.926,11 | 413.105,7 | 6.465,041 | 271.326,8 | .540,158,7 | 89.799,07 |
| 63 | 53.208,65 | 40.282,96 | 419.454,3 | 19.777,72 | 373.270,7 | 23.293,49 | 424.018,5 | 6.881,520 | 279.455,8 | .575,632,8 | 92.663,24 |
| 64 | 55.709,07 | 42.637,10 | 427.466,0 | 21.315,16 | 389.183,4 | 24.719,78 | 434.955,7 | 7.317,261 | 287.671,6 | .612,814,2 | 95.569,64 |
| 65 | 58.285,29 | 45.080,04 | 435.303,1 | 22.942,67 | 405.442,2 | 26.206,29 | 445.911,3 | 7.772,801 | 295.972,4 | .651,756,5 | 98.518,14 |
| 66 | 60.936,13 | 47.612,82 | 442.955,3 | 24.663,72 | 422.043,6 | 27.754,28 | 456.879,3 | 8.248,676 | 304.356,2 | .692,513,1 | 101.508,5 |
| 67 | 63.662,39 | 50.236,43 | 450.412,2 | 26.481,84 | 438.984,0 | 29.365,03 | 467.853,7 | 8.745,429 | 312.821,2 | .735,138,1 | 104.540,7 |
| 68 | 66.464,85 | 52.951,79 | 457.663,8 | 28.400,57 | 456.259,6 | 31.039.77 | 478.828,4 | 9.263,600 | 321.365,4 | .779,666,8 | 107.614,4 |
| 69 | 69.344,27 | 55.759,71 | 464.700,1 | 30.423,53 | 473.866,1 | 32.779.76 | 489.797,4 | 9.803,739 | 329.986,7 | .826,214,6 | 110.729,5 |
| 70 | 72.301,40 | 58.660,94 | 471.511,0 | 32.554,38 | 491.799,2 | 34.586,18 | 500.754,6 | 10.366,39 | 338.683,3 | .874,778,2 | 113.885,9 |
| 71 | 75.336,98 | 61.656,14 | 478.086,7 | 34.796,79 | 510.054,1 | 36.460,26 | 511.694,0 | 10.952,11 | 347.453,0 | .925,434,5 | 117.083,2 |
| 72 | 78.451,73 | 64.745,90 | 484.417,6 | 37.154,51 | 528.625,7 | 38.403,14 | 522.609,6 | 11.561,45 | 356.294,0 | .978,241,4 | 120.321,4 |
| 73 | 81.646,34 | 67.930,69 | 490.494,2 | 39.631.29 | 547.508,8 | 40.415,98 | 533.495,3 | 12.194,97 | 365.204,0 | 1.033,257 | 123.600,3 |
| 74 | 84.921,52 | 71.210,92 | 496.307,0 | 42.230,92 | 566.697,9 | 42.499,91 | 544.345,2 | 12.853,22 | 374.181,2 | 1.090,541 | 126.919,7 |
| 75 | 88.277,93 | 74.586,90 | 501.846,9 | 44.957,24 | 586.187,1 | 44.656,04 | 555.153,2 | 13.536,76 | 383.223,3 | 1.150,154 | 130.279,3 |
| 76 | 91.716,22 | 78.058,83 | 507.104,8 | 47.814,09 | 605.970,5 | 46.885,43 | 565.913,4 | 14.246,15 | 392.328,4 | 1.212,154 | 133.679,0 |
| 77 | 95.237,05 | 81.626,82 | 512.071,8 | 50.805,35 | 626.041,5 | 49.189,14 | 576.619,8 | 14.981,95 | 401.494,3 | 1.276,605 | 137.118,6 |
| 78 | 98.841,01 | 85.290,91 | 516.739,2 | 53.934,90 | 646.393,8 | 51.568,20 | 587.266,6 | 15.744,73 | 410.719,0 | 1.343,566 | 140.598,0 |
| 79 | 102.528,7 | 89.050,99 | 521.098,5 | 57.206,67 | 667.020,5 | 54.023,58 | 597.847,8 | 16.535,04 | 420.000,3 | 1.413,102 | 144.116,8 |
| 80 | 106.300,8 | 92.906,89 | 525.141,3 | 60.624,57 | 687.914,6 | 56.556,26 | 608.357,5 | 17.353,44 | 429.336,1 | 1.485,275 | 147.674,9 |
| 81 | 110.157,8 | 96.858,32 | 528.859,5 | 64.192,55 | 709.068,8 | 59.167,16 | 618.789,9 | 18.200,51 | 438.724,2 | 1.560,149 | 151.272,1 |
| 82 | 114.100,3 | 100.904,9 | 532.245,2 | 67.914,54 | 730.475,7 | 61.857,18 | 629.139,5 | 19.076,80 | 448.162,6 | 1.637,787 | 154.908,2 |
| 83 | 118.128,7 | 105.046,1 | 535.290,6 | 71.794,49 | 752.127,4 | 64.627,19 | 639.400,1 | 19.982,87 | 457.648,9 | 1.718,256 | 158.583,0 |
| 84 | 122.243,8 | 109.281,3 | 537.988,2 | 75.836.36 | 774.016,2 | 67.478.02 | 649.566,3 | 20.919,29 | 467.181,2 | 1.801,620 | 162.296,3 |
| 85 | 126.445,8 | 113.609,9 | 540.330,8 | 80.044,07 | 796.134,0 | 70.410,45 | 659.632,3 | 21.886,61 | 476.757,2 | 1.887,947 | 166.047,8 |
| 86 | 130.735,4 | 118.030,9 | 542.311,2 | 84.421,60 | 818.472,4 | 73.425,26 | 669.592,5 | 22.885,39 | 486.374,7 | 1.977.302 | 169.837,4 |
| 87 | 135.113,0 | 122.543,6 | 543.922,8 | 88.972,85 | 841.022,8 | 76.523,14 | 679.441,5 | 23.916,20 | 496.031,5 | 2069,754 | 173.664,7 |
| 88 | 139.579,1 | 127.146,7 | 545.158,8 | 93.701,74 | 863.776,8 | 79.704,79 | 689.173,6 | 24.979.59 | 505.725,5 | 2.165,369 | 177.529,9 |
| 89 | 144.134,1 | 131.839,2 | 546.012,9 | 98.612,18 | 886.725,3 | 82.970,85 | 698.783,5 | 26.076,11 | 515.454,4 | 2.264,218 | 181.432,1 |
| 90 | 148.778,4 | 136.619,8 | 546.479,1 | 103.708,1 | 909.859,3 | 86.321,91 | 708.265,7 | 27.206,32 | 525.216,0 | 2.366,368 | 185.371,7 |

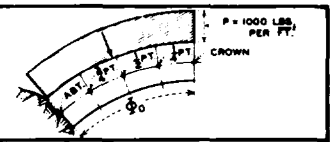
SHEET 5 OF 5

205-D-398

Table H-9.–*Load constants for circular arches–twist and tangential deflections* (sheet 1).

UNIT RADIAL LOAD NO. 1

TWIST AND TANGENTIAL DEFLECTIONS



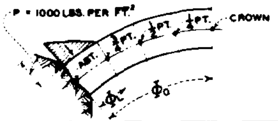
| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .884,747,5 | .008,068,29 | 4.407,601 | .373,501,7 | .001,917,68 | 1.863,673 | .110,719,9 | .0$^{3}$252,82 | .553,093,6 | .013,843,94 | .0$^{5}$7,907 | .069,203,88 |
| 11 | 1.177,222 | .012,984,1 | 5.860,143 | .497,041,3 | .003,087,13 | 2.479,032 | .147,356,3 | .0$^{3}$407,09 | .735,967,5 | .018,425,91 | .0$^{4}$12,733 | .092,104,07 |
| 12 | 1.527,819 | .020,044,5 | 7.599,008 | .645,167,6 | .004,767,50 | 3.216,303 | .191,291,9 | .0$^{3}$628,85 | .955,201,6 | .023,921,32 | .0$^{4}$19,671 | .119,567,2 |
| 13 | 1.941,748 | .029,882,1 | 9.648,978 | .820,098,2 | .007,110,21 | 4.086,271 | .243,187,6 | .0$^{3}$938,12 | 1.214,062 | .030,413,13 | .0$^{4}$29,350 | .152,008,9 |
| 14 | 2.424,200 | .043,242,4 | 12.034,51 | 1.024,046 | .010,293,6 | 5.099,643 | .303,704,2 | .001,358,54 | 1.515,804 | .037,984,29 | .0$^{4}$42,511 | .189,836,4 |
| 15 | 2.980,343 | .060,991,5 | 14.779,73 | 1.259,219 | .014,525,3 | 6.267,043 | .373,501,7 | .001,917,68 | 1.863,673 | .046,717,72 | .0$^{4}$60,020 | .233,468,6 |
| 16 | 3.615,325 | .084,124,9 | 17.908,37 | 1.527,819 | .020,044,5 | 7.599,008 | .453,239,2 | .002,647,26 | 2.260,901 | .056,696,34 | .0$^{4}$82,872 | .283,315,9 |
| 17 | 4.334,268 | .113,775,6 | 21.443,79 | 1.832,045 | .027,123,6 | 9.105,976 | .543,575,3 | .003,583,55 | 2.710,709 | .068,003,01 | .0$^{3}$,112,206 | .339,790,7 |
| 18 | 5.142,271 | .151,222,4 | 25.408,91 | 2.174,085 | .036,070,8 | 10.798,28 | .645,167,6 | .004,767,50 | 3.216,303 | .080,720,61 | .0$^{3}$.149,314 | .403,304,4 |
| 19 | 6.044,403 | .197,896,7 | 29.826,22 | 2.556,125 | .047,231,6 | 12.686,16 | .758,673,1 | .006,245,30 | 3.780,875 | .094,931,96 | .0$^{3}$,195,646 | .474,268,5 |
| 20 | 7.045,707 | .255,391,4 | 34.717,75 | 2.980,343 | .060,991,5 | 14.779,73 | .884,747,5 | .008,068,29 | 4.407,601 | .110,719,9 | .0$^{3}$252,82 | .553,093,6 |
| 21 | 8.151,193 | .325,466,9 | 40.105,03 | 3.448,907 | .077,777,2 | 17.088,98 | 1.024,046 | .010,293,6 | 5.099,643 | .128,167,1 | .0$^{3}$,322,65 | .640,190,1 |
| 22 | 9.365,842 | .410,058,9 | 46.009,09 | 3.963,982 | .098,059,2 | 19.623,79 | 1.177,222 | .012,984,1 | 5.860,143 | .147,356,3 | .0$^{3}$,407,09 | .735,967,5 |
| 23 | 10.694,60 | .511,285 | 52.450,43 | 4.527,721 | .122,353,2 | 22.393,90 | 1.344,930 | .016,209,3 | 6.692,230 | .168,370,4 | .0$^{3}$,508,37 | .840,835,2 |
| 24 | 12.142,38 | .631,451 | 59.448,99 | 5.142,271 | .151,222,4 | 25.408,91 | 1.527,819 | .020,044,5 | 7.599,008 | .191,291,9 | .0$^{3}$,628,85 | .955,201,6 |
| 25 | 13.714,05 | .773,057 | 67.024,14 | 5.809,769 | .185,278,7 | 28.678,29 | 1.726,543 | .024,572,3 | 8.583,568 | .216,203,4 | .0$^{3}$.771,16 | 1.079,475 |
| 26 | 15.414,46 | .938,803 | 75.194,69 | 6.532,345 | .225,185,5 | 32.211,35 | 1.941,748 | .029,882,1 | 9.648,978 | .243,187,6 | .0$^{3}$938,12 | 1.241,062 |
| 27 | 17.248,40 | 1.131,596 | 83.978,80 | 7.312,117 | .271,658,3 | 36.017,27 | 2.174,085 | .036,070,8 | 10.798,28 | .272,327,1 | .001,132,81 | 1.359,370 |
| 28 | 19.220,63 | 1.354,554 | 93.394,03 | 8.151,193 | .325,466,9 | 40.105,03 | 2.424,200 | .043,242,4 | 12.034,51 | .303,704,2 | .001,358,54 | 1.515,804 |
| 29 | 21.335,86 | 1.611,008 | 103.457,3 | 9.051,675 | .387,437,4 | 44.483,50 | 2.692,738 | .051,509,3 | 13.360,67 | .337,401,6 | .001,618,89 | 1.683,770 |
| 30 | 23.598,78 | 1.904,515 | 114.184,9 | 10.015,65 | .458,452,8 | 49.161,34 | 2.980,343 | .060,991,5 | 14.779,73 | .373,501,7 | .001,917,68 | 1.863,673 |
| 31 | 26.013,99 | 2.238,850 | 125.592,3 | 11.045,20 | .539,457 | 54.147,07 | 3.287,658 | .071,817,6 | 16.294,65 | .412,086,7 | .002,258,95 | 2.055,916 |
| 32 | 28.586,10 | 2.618,024 | 137.694,4 | 12.142,38 | .631,451 | 59.448,99 | 3.615,325 | .084,124,9 | 17.908,37 | .453,239,2 | .002,647,26 | 2.260,901 |
| 33 | 31.319,62 | 3.046,274 | 150.505,5 | 13.309,25 | .735,501 | 65.075,26 | 3.963,982 | .098,059,2 | 19.623,79 | .497,041,3 | .003,087,13 | 2.479,032 |
| 34 | 34.219,04 | 3.528,075 | 164.039,1 | 14.547,86 | .852,736 | 71.033,84 | 4.334,268 | .113,775,6 | 21.443,79 | .543,575,3 | .003,583,55 | 2.710,709 |
| 35 | 37.288,80 | 4.068,140 | 178.307,7 | 15.860,24 | .984,347 | 77.332,50 | 4.726,820 | .131,438,7 | 23.371,22 | .592,923,4 | .004,141,82 | 2.956,333 |
| 36 | 40.533,28 | 4.671,420 | 193.323,6 | 17.248,40 | 1.131,596 | 83.978,80 | 5.142,271 | .151,222,4 | 25.408,91 | .645,167,6 | .004,767,50 | 3.216,303 |
| 37 | 43.956,80 | 5.343,112 | 209.097,8 | 18.714,35 | 1.295,808 | 90.980,12 | 5.561,255 | .173,310,4 | 27.559,66 | .700,390,2 | .005,466,60 | 3.491,018 |
| 38 | 47.563,64 | 6.088,654 | 225.640,9 | 20.260,08 | 1.478,377 | 98.343,63 | 6.044,403 | .197,896,7 | 29.826,22 | .758,673,1 | .006,245,30 | 3.780,875 |
| 39 | 51.358,02 | 6.913,730 | 242.962,6 | 21.887,57 | 1.680,768 | 106.076,3 | 6.532,345 | .225,185,5 | 32.211,35 | .820,098,2 | .007,110,21 | 4.086,271 |
| 40 | 55.344,09 | 7.824,269 | 261.071,9 | 23.598,78 | 1.904,515 | 114.184,9 | 7.045,707 | .255,391,4 | 34.717,75 | .884,747,5 | .008,068,29 | 4.407,601 |
| 41 | 59.525,96 | 8.826,449 | 279.976,9 | 25.395,66 | 2.151,224 | 122.675,6 | 7.585,115 | .288,739,7 | 37.348,10 | .952,702,9 | .009,126,87 | 4.745,261 |
| 42 | 63.907,68 | 9.926,690 | 299.685,0 | 27.280,15 | 2.422,574 | 131.555,6 | 8.151,193 | .325,466,9 | 40.105,03 | 1.024,046 | .010,293,6 | 5.099,643 |
| 43 | 68.493,22 | 11.131,66 | 320.202,8 | 29.254,17 | 2.720,315 | 140.830,2 | 8.744,563 | .365,820,4 | 42.991,17 | 1.098,859 | .011,576,3 | 5.471,140 |
| 44 | 73.286,50 | 12.448,27 | 341.536,0 | 31.319,62 | 3.046,274 | 150.505,5 | 9.365,842 | .410,058,9 | 46.009,09 | 1.177,222 | .012,984,1 | 5.860,143 |
| 45 | 78.291,36 | 13.883,68 | 363.689,5 | 33.478,39 | 3.402,351 | 160.587,2 | 10.015,65 | .458,452,8 | 49.161,34 | 1.259,219 | .014,525,3 | 6.267,043 |
| 46 | 83.511,66 | 15.445,29 | 386.667,7 | 35.732,39 | 3.790,520 | 171.080,7 | 10.694,60 | .511,285 | 52.450,43 | 1.344,930 | .016,209,3 | 6.692,230 |
| 47 | 88.951,05 | 17.140,73 | 410.473,6 | 38.083,37 | 4.212,834 | 181.991,2 | 11.403,31 | .568,849 | 55.878,83 | 1.434,436 | .018,045,5 | 7.136,089 |
| 48 | 94.613,22 | 18.977,88 | 435.110,3 | 40.533,28 | 4.671,420 | 193.323,6 | 12.142,38 | .631,451 | 59.448,99 | 1.527,819 | .020,044,5 | 7.599,008 |
| 49 | 100.501,8 | 20.964,86 | 460.579,0 | 43.083,90 | 5.168,483 | 205.082,5 | 12.912,42 | .699,410 | 63.163,30 | 1.625,161 | .022,216,4 | 8.081,375 |
| 50 | 106.620,2 | 23.110,00 | 486.880,9 | 45.737,04 | 5.706,302 | 217.272,6 | 13.714,05 | .773,057 | 67.024,14 | 1.726,543 | .024,572,3 | 8.583,568 |
| 51 | 112.972,0 | 25.421,86 | 514.016,1 | 48.494,49 | 6.287,240 | 229.898,0 | 14.547,86 | .852,735 | 71.033,84 | 1.832,045 | .027,123,6 | 9.105,976 |
| 52 | 119.560,5 | 27.909,24 | 541.983,8 | 51.358,02 | 6.913,730 | 242.962,6 | 15.414,46 | .938,803 | 75.194,69 | 1.941,748 | .029,882,1 | 9.648,978 |
| 53 | 126.389,0 | 30.581,16 | 570.782,7 | 54.329,38 | 7.588,289 | 256.470,3 | 16.314,44 | 1.031,629 | 79.508,93 | 2.055,735 | .032,860,3 | 10.212,95 |
| 54 | 133.460,8 | 33.446,84 | 600.410,3 | 57.410,30 | 8.313,509 | 270.424,5 | 17.248,40 | 1.131,596 | 83.978,80 | 2.174,085 | .036,070,8 | 10.798,28 |
| 55 | 140.779,1 | 36.515,71 | 630.863,8 | 60.802,50 | 9.092,059 | 284.828,4 | 18.216,93 | 1.239,101 | 88.606,45 | 2.298,880 | .039,526,9 | 11.405,35 |
| 56 | 148.346,8 | 39.797,39 | 662.139,3 | 63.907,68 | 9.926,690 | 299.685,0 | 19.220,63 | 1.354,554 | 93.394,03 | 2.424,200 | .043,242,4 | 12.034,51 |
| 57 | 156.167,1 | 43.301,73 | 694.232,0 | 67.327,51 | 10.820,23 | 314.997,1 | 20.260,08 | 1.478,377 | 98.343,63 | 2.556,125 | .047,231,6 | 12.686,16 |
| 58 | 164.242,9 | 47.038,77 | 727.136,8 | 70.863,65 | 11.775,57 | 330.767,1 | 21.335,86 | 1.611,008 | 103.457,3 | 2.692,738 | .051,509,3 | 13.360,67 |
| 59 | 172.577,0 | 51.018,69 | 760.847,4 | 74.517,73 | 12.796,70 | 348.997,3 | 22.448,57 | 1.752,899 | 108.737,0 | 2.834,117 | .056,090,7 | 14.058,40 |
| 60 | 181.172,2 | 55.251,87 | 795.357,0 | 78.291,38 | 13.883,68 | 363.689,5 | 23.598,78 | 1.904,515 | 114.184,9 | 2.980,343 | .060,991,5 | 14.779,73 |
| 61 | 190.031,1 | 59.748,87 | 830.657,9 | 82.186,19 | 15.042,64 | 380.845,7 | 24.787,06 | 2.066,334 | 119.802,6 | 3.131,497 | .066,226,2 | 15.525,03 |
| 62 | 199.156,6 | 64.520,41 | 866.741,9 | 86.203,73 | 16.275,80 | 398.467,0 | 26.013,99 | 2.238,850 | 125.592,3 | 3.287,658 | .071,817,6 | 16.294,65 |
| 63 | 208.550,9 | 69.577,35 | 903.599,8 | 90.345,56 | 17.566,42 | 416.555,0 | 27.280,15 | 2.422,574 | 131.555,6 | 3.448,907 | .077,777,2 | 17.088,98 |
| 64 | 218.216,7 | 74.930,67 | 941.222,0 | 94.613,22 | 18.977,88 | 435.110,3 | 28.586,10 | 2.618,024 | 137.694,4 | 3.615,325 | .084,124,9 | 17.908,37 |
| 65 | 228.156,2 | 80.591,55 | 979.598,0 | 99.008,20 | 20.453,62 | 454.133,8 | 29.932,40 | 2.825,741 | 144.010,5 | 3.786,989 | .090,879,2 | 18.753,19 |
| 66 | 238.371,9 | 86.571,25 | 1,018.717 | 103.532,0 | 22.017,12 | 473.625,8 | 31.319,62 | 3.046,274 | 150.505,5 | 3.963,982 | .098,059,2 | 19.623,79 |
| 67 | 248.865,8 | 92.881,15 | 1,058.567 | 108.188,1 | 23.671,99 | 493.586,6 | 32.748,31 | 3.260,192 | 157.181,2 | 4.146,382 | .105,684,6 | 20.520,54 |
| 68 | 259.640,0 | 99.532,72 | 1,099.135 | 112.972,0 | 25.421,86 | 514.016,1 | 34.219,04 | 3.528,075 | 164.039,1 | 4.334,268 | .113,775,6 | 21.443,79 |
| 69 | 270.696,8 | 106.537,6 | 1,140.409 | 117.891,0 | 27.270,45 | 534.913,9 | 35.732,35 | 3.790,520 | 171.080,7 | 4.527,721 | .122,353,2 | 22.393,90 |
| 70 | 282.037,9 | 113.907,4 | 1,182.375 | 122.944,5 | 29.221,56 | 556.279,5 | 37.288,80 | 4.068,140 | 178.307,7 | 4.726,820 | .131,438,7 | 23.371,22 |
| 71 | 293.665,2 | 121.653,9 | 1,225.018 | 128.134,0 | 31.279,05 | 578.112,0 | 38.888,93 | 4.361,558 | 185.721,5 | 4.931,643 | .141,054,2 | 24.376,11 |
| 72 | 305.580,5 | 129.786,9 | 1,268.325 | 133.460,8 | 33.446,84 | 600.410,3 | 40.533,28 | 4.671,420 | 193.323,6 | 5.142,271 | .151,222,4 | 25.408,91 |
| 73 | 317.785,6 | 138.324,2 | 1,312.280 | 138.926,2 | 35.728,91 | 623.173,2 | 42.222,39 | 4.998,381 | 201.115,2 | 5.358,782 | .161,966,4 | 26.469,98 |
| 74 | 330.282,0 | 147.271,9 | 1,356.866 | 144.531,5 | 38.129,33 | 646.399,1 | 43.956,80 | 5.343,112 | 209.097,8 | 5.581,255 | .173,310,4 | 27.559,66 |
| 75 | 343.071,1 | 156.643,8 | 1,402.068 | 150.278,1 | 40.652,21 | 670.086,0 | 45.737,04 | 5.706,302 | 217.272,6 | 5.809,769 | .185,278,7 | 28.678,29 |
| 76 | 356.154,5 | 166.451,8 | 1,447.869 | 156.167,1 | 43.301,73 | 694.232,0 | 47.563,64 | 6.088,654 | 225.640,9 | 6.044,403 | .197,896,7 | 29.826,22 |
| 77 | 369.533,5 | 176.707,9 | 1,494.251 | 162.199,8 | 46.082,14 | 718.834,8 | 49.437,13 | 6.490,886 | 234.203,9 | 6.285,236 | .211,190,1 | 31.003,80 |
| 78 | 383.209,2 | 187.424,2 | 1,541.198 | 168.377,5 | 48.997,72 | 743.891,8 | 51.358,02 | 6.913,730 | 242.962,5 | 6.532,345 | .225,185,5 | 32.211,35 |
| 79 | 397.182,9 | 198.612,5 | 1,588.690 | 174.701,2 | 52.052,83 | 769.400,2 | 53.326,83 | 7.357,937 | 251.918,3 | 6.785,809 | .239,910,0 | 33.449,23 |
| 80 | 411.455,7 | 210.284,6 | 1,636.709 | 181.172,2 | 55.251,87 | 795.357,0 | 55.344,09 | 7.824,269 | 261.071,9 | 7.045,707 | .255,391,4 | 34.717,75 |
| 81 | 426.028,4 | 222.452,6 | 1,685.237 | 187.791,5 | 58.599,30 | 821.759,0 | 57.410,30 | 8.313,509 | 270.424,5 | 7.312,117 | .271,658,3 | 36.017,27 |
| 82 | 440.901,9 | 235.128,2 | 1,734.253 | 194.560,4 | 62.099,65 | 848.602,6 | 59.525,96 | 8.826,449 | 279.976,9 | 7.585,115 | .288,739,7 | 37.348,10 |
| 83 | 456.077,2 | 248.323,1 | 1,783.740 | 201.479,8 | 65.757,46 | 875.884,2 | 61.691,59 | 9.363,900 | 289.730,2 | 7.864,782 | .306,665,7 | 38.710,58 |
| 84 | 471.554,7 | 262.049,0 | 1,833.675 | 208.550,9 | 69.577,35 | 903.599,8 | 63.907,68 | 9.926,690 | 299.685,0 | 8.151,193 | .325,466,9 | 40.105,03 |
| 85 | 487.335,2 | 276.317,4 | 1,884.[illegible] | 215.774,7 | 73.563,95 | 931.745,4 | 66.174,72 | 10.515,66 | 309.842,3 | 8.444,428 | .345,174,4 | 41.531,79 |
| 86 | 503.419,1 | 291.139,9 | 1,934.816 | 223.152,1 | 77.721,98 | 960.318,5 | 68.493,22 | 11.131,66 | 320.202,8 | 8.744,563 | .365,820,4 | 42.991,17 |
| 87 | 519.806,9 | 306.527,7 | 1,985.979 | 230.684,2 | 82.056,16 | 989.308,5 | 70.863,65 | 11.775,57 | 330.767,1 | 9.051,675 | .387,437,4 | 44.483,50 |
| 88 | 536.498,9 | 322.492,1 | 2,037.510 | 238.371,9 | 86.571,25 | 1,018.717 | 73.286,50 | 12.448,27 | 341.536,0 | 9.365,842 | .410,058,9 | 46.009,09 |
| 89 | 553.495,3 | 339.044,0 | 2,089.389 | 246.216,1 | 91.272,09 | ,048.536 | 75.762,25 | 13.150,67 | 352.509,9 | 9.687,141 | .433,719,2 | 47.568,27 |
| 90 | 570.796,3 | 356.194,5 | 2,141.593 | 254.217,7 | 96.163,5 | 1,078.762 | 78.291,38 | 13.883,68 | 363.689,5 | 10.015,65 | .458,452,8 | 49.161,34 |

SHEET 1 OF 5

205-D-399

Table H-9.–*Load constants for circular arches–twist and tangential deflections* (sheet 2).

## UNIT RADIAL LOAD NO. 2
## TWIST AND TANGENTIAL DEFLECTIONS



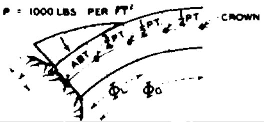
| $\phi$ ° | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | |
| | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .003,461,094 | .0$^4$47,553,5 | .151,630,9 | .003,461,094 | .0$^4$25,885,8 | .110,499,1 | .003,461,094 | .0$^4$10,757,0 | .069,158,87 | .003,461,094 | .0$^5$2,196,3 | .027,883,04 |
| 11 | .004,606,654 | .0$^4$76,546,9 | .201,634,6 | .004,606,654 | .0$^4$41,677,3 | .147,000,8 | .004,606,654 | .0$^4$17,321,8 | .092,028,41 | .004,606,654 | .0$^5$3,536,7 | .036,844,04 |
| 12 | .005,980,602 | .0$^3$,118,204,3 | .261,512,2 | .005,980,602 | .0$^4$64,374,0 | .190,742,7 | .005,980,602 | .0$^4$26,759.7 | .119,450,3 | .005,980,602 | .0$^5$5,464.3 | .047,830,61 |
| 13 | .007,603,689 | .0$^3$,176,272,8 | .332,124,5 | .007,603,689 | .0$^4$96,023,1 | .242,368,3 | .007,603,689 | .0$^4$39,923,2 | .151,832,5 | .007,603,689 | .0$^5$8,153,1 | .060,808,32 |
| 14 | .009,496,661 | .0$^3$,255,170,3 | .414,323,3 | .009,496,661 | .0$^3$,139,042 | .302,517,8 | .009,496,661 | .0$^4$57,821,0 | .189,583,8 | .009,496,661 | .0$^4$11,810 | .075,942,59 |
| 15 | .011,680,26 | .0$^3$,360,036 | .508,949,7 | .011,680,26 | .0$^3$,196,242 | .371,826,9 | .011,680,26 | .0$^4$81,624 | .233,112,0 | .011,680,26 | .0$^4$16,673 | .093,398,76 |
| 16 | .014,175,24 | .0$^3$,496,787 | .616,834,0 | .014,175,24 | .0$^3$,270,868 | .450,927,2 | .014,175,24 | .0$^3$,112,690 | .282,823,6 | .014,175,24 | .0$^4$23,022 | .113,342,0 |
| 17 | .017,002,31 | .0$^3$,672,161 | .738,795,0 | .017,002,31 | .0$^3$,366,615 | .540,445,6 | .017,002,31 | .0$^3$,152,560 | .339,124,1 | .017,002,31 | .0$^4$31,171 | .135,937,5 |
| 18 | .020,182,23 | .0$^3$,893,778 | .875,638,4 | .020,182,23 | .0$^3$,487,670 | .641,003,9 | .020,182,23 | .0$^3$,202,986 | .402,417,5 | .020,182,23 | .0$^4$41,480 | .161,350,0 |
| 19 | .023,735,71 | .001,170,180 | 1,028,157 | .023,735,71 | .0$^3$,638,731 | .753,218,7 | .023,735,71 | .0$^3$,265,935 | .473,106,4 | .023,735,71 | .0$^4$54,352 | .189,744,3 |
| 20 | .027,683,48 | .001,510,887 | 1.197,130 | .027,683,48 | .0$^3$,825,041 | .877,700,9 | .027,683,48 | .0$^3$,343,603 | .551,592,0 | .027,683,48 | .0$^4$70,239 | .221,285,2 |
| 21 | .032,046,25 | .001,926,434 | 1.383,321 | .032,046,25 | .001,052,408 | 1.015,056 | .032,046,25 | .0$^3$,438,425 | .638,274,0 | .032,046,25 | .0$^4$89,636 | .256,136,9 |
| 22 | .036,844,74 | .002,428,435 | 1.587,479 | .036,844,74 | .001,327,250 | 1.165,882 | .036,844,74 | .0$^3$,553,096 | .733,550,1 | .036,844,74 | .0$^3$,113,101 | .294,463,9 |
| 23 | .042,099,66 | .003,029,610 | 1.810,337 | .042,099,66 | .001,656,602 | 1.330,772 | .042,099,66 | .0$^3$,690,572 | .837,616,6 | .042,099,66 | .0$^3$,141,239 | .336,430,0 |
| 24 | .047,831,71 | .003,743,841 | 2.052,809 | .047,831,71 | .002,048,159 | 1.510,312 | .047,831,71 | .0$^3$,854,090 | .951,468,0 | .047,831,71 | .0$^3$,174,717 | .382,199,3 |
| 25 | .054,061,57 | .004,586,214 | 2.314,995 | .054,061,57 | .002,510,291 | 1.705,080 | .054,061,57 | .001,047,175 | 1,074,897 | .054,061,57 | .0$^3$,214,258 | .431,935,4 |
| 26 | .060,809,95 | .005,573,052 | 2.598,175 | .060,809,95 | .003,052,080 | 1.915,648 | .060,809,95 | .001,273,658 | 1.208,493 | .060,809,95 | .0$^3$,260,653 | .485,801,8 |
| 27 | .068,097,52 | .006,721,967 | 2.902,809 | .068,097,52 | .003.683,335 | 2.142,579 | .068,097,52 | .001,537,683 | 1.352,646 | .068,097,52 | .0$^3$,314,754 | .543,961,6 |
| 28 | .075,944,95 | .008,051,894 | 3.229,540 | .075,944,95 | .004,414,629 | 2.386,429 | .075,944,95 | .001,843,716 | 1.507,742 | .075,944,95 | .0$^3$,377,481 | .606,577,9 |
| 29 | .084,372,92 | .009,583.134 | 3.578,986 | .084,372,92 | .005,257,311 | 2.647,745 | .084,372,92 | .002,196,563 | 1.674,164 | .084,372,92 | .0$^3$,449,823 | .673,813,5 |
| 30 | .093,402,09 | .011,337,37 | 3.951,755 | .093,402,09 | .006,223,558 | 2.927,067 | .093,402,09 | .002,601,396 | 1.852,296 | .093,402,09 | .0$^3$,532,858 | .745,831,0 |
| 31 | .103,053,1 | .013,337,75 | 4.348,420 | .103,053,1 | .007,326,362 | 3.224,924 | .103,053,1 | .003,063,729 | 2.042,515 | .103,053,1 | .0$^3$,627,72 | .822,792,4 |
| 32 | .113,346,6 | .015,608,84 | 4.769,537 | .113,346,6 | .008,579,557 | 3.541,839 | .113,346,6 | .003,58945 | 2.245,200 | .113,346,6 | .0$^3$,735,63 | .904,859,8 |
| 33 | .124,303,3 | .018,176,77 | 5.215,651 | .124,303,3 | .009,997,893 | 3.878,323 | .124,303,3 | .004,184,82 | 2.460,724 | .124,303,3 | .0$^3$,857,87 | .992,195,1 |
| 34 | .135,943,7 | .021,069,12 | 5.687,261 | .135,943,6 | .011,596,99 | 4.234,877 | .135,943,6 | .004,856,55 | 2.689,460 | .135,943,6 | .0$^3$,995,86 | 1.084,960 |
| 35 | .148,288,5 | .024,315,11 | 6.184,864 | .148,288,5 | .013,393,42 | 4.611,997 | .148,288,5 | .005,611,67 | 2.931,777 | .148,288,5 | .001,151,02 | 1.183,314 |
| 36 | .161,358,3 | .027,945,47 | 6.708,918 | .161,358,3 | .015,404,66 | 5.010,163 | .161,358,3 | .006,457,71 | 3.188,042 | .161,358,3 | .001,324,92 | 1.287,420 |
| 37 | .175,173,6 | .031,992,62 | 7.259,863 | .175,173,6 | .017,649,22 | 5.429,848 | .175,173,6 | .007,402,61 | 3.458,617 | .175,173,6 | .001,519,25 | 1.397,438 |
| 38 | .189,755,2 | .036,490,54 | 7.838,113 | .189,755,2 | .020,146,54 | 5.871,513 | .189,755,2 | .008,454,70 | 3.743,865 | .189,755,2 | .001,735,71 | 1.513,527 |
| 39 | .205.123,6 | .041,474,91 | 8.444,055 | .205,123,6 | .022,917,08 | 6.335,810 | .205,123,6 | .009,622,81 | 4.044,142 | .205,123,6 | .001,976,15 | 1.635,848 |
| 40 | .221,299,2 | .046,983,06 | 9.078,050 | .221,299,2 | .025,982,34 | 6.822,577 | .221,299,2 | .010,916,23 | 4.359,804 | .221,299,2 | .002,242,50 | 1.764,561 |
| 41 | .238,302,8 | .053,054,05 | 9.740,432 | .238,302,8 | .029,364,87 | 7.332,844 | .238,302,8 | .012,344,69 | 4.691,202 | .238,302,8 | .002,536,78 | 1.899,824 |
| 42 | .256,154,9 | .059,728,64 | 10.431,51 | .256,154,9 | .033,088,27 | 7.866,826 | .256,154,9 | .013,918,44 | 5.038,685 | .256,154,9 | .002,861,16 | 2.041,796 |
| 43 | .274,875,9 | .067,049,28 | 11.151,58 | .274,875,9 | .037,177,21 | 8.424,930 | .274,875,9 | .015,648,19 | 5.402,598 | .274,875,9 | .003,217,86 | 2.190,836 |
| 44 | .294,486,5 | .075,060,18 | 11.900,83 | .294,486,5 | .041,657,51 | 9.007,548 | .294,486,5 | .017,545,19 | 5.783,281 | .294,486,5 | .003,609,25 | 2.346,503 |
| 45 | .315,007,1 | .083,807,36 | 12.679,54 | .315,007.1 | .046,556,05 | 9.615.060 | .315,007,1 | .019,621,15 | 6.181,074 | .315,007,1 | .004,037,78 | 2.509,553 |
| 46 | .336,458,3 | .093,338,54 | 13.487,90 | .336,458,3 | .051,900,88 | 10.247.83 | .336,458,3 | .021,888,37 | 6.596,312 | .336,458,3 | .004,506,02 | 2.679,943 |
| 47 | .358,860,5 | .103,703,3 | 14.326,06 | .358,860,5 | .057,721,28 | 10.906,23 | .358,860,5 | .024,359,71 | 7.029,326 | .358,860,5 | .005,016,74 | 2.857,832 |
| 48 | .382,234,3 | .114,952,8 | 15.194,15 | .382,234,3 | .064,047,23 | 11.590,58 | .382,234,3 | .027,048,23 | 7.480,444 | .382,234,3 | .005,572,56 | 3.043,376 |
| 49 | .406,800,0 | .127,139,8 | 16.092,29 | .406,600,0 | .070,910,75 | 12.301,22 | .406,800,0 | .029,968,25 | 7.949,989 | .406,600,0 | .006,176,74 | 3.236,730 |
| 50 | .431,978,3 | .140,319,4 | 17.020,54 | .431,978,3 | .078,344,04 | 13.038,47 | .431,978,3 | .033,133,70 | 8.438,282 | .431,978,3 | .006,831,83 | 3.438,051 |
| 51 | .458,389,4 | .154,548,4 | 17.978.94 | .458,389,4 | .086,381,35 | 13.802,83 | .458,389,4 | .036,560,05 | 8.945,641 | .458,389,4 | .007,541,42 | 3.647,493 |
| 52 | .485,853,9 | .169,884,9 | 18.967,51 | .485,853,9 | .095,057,74 | 14.593,98 | .485,853,9 | .040,262,80 | 9.472,375 | .485,853,9 | .008,308,77 | 3.865,212 |
| 53 | .514,392,2 | .186,389,0 | 19.986,23 | .514,392,2 | .104,409,4 | 15.412,81 | .514,392,2 | .044,257,94 | 10.018,79 | .514,392,2 | .009,137,14 | 4.091,362 |
| 54 | .544,024,6 | .204,122,7 | 21.035,04 | .544,024,6 | .114,474,1 | 16.259,37 | .544,024,6 | .048,562,54 | 10.585,20 | .544,024,6 | .010,030,28 | 4.326,096 |
| 55 | .574,771,6 | .223,149,7 | 22.113,87 | .574,771,6 | .125,290,7 | 17.133,91 | .574,771,6 | .053,193,99 | 11.171,90 | .574,771,6 | .010,991,96 | 4.569,569 |
| 56 | .606,853,5 | .243,535,0 | 23.222,60 | .606,653,5 | .136,898,7 | 18.036,66 | .606,653,5 | .058,169,75 | 11.779,19 | .606,653,5 | .012,025,49 | 4.821,934 |
| 57 | .639,690,7 | .265,346,0 | 24.361,08 | .639,690,7 | .149,339,9 | 18.967,84 | .639,690,7 | .063,509,05 | 12.407,35 | .639,690,7 | .013,135,37 | 5.083,343 |
| 58 | .673,903,6 | .288,651,5 | 25.529,12 | .673,903,6 | .162,656,8 | 19.927,65 | .673,903,6 | .069,230,92 | 13.056,68 | .673,903,6 | .014,325,56 | 5.353,947 |
| 59 | .709,312,5 | .313,521,9 | 26.726,53 | .709,312,5 | .176,893,3 | 20.916,27 | .709,312,5 | .075,355,36 | 13.727,46 | .709,312,5 | .015,800,42 | 5.633,899 |
| 60 | .745,937,6 | .340,029,5 | 27.953,05 | .745,937,6 | .192,094,4 | 21.933,89 | .745,937,6 | .081,902,86 | 14.419,97 | .745,937,6 | .016,964,25 | 5.923,349 |
| 61 | .783,799,4 | .368,247,7 | 29.208,41 | .783,799,4 | .208,306,5 | 22.980,65 | .783,799,4 | .088,894,57 | 15.134,48 | .783,799,4 | .018,421,55 | 6.222,449 |
| 62 | .822,918,1 | .398,252,4 | 30.492,28 | .822,918,1 | .225,577,4 | 24.056,70 | .822,918,1 | .098,352,42 | 15.871,25 | .822,918,1 | .019,977,19 | 6.531,348 |
| 63 | .863,313,9 | .430,120,3 | 31.804,34 | .863,313,9 | .243,955,9 | 25.162,16 | .863,313,9 | .104,298,9 | 16.630,57 | .863,313,9 | .021,635,91 | 6.850,195 |
| 64 | .905,007,1 | .463,930,1 | 33.144,21 | .905,007,1 | .263,492,5 | 26.297,15 | .905,007,1 | .112,757,0 | 17.412,68 | .905,007,1 | .023,402,75 | 7.179,140 |
| 65 | .948,018,1 | .499,762,0 | 34.511,47 | .948,018,1 | .284,238,6 | 27.461,76 | .948,018,1 | .121,751,2 | 18.217,85 | .948,018,1 | .025,282,92 | 7.518,330 |
| 66 | .992,366,9 | .537,697,5 | 35.905,67 | .992,366,9 | .306,247,1 | 28.656,06 | .992,366,9 | .131,305,7 | 19.046,32 | .992,366,9 | .027,281,75 | 7.867,915 |
| 67 | 1.038,074 | .577,819,8 | 37.326,36 | 1.038,074 | .329,571,9 | 29.880,12 | 1.038,074 | .141,445,6 | 19.898,34 | 1.038,074 | .029,404,60 | 8.228,041 |
| 68 | 1.085,159 | .620,213,4 | 38.773,00 | 1.085,159 | .354,268,9 | 31.133,98 | 1.085,159 | .152,197,2 | 20.774.15 | 1.085,159 | .031,657,4 | 8.598,854 |
| 69 | 1.133,643 | .664,963,9 | 40.245,07 | 1.133,642 | .380,394,2 | 32.417,68 | 1.133,642 | .163,586,8 | 21.673,99 | 1.133,642 | .034,045,7 | 8.980,501 |
| 70 | 1.183,545 | .712,159,3 | 41.741,99 | 1.183,545 | .408,006,5 | 33.731,22 | 1.183,545 | .175,642,3 | 22.598,08 | 1.183,545 | .036,575,7 | 9.373,128 |
| 71 | 1.234,886 | .761,887,3 | 43.263,15 | 1.234,886 | .437,164,5 | 35.074,60 | 1.234,886 | .188,391,4 | 23.546,66 | 1.234,886 | .039,253,4 | 9.776,880 |
| 72 | 1.287,685 | .814,238,3 | 44.807,91 | 1.287,685 | .467,929,0 | 36.447,81 | 1.287,685 | .201,863,2 | 24.519,95 | 1.287,685 | .042,085,2 | 10.191,90 |
| 73 | 1.341,964 | .869,303,2 | 46.375,59 | 1.341,964 | .500,361,9 | 37.850,80 | 1.341,964 | .216,087,1 | 25.518,15 | 1.341,964 | .045,077,7 | 10.618,33 |
| 74 | 1.397,742 | .927,174,2 | 47.965,49 | 1.397,742 | .534,526,1 | 39.283,52 | 1.397,742 | .231,093,8 | 26.541,49 | 1.397,742 | .048,237,5 | 11.056,32 |
| 75 | 1.455,039 | .987,944,7 | 49.576,87 | 1.455,039 | .570,486,1 | 40.745,90 | 1.455,039 | .246,913,8 | 27.590,16 | 1.455,039 | .051,571,5 | 11.506,01 |
| 76 | 1.513,875 | 1.051,709 | 51.208,97 | 1.513,875 | .608,307,5 | 42.237,85 | 1.513,875 | .263,579,2 | 28.664,36 | 1.513,875 | .055,086,6 | 11.967,54 |
| 77 | 1.574,270 | 1.118,563 | 52.860,97 | 1.574,270 | .648,057,0 | 43.759,26 | 1.574,270 | .281,122,5 | 29.764,31 | 1.574,270 | .058,790,2 | 12.441,05 |
| 78 | 1.636,244 | 1.188,802 | 54.532,04 | 1.636,244 | .689,802,8 | 45.310,01 | 1.636,244 | .299,577,1 | 30.890,16 | 1.636,244 | .062,689,7 | 12.926,68 |
| 79 | 1.699,818 | 1.261,923 | 56.221,32 | 1.699,818 | .733,614,3 | 46.889,97 | 1.699,818 | .318,976,9 | 32.042,12 | 1.699,818 | .066,792,7 | 13.424,57 |
| 80 | 1.765,010 | 1.338,626 | 57.927,91 | 1.765,010 | .779,561,7 | 48.498,98 | 1.765,010 | .339,357,0 | 33.220,36 | 1.765,010 | .071,106,9 | 13.934,87 |
| 81 | 1.831,842 | 1.418,809 | 59.650,89 | 1.831,842 | .827,717,0 | 50.138,86 | 1.831,842 | .360,752,6 | 34.425,05 | 1.831,842 | .075,640,4 | 14.457,70 |
| 82 | 1.900,332 | 1.502,570 | 61.389,29 | 1.900,332 | .878,152,9 | 51.803,42 | 1.900,332 | .383,200,5 | 35.656,36 | 1.900,332 | .080,401,3 | 14.993,21 |
| 83 | 1.970,502 | 1.590,011 | 63.142,13 | 1.970,502 | .930,943,7 | 53.498,45 | 1.970,502 | .406,737,6 | 36.914,45 | 1.970,502 | .085,398,0 | 15.541,54 |
| 84 | 2.042,370 | 1.681,232 | 64.908,38 | 2.042,370 | .986,164,3 | 55.221,73 | 2.042,370 | .431,402,0 | 38.199,47 | 2.042,370 | .090,639,2 | 16.102,82 |
| 85 | 2.115,956 | 1.776,334 | 66.687,01 | 2.115,956 | 1.043,891 | 56.973,01 | 2.115,956 | .457,232,1 | 39.511,58 | 2.115,956 | .096,133,4 | 16.677,19 |
| 86 | 2.191,281 | 1.875,420 | 68.476,93 | 2.191,281 | 1.104,202 | 58.752,03 | 2:191,281 | .484,267,7 | 40.850,91 | 2.191,281 | .101,889,7 | 17.264,78 |
| 87 | 2.268,365 | 1.978,590 | 70.277.04 | 2.268,365 | 1.167,175 | 60.558,51 | 2.268,365 | .512,549,0 | 42.217,61 | 2.268,365 | .107,917,4 | 17.865,73 |
| 88 | 2.347,226 | 2.085,947 | 72.086.20 | 2.347,226 | 1.232,890 | 62.392,16 | 2.347,226 | .542,117,0 | 43.611,81 | 2.347,226 | .114,225,7 | 18.480,17 |
| 89 | 2.427,885 | 2.197,594 | 73.903,25 | 2.427,885 | 1.301,428 | 64.252,86 | 2.427,885 | .573,013,4 | 45.033,63 | 2.427,885 | .120,824,2 | 19.108,23 |
| 90 | 2.510,362 | 2.313,633 | 75.727,01 | 2.510,362 | 1.372,871 | 66.139,68 | 2.510,362 | .605,281,3 | 46.483,19 | 2.510,362 | .127,722,8 | 19.750,05 |

SHEET 2 OF 5

205-D-400

Table H-9.–*Load constants for circular arches–twist and tangential deflections* (sheet 3).

UNIT RADIAL LOAD NO. 3
TWIST AND TANGENTIAL DEFLECTIONS



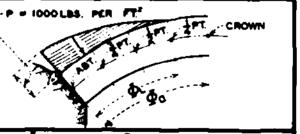
| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | |
| ° | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term |
| 10 | .027,683,48 | .0$^{3}$,343,603 | .551,592,0 | .027,683,48 | .0$^{3}$,180,744 | .386,806,7 | .027,683,48 | .0$^{4}$70,239 | .221,285,1 | .008,652,514 | .0$^{5}$5,052 | .048,443,45 |
| 11 | .036,844,74 | .0$^{3}$,553,096 | .733,550,1 | .036,844,74 | .0$^{3}$,291,002 | .514,599,6 | .036,844,74 | .0$^{3}$,113,101 | .294,463,9 | .011,516,28 | .0$^{5}$8,135 | .064,474,06 |
| 12 | .047,831,71 | .0$^{3}$,854,090 | .951,468,0 | .047,831,71 | .0$^{3}$,449,468 | .667,748,8 | .047,831,71 | .0$^{3}$,174,717 | .382,199,3 | .014,950,96 | .0$^{4}$12,568 | .083,698,93 |
| 13 | .060,809,95 | .001,273,658 | 1.208,493 | .060,809,95 | .0$^{3}$,670,432 | .848,512,2 | .060,809,95 | .0$^{3}$,260,653 | .485,801,8 | .019,008,41 | .0$^{4}$18,751 | .106,407,6 |
| 14 | .075,944,95 | .001,843,716 | 1.507,742 | .075,944,95 | .0$^{3}$,970,757 | 1.059,136 | .075,944,95 | .0$^{3}$,377,481 | .606,577,9 | .023,740,47 | .0$^{4}$27,160 | .132,889,5 |
| 15 | .093,402,09 | .002,601,396 | 1.652,296 | .093,402,09 | .001,370,08 | 1.301,851 | .093,402,09 | .0$^{3}$,532,858 | .745,831,0 | .029,198,92 | .0$^{4}$38,347 | .163,433,7 |
| 16 | .113,346,6 | .003,589,45 | 2.245,200 | .113,346,6 | .001,891,04 | 1.578,876 | .113,346,6 | .0$^{3}$,735,63 | .904,859,8 | .035,435,79 | .0$^{4}$52,947 | .198,329,0 |
| 17 | .135,943,7 | .004,856,55 | 2.689,460 | .135,943,7 | .002,559,42 | 1.892,414 | .135,943,7 | .0$^{3}$,995,86 | 1.084,960 | .042,502,66 | .0$^{4}$71,689 | .237,864,0 |
| 18 | .161,358,3 | .006,457,71 | 3.188,042 | .161,358,3 | .003,404,40 | 2.244,650 | .161,358,3 | .001,324,92 | 1.287,420 | .050,451,42 | .0$^{4}$95,397 | .282,327,2 |
| 19 | .189,755,2 | .008,454,70 | 3.743,865 | .189,755,2 | .004,458,81 | 2.637,755 | .189,755,2 | .001,735,71 | 1.513,527 | .059,333,83 | .0$^{3}$,124,998 | .332,006,4 |
| 20 | .221,299,2 | .010,916,23 | 4.359,804 | .221,299,2 | .005,759,17 | 3.073,879 | .221,299,2 | .002,242,50 | 1.764,561 | .069,201,66 | .0$^{3}$,161,529 | .387,189,4 |
| 21 | .256,154,9 | .013,918,44 | 5.038,685 | .256,154,9 | .007,346,03 | 3.555,155 | .256,154,9 | .002,861,16 | 2.041,796 | .080,106,66 | .0$^{3}$,206,143 | .448,163,5 |
| 22 | .294,486,5 | .017,545,19 | 5.783,281 | .294,486,5 | .009,264,11 | 4.083,692 | .294,486,5 | .003,609,25 | 2.346,503 | .092,100,54 | .0$^{3}$,260,095 | .515,215,7 |
| 23 | .336,458,3 | .021,868,37 | 6.596,312 | .336,458,3 | .011,562,83 | 4.661,583 | .336,458,3 | .004,508,02 | 2.679,943 | .105,235,0 | .0$^{3}$,324,80 | .586,632,6 |
| 24 | .382,234,3 | .027,048,23 | 7.480,444 | .382,234,3 | .014,294,78 | 5.290,894 | .382,234,3 | .005,572,56 | 3.043,376 | .119,561,8 | .0$^{3}$,401,79 | .668,700,5 |
| 25 | .431,978,3 | .033,133,70 | 8.438,282 | .431,978,3 | .017,519,37 | 5.973,672 | .431,978,3 | .006,831,83 | 3.438,051 | .135,132,5 | .0$^{3}$,492,71 | .755,705,0 |
| 26 | .485,853,9 | .040,262,80 | 9.472,375 | .485,853,9 | .021,299,53 | 6.711,939 | .485,853,9 | .008,308,77 | 3.865,212 | .151,998,8 | .0$^{3}$,599,38 | .849,931,7 |
| 27 | .544,024,6 | .048,562,54 | 10.585,20 | .544,024,6 | .025,703,65 | 7.507,690 | .544,024,6 | .010,030,28 | 4.326,096 | .170,212,3 | .0$^{3}$,723,78 | .951,665,6 |
| 28 | .606,653,5 | .058,169,75 | 11.779,19 | .606,653,5 | .030,805,32 | 8.362,897 | .606,653,5 | .012,025,49 | 4.821,934 | .189,824,6 | .0$^{3}$,868,01 | 1.061,191 |
| 29 | .673,903,6 | .069,230,92 | 13.056,68 | .673,903,6 | .036,683,64 | 9.279,505 | .673,903,6 | .014,325,56 | 5.353,947 | .210,887,3 | .001,034,35 | 1.178,792 |
| 30 | .745,937,6 | .081,902,86 | 14.419,97 | .745,937,6 | .043,423,47 | 10.259,43 | .745,937,6 | .016,964,25 | 5.923,349 | .233,451,9 | .001,225,27 | 1.304,752 |
| 31 | .822,918,1 | .096,352,42 | 15.871,25 | .822,918,1 | .051,115,08 | 11.304,56 | .822,918,1 | .019,977,19 | 6.531,348 | .257,569,9 | .001,443,34 | 1.439,354 |
| 32 | .905,007,1 | .112,757,0 | 17.412,68 | .905,007,1 | .059,854,87 | 12.416,75 | .905,007,1 | .023,402,75 | 7.179,140 | .283,292,9 | .001,691,44 | 1.582,881 |
| 33 | .992,366,9 | .131,305,7 | 19.046,32 | .992,366,9 | .069,745,85 | 13.597,84 | .992,366,9 | .027,281,75 | 7.867,915 | .310,672,3 | .001,972,50 | 1.735,614 |
| 34 | 1.085,159 | .152,197,2 | 20.774,15 | 1.085,159 | .080,896,32 | 14.849,61 | 1.085,159 | .031,657,4 | 8.598,854 | .339,759,4 | .002,289,70 | 1.897,834 |
| 35 | 1.183,545 | .175,642,3 | 22.598,08 | 1.183,545 | .093,421,60 | 16.173,85 | 1.183,545 | .036,575,7 | 9.373,128 | .370,605,9 | .002,646,42 | 2.069,824 |
| 36 | 1.287,685 | .201,863,2 | 24.519,95 | 1.287,685 | .107,443,5 | 17.572,27 | 1.287,685 | .042,085,2 | 10.191,90 | .403,262,9 | .003,048,21 | 2.251,862 |
| 37 | 1.397,742 | .231,093,8 | 26.541,49 | 1.397,742 | .123,090,5 | 19.046,58 | 1.397,742 | .048,237,5 | 11.056,32 | .437,781,9 | .003,492,93 | 2.444,228 |
| 38 | 1.513,875 | .263,579,2 | 28.664,36 | 1.513,875 | .140,497,8 | 20.598,45 | 1.513,875 | .055,086,6 | 11.967,54 | .474,214,1 | .003,990,51 | 2.647,201 |
| 39 | 1.636,244 | .299,577,1 | 30.890,16 | 1.636,244 | .159,807,8 | 22.229,50 | 1.636,244 | .062,689,7 | 12.926,68 | .512,610,9 | .004,543,18 | 2.861,059 |
| 40 | 1.765,010 | .339,357,0 | 33.220,36 | 1.765,010 | .181,169,8 | 23.941,33 | 1.765,010 | .071,106,9 | 13.934,87 | .553,023,4 | .005,155,40 | 3.086,081 |
| 41 | 1.900,332 | .383,200,5 | 35.656,36 | 1.900,332 | .204,740,5 | 25.735,51 | 1.900,332 | .080,401,3 | 14.993,21 | .595,502,8 | .005,831,83 | 3.322,542 |
| 42 | 2.042,370 | .431,402,0 | 38.199,47 | 2.042,370 | .230,683,7 | 27.613,54 | 2.042,370 | .090,639,2 | 16.102,82 | .640,100,4 | .006,577,38 | 3.570,719 |
| 43 | 2.191,281 | .484,267,7 | 40.850,91 | 2.191,281 | .259,170,9 | 29.576,91 | 2.191,281 | .101,889,7 | 17.264,78 | .686,867,3 | .007,397,23 | 3.830,888 |
| 44 | 2.347,226 | .542,117,0 | 43.611,81 | 2.347,226 | .290,381,3 | 31.627,07 | 2.347,226 | .114,225,7 | 18.480,17 | .735,854,4 | .008,296,67 | 4.103,323 |
| 45 | 2.510,362 | .605,281,3 | 46.483,19 | 2.510,362 | .324,501,3 | 33.765,42 | 2.510,362 | .127,722,8 | 19.750,05 | .787,112,9 | .009,281,54 | 4.388,298 |
| 46 | 2.680,847 | .674,105,1 | 49.465,99 | 2.680,847 | .361,725,6 | 35.993,32 | 2.680,847 | .142,460,3 | 21.075,49 | .840,693,9 | .010,357,63 | 4.686,086 |
| 47 | 2.858,839 | .748,945,5 | 52.561,11 | 2.858,839 | .402,256,7 | 38.312,09 | 2.858,839 | .158,520,8 | 22.457,52 | .896,648,2 | .011,531,12 | 4.996,960 |
| 48 | 3.044,494 | .830,172,4 | 55.769,09 | 3.044,494 | .446,304,8 | 40.723,03 | 3.044,494 | .175,990,2 | 23.897,18 | .955,026,8 | .012,808,54 | 5.321,192 |
| 49 | 3.237,969 | .918,168,1 | 59.090,77 | 3.237,969 | .494,088,1 | 43.227,36 | 3.237,969 | .194,958,4 | 25.395,48 | 1.015,891 | .014,196,6 | 5.659,052 |
| 50 | 3.439,421 | 1.013,329 | 62.526,82 | 3.439,421 | .545,833,8 | 45.826,29 | 3.439,421 | .215,518,3 | 26.953,43 | 1.079,260 | .015,702,0 | 6.010,809 |
| 51 | 3.649,006 | 1.116,027 | 66.077,05 | 3.649,006 | .601,776,6 | 48.520,97 | 3.649,006 | .237,766,7 | 28.572,03 | 1.145,217 | .017,332,5 | 6.376,735 |
| 52 | 3.866,879 | 1.226,755 | 69.742,49 | 3.866,879 | .662,159,4 | 51.312,50 | 3.866,879 | .261,803,8 | 30.252,25 | 1.213,801 | .019,095,4 | 6.757,095 |
| 53 | 4.093,196 | 1.345,942 | 73.523,06 | 4.093,196 | .727,233,7 | 54.201,96 | 4.093,196 | .287,733,8 | 31.995,07 | 1.285,064 | .020,998,7 | 7.152,158 |
| 54 | 4.328,110 | 1.473,970 | 77.418,94 | 4.328,110 | .797,259,8 | 57.190,36 | 4.328,110 | .315,664,9 | 33.801,43 | 1.359,055 | .023,050,5 | 7.562,190 |
| 55 | 4.571,776 | 1.611,330 | 81.430,14 | 4.571,776 | .872,506,0 | 60.278,67 | 4.571,776 | .345,708,6 | 35.672,27 | 1.435,826 | .025,259,4 | 7.987,457 |
| 56 | 4.824,348 | 1.758,495 | 85.556,58 | 4.824,348 | .953,249,4 | 63.467,82 | 4.824,348 | .377,980,6 | 37.608,53 | 1.515,427 | .027,633,9 | 8.428,224 |
| 57 | 5.085,980 | 1.915,947 | 89.798,07 | 5.085,980 | 1.039,776 | 66.758,69 | 5.085,980 | .412,600,8 | 39.611,11 | 1.597,908 | .030,183,5 | 8.884,753 |
| 58 | 5.356,824 | 2.084,183 | 94.154,32 | 5.356,824 | 1.132,379 | 70.152,13 | 5.356,824 | .449,692,1 | 41.680,91 | 1.683,321 | .032,917,4 | 9.357,308 |
| 59 | 5.637,032 | 2.263,711 | 98.624,92 | 5.637,032 | 1.231,362 | 73.648,90 | 5.637,032 | .489,381,9 | 43.818,82 | 1.771,714 | .035,845,6 | 9.846,150 |
| 60 | 5.926,757 | 2.455,053 | 103.209,4 | 5.926,757 | 1.337,037 | 77.249,75 | 5.926,757 | .531,802,4 | 46.025,69 | 1.863,140 | .038,977,8 | 10.351,54 |
| 61 | 6.226,150 | 2.658,738 | 107.907,0 | 6.226,150 | 1.449,723 | 80.955,37 | 6.226,150 | .577,089,0 | 48.302,40 | 1.957,648 | .042,324,8 | 10.873,74 |
| 62 | 6.535,362 | 2.875,313 | 112.717,2 | 6.535,362 | 1.569,750 | 84.766,40 | 6.535,362 | .625,381,4 | 50.649,77 | 2.055,288 | .045,897,4 | 11.413,00 |
| 63 | 6.854,543 | 3.105,332 | 117.639,0 | 6.854,543 | 1.697,456 | 88.683,42 | 6.854,543 | .676,823,7 | 53.068,63 | 2.156,111 | .049,706,6 | 11.969,59 |
| 64 | 7.183,843 | 3.349,362 | 122.671,6 | 7.183,843 | 1.833,188 | 92.706,99 | 7.183,843 | .731,564,2 | 55.559,79 | 2.260,166 | .053,763,8 | 12.543,76 |
| 65 | 7.523,413 | 3.607,980 | 127.813,8 | 7.523,413 | 1.977,299 | 96.837,59 | 7.523,413 | .789,754,8 | 58.124,04 | 2.367,504 | .058,081,0 | 13.135,76 |
| 66 | 7.873,400 | 3.881,776 | 133.064,5 | 7.873,400 | 2.130,156 | 101.075,6 | 7.873,400 | .851,552,8 | 60.762,15 | 2.478,174 | .062,670,5 | 13.745,85 |
| 67 | 8.233,954 | 4.171,348 | 138.422,4 | 8.233,954 | 2.292,130 | 105.421,6 | 8.233,954 | .917,119,0 | 63.474,89 | 2.592,228 | .067,544,6 | 14.374,29 |
| 68 | 8.605,221 | 4.477,306 | 143.886,2 | 8.605,221 | 2.463,603 | 109.875,7 | 8.605,221 | .986,618,8 | 66.263,00 | 2.709,713 | .072,716,5 | 15.021,32 |
| 69 | 8.987,350 | 4.800,269 | 149.454,4 | 8.987,350 | 2.644,964 | 114.438,3 | 8.987,350 | 1.060,222 | 69.127,22 | 2.830,682 | .078,199,5 | 15.687,20 |
| 70 | 9.380,488 | 5.140,868 | 155.125,4 | 9.380,488 | 2.836,815 | 119.109,6 | 9.380,488 | 1.138,103 | 72.068,25 | 2.955,182 | .084,007,2 | 16.372,17 |
| 71 | 9.784,780 | 5.499,740 | 160.897,4 | 9.784,780 | 3.038,961 | 123.889,8 | 9.784,780 | 1.220,440 | 75.086,79 | 3.083,264 | .090,153,8 | 17.076,49 |
| 72 | 10.200,37 | 5.877,534 | 166.768,7 | 10.200,37 | 3.252,418 | 128.779,0 | 10.200,37 | 1.307,415 | 78.183,52 | 3.214,978 | .096,653,8 | 17.800,41 |
| 73 | 10.627,41 | 6.274,907 | 172.737,3 | 10.627,41 | 3.477,411 | 133.777,3 | 10.627,41 | 1.399,218 | 81.359,10 | 3.350,373 | .103,522,1 | 18.544,16 |
| 74 | 11.066,04 | 6.692,523 | 178.801,2 | 11.066,04 | 3.714,374 | 138.884,8 | 11.066,04 | 1.496,039 | 84.614,18 | 3.489,499 | .110,774,0 | 19.307,99 |
| 75 | 11.516,40 | 7.131,056 | 184.958,8 | 11.516,40 | 3.963,748 | 144.101,3 | 11.516,40 | 1.598,076 | 87.949,39 | 3.632,404 | .118,425,1 | 20.092,15 |
| 76 | 11.978,63 | 7.591,187 | 191.206,5 | 11.978,63 | 4.225,983 | 149.426,9 | 11.978,63 | 1.705,530 | 91.365,33 | 3.779,139 | .126,491,7 | 20.896,88 |
| 77 | 12.452,89 | 8.073,604 | 197.543,2 | 12.452,89 | 4.501,537 | 154.861,3 | 12.452,89 | 1.818,605 | 94.862,80 | 3.929,753 | .134,990,2 | 21.722,42 |
| 78 | 12.939,31 | 8.579,004 | 203.966,1 | 12.939,31 | 4.790,876 | 160.404,5 | 12.939,31 | 1.937,513 | 98.441,78 | 4.084,295 | .143,937,6 | 22.569,02 |
| 79 | 13.438,03 | 9.108,085 | 210.472,6 | 13.438,03 | 5.094,475 | 166.056,1 | 13.438,03 | 2.062,467 | 102.103,4 | 4.242,813 | .153,351,3 | 23.436,90 |
| 80 | 13.949,20 | 9.661,558 | 217.060,1 | 13.949,20 | 5.412,817 | 171.815,8 | 13.949,20 | 2.193,688 | 105.848,1 | 4.405,359 | .163,249,2 | 24.328,31 |
| 81 | 14.472,95 | 10.240,14 | 223.725,8 | 14.472,95 | 5.746,392 | 177.683,4 | 14.472,95 | 2.331,402 | 109.676,3 | 4.571,979 | .173,649,4 | 25.237,48 |
| 82 | 15.009,43 | 10.844,54 | 230.466,9 | 15.009,43 | 6.095,698 | 183.658,4 | 15.009,43 | 2.475,834 | 113.588,5 | 4.742,724 | .184,570,5 | 26.170,66 |
| 83 | 15.558,77 | 11.475,48 | 237.280,4 | 15.558,77 | 6.461,243 | 189.740,3 | 15.558,77 | 2.627,221 | 117.585,3 | 4.917,642 | .196,031,9 | 27.126,06 |
| 84 | 16.121,11 | 12.133,70 | 244.163,4 | 16.121,11 | 6.843,538 | 195.928,7 | 16.121,11 | 2.785,797 | 121.667,0 | 5.096,781 | .208,053,0 | 28.103,93 |
| 85 | 16.696,59 | 12.819,93 | 251.112,5 | 16.696,59 | 7.242,106 | 202.222,9 | 16.696,59 | 2.951,807 | 125.834,2 | 5.280,192 | .220,653,9 | 29.104,49 |
| 86 | 17.285,35 | 13.534,90 | 258.124,7 | 17.285,35 | 7.660,475 | 208.622,4 | 17.285,35 | 3.125,498 | 130.087,3 | 5.467,922 | .233,855,1 | 30.127,96 |
| 87 | 17.887,52 | 14.279,34 | 265.196,5 | 17.887,52 | 8.096,183 | 215.126,5 | 17.887,52 | 3.307,124 | 134.426,6 | 5.660,020 | .247,677,3 | 31.174,62 |
| 88 | 18.503,23 | 15.054,00 | 272.324,6 | 18.503,23 | 8.550,763 | 221.734,5 | 18.503,23 | 3.496,927 | 138.852,7 | 5.856,534 | .262,142,3 | 32.244,63 |
| 89 | 19.132,63 | 15.859,62 | 279.505,4 | 19.132,63 | 9.024,785 | 228.445,6 | 19.132,63 | 3.695,204 | 143.365,8 | 6.057,513 | .277,271,7 | 33.338,25 |
| 90 | 19.775,85 | 16.696,94 | 286.735,4 | 19.775,85 | 9.518,790 | 235.258,9 | 19.775,85 | 3.902,186 | 147.965,4 | 6.263,006 | .293,087,8 | 34.455,70 |

SHEET 3 OF 5

205-D-401

Table H-9.–*Load constants for circular arches–twist and tangential deflections* (sheet 4).

UNIT RADIAL LOAD NO. 4
TWIST AND TANGENTIAL DEFLECTIONS



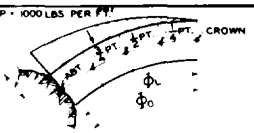
| Φ | Crown | | | ¼ Point | | | ½ Point | | | ¾ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | |
| ° | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term |
| 10 | .093,402,09 | .001,047,005 | 1.117,013 | .093,402,09 | .0$^3$532,858 | .745,831,0 | .055,362,27 | .0$^3$131,099 | .331,888,0 | .010,382,99 | .0$^5$6,003,5 | .055,363,59 |
| 11 | .124,303,3 | .001,685,29 | 1.485,502 | .124,303,3 | .0$^3$857,87 | .992,195,1 | .073,681,94 | .0$^3$211,097 | .441,631,7 | .013,819,49 | .0$^5$9,888 | .073,684,06 |
| 12 | .161,358,3 | .002,602,28 | 1.926,801 | .161,358,3 | .001,324,92 | 1.287,420 | .095,651,75 | .0$^3$326,094 | .573,200,1 | .017,941,08 | .0$^4$14,936 | .095,655,03 |
| 13 | .205,123,6 | .003,880,45 | 2.447,347 | .205,123,6 | .001,976,15 | 1.635,848 | .121,602,5 | .0$^3$486,47 | .728,555,1 | .022,809,98 | .0$^4$22,284 | .121,607,4 |
| 14 | .256,154,9 | .005,616,91 | 3.053,393 | .256,154,9 | .002,861,16 | 2.041,796 | .151,864,7 | .0$^3$704,50 | .909,653,3 | .028,488,41 | .0$^4$32,278 | .151,871,8 |
| 15 | .315,007,1 | .007,924,69 | 3.751,206 | .315,007,1 | .004,037,78 | 2.509,553 | .186,768,6 | .0$^3$994,47 | 1.118,445 | .035,038,57 | .0$^4$45,571 | .186,778,6 |
| 16 | .382,234,3 | .010,933,83 | 4.546,957 | .382,234,3 | .005,572,56 | 3.043,376 | .226,644,2 | .001,372,84 | 1.356,874 | .042,522,64 | .0$^4$62,922 | .226,658,0 |
| 17 | .458,389,4 | .014,792,45 | 5.446,741 | .458,389,4 | .007,541,42 | 3.647,493 | .271,820,9 | .001,858,42 | 1.626,876 | .051,002,78 | .0$^4$85,194 | .271,839,6 |
| 18 | .544,024,6 | .019,667,98 | 6.456,569 | .544,024,6 | .010,030,28 | 4.326,096 | .322,628,0 | .002,472,45 | 1.930,381 | .060,541,15 | .0$^3$113,370 | .322,652,9 |
| 19 | .639,690,7 | .025,747,95 | 7.582,365 | .639,690,7 | .013,135,37 | 5.083,343 | .379,394,5 | .003,238,91 | 2.269,310 | .071,199,88 | .0$^3$148,548 | .379,427,1 |
| 20 | .745,937,6 | .033,241,50 | 8.829,960 | .745,937,6 | .016,964,25 | 5.923,349 | .442,448,7 | .004,184,43 | 2.645,574 | .083,041,06 | .0$^3$191,960 | .442,490,8 |
| 21 | .863,313,9 | .042,379,78 | 10.205,09 | .863,313,9 | .021,635,91 | 6.850,195 | .512,118,6 | .005,338,65 | 3.061,078 | .096,126,80 | .0$^3$244,979 | .512,172,3 |
| 22 | .992,366,9 | .053,417,65 | 11.713,37 | .992,366,9 | .027,281,75 | 7.867,915 | .588,731,8 | .006,734,20 | 3.517,716 | .110,519,1 | .0$^3$309,09 | .588,799,6 |
| 23 | 1.133,643 | .066,635,2 | 13.360,35 | 1.133,643 | .034,045,7 | 8.980,501 | .672,815,4 | .008,407,17 | 4.017,372 | .126,280,1 | .0$^3$385,99 | .672,700,1 |
| 24 | 1.287,665 | .082,333,7 | 15.151,43 | 1.287,665 | .042,065,2 | 10.191,90 | .764,096,0 | .010,396,54 | 4.561,920 | .143,471,8 | .0$^3$477,47 | .764,200,8 |
| 25 | 1.455,039 | .100,846,9 | 17.091,90 | 1.455,039 | .051,571,5 | 11.506,01 | .863,499,7 | .012,745,33 | 5.153,223 | .162,156,1 | .0$^3$585,52 | .863,626,2 |
| 26 | 1.636,244 | .122,531,2 | 19.166,95 | 1.636,244 | .062,669,7 | 12.926,66 | .971,152,1 | .015,499,86 | 5.793,134 | .182,395,1 | .0$^3$712,30 | .971,306,2 |
| 27 | 1.831,842 | .147,772,6 | 21.441,63 | 1.831,842 | .075,640,4 | 14.457,70 | 1.087,376 | .018,710,4 | 6.483,493 | .204,250,5 | .0$^3$860,13 | 1.087,567 |
| 28 | 2.042,370 | .176,965,5 | 23.860,87 | 2.042,370 | .090,639,2 | 16.102,82 | 1.212,502 | .022,431,2 | 7.226,127 | .227,784,5 | .001,031,52 | 1.212,729 |
| 29 | 2.268,365 | .210,613,6 | 26.449,45 | 2.268,365 | .107,917,4 | 17.865,73 | 1.346,846 | .026,720,1 | 8.022,854 | .253,056,7 | .001,229,20 | 1.347,116 |
| 30 | 2.510,362 | .249,131,7 | 29.212,04 | 2.510,362 | .127,722,8 | 19.750,05 | 1.490,739 | .031,640,0 | 8.875,476 | .280,135,2 | .001,456,07 | 1.491,059 |
| 31 | 2.766,895 | .293,045,1 | 32.153,15 | 2.766,895 | .150,319,7 | 21.759,37 | 1.644,496 | .037,257,36 | 9.785,783 | .309,075,5 | .001,715,206 | 1.644,875 |
| 32 | 3.044,494 | .342,891,2 | 35.277,15 | 3.044,494 | .175,990,2 | 23.897,18 | 1.808,446 | .043,643,47 | 10.755,55 | .339,941,7 | .002,010,049 | 1.808,888 |
| 33 | 3.337,688 | .399,239,9 | 38.586,26 | 3.337,688 | .205,033,5 | 26.166,93 | 1.982,905 | .050,874,24 | 11.786,54 | .372,795,2 | .002,344,044 | 1.983,420 |
| 34 | 3.649,006 | .462,693,6 | 42.090,59 | 3.649,006 | .237,766,7 | 28.572,03 | 2.168,195 | .059,030,13 | 12.880,50 | .407,696,2 | .002,721,162 | 2.168,792 |
| 35 | 3.978,972 | .533,886,4 | 45.788,02 | 3.978,972 | .274,525,4 | 31.115,77 | 2.364,636 | .068,196,70 | 14.039,16 | .444,711,7 | .003,144,89 | 2.365,327 |
| 36 | 4.328,110 | .613,495,8 | 49.684,31 | 4.328,110 | .315,664,9 | 33.801,43 | 2.572,547 | .078,464,30 | 15.264,24 | .483,897,6 | .003,619,98 | 2.573,342 |
| 37 | 4.696,940 | .702,220,4 | 53.783,07 | 4.696,940 | .361,556,9 | 36.632,17 | 2.792,246 | .089,928,47 | 16.557,43 | .525,318,0 | .004,150,82 | 2.793,158 |
| 38 | 5.085,980 | .800,802,8 | 58.087,74 | 5.085,980 | .412,600,8 | 39.611,11 | 3.024,051 | .102,690,0 | 17.920,43 | .569,033,8 | .004,742,10 | 3.025,092 |
| 39 | 5.495,748 | .910,018,9 | 62.601,57 | 5.495,748 | .469,204,0 | 42.741,29 | 3.268,278 | .116,855,0 | 19.354,90 | .615,106,7 | .005,398,86 | 3.269,463 |
| 40 | 5.926,757 | 1.030,880 | 67.327,66 | 5.926,757 | .531,802,4 | 46.025,69 | 3.525,242 | .132,535,0 | 20.862,50 | .663,598,1 | .006,128,36 | 3.526,587 |
| 41 | 6.379,519 | 1.163,636 | 72.268,95 | 6.379,519 | .600,850,5 | 49.467,20 | 3.795,260 | .149,847,4 | 22.444,84 | .714,569,5 | .006,930,17 | 3.796,782 |
| 42 | 6.854,543 | 1.309,770 | 77.428,16 | 6.854,543 | .676,823,7 | 53.068,63 | 4.078,644 | .168,915,0 | 24.103,56 | .768,082,2 | .007,816,12 | 4.080,360 |
| 43 | 7.352,335 | 1.470,004 | 82.807,86 | 7.352,335 | .760,218,5 | 56.832,73 | 4.375,706 | .189,866,6 | 25.840,25 | .824,197,7 | .008,790,35 | 4.377,639 |
| 44 | 7.873,400 | 1.645,297 | 88.410,49 | 7.873,400 | .851,552,8 | 60.762,15 | 4.686,765 | .212,836,8 | 27.656,48 | .882,977,0 | .009,859,15 | 4.688,930 |
| 45 | 8.418,239 | 1.836,644 | 94.238,16 | 8.418,239 | .951,366,8 | 64.859,48 | 5.012,124 | .237,966,2 | 29.553,82 | .944,481,6 | .011,029,46 | 5.014,546 |
| 46 | 8.987,350 | 2.045,079 | 100.293,0 | 8.987,350 | 1.060,222 | 69.127,22 | 5.352,098 | .265,401,8 | 31.533,80 | 1.008,772 | .012,308,13 | 5.354,800 |
| 47 | 9.581,230 | 2.271,673 | 106.576,7 | 9.581,230 | 1.178,703 | 73.567,79 | 5.706,994 | .295,296,9 | 33.597,96 | 1.075,910 | .013,702,58 | 5.710,003 |
| 48 | 10.200,37 | 2.517,532 | 113.091,1 | 10.200,37 | 1.307,415 | 78.183,72 | 6.077,121 | .327,810,6 | 35.747,78 | 1.145,958 | .015,220,50 | 6.080,464 |
| 49 | 10.845,26 | 2.783,803 | 119.837,4 | 10.845,26 | 1.448,989 | 82.976,66 | 6.462,788 | .363,108,9 | 37.984,75 | 1.218,974 | .016,869,85 | 6.466,493 |
| 50 | 11.516,40 | 3.071,666 | 126.817,1 | 11.516,40 | 1.598,076 | 87.949,39 | 6.864,298 | .401,364,5 | 40.310,34 | 1.295,021 | .018,658,81 | 6.868,396 |
| 51 | 12.214,25 | 3.382,348 | 134.031,1 | 12.214,25 | 1.761,351 | 93.103,76 | 7.281,958 | .442,756,1 | 42.725,97 | 1.374,160 | .020,596,21 | 7.286,482 |
| 52 | 12.939,31 | 3.717,096 | 141.480,3 | 12.939,31 | 1.937,513 | 98.441,78 | 7.716,073 | .487,470,2 | 45.233,06 | 1.456,450 | .022,690,99 | 7.721,055 |
| 53 | 13.692,05 | 4.077,212 | 149.165,4 | 13.692,05 | 2.127,281 | 103.965,4 | 8.166,943 | .535,696,9 | 47.833,02 | 1.541,954 | .024,952,6 | 8.172,424 |
| 54 | 14.472,95 | 4.464,022 | 157.086,8 | 14.472,95 | 2.331,402 | 109.676,3 | 8.634,872 | .587,642,0 | 50.527,22 | 1.630,732 | .027,390,6 | 8.640,888 |
| 55 | 15.282,46 | 4.878,895 | 165.244,9 | 15.282,46 | 2.550,644 | 115.576,3 | 9.120,161 | .643,506,0 | 53.317,00 | 1.722,843 | .030,015,2 | 9.126,753 |
| 56 | 16.121,11 | 5.323,233 | 173.639,7 | 16.121,11 | 2.785,797 | 121.667,0 | 9.623,108 | .703,505,1 | 56.203,70 | 1.818,351 | .032,836,6 | 9.630,320 |
| 57 | 16.989,30 | 5.798,470 | 182.271,1 | 16.989,30 | 3.037,676 | 127.950,0 | 10.144,01 | .767,859,5 | 59.188,61 | 1.917,314 | .035,866,2 | 10.151,89 |
| 58 | 17.887,52 | 6.306,066 | 191.138,8 | 17.887,52 | 3.307,124 | 134.426,6 | 10.683,17 | .836,797 | 62.273,04 | 2.019,793 | .039,114,8 | 10.691,76 |
| 59 | 18.816,21 | 6.847,567 | 200.242,1 | 18.816,21 | 3.594,997 | 141.098,4 | 11.240,88 | .910,554 | 65.458,23 | 2.125,848 | .042,593,9 | 11.250,23 |
| 60 | 19.775,85 | 7.424,515 | 209.579,4 | 19.775,85 | 3.902,186 | 147.966,4 | 11.817,43 | .989,374 | 68.745,41 | 2.235,541 | .046,315,6 | 11.827,60 |
| 61 | 20.766,90 | 8.036,460 | 219.152,8 | 20.766,90 | 4.229,599 | 155.031,8 | 12.413,12 | 1.073,504 | 72.135,86 | 2.348,931 | .050,292,8 | 12.424,17 |
| 62 | 21.789,75 | 8.690,986 | 228.958,2 | 21.789,75 | 4.578,169 | 162.295,9 | 13.028,24 | 1.163,205 | 75.630,60 | 2.466,078 | .054,537,5 | 13.040,22 |
| 63 | 22.844,90 | 9.383,777 | 238.995,1 | 22.844,90 | 4.948,851 | 169.759,4 | 13.663,08 | 1.258,740 | 79.230,96 | 2.587,043 | .059,063,4 | 13.676,05 |
| 64 | 23.932,77 | 10.118,49 | 249.262,1 | 23.932,77 | 5.342,629 | 177.423,3 | 14.317,93 | 1.360,384 | 82.938,00 | 2.711,885 | .063,884,2 | 14.331,96 |
| 65 | 25.053,81 | 10.898,84 | 259.757,3 | 25.053,81 | 5.760,502 | 185.268,4 | 14.993,08 | 1.468,417 | 86.752,86 | 2.840,666 | .069,013,8 | 15.008,24 |
| 66 | 26.208,44 | 11.720,55 | 270.479,0 | 26.208,44 | 6.203,500 | 193.355,3 | 15.688,81 | 1.583,126 | 90.676,61 | 2.973,444 | .074,468,7 | 15.705,16 |
| 67 | 27.397,09 | 12.591,38 | 281.424,8 | 27.397,09 | 6.672,670 | 201.624,6 | 16.405,41 | 1.704,810 | 94.710,32 | 3.110,279 | .080,257,9 | 16.423,04 |
| 68 | 28.620,19 | 13.511,12 | 292.592,6 | 28.620,19 | 7.169,085 | 210.096,9 | 17.143,16 | 1.833,771 | 98.855,02 | 3.251,232 | .086,402,8 | 17.162,14 |
| 69 | 29.878,16 | 14.481,60 | 303.979,7 | 29.878,16 | 7.693,842 | 218.772,5 | 17.902,35 | 1.970,321 | 103.111,7 | 3.398,361 | .092,917,4 | 17.922,77 |
| 70 | 31.171,42 | 15.504,66 | 315.583,5 | 31.171,42 | 8.246,057 | 227.651,7 | 18.683,26 | 2.114,782 | 107.481,4 | 3.545,728 | .099,817,7 | 18.705,19 |
| 71 | 32.500,39 | 16.582,16 | 327.400,9 | 32.500,39 | 8.832,871 | 236.734,7 | 19.486,16 | 2.267,479 | 111.965,0 | 3.699,391 | .107,120,6 | 19.509,70 |
| 72 | 33.865,47 | 17.716,00 | 339.428,9 | 33.865,47 | 9.449,447 | 246.021,7 | 20.311,34 | 2.428,751 | 116.563,5 | 3.857,409 | .114,843,4 | 20.336,56 |
| 73 | 35.267,06 | 18.906,10 | 351.664,2 | 35.267,06 | 10.098,96 | 255.512,4 | 21.159,07 | 2.598,939 | 121.277,8 | 4.019,843 | .123,003,5 | 21.186,10 |
| 74 | 36.705,56 | 20.160,40 | 364.103,3 | 36.705,56 | 10.782,64 | 265.207,1 | 22.029,62 | 2.778,397 | 126.108,7 | 4.186,751 | .131,619,5 | 22.058,54 |
| 75 | 38.181,41 | 21.474,86 | 376.742,4 | 38.181,41 | 11.501,68 | 275.105,3 | 22.923,28 | 2.967,485 | 131.057,1 | 4.358,192 | .140,709,6 | 22.954,20 |
| 76 | 39.694,95 | 22.853,45 | 389.577,7 | 39.694,95 | 12.257,36 | 285.206,8 | 23.840,30 | 3.166,571 | 136.123,8 | 4.534,227 | .150,293,4 | 23.873,33 |
| 77 | 41.246,60 | 24.298,18 | 402.605,1 | 41.246,60 | 13.050,93 | 295.511,2 | 24.780,97 | 3.376,032 | 141.309,7 | 4.714,914 | .160,390,2 | 24.816,21 |
| 78 | 42.836,72 | 25.811,06 | 415.820,4 | 42.836,72 | 13.883,68 | 306.018,1 | 25.745,55 | 3.596,252 | 146.615,4 | 4.900,311 | .171,020,2 | 25.783,13 |
| 79 | 44.465,71 | 27.394,12 | 429.219,2 | 44.465,71 | 14.756,93 | 316.726,8 | 26.734,30 | 3.827,624 | 152.041,7 | 5.090,479 | .182,204,2 | 26.774,34 |
| 80 | 46.133,94 | 29.049,41 | 442.796,8 | 46.133,94 | 15.671,99 | 327.636,7 | 27.747,50 | 4.070,550 | 157.589,4 | 5.285,475 | .193,963,3 | 27.790,12 |
| 81 | 47.841,79 | 30.776,96 | 456.548,5 | 47.841,79 | 16.630,23 | 338.747,1 | 28.785,40 | 4.325,438 | 163.259,0 | 5.485,358 | .206,319,0 | 28.830,74 |
| 82 | 49.589,62 | 32.584,92 | 470.469,1 | 49.589,62 | 17.633,00 | 350.056,7 | 29.848,27 | 4.592,706 | 169.051,3 | 5.690,188 | .219,293,5 | 29.896,47 |
| 83 | 51.377,78 | 34.469,29 | 484.553,6 | 51.377,78 | 18.681,70 | 361.564,9 | 30.936,38 | 4.872,761 | 174.966,9 | 5.900,022 | .232,909,6 | 30.987,57 |
| 84 | 53.206,65 | 36.434,18 | 498.797,2 | 53.206,65 | 19.777,72 | 373.270,7 | 32.049,97 | 5.166,095 | 181.006,3 | 6.114,918 | .247,191,0 | 32.104,30 |
| 85 | 55.076,57 | 38.481,70 | 513.193,8 | 55.076,57 | 20.922,49 | 385.172,7 | 33.189,30 | 5.473,090 | 187.170,2 | 6.334,939 | .262,160,7 | 33.246,92 |
| 86 | 56.987,90 | 40.613,94 | 527.738,0 | 56.987,90 | 22.117,44 | 397.269,8 | 34.354,64 | 5.794,219 | 193.459,1 | 6.560,136 | .277,843,5 | 34.415,71 |
| 87 | 58.940,97 | 42.833,01 | 542.424,0 | 58.940,97 | 23.364,02 | 409.560,6 | 35.546,23 | 6.129,939 | 199.873,5 | 6.790,572 | .294,264,1 | 35.610,91 |
| 88 | 60.936,13 | 45.141,02 | 557.246,0 | 60.936,13 | 24.663,72 | 422.043,6 | 36.764,32 | 6.480,715 | 206.413,8 | 7.026,303 | .311,447,8 | 36.832,78 |
| 89 | 62.973,71 | 47.540,07 | 572.197,7 | 62.973,71 | 26.018,01 | 434.717,3 | 38.009,17 | 6.847,027 | 213.080,5 | 7.267,389 | .329,420,8 | 38.081,59 |
| 90 | 65.054,05 | 50.032,28 | 587.273,0 | 65.054,05 | 27.428,40 | 447.580,1 | 39.281,03 | 7.229,352 | 219.874,1 | 7.513,867 | .348,209,5 | 39.357,56 |

SHEET 4 OF 5

205-D-402

Table H-9.–*Load constants for circular arches–twist and tangential deflections* (sheet 5).

UNIT RADIAL LOAD NO. 5
TWIST AND TANGENTIAL DEFLECTIONS



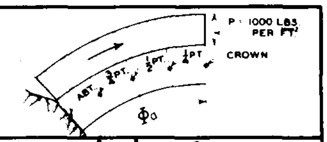
| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | | $D_1$ | $D_3$ | |
| ° | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term |
| 10 | .221,299,2 | .002,242,50 | 1.764,561 | .163427,0 | .0$^3$,87907 | 1.025,292 | .069,201,66 | .0$^3$,161,529 | .387,189,4 | .011,248,23 | .0$^5$6,480 | .058,823,66 |
| 11 | .294,486.5 | .003,609,25 | 2.346,503 | .217,487,8 | .001,415,19 | 1.363,904 | .092,100,54 | .0$^3$260,095 | .515,215,7 | .014,971,09 | .0$^4$10,433 | .078,289,07 |
| 12 | .382,234,3 | .005,572,56 | 3.043,376 | .282,310,6 | .002,185,57 | 1.769,641 | .119,561,8 | .0$^3$401,79 | .668,700,5 | .019,436,14 | .0$^4$16,119 | .101,633,1 |
| 13 | .485,853,9 | .008,308,77 | 3.865,212 | .358,867,2 | .003,259,67 | 2.248,454 | .151,998,8 | .0$^3$,599,39 | .849,931,7 | .024,710,77 | .0$^4$24,051 | .129,207,3 |
| 14 | .606,653,5 | .012,025,49 | 4.821,934 | .448,127,7 | .004,719,28 | 2.806,258 | .189,824,6 | .0$^3$868,01 | 1.061,191 | .030,862,38 | .0$^4$34,836 | .161,363,0 |
| 15 | .745,937,6 | .016,964,25 | 5.923,349 | .551,060,0 | .006,659,66 | 3.448,925 | .233,451,9 | .001,225,26 | 1.304,752 | .037,958,36 | .0$^4$49,183 | .198,451,1 |
| 16 | .905,007,1 | .023,402,75 | 7.179,140 | .668,630,6 | .009,190,56 | 4.182,284 | .283,292,9 | .001,691,44 | 1.582,881 | .046,066,06 | .0$^4$67,910 | .240,822,5 |
| 17 | 1.085,159 | .031,657,4 | 8.598,854 | .801,803,2 | .012,436,97 | 5.012,114 | .339,759,4 | .002,289,70 | 1.897,834 | .055,252,84 | .0$^4$91,948 | .288,827,4 |
| 18 | 1.287,685 | .042,085,2 | 10.191,90 | .951,539,8 | .016,540,41 | 5.944,144 | .403,262,9 | .003,046,21 | 2.251,862 | .065,586,02 | .0$^3$122,356 | .342,815,8 |
| 19 | 1.513,875 | .055,086,6 | 11.967,54 | 1.118,799 | .021,659,4 | 6.984,048 | .474,214,1 | .003,990,51 | 2.647,201 | .077,132,90 | .0$^3$180,322 | .403,137,5 |
| 20 | 1.765,010 | .071,106,9 | 13.934,87 | 1.304,539 | .027,971,0 | 8.137,445 | .553,023,4 | .005,155,40 | 3.086,081 | .089,960,76 | .0$^3$207,175 | .470,141,5 |
| 21 | 2.042,370 | .090,639,2 | 16.102,82 | 1.509,712 | .035,671,3 | 9.409,892 | .640,100,4 | .006,577,38 | 3.570,719 | .104,136,9 | .0$^3$264,40 | .544,176,8 |
| 22 | 2.347,226 | .114,225,7 | 18.480,17 | 1.735,271 | .044,976,1 | 10.806,88 | .735,854,4 | .008,296,67 | 4.103,323 | .119,728,4 | .0$^3$333,59 | .625,591,6 |
| 23 | 2.680,847 | .142,460,3 | 21.075,49 | 1.982,162 | .056,122,6 | 12.333,85 | .840,893,9 | .010,357,63 | 4.686,086 | .136,802,7 | .0$^3$416,59 | .714,733,9 |
| 24 | 3.044,494 | .175,990,2 | 23.897,18 | 2.251,332 | .069,369,6 | 13.996,15 | .955,026,8 | .012,808,54 | 5.321,192 | .155,426,8 | .0$^3$515,31 | .811,951,0 |
| 25 | 3.439,421 | .215,518,3 | 26.953,43 | 2.543,721 | .084,998,2 | 15.799,08 | 1.079,260 | .015,702,0 | 6.010,809 | .175,667,9 | .0$^3$631,93 | .917,589,8 |
| 26 | 3.866,879 | .261,803,8 | 30.252,25 | 2.860,269 | .103,313,6 | 17.747,85 | 1.213,801 | .019,095,4 | 6.757,095 | .197,593,2 | .0$^3$768,75 | 1.031,996 |
| 27 | 4.328,110 | .315,664,9 | 33.801,43 | 3.201,910 | .124,644,8 | 19.847,59 | 1.359,055 | .023,050,5 | 7.562,190 | .221,269,7 | .0$^3$928,30 | 1.155,518 |
| 28 | 4.824,348 | .377,980,6 | 37.608,53 | 3.569,576 | .149,346,1 | 22.103,38 | 1.515,427 | .027,633,9 | 8.428,224 | .246,764,4 | .001,113,28 | 1.288,498 |
| 29 | 5.356,824 | .449,692,1 | 41.680,91 | 3.964,192 | .177,797,4 | 24.520,17 | 1.683,321 | .032,917,4 | 9.357,308 | .274,144,5 | .001,326,62 | 1.431,281 |
| 30 | 5.926,757 | .531,802,4 | 46.025,69 | 4.386,684 | .210,405,3 | 27.102,88 | 1.863,140 | .038,977,8 | 10.351,54 | .303,476,8 | .001,571,47 | 1.584,213 |
| 31 | 6.535,362 | .625,381,4 | 50.649,77 | 4.837,970 | .247,604,1 | 29.856,29 | 2.055,288 | .045,897,4 | 11.413,00 | .334,828,3 | .001,851,14 | 1.747,635 |
| 32 | 7.183,843 | .731,564,2 | 55.559,79 | 5.318,964 | .289,855,4 | 32.785,13 | 2.260,166 | .053,763,8 | 12.543,76 | .368,266,1 | .002,169,35 | 1.921,891 |
| 33 | 7.873,400 | .851,552,8 | 60.762,15 | 5.830,579 | .337,650,6 | 35.894,02 | 2.478,174 | .062,670,5 | 13.745,85 | .403,856,8 | .002,529,81 | 2.107,323 |
| 34 | 8.605,221 | .986,618,8 | 66.263,00 | 6.373,720 | .391,509,0 | 39.187,48 | 2.709,713 | .072,716,5 | 15.021,32 | .441,667,3 | .002,936,62 | 2.304,271 |
| 35 | 9.380,488 | 1.138,103 | 72.068,25 | 6.949,289 | .451,980,8 | 42.669,95 | 2.955,182 | .084,007,2 | 16.372,17 | .481,764,6 | .003,394,12 | 2.513,078 |
| 36 | 10.200,37 | 1.307,415 | 78.183,52 | 7.558,182 | .519,647,7 | 46.345,77 | 3.214,978 | .096,653,8 | 17.800,41 | .524,215,3 | .003,906,86 | 2.734,082 |
| 37 | 11.066,04 | 1.496,039 | 84.614,18 | 8.201,291 | .595,121,2 | 50.219,16 | 3.489,499 | .110,774,0 | 19.307,99 | .569,086,1 | .004,479,76 | 2.967,623 |
| 38 | 11.978,63 | 1.705,530 | 91.365,33 | 8.879,504 | .679,044,8 | 54.294,24 | 3.779,139 | .126,491,7 | 20.896,88 | .616,443,6 | .005,117,90 | 3.214,038 |
| 39 | 12.939,31 | 1.937,513 | 98.441,78 | 9.593,702 | .772,095,0 | 58.575,04 | 4.084,295 | .143,937,6 | 22.569,02 | .666,354,6 | .005,826,70 | 3.473,865 |
| 40 | 13.949,20 | 2.193,688 | 105.848,1 | 10.344,76 | .874,980,3 | 63.065,48 | 4.405,359 | .163,249,2 | 24.326,31 | .718,885,4 | .006,611,85 | 3.746,841 |
| 41 | 15.009,43 | 2.475,834 | 113.588,5 | 11.133,55 | .988,443,9 | 67.769,36 | 4.742,724 | .184,570,5 | 26.170,66 | .774,102,8 | .007,479,35 | 4.033,901 |
| 42 | 16.121,11 | 2.785,797 | 121.667,0 | 11.960,94 | 1.113,261 | 72.690,37 | 5.096,782 | .208,053,0 | 28.103,93 | .832,073,1 | .008,435,49 | 4.335,181 |
| 43 | 17.285,35 | 3.125,498 | 130.087,3 | 12.827,79 | 1.250,243 | 77.832,10 | 5.467,922 | .233,855,1 | 30.127,98 | .892,862,9 | .009,486,93 | 4.651,014 |
| 44 | 18.503,23 | 3.496,937 | 138.852,7 | 13.734,95 | 1.400,233 | 83.198,00 | 5.856,534 | .262,142,3 | 32.244,63 | .956,538,3 | .010,640,4 | 4.981,733 |
| 45 | 19.775,85 | 3.902,186 | 147.966,4 | 14.683,28 | 1.564,113 | 88.791,42 | 6.263,006 | .293,087,8 | 34.455,70 | 1.023,166 | .011,903,4 | 5.327,671 |
| 46 | 21.104,28 | 4.343,392 | 157.431,1 | 15.673,60 | 1.742,795 | 94.615,59 | 6.687,723 | .326,872,6 | 36.762,96 | 1.092,812 | .013,283,4 | 5.689,158 |
| 47 | 22.489,57 | 4.822,773 | 167.249,3 | 16.706,77 | 1.937,236 | 100.673,6 | 7.131,072 | .363,684,9 | 39.168,17 | 1.165,542 | .014,788,3 | 6.066,524 |
| 48 | 23.932,77 | 5.342,629 | 177.423,3 | 17.783,60 | 2.148,417 | 106.968,5 | 7.593,435 | .403,720,7 | 41.673,08 | 1.241,423 | .016,426,5 | 6.460,100 |
| 49 | 25.434,93 | 5.905,325 | 187.954,9 | 18.904,92 | 2.377,363 | 113.503,1 | 8.075,197 | .447,184,2 | 44.279,39 | 1.320,521 | .018,206,5 | 6.870,214 |
| 50 | 26.997,07 | 6.513,305 | 198.845,7 | 20.071,56 | 2.625,133 | 120.280,2 | 8.576,736 | .494,287,7 | 46.988,79 | 1.402,902 | .020,137,2 | 7.297,189 |
| 51 | 28.620,19 | 7.169,085 | 210.096,9 | 21.284,31 | 2.892,823 | 127.302,3 | 9.098,434 | .545,250,8 | 49.802,93 | 1.488,631 | .022,228,1 | 7.741,355 |
| 52 | 30.305,31 | 7.875,252 | 221.709,6 | 22.543,99 | 3.181,567 | 134.572,0 | 9.640,669 | .600,303,3 | 52.723,47 | 1.577,775 | .024,488,8 | 8.203,036 |
| 53 | 32.053,41 | 8.634,462 | 233.684,4 | 23.851,38 | 3.492,533 | 142.091,6 | 10.203,82 | .659,681,1 | 55.752,00 | 1.670,399 | .026,929,5 | 8.682,556 |
| 54 | 33.865,47 | 9.449,447 | 246.021,7 | 25.207,29 | 3.826,928 | 149.863,3 | 10.788,25 | .723,631 | 58.890,12 | 1.766,570 | .029,560,6 | 9.180,237 |
| 55 | 35.742,44 | 10.323,00 | 258.721,4 | 26.612,49 | 4.185,999 | 157.889,3 | 11.394,35 | .792,405 | 62.139,36 | 1.866,353 | .032,393,2 | 9.696,401 |
| 56 | 37.685,29 | 11.257,99 | 271.783,3 | 28.067,75 | 4.571,021 | 166.171,5 | 12.022,49 | .866,267 | 65.501,28 | 1.969,813 | .035,438,2 | 10.231,37 |
| 57 | 39.694,95 | 12.257,36 | 285.206,8 | 29.573,85 | 4.983,314 | 174.711,7 | 12.673,03 | .945,489 | 68.977,37 | 2.077,017 | .038,707,5 | 10.785,46 |
| 58 | 41.772,34 | 13.324,09 | 298.991,1 | 31.131,55 | 5.424,235 | 183.511,8 | 13.346,34 | 1.030,350 | 72.569,10 | 2.188,029 | .042,213,3 | 11.358,99 |
| 59 | 43.918,37 | 14.461,26 | 313.134,8 | 32.741,59 | 5.895,174 | 192.573,1 | 14.042,80 | 1.121,140 | 76.277,93 | 2.302,915 | .045,968,1 | 11.952,28 |
| 60 | 46.133,94 | 15.671,99 | 327.636,7 | 34.404,74 | 6.397,561 | 201.897,2 | 14.762,77 | 1.218,158 | 80.105,27 | 2.421,741 | .049,984,5 | 12.565,63 |
| 61 | 48.419,94 | 16.959,47 | 342.494,7 | 36.121,71 | 6.932,860 | 211.484,1 | 15.506,60 | 1.321,711 | 84.052,51 | 2.544,572 | .054,276,5 | 13.199,38 |
| 62 | 50.777,22 | 18.326,95 | 357.706,8 | 37.893,24 | 7.502,574 | 221.338,2 | 16.274,68 | 1.432,116 | 88.121,01 | 2.671,473 | .058,857,5 | 13.853,83 |
| 63 | 53.206,65 | 19.777,72 | 373.270,7 | 39.720,07 | 8.108,244 | 231.458,3 | 17.067,35 | 1.549,698 | 92.312,12 | 2.802,509 | .063,741,8 | 14.529,29 |
| 64 | 55.709,07 | 21.315,16 | 389.183,4 | 41.602,88 | 8.751,442 | 241.845,0 | 17.884,97 | 1.674,794 | 96.627,11 | 2.937,745 | .068,944,3 | 15.226,06 |
| 65 | 58.285,29 | 22.942,67 | 405.442,2 | 43.542,41 | 9.433,781 | 252.499,7 | 18.727,91 | 1.807,748 | 101.067,3 | 3.077,247 | .074,480,2 | 15.944,47 |
| 66 | 60.936,13 | 24.663,72 | 422.043,6 | 45.539,33 | 10.156,91 | 263.422,9 | 19.596,51 | 1.948,913 | 105.633,8 | 3.221,078 | .080,364,8 | 16.684,82 |
| 67 | 63.662,39 | 26.481,84 | 438.984,0 | 47.594,35 | 10.922,50 | 274.615,1 | 20.491,13 | 2.098,656 | 110.328,0 | 3.369,305 | .086,614,6 | 17.447,41 |
| 68 | 66.464,85 | 28.400,57 | 456.259,6 | 49.708,13 | 11.732,28 | 286.076,7 | 21.412,13 | 2.257,347 | 115.151,0 | 3.521,991 | .093,246,0 | 18.232,56 |
| 69 | 69.344,27 | 30.423,53 | 473.866,1 | 51.881,36 | 12.588,00 | 297.807,9 | 22.359,85 | 2.425,371 | 120.104,0 | 3.679,201 | .100,276,3 | 19.040,55 |
| 70 | 72.301,40 | 32.554,38 | 491.799,2 | 54.114,70 | 13.491,43 | 309.808,7 | 23.334,64 | 2.603,121 | 125.188,0 | 3.841,001 | .107,723,0 | 19.871,70 |
| 71 | 75.336,98 | 34.796,79 | 510.054,1 | 56.408,80 | 14.444,42 | 322.079,0 | 24.336,85 | 2.790,999 | 130.404,2 | 4.007,454 | .115,604,0 | 20.726,30 |
| 72 | 78.451,73 | 37.154,51 | 528.625,7 | 58.764,30 | 15.448,79 | 334.618,8 | 25.366,83 | 2.989,418 | 135.753,5 | 4.178,625 | .123,938,1 | 21.604,66 |
| 73 | 81.646,34 | 39.631,29 | 547.508,8 | 61.181,85 | 16.506,45 | 347.427,7 | 26.424,90 | 3.198,799 | 141.237,2 | 4.354,578 | .132,744,2 | 22.507,07 |
| 74 | 84.921,52 | 42.230,92 | 566.697,9 | 63.662,07 | 17.619,31 | 360.505,1 | 27.511,42 | 3.419,576 | 146.856,0 | 4.535,377 | .142,042,2 | 23.433,82 |
| 75 | 88.277,93 | 44.957,24 | 586.187,1 | 66.205,58 | 18.789,32 | 373.850,5 | 28.626,72 | 3.652,189 | 152.611,0 | 4.721,087 | .151,851,8 | 24.385,22 |
| 76 | 91.716,22 | 47.814,09 | 605.970,5 | 68.812,99 | 20.018,45 | 387.463,1 | 29.771,14 | 3.897,092 | 158.503,1 | 4.911,771 | .162,194,1 | 25.361,55 |
| 77 | 95.237,05 | 50.805,35 | 626.041,5 | 71.484,90 | 21.308,74 | 401.342,1 | 30.945,01 | 4.154,745 | 164.533,2 | 5.107,494 | .173,090,1 | 26.363,11 |
| 78 | 98.841,01 | 53.934,90 | 646.393,8 | 74.221,90 | 22.662,19 | 415.486,5 | 32.148,66 | 4.425,622 | 170.702,2 | 5.308,320 | .184,561,5 | 27.390,19 |
| 79 | 102.528,7 | 57.206,67 | 667.020,5 | 77.024,58 | 24.080,90 | 429.895,1 | 33.382,43 | 4.710,203 | 177.010,9 | 5.514,311 | .196,630,7 | 28.443,06 |
| 80 | 106.300,8 | 60.624,57 | 687.914,6 | 79.893,50 | 25.566,96 | 444.566,7 | 34.646,65 | 5.008,979 | 183.460,0 | 5.725,533 | .209,320,2 | 29.522,03 |
| 81 | 110.157,8 | 64.192,55 | 709.068,8 | 82.829,22 | 27.122,50 | 459.497,6 | 35.941,63 | 5.322,456 | 190.050,4 | 5.942,048 | .222,653,8 | 30.627,38 |
| 82 | 114.100,3 | 67.914,54 | 730.475,7 | 85.832,31 | 28.749,67 | 474.693,1 | 37.267,70 | 5.651,142 | 196.782,7 | 6.163,920 | .236,655,1 | 31.759,38 |
| 83 | 118.128,7 | 71.794,49 | 752.127,4 | 88.903,29 | 30.450,64 | 490.144,7 | 38.625,18 | 5.995,561 | 203.657,7 | 6.391,212 | .251,348,8 | 32.918,32 |
| 84 | 122.243,8 | 75.836,36 | 774.016,2 | 92.042,72 | 32.227,63 | 505.853,0 | 40.014,39 | 6.356,243 | 210.676,0 | 6.623,988 | .266,760,0 | 34.104,48 |
| 85 | 126.445,8 | 80.044,07 | 796.134,0 | 95.251,10 | 34.082,85 | 521.815,9 | 41.435,66 | 6.733,732 | 217.838,2 | 6.862,310 | .282,914,1 | 35.318,14 |
| 86 | 130.735,4 | 84.421,60 | 818.472,4 | 98.528,95 | 36.018,57 | 538.031,5 | 42.889,28 | 7.128,579 | 225.145,0 | 7.106,243 | .299,837,7 | 36.559,58 |
| 87 | 135.113,0 | 88.972,85 | 841.022,8 | 101.876,8 | 38.037,06 | 554.497,5 | 44.375,58 | 7.541,346 | 232.596,9 | 7.355,847 | .317,557,4 | 37.829,06 |
| 88 | 139.579,1 | 93.701,74 | 863.776,8 | 105.295,1 | 40.140,60 | 571.211,9 | 45.894,87 | 7.972,605 | 240.194,3 | 7.611,188 | .336,100,6 | 39.126,86 |
| 89 | 144.134,1 | 98.612,18 | 886.725,3 | 108.784,3 | 42.331,54 | 588.172,0 | 47.447,44 | 8.422,938 | 247.937,9 | 7.872,327 | .355,495,4 | 40.453,26 |
| 90 | 148.778,4 | 103.708,1 | 909.859,3 | 112.345,0 | 44.612,18 | 605.375,5 | 49.033,62 | 8.892,935 | 255.828,0 | 8.139,327 | .375,770,3 | 41.808,52 |

SHEET 5 OF 5

205-D-403

Table H-10.—*Load constants for circular arches—tangential deflections* (sheet 1).

UNIT TANGENTIAL LOAD NO. I
TANGENTIAL DEFLECTIONS



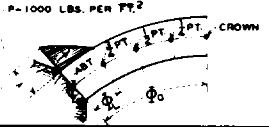
| $\Phi$ | Crown | | | | | ¼ Point | | ½ Point | | ¾ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | | $D_3$ | | $D_3$ | | $D_3$ | | $D_3$ | |
| | | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term |
| 10 | .038,624,00 | .005,373,006 | .868,610,9 | $0^3$391,390 | 15.423,05 | $0^4$69,751 | 8 628,329 | $0^5$6,129,5 | 3 819,782 | $0^7$95,83 | 952,684,3 |
| 11 | 056,537,34 | 008,644,710 | 1 151,254 | .$0^3$,692,926 | 18 710,38 | .$0^3$.123,525 | 10.455,71 | $0^4$10,856,9 | 4.624,997 | $0^6$,169,75 | 1 152,940 |
| 12 | 080,054,96 | 013,342,03 | 1 487,730 | .001,167,12 | 22 329,93 | .$0^3$,208,119 | 12463,24 | $0^4$18,296,3 | 5 508,114 | $0^6$,286,11 | 1 372,344 |
| 13 | .110,236,8 | .019,884,74 | 1.881,984 | 001.885,20 | 26 286,83 | $0^3$,336,281 | 14.652,57 | $0^4$29,570,2 | 6 469,465 | $0^6$462,47 | 1.610,916 |
| 14 | .148,233,4 | .028,766,68 | 2 337,715 | .002,938,38 | 30 586.62 | .$0^3$524.336 | 17.025,49 | $0^4$46,118,1 | 7 509,408 | .$0^6$721,45 | 1.868,681 |
| 15 | 195,286,0 | 040,561,22 | 2 858,360 | 004,441,23 | 35.235,22 | .$0^3$,792,816 | 19.583,93 | .$0^4$69,751 | 8.628,329 | .$0^5$1,091,27 | 2 145,661 |
| 16 | .252,725,7 | .055,926,64 | 3 447,075 | 006,535,28 | 40.238,96 | 001,167,12 | 22.329,93 | $0^3$,102,713 | 9 826,642 | $0^5$1,607,24 | 2 441,884 |
| 17 | 321,973,1 | 075,611,22 | 4.106,716 | 009,392,94 | 45.604.52 | .001,678,19 | 25 265,68 | $0^3$,147,738 | 11 104,79 | $0^5$2,312,2 | 2 757,376 |
| 18 | .404,538,3 | 100,458,3 | 4.839,826 | 013,221,47 | 51 338,95 | 002,363,33 | 28 393,50 | $0^3$,208,119 | 12.463,24 | $0^5$3,257,9 | 3 092,169 |
| 19 | 502,019,9 | 131,411,0 | 5.648,610 | .018.267,41 | 57.449,65 | 003,266,89 | 31.715,83 | .$0^3$,287,792 | 13 902,48 | .$0^5$4,506,0 | 3 446,293 |
| 20 | 616,104,7 | .169,516,8 | 6 534,924 | 024,820,99 | 63 944.36 | .004,441,23 | 35.235,22 | $0^3$,391,390 | 15 423,05 | .$0^5$6,129,5 | 3.819,782 |
| 21 | 748,567,6 | .215,931,9 | 7 500.260 | 033,220,98 | 70 831.13 | 005,947,48 | 38.954,37 | $0^3$,524,336 | 17 025,49 | $0^5$8,213,4 | 4 212,671 |
| 22 | .901,270,2 | 271,925,6 | 8 545,724 | 043,859,52 | 78 118.34 | 007,856,58 | 42 876,08 | $0^3$,692,926 | 18 710,38 | $0^4$10,856,9 | 4 624,997 |
| 23 | 1.076,161 | .338,883,7 | 9 672,028 | .057,187,2 | 85 814.62 | .010,250,13 | 47.003,26 | .$0^3$,904,418 | 20 478,31 | .$0^4$14,174,3 | 5.056,798 |
| 24 | 1 275,275 | .418,312,8 | 10.879,47 | .073,718,6 | 93 928.93 | .013,221,47 | 51 338,95 | .001,167,12 . | 22 329,93 | $0^4$18,296,3 | 5 508,114 |
| 25 | 1.500,731 | .511,843,0 | 12 167,94 | 094,037,5 | 102.470,4 | .016,876,71 | 55.886,28 | .001,490,47 | 24 265,87 | $0^4$23,372,0 | 5 978,989 |
| 26 | 1.754.734 | 621.231.9 | 13 536,85 | 118,802,8 | 111.448,6 | .021,335,77 | 60 648,51 | 001,885,20 | 26.286,83 | $0^4$29,570,2 | 6 469,465 |
| 27 | 2.039,574 | .748.366.7 | 14.985,21 | .148,753,6 | 120 873,1 | 026,733,52 | 65.628,98 | .002,363,33 | 28.393.50 | .$0^4$37,081,1 | 6 979,589 |
| 28 | 2357,621 | .895.267,4 | 16 511,52 | 184.716,2 | 130 753,7 | 033,220,98 | 70 831.13 | 002,938,38 | 30 586,62 | $0^4$46,118,1 | 7 509,408 |
| 29 | 2711,332 | 1.064.089 | 18.113,85 | .227,609,6 | 141.100,5 | .040,966,41 | 76.258,53 | .003,625,41 | 32 866,93 | $0^4$56,919 | 8.058,971 |
| 30 | 3.103,243 | 1 257.122 | 19.789,75 | .278,451,1 | 151 923,8 | .050,156,62 | 81.914,79 | 004,441,23 | 35.235,22 | .$0^4$69,751 | 8.628,329 |
| 31 | 3 535,971 | 1.476.798 | 21.536,29 | 338,363.9 | 163.233.9 | .060,998,1 | 87 803,66 | .005,404,35 | 37.692,29 | $0^4$84,908 | 9 217,534 |
| 32 | 4012,215 | 1.725,688 | 23 350.05 | 408,582,5 | 175 041,3 | .073,718,6 | 93.928,93 | .006,535,28 | 40 238,96 | $0^3$,102,713 | 9.826,642 |
| 33 | 4 534,753 | 2.006,502 | 25.227.07 | 490,459,2 | 187.356.6 | .088,568,1 | 100.294.5 | 007,856,58 | 42 876,08 | $0^3$,123,525 | 10.455,71 |
| 34 | 5 106.441 | 2322,095 | 27 162,89 | 585,471,4 | 200 190,6 | 105,820.2 | 106 904,3 | 009,392,94 | 45 604,52 | $0^3$,147,738 | 11.104,79 |
| 35 | 5730.214 | 2675,464 | 29 152,52 | 695.227,4 | 213 553,9 | .125.773,5 | 113 762,5 | 011,171,41 | 48 425,17 | $0^3$,175,779 | 11.773,94 |
| 36 | 6409,082 | 3 069,747 | 31 190,44 | 821,473,3 | 227 457,5 | .148.753,6 | 120 873,1 | .013,221,47 | 51 338,95 | $0^3$208,119 | 12 463,24 |
| 37 | 7 146.134 | 3 508,226 | 33 270,58 | 966.100,1 | 241.912,2 | 175,113,4 | 128 240,2 | 015,575,21 | 54 346,79 | $0^3$245,273 | 13 172,73 |
| 38 | 7944.531 | 3 994.326 | 35 386.33 | 1 131,150 | 256 928,7 | 205,235.4 | 135 868,2 | 018,267,41 | 57.449,65 | $0^3$287,792 | 13 902.48 |
| 39 | 8807,509 | 4 531,611 | 37 530,56 | 1.318,823 | 272 518,0 | .239,532,8 | 143.761,3 | .021,335,77 | 60 648,51 | $0^3$,336,281 | 14 652,57 |
| 40 | 9.738,379 | 5.123,788 | 39.695,55 | 1.531,484 | 288.690,8 | 278,451,1 | 151 923,8 | 024,820,99 | 63 944,36 | $0^3$,391,390 | 15 423,05 |
| 41 | 10 740,52 | 5.774,700 | 41 873,07 | 1 771,670 | 305.457,8 | 322,469,7 | 160 360,1 | 028,766,99 | 67 338,22 | $0^3$,453,821 | 16 214,00 |
| 42 | 11 817,39 | 6.488,328 | 44.054,30 | 2.042,096 | 322.829,8 | .372,103,2 | 169 074,8 | 033,220,98 | 70 831,13 | .$0^3$,524,336 | 17.025,49 |
| 43 | 12.972,51 | 7.268,790 | 46 229,89 | 2 345,660 | 340 817,1 | 427,903,2 | 178 072,1 | 038,233,69 | 74 424,15 | $0^3$,603,744 | 17 857,59 |
| 44 | 14 209,46 | 8.120,332 | 48.389,95 | 2 685,455 | 359 430.4 | .490.459,2 | 187 356,6 | 043,859,52 | 78 118,34 | $0^3$,692,926 | 18 710,38 |
| 45 | 15 531,92 | 9047.332 | 50.524,02 | 3 064,770 | 378 679 7 | 560,401,3 | 196.932,9 | 050,156,62 | 81 914,79 | $0^3$,792,816 | 19 583,93 |
| 46 | 16 943.60 | 10.054,30 | 52 621,08 | 3.487.098 | 398.575,1 | .638,400.6 | 206.805,4 | .057,187,2 | 85 814,62 | $0^3$,904,418 | 20 478,31 |
| 47 | 18448.30 | 11 145,85 | 54 669,59 | 3.956,144 | 419 126,7 | .725.171,6 | 216.978,8 | 065,017,7 | 89 818,96 | 001,028,80 | 21 393,62 |
| 48 | 20 049,87 | 12 326,73 | 56 657,44 | 4 475,830 | 440 344.0 | 821,473.3 | 227 457,5 | 073,718,6 | 93 928,93 | 001,167,12 | 22 329,93 |
| 49 | 21 752,24 | 13.601,81 | 58 572,04 | 5 050.301 | 462 236.4 | 928,111,2 | 238 246,2 | 083,365,3 | 98 145,70 | 001,320,57 | 23 287,32 |
| 50 | 23 559,38 | 14.976,06 | 60 400,20 | 5.683,932 | 484.813.1 | 1.045,939 | 249.349,5 | 094.037,5 | 102 470,4 | 001,490,47 | 24 265,87 |
| 51 | 25475,35 | 16 454,55 | 62 128,25 | 6.381,331 | 508 083,0 | 1 175,858 | 260 771,9 | 105,820,2 | 106 904,3 | 001,678,19 | 25.265,68 |
| 52 | 27 504,23 | 18 042,45 | 63 741,98 | 7 147,352 | 532 054,6 | 1 318,823 | 272 518,0 | 118,802,8 | 111 448,6 | 001,885,20 | 26 286,83 |
| 53 | 29.650,19 | 19 745.04 | 65 226.67 | 7.987,089 | 556 736,3 | 1 475,841 | 284 592,3 | 133,080,4 | 116 104,4 | 002,113,02 | 27.329,41 |
| 54 | 31 917,45 | 21.567,68 | 66 567,14 | 8.905,892 | 582.135,8 | 1.647,971 | 296.999,3 | 148,753,6 | 120 873,1 | 002,363,33 | 28 393,50 |
| 55 | 34 310.28 | 23.515,80 | 67.747,63 | 9 909.370 | 608 260,6 | 1 836,332 | 309 743,7 | 165,928,2 | 125 755,7 | 002,637,88 | 29 479,21 |
| 56 | 36 833.02 | 25 594,94 | 68 752,03 | 11 003,39 | 635 118,0 | 2042,096 | 322 829,8 | 184,716,2 | 130 753,7 | 002.938,38 | 30.586,62 |
| 57 | 39.490,03 | 27.810,67 | 69 563,64 | 12 194,08 | 662 714,6 | 2 266.496 | 336 262,0 | 205,235,4 | 135.868,2 | 003,266,89 | 31 715,83 |
| 58 | 42285,76 | 30 188,64 | 70 165,33 | 13 487,86 | 691 056.6 | 2 510,829 | 350 044,9 | 227,609,6 | 141 100,5 | 003,625,41 | 32 866,93 |
| 59 | 45.224,69 | 32 674,57 | 70 539,58 | 14 891,40 | 720.150,0 | 2.776,446 | 364 182,6 | 251,969,1 | 146 452,0 | 004,016,11 | 34 040,03 |
| 60 | 48311,36 | 35 334,20 | 70 668,42 | 16 411,67 | 750 000,0 | 3064,770 | 378 679,7 | 278,451,1 | 151 923,8 | 004,441,23 | 35 235,22 |
| 61 | 51 550,33 | 38 153,34 | 70 533,38 | 18 055,92 | 780 611,5 | 3 377,285 | 393 540,2 | 307,199,1 | 157 517,3 | 004,903,13 | 36.452 61 |
| 62 | 54 946,24 | 41 137,80 | 70.115,72 | 19 831,67 | 811.988.9 | 3 715,542 | 408 768,3 | 338,363,9 | 163 233,9 | 005,404,35 | 37 692,29 |
| 63 | 58 503,77 | 44 293,43 | 69 396,21 | 21 746,74 | 844 135,9 | 4 081,160 | 424 368,2 | 372,103,2 | 169 074,8 | 005,947,48 | 38 954,37 |
| 64 | 62 227,62 | 47 626,10 | 68 355.33 | 23809,26 | 877 055,8 | 4.475,830 | 440 344,0 | 408,582,5 | 175 041,3 | 006,535,28 | 40 238,96 |
| 65 | 66.122,56 | 51 141,68 | 66 973,11 | 26 027,63 | 910 751,4 | 4 901,312 | 456 699,5 | .447.974,2 | 181 134,8 | 007,170,64 | 41 546,16 |
| 66 | 70.193,38 | 54 846,03 | 65 229,35 | 28.410,57 | 945 224,8 | 5 359,439 | 473 438,6 | 490,459,2 | 187 356,6 | 007.856.58 | 42 876,08 |
| 67 | 74.444,93 | 58 745.01 | 63 103,45 | 30 967,08 | 980.477.4 | 5 852,118 | 490 565,2 | .536,226,0 | 193 708,1 | 008,596,23 | 44 228,83 |
| 68 | 78882,07 | 62 844,45 | 60 574,61 | 33 706,47 | 1,016 510 | 6 381,331 | 508 083.0 | 585,471,4 | 200 190,6 | 009,392,94 | 45.604,52 |
| 69 | 83 509,71 | 67.150,14 | 57 621,60 | 36 638,37 | 1,053 324 | 6 949,139 | 525.995,5 | 638,400,6 | 206 805,4 | 010,250,13 | 47 003,26 |
| 70 | 88 332,82 | 71 667,85 | 54 223,10 | 39 772,68 | 1.090 917 | 7 557,677 | 544 306,2 | .695,227,4 | 213 553,9 | 011,171,41 | 48.425,17 |
| 71 | 93 356,36 | 76 403,29 | 50 357,44 | 43 119,62 | 1,129 290 | 8 209,163 | 563 018,5 | 756,174,4 | 220 437,5 | 012,160,55 | 49 870,36 |
| 72 | 98 585,35 | 81 362,09 | 46 002.79 | 46 689,73 | 1,168 441 | 8 905,892 | 582 135,8 | 821,473,3 | 227 457,5 | 013,221,47 | 51.338,95 |
| 73 | 104 024,8 | 86 549.84 | 41 137,10 | 50.493,84 | 1.208 366 | 9 650,246 | 601 661,0 | .891,365,1 | 234 615.3 | 014,358,27 | 52 831,05 |
| 74 | 109 679,9 | 91 972,01 | 35 738,17 | 54 543,08 | 1,249 064 | 10444,68 | 621.597,3 | 966,100,1 | 241 912,2 | 015,575,21 | 54 346,79 |
| 75 | 115.555,5 | 97 634,02 | 29.783,61 | 58 848,90 | 1,290 530 | 11 291,75 | 641 947,6 | 1 045,939 | 249.349,5 | 016,876,71 | 55 886,28 |
| 76 | 121.657,0 | 103 541,2 | 23 250,93 | 63 423,01 | 1,332 761 | 12 194,08 | 662 714,6 | 1 131,150 | 256.928,7 | 018,267,41 | 57 449,65 |
| 77 | 127 989,4 | 109 698,6 | 16 117,58 | 68 277,49 | 1,375 750 | 13 154,40 | 683 900,9 | 1 222,015 | 264 651,1 | 019,752,09 | 59 037,02 |
| 78 | 134 557,9 | 116 111,4 | 8 360,84 | 73 424,65 | 1,419 492 | 14 175,48 | 705 509,0 | 1 318,823 | 272 518,0 | 021,335,77 | 60 648,51 |
| 79 | 141.367,7 | 122.784.4 | -041,97 | 78 877, 3 | 1.463 981 | 15 260,25 | 727 541,4 | 1 421,876 | 280 530,8 | 023,023,60 | 62 284,25 |
| 80 | 148423,9 | 129.722,5 | - 9113,60 | 84 647,87 | 1,509 209 | 16 411,67 | 750 000,0 | 1 531,484 | 288 690,8 | 024,820,99 | 63 944,36 |
| 81 | 155.731,9 | 136 930,2 | -18.876,88 | 90 750,08 | 1,555 168 | 17 632,83 | 772 887,0 | 1 647,971 | 296 999,3 | 026,733,52 | 65 628,98 |
| 82 | 163 296,9 | 144.412,1 | -29 354,43 | 97 197,25 | 1,601 850 | 18 926,87 | 796 204,2 | 1.771,670 | 305.457,8 | 028,766,99 | 67 338,22 |
| 83 | 171 124,1 | 152172,2 | -40.569,05 | 104 003,2 | 1,649 244 | 20 297,06 | 819 953,3 | 1 902,927 | 314 067,5 | 030,927,40 | 69 072,23 |
| 84 | 179 218,7 | 160214,8 | -52 543,29 | 111 181.9 | 1,697 341 | 21 746,74 | 844 135,9 | 2042,096 | 322 829,8 | 033,220,98 | 70 831,13 |
| 85 | 187586,2 | 168543,7 | -65 299,73 | 118 747,8 | 1,746 129 | 23 279,35 | 868 753,2 | 2189,547 | 331 745,9 | 035,654,18 | 72 615,06 |
| 86 | 196 231,7 | 177162,5 | -78 860,70 | 126 715.4 | 1,795 597 | 24 898,43 | 893 806,5 | 2 345,660 | 340 817,1 | 038,233,69 | 74 424,15 |
| 87 | 205 160,6 | 186 074,6 | -93 248,54 | 135 099.6 | 1,845 731 | 26 607,59 | 919 296,8 | 2 510,829 | 350.044.9 | 040,966,41 | 76 258,53 |
| 88 | 214 378,2 | 195.283,3 | -108 485,2 | 143.915.6 | 1,896.519 | 28 410,57 | 945 224,8 | 2685,455 | 359.430.4 | 043,859,52 | 78 118,34 |
| 89 | 223.889,7 | 204 791,5 | -124 592,7 | 153 178,6 | 1,947.947 | 30 311,18 | 971 591,1 | 2869,960 | 368.974,9 | 046,920,34 | 80 003,71 |
| 90 | 233 700,6 | 214 601,8 | -141 592,6 | 162.904,2 | 2,000.000 | 32 313,32 | 998 396,3 | 3064,770 | 378 679.7 | 050,156,62 | 81 914,79 |

SHEET 1 OF 5

205-D-404

Table H-10.—*Load constants for circular arches—tangential deflections* (sheet 2).

UNIT TANGENTIAL LOAD NO. 2
TANGENTIAL DEFLECTIONS



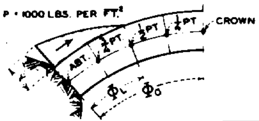
| $\phi$ | Crown | | | | | ¼ Point | | ½ Point | | ¾ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_3$ | | $D_3$ | | $D_3$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | .0$^{4}$30,204,29 | .0$^{5}$5,028,372 | -.041,440,86 | .0$^{6}$422,078 | .314,744,3 | .0$^{6}$231,089 | .316,252,4 | .0$^{7}$97,157 | .317,158,4 | .0$^{7}$20,534 | .317,460,8 |
| 11 | .0$^{4}$44,221,67 | .0$^{5}$8,090,191 | -.055,131,26 | .0$^{6}$,747,362 | .380,189,4 | .0$^{6}$409,275 | .382,396,6 | .0$^{6}$,172,298 | .383,723,1 | .0$^{7}$36,378 | .384,165,6 |
| 12 | .0$^{4}$62,630,36 | .0$^{4}$12,486,14 | -.071,537,67 | .0$^{5}$1,258,992 | .451,608,2 | .0$^{6}$689,625 | .454,733,3 | .0$^{6}$290,034 | .456,612,0 | .0$^{7}$61,312 | .457,239,1 |
| 13 | .0$^{4}$86,263,76 | .0$^{4}$18,609,04 | -.090,901,73 | .0$^{5}$2,033,928 | .528,930,3 | .0$^{5}$1,114,928 | .533,233,1 | .0$^{6}$,468,766 | .535,820,6 | .0$^{7}$99,106 | .536,684,6 |
| 14 | .0$^{3}$,116,027,7 | .0$^{4}$26,921,01 | -.113,463,8 | .0$^{5}$3,170,754 | .612,079,1 | .0$^{5}$1,737,767 | .617,864,2 | .0$^{6}$,731,127 | .621,344,5 | .0$^{6}$,154,59 | .622,506,9 |
| 15 | .0$^{3}$,152,900,5 | .0$^{4}$37,958,56 | -.139,462,6 | .0$^{5}$4,793,374 | .700,972,1 | .0$^{5}$2,627,875 | .708,592,6 | .0$^{5}$1,105,87 | .713,178,7 | .0$^{6}$,233,87 | .714,710,9 |
| 16 | .0$^{3}$,197,932,8 | .0$^{4}$52,337,63 | -.169,135,5 | .0$^{5}$7,054,889 | .795,520,6 | .0$^{5}$3,868,964 | .805,381,1 | .0$^{5}$1,628,51 | .811,317,8 | .0$^{6}$,344,42 | .813,301,6 |
| 17 | .0$^{3}$,252,247,6 | .0$^{4}$70,758,38 | -.202,717,9 | .0$^{4}$10,141,92 | .895,630,0 | .0$^{5}$5,563,872 | .908,190,4 | .0$^{5}$2,342,49 | .915,756,0 | .0$^{6}$,495,49 | .918,285,2 |
| 18 | .0$^{3}$,317,040,3 | .0$^{4}$94,009,81 | -.240,443,6 | .0$^{4}$14,279,00 | 1.001,200 | .0$^{5}$7,836,392 | 1.016,978 | .0$^{5}$3,300,12 | 1.028,487 | .0$^{6}$,698,17 | 1.029,667 |
| 19 | .0$^{3}$,393,578,7 | .0$^{3}$,122,974,3 | -.282,544,3 | .0$^{4}$19,733,27 | 1.112,123 | .0$^{4}$10,833,96 | 1.131,701 | .0$^{5}$4,563,73 | 1.143,505 | .0$^{6}$,965,63 | 1.147,454 |
| 20 | .0$^{3}$,483,202,9 | .0$^{3}$,158,631,7 | -.329,249,8 | .0$^{4}$28,819,57 | 1.228,288 | .0$^{4}$14,730,58 | 1.252,310 | .0$^{5}$6,206,95 | 1.266,802 | .0$^{5}$1,313,55 | 1.271,652 |
| 21 | .0$^{3}$,587,325,1 | .0$^{3}$,202,063,6 | -.380,787,5 | .0$^{4}$35,905,46 | 1.349,576 | .0$^{4}$19,729,56 | 1.378,757 | .0$^{5}$8,315,86 | 1.396,370 | .0$^{5}$1,760,15 | 1.402,268 |
| 22 | .0$^{3}$,707,431,1 | .0$^{3}$,254,457,4 | -.437,382.9 | .0$^{4}$47,416,94 | 1.475,863 | .0$^{4}$26,066,85 | 1.510,990 | .0$^{4}$10,990,5 | 1.532,204 | .0$^{5}$2,326,89 | 1.539,310 |
| 23 | .0$^{3}$,845,075.3 | .0$^{3}$,317,108.5 | -.499,258.7 | .0$^{4}$61,843,66 | 1.607,019 | .0$^{4}$34,014,00 | 1.648,953 | .0$^{4}$14,346,0 | 1.674,294 | .0$^{5}$3,037,62 | 1.682,786 |
| 24 | .001,001.890 | .0$^{3}$,391.426,4 | -.566,635.5 | .0$^{4}$79,745,51 | 1.742,907 | .0$^{4}$43,881,89 | 1.792,589 | .0$^{4}$18,514,5 | 1.822,630 | .0$^{5}$3,921,04 | 1.832,703 |
| 25 | .001,179,575 | .0$^{3}$,478,935,3 | -.639,730,8 | .0$^{3}$,101,757,8 | 1.883,386 | .0$^{4}$56,023,81 | 1.941,838 | .0$^{4}$23,645,8 | 1.977,206 | .0$^{5}$5,008,75 | 1.989,070 |
| 26 | .001,379,903 | .0$^{3}$,581,277,7 | -.718,759,9 | .0$^{3}$,128,598,7 | 2.028,310 | .0$^{4}$70,839,79 | 2.096,638 | .0$^{4}$29,910,5 | 2.138,010 | .0$^{5}$6,337,18 | 2.151,896 |
| 27 | .001,604.722 | .0$^{3}$,700,218.1 | -.803,934,9 | .0$^{3}$,161,074,4 | 2.177,524 | .0$^{4}$88,779,35 | 2.256,923 | .0$^{4}$37,499,8 | 2.305,034 | .0$^{5}$7,946,97 | 2.321,190 |
| 28 | .001,855,949 | .0$^{3}$,837,644,0 | -.895,465,1 | .0$^{3}$,200,086,9 | 2.330,870 | .0$^{3}$,110,346,3 | 2.422,626 | .0$^{4}$46,628,6 | 2.478,266 | .0$^{5}$9,883,82 | 2.496,961 |
| 29 | .002,135,574 | .0$^{3}$,995,568,7 | -.993,556,6 | .0$^{3}$,246,640,2 | 2.488,184 | .0$^{3}$,136,102,6 | 2.593,677 | .0$^{4}$57,536,7 | 2.657,697 | .0$^{4}$12,198,9 | 2.679,219 |
| 30 | .002,445,660 | .001,176,133 | -1.098,412 | .0$^{3}$,301,847,9 | 2.649,296 | .0$^{3}$,166,672,3 | 2.770,003 | .0$^{4}$70,490,9 | 2.843,314 | .0$^{4}$14,949,2 | 2.867,975 |
| 31 | .002,788,341 | .001,381,606 | -1.210,232 | .0$^{3}$,366,939,1 | 2.814,030 | .0$^{3}$202,745,6 | 2.951,528 | .0$^{4}$85,786,1 | 3.035,106 | .0$^{4}$18,197,6 | 3.063,238 |
| 32 | .003,165,823 | .001,614,222 | -1.329,212 | .0$^{3}$,443,267,6 | 2.982,206 | .0$^{3}$,245,084,0 | 3.138,174 | .0$^{3}$,103,749,1 | 3.233,061 | .0$^{4}$22,014,0 | 3.265,020 |
| 33 | .003,580,384 | .001,877,011 | -1.455,546 | .0$^{3}$,532,318,1 | 3.153,636 | .0$^{3}$,294,524,1 | 3.329,861 | .0$^{3}$,124,738,3 | 3.437,166 | .0$^{4}$26,474,7 | 3.473,332 |
| 34 | .004,034,376 | .002,172,136 | -1.589,422 | .0$^{3}$,635,713,3 | 3.328,129 | .0$^{3}$,351,982,5 | 3.526,504 | .0$^{3}$,149,147,6 | 3.647,406 | .0$^{4}$31,664,4 | 3.688,185 |
| 35 | .004,530,220 | .002,502.557 | -1.731,026 | .0$^{3}$,755,222,8 | 3.505,490 | .0$^{3}$,418,460,5 | 3.728,020 | .0$^{3}$,177,407,6 | 3.863,774 | .0$^{4}$37,675,0 | 3.909,591 |
| 36 | .005,070,410 | .002,871,202 | -1.880,539 | .0$^{3}$,892,770,4 | 3.685,512 | .0$^{3}$,495,049,1 | 3.934,319 | .0$^{3}$,209,988,5 | 4.086,249 | .0$^{4}$44,607,5 | 4.137,562 |
| 37 | .005,657,514 | .003,281,127 | -2.038,138 | .001,050,442 | 3.867,992 | .0$^{3}$,582,933,6 | 4.145,309 | .0$^{3}$,247,401,4 | 4.314,819 | .0$^{4}$52,571,4 | 4.372,112 |
| 38 | .006,294,169 | .003,735,523 | -2.203,999 | .001,230,492 | 4.052,714 | .0$^{3}$,683,398,0 | 4.360,898 | .0$^{3}$,290,200,8 | 4.549,468 | .0$^{4}$61,685,2 | 4.613,252 |
| 39 | .006,983,084 | .004,237,706 | -2.378,289 | .001,435,355 | 4.239,462 | .0$^{3}$,797,832,5 | 4.580,989 | .0$^{3}$,338,988,9 | 4.790,182 | .0$^{4}$72,079.3 | 4.860,996 |
| 40 | .007,727,042 | .004,791,123 | -2.561,173 | .001,667,650 | 4.428,012 | .0$^{3}$,927,735,3 | 4.805,483 | .0$^{3}$,394,414,9 | 5.036,943 | .0$^{4}$83,892,5 | 5.115,357 |
| 41 | .008,528,895 | .005,399,365 | -2.752,814 | .001,930,190 | 4.618,135 | .001,074,710 | 5.034,279 | .0$^{3}$,457,179,9 | 5.289,735 | .0$^{4}$97,276,3 | 5.376,350 |
| 42 | .009,391.569 | .006,066,080 | -2.953,366 | .002,225,990 | 4.809,601 | .001,240,519 | 5.267,272 | .0$^{3}$,528,037,6 | 5.548,541 | .0$^{3}$,112,393 | 5.643,988 |
| 43 | .010,318,06 | .006,795,130 | -3.162,981 | .002,558,278 | 5.002,169 | .001,427,004 | 5.504,355 | .0$^{3}$,607,794,1 | 5.813,342 | .0$^{3}$,129,416 | 5.918,286 |
| 44 | .011,311,44 | .007,590,447 | -3.381,806 | .002,930,496 | 5.195,599 | .001,636,152 | 5.745,420 | .0$^{3}$,697,332,5 | 6.084,122 | .0$^{3}$,148,553 | 6.199,259 |
| 45 | .012,374,84 | .008,456,082 | -3.609,982 | .003,346,316 | 5.389,642 | .001,870,097 | 5.990,354 | .0$^{3}$,797,569,2 | 6.360,860 | .0$^{3}$,169,950 | 6.486,922 |
| 46 | .013,511,49 | .009,396,201 | -3.847,647 | .003,809,642 | 5.584,046 | .002,131,109 | 6.239,042 | .0$^{3}$,909,504,1 | 6.643,537 | .0$^{3}$,193,876 | 6.781,290 |
| 47 | .014,724,65 | .010,415,08 | -4.094,933 | .004,324,625 | 5.778,555 | .002,421,608 | 6.491,368 | .001,034,199 | 6.932,133 | .0$^{3}$,220,544 | 7.082,379 |
| 48 | .016,017,68 | .011,517,10 | -4.351,966 | .004,895,661 | 5.972,908 | .002,744,165 | 6.747,210 | .001,172,784 | 7.226,627 | .0$^{3}$,250,198 | 7.390,205 |
| 49 | .017,394,02 | .012,706,73 | -4.618,868 | .005,527,407 | 6.166,839 | .003,101,512 | 7.006,447 | .001,326,465 | 7.526,998 | .0$^{3}$,283,100 | 7.704,785 |
| 50 | .018,857.16 | .013,988,56 | -4.895.755 | .006,224,790 | 6.360,078 | .003,496,548 | 7.268,954 | .001,496,521 | 7.833,224 | .0$^{3}$,319,528 | 8.026,136 |
| 51 | .020,410,66 | .015,367,25 | -5.182,737 | .006,993,006 | 6.552,352 | .003,932,342 | 7.534,602 | .001,684,312 | 8.145,282 | .0$^{3}$,359,778 | 8.354,273 |
| 52 | .022,058,17 | .016,847,54 | -5.479,919 | .007,837,536 | 6.743,380 | .004,412,139 | 7.803,262 | .001,891,275 | 8.463,149 | .0$^{3}$,404,162 | 8.689,216 |
| 53 | .023,803.39 | .018,434,25 | -5.787,401 | .008.764,150 | 6.932,883 | .004,939,368 | 8.074,799 | .002,118,935 | 8.786,801 | .0$^{3}$,453,015 | 9.030,980 |
| 54 | .025,650,11 | .020.132,26 | -6.105,276 | .009,778,913 | 7.120,572 | .005,517,648 | 8.349,078 | .002,368,906 | 9.116,213 | .0$^{3}$,506,686 | 9.379,585 |
| 55 | .027,602,19 | .021,946,60 | -6.433,632 | .010,888,197 | 7.306,156 | .006,150,789 | 8.625,961 | .002,642,887 | 9.451,361 | .0$^{3}$,565,553 | 9.735,048 |
| 56 | .029,663.55 | .023,882,20 | -6.772,551 | .012,098,686 | 7.489,344 | .006,842,817 | 8.905,307 | .002,942,666 | 9.792,218 | .0$^{3}$,630,007 | 10.097,39 |
| 57 | .031,838,18 | .025,944,17 | -7.122,106 | .013,417,380 | 7.669,835 | .007,597,928 | 9.186,971 | .003,270,177 | 10.138,76 | .0$^{3}$,700,448 | 10.466,62 |
| 58 | .034,130.14 | .028,137,60 | -7.482,369 | .014,851,598 | 7.847,329 | .008,420,572 | 9.470,808 | .003,627,368 | 10.490,95 | .0$^{3}$,777,334 | 10.842,77 |
| 59 | .036.543,58 | .030,467,63 | -7.853,402 | .016,409,018 | 8.021,519 | .009,315,414 | 9.756,669 | .004,016,368 | 10.848,78 | .0$^{3}$,861,125 | 11.225,86 |
| 60 | .039,082.71 | .032,939,44 | -8.235,260 | .018,097,61 | 8.192,099 | .010,287,302 | 10.044,40 | .004,439,356 | 11.212,20 | .0$^{3}$,952,298 | 11.615,90 |
| 61 | .041.751,80 | .035,558,19 | -8.627,997 | .019,925,72 | 8.358,755 | .011,341,36 | 10.333,85 | .004,898,663 | 11.581,19 | .001,051,360 | 12.012,91 |
| 62 | .044.555,20 | .039.329,08 | -9.031.653 | .021.902,04 | 8.521,171 | .012,482,92 | 10.624,86 | .005,396,713 | 11.955,71 | .001,158,857 | 12.416,92 |
| 63 | .047,497,33 | .041.257,28 | -9.446,265 | .024.035,61 | 8.679,032 | .013,717,58 | 10.917,28 | .005,936,058 | 12.335,75 | .001,275,350 | 12.827,94 |
| 64 | .050,582,69 | .044,347,97 | -9.871,864 | .026,335,85 | 8.832,013 | .015,051.18 | 11.210.93 | .006,519,366 | 12.721,26 | .001,401,430 | 13.246,00 |
| 65 | .053,815.83 | .047.606.29 | -10.308,47 | .028,812,53 | 8.979.792 | .016,489.80 | 11.505.66 | .007,149,424 | 13.112,22 | .001,537,709 | 13.671,11 |
| 66 | .057,201.39 | .051.037.34 | -10.756.11 | .031,475,78 | 9.122,041 | .018.039,78 | 11.801,29 | .007,829,149 | 13.508,58 | .001,684,843 | 14.103,30 |
| 67 | .060.744.07 | .054,646,21 | -11.214,77 | .034,336.12 | 9.258,430 | .019.707,74 | 12.097.66 | .008,561,595 | 13.910,32 | .001,843,508 | 14.542,59 |
| 68 | .064,448.64 | .058,437,90 | -11.684,48 | .037,404,46 | 9.388,627 | .021,500,57 | 12.394.60 | .009,349,941 | 14.317,40 | .002,014,411 | 14.989,00 |
| 69 | .068,319.95 | .062,417,34 | -12.165,22 | .040,692,07 | 9.512,298 | .023,425,44 | 12.691.93 | .010,197,52 | 14.729,79 | .002,198,301 | 15.442,56 |
| 70 | .072,362,90 | .066,589,41 | -12.656,97 | .044,210,61 | 9.629,105 | .025,489,76 | 12.989,47 | .011,107,78 | 15.147,44 | .002,395,940 | 15.903,28 |
| 71 | .076,582,48 | .070,958,88 | -13.159,72 | .047,972,16 | 9.738,709 | .027,701,30 | 13.287,04 | .012,084,35 | 15.570,32 | .002,608,152 | 16.371,19 |
| 72 | .080,983,75 | .075,530,42 | -13.673,44 | .051,989,16 | 9.840,768 | .030,068,07 | 13.584,45 | .013,130,97 | 15.998,40 | .002,835,766 | 16.846,31 |
| 73 | .085,571,84 | .080,308,58 | -14.198,10 | .056,274,46 | 9.934,940 | .032,598,40 | 13.881,54 | .014,251,57 | 16.431,62 | .003,079,675 | 17.328,66 |
| 74 | .090,351.91 | .085,297.78 | -14.733,65 | .060,841,27 | 10.020,88 | .035,300,89 | 14.178,09 | .015,450,18 | 16.869,96 | .003,340,762 | 17.818,28 |
| 75 | .095,329.25 | .090,502,31 | -15.280,04 | .065.703,27 | 10.098,24 | .038.184,51 | 14.473,94 | .016,731,07 | 17.313,37 | .003,620,02 | 18.315,17 |
| 76 | .100,509,2 | .095,926,29 | -15.837,21 | .070,874,49 | 10.166,67 | .041,258,49 | 14.768,87 | .018,098,61 | 17.761,81 | .003,918,43 | 18.819,37 |
| 77 | .105,897,1 | .101,573.7 | -16.405,10 | .076,369,33 | 10.225,83 | .044,532,43 | 15.062.70 | .019,557,38 | 18.215,23 | .004,237,01 | 19.330,90 |
| 78 | .111,498,6 | .107,448,3 | -16.983,63 | .082,202,79 | 10.275,35 | .048,016,18 | 15.355.22 | .021,112,08 | 18.673,59 | .004,576,85 | 19.849,78 |
| 79 | .117,319.0 | .113,553,6 | -17.572,72 | .088,389,92 | 10.314,90 | .051,719,98 | 15.646.25 | .022,767,63 | 19.136,85 | .004,939,06 | 20.376,05 |
| 80 | .123,364,0 | .119,893.1 | -18.172,28 | .094,946,48 | 10.344,11 | .055,654,41 | 15.935,57 | .024,529,12 | 19.604,96 | .005,324,78 | 20.909,71 |
| 81 | .129,639,4 | .126,469,9 | -18.782,21 | .101,888,5 | 10.362,63 | 059,830,33 | 16.222.97 | .026,401,81 | 20.077,88 | .005,735,24 | 21.450,60 |
| 82 | .136,150.8 | .133,286,9 | -19.402,40 | .109,232,3 | 10.370.12 | 064,258,97 | 16.508,26 | .028,391,14 | 20.555,54 | .006,171,67 | 21.999,35 |
| 83 | .142,904.1 | .140.346,7 | -20.032,74 | .116,994,9 | 10.366,20 | 068,951,96 | 16.791,22 | .030,502,76 | 21.037,66 | .006,635,35 | 22.555,36 |
| 84 | .149,905,2 | .147,651,8 | -20.673.10 | .125,193,5 | 10.350,54 | 073,920,82 | 17.071,63 | .032,742,50 | 21.524,95 | .007,127,64 | 23.118,91 |
| 85 | .157,159.9 | .155,204,2 | -21.323,35 | .133,845,6 | 10.322.76 | 079,178,90 | 17.349,29 | .035,116,40 | 22.016,58 | .007,649,91 | 23.689,97 |
| 86 | .164,674,5 | .163,005,8 | -21.983,35 | .142,969,3 | 10.282,53 | 084,737,82 | 17.623.97 | .037,630,70 | 22.512,77 | .008,203,61 | 24.268,59 |
| 87 | .172,454,9 | .171,058.0 | -22.652,95 | .152,583,0 | 10.229,47 | 090,610,91 | 17.895,45 | .040,291,81 | 23.013,47 | .008,790,23 | 24.854,79 |
| 88 | .180,507.3 | .179,362,1 | -23.331.99 | .162,705.5 | 10.163,25 | 096,811.58 | 18.163,51 | .043,106,43 | 23.518,59 | .009,411,29 | 25.448,60 |
| 89 | .188,838.0 | .187,918,8 | -24.020,31 | .173,355.9 | 10.083,49 | 103,353,45 | 18.427,93 | .046,081,33 | 24.028,11 | .010,068,38 | 26 050,04 |
| 90 | .197,453,2 | .196,728.7 | -24.717.73 | .184,552,7 | 9.989,85 | 110,250,77 | 18.688,49 | .049,223,68 | 24.541,96 | .010,763,18 | 26 659,15 |

SHEET 2 OF 5

205-D-405

Table H-10.—*Load constants for circular arches—tangential deflections* (sheet 3).

UNIT TANGENTIAL LOAD NO. 3
TANGENTIAL DEFLECTIONS



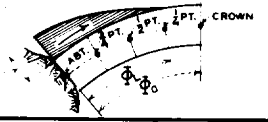
| Φ | Crown |  |  |  |  | ¼ Point |  | ½ Point |  | ¾ Point |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  | D1 | D2 |  | D3 |  | D3 |  | D3 |  | D3 |  |
| ° |  | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term |
| 10 | .0$^{3}$483,202,9 | .0$^{4}$76,971,58 | -.110,957,7 | .0$^{5}$6,206,95 | 1.266.802 | .0$^{5}$3,303,49 | 1.270,436 | .0$^{5}$1,313,55 | 1.271,652 | .0$^{7}$58,182 | .635,072,5 |
| 11 | .0$^{3}$707,431,1 | .0$^{3}$123,845,7 | -.147,739,0 | .0$^{4}$10,990,5 | 1.532,204 | .0$^{5}$5,850,64 | 1.537,528 | .0$^{5}$2,326,69 | 1.539,310 | .0$^{6}$103,064 | .768,552,8 |
| 12 | .001,001,890 | .0$^{3}$191,148,2 | -.191,882,4 | .0$^{4}$18,514,5 | 1,822,630 | .0$^{5}$9,858,196 | 1.830,175 | .0$^{5}$3,921,04 | 1.832,703 | .0$^{6}$173,711 | .914,791,3 |
| 13 | .001,379.903 | .0$^{3}$284,897,3 | -.244,067,7 | .0$^{4}$29,910,5 | 2.138,010 | .0$^{4}$15,930,13 | 2.148,409 | .0$^{5}$6,337,18 | 2.151,896 | .0$^{6}$280,789 | 1.073,800 |
| 14 | .001,855,949 | .0$^{3}$412,173,0 | -.304,977,3 | .0$^{4}$46,628,6 | 2,478,266 | .0$^{4}$24,840,80 | 2,492,262 | .0$^{5}$9,883,82 | 2.496,961 | .0$^{6}$438,020 | 1.245,593 |
| 15 | .002,445,660 | .0$^{3}$581,197,7 | -.375,296,6 | .0$^{4}$70,490,9 | 2.843.314 | .0$^{4}$37,564,14 | 2.861,772 | .0$^{4}$14,949,2 | 2.867,975 | .0$^{6}$662,565 | 1.430,186 |
| 16 | .003,165.823 | .0$^{3}$801,412,3 | -.455,714,0 | .0$^{3}$103,749,1 | 3.233,061 | .0$^{4}$55,304,52 | 3.256,974 | .0$^{4}$22,014,0 | 3.265,020 | .0$^{6}$975,830 | 1.627,593 |
| 17 | .004,034,376 | .001,083,551 | -.546,921,0 | .0$^{3}$149,147,6 | 3.647,408 | .0$^{4}$79,530,88 | 3.677,910 | .0$^{4}$31,664,4 | 3.688,185 | .0$^{5}$1,403,85 | 1.837,831 |
| 18 | .005,070,410 | .001,439,714 | -.649,612,5 | .0$^{3}$209,988,5 | 4.086,249 | .0$^{3}$112,012,9 | 4.124,620 | .0$^{4}$44,607,5 | 4.137,562 | .0$^{5}$1,978,04 | 2.080,918 |
| 19 | .006,294,169 | .001,883,435 | -.764,487,0 | .0$^{3}$290,200,8 | 4.549,468 | .0$^{3}$154,857,6 | 4.597,149 | .0$^{4}$61,685,2 | 4.613,252 | .0$^{5}$2,735,82 | 2.296,874 |
| 20 | .007,727,042 | .002,429,749 | -.892,246,7 | .0$^{3}$394,414,9 | 5.036,943 | .0$^{3}$210,551,1 | 5.095,540 | .0$^{4}$83,892,5 | 5.115,357 | .0$^{5}$3,721,5 | 2.545,717 |
| 21 | .009,391,569 | .003,095,256 | -1.033,598 | .0$^{3}$528,037,6 | 5.548,541 | .0$^{3}$282,005,3 | 5.619,840 | .0$^{3}$112,393 | 5.643,988 | .0$^{5}$4,986,8 | 2.807,470 |
| 22 | .011,311,44 | .003,898,183 | -1.189,250 | .0$^{3}$697,332,5 | 6.084,122 | .0$^{3}$372,590 | 6.170,096 | .0$^{3}$148,553 | 6.199.259 | .0$^{5}$6,591,8 | 3.082,153 |
| 23 | .013,511,49 | .004,858,436 | -1,359,918 | .0$^{3}$909.503,9 | 6.643,537 | .0$^{3}$488,156 | 6.746,358 | .0$^{3}$193,876 | 6.781,290 | .0$^{5}$8,606,0 | 3.369,792 |
| 24 | .016,017,68 | .005,997,658 | -1,546,319 | .001,172,784 | 7.226,627 | .0$^{3}$627,189 | 7.348,673 | .0$^{3}$250,198 | 7.390,205 | .0$^{4}$11,108,7 | 3.670,409 |
| 25 | .018,857,16 | .007,339,277 | -1.749,176 | .001,496,521 | 7.833,224 | .0$^{3}$800.707 | 7.977,093 | .0$^{3}$319,528 | 8.026,136 | .0$^{4}$14,190,4 | 3.984,030 |
| 26 | .022,058,17 | .008,908,552 | -1.969,215 | .001,891,275 | 8.463,149 | .001,012,427 | 8.631,668 | .0$^{3}$404,162 | 8.689,216 | .0$^{4}$17,953,7 | 4.310,681 |
| 27 | .025,650,11 | .010,732,62 | -2.207,169 | .002,368,906 | 9.116,213 | .001,268,796 | 9.312,448 | .0$^{3}$506,686 | 9.379,585 | .0$^{4}$22,514,0 | 4.650,390 |
| 28 | .029,663,55 | .012,840,53 | -2463,773 | .002,942,688 | 9.792,218 | .001,576,995 | 10.019,487 | .0$^{3}$630,007 | 10.097,39 | .0$^{4}$28,001,0 | 5.003,185 |
| 29 | .034,130,14 | .015,263,28 | -2.739,767 | .003,627,368 | 10.490,95 | .001,945,030 | 10.752,833 | .0$^{3}$777,334 | 10.842,77 | .0$^{4}$34,558,9 | 5.369,095 |
| 30 | .039,082,71 | .018,033,84 | -3035,896 | .004,439,356 | 11.212,20 | .002,381,846 | 11,512,54 | .0$^{3}$952,298 | 11.615,90 | .0$^{4}$42,350,1 | 5.748,152 |
| 31 | .044,555,20 | .021,187,20 | -3352,910 | .005,396,713 | 11.955,71 | .002,897,277 | 12.298,65 | .001,158,857 | 12.416,92 | .0$^{4}$51,552,8 | 6.140,386 |
| 32 | .050,582,69 | .024,760,34 | -3.691,562 | .006,519,366 | 12.721,26 | .003,502,214 | 13.111,23 | .001,401,430 | 13.246,00 | .0$^{4}$62,363,4 | 6.545,831 |
| 33 | .057,201,39 | .028,792,34 | -4.052,610 | .007,829,149 | 13.508,58 | .004,208,594 | 13,950,31 | .001,684,843 | 14.103,30 | .0$^{4}$75,000 | 6.964,519 |
| 34 | .064,448,64 | .033,324,30 | -4.436,819 | .009,349,941 | 14.317,40 | .005,029,507 | 14.815,94 | .002,014,411 | 14.989.00 | .0$^{4}$89,701 | 7.396,486 |
| 35 | .072,362,90 | .038,399,36 | -4.844,955 | .011,107,78 | 15.147,44 | .005,979,252 | 15.708,18 | .002,395,940 | 15.903,28 | .0$^{3}$106,727 | 7.841,767 |
| 36 | .080,983,75 | .044,062,80 | -5.277,790 | .013,130,97 | 15.998,40 | .007,073,407 | 16.627,06 | .002,835,766 | 16.846,31 | .0$^{3}$126,363 | 8.300,399 |
| 37 | .090,351,91 | .050,361,89 | -5.736,101 | .015,450,18 | 16.869,96 | .008,328,871 | 17.572,63 | .003,340,762 | 17.818,28 | .0$^{3}$148,922 | 8.772,420 |
| 38 | .100,509,2 | .057,346,00 | -6.220,668 | .018,098,61 | 17.761,81 | .009,764,00 | 18.544,92 | .003,918,43 | 18.819,37 | .0$^{3}$174,739 | 9.257,867 |
| 39 | .111,498,6 | .065,066,56 | -6.732,275 | .021,112,08 | 18.673,59 | .011,398,63 | 19.543,98 | .004,576,85 | 19.849,78 | .0$^{3}$204,180 | 9.756,782 |
| 40 | .123,364,0 | .073,576,99 | -7.271,711 | .024,529,12 | 19.604,96 | .013,254,14 | 20.569,84 | .005,324,78 | 20.909,71 | .0$^{3}$237.641 | 10.269,21 |
| 41 | .136,150,8 | .082,932,74 | -7.839,768 | .028,391,14 | 20.555,54 | .015,353,56 | 21.622,53 | .006,171,67 | 21.999,35 | .0$^{3}$275,548 | 10.795,18 |
| 42 | .149,905.2 | .093,191,26 | -8.437,243 | .032,742,50 | 21.524,95 | .017,721,75 | 22.702,08 | .007,127,64 | 23.118,91 | .0$^{3}$318,367 | 11.334,74 |
| 43 | .164,674.5 | .104,411,9 | -9.064,934 | .037,630,70 | 22.512,77 | .020,384,93 | 23.808,51 | .008,203,61 | 24.268,59 | .0$^{3}$366,579 | 11.887,94 |
| 44 | .180,507.3 | .116,656,1 | -9.723,645 | .043,106,43 | 23.518,59 | .023,371,84 | 24.941,84 | .009,411,29 | 25.448,60 | .0$^{3}$420,737 | 12.454,82 |
| 45 | .197,453,2 | .129,986,8 | -10.414,18 | .049,223,68 | 24.541,96 | .026,712,70 | 26.102,10 | .010,763,18 | 26.659,15 | .0$^{3}$481,384 | 13.035,42 |
| 46 | .215,562,8 | .144,469,1 | -11.137,35 | .056,039,98 | 25.582,44 | .030,439,93 | 27.289,29 | .012,272,67 | 27.900,47 | .0$^{3}$549,146 | 13.629,80 |
| 47 | .234,888,0 | .160,169,9 | -11.893,97 | .063,616,40 | 26.639,53 | .034,588,03 | 28.503,42 | .013,954,05 | 29.172,76 | .0$^{3}$624,674 | 14.238,00 |
| 48 | .255,481,5 | .177,157,5 | -12.684,84 | .072,017,65 | 27.712,76 | .039,193,66 | 29.744,50 | .015,822,49 | 30.476,26 | .0$^{3}$708,658 | 14.860,07 |
| 49 | .277,397,5 | .195,502,1 | -13.510,79 | .081,312,37 | 28.801,60 | .044,295,83 | 31.012,51 | .017,894,22 | 31.811,18 | .0$^{3}$801,836 | 15.496,05 |
| 50 | .300,691,0 | .215,275,4 | -14.372,63 | .091,572,99 | 29.905,53 | .049,935.80 | 32.307,45 | .020,186,41 | 33.177,75 | .0$^{3}$905,002 | 16.146,00 |
| 51 | .325,418,0 | .236,550,6 | -15.271,18 | .102,876,1 | 31.023,98 | .056,157,35 | 33.629,31 | .022,717,29 | 34.576,20 | .001,018,985 | 16.809,98 |
| 52 | .351,635,9 | .259,402,3 | -16.207,25 | .115,302,4 | 32.156,39 | .063,006,75 | 34.978,06 | .025,506,23 | 36.006,77 | .001,144,678 | 17.488,02 |
| 53 | .379,402,9 | .283,906,6 | -17.181,68 | .128,936,8 | 33.302,16 | .070,532,87 | 36.353,68 | .028,573,66 | 37.469,68 | .001,283,016 | 18.180,19 |
| 54 | .408,778,5 | .310,140,6 | -18.195,27 | .143,868,7 | 34.460.67 | .078,787,30 | 37.756,13 | .031,941,27 | 38.965,17 | .001,435,007 | 18.886,54 |
| 55 | .439,823,1 | .338,182,7 | -19.248,86 | .160,191,9 | 35.631,30 | .087,824,32 | 39.185,36 | .035,631,83 | 40.493,48 | .001,601,714 | 19.607,13 |
| 56 | .472,598,3 | .368,112,4 | -20.343,25 | .178,005,1 | 36.813,36 | .097,701,33 | 40.641,33 | .039,669,65 | 42.054,85 | .001,784,194 | 20.342,00 |
| 57 | .507,166,4 | .400,010,0 | -21.479,27 | .197,410,9 | 38.006,20 | .108,478,0 | 42.123,98 | .044,079,66 | 43.649,52 | .001,983,869 | 21.091,23 |
| 58 | .543,591,4 | .433,956,8 | -22.657,73 | .218,517,6 | 39.209,10 | .120,217,9 | 43.633,24 | .048,889,03 | 45.277,73 | .002,201,37 | 21.854,85 |
| 59 | .581,937,8 | .470,034,6 | -23.879,45 | .241,438,0 | 40.421,32 | .132,987,1 | 45.169,03 | .054,125,75 | 46.939,73 | .002,438,62 | 22.632,95 |
| 60 | .622,271.5 | .508,326,0 | -25.145,24 | .266,289,9 | 41.642,13 | .146,855,2 | 46.731,26 | .059,819,38 | 48.635,77 | .002,696,76 | 23,425,56 |
| 61 | .664,659,3 | .548,914,1 | -26,455,91 | .293,196,2 | 42.870,75 | .161,894,9 | 48.319,88 | .066,000,89 | 50.366,08 | .002,977,24 | 24.232,76 |
| 62 | .709,169,0 | .591,882,0 | -27.812,27 | .322,284,9 | 44.106,37 | .178,182,1 | 49.934,74 | .072,702,83 | 52.130,91 | .003,281,60 | 25.054,60 |
| 63 | .755,869,6 | .637,313,5 | -29.215,11 | .353,689,5 | 45.348,16 | .195,796,6 | 51.575,74 | .079,959,26 | 53.930,52 | .003,611,41 | 25.891,16 |
| 64 | .804,831,0 | .685,292,1 | -30.665,24 | .387,548,5 | 46.595,29 | .214,821,2 | 53.242,76 | .087,805,90 | 55.765,15 | .003,968,35 | 26.742,48 |
| 65 | .856,124,2 | .735,901,3 | -32.163,44 | .424,006,2 | 47.846,88 | .235,342,8 | 54.935,65 | .096,280,08 | 57.635,05 | .004,354,17 | 27.608,63 |
| 66 | .909,821,2 | .789,224,3 | -33.710,50 | .463,211,9 | 49.102,02 | .257,451,6 | 56.654,29 | .105,420,7 | 59.540,47 | .004,770,72 | 28.489,69 |
| 67 | .965,995,1 | .845,344,2 | -35.307,19 | .505,320,9 | 50.359,79 | .281,241,7 | 58.398,50 | .115,268,7 | 61.481,66 | .005,219,86 | 29.385,71 |
| 68 | 1.024,720 | .904,343,2 | -36.954,30 | .550,494,0 | 51.619,23 | .306,811,1 | 60.168,11 | .125,866,6 | 63.458,87 | .005,703,67 | 30.296,76 |
| 69 | 1.086,071 | .966,303,0 | -38.852,59 | .598,897,6 | 52.879,38 | .334,261,7 | 61.962,96 | .137,258,6 | 65.472,35 | .006,224,22 | 31.222,91 |
| 70 | 1.150,124 | 1.031,304 | -40.402,80 | .650,703,9 | 54.139,22 | .363,699,4 | 63.782,85 | .149,491,2 | 67.522,35 | .006,783,67 | 32.164,22 |
| 71 | 1.216,957 | 1.099,427 | -42.205,69 | .706,090,7 | 55.397,71 | .395,234,1 | 65.627,56 | .162,612,3 | 69.609,11 | .007,384,35 | 33.120,77 |
| 72 | 1.286.646 | 1.170,749 | -44.061,99 | .765,242,0 | 56.653,81 | .428,980,0 | 67.496,88 | .176,672,3 | 71.732,88 | .008,028,62 | 34.092,63 |
| 73 | 1.359,273 | 1.245,348 | -45.972,43 | .828,347,4 | 57.906,42 | .465,055,4 | 69.390,57 | .191,723,2 | 73.893,92 | .008,718,98 | 35.079,86 |
| 74 | 1.434,916 | 1.323,300 | -47.937,71 | .895,602,4 | 59.154,43 | .503,583,0 | 71.308,40 | .207,819,3 | 76.092,46 | .009,457,99 | 36.082,53 |
| 75 | 1.513,657 | 1.404,677 | -49.958,55 | .967,208,4 | 60.396,69 | .544,689,7 | 73.250,09 | .225,016,9 | 78.328,75 | .010,248,36 | 37.100,73 |
| 76 | 1.595,579 | 1.489,553 | -52.035,63 | 1.043,373 | 61.632,04 | .588,507,1 | 75.215,39 | .243,374,4 | 80.603,05 | .011,092,92 | 38.134,51 |
| 77 | 1.680.763 | 1.577,996 | -54.169,63 | 1.124,310 | 62.859,28 | .635,171,4 | 77.203,97 | .262,952,9 | 82.915,57 | .011,994,55 | 39.183.96 |
| 78 | 1.769,295 | 1.670,073 | -56.361,20 | 1.210,238 | 64.077,19 | .684,822,6 | 79.215,57 | .283,814,6 | 85.266,58 | .012,956,31 | 40.249,15 |
| 79 | 1.861,259 | 1.765,851 | -58.611,00 | 1.301,383 | 65.284,50 | .737,606,5 | 81.249,85 | .306,025,6 | 87.656,31 | .013,981,33 | 41.330,15 |
| 80 | 1.956,741 | 1.865,389 | -60.919,64 | 1.397,976 | 66.479,95 | .793,672,7 | 83.306,46 | .329,653,0 | 90.084,99 | .015,072,89 | 42.427,04 |
| 81 | 2.055,829 | 1.968.748 | -63.287,75 | 1.500,254 | 67.662,21 | .853,175,9 | 85.385,07 | .354,766,7 | 92.552,85 | .016,234,38 | 43.539,89 |
| 82 | 2.158,610 | 2.075,983 | -65.715,92 | 1.608,461 | 68.829,95 | .916,275,7 | 87.485,30 | .381,439,4 | 95.060,14 | .017,469,32 | 44.668,79 |
| 83 | 2.265,174 | 2.187,148 | -68.204,72 | 1.722,845 | 69.981,81 | .983,136,7 | 89.606,78 | .409,746,0 | 97.607,09 | .018,781,38 | 45.315,60 |
| 84 | 2.375,609 | 2.302,290 | -70.754,70 | 1.843,663 | 71.116,39 | 1.053,928 | 91.749,09 | .439,763,7 | 100.193,9 | .020,174,36 | 46.975,02 |
| 85 | 2.490,008 | 2.421,455 | -73.327,99 | 1.971,175 | 72.232,26 | 1.128,824 | 93.911,82 | .471,572,8 | 102.820,8 | .021,652,04 | 48.152,52 |
| 86 | 2.608,462 | 2.544,684 | -76.040,36 | 2.105,647 | 73.327,99 | 1.208,004 | 96.094,53 | .505,256,0 | 105.488,1 | .023,218,65 | 49.346,37 |
| 87 | 2.731,064 | 2.672,015 | -78.777,04 | 2.247,353 | 74.402,08 | 1.291,653 | 98.296,78 | .540,898,5 | 108.195,9 | .024,878,32 | 50.556,66 |
| 88 | 2.857,906 | 2.803,479 | -81.576,92 | 2.396,570 | 75.453,03 | 1.379,961 | 100.518,1 | .578,588,4 | 110.944,4 | .026,635,41 | 51.783,47 |
| 89 | 2.989,085 | 2.939,105 | -84.440,44 | 2.553,583 | 76.479,31 | 1.473,122 | 102.758,0 | .618,416,7 | 113.734,0 | .028,494,37 | 53.026,88 |
| 90 | 3.124,695 | 3.078,916 | -87.368,02 | 2.718,680 | 77.479,35 | 1.571,336 | 105.015,9 | .660,476,8 | 116.564,7 | .030,459,90 | 54.286,97 |

SHEET 3 OF 5

205-D-406

Table H-10.–*Load constants for circular arches–tangential deflections* (sheet 4).

UNIT TANGENTIAL LOAD NO. 4
TANGENTIAL DEFLECTIONS



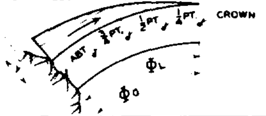
| $\phi$ | | Crown | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_3$ | | $D_3$ | | $D_3$ | |
| ° | | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term | 1ST Term | 2ND Term |
| 10 | .002,445,660 | .0$^{3}$371,976,5 | -.126,057,5 | .0$^{4}$28,874,4 | 2.865,203 | .0$^{4}$14,949,2 | 2.867,975 | .0$^{5}$2,918,8 | 2.121,028 | .0$^{7}$70,73 | .740,943,1 |
| 11 | .003,580,384 | .0$^{3}$598,508,7 | -.168,186,1 | .0$^{4}$51,125,4 | 3.469,258 | .0$^{4}$26,474,7 | 3.473,332 | .0$^{5}$5,170,0 | 2.567,872 | .0$^{6}$125,30 | .896,681,7 |
| 12 | .005,070,410 | .0$^{3}$923,774,2 | -.218,924,5 | .0$^{4}$86,123,3 | 4.131,767 | .0$^{4}$44,607,5 | 4.137,562 | .0$^{5}$8,712,8 | 3.057,840 | .0$^{6}$211,17 | 1.067,309 |
| 13 | .006,983,084 | .001,376,860 | -.279,134,1 | .0$^{3}$139,130,0 | 4.852,976 | .0$^{4}$72,079,3 | 4.860,996 | .0$^{4}$14,081,6 | 3.591,086 | .0$^{6}$341,35 | 1.252,839 |
| 14 | .009,391,569 | .001,991,993 | -.349,696,5 | .0$^{3}$216,888,1 | 5.633,147 | .0$^{3}$112,393 | 5.643,988 | .0$^{4}$21,961,9 | 4.167,777 | .0$^{6}$532,50 | 1.453,289 |
| 15 | .012,374,84 | .002,808,912 | -.431,515,4 | .0$^{3}$327,870 | 6.472,558 | .0$^{3}$169,950 | 6.486,922 | .0$^{4}$33,218,5 | 4.788,093 | .0$^{6}$805,47 | 1.668,678 |
| 16 | .016,017,68 | .003,873,279 | -.525,517,7 | .0$^{3}$482,541 | 7.371,504 | .0$^{3}$250,198 | 7.390,205 | .0$^{4}$48,913,6 | 5.452,227 | .0$^{5}$1,186,30 | 1.899,025 |
| 17 | .020,410.66 | .005,236,977 | -.632,655,1 | .0$^{3}$693,664 | 8.330,295 | .0$^{3}$359,778 | 8.354,273 | .0$^{4}$70,355,6 | 6.160,386 | .0$^{5}$1,706,63 | 2.144,346 |
| 18 | .025,650,11 | .006,958,505 | -.753,905,5 | .0$^{3}$976,585 | 9.349,255 | .0$^{3}$506,686 | 9.379,585 | .0$^{4}$99,111,0 | 6.912,787 | .0$^{5}$2,404,61 | 2.404,668 |
| 19 | .031,838,18 | .009,103,312 | -.890,274,5 | .001,349,564 | 10.428,72 | .0$^{3}$700,448 | 10.466,62 | .0$^{3}$137,054 | 7.709,662 | .0$^{5}$3,325,8 | 2.680,013 |
| 20 | .039,082,71 | .011,744,11 | -1.042,798 | .001,834,123 | 11.569,04 | .0$^{3}$952,298 | 11.615,90 | .0$^{3}$186,391 | 8.551,255 | .0$^{5}$4,524,2 | 2.970,405 |
| 21 | .047,497,33 | .014,961,18 | -1.212,537 | .002,455,367 | 12.770,56 | .001,275,350 | 12.827,94 | .0$^{3}$249,707 | 9.437,822 | .0$^{5}$6,062,3 | 3.275,870 |
| 22 | .057,201.39 | .018,842,68 | -1.400,594 | .003,242,418 | 14.033,67 | .001,684,843 | 14.103,30 | .0$^{3}$329,998 | 10.369,63 | .0$^{5}$8,013,5 | 3.596,434 |
| 23 | .068,319,95 | .023,484,90 | -1.608,094 | .004,228,734 | 15.358,72 | .002,198,301 | 15.442,56 | .0$^{3}$430,723 | 11.346,96 | .0$^{4}$10,462,1 | 3.932,127 |
| 24 | .080,983.75 | .028,992,53 | -1.836,204 | .005,452,542 | 16.746,11 | .002,835,766 | 16.846,31 | .0$^{3}$555,837 | 12.370,11 | .0$^{4}$13,504,6 | 4.282,977 |
| 25 | .095,329,25 | .035,478,95 | -2.086,121 | .006,957,250 | 18.196,21 | .003,620,02 | 18.315,17 | .0$^{3}$709,843 | 13.439,38 | .0$^{4}$17,250,9 | 4.649,016 |
| 26 | .111,498,6 | .043,066,38 | -2.359,082 | .008,791,891 | 19.709,42 | .004,576,85 | 19.849,78 | .0$^{3}$897,841 | 14.555,09 | .0$^{4}$21,825,8 | 5.030,276 |
| 27 | .129,639,4 | .051,886,15 | -2.656,360 | .011,011,53 | 21.286,13 | .005,735,24 | 21.450,80 | .001,125,57 | 15.717,56 | .0$^{4}$27,369,7 | 5.426,790 |
| 28 | .149,905,2 | .062,078,86 | -2.979,265 | .013,677,73 | 22.926,72 | .007,127,64 | 23.118,91 | .001,399,47 | 16.927,13 | .0$^{4}$34,040,0 | 5.838,592 |
| 29 | .172,454,9 | .073,794,59 | -3.329,147 | .016,859,04 | 24.631,58 | .008,790,23 | 24.854,79 | .001,726,69 | 18.184,16 | .0$^{4}$42,012,3 | 6.265,721 |
| 30 | .197,453,2 | .087,192,98 | -3.707,399 | .020,631,41 | 26.401,10 | .010,763,18 | 26.659,15 | .002,115,28 | 19.489,01 | .0$^{4}$51,484,1 | 6.708,211 |
| 31 | .225.070,2 | .102,443,4 | -4.115,452 | .025,078,80 | 28.235,66 | .013,090,95 | 28.532,73 | .002,574,02 | 20.842,04 | .0$^{4}$62,671 | 7.166,102 |
| 32 | .256,481,5 | .119,725,2 | -4.554,779 | .030,293.51 | 30.135,63 | .015.822,49 | 30.476,26 | .003,112,71 | 22.243,65 | .0$^{4}$75,814 | 7.639,435 |
| 33 | .288,868,6 | .139,227,5 | -5.026,897 | .036,376,82 | 32.101,37 | .019.011,64 | 32.490,49 | .003,742,09 | 23.694,23 | .0$^{4}$91,174 | 8.128,248 |
| 34 | .325,418,0 | .161,149,5 | -5.533,363 | .043,439,42 | 34.133,24 | .022,717,29 | 34.576,20 | .004,473,92 | 25.194,17 | .0$^{3}$109,047 | 8.632,587 |
| 35 | .365,322,1 | .185,700,4 | -6.075,782 | .051,601,98 | 36.231,57 | .027.003,82 | 36.734,16 | .005,321,10 | 26.743,91 | .0$^{3}$129,745 | 9.152,493 |
| 36 | .408,778,5 | .213,099,6 | -6.655,800 | .060,995,60 | 38.396,70 | .031,941,27 | 38.365,17 | .006,297,67 | 28.343,86 | .0$^{3}$153,616 | 9.688,012 |
| 37 | .455,990,4 | .243,576,4 | -7.275,107 | .071,762,45 | 40.626,92 | .037,605,73 | 41.270,02 | .007,418,91 | 29.994,45 | .0$^{3}$181,039 | 10.239,19 |
| 38 | .507,166,4 | .277,370,3 | -7.935,439 | .084,056,27 | 42.928,53 | .044,079,66 | 43.649.52 | .008,701,42 | 31.696,13 | .0$^{3}$212,423 | 10.806,07 |
| 39 | .562,520,3 | .314,730,8 | -8.638,579 | .098,342,78 | 45.295,81 | .051,452,16 | 46.104,49 | .010,163,2 | 33.449,36 | .0$^{3}$248,213 | 11.388,71 |
| 40 | .622,271,5 | .355,917,3 | -9.386,350 | .113,900,5 | 47.730,98 | .059,819,38 | 48.635,77 | .011,823,5 | 35.254,59 | .0$^{3}$288,891 | 11.987,15 |
| 41 | .686,644,6 | .401,199,0 | -10.180,63 | .131,820,9 | 50.234,29 | .069,284,72 | 51.244,16 | .013,703,4 | 37.112,31 | .0$^{3}$334,973 | 12.601,45 |
| 42 | .755,869,6 | .450,855,0 | -11.023,33 | .152,009,4 | 52.805,92 | .079,959,26 | 53.930,52 | .015,825,4 | 39.022,98 | .0$^{3}$387,020 | 13.231,66 |
| 43 | .830,181,6 | .505,173,7 | -11.916,41 | .174,685,5 | 55.446,03 | .091,962,15 | 56.695,68 | .018,213,6 | 40.987,11 | .0$^{3}$445,634 | 13.877,82 |
| 44 | .909,821,2 | .564,453,2 | -12.861,89 | .200,083,6 | 58.154,78 | .105,420,7 | 59.540,47 | .020.894,0 | 43.005,18 | .0$^{3}$511,463 | 14.540,00 |
| 45 | .995,034,0 | .629,000,8 | -13.861,81 | .228,453,5 | 60.932,24 | .120,471,2 | 62.465,75 | .023.894,3 | 45.077,70 | .0$^{3}$585,194 | 15.218,26 |
| 46 | 1.086,071 | .699,132,8 | -14.918,28 | .260,060,9 | 63.778,50 | .137,258,6 | 65.472,35 | .027,244,2 | 47.205,19 | .0$^{3}$667,569 | 15.912,64 |
| 47 | 1.183,188 | .775,174,1 | -16.033,43 | .295,187,7 | 66.693,57 | .155,937,5 | 68.561,11 | .030,975,3 | 49.388,16 | .0$^{3}$759,384 | 16.623,21 |
| 48 | 1.286,646 | .857,458,3 | -17.209,45 | .334,133,2 | 69.677,45 | .176,872,3 | 71.732,88 | .035,121,2 | 51.627,15 | .0$^{3}$861,477 | 17.350,02 |
| 49 | 1.396,712 | .946,327,4 | -18.448,58 | .377,213,8 | 72.730,09 | .199,637,2 | 74.988,48 | .039,717,9 | 53.922,68 | .0$^{3}$974,746 | 18.093,14 |
| 50 | 1.513,657 | 1.042,131 | -19.753,08 | .424,764,1 | 75.851,39 | .225,016,9 | 78.328,75 | .044,803,4 | 56.275,31 | .001,100,159 | 18.852,63 |
| 51 | 1.637,758 | 1.145,226 | -21.125,28 | .477,137,5 | 79.041,19 | .253,007,2 | 81.754,51 | .050,418,2 | 58.685,58 | .001,238,719 | 19.628,54 |
| 52 | 1.769,295 | 1.255,978 | -22.567,52 | .534,705,8 | 82.299,32 | .283,814,6 | 85.266,58 | .056,605,1 | 61.154,04 | .001,391,533 | 20.420,96 |
| 53 | 1.908,555 | 1.374,757 | -24.082,22 | .597,861,1 | 85.625,52 | .317,657,9 | 88.865,76 | .063,409,2 | 63.681,26 | .001,559,678 | 21.229.93 |
| 54 | 2.055,829 | 1.501,941 | -25.671,80 | .667,014,9 | 89.019,51 | .354,766,7 | 92.552,85 | .070,878,7 | 66.267,80 | .001,744,449 | 22.055,53 |
| 55 | 2.211,413 | 1.637,915 | -27.338.75 | .742,599,9 | 92.480,92 | .395,363,6 | 96.328,65 | .079,064,0 | 68.914,22 | .001,947,10 | 22.897,82 |
| 56 | 2.375,609 | 1.783,068 | -29.085,57 | .825,069.1 | 96.009,37 | .439,763,7 | 100.193,9 | .088,018,5 | 71.621,12 | .002,168,92 | 23.756,87 |
| 57 | 2.546,722 | 1.937,793 | -30.914,83 | .914,897,6 | 99.604,36 | .488,174,9 | 104.149,4 | .097,798,2 | 74.389,07 | .002,411,41 | 24.632,76 |
| 58 | 2.731,064 | 2.102,489 | -32.829,11 | 1.012,582 | 103.265,4 | .540,898,5 | 108.195,9 | .108,462,5 | 77.218,66 | .002,676,05 | 25.525,55 |
| 59 | 2.922,948 | 2.277,560 | -34.831,03 | 1.118,641 | 106.991,8 | .598,229,4 | 112.334,1 | .120,073,5 | 80.110,47 | .002,964,45 | 26.435,31 |
| 60 | 3.124,695 | 2.463,412 | -36.923,25 | 1.233,618 | 110.783,1 | .660,476,8 | 116.564,7 | .132,696,6 | 83.065,11 | .003,278,25 | 27.362,11 |
| 61 | 3.336,631 | 2.660,454 | -39.108,45 | 1.358,076 | 114.638,4 | .727,963,6 | 120.868,4 | .146,400,3 | 86.083,16 | .003,619,20 | 28.306,04 |
| 62 | 3.559,084 | 2.869,097 | -41.369,35 | 1.492,604 | 118.557,0 | .801,028,1 | 125.305,9 | .161,256,5 | 89.165,24 | .003,989,18 | 29.267,17 |
| 63 | 3.792,388 | 3.089,755 | -43.768,71 | 1.637,815 | 122.537,9 | .880,023,6 | 129.817,9 | .177,340,5 | 92.311,93 | .004,390,11 | 30.245,56 |
| 64 | 4.036,882 | 3.322,842 | -46.249,27 | 1.794,346 | 126.580,4 | .965,318,6 | 134.424,9 | .194,731,4 | 95.523,86 | .004,824,00 | 31.241,31 |
| 65 | 4.292,909 | 3.568,771 | -48.833,86 | 1.962,857 | 130.683,3 | 1.057,297 | 139.127,6 | .213,511,4 | 98.801,63 | .005,293,00 | 32.254,48 |
| 66 | 4.560,817 | 3.827,959 | -51.525,28 | 2.144,035 | 134.845,5 | 1.156,360 | 143.926,5 | .233,766,9 | 102.145,9 | .005,799,33 | 33.285,15 |
| 67 | 4.840,956 | 4.100,816 | -54.326,38 | 2.338,592 | 139.066,0 | 1.262,924 | 148.822,2 | .255,587,6 | 105.557,1 | .006,345,32 | 34.333,42 |
| 68 | 5.133,684 | 4.387,756 | -57.240,02 | 2.547,264 | 143.343,3 | 1.377,422 | 153.815,3 | .279,068,2 | 109.036,1 | .006,933,42 | 35.399,35 |
| 69 | 5.439,361 | 4.689,187 | -60.269,07 | 2.770,813 | 147.676,4 | 1.500,305 | 158.906,2 | .304,306,0 | 112.583,4 | .007,566,18 | 36.483,03 |
| 70 | 5.758,352 | 5.005,514 | -63.416,44 | 3.010,028 | 152.063,6 | 1.632,042 | 164.095,3 | .331,403,2 | 116.199,5 | .008,246,26 | 37.584,54 |
| 71 | 6.091,027 | 5.337,140 | -66.685,02 | 3.265,724 | 156.503,5 | 1.773,117 | 169.383,2 | .360,466,4 | 119.885,2 | .008,976,42 | 38.703,97 |
| 72 | 6.437,757 | 5.684,461 | -70.077,73 | 3.538,742 | 160.994,6 | 1.924,035 | 174.770,2 | .391,606,0 | 123.641,1 | .009,759,57 | 39.841,40 |
| 73 | 6.798,922 | 6.047,869 | -73.597,52 | 3.829,948 | 165.535,1 | 2.085,317 | 180.256,6 | .424,937,2 | 127.467,7 | .010,598,74 | 40.996,92 |
| 74 | 7.174,903 | 6.427,749 | -77.247,29 | 4.140,238 | 170.123,3 | 2.257,503 | 185.842,9 | .460,579,6 | 131.365,7 | .011,497,06 | 42.170,62 |
| 75 | 7.566,085 | 6.824,478 | -81.029,99 | 4.470,530 | 174.757,4 | 2.441,154 | 191.529,3 | .498,657,4 | 135.335,7 | .012,457,81 | 43.362,58 |
| 76 | 7.972,858 | 7.238,425 | -84.948,55 | 4.821,773 | 179.435,3 | 2.636,848 | 197.316,0 | .539,299,6 | 139.378,3 | .013,484,41 | 44.572,89 |
| 77 | 8.395,616 | 7.669,957 | -89.005,93 | 5.194,943 | 184.155,1 | 2.845,183 | 203.203,3 | .582,640,2 | 143.494,1 | .014,580,40 | 45.801,64 |
| 78 | 8.834,756 | 8.119,409 | -93.205,03 | 5.591,034 | 188.914,7 | 3.066,775 | 209.191,3 | .628,817,4 | 147.683,7 | .015,749,46 | 47.048,93 |
| 79 | 9.290,681 | 8.587,137 | -97.548,80 | 6.011,081 | 193.711,7 | 3.302,263 | 215.280,2 | .677,975,6 | 151.947,8 | .016,995,42 | 48.314,85 |
| 80 | 9.763,795 | 9.073,462 | -102.040,1 | 6.456,135 | 198.543,9 | 3.552,305 | 221.470,0 | .730,263,4 | 156.286,9 | .018,322,25 | 49.599,48 |
| 81 | 10.254,51 | 9.578,703 | -106.682,0 | 6.927,278 | 203.408,9 | 3.817,577 | 227.760,9 | .785,834,8 | 160.701,7 | .019,734,09 | 50.902,92 |
| 82 | 10.763,23 | 10.103,16 | -111.477,2 | 7.425,617 | 208.304,1 | 4.098,779 | 234.152,8 | .844,849,5 | 165.192,7 | .021,235,21 | 52.225,27 |
| 83 | 11.290,36 | 10.647,12 | -116.428,6 | 7.952,285 | 213.226,9 | 4.396,630 | 240.645,6 | .907,472,9 | 169.760,6 | .022,830,05 | 53.566,61 |
| 84 | 11.836,36 | 11.210,86 | -121.539,1 | 8.508,445 | 218.174,7 | 4.711,871 | 247.239,3 | .973,874,5 | 174.405,9 | .024,523,20 | 54.927,06 |
| 85 | 12.401,65 | 11.794,64 | -126.811,6 | 9.095,281 | 223.144,5 | 5.045,262 | 253.933,8 | 1.044,231 | 179.129,2 | .026,319,42 | 56.306,70 |
| 86 | 12.986,62 | 12.398,69 | -132.248,8 | 9.714,006 | 228.133,5 | 5.397,588 | 260.728,8 | 1.118,724 | 183.931,1 | .028,223,66 | 57.705,63 |
| 87 | 13.591,72 | 13.023,23 | -137.853,4 | 10.365,86 | 233.138,6 | 5.769,653 | 267.624,0 | 1.197,542 | 188.812,2 | .030,241,02 | 59.123,95 |
| 88 | 14.217,38 | 13.668,47 | -143.628,4 | 11.052,10 | 238.156,8 | 6.162,281 | 274.619,3 | 1.280,877 | 193.773,1 | .032,376,78 | 60.561,76 |
| 89 | 14.864,05 | 14.334,60 | -149.576,2 | 11.774,02 | 243.184,8 | 6.576,322 | 281.714,2 | 1.368,931 | 198.814,3 | .034,536,35 | 62.013,16 |
| 90 | 15.532,16 | 15.021,84 | -155.699,7 | 12.532,96 | 248.219,2 | 7.012,646 | 288.908,4 | 1.461,908 | 203.938,4 | .037,025,47 | 63.496,25 |

SHEET 4 OF 5

205-D-407

Table H-10.–*Load constants for circular arches–tangential deflection* (sheet 5).

UNIT TANGENTIAL LOAD NO. 5
TANGENTIAL DEFLECTIONS



| $\Phi$ | Crown | | | | | $\frac{1}{4}$ Point | | $\frac{1}{2}$ Point | | $\frac{3}{4}$ Point | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_3$ | | $D_3$ | | $D_3$ | | $D_3$ | |
| ° | | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term | $1^{ST}$ Term | $2^{ND}$ Term |
| 10 | .007,727,042 | .001,119,538 | -.004,035,59 | .0$^4$83,892,5 | 5.115,357 | .0$^4$28,649,7 | 4.308,063 | .0$^5$3,721,5 | 2.545,717 | .0$^7$77,01 | .793,878,4 |
| 11 | .011,311,44 | .001,801,296 | -.006,494,89 | .0$^3$,148,553 | 6.199,259 | .0$^4$50,737,3 | 5.218,925 | .0$^5$6,591,8 | 3.082,153 | .0$^6$,136,41 | .960,746,3 |
| 12 | .016,017,68 | .002,780,168 | -.010,027,5 | .0$^3$,250,198 | 7.390,205 | .0$^4$85,485 | 6.218,981 | .0$^4$11,108,7 | 3.670,409 | .0$^6$,229,91 | 1.143,567 |
| 13 | .022,058,17 | .004,143,664 | -.014,950,3 | .0$^3$,404,162 | 8.689,216 | .0$^3$,138,130 | 7.308,889 | .0$^4$17,953,7 | 4.310,681 | .0$^6$,371,63 | 1.342,358 |
| 14 | .029,663,55 | .005,994,747 | -.021,636,6 | .0$^3$,630,007 | 10.097,39 | .0$^3$,215,371 | 8.489,364 | .0$^4$28,001.0 | 5.003,185 | .0$^6$,579,73 | 1.557,137 |
| 15 | .039,082,71 | .008,452,995 | -.030,520,7 | .0$^3$,952,298 | 11.615,90 | .0$^3$,325,668 | 9.761,173 | .0$^4$42,350,1 | 5.748,152 | .0$^6$,876,92 | 1.787,924 |
| 16 | .050,582,69 | .011,655,67 | -.042,101,4 | .001,401,430 | 13.246,00 | .0$^3$,479,433 | 11.125,14 | .0$^4$62,363,4 | 6.545,831 | .0$^5$1,291,54 | 2.034,738 |
| 17 | .064,448,64 | .015,756,88 | -.056,947,4 | .002,014,411 | 14.989,00 | .0$^3$,689,384 | 12.582,13 | .0$^4$89,701,2 | 7.396,486 | .0$^5$1,858.03 | 2.297,604 |
| 18 | .080,983,75 | .020,938,55 | -.075,699,8 | .002,835,766 | 16.846,31 | .0$^3$,970,847 | 14.133,06 | .0$^3$,126,363 | 8.300,399 | .0$^5$2,618,0 | 2.576,544 |
| 19 | .100,509,2 | .027,391,45 | -.099,077,7 | .003,918,43 | 18.819,37 | .001,342,058 | 15.778,92 | .0$^3$,174,739 | 9.257,867 | .0$^5$3,620,9 | 2.871,583 |
| 20 | .123,364,0 | .035,336,22 | -.127,881 | .005,324,78 | 20.909,71 | .001,824,531 | 17.520,73 | .0$^3$,237,641 | 10.269,21 | .0$^5$4,925,5 | 3.182,749 |
| 21 | .149,905,2 | .045,014,18 | -.162,993 | .007,127,64 | 23.118,91 | .002,443,382 | 19.359,55 | .0$^3$,318,367 | 11.334,74 | .0$^5$6,600,1 | 3.510,070 |
| 22 | .180,507,3 | .056,690,37 | -.205,389 | .009,411,29 | 25.448,60 | .003,227,78 | 21.296,49 | .0$^3$,420,739 | 12.454,82 | .0$^5$8,724,3 | 3.853,575 |
| 23 | .215,562,8 | .070,654,03 | -.256,132 | .012,272,67 | 27.900,47 | .004,211,26 | 23.332,73 | .0$^3$,549,149 | 13.629,80 | .0$^4$11,390,1 | 4.213,295 |
| 24 | .255,481,5 | .087,219,96 | -.316,383 | .015,822,49 | 30.476,26 | .005,432,19 | 25.469,47 | .0$^3$,708,661 | 14.860,07 | .0$^4$14,702,4 | 4.589,261 |
| 25 | .300,691,0 | .106,728,7 | -.387,403 | .020,186,41 | 33.177,75 | .006,934,20 | 27.707,95 | .0$^3$,905,003 | 16.146,00 | .0$^4$18,781,2 | 4.981,509 |
| 26 | .351,635,9 | .129,547,5 | -.470,548 | .025,506,23 | 36.006,77 | .008,766,58 | 30.049,46 | .001,144,678 | 17.488,02 | .0$^4$23,761,9 | 5.390,073 |
| 27 | .408,778,5 | .156,070,7 | -.567,288 | .031,941,27 | 38.965,17 | .010,984,81 | 32.495,35 | .001,435,007 | 18.886,54 | .0$^4$29,797,6 | 5.814,990 |
| 28 | .472,598,3 | .186,720,8 | -.679,195 | .039,669,65 | 42.054,85 | .013,650,97 | 35.046,96 | .001,784,194 | 20.342,00 | .0$^4$37,059,5 | 6.256,296 |
| 29 | .543,591,4 | .221,948,2 | -.807,952 | .048,889,03 | 45.277,73 | .016,834,27 | 37.705,72 | .002,201,37 | 21.854,85 | .0$^4$45,738,9 | 6.714,033 |
| 30 | .622,271,5 | .262,232,4 | -.955,352 | .059,819,38 | 48.635,77 | .020,611,54 | 40.473,06 | .002,696,76 | 23.425,56 | .0$^4$56,050,6 | 7.188,240 |
| 31 | .709,169,0 | .308,081,9 | -1.123,310 | .072,702,83 | 52.130,91 | .025,067,73 | 43.350,46 | .003,281,60 | 25.054,60 | .0$^4$68,230 | 7.678,960 |
| 32 | .804,831,0 | .360,034,6 | -1.313,852 | .087,805,90 | 55.765,15 | .030,296,52 | 46.339,42 | .003,968,35 | 26.742,48 | .0$^4$82,538 | 8.186,236 |
| 33 | .909,821,2 | .418,658,3 | -1.529,123 | .105,420,7 | 59.540,47 | .036,400,75 | 49.441,49 | .004,770,72 | 28.489,69 | .0$^4$99,262 | 8.710,113 |
| 34 | 1.024,720 | .484,550,7 | -1.771,394 | .125,866,6 | 63.458,87 | .043,493,01 | 52.658,24 | .005,703,67 | 30.296,76 | .0$^3$,118,720 | 9.250,637 |
| 35 | 1.150,124 | .558,339,4 | -2.043,057 | .149,491,2 | 67.522,35 | .051,696,24 | 55.991,24 | .006,783,67 | 32.164,22 | .0$^3$,141,253 | 9.807,855 |
| 36 | 1.286,646 | .640,682,2 | -2.346,623 | .176,672,3 | 71.732,88 | .061,144,36 | 59.442,14 | .008,028,62 | 34.092,63 | .0$^3$,167,248 | 10.381,82 |
| 37 | 1.434,916 | .732,267,1 | -2.684,737 | .207,819,3 | 76.092,46 | .071,982,64 | 63.012,56 | .009,457,99 | 36.082,53 | .0$^3$,197,098 | 10.972,57 |
| 38 | 1.595,579 | .833,812,1 | -3.060,165 | .243,374,4 | 80.603,05 | .084,368,6 | 66.704,18 | .011,092,92 | 38.134,51 | .0$^3$,231,265 | 11.580,18 |
| 39 | 1.769,295 | .946,065,0 | -3.475,801 | .283,814,6 | 85.266,58 | .098,472,4 | 70.518,69 | .012,956,31 | 40.249,15 | .0$^3$,270,231 | 12.204,68 |
| 40 | 1.956,741 | 1.069,803 | -3.934,869 | .329,653,0 | 90.084,99 | .114,477,3 | 74.457,77 | .015,072,89 | 42.427,04 | .0$^3$,314,516 | 12.846,13 |
| 41 | 2.158,610 | 1.205,834 | -4.439,920 | .381,439,4 | 95.060,14 | .132,581,0 | 78.523,16 | .017,469,32 | 44.668,79 | .0$^3$,364,685 | 13.504,59 |
| 42 | 2.375,609 | 1.354,993 | -4.994,838 | .439,763,7 | 100.193,9 | .152,995,2 | 82.716,58 | .020,174,36 | 46.975,02 | .0$^3$,421,347 | 14.180,11 |
| 43 | 2.608,462 | 1.518,145 | -5.602,833 | .505,256,0 | 105.488,1 | .175,947,4 | 87.039,78 | .023,218,65 | 49.346,37 | .0$^3$,485,161 | 14.872,76 |
| 44 | 2.857,906 | 1.696,183 | -6.267,446 | .578,588,4 | 110.944,4 | .201,680,4 | 91.494,51 | .026,635,41 | 51.783,47 | .0$^3$,556,829 | 15.582,60 |
| 45 | 3.124,695 | 1.890,027 | -6.992,345 | .660,478,8 | 116.564,7 | .230,453,7 | 96.082,53 | .030,459,90 | 54.286,97 | .0$^3$,637,099 | 16.309,67 |
| 46 | 3.409,597 | 2.100,623 | -7.781,327 | .751,681,5 | 122.350,5 | .262,544,4 | 100.805,6 | .034,730,10 | 56.857,55 | .0$^3$,726,781 | 17.054,06 |
| 47 | 3.713,394 | 2.328,946 | -8.638,319 | .853,010,4 | 128.303,4 | .298,246,0 | 105.665,5 | .039,485,87 | 59.495,86 | .0$^3$,826,739 | 17.815,81 |
| 48 | 4.036,882 | 2.575,994 | -9.567,376 | .965,318,6 | 134.424,9 | .337,872,7 | 110.664,0 | .044,770,55 | 62.202,59 | .0$^3$,937,887 | 18.595,00 |
| 49 | 4.380,874 | 2.842,790 | -10.572,68 | 1.089,511 | 140.716,5 | .381,755,6 | 115.802,9 | .050,629,76 | 64.978,44 | .001,061,206 | 19.391,68 |
| 50 | 4.746,195 | 3.130,381 | -11.658,53 | 1.226,543 | 147.179,5 | .430,247,4 | 121.083,9 | .057,111,95 | 67.824,10 | .001,197,738 | 20.205,94 |
| 51 | 5.133,684 | 3.439,836 | -12.829,36 | 1.377,422 | 153.815,3 | .483,719,8 | 126.508,9 | .064,268,74 | 70.740,27 | .001,348,590 | 21.037,83 |
| 52 | 5.544,194 | 3.772,247 | -14.089,72 | 1.543,210 | 160.624,9 | .542,566,7 | 132.079,4 | .072,154,5 | 73.727,68 | .001,514,936 | 21.887,42 |
| 53 | 5.978,592 | 4.128,724 | -15.444,28 | 1.725,024 | 167.609.6 | .607,203,5 | 137.797,4 | .080,827,0 | 76.787,05 | .001,698,01 | 22.754,80 |
| 54 | 6.437,757 | 4.510,399 | -16.897,83 | 1.924,035 | 174.770,2 | .678,067,8 | 143.664.5 | .090,347,4 | 79.919,11 | .001,899,16 | 23.640,02 |
| 55 | 6.922,584 | 4.918,420 | -18.455,27 | 2.141,473 | 182.107,6 | .755,620,7 | 149.682,4 | .100,780,0 | 83.124,60 | .002,119,80 | 24.543,17 |
| 56 | 7.433,978 | 5.353,951 | -20.121,62 | 2.378,628 | 189.622,7 | .840,346,7 | 155.852,9 | .112,193,0 | 86.404,26 | .002,361,29 | 25.464,31 |
| 57 | 7.972,858 | 5.818,173 | -21.902,01 | 2.636,848 | 197.316,0 | .932,755,2 | 162.177,6 | .124,657,5 | 89.758,85 | .002,625,28 | 26.403,53 |
| 58 | 8.540,190 | 6.312,279 | -23.801,69 | 2.917,541 | 205.188,1 | 1.033,381 | 168.658,1 | .138,249,3 | 93.189,12 | .002,913,39 | 27.360,89 |
| 59 | 9.136,816 | 6.837,476 | -25.825,99 | 3.222,183 | 213.239,3 | 1.142,784 | 175.296,2 | .153,047,5 | 96.695,84 | .003,227,36 | 28.336,49 |
| 60 | 9.763,795 | 7.394,979 | -27.980,35 | 3.552,305 | 221.470,0 | 1.261,550 | 182.093,4 | .169,135,2 | 100.279,8 | .003,569,00 | 29.330,39 |
| 61 | 10.422,06 | 7.986,015 | -30.270,32 | 3.909,506 | 229.880,3 | 1.390,294 | 189.051,4 | .186,600,0 | 103.941,7 | .003,940,19 | 30.342,68 |
| 62 | 11.112,59 | 8.611,815 | -32.701,54 | 4.295,451 | 238.470,1 | 1.529,657 | 196.171,5 | .205,533,4 | 107.682,4 | .004,342,98 | 31.373,45 |
| 63 | 11.836,38 | 9.273,617 | -35.279,74 | 4.711,871 | 247.239,3 | 1.680,308 | 203.455,6 | .226,031,3 | 111.502,6 | .004,779,45 | 32.422,76 |
| 64 | 12.594,43 | 9.972,662 | -38.010,73 | 5.160,562 | 256.187,6 | 1.842,946 | 210.904,7 | .248,194,3 | 115.403,2 | .005,251,82 | 33.490,72 |
| 65 | 13.387,76 | 10.710,19 | -40.900,41 | 5.643,387 | 265.314,5 | 2.018,301 | 218.520,6 | .272,127,1 | 119.384,9 | .005,762,41 | 34.577,40 |
| 66 | 14.217,38 | 11.487,45 | -43.954,78 | 6.162,281 | 274.619,3 | 2.207,130 | 226.304,5 | .297,940,0 | 123.448,5 | .006,313,64 | 35.682,88 |
| 67 | 15.084,35 | 12.305,68 | -47.179,85 | 6.719,244 | 284.101,2 | 2.410,222 | 234.258,0 | .325,747,4 | 127.594,9 | .006,908,05 | 36.807,27 |
| 68 | 15.989,69 | 13.166,11 | -50.581,79 | 7.316,348 | 293.759,4 | 2.628,399 | 242.382,0 | .355,669,0 | 131.824,7 | .007,548,30 | 37.950,64 |
| 69 | 16.934,48 | 14.069,97 | -54.166,76 | 7.955,734 | 303.592,6 | 2.862,514 | 250.678,5 | .387,829,6 | 136.138,9 | .008,237,17 | 39.113,09 |
| 70 | 17.919,77 | 15.018,49 | -57.941,02 | 8.639,612 | 313.599,5 | 3.113,451 | 259.148,0 | .422,358,8 | 140.538,1 | .008,977,57 | 40.294,70 |
| 71 | 18.946,64 | 16.012,86 | -61.910,87 | 9.370,266 | 323.778,6 | 3.382,129 | 267.792,0 | .459,393,4 | 145.023,3 | .009,772,45 | 41.495,57 |
| 72 | 20.016,18 | 17.054,30 | -66.082,66 | 10.150,05 | 334.128,3 | 3.689,499 | 276.611,8 | .499,072,8 | 149.595,2 | .010,625,04 | 42.715,79 |
| 73 | 21.129,48 | 18.143,98 | -70.462,79 | 10.981,39 | 344.646,6 | 3.976,548 | 285.607,7 | .541,544,2 | 154.254,6 | .011,538,63 | 43.955,46 |
| 74 | 22.287,65 | 19.283,05 | -75.057,71 | 11.866,77 | 355.331,5 | 4.304,298 | 294.781,5 | .586,959,7 | 159.002,3 | .012,516,59 | 45.214,66 |
| 75 | 23.491,81 | 20.472,68 | -79.873,88 | 12.808,77 | 366.180,7 | 4.653,804 | 304.133,8 | .635,477,7 | 163.839,1 | .013,562,54 | 46.493,50 |
| 76 | 24.743,06 | 21.713,97 | -84.917,81 | 13.810,03 | 377.191,8 | 5.026,157 | 313.665,6 | .687,262,3 | 168.765,9 | .014,680,16 | 47.792,08 |
| 77 | 26.042,55 | 23.008,02 | -90.195,98 | 14.873,25 | 388.362,1 | 5.422,486 | 323.377,7 | .742,483,8 | 173.783,3 | .015,873,32 | 49.110,49 |
| 78 | 27.391,41 | 24.355,90 | -95.714,99 | 16.001,22 | 399.688,8 | 5.843,951 | 333.270,7 | .801,318,8 | 178.892,3 | .017,146,04 | 50.448,83 |
| 79 | 29.790,79 | 25.758,66 | -101.481,3 | 17.196,79 | 411.168,8 | 6.291,759 | 343.345,5 | .863,950,6 | 184.093,5 | .018,502,46 | 51.807,20 |
| 80 | 30.241,85 | 27.217,29 | -107.501,6 | 18.462,88 | 422.798,8 | 6.767,147 | 353.602,5 | .930,568,6 | 189.387,9 | .019,946,93 | 53.185,70 |
| 81 | 31.745,73 | 28.732,77 | -113.782,3 | 19.802,49 | 434.575,1 | 7.271,389 | 364.042,4 | 1.001,369 | 194.776,1 | .021,483,95 | 54.584,43 |
| 82 | 33.303,62 | 30.306,03 | -120.330,0 | 21.218,67 | 446.494,8 | 7.805,802 | 374.665,7 | 1.076,555 | 200.259,0 | .023,118,16 | 56.003,50 |
| 83 | 34.916,69 | 31.937,98 | -127.151,2 | 22.714,56 | 458.553,1 | 8.371,737 | 385.472,6 | 1.156,336 | 205.837,3 | .024,854,39 | 57.443,02 |
| 84 | 36.586,12 | 33.629,46 | -134.252,3 | 24.293,36 | 470.746,3 | 8.970,588 | 396.463,5 | 1.240,930 | 211.511,8 | .026,697,64 | 58.903,08 |
| 85 | 38.313,11 | 35.381,28 | -141.640,0 | 25.958,33 | 483.070,0 | 9.603,785 | 407.638,6 | 1.330,560 | 217.283,4 | .028,653,12 | 60.383,79 |
| 86 | 40.098,83 | 37.194,20 | -149.320,6 | 27.712,80 | 495.519,6 | 10.272,80 | 418.998,2 | 1.425,458 | 223.152,6 | .030,726,17 | 61.885,26 |
| 87 | 41.944,50 | 39.068,93 | -157.300,4 | 29.560,18 | 508.090,4 | 10.979,14 | 430.542,2 | 1.525,864 | 229.120,4 | .032,922,37 | 63.407,59 |
| 88 | 43.851,33 | 41.006,14 | -165.585,9 | 31.503,92 | 520.777,5 | 11.724,35 | 442.270,7 | 1.632,022 | 235.187,4 | .035,247,47 | 64.950,90 |
| 89 | 45.820,52 | 43.006,43 | -174.183,3 | 33.547,56 | 533.575,5 | 12.510,04 | 454.183,4 | 1.744,188 | 241.354,4 | .037,707,35 | 66.515,30 |
| 90 | 47.853,28 | 45.070,34 | -183.098,9 | 35.694,66 | 546.479,1 | 13.337,82 | 466.280,3 | 1.862,623 | 247.622,2 | .040,308,26 | 68.100,88 |

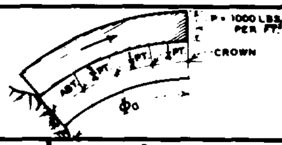
SHEET 5 OF 5

205-D-408

Table H-11.–*Load constants for circular arches–radial and twist deflections* (sheet 1).

UNIT TANGENTIAL LOAD NO. 1

RADIAL AND TWIST DEFLECTIONS



| $\phi$ ° | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | |
| | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term |
| 10 | .038,624,00 | .005,373,006 | .868,610,9 | .012,226,31 | .001,277,674 | .369,666,3 | .002,415,839 | $0^3$168,501,5 | .110,214,2 | $0^3$151,018,7 | $0^5$5,270,328 | .013,828,12 |
| 11 | .056,537,34 | .008,644,710 | 1.151,254 | .017,898,39 | .002,056,560 | .490,867,1 | .003,536,841 | $0^3$271,306,4 | .146,542,2 | $0^3$221,103,5 | $0^5$8,487,391 | .018,400,44 |
| 12 | .080,054,96 | .013,342,03 | 1.487,730 | .025,346,10 | .003,175,552 | .635,632,5 | .005,008,925 | $0^3$419,068,2 | .190,034,2 | $0^3$313,143,6 | $0^4$13,112,56 | .023,881,98 |
| 13 | .110,236,8 | .019,884,74 | 1.881,984 | .034,905,79 | .004,735,236 | .805,877,8 | .006,898,670 | $0^3$625,125,5 | .241,311,4 | $0^3$431,305,8 | $0^4$19,564,38 | .030,354,43 |
| 14 | .148,233,4 | .028,766,68 | 2.337,715 | .046,942,83 | .006,854,153 | 1.003,459 | .009,278,426 | $0^3$905,216,3 | .300,987,2 | $0^3$580,118,1 | $0^4$28,337,07 | .037,899,26 |
| 15 | .195,286,0 | .040,561,22 | 2.858,360 | .061,851,50 | .009,670,204 | 1.230,168 | .012,226,31 | .001,277,674 | .369,666,3 | $0^3$764,471,5 | $0^4$40,006,82 | .046,597,88 |
| 16 | .252,725,7 | .055,926,64 | 3.447,075 | .080,054,96 | .013,342,03 | 1.487,730 | .015,826,18 | .001,763,619 | .447,944,7 | $0^3$989,618,1 | $0^4$55,237,98 | .056,530,59 |
| 17 | .321,973,1 | .075,611,22 | 4.106,716 | .102,005,2 | .018,050,37 | 1.777,797 | .020,167,65 | .002,387,155 | .536,408,2 | .001,261,172 | $0^4$74,789,45 | .067,778,80 |
| 18 | .404,538,3 | .100,458,3 | 4.839,826 | .128,182,8 | .023,999,40 | 2.101,944 | .025,346,10 | .003,175,552 | .635,632,5 | .001,585,109 | $0^4$99,520,73 | .080,421,98 |
| 19 | .502,019,9 | .131,411,0 | 5.648,610 | .159,097,1 | .031,418,03 | 2.461,662 | .031,462,61 | .004,159,443 | .746,182,4 | .001,967,765 | $0^3$130,398,2 | .094,540,67 |
| 20 | .616,104,7 | .169,516,8 | 6.534,924 | .195,286,0 | .040,561,22 | 2.858,360 | .038,624,00 | .005,373,006 | .868,610,9 | .002,415,839 | $0^3$168,501,5 | .110,214,2 |
| 21 | .748,567,6 | .215,931,9 | 7.500,260 | .237,315,8 | .051,711,18 | 3.293,353 | .046,942,83 | .006,854,153 | 1.003,459 | .002,936,391 | $0^3$215,029,3 | .127,521,8 |
| 22 | .901,270,2 | .271,925,6 | 8.545,724 | .285,781,1 | .065,178,61 | 3.767,864 | .056,537,34 | .008,644,710 | 1.151,254 | .003,536,841 | $0^3$271,306,4 | .146,542,2 |
| 23 | 1.076,161 | .338,883,7 | 9.672,028 | .341,305,0 | .081,303,91 | 4.283,014 | .067,531,50 | .010,790,80 | 1.312,511 | .004,224,972 | $0^3$338,787,8 | .167,353,6 |
| 24 | 1.275,275 | .418,312,8 | 10.879,47 | .404,538,3 | .100,458,3 | 4.839,826 | .080,054,96 | .013,342,03 | 1.487,730 | .005,008,925 | $0^3$419,068,2 | .190,034,2 |
| 25 | 1.500,731 | .511,843,0 | 12.167,94 | .476,160,3 | .123,044,9 | 5.439,212 | .094,243,04 | .016,353,65 | 1.677,398 | .005,897,203 | $0^3$513,885,0 | .214,661,1 |
| 26 | 1.754,734 | .621,231,9 | 13.536,85 | .556,878,0 | .149,500,1 | 6.081,974 | .110,236,8 | .019,884,74 | 1.881,984 | .006,898,670 | $0^3$625,125,5 | .241,311,4 |
| 27 | 2.039,574 | .748,366,7 | 14.985,21 | .647,426,3 | .180,293,9 | 6.768,800 | .128,182,8 | .023,999,40 | 2.101,944 | .008,022,549 | $0^3$754,833,9 | .270,061,4 |
| 28 | 2.357,621 | .895,267,4 | 16.511,52 | .748,567,6 | .215,931,9 | 7.500,260 | .148,233,4 | .028,766,68 | 2.337,715 | .009,278,426 | $0^3$905,216,3 | .300,987,2 |
| 29 | 2.711,332 | 1.064,089 | 18.113,85 | .861,092,0 | .256,955,4 | 8.276,800 | .170,546,6 | .034,260,78 | 2.589,718 | .010,676,24 | .001,078,647 | .334,163,8 |
| 30 | 3.103,243 | 1.257,122 | 19.789,75 | .985,816,9 | .303,942,8 | 9.098,743 | .195,286,0 | .040,561,22 | 2.858,360 | .012,226,31 | .001,277,674 | .369,666,3 |
| 31 | 3.535,971 | 1.476,798 | 21.536,29 | 1.123,587 | .357,510,3 | 9.966,281 | .222,620,8 | .047,752,97 | 3.144,023 | .013,939,28 | .001,505,026 | .407,568,7 |
| 32 | 4.012,215 | 1.725,688 | 23.350,05 | 1.275,275 | .418,312,8 | 10.879,47 | .252,725,7 | .055,926,64 | 3.447,075 | .015,826,18 | .001,763,619 | .447,944,7 |
| 33 | 4.534,753 | 2.006,502 | 25.227,07 | 1.441,778 | .487,044,6 | 11.838,25 | .285,781,1 | .065,178,61 | 3.767,864 | .017,898,39 | .002,056,560 | .490,867,1 |
| 34 | 5.106,441 | 2.322,095 | 27.162,89 | 1.624,023 | .564,440,3 | 12.842,39 | .321,973,1 | .075,611,22 | 4.106,716 | .020,167,65 | .002,387,155 | .536,408,2 |
| 35 | 5.730,214 | 2.675,464 | 29.152,52 | 1.822,962 | .651,275,3 | 13.891,54 | .361,493,2 | .087,332,90 | 4.463,942 | .022,646,07 | .002,758,912 | .584,639,7 |
| 36 | 6.409,082 | 3.069,747 | 31.190,44 | 2.039,574 | .748,366,7 | 14.985,21 | .404,538,3 | .100,458,3 | 4.839,826 | .025,346,10 | .003,175,552 | .635,632,5 |
| 37 | 7.146,134 | 3.508,226 | 33.270,58 | 2.274,863 | .856,573,6 | 16.122,73 | .451,311,2 | .115,108,5 | 5.234,634 | .028,280,55 | .003,641,010 | .689,457,0 |
| 38 | 7.944,531 | 3.994,326 | 35.386,33 | 2.529,862 | .976,797,9 | 17.303,32 | .502,019,9 | .131,411,0 | 5.648,610 | .031,462,61 | .004,159,443 | .746,182,4 |
| 39 | 8.807,509 | 4.531,611 | 37.530,56 | 2.805,628 | 1.109,985 | 18.526,03 | .556,878,0 | .149,500,1 | 6.081,974 | .034,905,79 | .004,735,236 | .805,877,8 |
| 40 | 9.738,379 | 5.123,788 | 39.695,55 | 3.103,243 | 1.257,122 | 19.789,75 | .616,104,7 | .169,516,8 | 6.534,924 | .038,624,00 | .005,373,006 | .868,610,9 |
| 41 | 10.740,52 | 5.774,700 | 41.873,07 | 3.423,816 | 1.419,243 | 21.093,21 | .679,924,6 | .191,609,0 | 7.007,636 | .042,631,48 | .006,077,612 | .934,449,1 |
| 42 | 11.817,39 | 6.488,328 | 44.054,30 | 3.768,482 | 1.597,425 | 22.435,00 | .748,567,6 | .215,931,9 | 7.500,260 | .046,942,83 | .006,854,153 | 1.003,459 |
| 43 | 12.972,51 | 7.268,790 | 46.229,89 | 4.138,399 | 1.792,789 | 23.813,54 | .822,269,1 | .242,647,6 | 8.012,921 | .051,573,01 | .007,707,980 | 1.075,705 |
| 44 | 14.209,46 | 8.120,332 | 48.389,95 | 4.534,753 | 2.006,502 | 25.227,07 | .901,270,2 | .271,925,6 | 8.545,724 | .056,537,34 | .008,644,710 | 1.151,254 |
| 45 | 15.531,92 | 9.047,332 | 50.524,02 | 4.958,752 | 2.239,775 | 26.673,69 | .985,816,9 | .303,942,8 | 9.098,743 | .061,851,50 | .009,670,204 | 1.230,168 |
| 46 | 16.943,60 | 10.054,30 | 52.621,08 | 5.411,631 | 2.493,865 | 28.151,31 | 1.076,161 | .338,883,7 | 9.672,028 | .067,531,50 | .010,790,80 | 1.312,511 |
| 47 | 18.448,30 | 11.145,85 | 54.669,59 | 5.894,647 | 2.770,073 | 29.657,70 | 1.172,559 | .376,940,4 | 10.265,61 | .073,593,74 | .012,012,31 | 1.398,345 |
| 48 | 20.049,87 | 12.326,73 | 56.657,44 | 6.409,082 | 3.069,747 | 31.190,44 | 1.275,275 | .418,312,8 | 10.879,47 | .080,054,96 | .013,342,03 | 1.487,730 |
| 49 | 21.752,24 | 13.601,81 | 58.572,04 | 6.956,245 | 3.394,278 | 32.746,94 | 1.384,574 | .463,208,4 | 11.513,60 | .086,932,24 | .014,786,72 | 1.580,728 |
| 50 | 23.559,38 | 14.976,06 | 60.400,20 | 7.537,464 | 3.745,104 | 34.324,44 | 1.500,731 | .511,843,0 | 12.167,94 | .094,243,04 | .016,353,65 | 1.677,398 |
| 51 | 25.475,35 | 16.454,55 | 62.128,25 | 8.154,093 | 4.123,706 | 35.920,01 | 1.624,023 | .564,440,3 | 12.842,39 | .102,005,2 | .018,050,37 | 1.777,797 |
| 52 | 27.504,23 | 18.042,45 | 63.741,98 | 8.807,509 | 4.531,611 | 37.530,56 | 1.754,734 | .621,231,9 | 13.536,85 | .110,236,8 | .019,884,74 | 1.881,984 |
| 53 | 29.850,19 | 19.745,04 | 65.226,67 | 9.499,113 | 4.970,390 | 39.152,80 | 1.893,153 | .682,458,0 | 14.251,18 | .118,956,3 | .021,864,92 | 1.990,014 |
| 54 | 31.917,45 | 21.567,68 | 66.567,14 | 10.230,33 | 5.441,656 | 40.783,28 | 2.039,574 | .748,366,7 | 14.985,21 | .128,182,8 | .023,999,40 | 2.101,944 |
| 55 | 34.310,28 | 23.515,80 | 67.747,63 | 11.002,60 | 5.947,069 | 42.418,39 | 2.194,295 | .819,214,8 | 15.738,73 | .137,935,3 | .026,296,94 | 2.217,826 |
| 56 | 36.833,02 | 25.594,94 | 68.752,03 | 11.817,39 | 6.488,328 | 44.054,30 | 2.357,621 | .895,267,4 | 16.511,52 | .148,233,4 | .028,766,68 | 2.337,715 |
| 57 | 39.490,03 | 27.810,67 | 69.563,64 | 12.676,20 | 7.067,179 | 45.687,06 | 2.529,862 | .976,797,9 | 17.303,32 | .159,097,1 | .031,418,03 | 2.461,662 |
| 58 | 42.285,76 | 30.168,64 | 70.165,33 | 13.580,53 | 7.685,405 | 47.312,51 | 2.711,332 | 1.064,089 | 18.113,85 | .170,546,6 | .034,260,78 | 2.589,718 |
| 59 | 45.224,69 | 32.674,57 | 70.539,56 | 14.531,92 | 8.344,834 | 48.926,33 | 2.902,351 | 1.157,430 | 18.942,77 | .182,602,6 | .037,305,03 | 2.721,935 |
| 60 | 48.311,36 | 35.334,20 | 70.668,42 | 15.531,92 | 9.047,332 | 50.524,02 | 3.103,243 | 1.257,122 | 19.789,75 | .195,286,0 | .040,561,22 | 2.858,360 |
| 61 | 51.550,33 | 38.153,34 | 70.553,38 | 16.582,11 | 9.794,807 | 52.100,90 | 3.314,338 | 1.363,473 | 20.654,39 | .208,618,2 | .044,040,15 | 2.999,040 |
| 62 | 54.946,24 | 41.137,80 | 70.115,72 | 17.684,08 | 10.589,20 | 53.652,14 | 3.535,971 | 1.476,798 | 21.536,29 | .222,620,8 | .047,752,97 | 3.144,023 |
| 63 | 58.503,77 | 44.293,43 | 69.396,21 | 18.839,46 | 11.432,51 | 55.172,72 | 3.768,482 | 1.597,425 | 22.435,00 | .237,315,8 | .051,711,18 | 3.293,353 |
| 64 | 62.227,62 | 47.626,10 | 68.355,33 | 20.049,87 | 12.326,73 | 56.657,44 | 4.012,215 | 1.725,688 | 23.350,05 | .252,725,7 | .055,926,64 | 3.447,075 |
| 65 | 66.122,56 | 51.141,68 | 66.973,11 | 21.316,98 | 13.273,94 | 58.100,97 | 4.267,521 | 1.861,929 | 24.280,92 | .268,873,1 | .060,411,58 | 3.605,231 |
| 66 | 70.193,38 | 54.846,03 | 65.229,35 | 22.642,47 | 14.278,23 | 59.497,77 | 4.534,753 | 2.006,502 | 25.227,07 | .285,781,1 | .065,178,61 | 3.767,864 |
| 67 | 74.444,93 | 58.745,01 | 63.103,45 | 24.028,01 | 15.335,71 | 60.842,14 | 4.814,272 | 2.159,767 | 26.187,93 | .303,473,2 | .070,240,71 | 3.935,012 |
| 68 | 78.882,07 | 62.844,45 | 60.574,61 | 25.475,35 | 16.454,55 | 62.128,25 | 5.106,441 | 2.322,095 | 27.162,89 | .321,973,1 | .075,611,22 | 4.106,716 |
| 69 | 83.509,71 | 67.150,14 | 57.621,60 | 26.986,19 | 17.634,93 | 63.350,06 | 5.411,631 | 2.493,865 | 28.151,31 | .341,305,0 | .081,303,91 | 4.283,014 |
| 70 | 88.332,82 | 71.667,85 | 54.223,10 | 28.562,31 | 18.879,08 | 64.501,40 | 5.730,214 | 2.675,464 | 29.152,52 | .361,493,2 | .087,332,90 | 4.463,942 |
| 71 | 93.356,36 | 76.403,29 | 50.357,44 | 30.205,47 | 20.189,23 | 65.575,92 | 6.062,570 | 2.867,289 | 30.165,81 | .382,562,6 | .093,712,72 | 4.649,535 |
| 72 | 98.585,35 | 81.362,09 | 46.002,79 | 31.917,45 | 21.567,68 | 66.567,14 | 6.409,082 | 3.069,747 | 31.190,44 | .404,538,3 | .100,458,3 | 4.839,826 |
| 73 | 104.024,8 | 86.549,84 | 41.137,10 | 33.700,07 | 23.016,71 | 67.468,39 | 6.770,139 | 3.283,251 | 32.225,63 | .427,445,9 | .107,585,0 | 5.034,849 |
| 74 | 109.679,9 | 91.972,01 | 35.738,17 | 35.555,14 | 24.538,65 | 68.272,87 | 7.146,134 | 3.508,226 | 33.270,58 | .451,311,2 | .115,108,5 | 5.234,634 |
| 75 | 115.555,5 | 97.634,02 | 29.783,61 | 37.484,51 | 26.135,84 | 68.973,67 | 7.537,464 | 3.745,104 | 34.324,44 | .476,160,3 | .123,044,9 | 5.439,212 |
| 76 | 121.657,0 | 103.541,2 | 23.250,93 | 39.490,03 | 27.810,67 | 69.563,64 | 7.944,531 | 3.994,326 | 35.386,33 | .502,019,9 | .131,411,0 | 5.648,610 |
| 77 | 127.989,4 | 109.698,6 | 16.117,58 | 41.573,58 | 29.565,50 | 70.035,54 | 8.367,742 | 4.256,342 | 36.455,35 | .528,916,7 | .140,223,5 | 5.862,856 |
| 78 | 134.557,9 | 116.111,4 | 8.360,84 | 43.737,05 | 31.402,75 | 70.382,01 | 8.807,509 | 4.531,611 | 37.530,56 | .556,878,0 | .149,500,1 | 6.081,974 |
| 79 | 141.367,7 | 122.784,4 | -.041,97 | 45.982,33 | 33.324,84 | 70.595,51 | 9.264,248 | 4.820,601 | 38.610,96 | .585,931,4 | .159,258,4 | 6.305,989 |
| 80 | 148.423,9 | 129.722,5 | -9.113,60 | 48.311,36 | 35.334,20 | 70.668,42 | 9.738,379 | 5.123,788 | 39.695,55 | .616,104,7 | .169,516,8 | 6.534,924 |
| 81 | 155.731,9 | 136.930,2 | -18.876,88 | 50.726,06 | 37.433,28 | 70.592,91 | 10.230,33 | 5.441,656 | 40.783,28 | .647,426,3 | .180,293,9 | 6.768,800 |
| 82 | 163.296,9 | 144.412,1 | -29.354,43 | 53.228,38 | 39.624,54 | 70.361,09 | 10.740,52 | 5.774,700 | 41.873,07 | .679,924,6 | .191,609,0 | 7.007,636 |
| 83 | 171.124,1 | 152.172,2 | -40.569,05 | 55.820,29 | 41.910,43 | 69.964,90 | 11.269,40 | 6.123,420 | 42.963,79 | .713,628,6 | .203,481,7 | 7.251,450 |
| 84 | 179.218,7 | 160.214,8 | -52.543,29 | 58.503,77 | 44.293,43 | 69.396,21 | 11.817,39 | 6.488,328 | 44.054,30 | .748,567,6 | .215,931,9 | 7.500,260 |
| 85 | 187.586,2 | 168.543,7 | -65.299,73 | 61.280,81 | 46.776,02 | 68.646,77 | 12.384,94 | 6.869,942 | 45.143,41 | .784,771,1 | .228,980,2 | 7.754,079 |
| 86 | 196.231,7 | 177.162,5 | -78.860,70 | 64.153,41 | 49.360,66 | 67.708,14 | 12.972,51 | 7.268,790 | 46.229,89 | .822,269,1 | .242,647,6 | 8.012,921 |
| 87 | 205.160,6 | 186.074,6 | -93.248,54 | 67.123,59 | 52.049,84 | 66.571,85 | 13.580,53 | 7.685,405 | 47.312,51 | .861,092,0 | .256,955,4 | 8.276,800 |
| 88 | 214.376,2 | 195.283,3 | -108.485,2 | 70.193,38 | 54.846,03 | 65.229,35 | 14.209,46 | 8.120,332 | 48.389,95 | .901,270,2 | .271,925,6 | 8.545,724 |
| 89 | 223.889,7 | 204.791,5 | -124.592,7 | 73.364,83 | 57.751,70 | 63.671,89 | 14.859,77 | 8.574,121 | 49.460,92 | .942,834,7 | .287,580,4 | 8.819,703 |
| 90 | 233.700,6 | 214.601,8 | -141.592,6 | 76.639,99 | 60.769,31 | 61.890,70 | 15.531,92 | 9.047,332 | 50.524,02 | .985,816,9 | .303,942,8 | 9.098,743 |

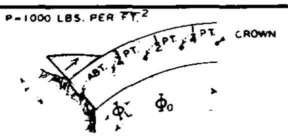
SHEET 1 OF 5

205-D-409

Table H-11.—*Load constants for circular arches—radial and twist deflections* (sheet 2).

UNIT TANGENTIAL LOAD NO. 2

RADIAL AND TWIST DEFLECTIONS

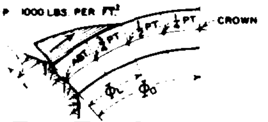


| $\Phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | |
| ° | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term | | 1$^{ST}$ Term | 2$^{ND}$ Term |
| 10 | $.0^{4}$30.204,29 | $.0^{5}$5,028,372 | -.041,440,86 | $.0^{4}$30,204,29 | $.0^{5}$3,724,504 | -.027,672,47 | $.0^{4}$30,204,29 | $.0^{5}$2,413,317 | -.013,851,39 | $.0^{4}$30,204,29 | $.0^{5}$1,097,995 | -$.0^{5}$3,954 |
| 11 | $.0^{4}$44.221.67 | $.0^{5}$8,090,191 | -.055.131,26 | $.0^{4}$44,221,67 | $.0^{5}$5,995,058 | -.036,827,00 | $.0^{4}$44,221,67 | $.0^{5}$3,885,931 | -.018,437,92 | $.0^{4}$44,221,67 | $.0^{5}$1,768,226 | -$.0^{5}$6,360 |
| 12 | $.0^{4}$62.630.36 | $.0^{4}$12,486,14 | -.071.537.67 | $.0^{4}$62.630,36 | $.0^{5}$9.257.101 | -.047,804,28 | $.0^{4}$62,630,36 | $.0^{5}$6,002.687 | -.023.939,87 | $.0^{4}$62.630.36 | $.0^{5}$2,731,820 | -$.0^{5}$9,836 |
| 13 | $.0^{4}$86.263.76 | $.0^{4}$18.609,04 | -.090.901.73 | $.0^{4}$86,263.76 | $.0^{4}$13,803.89 | -.060.769.00 | $.0^{4}$86,263.76 | $.0^{5}$8,954,334 | -.030,440,80 | $.0^{4}$86,263,76 | $.0^{5}$4,075,975 | -$.0^{4}$14,676 |
| 14 | $.0^{3}$116.027 7 | $.0^{4}$26.921,01 | -.113,463.8 | $.0^{3}$116,027.7 | $.0^{4}$19,981.04 | -.075.885,61 | $.0^{3}$116,027,7 | $.0^{4}$12,966.54 | -.038,024,36 | $.0^{3}$116,027,7 | $.0^{5}$5,903.662 | -$.0^{4}$21,259 |
| 15 | $.0^{3}$152.900.5 | $.0^{4}$37.958.56 | -.139,462.6 | $.0^{3}$152,900.5 | $.0^{4}$28,190,62 | -.093,318,25 | $.0^{3}$152,900.5 | $.0^{4}$18.301.96 | -.046,774,28 | $.0^{3}$152,900.5 | $.0^{5}$8,334,923 | -$.0^{4}$30,012 |
| 16 | $.0^{3}$197.932.8 | $.0^{4}$52.337,63 | -.169,135.5 | $.0^{3}$197,932,8 | $.0^{4}$38.895,17 | -.113.230,8 | $.0^{3}$197,932,8 | $.0^{4}$25.263,22 | -.056,774,39 | $.0^{3}$197,932,8 | $.0^{4}$11,508,18 | -$.0^{4}$41,426 |
| 17 | $.0^{3}$252.247,6 | $.0^{4}$70.758,38 | -.202.717.9 | $.0^{3}$252.247.6 | $.0^{4}$52,621.72 | -.135.786.7 | $.0^{3}$252,247,6 | $.0^{4}$34.195,66 | -.068.108,68 | $.0^{3}$252,247,6 | $.0^{4}$15,581,54 | -$.0^{4}$56,108 |
| 18 | $.0^{3}$317.040.3 | $.0^{4}$94,009.81 | -.240,443.6 | $.0^{3}$317.040,3 | $.0^{4}$69,965.64 | -.161.149.1 | $.0^{3}$317,040,3 | $.0^{4}$45.490,09 | -.080,861.18 | $.0^{3}$317,040,3 | $.0^{4}$20.734,09 | -$.0^{4}$74,658 |
| 19 | $.0^{3}$393,578.7 | $.0^{3}$122,974.3 | -.282,544,3 | $.0^{3}$393,578,7 | $.0^{4}$91,594.45 | -.189,481.0 | $.0^{3}$393,578,7 | $.0^{4}$59.585,48 | -.095,116,07 | $.0^{3}$393,578,7 | $.0^{4}$27,167,18 | -$.0^{4}$97,827 |
| 20 | $.0^{3}$483.202.9 | $.0^{3}$158.631.7 | -.329,249,8 | $.0^{3}$483,202,9 | $.0^{3}$118,251,6 | -.220.944,5 | $.0^{3}$483,202,9 | $.0^{4}$76.971.58 | -.110,957.7 | $.0^{3}$483,202,9 | $.0^{4}$35,105,75 | -$.0^{3}$126,42 |
| 21 | $.0^{3}$587 325.1 | $.0^{3}$202,063,6 | -.380.787,5 | $.0^{3}$587,325,1 | $.0^{3}$150,760.1 | -.255.701,7 | $.0^{3}$587,325,1 | $.0^{4}$98,191,77 | -.128,470,5 | $.0^{3}$587,325,1 | $.0^{4}$44,799,56 | -$.0^{3}$161,33 |
| 22 | $.0^{3}$707.431 1 | $.0^{3}$254.457.4 | -.437,382.9 | $.0^{3}$707,431.1 | $.0^{3}$190.026,4 | -.293.914.1 | $.0^{3}$707,431,1 | $.0^{3}$123,845,7 | -.147,739.0 | $.0^{3}$707,431 1 | $.0^{4}$56.524.65 | -$.0^{3}$203.57 |
| 23 | $.0^{3}$845.075.3 | $.0^{3}$317.108,5 | -.499.258.7 | $.0^{3}$845.075,3 | $.0^{3}$237.042,5 | -.335.742.6 | $.0^{3}$845,075,3 | $.0^{3}$154,591,1 | -.168,847.9 | $.0^{3}$845,075,3 | $.0^{4}$70,584,15 | -$.0^{3}$254,21 |
| 24 | .001.001,890 | $.0^{3}$391,426,4 | -.566.635.5 | .001,001.890 | $.0^{3}$292,891.8 | -.381,348.0 | .001,001.890 | $.0^{3}$191,148,2 | -.191,882.4 | .001.001.890 | $.0^{4}$87,310,42 | -$.0^{3}$314,47 |
| 25 | .001.179.575 | $.0^{3}$478.935,3 | -.639.730.8 | .001,179,575 | $.0^{3}$358,750,1 | -.430.890.1 | .001,179,575 | $.0^{3}$234,300,4 | -.216,927,2 | .001,179,575 | $.0^{3}$107,065,4 | -$.0^{3}$385,64 |
| 26 | .001.379.903 | $.0^{3}$581,277,7 | -.718.759.9 | .001,379.903 | $.0^{3}$435,889,4 | -.484,528,4 | .001,379,903 | $.0^{3}$284,897,3 | -.244,067,7 | .001,379,903 | $.0^{3}$130,242,4 | -$.0^{3}$469,13 |
| 27 | .001.604.722 | $.0^{3}$700,218.1 | -.803,934.9 | .001,604.722 | $.0^{3}$525.681,9 | -.542,421,9 | .001,604,722 | $.0^{3}$343,858,1 | -.273,389,2 | .001,604,722 | $.0^{3}$157,267,4 | -$.0^{3}$566,50 |
| 28 | .001.855,949 | $.0^{3}$837.644.0 | -.895,465.1 | .001,855,949 | $.0^{3}$629.601,5 | -.604,728.8 | .001,855,949 | $.0^{3}$412,173,0 | -.304,977,3 | .001,855,949 | $.0^{3}$188,600,1 | -$.0^{3}$679,39 |
| 29 | .002.135,574 | $.0^{3}$995,568,7 | -.993,556.6 | .002,135,574 | $.0^{3}$749,227,5 | -.671,608.7 | .002,135,574 | $.0^{3}$490,906,2 | -.338,917,8 | .002,135,574 | $.0^{3}$224,735,2 | -$.0^{3}$809,59 |
| 30 | .002,445,660 | .001,176.133 | -1.098,412 | .002,445,660 | $.0^{3}$886,247,4 | -.743,212,9 | .002,445,660 | $.0^{3}$581,197,7 | -.375,296,6 | .002,445,660 | $.0^{3}$266,203,7 | -$.0^{3}$959,031 |
| 31 | .002,788,341 | .001,381,606 | -1.210,232 | .002,788,341 | .001,042,458 | -.819,703,5 | .002,788,341 | $.0^{3}$684,266,6 | -.414,200,0 | .002,788,341 | $.0^{3}$313,574,4 | -.001,129,76 |
| 32 | .003,165,823 | .001,614,388 | -1.329,212 | .003,165,823 | .001,219,771 | -.901,234,2 | .003,165,823 | $.0^{3}$801,412,3 | -.455,714,0 | .003,165,823 | $.0^{3}$367,454,7 | -.001,323,94 |
| 33 | .003,580,384 | .001,877,011 | -1.455.548 | .003.580,384 | .001,420,212 | -.987,959,9 | .003,580.384 | $.0^{3}$934,017,7 | -.499,925,4 | .003.580.384 | $.0^{3}$428,492,3 | -.001,543,94 |
| 34 | .004,034,376 | .002.172.136 | -1.589,422 | .004.034.376 | .001,645,922 | -1.080,035 | .004,034,376 | .001,083,551 | -.546,921,0 | .004,034,376 | $.0^{3}$497,375,9 | -.001,792,26 |
| 35 | .004,530,220 | .002,502,557 | -1.731,026 | .004.530,220 | .001.899.166 | -1.177,612 | .004,530,220 | .001,251,568 | -.596,787,6 | .004.530,220 | $.0^{3}$574,837,0 | -.002,071,48 |
| 36 | .005.070,410 | .002,871,202 | -1.880,539 | .005,070,410 | .002,182,326 | -1.280,845 | .005,070,410 | .001,439,714 | -.849,612,5 | .005.070,410 | $.0^{3}$661,650,8 | -.002,384,46 |
| 37 | .005,657.514 | .003,281,127 | -2.036,138 | .005,657.514 | .002,497,908 | -1.389,884 | .005,657,514 | .001,649,725 | -.705,483,0 | .005,657.514 | $.0^{3}$758,637,5 | -.002,734,15 |
| 38 | .006.294,169 | .003.735.523 | -2.203,999 | .006,294,169 | .002,848,545 | -1.504,881 | .006,294.169 | .001,883,435 | -.764,487,0 | .006,294,169 | $.0^{3}$866,863,9 | -.003,123,67 |
| 39 | .006.983.084 | .004.237.706 | -2.378.289 | .006,983,084 | .003,236.991 | -1.625,986 | .006,983,084 | .002,142,767 | -.826,712,2 | .006,983,084 | $.0^{3}$986,643,2 | -.003,556,35 |
| 40 | .007.727,042 | .004.791.123 | -2.561.173 | .007,727,042 | .003,666.132 | -1.753,347 | .007.727,042 | .002,429,749 | -.892,246,7 | .007,727,042 | .001,119,538 | -.004,035,59 |
| 41 | .008,528,895 | .005,399.365 | -2.752,814 | .008,528,895 | .004,138,992 | -1.887,113 | .008,528,895 | .002,746,509 | -961,179,1 | .008,528,895 | .001,266,361 | -.004,565,19 |
| 42 | .009,391,569 | .006.066.080 | -2.953,366 | .009,391,569 | .004,658,679 | -2.027,431 | .009,391.569 | .003,095,258 | -1.033,598 | .009,391,569 | .001,428,176 | -.005,148,86 |
| 43 | .010,318.06 | .006.795,130 | -3.162,981 | .010,318,06 | .005,228,493 | -2.174,447 | .010,318,06 | .003,478,339 | -1.109,592 | .010,318,06 | .001,606,098 | -.005,790,67 |
| 44 | .011,311,44 | .007.590.447 | -3.381.806 | .011.311.44 | .005,851,830 | -2.328,305 | .011,311,44 | .003,898,183 | -1.189,250 | .011,311,44 | .001,801,296 | -.006,494,89 |
| 45 | .012,374,84 | .008,456,082 | -3.609,982 | .012,374,84 | .006,532,222 | -2.489,150 | .012,374,84 | .004,357,333 | -1.272,662 | .012,374,84 | .002,014,994 | -.007,266,02 |
| 46 | .013,511,49 | .009.396.201 | -3.847,647 | .013,511,49 | .007,273,333 | -2.657,125 | .013,511,49 | .004,858,436 | -1.359,918 | .013,511,49 | .002,248.471 | -.008,108,5 |
| 47 | .014,724,65 | .010,415,08 | -4.094,933 | .014,724,65 | .008,078,957 | -2.832,371 | .014,724,65 | .005,404,254 | -1.451,106 | .014,724,65 | .002,503,064 | -.009,027,3 |
| 48 | .016,017.68 | .011,517,10 | -4.351,966 | .016,017.68 | .008,953,022 | -3.015,028 | .016,017,68 | .005,997,658 | -1.546,319 | .016,017,68 | .002,760,168 | -.010,027,5 |
| 49 | .017,394,02 | .012.706.73 | -4.618,868 | .017,394,02 | .009,899.586 | -3.205,236 | .017,394,02 | .006,641,632 | -1,645,645 | .017,394,02 | .003,081,233 | -.011,114,3 |
| 50 | .018,857.16 | .013,988,56 | -4.895,755 | .018,857,16 | .010,922,84 | -3,403,133 | .018.857,16 | .007,339,277 | -1.749,176 | .018,857,16 | .003,407,779 | -.012,293,2 |
| 51 | .020,410.66 | .015,367.25 | -5.182,737 | .020,410,66 | .012,027,09 | -3.608,855 | .020,410,66 | .008,093,806 | -1.857,002 | .020,410,66 | .003,761,375 | -.013,569,7 |
| 52 | .022,058.17 | .016,847,54 | -5.479,919 | .022,058.17 | .013,216.79 | -3.822,539 | .022,058,17 | .008,908,552 | -1.969,215 | .022,058,17 | .004,143,664 | -.014,950,3 |
| 53 | .023.803.39 | .018,434,25 | -5.787,401 | .023,803,39 | .014,496,52 | -4.044,317 | .023,803,39 | .009,786,967 | -2.085,907 | .023,803,39 | .004,556,346 | -.016,440,5 |
| 54 | .025,650.11 | .020,132,28 | -6.105,276 | .025,650,11 | .015.870,97 | -4.274,323 | .025.650,11 | .010,732,62 | -2.207.169 | .025,650,11 | .005,001,188 | -.018,047,4 |
| 55 | .027,602,19 | .021,946.60 | -6.433,632 | .027,602,19 | .017,344,97 | -4.512,698 | .027,602,19 | .011,749,21 | -2.333,094 | .027,602,19 | .005,480,023 | -.019,777,4 |
| 56 | .029,663.55 | .023,882,20 | -6.772,551 | .029,663,55 | .018,923,47 | -4.759,540 | .029,663,55 | .012.840,53 | -2.463,773 | .029,663,55 | .005,994,747 | -.021,636,6 |
| 57 | .031.838.18 | .025,944,17 | -7.122,106 | .031,838,18 | .020,611,55 | -5.015,010 | .031,838,18 | .014,010,54 | -2.599,300 | .031,838,18 | .006,547,340 | -.023,633,3 |
| 58 | .034,130,14 | .028.137.60 | -7.482,369 | .034.130,14 | .022,414,39 | -5.279,224 | .034,130,14 | .015,263,28 | -2.739,767 | .034,130,14 | .007,139,821 | -.025,774,2 |
| 59 | .036.543,58 | .030,467,63 | -7.853,402 | .036,543,58 | .024,337,30 | -5.552,306 | .036,543,58 | .016,602,95 | -2.885,268 | .036,543,58 | .007,774,319 | -.028,067,4 |
| 60 | .039.082,71 | .032.939.44 | -8.235,260 | .039,082,71 | .026,385.71 | -5.834,380 | .039.082,71 | .018,033,84 | -3.035,896 | .039,082,71 | .008,452,995 | -.030,520,7 |
| 61 | .041.751.80 | .035.558.19 | -8.627.997 | .041,751,80 | .028.565.15 | -6.125,569 | .041.751.80 | .019.560,40 | -3.191,745 | .041.751,80 | .009,178,104 | -.033,142.0 |
| 62 | .044.555.20 | .038.329.08 | -9.031.653 | .044.555.20 | .030.881.28 | -6.425.991 | .044.555.20 | .021.187.20 | -3.352.910 | .044.555.20 | .009.951.977 | -.035.940.1 |
| 63 | .047,497.33 | .041.257.28 | -9.446.265 | .047,497.33 | .033,339,83 | -6.735.767 | .047,497.33 | .022,918,90 | -3.519,483 | .047,497.33 | .010.777.01 | -.038,923.5 |
| 64 | .050.582.69 | .044.347.97 | -9.871,864 | .050.582.69 | .035.946.67 | -7.055,012 | .050.582,69 | .024.760.34 | -3.691,562 | .050,582,69 | .011.655.67 | -.042,101,4 |
| 65 | .053.815.83 | .047.606.29 | -10.308.47 | .053.815.83 | .038,707,76 | -7.383.841 | .053.815.83 | .026,716,47 | -3.869.239 | .053.815.83 | .012.590.53 | -.045,483,2 |
| 66 | .057.201.39 | .051.037.34 | -10.756.11 | .057.201.39 | .041,629.14 | -7.722,368 | .057,201,39 | .028,792.34 | -4.052.610 | .057,201,39 | .013.584.19 | -.049,078,0 |
| 67 | .060.744.07 | .054,646.21 | -11.214,77 | .060.744.07 | .044.716.97 | -8.070,702 | .060,744,07 | .030,993,18 | -4.241.772 | .060.744,07 | .014,639.39 | -.052,895,8 |
| 68 | .064.448.64 | .058,437.90 | -11.684,48 | .064,448.64 | .047.977.48 | -8.428,954 | .064,448,64 | .033,324,30 | -4.436.819 | .064,448,64 | .015,758,88 | -.056,947,4 |
| 69 | .068.319.95 | .062,417,34 | -12.165.22 | .068,319.95 | .051,416,99 | -8.797,231 | .068,319,95 | .035,791,16 | -4.637,848 | .068,319,95 | .016,945,56 | -.061,242,5 |
| 70 | .072.362.90 | .066,589.41 | -12.656.97 | .072.362.90 | .055,041,89 | -9.175,639 | .072,362,90 | .038,399,36 | -4.844.955 | .072,362,90 | .018,202,36 | -.065,792,2 |
| 71 | .076.582,48 | .070.958.88 | -13.159.72 | .076,582,48 | .058.858,67 | -9.564,279 | .076,582,48 | .041,154,63 | -5.058,237 | .076,582,48 | .019,532,31 | -.070,607,5 |
| 72 | .080.983.75 | .075.530,42 | -13,673,44 | .080,983,75 | .062,873,87 | -9.963,253 | .080,983,75 | .044.062,80 | -5.277.790 | .080.983.75 | .020,938,55 | -.075,699,8 |
| 73 | .085.571,84 | .080.308.58 | -14.198.10 | .085,571,84 | .067.094,11 | -10.372,66 | .085,571,84 | .047.129,85 | -5.503.713 | .085.571,84 | .022,424,25 | -.081,081,0 |
| 74 | .090.351.91 | .085.297.78 | -14.733.65 | .090.351,91 | .071.526,04 | -10.792,60 | .090,351,91 | .050.361,89 | -5.736,101 | .090,351,91 | .023,992,70 | -.086.762,6 |
| 75 | .095.329.25 | .090.502.31 | -15.280.04 | .095,329,25 | .076,176,40 | -11.223.16 | .095,329,25 | .053,765,16 | -5.975,053 | .095,329.25 | .025,647,29 | -.092.757,3 |
| 76 | .100.509.2 | .095.926.29 | -15.837,21 | .100.509,2 | .081,051,97 | -11.664.44 | .100,509,2 | .057,346,00 | -6.220.668 | .100,509,2 | .027,391,45 | -.099,077,7 |
| 77 | .105.897.1 | 101,573,7 | -16.405.10 | .105.897.1 | .086,159,58 | -12.116,52 | .105,897.1 | .061,110,94 | -6.473.042 | .105,897,1 | .029.228,76 | -.105,736,7 |
| 78 | .111.498.6 | .107.448,3 | -16.983.63 | .111,498,6 | .091,506,06 | -12.579,49 | .111.498.6 | .065,066.56 | -6.732.275 | .111,498.6 | .031,162,82 | -.112,747,8 |
| 79 | .117.319.0 | .113.553,6 | -17.572,72 | .117.319,0 | .097,098,33 | -13.053,45 | .117,319.0 | .069,219,62 | -6.998.465 | .117,319,0 | .033,197,36 | -.120,124,3 |
| 80 | .123,364.0 | .119.893,1 | -18.172,28 | .123,364,0 | .102,943,3 | -13.538,47 | .123,364,0 | .073,576,99 | -7.271,711 | .123,364,0 | .035,336,22 | -.127,880,6 |
| 81 | .129.639,4 | .126,469,9 | -18.782.21 | .129,639,4 | .109,047,9 | -14.034,62 | .129.639.4 | .078,145,65 | -7.552,111 | .129,639,4 | .037,583,28 | -.136,030,8 |
| 82 | .136.150.8 | .133.296,9 | -19.402,40 | .136,150,8 | .115,419,0 | -14.542,00 | .136,150,8 | .082,932,74 | -7.839,768 | .136.150.8 | .039.942,55 | -.144,589.5 |
| 83 | .142.904.1 | .140.346,7 | -20.032,74 | .142,904,1 | .122.063,7 | -15.060,66 | .142.904.1 | .087,945,50 | -8.134,778 | .142.904.1 | .042.418,13 | -.153,571,8 |
| 84 | .149.905.2 | .147.651.8 | -20.673,10 | .149,905,2 | .128.988.9 | -15.590,70 | .149,905,2 | .093,191,26 | -8.437,243 | .149,905,2 | .045.014,18 | -.162,993.3 |
| 85 | .157.159.9 | .155,204,2 | -21.323.35 | .157,159,9 | .136.201,5 | -16.132,16 | .157,159,9 | .098,677,55 | -8.747,262 | .157,159,9 | .047,735,01 | -.172,869,2 |
| 86 | .164.674.5 | .163.005,8 | -21.983.35 | .164,674,5 | .143,708,4 | -16.685,13 | .164,674.5 | .104,411,9 | -9.064,934 | .164,674.5 | .050.584,98 | -.183,216.1 |
| 87 | .172.454.9 | .171.058,0 | -22.652.95 | .172,454,9 | .151,516,6 | -17.249,67 | .172,454,9 | .110,402,2 | -9.390.363 | .172,454,9 | .053.568,56 | -.194,050,3 |
| 88 | .180.507.3 | .179.362,1 | -23.331,99 | .180,507,3 | .159,632,9 | -17.825,83 | .180,507,3 | .116,656,1 | -9.723,645 | .180,507,3 | .056,690,37 | -.205,388,7 |
| 89 | .188.838.0 | .187.918.8 | -24.020.31 | .188,838,0 | .168,064,2 | -18.413,67 | .188,838,0 | .123,181,6 | -10.064,89 | .188,838,0 | .059,954,93 | -.217,248,1 |
| 90 | .197.453.2 | .196.728.7 | -24.717.73 | .197,453,2 | .176,817,1 | -19.013,25 | .197,453,2 | .129,986,8 | -10.414,18 | .197,453,2 | .063,367,16 | -.229,646,5 |

Table H-11.—*Load constants for circular arches—radial and twist deflections* (sheet 3).

UNIT TANGENTIAL LOAD NO. 3

RADIAL AND TWIST DEFLECTIONS



| $\Phi$ | Crown | | | ¼ Point | | | ½ Point | | | ¾ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | |
| ° | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term | | 1ST Term | 2ND Term |
| 10 | $.0^3483,2029$ | $.0^476,971,58$ | -.110,957,7 | $.0^3483,2029$ | $.0^456,092,05$ | -.055,594,95 | $.0^3483,2029$ | $.0^435,105,75$ | $-.0^3,126,424$ | $.0^490,611,49$ | $.0^53,183,161$ | .006,912,084 |
| 11 | $.0^3,707,431,1$ | $.0^3,123,845,7$ | -.147,739,0 | $.0^3,707,431,1$ | $.0^490,289,13$ | -.074,056,56 | $.0^3,707,431,1$ | $.0^456,524,65$ | $-.0^3,203,57$ | $.0^3,132,626,6$ | $.0^55,127,808$ | .009,197,038 |
| 12 | .001,001,890 | $.0^3,191,148,2$ | -.191,882,4 | .001,001,890 | $.0^3,139,420,4$ | -.096,230,30 | .001,001,890 | $.0^487,310,42$ | $-.0^3,314,47$ | $.0^3,187,887,0$ | $.0^57,922,192$ | .011,936,07 |
| 13 | .001,379,903 | $.0^3,284,897,3$ | -.244,067,7 | .001,379,903 | $.0^3,207,904,2$ | -.122,465,4 | .001,379,903 | $.0^3,130,242,4$ | $-.0^3,469,13$ | $.0^3,258,784,7$ | $.0^411,820,18$ | .015,169,88 |
| 14 | .001,855,949 | $.0^3,412,173,0$ | -.304,977,3 | .001,855,949 | $.0^3,300,947,9$ | -.153,113,9 | .001,855,949 | $.0^3,188,600,1$ | $-.0^3,679,39$ | $.0^3,348,072,9$ | $.0^417,120,37$ | .018,939,00 |
| 15 | .002,445,660 | $.0^3,581,197,7$ | -.375,296,6 | .002,445,660 | $.0^3,424,609,9$ | -.188,531,5 | .002,445,660 | $.0^3,266,203,7$ | $-.0^3,959,031$ | $.0^3,458,686,0$ | $.0^424,170,87$ | .023,283,84 |
| 16 | .003,165,823 | $.0^3,801,412,3$ | -.455,714,0 | .003,165,823 | $.0^3,585,860,7$ | -.229,077,0 | .003,165,823 | $.0^3,367,454,7$ | -.001,323,94 | $.0^3,593,775,4$ | $.0^433,373,08$ | .028,244,58 |
| 17 | .004,034,376 | .001,083,551 | -.546,921,0 | .004,034,376 | $.0^3,792,643,1$ | -.275,113,1 | .004,034,376 | $.0^3,497,375,9$ | -.001,792,26 | $.0^3,756,709,7$ | $.0^445,185,50$ | .033,861,25 |
| 18 | .005,070,410 | .001,439,714 | -.649,612,5 | .005,070,410 | .001,053,931 | -.327,006,5 | .005,070,410 | $.0^3,661,650,8$ | -.002,384,46 | $.0^3,951,074,4$ | $.0^460,127,41$ | .040,173,66 |
| 19 | .006,294,169 | .001,883,435 | -.764,487,0 | .006,294,169 | .001,379,788 | -.385,128,1 | .006,294,169 | $.0^3,866,663,9$ | -.003,123,67 | .001,180,672 | $.0^478,782,70$ | .047,221,42 |
| 20 | .007,727,042 | .002,429,749 | -.892,246,7 | .007,727,042 | .001,781,422 | -.449,853,0 | .007,727,042 | .001,119,538 | -.004,035,59 | .001,449,521 | $.0^3,101,803,6$ | .055,043,89 |
| 21 | .009,391,569 | .003,095,258 | -1.033,598 | .009,391,569 | .002,271,245 | -.521,561,3 | .009,391,569 | .001,428,176 | -.005,148,86 | .001,761,858 | $.0^3,129,914,5$ | .063,680,22 |
| 22 | .011,311,44 | .003,898,183 | -1.189,250 | .011,311,44 | .002,862,920 | -.600,637,7 | .011,311,44 | .001,801,296 | -.006,494,89 | .002,122,136 | $.0^3,163,915,5$ | .073,169,30 |
| 23 | .013,511,49 | .004,858,436 | -1.359,918 | .013,511,49 | .003,571,422 | -.687,471,9 | .013,511,49 | .002,248,471 | -.008,108,46 | .002,535,024 | $.0^3,204,686,0$ | .083,549,72 |
| 24 | .016,017,68 | .005,997,658 | -1.546,319 | .016,017,68 | .004,413,088 | -.782,459,3 | .016,017,68 | .002,780,168 | -.010,027,5 | .003,005,407 | $.0^3,253,189,3$ | .094,859,85 |
| 25 | .018,857,16 | .007,339,277 | -1.749,176 | .018,857,16 | .005,405,657 | -.886,000,4 | .018,857,16 | .003,407,779 | -.012,293,2 | .003,538,389 | $.0^3,310,475,2$ | .107,137,7 |
| 26 | .022,058,17 | .008,908,552 | -1.969,215 | .022,058,17 | .006,568,329 | -.998,501,2 | .022,058,17 | .004,143,664 | -.014,950,3 | .004,139,288 | $.0^3,377,683,9$ | .120,421,1 |
| 27 | .025,650,11 | .010,732,62 | -2.207,169 | .025,650,11 | .007,921,818 | -1.120,374 | .025,650,11 | .005,001,188 | -.018,047,4 | .004,813,636 | $.0^3,456,050,7$ | .134,747,5 |
| 28 | .029,663,55 | .012,840,53 | -2.463,773 | .029,663,55 | .009,488,361 | -1.252,037 | .029,663,55 | .005,994,747 | -.021,636,8 | .005,567,188 | $.0^3,546,908,2$ | .150,153,9 |
| 29 | .034,130,14 | .015,263,28 | -2.739,767 | .034,130,14 | .011,291,83 | -1.393,915 | .034,130,14 | .007,139,821 | -.025,774,2 | .006,405,909 | $.0^3,651,690,9$ | .166,677,1 |
| 30 | .039,082,71 | .018,033,84 | -3.035,896 | .039,082,71 | .013,357,70 | -1.546,438 | .039,082,71 | .008,452,995 | -.030,520,7 | .007,335,982 | $.0^3,771,938,8$ | .184,353,6 |
| 31 | .044,555,20 | .021,187,20 | -3.352,910 | .044,555,20 | .015,713,11 | -1.710,045 | .044,555,20 | .009,951,977 | -.035,940,1 | .008,363,808 | $.0^3,909,300,4$ | .203,219,5 |
| 32 | .050,582,69 | .024,760,34 | -3.691,562 | .050,582,69 | .018,386,95 | -1.885,178 | .050,582,69 | .011,655,67 | -.042,101,4 | .009,495,999 | .001,065,537 | .223,310,4 |
| 33 | .057,201,39 | .028,792,34 | -4.052,610 | .057,201,39 | .021,409,83 | -2.072,289 | .057,201,39 | .013,584,19 | -.049,078,0 | .010,739,39 | .001,242,526 | .244,661,5 |
| 34 | .064,448,64 | .033,324,30 | -4.436,819 | .064,448,64 | .024,814,15 | -2.271,837 | .064,448,64 | .015,758,88 | -.056,947,4 | .012,101,01 | .001,442,265 | .267,308,0 |
| 35 | .072,362,90 | .038,399,36 | -4.844,955 | .072,362,90 | .028,634,12 | -2.484,287 | .072,362,90 | .018,202,36 | -.065,792,2 | .013,588,14 | .001,666,874 | .291,284,1 |
| 36 | .080,983,75 | .044,062,80 | -5.277,790 | .080,983,75 | .032,905,80 | -2.710,111 | .080,983,75 | .020,938,55 | -.075,699,8 | .015,208,25 | .001,918,601 | .316,624,0 |
| 37 | .090,351,91 | .050,361,89 | -5.736,101 | .090,351,91 | .037,667,11 | -2.949,790 | .090,351,91 | .023,992,70 | -.086,762,6 | .016,969,03 | .002,199,824 | .343,361,4 |
| 38 | .100,509,2 | .057,346,00 | -6.220,668 | .100,509,2 | .042,957,87 | -3.203,811 | .100,509,2 | .027,391,45 | -.099,077,7 | .018,878,39 | .002,513,053 | .371,529,4 |
| 39 | .111,498,6 | .065,066,56 | -6.732,275 | .111,498,6 | .048,819,84 | -3.472,670 | .111,498,6 | .031,162,82 | -.112,748 | .020,944,44 | .002,860,939 | .401,160,7 |
| 40 | .123,364,0 | .073,576,99 | -7.271,711 | .123,364,0 | .055,296,69 | -3.756,871 | .123,364,0 | .035,336,22 | -.127,881 | .023,175,52 | .003,246,272 | .432,287,6 |
| 41 | .136,150,8 | .082,932,74 | -7.839,768 | .136,150,8 | .062,434,05 | -4.056,925 | .136,150,8 | .039,942,55 | -.144,589 | .025,580,19 | .003,671,986 | .464,941,9 |
| 42 | .149,905,2 | .093,191,26 | -8.437,243 | .149,905,2 | .070,279,56 | -4.373,350 | .149,905,2 | .045,014,18 | -.162,993 | .028,167,21 | .004,141,165 | .499,155,0 |
| 43 | .164,674,5 | .104,411,9 | -9.064,934 | .164,674,5 | .078,882,82 | -4.706,676 | .164,674,5 | .050,584,98 | -.183,216 | .030,945,54 | .004,657,041 | .534,957,4 |
| 44 | .180,507,3 | .116,656,1 | -9.723,645 | .180,507,3 | .088,295,47 | -5.057,436 | .180,507,3 | .056,690,37 | -.205,389 | .033,924,40 | .005,223,003 | .572,379,6 |
| 45 | .197,453,2 | .129,986,8 | -10.414,18 | .197,453,2 | .098,570,99 | -5.426,177 | .197,453,2 | .063,367,16 | -.229,646 | .037,113,17 | .005,842,599 | .611,451,1 |
| 46 | .215,562,8 | .144,469,1 | -11.137,35 | .215,562,8 | .109,765,1 | -5.813,449 | .215,562,8 | .070,654,03 | -.256,132 | .040,521,49 | .006,519,536 | .652,201,2 |
| 47 | .234,888,0 | .160,169,9 | -11.893,97 | .234,888,0 | .121,935,6 | -6.219,813 | .234,888,0 | .078,591,05 | -.284,993 | .044,159,19 | .007,257,689 | .694,658,8 |
| 48 | .255,481,5 | .177,157,5 | -12.684,84 | .255,481,5 | .135,141,9 | -6.645,839 | .255,481,5 | .087,219,96 | -.316,383 | .048,036,32 | .008,061,098 | .738,851,4 |
| 49 | .277,397,5 | .195,502,1 | -13.510,79 | .277,397,5 | .149,445,9 | -7.092,106 | .277,397,5 | .096,584,16 | -.350,464 | .052,163,13 | .008,933,976 | .784,806,7 |
| 50 | .300,691,0 | .215,275,4 | -14.372,63 | .300,691,0 | .164,911,1 | -7.559,198 | .300,691,0 | .106,728,7 | -.387,403 | .056,550,10 | .009,880,711 | .832,552,5 |
| 51 | .325,418,0 | .236,550,6 | -15.271,18 | .325,418,0 | .181,603,4 | -8.047,712 | .325,418,0 | .117,700,3 | -.427,371 | .061,207,91 | .010,905,87 | .882,113,8 |
| 52 | .351,635,9 | .259,402,3 | -16.207,25 | .351,635,9 | .199,590,4 | -8.558,248 | .351,635,9 | .129,547,5 | -.470,548 | .066,147,46 | .012,014,20 | .933,517,0 |
| 53 | .379,402,9 | .283,906,6 | -17.181,68 | .379,402,9 | .218,941,8 | -9.091,422 | .379,402,9 | .142,320,3 | -.517,124 | .071,379,86 | .013,210,63 | .986,786,9 |
| 54 | .408,778,5 | .310,140,6 | -18.195,27 | .408,778,5 | .239,729,4 | -9.647,852 | .408,778,5 | .156,070,7 | -.567,288 | .076,916,45 | .014,500,29 | 1.041,948 |
| 55 | .439,823,1 | .338,182,7 | -19.248,86 | .439,823,1 | .262,026,7 | -10.228,17 | .439,823,1 | .170,852,4 | -.621,243 | .082,768,74 | .015,888,48 | 1.099,024 |
| 56 | .472,598,3 | .368,112,4 | -20.343,25 | .472,598,3 | .285,909,3 | -10.833,01 | .472,598,3 | .186,720,8 | -.679,195 | .088,948,48 | .017,380,71 | 1.158,039 |
| 57 | .507,166,4 | .400,010,0 | -21.479,27 | .507,166,4 | .311,454,7 | -11.463,02 | .507,166,4 | .203,733,1 | -.741,358 | .095,467,64 | .018,982,69 | 1.219,015 |
| 58 | .543,591,4 | .433,956,8 | -22.657,73 | .543,591,4 | .338,742,2 | -12.118,86 | .543,591,4 | .221,948,2 | -.807,952 | .102,338,4 | .020,700,30 | 1.281,972 |
| 59 | .581,937,8 | .470,034,6 | -23.879,45 | .581,937,8 | .367,853,1 | -12.801,18 | .581,937,8 | .241,427,2 | -.879,205 | .109,573,1 | .022,539,67 | 1.346,934 |
| 60 | .622,271,5 | .508,326,0 | -25.145,24 | .622,271,5 | .398,870,4 | -13.510,66 | .622,271,5 | .262,232,4 | -.955,352 | .117,184,4 | .024,507,11 | 1.413,920 |
| 61 | .664,659,3 | .548,914,1 | -26.455,91 | .664,659,3 | .431,878,9 | -14.247,98 | .664,659,3 | .284,428,6 | -1.036,637 | .125,185,0 | .026,609,13 | 1.482,949 |
| 62 | .709,169,0 | .591,882,0 | -27.812,27 | .709,169,0 | .466,965,3 | -15.013,84 | .709,169,0 | .308,081,9 | -1.123,310 | .133,588,0 | .028,852,47 | 1.554,041 |
| 63 | .755,869,6 | .637,313,5 | -29.215,11 | .755,869,6 | .504,217,8 | -15.808,91 | .755,869,6 | .333,260,6 | -1.215,626 | .142,406,6 | .031,244,09 | 1.627,215 |
| 64 | .804,831,0 | .685,232,1 | -30.665,24 | .804,831,0 | .543,726,4 | -16.633,91 | .804,831,0 | .360,034,6 | -1.313,852 | .151,654,2 | .033,791,15 | 1.702,487 |
| 65 | .856,124,2 | .735,901,3 | -32.163,44 | .856,124,2 | .585,582,8 | -17.489,56 | .856,124,2 | .388,476,0 | -1.418,257 | .161,344,5 | .036,501,06 | 1.779,874 |
| 66 | .909,821,2 | .789,224,3 | -33.710,50 | .909,821,2 | .629,879,9 | -18.376,56 | .909,821,2 | .418,658,3 | -1.529,123 | .171,491,3 | .039,381,40 | 1.859,393 |
| 67 | .965,995,1 | .845,344,2 | -35.307,19 | .965,995,1 | .676,712,8 | -19.295,65 | .965,995,1 | .450,657,4 | -1.646,737 | .182,108,6 | .042,440,05 | 1.941,058 |
| 68 | 1.024,720 | .904,343,2 | -36.954,30 | 1.024,720 | .726,177,4 | -20.247,57 | 1.024,720 | .484,550,7 | -1.771,394 | .193,210,9 | .045,685,05 | 2.024,885 |
| 69 | 1.086,071 | .966,303,0 | -38.652,59 | 1.086,071 | .778,371,5 | -21.233,06 | 1.086,071 | .520,417,6 | -1.903,397 | .204,812,5 | .049,124,73 | 2.110,886 |
| 70 | 1.150,124 | 1.031,304 | -40.402,80 | 1.150,124 | .833,393,8 | -22.252,86 | 1.150,124 | .558,339,4 | -2.043,057 | .216,928,0 | .052,767,63 | 2.199,075 |
| 71 | 1.216,957 | 1.099,427 | -42.205,69 | 1.216,957 | .891,344,8 | -23.307,73 | 1.216,957 | .598,399,2 | -2.190,689 | .229,572,5 | .056,622,51 | 2.289,484 |
| 72 | 1.286,646 | 1.170,749 | -44.061,99 | 1.286,646 | .952,325,8 | -24.398,45 | 1.286,646 | .640,682,2 | -2.346,623 | .242,761,0 | .060,698,42 | 2.382,063 |
| 73 | 1.359,273 | 1.245,348 | -45.972,43 | 1.359,273 | 1.016,440 | -25.525,78 | 1.359,273 | .685,275,2 | -2.511,194 | .256,508,9 | .065,004,61 | 2.476,884 |
| 74 | 1.434,916 | 1.323,300 | -47.937,71 | 1.434,916 | 1.083,790 | -26.690,49 | 1.434,916 | .732,267,1 | -2.684,737 | .270,831,5 | .069,550,58 | 2.573,936 |
| 75 | 1.513,657 | 1.404,677 | -49.958,55 | 1.513,657 | 1.154,481 | -27.893,38 | 1.513,657 | .781,748,6 | -2.867,609 | .285,744,8 | .074,346,11 | 2.873,227 |
| 76 | 1.595,579 | 1.489,553 | -52.035,63 | 1.595,579 | 1.228,620 | -29.135,23 | 1.595,579 | .833,812,1 | -3.060,165 | .301,264,5 | .079,401,21 | 2.774,766 |
| 77 | 1.680,763 | 1.577,996 | -54.169,63 | 1.680,763 | 1.306,311 | -30.416,83 | 1.680,763 | .888,552,2 | -3.262,769 | .317,406,9 | .084,726,15 | 2.878,560 |
| 78 | 1.769,295 | 1.670,073 | -56.361,20 | 1.769,295 | 1.387,664 | -31.739,01 | 1.769,295 | .946,065,0 | -3.475,801 | .334,188,3 | .090,331,44 | 2.984,613 |
| 79 | 1.861,259 | 1.765,851 | -58.611,00 | 1.861,259 | 1.472,785 | -33.102,54 | 1.861,259 | 1.006,449 | -3.699,637 | .351,625,2 | .096,227,87 | 3.092,932 |
| 80 | 1.956,741 | 1.865,389 | -60.919,64 | 1.956,741 | 1.561,783 | -34.508,26 | 1.956,741 | 1.069,803 | -3.934,669 | .369,734,4 | .102,426,5 | 3.203,522 |
| 81 | 2.055,829 | 1.968,748 | -63.287,75 | 2.055,829 | 1.654,768 | -35.956,98 | 2.055,829 | 1.136,231 | -4.181,296 | .388,532,8 | .108,938,6 | 3.316,384 |
| 82 | 2.158,610 | 2.075,983 | -65.715,92 | 2.158,610 | 1.751,849 | -37.449,51 | 2.158,610 | 1.205,834 | -4.439,920 | .408,037,7 | .115,775,8 | 3.431,524 |
| 83 | 2.265,174 | 2.187,148 | -68.204,72 | 2.265,174 | 1.853,137 | -38.986,70 | 2.265,174 | 1.278,719 | -4.710,962 | .428,266,4 | .122,949,9 | 3.548,939 |
| 84 | 2.375,609 | 2.302,290 | -70.754,70 | 2.375,609 | 1.958,740 | -40.569,37 | 2.375,609 | 1.354,993 | -4.994,839 | .449,236,4 | .130,473,1 | 3.668,634 |
| 85 | 2.490,008 | 2.421,455 | -73.327,99 | 2.490,008 | 2.068,770 | -42.198,36 | 2.490,008 | 1.434,765 | -5.291,984 | .470,965,5 | .138,357,6 | 3.790,604 |
| 86 | 2.608,462 | 2.544,684 | -76.040,36 | 2.608,462 | 2.183,337 | -43.874,49 | 2.608,462 | 1.518,145 | -5.602,833 | .493,471,8 | .146,616,3 | 3.914,853 |
| 87 | 2.731,064 | 2.672,015 | -78.777,04 | 2.731,064 | 2.302,550 | -45.596,52 | 2.731,064 | 1.605,247 | -5.927,836 | .516,773,4 | .155,262,0 | 4.041,375 |
| 88 | 2.857,906 | 2.803,479 | -81.576,92 | 2.857,906 | 2.426,521 | -47.371,60 | 2.857,906 | 1.696,183 | -6.267,446 | .540,888,8 | .164,308,0 | 4.170,167 |
| 89 | 2.989,085 | 2.939,105 | -84.440,44 | 2.989,085 | 2.555,359 | -49.194,26 | 2.989,085 | 1.791,070 | -6.622,125 | .565,836,4 | .173,767,7 | 4.301,228 |
| 90 | 3.124,695 | 3.078,916 | -87.368,02 | 3.124,695 | 2.689,173 | -51.067,46 | 3.124,695 | 1.890,027 | -6.992,345 | .591,635,1 | .183,655,0 | 4.434,548 |

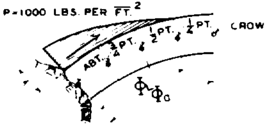
SHEET 3 OF 5

205-D-411

Table H-11.—*Load constants for circular arches—radial and twist deflections* (sheet 4).

## UNIT TANGENTIAL LOAD NO. 4

## RADIAL AND TWIST DEFLECTIONS



| $\Phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | |
| ° | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term |
| 10 | .002,445.660 | .$0^3$371.9765 | -.126,057.5 | .002,445.660 | .$0^3$266.203.7 | -.$0^3$959,031 | .001,127,415 | .$0^3$079.571.0 | .036,653,80 | .$0^3$,110,747,2 | .$0^5$3,879,550 | .009,217,426 |
| 11 | .003,580.384 | .$0^3$598.508.7 | -.168,186.1 | .003,580,384 | .$0^3$428,492.3 | -.001,543,94 | .001,650.568 | .$0^3$,128,118,6 | .048,711,63 | .$0^3$,162,142.9 | .$0^5$6,247.669 | .012,264,84 |
| 12 | .005,070,410 | .$0^3$.923.774,2 | -.218,924.5 | .005.070.410 | .$0^3$661,650.8 | -.002,384,46 | .002.337.568 | .$0^3$,197.896.3 | .063.135.07 | .$0^3$229.639.2 | .$0^5$9.652,315 | .015,918,04 |
| 13 | .006,983,084 | .001.376.860 | -.279,134,1 | .006.983,084 | .$0^3$,986.643,2 | -.003.556.35 | .003,219,492 | .$0^3$.295.203,4 | .080,124,36 | .$0^3$,316,291,7 | .$0^4$14,401.58 | .020,231,39 |
| 14 | .009,391.569 | .001.991.993 | -349,696,5 | .009.391.569 | .001,428,176 | -.005.148,86 | .004,330,108 | .$0^3$427,472,1 | .099.876.11 | .$0^3$425,421.3 | .$0^4$20.859,27 | .025,259,09 |
| 15 | .012,374.84 | .002,808,912 | -431,515,4 | .012,374,84 | .002,014,994 | -.007,266,02 | .005.705,875 | .$0^3$,603,3604 | .122,582,7 | .$0^3$,560,614,5 | .$0^4$29,449,46 | .031,055,12 |
| 16 | .016,017,68 | .003,873,279 | -.525,517,7 | .016.017.68 | .002.780,168 | -.010,027,5 | .007,385,940 | .$0^3$.832,843.0 | .148,432.3 | .$0^3$,725,723,0 | .$0^4$40,661,38 | .037,673,25 |
| 17 | .020,410.66 | .005,236,977 | -.632.655.1 | .020,410.66 | .003,761,375 | -.013,569.7 | .009,412,135 | .001.127.302 | .177,607,9 | .$0^3$,924,863,8 | .$0^4$55,053,48 | .045,167,03 |
| 18 | .025,650.11 | .006,958,505 | -.753,905,5 | .025,650.11 | .005,001,188 | -.018,047,4 | .011.828,97 | .001,499,618 | .210,287,9 | .001,162,419 | .$0^4$73,258,51 | .053,589,77 |
| 19 | .031,838.18 | .009,103,312 | -.890,274,5 | .031,838.18 | .006.547,340 | -.023,633,3 | .014,683,65 | .001,964,257 | .246,645,0 | .001,443,036 | .$0^4$95,987.87 | .062,994,50 |
| 20 | .039,082,71 | .011,744,11 | -1.042.798 | .039.082.71 | .008,452,995 | -.030,520,7 | .018,026.03 | .002,537,361 | .286,846.6 | .001.771,627 | .$0^3$,124,036,2 | .073,434,00 |
| 21 | .047,497,33 | .014,961,18 | -1.212,537 | .047,497,33 | .010,777,01 | -.038,923,5 | .021,908,66 | .003,236,835 | .331,053,7 | .002,153,369 | .$0^3$,158,286,1 | .084,960,78 |
| 22 | .057.201.39 | .018.842.68 | -1.400.594 | .057,201.39 | .013,584,19 | -.049,078,0 | .026.386,75 | .004,082,434 | .379,421,4 | .002,593,705 | .$0^3$,199.712,5 | .097,626.91 |
| 23 | .068,319.95 | .023,484,90 | -1.608,094 | .068.319.95 | .016,945,56 | -.061,242,5 | .031,518.16 | .005,095.850 | .432,098,0 | .003,098,340 | .$0^3$,249,386,6 | .111,484,5 |
| 24 | .080.983,75 | .028,992,53 | -1.836,204 | .080.983.75 | .020,938,55 | -.075,699,8 | .037,363,44 | .006,300,790 | .489,225,1 | .003,673,246 | .$0^3$,308,482,3 | .126,584,6 |
| 25 | .095.329.25 | .035,478.95 | -2.086,121 | .095,329.25 | .025,647.29 | -.092,757,3 | .043,985,78 | .007,723,068 | .550.937,3 | .004.324.660 | .$0^3$,378,278,5 | .142,978.8 |
| 26 | .111,498.6 | .043,066,38 | -2.359,082 | .111,498,6 | .031,162,82 | -.112,748 | .051,451,03 | .009,390,686 | .617,361,2 | .005.059,081 | .$0^3$,460,164,4 | .160,717,9 |
| 27 | .129,639.4 | .051,886,15 | -2.656,360 | .129,639,4 | .037.583,28 | -.136.031 | .059.827,67 | .011,333.93 | .688,616,4 | .005.883,274 | .$0^3$,555,645,1 | .179,852,1 |
| 28 | .149,905.2 | .062.078.86 | -2.979.265 | .149,905,2 | .045,014.18 | -.162,993 | .069.186.84 | .013,585,39 | .764,813,9 | .006,804,267 | .$0^3$,666,344,2 | .200,431,6 |
| 29 | .172,454,9 | .073,794,59 | -3.329,147 | .172,454,9 | .053,568,56 | -.194,050 | .079,602,30 | .016,180,14 | .846,056,9 | .007,829,353 | .$0^3$,794,009,5 | .222,506,0 |
| 30 | .197,453,2 | .087,192,98 | -3.707,399 | .197,453.2 | .063,367,16 | -.229,646 | .091,150,48 | .019,155.74 | .932,439,5 | .008,966,090 | .$0^3$,940,517,2 | .246,124,5 |
| 31 | .225,070,2 | .102,443.4 | -4.115,452 | .225,070.2 | .074.538.70 | -.270,256 | .103.910.4 | .022,552,31 | 1.024,047 | .010.222.30 | .001,107,876 | .271,335,9 |
| 32 | .255,481,5 | .119,725,2 | -4.554,779 | .255,481.5 | .087.219.96 | -.316.383 | .117,963.7 | .026,412.66 | 1.120,957 | .011,606.06 | .001,298,231 | .298,188.5 |
| 33 | .288,868.6 | .139,227,5 | -5.026,897 | .288,868.6 | .101,556.0 | -.368,566 | .133,394.6 | .030,782,33 | 1.223.236 | .013,125,72 | .001,513,871 | .326.730,1 |
| 34 | .325,418.0 | .161,149,5 | -5.533.363 | .325,418.0 | .117,700.3 | -.427,371 | .150.290.1 | .035.709.66 | 1.330.940 | .014,789.89 | .001,757.228 | .357.008.0 |
| 35 | .365.322.1 | .185,700.4 | -6.075,782 | .365,322,1 | .135.815.0 | -.493,399 | .168.739.6 | .041,245.90 | 1,444,119 | .016.607,45 | .002,030,887 | .389.069.3 |
| 36 | .408.778,5 | .213,099,6 | -6.655,800 | .408.778.5 | .156,070.7 | -.567,288 | .188,835.3 | .047,445.13 | 1.562.809 | .018.587,53 | .002.337.585 | .422,960.2 |
| 37 | .455,990.4 | .243.576.4 | -7.275,107 | .455.990.4 | .178,647,2 | -.649,707 | .210,671,7 | .054,364,62 | 1.687.036 | .020.739.54 | .002,680,219 | .458.726.6 |
| 38 | .507.166.4 | .277.370.3 | -7.935,439 | .507,166,4 | .203,733.1 | -.741.358 | .234,346.1 | .062,064,62 | 1.816,818 | .023.073.13 | .003.061.850 | .496,413.7 |
| 39 | .562,520,3 | .314.730.8 | -8.638,579 | .562.520,3 | .231.525.9 | -.842,981 | .259,958,4 | .070.608,57 | 1.952,159 | .025,598.22 | .003.485.705 | .536,066,4 |
| 40 | .622,271.5 | .355.317,3 | -9.386.350 | .622,271.5 | .262.232,4 | -.955,352 | .287,610.9 | .080.063.07 | 2,093.054 | .028.325.01 | .003,955.183 | .577.728.7 |
| 41 | .686,644,6 | .401,199.0 | -10.180,63 | .686.644.6 | .296,068,9 | -1.079,284 | .317,408,7 | .090,498,05 | 2.239,485 | .031,263,95 | .004,473,862 | .621,444,3 |
| 42 | .755,869.6 | .450,855,0 | -11.023,33 | .755,869.6 | .333,260,6 | -1.215,626 | .349,459,3 | .101,986,8 | 2.391,425 | .034,425,74 | .005.045,494 | .667,256.3 |
| 43 | .830.181,6 | .505.173.7 | -11.916,41 | .830,181,6 | .374,042.3 | -1.365,265 | .383,872,7 | .114,605.8 | 2.548,829 | .037.821.36 | .005.674.021 | .715,206.7 |
| 44 | .909.821,2 | .564,453,2 | -12.861.89 | .909,821,2 | .418.658,3 | -1.529.123 | .420.761,6 | .128,435.4 | 2.711,649 | .041,462,04 | .006.363,572 | .765.337,7 |
| 45 | .995.034.0 | .629,000.8 | -13.861.81 | .995,034,0 | .467,362.3 | -1.708,167 | .460,241.1 | .143.559,0 | 2.879,817 | .045.359,28 | .007,118,467 | .817.690.1 |
| 46 | 1.086.071 | .699.132.8 | -14.918.28 | 1.086.071 | .520,417.6 | -1.903,397 | .502,428,8 | .160.064.0 | 3.053.255 | .049.524.83 | .007.943,225 | .872.304.5 |
| 47 | 1.183.188 | .775,174,1 | -16.033,43 | 1.183.188 | .578,096,8 | -2.115,857 | .547,445.1 | .178,040.8 | 3.231.873 | .053.970.71 | .008,842,563 | .929.220.8 |
| 48 | 1.286.646 | .857,458,3 | -17.209,45 | 1.286.646 | .640,682.2 | -2.346,623 | .595,412,5 | .197,584,2 | 3.415,570 | .058.709.20 | .009,821,409 | .988,477.7 |
| 49 | 1.396.712 | .946,327.4 | -18.448,58 | 1.396.712 | .708,465,7 | -2.596,820 | .646,456,3 | .218,792,2 | 3.604.226 | .063.752.83 | .010,884,89 | 1.050.114 |
| 50 | 1.513,657 | 1.042,131 | -19.753,08 | 1.513.657 | .781,748,6 | -2.867,609 | .700,704,2 | .241.766,8 | 3.797.711 | .069.114,41 | .012,038,36 | 1.114,168 |
| 51 | 1.637.758 | 1.145,226 | -21.125,28 | 1.637,758 | .860.841.6 | -3.160,188 | .758,286,3 | .266,613.6 | 3.995,883 | .074,806,99 | .013,287,37 | 1.180,675 |
| 52 | 1.769.295 | 1.255,978 | -22.567,52 | 1.769.295 | .946,065,0 | -3.475.801 | .819,335.3 | .293,442.3 | 4.198,585 | .080,843.89 | .014,637,71 | 1,249,673 |
| 53 | 1.908,555 | 1,374,757 | -24.082.22 | 1.908.555 | 1.037,748 | -3.815.728 | .883.986.3 | .322,366,2 | 4.405,645 | .087.238.69 | .016,095,40 | 1.321,196 |
| 54 | 2.055,829 | 1.501,941 | -25.871.80 | 2.055.829 | 1.136,231 | -4.181,296 | .952.376.9 | .353.502.7 | 4.616,876 | .094,005.23 | .017,666,66 | 1.395.280 |
| 55 | 2 211.413 | 1.637,915 | -27.338.75 | 2.211,413 | 1.241,860 | -4.573,864 | 1.024,647 | .386,973.2 | 4.832.081 | .101.157,6 | .019,357,97 | 1.471,958 |
| 56 | 2.375,609 | 1.783,068 | -29.085.57 | 2.375,609 | 1.354.993 | -4.994,839 | 1.100.939 | .422,903.0 | 5.051.043 | .108,710.1 | .021,178,03 | 1.551,264 |
| 57 | 2,548.722 | 1.937.793 | -30.914.83 | 2.548,722 | 1.475,997 | -5,445,668 | 1.181.398 | .461,421,3 | 5.273.536 | .116,677.5 | .023,127,80 | 1.633,230 |
| 58 | 2.731.064 | 2.102,489 | -32.829.11 | 2.731,064 | 1.605.247 | -5.927.836 | 1.266,172 | .502,661,7 | 5.499.315 | .125,074.5 | .025.220,46 | 1.717.888 |
| 59 | 2.922.948 | 2.277.560 | -34.831.03 | 2.922,948 | 1.743,126 | -6.442.874 | 1.355,409 | .546.761.5 | 5.728,121 | .133,916.3 | .027,461.46 | 1.805.268 |
| 60 | 3.124,695 | 2,463,412 | -36.923.25 | 3.124.695 | 1.890,027 | -6.992,345 | 1,449,262 | .593.862.3 | 5.959.680 | .143,218,2 | .029,858,48 | 1.895,400 |
| 61 | 3.336.631 | 2.660,454 | -39.108,45 | 3.336,631 | 2.046,351 | -7.577,860 | 1.547,886 | .644,109,9 | 6.193.706 | .152,996.0 | .032,419,47 | 1.988,313 |
| 62 | 3.559.084 | 2.869.097 | -41.389,35 | 3.559,084 | 2 212.507 | -8.201,070 | 1.651,436 | .697,654.0 | 6,429.891 | .163.265.6 | .035,152,64 | 2.084.035 |
| 63 | 3.792.388 | 3.089.755 | -43.768.71 | 3.792,388 | 2.388.913 | -8.863.668 | 1.760,074 | .754,648.8 | 6.667,916 | .174.043,0 | .038,066,45 | 2.182.594 |
| 64 | 4.036.882 | 3.322,842 | -46.249.27 | 4.036.882 | 2.575.994 | -9.567.376 | 1.873,959 | .815.252.3 | 6.907.448 | .185.344.7 | .041,169,65 | 2.284,016 |
| 65 | 4.292.909 | 3.568.771 | -48.833.86 | 4.292,909 | 2 774.182 | -10.313,97 | 1.993,256 | .879.627,1 | 7.148.135 | .197.187.3 | .044,471,23 | 2.388.327 |
| 66 | 4.560.817 | 3.827.959 | -51.525,28 | 4.560.817 | 2.983.919 | -11.105.26 | 2.118.132 | .947.939,6 | 7.389.608 | .209,587,9 | .047,980,47 | 2.495.550 |
| 67 | 4.840.956 | 4.100.816 | -54.326.38 | 4.840.956 | 3.205.652 | -11,943.10 | 2.248,754 | 1.020.361 | 7.631,485 | .222,563.5 | .051,706,93 | 2.605,710 |
| 68 | 5.133.684 | 4.387,756 | -57.240.02 | 5.133,684 | 3.439.836 | -12.829,36 | 2.385,294 | 1.097.065 | 7.873,368 | .236,131.6 | .055,660,44 | 2.718,829 |
| 69 | 5.439.361 | 4.889.187 | -60.269.07 | 5.439.361 | 3.686.931 | -13.765,98 | 2.527,924 | 1,178,233 | 8.114,840 | .250,310,0 | .059,851,13 | 2.834,929 |
| 70 | 5.758,352 | 5.005.514 | -63.416,44 | 5.758.352 | 3.947,407 | -14.754,92 | 2.676,821 | 1.264,048 | 8.355,469 | .265,116,4 | .064,289,38 | 2.954.031 |
| 71 | 6.091,027 | 5.337,140 | -66.685,02 | 6.091,027 | 4.221.736 | -15.798,20 | 2.832.161 | 1.354.696 | 8.594,811 | .280,569.2 | .068.985.92 | 3.076,154 |
| 72 | 6.437.757 | 5.684,461 | -70.077,73 | 6.437,757 | 4.510,399 | -16.897,83 | 2.994.125 | 1.450,370 | 8.832,396 | .296,686.8 | .073,951,72 | 3.201.317 |
| 73 | 6.798.922 | 6.047,869 | -73.597.52 | 6.798,922 | 4.813,881 | -18.055.90 | 3.162,895 | 1.551,267 | 9.067,748 | .313,487,9 | .079,198.06 | 3.329.539 |
| 74 | 7.174,903 | 6.427,749 | -77,247,29 | 7.174,903 | 5.132,673 | -19.274,51 | 3.338,655 | 1 657.587 | 9.300,367 | .330,991,4 | .084.736,54 | 3.460,835 |
| 75 | 7.566,085 | 6.824,478 | -81.029,99 | 7.566,085 | 5.467,270 | -20.555.83 | 3.521.593 | 1.769,534 | 9.529,739 | .349,216,6 | .090.579.06 | 3.595,222 |
| 76 | 7.972,858 | 7.238,425 | -84.948,55 | 7.972,858 | 5.818,173 | -21.902.01 | 3.711,896 | 1.887,317 | 9.755,335 | .368.183.0 | .096.737,80 | 3.732,715 |
| 77 | 8.395,616 | 7.669.957 | -89.005.93 | 8.395,616 | 6.185.885 | -23.315.30 | 3.909,756 | 2.011,149 | 9.976,604 | .387,910,2 | .103.225.3 | 3.873.325 |
| 78 | 8.834,756 | 8.119,409 | -93,205,03 | 8.834.756 | 6.570,915 | -24.797.92 | 4.115,366 | 2.141,247 | 10.192,99 | .408,418.2 | .110.054.3 | 4.017.067 |
| 79 | 9.290.681 | 8.587,137 | -97.548,80 | 9.290.681 | 6.973.775 | -26.352,17 | 4.328,922 | 2.277.833 | 10,403.90 | .429,727,3 | .117,238,0 | 4.163,951 |
| 80 | 9.763.795 | 9.073,462 | -102,040,1 | 9.763.795 | 7.394.979 | -27.980.35 | 4.550.620 | 2.421.132 | 10.608.74 | .451,857.8 | .124.789.9 | 4.313,990 |
| 81 | 10.254,51 | 9.578.703 | -106.682.0 | 10.254,51 | 7.835,045 | -29.684,82 | 4.780,661 | 2.571,372 | 10.806,89 | .474,830,7 | .132,723.7 | 4.467.191 |
| 82 | 10.763,23 | 10.103.16 | -111,477.2 | 10.763.23 | 8.294,492 | -31,467.91 | 5.019,247 | 2.728,789 | 10,997,74 | .498,666.7 | .141,053,5 | 4.623.561 |
| 83 | 11.290.38 | 10.647,12 | -116.428.6 | 11.290,38 | 8.773,842 | -33.332,09 | 5.266.581 | 2.893,620 | 11.180,62 | .523,387,1 | .149,793,8 | 4.783,110 |
| 84 | 11.836.38 | 11.210.86 | -121.539.1 | 11.836,38 | 9.273,617 | -35.279,74 | 5.522,870 | 3.066,105 | 11.354,87 | .549,013,4 | .158,959,3 | 4.945.842 |
| 85 | 12.401,65 | 11.794,64 | -126.811.6 | 12.401.65 | 9.794,341 | -37.313,33 | 5.788.320 | 3.246,491 | 11.519,82 | .575,567,4 | .168,565,2 | 5.111,763 |
| 86 | 12.986.62 | 12.398,69 | -132.248,8 | 12.986,62 | 10.336,54 | -39.435,37 | 6.063,143 | 3.435,027 | 11.674.74 | .603,070,9 | .178,626,7 | 5.280.876 |
| 87 | 13.591,72 | 13.023,23 | -137.853,4 | 13.591.72 | 10.300,73 | -41.648,33 | 6.347.552 | 3.631,966 | 11.818.95 | .631,546.3 | .189,159,8 | 5.453.183 |
| 88 | 14.217,38 | 13.668.47 | -143.628,4 | 14.217.38 | 11.487,45 | -43.954,78 | 6.641,758 | 3.837.566 | 11.951.69 | .661,015,9 | .200.180,5 | 5.628,687 |
| 89 | 14.864.05 | 14.334.60 | -149.576,2 | 14.864,05 | 12.097.21 | -46.357,25 | 6.945,981 | 4.052.087 | 12.072,22 | .691,502,5 | .211,705.3 | 5.807.386 |
| 90 | 15.532.16 | 15.021.84 | -155.699.7 | 15.532,16 | 12.730,63 | -48.858,32 | 7.260.436 | 4.275.795 | 12.179.78 | .723,029.0 | .223.750.3 | 5.989,280 |

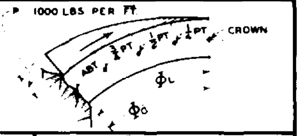
SHEET 4 OF 5

205-D-412

Table H-11.–*Load constants for circular arches–radial and twist deflections* (sheet 5).

UNIT TANGENTIAL LOAD NO. 5

RADIAL AND TWIST DEFLECTIONS



| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | | $D_1$ | $D_2$ | |
| | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term | | $1^{ST}$ Term | $2^{ND}$ Term |
| 10 | .007,727,042 | .001,119,538 | -.004,035,59 | .004,890,821 | .0$^3$,519,071,3 | .091,697,30 | .001,449,521 | .0$^3$,101,803,6 | .055,043,89 | .0$^3$,120,815,1 | .0$^5$4,227,445 | .010,370,10 |
| 11 | .011,311,44 | .001,801,296 | -.006,494.89 | .007,159.885 | .0$^3$,835,509,2 | .121,558,8 | .002,122,136 | .0$^3$,163,915,5 | .073,169,30 | .0$^3$,176,883,1 | .0$^5$6,807.600 | .013,798,74 |
| 12 | .016,017,68 | .002,780,168 | -.010,027,5 | .010,139,33 | .001,290,126 | .157,119,8 | .003,005,407 | .0$^3$,253,189,3 | .094,859,85 | .0$^3$,250,515,3 | .0$^4$10,517,38 | .017,909,02 |
| 13 | .022,058,17 | .004,143,664 | -.014,950,3 | .013,963,76 | .001,923,791 | .198,802,2 | .004,139,286 | .0$^3$,377,683,9 | .120,421,1 | .0$^3$,345,045,2 | .0$^4$15,692,28 | .022,762,15 |
| 14 | .029,663,55 | .005,994,747 | -.021,636,6 | .018,779,38 | .002,784,671 | 247,003,1 | .005,567,188 | .0$^3$,546,908,2 | .150,153,9 | .0$^3$,464,095,5 | .0$^4$22,728,72 | .028,419,13 |
| 15 | .039,082,71 | .008,452,995 | -.030,520,7 | .024,744,01 | .003,928,792 | 302,092,5 | .007,335,982 | .0$^3$,771,938,8 | .184,353,6 | .0$^3$,611,578,7 | .0$^4$32,086,84 | .034,940,76 |
| 16 | .050,582,69 | .011,655,67 | -.042,101,4 | .032,027,00 | .005,420,633 | 364,412,0 | .009,495,999 | .001,065,537 | .223,310,4 | .0$^3$,791,696,8 | .0$^4$44,305,53 | .042,387,59 |
| 17 | .064,448,64 | .015,758,88 | -.056,947,4 | .040,809,28 | .007,333,627 | .434,272,0 | .012,101,01 | .001,442,265 | .267,308,0 | .001,008,941 | .0$^4$59,987,47 | .050,819,92 |
| 18 | .080,983,75 | .020,938,55 | -.075,699,8 | .051,283,28 | .009,750,742 | .511,950,4 | .015,208,25 | .001,918,601 | .316,624,0 | .001,268,092 | .0$^4$79,824,07 | .060,297,82 |
| 19 | .100,509,2 | .027,391,45 | -.099,077,7 | .063,652,91 | .012,765,01 | .597,690,7 | .018,878,39 | .002,513,053 | .371,529,4 | .001,574,218 | .0$^3$,104,590,5 | .070,881,05 |
| 20 | .123,364,0 | .035,336,22 | -.127,881 | .078,133,54 | .016,480,05 | 691,699,4 | .023,175,52 | .003,246,272 | .432,287,6 | .001,932,680 | .0$^3$,135,152,5 | .082,629,06 |
| 21 | .149,905,2 | .045,014,18 | -.162,993 | .094,951,95 | .021,010,55 | .794,145,5 | .028,167,21 | .004,141,165 | .499,155,0 | .002,349,125 | .0$^3$,172,471,9 | .095,600,96 |
| 22 | .180,507,3 | .056,690,37 | -.205,389 | .114,346,3 | .026,482,80 | .905,157,4 | .033,924,40 | .005,223,003 | .572,379,6 | .002,829,489 | .0$^3$,217,611,0 | .109,855,7 |
| 23 | .215,562,8 | .070,654,03 | -.256,[illegible]32 | .[illegible]36,566,2 | .033,035,15 | .024,822 | .040,521,49 | .006,519,536 | .652,201,2 | .003,379,998 | .0$^3$,271,736,9 | .125,451,7 |
| 24 | .255,481,5 | .087,219,96 | -.316,383 | .161,872,4 | .040,818,48 | 1.153,182 | .048,036,32 | .008,061,098 | .738,851,4 | .004,007,166 | .0$^3$,336,128,7 | .142,447,0 |
| 25 | .300,691,0 | .106,728,7 | -.387,403 | .190,537,0 | .049,996,70 | 1.290,235 | .056,550,10 | .009,880,711 | .832,552,5 | .004,717,796 | .0$^3$,412,180,1 | .160,899,4 |
| 26 | .351,635,9 | .129,547,5 | -.470,548 | .222,843,4 | .060,747,13 | 1.435,933 | .066,147,46 | .012,014,20 | .933,517,0 | .005,518,978 | .0$^3$,501,404,7 | .180,866,2 |
| 27 | .408,778,5 | .156,070,7 | -.567,288 | .259,086,1 | .073,260,95 | 1.590,177 | .076,916,45 | .014,500,29 | 1.041,948 | .006,418,093 | .0$^3$,605,442,3 | .202,404,5 |
| 28 | .472,598,3 | .186,720,8 | -.679,195 | .299,570,8 | .087,743,62 | 1.752,820 | .088,948,48 | .017,380,71 | 1.158,039 | .007,422,807 | .0$^3$,726,062,2 | .225,570,5 |
| 29 | .543,591,4 | .221,948,2 | -.807,952 | .344,614,2 | .104,415,3 | 1.923,663 | .102,338,4 | .020,700,30 | 1.281,972 | .008,541,076 | .0$^3$,865,168,8 | .250,420,5 |
| 30 | .622,271,5 | .262,232,4 | -.955,352 | .394,544,1 | .123,511,1 | 2.102,451 | .117,184,4 | .024,507,11 | 1.413,920 | .009,781,144 | .001,024,806 | .277,010,0 |
| 31 | .709,[illegible]69,0 | .308,08[illegible]9 | -1.123,310 | .449,699,4 | .145,281,6 | 2.288,879 | .133,588,0 | .028,852,47 | 1.554,041 | .011,151,54 | .001,207,163 | .305,394,1 |
| 32 | .804,831,0 | .360,034,6 | -1.313,852 | .510,429,8 | .169,993,2 | 2.482,581 | .151,654,2 | .033,791,15 | 1.702,487 | .012,661,09 | .001,414,578 | .335,627,5 |
| 33 | .909,821,2 | .418,658,3 | -1.529,123 | .577,095,9 | .197,928,2 | 2.683,138 | .171,491,3 | .039,381,40 | 1.859,393 | .014,318,89 | .001,649,543 | .367,764,3 |
| 34 | .024,720 | .484,550,7 | -1.771,394 | .650,069,2 | .229,385,3 | 2.890,071 | .193,210,9 | .045,685,05 | 2.024,885 | .016,134,33 | .001,914,710 | .401,858,1 |
| 35 | 1.150,124 | .558,339,4 | -2.043,057 | .729,732,0 | .264,680,1 | 3.102,836 | .216,928,0 | .052,767,63 | 2.199,075 | .018,117,11 | .002,212,893 | .437,961,9 |
| 36 | 1.286,646 | .640,682,2 | -2.346,623 | .816,477,3 | .304,144,7 | 3.320,834 | .242,761,0 | .060,698,42 | 2.382,063 | .020,277,17 | .002,547,077 | .476,128,2 |
| 37 | 1.434,916 | .732,267,1 | -2.684,737 | .910,708,7 | .348,128,8 | 3.543,403 | .270,831,5 | .069,550,58 | 2.573,936 | .022,624,79 | .002,920,417 | .516,409,2 |
| 38 | 1.595,579 | .833,812,1 | -3.060,165 | 1.012,840 | .396,999,3 | 3.769,812 | .301,264,5 | .079,401,21 | 2.774,766 | .025,170,50 | .003,336,248 | .558,855,9 |
| 39 | 1.769,295 | .946,065,0 | -3.475,801 | 1.123,297 | .451,140,5 | 3.999,271 | .334,188,3 | .090,331,44 | 2.984,613 | .027,925,12 | .003,798,088 | .603,519,2 |
| 40 | 1.956,741 | 1.069,803 | -3.934,669 | 1.242,514 | .510,954,9 | 4.230,923 | .369,734,4 | .102,426,5 | 3.203,522 | .030,899,76 | .004,309,639 | .650,449,3 |
| 41 | 2.158,610 | 1.205,834 | -4.439,920 | 1.370,938 | .576,862,5 | 4.463,839 | .408,037,7 | .115,775,8 | 3.431,524 | .034,105,83 | .004,874,799 | .699,695,5 |
| 42 | 2.375,609 | 1.354,993 | -4.994,838 | 1.509,023 | .649,301,7 | 4.697,031 | .449,236,4 | .130,473,1 | 3.668,634 | .037,555,01 | .005,497,659 | .751,306,8 |
| 43 | 2.608,462 | 1.518,145 | -5.602,833 | 1.657,236 | .728,729,1 | 4.929,437 | .493,471,8 | .146,616,3 | 3.914,853 | .041,259,27 | .006,182,514 | .805,331,3 |
| 44 | 2.857,906 | 1.696,183 | -6.267,446 | 1.816,054 | .815,619,3 | 5.159,926 | .540,888,8 | .164,308,0 | 4.170,167 | .045,230,87 | .006,933,857 | .861,816,9 |
| 45 | 3.124,695 | 1.890,027 | -6.992,345 | 1.985,964 | .910,465,5 | 5.387,297 | .591,635,1 | .183,655,0 | 4.434,548 | .049,482,34 | .007,756,400 | .920,809,5 |
| 46 | 3.409,597 | 2.100,623 | -7.781,327 | 2.167,461 | 1.013,779 | 5.610,281 | .645,861,9 | .204,768,9 | 4.707,948 | .054,026,50 | .008,655,073 | .982,356,2 |
| 47 | 3.713,394 | 2.328,946 | -8.638,319 | 2.361,053 | 1.126,09 | 5.827,533 | .703,723,7 | .227,765,7 | 4.990,306 | .058,876,51 | .009,635,001 | 1.046,502 |
| 48 | 4.036,882 | 2.575,994 | -9.567,376 | 2.567,255 | 1.247,948 | 6.037,641 | .765,378,0 | .252,766,4 | 5.281,546 | .064,045,64 | .010,701,56 | 1.113,291 |
| 49 | 4.380,874 | 2.842,790 | -10.572,68 | 2.786,596 | 1.379,919 | 6.239,118 | .830,985,8 | .279,896,3 | 5.581,570 | .069,547,68 | .011,860,35 | 1.182,767 |
| 50 | 4.746,195 | 3.130,381 | -11.658,53 | 3.019,609 | 1.522,587 | 6.430,401 | .900,710,8 | .309,285,9 | 5.890,268 | .075,396,57 | .013,117,18 | 1.254,975 |
| 51 | 5.133,684 | 3.439,836 | -12.829,36 | 3.266,841 | 1.676,558 | 6.609,862 | .974,720,5 | .341,070,3 | 6.207,511 | .081,606,53 | .014,478,12 | 1.329,956 |
| 52 | 5.544,194 | 3.772,247 | -14.089,72 | 3.528,848 | 1.842,452 | 6.775,789 | 1.053,185 | .375,389,7 | 6.533,152 | .088,192,11 | .015,949,47 | 1.407,751 |
| 53 | 5.978,592 | 4.128,724 | -15.444,28 | 3.806,194 | 2.020,909 | 6.926,405 | 1.136,278 | .412,389,1 | 6.867,030 | .095,168,1 | .017,537,78 | 1.488,401 |
| 54 | 6.437,757 | 4.510,399 | -16.897,83 | 4.099,453 | 2.212,587 | 7.059,848 | 1.224,176 | .452,218,7 | 7.208,958 | .102,549,6 | .019,249,84 | 1.571,946 |
| 55 | 6.922,584 | 4.918,420 | -18.455,27 | 4.409,209 | 2.418,162 | 7.174,199 | 1.317,059 | .495,033,6 | 7.558,742 | .110,352,0 | .021,092,76 | 1.658,425 |
| 56 | 7.433,978 | 5.353,951 | -20.121,62 | 4.736,054 | 2.638,327 | 7.267,446 | 1.415,110 | .540,994,1 | 7.916,163 | .118,591,0 | .023,073,69 | 1.747,877 |
| 57 | 7.972,858 | 5.818,173 | -21.902,01 | 5.080,591 | 2.873,793 | 7.337,514 | 1.518,514 | .590,265,5 | 8.280,983 | .127,282,4 | .025,200,36 | 1.840,339 |
| 58 | 8.540,190 | 6.312,279 | -23.801,69 | 5.443,430 | 3.125,286 | 7.382,251 | 1.627,462 | .643,018,4 | 8.652,947 | .136,442,5 | .027,480,54 | 1.935,847 |
| 59 | 9.136,816 | 6.837,476 | -25.825,99 | 5.825,190 | 3.393,553 | 7.399,427 | 1.742,144 | .699,428,7 | 9.031,783 | .146,087,9 | .029,922,35 | 2.034,434 |
| 60 | 9.763,795 | 7.394,979 | -27.980,35 | 6.226,501 | 3.679,353 | 7.386,744 | 1.862,757 | .759,677,3 | 9.417,197 | .156,235,2 | .032,534,16 | 2.136,139 |
| 61 | 10.422,06 | 7.986,015 | -30.270,32 | 6.648,001 | 3.983,465 | 7.341,830 | 1.989,499 | .823,950,5 | 9.808,877 | .166,901,6 | .035,324,64 | 2.240,995 |
| 62 | 11.112,59 | 8.611,815 | -32.701,54 | 7.090,334 | 4.306,681 | 7.262,231 | 2.122,570 | .892,440,1 | 10.206,49 | .178,104,4 | .038,302,72 | 2.349,032 |
| 63 | 11.836,38 | 9.273,617 | -35.279,74 | 7.554,156 | 4.649,811 | 7.145,429 | 2.262,176 | .965,342,9 | 10.609,69 | .189,861,2 | .04[illegible],477,63 | 2.460,284 |
| 64 | 12.594,43 | 9.972,662 | -38.010,73 | 8.040,130 | 5.013,679 | 6.988,828 | 2.408,523 | 1.042,861 | 11.018,10 | .202,189,9 | .044,858,90 | 2.574,781 |
| 65 | 13.387,76 | 10.710,19 | -40.900,41 | 8.548,928 | 5.399,123 | 6.789,763 | 2.561,822 | 1.125,203 | 11.431,33 | .215,108,8 | .048,456,32 | 2.692,553 |
| 66 | 14.217,38 | 11.487,45 | -43.954,78 | 9.081,229 | 5.806,996 | 6.545,497 | 2.722,287 | 1.212,580 | 11.848,97 | .228,636,2 | .052,280,01 | 2.813,628 |
| 67 | 5.084,35 | 12.305,68 | -47.179,85 | 9.637,721 | 6.238,166 | 6.253,21 | 2.890,133 | 1.305,212 | 12.270,60 | .242,790,9 | .056,340,38 | 2.938,035 |
| 68 | 5.989,69 | 13.166,11 | -50.581,79 | 10.219,10 | 6.693,513 | 5.910,05 | 3.065,581 | 1.403,320 | 12.695,75 | .257,592,0 | .060,648,17 | 3.065,800 |
| 69 | 16.934,48 | 14.069,37 | -54.166,76 | 10.826,07 | 7.173,931 | 5.513,03 | 3.248,851 | 1.507,141 | 13.123,96 | .273,058,7 | .065,214,32 | 3.196,950 |
| 70 | 7.919,77 | 15.018,49 | -57.941,02 | 1.459,34 | 7.680,324 | 5.059,15 | 3.440,169 | 1.616,902 | 13.554,73 | .289,210,6 | .070,050,26 | 3.331,508 |
| 71 | 18.946,64 | 16.012,86 | -61.910,87 | 12.119,64 | 8.213,610 | 4.545,33 | 3.639,763 | 1.732,844 | 13.987,56 | .306,067,5 | .075,167,62 | 3.469,499 |
| 72 | 20.016,18 | 17.054,30 | -66.082,06 | 12.807,68 | 8.774,718 | 3.968,42 | 3.847,864 | 1.855,215 | 14.421,91 | .323,649,7 | .080,578,36 | 3.610,945 |
| 73 | 21.129,48 | 18.143,98 | -70.462,79 | 13.524,21 | 9.364,587 | 3.325,18 | 4.064,706 | 1.984,263 | 14.857,22 | .341,977,4 | .086,294,78 | 3.755,867 |
| 74 | 22.287,65 | 9.283,05 | -75.057,71 | 14.269,96 | 9.984,[illegible]66 | 2.612,33 | 4.290,525 | 2.120,247 | 15.292,92 | .361,071,4 | .092,329,52 | 3.904,285 |
| 75 | 23.491,81 | 20.472,68 | -79.873,88 | 15.045,69 | 10.634,4 | 1.826,55 | 4.525,561 | 2.263,426 | 15.728,41 | .330,952,6 | .098,695,53 | 4.056,219 |
| 76 | 24.743,06 | 21.713,97 | -84.917,81 | 15.852,15 | 11.316,30 | .964,40 | 4.770,055 | 2.414,069 | 16.163,08 | .401,642,2 | .105,406,1 | 4.211,688 |
| 77 | 26.042,55 | 23.008,02 | -90.195,98 | 16.690,11 | 12.030,79 | .022,41 | 5.024,253 | 2.572,447 | 16.596,29 | .423,161,8 | .112,474,8 | 4.370,707 |
| 78 | 27.391,41 | 24.355,90 | -95.714,99 | 17.560,33 | 12.778,88 | -1.002,94 | 5.288,402 | 2.738,838 | 17.027,38 | .445,533,2 | .119,915,8 | 4.533,293 |
| 79 | 29.790,79 | 25.758,66 | -101.481,3 | 18.463,59 | 13.561,54 | -2.115,24 | 5.562,753 | 2.913,525 | 17.455,66 | .468,778,3 | .127,743,9 | 4.699,461 |
| 80 | 30.241,85 | 27.217,29 | -107.501,6 | 19.400,69 | 14.379,79 | -3.318,16 | 5.847,560 | 3.096,800 | 17.880,44 | .492,919,6 | .135,971,6 | 4.869,223 |
| 81 | 3[illegible].745,73 | 28.732,77 | -113.782,3 | 20.372,39 | 15.234,61 | -4.615,38 | 6.143,077 | 3.288,943 | 18.300,99 | .517,979,6 | .144,616,3 | 5.042,592 |
| 82 | 33.303,62 | 30.306,03 | -120.330,0 | 21.379,52 | 16.127,00 | -6.0[illegible],67 | 6.449,565 | 3.490,267 | 18.716,57 | .543,981,1 | .153,692,4 | 5.219,579 |
| 83 | 34.9[illegible],69 | 31.937,98 | -127.[illegible] | 22.422,86 | 17.057,94 | -7.507,84 | 6.767,285 | 3.701,070 | 19.126,42 | .570,947,5 | .163,215,8 | 5.400,195 |
| 84 | 36.586,12 | 33.629,46 | -134.252,3 | 23.503,23 | 18.028,57 | -9.110,76 | 7.096,500 | 3.921,661 | 19.529,73 | .598,902,0 | .173,202,5 | 5.584,447 |
| 85 | 38.313,11 | 35.381,28 | -141.640,0 | 24.621,44 | 19.039,76 | -0.823,31 | 7.437,476 | 4.152,354 | 19.925,71 | .627,868,3 | .183,668,9 | 5.772,342 |
| 86 | 40.098,83 | 37.194,20 | -149.320,6 | 25.778,32 | 20.092,57 | -12.649,49 | 7.790,484 | 4.393,408 | 20.313,53 | .657,870,5 | .194,631,9 | 5.963,887 |
| 87 | 4[illegible].944,50 | 39.068,93 | -57.300,4 | 26.974,69 | 21.[illegible]88,0[illegible] | -[illegible]593,29 | 8.155,796 | 4.645,326 | 20.692,34 | .688,932,7 | .206,108,7 | 6.159,088 |
| 88 | 43.85[illegible],33 | 4[illegible].006,14 | -165.585,9 | 28.211,38 | 2[illegible].3[illegible]7,10 | -16.658,74 | 8.533,684 | 4.908,257 | 21.061,25 | .721,079,5 | .218,116,8 | 6.357,946 |
| 89 | 45.820,52 | 43.[illegible]6,43 | -174.183,3 | 29.489,25 | 23.510,83 | -18.849,96 | 8.924,429 | 5.182,596 | 21.419,40 | .754,335,5 | .230,674,1 | 6.560,465 |
| 90 | 47.853,28 | 45.070,34 | 83.098,9 | 30.809,12 | 24.740,30 | -21.171,07 | 9.328,307 | 5.468,679 | 21.765,84 | .788,726,0 | .243,798,9 | 6.766,645 |

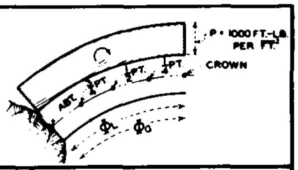
SHEET 5 OF 5

205 - D - 413

Table H-12.–*Load constants for circular arches–all deflections* (sheet 1).

UNIT TWIST LOAD NO. 1

TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



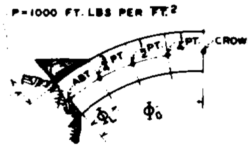
| $\Phi$ ° | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
| 10 | 15.230,87 | 1.766,800 | .115,794 | 8.567,365 | .746,363,4 | .036,664,9 | 3.807,718 | .221,355,4 | .007,246,2 | .951,929,4 | .027,685,23 | .0$^3$453,0 |
| 11 | 18.429,35 | 2.350,105 | .169,473 | 10.366,51 | .993,052,0 | .053,670 | 4.607,339 | .294,576,9 | .010,608,4 | 1.151,835 | .036,847,57 | .0$^3$663,3 |
| 12 | 21.932,45 | 3.048,936 | .239,931 | 12.337,01 | 1.288,743 | .075,997 | 5.483,114 | .382,373,9 | .015,023,1 | 1.370,778 | .047,836,08 | .0$^3$939,4 |
| 13 | 25.740,17 | 3.873,499 | .330,332 | 14.478,85 | 1.637,822 | .104,650 | 6.435,043 | .486,062,2 | .020,690,1 | 1.608,761 | .060,816,47 | .001,293,8 |
| 14 | 29.852,51 | 4.833,924 | .444,110 | 16.792,04 | 2.044,653 | .140,723 | 7.463,127 | .606,955,1 | .027,826,1 | 1.865,782 | .075,954,40 | .001,740,2 |
| 15 | 34.269,46 | 5.940,255 | .584,966 | 19.278,57 | 2.513,583 | .185,395 | 8.567,365 | .746,363,4 | .036,664,9 | 2.141,841 | .093,415,43 | .002,293,2 |
| 16 | 38.991,03 | 7.202,451 | .756,864 | 21.932,45 | 3.048,936 | .239,931 | 9.747,757 | .905,594,8 | .047,457,9 | 2.436,939 | .113,365,0 | .002,968,5 |
| 17 | 44.017,22 | 8.630,372 | .964,030 | 24.759,69 | 3.655,016 | .305,679 | 11.004,30 | 1.085,954 | .060,473 | 2.751,076 | .135,968,6 | .003,783,1 |
| 18 | 49.348,02 | 10.233,78 | 1.210,954 | 27.758,26 | 4.336,099 | .384,074 | 12.337,01 | 1.288,743 | .075,997 | 3.084,251 | .161,391,4 | .004,754,7 |
| 19 | 54.983,44 | 12.022,32 | 1.502,380 | 30.928,19 | 5.096,437 | .476,635 | 13.745,86 | 1.515,260 | .094,330 | 3.436,465 | .189,798,7 | .005,902,4 |
| 20 | 60.923,48 | 14.005,54 | 1.843,311 | 34.269,46 | 5.940,255 | .584,966 | 15.230,87 | 1.766,800 | .115,794 | 3.807,718 | .221,355,4 | .007,246,2 |
| 21 | 67.168,14 | 16.192,85 | 2.239,001 | 37.782,08 | 6.871,749 | .710,752 | 16.792,04 | 2.044,653 | .140,723 | 4.198,009 | .256,226,5 | .008,807,5 |
| 22 | 73.717,42 | 18.593,55 | 2.694,954 | 41.466,05 | 7.895,083 | .855,763 | 18.429,35 | 2.350,105 | .169,473 | 4.607,339 | .294,576,9 | .010,608,4 |
| 23 | 80.571,31 | 21.216,80 | 3.216,927 | 45.321,36 | 9.014,393 | 1.021,852 | 20.142,83 | 2.684,440 | .202,413 | 5.035,707 | .336,571,2 | .012,672,1 |
| 24 | 87.729,82 | 24.071,62 | 3.810,913 | 49.348,02 | 10.233,78 | 1.210,954 | 21.932,45 | 3.048,936 | .239,931 | 5.483,114 | .382,373,9 | .015,023,1 |
| 25 | 95.192,94 | 27.166,89 | 4.483,153 | 53.546,03 | 11.557,31 | 1.425,082 | 23.798,24 | 3.444,867 | .282,430 | 5.949,559 | .432,149,5 | .017,687,0 |
| 26 | 102.960,7 | 30.511,35 | 5.240,13 | 57.915,39 | 12.989,00 | 1.666,335 | 25.740,17 | 3.873,499 | .330,332 | 6.435,043 | .486,062,2 | .020,690,1 |
| 27 | 111.033,1 | 34.113,57 | 6.088,54 | 62.456,09 | 14.532,87 | 1.936,890 | 27.758,26 | 4.336,099 | .384,074 | 6.939,566 | .544,276,1 | .024,060,2 |
| 28 | 119.410,0 | 37.981,97 | 7.035,34 | 67.168,14 | 16.192,85 | 2.239,001 | 29.852,51 | 4.833,924 | .444,110 | 7.463,127 | .606,955,1 | .027,826,1 |
| 29 | 128.091,6 | 42.124,81 | 8.087,71 | 72.051,54 | 17.972,87 | 2.575,006 | 32.022,91 | 5.368,226 | .510,912 | 8.005,727 | .674,263,0 | .032,017,4 |
| 30 | 137.077,8 | 46.550,16 | 9.253,05 | 77.106,29 | 19.876,79 | 2.947,320 | 34.269,46 | 5.940,255 | .584,966 | 8.567,365 | .746,363,4 | .036,664,9 |
| 31 | 146.368,7 | 51.265,93 | 10.538,95 | 82.332,38 | 21.908,44 | 3.358,432 | 36.592,17 | 6.551,251 | .666,777 | 9.148,042 | .823,419,5 | .041,800,8 |
| 32 | 155.964,1 | 56.279,86 | 11.953,27 | 87.729,82 | 24.071,62 | 3.810,913 | 38.991,03 | 7.202,451 | .756,864 | 9.747,757 | .905,594,8 | .047,457,9 |
| 33 | 165.864,2 | 61.599,46 | 13.504,05 | 93.298,80 | 26.370,05 | 4.307,406 | 41.466,05 | 7.895,083 | .855,763 | 10.366,51 | .993,052,0 | .053,670 |
| 34 | 176.068,9 | 67.232,11 | 15.199,55 | 99.038,74 | 28.807,43 | 4.850,633 | 44.017,22 | 8.630,372 | .964,030 | 11.004,30 | 1.085,954 | .060,473 |
| 35 | 186.578,2 | 73.184,93 | 17.048,22 | 104.950,2 | 31.387,41 | 5.443,39 | 46.644,54 | 9.409,533 | 1.082,231 | 11.661,14 | 1.184,464 | .067,903 |
| 36 | 197.392,1 | 79.464,88 | 19.058,73 | 111.033,1 | 34.113,57 | 6.088,54 | 49.348,02 | 10.233,78 | 1.210,954 | 12.337,01 | 1.288,743 | .075,997 |
| 37 | 208.510,6 | 86.078,71 | 21.239,93 | 117.287,2 | 36.989,46 | 6.789,04 | 52.127,66 | 11.104,31 | 1.350,798 | 13.031,91 | 1.398,955 | .084,792 |
| 38 | 219.933,8 | 93.032,95 | 23.600,87 | 123.712,8 | 40.018,57 | 7.547,88 | 54.983,44 | 12.022,32 | 1.502,380 | 13.745,86 | 1.515,260 | .094,330 |
| 39 | 231.661,6 | 100.333,9 | 26.150,79 | 130.309,6 | 43.204,35 | 8.368,17 | 57.915,39 | 12.989,00 | 1.666,335 | 14.478,85 | 1.637,822 | .104,650 |
| 40 | 243.693,9 | 107.987,7 | 28.899,09 | 137.077,8 | 46.550,16 | 9.253,05 | 60.923,48 | 14.005,54 | 1.843,311 | 15.230,87 | 1.766,800 | .115,794 |
| 41 | 256.030,9 | 116.000,2 | 31.855,35 | 144.017,4 | 50.059,34 | 10.205,74 | 64.007,74 | 15.073,10 | 2.033,972 | 16.001,93 | 1.902,357 | .127,804 |
| 42 | 268.672,6 | 124.377,0 | 35.029,38 | 151.128,3 | 53.735,15 | 11.229,57 | 67.168,14 | 16.192,85 | 2.239,001 | 16.792,04 | 2.044,653 | .140,723 |
| 43 | 281.618,8 | 133.123,6 | 38.431,07 | 158.410,6 | 57.580,81 | 12.327,86 | 70.404,70 | 17.365,95 | 2.459,091 | 17.601,18 | 2.193,849 | .154,598 |
| 44 | 294.869,7 | 142.245,1 | 42.070,53 | 165.864,2 | 61.599,46 | 13.504,05 | 73.717,42 | 18.593,55 | 2.694,954 | 18.429,35 | 2.350,105 | .169,473 |
| 45 | 308.425,1 | 151.746,4 | 45.957,99 | 173.489,1 | 65.794,20 | 14.761,65 | 77.106,29 | 19.876,79 | 2.947,320 | 19.276,57 | 2.513,583 | .185,395 |
| 46 | 322.285,2 | 161.632,3 | 50.103,86 | 181.285,4 | 70.168,05 | 16.104,20 | 80.571,31 | 21.216,80 | 3.216,926 | 20.142,83 | 2.684,440 | .202,413 |
| 47 | 336.449,9 | 171.907,2 | 54.518,67 | 189.253,1 | 74.723,98 | 17.535,33 | 84.112,49 | 22.614,70 | 3.504,533 | 21.028,12 | 2.862,839 | .220,575 |
| 48 | 350.919,3 | 182.575,3 | 59.213,10 | 197.392,1 | 79.464,88 | 19.058,73 | 87.729,82 | 24.071,62 | 3.810,913 | 21.932,45 | 3.048,936 | .239,931 |
| 49 | 365.693,2 | 193.640,5 | 64.197,99 | 205.702,4 | 84.393,59 | 20.878,13 | 91.423,30 | 25.588,65 | 4.136,852 | 22.855,83 | 3.242,893 | .260,532 |
| 50 | 380.771,8 | 205.106,4 | 69.484,27 | 214.184,1 | 89.512,88 | 22.397,36 | 95.192,94 | 27.166,89 | 4.483,153 | 23.798,24 | 3.444,867 | .282,430 |
| 51 | 396.155,0 | 216.976,6 | 75.083,02 | 222.837,2 | 94.825,45 | 24.220,27 | 99.038,74 | 28.807,43 | 4.850,633 | 24.759,69 | 3.655,016 | .305,679 |
| 52 | 411.842,8 | 229.254,1 | 81.005,40 | 231.661,6 | 100.333,9 | 26.150,79 | 102.960,7 | 30.511,35 | 5.240,13 | 25.740,17 | 3.873,499 | .330,332 |
| 53 | 427.835,2 | 241.941,9 | 87.262,73 | 240.657,3 | 106.040,9 | 28.192,89 | 106.958,8 | 32.279,71 | 5.652,47 | 26.739,70 | 4.100,475 | .356,445 |
| 54 | 444.132,2 | 255.042,5 | 93.866,39 | 249.824,4 | 111.948,7 | 30.350,62 | 111.033,1 | 34.113,57 | 6.088,54 | 27.758,26 | 4.336,099 | .384,074 |
| 55 | 460.733,9 | 268.558,2 | 100.827,9 | 259.162,8 | 118.060,0 | 32.628,07 | 115.183,5 | 36.013,98 | 6.549,20 | 28.795,87 | 4.580,530 | .413,276 |
| 56 | 477.640,1 | 282.491,2 | 108.158,6 | 268.672,6 | 124.377,0 | 35.029,38 | 119.410,0 | 37.981,97 | 7.035,34 | 29.852,51 | 4.833,924 | .444,110 |
| 57 | 494.851,0 | 296.843,1 | 115.870,9 | 278.353,7 | 130.902,0 | 37.558,76 | 123.712,8 | 40.018,57 | 7.547,88 | 30.928,19 | 5.096,437 | .476,635 |
| 58 | 512.366,5 | 311.615,6 | 123.975,8 | 288.206,2 | 137.637,1 | 40.220,45 | 128.091,6 | 42.124,81 | 8.087,71 | 32.022,91 | 5.368,226 | .510,912 |
| 59 | 530.186,6 | 326.809,8 | 132.485,4 | 298.230,0 | 144.584,6 | 43.018,74 | 132.546,7 | 44.301,67 | 8.655,79 | 33.136,66 | 5.649,447 | .547,002 |
| 60 | 548.311,4 | 342.426,6 | 141.411,7 | 308.425,1 | 151.746,4 | 45.957,99 | 137.077,8 | 46.550,16 | 9.253,05 | 34.269,46 | 5.940,255 | .584,966 |
| 61 | 566.740,7 | 358.466,7 | 150.766,5 | 318.791,7 | 159.124,5 | 49.042,59 | 141.685,2 | 48.871,26 | 9.880,45 | 35.421,29 | 6.240,805 | .624,869 |
| 62 | 585.474,7 | 374.930,5 | 160.561,9 | 329.329,5 | 166.720,9 | 52.276,98 | 146.368,7 | 51.265,93 | 10.538,95 | 36.592,17 | 6.551,251 | .666,777 |
| 63 | 604.513,3 | 391.817,9 | 170.809,9 | 340.038,7 | 174.537,2 | 55.665,84 | 151.128,3 | 53.735,15 | 11.229,57 | 37.782,08 | 6.871,749 | .710,752 |
| 64 | 623.856,5 | 409.128,8 | 181.522,7 | 350.919,3 | 182.575,3 | 59.213,10 | 155.964,1 | 56.279,86 | 11.953,27 | 38.991,03 | 7.202,451 | .756,864 |
| 65 | 643.504,3 | 426.862,6 | 192.712,5 | 361.971,2 | 190.836,7 | 62.923,95 | 160.876,1 | 58.900,99 | 12.711,09 | 40.219,02 | 7.543,511 | .805,178 |
| 66 | 663.456,7 | 445.018,5 | 204.391,3 | 373.194,4 | 199.323,1 | 66.802,78 | 165.864,2 | 61.599,46 | 13.504,05 | 41.466,05 | 7.895,083 | .855,763 |
| 67 | 683.713,8 | 463.595,4 | 216.571,4 | 384.589,0 | 208.036,0 | 70.854,23 | 170.928,5 | 64.376,20 | 14.333,19 | 42.732,11 | 8.257,319 | .908,691 |
| 68 | 704.275,5 | 482.591,8 | 229.264,9 | 396.155,0 | 216.976,6 | 75.083,02 | 176.068,9 | 67.232,11 | 15.199,55 | 44.017,22 | 8.630,372 | .964,030 |
| 69 | 725.141,8 | 502.006,1 | 242.484,2 | 407.892,2 | 226.146,4 | 79.493,84 | 181.285,4 | 70.168,05 | 16.104,20 | 45.321,36 | 9.014,393 | 1.021,852 |
| 70 | 746.312,7 | 521.836,2 | 256.241,4 | 419.800,9 | 235.546,6 | 84.091,49 | 186.578,2 | 73.184,93 | 17.048,22 | 46.644,54 | 9.409,533 | 1.082,231 |
| 71 | 767.788,2 | 542.079,8 | 270.548,8 | 431.880,9 | 245.178,2 | 88.880,73 | 191.947,1 | 76.283,59 | 18.032,69 | 47.986,76 | 9.815,945 | 1.145,241 |
| 72 | 789.568,4 | 562.734,3 | 285.418,5 | 444.132,2 | 255.042,5 | 93.866,39 | 197.392,1 | 79.464,88 | 19.058,73 | 49.348,02 | 10.233,78 | 1.210,954 |
| 73 | 811.653,1 | 583.796,8 | 300.862,7 | 456.554,9 | 265.140,2 | 99.053,34 | 202.913,3 | 82.729,65 | 20.127,43 | 50.728,32 | 10.663,18 | 1.279,448 |
| 74 | 834.042,5 | 605.264,0 | 316.893,7 | 469.148,9 | 275.472,4 | 104.446,5 | 208.510,6 | 86.078,71 | 21.239,93 | 52.127,66 | 11.104,31 | 1.350,798 |
| 75 | 856.736,5 | 627.132,5 | 333.523,5 | 481.914,3 | 286.039,8 | 110.050,7 | 214.184,1 | 89.512,88 | 22.397,36 | 53.546,03 | 11.557,31 | 1.425,082 |
| 76 | 879.735,1 | 649.398,4 | 350.764,2 | 494.851,0 | 296.843,1 | 115.870,9 | 219.933,8 | 93.032,95 | 23.600,87 | 54.983,44 | 12.022,32 | 1.502,380 |
| 77 | 903.038,3 | 672.057,6 | 368.627,9 | 507.959,1 | 307.883,0 | 121.912,1 | 225.759,6 | 96.639,71 | 24.851,62 | 56.439,90 | 12.499,51 | 1.582,771 |
| 78 | 926.646,2 | 695.105,6 | 387.126,6 | 521.238,5 | 319.159,9 | 128.179,3 | 231.661,6 | 100.333,9 | 26.150,79 | 57.915,39 | 12.989,00 | 1.666,335 |
| 79 | 950.558,7 | 718.537,8 | 406.272,2 | 534.689,2 | 330.674,4 | 134.677,5 | 237.639,7 | 104.116,3 | 27.499,54 | 59.409,92 | 13.490,97 | 1.753,154 |
| 80 | 974.775,7 | 742.349,2 | 426.076,5 | 548.311,4 | 342.426,6 | 141.411,7 | 243.693,9 | 107.987,7 | 28.899,09 | 60.923,48 | 14.005,54 | 1.843,311 |
| 81 | 999.297,5 | 766.534,3 | 446.551,5 | 562.104,8 | 354.417,0 | 148.386,9 | 249.824,4 | 111.948,7 | 30.350,62 | 62.456,09 | 14.532,87 | 1.936,890 |
| 82 | 1,024.124 | 791.087,7 | 467.708,7 | 576.069,6 | 366.645,6 | 155.608,4 | 256.030,9 | 116.000,2 | 31.855,35 | 64.007,74 | 15.073,10 | 2.033,972 |
| 83 | 1,049.255 | 816.003,4 | 489.559,9 | 590.205,8 | 379.112,6 | 163.081,0 | 262.313,7 | 120.142,7 | 33.414,55 | 65.578,42 | 15.626,38 | 2.134,648 |
| 84 | 1,074.690 | 841.275,2 | 512.116,6 | 604.513,3 | 391.817,9 | 170.809,9 | 268.672,6 | 124.377,0 | 35.029,38 | 67.168,14 | 16.192,85 | 2.239,001 |
| 85 | 1,100.430 | 866.896,6 | 535.390,1 | 618.992,1 | 404.761,4 | 178.800,3 | 275.107,6 | 128.703,7 | 36.701,15 | 68.776,90 | 16.772,66 | 2.347,119 |
| 86 | 1,126.475 | 892.860,8 | 559.391,9 | 633.642,3 | 417.942,9 | 187.057,2 | 281.618,8 | 133.123,6 | 38.431,07 | 70.404,70 | 17.365,95 | 2.459,091 |
| 87 | 1,152.825 | 919.160,7 | 584.133,2 | 648.463,9 | 431.362,0 | 195.585,9 | 288.206,2 | 137.637,1 | 40.220,45 | 72.051,54 | 17.972,87 | 2.575,006 |
| 88 | 1,179.479 | 945.789,1 | 609.625,0 | 663.456,7 | 445.018,5 | 204.391,3 | 294.869,7 | 142.245,1 | 42.070,53 | 73.717,42 | 18.593,55 | 2.694,954 |
| 89 | 1,206.437 | 972.738,1 | 635.878,4 | 678.621,0 | 458.911,8 | 213.478,7 | 301.609,3 | 146.948,0 | 43.982,61 | 75.402,33 | 19.228,14 | 2.819,031 |
| 90 | 1,233.701 | 1,000.000 | 662.904,2 | 693.956,6 | 473.041,2 | 222.853,2 | 308.425,1 | 151.746,4 | 45.957,99 | 77.106,29 | 19.876,79 | 2.947,320 |

SHEET 1 OF 5

205-D-414

Table H-12.–*Load constants for circular arches–all deflections* (sheet 2).

UNIT TWIST LOAD NO. 2
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



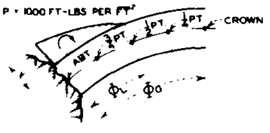
| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
| 10 | .317,309,8 | .051,686,51 | .004,249,36 | .317,309,8 | .037,981,82 | .002,292,79 | .317,309,8 | .024,204,82 | .0$^3$935,87 | .317,309,8 | .010,381,74 | .0$^3$181,19 |
| 11 | .383,944,8 | .068,729,68 | .006,218,56 | .383,944,8 | .050,527,93 | .003,356,04 | .383,944,8 | .032,209,80 | .001,370,09 | .383,944,8 | .013,817,49 | .0$^3$265,31 |
| 12 | .456,926,1 | .089,137,35 | .008,802,75 | .456,926,1 | .065,562,22 | .004,751,80 | .456,926,1 | .041,807,40 | .001,940,23 | .456,926,1 | .017,937,98 | .0$^3$375,74 |
| 13 | .536,253,6 | .113,202,8 | .012,117,75 | .536,253,6 | .083,305,81 | .006,542,96 | .536,253,6 | .053,141,05 | .002,672,07 | .536,253,6 | .022,805,36 | .0$^3$517,53 |
| 14 | .621,927,2 | .141,215,2 | .016,289,03 | .621,927,2 | .103,978,4 | .008,797,69 | .621,927,2 | .066,353,85 | .003,593,57 | .621,927,2 | .028,481,72 | .0$^3$696,08 |
| 15 | .713,947,1 | .173,461,3 | .021,451,78 | .713,947,1 | .127,798,5 | .011,589,56 | .713,947,1 | .081,588,50 | .004,734,94 | .713,947,1 | .035,029,12 | .0$^3$917,27 |
| 16 | .812,313,1 | .210.223,2 | .027,750,67 | .812,313,1 | .154,982,8 | .014,997,39 | .812,313,1 | .098,987,31 | .006,128,56 | .812,313,1 | .042,509,59 | .001,187,4 |
| 17 | .917,025,4 | .251,779,1 | .035,339,88 | .917,025,4 | .185,746,4 | .019,105,39 | .917,025,4 | .118,692,1 | .007,809,11 | .917,025,4 | .050,985,11 | .001,513,2 |
| 18 | 1.028,084 | .298,402,9 | .044,382,88 | 1.028,084 | .220,302,8 | .024,002,88 | 1.028,084 | .140,844,4 | .009,813,37 | 1.028,084 | .060,517,64 | .001,901,8 |
| 19 | 1.145,488 | .350,363,9 | .055,052,4 | 1.145,488 | .258,863,5 | .029,784,5 | 1.145,488 | .165,585,0 | .012,180,3 | 1.145,488 | .071,169,06 | .002,360,9 |
| 20 | 1.269,239 | .407,926,2 | .067.530,0 | 1.269,239 | .301,638,0 | .036,549,8 | 1.269,239 | .193,054,3 | .014,951,1 | 1.269,239 | .083,001,24 | .002,898,3 |
| 21 | 1.399,336 | .471,349,0 | .082,007,2 | 1.399,336 | .348,834,0 | .044,404,3 | 1.399,336 | .223,392,1 | .018,169,5 | 1.399,336 | .096,075,98 | .003,522,8 |
| 22 | 1.535,780 | .540.888,3 | .098,683,3 | 1.535,780 | .400,656,6 | .053,457,8 | 1.535,780 | .256,737,8 | .021,880,7 | 1.535,780 | .110,455,0 | .004,243,2 |
| 23 | 1.678,569 | .616,786,0 | .117,766,2 | 1.678,569 | .457,308,8 | .063,824,8 | 1.678,569 | .293,229,8 | .026,132,4 | 1.678,569 | .126,200,1 | .005,068,5 |
| 24 | 1.827,705 | .699,290,5 | .139,473,3 | 1.827,705 | .518,991,5 | .075,626,3 | 1.827,705 | .333,006,4 | .030,974,9 | 1.827,705 | .143,372,8 | .006,009,0 |
| 25 | 1.983,186 | .788.635,8 | .164,030,8 | 1.983,186 | .585,902,6 | .088,986,9 | 1,983,186 | .376,204,7 | .036,460,0 | 1.983,186 | .162,034,6 | .007,074,5 |
| 26 | 2.145,014 | .885,051,9 | .191,671,2 | 2.145,014 | .658,237,9 | .104,036,9 | 2.145,014 | .422,961,5 | .042,641,9 | 2.145,014 | .182,247,3 | .008,275,7 |
| 27 | 2.313,189 | .988,761,8 | .222,636,6 | 2.313,189 | .736,190,1 | .120,910,9 | 2,313,189 | .473,412,5 | .049,577,0 | 2313,189 | .204,072,1 | .009,623,5 |
| 28 | 2.487.709 | 1.099,982 | .257,176,9 | 2.487,709 | .819,949,3 | .139,749,0 | 2.487.709 | .527,693,2 | .057,323,6 | 2487,709 | .227,570,4 | .011,129,7 |
| 29 | 2.668,576 | 1.218,922 | .295,549,8 | 2.668,576 | .909,702,8 | .160,695,6 | 2.668,576 | .585,937,8 | .065,942,7 | 2.668,576 | .252,803,6 | .012,806,1 |
| 30 | 2.855,788 | 1.345,783 | .338,020,1 | 2.855,788 | 1.005,635 | .183,900,0 | 2.855,788 | .648,279,9 | .075,496,7 | 2.855,788 | .279,832,9 | .014,664,9 |
| 31 | 3.049,347 | 1,480,760 | .384,860,3 | 3.049,347 | 1.107,926 | .209,516,1 | 3.049,347 | .714,852,4 | .086,050,5 | 3.049,347 | .308,719,5 | .016,719,1 |
| 32 | 3.249.252 | 1.624.039 | .436,350,2 | 3.249,252 | 1.216,755 | .237,702,5 | 3.249,252 | .785,787,0 | .097,671,1 | 3.249,252 | .339,524,4 | .018,981,7 |
| 33 | 3.455.504 | 1,775,803 | .492,776,6 | 3.455,504 | 1.332,296 | .268,622,3 | 3.455,504 | .861,214,8 | .110,427,4 | 3.455,504 | .372,308,6 | .021,486,4 |
| 34 | 3.668,101 | 1.936,218 | .554,433,4 | 3.668,101 | 1.454,721 | .302,443,1 | 3.668,101 | .941,265,9 | .124,390,5 | 3.668,101 | .407,133,0 | .024,187,1 |
| 35 | 3.887,045 | 2.105,448 | .621,620,3 | 3.887,045 | 1.584,197 | .339,337,0 | 3.887,045 | 1.028,070 | .139,633,6 | 3.887,045 | .444,058,7 | .027,158,6 |
| 36 | 4.112,335 | 2.283.648 | .694,644,0 | 4.112,335 | 1.720,888 | .379,480,1 | 4.112,335 | 1.115,754 | .156,231,7 | 4.112,335 | .483,146,1 | .030,395,5 |
| 37 | 4.343,971 | 2,470,983 | .773,817,7 | 4.343.971 | 1.864,955 | .423,053,8 | 4.343,971 | 1.210,446 | .174,262,3 | 4.343,971 | .524,455,9 | .033,913,5 |
| 38 | 4.581,954 | 2.667,528 | .859,459,8 | 4.581,954 | 2.016,556 | .470,242,4 | 4.581,954 | 1.310,272 | .193,804,3 | 4.581,954 | .568,048,8 | .037,727,9 |
| 39 | 4.826,282 | 2.873,472 | .951,894,9 | 4.826,282 | 2.175.842 | .521,235,3 | 4.826,282 | 1,415,357 | .214,939,3 | 4.826,282 | .613,985,2 | .041,855,2 |
| 40 | 5.076,958 | 3.088,913 | 1.051,453 | 5.076,958 | 2.342,964 | .576,225,5 | 5.076,958 | 1.525,826 | .237,750,2 | 5.076,958 | .662,325,4 | .046,311,8 |
| 41 | 5.333,978 | 3.313,960 | 1.158,470 | 5.333,978 | 2.518,066 | .635,410,2 | 5.333,978 | 1.641,800 | .262,322,5 | 5.333,978 | .713,129,6 | .051,114,8 |
| 42 | 5.597,345 | 3.548,711 | 1.273,286 | 5.597,345 | 2.701,290 | .698.991,5 | 5.597,345 | 1.763,402 | .288,743,5 | 5.597,345 | .766,458,1 | .056,281,9 |
| 43 | 5.867,059 | 3.793,256 | 1.396,245 | 5.867,059 | 2.892,772 | .767,172,3 | 5.867,059 | 1,890,753 | .317,101,8 | 5.867,059 | .822,370,9 | .061,830,4 |
| 44 | 6.143,118 | 4.047,675 | 1.527,698 | 6.143,118 | 3.092,644 | .840,163,5 | 6.143,118 | 2.023,972 | .347,489,4 | 6.143,118 | .880,927,9 | .067,779,3 |
| 45 | 6.425,524 | 4.312,037 | 1.667,999 | 6.425,524 | 3.301,036 | .918,176,8 | 6.425,524 | 2.163,177 | .379,998.7 | 6.425,524 | .942,189,0 | .074,146,9 |
| 46 | 6.714,276 | 4.586,403 | 1.817,506 | 6.714,276 | 3.518,071 | 1.001,429 | 6.714,276 | 2.308,486 | .414,725,2 | 6.714,276 | 1.006,214 | .080,952,5 |
| 47 | 7.009,374 | 4.870,820 | 1.978,580 | 7.009,374 | 3.743,868 | 1.090,138 | 7.009,374 | 2.460,013 | .451,764,8 | 7.009,374 | 1.073,062 | .088,215,0 |
| 48 | 7.310,818 | 5.165,329 | 2.145,589 | 7.310,818 | 3.978,542 | 1.184,529 | 7.310,818 | 2.617,874 | .491,217,7 | 7.310,818 | 1.142,793 | .095,955,6 |
| 49 | 7.618,609 | 5.469,956 | 2.324,898 | 7.618,609 | 4.222,204 | 1.284,827 | 7.618,609 | 2.782,182 | .533,183,2 | 7.618,609 | 1.215,466 | .104,193,7 |
| 50 | 7.932,746 | 5.784,720 | 2.514,881 | 7.932,746 | 4.474,958 | 1.391,263 | 7.932,746 | 2.953,048 | .577,765,2 | 7.932,746 | 1.291,140 | .112,951,1 |
| 51 | 8.253,228 | 6.109,627 | 2.715,913 | 8.253,228 | 4.736,907 | 1.504,071 | 8.253,228 | 3.130,584 | .625,066,7 | 8.253,228 | 1.369,875 | .122,247,6 |
| 52 | 8.580,058 | 6.444,674 | 2.928,369 | 8.580,058 | 5.008,144 | 1.623,485 | 8.580,058 | 3.314,897 | .675,194,6 | 8.580,058 | 1.451,729 | .132,106,0 |
| 53 | 8.913,232 | 6.789,845 | 3.152,629 | 8.913.232 | 5.288,762 | 1.749,745 | 8.913,232 | 3.506,097 | .728,256,2 | 8.913,232 | 1.536,762 | .142,547,6 |
| 54 | 9.252.754 | 7.145,114 | 3.389,073 | 9.252.754 | 5.578,845 | 1.883,093 | 9.252,754 | 3.704,288 | .784,362,1 | 9.252,754 | 1.625,032 | .153,595,6 |
| 55 | 9.598.622 | 7.510,445 | 3.638,085 | 9.598.622 | 5.878,476 | 2.023,775 | 9.598,622 | 3.909,576 | .843,623,7 | 9.598,622 | 1.716,597 | .165,272,7 |
| 56 | 9.950.836 | 7.885,788 | 3.900,048 | 9.950,836 | 6.187,729 | 2,172,037 | 9.950,836 | 4.122,065 | .906,154,2 | 9.950,836 | 1.811,515 | .177,602,2 |
| 57 | 10,309,40 | 8.271,085 | 4.175,347 | 10.309,40 | 6.506,675 | 2.328,131 | 10.309,40 | 4.341,855 | .972,068,2 | 10.309,40 | 1.909,847 | .190,607,2 |
| 58 | 10.674,30 | 8.666,263 | 4.464,367 | 10.674,30 | 6.835,379 | 2.492,309 | 10.674,30 | 4.569,048 | 1.041,483 | 10.674,30 | 2.011,648 | .204,312 |
| 59 | 11,045,55 | 9.071,241 | 4.767,496 | 11.045.55 | 7.173,901 | 2.664.827 | 11.045,55 | 4.803,742 | 1.114,517 | 11.045,55 | 2.116,978 | .218,743 |
| 60 | 11.423,15 | 9.485,926 | 5.085,117 | 11.423.15 | 7.522,296 | 2.845,942 | 11.423.15 | 5.046,034 | 1.191,290 | 11,423,15 | 2.225,894 | .233,922 |
| 61 | 11.807,10 | 9.910,210 | 5.417,619 | 11.807,10 | 7.880,613 | 3.035,915 | 11.807.10 | 5.296,020 | 1,271,924 | 11.807,10 | 2.338,454 | .249,877 |
| 62 | 12.197,39 | 10.343,98 | 5.765,385 | 12.197,39 | 8.248,895 | 3.235,006 | 12.197,39 | 5.553,794 | 1.356,543 | 12.197,39 | 2.454,715 | .266,633 |
| 63 | 12.594,03 | 10.787,10 | 6.128,803 | 12.594.03 | 8.627,181 | 3.443,482 | 12.594,03 | 5.819,447 | 1.445,271 | 12.594,03 | 2.574,735 | .284,215 |
| 64 | 12,997,01 | 11.239,44 | 6.508,255 | 12.997.01 | 9.015,502 | 3.661,608 | 12.997,01 | 6.093,072 | 1.538,236 | 12,997,01 | 2.698,570 | .302,652 |
| 65 | 13.406,34 | 11.700,85 | 6.904,122 | 13.406,34 | 9.413,887 | 3.889,651 | 13.406,34 | 6.374,755 | 1.635,565 | 13.406,34 | 2.826,279 | .321,968 |
| 66 | 13.822,02 | 12.171,15 | 7.316,789 | 13.822,02 | 9.822,356 | 4.127,882 | 13.822,02 | 6.664,586 | 1.737,389 | 13.822,02 | 2.957,918 | .342,192 |
| 67 | 14.244,04 | 12.650,18 | 7.746,632 | 14.244.04 | 10.240,93 | 4.376,572 | 14.244.04 | 6.962,649 | 1.843,639 | 14.244,04 | 3.093.543 | .363,353 |
| 68 | 14.672,41 | 13.137,75 | 8.194,027 | 14.672,41 | 10.669,60 | 4.635,994 | 14.672,41 | 7.269,028 | 1.955,048 | 14.672,41 | 3.233,211 | .385,478 |
| 69 | 15.107,12 | 13.633,67 | 8.659,350 | 15.107,12 | 11.108,39 | 4.906,421 | 15.107,12 | 7.583.806 | 2.071,149 | 15.107,12 | 3.376,978 | .408,594 |
| 70 | 15.548,18 | 14.137,72 | 9.142,971 | 15.548,18 | 11.557.30 | 5.188,131 | 15.548,18 | 7.907,061 | 2.192,280 | 15.548,18 | 3.524,902 | .432,732 |
| 71 | 15.995,59 | 14.649,68 | 9.645,261 | 15.995,59 | 12.016,30 | 5.481,400 | 15.995.59 | 8.238,873 | 2.318,578 | 15.995,59 | 3.677,037 | .457,923 |
| 72 | 16.449,34 | 15.169,32 | 10.166,58 | 16.449,34 | 12.485,40 | 5.786,503 | 16.449,34 | 8.579,319 | 2.450,180 | 16.449,34 | 3.833,439 | .484,192 |
| 73 | 16.909,44 | 15.696,39 | 10.707,29 | 16.909,44 | 12.964,56 | 6.103,722 | 16.909,44 | 8.928,473 | 2.587,227 | 16.909,44 | 3.994,165 | .511,572 |
| 74 | 17.375,89 | 16.230,65 | 11.267,75 | 17.375,89 | 13.453,77 | 6.433,336 | 17.375.89 | 9.286,407 | 2.729,862 | 17.375,89 | 4.159,269 | .540,095 |
| 75 | 17.848,68 | 16.771,83 | 11.848,31 | 17.848.68 | 13.952.99 | 6.775,625 | 17.848,68 | 9.653,193 | 2.878,228 | 17.848,68 | 4.328,806 | .569,790 |
| 76 | 18.327,82 | 17.319,64 | 12.449.32 | 18.327,82 | 14.462,19 | 7.130,870 | 18.327.82 | 10.028,90 | 3.032,468 | 18.327,82 | 4.502,834 | .600,690 |
| 77 | 18.813,30 | 17.873,80 | 13.071.13 | 18.813.30 | 14.981,32 | 7.499,354 | 18.813.30 | 10.413,60 | 3.192,728 | 18.813,30 | 4.681,404 | .632,825 |
| 78 | 19.305.13 | 18.434.01 | 13.714,06 | 19.305.13 | 15.510,33 | 7.881,357 | 19.305.13 | 10.807,34 | 3.359,155 | 19.305,13 | 4.864,574 | .666,227 |
| 79 | 19.803.31 | 18.999.96 | 14.378,45 | 19.803,31 | 16.049,16 | 8.277,162 | 19.803,31 | 11.210,21 | 3.531,897 | 19.803,31 | 5.052,396 | .700,930 |
| 80 | 20.307.83 | 19.571.33 | 15.064.64 | 20.307.83 | 16.597.75 | 8.687,052 | 20.307,83 | 11.622,25 | 3.711,105 | 20.307,83 | 5.244,926 | .736,967 |
| 81 | 20.818,70 | 20.147.78 | 15.772.93 | 20.818.70 | 17.156,04 | 9.111,309 | 20.818.70 | 12.043,52 | 3.896,928 | 20.818,70 | 5.442,218 | .774,371 |
| 82 | 21.335,91 | 20.728,97 | 16.503,64 | 21.335,91 | 17.723,95 | 9.550,216 | 21.335,91 | 12.474,10 | 4.089,519 | 21.335,91 | 5.644,325 | .813,175 |
| 83 | 21.859,47 | 21.314,55 | 17.257,08 | 21.859,47 | 18.301,40 | 10.004,05 | 21.859,47 | 12.914,02 | 4.289,031 | 21.859,47 | 5.851,300 | .853,414 |
| 84 | 22.389,38 | 21.904,16 | 18.033.54 | 22.389,38 | 18.888,30 | 10.473,11 | 22.389,38 | 13.363,34 | 4.495,617 | 22.389,38 | 6.063,199 | .895,122 |
| 85 | 22.925,63 | 22.497,42 | 18.833,33 | 22.925.63 | 19.484.56 | 10.957,65 | 22.925,63 | 13.822,11 | 4.709,435 | 22.925,63 | 6.280,073 | .938,334 |
| 86 | 23.468,23 | 23.093,94 | 19.656.72 | 23.468,23 | 20.090,08 | 11.457,97 | 23.468,23 | 14.290,39 | 4.930,639 | 23.468,23 | 6.501,976 | .983,086 |
| 87 | 24.017,18 | 23.693.33 | 20.503,99 | 24.017,18 | 20.704,75 | 11.374.35 | 24.017.18 | 14.768,21 | 5.159,389 | 24.017,18 | 6.728,961 | 1.029,412 |
| 88 | 24.572,47 | 24.295,19 | 21.375,41 | 24.572,47 | 21.328,47 | 12.507,07 | 24.572,47 | 15.255,63 | 5.395,841 | 24.572,47 | 6.961,080 | 1.077,350 |
| 89 | 25.134,11 | 24.899,10 | 22.271,24 | 25.134,11 | 21.961.10 | 13.056,40 | 25.134,11 | 15.752,88 | 5.640,156 | 25.134,11 | 7.198,367 | 1.126,935 |
| 90 | 25.702.10 | 25.504,64 | 23.191.73 | 25.702.10 | 22.602,54 | 13.622.62 | 25.702.10 | 16.259,41 | 5.892,496 | 25.702,10 | 7.440,932 | 1.178,205 |

SHEET 2 OF 5

205-D-415

Table H-12.–*Load constants for circular arches–all deflections* (sheet 3).

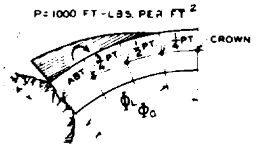
## UNIT TWIST LOAD NO. 3
## TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



| $\phi$ | Crown | | | $\frac{1}{4}$ Point | | | $\frac{1}{2}$ Point | | | $\frac{3}{4}$ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
| 10 | 1.269.239 | .193054,3 | .014.951,1 | 1.269,239 | .138.159,3 | .007,724,06 | 1.269,239 | .083,001,24 | .002,898.3 | .634,619,6 | .019,033,49 | .0$^3$317,11 |
| 11 | 1.535.780 | .256,737,8 | .021.880.7 | 1.535,780 | .183,808.1 | .011,306,3 | 1.535.780 | .110,455,0 | .004,243,2 | .767,889,7 | .025,332,53 | .0$^3$464,31 |
| 12 | 1.827.705 | .333,0064 | .030974.9 | 1.827,705 | .238,516,4 | .016,009.1 | 1.827,705 | .143,372,8 | .006.009,0 | .913,852,3 | .032,887,03 | .0$^3$657,57 |
| 13 | 2.145,014 | .422,961,5 | .042.641.9 | 2.145,014 | .303,091,9 | .022,0444 | 2.145,014 | .182,247,3 | .008,275.7 | 1.072,507 | .041,810,92 | .0$^3$905,66 |
| 14 | 2487.709 | .527,693,2 | .057.323.6 | 2.487,709 | .378,337.5 | .029,641,9 | 2487.709 | .227,570,4 | .011,129,7 | 1.243,855 | .052,218,05 | .001,218,2 |
| 15 | 2.855.788 | .648,279.9 | .075.496.7 | 2.855,788 | .465,052,1 | .039,050,0 | 2.855.788 | .279,832,9 | .014.664,9 | 1427,894 | .064,222,27 | .001,605,2 |
| 16 | 3.249.252 | .785.787.0 | .097.671.1 | 3.249,252 | .564,029,6 | .050,534,4 | 3.249.252 | .339,524,4 | .018,981,7 | 1.624.626 | .077,937,31 | .002,077.9 |
| 17 | 3.668.101 | .941.265.9 | .124,390,5 | 3.668,101 | .676,058,6 | .064.379,3 | 3.668.101 | .407,133,0 | .024,187,1 | 1.834,051 | .093,476,86 | .002,648,1 |
| 18 | 4.112.335 | 1.115.754 | .156,231,7 | 4.112,335 | .801,922,0 | .080,886,0 | 4.112,335 | .483,146,1 | .030,395.5 | 2.056,168 | .110,954,5 | .003,328,3 |
| 19 | 4.581.954 | 1.310.272 | .193,804,3 | 4.581,954 | .942,397.1 | .100,374,1 | 4.581,954 | .568,048,8 | .037,727.9 | 2290,977 | .130,483,9 | .004,131,7 |
| 20 | 5.076.958 | 1.525.826 | 237,750,2 | 5.076,958 | 1.098,255 | .1231804 | 5.076,958 | .662,325,4 | .046,311,8 | 2538,478 | .152,178,3 | .005,072,3 |
| 21 | 5.597.345 | 1.763.402 | .288,743,5 | 5.597,345 | 1.270,259 | .149,659,3 | 5.597.345 | .766,458.1 | .056,281,9 | 2.798.673 | .176,151,3 | .006,165,2 |
| 22 | 6.143.118 | 2.023.972 | .347489,4 | 6.143,118 | 1.459.168 | .180,182,2 | 6.143,118 | .880,927,9 | .067,779,3 | 3.071,559 | .202,516,0 | .007,425,8 |
| 23 | 6 714.276 | 2.308.486 | .414,725,2 | 6.714,276 | 1.665.731 | .215,138,5 | 6.714,276 | 1.006.214 | .080,952,5 | 3.357,138 | .231,385,6 | .008,807,3 |
| 24 | 7.310.818 | 2.617.874 | .491,217,7 | 7.310,818 | 1.890,691 | .254,933,8 | 7.310,818 | 1.142,793 | .095,955,6 | 3.655,409 | .262,873,3 | .010,516,0 |
| 25 | 7.932,746 | 2.953,048 | .577,765,2 | 7.932,746 | 2.134,783 | .299,991,6 | 7.932,746 | 1.291,140 | .112,951 | 3.966,373 | .297,092,1 | .012,380,8 |
| 26 | 8.580,058 | 3.314.897 | .675,194,6 | 8.580,058 | 2.398,733 | .350,751,0 | 8.580,058 | 1.451,729 | .132,106 | 4.290,029 | .334,154,7 | .014,482,9 |
| 27 | 9.252.754 | 3.704.288 | .784,362,1 | 9.252,754 | 2.683,259 | .407,668,7 | 9.252,754 | 1.625,032 | .153,596 | 4.626,377 | .374,174,1 | .016,841,9 |
| 28 | 9.950,836 | 4.122,065 | .906,154,2 | 9.950,836 | 2.989,070 | .471,218,6 | 9.950,836 | 1.811,515 | .177,602 | 4.975,418 | .417,262,8 | .019,477,9 |
| 29 | 10.674.30 | 4.569,048 | 1.041,483 | 10.674,30 | 3.316.867 | .541,889 | 10.674,30 | 2.011,648 | .204,312 | 5.337,151 | .463,533,3 | .022,411,8 |
| 30 | 11,423.15 | 5.046,034 | 1.191,290 | 11,423,15 | 3.667,339 | .620,185 | 11,423,15 | 2.225,894 | .233,922 | 5.711,577 | .513,098,1 | .025,664,9 |
| 31 | 12.197.39 | 5.553,794 | 1.356,543 | 12.197,39 | 4.041,167 | .706,630 | 12.197,39 | 2454,715 | .266,633 | 6.098,695 | .566,069,5 | .029,259,9 |
| 32 | 12.997,01 | 6.093,072 | 1.538,236 | 12.997,01 | 4439.021 | .801,761 | 12.997,01 | 2.698,570 | .302,652 | 6498,505 | .622,559,6 | .033,219.7 |
| 33 | 13.822,02 | 6.664.586 | 1,737,389 | 13.822,02 | 4.861,562 | .906,129 | 13.822,02 | 2.957,918 | .342,192 | 6.911,007 | .682,680,3 | .037,568,2 |
| 34 | 14.672,41 | 7.269.028 | 1.955,048 | 14.672,41 | 5.309,439 | 1.020,306 | 14.672,41 | 3.233,211 | .385,478 | 7.336,203 | .746,543,7 | .042,330,6 |
| 35 | 15.548.18 | 7.907,061 | 2.192,280 | 15.548,18 | 5.783,290 | 1.144,873 | 15.548,18 | 3.524,902 | .432,732 | 7.774,091 | .814,261,3 | .047,530,8 |
| 36 | 16.449,34 | 8.579,319 | 2450,180 | 16.449,34 | 6.283,742 | 1.280,431 | 16.449,34 | 3.833,439 | .484,192 | 8.224,671 | .885,944,7 | .053,196,2 |
| 37 | 17.375,89 | 9.286,407 | 2.729,862 | 17.375,89 | 6.811,411 | 1.427,593 | 17.375,89 | 4.159,269 | .540,095 | 8.687,943 | .961,705,4 | .059,353,0 |
| 38 | 18.327.82 | 10.028,90 | 3.032,468 | 18.327,82 | 7.366,900 | 1.586,988 | 18.327,82 | 4.502,834 | .600,690 | 9.163,907 | 1.041,655 | .066,028,9 |
| 39 | 19.305,13 | 10.807.34 | 3.359,155 | 19.305,13 | 7.950,799 | 1.759,260 | 19.305,13 | 4.864,574 | .666,227 | 9.652,565 | 1.125,903 | .073,252,5 |
| 40 | 20.307,83 | 11,622.25 | 3.711,105 | 20.307,83 | 8.563,689 | 1.945,064 | 20.307,83 | 5.244,926 | .736,967 | 10.153,91 | 1.214,563 | .081,052 |
| 41 | 21.335.91 | 12474.10 | 4.089,519 | 21.335,91 | 9.206,134 | 2.145,075 | 21.335,91 | 5.644,325 | .813,175 | 10.667,96 | 1.307,743 | .089,459 |
| 42 | 22.389,38 | 13.363.34 | 4.495,617 | 22.389,38 | 9.878,688 | 2.359,975 | 22.389,38 | 6.063,199 | .895,122 | 11.194,69 | 1.405,555 | .098,502 |
| 43 | 23468.23 | 14.290.39 | 4.930,639 | 23.468,23 | 10.581,89 | 2.590,466 | 23.468,23 | 6.501,976 | .983,086 | 11.734,12 | 1.508,110 | .108,214 |
| 44 | 24.572.47 | 15.255.63 | 5.395,841 | 24.572,47 | 11.316,27 | 2.837,259 | 24.572,47 | 6.961,080 | 1.077,350 | 12.286,24 | 1.615,517 | .118,626 |
| 45 | 25.702.10 | 16.259.41 | 5.892.496 | 25.702,10 | 12.082,33 | 3.101,078 | 25.702,10 | 7.440,932 | 1.178,205 | 12.851,05 | 1.727,886 | .129,771 |
| 46 | 26.857.10 | 17.302,05 | 6.421,893 | 26.857,10 | 12.880,58 | 3.382,662 | 26.857,10 | 7.941,946 | 1.285,946 | 13.428,55 | 1.845,327 | .141,682 |
| 47 | 28.037.50 | 18.383.82 | 6.985.340 | 28.037,50 | 13.711,50 | 3.682,766 | 28.037,50 | 8.464,537 | 1.400,875 | 14.018,75 | 1.967,950 | .154,395 |
| 48 | 29.243.27 | 19.504.96 | 7.584,153 | 29.243,27 | 14.575,55 | 4.002,147 | 29.243,27 | 9.009,113 | 1.523,299 | 14.621,64 | 2.095,865 | .167,943 |
| 49 | 30.474.43 | 20.665,69 | 8.219,670 | 30.474,43 | 15.473,19 | 4.341,588 | 30.474,43 | 9.576,080 | 1.653,537 | 15.237,22 | 2.229,179 | .182,363 |
| 50 | 31.730.98 | 21.866,18 | 8.893,228 | 31.730,98 | 16.404,87 | 4.701,866 | 31.730,98 | 10.165,84 | 1.791,900 | 15.865,49 | 2.368,003 | .197,691 |
| 51 | 33.012.91 | 23.106.56 | 9.606,198 | 33.012,91 | 17.371,00 | 5.083,794 | 33.012,91 | 10.778,79 | 1.938,725 | 16.506,46 | 2.512,446 | .213,964 |
| 52 | 34.320.23 | 24.386.93 | 10.359,93 | 34.320,23 | 18.372,00 | 5.488,167 | 34.320,23 | 11.415,32 | 2.094,331 | 17.180,12 | 2.662,614 | .231,219 |
| 53 | 35.652.94 | 25.707.35 | 11.155,83 | 35.652,94 | 19.408,25 | 5.915,825 | 35.652,94 | 12.075,83 | 2.259,070 | 17.826,47 | 2.818,618 | .249,496 |
| 54 | 37.011.02 | 27.067.83 | 11.995.25 | 37.011,02 | 20.480,13 | 6.367,573 | 37.011,02 | 12.760,69 | 2.433,262 | 18.505,51 | 2.980,565 | .268,835 |
| 55 | 38.394.49 | 28.468.38 | 12.879,61 | 38.394,49 | 21.588,01 | 6.844,278 | 38.394,49 | 13.470,30 | 2.617,277 | 19.197,24 | 3.148,563 | .289,274 |
| 56 | 39.803.34 | 29.908.92 | 13.810,30 | 39.803,34 | 22.732,21 | 7.346,785 | 39.803,34 | 14.205,02 | 2.811,463 | 19.901,67 | 3.322,720 | .310,856 |
| 57 | 41.237.58 | 31.389.38 | 14.788,75 | 41.237,58 | 23.913,09 | 7.875,955 | 41.237,58 | 14.965,23 | 3.016,177 | 20.618,79 | 3.503,142 | .333,621 |
| 58 | 42.697.21 | 32.909.61 | 15.816,35 | 42.697,21 | 25.130,93 | 8.432,660 | 42.697,21 | 15.751,30 | 3.231,784 | 21.348,60 | 3.689,937 | .357,612 |
| 59 | 44.182.22 | 34.469.45 | 16.894.54 | 44.182,22 | 26.386,04 | 9.017,786 | 44.182,22 | 16.563,58 | 3.458,659 | 22.091,11 | 3.883,213 | .382,872 |
| 60 | 45.692.61 | 36.068,68 | 18.024,72 | 45.692,61 | 27.678,69 | 9.632,217 | 45.692,61 | 17.402,45 | 3.697,172 | 22.846,31 | 4083,075 | .409,445 |
| 61 | 47.228,39 | 37.707.06 | 19.208,33 | 47.228,39 | 29.009,14 | 10.276,86 | 47.228,39 | 18.268,25 | 3.947,713 | 23.614,20 | 4.289,629 | .437,373 |
| 62 | 48.789.56 | 39.384,29 | 20.446,80 | 48.789.56 | 30.377,63 | 10.952,62 | 48.789,56 | 19.161,33 | 4.210,661 | 24.394.78 | 4.502,983 | .466,704 |
| 63 | 50.376.11 | 41.100,06 | 21.741,55 | 50.376,11 | 31 784.38 | 11.660,41 | 50.376,11 | 20.082,04 | 4.486,410 | 25.188,05 | 4.723,242 | .497,484 |
| 64 | 51.988.04 | 42.853,97 | 23.094,00 | 51.988,04 | 33.229,61 | 12.401,15 | 51.988,04 | 21.030,72 | 4.775.360 | 25.994,02 | 4.950,511 | .529,758 |
| 65 | 53.625,36 | 44.645,64 | 24.505,59 | 53.625,36 | 34.713,48 | 13.175,78 | 53.625,36 | 22.007,71 | 5.077,910 | 26.812,69 | 5.184,895 | .563,582 |
| 66 | 55.288,06 | 46.474.60 | 25.977.73 | 55.288,06 | 36.236,17 | 13.985,24 | 55.288,06 | 23.013,32 | 5.394,469 | 27.644,03 | 5.426,500 | .598,978 |
| 67 | 56.976.15 | 48.340,36 | 27.511.85 | 56.976,15 | 37.797,84 | 14.830,47 | 56.976,15 | 24.047,90 | 5.725,452 | 28.488,08 | 5.675,431 | .636,022 |
| 68 | 58.689.62 | 50.242,39 | 29.109,35 | 58.689,62 | 39.398,60 | 15.712,41 | 58.689,62 | 25.111,75 | 6.071,275 | 29.344,81 | 5.931,791 | .674,753 |
| 69 | 60.428,48 | 52.180,13 | 30.771,65 | 60.428,48 | 41.038,58 | 16.632,03 | 60.428,48 | 26.205,20 | 6.432,354 | 30.214,24 | 6.195,686 | .715,223 |
| 70 | 62.192,72 | 54.152,96 | 32.500,15 | 62.192,72 | 42.717,87 | 17.590,30 | 62.192,72 | 27.328,55 | 6.809,127 | 31.098,36 | 6.467,217 | .757,481 |
| 71 | 63.982.35 | 56.160.23 | 34.296,25 | 63.982,35 | 44.436,54 | 18.588,18 | 63.982,35 | 28.482,11 | 7.202,019 | 31.991,18 | 6.746,491 | .801,582 |
| 72 | 65.797.36 | 58.201,25 | 36.161,33 | 65.797,36 | 46.194,64 | 19.626,84 | 65.797,36 | 29.666,17 | 7.611,469 | 32.898,68 | 7.033,608 | .847,572 |
| 73 | 67.637.76 | 60.275,28 | 38.096,77 | 67.637.76 | 47.992,21 | 20.706,67 | 67.637,76 | 30.881,04 | 8.037,920 | 33.818,88 | 7.328,674 | .895,510 |
| 74 | 69.503,54 | 62.381.55 | 40.103.93 | 69.503,54 | 49.829,27 | 21.829,24 | 69.503,54 | 32.126,99 | 8.481,813 | 34.751,77 | 7.631,788 | .945,447 |
| 75 | 71.394,71 | 64.519,25 | 42.184,18 | 71.394,71 | 51.705,80 | 22.995,34 | 71.394,71 | 33.404.32 | 8.943,603 | 35.697,35 | 7.943,056 | .997,435 |
| 76 | 73.311.26 | 66.687,52 | 44.338,85 | 73.311,26 | 53.621,80 | 24.205,97 | 73.311,26 | 34.713,29 | 9.423,749 | 36.655,63 | 8.262,577 | 1.051,534 |
| 77 | 75.253,19 | 68.885,49 | 46.569,27 | 75.253,19 | 55.577,21 | 25.462,12 | 75.253,19 | 36.054,18 | 9.922,701 | 37.626,60 | 8.590,455 | 1.107,797 |
| 78 | 77.220,52 | 71.112,20 | 48.876.77 | 77.220,52 | 57.571,97 | 26.764,79 | 77.220,52 | 37.427,26 | 10.440,93 | 38.610,26 | 8.926,789 | 1.166,281 |
| 79 | 79.213,22 | 73.366,69 | 51.262.63 | 79.213,22 | 59.606,00 | 28.114,96 | 79.213,22 | 38.832,78 | 10.978,90 | 39.606,61 | 9.271,682 | 1.227,043 |
| 80 | 81.231,31 | 75.647,96 | 53.728,15 | 81.231,31 | 61.679,19 | 29.513,87 | 81.231,31 | 40.271,01 | 11.537,10 | 40.615,66 | 9.625,233 | 1.290,139 |
| 81 | 83.274.79 | 77.954,95 | 56.274,58 | 83.274,79 | 63.791,43 | 30.961,88 | 83.274,79 | 41.742,18 | 12.115,98 | 41.637,39 | 9.987,543 | 1.355.830 |
| 82 | 85.343.65 | 80.286.58 | 58.903.17 | 85.343,65 | 65.942,56 | 32.460,64 | 85.343,65 | 43.246,54 | 12.716,03 | 42.671,82 | 10.358,71 | 1.423,573 |
| 83 | 87.437.89 | 82.641.72 | 61.615,15 | 87.437.89 | 68.132,42 | 34.010,92 | 87.437,89 | 44.784,32 | 13.337,75 | 43.718,95 | 10.738,84 | 1.494,031 |
| 84 | 89.557.52 | 85.019.21 | 64.411,73 | 89.557,52 | 70.360,82 | 35.613,75 | 89.557,52 | 46.355.77 | 13.981,61 | 44.778,76 | 11.128,03 | 1.567,062 |
| 85 | 91.702.54 | 87.417.85 | 67.294.10 | 91.702,54 | 72.627,57 | 37.270.13 | 91.702,54 | 47.961,09 | 14.648,11 | 45.851,27 | 11.526,37 | 1.642,726 |
| 86 | 93.872,94 | 89.836,39 | 70.263,41 | 93.872,94 | 74.932,44 | 38.981,07 | 93.872,94 | 49.600,52 | 15.337,75 | 46.936,47 | 11.933,96 | 1.721,088 |
| 87 | 96.068.72 | 92.273.58 | 73.320,80 | 96.068.72 | 77.275,17 | 40.747,58 | 96.068,72 | 51.274,26 | 16.051,02 | 48.034,36 | 12.350,92 | 1.802.209 |
| 88 | 98.289.89 | 94.728,09 | 76.467,40 | 98.289,89 | 79.655,51 | 42.570,66 | 98.289,89 | 52.982,51 | 16.788,44 | 49.144,94 | 12.777,32 | 1.886,152 |
| 89 | 100.536,4 | 97.198,59 | 79.704,28 | 100.536,4 | 82.073,16 | 44.451,33 | 100.536,4 | 54.725,49 | 17.550,50 | 50.268,22 | 13.213,26 | 1.972,982 |
| 90 | 102.808.4 | 99.683.68 | 83.032,53 | 102.808,4 | 84.527,82 | 46.390,58 | 102.808,4 | 56.503,37 | 18.337,73 | 51.404.19 | 13.658,86 | 2.062,762 |

Table H-12.–*Load constants for circular arches–all deflections* (sheet 4).

UNIT TWIST LOAD NO 4
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



| Φ | Crown | | | ¼ Point | | | ½ Point | | | ¾ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
| 10 | 2.855,788 | .403,497,7 | .029,575,3 | 2.855,788 | .279,832,9 | .014,664,9 | 2.115,399 | .129,119,3 | .004,347,6 | .740,389,6 | .021,917,40 | .0$^3$362,42 |
| 11 | 3.455,504 | .536,638,6 | .043,283,7 | 3.455,504 | .372,308,6 | .021,466,4 | 2.559,633 | .171,829,0 | .006,364,9 | .895,871,3 | .029,170,88 | .0$^3$530,64 |
| 12 | 4.112,335 | .696,116,2 | .061,275,6 | 4.112,335 | .483,146,1 | .030,395,5 | 3.046,174 | .223,039,8 | .009,013,7 | 1.066,161 | .037,870,04 | .0$^3$751,51 |
| 13 | 4.826,282 | .884,240,2 | .084,358,6 | 4.826,282 | .613,985,2 | .041,855,2 | 3.575,024 | .283,518,9 | .012,413,8 | 1.251,258 | .048,146,10 | .001,035,0 |
| 14 | 5.597,345 | 1.103,302 | .113,408,3 | 5.597,345 | .766,458,1 | .056,281,9 | 4.146,182 | .354,032,0 | .016,695,1 | 1.451,164 | .060,130,17 | .001,392,2 |
| 15 | 6.425,524 | 1.355,572 | .149,367,8 | 6.425,524 | .942,189,0 | .074,146,9 | 4.759,647 | .435,343,1 | .021,998,3 | 1.665,876 | .073,953,32 | .001,334,6 |
| 16 | 7.310,818 | 1.643,292 | .193,247,9 | 7.310,818 | 1.142,793 | .095,955,6 | 5.415,421 | .528,214,5 | .028,473,8 | 1.895,397 | .089,746,56 | .002,374,8 |
| 17 | 8.253,228 | 1.968,683 | .246,125,7 | 8.253,228 | 1.369,875 | .122,247,6 | 6.113,502 | .633,406,8 | .036,282,8 | 2.139,726 | .107,640,8 | .003,026,5 |
| 18 | 9.252,754 | 2.333,934 | .309,143,8 | 9.252,754 | 1.625,032 | .153,595,6 | 6.853,892 | .751,678,5 | .045,596,0 | 2.398,962 | .127,766,8 | .003,803,7 |
| 19 | 10.309,40 | 2.741,206 | .383,511,5 | 10.309,40 | 1.909,847 | .190,607,2 | 7.636,589 | .883,786,0 | .056,595,3 | 2.672,806 | .150,255,5 | .004,721,9 |
| 20 | 11.423,15 | 3.192,629 | .470,500 | 11.423,15 | 2.225,894 | .233,922 | 8.461,595 | 1.030,483 | .069,472,5 | 2.961,558 | .175,237,3 | .005,796,8 |
| 21 | 12.594,03 | 3.890,301 | .571,448 | 12.594,03 | 2.574,735 | .284,215 | 9.328,908 | 1.192,523 | .084,428,9 | 3.265,118 | .202,843,0 | .007,045,9 |
| 22 | 13.822,02 | 4.236,284 | .687,755 | 13.822,02 | 2.957,918 | .342,192 | 10.238,53 | 1.370,654 | .101,677,2 | 3.583,485 | .233,202,9 | .008,486,7 |
| 23 | 15.107,12 | 4.832,604 | .820,882 | 15.107,12 | 3.376,978 | .408,594 | 11.190,46 | 1.565,623 | .121,439,3 | 3.916,661 | .266,447,5 | .010,137,5 |
| 24 | 16.449,34 | 5.481,251 | .972,354 | 16.449,34 | 3.833,439 | .484,192 | 12.184,70 | 1.778,174 | .143,947,4 | 4.264,644 | .302,706,9 | .012,018,4 |
| 25 | 17.848,68 | 6.184,176 | 1.143,754 | 17.848,68 | 4.328,806 | .569,790 | 13.221,24 | 2.009,049 | .169,444,0 | 4.627,435 | .342,111,2 | .014,149,5 |
| 26 | 19.305,13 | 6.943,287 | 1.336,727 | 19.305,13 | 4.864,574 | .666,227 | 14.300,10 | 2.258,986 | .198,182 | 5.005,033 | .384,790,6 | .016,551,9 |
| 27 | 20.818,70 | 7.760,453 | 1.552,973 | 20.818,70 | 5.442,218 | .774,371 | 15.421,26 | 2.528,721 | .230,422 | 5.397,440 | .430,874,8 | .019,248,0 |
| 28 | 22.389,38 | 8.637,496 | 1.794,255 | 22.389,38 | 6.063,199 | .895,122 | 16.584,73 | 2.818,985 | .266,438 | 5.804,654 | .480,493,6 | .022,260,6 |
| 29 | 24.017,18 | 9.576,196 | 2.062,389 | 24.017,18 | 6.728,961 | 1.029,412 | 17.790,50 | 3.130,508 | .306,512 | 6.226,676 | .533,776,6 | .025,613,5 |
| 30 | 25.702,10 | 10.578,28 | 2.359,246 | 25.702,10 | 7.440,932 | 1.178,205 | 19.038,59 | 3.464,014 | .350,937 | 6.663,506 | .590,853,2 | .029,331,6 |
| 31 | 27.444,13 | 11.645,44 | 2.686,755 | 27.444,13 | 8.200,519 | 1.342,493 | 20.328,98 | 3.820,227 | .400,014 | 7.115,144 | .651,852,8 | .033,440,2 |
| 32 | 29.243,27 | 12.779,31 | 3.046,894 | 29.243,27 | 9.009,113 | 1.523,299 | 21.661,68 | 4.199,864 | .454,055 | 7.581,589 | .716,904,6 | .037,965,7 |
| 33 | 31.099,54 | 13.981,47 | 3.441,702 | 31.099,54 | 9.868,086 | 1.721,684 | 23.036,69 | 4.603,639 | .513,382 | 8.062,842 | .786,137,0 | .042,935,5 |
| 34 | 33.012,91 | 15.253,45 | 3.873,255 | 33.012,91 | 10.778,79 | 1.938,725 | 24.454,01 | 5.032,264 | .578,329 | 8.558,903 | .859,678,5 | .048,378,4 |
| 35 | 34.983,41 | 16.596,73 | 4.343,686 | 34.983,41 | 11.742,56 | 2.175,535 | 25.913,64 | 5.486,445 | .649,232 | 9.069,772 | .937,662,0 | .054,321,4 |
| 36 | 37.011,02 | 18.012,74 | 4.855,184 | 37.011,02 | 12.760,69 | 2.433,262 | 27.415,57 | 5.966,885 | .726,446 | 9.595,449 | 1.020,211 | .060,795,9 |
| 37 | 39.095,74 | 19.502,84 | 5.409,975 | 39.095,74 | 13.834,50 | 2.713,076 | 28.959,81 | 6.474,282 | .810,329 | 10.135,93 | 1.107,455 | .067,832 |
| 38 | 41.237,58 | 21.068,34 | 6.010,336 | 41.237,58 | 14.965,23 | 3.016,177 | 30.546,36 | 7.009,329 | .901,253 | 10.691,23 | 1.199,523 | .075,462 |
| 39 | 43.436,54 | 22.710,50 | 6.658,582 | 43.436,54 | 16.154,14 | 3.343,790 | 32.175,22 | 7.572,717 | .999,596 | 11.261,33 | 1.296,543 | .083,718 |
| 40 | 45.692,61 | 24.430,50 | 7.357,083 | 45.692,61 | 17.402,45 | 3.697,172 | 33.846,38 | 8.165,131 | 1.105,749 | 11.846,23 | 1.398,642 | .092,634 |
| 41 | 48.005,80 | 26.229,48 | 8.108,236 | 48.005,80 | 18.711,36 | 4.077,607 | 35.559,85 | 8.787,250 | 1.220,107 | 12.445,95 | 1.505,948 | .102,241 |
| 42 | 50.376,11 | 28.108,50 | 8.914,502 | 50.376,11 | 20.082,04 | 4.486,410 | 37.315,63 | 9.439,750 | 1.343,080 | 13.060,47 | 1.618,588 | .112,576 |
| 43 | 52.803,53 | 30.068,58 | 9.778,347 | 52.803,53 | 21.515,66 | 4.924,909 | 39.113,72 | 10.123,30 | 1.475,087 | 13.689,80 | 1.736,689 | .123,675 |
| 44 | 55.288,06 | 32.110,65 | 10.702,30 | 55.288,06 | 23.013,32 | 5.394,489 | 40.954,12 | 10.838,57 | 1.616,551 | 14.333,94 | 1.860,379 | .135,575 |
| 45 | 57.829,71 | 34.235,59 | 11.688,93 | 57.829,71 | 24.576,15 | 5.896,481 | 42.836,83 | 11.586,22 | 1.767,910 | 14.992,89 | 1.989,785 | .148,313 |
| 46 | 60.428,48 | 36.444,22 | 12.740,82 | 60.428,48 | 26.205,20 | 6.432,354 | 44.761,84 | 12.366,90 | 1.929,606 | 15.666,64 | 2.125,032 | .161,926 |
| 47 | 63.084,36 | 38.737,27 | 13.860,60 | 63.084,36 | 27.901,53 | 7.003,530 | 46.729,16 | 13.181,26 | 2.102,097 | 16.355,21 | 2.266,246 | .176,455 |
| 48 | 65.797,36 | 41.115,42 | 15.050,91 | 65.797,36 | 29.666,17 | 7.611,469 | 48.738,79 | 14.029,95 | 2.285,837 | 17.058,58 | 2.413,555 | .191,939 |
| 49 | 68.567,48 | 43.579,29 | 16.314,46 | 68.567,48 | 31.500,11 | 8.257,656 | 50.790,72 | 14.913,60 | 2.481,309 | 17.776,75 | 2.567,084 | .208,419 |
| 50 | 71.394,71 | 46.129,39 | 17.653,96 | 71.394,71 | 33.404,32 | 8.943,603 | 52.884,97 | 15.832,86 | 2.688,984 | 18.509,74 | 2.726,958 | .225,938 |
| 51 | 74.279,05 | 48.766,21 | 19.072,15 | 74.279,05 | 35.379,73 | 9.670,845 | 55.021,52 | 16.788,34 | 2.909,361 | 19.257,53 | 2.893,302 | .244,535 |
| 52 | 77.220,52 | 51.490,14 | 20.571,79 | 77.220,52 | 37.427,26 | 10,440,93 | 57.200,38 | 17.780,66 | 3.142,929 | 20.020,13 | 3.066,243 | .264,257 |
| 53 | 80.219,09 | 54.301,49 | 22.155,68 | 80.219,09 | 39.547,79 | 11.255,44 | 59.421,55 | 18.810,45 | 3 390,201 | 20.797,54 | 3.245,904 | .285,145 |
| 54 | 83.274,79 | 57.200,53 | 23.826,62 | 83.274,79 | 41.742,18 | 12.115,98 | 61.685,03 | 19.878,32 | 3.651,689 | 21.589,76 | 3.432,410 | .307,247 |
| 55 | 86.387,60 | 60.187,42 | 25.587,45 | 86.387,60 | 44.011,24 | 13.024,15 | 63.990,81 | 20.984,86 | 3.927,919 | 22.396,78 | 3.625,885 | .330,608 |
| 56 | 89.557,52 | 63.282,27 | 27.441,01 | 89.557,52 | 46.355,77 | 13.981,61 | 66.338,91 | 22.130,67 | 4.219,424 | 23.218,62 | 3.826,454 | .355,273 |
| 57 | 92.784,56 | 66.425,11 | 29.390,18 | 92.784,56 | 48.776,53 | 14.990,01 | 68.729,31 | 23.316,34 | 4.526,745 | 24.055,26 | 4.034,241 | .381,292 |
| 58 | 96.068,72 | 69.675,88 | 31.437,82 | 96.068,72 | 51.274,26 | 16.051,02 | 71.162,02 | 24.542,47 | 4.850,432 | 24.906,71 | 4.249,367 | .408,713 |
| 59 | 99.409,99 | 73.014,47 | 33.586,84 | 99.409,99 | 53.849,65 | 17.166,36 | 73.637,03 | 25.809,61 | 5.191,036 | 25.772,96 | 4.471,957 | .437,582 |
| 60 | 102.808,4 | 76.440,68 | 35.840,15 | 102.808,4 | 56.503,37 | 18.337,73 | 76.154,36 | 27.118,35 | 5.549,132 | 26.654,02 | 4.702,135 | .467,951 |
| 61 | 106.263,9 | 79.954,22 | 38.200,63 | 106.263,9 | 59.236,06 | 19.566,87 | 78.713,99 | 28.469,25 | 5.925,292 | 27.549,89 | 4.940,021 | .499,871 |
| 62 | 109.776,5 | 83.554,75 | 40.671,24 | 109.776,5 | 62.048,32 | 20.855,53 | 81.315,93 | 29.862,87 | 6.320,090 | 28.460,57 | 5.185,739 | .533,396 |
| 63 | 113.346,2 | 87.241,83 | 43.254,88 | 113.346,2 | 64.940,72 | 22.205,48 | 83.960,18 | 31.299,75 | 6.734,131 | 29.386,06 | 5.439,411 | .568,573 |
| 64 | 116.973,1 | 91.014,95 | 45.954,48 | 116.973,1 | 67.913,81 | 23.618,50 | 86.646,73 | 32.760,43 | 7.167,996 | 30.326,36 | 5.701,157 | .605,460 |
| 65 | 120.657,1 | 94.873,53 | 48.772,98 | 120.657,1 | 70.968,07 | 25.096,40 | 89.375,59 | 34.305,47 | 7.622,303 | 31.281,46 | 5.971,100 | .644,108 |
| 66 | 124.398,1 | 98.816,91 | 51.713,31 | 124.398,1 | 74.104,00 | 26.640,97 | 92.146,77 | 35.875,37 | 8.097,663 | 32.251,37 | 6.249,361 | .684,573 |
| 67 | 128.196,3 | 102.844,3 | 54.778,39 | 128.196,3 | 77.322,01 | 28.254,07 | 94.960,25 | 37.490,67 | 8.594,698 | 33.236,09 | 6.536,060 | .726,912 |
| 68 | 132.051,7 | 106.955,0 | 57.971,16 | 132.051,7 | 80.622,52 | 29.937,52 | 97.816,04 | 39.151,87 | 9.114,033 | 34.235,61 | 6.831,318 | .771,179 |
| 69 | 135.964,1 | 111.148,0 | 61.294,53 | 135.964,1 | 84.005,88 | 31.893,19 | 100.714,1 | 40.859,48 | 9.656,303 | 35.249,95 | 7.135,255 | .817,433 |
| 70 | 139.933,6 | 115.422,3 | 64.751,42 | 139.933,6 | 87.472,44 | 33.522,94 | 103.654,5 | 42.614,01 | 10.222,16 | 36.279,09 | 7.447,989 | .865,731 |
| 71 | 143.960,3 | 119.776,9 | 68.344,72 | 143.960,3 | 91.022,49 | 35.428,66 | 106.637,2 | 44.415,94 | 10.812,24 | 37.323,04 | 7.769,642 | .916,135 |
| 72 | 148.044,1 | 124.210,6 | 72.077,35 | 148.044,1 | 94.656,29 | 37.412,23 | 109.662,3 | 46.265,75 | 11.427,22 | 38.381,80 | 8.100,332 | .968,700 |
| 73 | 152.185,0 | 128.722,3 | 75.952,14 | 152.185,0 | 98.374,06 | 39.475,56 | 112.729,6 | 48.163,91 | 12.067,76 | 39.455,36 | 8.440,177 | 1.023,489 |
| 74 | 156.383,0 | 133.310,6 | 79.971,99 | 156.383,0 | 102.176,0 | 41.620,56 | 115.839,2 | 50.110,90 | 12.734,51 | 40.543,73 | 8.789,295 | 1.080,564 |
| 75 | 160.638,1 | 137.974,1 | 84.139,72 | 160.638,1 | 106.062,3 | 43.849,14 | 118.991,2 | 52.107,17 | 13.428,19 | 41.646,91 | 9.147,805 | 1.139,985 |
| 76 | 164.950,3 | 142.711,5 | 88.458,21 | 164.950,3 | 110.033,0 | 46.163,29 | 122.185,4 | 54.153,18 | 14.149,45 | 42.764,90 | 9.515,825 | 1.201,817 |
| 77 | 169.319,7 | 147.521,1 | 92.930,20 | 169.319,7 | 114.088,2 | 48.564,87 | 125.422,0 | 56.249,36 | 14.899,01 | 43.897,70 | 9.893,471 | 1.266,122 |
| 78 | 173.746,2 | 152.401,4 | 97.558,51 | 173.746,2 | 118.228,0 | 51.055,88 | 128.700,9 | 58.396,15 | 15.677,55 | 45.045,30 | 10.280,86 | 1.332,965 |
| 79 | 178.229,7 | 157.350,7 | 102.345,9 | 178.229,7 | 122.452,3 | 53.638,24 | 132.022,0 | 60.593,97 | 16.485,78 | 46.207,71 | 10.678,11 | 1.402,412 |
| 80 | 182.770,5 | 162.367,2 | 107.295,1 | 182.770,5 | 126.761,2 | 56.313,96 | 135.385,5 | 62.843,24 | 17.324,42 | 47.384,93 | 11.085,33 | 1.474,530 |
| 81 | 187.368,3 | 167.449,1 | 112.408,6 | 187.368,3 | 131.154,5 | 59.084,95 | 138.791,3 | 65.144,37 | 18.194,19 | 48.576,96 | 11.502,65 | 1.549,384 |
| 82 | 192.023,2 | 172.594,4 | 117.689,7 | 192.023,2 | 135.632,2 | 61.953,22 | 142.239,4 | 67.497,75 | 19.095,80 | 49.783,79 | 11.930,17 | 1.627,040 |
| 83 | 196.735,3 | 177.801,2 | 123.140,4 | 196.735,3 | 140.194,2 | 64.920,72 | 145.729,8 | 69.903,78 | 20.030,01 | 51.005,44 | 12.368,02 | 1.707,570 |
| 84 | 201.504,4 | 183.067,4 | 128.763,7 | 201.504,4 | 144.840,1 | 67.989,47 | 149.262,5 | 72.362,84 | 20.997,54 | 52.241,89 | 12.816,30 | 1.791,041 |
| 85 | 206.330,7 | 188.390,9 | 134.561,7 | 206.330,7 | 149.569,9 | 71.161,39 | 152.837,6 | 74.875,31 | 21.999,12 | 53.493,15 | 13.275,13 | 1.877,524 |
| 86 | 211.214,1 | 193.769,3 | 140.537,4 | 211.214,1 | 154.383,3 | 74.438,53 | 156.454,9 | 77.441,54 | 23.035,55 | 54.759,21 | 13.744,63 | 1.967,089 |
| 87 | 216.154,6 | 199.200,5 | 146.693,1 | 216.154,6 | 159.279,9 | 77.822,85 | 160.114,5 | 80.061,88 | 24.107,50 | 56.040,09 | 14.224,90 | 2.059,808 |
| 88 | 221.152,3 | 204.682,2 | 153.031,3 | 221.152,3 | 164.259,5 | 81.318,34 | 163.816,5 | 82.736,69 | 25.215,80 | 57.335,77 | 14.718,06 | 2.155,752 |
| 89 | 226.207,0 | 210.211,7 | 159.554,4 | 226.207,0 | 169.321,5 | 84.920,97 | 167.560,7 | 85.466,31 | 26.361,20 | 58.646,26 | 15.218,22 | 2.254,997 |
| 90 | 231.318,9 | 215.786,7 | 166.264,8 | 231.318,9 | 174.465,8 | 88.638,76 | 171.347,3 | 88.251,05 | 27.544,48 | 59.971,56 | 15.731,50 | 2.357,615 |

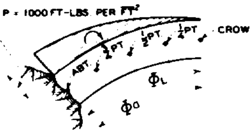
SHEET 4 OF 5

205-D-417

Table H-12.—*Load constants for circular arches—all deflections* (sheet 5).

UNIT TWIST LOAD NO. 5

TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



| Φ | Crown | | | ¼ Point | | | ½ Point | | | ¾ Point | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ | $D_1$ | $D_2$ | $D_3$ |
| 10 | 5.076,958 | 662,325,4 | .046,311,8 | -.283,682 | .396,465,5 | .020,164,9 | 2.538,478 | .152,178,3 | .005,072,3 | .793,274,5 | .023,359,36 | .0$^3$385,1 |
| 11 | 6.143,118 | .880,927,9 | .067,779,3 | 5.183,256 | .527,494,5 | .029,517,3 | 3.071,559 | .202,516,0 | .007,425,8 | .959,862,2 | .031,090,05 | .0$^3$563,8 |
| 12 | 7.310,818 | 1.142,793 | .095,955,6 | 6.168,503 | .684,545,4 | .041,795,8 | 3.655,409 | .262,873,3 | .010,516,0 | 1.142,315 | .040,361,55 | .0$^3$798,4 |
| 13 | 8.580,058 | 1.451,729 | .132,106,0 | 7.239,423 | .869,944,2 | .057,553,9 | 4.290,029 | .334,154,7 | .014,482,9 | 1.340,634 | .051,313,69 | .001,099,7 |
| 14 | 9.950,836 | 1.811,515 | .177,602,2 | 8.396,018 | 1.086,007 | .077,392,2 | 4.975,418 | .417,262,8 | .019,477,9 | 1.554,818 | .064,086,22 | .001,479,2 |
| 15 | 11.423,15 | 2.225,894 | .233,922 | 9.638,285 | 1.335,037 | .101,958,9 | 5.711,577 | .513,098,1 | .025,664,9 | 1.784,868 | .078,818,85 | .001,949,2 |
| 16 | 12.997,01 | 2.698,570 | .302,652 | 10.966,23 | 1.619,329 | .131,949 | 6.498,505 | .622,559,6 | .033,219,7 | 2.030,783 | .095,651,18 | .002,523,3 |
| 17 | 14.672,41 | 3.233,211 | .385,478 | 12.379,84 | 1.941,160 | .168,105 | 7.336,203 | .746,543,7 | .042,330,6 | 2.292,563 | .114,722,7 | .003,215,6 |
| 18 | 16.449,34 | 3.833,439 | .484,192 | 13.879,13 | 2.302,798 | .211,215 | 8.224,671 | .885,944,7 | .053,196,2 | 2.570,210 | .136,173,0 | .004,041,5 |
| 19 | 18.327,82 | 4.502,834 | .600,690 | 15.464,09 | 2.706,494 | .262,114 | 9.163,907 | 1.041,655 | .066,028,9 | 2.863,721 | .160,141,3 | .005,017,0 |
| 20 | 20.307,83 | 5.244,926 | .736,967 | 17.134,73 | 3.154,484 | .321,686 | 10.153,91 | 1.214,563 | .081,052 | 3.173,098 | .186,766,8 | .006,159,2 |
| 21 | 22.389,38 | 6.063,199 | .895,122 | 18.891,04 | 3.648,988 | .390,850 | 11.194,69 | 1.405,555 | .098,502 | 3.498,341 | .216,188,9 | .007,486,4 |
| 22 | 24.572,47 | 6.961,080 | 1.077,350 | 20.733,02 | 4.192,209 | .470,585 | 12.286,24 | 1.615,517 | .118,626 | 3.839,449 | .248,546,4 | .009,017,2 |
| 23 | 26.857,10 | 7.941,946 | 1.285,946 | 22.660,68 | 4.786,332 | .561,908 | 13.428,55 | 1.845,327 | .141,682 | 4.196,422 | .283,978,4 | .010,771,2 |
| 24 | 29.243,27 | 9.009,113 | 1.523,299 | 24.674,01 | 5.433,524 | .665,882 | 14.621,64 | 2.095,865 | .167,943 | 4.569,261 | .322,623,6 | .012,769,6 |
| 25 | 31.730,98 | 10.165,84 | 1.791,900 | 26.773,02 | 6.135,931 | .783,613 | 15.865,49 | 2.368,003 | .197,691 | 4.957,966 | .364,620,8 | .015,033,8 |
| 26 | 34.320,23 | 11.415,32 | 2.094,331 | 28.957,69 | 6.895,681 | .916,254 | 17.160,12 | 2.662,614 | .231,219 | 5.362,536 | .410,108,5 | .017,586,4 |
| 27 | 37.011,02 | 12.760,69 | 2.433,262 | 31.228,05 | 7.714,881 | 1.065,000 | 18.505,51 | 2.980,565 | .268,835 | 5.782,971 | .459,225,1 | .020,451,0 |
| 28 | 39.803,34 | 14.205,02 | 2.811,463 | 33.584,07 | 8.595,612 | 1.231,091 | 19.901,67 | 3.322,720 | .310,856 | 6.219,272 | .512,109,0 | .023,652,0 |
| 29 | 42.697,21 | 15.751,30 | 3.231,784 | 36.025,77 | 9.539,938 | 1.415,810 | 21.348,60 | 3.689,937 | .357,612 | 6.671,439 | .568,898,2 | .027,214,6 |
| 30 | 45.692,61 | 17.402,45 | 3.697,172 | 38.553,14 | 10.549,90 | 1.620,483 | 22.846,31 | 4.083,075 | .409,445 | 7.139,471 | .629,730,7 | .031,165,0 |
| 31 | 48.789,56 | 19.161,33 | 4.210,661 | 41.166,19 | 11.627,50 | 1.846,478 | 24.394,78 | 4.502,983 | .466,704 | 7.623,368 | .694,744,5 | .035,530,4 |
| 32 | 51.988,04 | 21.030,72 | 4.775,360 | 43.864,91 | 12.774,74 | 2.095,202 | 25.994,02 | 4.950,511 | .529,758 | 8.123,131 | .764,077,2 | .040,338,9 |
| 33 | 55.288,06 | 23.013,32 | 5.394,469 | 46.649,30 | 13.993,58 | 2.368,115 | 27.644,03 | 5.426,500 | .598,978 | 8.638,759 | .837,866,2 | .045,619,1 |
| 34 | 58.689,62 | 25.111,75 | 6.071,275 | 49.519,37 | 15.285,95 | 2.666,700 | 29.344,81 | 5.931,791 | .674,753 | 9.170,253 | .916,247,5 | .051,402,3 |
| 35 | 62.192,72 | 27.328,55 | 6.809,127 | 52.475,11 | 16.653,77 | 2.992,499 | 31.096,36 | 6.467,217 | .757,481 | 9.717,613 | .999,362,5 | .057,716,9 |
| 36 | 65.797,36 | 29.666,17 | 7.611,469 | 55.516,53 | 18.098,97 | 3.347,081 | 32.898,68 | 7.033,608 | .847,572 | 10.280,84 | 1.087,344 | .064,597 |
| 37 | 69.503,54 | 32.126,99 | 8.481,813 | 58.643,61 | 19.623,24 | 3.732,067 | 34.751,77 | 7.631,788 | .945,447 | 10.859,93 | 1.180,330 | .072,073 |
| 38 | 73.311,26 | 34.713,29 | 9.423,749 | 61.856,37 | 21.228,57 | 4.149,102 | 36.655,63 | 8.262,577 | 1.051,534 | 11.454,88 | 1.278,457 | .080,179 |
| 39 | 77.220,52 | 37.427,26 | 10.440,93 | 65.154,81 | 22.916,69 | 4.599,885 | 38.610,26 | 8.926,789 | 1.166,281 | 12.065,71 | 1.381,862 | .088,951 |
| 40 | 81.231,31 | 40.271,01 | 11.537,10 | 68.538,92 | 24.689,38 | 5.086,142 | 40.615,66 | 9.625,233 | 1.290,139 | 12.692,39 | 1.490,681 | .098,423 |
| 41 | 85.343,65 | 43.246,54 | 12.716,03 | 72.008,70 | 26.548,35 | 5.609,640 | 42.671,82 | 10.358,71 | 1.423,573 | 13.334,95 | 1.605,050 | .108,632 |
| 42 | 89.557,52 | 46.355,77 | 13.981,61 | 75.564,16 | 28.495,32 | 6.172,201 | 44.778,76 | 11.129,03 | 1.567,062 | 13.993,36 | 1.725,104 | .119,613 |
| 43 | 93.872,94 | 49.600,52 | 15.337,75 | 79.205,29 | 30.531,95 | 6.775,646 | 46.936,47 | 11.933,96 | 1.721,088 | 14.667,65 | 1.850,979 | .131,406 |
| 44 | 98.289,89 | 52.982,51 | 16.788,44 | 82.932,09 | 32.659,86 | 7.421,864 | 49.144,94 | 12.777,32 | 1.886,152 | 15.357,80 | 1.982,811 | .144,050 |
| 45 | 102.808,4 | 56.503,37 | 18.337,73 | 86.744,57 | 34.880,66 | 8.112,773 | 51.404,19 | 13.658,86 | 2.062,762 | 16.063,81 | 2.120,734 | .157,583 |
| 46 | 107.428,4 | 60.164,61 | 19.989,72 | 90.642,72 | 37.195,91 | 8.850,317 | 53.714,20 | 14.579,37 | 2.251,435 | 16.785,69 | 2.264,884 | .172,048 |
| 47 | 112.150,0 | 63.967,65 | 21.748,58 | 94.626,54 | 39.607,15 | 9.636,480 | 56.074,99 | 15.539,62 | 2.452,703 | 17.523,43 | 2.415,395 | .187,485 |
| 48 | 116.973,1 | 67.913,81 | 23.618,50 | 98.696,04 | 42.115,85 | 10.473,28 | 58.486,54 | 16.540,36 | 2.667,106 | 18.277,05 | 2.572,400 | .203,937 |
| 49 | 121.897,7 | 72.004,29 | 25.603,76 | 102.851,2 | 44.723,48 | 11.362,77 | 60.948,87 | 17.582,36 | 2.895,195 | 19.046,52 | 2.736,036 | .221,448 |
| 50 | 126.923,9 | 76.240,19 | 27.708,64 | 107.092,1 | 47.431,46 | 12.307,04 | 63.461,96 | 18.666,36 | 3.137,527 | 19.831,86 | 2.906,435 | .240,060 |
| 51 | 132.051,7 | 80.622,52 | 29.937,52 | 111.418,6 | 50.241,16 | 13.308,20 | 66.025,83 | 19.793,11 | 3.394,679 | 20.633,07 | 3.083,731 | .259,821 |
| 52 | 137.280,9 | 85.152,14 | 32.294,79 | 115.830,8 | 53.153,92 | 14.368,40 | 68.640,46 | 20.963,34 | 3.667,229 | 21.450,14 | 3.268,057 | .280,775 |
| 53 | 142.611,7 | 89.829,85 | 34.784,86 | 120.328,6 | 56.171,06 | 15.489,80 | 71.305,86 | 22.177,77 | 3.955,766 | 22.283,08 | 3.459,546 | .302,971 |
| 54 | 148.044,1 | 94.656,29 | 37.412,23 | 124.912,2 | 59.293,82 | 16.674,64 | 74.022,03 | 23.437,13 | 4.260,902 | 23.131,88 | 3.658,332 | .326,454 |
| 55 | 153.578,0 | 99.632,02 | 40.181,39 | 129.581,4 | 62.523,43 | 17.925,13 | 76.788,97 | 24.742,14 | 4.583,239 | 23.996,55 | 3.864,546 | .351,275 |
| 56 | 159.213,4 | 104.757,5 | 43.096,88 | 134.336,3 | 65.861,08 | 19.243,55 | 79.606,69 | 26.093,49 | 4.923,407 | 24.877,09 | 4.078,322 | .377,483 |
| 57 | 164.950,3 | 110.033,0 | 46.163,29 | 139.176,8 | 69.307,89 | 20.632,19 | 82.475,17 | 27.491,90 | 5.282,028 | 25.773,49 | 4.299,790 | .405,128 |
| 58 | 170.788,9 | 115.458,7 | 49.385,21 | 144.103,1 | 72.864,98 | 22.093,38 | 85.394,41 | 28.938,05 | 5.659,747 | 26.685,76 | 4.529,082 | .434,262 |
| 59 | 176.728,9 | 121.034,8 | 52.767,19 | 149.115,0 | 76.533,39 | 23.629,46 | 88.364,43 | 30.432,63 | 6.057,224 | 27.613,89 | 4.766,330 | .464,938 |
| 60 | 182.770,5 | 126.761,2 | 56.313,96 | 154.212,6 | 80.314,13 | 25.242,80 | 91.385,23 | 31.976,30 | 6.475,111 | 28.557,88 | 5.011,665 | .497,205 |
| 61 | 188.913,6 | 132.637,7 | 60.030,13 | 159.395,8 | 84.208,18 | 26.935,81 | 94.456,79 | 33.569,75 | 6.914,081 | 29.517,75 | 5.265,217 | .531,121 |
| 62 | 195.158,2 | 138.664,2 | 63.920,41 | 164.664,8 | 88.216,46 | 28.710,90 | 97.579,11 | 35.213,63 | 7.374,806 | 30.493,47 | 5.527,117 | .566,741 |
| 63 | 201.504,4 | 144.840,1 | 67.989,47 | 170.019,4 | 92.339,84 | 30.570,52 | 100.752,21 | 36.908,60 | 7.857,990 | 31.485,07 | 5.797,495 | .604,118 |
| 64 | 207.952,2 | 151.165,1 | 72.241,99 | 175.459,6 | 96.579,18 | 32.517,16 | 103.976,1 | 38.655,29 | 8.364,315 | 32.492,53 | 6.076,481 | .643,311 |
| 65 | 214.501,4 | 157.638,5 | 76.682,71 | 180.985,6 | 100.935,2 | 34.553,29 | 107.250,7 | 40.454,35 | 8.894,500 | 33.515,85 | 6.364,203 | .684,375 |
| 66 | 221.152,3 | 164.259,5 | 81.316,34 | 186.597,2 | 105.408,8 | 36.681,42 | 110.576,1 | 42.306,39 | 9.449,26 | 34.555,04 | 6.660,792 | .727,371 |
| 67 | 227.904,6 | 171.027,2 | 86.147,56 | 192.294,5 | 110.000,5 | 38.904,10 | 113.952,3 | 44.212,05 | 10.029,32 | 35.610,09 | 6.966,375 | .772,356 |
| 68 | 234.758,5 | 177.940,7 | 91.181,12 | 198.077,5 | 114.711,0 | 41.223,90 | 117.379,3 | 46.171,93 | 10.635,42 | 36.681,01 | 7.281,082 | .819,392 |
| 69 | 241.713,9 | 184.998,9 | 96.421,72 | 203.946,1 | 119.541,0 | 43.643,35 | 120.857,0 | 48.186,63 | 11.268,28 | 37.767,80 | 7.605,039 | .868,537 |
| 70 | 248.770,9 | 192.200,6 | 101.874,1 | 209.900,4 | 124.491,0 | 46.165,08 | 124.385,5 | 50.256,74 | 11.928,68 | 38.870,45 | 7.938,375 | .919,857 |
| 71 | 255.929,4 | 199.544,3 | 107.542,9 | 215.940,4 | 129.561,4 | 48.791,67 | 127.964,7 | 52.382,85 | 12.617,35 | 39.988,97 | 8.281,218 | .973,411 |
| 72 | 263.189,5 | 207.028,6 | 113.432,8 | 222.066,1 | 134.752,8 | 51.525,77 | 131.594,7 | 54.565,53 | 13.335,10 | 41.123,35 | 8.633,693 | 1.029,264 |
| 73 | 270.551,0 | 214.651,9 | 119.548,4 | 228.277,4 | 140.065,6 | 54.370,01 | 135.275,5 | 56.805,34 | 14.082,67 | 42.273,60 | 8.995,928 | 1.087,479 |
| 74 | 278.014,2 | 222.412,6 | 125.894,6 | 234.574,5 | 145.500,1 | 57.327,04 | 139.007,1 | 59.102,85 | 14.860,87 | 43.439,71 | 9.368,048 | 1.148,122 |
| 75 | 285.578,8 | 230.308,6 | 132.475,8 | 240.957,1 | 151.056,7 | 60.399,52 | 142.789,4 | 61.458,60 | 15.670,48 | 44.621,69 | 9.750,180 | 1.211,260 |
| 76 | 293.245,0 | 238.338,2 | 139.296,7 | 247.425,5 | 156.735,5 | 63.590,19 | 146.622,5 | 63.873,12 | 16.512,31 | 45.819,54 | 10.142,45 | 1.276,958 |
| 77 | 301.012,8 | 246.499,1 | 146.361,8 | 253.979,5 | 162.536,9 | 66.901,68 | 150.506,4 | 66.346,94 | 17.387,16 | 47.033,25 | 10.544,98 | 1.345,285 |
| 78 | 308.882,1 | 254.789,2 | 153.675,8 | 260.619,2 | 168.461,0 | 70.336,73 | 154.441,0 | 68.880,59 | 18.295,86 | 48.262,82 | 10.957,90 | 1.416,308 |
| 79 | 316.852,9 | 263.206,2 | 161.243,1 | 267.344,6 | 174.507,8 | 73.898,06 | 158.426,4 | 71.474,56 | 19.239,22 | 49.508,26 | 11.381,32 | 1.490,098 |
| 80 | 324.925,3 | 271.747,6 | 169.068,2 | 274.155,7 | 180.677,5 | 77.588,39 | 162.462,6 | 74.129,35 | 20.218,10 | 50.769,57 | 11.815,39 | 1.566,725 |
| 81 | 333.099,2 | 280.410,8 | 177.155,5 | 281.052,4 | 186.970,2 | 81.410,46 | 166.549,6 | 76.845,46 | 21.233,29 | 52.046,74 | 12.260,20 | 1.646,260 |
| 82 | 341.374,6 | 289.193,2 | 185.509,6 | 288.034,8 | 193.385,6 | 85.367,00 | 170.687,3 | 79.623,36 | 22.285,69 | 53.339,78 | 12.715,91 | 1.728,772 |
| 83 | 349.751,6 | 298.091,9 | 194.134,6 | 295.102,9 | 199.923,8 | 89.460,78 | 174.875,8 | 82.463,51 | 23.376,15 | 54.648,68 | 13.182,61 | 1.814,340 |
| 84 | 358.230,1 | 307.104,0 | 203.035,0 | 302.256,6 | 206.584,6 | 93.694,58 | 179.115,0 | 85.366,38 | 24.505,49 | 55.973,45 | 13.660,44 | 1.903,031 |
| 85 | 366.810,2 | 316.226,5 | 212.215,1 | 309.496,1 | 213.367,8 | 98.071,12 | 183.405,1 | 88.332,41 | 25.674,63 | 57.314,09 | 14.149,51 | 1.994,923 |
| 86 | 375.491,8 | 325.456,3 | 221.679,0 | 316.821,2 | 220.273,2 | 102.593,2 | 187.745,9 | 91.362,04 | 26.884,41 | 58.670,58 | 14.649,96 | 2.090,089 |
| 87 | 384.274,9 | 334.790,0 | 231.430,8 | 324.231,9 | 227.300,4 | 107.263,6 | 192.137,4 | 94.455,69 | 28.135,74 | 60.042,95 | 15.161,89 | 2.188,607 |
| 88 | 393.159,6 | 344.224,2 | 241.474,8 | 331.728,4 | 234.449,2 | 112.085,1 | 196.579,8 | 97.613,79 | 29.429,48 | 61.431,18 | 15.685,43 | 2.290,553 |
| 89 | 402.145,8 | 353.755,5 | 251.814,9 | 339.310,5 | 241.719,1 | 117.060,4 | 201.072,9 | 100.836,7 | 30.766,56 | 62.835,28 | 16.220,71 | 2.396,006 |
| 90 | 411.233,5 | 363.380,2 | 262.455,1 | 346.978,3 | 249.109,7 | 122.192,4 | 205.616,8 | 104.124,9 | 32.147,85 | 64.255,24 | 16.767,82 | 2.505,041 |

SHEET 5 OF 5

205-D-418

Table H-13.–*Trigonometric table for arch analysis* (sheet 1).

| Φ Degrees | Φ Radians | SIN Φ | COS Φ | VERS Φ | Φ-SIN Φ | $\frac{\Phi^2}{2}$-VERS Φ | $\frac{\Phi^2}{2}$ | SIN$^2$ Φ | COS$^2$ Φ |
|---|---|---|---|---|---|---|---|---|---|
| 0 15 | .004.363.323 | .004,363,309 | .999,990,5 | .0$^{5}$9,519,279 | .0$^{7}$13,845,24 | .0$^{10}$15,102,82 | .0$^{5}$9,519,294 | .0$^{4}$19,038,47 | .999,981,0 |
| 0 30 | .008.726,646 | .008,726,535 | .999,961,9 | .0$^{4}$38,076,94 | .0$^{6}$110,761,6 | .0$^{9}$241,644,6 | .0$^{4}$38,077,18 | .0$^{4}$76,152,42 | .999,923,8 |
| 0 45 | .013,089,97 | .013.089,60 | .999,914,3 | .0$^{4}$85,672,43 | .0$^{6}$373,818,6 | .0$^{8}$1,223,322 | .0$^{4}$85,673,65 | .0$^{3}$171,337,5 | .999,828,7 |
| 1 00 | .017,453,29 | .017,452,41 | .999,847,7 | .0$^{3}$152,304,8 | .0$^{6}$886,082,6 | .0$^{8}$3,866,284 | .0$^{3}$152,308,7 | .0$^{3}$304,586,5 | .999,695,4 |
| 1 15 | .021,816,62 | .021,814,89 | .999,762,0 | .0$^{3}$237,972,9 | .0$^{5}$1,730,615 | .0$^{8}$9,439,117 | .0$^{3}$237,982,4 | .0$^{3}$475,889,2 | .999,524,1 |
| 1 30 | .026,179.94 | .026,176,95 | .999,657,3 | .0$^{3}$342,675,0 | .0$^{5}$2,990,472 | .0$^{7}$19,572,82 | .0$^{3}$342,694,6 | .0$^{3}$685,232,6 | .999,314,8 |
| 1 45 | .030,543,26 | .030,538,51 | .999,533,6 | .0$^{3}$466,409,2 | .0$^{5}$4,748,700 | .0$^{7}$36,260,76 | .0$^{3}$466,445,4 | .0$^{3}$932,600,8 | .999,067,4 |
| 2 00 | .034,906,59 | .034,899,50 | .999,390.8 | .0$^{3}$609,173,0 | .0$^{5}$7,088,337 | .0$^{7}$61,858,67 | .0$^{3}$609,234,8 | .001,217,975 | .998,782,0 |
| 2 15 | .039.269.91 | .039,259,82 | .999,229,0 | .0$^{3}$770,963,8 | .0$^{4}$10,092,41 | .0$^{7}$99,084,56 | .0$^{3}$771,062,8 | .001,541,333 | .998,458,7 |
| 2 30 | .043,633,23 | .043,619,39 | .999,048,2 | .0$^{3}$951,778,4 | .0$^{4}$13.843,93 | .0$^{6}$151,018,7 | .0$^{3}$951,929,4 | .001,902,651 | .998,097,3 |
| 2 45 | .047,996,55 | .047,978,13 | .998,848,4 | .001,151,614 | .0$^{4}$18,425,91 | .0$^{6}$221,103,5 | .001,151,835 | .002,301,901 | .997,698,1 |
| 3 00 | .052,359,88 | .052,335,96 | .998,629,5 | .001,370,465 | .0$^{4}$23,921,32 | .0$^{6}$313,143,6 | .001,370,778 | .002,739,052 | .997,260,9 |
| 3 15 | .056,723,20 | .056,692,79 | .998,391,7 | .001,608,329 | .0$^{4}$30,413.13 | .0$^{6}$431,305,6 | .001,608,761 | .003,214,072 | .996,785,9 |
| 3 30 | .061,086,52 | .061,048.54 | .998,134,8 | .001,865,202 | .0$^{4}$37,984,28 | .0$^{6}$580,118,1 | .001,865,782 | .003,726,924 | .996,273,1 |
| 3 45 | .065,449,85 | .065,403,13 | .997,858,9 | .002,141,077 | .0$^{4}$46,717,72 | .0$^{6}$764,471,5 | .002,141,841 | .004,277,569 | .995,722,4 |
| 4 00 | .069,813,17 | .069,756,47 | .997,564.1 | .002,435,950 | .0$^{4}$56,693,34 | .0$^{6}$989,618,1 | .002,436,939 | .004,865,966 | .995,134,0 |
| 4 15 | .074.176,49 | .074,108,49 | .997,250,2 | .002,749,815 | .0$^{4}$68,003,01 | .0$^{5}$1,261,172 | .002,751.076 | .005,492,068 | .994,507,9 |
| 4 30 | .078,539,82 | .078,459,10 | .996,917,3 | .003,082,666 | .0$^{4}$80,720,61 | .0$^{5}$1,585,108 | .003,084.251 | .006,155,830 | .993,844,2 |
| 4 45 | .082,903,14 | .082,808,21 | .996,565,5 | .003,434,498 | .0$^{4}$94,931,96 | .0$^{5}$1,967,765 | .003,436,465 | .006,857,199 | .993,142,8 |
| 5 00 | .087,266,46 | .087,155,74 | .996,194,7 | .003,805,302 | .0$^{3}$110.719,9 | .0$^{5}$2,415,839 | .003,807,718 | .007,596,124 | .992,403,9 |
| 5 15 | .091,629,79 | .091,501,62 | .995,804,9 | .004,195,072 | .0$^{3}$128,187,1 | .0$^{5}$2,936,391 | .004,198,009 | .008,372,546 | .991,627,5 |
| 5 30 | .095,993,11 | .095,845,75 | .995,396,2 | .004,603,802 | .0$^{3}$147,356,3 | .0$^{5}$3,536,841 | .004,607,338 | .009,186,408 | .990,813,6 |
| 5 45 | .100,356,4 | .100,188,1 | .994,968,5 | .005,031,482 | .0$^{3}$168,370,4 | .0$^{5}$4,224,972 | .005.035,707 | .010,037,65 | .989,962,4 |
| 6 00 | .104,719,8 | .104,528,5 | .994,521,9 | .005,478,105 | .0$^{3}$191,291,9 | .0$^{5}$5,008,924 | .005,483,114 | .010,926,20 | .989,073,8 |
| 6 15 | .109,083,1 | .108,866,9 | .994,056,3 | .005,943,662 | .0$^{3}$216,203,4 | .0$^{5}$5,897,203 | .005,949,559 | .011,852,00 | .988,148,0 |
| 6 30 | .113,446,4 | .113,203,2 | .993,571,9 | .006,428,144 | .0$^{3}$243,187,6 | .0$^{5}$6,898,670 | .006,435,043 | .012,814,97 | .987,185,0 |
| 6 45 | .117,809,7 | .117,537,4 | .993,068,5 | .006,931,543 | .0$^{3}$272.327,1 | .0$^{5}$8,022,549 | .006,939,566 | .013,815,04 | .986,185,0 |
| 7 00 | .122,173,0 | .121,869,3 | .992,546,2 | .007,453,848 | .0$^{3}$303.704,2 | .0$^{5}$9,278,426 | .007,463,127 | .014,852,14 | .985,147,9 |
| 7 15 | .126,536,4 | .126,199,0 | .992,004,9 | .007,995,050 | .0$^{3}$337.401,6 | .0$^{4}$10,676,24 | .008,005,727 | .015,926,18 | .984,073,8 |
| 7 30 | .130,899,7 | .130,526,2 | .991,444,9 | .008,555,139 | .0$^{3}$373.501,7 | .0$^{4}$12,226,31 | .008,567,365 | .017,037,09 | .982,962,9 |
| 7 45 | .135,263,0 | .134,850,9 | .990,865,9 | .009,134.103 | .0$^{3}$412,086,8 | .0$^{4}$13,939,27 | .009,148,042 | .018,184,77 | .981,815,2 |
| 8 00 | .139,626,3 | .139,173,1 | .990,268,1 | .009,731,931 | .0$^{3}$453,239,2 | .0$^{4}$15,826,17 | .009,747,757 | .019,369,15 | .980,630,9 |
| 8 15 | .143.989,7 | .143,492,6 | .989,651,4 | .010,348,61 | .0$^{3}$497,041,3 | .0$^{4}$17,898,39 | .010,366,51 | .020,590,13 | .979,409,9 |
| 8 30 | .148.353,0 | .147,809,4 | .989,015.9 | .010,984,14 | .0$^{3}$543,575,3 | .0$^{4}$20.167,65 | .011,004,30 | .021,847,62 | .978,152,4 |
| 8 45 | .152,716,3 | .152,123.4 | .988,361,5 | .011,638,49 | .0$^{3}$592,923,4 | .0$^{4}$22,646,07 | .011,661,14 | .023,141,52 | .976,858,5 |
| 9 00 | .157,079,6 | .156,434,5 | .987,688,3 | .012,311,66 | .0$^{3}$645,167,6 | .0$^{4}$25,346,10 | .012,337,01 | .024,471,74 | .975,528,3 |
| 9 15 | .161,443,0 | .160,742,6 | .986,996,4 | .013,003,63 | .0$^{3}$700,390,2 | .0$^{4}$28,280,55 | .013,031,91 | .025,838,17 | .974,161,8 |
| 9 30 | .165,806,3 | .165.047,6 | .986,285,6 | .013,714,40 | .0$^{3}$758,673,1 | .0$^{4}$31,462,61 | .013,745,86 | .027,240,71 | .972,759,3 |
| 9 45 | .170,169,6 | .169,349,5 | .985,556,1 | .014,443,94 | .0$^{3}$820,098,2 | .0$^{4}$34,905,79 | .014,478,85 | .028,679,25 | .971,320,7 |
| 10 00 | .174,532,9 | .173,648,2 | .984,807,8 | .015,192,25 | .0$^{3}$884,747,5 | .0$^{4}$38,624,00 | .015,230,87 | .030,153,69 | .969,846,3 |
| 10 15 | .178,896,2 | .177,943,5 | .984,040,7 | .015,959,30 | .0$^{3}$952,702,9 | .0$^{4}$42,631,48 | .016,001,93 | .031,663,91 | .968,336,1 |
| 10 30 | .183,259,6 | .182,235.5 | .983,254,9 | .016,745,09 | .001,024,046 | .0$^{4}$46.942,83 | .016,792,04 | .033,209,79 | .966,790,2 |
| 10 45 | .187,622,9 | .186.524.0 | .982,450,4 | .017.549,60 | .001.098,859 | .0$^{4}$51.573,02 | .017,601,18 | .034,791,22 | .965,208,8 |
| 11 00 | .191,986,2 | .190,809.0 | .981,627,2 | .018,372,82 | .001,177,222 | .0$^{4}$56,537,34 | .018,429,35 | .036,408,07 | .963,591,9 |
| 11 15 | .196,349,5 | .195,090,3 | .980,785,3 | .019,214,72 | .001,259,219 | .0$^{4}$61,851,50 | .019,276,57 | .038,060,23 | .961,939,8 |
| 11 30 | .200,712,9 | .199,367,9 | .979,924,7 | .020,075,30 | .001,344,930 | .0$^{4}$67,531,50 | .020,142,83 | .039,747,57 | .960,252,4 |
| 11 45 | .205,076,2 | .203,641,8 | .979,045,5 | .020,954,53 | .001,434,436 | .0$^{4}$73,593,74 | .021,028,12 | .041,469,96 | .958,530,0 |
| 12 00 | .209,439,5 | .207,911,7 | .978,147,6 | .021,852,40 | .001,527,819 | .0$^{4}$80,054,96 | .021,932,45 | .043,227,27 | .956,772,7 |
| 12 15 | .213,802,8 | .212,177,7 | .977,231,1 | .022,768,89 | .001,625,161 | .0$^{4}$86,932,24 | .022,855,83 | .045,019,36 | .954,980,6 |
| 12 30 | .218.166.2 | .216,439,6 | .976,296,0 | .023,703,99 | .001,726,543 | .0$^{4}$94,243,04 | .023,798,24 | .046,846,11 | .953,153,9 |
| 12 45 | .222,529,5 | .220,697,4 | .975,342,3 | .024,657,68 | .001,832,045 | .0$^{3}$102,005,2 | .024,759,68 | .048,707,36 | .951,292,6 |
| 13 00 | .226,892,8 | .224,951,1 | .974,370,1 | .025,629,94 | .001,941,748 | .0$^{3}$110,236,8 | .025,740,17 | .050,602,98 | .949,397,0 |
| 13 15 | .231,256,1 | .229,200,4 | .973,379.3 | .026,620,74 | .002,055,735 | .0$^{3}$118,956,3 | .026,739,70 | .052,532,82 | .947,467,2 |
| 13 30 | .235,619,4 | .233,445,4 | .972,369,9 | .027,630,08 | .002,174,085 | .0$^{3}$128,182,8 | .027,758,26 | .054,496,74 | .945,503,3 |
| 13 45 | .239,982,8 | .237,685.9 | .971,342,1 | .028,657,93 | .002,296,880 | .0$^{3}$137,935,3 | .028,795,87 | .056,494,58 | .943,505,4 |
| 14 00 | .244,346,1 | .241,921,9 | .970,295,7 | .029,704,27 | .002,424,200 | .0$^{3}$148,233,4 | .029,852,51 | .058,526,20 | .941,473,8 |
| 14 15 | .248,709.4 | .246.153,3 | .969,230,9 | .030,769,09 | .002.556,125 | .0$^{3}$159,097,1 | .030,928,19 | .060,591,44 | .939,408,6 |
| 14 30 | .253.072,7 | .250,380.0 | .968,147,6 | .031,852,36 | .002,692,737 | .0$^{3}$170,546,6 | .032,022,91 | .062,690,15 | .937,309,9 |
| 14 45 | .257,436,1 | .254,601,9 | .967,045,9 | .032,954,06 | .002,834,117 | .0$^{3}$182,602,6 | .033,136,66 | .064.822,15 | .935,177,8 |
| 15 00 | .261,799,4 | .258,819,0 | .965,925,8 | .034,074,17 | .002,980.343 | .0$^{3}$195,286,0 | .034,269,46 | .066,987,30 | .933,012,7 |
| 15 15 | .266,162,7 | .263,031,2 | .964,787,3 | .035,212,68 | .003,131,496 | .0$^{3}$208,618,2 | .035,421,29 | .069,185,42 | .930,814,6 |
| 15 30 | .270,526,0 | .267,238,4 | .963,630,5 | .036,369,55 | .003,287,658 | .0$^{3}$222,620,8 | .036,592,17 | .071,416,35 | .928,583,7 |
| 15 45 | .274,889,4 | .271,440,4 | .962,455,2 | .037,544,76 | .003,448,907 | .0$^{3}$237,315,8 | .037,782,06 | .073,679,92 | .926,320,1 |
| 16 00 | .279,252,7 | .275,637,4 | .961,261,7 | .038,738,30 | .003,615,325 | .0$^{3}$252,725,7 | .038,991,03 | .075,975,95 | .924,024,1 |
| 16 15 | .283,616,0 | .279,829,0 | .960,049,9 | .039,950,15 | .003,786,989 | .0$^{3}$268,873,1 | .040,219,02 | .078,304,28 | .921,695,7 |
| 16 30 | .287,979,3 | .284,015,3 | .958,819,7 | .041,180,27 | .003,963,982 | .0$^{3}$285,781,1 | .041,466,05 | .080,664,72 | .919,335,3 |
| 16 45 | .292.342,6 | .288,196,3 | .957,571,4 | .042,428,64 | .004.146,382 | .0$^{3}$303,473,2 | .042,732,11 | .083,057,09 | .916,942,9 |
| 17 00 | .296,706,0 | .292,371,7 | .956,304,8 | .043,695,24 | .004,334,268 | .0$^{3}$321,973,1 | .044,017,22 | .085,481,21 | .914,518,8 |
| 17 15 | .301,069,3 | .296,541,6 | .955,019,9 | .044,980,06 | .004,527,721 | .0$^{3}$341,304,9 | .045,321,36 | .087,936,91 | .912,063,1 |
| 17 30 | .305,432,6 | .300.705,8 | .953,717,0 | .046,283,05 | .004,726,820 | .0$^{3}$361,493,2 | .046,644,54 | .090,423,98 | .909,576,0 |
| 17 45 | .309,795,9 | .304,864,3 | .952,395,8 | .047,604,20 | .004,931,643 | .0$^{3}$382,562,6 | .047,986,76 | .092,942,24 | .907,057,8 |
| 18 00 | .314,159,3 | .309,017,0 | .951,056,5 | .048,943,48 | .005,142,271 | .0$^{3}$404,538,3 | .049,348,02 | .095,491,50 | .904,508,5 |
| 18 15 | .318,522,6 | .313,163,8 | .949,699,1 | .050,300,87 | .005,358,782 | .0$^{3}$427,445,9 | .050,728,32 | .098,071,57 | .901,928,4 |
| 18 30 | .322.885,9 | .317,304.7 | .948,323,7 | .051,676,34 | .005,581,255 | .0$^{3}$451,311,2 | .052,127,66 | .100,882,2 | .899,317,8 |
| 18 45 | .327,249,2 | .321,439,5 | .946,930,1 | .053,069.87 | .005,809,769 | .0$^{3}$476,160,3 | .053,546,03 | .103,323,3 | .896,676,7 |
| 19 00 | .331,612,6 | .325,568,2 | .945,518,6 | .054,481,42 | .006,044,403 | .0$^{3}$502,019,9 | .054,983,44 | .105,994,6 | .894,0054 |
| 19 15 | .335,975,9 | .329,690,6 | .944,089,0 | .055,910,98 | .006,285,236 | .0$^{3}$528,916,7 | .056,439,90 | .108,695,9 | .891,304,1 |
| 19 30 | .340,339,2 | .333,806,9 | .942,641,5 | .057,358,51 | .006,532,345 | .0$^{3}$556,878,0 | .057,915,39 | .111,427,0 | .888,573,0 |
| 19 45 | .344,702,5 | .337,916,7 | .941,178,0 | .058,823,98 | .006,785,809 | .0$^{3}$585,931,4 | .059,409,92 | .114,187,7 | .885,812,3 |
| 20 00 | .349,065,9 | .342,020,1 | .939,692,6 | .060,307,38 | .007,045,707 | .0$^{3}$616,104,7 | .060,923,48 | .116,977,8 | .883,022,2 |
| 20 15 | .353,429,2 | .346,117,1 | .938,191,3 | .061,808,66 | .007,312,116 | .0$^{3}$647,426,3 | .062,456,09 | .119,797,0 | .880,203,0 |
| 20 30 | .357,792.5 | .350,207,4 | .936,672,2 | .063,327,81 | .007,585,115 | .0$^{3}$679,924,6 | .064,007,74 | .122,645,2 | .877,354,8 |
| 20 45 | .362,155,8 | .354,291,0 | .935,135,2 | .064,864,79 | .007,864,782 | .0$^{3}$713,628,6 | .065,578,42 | .125,522,1 | .874,477,9 |
| 21 00 | .366,519,1 | .358,367,9 | .933,580,4 | .066,419,57 | .008,151,193 | .0$^{3}$748,567,6 | .067,168,14 | .128,427,6 | .871,572,4 |
| 21 15 | .370,882,5 | .362,438,0 | .932,007,9 | .067,992,13 | .008,444,428 | .0$^{3}$784,771,1 | .068,776,90 | .131,361,3 | .868,638,7 |
| 21 30 | .375,245,8 | .366,501,2 | .930,417,6 | .069,582,43 | .008,744,562 | .0$^{3}$822,269,1 | .070,404,70 | .134,323,1 | .865,676,9 |
| 21 45 | .379,609,1 | .370,557,4 | .928,809,6 | .071,190,45 | .009,051,675 | .0$^{3}$861,091,9 | .072,051,54 | .137,312,8 | .862,687,2 |
| 22 00 | .383,972,4 | .374,606,6 | .927,183,9 | .072,816,15 | .009,365,842 | .0$^{3}$901,270,2 | .073,717,42 | .140,330,1 | .859,669,9 |
| 22 15 | .388,335,8 | .378,648,6 | .925,540,5 | .074,459,50 | .009,687,141 | .0$^{3}$942,834,7 | .075,402,33 | .143,374,8 | .856,625,2 |
| 22 30 | .392,699,1 | .382,683,4 | .923,879,5 | .076,120,47 | .010,015,65 | .0$^{3}$985,816,9 | .077,108,28 | .146,446,6 | .853,553,4 |

.0$^{4}$10,092,41 = 000,010,092,41

Table H-13.—*Trigonometric table for arch analysis* (sheet 2).

| Φ Degrees | Φ Radians | SIN Φ | COS Φ | VERS Φ | Φ-SIN Φ | $\frac{\Phi^2}{2}$-VERS Φ | $\frac{\Phi^2}{2}$ | SIN² Φ | COS² Φ |
|---|---|---|---|---|---|---|---|---|---|
| 22 45 | .397,062,4 | .386,711,0 | .922,201,0 | .077,799,03 | .010.351,44 | .001,030.248 | 078.829.28 | .149.545,4 | 850,454,6 |
| 23 00 | .401,425.7 | .390,731,1 | .920,504,9 | .079,495,15 | .010,694,60 | .001.076.161 | .080,571,31 | .152,670,8 | .847.329,2 |
| 23 15 | .405.789.1 | .394,743,9 | .918,791,2 | .081,208.79 | .011,045,19 | .001,123,587 | .082,332.38 | .155,822.7 | .844,177,3 |
| 23 30 | .410,152,4 | .398,749,1 | .917,080.1 | .082,939,93 | .011,403.31 | .001.172.559 | 084,112.49 | .159.000,8 | .840,999,2 |
| 23 45 | .414.515.7 | .402,746.7 | .915,311.5 | .084.688.52 | .011.769,01 | .001,223.111 | .085,911,63 | .162,204.9 | .837.795,1 |
| 24 00 | .418,879,0 | .406,736,6 | .913,545,5 | .086,454,54 | .012,142.38 | .001,275.275 | .087,729,82 | .165.434.7 | .834,565,3 |
| 24 15 | .423,242,3 | .410,718,9 | .911,762,0 | .088,237,96 | .012,523,49 | .001,329,084 | .089.567,04 | .168,690.0 | .831,310,0 |
| 24 30 | .427,605,7 | .414,693,2 | .909,961,3 | .090,038,73 | .012,912,42 | .001,384,574 | .091,423.30 | .171,970,5 | .828.029.5 |
| 24 45 | .431,969.0 | .418,659,7 | .908,143,2 | .091,856,83 | .013,309,25 | .001,441,778 | .093,298,60 | .175,276,0 | .824.724,0 |
| 25 00 | .436,332,3 | .422,618,3 | .906,307,8 | .093,692,21 | .013,714.05 | .001,500,731 | .095,192,94 | .178,606,2 | .821,393,8 |
| 25 15 | .440,695,6 | .426,568,7 | .904,455,1 | .095,544,85 | .014,126,90 | .001,561,467 | .097,106,32 | .181,960,9 | .818,039,1 |
| 25 30 | .445.059,0 | .430,511,1 | .902,585,3 | .097,414,72 | .014,547,86 | .001,624,023 | .099,038,74 | .185,339,8 | .814,660,2 |
| 25 45 | .449,422,3 | .434,445,3 | .900,698,2 | .099,301,76 | .014,977,02 | .001,688,433 | .100,990,2 | .188,742,7 | .811,257,3 |
| 26 00 | .453,785.6 | .438,371,1 | .898,794,0 | .101,206,0 | .015,414,46 | .001,754,734 | .102,960,7 | .192,169,3 | .807,830,7 |
| 26 15 | .458,148,9 | .442,288,7 | .896,872,7 | .103,127,3 | .015.860,24 | .001,822,962 | .104,950,2 | .195,619,3 | .804.380,7 |
| 26 30 | .462,512,3 | .446,197,8 | .894,934,4 | .105,065,6 | .016,314,44 | .001,893,153 | .106,958,8 | .199,092,5 | .800,907.5 |
| 26 45 | .466,875,6 | .450,098,4 | .892,978,9 | .107,021,1 | .016,777,13 | .001,965.345 | .108,986,4 | .202,588,6 | .797,411,4 |
| 27 00 | .471,238,9 | .453,990,5 | .891,006,5 | .108,993,5 | .017,248,40 | .002,039,574 | .111,033,0 | .206,107,4 | .793,892,6 |
| 27 15 | .475,602,2 | .457,873,9 | .889,017,1 | .110,982,9 | .017,728,31 | .002,115,878 | .113,096,7 | .209,648,5 | .790,351,5 |
| 27 30 | .479,965,5 | .461,748,6 | .887,010,8 | .112,989,2 | .018,216,93 | .002,194,295 | .115,183,5 | .213,211,8 | .786,788,2 |
| 27 45 | .484,328,9 | .465,614,5 | .884.987,6 | .115,012,4 | .018.714,35 | .002,274,863 | .117,287,2 | .216,796,9 | .783,203,1 |
| 28 00 | .488,692,2 | .469,471,6 | .882,947,6 | .117,052,4 | .019,220,63 | .002.357,621 | .119,410,0 | .220,403,5 | .779,596,5 |
| 28 15 | .493,055,5 | .473,319,7 | .880,890,7 | .119,109,3 | .019,735,85 | .002,442,608 | .121,551,9 | .224,031,5 | .775,968,5 |
| 28 30 | .497,418,8 | .477,158,8 | .878,817,1 | .121,182,9 | .020,260,08 | .002,529,862 | .123,712,7 | .227,680,5 | .772,319,5 |
| 28 45 | .501,782,2 | .480,988,8 | .876,726,8 | .123,273,2 | .020,793,39 | .002,619,424 | .125,892,7 | .231,350,2 | .768,649,8 |
| 29 00 | .506,145,5 | .484,809,6 | .874,619,7 | .125,380,3 | .021,335,86 | .002,711,332 | .128,091,6 | .235,040,4 | .764,959,6 |
| 29 15 | .510,508,8 | .488,621,2 | .872,496,0 | .127,504,0 | .021,887,56 | .002,805,628 | .130,309.6 | .238,750,7 | .761,249,3 |
| 29 30 | .514,872,1 | .492,423,6 | .870,355,7 | .129,644,3 | .022,448,57 | .002,902,351 | .132,546,7 | .242,481,0 | .757,519,0 |
| 29 45 | .519,235,5 | .496,216,5 | .868,198,8 | .131,801,2 | .023.018,95 | .003,001,542 | .134,802,7 | .246,230,8 | .753,769,2 |
| 30 00 | .523,598,8 | .500,000,0 | .866,025,4 | .133,974,6 | .023,598,78 | .003,103,243 | .137,077,8 | .250.000,0 | .750.000,0 |
| 30 15 | .527,962,1 | .503,774,0 | .863,835,5 | .136,164,5 | .024,188,12 | .003,207,494 | .139,372,0 | .253,788,2 | .746,211,8 |
| 30 30 | .532,325,4 | .507,538,4 | .861,629,2 | .138,370,8 | .024,787,06 | .003,314,338 | .141,685,2 | .257,595,2 | .742,404,8 |
| 30 45 | .536,688,7 | .511,293,1 | .859,406,4 | .140,593,6 | .025,395,66 | .003,423,816 | .144,017,4 | .261,420,6 | .738,579,4 |
| 31 00 | .541,052,1 | .515,038,1 | .857,167,3 | .142,832,7 | .026,013,99 | .003,535,971 | .146,368,7 | .265,264,2 | .734,735,8 |
| 31 15 | .545,415,4 | .518,773,3 | .854,911,9 | .145,088,1 | .026,642,13 | .003,650,845 | .148.739,0 | .269,125,7 | .730,874,3 |
| 31 30 | .549,778,7 | .522,498,6 | .852,640,2 | .147,359,8 | .027,280,15 | .003,768,482 | .151,128,3 | .273.004,8 | .726,995,2 |
| 31 45 | .554.142,0 | .526,213,9 | .850.352,2 | .149,647,8 | .027,928,11 | .003,888,924 | .153,536,7 | .276.901.1 | .723,098,9 |
| 32 00 | .558,505,4 | .529,919,3 | .848,048,1 | .151,951,9 | .028,586,10 | .004,012,215 | .155,964,1 | .280,814,4 | .719,185,6 |
| 32 15 | .562.868,7 | .533,614,5 | .845,727,8 | .154,272,2 | .029,254,17 | .004,138,399 | .158,410,6 | .284,744.5 | .715,255,5 |
| 32 30 | .567.232.0 | .537.299,6 | .843.391,4 | .156.608,6 | .029,932,40 | .004,267,521 | .160,876,1 | .288,690,9 | .711,309,1 |
| 32 45 | .571,595,3 | .540,974,5 | .841,039,0 | .158,961,0 | .030,620,86 | .004,399,624 | .163,360,6 | .292,653,4 | .707,346,6 |
| 33 00 | .575,958,7 | .544,639,0 | .838,670,6 | .161,329,4 | .031.319,62 | .004,534,753 | .165,864,2 | .296,631,7 | .703,368,3 |
| 33 15 | .580,322,0 | .548,293,2 | .836,286,2 | .163,713,8 | .032,028,75 | .004,672,954 | .168,386,8 | .300,625,5 | .699,374,5 |
| 33 30 | .584,685,3 | .551,937,0 | .833,885,8 | .166,114,2 | .032,748,31 | .004,814,272 | .170,928,4 | .304,634,4 | .695,365,6 |
| 33 45 | .589,048,6 | .555,570,2 | .831,469,6 | .168,530,4 | .033,478,39 | .004,958,752 | .173,489,1 | .308,658,3 | .691,341,7 |
| 34 00 | .593,411,9 | .559,192,9 | .829,037,6 | .170,962,4 | .034,219,04 | .005,106,441 | .176,068,9 | .312,696,7 | .687,303,3 |
| 34 15 | .597,775,3 | .562,804,9 | .826.589,7 | .173,410,3 | .034,970,34 | .005,257,385 | .178,667,6 | .316,749,4 | .683,250,6 |
| 34 30 | .602,138,6 | .566,406,2 | .824,126,2 | .175,873,8 | .035,732,36 | .005,411,631 | .181,285,4 | .320,816,0 | .679,184,0 |
| 34 45 | .606,501,9 | .569,996,8 | .821.646,9 | .178,353,1 | .036,505,15 | .005,569,224 | .183.922,3 | .324.896,3 | .675,103,7 |
| 35 00 | .610,865,2 | .573,576,4 | .819,152,0 | .180,848,0 | .037,288,80 | .005,730,214 | .186,578,2 | .328,989,9 | .671,010,1 |
| 35 15 | .615,228,6 | .577,145,2 | .816,641,6 | .183,358,4 | .038,083,37 | .005,894,646 | .189,253,1 | .333,096,6 | 666,903,4 |
| 35 30 | .619,591,9 | .580,703,0 | .814,115,5 | .185,884,5 | .038,888,93 | .006,062,570 | .191,947,1 | .337.215,9 | .662,784,1 |
| 35 45 | .623,955,2 | .584.249,7 | .811,574,0 | .188,426,0 | .039,705,54 | .006,234,033 | .194,660,1 | .341,347,7 | .658,652,3 |
| 36 00 | .628,318,5 | .587,785,3 | .809,017,0 | .190,983,0 | .040.533,28 | .006,409,082 | .197,392,1 | .345,491,5 | 654,508,5 |
| 36 15 | .632,681,9 | .591,309,6 | .806,444,6 | .193.555.4 | .041,372,21 | .006,587,768 | .200,143,2 | .349,647,1 | .650,352,9 |
| 36 30 | .637,045,2 | .594,822,8 | .803,856.9 | .196.143,1 | .042,222,39 | .006,770,139 | .202,913,3 | .353,814,1 | .646,185,9 |
| 36 45 | .641,408.5 | .598.324,6 | .801,253,8 | .198.746.2 | .043,083,90 | .006,956,245 | .205,702,4 | .357,992,3 | .642,007,7 |
| 37 00 | .645,771,8 | .601,815,0 | .798,635,5 | .201,364,5 | .043,956,80 | .007,146,134 | .208,510,6 | .362,181,3 | .637,818,7 |
| 37 15 | .650,135,1 | .605.294,0 | .796,002,0 | .203.998,0 | .044,841,16 | .007,339,857 | .211,337,9 | .366,380.8 | .633.619,2 |
| 37 30 | .654,498,5 | .608,761,4 | .793,353,3 | .206,646,7 | .045,737,04 | .007,537,464 | .214,184,1 | .370,590,5 | .629,409,5 |
| 37 45 | .658,861.8 | .612,217,3 | .790,689.6 | 209.310,4 | .046,644,51 | .007,739,005 | .217.049.4 | .374.810,0 | .625,190,0 |
| 38 00 | .663,225,1 | .615,661,5 | .788,010.8 | .211,989.2 | .047,563,64 | .007,944,531 | .219,933,8 | .379.039,1 | 620.960,9 |
| 38 15 | .667,588,4 | .619,093,9 | .785,316,9 | .214,683,1 | .048,494,49 | .008,154,093 | .222,837.2 | .383,277,3 | .616,722,7 |
| 38 30 | .671,951,8 | .622,514,6 | .782.608.2 | .217,391.8 | .049,437,13 | .008.367,742 | .225.759.6 | .387,524,5 | 612,475,5 |
| 38 45 | .676,315.1 | .625,923,5 | .779,884.5 | .220.115,5 | .050,391,61 | .008,585,530 | .228.701.0 | 391.780,2 | 608,219,8 |
| 39 00 | .680,678,4 | .629,320,4 | .777,146,0 | .222,854.0 | .051,358,02 | .008,807,509 | .231,661.5 | .396.044,2 | 603,955,8 |
| 39 15 | .685,041.7 | .632,705,3 | .774,392,6 | .225,607.4 | .052,336,40 | .009,033,731 | .234,641,1 | .400.316,0 | .599,684.0 |
| 39 30 | .689,405.1 | .636,078.2 | .771,624,6 | .228.375.4 | .053,326,83 | .009,264,248 | .237,639.7 | 404,595,5 | .595,404,5 |
| 39 45 | .693,768,4 | .639,439,0 | .768,841,8 | 231.158.2 | .054,329.38 | .009,499,113 | .240,657,3 | .408.882.2 | 591,117,8 |
| 40 00 | .698,131,7 | .642,787,6 | .766,044,4 | .233,955.6 | .055,344,09 | .009,738,379 | .243,693,9 | 413,175,9 | .586,824,1 |
| 40 15 | .702,495.0 | .646.124.0 | .763,232,5 | .236.767.5 | 056.371,04 | .009,982,099 | .246,749.6 | 417,476,2 | .582,523,8 |
| 40 30 | .706,858,3 | .649,448,0 | .760,406.0 | .239.594.0 | .057,410,30 | .010,230.33 | .249,824,4 | 421.782,8 | .578,217.2 |
| 40 45 | .711,221,7 | .652.759,8 | .757,565,0 | .242.435,0 | .058,461,92 | .010,483,12 | .252,918.1 | 426.095,3 | .573,904.7 |
| 41 00 | .715,585,0 | .656,059,0 | .754,709,6 | .245.290,4 | .059,525,96 | .010,740,52 | .256,030,9 | 430,413,4 | .569,586,6 |
| 41 15 | .719,948,3 | .659.345,8 | .751,839,8 | .248,160,2 | .060,602,50 | .011,002,60 | .259,162,8 | 434,736,9 | 565,263,1 |
| 41 30 | .724,311,6 | .662,620,0 | .748,955,7 | .251.044,3 | .061,691.59 | .011.269,40 | .262.313.7 | 439,065,3 | 560,934,7 |
| 41 45 | .728.675,0 | .665,881.7 | .746,057.4 | .253,942,6 | .062,793,30 | .011,540,98 | .265,483,6 | 443,398,4 | 556,601,6 |
| 42 00 | .733,038,3 | .669,130,6 | .743,144,8 | .256,855.2 | .063.907,68 | .011,817,39 | .268,672.6 | 447.735,8 | .552,264,2 |
| 42 15 | .737,401.6 | .672.366,8 | .740,218,1 | 259,781,9 | .065,034,80 | .012,098.69 | .271,880,6 | 452.077,1 | .547,922,9 |
| 42 30 | .741,764,9 | .675,590,2 | .737,277.3 | .262,722.7 | 066,174.72 | .012,384,94 | .275,107,6 | 456,422,1 | 543,577,9 |
| 42 45 | .746,128,3 | .678,800,7 | .734,322,5 | .265,677.5 | .067,327,51 | .012,676,20 | .278,353.7 | 460.770,5 | .539,229,5 |
| 43 00 | .750,491,6 | .681,998,4 | .731,353,7 | .268,646.3 | .068,493,22 | .012,972,51 | .281,618,8 | 465,121.8 | .534.878,2 |
| 43 15 | .754,854,9 | .685,183,0 | .728,371,0 | .271,629.0 | .069,671,91 | .013.273,93 | .284.903,0 | 469,475.7 | 530.524,3 |
| 43 30 | .759.218.2 | .688,354,6 | .725,374,4 | .274,625.6 | .070,863,65 | .013,580,53 | .288.206.2 | 473,832,0 | 526.168,0 |
| 43 45 | .763,581,5 | .691,513,1 | .722,364,0 | .277,636,0 | 072,068,49 | .013,892,35 | .291.528,4 | 478,190,3 | 521,809,7 |
| 44 00 | .767,944,9 | .694,658,4 | .719,339,8 | .280,660,2 | .073,286,50 | .014,209,46 | .294,869,7 | 482.550,3 | 517,449,7 |
| 44 15 | .772,308,2 | .697,790,5 | .716,301,9 | .283,698,1 | .074,517,73 | .014,531,92 | .298,230,0 | 486.911,5 | .513,088,5 |
| 44 30 | .776,671,5 | .700,909,3 | .713,250,4 | .286,749,6 | .075,762,25 | .014,859,77 | .301,609,3 | 491,273,8 | .508,726,2 |
| 44 45 | .781,034,8 | .704,014,7 | .710,185,4 | .289,814,6 | .077,020,12 | .015,193,09 | .305.007,7 | 495,636.7 | .504,363,3 |
| 45 00 | .785,398.2 | .707,106.8 | .707,106.8 | .292,893,2 | .078,291,38 | .015.531,92 | 308,425.1 | .500,000,0 | .500,000,0 |

Table H-13.–*Trigonometric table for arch analysis* (sheet 3).

| $\Phi$ Degrees | $\Phi$ Radians | SIN $\Phi$ | COS $\Phi$ | VERS $\Phi$ | $\Phi$-SIN $\Phi$ | $\frac{\Phi^2}{2}$-VERS $\Phi$ | $\frac{\Phi^2}{2}$ | SIN$^2$ $\Phi$ | COS$^2$ $\Phi$ |
|---|---|---|---|---|---|---|---|---|---|
| 45 15 | .789.761.5 | .710,185,4 | .704,014,7 | .295,985.3 | .079,578,11 | .015,876,33 | .311,861,6 | .504,363,3 | .495,636,7 |
| 45 30 | .794,124,8 | .713,250,4 | .700,909,3 | .299,090,7 | .080,874,36 | .016,226,37 | .315,317,1 | .508,726,2 | .491,273,8 |
| 45 45 | .798,488,1 | .716,301,9 | .697,790,5 | .302,209,5 | .082,186,19 | .016,582,11 | .318,791,6 | .513,088,5 | .486,911,5 |
| 46 00 | .802,851,5 | .719,339,8 | .694,658,4 | .305,341,6 | .083,511,66 | .016,943,60 | .322,285,2 | .517,449,7 | .482,550,3 |
| 46 15 | .807,214,8 | .722,364,0 | .691,513,1 | .308,486.9 | .084,850,82 | .017,310,91 | .325,797,9 | .521,809,7 | .478,190,3 |
| 46 30 | .811,578,1 | .725,374,4 | .688,354,6 | .311,645,4 | .086,203,73 | .017,684,08 | .329,329,5 | .526,168,0 | .473,832,0 |
| 46 45 | .815,941,4 | .728,371,0 | .685,183,0 | .314,817,0 | .087,570,46 | .018,063,20 | .332,880,2 | .530,524,3 | .469,475,7 |
| 47 00 | .820,304,7 | .731,353,7 | .681,998,4 | .318,001,6 | .088,951,05 | .018,448,30 | .336,449,9 | .534,878,2 | .465,121,8 |
| 47 15 | .824.668,1 | .734,322,5 | .678,800.7 | .321,199,3 | .090,345,56 | .018.839,46 | .340,038,7 | .539,229,5 | .460,770,5 |
| 47 30 | .829,031,4 | .737,277,3 | .675,590,2 | .324,409,8 | .091,754,06 | .019,236,73 | .343,646,5 | .543,577.9 | .456,422,1 |
| 47 45 | .833,394,7 | .740,218,1 | .672,366,8 | .327.633,2 | .093,176,59 | .019.640.19 | .347,273,4 | .547,922,9 | .452,077,1 |
| 48 00 | .837,758,0 | .743,144,8 | .669,130,6 | .330,869,4 | .094,613,22 | .020,049,87 | .350,919,3 | .552,264,2 | .447,735,8 |
| 48 15 | .842,121,4 | .746,057,4 | .665,881.7 | .334,118,3 | .096,063,99 | .020,465.86 | .354,584,2 | .556,601,6 | .443,398,4 |
| 48 30 | .846,484,7 | .748,955,7 | .662,620,0 | .337,380,0 | .097,528,97 | .020.888,22 | .358,268,2 | .560,934,7 | .439,065,3 |
| 48 45 | .850,848,0 | .751,839.8 | .659,345,8 | .340.654,2 | .099.008,20 | .021.316,98 | .361,971,2 | .565,263,1 | .434,736,9 |
| 49 00 | .855,211,3 | .754,709,6 | .656,059,0 | .343,941.0 | .100,501,8 | .021,752,24 | .365,693,2 | .569,586,6 | .430,413,4 |
| 49 15 | .859,574,7 | .757,565,0 | .652,759,8 | .347,240,2 | .102,009,7 | .022,194,05 | .369,434,3 | .573,904,7 | .426,095,3 |
| 49 30 | .863,938.0 | .760,406,0 | .649,448,0 | .350,552,0 | .103,532,0 | .022,642,46 | .373,194,4 | .578,217,2 | .421,782,8 |
| 49 45 | .868.301.3 | .763,232,5 | .646,124,0 | .353,876.0 | .105,068,8 | .023,097,56 | .376,973,6 | .582,523,8 | .417,476,2 |
| 50 00 | .872.664.6 | .766,044,4 | .642.787,6 | .357,212,4 | .106,620,2 | .023.559.38 | .380,771,8 | .586,824,1 | .413,175,9 |
| 50 15 | .877,027,9 | .768,841,8 | .639,439,0 | .360,561,0 | .108,186,1 | .024,028,01 | .384.589,0 | .591,117,8 | .408,882,2 |
| 50 30 | .881,391,3 | .771,624,6 | .636,078,2 | .363,921,8 | .109,766,7 | .024,503,51 | .388,425,3 | .595,404,5 | .404,595,5 |
| 50 45 | .885.754,6 | .774,392,6 | .632,705,3 | .367,294,7 | .111.362,0 | .024.985,93 | .392,280,6 | .599,684,0 | .400,316,0 |
| 51 00 | .890,117,9 | .777,146,0 | .629,320,4 | .370,679,6 | .112,972,0 | .025,475,35 | .396,155.0 | .603,955,8 | .396,044,2 |
| 51 15 | .894,481,2 | .779,884,5 | .625,923,5 | .374,076,5 | .114,596,8 | .025,971,82 | .400.048,3 | .608.219,8 | .391,780,2 |
| 51 30 | .898.844.6 | .782,608.2 | .622,514,6 | .377,485,4 | .116,236,4 | .026,475,41 | .403,960,8 | .612,475,5 | .387,524,5 |
| 51 45 | .903,207.9 | .785,316.9 | .619,093,9 | .380,906,1 | .117,891,0 | .026,986,19 | .407,892.2 | .616,722,7 | .383,277,3 |
| 52 00 | .907,571,2 | .788,010,8 | .615,661,5 | .384,338.5 | .119,560,5 | .027,504,23 | .411,842,8 | .620,960,9 | .379,039,1 |
| 52 15 | .911,934,5 | .790,689,6 | .612,217,3 | .387,782,7 | .121,245,0 | .028,029,58 | .415,812,3 | .625,190,0 | .374,810,0 |
| 52 30 | .916,297,9 | .793,353,3 | .608,761,4 | .391,238,6 | .122,944,5 | .028,562,31 | .419,800,9 | .629,409,5 | .370,590,5 |
| 52 45 | .920,661,2 | .796,002.0 | .605,294,0 | .394,706,0 | .124,659,2 | .029,102,49 | .423,808,5 | .633,619,2 | .366,380,8 |
| 53 00 | .925,024.5 | .798,635,5 | .601,815,0 | .398,185,0 | .126,389.0 | .029,650,19 | .427,835,2 | .637,818,7 | .362,181,3 |
| 53 15 | .929,387,8 | .801,253.8 | .598,324,6 | .401,675,4 | .128,134,0 | .030.205,47 | .431,880,9 | .642,007,7 | .357,992,3 |
| 53 30 | .933,751,1 | .803,856,9 | .594,822,8 | .405,177,2 | .129.894,3 | .030,768,39 | .435,945,6 | .646,185,9 | .353,814,1 |
| 53 45 | .938,114,5 | .806,444,6 | .591,309,6 | .408.690.4 | .131,669,9 | .031,339,03 | .440,029.4 | .650,352,9 | .349,647,1 |
| 54 00 | .942,477,8 | .809,017,0 | .587,785,3 | .412,214,7 | .133,460,8 | .031,917,45 | .444,132,2 | .654,508,5 | .345,491,5 |
| 54 15 | .946,841,1 | .811,574,0 | .584,249,7 | .415,750.3 | .135,267,1 | .032,503,72 | .448,254,1 | .658,652,3 | .341,347,7 |
| 54 30 | .951,204,4 | .814,115,5 | .580,703,0 | .419,297,0 | .137,088,9 | .033,097,90 | .452,394,9 | .662,784,1 | .337,215,9 |
| 54 45 | .955,567,8 | .816,641,6 | .577,145,2 | .422,854,8 | .138,926,2 | .033,700,07 | .456,554,9 | .666.903,4 | .333,096,6 |
| 55 00 | .959,931,1 | .819,152,0 | .573,576,4 | .426,423.6 | .140,779,0 | .034,310,28 | .460,733,8 | .671,010,1 | .328,989,9 |
| 55 15 | .964,294,4 | .821,646,9 | .569.996,8 | .430,003,2 | .142,847,5 | .034,928,82 | .464,931,9 | .675,103,7 | .324,896,3 |
| 55 30 | .968,657,7 | .824,126,2 | .566,406,2 | .433,593,8 | .144,531,5 | .035,555,14 | .469,148,9 | .679,184,0 | .320,816,0 |
| 55 45 | .973,021,1 | .826,589,7 | .562,804,9 | .437,195,1 | .146,431,3 | .036,189,92 | .473,385,0 | .683,250,6 | .316,749,4 |
| 56 00 | .977,384,4 | .829,037.6 | .559,192,9 | .440,807,1 | .148,346,8 | .036,833,02 | .477,640,1 | .687,303,3 | .312,696,7 |
| 56 15 | .981.747.7 | .831,469,6 | .555,570,2 | .444,429,8 | .150,278,1 | .037,484,51 | .481.914,3 | .691,341,7 | .308,658,3 |
| 56 30 | .986,111,0 | .833,885,8 | .551,937,0 | .448,063,0 | .152,225,2 | .038,144,46 | .486,207,5 | .695,365,6 | .304,634,4 |
| 56 45 | .990,474,4 | .836,286,2 | .548,293,2 | .451,706,8 | .154,188,2 | .038,812,95 | .490,519,7 | .699,374,5 | .300,625,5 |
| 57 00 | .994,837,7 | .838,670.6 | .544,639,0 | .455,361,0 | .156,167,1 | .039,490,03 | .494,851,0 | .703,368,3 | .296,631,7 |
| 57 15 | .999,201,0 | .841,039,0 | .540,974,5 | .459,025.5 | .158,162,0 | .040,175,79 | .499,201,3 | .707,346,6 | .292,653,4 |
| 57 30 | 1.003,564,3 | .843,391,4 | .537,299,6 | .462,700.4 | .160,172,9 | .040,870,28 | .503,570,7 | .711,309,1 | .288,690.9 |
| 57 45 | 1.007,927,6 | .845,727.8 | .533,614,5 | .466,385.5 | .162,199,8 | .041,573,58 | .507,959.1 | .715,255,5 | .284,744.5 |
| 58 00 | 1.012,291,0 | .848,048,1 | .529,919,3 | .470,080,7 | .164,242,9 | .042,285,76 | .512,366,5 | .719,185,6 | .280,814,4 |
| 58 15 | 1.016,654.3 | .850,352,2 | .526,213,9 | .473,786,1 | .166,302,1 | .043,006,90 | .516,793,0 | .723,098.9 | .276,901,1 |
| 58 30 | 1.021,017,6 | .852,640,2 | .522,498,6 | .477,501,4 | .168.377,4 | .043,737.05 | .521,238,5 | .726,995,2 | .273,004,8 |
| 58 45 | 1.025.380.9 | .854,911,9 | .518,773,3 | .481.226,7 | .170,469,1 | .044,476,29 | .525,703.0 | .730,874.3 | .269,125,7 |
| 59 00 | 1.029,744.3 | .857,167.3 | .515,038,1 | .484,961,9 | .172,577,0 | .045,224,69 | .530,186.6 | .734.735,8 | .265,264.2 |
| 59 15 | 1.034,107.6 | .859,406,4 | .511,293,1 | .488,706,9 | .174.701,2 | .045,982.33 | .534,689,2 | .738,579,4 | .261,420,6 |
| 59 30 | 1.038,470,9 | .861,629,2 | .507,538,4 | .492,461,6 | .176,841,7 | .046,749,27 | .539,210,9 | .742,404,8 | .257,595,2 |
| 59 45 | 1.042,834,2 | .863.835.5 | .503,774,0 | .496,226,0 | .178,998,7 | .047,525,59 | .543,751,6 | .746,211,8 | .253,788,2 |
| 60 00 | 1.047,197,6 | .866,025,4 | .500,000,0 | .500,000,0 | .181,172,1 | .048,311,36 | .548,311,4 | .750,000,0 | .250,000,0 |
| 60 15 | 1.051.560,9 | .868,198,8 | .496,218,5 | .503,783.5 | .183.362,1 | .049,106,64 | .552,890,1 | .753,769,2 | .246,230,8 |
| 60 30 | 1.055,924,2 | .870,355.7 | .492,423.6 | .507,576,4 | .185,568,5 | .049,911.52 | .557,488,0 | .757,519,0 | .242,481,0 |
| 60 45 | 1.060,287,5 | .872,496,0 | .488,621,2 | .511,378,8 | .187,791,5 | .050,726,05 | .562,104,8 | .761,249,3 | .238,750,7 |
| 61 00 | 1.064,850,8 | .874,619,7 | .484,809,6 | .515,190,4 | .190,031,1 | .051,550,33 | .566,740,7 | .764,959,6 | .235,040,4 |
| 61 15 | 1.069,014,2 | .876.726,8 | .480,988,8 | .519,011,2 | .192,287,4 | .052,384,41 | .571,395,6 | .768,649,8 | .231,350,2 |
| 61 30 | 1.073,377,5 | .878,817,1 | .477,158,8 | .522,841,2 | .194,560,4 | .053,228,38 | .576,069,6 | .772,319,5 | .227,680,5 |
| 61 45 | 1.077,740,8 | .880,890,7 | .473,319,7 | .526,680,3 | .196,850,1 | .054,082,30 | .580,762,6 | .775,968,5 | .224,031,5 |
| 62 00 | 1.082,104,1 | .882,947,6 | .469,471,6 | .530,528,4 | .199,156,5 | .054,946,24 | .585,474,7 | .779,596,5 | .220,403,5 |
| 62 15 | 1.086,467,5 | .884,987,6 | .465,614,5 | .534,385,5 | .201,479,8 | .055,820,29 | .590,205,8 | .783,203,1 | .216,796,9 |
| 62 30 | 1.090,830,8 | .887,010,8 | .461,748,6 | .538,251,4 | .203,819.9 | .056,704,51 | .594,955,9 | .786,788,2 | .213,211,8 |
| 62 45 | 1.095,194,1 | .889,017,1 | .457,873,9 | .542,126,1 | .206,177,0 | .057,598,98 | .599,725,1 | .790,351,5 | .209,648,5 |
| 63 00 | 1.099,557,4 | .891,006.5 | .453,990,5 | .546,009,5 | .208,550,9 | .058,503,77 | .604,513,3 | .793,892,6 | .206,107,4 |
| 63 15 | 1.103,920,8 | .892,978,9 | .450.098,4 | .549,901,6 | .210,941,8 | .059,418,95 | .609,320,5 | .797,411,4 | .202,588,6 |
| 63 30 | 1.108,284,1 | .894,934,4 | .446,197,8 | .553,802,2 | .213,349,7 | .060,344,61 | .614,146,8 | .800,907,5 | .199,092,5 |
| 63 45 | 1.112,647,4 | .896,872,7 | .442,288,7 | .557,711,3 | .215.774,7 | .061.280,81 | .618,992,1 | .804,380,7 | .195,619,3 |
| 64 00 | 1.117,010,7 | .898,794,0 | .438,371,1 | .561,628.9 | .218,216,7 | .062,227,62 | .623,856,5 | .807,830,7 | .192,169,3 |
| 64 15 | 1.121,374,0 | .900,698,2 | .434,445,3 | .565.554,7 | .220,675,8 | .063,185,13 | .628,739,9 | .811,257,3 | .188,742,7 |
| 64 30 | 1.125,737,4 | .902,585.3 | .430,511,1 | .569,488.9 | .223,152,1 | .064,153,41 | .633,642,3 | .814,660,2 | .185,339,8 |
| 64 45 | 1.130,100,7 | .904,455.1 | .426,568,7 | .573,431,3 | .225,645,5 | .065,132,53 | .638,563,8 | .818,039,1 | .181,960,9 |
| 65 00 | 1.134,464,0 | .906,307.8 | .422,618,3 | .577,381,7 | .228,156,2 | .066,122,56 | .643,504,3 | .821,393,8 | .178,606,2 |
| 65 15 | 1.138,827,3 | .908,143,2 | .418,659,7 | .581,340,3 | .230,684,2 | .067,123,59 | .648,463,9 | .824,724,0 | .175,276,0 |
| 65 30 | 1.143,190,7 | .909,961,3 | .414,693,2 | .585,306,8 | .233,229,4 | .068,135,69 | .653,442,4 | .828,029,5 | .171,970,5 |
| 65 45 | 1.147.554,0 | .911,762.0 | .410,718.9 | .589,281,1 | .235,791,9 | .069,158.92 | .658,440,1 | .831,310.0 | .168,690,0 |
| 66 00 | 1.151,917,3 | .913,545.5 | .406,736,6 | .593,263,4 | .238,371,8 | .070,193,38 | .663,456,7 | .834,565,3 | .165,434,7 |
| 66 15 | 1.156.280,6 | .915,311,5 | .402,746,7 | .597,253,3 | .240,969,2 | .071,239,14 | .668,492,4 | .837,795,1 | .162,204,9 |
| 66 30 | 1.160,644,0 | .917,060,1 | .398,749,1 | .601,250.9 | .243,583,9 | .072,296,26 | .673,547,2 | .840,999,2 | .159,000,8 |
| 66 45 | 1.165,007,3 | .918,791,2 | .394,743,9 | .605,256,1 | .246,216,1 | .073,364,83 | .678,621,0 | .844,177.3 | .155,822,7 |
| 67 00 | 1.169,370,6 | .920,504,9 | .390,731,1 | .609,268,9 | .248,865,7 | .074,444,93 | .683,713,8 | .847,329,2 | .152,670,8 |
| 67 15 | 1.173,733,9 | .922,201,0 | .386,711,0 | .613,289,0 | .251,533,0 | .075,536,62 | .688,825,7 | .850,454,6 | .149,545,4 |
| 67 30 | 1.178,097.2 | .923.879.5 | .382.683,4 | .617,316,6 | .254,217.7 | .076,639,99 | .693,956.6 | .853,553,4 | .146,446,6 |

Table H-13.—*Trigonometric table for arch analysis* (sheet 4).

| $\phi$ Degrees | $\phi$ Radians | SIN $\phi$ | COS $\phi$ | VERS $\phi$ | $\phi$-SIN $\phi$ | $\frac{\phi^2}{2}$-VERS $\phi$ | $\frac{\phi^2}{2}$ | SIN$^2$ $\phi$ | COS$^2$ $\phi$ |
|---|---|---|---|---|---|---|---|---|---|
| 67 45 | 1.182,460.6 | .925,540,5 | .378,648,6 | .621,351,4 | .256,920,1 | .077,755,12 | .699,106,5 | .856,625,2 | .143,374,8 |
| 68 00 | 1.186.823,9 | .927,183,9 | .374,606,6 | .625,393,4 | .259,640,0 | .078,882,07 | .704,275,5 | .859,669,9 | .140,330,1 |
| 68 15 | 1.191,187,2 | .328,809,6 | .370,557,4 | .629,442,6 | .262,377,7 | .080,020,93 | .709,463,5 | .862,687,2 | .137,312,8 |
| 68 30 | 1.195,550,5 | .930,417,6 | .366,501,2 | .633,498,8 | .265,133,0 | .081,171,77 | .714,670,5 | .865,676,9 | .134,323,1 |
| 68 45 | 1,199,913,9 | .932,007,9 | .362,438,0 | .637,562,0 | .267,906,0 | .082,334,67 | .719,896,6 | .868,638,7 | .131,361,3 |
| 69 00 | 1.204,277,2 | .933,580,4 | .358,367,9 | .641,632,1 | .270,696,8 | .083,509,72 | .725,141,8 | .871,572,4 | .128,427,6 |
| 69 15 | 1.208,640,5 | .935,135,2 | .354.291,0 | .645,709,0 | .273,505,3 | .084,696,98 | .730,405,9 | .874,477,9 | .125,522,1 |
| 69 30 | 1.213,003,8 | .936.672,2 | .350,207,4 | .649,792,6 | .276,331,6 | .085,896,53 | .735,689,1 | .877,354,8 | .122,645,2 |
| 69 45 | 1.217,367.2 | .938,191,3 | .346,117,1 | .653,882,9 | .279,175,8 | .087,108,45 | .740,991,4 | .880,203,0 | .119,797,0 |
| 70 00 | 1.221,730,5 | .339.692,6 | .342,020,1 | .657,979,9 | .282,037,9 | .088,332,82 | .746,312,7 | .883,022,2 | .116,977,8 |
| 70 15 | 1.226,093,8 | .941.176,0 | .337,916,7 | .662,083,3 | .284,917,8 | .089,569,72 | .751,653,0 | .885,812,3 | .114,187,7 |
| 70 30 | 1.230,457,1 | .942,641,5 | .333,806,9 | .666,193,1 | .287,815,6 | .090,819,22 | .757,012,4 | .888,573,0 | .111,427,0 |
| 70 45 | 1.234,820,4 | .944,069.0 | .329,690,6 | .670,309,4 | .290,731,4 | .092,081,41 | .762,390,8 | .891,304,1 | .108,695,9 |
| 71 00 | 1.239,183.8 | .945.518,6 | .325,568,2 | .674.431,8 | .293,665,2 | .093,356,36 | .767,788,2 | .894,005,4 | .105,994,6 |
| 71 15 | 1.243,547,1 | .946.930.1 | .321,439,5 | .678,560,5 | .296,617,0 | .094,644,15 | .773,204,7 | .896,676,7 | .103,323,3 |
| 71 30 | 1.247,910,4 | .948.323,7 | .317,304,7 | .682,695,3 | .299,586,6 | .095,944,86 | .778,640,2 | .899,317,8 | .100,682,2 |
| 71 45 | 1.252,273,7 | .949,699,1 | .313,163,8 | .686,836,2 | .302,574,6 | .097,258,56 | .784,094,8 | .901,928,4 | .098,071,57 |
| 72 00 | 1.256,637,1 | .951,056,5 | .309,017,0 | .690,983,0 | .305,580,5 | .098,585,35 | .789,568,4 | .904,508,5 | .095,491,50 |
| 72 15 | 1.261,000,4 | .952,395.8 | .304.864,3 | .695,135,7 | .308,604,6 | .099,925,28 | .795,061,0 | .907,057,8 | .092,942,24 |
| 72 30 | 1.265,383,7 | .953,717,0 | .300,705,8 | .699,294,2 | .311,646,8 | .101,278,5 | .800,572,7 | .909,576,0 | .090,423,98 |
| 72 45 | 1.269,727.0 | .955,019,9 | .296,541,6 | .703,458,4 | .314,707,1 | .102,644,9 | .806,103,4 | .912,063,1 | .087,936,91 |
| 73 00 | 1.274,090,4 | .956,304,8 | .292,371,7 | .707,628,3 | .317,785,6 | .104,024,8 | .811,653,1 | .914,518,8 | .085,481,21 |
| 73 15 | 1.278,453,7 | .957,571,4 | .288,196,3 | .711,803,7 | .320,882,3 | .105,418,2 | .817,221,9 | .916,942,9 | .083,057,09 |
| 73 30 | 1.282,817,0 | .958,819,7 | .284,015,3 | .715,984,7 | .323,997,3 | .106,825,1 | .822,809,7 | .919,335,3 | .080,664,72 |
| 73 45 | 1.287,180,3 | .960,049,9 | .279,829,0 | .720,171,0 | .327,130,5 | .108,245,6 | .828,416,6 | .921,695,7 | .078,304,28 |
| 74 00 | 1.291,543,6 | .961,261,6 | .275,637,4 | .724,362,6 | .330,282,0 | .109,679,9 | .834,042,5 | .924,024,1 | .075,975,95 |
| 74 15 | 1.295,907,0 | .962,455,2 | .271,440,4 | .728,559,6 | .333,451,7 | .111,127,9 | .839,687,4 | .926,320,1 | .073,679,92 |
| 74 30 | 1.300.270,3 | .963,630,5 | .267,238,4 | .732,761,6 | .336,639,8 | .112,589,8 | .845,351,4 | .928,583,7 | .071,416,35 |
| 74 45 | 1.304,633,6 | .964,787,3 | .263,031,2 | .736,968,8 | .339,846,3 | .114,065,7 | .851,034,4 | .930,814,6 | .069,185,42 |
| 75 00 | 1.308,996,9 | .965,925,8 | .258,819,0 | .741,181,0 | .343,071,1 | .115,555,5 | .856,736,5 | .933,012,7 | .066,987,30 |
| 75 15 | 1.313,360,3 | .967,045,9 | .254,601,9 | .745,398,1 | .346,314,3 | .117,059,5 | .862,457,6 | .935,177,8 | .064,822,15 |
| 75 30 | 1.317,723,6 | .968,147,6 | .250,380,0 | .749.620,0 | .349,575,9 | .118,577,7 | .868,197,7 | .937,309,9 | .062,690,15 |
| 75 45 | 1.322,086,9 | .969,230,9 | .246,153,3 | .753,846,7 | .352,856,0 | .120,110,2 | .873,956,9 | .939,408,6 | .060,591,44 |
| 76 00 | 1.326,450,2 | .970.295,7 | .241,921,9 | .758,078,1 | .356,154,5 | .121,657,0 | .879,735,1 | .941,473,8 | .058,526.20 |
| 76 15 | 1.330,813.6 | .971,342.1 | .237,685.9 | .762,314,1 | .359,471,5 | .123,218,3 | .885,532,4 | .943,505,4 | .056,494,58 |
| 76 30 | 1.335.176,9 | .972,369,9 | .233,445,4 | .766.554,6 | .362,807,0 | .124,794,0 | .891,348,6 | .945,503,3 | .054,496,74 |
| 76 45 | 1.339.540.2 | .973,379,3 | .229,200,4 | .770,799,6 | .366,160,9 | .126,384,4 | .897,164,0 | .947,467,2 | .052,532,82 |
| 77 00 | 1.343.903.5 | .974,370,1 | .224,951.1 | .775,048,9 | .369,533,5 | .127,989,4 | .903,038,3 | .949,397,0 | .050,602,98 |
| 77 15 | 1.348,266,8 | .975,342,3 | .220,697,4 | .779,302,6 | .372,924,5 | .129,609,2 | .908,911,7 | .951,292,6 | .048,707,36 |
| 77 30 | 1.352,630,2 | .976,296,0 | .216,439,6 | .783,560,4 | .376,334,2 | .131,243,8 | .914,804,2 | .953,153,9 | .046,846,11 |
| 77 45 | 1.356,993,5 | .977,231,1 | .212,177,7 | .787,822,3 | .379,762,4 | .132,893,3 | .920,715,7 | .954,980,6 | .045,019,36 |
| 78 00 | 1.361,356,8 | .978,147,6 | .207.911,7 | .792,088,3 | .383,209,2 | .134,557,9 | .926,646,2 | .956,772,7 | .043,227,27 |
| 78 15 | 1.365,720,1 | .979,045,5 | .203,641.8 | .796,358,2 | .386,674,7 | .136,237,5 | .932,595,7 | .958,530,0 | .041,469,96 |
| 78 30 | 1.370,083,5 | .979,924,7 | .199,367,9 | .800,632,1 | .390,158.8 | .137,932,3 | .938,564,3 | .960,252,4 | .039,747,57 |
| 78 45 | 1.374,446,8 | .980,765,3 | .195,090,3 | .804,909,7 | .393,661,5 | .139,642,3 | .944,552,0 | .961,939,6 | .038,060,23 |
| 79 00 | 1.378,810,1 | .981,627,2 | .190,809,0 | .809,191,0 | .397,182,9 | .141,367,7 | .950,558,7 | .963,591,9 | .036,408,07 |
| 79 15 | 1.383,173,4 | .982,450,4 | .186,524,0 | .813,476,0 | .400,723,0 | .143,108,4 | .956,584,4 | .965,208,8 | .034,791,22 |
| 79 30 | 1.387,536,8 | .983,254,9 | .182,235.5 | .817,764,5 | .404,281,8 | .144,864,6 | .962,629,1 | .966,790,2 | .033,209,79 |
| 79 45 | 1.391,900,1 | .984,040,7 | .177,943,5 | .822,056,5 | .407,859,4 | .146,636,5 | .968,692,9 | .968,336,1 | .031,663,91 |
| 80 00 | 1.396,263,4 | .984,807,8 | .173,648,2 | .826,351,8 | .411,455,6 | .148,423,9 | .974,775,7 | .969,846,3 | .030,153,69 |
| 80 15 | 1.400,626,7 | .985,556,1 | .169,349,5 | .830,650,5 | .415,070,7 | .150,227,1 | .980,877,6 | .971,320,7 | .028,679,25 |
| 80 30 | 1.404,990,0 | .986.285,6 | .165,047,6 | .834,952,4 | .418,704,4 | .152,046,1 | .986,998,5 | .972,759,3 | .027,240,71 |
| 80 45 | 1.409,353,4 | .986,996,4 | .160,742.6 | .839,257,4 | .422,357,0 | .153,881,0 | .993,138,5 | .974,161,8 | .025,838,17 |
| 81 00 | 1.413,716,7 | .987,688,3 | .156,434,5 | .843,565,5 | .426,028,4 | .155,731,9 | .999,297,4 | .975,528,3 | .024,471,74 |
| 81 15 | 1.418,080,0 | .988,361,5 | .152,123,4 | .847,876,6 | .429,718,5 | .157,596,9 | 1.005,475,5 | .976,858,5 | .023,141,52 |
| 81 30 | 1.422,443,3 | .989,015.9 | .147,809,4 | .852,190,6 | .433,427,5 | .159,481,9 | 1.011,672,5 | .978,152,4 | .021,847,62 |
| 81 45 | 1.426.806,7 | .989,651,4 | .143,492,6 | .856,507.4 | .437,155,3 | .161,381,2 | 1.017,888,6 | .979,409,9 | .020,590,13 |
| 82 00 | 1.431,170,0 | .990,268,1 | .139,173,1 | .860,826,9 | .440,901,9 | .163,296,9 | 1.024,123,8 | .980,630,9 | .019,369,15 |
| 82 15 | 1.435,533,3 | .990,865,9 | .134,850,9 | .865,149,1 | .444,667,4 | .165,228,9 | 1.030,377,9 | .981,815,2 | .018,184,77 |
| 82 30 | 1.439,896,6 | .991,444,9 | .130,526,2 | .869,473,8 | .448,451,8 | .167,177,3 | 1.036,651,2 | .982,962,9 | .017,037,09 |
| 82 45 | 1.444,260,0 | .992,004,9 | .126,199,0 | .873,801,0 | .452,255,0 | .169,142,4 | 1.042,943,4 | .984,073,8 | .015,926,18 |
| 83 00 | 1.448,623,3 | .992,546,2 | .121,869,3 | .878,130,7 | .456,077,1 | .171,124,0 | 1.049,254,7 | .985,147,9 | .014,852,14 |
| 83 15 | 1.452.986,6 | .993,068,5 | .117,537,4 | .882,462,6 | .459,918,1 | .173,122,4 | 1.055,585,0 | .986,185,0 | .013,815,04 |
| 83 30 | 1.457,349,9 | .993,571,9 | .113,203,2 | .886,796,8 | .463,778,1 | .175,137,6 | 1.061,934,4 | .987,185,0 | .012,814,97 |
| 83 45 | 1.461,713.2 | .994,056.3 | .108,866,9 | .891,133,1 | .467,656,9 | .177,169,7 | 1.068,302,8 | .988,148,0 | .011,852,00 |
| 84 00 | 1.466,076.6 | .994,521,9 | .104.528,5 | .895,471,5 | .471,554,7 | .179,218,7 | 1.074,690,3 | .989,073,8 | .010,926,20 |
| 84 15 | 1.470,439.9 | .994,986,5 | .100,188,1 | .899,811,9 | .475,471,4 | .181,284,8 | 1.081,096,7 | .989,962,4 | .010,037,65 |
| 84 30 | 1.474,803.2 | .995,396.2 | .095,845,75 | .904,154,2 | .479,407,0 | .183,368,0 | 1.087,522,3 | .990,813,6 | .009,186,408 |
| 84 45 | 1.479,166.5 | .995.804,9 | .091,501,62 | .908,498,4 | .483,361,6 | .185,468,4 | 1.093,966,8 | .991,627,5 | .008,372,546 |
| 85 00 | 1.483,529,9 | .996,194,7 | .087,155,74 | .912,844,3 | .487,335,2 | .187,586,2 | 1.100,430,4 | .992,403,9 | .007,596,124 |
| 85 15 | 1.487,893.2 | .996,565,5 | .082,808,21 | .917,191,8 | .491,327,7 | .189,721,3 | 1.106,913,1 | .993,142,8 | .006,857,199 |
| 85 30 | 1.492,256,5 | .996,917,3 | .078,459,10 | .921,540,9 | .495,339,2 | .191,873,8 | 1.113,414,7 | .993,844,2 | .006,155,830 |
| 85 45 | 1.496,619,8 | .997,250,2 | .074,108,49 | .925,891,5 | .499,369.6 | .194,044,0 | 1.119,935,5 | .994,507,9 | .005,492,068 |
| 86 00 | 1.500.983,2 | .997,564,1 | .069,756,47 | .930,243,5 | .503,419,1 | .196,231,7 | 1.126,475,2 | .995,134,0 | .004,865,966 |
| 86 15 | 1.505,346.5 | .997,858,9 | .065,403,13 | .934,596,9 | .507,487,6 | .198,437,1 | 1.133,034,0 | .995,722,4 | .004,277,569 |
| 86 30 | 1.509,709,8 | .998,134,8 | .061,048,54 | .938,951,5 | .511,575,0 | .200,660,4 | 1.139,611,8 | .996,273,1 | .003,726,924 |
| 86 45 | 1.514,073.1 | .998,391,7 | .056,692,79 | .943,307,2 | .515,681,5 | .202,901,5 | 1.146,208,7 | .996,785,9 | .003,214,072 |
| 87 00 | 1.518,436,4 | .998,629,4 | .052,335,96 | .947,664,0 | .519,806,9 | .205,160,6 | 1.152,824,6 | .997,260,9 | .002,739,052 |
| 87 15 | 1.522,799,8 | .998,848,4 | .047,978,13 | .952.021,9 | .523,951,4 | .207,437,7 | 1.159,459,6 | .997,698,1 | .002,301,901 |
| 87 30 | 1.527,163,1 | .999,048,2 | .043,619,39 | .956,380,6 | .528,114,9 | .209,733,0 | 1.166,113,6 | .998,097,3 | .001,902,851 |
| 87 45 | 1.531,526,4 | .999,229,0 | .039,259,82 | .960,740,2 | .532,297,4 | .212,046,4 | 1.172,786,6 | .998,458,7 | .001,541,333 |
| 88 00 | 1.535,889,7 | .999.390,8 | .034,899,50 | .965,100,5 | .536,498,9 | .214,378,1 | 1.179,478,6 | .998,782,0 | .001,217,975 |
| 88 15 | 1.540,253,1 | .999,533,6 | .030,538,51 | .969,461,5 | .540,719,5 | .216,728,3 | 1.186,189,8 | .999,067,4 | .000,932,600,8 |
| 88 30 | 1.544,616,4 | .999,657,3 | .026,176,95 | .973,823,1 | .544,959,1 | .219,096,8 | 1.192,919,9 | .999,314,8 | .000,685,232,6 |
| 88 45 | 1.548,979,[illegible] | .999,762,0 | .021,814,89 | .978,185,1 | .549,217,7 | .221,484,0 | 1.199,669,1 | .999,524,1 | .000,475,889,2 |
| 89 00 | 1.553,343,0 | .999,847,7 | .017,452,41 | .982,547,6 | .553,495,3 | .223,889,7 | 1.206,437,3 | .999,695,4 | .000,304,586,5 |
| 89 15 | 1.557,706,4 | .999,914,3 | .013,089,60 | .986,910,4 | .557,792,0 | .226,314,1 | 1.213,224,5 | .999,828,7 | .000,171,337,5 |
| 89 30 | 1.562,069,7 | .999,981,9 | .008,726,535 | .991,273,5 | .562,107,8 | .228,757,4 | 1.220,030,8 | .999,923,8 | $.0^{4}76,152,42$ |
| 89 45 | 1.566.433,0 | .999,990.5 | .004,363,309 | .995,636,7 | .566,442,5 | .231,219,5 | 1.226,856,2 | .999,981,0 | $.0^{4}19,038,47$ |
| 90 00 | 1.570,796,3 | 1.000,000,0 | .000,000,000 | 1.000,000,0 | .570,796,3 | .233,700,6 | 1.233,700,6 | 1.000,000,0 | 0 |

SHEET 4 OF 4

<<Appendix I

# Special Studies

I-1. *Introduction.*—Special investigations are sometimes required to determine the magnitude of stresses at corners, at the junction of the face of the dam with the foundation, around openings, and at foundation seams, due to special conditions affecting the structure, or caused by any structural discontinuity. For dams of low height for which computed stresses are low and well within allowable limits, the consideration of stress concentrations may not be important. On the other hand, for high dams with higher computed stresses which are close to the allowable, the determination of stress concentrations is of great importance. This is one of the reasons for the comprehensive stress studies for Hoover Dam, which are described in one of the Boulder Canyon Project Final Reports [1].[1] Owing to the unusual height and size of Hoover Dam, it was considered imperative that stress distributions should be determined as accurately as possible.

As discussed in chapter IV, the trial-load method is based on the assumption that normal stresses on a horizontal plane vary linearly from upstream face to downstream face of the dam at all elevations. However, it is known that this assumption does not hold for horizontal sections at or near the base of the dam. The nonlinear stress analysis described on the following pages is an analytical approach to a solution of these stresses.

The studies of stress changes due to water soaking of concrete at the upstream face of Hoover Dam showed that such effects are primarily of a surface nature. They do not extend any appreciable distance, because of the dense concrete used and the efficient system of vertical concrete drains installed near the upstream face. Stress conditions caused by daily temperature variations are of a similar nature, although daily temperature variations at the faces may be of appreciable magnitude above the water surface. Temperatures due to these conditions can be included in the trial-load analysis.

Studies of the effects of spreading of the canyon walls at Hoover Dam showed that no crack in the foundation would be developed, and that such effects have a tendency to reduce compressive stresses in the foundation and could be adequately allowed for by utilizing a few additional degrees of seasonal temperature drop after grouting of construction joints or by providing a few additional degrees of subcooling during construction of the dam.

Studies of the settlement of the reservoir bed at Hoover Dam due to the great weight of stored water, more than 41 billion tons, indicated that flow effects and elastic deformations at the dam might be appreciable and settlement at the dam might possibly amount to 0.9 foot. However, actual measurements taken later showed a settlement of only about 2½ inches at the dam, indicating that little or no change in stress conditions had occurred at the dam due to the effect of reservoir bed settlement.

If there are seams in the foundation which contain relatively soft material, it is necessary to study the effect on the structure. So far, the Bureau has reduced such effects by placing sufficient blocks of concrete in these seams so

---

[1] Numbers in brackets refer to items in the bibliography, sec. I-18.

that elastic deformations in the foundation are within allowable limits. This was done for Seminoe, Friant, and Shasta Dams. Separate studies are required to determine the size and extent of the concrete mass placed in the seams. These studies are usually made by analyses based on the theory of elasticity, which are shown on the following pages.

I-2. *Introduction to Nonlinear Stress Analysis.*—The "Nonlinear Method of Stress Analysis" is presented in the manual, "Design of Gravity Dams" [2]. The analysis of an arch dam can be made by the same method, except that the load on the element is determined by the results of the trial-load analysis, and corrections of nonlinear stresses in the cantilever must be made for a cantilever with radial sides and including the effects of twist action.

I-3. *Cantilever with Radial Sides.*—The correction of nonlinear stresses in the cantilever is made on the assumption that the relation between linear and nonlinear stresses in a cantilever with radial sides is the same as the relation between linear and nonlinear stresses in a cantilever with parallel sides, when the two elements occupy the same position in an arch dam and are subjected to the same water pressures at the upstream face. In the analysis of a cantilever in Hoover Dam, this change in stress for a cantilever with radial sides for the trial-waterload analysis amounted to a decrease in vertical stress of 82 pounds per square inch at the upstream edge of the base, an increase of 166 pounds per square inch at the downstream edge of the base, a decrease in shear stress of 7 pounds per square inch at the upstream edge of the base, and an increase of 84 pounds per square inch at the downstream edge of the base.

I-4. *Twist Effects.*—Effects of twist action on nonlinear cantilever stresses at the faces of the dam may be assumed to be the same as those determined by the trial-load analysis. That is to say, the change in stress from those stresses calculated for the radial adjustment to those calculated for the twist adjustment (neglecting the effects of the tangential adjustment) is the change in stress to be used in correcting, or adjusting, the nonlinear stresses.

For Hoover Dam, stress changes due to twist effects increased nonlinear vertical and shearing stresses at the upstream face and decreased these nonlinear stresses at the downstream face. In other words, twist adjustments of nonlinear stresses tend to balance radial cantilever adjustments. At the base of a cantilever in Hoover Dam, twist effects increased the vertical stress by 57 pounds per square inch at the upstream face and decreased the vertical stress by 76 pounds per square inch at the downstream face. Shear stresses were increased at the upstream edge of the base by 17 pounds per square inch and decreased by 59 pounds per square inch at the downstream edge of the base.

I-5. *General Discussion on Foundations.*—Special studies are sometimes required for arch dams, such as Hoover Dam, to determine the effects of movements, structural defects, and stress concentrations in the rock formations. Among the factors studied are the extent and effect of movement of the abutments and foundation, the effect of seams in the foundation, and the effect of other conditions which are not in agreement with the basic assumptions made in the trial-load analysis.

Initial stress conditions in the canyon floor beneath the base of the dam are important in considering possible effects of the spreading of the abutments due to reservoir pressure. If these initial stresses are compressive and are also greater than the tensile stresses produced by the spreading of the canyon walls, the formation of a longitudinal crack would not be expected to occur along the bottom of the canyon. If such a crack did develop, it would affect the structure as well as movement of the canyon walls. Outward movements of the canyon walls would lengthen the horizontal distance between the walls and would obviously affect arch stresses.

The effect on stress conditions in the dam produced by the movement of the canyon floor due to weight of stored water is probably negligible. As indicated earlier, measurements made at Hoover Dam indicate that a depression of about 2½ inches occurred at the dam. It is not believed that this would affect the structure to any important degree.

For the study of foundation and abutment effects it was necessary in the case of Hoover Dam to determine the original stresses in the rock on which the dam was built, the modulus of elasticity and ultimate strength of various rock formations, the permeability of the rock, and the effect of absorbed water on the elasticity of the rock. Samples were taken from an experimental tunnel in the canyon floor and tests were made to determine Young's modulus, Poisson's ratio, and the ultimate strength of the samples.

**I-6.** ***Two-Dimensional Study of Spreading of Abutments at Hoover Dam.***—A two-dimensional analysis of the possible spreading of the abutments at Hoover Dam was made as a preliminary study. Although such an analysis is only applicable to a canyon of uniform cross section and great length, subjected to uniform loading conditions, the results of the study serve to indicate the general nature of the displacements to be expected.

Figure I-1(a) shows the cross section of a semicircular canyon filled with water. If $R$, $r$, and $\theta$ are as shown in the figure and $p$ is the unit weight of water, the stress condition for this case is represented by the Airy's function,

$$F = -\frac{p R^2 r \theta \sin \theta}{2} \qquad (1)$$

Stresses, strains, and displacements can be evaluated for the above function by standard methods. With the reservoir full, spreading of the sides of the semicircular canyon is practically negligible. From a physical viewpoint, spreading due to the horizontal thrust of the water is counterbalanced by closing due to vertical water pressure on the bottom. Vertical pressures on the canyon floor cause the sides to tilt inward. The action is similar to that observed when pressure is applied to the bottom of a groove in a block of rubber.

The investigation of spreading displacements for a semicircular canyon was followed by an approximate analysis for a U-shaped canyon of the form shown on figure I-1(b). In this case, computations indicated that wall displacements of appreciable magnitude would occur for larger values of the height, $a$.

An indication of how closely the analyzed sections conform to the actual canyon may be obtained by comparing figures I-1(a) and I-1(b) with figure I-2. Figure I-2 shows the average cross section of the canyon from the dam to the first bend, about 6,000 feet upstream. The canyon section at the dam is indicated by the dashed line.

**I-7.** ***Three-Dimensional Study of Spreading of Abutments at Hoover Dam.***—At this point, it was thought advisable to continue the investigation with the use of three-dimensional methods. The canyon was represented by two infinitely large masses of rock with vertical faces, to which were applied hydrostatic loads representing reservoir water pressure. A section of the structure first examined is shown on figure I-3. The loaded areas had a length of 6,000 feet along the canyon, and the height of the loaded area was taken as 635 feet, which is about the average depth of water in this reach of the reservoir.

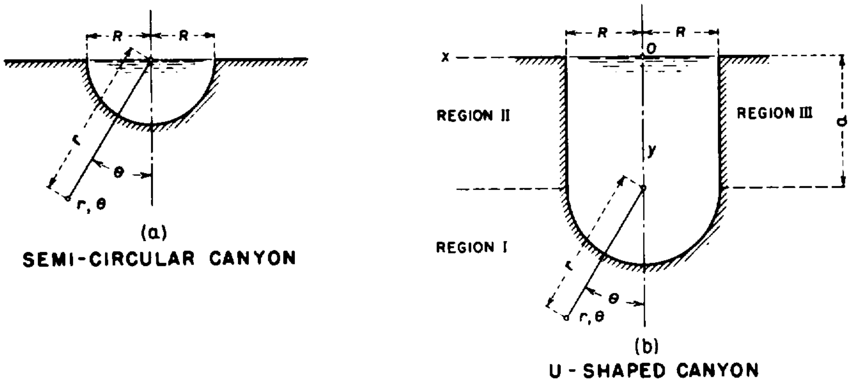
Surface displacements of the two rock masses shown on figure I-3 may be computed from the Boussinesq formulas,

$$\Delta z_0 = \frac{(1-\mu^2)}{\pi E}\left[\frac{P}{\sqrt{x^2+y^2}}\right] \qquad (2)$$

$$\Delta y_0 = \frac{(1-2\mu)(1+\mu)}{2\pi E}\left[\frac{Py}{x^2+y^2}\right] \qquad (3)$$

In the above equations $x$ and $y$ are coordinates in the surface plane, $z$ is the coordinate normal to that plane, $\Delta z_0$ and $\Delta y_0$ are surface displacements in the $z$ and $y$ directions, $P$ is a concentrated load, $\mu$ is Poisson's ratio, and $E$ is the modulus of elasticity. In calculations based on the above equations, $\mu$ was taken equal to 0.216 and $E$ equal to 868,320,000 pounds per square foot.

These equations may be integrated to obtain displacement formulas for various load patterns. For example, equation (2) may be integrated to obtain the formula,



(a)
SEMI-CIRCULAR CANYON

(b)
U-SHAPED CANYON

*Figure I-1.* **Hoover Dam studies–canyon cross sections.–DS2-1(283)**

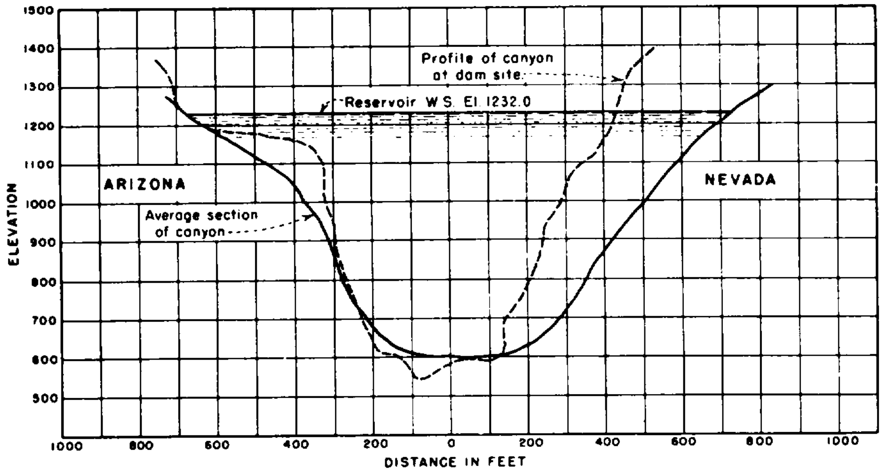


*Figure I-2.* **Hoover Dam studies–cross section of canyon above dam.–DS2-1(284)**

$$\Delta z_0 = \frac{(1-\mu^2)}{\pi E} \bullet p \left[ b \sinh^{-1} \frac{a}{b} + a \sinh^{-1} \frac{b}{a} \right] \quad (4)$$

This gives the displacement in the $z$ direction of the corner of a rectangle of length $a$ and width $b$ loaded with a uniform pressure $p$. For the same loading, equation (3) may be integrated to give the $y$ displacement, or

$$\Delta y_0 = \frac{(1-2\mu)(1+\mu)}{2\pi E} \bullet p \left[ b \tan^{-1} \frac{a}{b} + \frac{a}{2} \ln \frac{a^2+b^2}{a^2} \right] \quad (5)$$

Note that $\Delta y_0$ indicates a shortening of width $b$ of the loaded rectangle.

In a similar manner equations of displacements for a hydrostatic load may be obtained. At the time the study was made, hydrostatic loads were represented by a stepped series of uniformly loaded rectangles. Results obtained by this method were later checked with displacements computed for the true triangular load. Differences between computed values were negligible. Displacements were computed for various values of $y$ along the line forming one end of the loaded area. This line corresponds to an abutment of the dam. Computed displacements of the abutments are given in the following tabulation, $y$ being expressed in terms of elevations at the dam.

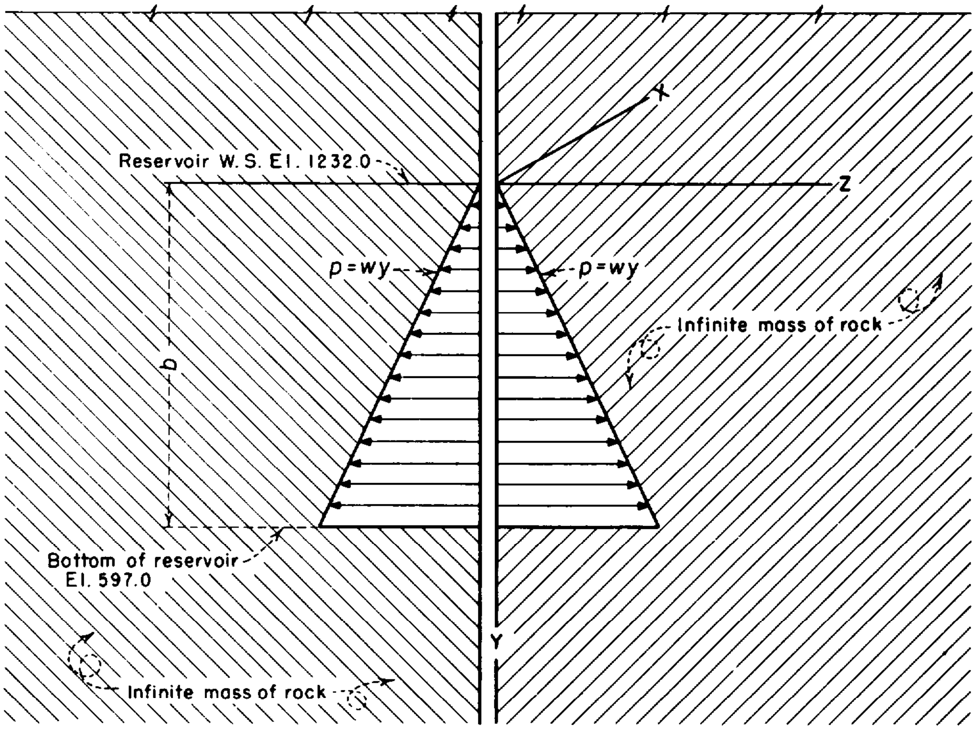


*Figure I-3.* **Hoover Dam studies—canyon with vertical walls in infinite rock formations.—DS2-1(285)**

| Elevation | Horizontal movement of each abutment, feet |
|---|---|
| 1232 | 0.0151 |
| 1100 | .0173 |
| 900 | .0206 |
| 700 | .0218 |
| 505 | .0168 |
| 432 | .0156 |
| 382 | .0150 |

Displacements in the tabulation are for each abutment. The total spreading of the canyon walls would be twice the tabulated values. These movements increase the distances between abutments. Figure I-3 shows the section considered during this stage of the analysis. Hydrostatic loads were applied to the abutment faces over a height, *b*, equal to the reservoir depth of 635 feet, and over a canyon length of 6,000 feet. The result indicated a crack opening of 0.034 foot in the foundation rock at the dam.

I-8. ***Foundation Closing Loads at Hoover Dam.***—It was considered probable that compressive stresses great enough to prevent the development of a crack might naturally exist in the rock strata along the bottom of the canyon. In this case, spreading of the canyon walls would merely result in a reduction of compression in the foundation rather than in the formation of a crack. Subsequent investigation at the dam confirmed this assumption. The two-walled structure of figure I-3 was therefore modified as shown on figure I-5 to include closing of the two walls at the elevation of the base of the dam.

In the modified structure, tensile loads were applied to the abutment walls below the elevation of the reservoir bottom. Loads required to bring the walls back into contact at elevation 505 were determined by trial, allowances being made for the lateral contraction of the reservoir bottom as indicated on figure I-7. A bottom width of 630 feet was used in the computations. With a water depth of 635 feet, the lateral contraction of the bottom, as computed by equation (4) was 0.0048 foot. This computation, as well as all others in the analysis, was made for the location of the dam which is at one end of the loaded area. Movements would be somewhat greater for the center of the loaded area, but such conditions would have no material effect on the balancing of loads at the dam.

Figure I-4(a) shows the loading required to close the walls at the foundation level. Figure I-4(b) shows deflections resulting from combined water and foundation closing loads. The stepped loads used for water and foundation closing loads represent conditions slightly different from those used in the remainder of the investigation, but the results are affected only in a minor degree by this discrepancy. Figure I-4(a) shows that the diagram for closing loads is roughly parabolic in shape, as would be expected for the structure represented.

It is evident from figure I-5 that the modified structure is essentially a stressed crack in a large mass of rock, modified slightly by the inclusion of bottom contraction. From the theory of elasticity, an approximately parabolic stress distribution is known to apply adjacent to a crack in a stressed body. This study, therefore, both as to formulation and result, represents an extremely narrow canyon approaching a crack in form.

I-9. ***Strains in Canyon Floor at Hoover Dam.***—The next study of abutment spreading included, as additional factors, stretching of the canyon floor due to closing loads and tilting of canyon walls due to the reservoir water mass pressing on the floor. Stretching of the floor is easily demonstrated. Tensile loads of the previous section must be transmitted between abutment walls by the rock below the canyon floor. Extensional strains corresponding to these tensions are present in the rock, causing stretching of the canyon floor. To avoid confusion, it may be reiterated that tensions here referred to are actually reductions of existing compressive stresses. For convenience of reference they have been called tensions. The strain condition is the same for either case.

Inclusion of canyon-floor stretching is simple. Operations are indicated in the reasoning just given. The actual closing movement of the walls in this case is reduced by the extension of the floor due to the closing loads acting as tensions. Expressed in equation form, at any point,

$$2\,\Delta z_c + \Delta z_f = 2\,\Delta z_w \qquad (6)$$

where $\Delta z_w$ is the movement of one wall due to waterload, $\Delta z_c$ is the movement of one wall due to closing loads, and $\Delta z_f$ is the extension of the floor due to closing loads considered as stresses. Taking $c$ equal to the closing load intensity at the point and $L$ equal to the width of the canyon floor,

$$\Delta z_f = \frac{cL}{E}$$

$$2\,\Delta z_c + \frac{cL}{E} = 2\,\Delta z_w \qquad (7)$$

Both water and closing loads were triangular in form for this study. Three triangular closing loads were used, extending from the canyon floor downward distances of 100, 250, and 727 feet, respectively. Choice of these distances, which was entirely arbitrary, was based partly on an examination of figure I-4(a) and partly on the availability of previously computed results for the 727-foot load.

By means of simultaneous equations, load intensities were evaluated to satisfy equation (7) at three elevations. A floor width of 630 feet was used as before. The computations resulted in the total closing loads shown on figure I-6. These loads are actually horizontal transverse stresses at a vertical section in the center of the canyon. Wall movements produced by these loads are given in table I-1. Near the sides of the canyon shown on figure I-7, stress concentrations due to the reentrant corners would be expected.

I-10. ***Canyon-Wall Tilting at Hoover Dam.***—Tilting of the canyon walls was approximated by placing the whole reservoir water mass on the plane surface of an infinite foundation. The weight of this mass will depress the plane into a lightly curved surface, the form of which can be computed by Boussinesq equations as before. If the canyon walls are assumed to rest on the foundation surface, they will be tilted inward as the

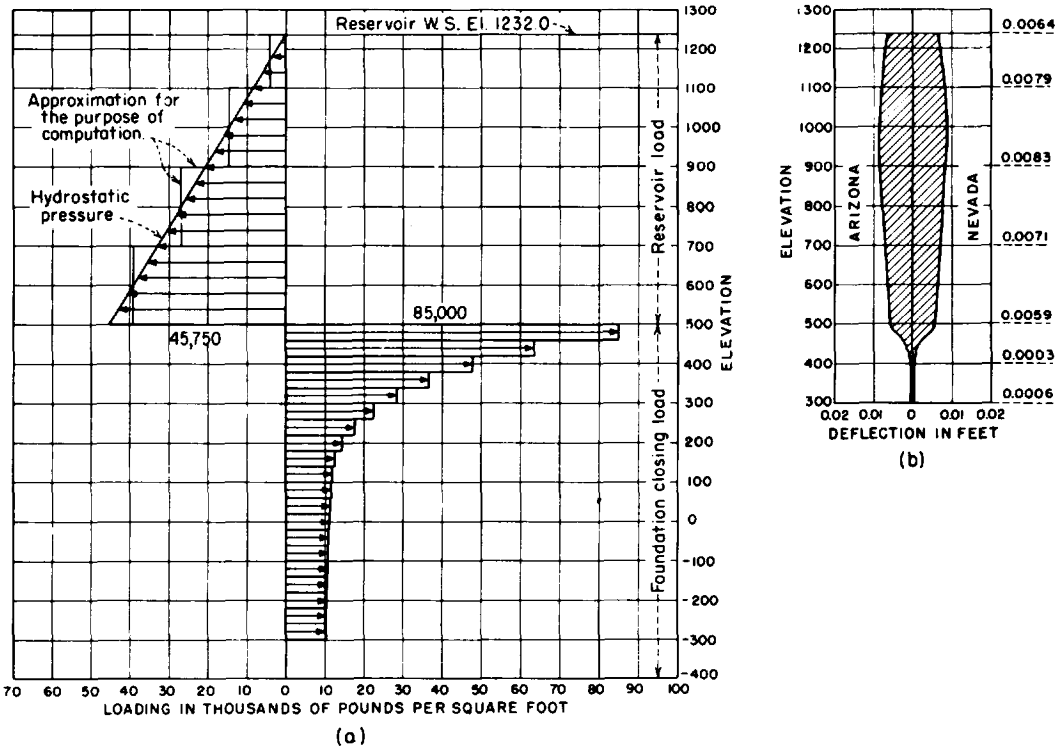


*Figure I-4.* **Hoover Dam studies—foundation load and resultant deflections.—DS2-1(286)**

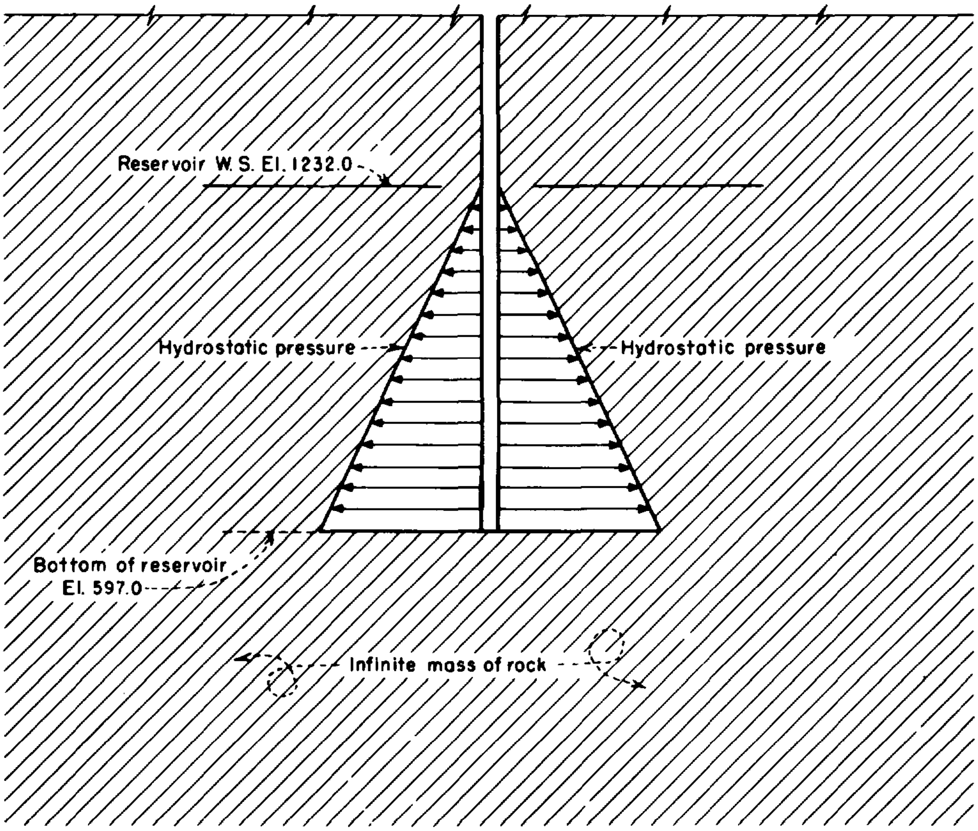


*Figure I-5.* **Hoover Dam studies—infinite abutment closed at base of dam.—DS2-1(287)**

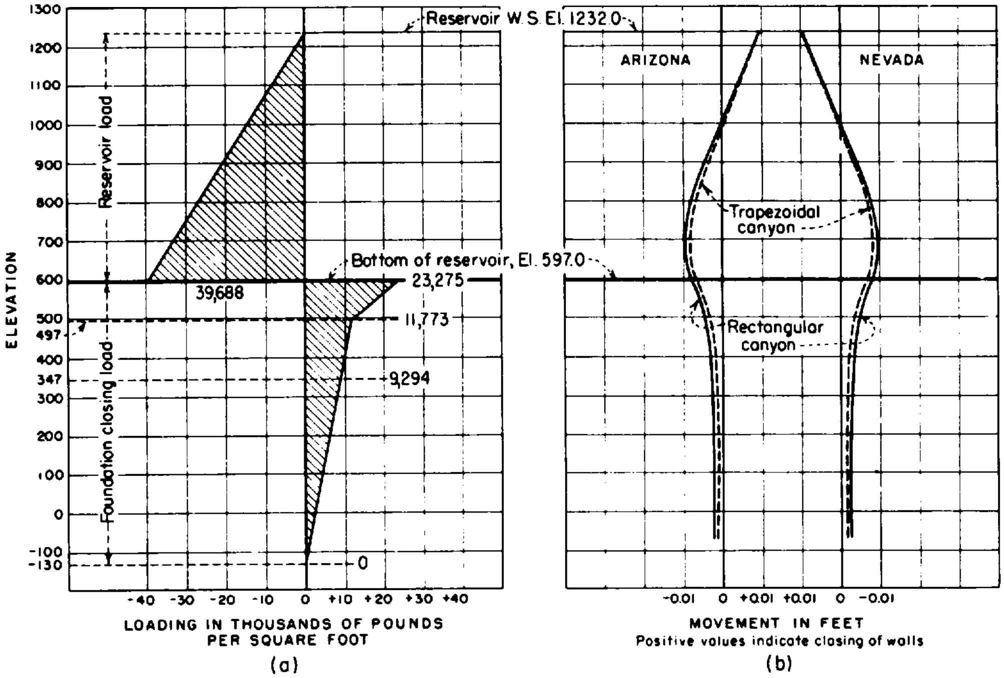


*Figure I-6.* **Hoover Dam studies–closing loads and canyon-wall movements.–DS2-1(288)**

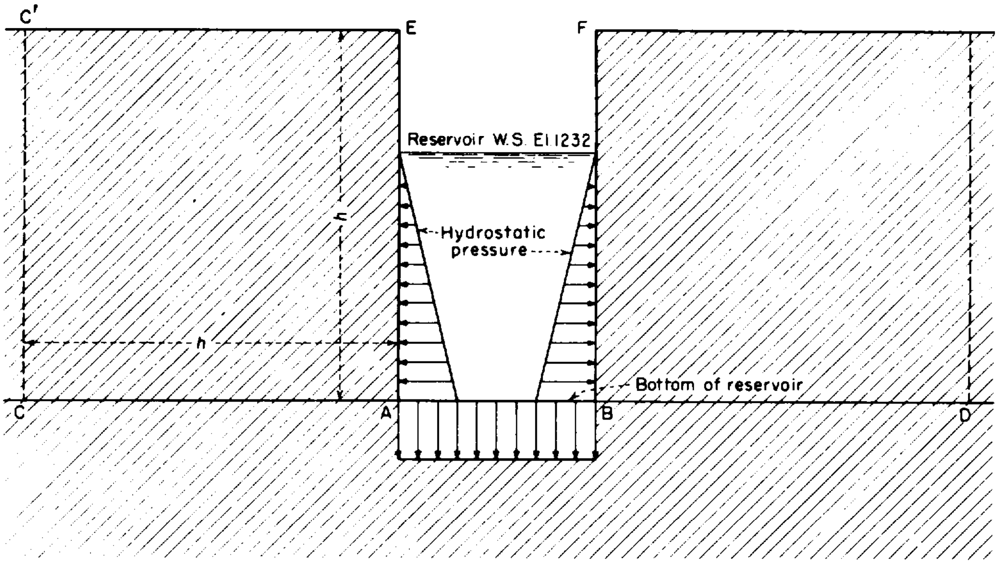


*Figure I-7.* **Hoover Dam studies–canyon as represented by final approximation.–DS2-1(289)**

Table I-1.–*Hoover Dam studies–computed canyon-wall movements in feet.*

| Elev | Movement due to reservoir waterload | Movement due to closing loads | Movement due to combined* closing and waterloads | Movement due to tilt 0.000,026,43 | Total movement |
|---|---|---|---|---|---|
| 1232 | –0.015,149 | 0.005,140 | –0.007,61 | 0.016,783 | +0.009,17 |
| 1100 | – .017,341 | .005,487 | – .009,45 | .013,294 | + .003,84 |
| 900 | – .020,594 | .006,155 | – .012,04 | .008,008 | – .004,03 |
| 700 | – .021,767 | .007,271 | – .012,10 | .002,722 | – .009,38 |
| 597 | – .019,265 | .008,654 | – .008,21 | 0 | – .008,21 |
| 574 | – .018,440 | .009,115 | – .006,93 | 0 | – .006,93 |
| 524 | – .017,145 | .009,342 | – .005,40 | 0 | – .005,40 |
| 332 | – .014,425 | .009,068 | – .002,96 | 0 | – .002,96 |
| 224 | – .013,445 | .008,545 | – .002,50 | 0 | – .002,50 |
| 132 | – .012,781 | .008,004 | – .002,38 | 0 | – .002,38 |
| –68 | – .011,606 | .006,702 | – .002,50 | 0 | – .002,50 |

*+0.004,8 x ½ is added to this total for floor movement.

surface is depressed. Any stiffness of the walls in resisting deformation of the surface is herein neglected in view of the much greater rigidity of the foundation mass.

On figure I-7, *CABD* represents the plane surface on which the reservoir water mass is assumed to be placed; *AE* and *BF* are canyon walls. Plane *CABD* is depressed by the waterload, causing sections *CA* and *BD* to slope inward. Distances *CA* and *BD* are made equal to *h*, the canyon-wall height. The tilt of the canyon walls was taken equal to the slope of the horizontal sections *CA* and *BD*. In other words, the canyon wall *AE* is the face of block *CAEC'*, which tilts as a rigid body. Consequently, rotation of *CA* produces an equal rotation of *AE*.

The area of the average canyon section, figure I-2, is 509,000 square feet. A water depth of 635 feet gives a load width of 802 feet. This is somewhat larger than the base width *AB* of the canyon due to the trapezoidal shape of the canyon. A load corresponding to the water depth of 635 feet was placed on an area 802 feet wide and 6,000 feet long. Displacements at *C* and *A* produced by this load were computed by a modification of equation (4); *AB* was 630 feet as before, and *h* was taken at 950 feet which is the average depth in this reach of the canyon. Using these data, the slope of *CA* is:

$$\alpha_{CA} = \frac{0.0505 - 0.0253}{950}$$

$$= 0.000{,}026{,}4 \text{ foot per foot.}$$

This is the tilt of the canyon walls. Calculated wall displacements due to tilt are given in table I-1. The last column of the tabulation gives the sum of all effects considered in the analysis. The greatest spreading of abutments, 0.009,38 foot, appears at elevation 700, while at the water surface there is actually a closing of 0.009,17 foot, which is practically the same as the maximum spreading. To clarify the analysis, the final method of computing displacements may be stated as follows. Horizontal movements were computed by considering the canyon as parallel vertical faces of two infinite masses of rock, spaced the base width of the canyon apart and connected by an elastic body of rock extending indefinitely downward from the canyon floor. Wall tilting was computed by representing the canyon as composed of two parallel blocks of rectangular section, as *CAEC'* on figure I-7, resting on the horizontal surface of the infinite mass of rock *CABD*.

I-11. ***Results of Abutment Movement Analysis.***–The results of the analysis seem compatible with the structure analyzed. The analysis was based on a rectangular canyon except for the minor point of including the total weight of the water mass in computations for tilting. With a rectangular canyon, as on figure I-7, deformations indicate a maximum spreading at elevation 700, decreasing to zero and changing to a closing value at the top of the canyon.

The structure examined departs somewhat from the prototype canyon. Variations from actuality may be expected to influence deformations in some degree. In order to ascertain any consequent trend in results, it is pertinent to examine briefly the major approximations and assumptions utilized.

The assumptions of elasticity and selection of elastic constants are essential to the mathematical solutions. Deviations would be expected to alter the magnitude of deformations but hardly their general nature. Aside from these, the first major assumption considered the canyon walls to extend infinitely in the vertical direction. The canyon walls actually extend only some 300 feet above the water surface. Under the true condition, spreading in the upper part of the canyon would be somewhat greater than computed. It should be borne in mind that the assumption of infinite wall height was only used for computation of direct wall displacements and canyon-floor stretching. Tilting computations were based on a finite height of canyon walls.

In computations for stretching of the canyon floor and wall tilting, the canyon was considered rectangular in section. As shown on figure I-2, the shape is approximately trapezoidal. It was considered that the principles of this analysis could be applied, with some modification, to a trapezoidal section. Operations were not so direct, however, and methods of computing deformations were less satisfactory. A brief investigation along this line indicated reduced spreading of the walls at all elevations, the greatest effect being at the canyon floor.

The third major approximation was the method of computing tilting of the walls due to water weight. In this study (see figure I-7), the walls were assumed to tilt the same amount as a horizontal section *CA* equal in width to the wall height. In the investigations, several other assumptions were considered. A considerable change in distance *CA* or the canyon width *AB* produced only a moderate change of tilt. The assumption used was, therefore, considered reasonable.

Summing up all factors, which in some degree compensate each other, it appears most probable that computed movements for the lower part of the canyon are somewhat too high. Those for the upper elevations should be nearly correct. The dotted curves on figure I-6 show displacements when the trapezoidal shape of the canyon is considered. Both two-dimensional and three-dimensional studies are in agreement in that spreading of canyon walls due to reservoir water pressure will be slight. Only the factor of reservoir water pressure has been here considered. Since the movements due to this factor appear small, it is likely that other influences will obscure these movements in field measurements at the dam.

The mathematical investigations showed that the spreading of the canyon walls due to reservoir water pressure could be adequately allowed for by considering a few additional degrees of seasonal temperature drop to take place after grouting of contraction joints, or by providing a few additional degrees of subcooling during the construction of the dam.

I-12. ***Stresses in Floor at Hoover Dam.***—The study of abutment spreading indicated the desirability of measuring stress in the canyon floor at Hoover Dam. If compressive stresses in the foundation, normal to the channel, should be lacking or low in intensity, the filling of the reservoir might cause the development of a crack in the bottom of the canyon. The possibility of such a development might require consideration in the construction program.

It was decided to investigate foundation stresses by measuring initial rock strains at the damsite, using existing facilities as far as possible. Proper investigation of foundation stresses required a cross-river tunnel underneath the rock gorge. Fortunately, a drainage tunnel partially fulfilling the requirement had already been excavated by the contractor. This tunnel, located just downstream from the toe of the dam penetrated about halfway under the gorge. A bore, 5 feet wide by 6 feet high, was extended from the drainage tunnel to the far side of the gorge. Plan and elevation views of the tunnel are shown on figure I-8. Although the tunnel was not constructed solely for research purposes, it provided an excellent means for measuring initial strains in the canyon floor.

I-13. ***Strain Measurements at Hoover Dam.***—The strain measurements herein described were made during the period from December 23, 1932, to January 15, 1933. The data secured represent strain conditions in the foundation prior to construction of the dam. Strain-gage points were set in the tunnel-wall rock, using a 20-inch gage length. The initial distance between points was measured with a strain gage. Following the initial measurements, a square block, containing the gage points, was cut free from the rock in the tunnel walls. A second measurement of the gage line gave the unstrained length of the line. The difference between initial and unstrained lengths was the strain in 20 inches. Specimens were cut from the blocks and tested in the Denver laboratory to determine values of the modulus of elasticity and Poisson's ratio for use in translating measured strains into stresses.

Strain-gage stations were established at approximately 50-foot intervals, as shown on figure I-8. Some variation in spacing was necessary in order to locate suitable faces of rock, free from seams, planes of cleavage, or projections which would interfere with strain measurements. Eight gage points were set at each station. These were arranged in pairs, one pair being horizontal, one vertical, and two diagonal. Ellipses of stress and strain were prepared from the vertical, the horizontal, and one of the diagonal measurements. The second diagonal measurement served as a check on the other observations.

Gage holes to receive brass inserts were spaced by a template and drilled five-eighths of an inch in diameter and 3 inches deep. Inserts were held accurately to position, by spacing bars, while being cemented in place.

Considerable difficulty was experienced in finding a suitable cementing material. The presence of moisture in the rock and of flowing water at many stations eliminated most common materials. Leadite was finally adopted for the purpose and proved very satisfactory. The drilled holes were kept dry by blowtorches, and inserts cemented as rapidly as possible to avoid expansion of the spacer bar from the heat of the torches. Too great an expansion would place the inserts beyond the limits of the gage.

After inserts were set, measurements of initial gage lengths were made and continued until a series of four consistent readings were obtained. Solid particles, washed into the gage holes in the inserts by flowing water, frequently caused erroneous readings. Cleaning of inserts immediately before placing the gage eliminated this difficulty. No measurements were made for a period of about one-half hour after the ventilating blower and air-drills were stopped. This period was required to allow the instruments and the air to reach a uniform temperature. Operation of a single air-drill lowered the temperature by about $5^{\circ}$ C.

After points were set and initial readings taken, the blocks were cut free. This was accomplished by drilling a continuous series of holes around each block, at a minimum distance of 14 inches from the inserts. This distance, which made the blocks about 4 feet square, avoided disturbance of inserts from drilling. Holes were drilled to a depth of 30 inches, which was found sufficient to give practically an unstressed condition at the face. The fins between successive holes were broken out by tilting the drill sideways. Strain measurements, made during the process of cutting out the blocks, showed very clearly the effect of Poisson's ratio. When a block was relieved in one direction, the strain line in that direction showed a definite increase in length, while the gage line at right angles showed a reduction in length. When the block was completely freed, the first line was found to have lost some of its increase and the second line to have regained its lost length plus an increment of expansion.

Only one pair of points did not show expected expansion. These were at station 3 where a vertical rock seam, close to the strain-gage points, probably allowed relief of shear along a vertical plane. This seam is shown

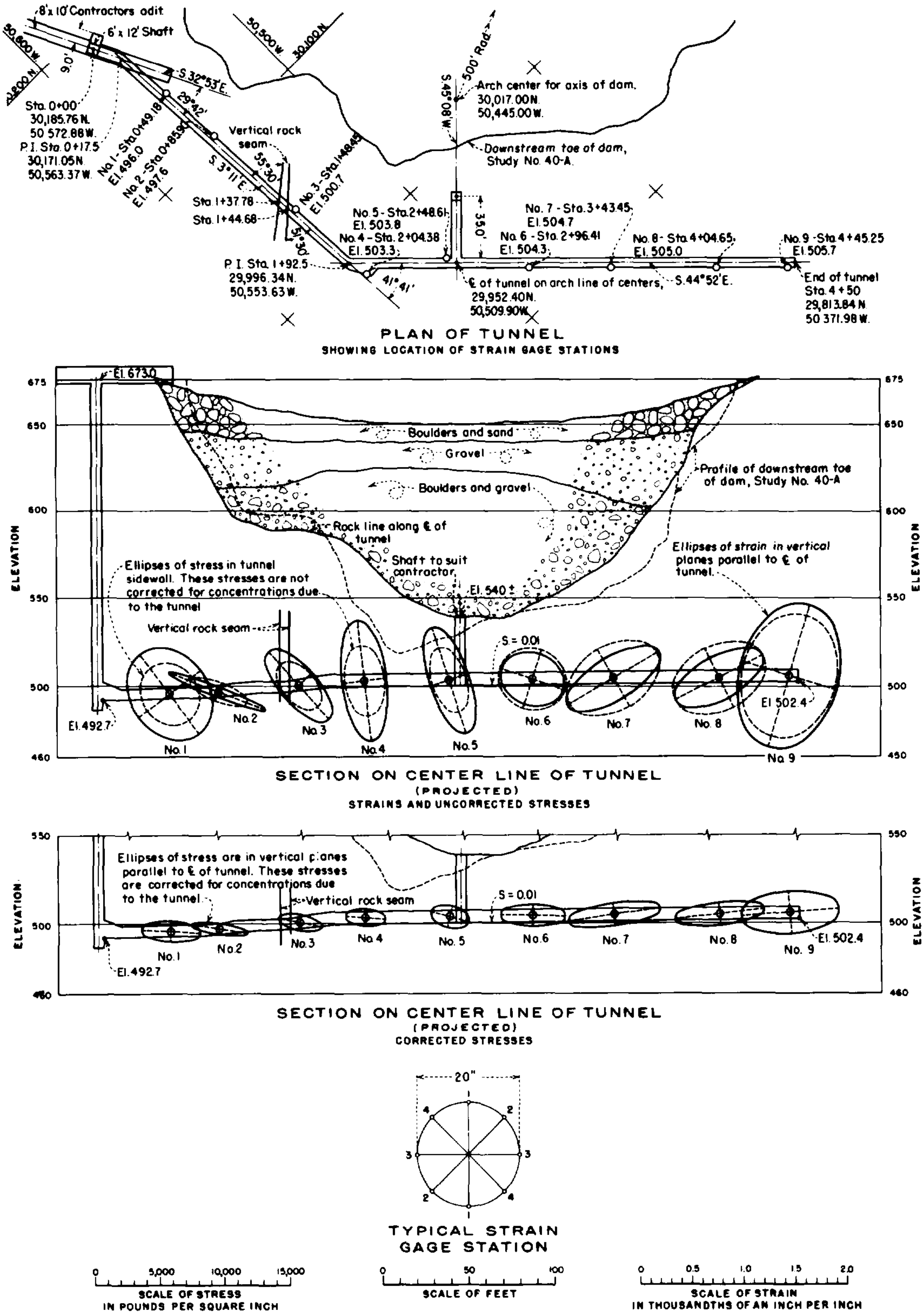


*Figure I-8.* Hoover Dam studies–stresses in canyon floor.–288-D-617 on figure I-8. The effect was apparently local, since it did not appear at adjacent stations.

Following final strain measurements, a core was taken from each block, except Nos. 4 and 5 where the rock fractured when the stress was relieved. Laboratory tests of the cores showed moduli of elasticity ranging from 4,700,000 to 6,800,000 pounds per square inch and Poisson's ratios ranging from 0.21 to 0.32. Values for individual blocks were used in computing stresses in the blocks.

**I-14.** ***Results of Measurements.***—In the center diagrams of figure I-8, ellipses of strain and uncorrected ellipses of stress are shown. In the lower diagrams of figure I-8, stress ellipses, corrected for the concentration effect of the tunnel, are shown. The stresses in the lower diagrams of figure I-6 are those probably existing in the undisturbed rock before the tunnel was excavated.

**I-15.** ***Types of Experimental Investigations.***—Experimental studies are generally made to supplement or check the results of analytical methods. These are usually of two types: (1) structural model tests and (2) photoelastic analyses. The following sections briefly describe these investigations.

**I-16.** ***Structural Model Tests.***—Several dams have been tested by the Bureau using three-dimensional structural models. Structural model tests are most useful in the following ways: (1) Effects of structural discontinuities can be readily determined; and (2) models can be tested to destruction, thus indicating a true safety factor.

Hoover Dam was studied by the use of models in a most comprehensive manner. The results of these studies are available for use and have been described in other publications [3,4]. Figures I-9 and I-10 are of the structural model made for Glen Canyon Dam. The Morrow Point Dam model is shown on figures I-11 and I-12.

**I-17.** ***Photoelastic Analyses.***—The general procedure for photoelastic analyses involves the use of polarized light for analyzing loaded transparent models. These tests are very effective in determining stress patterns and stress concentrations at corners, around openings, and in the vicinity of abutments.

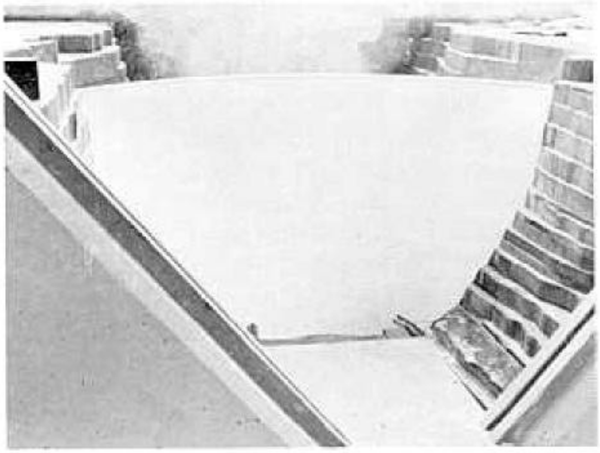


*Figure I-9.* **Structural model of Glen Canyon Dam.–PX-D-43725 NA**

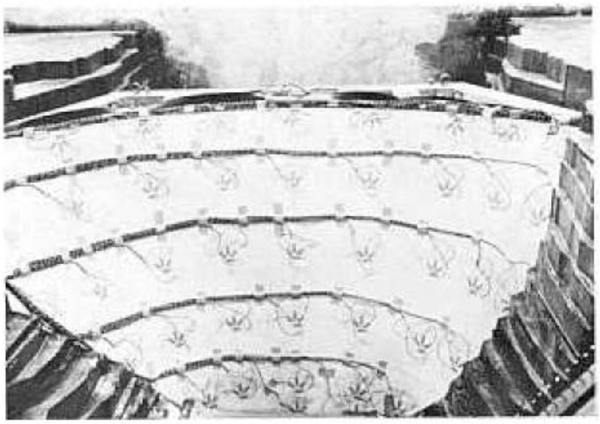


*Figure I-10.* **Structural model of Glen Canyon Dam with strain gages in place.–PX-D-43726 NA**

*Figure I-11.* **Structural model of Morrow Point Dam.–P622-D-40370**

Three-dimensional techniques have been developed. Earlier studies by the Bureau were limited to two-dimensional methods, which, of course, do not give a complete stress picture for a three-dimensional structure.

Seminoe Dam was studied by photoelastic analysis. Stress distributions near the abutments and throughout the lower arches of Glen Canyon Dam were investigated photoelastically. When the spillway design of Morrow Point Dam was being considered, stresses around these large openings were determined by photoelastic methods. Figures I-13, I-14, and I-15 are examples from the Glen Canyon studies. Examples from the Morrow Point study are shown on figures I-16, I-17, and I-18.

### I-18. *Bibliography*.

[1] Bureau of Reclamation, "Stress Studies for Boulder Dam," Bulletin 4 of Part V, Boulder Canyon Project Final Reports, 1939.

[2] Bureau of Reclamation, "Design of Gravity Dams," 1976.

[3] Houk, I. E., "Technical Design Studies for Hoover Dam," Western Construction News, April 10, 1932.

[4] Bureau of Reclamation, "Model Tests of Boulder Dam," Bulletins 2, 3, and 6 of Part V, Boulder Canyon Project Final Reports, 1938-1940.

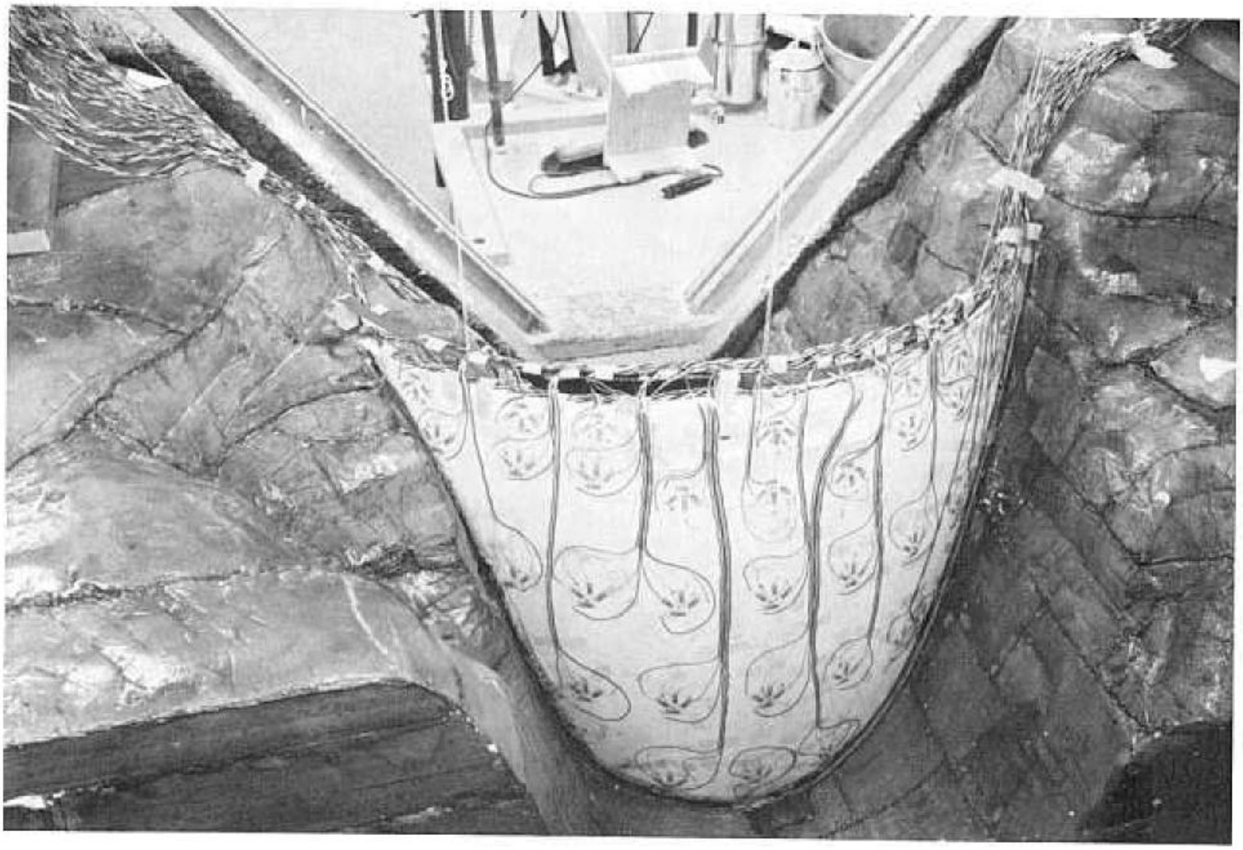


*Figure I-12.* **Structural model of Morrow Point Dam with strain gages in place.–P622-D-40367**

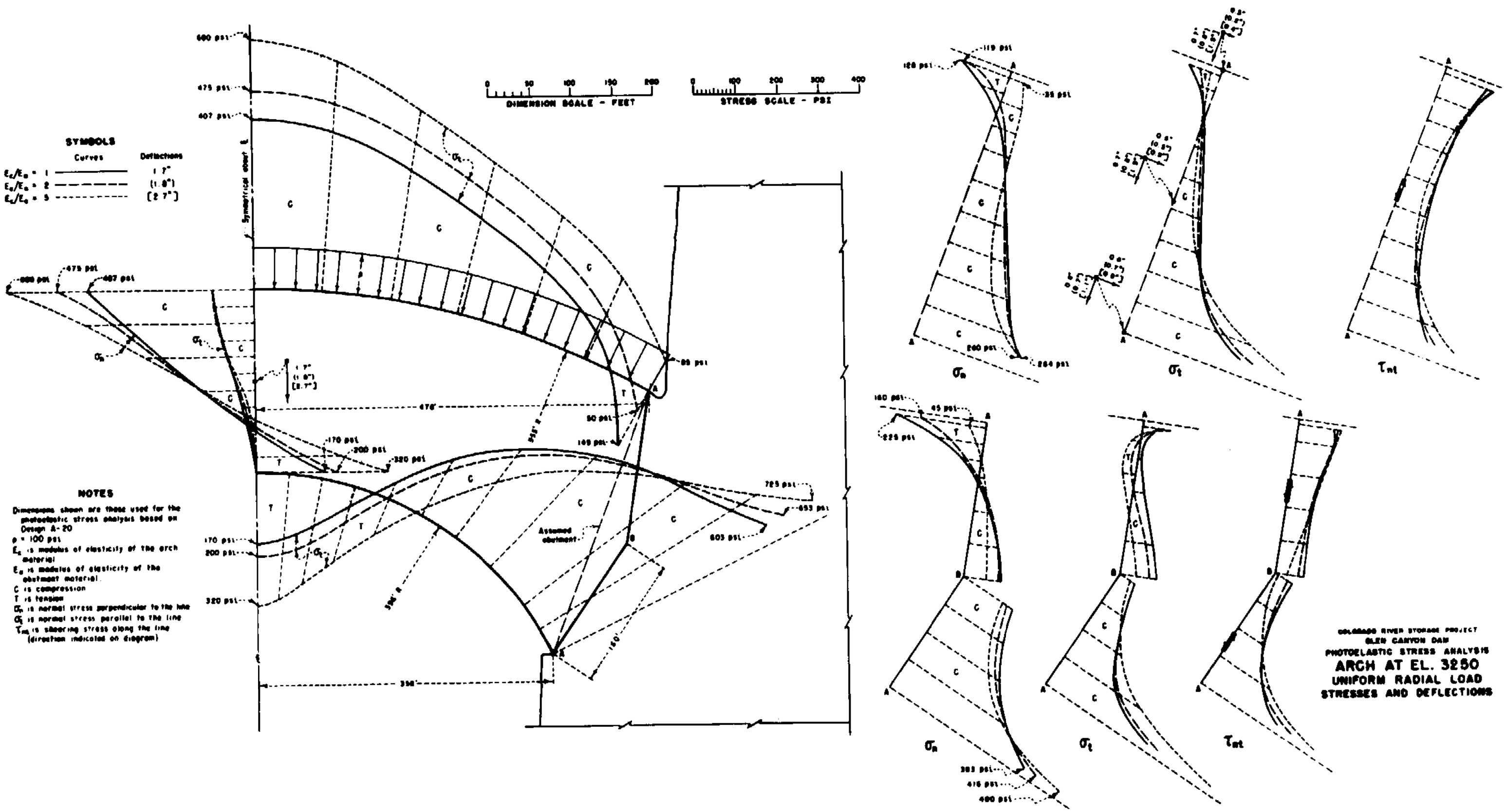


*Figure I-13.* **Stresses and deflections in Glen Canyon Dam obtained by photoelastic stress analysis.–DS2-1(297)**

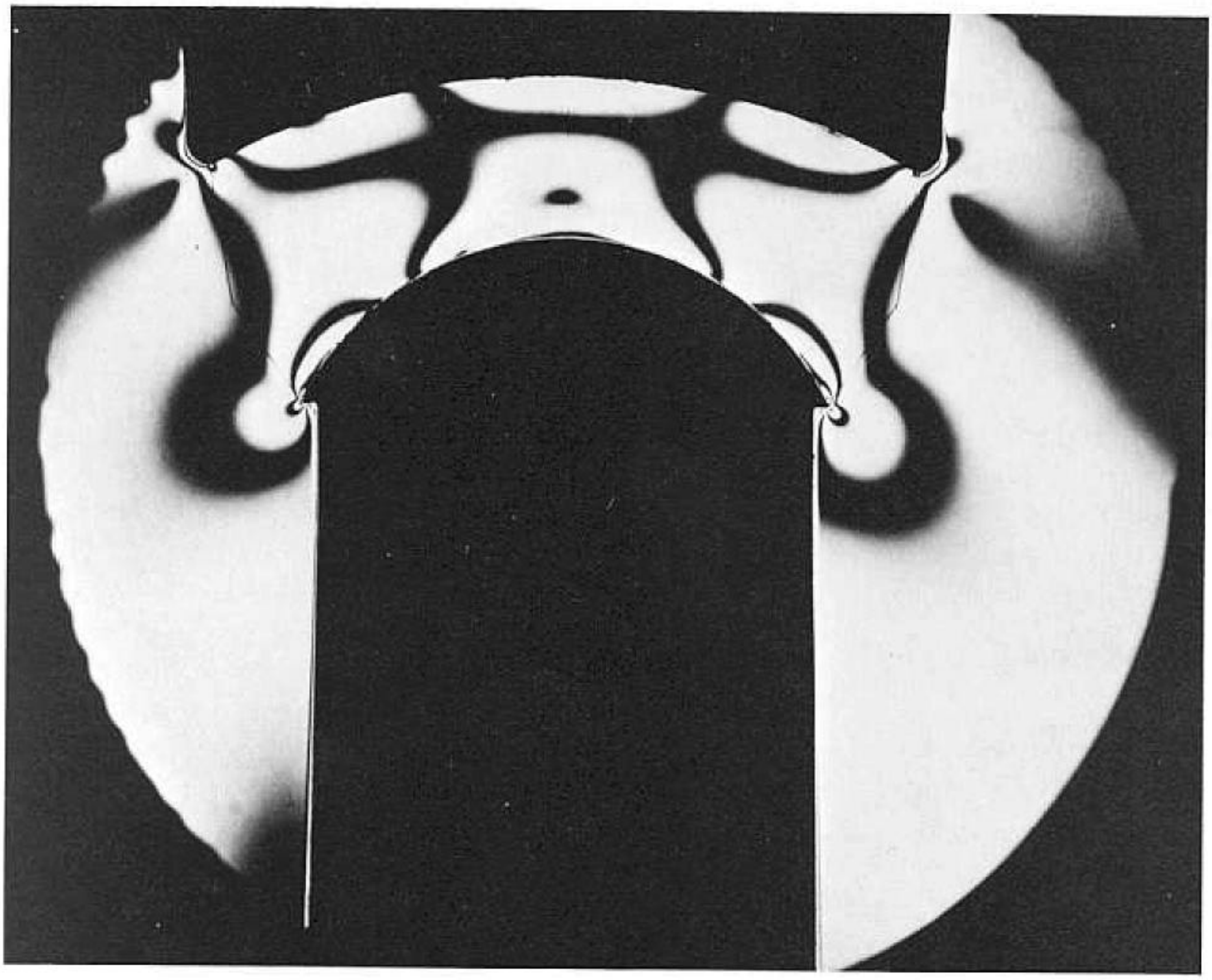


*Figure I-14.* **Isochromatic fringe pattern of two-dimensional photoelastic model of Glen Canyon Dam arch at elevation 3250. Arch is subject to a uniform radial load.–557-D-47828 NA**

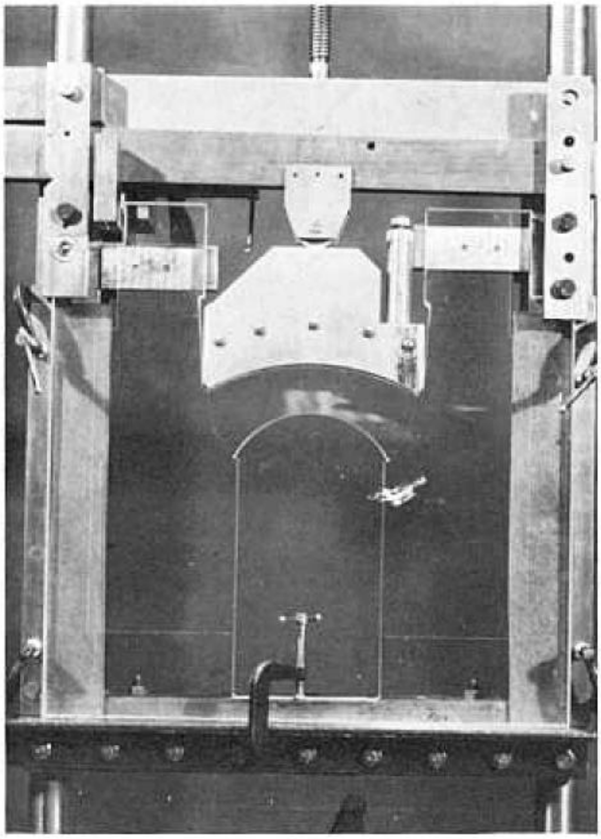


*Figure I-15.* **Two-dimensional photoelastic model and loading apparatus for determining stresses in arch and abutments of Glen Canyon Dam at elevation 3250.–P557-D-20838**

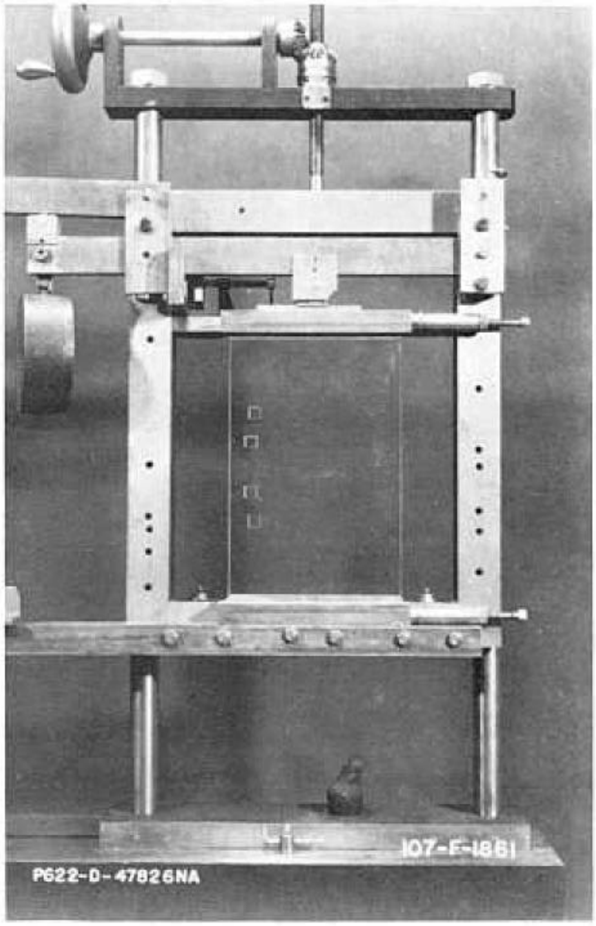


*Figure I-16.* **Two-dimensional photoelastic model and loading apparatus for determining stresses around spillway openings in Morrow Point Dam.–P622-D-47826 NA**

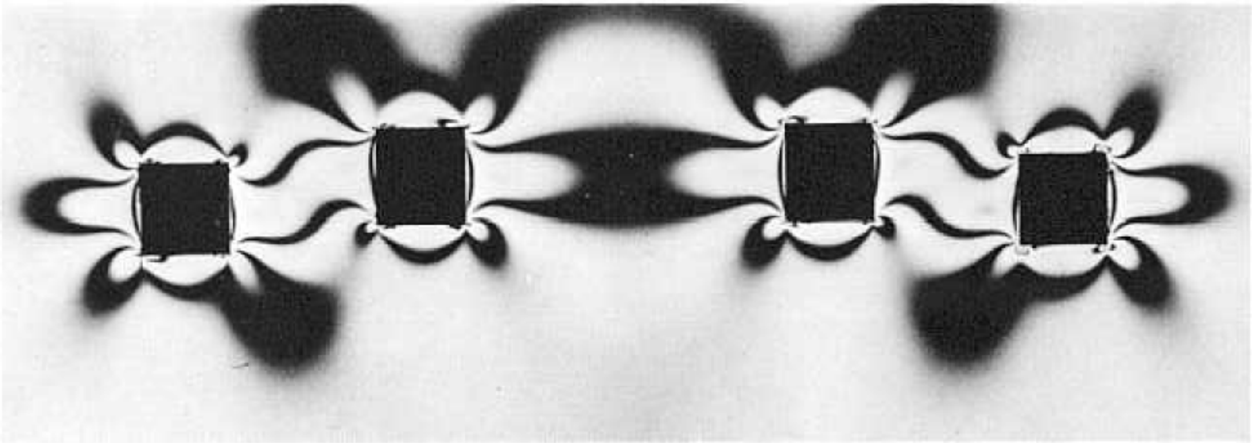


*Figure* ***I-1***: Isochromatic fringe pattern of two-dimensional photoelastic of Morrow Point spillway openings.–P622-D-47827 NA

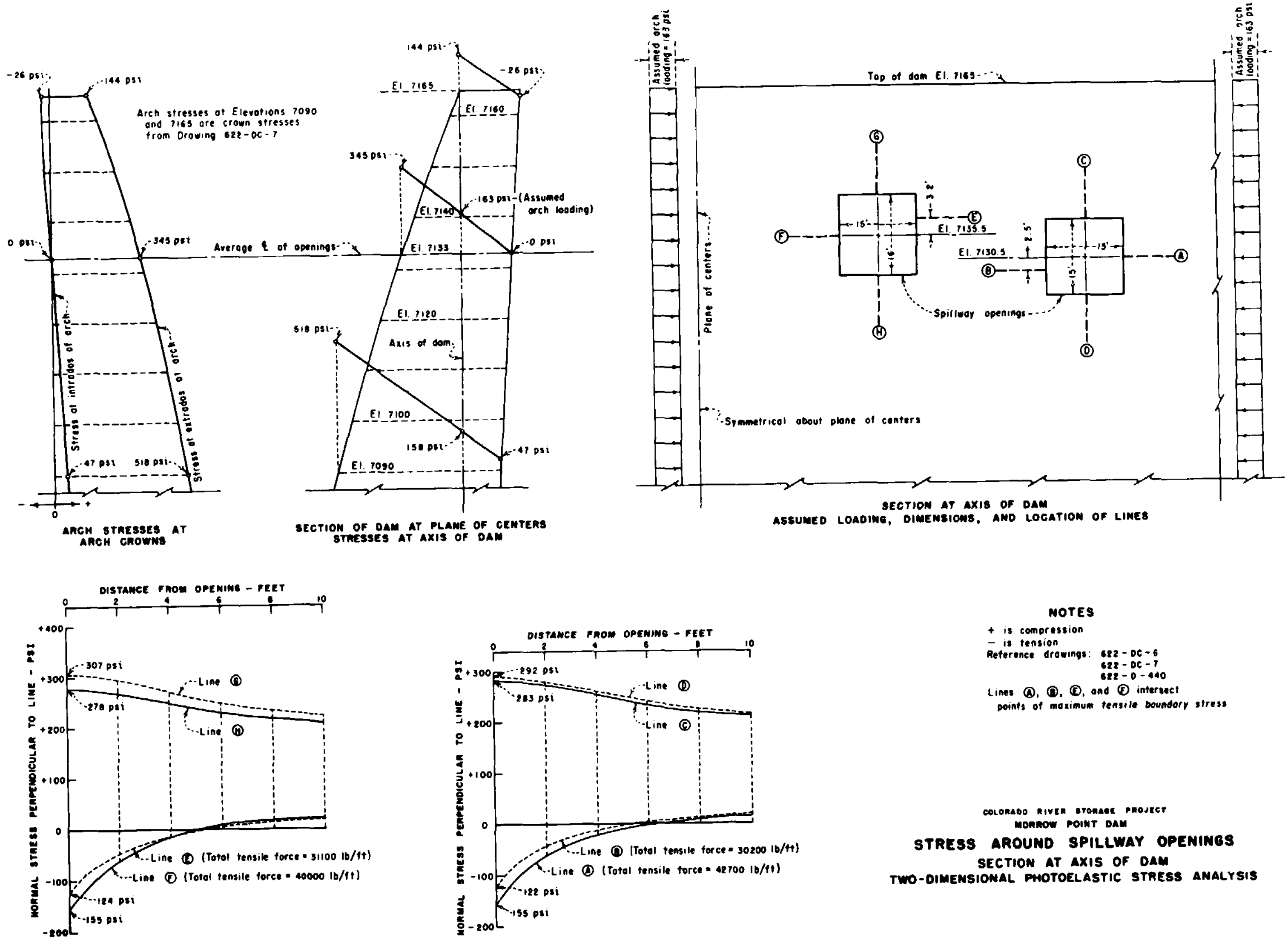


*Figure I-18.* **Stresses around spillway openings in Morrow Point Dam.–DS2-1(302)**

# Finite Element Method— Two-Dimensional Analysis

J-1. *Introduction.*—The two-dimensional finite element analysis is a system for solving problems involving plane strain and plane stress. The system is discussed in subchapter E of chapter IV. This example demonstrates the use of the system in analyzing a vertical section of an arch dam. Stress analyses indicated the desirability of using an abutment pad between the dam and the foundation to spread out and reduce the stresses transmitted by the dam. The finite element analysis was made using loads obtained from an Arch Dam Stress Analysis System (ADSAS) study to determine a shape of pad required for an acceptable stress distribution. Equations and basic methods used in the two-dimensional finite element analysis are discussed by Clough and Zienkiewicz in references [1] and [2], respectively.[1]

J-2. *Grid and Numbering.*—The grid drawn in the dam section to be analyzed and a portion of its foundation are shown on figure J-1. The nodal points are numbered consecutively from the top left, proceeding from left to right and downward from one elevation to the next. The elements are numbered in a pattern similar to that used for the nodal points. The element numbers for some of the elements are shown circled on the figure. An element is described by the nodal points enclosing it. For example, element 32 is formed by nodal points 38, 37, 43, and 44. The grid consists of 313 nodal points and 282 elements for the section.

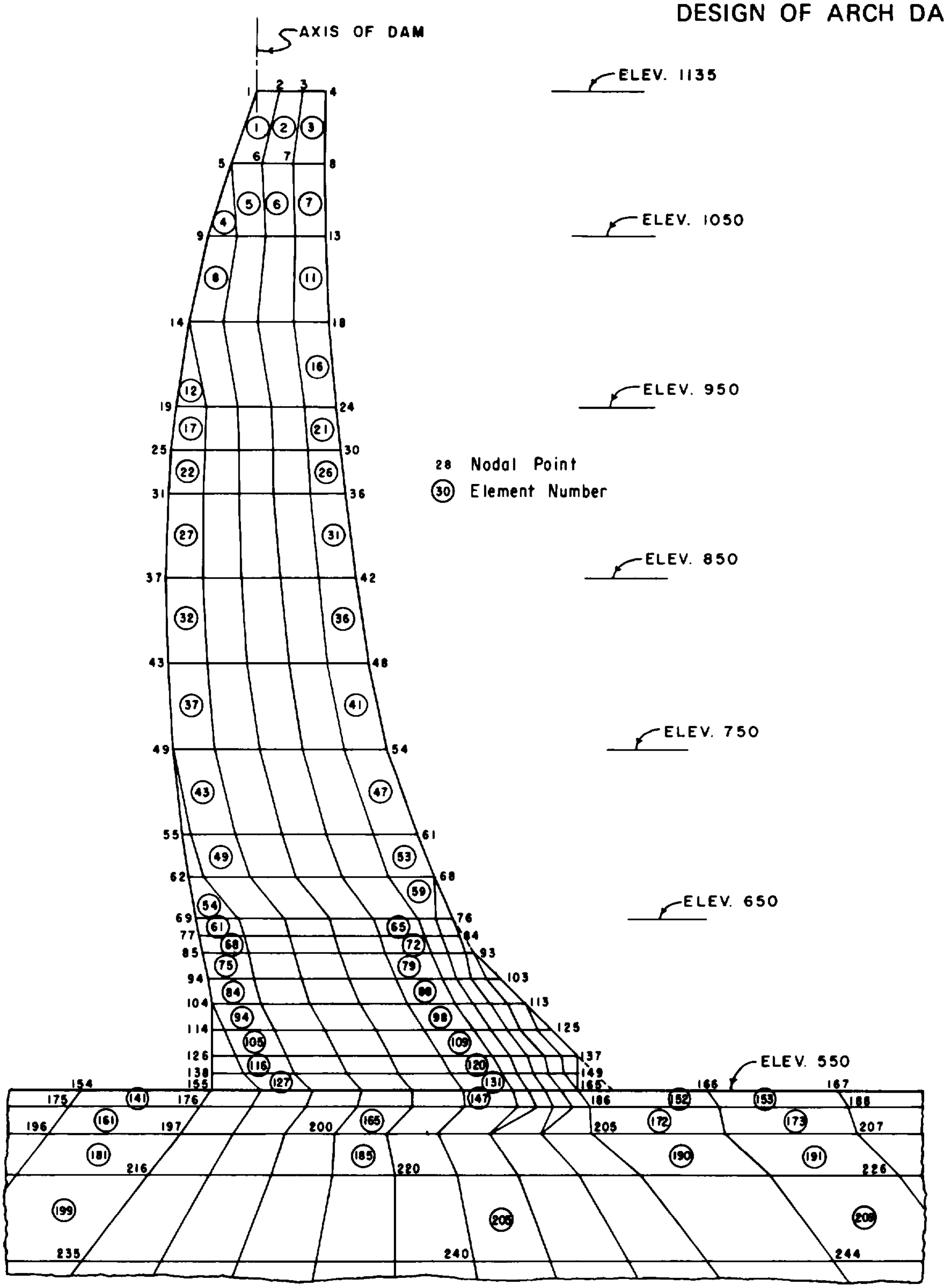
J-3. *Input.*—The required input is shown on figures J-1, J-2, J-3, and J-4. The printout shown on figure J-2 lists the numbers of nodal points, elements, and different materials. It also shows the accelerations in the $X$ and $Y$ directions and describes the properties of the materials for density, modulus of elasticity in compression $E(C)$ and tension $E(T)$, and Poisson's ratio $NU$. Figure J-3 is a printout describing each nodal point. The type number indicates the degree of restraint: 0 = free to move in either direction; 1 = fixed in the $X$ direction but free to move in the $Y$ direction; 2 = fixed in the $Y$ direction but free to move in the $X$ direction; and 3 indicates fixed in both $X$ and $Y$ directions. The $X$ ordinates and $Y$ ordinates give the distances from the origin located in the left corner of the base. Loads or displacements can be used for either or both the $X$ and $Y$ directions. An example of element description is shown on figure J-4. The number of the material making up the element is also listed.

J-4. *Output.*—The program computes the displacements of the nodal points in both directions and stresses at the centroids of the elements. The printout listing some of the displacements is shown on figure J-5. For example, point 43 is displaced 0.124 foot horizontally downstream and 0.016 foot vertically downward.

An example of the stress output is shown on figure J-6. The location of the stresses in each element is given by the $X$ and $Y$ coordinates of the centroid. Stresses are listed in kips per square foot and are tabulated for the horizontal and vertical directions, together

---

[1] Numbers in brackets refer to items in the bibliography, sec. J-5.



TWO-DIMENSIONAL FINITE ELEMENT ANALYSIS

*Figure J-1.* Section of dam showing grid system.—288-D-3144

```
FINITE ELEMENT STUDY OF CANTILEVER 23 ELEV 550 RIGHT ABUTMENT

 NUMBER OF NODAL POINTS------313

 NUMBER OF ELEMENTS----------282

 NUMBER OF DIFF. MATERIALS---  2

 NUMBER OF PRESSURE CARDS---- -0

 X-ACCELERATION-------------- -0.0000+000

 Y-ACCELERATION-------------- -1.0000+000

 REFERENCE TEMPERATURE------- -0.0000+000

 NUMBER OF APPROXIMATIONS----  1

MATERIAL NUMBER  =  1,     NUMBER OF TEMPERATURE CARDS  =  1,     MASS DENSITY  =  0.0000+000

TEMPERATURE        E(C)          NU           E(T)          G/H2        ALPHA     X-STRESS    Y-STRESS
  -0.000      360000.00000   0.20000   360000.00000   -0.00000    -0.00000    -0.00000    -0.00000

MATERIAL NUMBER  =  2,     NUMBER OF TEMPERATURE CARDS  =  1,     MASS DENSITY  =  1.5000-001

TEMPERATURE        E(C)          NU           E(T)          G/H2        ALPHA     X-STRESS    Y-STRESS
  -0.000      432000.00000   0.20000   432000.00000   -0.00000    -0.00000    -0.00000    -0.00000
```

*Figure J-2.* **Input data—control data and material properties.—288-D-3145**

```
NODAL POINT    TYPE   X-ORDINATE   Y-ORDINATE   X LOAD OR DISPLACEMENT   Y LOAD OR DISPLACEMENT   TEMPERATURE

      1       -0.00       0.000     1135.000        -5.1200000+001           -0.0000000+000          -0.000
      2        0.00      13.333     1135.000         0.0000000+000            0.0000000+000           0.000
      3        0.00      26.667     1135.000         0.0000000+000            0.0000000+000           0.000
      4       -0.00      40.000     1135.000        -0.0000000+000           -0.0000000+000          -0.000
      5       -0.00     -15.300     1092.500        -8.1000000+001           -0.0000000+000          -0.000

      6        0.00       2.833     1092.500         0.0000000+000            0.0000000+000           0.000
      7        0.00      20.967     1092.500         0.0000000+000            0.0000000+000           0.000
      8       -0.00      39.100     1092.500        -0.0000000+000           -0.0000000+000          -0.000
      9       -0.00     -27.700     1050.000        -5.0600000+001           -0.0000000+000          -0.000
     10        0.00     -10.875     1050.000         0.0000000+000            0.0000000+000           0.000

     11        0.00       5.950     1050.000         0.0000000+000            0.0000000+000           0.000
     12        0.00      22.775     1050.000         0.0000000+000            0.0000000+000           0.000
     13       -0.00      39.600     1050.000        -0.0000000+000           -0.0000000+000          -0.000
     14       -0.00     -38.700     1000.000         2.9000000+001           -0.0000000+000          -0.000
     15        0.00     -18.525     1000.000         0.0000000+000            0.0000000+000           0.000

     16        0.00       1.650     1000.000         0.0000000+000            0.0000000+000           0.000
     17        0.00      21.825     1000.000         0.0000000+000            0.0000000+000           0.000
     18       -0.00      42.000     1000.000        -0.0000000+000           -0.0000000+000          -0.000
     19       -0.00     -45.900      950.000         8.6200000+001           -0.0000000+000          -0.000
     20        0.00     -27.440      950.000         0.0000000+000            0.0000000+000           0.000

     21        0.00      -8.980      950.000         0.0000000+000            0.0000000+000           0.000
     22        0.00       9.480      950.000         0.0000000+000            0.0000000+000           0.000
     23        0.00      27.940      950.000         0.0000000+000            0.0000000+000           0.000
     24       -0.00      46.400      950.000        -0.0000000+000           -0.0000000+000          -0.000
     25       -0.00     -48.200      925.000         1.1440000+002           -0.0000000+000          -0.000

     26        0.00     -28.680      925.000         0.0000000+000            0.0000000+000           0.000
     27        0.00      -9.160      925.000         0.0000000+000            0.0000000+000           0.000
     28        0.00      10.360      925.000         0.0000000+000            0.0000000+000           0.000
     29        0.00      29.880      925.000         0.0000000+000            0.0000000+000           0.000
     30       -0.00      49.400      925.000        -0.0000000+000           -0.0000000+000          -0.000

     31       -0.00     -49.900      900.000         2.7040000+002           -0.0000000+000          -0.000
     32        0.00     -29.400      900.000         0.0000000+000            0.0000000+000           0.000
     33        0.00      -8.900      900.000         0.0000000+000            0.0000000+000           0.000
     34        0.00      11.600      900.000         0.0000000+000            0.0000000+000           0.000
     35        0.00      32.100      900.000         0.0000000+000            0.0000000+000           0.000

     36       -0.00      52.600      900.000        -0.0000000+000           -0.0000000+000          -0.000
     37       -0.00     -51.600      850.000         5.2500000+002           -0.0000000+000          -0.000
     38        0.00     -29.460      850.000         0.0000000+000            0.0000000+000           0.000
     39        0.00      -7.320      850.000         0.0000000+000            0.0000000+000           0.000
     40        0.00      14.820      850.000         0.0000000+000            0.0000000+000           0.000

     41        0.00      36.960      850.000         0.0000000+000            0.0000000+000           0.000
     42       -0.00      59.100      850.000        -0.0000000+000           -0.0000000+000          -0.000
     43       -0.00     -50.200      800.000         6.4220000+002           -0.0000000+000          -0.000
     44        0.00     -26.880      800.000         0.0000000+000            0.0000000+000           0.000
     45        0.00      -3.560      800.000         0.0000000+000            0.0000000+000           0.000

     46        0.00      19.760      800.000         0.0000000+000            0.0000000+000           0.000
     47        0.00      43.080      800.000         0.0000000+000            0.0000000+000           0.000
     48       -0.00      66.400      800.000        -0.0000000+000           -0.0000000+000          -0.000
     49       -0.00     -46.500      750.000         7.3950000+002           -0.0000000+000          -0.000
     50        0.00     -21.720      750.000         0.0000000+000            0.0000000+000           0.000

     51        0.00       3.060      750.000         0.0000000+000            0.0000000+000           0.000
     52        0.00      27.840      750.000         0.0000000+000            0.0000000+000           0.000
     53        0.00      52.620      750.000         0.0000000+000            0.0000000+000           0.000
     54       -0.00      77.400      750.000        -0.0000000+000           -0.0000000+000          -0.000
     55       -0.00     -41.900      700.000         5.8760000+002           -0.0000000+000          -0.000

     56       -0.00     -35.900      700.000        -0.0000000+000           -0.0000000+000          -0.000
     57        0.00      -9.720      700.000         0.0000000+000            0.0000000+000           0.000
     58        0.00      16.460      700.000         0.0000000+000            0.0000000+000           0.000
     59        0.00      42.640      700.000         0.0000000+000            0.0000000+000           0.000
```

*Figure J-3.* **Input data—loading and description of section by nodal points.—288-D-3146**

| ELEMENT NO. | I | J | K | L | MATERIAL |
|---|---|---|---|---|---|
| 1 | 1 | 5 | 6 | 2 | 2 |
| 2 | 2 | 6 | 7 | 3 | 2 |
| 3 | 3 | 7 | 8 | 4 | 2 |
| 4 | 5 | 9 | 10 | 10 | 2 |
| 5 | 5 | 10 | 11 | 6 | 2 |
| 6 | 6 | 11 | 12 | 7 | 2 |
| 7 | 7 | 12 | 13 | 8 | 2 |
| 8 | 9 | 14 | 15 | 10 | 2 |
| 9 | 10 | 15 | 16 | 11 | 2 |
| 10 | 11 | 16 | 17 | 12 | 2 |
| 11 | 12 | 17 | 18 | 13 | 2 |
| 12 | 14 | 19 | 20 | 20 | 2 |
| 13 | 14 | 20 | 21 | 15 | 2 |
| 14 | 15 | 21 | 22 | 16 | 2 |
| 15 | 16 | 22 | 23 | 17 | 2 |
| 16 | 17 | 23 | 24 | 18 | 2 |
| 17 | 19 | 25 | 26 | 20 | 2 |
| 18 | 20 | 26 | 27 | 21 | 2 |
| 19 | 21 | 27 | 28 | 22 | 2 |
| 20 | 22 | 28 | 29 | 23 | 2 |
| 21 | 23 | 29 | 30 | 24 | 2 |
| 22 | 25 | 31 | 32 | 26 | 2 |
| 23 | 26 | 32 | 33 | 27 | 2 |
| 24 | 27 | 33 | 34 | 28 | 2 |
| 25 | 28 | 34 | 35 | 29 | 2 |
| 26 | 29 | 35 | 36 | 30 | 2 |
| 27 | 31 | 37 | 38 | 32 | 2 |
| 28 | 32 | 38 | 39 | 33 | 2 |
| 29 | 33 | 39 | 40 | 34 | 2 |
| 30 | 34 | 40 | 41 | 35 | 2 |
| 31 | 35 | 41 | 42 | 36 | 2 |
| 32 | 37 | 43 | 44 | 38 | 2 |
| 33 | 38 | 44 | 45 | 39 | 2 |
| 34 | 39 | 45 | 46 | 40 | 2 |
| 35 | 40 | 46 | 47 | 41 | 2 |
| 36 | 41 | 47 | 48 | 42 | 2 |
| 37 | 43 | 49 | 50 | 44 | 2 |
| 38 | 44 | 50 | 51 | 45 | 2 |
| 39 | 45 | 51 | 52 | 46 | 2 |
| 40 | 46 | 52 | 53 | 47 | 2 |
| 41 | 47 | 53 | 54 | 48 | 2 |
| 42 | 49 | 55 | 56 | 56 | 2 |
| 43 | 49 | 56 | 57 | 50 | 2 |
| 44 | 50 | 57 | 58 | 51 | 2 |
| 45 | 51 | 58 | 59 | 52 | 2 |
| 46 | 52 | 59 | 60 | 53 | 2 |
| 47 | 53 | 60 | 61 | 54 | 2 |
| 48 | 55 | 62 | 63 | 56 | 2 |
| 49 | 56 | 63 | 64 | 57 | 2 |
| 50 | 57 | 64 | 65 | 58 | 2 |
| 51 | 58 | 65 | 66 | 59 | 2 |
| 52 | 59 | 66 | 67 | 60 | 2 |
| 53 | 60 | 67 | 68 | 61 | 2 |
| 54 | 62 | 69 | 70 | 63 | 2 |
| 55 | 63 | 70 | 71 | 64 | 2 |
| 56 | 64 | 71 | 72 | 65 | 2 |
| 57 | 65 | 72 | 73 | 66 | 2 |
| 58 | 66 | 73 | 74 | 67 | 2 |
| 59 | 67 | 74 | 75 | 68 | 2 |

*Figure J-4.* **Input data–elements defined by nodal points with material.–288-D-3147**

| N.P. NUMBER | UX | UY |
|---|---|---|
| 1 | 2.5655460-001 | -5.2718722-002 |
| 2 | 2.5662940-001 | -5.7295167-002 |
| 3 | 2.5666855-001 | -6.1848168-002 |
| 4 | 2.5668689-001 | -6.6374542-002 |
| 5 | 2.4215536-001 | -4.7000127-002 |
| 6 | 2.4222883-001 | -5.3308736-002 |
| 7 | 2.4228044-001 | -5.9611051-002 |
| 8 | 2.4231764-001 | -6.5811475-002 |
| 9 | 2.2749363-001 | -4.1256881-002 |
| 10 | 2.2761422-001 | -4.7444541-002 |
| 11 | 2.2772841-001 | -5.3590988-002 |
| 12 | 2.2780796-001 | -5.9606560-002 |
| 13 | 2.2786586-001 | -6.5487565-002 |
| 14 | 2.0968362-001 | -3.5031541-002 |
| 15 | 2.0978930-001 | -4.2757019-002 |
| 16 | 2.0989186-001 | -5.0464725-002 |
| 17 | 2.0999319-001 | -5.8066795-002 |
| 18 | 2.1007885-001 | -6.5499765-002 |
| 19 | 1.9053813-001 | -2.8980615-002 |
| 20 | 1.9065601-001 | -3.6539375-002 |
| 21 | 1.9080666-001 | -4.4104964-002 |
| 22 | 1.9093013-001 | -5.1535795-002 |
| 23 | 1.9104190-001 | -5.8863966-002 |
| 24 | 1.9113652-001 | -6.6060050-002 |
| 25 | 1.8051134-001 | -2.6254655-002 |
| 26 | 1.8059215-001 | -3.4490738-002 |
| 27 | 1.8068066-001 | -4.2650276-002 |
| 28 | 1.8080130-001 | -5.0723696-002 |
| 29 | 1.8093335-001 | -5.8710393-002 |
| 30 | 1.8104746-001 | -6.6640313-002 |
| 31 | 1.7004664-001 | -2.3603424-002 |
| 32 | 1.7007523-001 | -3.2520225-002 |
| 33 | 1.7012532-001 | -4.1275753-002 |
| 34 | 1.7021615-001 | -4.9932670-002 |
| 35 | 1.7034505-001 | -5.8545590-002 |
| 36 | 1.7048960-001 | -6.7197026-002 |
| 37 | 1.4795465-001 | -1.9006793-002 |
| 38 | 1.4778527-001 | -2.8976635-002 |
| 39 | 1.4768996-001 | -3.8676083-002 |
| 40 | 1.4769147-001 | -4.8226183-002 |
| 41 | 1.4780395-001 | -5.7786802-002 |
| 42 | 1.4801636-001 | -6.7531483-002 |
| 43 | 1.2445155-001 | -1.6002853-002 |
| 44 | 1.2412076-001 | -2.6597228-002 |
| 45 | 1.2385479-001 | -3.6724706-002 |
| 46 | 1.2371282-001 | -4.6588002-002 |
| 47 | 1.2375015-001 | -5.6426572-002 |
| 48 | 1.2403581-001 | -6.6592651-002 |
| 49 | 1.0058974-001 | -1.4646612-002 |
| 50 | 1.0004760-001 | -2.5559153-002 |
| 51 | 9.9570093-002 | -3.5745990-002 |
| 52 | 9.9232528-002 | -4.5468426-002 |
| 53 | 9.9119755-002 | -5.4990642-002 |
| 54 | 9.9362060-002 | -6.4533552-002 |
| 55 | 7.7666510-002 | -1.4441438-002 |
| 56 | 7.7501701-002 | -1.7048691-002 |
| 57 | 7.6747034-002 | -2.7423705-002 |
| 58 | 7.6056996-002 | -3.6897315-002 |
| 59 | 7.5481713-002 | -4.5712259-002 |

*Figure J-5.* **Displacement of nodal points.–288-D-3148**

| EL. NO. | X | Y | X-STRESS | Y-STRESS | XY-STRESS | MAX-STRESS | MIN-STRESS | ANGLE |
|---|---|---|---|---|---|---|---|---|
| 101 | 147.50 | 592.50 | -38.7184+000 | -54.8375+000 | 46.4215+000 | 33.7977-002 | -93.8939+000 | 40.08 |
| 102 | 157.00 | 592.50 | -40.7349+000 | -52.8835+000 | 45.7177+000 | -68.9774-002 | -92.9286+000 | 41.22 |
| 103 | 169.00 | 585.00 | -34.9679+000 | -30.1642+000 | 35.8947+000 | 34.0896-001 | -68.5410+000 | 46.91 |
| 104 | -19.30 | 577.50 | -17.2135+000 | -17.9262+000 | 13.3942-001 | -16.1838+000 | -18.9559+000 | 37.55 |
| 105 | 2.08 | 577.50 | -14.1872+000 | -21.7358+000 | 74.7868-001 | -95.8436-001 | -26.3386+000 | 31.61 |
| 106 | 31.24 | 577.50 | -13.5107+000 | -34.0889+000 | 11.7103+000 | -82.1146-001 | -39.3881+000 | 24.35 |
| 107 | 60.40 | 577.50 | -16.4139+000 | -47.1728+000 | 15.6864+000 | -98.2545-001 | -53.7613+000 | 22.78 |
| 108 | 89.56 | 577.50 | -20.5172+000 | -57.2947+000 | 23.4473+000 | -91.0790-001 | -68.7040+000 | 25.95 |
| 109 | 118.72 | 577.50 | -27.0215+000 | -60.0767+000 | 34.9564+000 | -48.8242-001 | -82.2158+000 | 32.35 |
| 110 | 138.00 | 577.50 | -32.8277+000 | -57.2780+000 | 42.7996+000 | -54.1487-002 | -89.5642+000 | 37.03 |
| 111 | 147.50 | 577.50 | -34.4593+000 | -55.7873+000 | 45.3645+000 | 14.7775-001 | -91.7244+000 | 38.39 |
| 112 | 157.00 | 577.50 | -33.4577+000 | -54.9057+000 | 46.5068+000 | 35.4545-001 | -91.9089+000 | 38.51 |
| 113 | 166.50 | 577.50 | -27.6556+000 | -51.6286+000 | 42.2168+000 | 42.4337-001 | -83.5276+000 | 37.07 |
| 114 | 176.00 | 577.50 | -22.7270+000 | -39.7018+000 | 29.3823+000 | -63.0838-002 | -61.7980+000 | 36.94 |
| 115 | -16.30 | 565.00 | -13.0061+000 | -15.6134+000 | 59.3425-001 | -82.3398-001 | -20.3855+000 | 38.80 |
| 116 | 7.48 | 565.00 | -65.4261-001 | -23.0323+000 | 10.1932+000 | -16.7724-001 | -27.8977+000 | 25.52 |
| 117 | 37.24 | 565.00 | -91.7106-001 | -37.9365+000 | 10.8294+000 | -55.4995-001 | -41.5577+000 | 18.49 |
| 118 | 67.00 | 565.00 | -13.3018+000 | -48.9672+000 | 15.3092+000 | -76.3175-001 | -54.6372+000 | 20.32 |
| 119 | 96.76 | 565.00 | -18.6736+000 | -56.0714+000 | 23.4989+000 | -73.4171-001 | -67.4033+000 | 25.74 |
| 120 | 126.52 | 565.00 | -27.8015+000 | -56.3459+000 | 34.1425+000 | -50.6821-001 | -79.0792+000 | 33.66 |
| 121 | 146.50 | 565.00 | -35.4493+000 | -53.3120+000 | 42.0007+000 | -14.4083-001 | -87.3205+000 | 39.00 |
| 122 | 155.90 | 565.00 | -38.3772+000 | -53.2629+000 | 46.4142+000 | 11.8706-001 | -92.8272+000 | 40.44 |
| 123 | 165.30 | 565.00 | -40.0260+000 | -55.6532+000 | 50.7502+000 | 35.0855-001 | -99.1878+000 | 40.62 |
| 124 | 174.70 | 565.00 | -32.2722+000 | -67.1032+000 | 52.0468+000 | 51.9557-001 | -10.4571+001 | 35.75 |
| 125 | 184.10 | 565.00 | -71.6023-001 | -48.5135+000 | 24.9028+000 | 45.3094-001 | -60.2047+000 | 25.15 |
| 126 | -14.30 | 555.00 | 51.9255-001 | -12.1829+000 | 16.1395+000 | 14.8341+000 | -21.8245+000 | 30.85 |
| 127 | 15.16 | 555.00 | -11.4446-001 | -28.9876+000 | 78.5241-001 | 91.7419-002 | -31.0494+000 | 14.71 |
| 128 | 44.63 | 555.00 | -66.7121-001 | -41.1900+000 | 10.0835+000 | -39.4153-001 | -43.9197+000 | 15.15 |
| 129 | 74.09 | 555.00 | -11.5609+000 | -50.0402+000 | 15.1769+000 | -62.9543-001 | -55.3057+000 | 19.13 |
| 130 | 103.55 | 555.00 | -17.9596+000 | -54.8249+000 | 22.9726+000 | -69.3883-001 | -65.8457+000 | 25.63 |
| 131 | 133.02 | 555.00 | -29.2908+000 | -53.3733+000 | 31.6440+000 | -74.7451-001 | -75.1896+000 | 34.58 |
| 132 | 151.90 | 555.00 | -40.9334+000 | -50.0114+000 | 39.1331+000 | -60.7693-001 | -84.8679+000 | 41.69 |
| 133 | 161.05 | 555.00 | -49.8337+000 | -51.1744+000 | 42.4738+000 | -80.2500-001 | -92.9831+000 | 44.55 |
| 134 | 170.20 | 555.00 | -59.9248+000 | -54.1311+000 | 47.7709+000 | -91.6923-001 | -10.4887+001 | 46.74 |
| 135 | 179.35 | 555.00 | -84.5316+000 | -73.0453+000 | 68.6254+000 | -99.2319-001 | -14.7654+001 | 47.39 |
| 136 | 184.10 | 555.00 | -45.6669+000 | -11.3921+001 | 91.6428+000 | 17.9968+000 | -17.7585+001 | 34.79 |
| 137 | -364.55 | 545.00 | 18.6030+000 | -66.8198-002 | -18.2193-001 | 18.7737+000 | -83.8934-002 | -5.35 |
| 138 | -289.55 | 545.00 | 16.2613+000 | 44.9056-002 | 22.4027-002 | 16.2645+000 | 44.5883-002 | 0.81 |
| 139 | -214.55 | 545.00 | 16.9512+000 | -18.2852-002 | 26.4459-002 | 16.9553+000 | -18.6933-002 | 0.88 |
| 140 | -139.55 | 545.00 | 19.3445+000 | -78.8437-002 | 20.4992-002 | 19.3466+000 | -79.0524-002 | 0.58 |
| 141 | -64.55 | 545.00 | 28.3066+000 | -48.2818-002 | 39.1700-001 | 28.8300+000 | -10.0624-001 | 7.61 |
| 142 | -12.57 | 545.00 | 14.0883+000 | -35.3737-001 | 27.4477-001 | 14.5059+000 | -39.5491-001 | 8.65 |
| 143 | 17.32 | 545.00 | 12.8053-001 | -32.9154+000 | 59.3783-001 | 22.8224-001 | -33.9171+000 | 9.58 |
| 144 | 47.20 | 545.00 | -47.0054-001 | -41.3984+000 | 88.4750-001 | -26.7887-001 | -43.4201+000 | 12.87 |
| 145 | 77.08 | 545.00 | -99.4314-001 | -49.3715+000 | 14.1899+000 | -53.6736-001 | -53.9473+000 | 17.87 |
| 146 | 106.97 | 545.00 | -16.4993+000 | -53.6290+000 | 21.7332+000 | -64.8120-001 | -63.6471+000 | 24.75 |
| 147 | 136.85 | 545.00 | -25.5736+000 | -51.3802+000 | 27.1999+000 | -83.7158-001 | -68.5822+000 | 32.31 |
| 148 | 157.00 | 545.00 | -43.4720+000 | -49.0305+000 | 33.7009+000 | -12.4359+000 | -80.0666+000 | 42.64 |
| 149 | 167.00 | 545.00 | -44.5604+000 | -52.1602+000 | 32.0148+000 | -16.1208+000 | -80.5998+000 | 41.62 |
| 150 | 177.00 | 545.00 | -64.1156+000 | -55.1900+000 | 34.9148+000 | -24.4540+000 | -94.8517+000 | 48.64 |

*Figure J-6.* Stresses in elements, in thousands of pounds (kips) per square foot.–288-D-3149 with shear stress for the $XY$ plane. These stresses are then combined to obtain principal stresses. The tabulated angle is measured to the maximum principal stress from the horizontal. A positive angle is measured in a counterclockwise direction. As an example, the centroid of element 108 is located 89.6 feet downstream from the axis of the dam at elevation 577.5. Stress in the $X$ direction is 20,517 p.s.f. (142 p.s.i.) compression, and in the $Y$ direction 57,295 p.s.f. (398 p.s.i.) compression. Shear stress in the $XY$ plane is 23,447 p.s.f. (163 p.s.i.). The maximum principal stress[2] is 9,108 p.s.f. (63 p.s.i.) compression, and the minimum principal stress is 68,704 p.s.f. (477 p.s.i.) compression. The angle to the maximum stress from the horizontal is 25.95° measured in a counterclockwise direction.

Vertical stresses in the pad for the section on figure J-1 are shown in the elements on figure J-7. The maximum compressive stress computed is 851 p.s.i. located on the downstream face of the dam at its contact with the pad (elev. 630). A compressive stress of 791 p.s.i. occurs at the toe of the pad (elev. 550). The geometric configuration was revised by adding wedges as shown by dashed lines on figure J-7, which reduced the compressive stresses in these areas to 699 p.s.i. and 408 p.s.i., respectively (see fig. J-8).

## J-5. *Bibliography.*

[1] Clough, R. W., "The Finite Element Method in Plane Stress Analysis," ASCE Conference Papers (Second Conference on Electronic Computation, September 1960).

[2] Zienkiewicz, O. C., "The Finite Element in Structural and Continuum Mechanics," McGraw-Hill, London, 1967.

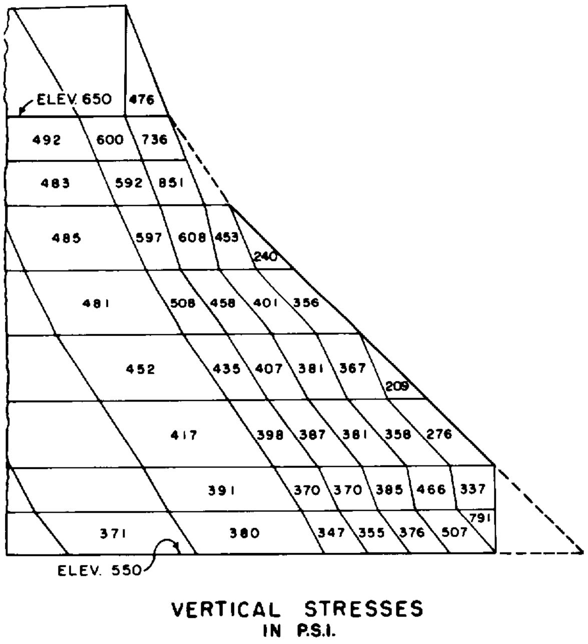


*Figure J-7.* **Enlarged portion of figure J-1 showing vertical stresses.–288-D-3150**

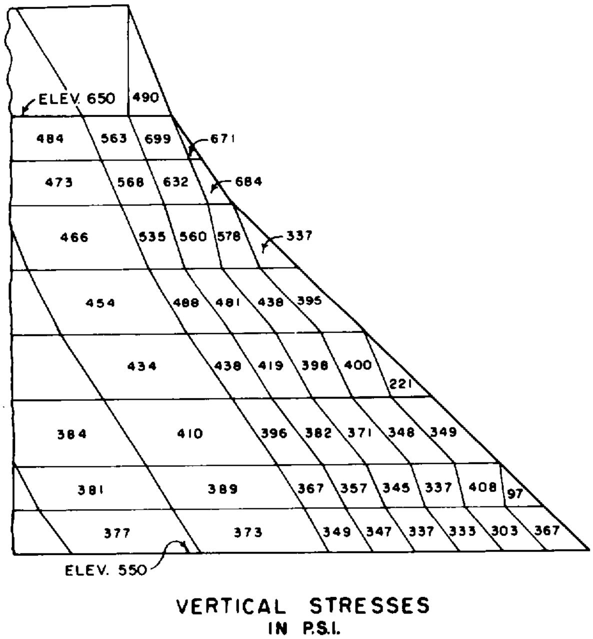


*Figure J-8.* **Enlarged portion of figure J-1 showing vertical stresses, with additional concrete in areas indicated by dashed lines on figure J-7.–288-D-3151**

---

[2] Maximum tension or minimum compression.

## <<Appendix K

# Hydraulic Data and Tables

K-1. *Lists of Symbols and Conversion Factors.*—The following list includes symbols used in hydraulic formulas given in chapters IX and X and in this appendix. Standard mathematical notations and symbols having only very limited applications have been omitted.

| *Symbol* | *Description* |
|---|---|
| $A, a$ | An area; area of a surface; cross-sectional area of flow in an open channel; cross-sectional area of a closed conduit |
| $a_g$ | Gross area of a trashrack |
| $a_n$ | Net area of a trashrack |
| $b$ | Bottom width of a channel |
| $C$ | A coefficient; coefficient of discharge |
| $C_d$ | Coefficient of discharge through an orifice |
| $C_i$ | Coefficient of discharge for an ogee crest with inclined upstream face |
| $C_o$ | Coefficient of discharge for a nappe-shaped ogee crest designed for an $H_o$ head |
| $C_s$ | Coefficient of discharge for a partly submerged crest |
| $D$ | Diameter; conduit diameter; height of a rectangular conduit or passageway; height of a square or rectangular orifice |
| $d$ | Depth of flow in an open channel; height of an orifice or gate opening |
| $d_c$ | Critical depth |
| $d_H$ | Depth for high (subcritical) flow stage (alternate to $d_L$) |
| $d_j$ | Height of a hydraulic jump (difference in the conjugate depths) |
| $d_L$ | Depth for low (supercritical) flow stage (alternate to $d_H$) |
| $d_m$ | Mean depth of flow |
| $d_{m_c}$ | Critical mean depth |
| $d_n$ | Depth of flow measured normal to channel bottom |
| $d_s$ | Depth of scour below tailwater in a plunge pool |
| $d_t$ | Depth of flow in a chute at tailwater level |
| $E$ | Energy |
| $E_m$ | Energy of a particle of mass |
| $F$ | Froude number parameter for defining flow conditions in a channel, $F = \frac{v}{\sqrt{gd}}$ |
| $F_t$ | Froude number parameter for flow in a chute at the tailwater level |
| $f$ | Friction loss coefficient in the Darcy-Weisbach formula $h_f = \frac{fL}{D}\frac{v^2}{2g}$ |
| $g$ | Acceleration due to the force of gravity |
| $H$ | Head over a crest; head on center of an orifice opening; head difference at a gate (between the upstream and downstream water surface levels) |
| $H_A$ | Absolute head above a datum plane, in channel flow |
| $H_a$ | Head above a section in the transition of a drop inlet spillway |
| $H_1$ | Head measured to bottom of an orifice opening |
| $H_2$ | Head measured to top of an orifice opening |
| $h$ | Head; height of baffle block; height of end sill |
| $h_a$ | Approach velocity head |
| $h_b$ | Head loss due to bend |
| $h_c$ | Head loss due to contraction |
| $H_D$ | Head from reservoir water surface to water surface at a given point in the downstream channel |
| $h_d$ | Difference in water surface level, measured from reservoir water surface to the downstream channel water surface |
| $H_E$ | Specific energy head |
| $H_{E_C}$ | Specific energy head at critical flow |
| $H_e$ | Total head on a crest, including velocity of approach |

| *Symbol* | *Description* |
|---|---|
| $h_e$ | Head loss due to entrance |
| $h_{ex}$ | Head loss due to expansion |
| $h_f$ | Head loss due to friction |
| $\Delta h_f$ | Incremental head loss due to friction |
| $h_g$ | Head loss due to gates or valves |
| $h_L$ | Head losses from all causes |
| $\Sigma h_{L_u}$ | Sum of head losses upstream from a section |
| $\Delta h_L$ | Incremental head loss from all causes |
| $\Sigma(\Delta h_L)$ | Sum of incremental head losses from all causes |
| $H_o$ | Design head over ogee crest |
| $h_o$ | Head measured from the crest of an ogee to the reservoir surface immediately upstream, not including the velocity of approach (crest shaped for design head $H_o$) |
| $H_s$ | Total head over a sharp-crested weir |
| $h_s$ | Head over a sharp-crested weir, not including velocity of approach |
| $H_T$ | Total head from reservoir water surface to tailwater, or to center of outlet of a free-discharging pipe |
| $h_t$ | Head loss due to trashrack |
| $h_v$ | Velocity head; head loss due to exit |
| $h_{v_c}$ | Critical velocity head |
| $K$ | A constant factor for various equations; a coefficient |
| $k$ | A constant |
| $K_a$ | Abutment contraction coefficient |
| $K_b$ | Bend loss coefficient |
| $K_c$ | Contraction loss coefficient |
| $K_e$ | Entrance loss coefficient |
| $K_{ex}$ | Expansion loss coefficient |
| $K_g$ | Gate or valve loss coefficient |
| $K_L$ | A summary loss coefficient for losses due to all causes |
| $K_p$ | Pier contraction coefficient |
| $K_t$ | Trashrack loss coefficient |
| $K_v$ | Velocity head loss coefficient |
| $L$ | Length; length of a channel or a pipe; effective length of a crest; length of a hydraulic jump; length of a stilling basin; length of a transition |
| $\Delta L$ | Incremental length; incremental channel length |
| $L_I, L_{II}, L_{III}$ | Stilling basin lengths for different hydraulic jump stilling basins |
| $L'$ | Net length of a crest |
| $M$ | Momentum |
| $M_d$ | Momentum in a downstream section |
| $M_u$ | Momentum in an upstream section |
| $\Delta M$ | Difference in momentum between successive sections |

| *Symbol* | *Description* |
|---|---|
| $m$ | Mass |
| $N$ | Number of piers on an overflow crest; number of slots in a slotted grating dissipator |
| $n$ | Exponential constant used in equation for defining crest shapes; coefficient of roughness in the Manning equation |
| $P$ | Approach height of an ogee weir, hydrostatic pressure of a water prism cross section |
| $p$ | Unit pressure intensity; unit dynamic pressure on a spillway floor; wetted perimeter of a channel or conduit cross section |
| $Q$ | Discharge; volume rate of flow |
| $\Delta Q$ | Incremental change in rate of discharge |
| $q$ | Unit discharge |
| $Q_c$ | Critical discharge |
| $q_c$ | Critical discharge per unit of width |
| $Q_i$ | Average rate of inflow |
| $Q_o$ | Average rate of outflow |
| $R$ | Radius; radius of a cross section; crest profile radius; vertical radius of curvature of the channel floor profile; radius of a terminal bucket profile |
| $r$ | Hydraulic radius; radius of abutment rounding |
| $R_b$ | Radius of a bend in a channel or pipe |
| $R_s$ | Radius of a circular sharp-crested weir |
| $S$ | Storage |
| $\Delta S$ | Incremental storage |
| $s$ | Friction slope in the Manning equation; spacing |
| $s_b$ | Slope of the channel floor, in profile |
| $s_{ws}$ | Slope of the water surface |
| $T$ | Tailwater depth; width at the water surface in a cross section of an open channel |
| $T_{max}$ | Limiting maximum tailwater depth |
| $T_{min}$ | Limiting minimum tailwater depth |
| $t$ | Time |
| $\Delta t$ | Increment of time |
| $T_s$ | Tailwater sweep-out depth |
| $T.W.$ | Tailwater; tailwater depth |
| $U$ | A parameter for defining flow conditions in a closed waterway, $U = \frac{v}{\sqrt{gD}}$ |
| $v$ | Velocity |
| $\Delta v$ | Incremental change in velocity |
| $v_a$ | Velocity of approach |
| $v_c$ | Critical velocity |
| $v_t$ | Velocity of flow in a channel or chute, at tailwater depth |
| $W$ | Weight of a mass; width of a stilling basin |
| $w$ | Unit weight of water; width of chute and baffle blocks in a stilling basin |

| *Symbol* | *Description* |
|---|---|
| $x$ | A coordinate for defining a crest profile; a coordinate for defining a channel profile; a coordinate for defining a conduit entrance |
| $\Delta x$ | Increment of length |
| $x_c$ | Horizontal distance from the break point, on the upstream face of an ogee crest, to the apex of the crest |
| $x_s$ | Horizontal distance from the vertical upstream face of a circular sharp-crested weir to the apex of the undernappe of the overflow sheet |
| $Y$ | Drop distance measured from the crest of the overflow to the basin floor, for a free overfall spillway |
| $y$ | A coordinate for defining a crest profile; a coordinate for defining a channel profile; a coordinate for defining a conduit entrance |
| $\bar{y}$ | Depth from water surface to the center of gravity of a water prism cross section |
| $\Delta y$ | Difference in elevation of the water surface profile between successive sections in a side channel trough |
| $y_c$ | Vertical distance from the break point, on the upstream face of an ogee crest, to the apex of the crest |
| $y_s$ | Vertical distance from the crest of a circular sharp-crested weir to the apex of the undernappe of the overflow sheet |
| $Z$ | Elevation above a datum plane |
| $\Delta Z$ | Elevation difference of the bottom profile between successive sections in an open channel |
| $z$ | Ratio, horizontal to vertical, of the slope of the sides of a channel cross section |
| $\alpha$ | A coefficient; angular variation of the side wall with respect to the structure centerline |
| $\beta$ | Deflection angle of bend in a conduit |
| $\theta$ | Angle from the horizontal; angle from vertical of the position of an orifice; angle from the horizontal of the edge of the lip of a deflector bucket |

Table K-1 presents conversion factors most frequently used by the designer of concrete dams to convert from one set of units to another—for example, to convert from cubic feet per second to acre-feet. Also included are some basic conversion formulas such as the ones for converting flow for a given time to volume.

K-2. *Flow in Open Channels.*—(a) *Energy and Head.*—If it is assumed that streamlines of flow in an open channel are parallel and that velocities at all points in a cross section are equal to the mean velocity $v$, the energy possessed by the water is made up of two parts: kinetic (or motive) energy and potential (or latent) energy. Referring to figure K-1, if $W$ is the weight of a mass $m$, the mass possesses $Wh_2$ foot-pounds of energy with reference to the datum. Also, it possesses $Wh_1$ foot-pounds of energy because of the pressure exerted by the water above it. Thus, the potential energy of the mass $m$ is $W(h_1 + h_2)$. This value is the same for each particle of mass in the cross section. Assuming uniform velocity, the kinetic energy of $m$ is $W\left(\frac{v^2}{2g}\right)$.

Thus, the total energy of each mass particle is:

$$E_m = W\left(h_1 + h_2 + \frac{v^2}{2g}\right) \qquad (1)$$

Applying the above relationship to the whole discharge $Q$ of the cross section in terms of the unit weight of water $w$,

$$E = Qw\left(d + Z + \frac{v^2}{2g}\right) \qquad (2)$$

where $E$ is total energy per second at the cross section.

The portion of equation (2) in the parentheses is termed the absolute head, and is written:

$$H_A = d + Z + \frac{v^2}{2g} \qquad (3)$$

Equation (3) is called the Bernoulli equation.

The energy in the cross section, referred to the bottom of the channel, is termed the specific energy. The corresponding head is referred to as the specific energy head and is expressed as:

$$H_E = d + \frac{v^2}{2g} \qquad (4)$$

Where $Q = av$, equation (4) can be stated:

Table K-1.—*Conversion factors and formulas.*—288-D-3199(1/2)

[To reduce units in column 1 to units in column 4, multiply column 1 by column 2
To reduce units in column 4 to units in column 1, multiply column 4 by column 3]

| CONVERSION FACTORS | | | |
|---|---|---|---|
| **Column 1** | **Column 2** | **Column 3** | **Column 4** |
| **LENGTH** | | | |
| In. | 2.54 | 0.3937 | Cm. |
| | 0.0254 | 39.37 | M. |
| Ft. | 0.3048 | 3.2808 | M. |
| Miles | 1.609 | 0.621 | Km. |
| **AREA** | | | |
| Sq. in. | 6.4516 | 0.1550 | Sq. cm. |
| Sq. m. | 10.764 | .0929 | Sq. ft. |
| Sq. miles | $27.8784\times10^6$ | $0.3587\times10^{-7}$ | Sq. ft. |
| | 640.0 | $.15625\times10^{-2}$ | Acres (1 section). |
| | $30.976\times10^5$ | $.3228\times10^{-6}$ | Sq. yd. |
| | 2.59 | .386 | Sq. km. |
| Acre | 43,56C 0 | $0.22957\times10^{-4}$ | Sq. ft. |
| | 4,046 9 | $.2471\times10^{-3}$ | Sq. m. |
| | 4,840.0 | $.2066\times10^{-3}$ | Sq. yd. |
| **VOLUME** | | | |
| Cu. ft. | 1,728.0 | $0.5787\times10^{-3}$ | Cu. in. |
| | 7.4805 | .13368 | Gal. |
| | 6.2321 | .16046 | Imperial gal. |
| Cu. m. | 35.3145 | 0.028317 | Cu. ft. |
| | 1.3079 | .76456 | Cu. yd. |
| Gal. | 231.0 | $0.4329\times10^{-2}$ | Cu. in. |
| | 3.7854 | .26417 | Liters. |
| Million gal. | 133,681.0 | $0.74805\times10^{-5}$ | Cu. ft. |
| | 3.0689 | .32585 | Acre-ft. |
| Imperial gal. | 1.2003 | 0.83311 | Gal. |
| Acre-in. | 3,630.0 | $.27548\times10^{-3}$ | Cu. ft. |
| Acre-ft. | 1,233.5 | $0.81071\times10^{-3}$ | Cu. m. |
| | 43,560.0 | $.22957\times10^{-4}$ | Cu. ft. |
| In. on 1 sq. mile | $232.32\times10^4$ | $0.43044\times10^{-6}$ | Cu. ft. |
| | 53.33 | .01875 | Acre-ft. |
| Ft. on 1 sq. mile | $278.784\times10^5$ | $0.3587\times10^{-7}$ | Cu. ft. |
| | 640.0 | $.15625\times10^{-2}$ | Acre-ft. |
| **VELOCITY AND GRADE** | | | |
| Miles/hr. | 1.4667 | 0.68182 | Ft./sec. |
| M./sec. | 3.2808 | .3048 | Ft./sec. |
| | 2.2369 | .44704 | Miles/hr. |
| Fall in ft./mile | $189.39\times10^{-6}$ | $5.28\times10^3$ | Fall/ft. |

| CONVERSION FACTORS | | | |
|---|---|---|---|
| **Column 1** | **Column 2** | **Column 3** | **Column 4** |
| **FLOW** | | | |
| Cu. ft./sec. (c.f.s.) (second-feet) (sec.-ft.). | 60.0 | 0.016667 | Cu. ft./min. |
| | 86,400.0 | $.11574\times10^{-4}$ | Cu. ft./day. |
| | $31.536\times10^6$ | $.31709\times10^{-7}$ | Cu. ft./yr. |
| | 448.83 | $.2228\times10^{-2}$ | Gal./min. |
| | 646,317.0 | $.15472\times10^{-5}$ | Gal./day. |
| | 1.98347 | .50417 | Acre-ft./day. |
| | 723.98 | $.13813\times10^{-2}$ | Acre-ft./365 days. |
| | 725.78 | $.13778\times10^{-2}$ | Acre-ft./366 days. |
| | 55.54 | .018006 | Acre-ft./28 days. |
| | 57.52 | .017385 | Acre-ft./29 days. |
| | 59.50 | .016806 | Acre-ft./30 days. |
| | 61.49 | .016262 | Acre-ft./31 days. |
| | 50.0 | .020 | Miner's inch in Idaho, Kans., Nebr., N. Mex., N. Dak., S. Dak., and Utah. |
| | 40.0 | .025 | Miner's inch in Ariz., Calif., Mont., Nev., and Oreg. |
| | 38.4 | .026042 | Miner's inch in Colo. |
| | 35.7 | .028011 | Miner's inch in British Columbia. |
| | 0.028317 | 35.31 | Cu. m./sec. |
| | 1.699 | .5886 | Cu. m./min. |
| | 0.99173 | 1.0083 | Acre-in./hr. |
| Cu. ft./min. | 7.4805 | 0.13368 | Gal./min. |
| | 10,772.0 | $.92834\times10^{-4}$ | Gal./day. |
| $10^6$ gal./day | 1.5472 | 0.64632 | C.f.s. |
| | 694.44 | $.1440\times10^{-2}$ | Gal./min. |
| | 3.0689 | .32585 | Acre-ft./day. |
| In. depth/hr. | 645.33 | $0.15496\times10^{-2}$ | C.f.s./sq. mile. |
| In. depth/day | 26.889 | 0.03719 | C.f.s./sq. mile. |
| | 53.33 | .01878 | Acre-ft./sq. mile. |
| C.f.s./sq. mile | 1.0413 | 0.96032 | In. depth/28 days. |
| | 1.0785 | .92720 | In. depth/29 days. |
| | 1.1157 | .89630 | In. depth/30 days. |
| | 1.1529 | .86738 | In. depth/31 days. |
| | 13.574 | .073668 | In. depth/365 days. |
| | 13.612 | .073467 | In. depth/366 days. |
| Acre-ft./day | 226.24 | $0.442\times10^{-2}$ | Gal./min. |
| | 20.17 | .0496 | Miner's inch in Calif. |
| | 19.36 | .0517 | Miner's inch in Colo. |
| Gal./sec. | 5.347 | 0.187 | Miner's inch in Calif. |
| | 5.128 | .195 | Miner's inch in Colo. |
| **PERMEABILITY** | | | |
| Meinzer (gal./day through 1 sq. ft. under unit gradient). | 48.8 | 0.02049 | Bureau of Reclamation (cu. ft./yr. through 1 sq. ft. under unit gradient). |

Table K-1.—*Conversion factors and formulas.*—Continued.—288-D-3199(2/2)

## CONVERSION FACTORS

| Column 1 | Column 2 | Column 3 | Column 4 |
|---|---|---|---|
| POWER AND ENERGY | | | |
| Hp | 555.0 | $0.18182\times10^{-2}$ | Ft.-lb./sec. |
| | 0.746 | 1.3405 | Kw. |
| | 6,535. | $0.15303\times10^{-3}$ | Kw.-hr./yr. |
| | 42.4 | .0236 | B.t.u./min. |
| | 1.0 | 1.0 | C.f.s. falling 8.8 ft. |
| Hp.-hr | 0.746 | 1.3405 | Kw.-hr. |
| | $198.0\times10^{4}$ | $0.505\times10^{-6}$ | Ft.-lb. |
| | 2.545.0 | $.393\times10^{-3}$ | B.t.u. |
| Kw | 8,760.0 | $0.11416\times10^{-3}$ | Kw.-hr./yr. |
| | 737.56 | $.1354\times10^{-2}$ | Ft.-lb./sec. |
| | 11.8 | .0846 | C.f.s. falling 1 ft. |
| | 3,412.0 | $.29308\times10^{-3}$ | B.t.u./hr. |
| Kw.-hr | 0.975 | 1.025 | Acre-ft. falling 1 ft. |
| B.t.u | 778.0 | $0.1285\times10^{-2}$ | Ft.-lb. |
| | $0.1\times10^{-3}$<br>to<br>$.834\times10^{-4}$ | 10,000<br>to<br>12,000 | Lb. of coal. |
| PRESSURE | | | |
| Ft. water at max. density | 62.425 | 0.01602 | Lb./sq./ft. |
| | 0.4335 | 2.3067 | Lb./sq. in. |
| | .0295 | 33.93 | Atm. |
| | .8826 | 1.133 | In. Hg at 30° F. |
| | 773.3 | $0.1293\times10^{-2}$ | Ft. air at 32° F. and atm. pressure. |
| Ft. avg. sea water | 1.026 | 0.9746 | Ft. pure water. |
| Atm., sea level, 32° F | 14.697 | .06804 | Lb./sq. in. |
| Millibars | $295.299\times10^{-4}$ | 33.863 | In. Hg. |
| | $75.008\times10^{-2}$ | 1.3331 | Mm. Hg. |
| Atm | 29.92 | $33.48\times10^{-3}$ | In. Hg. |
| WEIGHT | | | |
| P.p.m | 0.00136 | 735.29 | Tons/acre-ft. |
| | .0584 | 17.123 | Gr./gal. |
| | 8.345 | 0.1198 | Lb./$10^{6}$ gal. |
| Lb | $7.0\times10^{3}$ | $0.14286\times10^{-3}$ | Gr. |
| Gm | 15.432 | .064799 | Gr. |
| Kg | 2.2046 | .45359 | Lb. |
| Lb. water at 39.1° F | 27.6812 | 0.03612 | Cu. in. |
| | 0.11983 | 8.345 | Gal. |
| | .09983 | 10.016 | Imperial gal. |
| | .453617 | 2.204 | Liters. |
| | .01602 | 62.425 | Cu. ft. pure water. |
| | .01560 | 64.048 | Cu. ft. sea water. |
| Lb. water at 62° F | 0.01604 | 62.355 | Cu. ft. pure water. |
| | .01563 | 63.976 | Cu. ft. sea water. |

## FORMULAS

### VOLUME

Average depth in inches, or acre-inch per acre

$$=\frac{(\text{c.f.s.})\ (\text{hr.})}{\text{acres}}$$

$$=\frac{(\text{gal./min.})\ (\text{hr.})}{450\ (\text{acres})}$$

$$=\frac{(\text{miner's in.})\ (\text{hr.})}{(40^{*})\ (\text{acres})}$$

*Where 1 miner's in. = 1/40 c.f.s.

Use 50 where 1 miner's in. = 1/50 c.f.s.

Conversion of inches depth on area to c.f.s.

$$\text{c.f.s.} = \frac{(645)\ (\text{sq. miles})\ (\text{in. on area})}{(\text{time in hr.})}$$

### POWER AND ENERGY

$$\text{hp.}=\frac{(\text{c.f.s.})\ (\text{head in ft.})}{8.8}$$

$$=\frac{(\text{c.f.s.})\ (\text{pressure in lb./sq. in.})}{3.8}$$

$$=\frac{(\text{gal./min.})\ (\text{head in ft.})}{3{,}960}$$

$$=\frac{(\text{gal./min.})\ (\text{pressure in lb./sq. in.})}{1{,}714}$$

$$\text{b. hp.}=\frac{\text{water hp.}}{\text{pump efficiency}}$$

kw.-hr./1,000 gal. pumped/hr.

$$=\frac{(\text{head in ft.})\ (0.00315)}{(\text{pump efficiency})\ (\text{motor efficiency})}$$

Kw.-hr. = (plant efficiency) (1.025) (head in ft.) (water in acre-ft.)

$$\text{Load factor}=\frac{(\text{kw.-hr. in time t})}{(\text{kw. peak load})\ (\text{time t in hr.})}$$

### SEDIMENTATION

Tons/acre-ft. = (unit weight/cu. ft.) (21.78)

Tons/day = (c.f.s.) (p.p.m.) (0.0027)

### TEMPERATURE

$$°\text{C.}=\frac{5}{9}\ (°\text{F.}-32°) \qquad °\text{F.}=\frac{9}{5}\ °\text{C.}+32°$$

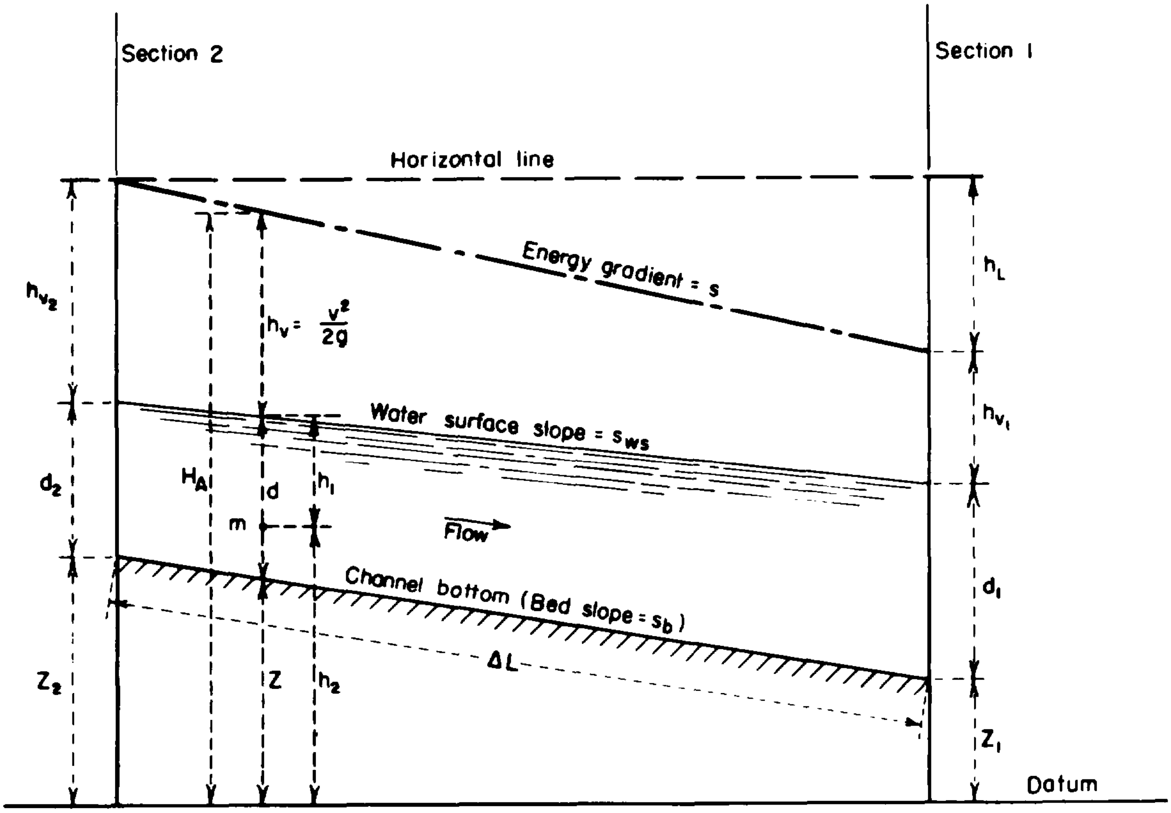


*Figure K-1.* **Characteristics of open-channel flow.–288-D-2550**

$$H_E = d + \frac{Q^2}{2ga^2} \qquad (5)$$

For a trapezoidal channel where $b$ is the bottom width and $z$ defines the side slope, if $q$ is expressed as $\frac{Q}{b}$ and $a$ is expressed as $d(b + zd)$, equation (5) becomes:

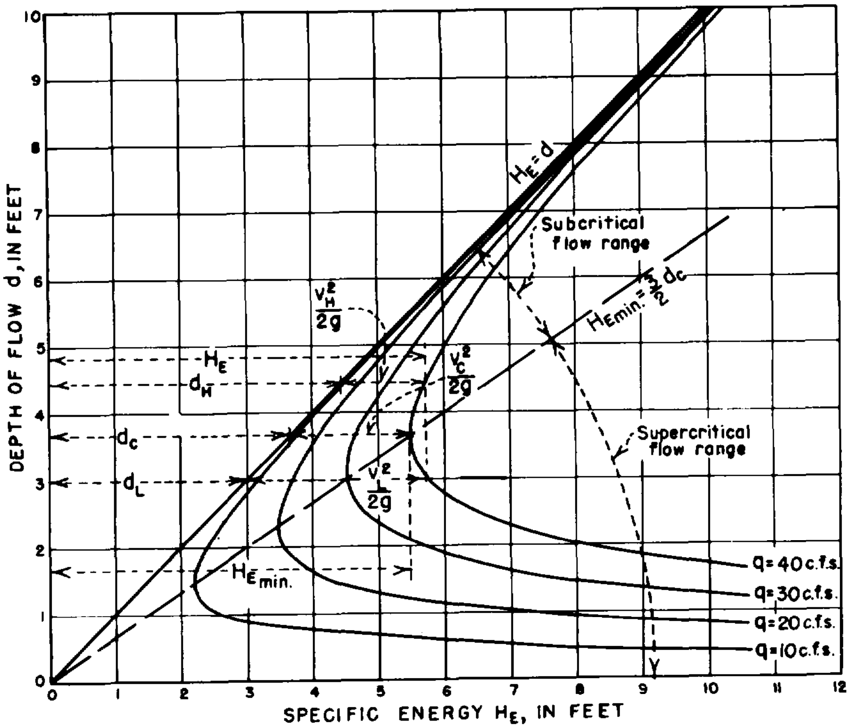
$$H_E = d + \frac{q^2}{2gd^2\left(1 + \frac{zd}{b}\right)^2} \qquad (6)$$

Equation (5) is represented in diagrammatic form on figure K-2 to show the relationships between discharge, energy, and depth of flow in an open channel. The diagram is drawn for several values of unit discharge in a rectangular channel.

It can be seen that there are two values of $d$, $d_H$, and $d_L$ for each value of $H_E$, except at the point where $H_E$ is minimum, where only a single value exists. The depth at energy $H_{E_{min}}$ is called the critical depth, and the depths for other values of $H_E$ are called alternate depths. Those depths lying above the trace through the locus of minimum depths are in the subcritical flow range and are termed subcritical depths, while those lying below the trace are in the supercritical flow range and are termed supercritical depths.

Figure K-3 plots the relationships of $d$ to $H_E$ as stated in equation (6), for various values of unit discharge $q$ and side slope $z$. The curves can be used to quickly determine alternate depths of flow in open channel spillways.

(b) *Critical Flow.*–Critical flow is the term used to describe open channel flow when certain relationships exist between specific energy and discharge and between specific energy and depth. As indicated in section K-2(a) and as demonstrated on figure K-2, critical flow terms can be defined as follows:



$H_E = d + \frac{v^2}{2g} = d + \frac{q^2}{2gd^2}$ where q = discharge per unit width.

$d_c = \left(\frac{q_c}{\sqrt{g}}\right)^{\frac{2}{3}} = \frac{2}{3} H_{E_{min.}}$ where $d_c$ = critical depth

$q_c$ = critical discharge per unit width

$H_{E_{min.}}$ = minimum energy content.

*Figure K-2.* **Depth of flow and specific energy for rectangular section in open channel.–288-D-2551**

(1) *Critical discharge.*–The maximum discharge for a given specific energy, or the discharge which will occur with minimum specific energy.

(2) *Critical depth.*–The depth of flow at which the discharge is maximum for a given specific energy, or the depth at which a given discharge occurs with minimum specific energy.

(3) *Critical velocity.*–The mean velocity when the discharge is critical.

(4) *Critical slope.*–That slope which will sustain a given discharge at uniform critical depth in a given channel.

(5) *Subcritical flow.*–Those conditions of flow for which the depths are greater than critical and the velocities are less than critical.

(6) *Supercritical flow.*–Those conditions of flow for which the depths are less than critical and the velocities are greater than critical.

More complete discussions of the critical flow theory in relationship to specific energy are given in most hydraulic textbooks [1, 2, 3, 4, 5].[1] The relationship between cross section and discharge which must exist in order that flow may occur at the critical stage is:

$$\frac{Q^2}{g} = \frac{a^3}{T} \qquad (7)$$

where:

$a$ = cross-sectional area in square feet, and

$T$ = water surface width in feet.

---

[1] Numbers in brackets refer to items in the bibliography, sec. K-5.

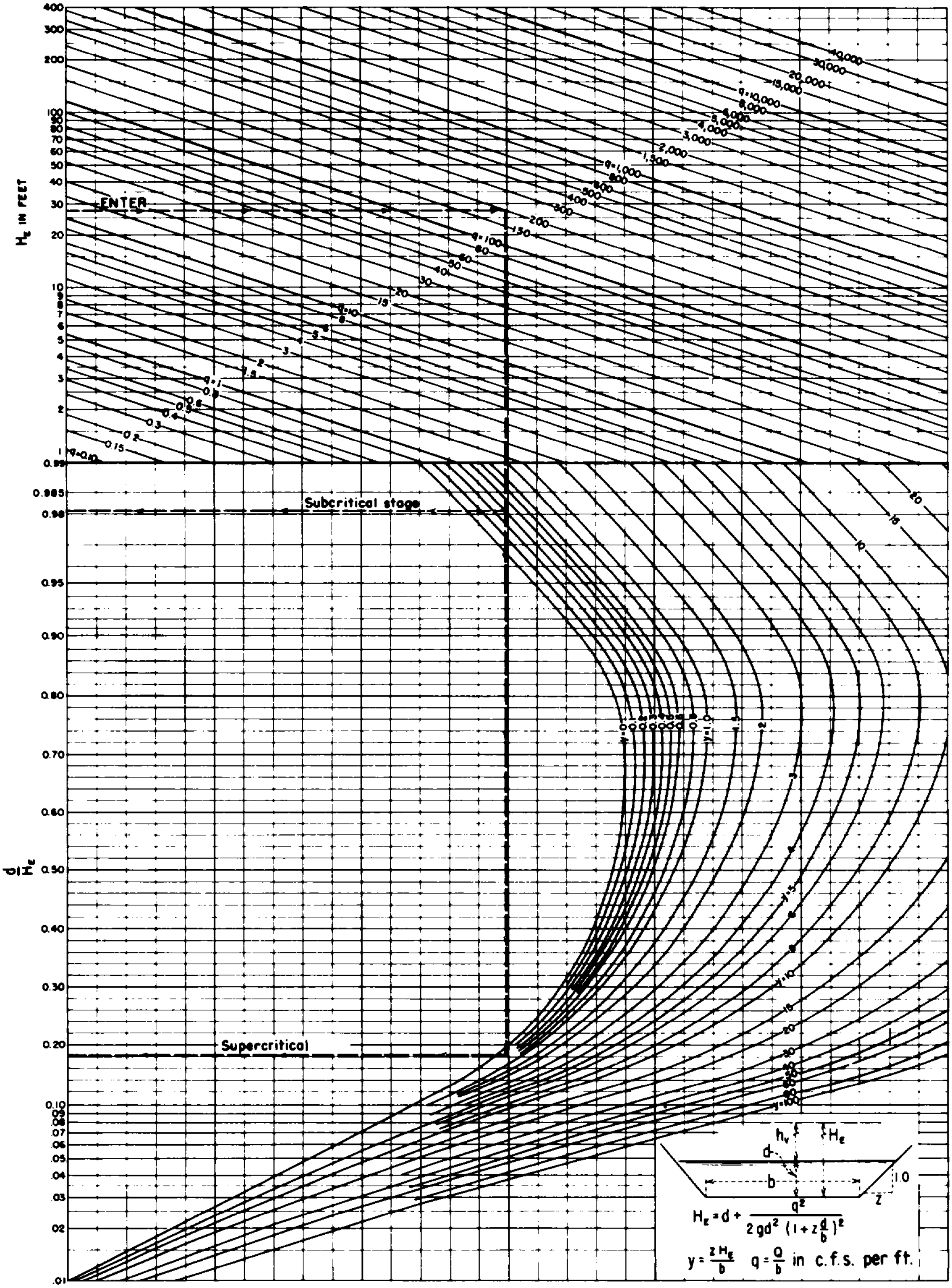


*Figure K-3.* Energy-depth curves for rectangular and trapezoidal channels.—288-D-3193

Since $Q^2 = a^2 v^2$, equation (7) can be written:

$$\frac{v_c{}^2}{2g} = \frac{a}{2T} \tag{8}$$

Also, since $a = d_m T$, where $d_m$ is the mean depth of flow at the section, and $\frac{v_c{}^2}{2g} = h_{v_c}$, equation (8) can be rewritten:

$$h_{v_c} = \frac{d_{m_c}}{2} \tag{9}$$

Then equation (4) can be stated

$$H_E = d_c + \frac{d_{m_c}}{2} \tag{10}$$

From the foregoing, the following additional relations can be stated:

$$d_{m_c} = \frac{v_c{}^2}{g} \tag{11}$$

$$d_{m_c} = \frac{Q_c{}^2}{a^2 g} \tag{12}$$

$$v_c = \sqrt{g d_{m_c}} \tag{13}$$

$$v_c = \sqrt{\frac{ag}{T}} = 5.67\sqrt{\frac{a}{T}} \tag{14}$$

$$Q_c = a\sqrt{g d_{m_c}} \tag{15}$$

For rectangular sections, if $q$ is the discharge per foot width of channel, the various critical flow formulae are:

$$H_{E_c} = \frac{3}{2} d_c \tag{16}$$

$$d_c = \frac{2}{3} H_{E_c} \tag{17}$$

$$d_c = \frac{v_c{}^2}{g} \tag{18}$$

$$d_c = \sqrt[3]{\frac{q_c{}^2}{g}} \tag{19}$$

$$d_c = \sqrt[3]{\frac{Q_c{}^2}{b^2 g}} \tag{20}$$

$$v_c = \sqrt{g d_c} \tag{21}$$

$$v_c = \sqrt[3]{g q_c} \tag{22}$$

$$v_c = \sqrt[3]{\frac{g Q_c}{b}} \tag{23}$$

$$q_c = d_c{}^{3/2}\sqrt{g} \tag{24}$$

$$Q_c = 5.67 b d_c{}^{3/2} \tag{25}$$

$$Q_c = 3.087 b H_{E_c}{}^{3/2} \tag{26}$$

The critical depth for trapezoidal sections is given by the equation:

$$d_c = \frac{v_c{}^2}{g} - \frac{b}{2z} + \sqrt{\frac{v_c{}^4}{g^2} + \frac{b^2}{4z^2}} \tag{27}$$

where $z$ = the ratio, horizontal to vertical, of the slope of the sides of the channel.

Similarly, for the trapezoidal section,

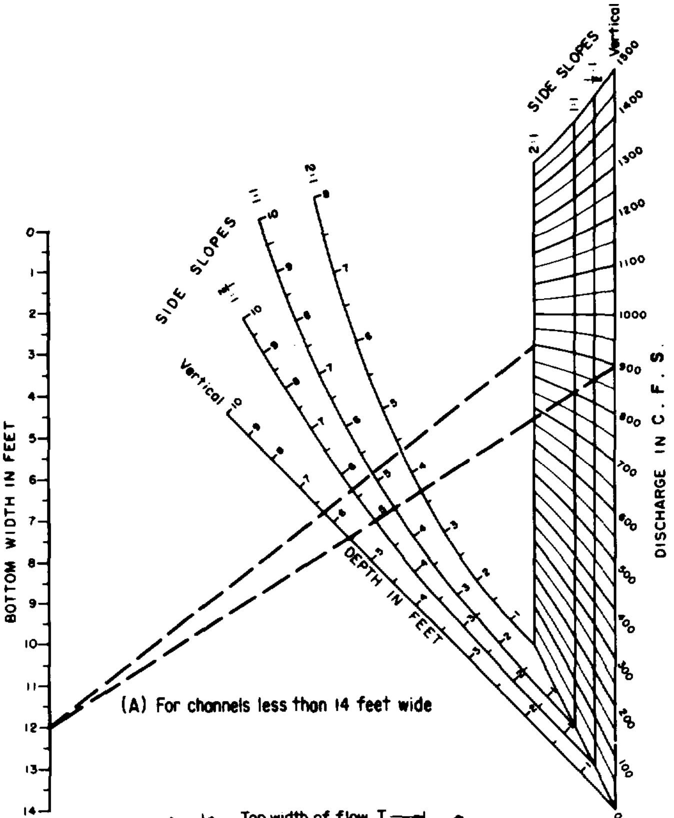
$$v_c = \sqrt{\left(\frac{b + z d_c}{b + 2z d_c}\right) d_c g} \tag{28}$$

and

$$Q_c = d_c{}^{3/2}\sqrt{\frac{g(b + z d_c)^3}{b + 2z d_c}} \tag{29}$$

The solutions of equations (25) and (29) are simplified by use of figure K-4.

(c) *Manning Formula.*—The formula developed by Manning for flow in open channels is used in most of the hydraulic analyses discussed in this text. It is a special form of Chezy's formula; the complete development is contained in most textbooks on elementary fluid mechanics. The formula is written as follows:



(A) For channels less than 14 feet wide

Example No. 1
$Q_c$ = 900 c.f.s.
Bottom width "b" = 12'

| Side slope | Critical depth "$d_c$" (feet) |
|---|---|
| 2:1 | 4.4 |
| Vertical | 5.6 |

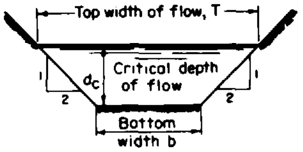
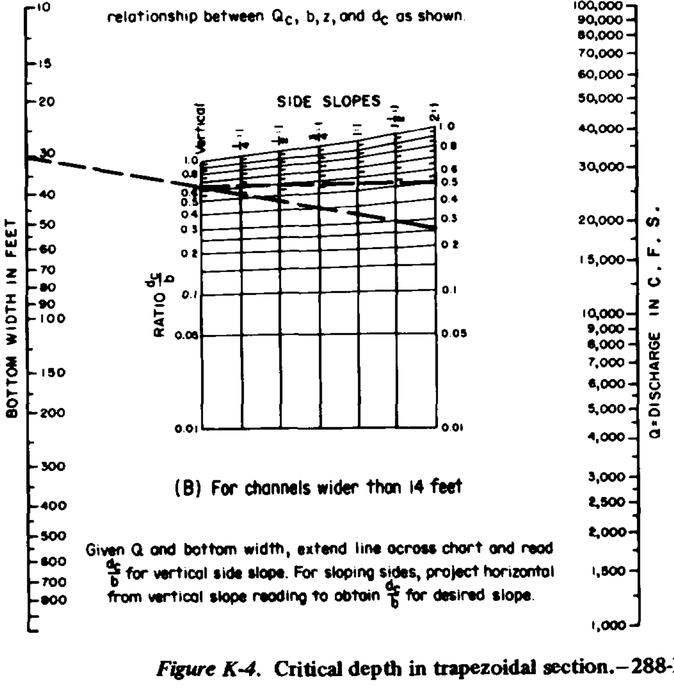


Chart gives values of $d_c$ for known values of $Q_c$ in the relationship $Q_c=(\frac{A^3g}{T})^{\frac{1}{2}}$. Single solution line gives relationship between $Q_c$, b, z, and $d_c$ as shown.



(B) For channels wider than 14 feet

Given Q and bottom width, extend line across chart and read $\frac{d_c}{b}$ for vertical side slope. For sloping sides, project horizontal from vertical slope reading to obtain $\frac{d_c}{b}$ for desired slope.

Example No 2
Q = 15,000 c.f.s.
Bottom width "b" = 30'
Side slope = 2:1
Vertical "$d_c$" = .68b = 20'+
"$d_c$" for 2:1 = (0.5)(30) = 15'

*Figure K-4.* **Critical depth in trapezoidal section.–288-D-3194**

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (30)$$

or

$$Q = \frac{1.486}{n} ar^{2/3} s^{1/2} \qquad (31)$$

where:

$Q$ = discharge in cubic feet per second (c.f.s.),
$a$ = the cross section of flow area in square feet,
$v$ = the velocity in feet per second,
$n$ = a roughness coefficient,
$r$ = the hydraulic radius $= \frac{\text{area } (a)}{\text{wetted perimeter } (p)}$, and
$s$ = the slope of the energy gradient.

The value of the roughness coefficient, $n$, varies according to the physical roughness of the sides and bottom of the channel and is influenced by such factors as channel curvature, size and shape of cross section, alinement, and type and condition of the material forming the wetted perimeter.

Values of $n$ commonly used in the design of artificial channels are as follows:

| Description of channel | Values of $n$ | | |
|---|---|---|---|
| | Minimum | Maximum | Average |
| Earth channels, straight and uniform ...... | 0.017 | 0.025 | 0.0225 |
| Dredged earth channels ......... | .025 | .033 | .0275 |
| Rock channels, straight and uniform ...... | .025 | .035 | .033 |
| Rock channels, jagged and irregular ...... | .035 | .045 | .045 |
| Concrete lined ...... | .012 | .018 | .014 |
| Neat cement lined ... | .010 | .013 | ...... |
| Grouted rubble paving ........... | .017 | .030 | ...... |
| Corrugated metal ... | .023 | .025 | .024 |

(d) *Bernoulli Theorem.*—The Bernoulli theorem, which is the principle of conservation of energy applied to open channel flow, may be stated: The absolute head at any section is equal to the absolute head at a section downstream plus intervening losses of head. Referring to figure K-1, the energy equation (3) can be written:

$$Z_2 + d_2 + h_{v_2} = Z_1 + d_1 + h_{v_1} + h_L \qquad (32)$$

where $h_L$ represents all losses in head between section 2 (subscript 2) and section 1 (subscript 1). Such head losses will consist largely of friction loss, but may include minor other losses such as those due to eddy, transition, obstruction, impact, etc.

When the discharge at a given cross section of a channel is constant with respect to time, the flow is steady. If steady flow occurs at all sections in a reach, the flow is continuous and

$$Q = a_1 v_1 = a_2 v_2 \qquad (33)$$

Equation (33) is termed the equation of continuity. Equations (32) and (33), solved simultaneously, are the basic formulas used in solving problems of flow in open channels.

(e) *Hydraulic and Energy Gradients.*—The hydraulic gradient in open channel flow is the water surface. The energy gradient is above the hydraulic gradient a distance equal to the velocity head. The fall of the energy gradient for a given length of channel represents the loss of energy, either from friction or from friction and other influences. The relationship of the energy gradient to the hydraulic gradient reflects not only the loss of energy, but also the conversion between potential and kinetic energy. For uniform flow the gradients are parallel and the slope of the water surface represents the friction loss gradient. In accelerated flow the hydraulic gradient is steeper than the energy gradient, indicating a progressive conversion from potential to kinetic energy. In retarded flow the energy gradient is steeper than the hydraulic gradient, indicating a conversion from kinetic to potential energy. The Bernoulli theorem defines the progressive relationships of these energy gradients.

For a given reach of channel $\Delta L$, the average slope of the energy gradient is $\frac{\Delta h_L}{\Delta L}$, where $\Delta h_L$ is the cumulative losses through the reach. If these losses are solely from friction, $\Delta h_L$ will become $\Delta h_f$ and

$$\Delta h_f = \left(\frac{s_2 + s_1}{2}\right) \Delta L \qquad (34)$$

Expressed in terms of the hydraulic properties at each end of the reach and of the roughness coefficient,

$$\Delta h_f = \frac{n^2}{4.41} \left[\left(\frac{v_2}{r_2{}^{2/3}}\right)^2 + \left(\frac{v_1}{r_1{}^{2/3}}\right)^2\right] \Delta L \quad (35)$$

If the average friction slope, $s_f$, is equal to $\frac{s_2 + s_1}{2} = \frac{\Delta h_f}{\Delta L}$, and $s_b$ is the slope of the channel floor, by substituting $s_b \Delta L$ for $Z_2 - Z_1$, and $H_E$ for $(d + h_v)$, equation (32) may be written:

$$\Delta L = \frac{H_{E_1} - H_{E_2}}{s_b - s_f} \qquad (36)$$

(f) *Chart for Approximating Friction Losses in Chutes.*—Figure 9-26 is a nomograph from which approximate friction losses in a channel can be evaluated. To generalize the chart so that it can be applied for differing channel conditions, several approximations are made. First, the depth of flow in the channel is assumed equal to the hydraulic radius; the results will therefore be most applicable to wide, shallow channels. Furthermore, the increase in velocity head is assumed to vary proportionally along the length of the channel. Thus, the data given in the chart are not exact and are intended to serve only as a guide in estimating channel losses.

The chart plots the solution of the equation $s = \frac{dh_f}{dx}$, integrated between the limits from zero to $L$, or

$$h_f = \int_o^L s\,dx,$$

where, from the Manning equation,

$$s = \frac{v^2}{\left(\frac{1.486}{n}\right)^2 r^{4/3}}$$

K-3. *Flow in Closed Conduits.*—(a) *Partly Full Flow in Conduits.*—The hydraulics of partly full flow in closed conduits is similar to that in open channels, and open channel flow formulas are applicable. Hydraulic properties for different flow depths in circular and horseshoe conduits are tabulated in tables K-2 through K-5 to facilitate hydraulic computations for these sections.

Tables K-2 and K-4 give data for determining critical depths, critical velocities, and hydrostatic pressures of the water prism cross section for various discharges and conduit diameters. If the area at critical flow, $a_c$, is represented as $k_1 D^2$ and the top width of the water prism, $T$, for critical flow is equal to $k_2 D$, equation (7) can be written:

$$\frac{Q_c{}^2}{g} = \frac{(k_1 D^2)^3}{k_2 D}, \text{ or } Q_c = k_3 D^{5/2} \qquad (37)$$

Values of $k_3$, for various flow depths, are tabulated in column 3. The hydrostatic pressure, $P$, of the water prism cross section is $wa\bar{y}$, where $\bar{y}$ is the depth from the water surface to the center of gravity of the cross section. If $a_c = k_1 D^2$ and $\bar{y} = k_4 D$, then

$$P = k_5 D^3 \qquad (38)$$

Values of $k_5$, for various flow depths, are tabulated in column 4. Column 2 gives the values of $h_{v_c}$ in relation to the conduit diameter, for various flow depths.

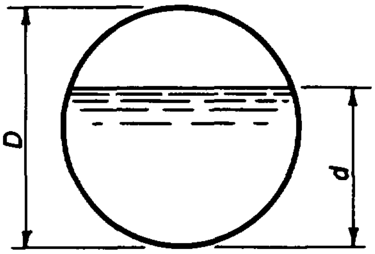
Tables K-3 and K-5 give areas and hydraulic radii for partly full conduits and coefficients which can be applied in the solution of the Manning equation. If $A = k_6 \frac{\pi D^2}{4}$ and $r = k_7 D$, Manning's equation can be written:

$$Q = \frac{1.486}{n}\left(k_6 \frac{\pi D^2}{4}\right)(k_7 D)^{2/3}\, s^{1/2},$$

or

Table K-2.—*Velocity head and discharge at critical depths and static pressures in circular conduits partly full.*—288-D-3195

$D$ = Diameter of pipe.
$d$ = Depth of flow.
$h_{v_c}$ = Velocity head for a critical depth of $d$.
$Q_c$ = Discharge when the critical depth is $d$.
$P$ = Pressure on cross section of water prism in cubic units of water. To get $P$ in pounds, when $d$ and $D$ are in feet, multiply by 62.5.



| $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ | $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ | $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 0.01 | 0.0033 | 0.0006 | 0.0000 | 0.34 | 0.1243 | 0.6657 | 0.0332 | 0.67 | 0.2974 | 2.4464 | 0.1644 |
| .02 | .0067 | .0025 | .0000 | .35 | .1284 | .7040 | .0356 | .68 | .3048 | 2.5182 | .1700 |
| .03 | .0101 | .0055 | .0001 | .36 | .1326 | .7433 | .0381 | .69 | .3125 | 2.5912 | .1758 |
| .04 | .0134 | .0098 | .0002 | .37 | .1368 | .7836 | .0407 | .70 | .3204 | 2.6656 | .1816 |
| .05 | .0168 | .0153 | .0003 | .38 | .1411 | .8249 | .0434 | .71 | .3286 | 2.7414 | .1875 |
| .06 | .0203 | .0220 | .0005 | .39 | .1454 | .8671 | .0462 | .72 | .3371 | 2.8188 | .1935 |
| .07 | .0237 | .0298 | .0007 | .40 | .1497 | .9103 | .0491 | .73 | .3459 | 2.8977 | .1996 |
| .08 | .0271 | .0389 | .0010 | .41 | .1541 | .9545 | .0520 | .74 | .3552 | 2.9783 | .2058 |
| .09 | .0306 | .0491 | .0013 | .42 | .1586 | .9996 | .0551 | .75 | .3648 | 3.0607 | .2121 |
| .10 | .0341 | .0605 | .0017 | .43 | .1631 | 1.0458 | .0583 | .76 | .3749 | 3.1450 | .2185 |
| .11 | .0376 | .0731 | .0021 | .44 | .1676 | 1.0929 | .0616 | .77 | .3855 | 3.2314 | .2249 |
| .12 | .0411 | .0868 | .0026 | .45 | .1723 | 1.1410 | .0650 | .78 | .3967 | 3.3200 | .2314 |
| .13 | .0446 | .1016 | .0032 | .46 | .1769 | 1.1899 | .0684 | .79 | .4085 | 3.4112 | .2380 |
| .14 | .0482 | .1176 | .0038 | .47 | .1817 | 1.2399 | .0720 | .80 | .4210 | 3.5050 | .2447 |
| .15 | .0517 | .1347 | .0045 | .48 | .1865 | 1.2908 | .0757 | .81 | .4343 | 3.6019 | .2515 |
| .16 | .0553 | .1530 | .0053 | .49 | .1914 | 1.3427 | .0795 | .82 | .4485 | 3.7021 | .2584 |
| .17 | .0589 | .1724 | .0061 | .50 | .1964 | 1.3955 | .0833 | .83 | .4638 | 3.8061 | .2653 |
| .18 | .0626 | .1928 | .0070 | .51 | .2014 | 1.4493 | .0873 | .84 | .4803 | 3.9144 | .2723 |
| .19 | .0662 | .2144 | .0080 | .52 | .2065 | 1.5041 | .0914 | .85 | .4982 | 4.0276 | .2794 |
| .20 | .0699 | .2371 | .0091 | .53 | .2117 | 1.5598 | .0956 | .86 | .5177 | 4.1465 | .2865 |
| .21 | .0736 | .2609 | .0103 | .54 | .2170 | 1.6164 | .0998 | .87 | .5392 | 4.2721 | .2938 |
| .22 | .0773 | .2857 | .0115 | .55 | .2224 | 1.6735 | .1042 | .88 | .5632 | 4.4056 | .3011 |
| .23 | .0811 | .3116 | .0128 | .56 | .2279 | 1.7327 | .1087 | .89 | .5900 | 4.5486 | .3084 |
| .24 | .0848 | .3386 | .0143 | .57 | .2335 | 1.7923 | .1133 | .90 | .6204 | 4.7033 | .3158 |
| .25 | .0887 | .3667 | .0157 | .58 | .2393 | 1.8530 | .1179 | .91 | .6555 | 4.8725 | .3233 |
| .26 | .0925 | .3957 | .0173 | .59 | .2451 | 1.9146 | .1227 | .92 | .6966 | 5.0603 | .3308 |
| .27 | .0963 | .4259 | .0190 | .60 | .2511 | 1.9773 | .1276 | .93 | .7459 | 5.2726 | .3384 |
| .28 | .1002 | .4571 | .0207 | .61 | .2572 | 2.0409 | .1326 | .94 | .8065 | 5.5183 | .3460 |
| .29 | .1042 | .4893 | .0226 | .62 | .2635 | 2.1057 | .1376 | .95 | .8841 | 5.8118 | .3537 |
| .30 | .1081 | .5225 | .0255 | .63 | .2699 | 2.1716 | .1428 | .96 | .9885 | 6.1787 | .3615 |
| .31 | .1121 | .5568 | .0266 | .64 | .2765 | 2.2386 | .1481 | .97 | 1.1410 | 6.6692 | .3692 |
| .32 | .1161 | .5921 | .0287 | .65 | .2833 | 2.3067 | .1534 | .98 | 1.3958 | 7.4063 | .3770 |
| .33 | .1202 | .6284 | .0309 | .66 | .2902 | 2.3760 | .1589 | .99 | 1.9700 | 8.8263 | .3848 |
|  |  |  |  |  |  |  |  | 1.00 | ------ | ------ | .3927 |

$$\frac{Qn}{D^{8/3}s^{1/2}} = k_6 \frac{1.486\pi}{4}(k_7)^{2/3} = k_8 \qquad (39)$$

Values of $k_8$, for various flow depths, are tabulated in column 4. If $D = k_9 d$, equation (39) can be written:

$$\frac{Qn}{d^{8/3}s^{1/2}} = \frac{1.486\pi}{4} k_6 (k_7)^{2/3} (k_9)^{8/3} = k_{10} \qquad (40)$$

Table K-3.–*Uniform flow in circular sections flowing partly full.*–288-D-3196

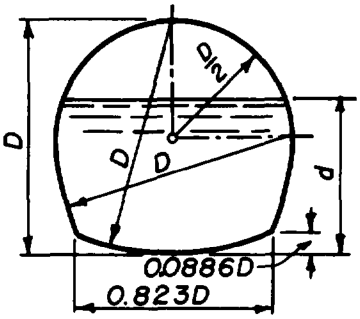
$d$ = Depth of flow.
$D$ = Diameter of pipe.
$A$ = Area of flow.
$r$ = Hydraulic radius.

$Q$ = Discharge in c.f.s. by Manning's formula.
$n$ = Manning's coefficient.
$s$ = Slope of the channel bottom and of the water surface.

| $\frac{d}{D}$ | $\frac{A}{D^2}$ | $\frac{r}{D}$ | $\frac{Qn}{D^{8/3}s^{1/2}}$ | $\frac{Qn}{d^{8/3}s^{1/2}}$ | $\frac{d}{D}$ | $\frac{A}{D^2}$ | $\frac{r}{D}$ | $\frac{Qn}{D^{8/3}s^{1/2}}$ | $\frac{Qn}{d^{8/3}s^{1/2}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| 0.01 | 0.0013 | 0.0066 | 0.00007 | 15.04 | 0.51 | 0.4027 | 0.2531 | 0.239 | 1.442 |
| .02 | .0037 | .0132 | .00031 | 10.57 | .52 | .4127 | .2562 | .247 | 1.415 |
| .03 | .0069 | .0197 | .00074 | 8.56 | .53 | .4227 | .2592 | .255 | 1.388 |
| .04 | .0105 | .0262 | .00138 | 7.38 | .54 | .4327 | .2621 | .263 | 1.362 |
| .05 | .0147 | 0325 | .00222 | 6.55 | .55 | .4426 | .2649 | .271 | 1.336 |
| .06 | .0192 | .0389 | .00328 | 5.95 | .56 | .4526 | .2676 | .279 | 1.311 |
| .07 | .0242 | .0451 | .00455 | 5.47 | .57 | .4625 | .2703 | .287 | 1.286 |
| .08 | .0294 | .0513 | .00604 | 5.09 | .58 | .4724 | .2728 | .295 | 1.262 |
| .09 | .0350 | .0575 | .00775 | 4.76 | .59 | .4822 | .2753 | .303 | 1.238 |
| .10 | .0409 | .0635 | .00967 | 4.49 | .60 | .4920 | .2776 | .311 | 1.215 |
| .11 | .0470 | .0695 | .01181 | 4.25 | .61 | .5018 | .2799 | .319 | 1.192 |
| .12 | .0534 | .0755 | .01417 | 4.04 | .62 | .5115 | .2821 | .327 | 1.170 |
| .13 | .0600 | .0813 | .01674 | 3.86 | .63 | .5212 | .2842 | .335 | 1.148 |
| .14 | .0668 | .0871 | .01952 | 3.69 | .64 | .5308 | .2862 | .343 | 1.126 |
| .15 | .0739 | .0929 | .0225 | 3.54 | .65 | .5404 | .2882 | .350 | 1.105 |
| .16 | .0811 | .0985 | .0257 | 3.41 | .66 | .5499 | .2900 | .358 | 1.084 |
| .17 | .0885 | .1042 | .0291 | 3.28 | .67 | .5594 | .2917 | .366 | 1.064 |
| .18 | .0961 | .1097 | .0327 | 3.17 | .68 | .5687 | .2933 | .373 | 1.044 |
| .19 | .1039 | .1152 | .0365 | 3.06 | .69 | .5780 | .2948 | .380 | 1.024 |
| .20 | .1118 | .1206 | .0406 | 2.96 | .70 | .5872 | .2962 | .388 | 1.004 |
| .21 | .1199 | .1259 | .0448 | 2.87 | .71 | .5964 | .2975 | .395 | 0.985 |
| .22 | .1281 | .1312 | .0492 | 2.79 | .72 | .6054 | .2987 | .402 | .965 |
| .23 | .1365 | .1364 | .0537 | 2.71 | .73 | .6143 | .2998 | .409 | .947 |
| .24 | .1449 | .1416 | .0585 | 2.63 | .74 | .6231 | .3008 | .416 | .928 |
| .25 | .1535 | .1466 | .0634 | 2.56 | .75 | .6319 | .3017 | .422 | .910 |
| .26 | .1623 | .1516 | .0686 | 2.49 | .76 | .6405 | .3024 | .429 | .891 |
| .27 | .1711 | .1566 | .0739 | 2.42 | .77 | .6489 | .3031 | 435 | .873 |
| .28 | .1800 | .1614 | .0793 | 2.36 | .78 | .6573 | .3036 | .441 | .856 |
| .29 | .1890 | .1662 | .0849 | 2.30 | .79 | .6655 | .3039 | .447 | .838 |
| .30 | .1982 | .1709 | .0907 | 2.25 | .80 | .6736 | .3042 | .453 | .821 |
| .31 | .2074 | .1756 | .0966 | 2.20 | .81 | .6815 | .3043 | .458 | .804 |
| .32 | .2167 | .1802 | .1027 | 2.14 | .82 | .6893 | .3043 | .463 | .787 |
| .33 | .2260 | .1847 | .1089 | 2.09 | .83 | .6969 | .3041 | .468 | .770 |
| .34 | .2355 | .1891 | .1153 | 2.05 | .84 | .7043 | .3038 | .473 | .753 |
| .35 | .2450 | .1935 | .1218 | 2.00 | .85 | .7115 | .3033 | .477 | .736 |
| .36 | .2546 | .1978 | .1284 | 1.958 | .86 | .7186 | .3026 | .481 | .720 |
| .37 | .2642 | .2020 | .1351 | 1.915 | .87 | .7254 | .3018 | .485 | .703 |
| .38 | .2739 | .2062 | .1420 | 1.875 | .88 | .7320 | .3007 | .488 | .687 |
| .39 | .2836 | .2102 | .1490 | 1.835 | .89 | .7384 | .2995 | .491 | .670 |
| .40 | .2934 | .2142 | .1561 | 1.797 | .90 | .7445 | .2980 | .494 | .654 |
| .41 | .3032 | .2182 | .1633 | 1.760 | .91 | .7504 | .2963 | .496 | .637 |
| .42 | .3130 | .2220 | .1705 | 1.724 | .92 | .7560 | .2944 | .497 | .621 |
| .43 | .3229 | .2258 | .1779 | 1.689 | .93 | .7612 | .2921 | .498 | .604 |
| .44 | .3328 | .2295 | .1854 | 1.655 | .94 | .7662 | .2895 | .498 | .588 |
| .45 | .3428 | .2331 | .1929 | 1.622 | .95 | .7707 | .2865 | .498 | .571 |
| .46 | .3527 | .2366 | .201 | 1.590 | .96 | .7749 | .2829 | .496 | .553 |
| .47 | .3627 | .2401 | .208 | 1.559 | .97 | .7785 | .2787 | .494 | .535 |
| .48 | .3727 | .2435 | .216 | 1.530 | .98 | .7817 | .2735 | .489 | .517 |
| .49 | .3827 | .2468 | .224 | 1.500 | .909 | .7841 | .2666 | .483 | .496 |
| .50 | .3927 | .2500 | .232 | 1.471 | 1.00 | .7854 | .2500 | .463 | .463 |

Table K-4.–*Velocity head and discharge at critical depths and static pressures in horseshoe conduits partly full.*–288-D-3197

**$D$**=Diameter of horseshoe.
**$d$**=Depth of flow.
**$h_{v_c}$**=Velocity head for a critical depth of $d$.
**$Q_c$**=Discharge when the critical depth is $d$.
**$P$**=Pressure on cross section of water prism in cubic units of water. To get $P$ in pounds, when $d$ and $D$ are in feet, multiply by 62.5.



| $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ | $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ | $\frac{d}{D}$ | $\frac{h_{v_c}}{D}$ | $\frac{Q_c}{D^{5/2}}$ | $\frac{P}{D^3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 0.01 | 0.0033 | 0.0009 | 0.0000 | 0.35 | 0.1472 | 0.8854 | 0.0449 | 0.69 | 0.3362 | 2.8922 | 0.1999 |
| .02 | .0067 | .0035 | .0000 | .36 | .1518 | .9296 | .0478 | .70 | .3443 | 2.9702 | .2062 |
| .03 | .0100 | .0079 | .0001 | .37 | .1563 | .9746 | .0508 | .71 | .3528 | 3.0499 | .2125 |
| .04 | .0134 | .0139 | .0002 | .38 | .1609 | 1.0205 | .0540 | .72 | .3615 | 3.1311 | .2190 |
| .05 | .0168 | .0217 | .0004 | .39 | .1655 | 1.0673 | .0572 | .73 | .3707 | 3.2140 | .2255 |
| .06 | .0201 | .0312 | .0007 | .40 | .1702 | 1.1148 | .0605 | .74 | .3802 | 3.2987 | .2321 |
| .07 | .0235 | .0425 | .0010 | .41 | .1749 | 1.1633 | .0639 | .75 | .3902 | 3.3853 | .2388 |
| .08 | .0269 | .0554 | .0014 | .42 | .1795 | 1.2125 | .0675 | .76 | .4006 | 3.4740 | .2457 |
| .09 | .0305 | .0703 | .0018 | .43 | .1843 | 1.2626 | .0711 | .77 | .4116 | 3.5650 | .2525 |
| .10 | .0351 | .0879 | .0024 | .44 | .1890 | 1.3135 | .0748 | .78 | .4232 | 3.6584 | .2595 |
| .11 | .0397 | .1069 | .0030 | .45 | .1938 | 1.3652 | .0786 | .79 | .4354 | 3.7544 | .2666 |
| .12 | .0443 | .1272 | .0037 | .46 | .1986 | 1.4178 | .0825 | .80 | .4484 | 3.8534 | .2737 |
| .13 | .0489 | .1487 | .0045 | .47 | .2035 | 1.4712 | .0865 | .81 | .4623 | 3.9557 | .2809 |
| .14 | .0534 | .1714 | .0054 | .48 | .2084 | 1.5253 | .0907 | .82 | .4771 | 4.0616 | .2882 |
| .15 | .0579 | .1963 | .0063 | .49 | .2133 | 1.5803 | .0949 | .83 | .4930 | 4.1716 | .2956 |
| .16 | .0624 | .2203 | .0074 | .50 | .2183 | 1.6361 | .0992 | .84 | .5102 | 4.2863 | .3030 |
| .17 | .0669 | .2465 | .0085 | .51 | .2234 | 1.6928 | .1036 | .85 | .5289 | 4.4063 | .3105 |
| .18 | .0714 | .2736 | .0098 | .52 | .2285 | 1.7505 | .1081 | .86 | .5494 | 4.5325 | .3181 |
| .19 | .0758 | .3019 | .0111 | .53 | .2337 | 1.8092 | .1127 | .87 | .5719 | 4.6660 | .3258 |
| .20 | .0803 | .3312 | .0125 | .54 | .2391 | 1.8688 | .1174 | .88 | .5969 | 4.8080 | .3335 |
| .21 | .0847 | .3615 | .0140 | .55 | .2445 | 1.9294 | .1223 | .89 | .6251 | 4.9605 | .3413 |
| .22 | .0891 | .3928 | .0156 | .56 | .2500 | 1.9911 | .1272 | .90 | .6570 | 5.1256 | .3492 |
| .23 | .0936 | .4251 | .0173 | .57 | .2557 | 2.0537 | .1322 | .91 | .6939 | 5.3065 | .3572 |
| .24 | .0980 | .4583 | .0191 | .58 | .2615 | 2.1174 | .1373 | .92 | .7371 | 5.5077 | .3653 |
| .25 | .1024 | .4926 | .0210 | .59 | .2674 | 2.1821 | .1425 | .93 | .7889 | 5.7354 | .3733 |
| .26 | .1069 | .5277 | .0229 | .60 | .2735 | 2.2479 | .1478 | .94 | .8528 | 5.9996 | .3813 |
| .27 | .1113 | .5638 | .0250 | .61 | .2797 | 2.3148 | .1532 | .95 | .9345 | 6.3157 | .3894 |
| .28 | .1158 | .6009 | 0271 | .62 | .2861 | 2.3828 | .1587 | .96 | 1.0446 | 6.7114 | .3976 |
| .29 | .1202 | .6389 | .0294 | .63 | .2926 | 2.4519 | .1643 | .97 | 1.2053 | 7.2417 | .4058 |
| .30 | .1247 | .6777 | .0317 | .64 | .2994 | 2.5221 | .1700 | .98 | 1.4742 | 8.0892 | .4140 |
| .31 | .1292 | .7175 | .0342 | .65 | .3063 | 2.5936 | .1758 | .99 | 2.0604 | 9.5780 | .4223 |
| .32 | .1337 | .7582 | .0367 | .66 | .3134 | 2.6663 | .1817 | 1.00 | - - - - - - | - - - - - - | .4306 |
| .33 | .1382 | .7997 | .0393 | .67 | .3208 | 2.7402 | .1877 | | | | |
| .34 | .1427 | .8421 | .0421 | .68 | .3283 | 2.8155 | .1937 | | | | |

Values of $k_{10}$, for various flow depths, are tabulated in column 5.

(b) *Pressure Flow in Conduits.*–Since factors affecting head losses in conduits are independent of pressure, the same laws apply to flow in both closed conduits and open channels, and the formulas for each take the same general form. Thus, the equation of continuity, equation (33), $Q = a_1 v_1 = a_2 v_2$, also applies to pressure flow in conduits.

Table K-5.—*Uniform flow in horseshoe sections flowing partly full.*—288-D-3198

$d$ = Depth of flow.
$D$ = Diameter.
$A$ = Area of flow.
$r$ = Hydraulic radius.

$Q$ = Discharge in c.f.s. by Manning's formula.
$n$ = Manning's coefficient.
$s$ = Slope of the channel bottom and of the water surface.

| $\frac{d}{D}$ 1 | $\frac{A}{D^2}$ 2 | $\frac{r}{D}$ 3 | $\frac{Qn}{D^{8/3}s^{1/2}}$ 4 | $\frac{Qn}{d^{8/3}s^{1/2}}$ 5 | $\frac{d}{D}$ 1 | $\frac{A}{D^2}$ 2 | $\frac{r}{D}$ 3 | $\frac{Qn}{D^{8/3}s^{1/2}}$ 4 | $\frac{Qn}{d^{8/3}s^{1/2}}$ 5 |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.0019 | 0.0066 | 0.00010 | 21.40 | 0.51 | 0.4466 | 0.2602 | 0.2705 | 1.629 |
| .02 | .0053 | .0132 | .00044 | 14.93 | .52 | .4566 | .2630 | .2785 | 1.593 |
| .03 | .0097 | .0198 | .00105 | 12.14 | .53 | .4666 | .2657 | .2866 | 1.558 |
| .04 | .0150 | .0264 | .00198 | 10.56 | .54 | .4766 | .2683 | .2946 | 1.524 |
| .05 | .0209 | .0329 | .00319 | 9.40 | .55 | .4865 | .2707 | .303 | 1.490 |
| .06 | .0275 | .0394 | .00473 | 8.58 | .56 | .4965 | .2733 | .311 | 1.458 |
| .07 | .0346 | .0459 | .00659 | 7.92 | .57 | .5064 | .2757 | .319 | 1.427 |
| .08 | .0421 | .0524 | .00876 | 7.37 | .58 | .5163 | .2781 | .327 | 1.397 |
| .09 | .0502 | .0590 | .01131 | 6.95 | .59 | .5261 | .2804 | .335 | 1.368 |
| .10 | .0585 | .0670 | .01434 | 6.66 | .60 | .5359 | .2824 | .343 | 1.339 |
| .11 | .0670 | .0748 | .01768 | 6.36 | .61 | .5457 | .2844 | .351 | 1.310 |
| .12 | .0753 | .0823 | .02117 | 6.04 | .62 | .5555 | .2864 | .359 | 1.283 |
| .13 | .0839 | .0895 | .02495 | 5.75 | .63 | .5651 | .2884 | .367 | 1.257 |
| .14 | .0925 | .0964 | .02890 | 5.47 | .64 | .5748 | .2902 | .374 | 1.231 |
| .15 | .1012 | .1031 | .0331 | 5.21 | .65 | .5843 | .2920 | .382 | 1.206 |
| .16 | .1100 | .1097 | .0375 | 4.96 | .66 | .5938 | .2937 | .390 | 1.181 |
| .17 | .1188 | .1161 | .0420 | 4.74 | .67 | .6033 | .2953 | .398 | 1.157 |
| .18 | .1277 | .1222 | .0467 | 4.52 | .68 | .6126 | .2967 | .405 | 1.133 |
| .19 | .1367 | .1282 | .0516 | 4.33 | .69 | .6219 | .2981 | .412 | 1.109 |
| .20 | .1457 | .1341 | .0567 | 4.15 | .70 | .6312 | .2994 | .420 | 1.087 |
| .21 | .1549 | .1398 | .0620 | 3.98 | .71 | .6403 | .3006 | .427 | 1.064 |
| .22 | .1640 | .1454 | .0674 | 3.82 | .72 | .6493 | .3018 | .434 | 1.042 |
| .23 | .1733 | .1508 | .0730 | 3.68 | .73 | .6582 | .3028 | .441 | 1.021 |
| .24 | .1825 | .1560 | .0786 | 3.53 | .74 | .6671 | .3036 | .448 | 1.000 |
| .25 | .1919 | .1611 | .0844 | 3.40 | .75 | .6758 | .3044 | .454 | 0.979 |
| .26 | .2013 | .1662 | .0904 | 3.28 | .76 | .6844 | .3050 | .461 | .958 |
| .27 | .2107 | .1710 | .0965 | 3.17 | .77 | .6929 | .3055 | .467 | .938 |
| .28 | .2202 | .1758 | .1027 | 3.06 | .78 | .7012 | .3060 | .473 | .918 |
| .29 | .2297 | .1804 | .1090 | 2.96 | .79 | .7094 | .3064 | .479 | .898 |
| .30 | .2393 | .1850 | .1155 | 2.86 | .80 | .7175 | .3067 | .485 | .879 |
| .31 | .2489 | .1895 | .1220 | 2.77 | .81 | .7254 | .3067 | .490 | .860 |
| .32 | .2586 | .1938 | .1287 | 2.69 | .82 | .7332 | .3066 | .495 | .841 |
| .33 | .2683 | .1981 | .1355 | 2.61 | .83 | .7408 | .3064 | .500 | .822 |
| .34 | .2780 | .2023 | .1424 | 2.53 | .84 | .7482 | .3061 | .505 | .804 |
| .35 | .2878 | .2063 | .1493 | 2.45 | .85 | .7554 | .3056 | .509 | .786 |
| .36 | .2975 | .2103 | .1563 | 2.38 | .86 | .7625 | .3050 | .513 | .768 |
| .37 | .3074 | .2142 | .1635 | 2.32 | .87 | .7693 | .3042 | .517 | .750 |
| .38 | .3172 | .2181 | .1708 | 2.25 | .88 | .7759 | .3032 | .520 | .732 |
| .39 | .3271 | .2217 | .1781 | 2.19 | .89 | .7823 | .3020 | .523 | .714 |
| .40 | .3370 | .2252 | .1854 | 2.13 | .90 | .7884 | .3005 | .526 | .696 |
| .41 | .3469 | .2287 | .1928 | 2.08 | .91 | .7943 | .2988 | .528 | .678 |
| .42 | .3568 | .2322 | .2003 | 2.02 | .92 | .7999 | .2969 | .529 | .661 |
| .43 | .3667 | .2356 | .2079 | 1.973 | .93 | .8052 | .2947 | .530 | .643 |
| .44 | .3767 | .2390 | .2156 | 1.925 | .94 | .8101 | .2922 | .530 | .625 |
| .45 | .3867 | .2422 | .2233 | 1.878 | .95 | .8146 | .2893 | .529 | .607 |
| .46 | .3966 | .2454 | .2310 | 1.832 | .96 | .8188 | .2858 | .528 | .589 |
| .47 | .4066 | .2484 | .2388 | 1.788 | .97 | .8224 | .2816 | .525 | .569 |
| .48 | .4166 | .2514 | .2466 | 1.746 | .98 | .8256 | .2766 | .521 | .550 |
| .49 | .4266 | .2544 | .2545 | 1.705 | .99 | .8280 | .2696 | .513 | .527 |
| .50 | .4366 | .2574 | .2625 | 1.667 | 1.00 | .8293 | .2538 | .494 | .494 |

A mass of water, as such, does not have pressure energy. Pressure energy is acquired by contact with other masses and is, therefore, transmitted to or through the mass under consideration. The pressure head $\frac{p}{w}$ (where $p$ is the pressure intensity in pounds per square foot and $w$ is unit weight in pounds per cubic foot), like velocity and elevation heads, also expresses energy. Thus, to be applicable to pressure flow in a conduit, the Bernoulli equation for flow in open channels, equation (3), can be rewritten:

$$H_A = \frac{p}{w} + Z + \frac{v^2}{2g} \qquad (41)$$

The Bernoulli theorem for flow in a reach of pressure conduit (as shown on fig. K-5) is:

$$\frac{p_1}{w} + Z_1 + h_{v_1} = \frac{p_2}{w} + Z_2 + h_{v_2} + \Delta h_L \qquad (42)$$

where $\Delta h_L$ represents the head losses within the reach from all causes. If $H_T$ is the total head and $v$ is the velocity at the outlet, Bernoulli's equation for the entire length is:

$$H_T = \Sigma(\Delta h_L) + h_v \qquad (43)$$

As in open channel flow, the Bernoulli theorem and the continuity equation are the basic formulas used in solving problems of pressure conduit flow.

(c) *Energy and Pressure Gradients.*—If piezometer standpipes were to be inserted at various points along the length of a conduit flowing under pressure, as illustrated on figure K-5, water would rise in each standpipe to a level equal to the pressure head in the conduit at those points. The pressure at any point may be equal to, greater than, or less than the local atmospheric pressure. The height to which the water would rise in a piezometer is termed the pressure gradient. The energy gradient is above the pressure gradient a distance equal to the velocity head. The fall of the energy gradient for a given length of conduit represents the loss of energy, either from friction or from friction and other influences. The relationship of the energy gradient to the pressure gradient reflects the variations between kinetic energy and pressure head.

(d) *Friction Losses.*—Many empirical formulas have been developed for evaluating the flow of fluids in conduits. Those in most common use are the Manning equation and the Darcy-Weisbach equation, previously given in this appendix and further discussed in chapter X.

The Manning equation assumes that the energy loss depends only on the velocity, the dimensions of the conduit, and the magnitude of wall roughness as defined by the friction coefficient $n$. The $n$ value is related to the physical roughness of the conduit wall and is independent of the size of the conduit or of the density and viscosity of the water.

The Darcy-Weisbach equation assumes the loss to be related to the velocity, the dimensions of the conduit, and the friction factor $f$. The factor $f$ is a dimensionless variable based on the viscosity and density of the fluid and on the roughness of the conduit walls as it relates to the size of the conduit.

Data and criteria for determining $f$ values for large pipe are given in a Bureau of Reclamation engineering monograph [6].

K-4. ***Hydraulic Jump.***—The hydraulic jump is an abrupt rise in water surface which may occur in an open channel when water flowing at high velocity is retarded. The formula for the hydraulic jump is obtained by equating the unbalanced forces acting to retard the mass of flow to the rate of change of the momentum of flow. The general formula for this relationship is:

$$v_1{}^2 = g \frac{a_2\overline{y}_2 - a_1\overline{y}_1}{a_1\left(1 - \frac{a_1}{a_2}\right)} \qquad (44)$$

where:

$v_1$ = the velocity before the jump,

$a_1$ and $a_2$ = the areas before and after the jump, respectively, and

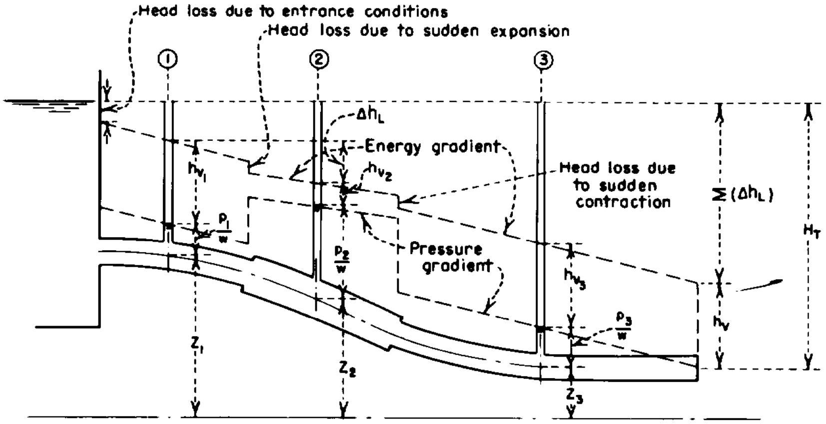


*Figure K-5.* **Characteristics of pressure flow in conduits.–288-D-2555**

$\bar{y}_1$ and $\bar{y}_2$ = the corresponding depths from the water surface to the center of gravity of the cross section.

The general formula expressed in terms of discharge is:

$$Q^2 = g\frac{a_2\bar{y}_2 - a_1\bar{y}_1}{\frac{1}{a_1} - \frac{1}{a_2}} \qquad (45)$$

or:

$$\frac{Q^2}{ga_1} + a_1\bar{y}_1 = \frac{Q^2}{ga_2} + a_2\bar{y}_2 \qquad (46)$$

For a rectangular channel, equation (44) can be reduced to $v_1{}^2 = \frac{gd_2}{2d_1}(d_2 + d_1)$, where $d_1$ and $d_2$ are the flow depths before and after the jump, respectively. Solving for $d_2$:

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{2v_1{}^2 d_1}{g} + \frac{d_1{}^2}{4}} \qquad (47)$$

Similarly, expressing $d_1$ in terms of $d_2$ and $v_2$:

$$d_1 = -\frac{d_2}{2} + \sqrt{\frac{2v_2{}^2 d_2}{g} + \frac{d_2{}^2}{4}} \qquad (48)$$

A graphic solution of equation (47) is shown on figure K-8.

If the Froude number $F_1 = \frac{v_1}{\sqrt{gd_1}}$ is substituted in the equation (47):

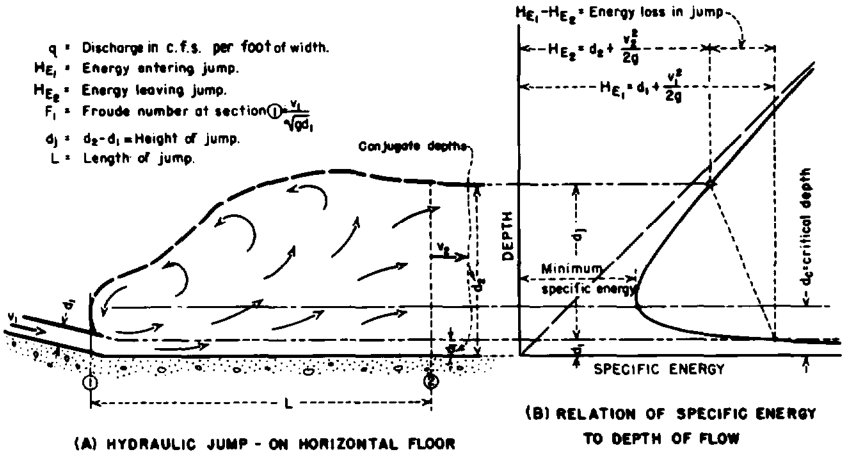
$$\frac{d_2}{d_1} = \frac{1}{2}(\sqrt{8F_1{}^2 + 1} - 1) \qquad (49)$$

Figure K-6 shows a graphical representation of the characteristics of the hydraulic jump. Figure K-7 shows the hydraulic properties of the jump in relation to the Froude number, as determined from experimental data [7]. And figure K-8 is a nomograph showing the relation between variables in the hydraulic jump.

Data are for jumps on a flat floor with no chute blocks, baffle piers, or end sills. Ordinarily, the jump length can be shortened by incorporation of such devices in the designs of a specific stilling basin.

### K-5. *Bibliography.*

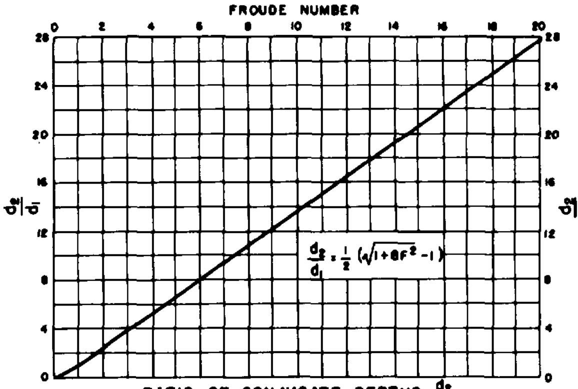
[1] King, H. W., revised by E. F. Brater, "Handbook of Hydraulics," fourth edition, McGraw-Hill Book Co., Inc., New York, N.Y., 1954.
[2] Woodward, S. B., and Posey, C. J., "Steady Flow in Open Channels," John Wiley & Sons, Inc., fourth printing, September 1949.
[3] Bakhmeteff, B. A., "Hydraulics of Open Channels," McGraw-Hill Book Co., Inc., New York, N.Y., 1932.
[4] Binder, R. C., "Fluid Mechanics," Prentice-Hall, Inc., Englewood Cliffs, N.J., third edition, 1955.
[5] Rouse, Hunter, "Engineering Hydraulics," John Wiley & Sons, Inc., New York, N.Y., 1950.
[6] Bradley, J. N., and Thompson, L. R., "Friction Factors for Large Conduits Flowing Full," Engineering Monograph No. 7, U.S. Department of the Interior, Bureau of Reclamation, March 1951.
[7] Bradley, J. N., and Peterka, A. J., "The Hydraulic Design of Stilling Basins," ASCE Proceedings, vol. 83, October 1957, Journal of Hydraulics Division, No. HY5, Papers No. 1401 to 1404, inclusive.



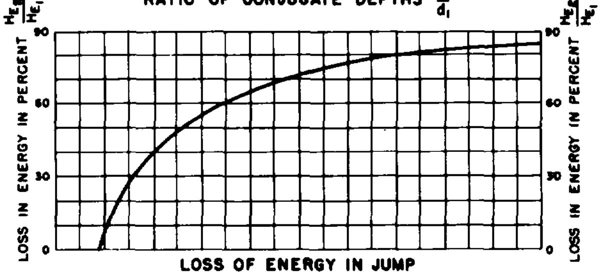
(A) HYDRAULIC JUMP - ON HORIZONTAL FLOOR

(B) RELATION OF SPECIFIC ENERGY TO DEPTH OF FLOW

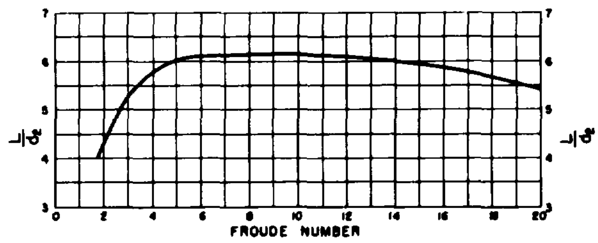
*Figure K-6.* **Hydraulic jump symbols and characteristics.—288-D-3190**



RATIO OF CONJUGATE DEPTHS $\frac{d_2}{d_1}$



LOSS OF ENERGY IN JUMP



LENGTH OF JUMP

*Figure K-7.* **Hydraulic jump properties in relation to Froude number.—288-D-2558**

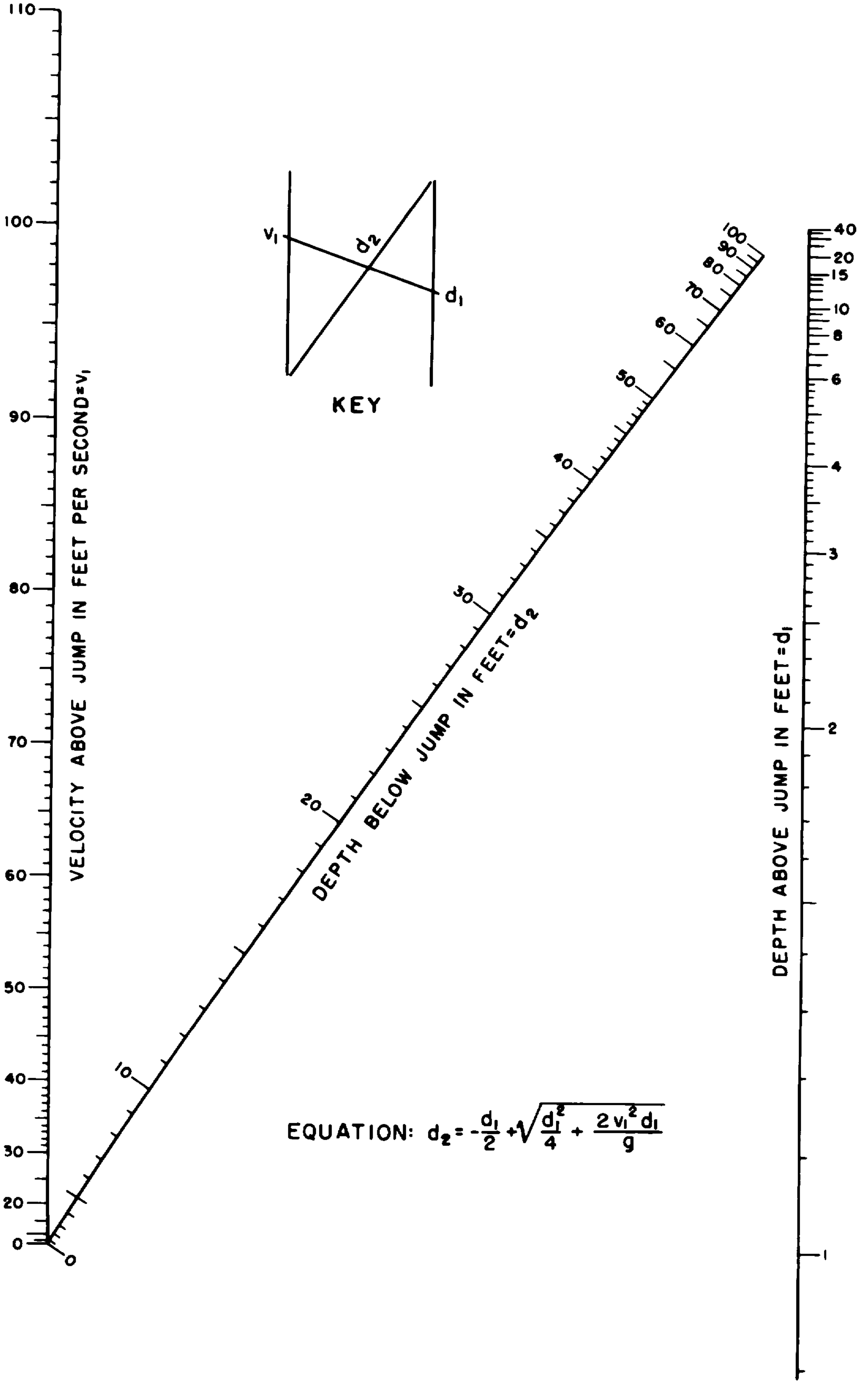


*Figure K-8.* **Relation between variables in the hydraulic jump.–288-D-2559**

# «Appendix L

# Inflow Design Flood Studies

L-1. ***Introduction.***—A 1970 report of the United States Committee on Large Dams (USCOLD) [1][1] gives a definition of an inflow design flood (IDF) as:

> "The reservoir inflow-discharge hydrograph used in estimating the maximum spillway discharge capacity and maximum surcharge elevation finally adopted as a basis for project design . . . ."

An inflow design flood selected for design of a dam impounding considerable storage located where partial or total failure would cause sudden release of water and create major hazards to life or property downstream should be equal to a probable maximum flood (PMF). The USCOLD report defines a probable maximum flood as:

> "Estimates of hypothetical flood characteristics (peak discharge, volume and hydrograph shape) that are considered to be the most severe *reasonably possible* at a particular location, based on relatively comprehensive hydrometeorological analyses of critical runoff producing precipitation (and snowmelt, if pertinent) and hydrologic factors favorable for maximum flood runoff."

This appendix discusses flood hydrology studies relating to estimates of an inflow design flood equal to a probable maximum flood, as defined in the USCOLD report. The phrase "relatively comprehensive hydrometeorological analyses" in the preceding definition refers to studies by hydrometeorologists directed towards estimation of *the physical upper limits of storm rainfall and maximum snow accumulation and melt rates.* The resulting estimates of the physical upper limits to storm rainfall in a basin or region are usually called the "probable maximum storm" or "probable maximum precipitation"[2]. Both of these terms are used in this text but with more precise meanings attached to each term as discussed in sections L-14 through L-17 on design storm studies.

Bureau of Reclamation policy in design of dams located where failure might create major hazards requires an inflow design flood estimated by evaluating the runoff from the most critical of the following situations:

(1) A probable maximum storm in conjunction with severe, but not uncommon, antecedent conditions.

(2) A probable maximum storm for the season of heavy snowmelt, in conjunction with a major snowmelt flood somewhat smaller than the probable maximum.

(3) A probable maximum snowmelt flood in conjunction with a major rainstorm less severe than the probable maximum storm for that season.

(a) *Items to be Evaluated.*—Depending on meteorological conditions for the basin above a damsite, on the size of the drainage area and, to a lesser extent, on the proposed size of reservoir and type of dam, it may be necessary to evaluate:

(1) Each of the above assumptions.

(2) Each of the two assumptions in which snowmelt is a factor.

(3) Where snowmelt is not a factor,

---

[1] Numbers in brackets refer to items in the bibliography, sec. L-32.

two probable maximum storms—a storm causing the maximum peak inflow, and a storm causing the maximum volume of inflow.

It is beyond the scope of this text to present a complete manual of all procedures used for estimating inflow design floods, because selection of procedures is dependent on available hydrological data and individual watershed characteristics.

(b) *Discussions in This Text.*—Discussions in this text will provide design engineers information about the problems encountered and some methods for their solution. Broad discussions accompany presentation of the information which concerns:

(1) Hydrologic data for estimating floodflows and data sources in the United States.

(2) Analyses of basic data.

(3) Unit hydrograph procedures for synthesizing the distribution of runoff of a basin above a damsite.

(4) Sources of generalized probable maximum precipitation values.

(5) An example of computation of a preliminary inflow design flood hydrograph and establishment of reservoir routing criteria for the flood.

Designers also need estimates of floodflows that may occur at the damsite during the construction period in order to estimate requirements for streamflow diversion. Such estimates are usually included in an inflow design flood study. Sections L-28 and L-29 discuss selected methods of estimating flood magnitudes and frequency of occurrence at the damsite.

Every damsite presents one or more unique problems to probable maximum flood estimates. An inflow design flood (IDF) used for final designs of a dam should be based on estimates by an experienced hydrometeorologist of probable maximum precipitation values *for the basin above the damsite*, not on generalized probable maximum precipitation values for a region. The methods of preparing a study which yields generalized estimates of probable maximum precipitation inherently result in values that are somewhat greater than values obtained from an individual basin study.

Sections L-14 through L-17 present a general discussion of methods and assumptions that a hydrometeorologist may use in the preparation of hydrometeorological studies for individual basins. The physical characteristics of a basin may vary as to: drainage area size, relatively small to extremely large; runoff characteristics, similar throughout the basin or including tributary areas with markedly dissimilar runoff producing conditions; contribution from snowmelt; etc. Sections L-23 through L-26 describe some methods of estimating the contribution of snowmelt runoff to inflow design floods.

The final IDF study converting probable maximum precipitation values to an IDF hydrograph should be prepared by experienced flood hydrologists. Remarks regarding considerations for development of a final IDF study are included throughout the text and a brief summary of these considerations is given in sections L-30 and L-31.

Computational procedures given in this text are oriented toward step by step "long-hand" solutions, recognizing that the ever-increasing advances in computer technology provide greatly expanded capability in all phases of flood hydrology studies. One should be mindful, though, as stated in World Meterological Organization (WMO) Technical Note No. 98 [2] that: "While the computer is a powerful tool, it must be recognized that it is simply that, and results are no better than the basic logic and methods of application."

The bibliography, section L-32, includes selected references to hydrometeorological studies in addition to those specifically referred to in the text.

# A. COLLECTION OF HYDROLOGIC DATA FOR USE IN ESTIMATING FLOODFLOWS

L-2. ***General.***—For all flood studies, compilation and judgment as to quality of all available streamflow, precipitation, and watershed data are most important. Mathematical procedures cannot improve the quality of input data, and analyses procedures must be compatible with the data available.

L-3. *Streamflow Data.*—The hydrologic data most directly useful in determining floodflows are actual streamflow records of considerable length at the location of the dam. Such records are rarely available. The engineer should obtain the streamflow records available for the general region in which the dam is to be situated. Locations of stream gaging stations and precipitation stations in the United States are shown on a series of maps entitled "River Basin Maps Showing Hydrologic Stations," edition 1961,[2] prepared under the supervision of the National Weather Service. Such data collecting stations are subject to change in location, discontinuation, or initiation of new stations. These maps cannot be kept current, and information thereon must be supplemented by additional investigations in order to be sure of the location and operation of stations in a given area. The engineer should consult the water supply papers, catalogs, maps, and indexes of the U.S. Geological Survey[2] and, if possible, confer with the Survey's district engineer. He should also make a search of the records of other Federal agencies which may have collected information in the region, and the records of State water conservation agencies or State geological surveys; and he should determine whether any information may be available from other State departments, from county engineer offices, from municipalities in the vicinity, or from utility companies. Where streamflow records are not available, some agencies or inhabitants of the vicinity may have information about high-water marks caused by specific historic floods.

With respect to the character of the streamflow data available, floodflows at the damsite may be determined under one of the following conditions:

(1) *Streamflow record at or near the damsite.*—If such a record is available and covers a period of 20 years or more, the floodflows shown by the record may be analyzed to provide flood frequency values. Hydrographs of outstanding flood events can be analyzed to provide runoff factors for use in determining the maximum probable flood.

If such a record is available but covers only a few years, it may not include any flood of great magnitude within its limits and, if used alone, it would give false indication of flood potential. Analysis may, however, give some or all of the runoff factors needed to compute the probable maximum flood. Frequency values obtained from a short record should not be used without analysis of data from nearby watersheds of comparable runoff characteristics.

(2) *Streamflow record available on the stream itself, but at a considerable distance from the damsite.*—Such a record may be analyzed to provide unitgraph characteristics and frequency data which may be transferred to the damsite by appropriate area and basin-characteristic coefficients. This transfer can be made directly from one drainage area to another if the areas have comparable characteristics. Often damsites are located within the transition zone from mountains to plains and the stream gaging stations are located well out on the plains; in such instances, special care must be exercised when using the plains record for determination of floodflows at the damsite.

---

[2] Published by the Government Printing Office and available in libraries designated as depositories of Government publications; most important libraries in the United States are so designated.

(3) *No adequate streamflow data available on the specific stream, but a satisfactory record for a drainage basin of similar characteristics in the same region.*—Such a record may be analyzed for unitgraph characteristics and frequency data, and these data transferred to the damsite by appropriate area and basin-characteristic coefficients.

(4) *Streamflow records in the region, but not satisfactorily useful for application and analysis under one of the above methods.*—These records may be assembled and analyzed as reference information on general runoff characteristics.

(5) *Use of high-water marks.*—High-water marks pointed out by inhabitants of the valley should be used with caution in estimating flood magnitudes. However, where there are a number of high-water marks in the vicinity of the project, and particularly if such marks are obtained from the records of public offices (such as State highway departments or county engineers), they may be used as the basis of a separate supplemental study. These records may be used to determine the water cross-sectional area and the water surface slope for the flood to which they refer, and from these data an estimate of that particular flood peak may be prepared using the slope-area method described in appendix B of the Bureau of Reclamation publication "Design of Small Dams" [31].

Whenever it appears that there will be one or more flood seasons between the selection of the damsite and construction of the dam, facilities for securing a streamflow record for the project should be set up as promptly as possible. This is of particular importance in order to obtain watershed data directly applicable to the computation of the inflow design flood for the dam, although a record usable for frequency computations cannot be secured. The facilities for obtaining such a record should be the best possible depending on the circumstances. A detailed discussion of these facilities, which may consist of either nonrecording or recording gages, is included in the following publications: "Equipment for Current-Meter Gaging Stations," U.S. Geological Survey Water Supply Paper 371; "Stream-Gaging Procedure," U.S. Geological Survey Water Supply Paper 888; and "Stream Flow," by Grover and Harrington, John Wiley & Sons, Inc., New York, 1943. The advice of Geological Survey engineers will be helpful in the site selection and installation, operation, and interpretation of records obtained.

A series of manuals "Techniques of Water-Resources Investigations of the United States Geological Survey," describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters; section A of book 3 is on surface water. The unit of publication, the chapter, is limited to a narrow field of subject matter. This format permits flexibility in revision and publication as the need arises.

Provisional drafts of chapters are distributed to field offices of the U.S. Geological Survey for their use. These drafts are subject to revision because of experience in use or because of advancement in knowledge, techniques, or equipment. After the technique described in a chapter is sufficiently developed, the chapter is published and is for sale by the Superintendent of Documents.[2]

The importance of utilizing records of runoff originating from the watershed above the damsite cannot be overemphasized. In the case of a damsite located on an ungaged stream, the establishment of measuring facilities as discussed above may produce basic data which would justify "eleventh hour" revision of the plans, thus improving the design of the dam.

L-4. ***Precipitation Data.***—In each of the situations outlined in the preceding section, precipitation data are needed to evaluate factors for use in computing the probable maximum flood. The engineer should assemble

---

[2] In loc. cit. p. 765.

the information with respect to precipitation during the greater storms in the region, and particularly for those storms for which runoff records are available. Such information can be obtained from publications of the National Weather Service[3] and Environmental Data Service. At present (1974), daily precipitation data for each month for each State are contained in the publication "Climatological Data." Hourly data for each month for each State obtained by recording precipitation gages are contained in the publication "Hourly Precipitation Data."[4] In areas where large storms have occurred, often precipitation data obtained by the National Weather Service precipitation stations have been supplemented by "bucket survey" data, i.e., information on rainfall amounts of unusual storms obtained from residents within the storm area by personnel of the National Weather Service and other Government agencies.

Locations of precipitation stations as of 1961 are shown on the series of maps "River Basin Maps showing Hydrologic Stations," previously referred to.

If plans are made to install streamflow measuring facilities as discussed in the preceding section, provision should also be made for obtaining precipitation records. An important item to consider is the selection of the location (or locations) of the precipitation gage, so that the catch will be a representative sample of average precipitation over the watershed. A comprehensive discussion of types of precipitation gages and observational procedures is contained in the National Weather Service publication "Instructions for Climatological Observers," Circular B, eleventh edition, January 1962.

L-5. ***Watershed Data.***—All available information concerning watershed characteristics should be assembled. A map of the area above the damsite should be prepared showing the drainage system, contours if available, drainage boundaries, and locations of any precipitation stations and streamflow gaging stations. Available data on soil types, cover, and land usage provide valuable guides to judgment of runoff potential. Soil maps prepared by the U.S. Department of Agriculture will prove helpful when the watershed lies within areas so mapped. These surveys (if in print) are available for purchase from the Superintendent of Documents, Washington, D.C. Out-of-print maps and other unpublished surveys may be available for examination from the U.S. Department of Agriculture, county extension agents, colleges, universities, and libraries.

The hydrologist preparing the flood study should make an inspection trip over the watershed to verify drainage area boundaries and soil and cover information, and to determine if any noncontributing areas are included within the drainage boundaries. The trip should also include visits to nearby watersheds if it is anticipated that records from nearby watersheds will be used in the study.

## B. ANALYSES OF BASIC HYDROLOGIC DATA

L-6. *General.*—A flood hydrologist first directs attention to individual large flood events, seeking procedures whereby a good estimate may be made of the hydrograph that will result from a given amount of precipitation. As floods which consist of combined snowmelt and rainfall runoff are difficult to separate into their two components, usually snowmelt floods and rain floods are analyzed separately. Analyses of rain floods only are discussed in these sections L-6 through L-8 with inclusion of examples of some mathematical computations. Considerations for runoff contribution from snowmelt are discussed separately in sections L-22 through L-26. Flood analyses of rainfall

---

[3] Official designation: U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service.

[4] Subscription to these publications may be made through the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

data are interrelated to analyses of respective runoff data, so that discussions of procedures for one must include some references to the other. In the discussion that follows, analysis of storm rainfall is described first and is followed by a description of the analysis of the resulting flood runoff. Procedures used to analyze streamflow data for estimating the frequency of occurrence of flood magnitudes are discussed in sections L-28 and L-29.

L-7. ***Estimating Runoff From Rainfall.***—(a) *General.*—The hydrometeorological approach to analyzing flood events and using the information obtained to estimate the magnitude of hypothetical floods requires a firm estimate of the difference between precipitation and the resulting runoff. From a flood determination point of view, this difference is considered loss, that is, loss from precipitation in the form of water over a given watershed. A simple solution to derive this loss value appears to be in finding the rate at which water will infiltrate the soil. If this infiltration rate is known, along with the amount of precipitation, a simple subtraction should give the amount of runoff. However, there are other precipitation losses in addition to infiltration, such as interception by vegetative cover, surface storage, and evaporation, that may have material effect on runoff amounts.

Various types of apparatus have been devised to test the infiltration rates of soils, and studies have been made of interception and evaporation losses. Although maps to an extremely large scale could define most of the surface storage area, it is apparent that an accurate volumetric evaluation of all the loss factors can be made only for a highly instrumented, small plot of ground and that such an evaluation is not practical for a natural watershed composed of many square miles of varying type soils, vegetative cover, and terrain features. For this reason, hydrologic literature contains arguments against the "infiltration rate approach" to determination of runoff amounts. However, the infiltration rate approach is applied on an empirical basis to obtain a practical solution to the problem of determining amounts of runoff, recognizing that the values used are of the nature of *index* values rather than *true* values.

Natural events are studied and the difference between rainfall and runoff determined. Since this difference includes all the losses described above, it is usually called a *retention loss* or a *retention rate*. Such retention rates derived from available records may be adjusted to ungaged watersheds by analogy of soil type and cover.

The characteristics of a hydrograph must be understood so that respective amounts of runoff and precipitation are compared for estimating retention rates (and for other comparisons described later). A hydrograph of storm runoff obtained at a streamflow gaging station represents one or more of the following types of runoff from the watershed: channel runoff, surface runoff, interflow, and base flow. Brief definitions of these types are:

> *Channel runoff.*—Caused by rain falling on the water surface of the stream. It begins with the start of precipitation and may be discernible from a slight rise of the hydrograph just after rainfall begins, but the quantity of channel runoff is so small that it is ignored in hydrograph analyses.
>
> *Surface runoff.*—Occurs only when the rainfall rate is greater than the retention loss rate. This type of runoff causes most floods and the computational procedures in this text consider this type of runoff dominant.
>
> *Interflow.*—Occurs when rainfall infiltrating the soil surface encounters an underground zone of lower permeability, travels above the zone to the surface downhill, and reappears to become surface runoff. This type of flow may also be called subsurface flow or *quick return flow.*
>
> *Base flow.*—The fairly steady flow of a stream from natural storage as shown by hydrographs during nonstorm (or nonactive snowmelt) periods.

In flood hydrology it is customary to deal separately with base flow and to combine all other types of flow into *direct runoff* in unknown proportions as assumed in this text.

Making studies to compare rainfall with

runoff requires a knowledge of the units of measurement used and the factors for conversion to common units. These conversion factors are given in appendix K. In the United States, precipitation is measured in inches and runoff is measured in cubic feet per second (abbreviated c.f.s.).

It is necessary to know the watershed area contributing the runoff at a given measuring point, in order to express the runoff volume of inches of depth over the watershed for comparison with precipitation amounts. When making such comparisons, the amount of runoff, expressed as inches, is termed *rainfall excess*, and the difference between the rainfall excess and the total precipitation is considered retention loss as just discussed.

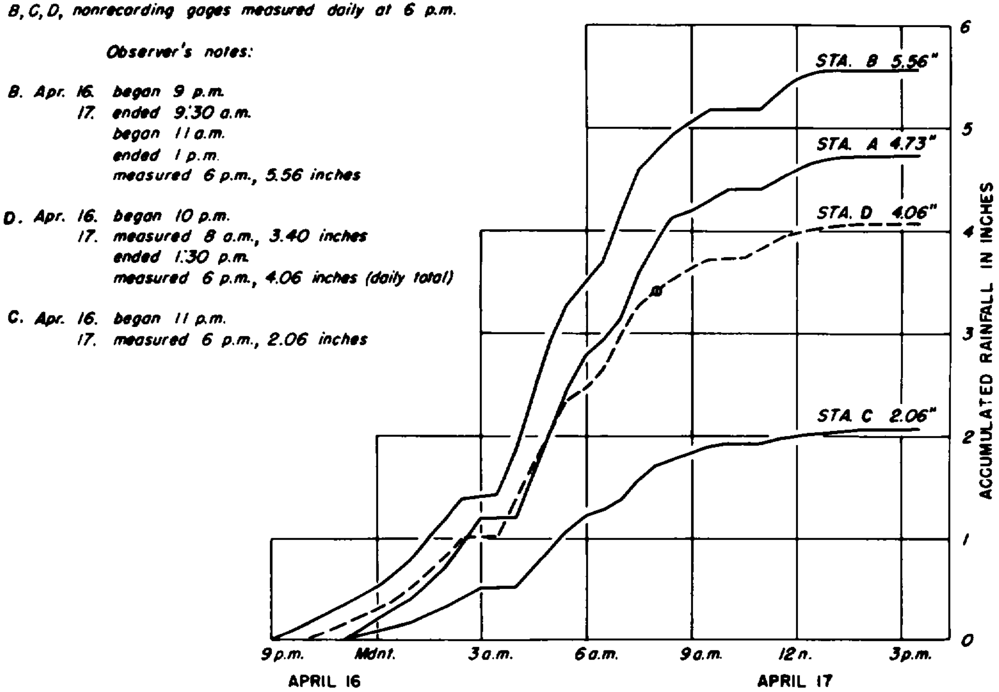
The following method of making a rainfall-runoff analysis has been selected for description in this text. The objectives of such analyses are: (1) the determination of a retention rate, and (2) the determination of the duration time interval of rainfall excess. A comparison of retention rates derived from several analyses leads to adoption of a rate for design flood computations. The determination of the duration of excess rainfall is necessary for the hydrograph analyses computations involving determinations of unitgraphs and lag-times, which are discussed later in this section and in sections beginning with L-9. In all such analyses, the runoff volume which is compared with precipitation amounts is that which relates directly to the rainfall under study. Therefore, the base flow of the streamflow hydrograph must be subtracted out before comparisons are made (see sec. L-8(c)).

(b) *Analysis of Observed Rainfall Data.*–

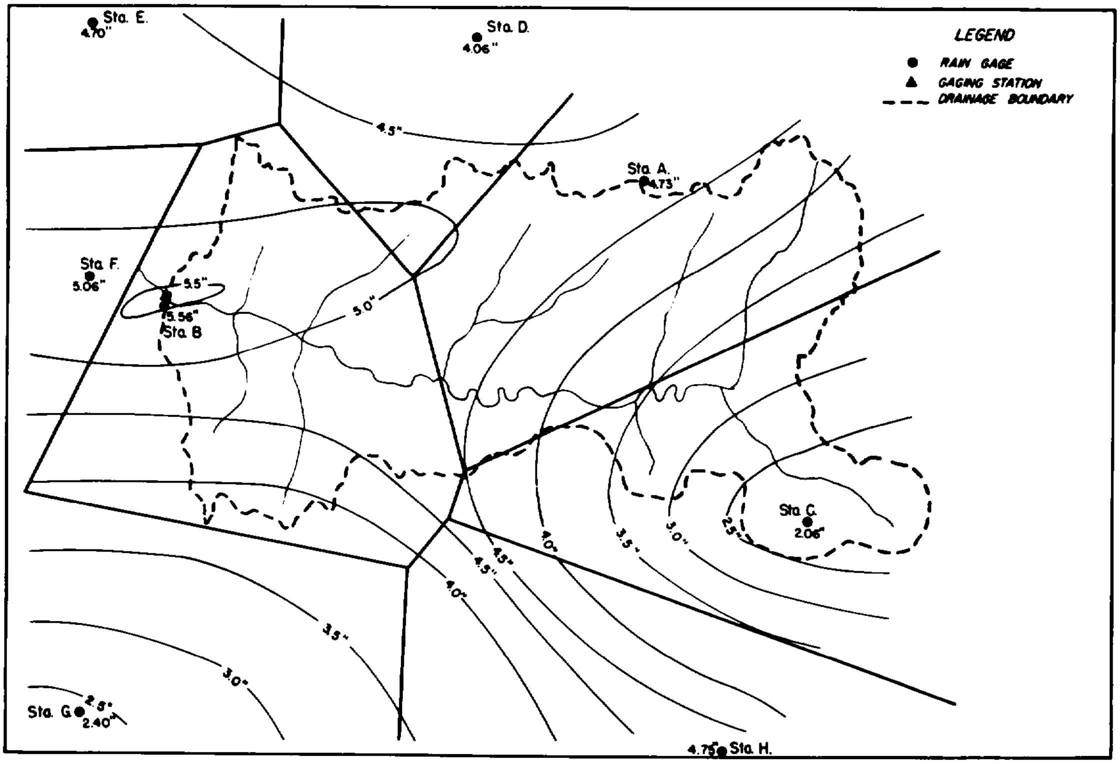
(1) *Mass curves of rainfall.*–Mass curves of cumulative rainfall during the storm period should be plotted for all precipitation stations in and near the basin as shown on figure L-1(A). To show clearly the relation of rainfall to runoff, it is sometimes desirable to plot the mass curves to the same time scale as the discharge hydrograph of storm runoff. Usually, however, the curves should be given a more expanded time scale than it is desirable to use for the hydrograph analysis. When only one recording station is located nearby, and in the absence of better information, the mass curve of precipitation at a nonrecording station is usually considered to be proportional in shape to that of the recording station, except as otherwise defined by the observer's readings and notes (fig. L-1(A)). The speed and direction of travel of the rainburst should be taken into account. Many rainfall observers enter the times of beginning and ending on the same line as the current daily reading. The notes may therefore refer to the previous day, especially when the gage is regularly read in the morning.

(2) *Isohyetal maps.*–The total amounts of rainfall occurring during the portion of the storm that produced the flood hydrograph under study should be determined from the mass curves for each station in and near the drainage area. For a flood hydrograph consisting of a single event, this will be the total depth of precipitation occurring during the storm period. For a compound hydrograph, in which individual portions of the hydrograph are studied separately, temporary cessations of rainfall will usually be indicated in the mass curves, and from inspection it usually will be apparent which of the increments of rainfall caused the runoff event under study. The appropriate depths of rainfall are then used to draw an isohyetal map, using standard procedures. A typical isohyetal map for plains-type terrain is shown on figure L-1(B). Isohyets are generally drawn smoothly, interpolating between precipitation stations. The interpolation should not be excessively mechanical.

Extreme caution should be used in drawing the isohyetal pattern in mountainous areas where the orographic effect is an important factor in the areal distribution of rainfall. For example, if there is a precipitation station in a valley on one side of a mountain range and another station in a valley on the opposite side of the range with no intervening station, it cannot be assumed that the rainfall during a storm would vary linearly between the two stations. It is likely that the rainfall would increase with increases in elevation on the windward side of the divide, whereas on the leeward side, precipitation would decrease



(A) MASS CURVES OF RAINFALL



(B) ISOHYETS AND THIESSEN POLYGONS

*Figure L-1.* **Analysis of observed rainfall data.–288-D-3158** rapidly with distance from the divide. This type of distribution can usually be verified in mountainous areas where there are sufficient precipitation stations to define the isohyetal pattern accurately.

A storm isohyetal pattern for mountainous terrain may be constructed by the isopercental technique, discussed in WMO Technical Note No. 98 [2] as follows:

> "In mountainous regions the simple interpolation technique would yield unsatisfactory isohyets. Yet to prepare a valid isohyetal pattern in a mountainous region is not easy. One commonly used procedure is the isopercental technique, excellent under certain limited conditions stated in the next paragraph. This method requires a base chart of either mean annual precipitation, or preferably mean precipitation for the season of the storm, such as winter, summer, or monsoon months. In this method the ratio of the storm precipitation to the mean annual or mean seasonal precipitation (base precipitation) is plotted at each station. Isolines are drawn smoothly to these numbers. The ratios on the lines are then multiplied by the original base chart values at a large number of points to yield the storm isohyetal chart. Thus the storm isohyetal gradients and locations of centers tend to resemble the features of the base chart, which in turn is influenced by terrain.
>
> "The first requirement for success of the isopercental technique is that a reasonably accurate mean annual or mean seasonal precipitation chart be available as a base. The base chart is of more value if it contains precipitation stations in addition to those reporting in the storm than if both charts are drawn exclusively from data observed at the same stations. The value of the base chart is also enhanced, in regions where the runoff of streams is a large percentage of the precipitation, if the precipitation shown on the chart has been adjusted not only for topographic factors, but also adjusted to agree with seasonal streamflow. In regions where a large percentage of the precipitation evaporates, adjustment to runoff volumes would be of dubious value.
>
> "An additional requirement for success of the isopercental technique is that most of the annual or seasonal precipitation in the region result from storms with relatively the same wind direction, and from storms with minimal convective activity. Under these circumstances an individual storm will have a strong resemblance to the mean chart, as the latter is an average of kindred storms.
>
> "In the Tropics with the dominance of convective activity and with lighter winds, the isopercental technique is of less value in analysis of an individual storm than in middle latitude locations that meet the other requirements."

After the preliminary hydrographs and the isohyetal maps have been drawn, the atypical flood events for unit hydrographs determination may readily be eliminated. *Those floods having a combination of large volume, uniform intensities, isolated periods of rainfall, and uniform areal distribution of rainfall, should be chosen for further study.*

(3) *Average rainfall by Thiessen polygons.*—The average rainfall on a drainage area can be determined from precipitation station records by the Thiessen polygon method. A sample computation of average hourly rainfall from the mass curves on figure L-1(A), using Thiessen polygons indicated on figure L-1(B), is given in table L-1.

The first step is to construct the Thiessen polygons, which are the areas bounded by the perpendicular bisectors of lines joining adjacent precipitation stations. The percentage of the drainage area controlled by each station's polygon is planimetered and entered in table L-1. Next, the average depth of rainfall over each station's polygon is determined by planimetering areas between isohyets on figure L-1(B). A factor to be used in weighing station rainfall values is obtained by multiplying the percentage of the drainage area controlled by each station's polygon by the ratio of the average depth of rainfall over each station's polygon to the observed rainfall at the station, and dividing by 100.

Hourly incremental rainfall values are determined for each precipitation station from

Table L-1.—*Computation of rainfall increments*.

COMPUTATION OF STATION WEIGHTS

| Station<br>(1) | Average rainfall over Thiessen polygon<br>(2) | Percent of basin area<br>(3) | Rainfall at station<br>(4) | Weight, col. (2) x col. (3) / 100 x col. (4)<br>(5) |
|---|---|---|---|---|
| A | 4.3 | 38.9 | 4.73 | 0.35 |
| B | 4.6 | 37.0 | 5.56 | .31 |
| C | 2.8 | 21.1 | 2.06 | .29 |
| D | 5.0 | 3.0 | 4.06 | .04 |

COMPUTATION OF WEIGHTED AVERAGE HOURLY RAINFALL OVER BASIN

| Time, hours | Station A | | | Station B | | | Station C | | | Station D | | | Weighted average, sum of cols. (3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mass rf.<br>(1) | Δ rf.<br>(2) | 0.35xΔrf.<br>(3) | Mass rf.<br>(1) | Δ rf.<br>(2) | 0.31xΔrf.<br>(3) | Mass rf.<br>(1) | Δ rf.<br>(2) | 0.29xΔrf.<br>(3) | Mass rf.<br>(1) | Δ rf.<br>(2) | 0.04xΔrf.<br>(3) | |
| 0 | | | | 0 | | | | | | | | | |
| 1 | | | | .17 | 0.17 | 0.053 | | | | 0 | | | 0.053 |
| 2 | 0 | | | .33 | .16 | .050 | 0 | | | .15 | 0.15 | 0.006 | .056 |
| 3 | .20 | 0.20 | 0.070 | .52 | .19 | .059 | .09 | 0.09 | 0.026 | .29 | .14 | .006 | .161 |
| 4 | .40 | .20 | .070 | .80 | .28 | .087 | .17 | .08 | .023 | .52 | .23 | .009 | .189 |
| 5 | .73 | .33 | .116 | 1.20 | .40 | .124 | .32 | .15 | .044 | .84 | .32 | .013 | .297 |
| 6 | 1.20 | .47 | .164 | 1.41 | .21 | .065 | .52 | .20 | .058 | 1.01 | .17 | .007 | .294 |
| 7 | 1.20 | 0 | 0 | 1.85 | .44 | .136 | .52 | 0 | 0 | 1.34 | .33 | .013 | .149 |
| 8 | 2.05 | .85 | .298 | 2.91 | 1.06 | .329 | .89 | .37 | .107 | 2.05 | .71 | .028 | .762 |
| 9 | 2.80 | .75 | .262 | 3.49 | .58 | .180 | 1.22 | .33 | .096 | 2.47 | .42 | .017 | .555 |
| 10 | 3.15 | .35 | .122 | 4.19 | .70 | .217 | 1.37 | .15 | .044 | 3.00 | .53 | .021 | .404 |
| 11 | 3.90 | .75 | .262 | 4.79 | .60 | .186 | 1.70 | .33 | .096 | 3.40 | .40 | .016 | .560 |
| 12 | 4.20 | .30 | .105 | 5.08 | .29 | .090 | 1.83 | .13 | .038 | 3.63 | .23 | .009 | .242 |
| 13 | 4.40 | .20 | .070 | 5.18 | .10 | .031 | 1.92 | .09 | .026 | 3.73 | .10 | .004 | .131 |
| 14 | 4.40 | 0 | 0 | 5.18 | 0 | 0 | 1.92 | 0 | 0 | 3.83 | .10 | .004 | .004 |
| 15 | 4.59 | .19 | .066 | 5.49 | .31 | .096 | 2.00 | .08 | .023 | 3.97 | .14 | .006 | .191 |
| 16 | 4.70 | .11 | .038 | 5.56 | .07 | .022 | 2.04 | .04 | .012 | 4.04 | .07 | .003 | .075 |
| 17 | 4.73 | .03 | .010 | 5.56 | 0 | 0 | 2.06 | .02 | .006 | 4.06 | .02 | .001 | .017 |
| Total | | 4.73 | 1.653 | | 5.56 | 1.725 | | 2.06 | .599 | | 4.06 | .163 | 4.140 |

the mass curves of figure L-1(A) and are multiplied by the appropriate weight factors as shown in table L-1, to obtain the total for the drainage area.

Additional information on determining average rainfall is given in "Cooperative Studies Technical Paper No. 1," published by the National Weather Service, and in references [2] and [17].

(4) *Determination of rainfall excess.*—Two methods may be used to determine rainfall excess: by assuming a constant average retention rate throughout the storm period, and by assuming a retention rate varying with time. The capacity rate of retention decreases progressively throughout the storm period until a constant minimum rate is reached if the rain is sufficiently prolonged. With dry antecedent conditions, the initial capacity rate will be greater and will decline faster. Because the use of a varying retention rate requires a complicated method of computation, it is often preferable to assume an average retention rate (sometimes referred to as infiltration index) with an estimate of initial loss being made if antecedent conditions are relatively dry.

The method of determining the period of rainfall excess, when an average retention rate is used, is a trial-and-error process in which a retention rate is assumed and subtracted from hourly rainfall increments determined as the average over the basin. Various retention rates are assumed until the total of the computed rainfall excess equals the measured storm runoff. An example of this procedure is given in table L-2. If the correct retention rate has not been assumed after two trials, a rainfall

Table L-2.—*Computation of rainfall excess.*

| Time, hours | Rainfall increment (basin average), inches | First trial | | Second trial | | Third trial | |
|---|---|---|---|---|---|---|---|
| | | Assumed retention rate, inches per hour | Rainfall excess, inches | Assumed retention rate, inches per hour | Rainfall excess, inches | Assumed retention rate, inches per hour | Rainfall excess, inches |
| 0 | | | | | | | |
| 1 | 0.05 | 0.25 | | 0.15 | | 0.17 | |
| 2 | .06 | | | | | | |
| 3 | .16 | | | | 0.01 | | |
| 4 | .19 | | | | .04 | | 0.02 |
| 5 | .30 | | 0.05 | | .15 | | .13 |
| 6 | .29 | | .04 | | .14 | | .12 |
| 7 | .15 | | 0 | | 0 | | 0 |
| 8 | .76 | | .51 | | .61 | | .59 |
| 9 | .56 | | .31 | | .41 | | .39 |
| 10 | .40 | | .15 | | .25 | | .23 |
| 11 | .56 | | .31 | | .41 | | .39 |
| 12 | .24 | | 0 | | .09 | | .07 |
| 13 | .13 | | | | 0 | | 0 |
| 14 | 0 | | | | 0 | | 0 |
| 15 | .19 | | | | .04 | | .02 |
| 16 | .08 | | | | | | |
| 17 | .02 | .25 | | .15 | | .17 | |
| Total | 4.14 | | 1.37 | | 2.15 | | 1.96 |

Total rainfall, 4.14 inches: observed runoff, 2.0 inches: total retention in 17 *hours, 2.1 inches. The average retention rate of 0.17 inches per hour assumed* in the third trial gives the best agreement of computed rainfall excess with measured runoff.

excess-retention curve will facilitate the solution. In the example of table L-2, the curve could be drawn through the two points represented by the coordinates 0.25, 1.37, and 0.15, 2.15. The correct retention rate corresponding to a rainfall excess of 2.0 inches would then be taken from this curve.

The duration time of excess rainfall is that time during which rainfall increments exceed the average retention rate. In the third trial, table L-2, the duration time may be taken as either 8 or 9 hours, or as two periods, one of 2 or 3 hours, and the other of 5 hours (the final 0.02 inch of precipitation being disregarded), according to the characteristics of the hydrograph. A small amount of excess rain in a marginal period is frequently assumed to have occurred within only a small part of that period and may be neglected.

(5) *Discussion of observed rainfall analyses procedures.*—The above classic procedure of rainfall-runoff analysis is simple and satisfactory, given rainfall data such as used in the illustration and a relatively homogeneous watershed not exceeding a few hundred square miles in area. As stated earlier in section L-7(a): "A comparison of retention rates derived from several analyses leads to adoption of a rate for design flood computations." Experienced judgment is needed for such comparison with due reconsideration given to the characteristics of the data for each analysis and of the watershed. The selected rate is not necessarily the minimum rate computed. Mass curves of rainfall and isohyetal patterns should always be constructed as described in sections L-7(b)(2) and (3) to obtain good results from any rainfall-runoff analysis.

The importance in flood computations of good estimates of retention losses is evident. As the ratio of retention loss to flood causative precipitation increases, the relative effect of retention loss estimates on resulting flood magnitudes increases. Research studies directed towards improved understanding and evaluation of all processes contributing to retention losses are increasing yearly. Many complex functions are being tested by electronic computer programs to model such processes. However, the most practical approach for estimating natural watershed retention losses continues to be use of empirically derived relationships, preferably from records within the watershed.

Often, relationships as percentages of runoff to rainfall, runoff coefficients, are obtained by analyses and judicially used in flood studies. This approach may be practical in cases where basic data are meager.

The following extract from WMO Technical Note No. 98 [2] gives information of a method that may be used.

> ". . . For a particular river basin with records of streamflow and precipitation, a common procedure is to develop multiple variable rainfall-runoff correlations. Such correlations may be derived either graphically or analytically. They usually involve at least four variables, (i) depth of storm rainfall over the basin, (ii) surface runoff volume from the storm event, (iii) an index of moisture conditions in the basin

prior to the storm, and (iv) a seasonal factor. In some cases storm duration is included as a fifth variable. The methods of determining these factors from the observational records in a basin or a region and graphical and analytic procedures for multiple-variable correlation analyses are outlined in the WMO Guide to Hydrometeorological Practices, Annex A, WMO 168.TP.82."

A hydrologist making an inflow design flood study seldom finds rainfall-runoff records for the watershed above a particular damsite adequate to establish a good estimate of retention loss for the watershed. Recourse is then made to information of analyses for other watersheds having similar runoff characteristics. For example, hydrologists of the Soil Conservation Service, U.S. Department of Agriculture, have made extensive analyses of runoff from small experimental watersheds having individually homogeneous soil and cover characteristics but such characteristics differing between watersheds. A procedure was developed from these studies for estimating runoff from precipitation for any watershed for which certain soil and cover data are known; such soil and cover data are usually obtainable or subject to reasonable approximations [3].

The SCS procedure with modifications to fit specific purposes is described in appendix A of the Bureau of Reclamation publication "Design of Small Dams," second edition [31]. An abridgement of that description is given in the following subsection. (The descriptive items have been renumbered for convenience.)

(6) *Method of estimating retention losses.*—This method consists of the following steps:

(I). Classification of watershed soils into hydrologic groups A, B, C, or D, and estimation of percent of areal extent of each in the watershed.

(II). Identification of land use characteristics dominant for each hydrologic group.

(III). The combination of a hydrologic group and its land use characteristics to give a *hydrologic soil-cover complex* identification for entering tables from which respective runoff curve numbers, CN, may be obtained.

(IV). Runoff values are obtained from a family of curves on a plot of rainfall versus runoff or by solution of the equation used to define the curves.

(V). Three antecedent moisture conditions, AMC, of a watershed are considered in relation to curve numbers; namely, AMC-I, AMC-II, AMC-III.

The mathematical procedure is given in this text with minimum definitions of the terms used in the procedure and without inclusion of a list of about 4,000 soil-type names and respective hydrologic group classification compiled by the Soil Conservation Service. A full discussion of the procedure including the list of soil-type names is given in "Design of Small Dams" [31]. Information on the development of the runoff curves may be found in the SCS National Engineering Handbook [3].

Further explanation of each of the above steps follows.

(I) *Hydrologic soil groups.*—Four major soil groups are used. The soils are classified on the basis of intake of water at the end of long-duration storms occurring after prior wetting and opportunity for swelling, and without the protective effects of vegetation.

In the definitions that follow, the *infiltration rate* is the rate at which water enters the soil at the surface and which is controlled by surface condition, and the *transmission rate* is the rate at which the water moves in the soil and which is controlled by the soil horizons. The hydrologic soil groups, as defined by SCS soil scientists, are as follows:

*Group A (low runoff potential).*—Soils having high infiltration rates even when thoroughly wetted and consisting chiefly of deep, well to excessively drained sands or gravels. These soils have a high rate of water transmission.

*Group B.*—Soils having moderate infiltration rates when thoroughly wetted and consisting chiefly of moderately deep to deep, moderately well to well drained soils with moderately fine to moderately coarse textures. These soils have a moderate rate of water transmission.

*Group C.*—Soils having slow infiltration rates when thoroughly wetted and consisting chiefly of soils with a layer that impedes downward movement of water, or soils with moderately fine to fine texture. These soils have a slow rate of water transmission.

*Group D (high runoff potential).*—Soils having very slow infiltration rates when thoroughly wetted and consisting chiefly of clay soils with a high swelling potential, soils with a permanent high water table, soils with a claypan or clay layer at or near the surface, and shallow soils over nearly impervious material. These soils have a very slow rate of water transmission.

(II) *Land use and treatment classes.*—These classes are used in the preparation of hydrologic soil-cover complexes (identified herein as item III), which in turn are used in estimating direct runoff. Types of land use and treatment are classified on a flood runoff-producing basis. The greater the ability of a given land use or treatment to increase total retention, the lower it is on a flood runoff-production scale. Land use or treatment types not described here may be classified by interpolation.

*Crop rotations.*—The sequence of crops on a watershed must be evaluated on the basis of its hydrologic effects. Rotations range from *poor* (or weak) to *good* (or strong) largely in proportion to the amount of dense vegetation in the rotation. *Poor rotations* are those in which a row crop or small grain is planted in the same field year after year. A poor rotation may combine row crops, small grains, or fallow, in various ways. *Good rotations* will contain alfalfa or other close-seeded legumes or grasses, to improve tilth and increase infiltration. For example, a 2-year rotation of wheat and fallow may be a good rotation for crop production where low annual rainfall is a limiting factor, but hydrologically it is a poor rotation.

*Native pasture and range.*—Three conditions are used, based on hydrologic considerations, not on forage production. *Poor pasture or range* is heavily grazed, has no mulch, or has plant cover on less than about 50 percent of the area. *Fair pasture or range* has between about 50 and 75 percent of the area with plant cover and is not heavily grazed. *Good pasture or range* has more than about 75 percent of the area with plant cover, and is lightly grazed.

*Farm woodlots.*—The classes are based on hydrologic factors, not on timber production. *Poor woodlots* are heavily grazed and regularly burned in a manner that destroys litter, small trees, and brush. *Fair woodlots* are grazed but not burned. These woodlots may have some litter, but usually these woods are not protected. *Good woodlots* are protected from grazing so that litter and shrubs cover the soil.

*Forests.*—See hydrologic soil-cover complex, item III following.

*Straight-row farming.*—This class includes up-and-down and cross-slope farming in straight rows. In areas of 1 or 2 percent slope, cross-slope farming in straight rows is almost the same as contour farming. Where the proportion of cross-slope farming is believed to be significant, it may be classed halfway between straight-row and contour farming in the table L-3(A).

*Contouring.*—Contour furrows used with small grains and legumes are made while planting, are generally small, and tend to disappear due to climatic action. Contour furrows, and beds on the contour, as used with row crops are generally large. They may be made in planting and later reduced in size by cultivation, or they may be insignificant after planting and become large from cultivation. Average conditions are used in table L-3(A).

Surface runoff reductions due to contour farming are greater as land slopes decrease. The curve numbers for contouring shown in table L-3(A) were obtained using data from experimental watersheds having slopes of 3 to 8 percent.

Contour furrows in pasture or range land are usually of the permanent type. Their dimensions and spacing generally vary with climate and topography. Table L-3(A) considers average conditions in the Great Plains.

*Terracing.*—Terraces may be graded, open-end level, or closed-end level. The effects of graded and open-end level terraces are considered in table L-3(A), and the effects of both contouring and the grass waterway outlets are included.

When considering land use and treatment classes for hydrologic soil groups within a large watershed, the above definitions should be applied broadly, estimating percentage of land use in each group, assigning proper CN and computing a weighed CN for each particular soil group.

(III) *Hydrologic soil-cover complexes.*—Combinations of hydrologic soil groups and land use and treatment classes into hydrologic soil-cover complexes with respective curve numbers are given in table L-3(A), (B), (C). The numbers show the relative value of the complexes as direct runoff-producers. The higher the number, the greater the amount of direct runoff to be expected from a storm. Table L-3(A) is applicable to farm lands and related areas, and table L-3(B) is applicable to forested watersheds. A more detailed method of estimating curve numbers for heavy forested land in humid regions is given in appendix A of "Design of Small Dams," second edition [31].

Table L-3(C) is applicable for forest-range areas in the Western United States. Descriptions of the types of cover listed are as follows:

*Herbaceous.*—Grass-weed-brush mixtures with brush the minor element.

*Oak-Aspen.*—Mountain brush mixtures of oak, aspen, mountain mahogany, bitter brush, maple, and other brush.

*Juniper-Grass.*—Juniper or pinon with an understory of grass.

*Sage-Grass.*—Sage with an understory of grass.

(IV) *Rainfall-runoff curves for estimating direct runoff amounts.*—The curves of figure L-2 are obtained using the equation:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \tag{1}$$

where:

$Q$ = direct runoff, in inches
$P$ = storm rainfall, in inches, and
$S$ = maximum potential difference between $P$ and $Q$, in inches, at time of storm's beginning.

There is some loss of rainfall before runoff begins due principally to interception, infiltration, and surface storage, so provision for an initial abstraction $I_a$ is included in the runoff equation (see diagram on figure L-2). With the condition that $I_a$ cannot be greater than $P$, an empirical relationship of $I_a = 0.2S$ was adopted in developing the equation, obtaining the empirical relationship of $I_a$ and $S$ from data from watersheds in various parts of the country.

For convenience in interpolation, the curves of figure L-2 are numbered from 100 to zero. The numbers are related to $S$ as follows:

$$\text{Curve number, CN} = \frac{1{,}000}{10 + S} \tag{2}$$

The procedure recommended in this text for estimating incremental rainfall excesses from design storm rainfall using appropriate CN and figure L-2 or the runoff equation is given in section L-19. In the process of hydrograph analyses, preliminary estimates of curve numbers for a watershed can be quickly obtained from figure L-2 by using total storm rainfall and runoff amounts. However, such preliminary estimates have to be revised by trial computations of rainfall excesses using the procedure given later in section L-19.

(V) *Antecedent moisture conditions.*—The following generalized criteria define three antecedent moisture conditions of watersheds used in the development of the runoff curve numbers.

*AMC-I.*—A condition of watershed soils where the soils are dry but not to the

Table L-3.–*Hydrologic soil-cover complexes and respective curve numbers (CN).*

### (A) RUNOFF CURVE NUMBERS (CN) FOR FARMLANDS AND RELATED AREAS

[FOR WATERSHED CONDITION AMC-II]

| Land use or cover | Treatment or practice | Hydrologic condition for infiltrating | Hydrologic soil group | | | |
|---|---|---|---|---|---|---|
| | | | A | B | C | D |
| Fallow | SR | | 77 | 86 | 91 | 94 |
| Row crops | SR | Poor | 72 | 81 | 88 | 91 |
| | SR | Good | 67 | 78 | 85 | 89 |
| | C | Poor | 70 | 79 | 84 | 88 |
| | C | Good | 65 | 75 | 82 | 86 |
| | C&T | Poor | 66 | 74 | 80 | 82 |
| | C&T | Good | 62 | 71 | 78 | 81 |
| Small grain | SR | Poor | 65 | 76 | 84 | 88 |
| | SR | Good | 63 | 75 | 83 | 87 |
| | C | Poor | 63 | 74 | 82 | 85 |
| | C | Good | 61 | 73 | 81 | 84 |
| | C&T | Poor | 61 | 72 | 79 | 82 |
| | C&T | Good | 59 | 70 | 78 | 81 |
| Close-seeded legumes [1] or rotation meadow. | SR | Poor | 66 | 77 | 85 | 89 |
| | SR | Good | 58 | 72 | 81 | 85 |
| | C | Poor | 64 | 75 | 83 | 85 |
| | C | Good | 55 | 69 | 78 | 83 |
| | C&T | Poor | 63 | 73 | 80 | 83 |
| | C&T | Good | 51 | 67 | 76 | 80 |
| Pasture or range | | Poor | 68 | 79 | 86 | 89 |
| | | Fair | 49 | 69 | 79 | 84 |
| | | Good | 39 | 61 | 74 | 80 |
| | C | Poor | 47 | 67 | 81 | 88 |
| | C | Fair | 25 | 59 | 75 | 83 |
| | C | Good | 6 | 35 | 70 | 79 |
| Meadow (*permanent*). | | do | 30 | 58 | 71 | 78 |
| Woods (farm woodlots). | | Poor | 45 | 66 | 77 | 83 |
| | | Fair | 36 | 60 | 73 | 79 |
| | | Good | 25 | 55 | 70 | 77 |
| Farmsteads | | | 59 | 74 | 82 | 86 |
| Roads (dirt)[2] (hard | | | 72 | 82 | 87 | 89 |
| surface).[2] | | | 74 | 84 | 90 | 92 |

[1] Close-drilled or broadcast.

[2] Including right-of-way.

(U.S. Soil Conservation Service.)

SR=Straight row.
*C=Contoured.*
T=Terraced.
C&T=Contoured and terraced.

### (B) RUNOFF CURVE NUMBERS (CN) FOR FORESTED WATERSHEDS

COMMERCIAL OR NATIONAL FOREST, FOR WATERSHED CONDITION AMC-II

| Hydrologic condition class | Hydrologic soil group | | | |
|---|---|---|---|---|
| | A | B | C | D |
| I. Poorest | 56 | 75 | 86 | 91 |
| II. Poor | 46 | 68 | 78 | 84 |
| III. Medium | 36 | 60 | 70 | 76 |
| IV. Good | 26 | 52 | 62 | 69 |
| V. Best | 15 | 44 | 54 | 61 |

### (C) RUNOFF CURVE NUMBERS (CN) FOR FOREST RANGE AREAS IN WESTERN UNITED STATES (AMC-II)

| Cover | Condition | Soil groups | | | |
|---|---|---|---|---|---|
| | | A | B | C | D |
| Herbaceous | Poor | --- | 78 | 85 | 92 |
| | Fair | --- | 68 | 81 | 88 |
| | Good | --- | 59 | 71 | 84 |
| Sage-Grass | Poor | --- | 64 | 78 | --- |
| | Fair | --- | 46 | 67 | --- |
| | Good | --- | 35 | 46 | --- |
| Oak-Aspen | Poor | --- | 63 | 71 | --- |
| | Fair | --- | 40 | 54 | --- |
| | Good | --- | 30 | 40 | --- |
| Juniper-Grass | Poor | --- | 73 | 84 | --- |
| | Fair | --- | 54 | 70 | --- |
| | Good | --- | 40 | 59 | --- |

wilting point, and when satisfactory plowing or cultivation takes place. (This condition is *not* considered applicable to the design flood computation methods presented in this text.)

*AMC-II.*–The average case for *annual floods*, that is, an average of the conditions which have preceded the occurrence of the maximum annual flood on numerous watersheds.

*AMC-III.*–Heavy rainfall has occurred during the 5 days previous to the given storm and the soil is nearly saturated.

Curve numbers in table L-3(A), (B), (C) for hydrologic soil-cover complexes all relate to AMC-II. Table L-4(A) lists curve numbers for AMC-II with respective *S* values (column (4)) and 0.2*S* values (column (5)) which may be used to solve the runoff equation on figure L-2.

Curve numbers for AMC-I and AMC-III respective to the CN for AMC-II in column (1) are listed in columns (2) and (3). This information is useful for estimating retention losses. If data are available for analyzing observed storms and resulting runoff, an estimate of antecedent moisture condition of a watershed may be made from table L-4(B).

L-8. *Analyses of Streamflow Data.*—Streamflow data at a given location may consist of: (1) a continuous hydrograph of discharges obtained from waterstage recording mechanisms; (2) mean (average) daily discharges computed from waterstage recorders or from once or twice daily observed water stages; or, in some instances (3) peak discharges computed from flood marks or crest stage gages. U.S. Geological Survey publications should be consulted for information about collection and processing these data for publication. However, one should be aware that U.S.G.S. publications give for each published station record an estimate of the degree of accuracy of field data and computed results for that record as follows:

> "*Excellent* means that about 95 percent of the daily discharges are within 5 percent; *good*, within 10 percent; and *fair*, within 15 percent. *Poor* means that daily discharges have less than *fair* accuracy."

Objectives of streamflow data analyses for inflow design flood computations are:

(1) Determinations of watershed retention losses (previously discussed).

(2) Determination of characteristic watershed response to precipitation; this is usually accomplished by deriving a unit hydrograph for the watershed. (Complex

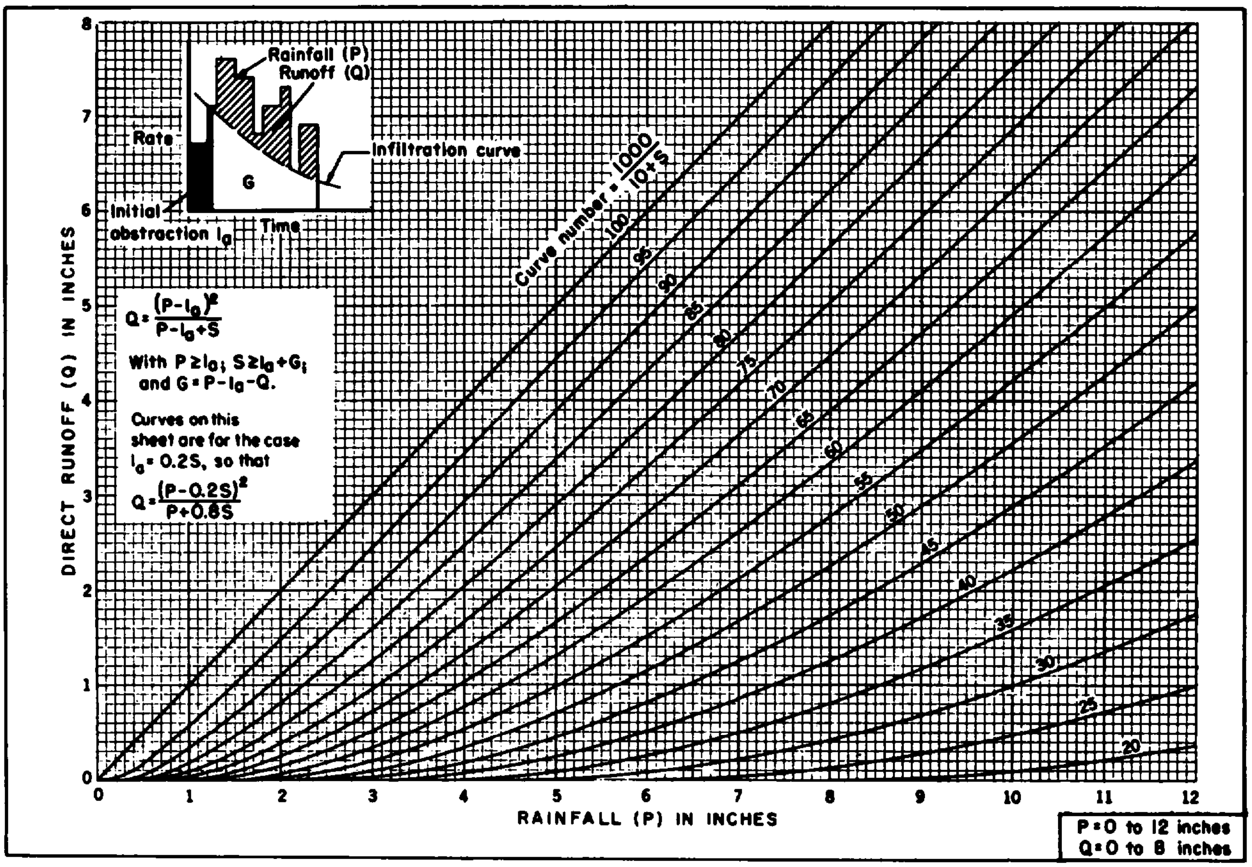


*Figure L-2.* Rainfall-runoff curves—solution of runoff equation, $Q = \frac{(P - 0.2S)^2}{P + 0.8S}$ (sheet 1 of 2) (U.S. Soil Conservation Service).—288-D-3178(1/2)

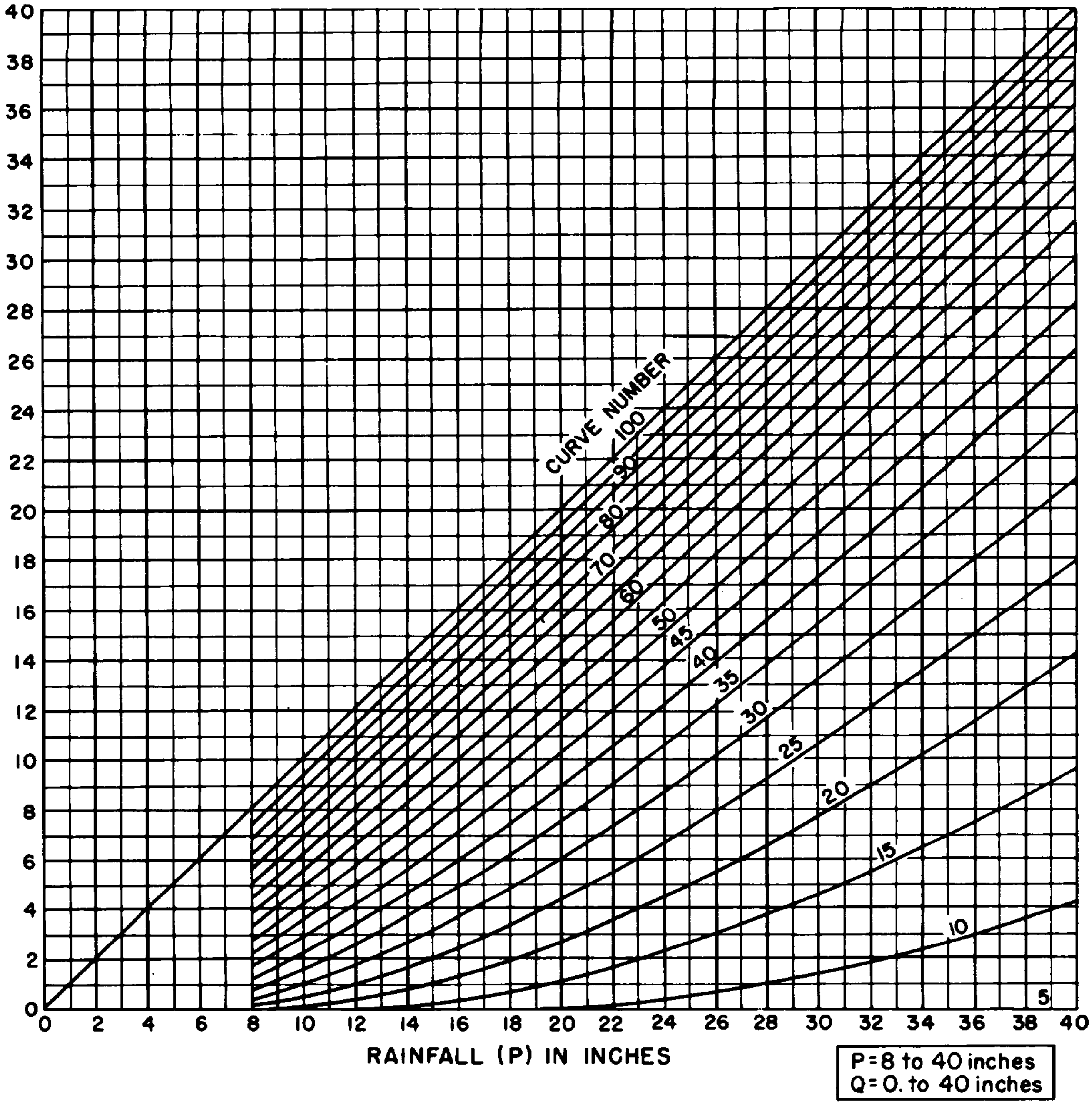


*Figure L-2.* **Rainfall-runoff curves–solution of runoff equation, $Q = \frac{(P - 0.2S)^2}{P + 0.8S}$ (sheet 2 of 2) (U.S. Soil Conservation Service).–288-D-3178(2/2) (*Note*: Curve designated by number is *below* number.)**

computer-programed watershed runoff models may use other means of estimating time distribution of runoff.)

Continuous hydrographs can provide for estimates of retention loss variations with time, with accumulative loss, or with accumulative precipitation. Mean daily discharges can provide ratio estimates of total retention loss to total storm precipitation.

Continuous hydrographs are essential to unit hydrograph derivations from recorded streamflow data. When mean daily discharges only are available, a continuous hydrograph is sketched for making unit hydrograph

Table L-4.—*Curve numbers, constants, and seasonal rainfall limits.*

(A) CURVE NUMBERS (CN) AND CONSTANTS FOR THE CASE $I_a = 0.2S$

| 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| CN for condition II | CN for conditions | | S values* | Curve* starts where P = | CN for condition II | CN for conditions | | S values* | Curve* starts where P = |
| | I | III | | | | I | III | | |
| | | | *inches* | *inches* | | | | *inches* | *inches* |
| 100 | 100 | 100 | 0 | 0 | 60 | 40 | 78 | 6.67 | 1.33 |
| 99 | 97 | 100 | .101 | .02 | 59 | 39 | 77 | 6.95 | 1.39 |
| 98 | 94 | 99 | .204 | .04 | 58 | 38 | 76 | 7.24 | 1.45 |
| 97 | 91 | 99 | .309 | .06 | 57 | 37 | 75 | 7.54 | 1.51 |
| 96 | 89 | 99 | .417 | .08 | 56 | 36 | 75 | 7.86 | 1.57 |
| 95 | 87 | 98 | .526 | .11 | 55 | 35 | 74 | 8.18 | 1.64 |
| 94 | 85 | 98 | .638 | .13 | 54 | 34 | 73 | 8.52 | 1.70 |
| 93 | 83 | 98 | .753 | .15 | 53 | 33 | 72 | 8.87 | 1.77 |
| 92 | 81 | 97 | .870 | .17 | 52 | 32 | 71 | 9.23 | 1.85 |
| 91 | 80 | 97 | .989 | .20 | 51 | 31 | 70 | 9.61 | 1.92 |
| 90 | 78 | 96 | 1.11 | .22 | 50 | 31 | 70 | 10.0 | 2.00 |
| 89 | 76 | 96 | 1.24 | .25 | 49 | 30 | 69 | 10.4 | 2.08 |
| 88 | 75 | 95 | 1.36 | .27 | 48 | 29 | 68 | 10.8 | 2.16 |
| 87 | 73 | 95 | 1.49 | .30 | 47 | 28 | 67 | 11.3 | 2.26 |
| 86 | 72 | 94 | 1.63 | .33 | 46 | 27 | 66 | 11.7 | 2.34 |
| 85 | 70 | 94 | 1.76 | .35 | 45 | 26 | 65 | 12.2 | 2.44 |
| 84 | 68 | 93 | 1.90 | .38 | 44 | 25 | 64 | 12.7 | 2.54 |
| 83 | 67 | 93 | 2.05 | .41 | 43 | 25 | 63 | 13.2 | 2.64 |
| 82 | 66 | 92 | 2.20 | .44 | 42 | 24 | 62 | 13.8 | 2.76 |
| 81 | 64 | 92 | 2.34 | .47 | 41 | 23 | 61 | 14.4 | 2.88 |
| 80 | 63 | 91 | 2.50 | .50 | 40 | 22 | 60 | 15.0 | 3.00 |
| 79 | 62 | 91 | 2.66 | .53 | 39 | 21 | 59 | 15.6 | 3.12 |
| 78 | 60 | 90 | 2.82 | .56 | 38 | 21 | 58 | 16.3 | 3.26 |
| 77 | 59 | 89 | 2.99 | .60 | 37 | 20 | 57 | 17.0 | 3.40 |
| 76 | 58 | 89 | 3.16 | .63 | 36 | 19 | 56 | 17.8 | 3.56 |
| 75 | 57 | 88 | 3.33 | .67 | 35 | 18 | 55 | 18.6 | 3.72 |
| 74 | 55 | 88 | 3.51 | .70 | 34 | 18 | 54 | 19.4 | 3.88 |
| 73 | 54 | 87 | 3.70 | .74 | 33 | 17 | 53 | 20.3 | 4.06 |
| 72 | 53 | 86 | 3.89 | .78 | 32 | 16 | 52 | 21.2 | 4.24 |
| 71 | 52 | 86 | 4.08 | .82 | 31 | 16 | 51 | 22.2 | 4.44 |
| 70 | 51 | 85 | 4.28 | .86 | 30 | 15 | 50 | 23.3 | 4.66 |
| 69 | 50 | 84 | 4.49 | .90 | | | | | |
| 68 | 48 | 84 | 4.70 | .94 | 25 | 12 | 43 | 30.0 | 6.00 |
| 67 | 47 | 83 | 4.92 | .98 | 20 | 9 | 37 | 40.0 | 8.00 |
| 66 | 46 | 82 | 5.15 | 1.03 | 15 | 6 | 30 | 56.7 | 11.34 |
| 65 | 45 | 82 | 5.38 | 1.08 | 10 | 4 | 22 | 90.0 | 18.00 |
| 64 | 44 | 81 | 5.62 | 1.12 | 5 | 2 | 13 | 190.0 | 38.00 |
| 63 | 43 | 80 | 5.87 | 1.17 | 0 | 0 | 0 | infinity | infinity |
| 62 | 42 | 79 | 6.13 | 1.23 | | | | | |
| 61 | 41 | 78 | 6.39 | 1.28 | | | | | |

*For CN in column 1 (value = 0.2*S*)

(B) SEASONAL RAINFALL LIMITS FOR AMC

| AMC group | Total 5-day antecedent rainfall, inches | |
|---|---|---|
| | Dormant season | Growing season |
| I | Less than 0.5 | Less than 1.4 |
| II | 0.5 to 1.1 | 1.4 to 2.1 |
| III | Over 1.1 | Over 2.1 |

estimates; the chance of introducing considerable error is obvious. Discussions which follow assume continuous hydrographs obtained from continuous recording waterstage records converted to discharges expressed as cubic feet per second (c.f.s.), the degree of accuracy of the records being *excellent* or *good*.

(a) *Unit Hydrograph (Unitgraph) Principles.*—The 1970 USCOLD report [1] states: "In general the unit hydrograph method, in conjunction with the estimated probable maximum precipitation, is used in estimating probable maximum floods. . . ." The unit hydrograph principle was originally developed by Sherman [4] in 1932. Although numerous refinements have been added by other investigators, the basic principles as presented by Sherman remain the same. These principles as now applied are given and illustrated on figure L-3.

Sherman's definition of unit hydrograph did not imply a specific volume of runoff, and the term was applied to the observed hydrograph as well as to a hydrograph of 1-inch volume computed from the observed graph. In present practice, observed hydrographs are usually identified as such, and the term *unitgraph* refers either to the 1-inch volume unitgraph derived from a specific observed hydrograph or to a 1-inch volume unitgraph representative of the watershed and used to compute synthetic floods from rainfall excess over the watershed. Random variations in rainfall rate in respect to time and area have a great effect on the shape of the runoff hydrograph. To minimize the effect of the time variations in rainfall rate, it has been found that the rainfall excess duration time of a basin unitgraph should not exceed one-fourth the basin lag-time as defined in section L-8(e), and the shorter the rainfall excess period with respect to lag-time, the better the unitgraph results are likely to be.

The term *unit hydrograph*, or *unitgraph*, as used in this text always means 1-inch volume of runoff; the volume notation is seldom included. The rainfall excess unit duration time is always given for a watershed representative unitgraph.

Natural flood hydrographs at a given stream gage are assumed to give integrated results of all interdependent effects on runoff such as watershed precipitation, retention losses, and routing effects of watershed vegetative cover and channel systems. A unit hydrograph which has been derived from recorded floods at a given stream location, and which will give close reconstruction of recorded flood hydrographs from recorded respective precipitation events as affected by retention losses, is considered representative of that particular watershed and also considered representative of other watersheds having similar runoff characteristics.

On this basis, synthetic unit hydrographs for ungaged basins are derived by judging comparative watershed characteristics and adjusting "representative" unit hydrographs to fit the size and lag-time of the ungaged watershed. Mathematical watershed runoff models are currently being developed by computer integration of meteorological, hydrological, and physiographical factors. Some hydrologists prefer to use these models rather than a unitgraph. However, each model includes constants related to watershed characteristics that must be empirically determined by trial analyses of recorded flows. As in the application of synthetic unitgraphs, transference of a mathematical model from a gaged to an ungaged watershed also requires experienced judgment of the effect from variations in watershed characteristics.

The use of the unit hydrograph is limited in the following ways:

(1) The principle of the unit hydrograph is applicable to basins of any size. However, it is desirable in the derivation of unitgraphs to use storms that are well distributed over the entire basin and produce runoff nearly concurrently from all parts of it. Such storms rarely occur over large areas. The extent of the basin for which a unitgraph may be derived from observed data is therefore limited in each case to the areal extent of rainfalls that have been observed.

(2) Hydrographs containing more than small amounts of snowmelt runoff are

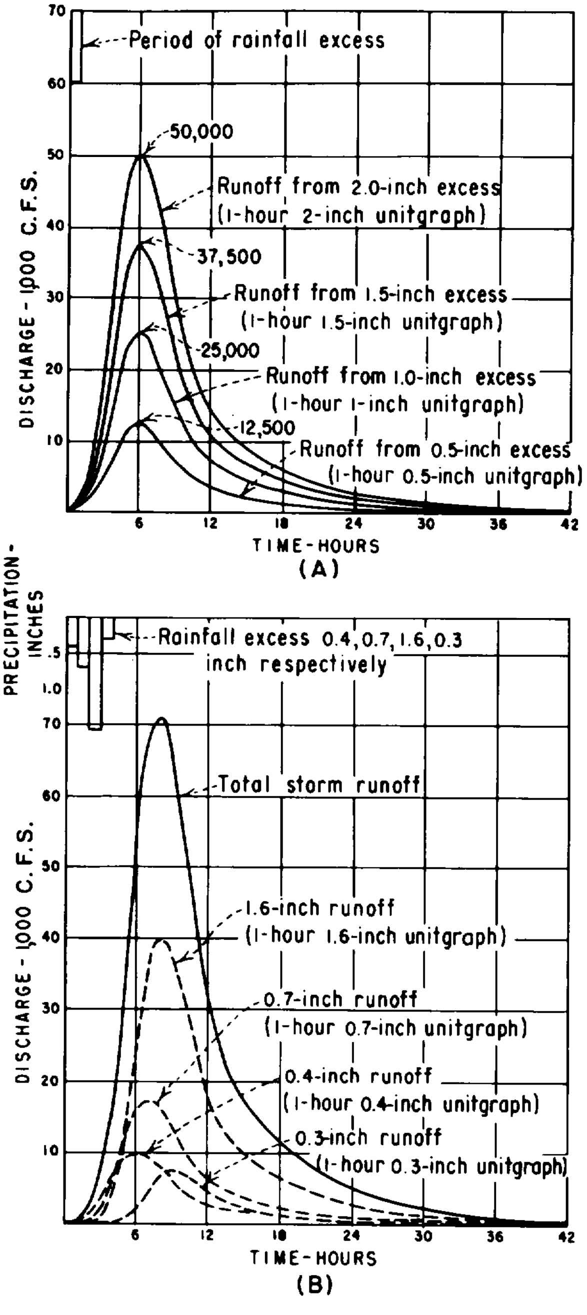


*Figure L-3.* **Unit hydrograph principles (sheet 1 of 2).–288-D-3179(1/2)**

**Definitions:** *

Unitgraph - A hydrograph of direct runoff* at a given point that will result from an isolated event of rainfall excess occurring within a unit of time and spread in an average pattern over the contributing drainage area. Identified by by the unit time and volume of the excess rainfall, that is 1-hour 1-inch unitgraph.

Rainfall excess - That portion of rainfall that enters a stream channel as direct runoff and produces the runoff hydrograph at the measuring point, base flow included

**Basic Assumptions:**

(1) The effects of all physical characteristics of a given drainage basin are reflected in the shape of the direct runoff hydrograph for that basin.

(2) At a given point on a stream, discharge ordinates of different unitgraphs of the same unit time of rainfall excess are mutually proportional to respective volumes. See (A) at left.

(3) A hydrograph of storm discharge that would result from a series of bursts of excess rain or from continuous excess rain of variable intensity may be constructed from a series of over-lapping unitgraphs each resulting from a single increment of excess rain of unit duration. See (B) at left.

**Practical Application:**

For a given runoff contributing area, a unitgraph representing exactly one inch of runoff (rainfall excess) for a selected unit time interval is computed. Increments of rainfall excess for the same unit time interval are determined for a storm. A total hydrograph of direct runoff from the storm is then computed using assumptions (2) and (3) above. See graph (B) at left.

**Note:* Direct runoff is defined in section L-8.

*Figure L-3.* **Unit hydrograph principles (sheet 2 of 2).–288-D-3179(2/2)**

usually unsuitable sources of unitgraphs.

(3) The observed hydrograph of storm discharge is a smooth curve, because it is actually made up of unitgraphs produced by infinitely short increments of excess rain. It cannot be reproduced perfectly by the use of rainfall increments of measurable duration. When unitgraphs are combined they produce a regular undulation similar to a harmonic with a period equal to that of the rainfall increments, superimposed upon the fundamental hydrograph. Another obstacle to exact reproduction is the fact that the successive rainfall increments do not have the same isohyetal pattern and a single form of unitgraph is not strictly applicable to all of them. These phenomena contradict, to a certain extent, the third basic assumption of the unit hydrograph (fig. L-3). They can be disregarded in the synthesis of hydrographs, but frequently cause difficulty in the use of arithmetical procedures for analyzing them.

An engineer attempting unitgraph analyses or researching literature regarding unitgraphs soon becomes aware that the three basic assumptions listed on figure L-3 are not theoretically supportable. However, experience has shown that this does not negate use of the method as a practical tool.

(b) *Selection of Hydrographs to Analyze.*—The statement made in section L-7(b)(2) bears enough importance to unit hydrograph studies to be repeated: "Those floods having a combination of large volume, uniform intensity, isolated periods of rainfall, and uniform areal distribution of rainfall, should be chosen for further study."

Streamflow discharge records and basin precipitation records must be examined jointly for selection of hydrographs to analyze for unit hydrograph derivation. Isolated floods likely to merit investigation are easily identified by a rapid rise to a single peak and a smooth curve recession to low flow. Preferably, volumes of selected hydrographs should be equivalent to about one-half inch or more of runoff from the watershed. Preliminary estimates of hydrograph volumes can be made by summing the daily mean daily discharges in c.f.s.-days for the flood period. A sum of c.f.s.-days equal in number to 15 times the drainage area size in square miles is equivalent to 0.56 inch of runoff from the area. A useful equation for converting discharge volume to equivalent inches of rainfall is:

$$P_e = \frac{V}{26.89\,A} \qquad (3)$$

where:

$P_e$ = rainfall excesses, inches, average depth over basin,
$V$ = volume of runoff, c.f.s.-days, and
$A$ = drainage area in square miles.

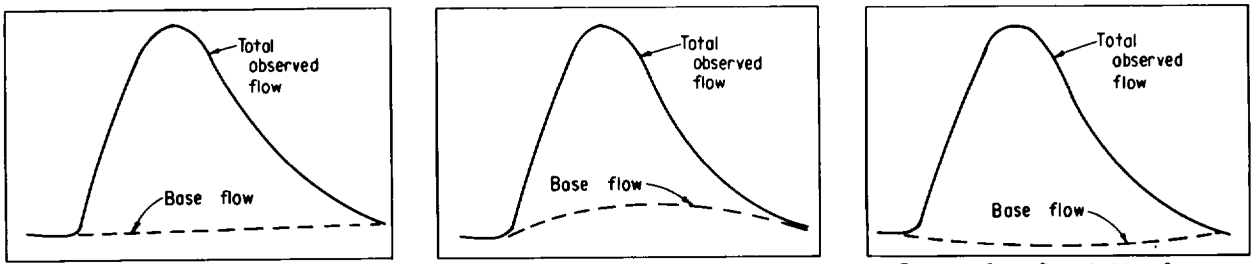
Hydrographs with volume sum of c.f.s.-days less than five times the drainage area size, 0.19 inch runoff, are almost always unsuitable for unit hydrograph analyses.

After noting dates of all flood hydrographs that satisfy preliminary volume criteria, rainfall records for respective flood events are examined for conformance with the ideal combination of short duration, uniform intensity, and uniform areal distribution of rainfall over the entire watershed. Those storms approaching nearest to the *ideal* criteria are analyzed as previously described in section L-7. If enough rainfall data are not available to do a good storm analysis for some of the isolated flood events having satisfactory volumes, the flood hydrographs may be analyzed for unitgraph comparisons as discussed in section L-8(e) by assuming that the beginning of rainfall excess coincides with the beginning of a sharp rise of the hydrograph, provided there is enough information available to reasonably assume the rainfall covered the total watershed.

Unit hydrograph derivations are difficult in regions where isolated flood events are rare and, instead, flood hydrographs commonly have two or more peaks caused by storms which usually persist for several days. Procedures for analyzing multipeaked flood hydrographs cannot be included in this text but can be found in publications listed in the bibliography, section L-32.

(c) *Hydrograph Analyses—Base Flow Separation.*—The purpose of flood hydrograph analyses is to determine for a watershed the time-distribution of the runoff which *quickly* reaches a particular point on a stream when rain falls on the watershed. The portion of the rainfall that infiltrates through the soil mantle into the ground-water supply will not reach the stream until days or months after the storm. Ground-water supply to a stream, base flow, may be a large proportion of that stream's total yearly discharge, but the base flow volume during an isolated flood is small in ratio to the total flood volume. However, base flow must be estimated and subtracted from the total discharge hydrograph in order to determine the direct runoff hydrograph. The schematic graphs on figure L-4 show three common approaches for estimating base flow discharges [6]. Base flow estimates are usually made graphically after plotting total flood discharges on linear or semilogarithmic graph paper.

(d) *Hydrograph Analysis of Direct Runoff—Need for Synthetic Unit Hydrographs.*—It is often necessary to use synthetic unit hydrographs for inflow design flood estimates and for obtaining indices for



Type A. – Straight line base flow between beginning and end of direct runoff.

Type B. – Base flow peaked at some time shortly after peak direct runoff.

Type C. – Base flow depletes for some time after beginning of direct runoff, then straight line to end of direct runoff.

*Figure L-4.* **Three common approaches for estimating base flow discharges.–288-D-3180**

synthetic unitgraph estimates. Suitable records of observed discharge are seldom available at the exact stream point for which a unitgraph is needed; in this discussion, at a proposed damsite. Even if such records are available, often the proposed reservoir will be large enough to inundate several miles of stream channels above the damsite, thus causing watershed runoff to enter a full reservoir more quickly than the respective runoff would arrive at the damsite through natural channels. Therefore, a unitgraph usable for estimating floods at the damsite under natural conditions must be properly adjusted to be usable to estimate inflow to a full reservoir.

The shape of a representative watershed unitgraph can be obtained by a proper average of several unitgraphs computed from observed discharge records at a gage, or occasionally by a single unitgraph from an intense rainburst, well centered and distributed. If there are available several isolated direct runoff hydrographs suitable for simple conversion to 1-inch volume unitgraphs by multiplying the hydrograph discharge ordinates by the ratio of 1 inch to the direct runoff volume in inches, only those unitgraphs having *equal duration times of rainfall excess* can be directly averaged. Most likely, rainfall excess duration time will be different for each 1-inch unitgraph. A general similarity in shape of the unitgraphs will be recognized, but they may show pronounced differences in their relative steepness and time of peak discharge.

It is possible to eliminate these differences to a large degree by adjusting the ordinates and abscissae of each unitgraph in proportion to some index related to both the duration of rainfall excess and to the average time interval between the rainfall excess and some representative point near the center of the respective runoff unitgraph. The index used for this purpose is known as *lag-time* which, for procedures to be described in this text is defined as: *The time interval between the mid-time of rainfall excess duration and the time of occurrence of one-half the volume of the hydrograph.*

Lag-time may be used as later described to convert each unitgraph into a dimensionless-graph form and the dimensionless-graphs can then be averaged. (*Note*: In this text, the hyphenated term dimensionless-graph refers to the particular form used within the Bureau of Reclamation. The two words, dimensionless graph(s) refer in general to graphs expressing time versus discharge as ratios.) Lag-time is also an index of time-of-concentration (time interval between end of rainfall excess and point of inflection on recession limb of direct runoff hydrograph) of runoff for a watershed, and can be correlated with certain measurable physical features common to all watersheds such as area, stream channel length, and slope. Correlations between lag-times derived from recorded floods and respective watershed features, in the form of *lag-time curves*, provide means for estimating lag-time at any desired ungaged stream point on the basis of watershed features above that point.

A synthetic unitgraph may be estimated for a watershed area, given a representative lag-time curve and dimensionless graph *based* *on the same lag-time definition*. Hydrology textbooks and published professional papers give many different definitions of lag-time, several different dimensionless graph forms, and many variations in correlations of basin features with lag-times.

Investigators are continually striving to improve estimates of time-distribution of runoff from rainfall. Only the lag-time versus basin factor relationships and related dimensionless-graph form used most often in Bureau of Reclamation inflow design flood studies will be described in detail in this text.

(e) *Hydrograph Analysis of Direct Runoff—Dimensionless-Graph Computations and Lag-Time Estimates.*—A direct runoff hydrograph may be converted to dimensionless-graph form using a function of lag-time. A lag-time for the flood event may also be computed if sufficient rainfall data are available to define the duration time of rainfall excess.

All hydrographs may be converted to dimensionless-graph form by the mathematical procedure to be described, but experienced judgment must be employed to select those that are suitable for further considerations. Lag-time is the basic index; however, a related value known as *lag-plus-semiduration* is the actual index used for dimensionless-graph computations. Lag-plus-semiduration is obtained by adding one-half of the duration time of rainfall excess to the lag-time. This addition provides a means of obtaining comparable dimensionless-graphs for unitgraphs of different rainfall excess durations, as, by definition, a unitgraph starts at the beginning of rainfall excess and the measurement of lag-time starts at the mid-time of rainfall excess duration. Lag-plus-semiduration is the elapsed time between the beginning of the major rise of the hydrograph and the point of 50 percent of runoff volume. Thus, in the analysis of an observed direct runoff hydrograph for which rainfall excess can be established and begins concurrently with the start of the major rise of the hydrograph, lag-time is computed as lag-plus-semiduration minus one-half of the rainfall excess duration.

When analyzing direct runoff hydrographs by the dimensionless-graph method, it is not necessary to first convert each hydrograph to a volume equivalent to 1 inch of runoff. In practice, selected observed direct runoff hydrographs are converted to dimensionless-graph form as follows. The elapsed time from the beginning of a hydrograph to the point of 50 percent volume is computed; this is the lag-plus-semiduration value for the hydrograph. The abscissae of the hydrograph is converted from actual hours into percent of the lag-plus-semiduration value. Each ordinate of the hydrograph, cubic feet per second (or c.f.s.), is multiplied by the lag-plus-semiduration value, and the product is divided by the total direct runoff hydrograph volume expressed as c.f.s.-days. The converted ordinates and abscissae are dimensionless and may be plotted for comparisons and averaging with other dimensionless-graphs similarly obtained.

The above method of eliminating the effect of rainfall excess duration time by lag-time relations is considered satisfactory in the comparison and averaging of a group of dimensionless-graphs when the maximum value of the rainfall excess duration, expressed in percent of lag-time, does not exceed about four times the minimum value found in the same group, expressed in the same way. When the duration of rainfall excess cannot be determined with reasonable accuracy, lag-plus-semiduration can frequently be measured directly from the start of rise of the direct runoff hydrograph. Thus, dimensionless-graphs may be obtained from recorded floods from watersheds where streamflows are gaged but precipitation data are meager or not collected. Use of this procedure increases the data available for synthetic unitgraph derivations.

To determine the average shape of a group of dimensionless-graphs, first determine the average of the peak ordinates and the average of the corresponding abscissae. These two values become the coordinates of the peak of the average graph. Points on the lower portions of the accession and recession are averaged *on the horizontal*, that is, an ordinate is assumed and the average of the abscissae corresponding to that ordinate is determined. If the plotting is on semilog paper and the recessions end in tangents, only two averages are needed to define the mean tangent. The *shoulder* portions of the mean graph are best sketched in by visual inspection. Arithmetical averages should not be used near the peak unless the ordinates of the points averaged are taken at a fixed percentage of the respective peak ordinates, or unless the individual peaks as plotted are at virtually the same height.

(1) *Procedures.*—A method of complete hydrograph analyses for obtaining a dimensionless-graph and lag-time estimate from a selected isolated flood event is given as a step-by-step outline with pertinent comments, graphically illustrated on figure L-5, and supplemented by a table of computation, table L-5. For illustrative purposes, computations included in table L-5 are more detailed than necessary in practice. An outline of procedures follows:

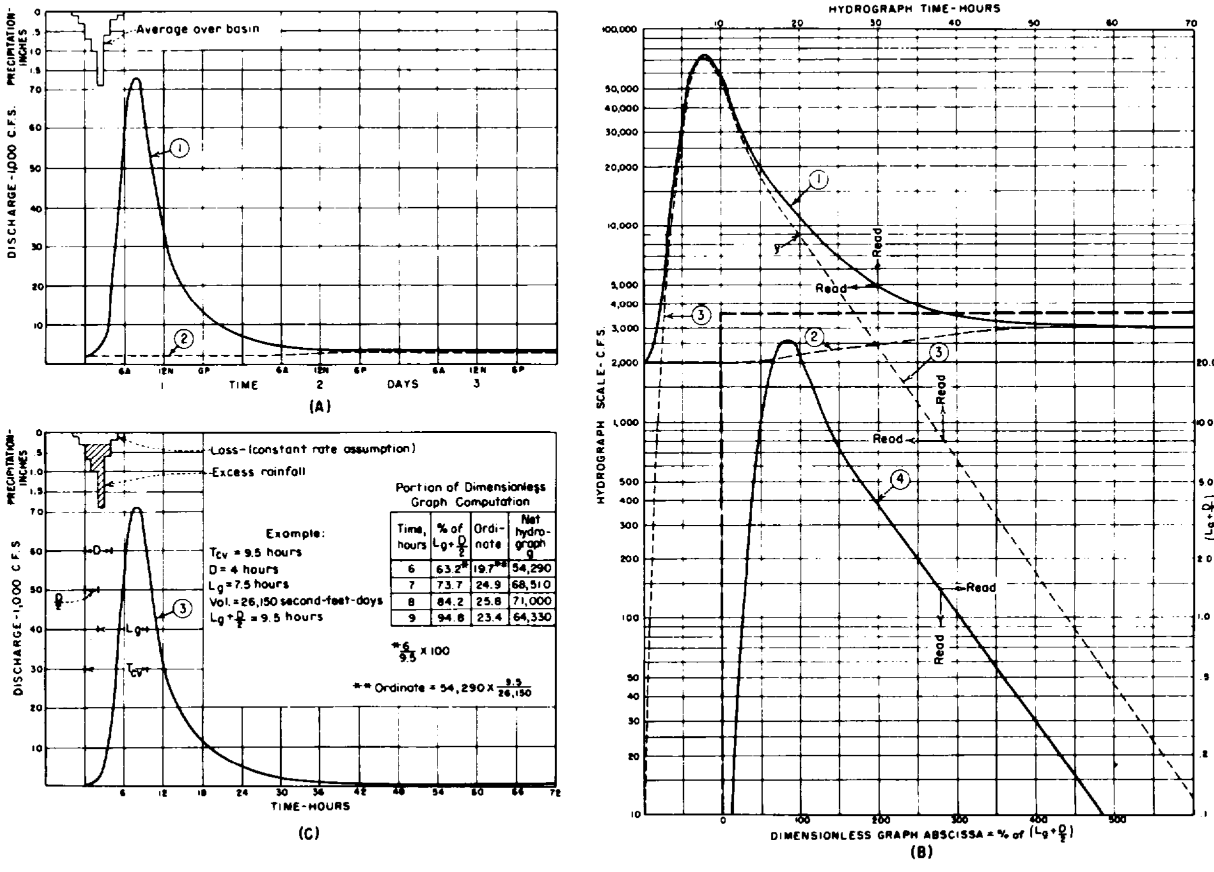
(a) Plot recorded hydrograph on cartesian coordinate paper and on semilog paper:

① on figure L-5(A), and

① on figure L-5(B)

Hypothetical total flood discharges are listed in table L-5. A hyetograph of average hourly basin rainfall, if obtainable, plotted as shown on the same coordinate paper with the total flood hydrograph, is helpful for determining the coincidence of beginning time of rainfall excess and direct runoff. The plot on semilog paper helps in making base flow estimates.

(b) Estimate base flow, ② on figure L-5(A) and (B), by trial and error. Subtract base flow from recorded



Example:
$T_{cv}$ = 9.5 hours
D = 4 hours
$L_g$ = 7.5 hours
Vol. = 26,150 second-feet-days
$L_g+\frac{D}{2}$ = 9.5 hours

Portion of Dimensionless Graph Computation

| Time, hours | % of $L_g+\frac{D}{2}$ | Ordinate | Net hydrograph q |
|---|---|---|---|
| 6 | 63.2* | 19.7** | 54,290 |
| 7 | 73.7 | 24.9 | 68,510 |
| 8 | 84.2 | 25.8 | 71,000 |
| 9 | 94.8 | 23.4 | 64,330 |

*$\frac{6}{9.5}$ x 100

** Ordinate = 54,290 x $\frac{9.5}{26,150}$

*Figure L-5.* **Hydrograph analysis.—288-D-3181** hydrograph and plot net hydrograph, ③ on figure L-5(B). If the base flow has been estimated correctly, the descending limb of hydrograph ③ on figure L-5(B) will be a straight line (exponential recession) [7]. (③ = ① minus ② on figure L-5(B).)

Large base flow discharges were used in this example to improve graphical illustration.

(c) Compute volume of net hydrograph ③ as follows (method 1, table L-5):

1. Add average hourly discharges (in c.f.s.-hours) to a point such as $y$ on the exponential recession, ③ on figure L-5(B).
2. Compute hourly recession constant, $k_{hr}$, from two points on exponential recession line by use of following equation:

$$k_{hr} = \sqrt[t]{\frac{q_t}{q_o}} \qquad (4)$$

where:

$q_o$ = discharge at first point,
$q_t$ = discharge at second point, and
$t$ = time interval, in hours, between points 1 and 2.

3. Storage, or volume after point $y$ (in c.f.s.-hours) equals:

$$\frac{-q_y}{\log_e k_{hr}} \qquad (5)$$

where:

$q_y$ = discharge in c.f.s. at point $y$, and
$\log_e k_{hr} = 2.3026 (\log_{10} k_{hr})$.

4. Total volume is sum of volume to $y$ plus volume after $y$.

(d) For comparison with rainfall data, convert volume of ③ to inches of runoff:

Inches of runoff =

$$\frac{\text{volume in c.f.s.-hours}}{(\text{area in sq. mi.}) \text{ X } 645.3^*} \qquad (6)$$

*(1 inch $P_{e_1}$/sq. mi. = 26.888 c.f.s.-days: (26.888)(24) = 645.3 c.f.s.-hrs.)

(e) Analyze rainfall data, if available; determine period $D$ of rainfall excess.

(f) Compute time of occurrence of one-half volume of hydrograph ③, figure L-5(C). The time to center of volume, $T_{cv}$, equals time from beginning of rise of net hydrograph to time one-half volume has passed measuring point.

(g) Find lag, $Lg$, time in hours from midpoint of excess rainfall period to time of occurrence of one-half volume.

(h) Compute dimensionless graph as follows and plot on semilog paper, ④ on figure L-5(B).

1. Abscissa–hours from beginning of excess rain expressed as percent of $(Lg + D/2)$.
2. Ordinates–discharge in c.f.s. of ③ (at respective abscissa) multiplied by $(Lg + D/2)$, all divided by net hydrograph volume expressed as

$$\text{c.f.s.-days} = \left(\frac{\text{c.f.s.-hours}}{24}\right).$$

(2) *Lag-time curves.*–Lag-time is a key function for estimating synthetic unitgraphs. An average lag-time value for a watershed is obtained by averaging the results of several good analyses of stream gage records. Such average values for different gages on a stream and/or different streams of similar runoff characteristics can be correlated empirically with certain measurable watershed features. The correlation equation most often used in the Bureau of Reclamation is of the form:

$$\text{Lag-time, hours} = C \left[\frac{LL_{ca}}{\sqrt{S}}\right]^{x} \qquad (7)$$

where: $C$ and $x$ are constants,

Table L-5.–*Hydrograph analysis computations.*

BASIC DATA:

Name of streamgage = (Hypothetical for this table)  A, drainage area, sq. mi. = 319

Date of flood = (Assume May 1-3, 1970)  Volume, c.f.s.-days, net = 26,150

Time, beginning of direct runoff–net hydrograph = 12:00 p.m., 30 April

Time, point of 50 percent volume of net hydrograph, $T_{cv}$ = 9:30 a.m., 1 May

Lag-plus-semiduration, hrs.; $\left(Lg + \frac{D}{2}\right) = 9.5$

Duration of rainfall excess, $D$, hrs. = 4 (obtained by storm analysis)

Lag-time, hrs. = $\left(Lg + \frac{D}{2}\right) - \left(\frac{D}{2}\right) = 7.5$

$Q$ = instantaneous discharge, c.f.s.

| Time | | Hydrographs | | | Net volume | | Dimensionless-graph | |
|---|---|---|---|---|---|---|---|---|
| Hour and day | Net $\Sigma$hr. | Total flood, $Q$ | Base flow $Q$ | Net $Q$ | Increm.[2] c.f.s.-hrs. | Accum. 1,000 c.f.s.-hrs. | Abscissae, percent of $Lg + \frac{D}{2}$ | Ordinates, net $Q$ x $\left[\frac{Lg + \frac{D}{2}}{\text{vol.}}\right]$ |
| 12P30 | 0 | 2,000 | 2,000 | 0 | 0 | 0 | 0 | 0 |
| 1A1 | 1 | 2,250 | 2,000 | 250 | 125 | .12 | 10.5 | 0.09 |
| 2A1 | 2 | 3,560 | 2,000 | 1,560 | 905 | 1.03 | 21.1 | 0.57 |
| 3A1 | 3 | 8,120 | 2,000 | 6,120 | 3,840 | 4.87 | 31.6 | 2.22 |
| 4A1 | 4 | 18,640 | 2,000 | 16,640 | 11,380 | 16.25 | 42.1 | 6.0 |
| 5A1 | 5 | 36,040 | 2,000 | 34,040 | 25,340 | 41.59 | 52.6 | 12.4 |
| 6A1 | 6 | 56,290 | 2,000 | 54,290 | 44,165 | 85.76 | 63.2 | 19.7 |
| 7A1 | 7 | 70,510 | 2,000 | 68,510 | 61,400 | 147.16 | 73.7 | 24.9 |
| 8A1 | 8 | 73,000 | 2,000 | 71,000 | 69,755 | 216.91 | 84.2 | 25.8 |
| 9A1 | 9 | 66,330 | 2,000 | 64,330 | 67,665 | 284.58 | 94.8 | 23.4 |
| 10A1 | 10 | 55,360 | 2,000 | 53,360 | 58,845 | 343.42 | 105.8 | 19.4 |
| 11A1 | 11 | 43,250 | 2,000 | 41,250 | 47,305 | 390.72 | 115.8 | 15.0 |
| 12N1 | 12 | 33,520 | 2,000 | 31,520 | 36,385 | 427.11 | 126.4 | 11.4 |
| 1P1 | 13 | 26,900 | 2,020 | 24,880 | 28,200 | 455.31 | 136.9 | 9.0 |
| 2P1 | 14 | 22,830 | 2,050 | 20,780 | 22,830 | 478.14 | 147.4 | 7.5 |
| 3P1 | 15 | 19,810 | 2,080 | 17,730 | 19,255 | 497.40 | 158.0 | 6.4 |
| 4P1 | 16 | 17,230 | 2,100 | 15,310 | 16,520 | 513.92 | 168.5 | 5.6 |
| 5P1 | 17 | 15,390 | 2,120 | 13,270 | 14,290 | 528.20 | 179.0 | 4.8 |
| 6P1 | 18 | 13,780 | 2,150 | 11,630 | 12,450 | 540.66 | 189.5 | 4.2 |
| 8P1 | [1]20 | 11,090 | 2,200 | 8,890 | (20,520) | (561.18) | 210.6 | 3.23 |
| 12P1 | 24 | 7,460 | 2,300 | 5,160 | (28,100) | (589.28) | | |
| 6A2 | 30 | 4,840 | 2,500 | 2,340 | (22,500) | (611.78) | [3]315.9 | [3].85 |
| 12N2 | 36 | 3,700 | 2,650 | 1,050 | (10,170) | (621.94) | | |
| 6P2 | 42 | 3,305 | 2,830 | 475 | ( 4,575) | (626.52) | [3]442.3 | [3].17 |
| 12P2 | 48 | 3,215 | 3,000 | 215 | ( 2,070) | (628.59) | | |
| 6A3 | 54 | 3,100 | 3,000 | 100 | ( 960) | (629.55) | | |
| 12N3 | 60 | 3,045 | 3,000 | 45 | ( 420) | (629.97) | | |
| 6P3 | 66 | 3,020 | 3,000 | 20 | ( 180) | (630.15) | | |
| 12P3 | 72 | 3,010 | 3,000 | 10 | ( 90) | (630.24) | | |
| 6A4 | 78 | 3,000 | 3,000 | 0 | ( 30) | (630.27) | | |

---

[1] Note variations in time intervals for listing discharges (optional).

[2] c.f.s-hrs. = $\left(\frac{Q_1 + Q_2}{2}\right)$ x (time interval, hrs.)

[3] For plot on semilog paper, only enough points to define a straight line need be computed.

Table L-5.—Continued

Equations for dimensionless-graph:

$$\text{Abscissae} = \frac{\text{net } \Sigma \text{ hr.}}{Lg + \frac{D}{2}} \times 100$$

$$\text{Ordinates} = \text{net } Q \times \frac{Lg + \frac{D}{2}}{\text{vol., c.f.s.-days}}$$

$$\left[ \text{c.f.s.-days} = \left( \frac{\text{c.f.s.-hours}}{24} \right) \right]$$

Lag-plus-semiduration:

1/2 volume is between net Σ hrs. 9 and 10
By linear interpolation:

Volume, method 1,

$$Lg + \frac{D}{2} = 9.50 \text{ hrs.}$$

Volume, method 2,

$$Lg + \frac{D}{2} = 9.52 \text{ hrs.}$$

Except for very small watersheds, lag-plus-semiduration values are rounded to nearest 1/10 hr.

For dimensionless-graph equations:

Use: $Lg + \frac{D}{2} = 9.5$

Volume = 26,150 c.f.s.-days

Lag estimate:

$D = 4$ hrs.

$\text{Lag} = 9.5 - \frac{D}{2} = 7.5$ hrs.

Net volume computations:

Method 1, by equations.

$q_o$: $Q$ at net Σ hr. 20 = 8,890 c.f.s.
$q_t$: $Q$ at net Σ hr. 30 = 2,340 c.f.s.
t: time interval, $q_o$ to $q_t$ = 10 hrs.

$$k_{hr} = \sqrt[t]{\frac{q_t}{q_o}} = \sqrt[10]{\frac{2,340}{8,890}}$$

$$k_{hr} = \sqrt[10]{0.263} = 0.875$$

$$\text{Volume after net } \Sigma \text{ hr. } 20 = \frac{-q_o}{\log_e k_{hr}}$$

$$= \frac{-8,890}{-0.1336}$$

= 66,540 c.f.s.-hrs.

Σ net volume, hrs. 0-20 = 561,180 c.f.s.-hrs.
Total net volume = 627,720 c.f.s.-hrs.
= 26,150 c.f.s.-days
½ total net volume = 313,860 c.f.s.-hrs.

Method 2.
Ordinates of total net hydrograph used as shown in table at left.
Discharges of recession limb read at time intervals for which recession curve can be approximated as a straight line.

Total volume = 630,270 c.f.s.-hrs.
= 26,260 c.f.s.-days
½ volume = 315,140 c.f.s.-hrs.

---

$L$ = length of longest watercourse from point of interest to watershed divide, measured in miles,
$ca$ = centroid of basin—usually found by vertically suspending a cardboard cutout of basin shape successively from two or more points and finding intersection of plumb lines from each point,
$L_{ca}$ = length of watercourse from point of interest to intersection of perpendicular from $ca$ to stream alinement, and
$S$ = overall slope in feet per mile of longest watercourse from point of interest to divide.

Values for the constants $C$ and $x$ are obtained empirically from plots on *log-log paper* of $\frac{LL_{ca}}{\sqrt{S}}$ values versus lag-time, hours, and fitting a straight line, either "by eye" or by least-squares computations. The lag-time indicated by the curve for an $\frac{LL_{ca}}{\sqrt{S}}$ value of 1.0 is the constant $C$, and the "slope" of the line on log-log paper is the constant $x$.

A lag-time curve for a watershed should be based on as many hydrograph analyses as can be obtained from the data available within the watershed and for other watersheds with similar runoff characteristics. When developing a lag-time curve, a consistent method of hydrograph analyses should be used and measurements of watercourse lengths should be made on maps of the same scale. If suitable data are limited to only one stream gage location, a lag-time curve can be constructed by drawing a line with slope of 0.33 through the point plotted on log-log paper of average lag-time versus $\frac{LL_{ca}}{\sqrt{S}}$ value.

In the absence of any runoff data suitable for hydrograph analyses, preliminary estimates of lag-times for *direct runoff* for watersheds having rapid runoff characteristics can be made by the following generalized equation:

$$\text{Lag-time, hours} = 1.6 \left[\frac{LL_{ca}}{\sqrt{S}}\right]^{0.33}$$

The above equation gives values acceptable as preliminary estimates of direct runoff lag-times for many streams in the plains and southwestern regions of the United States and for foothill streams of the Rocky Mountains. Certain types of watersheds have large variations in lag-times that are not adequately reflected by the generalized $C$ value given. These include watersheds which have physical features tending to retard surface runoff such as near level terrain, dense vegetative cover, etc.; and those in which the streams extend into high, well-forested mountains or whose streamflow records show pronounced interflow contribution. Lag-time estimates for such watersheds should be made by an experienced hydrologist.

## C. SYNTHETIC UNIT HYDROGRAPH

L-9. *Synthetic Unitgraphs by Lag-Time Dimensionless-Graph Method*.—Computation of a unitgraph for a watershed above a specific location by this method is done by reversing the mathematical process used to derive a dimensionless-graph. The important factors for obtaining a representative unitgraph for a given watershed are the selections of a proper lag-time curve and proper dimensionless-graph. An example of a unitgraph derivation for an ungaged watershed follows, given as a step-by-step outline with pertinent comments and graphically illustrated on figure L-6.

(1) Outline drainage boundary, determine area (fig. L-6(A)).

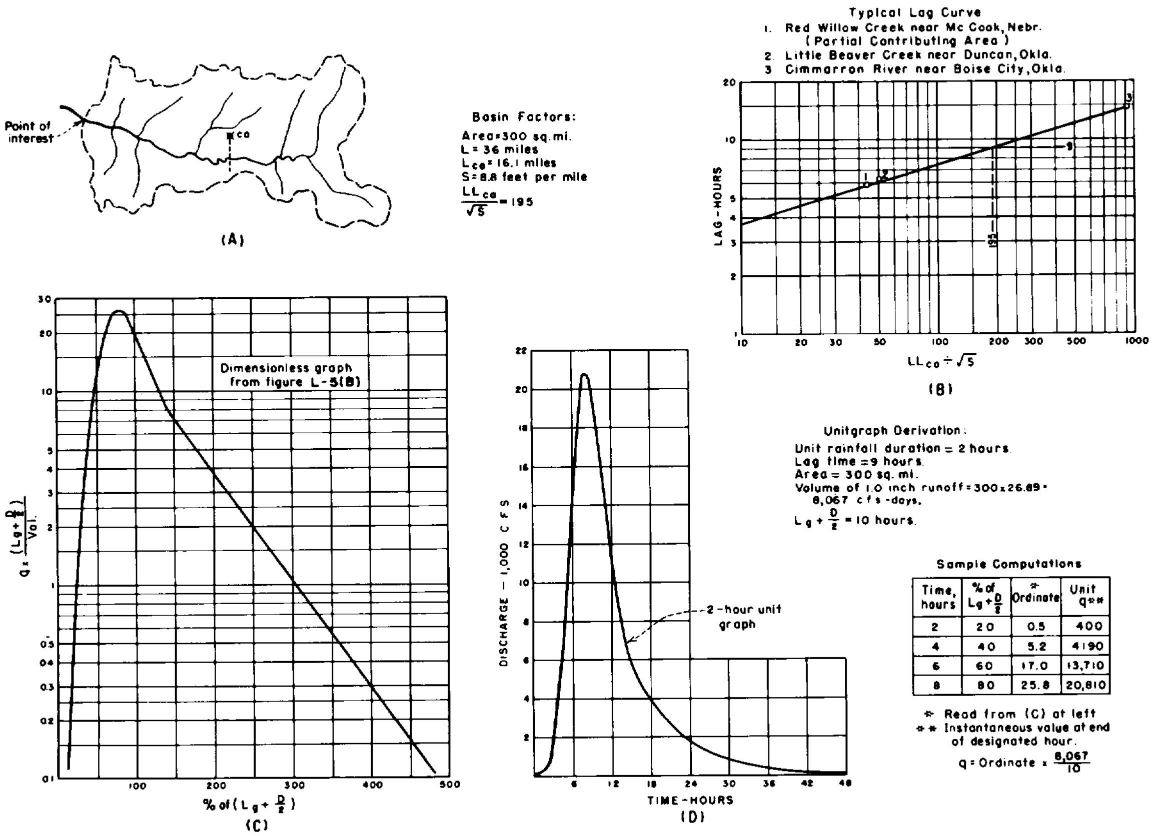
(2) Find basin center of area, *ca*, and project to the nearest point on the longest watercourse. Measure $L$ (to divide at head of longest watercourse) and $L_{ca}$ miles. (Refer to sec. L-8(e)(2).) Determine $S$ (for upper elevation, estimate average elevations along divide in vicinity of head of longest watercourse, not the specific elevation at the point of extention of longest watercourse to divide).

(3) Compute $\frac{LL_{ca}}{\sqrt{S}}$.

(4) Enter graph, lag-time curve (fig. L-6(B)), with $\frac{LL_{ca}}{\sqrt{S}}$ value and read the corresponding lag-time. (Lag-time curve (B) represents mean curve drawn "by eye" through plotted lag-times obtained from hydrograph analyses versus respective $\frac{LL_{ca}}{\sqrt{S}}$ for basins of similar runoff characteristics.)

(5) Select a dimensionless-graph (fig. L-6(C)) (usually an average dimensionless-graph of a number of dimensionless-graphs derived for the same stream or for streams of similar characteristics).

(6) Select a unit rainfall duration time; this should be one-fourth or less of lag-time for basin. (Unit times are selected for



Unitgraph Derivation:

Unit rainfall duration = 2 hours.
Lag time = 9 hours.
Area = 300 sq. mi.
Volume of 1.0 inch runoff = 300 x 26.89 = 8,067 c f s -days.
$L_g + \frac{D}{2} = 10$ hours.

Sample Computations

| Time, hours | % of $L_g+\frac{D}{2}$ | * Ordinate | Unit q** |
|---|---|---|---|
| 2 | 20 | 0.5 | 400 |
| 4 | 40 | 5.2 | 4190 |
| 6 | 60 | 17.0 | 13,710 |
| 8 | 80 | 25.8 | 20,810 |

* Read from (C) at left
** Instantaneous value at end of designated hour.

$q = \text{Ordinate} \times \frac{8{,}067}{10}$

*Figure L-6.* Unitgraph derivation for ungaged area.–288-D-3182

computational convenience, usually 1-, 2-, 3-, 4-, or 6-hour units for lag-times of 4 hours or greater. Unit times larger than 6 hours are seldom used. Units of one-half or one-quarter hour are used for lag-times less than 4 hours.)

(7) Compute unitgraph (fig. L-6(D)) using:

(a) Basin area, square miles.

(b) Lag-time plus one-half selected unit rainfall duration time.

(c) Dimensionless-graph.

(d) Notes regarding computational procedure:

1. Equations for deriving a dimensionless-graph are given in table L-5. Unitgraph computation requires solving for instantaneous discharges at end of successive unit time intervals.

2. Time, hours, accumulative by unit time intervals are listed, and each accumulative value expressed as percent of lag-plus-semiduration.

3. Dimensionless-graph (fig. L-6(C)) is entered with successive lag-plus-semiduration values, and respective ordinates read from the graph. Ordinates are substituted in the ordinate equation for solution of discharge values. When done by desk calculator, discharges are rounded.

(*Note*: Dimensionless-graph ordinates listed in the table of sample computations (fig. L-6) do not agree numerically at respective accumulative time values with dimensionless-graph ordinates in table L-5, because the dimensionless-graph ordinates in the table were derived at intervals of 10.5 percent of lag-plus-semiduration but the ordinates for 2-hour unitgraph derivation in figure L-6 were read at intervals of 20 percent of a different lag-plus-semiduration value.)

4. *Caution.*—The volume of a synthetic unitgraph should always be checked before being used, to be sure it has a

volume within 1 percent of 1-inch runoff volume for the watershed area. All of the ordinates of a unitgraph ((D) of fig. L-6) may be computed by reading the entire dimensionless-graph (C) and summing the ordinates to check the volume.

Another procedure may be used if the selected dimensionless-graph has an exponential recession limb such as on figure L-6(C). Unitgraph ordinates are obtained by reading the dimensionless-graph forward to an ordinate that is on the beginning portion of the exponential limb of the dimensionless-graph (see sec. L-8(e)(1)(b)). The volume of the unitgraph thus far obtained is computed and subtracted from the volume of 1 inch of runoff for the watershed area, giving the remaining volume, $V_x$. A recession constant, $k$, for the selected unit time interval can be computed by the equation,

$$\log_e k = -\frac{q}{V_x} \tag{8}$$

where:

$q$ = the discharge ordinate, c.f.s., on the exponential limb, and

$V_x$ = the remaining volume expressed in unit time (c.f.s.-hours).

The factor $k$ is used to compute the ordinates of the unitgraph following the last ordinate obtained by reading the dimensionless-graph. This procedure assures correct unitgraph volume.

L-10. ***Trial Reconstruction of Past Floods.***—Final decisions regarding appropriate lag-time, dimensionless-graph, and retention losses for a gaged watershed are made empirically by computing hydrographs of past recorded floods. Retention losses believed appropriate are applied to the observed storm precipitation data for each flood to be reconstructed to determine unit time increments of rainfall excess equivalent to the respective hydrograph volume. These increments are applied to a representative unitgraph according to basic assumption (3), figure L-3. The hydrograph thus computed is compared with the recorded hydrograph for *goodness of fit*; preliminary conclusions regarding appropriate factors are revised, if necessary, until an acceptable *fit* is obtained. These *test* trial reconstructions should be made for the large floods. Preferably, the largest flood of record should be excluded from the set of hydrographs selected for analyses and the parameters resulting from analyses tested by the *fit* achieved using them to reconstruct the largest flood.

L-11. ***Synthetic Unitgraphs by Other Methods.***—Descriptions of several different methods of estimating synthetic unitgraphs may be found in technical publications. Among those often used are the *S*-curve hydrograph [8], Snyder's method [9], and basin routing methods based on the Clark approach [5],[10],[11],[12].

## D. STREAMFLOW ROUTING

L-12. *General.*—Computation of an inflow design flood (IDF) hydrograph often requires that floodflows from several subareas within the drainage area be computed separately. Beginning with the farthest upstream subarea, hydrographs are transferred downstream by some method of streamflow routing, the flows being consecutively combined with other flood hydrographs, and the total inflow design flood hydrograph obtained for the proposed reservoir. Watershed features above a damsite which indicate the need to subdivide the basin into subareas include:

(1) Large tributary areas which have different sizes, shapes, and cover characteristics.

(2) Existing reservoirs or natural lakes which control runoff from significant portions

of the drainage area above a proposed damsite. The flood runoff from the portion of the design storm for the total drainage area that occurs above such an existing feature should be reservoir-routed through the feature to obtain an outflow hydrograph before routing on downstream. If an existing dam impounds a large-capacity reservoir, the capability of the existing dam to safely withstand the computed inflow flood must be determined. Should the upstream dam be found to have an inadequate spillway capacity (or structural weakness), steps should be taken to get the owners of the upstream dam to make modifications as necessary to safely pass the inflow design flood. Or as an alternative, failure of the structure should be assumed and provision made at the proposed downstream dam and reservoir to safely handle the flood wave surge that might be expected with failure and an additional inflow volume equivalent to the capacity of the upstream reservoir.

(3) Drainage areas in which storm potential varies to an extent that an assumption of average precipitation over the total area during a design storm is unreasonable.

(4) Drainage areas in which during design storm conditions some streams will have snowmelt runoff in addition to rainfall runoff and other streams have only rainfall runoff.

L-13. ***Practical Methods of Streamflow Routing Computations.***—Streamflow routing, the determination of a flood discharge hydrograph at any point on a stream from a discharge hydrograph at some point upstream, requires solution of the movement of flood waves in natural open channels which are extremely complex. A discussion of the theoretical and mathematical bases of flood routing methods is beyond the scope of this text. Many different methods and procedures have been described in engineering literature. If streamflow routing is necessary in the derivation of an inflow design flood hydrograph and the damsite is located on a stream that has discharge records at two or more locations, an applicable routing method may be selected from descriptions in publications, for example, "Hydrology for Engineers" [13].

Usually, inflow design flood derivations that include streamflow routing computations involve ungaged streams. Description of two practical methods of mathematical streamflow routing which can be used on the basis of an estimate of peak discharge travel time between two points on a reach of natural stream channel follows. These methods have been found to give acceptable results when tested by using recorded discharge hydrographs.

(a) *Tatum's Method* [14].—This method is also known as the *Method of Successive Averages*. Factors used when applying this method are travel time of peak discharge through the channel reach, $T$ in hours; selected routing interval between discharges of the upstream hydrograph to be routed, $t$ in hours; and routing constants listed in table L-6 for respective number of routing steps. Definite rules for selecting lengths of stream channel reaches for each routing computation cannot be set, but use of extremely long reaches may give very poor results. When computing an inflow design flood hydrograph, channel reaches are those on the main stream between points of inflow from subareas. Thus, inflow from a subarea can be added to the routed flow at the subarea inflow point to obtain a combined floodflow for routing through the next reach. After estimating travel time $T$ believed applicable for a reach, a routing interval $t$ is selected choosing an interval small enough to define well the hydrograph, and the number of routing steps for that reach computed by the equation:

$$\text{Number of routing steps} = 2T/t \qquad (9)$$

Computed fractional steps are rounded to the nearest whole number. The computational procedure is illustrated in table L-7. In actual practice when using a desk calculator, the routing constants are copied in a column on a separate sheet of paper and used as a slide beside the column of discharges to be routed. Products of the multiplications of constants and respective discharges are accumulated in the machine and only the total of each set of multiplications recorded. Constants for larger numbers of routing steps than given in table

Table L-6.—*Coefficients for floodrouting by Tatum's method.*

| Routing constants | Number of routing steps | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $C_1$ | 0.5000 | 0.2500 | 0.1250 | 0.0625 | 0.0313 | 0.0156 | 0.0078 | 0.0039 | 0.0020 | 0.0010 |
| $C_2$ | .5000 | .5000 | .3750 | .2500 | .1562 | .0937 | .0547 | .0313 | .0176 | .0098 |
| $C_3$ | | .2500 | .3750 | .3750 | .3125 | .2344 | .1641 | .1094 | .0703 | .0440 |
| $C_4$ | | | .1250 | .2500 | .3125 | .3126 | .2734 | .2187 | .1641 | .1172 |
| $C_5$ | | | | .0625 | .1562 | .2344 | .2734 | .2734 | .2460 | .2050 |
| $C_6$ | | | | | .0313 | .0937 | .1641 | .2187 | .2460 | .2460 |
| $C_7$ | | | | | | .0156 | .0547 | .1094 | .1641 | .2050 |
| $C_8$ | | | | | | | .0078 | .0313 | .0703 | .1172 |
| $C_9$ | | | | | | | | .0039 | .0176 | .0440 |
| $C_{10}$ | | | | | | | | | .0020 | .0098 |
| $C_{11}$ | | | | | | | | | | .0010 |

L-6 may be computed from the expression ($\frac{1}{2}$ + $\frac{1}{2}$)$^n$ by the general equation for each term of a binomial expansion, $n$ as the number of steps. Streamflow routing by Tatum's method using a desk calculator becomes tedious and time consuming when more than eight routing steps are used. The procedure may be easily programed for computer use.

(b) *Translation and Storage Method.*—In a paper describing a graphical reservoir-routing method, Wilson [15] also discusses streamflow routing, pointing out that it is partly analogous to reservoir routing but that natural channel storage produces less "flattening" effect on an inflow hydrograph than does reservoir storage. He suggested that in streamflow routing, the outflow (routed) hydrograph would lie between a hydrograph obtained by applying the graphical reservoir-routing method and the inflow hydrograph translated downstream a time interval equivalent to the reach travel time, and presented an example in which the routed hydrograph showed half translatory effect and half storage effect.

A report of the California Division of Water Resources [16] presented a streamflow routing method based on an adaptation of Wilson's graphical routing method showing that translation effect (travel time) and channel storage effect (attenuation) on the shape of a flood hydrograph moving downstream can be treated separately. In their studies, each effect was found to have approximately equal weight.

The translation and storage method of streamflow routing was devised[5] on the basis of evaluating separately the effects of travel time and channel storage and assuming equal weight for each effect in natural stream channels having "usual" storage characteristics. An equation for mathematical application of Wilson's graphical routing method was given in the U.S. Department of Agriculture, Soil Conservation Engineering Handbook, Supplement A, 1956. The given equation is used in the translation and storage method of streamflow routing as follows:

$$O_2 = O_1 + K(I_1 + I_2 - 2\,O_1) \qquad (10)$$

where:

$I_1$, $I_2$ = inflow, consecutive incremental instantaneous discharges at the head of a stream reach, and

$O_1$, $O_2$ = outflow, successive incremental instantaneous discharges at the end of a stream reach; $O_2$ is the outflow resulting from $I_1$ and $I_2$ and the preceding outflow $O_1$.

The routing constant, $K$, in the above equation, is obtained as follows:

$T$ = travel time, hours, of peak flow through the reach consisting of:

---

[5] Described in unpublished memoranda, Flood Hydrology Section, Engineering and Research Center, Bureau of Reclamation, Denver, Colo.

Table L-7.–*Illustrative example of streamflow routing by Tatum's method.*

**HYPOTHETICAL PROBLEM:** Streamflow-route total flood hydrograph, table L-5, through channel reach having travel time of 4 hours.

$$\text{If selected } t = 1 \text{ hr., routing steps} = \frac{(2)(4)}{1} = 8$$

$$\text{If selected } t = 2 \text{ hrs., routing steps} = \frac{(2)(4)}{2} = 4$$

| Hour and date | Upstream $Q$ 1,000 c.f.s. | $t$ = 1 hr., 8 routing steps | | | | Routed [3]$Q$ 1,000 c.f.s. | $t$ = 2 hrs., 4 routing steps | | | | Routed [3]$Q$ 1,000 c.f.s. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Illustrative positioning of routing constants[2] | | | | | Illustrative positioning of routing constants[2] | | | | |
| 4P30 | [1]2.0 | | | | | | | | | | |
| 5P | 2.0 | 0.0039 | | | | | | | | | |
| 6P | 2.0 | .0313 | | | | | 0.0625 | | | | |
| 7P | 2.0 | .1094 | | | | | | | | | |
| 8P | 2.0 | .2184 | | | | | .2500 | | | | |
| 9P | 2.0 | .2734 | | | | | | | | | |
| 10P | 2.0 | .2187 | | | | | .3750 | | | | |
| 11P | 2.0 | .1094 | | | | | | | | | |
| 12P30 | 2.0 | .0313 | | | | | .2500 | | | | |
| 1A1 | 2.3 | .0039 | | | | [4]2.0 | | | | | |
| 2A | 3.6 | | | | | | .0625 | 0.0625 | | | [4]2.1 |
| 3A | 8.1 | | 0.0039 | | | | | | | | |
| 4A | 18.6 | | .0313 | 0.0039 | | | | .2500 | 0.0625 | | |
| 5A | 36.0 | | .1094 | .0313 | 0.0039 | | | | | | |
| 6A | 56.3 | | .2187 | .1094 | .0313 | | | .3750 | .2500 | 0.0625 | |
| 7A | 70.5 | | .2734 | .2187 | .1094 | | | | | | |
| 8A | 73.0 | | .2187 | .2734 | .2187 | | | .2500 | .3750 | .2500 | |
| 9A | 66.3 | | .1094 | .2187 | .2734 | | | | | | |
| 10A | 55.4 | | .0313 | .1094 | .2187 | | | .0625 | .2500 | .3750 | 47.7 |
| 11A | 43.2 | | .0039 | .0313 | .1094 | 61.3 | | | | | |
| 12N1 | 33.5 | | | .0039 | .0313 | [5]64.8 | | | .0625 | .2500 | [6]58.6 |
| 1P1 | 26.9 | | | | .0039 | 61.7 | | | | | |
| 2P | 22.8 | | | | | | | | | .0625 | 37.8 |

---

[1] Constant base flow of 2,000 c.f.s. assumed to precede flood event.

[2] All routing constants are placed opposite respective $Q$'s at $t$ intervals.

[3] Discharge at bottom of reach; each $Q$ is instantaneous discharge at time given in column 1.

[4] Sum of products of each constant times respective $Q$.

[5] Peak discharge of routed hydrograph, occurs 4 hours later than upstream peak.

[6] Peak discharge of routed hydrograph, agrees in time with routing $t$ = 1 hr., but differs in magnitude because of longer routing interval.

---

$T_r$ = translation time component, hours (when assuming equal weight to storage effect, $T_r = 0.5T$)

$T_s$ = storage time component, hours (when assuming equal weight to translation effect, $T_s = 0.5T$)

and

$$T = T_r + T_s$$

Then for stream routing evaluation of storage time effect,

$$K = \frac{t}{2T_s + t}$$

where:

$t$ = routing time interval, hours, with $t \leq 0.5T_s$

Solving the equation for $O_2$ gives an instantaneous discharge value at the end of the incremental time interval designated by $I_2$. If $I_1$, $I_2$, etc., are designated by time at the head of a reach, the time of occurrence of $O_2$ at the bottom of the reach is obtained by adding the translation time component, $T_r$ to the time of respective $I_2$.

Use of the above equation with an assumption that the travel time for the reach is divided equally into translation time, $T_r$, and storage time, $T_s$, gives as acceptable results as those obtained by using Tatum's Method but requires less computational time when doing manual routing. A detailed example of application of the translation and storage method is shown in table L-8. Of course, in practice, such a detailed table is not necessary.

The translation and storage method, in addition to being easy to apply to stream reaches for which Tatum's method might be used, is also versatile enough to be applied to stream reaches having more or less storage effect than "usual." The relationship of storage time and translation time is not rigid, but may be varied depending on channel reach characteristics. If hydrographs are available at the head and bottom of a stream reach, a few trial routings will give an acceptable value for each component. Characteristics of ungaged stream channels are judged by comparison with characteristics of gaged streams when necessary to use streamflow routing methods.

(c) *Comparison of Methods.*—An illustration of results of applying the above two methods of streamflow routing is shown on figure L-7 on which the hypothetical flood hydrograph, with discharges listed in table L-5, is plotted. This hydrograph was routed downstream assuming a reach travel time of 4 hours: first, by Tatum's method assuming routing intervals of 1 hour and 2 hours; and secondly, by the translation and storage method using a routing interval of 1 hour. Routed (downstream) hydrographs are also plotted on figure L-7 (computations are not included). The two routed hydrographs obtained by Tatum's method differ because of different routing intervals; the routing by 1-hour intervals is the more representative because the upstream hydrograph is best defined in 1-hour intervals. The routed hydrograph obtained by the translation and storage method is acceptably similar to the hydrographs obtained by Tatum's method.

## E. DESIGN STORM STUDIES

L-14. ***General.***—Major floods, except those associated with dam failure, earthquakes, or landslides, result from a combination of severe meteorological and hydrological conditions. It follows that estimates of meteorological conditions which may approach the physical upper limits of rainfall or snow accumulation and melt rates must be considered where an inflow design flood (IDF) is required. This section is concerned only with rainfall studies. For the purpose of this text, the following terminology is used in regard to estimates of the physical upper limits of storm rainfall in a basin or region.

(a) *Probable Maximum Precipitation (PMP).*—Probable maximum precipitation values represent an envelopment of maximized intensity-duration values obtained from all types of storms. It is recognized that probable maximum precipitation values for all durations and all areas may not occur from only one type of storm. For example, a maximized thunderstorm is very likely to provide probable maximum precipitation over an area of 50 square miles for a duration of 6 hours or less, but the controlling values for longer durations or for larger areas generally will be obtained from general-type storms.

(b) *Probable Maximum Storm (PMS).*—The probable maximum storm values represent an envelopment of maximized intensity-duration values obtained from storms of a single type. Consideration is given to storm type and variations of precipitation with respect to

Table L-8.—*Translation and storage method of streamflow routing.*

Equation: $O_2 = O_1 + K(I_1 + I_2 - 2\,O_1)$

$T = 12$ hours $\qquad K = \frac{t}{2T_s + t}$

$T_r = 6$ hours $\qquad K = \frac{3}{12 + 3}$

$T_s = 6$ hours $\qquad K = 0.20$

$t = 3$ hours

(For definitions of symbols, see sec. L-13 (b).)

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|
| Time, hours[1] | Inflow, c.f.s. | $I_1 + I_2$, c.f.s. | $2\,O_1$ | (3) – (4) | (K)(5) | Outflow,[2] c.f.s. | Time, hours[4] |
| 0 | 300 | | | | | [3]300 | 6 |
| 3 | 300 | 600 | 600 | 0 | 0 | 300 | 9 |
| 6 | 415 | 715 | 600 | 115 | 23 | 323 | 12 |
| 9 | 1,604 | 2,019 | 646 | 1,373 | 275 | 598 | 15 |
| 12 | 5.458 | 7,062 | 1,196 | 5,866 | 1,173 | 1,771 | 18 |
| 15 | 10,093 | 15,551 | 3,542 | 12,009 | 2,402 | 4,173 | 21 |
| 18 | 16,567 | 26,660 | 8,346 | 18,314 | 3,663 | 7,836 | 24 |
| 21 | 17,924 | 34,491 | 15,672 | 18,819 | 3,764 | 11,600 | 27 |
| 24 | 18,608 | 26,532 | 23,200 | 13,332 | 2,666 | 14,266 | 30 |
| 27 | 19,244 | 37,852 | 28,532 | 9,320 | 1,864 | 16,130 | 33 |
| 30 | 19,772 | 39,016 | 32,260 | 6,756 | 1,351 | 17,481 | 36 |
| 33 | 25,913 | 45,685 | 34,962 | 10,723 | 2,145 | 19,626 | 39 |
| 36 | 23,499 | 49,412 | 39,252 | 10,160 | 2,032 | 21,658 | 42 |
| 39 | 20,552 | 44,051 | 43,316 | 735 | 147 | 21,805 | 45 |
| 42 | 17,377 | 37,929 | 43,610 | –5,681 | –1,136 | 20,669 | 48 |
| 45 | 14,703 | 32,080 | 41,338 | –9,258 | –1,852 | 18,817 | 51 |
| 48 | 12,054 | 26,757 | 37,634 | –10,877 | –2,175 | 16,642 | 54 |

[1] Time of instantaneous discharge at head of reach.

[2] Discharge at end of reach; (6) + preceding value in (7).

[3] Constant flow in reach assumed.

[4] Time of instantaneous discharge at end of reach. Translation time, $T_r$, added to time at head of reach.

location, areal coverage of a watershed, and storm duration.

(c) *Design Storm.*—The precipitation values selected for computing an inflow design flood are usually referred to as a design storm. These design storm values may or may not be equal to the PMP. The hydrometeorological report which describes the considerations and computations leading to the recommendation of a design storm for a particular watershed is usually called a "Design Storm Study."

(d) *Additional References.*—It is beyond the scope of this text to discuss in detail the meteorological considerations and computations involved in obtaining the "maximized intensity-duration values" cited in the above definitions. A comprehensive discussion of this subject is given in chapter 2, "Maximum Rainfall," of WMO Technical Note No. 98 [2]. A brief discussion on estimation of probable maximum storms is given in subsequent paragraphs. Also included in this section are generalized precipitation charts for estimating probable maximum precipitation values east of the 105° meridian and general-type design storm values west of the 105° meridian for watersheds in the 48 conterminous United States. These charts also are presented in chapter III of "Design of Small Dams," second edition [31], associated with procedures for

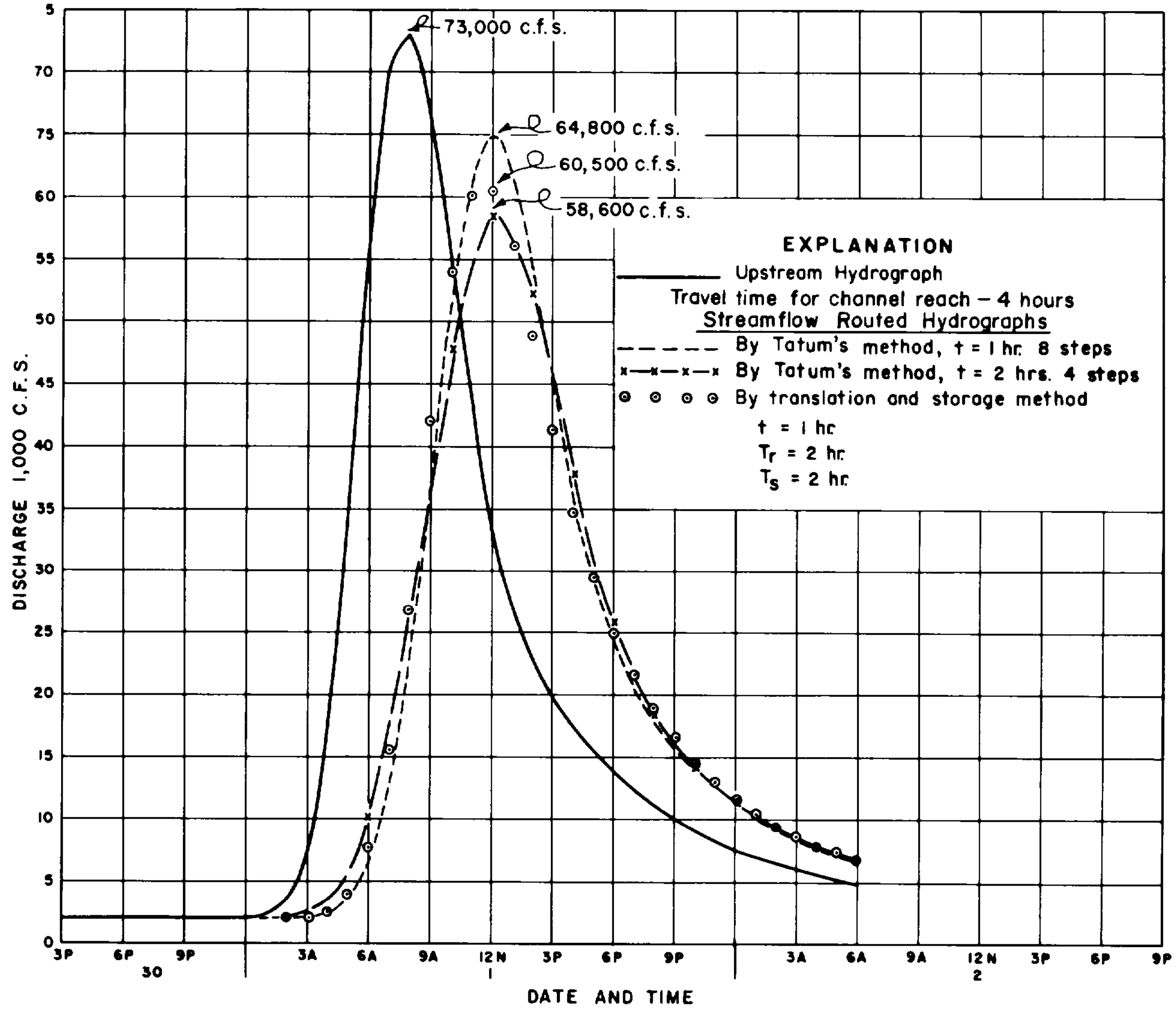


*Figure L-7.* **Comparison of results of streamflow routings.–288-D-3183**

estimating inflow design floods for small dams.

Discussion of design thunderstorm rainfall has been omitted in this text, anticipating that readers will be concerned generally with damsites controlling drainage areas large enough to preclude the use of thunderstorm rainfall. However, thunderstorm rainfall should never be ignored completely, as it may prove critical under some circumstances.

**L-15. *Probable Maximum Storm Considerations.***—Estimates of probable maximum storms (PMS) are based on analyses which consist of three steps: (1) determination of the areal and temporal distribution of the larger storms of record in the general area; (2) augmentation of these observed storms through moisture adjustment; and (3) consideration of storm transposition.

One objective of the first step cited above is the determination of maximum values of storm rainfall for selected durations and area. Depth-area-duration (DAD) values of each total storm are analyzed without regard to watershed boundaries [17]. Comparison of DAD values will indicate which storms are best suited for further analysis. If hydrographs of floods for specific watersheds associated with the storms are available for analyses, determination of rainfall data for these specific watersheds can be included as a part of the analyses.

Technical literature [2] should be consulted for a detailed discussion of the theoretical assumptions included in the computational procedures for storm maximization, step (2), and storm transposition, step (3). An abridged discussion of a procedure often used for maximization and transposition of storms in plains-type terrain follows. Discussion of procedures for storm maximization and limited transposition in mountainous terrain is beyond the scope of this text.

L-16. ***Procedure for Storm Maximization, Plains-Type Terrain.***—This procedure is based on assuming a saturated airmass with a pseudoadiabatic lapse rate. Moisture content under these circumstances is a unique function of surface dewpoint temperature, so that dewpoint temperatures may be used to quantitatively estimate total atmospheric water vapor or precipitable water values. Tables [18] have been published which list ambient temperatures for various elevations or pressures above a 1,000-mb. (1,000-millibar) surface, approximately equivalent to mean sea level, for selected temperatures in a saturated atmosphere with a pseudoadiabatic lapse rate.

Tables [18] also list, for each 1,000-mb. dewpoint temperature, values of precipitable water in inches for layers between the 1,000-mb. surface and various elevations to extreme heights in a saturated, pseudoadiabatic atmosphere. These precipitable water values may be used as an index to the moisture content of a unit column of air between sea level and the top of a moisture-bearing airmass. Maps with isotherms of maximum 12-hour persisting 1,000-mb. dewpoint temperatures ($^{\circ}$ F.) of record for each month for the 48 conterminous states are available in the "Climatic Atlas of the United States" [19].

Computational procedures for storm maximization and transposition, plains-type terrain, follow:

(a) *Maximization of a Storm in Place of Occurrence.*

(1) *Observed storm dewpoint.*—A representative 12-hour persisting surface dewpoint temperature is obtained for the storm period under study from temperature stations in the path of the inflowing moist air. If the rainfall is of a frontal type, the surface dewpoints within the rainfall area will be lower than those of the inflowing moist air, thus giving a low estimate of storm moisture content. Distance and direction from the storm center to the representative dewpoint station or stations should be recorded.

(2) *Adjustment to 1,000-mb. surface.*—Since during major storms the airmass will be saturated, the dewpoint temperature at the representative station can be adjusted to a 1,000-mb. surface temperature assuming a saturated, pseudoadiabatic lapse rate of temperature.

(3) *Precipitable water values.*—From the 1,000-mb. dewpoint temperature determined in (2) above, obtain two precipitable water values, $W_p$, for the observed storm:

(a) $W_{p\text{-}1}$ is the precipitable water between 1,000 mb. and the top of the moist layer for the storm system; an elevation of 40,000 feet, or pressure of 200 mb., is usually assumed.

(b) $W_{p\text{-}2}$ is the precipitable water between 1,000 mb. and the mean surface elevation of the central portion of the observed storm. If the inflowing moist air has passed over a topographical barrier with a higher elevation than at the central portion of the storm, $W_{p\text{-}2}$ is obtained using the inflow barrier elevation.

(4) *Observed storm's precipitable water, $W_s$.*—Compute the observed storm's moisture content or available precipitable water, $W_s$, as $W_{p\text{-}1}$ minus $W_{p\text{-}2}$.

(5) *Probable maximum precipitable water for the storm, $W_x$.*—An estimate of the probable maximum moisture content indicated for the storm is obtained as follows:

(a) From the "Climatic Atlas of the United States" [19], the maximum 12-hour persisting dewpoint temperature of record can be determined for the date of storm occurrence and the location of the representative dewpoint for the observed storm. Frequently, the maximum recorded dewpoint temperature within a period of plus or minus 15 days is used.

(b) From the maximum dewpoint of record, precipitable water is obtained for the same layers as used in $W_{p\text{-}1}$ and $W_{p\text{-}2}$ above. These precipitable water values are designated $W_{r\text{-}1}$ and $W_{r\text{-}2}$.

(c) The estimated probable maximum precipitable water, $W_x$, will be $W_{r\text{-}1}$ minus $W_{r\text{-}2}$.

(6) *Moisture maximization factor, $M_f$.*—The moisture maximization factor, $M_f$, is computed as the ratio of the probable maximum precipitable water to the precipitable water observed during the storm, or $M_f = W_x / W_s$.

(7) *Maximized storm values.*—Maximized storm values are computed by multiplying depth-area-duration (DAD) values of the observed storm by the maximization factor, $M_f$.

*Note*: This procedure assumes that the magnitude of rainfall in a storm is a function only of the inflow moisture charge. It also assumes that the most effective combination of storm efficiency and inflow wind has occurred or has been closely approached in the major storms of record. The procedure may not always prove adequate, particularly for regions where rainfall is strongly influenced by orographic effects [2].

(8) *Example of computations—maximization in place.*

(a) Dewpoint observation station: elevation 1000 feet.

Location: 100 miles southeast of storm center.
Representative 12-hour storm dewpoint: 69° F.
Sea level, 1,000 mb., dewpoint: 71° F.

(b) Surface elevation, storm center: 1500 feet.

$W_{p\text{-}1}$ = 2.38 inches (at 40,000 feet)
$W_{p\text{-}2}$ = 0.32 inch (at 1500 feet)
$W_s$ = 2.06 inches

(c) Maximum dewpoint of record, observed 100 miles southeast of storm center: 78° F. [19].

$W_{r\text{-}1}$ = 3.35 inches (at 40,000 feet)
$W_{r\text{-}2}$ = 0.41 inch (at 1500 feet)
$W_x$ = 2.94 inches

(d) Moisture maximization factor:

$$M_f = 2.94/2.06$$
$$M_f = 1.43$$

(b) *Maximization of Transposed Storm.*—When a storm is transposed and maximized for moisture content, the maximization factor is usually computed for the storm only at its transposed location. Computation of available precipitable water for the observed storm, $W_s$, remains the same as described above.

The moisture maximization factor is computed by determining the surface elevation at the center of the storm at its transposed position or the height of the mean inflow barrier to that location. The maximum dewpoint of record is obtained from the charts of dewpoints [19] at *the same distance from the transposed center and in the same direction as the observed storm dewpoint was obtained.*

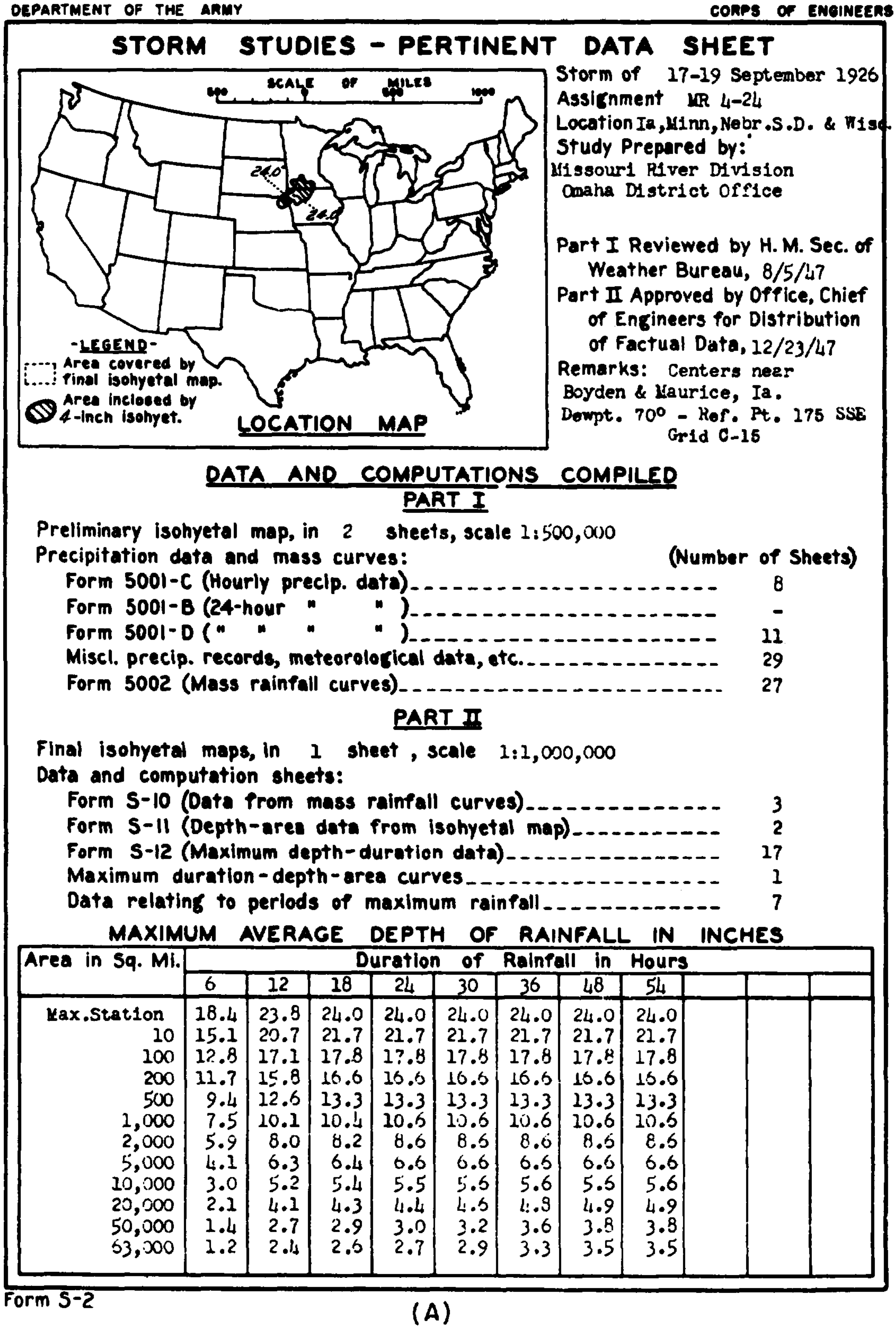
(1) *Example of computations—moisture maximization of transposed storm.*

(a) Assume that the storm used in the previous example is transposed to a location where the elevation of the storm center is 2500 feet and that there is not a higher inflow barrier between the transposed center and the moisture source.

(b) Mark the location of the transposed center on the charts of maximum recorded dewpoint temperatures and measure 100 miles southeast to determine the maximum dewpoint of record; for example 77° F.

(c) Observed storm precipitable water remains the same; $W_s$ = 2.06 inches.

(d) Maximum precipitable water for a dewpoint of 77° F:



DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

## STORM STUDIES - PERTINENT DATA SHEET

LOCATION MAP

Storm of 17-19 September 1926
Assignment MR 4-24
Location Ia, Minn, Nebr. S.D. & Wis.
Study Prepared by:
Missouri River Division
Omaha District Office

Part I Reviewed by H. M. Sec. of Weather Bureau, 8/5/47
Part II Approved by Office, Chief of Engineers for Distribution of Factual Data, 12/23/47
Remarks: Centers near Boyden & Maurice, Ia.
Dewpt. 70° - Ref. Pt. 175 SSE Grid C-15

### DATA AND COMPUTATIONS COMPILED

### PART I

Preliminary isohyetal map, in 2 sheets, scale 1:500,000

| Precipitation data and mass curves: | (Number of Sheets) |
|---|---|
| Form 5001-C (Hourly precip. data) | 8 |
| Form 5001-B (24-hour " " ) | - |
| Form 5001-D ( " " " " ) | 11 |
| Miscl. precip. records, meteorological data, etc. | 29 |
| Form 5002 (Mass rainfall curves) | 27 |

### PART II

Final isohyetal maps, in 1 sheet , scale 1:1,000,000

| Data and computation sheets: | |
|---|---|
| Form S-10 (Data from mass rainfall curves) | 3 |
| Form S-11 (Depth-area data from isohyetal map) | 2 |
| Form S-12 (Maximum depth-duration data) | 17 |
| Maximum duration-depth-area curves | 1 |
| Data relating to periods of maximum rainfall | 7 |

MAXIMUM AVERAGE DEPTH OF RAINFALL IN INCHES

| Area in Sq. Mi. | Duration of Rainfall in Hours | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6 | 12 | 18 | 24 | 30 | 36 | 48 | 54 | | | |
| Max.Station | 18.4 | 23.8 | 24.0 | 24.0 | 24.0 | 24.0 | 24.0 | 24.0 | | | |
| 10 | 15.1 | 20.7 | 21.7 | 21.7 | 21.7 | 21.7 | 21.7 | 21.7 | | | |
| 100 | 12.8 | 17.1 | 17.8 | 17.8 | 17.8 | 17.8 | 17.8 | 17.8 | | | |
| 200 | 11.7 | 15.8 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | | | |
| 500 | 9.4 | 12.6 | 13.3 | 13.3 | 13.3 | 13.3 | 13.3 | 13.3 | | | |
| 1,000 | 7.5 | 10.1 | 10.4 | 10.6 | 10.6 | 10.6 | 10.6 | 10.6 | | | |
| 2,000 | 5.9 | 8.0 | 8.2 | 8.6 | 8.6 | 8.6 | 8.6 | 8.6 | | | |
| 5,000 | 4.1 | 6.3 | 6.4 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 | | | |
| 10,000 | 3.0 | 5.2 | 5.4 | 5.5 | 5.6 | 5.6 | 5.6 | 5.6 | | | |
| 20,000 | 2.1 | 4.1 | 4.3 | 4.4 | 4.6 | 4.8 | 4.9 | 4.9 | | | |
| 50,000 | 1.4 | 2.7 | 2.9 | 3.0 | 3.2 | 3.6 | 3.8 | 3.8 | | | |
| 63,000 | 1.2 | 2.4 | 2.6 | 2.7 | 2.9 | 3.3 | 3.5 | 3.5 | | | |

Form S-2

(A)

*Figure L-8.* Example of summary sheet, "Storm Rainfall in the U.S." (sheet 1 of 2).–288-D-3184(1/2)

$W_{r\text{-}1}$ = 3.19 inches (at 40,000 feet)
$W_{r\text{-}2}$ = 0.64 inch (at 2500 feet)
$W_x$ = 2.55 inches

(e) Moisture maximization factor for the transposed storm:

$$M_f = 2.55/2.06$$
$$M_f = 1.24$$

*Note*: If an $M_f$ factor greater than 2.0 is computed, reexamine the computations and all meteorological aspects of the transposed storm. An $M_f$ factor greater than 2.0 has not been used in Bureau of Reclamation design storm studies.

(2) *Maximized transposed storm values.*—The maximized values for the transposed storm are computed by multiplying the DAD values of the observed storm by the maximization factor for the transposed location.

L-17. ***Design Storm—Probable Maximum Precipitation (PMP) or Probable Maximum Storm (PMS) Estimates for a Watershed.***—Estimates of PMP or PMS, whether made by storm transposition and procedure of dewpoint adjustment described above or by more detailed theoretical computations [20][6], are based generally on the results of analyses of observed storms. In the United States, passage of the Flood Control Act of 1936 led to the development of a National Storm Study Program under the primary sponsorship of the U.S. Army Corps of Engineers. Under this program more than 600 storms throughout the United States have been analyzed in a uniform manner and summary sheets distributed to Government agencies and the engineering profession [21]. An example of a storm analysis summary sheet from the publication "Storm Rainfall in the United States" [21] is shown on figure L-8. Each storm analyzed has been assigned a designation such as MR 4-24 on the figure. Unfortunately, not all of the summary sheets have a reference to the observed storm dewpoint, such as shown on figure L-8(A). Depth-area-duration (DAD) data for each storm analyzed are given in a table, such as the one at the bottom of figure L-8(A).

A storm location map and a few selected mass rainfall curves are given on figure L-8(B). Summaries of observed storm data such as presented in "Storm Rainfall in the United States," provide broad outlines of storm magnitudes and their seasonal and geographical variations.

A simplified example of the derivation of design storm values for a particular watershed follows. Sources of numerical values used are referenced when possible. The isohyetal patterns and watershed map are not presented. This example may provide the reader with information that will be useful in a better understanding of how preliminary design storm estimates are obtained from the generalized PMP charts given later.

(a) *Example of a Design Storm Study.*—(Final-type design storm studies should be prepared by experienced hydrometeorologists.) Let us assume that design storm values representing PMS estimates are required for a watershed with a 200-square-mile area at longitude 99°30′ west, latitude 41°00′ north, a region where storm transposition and maximization by dewpoint adjustment is an acceptable approach. Procedural steps are described first, then numerical computations are given.

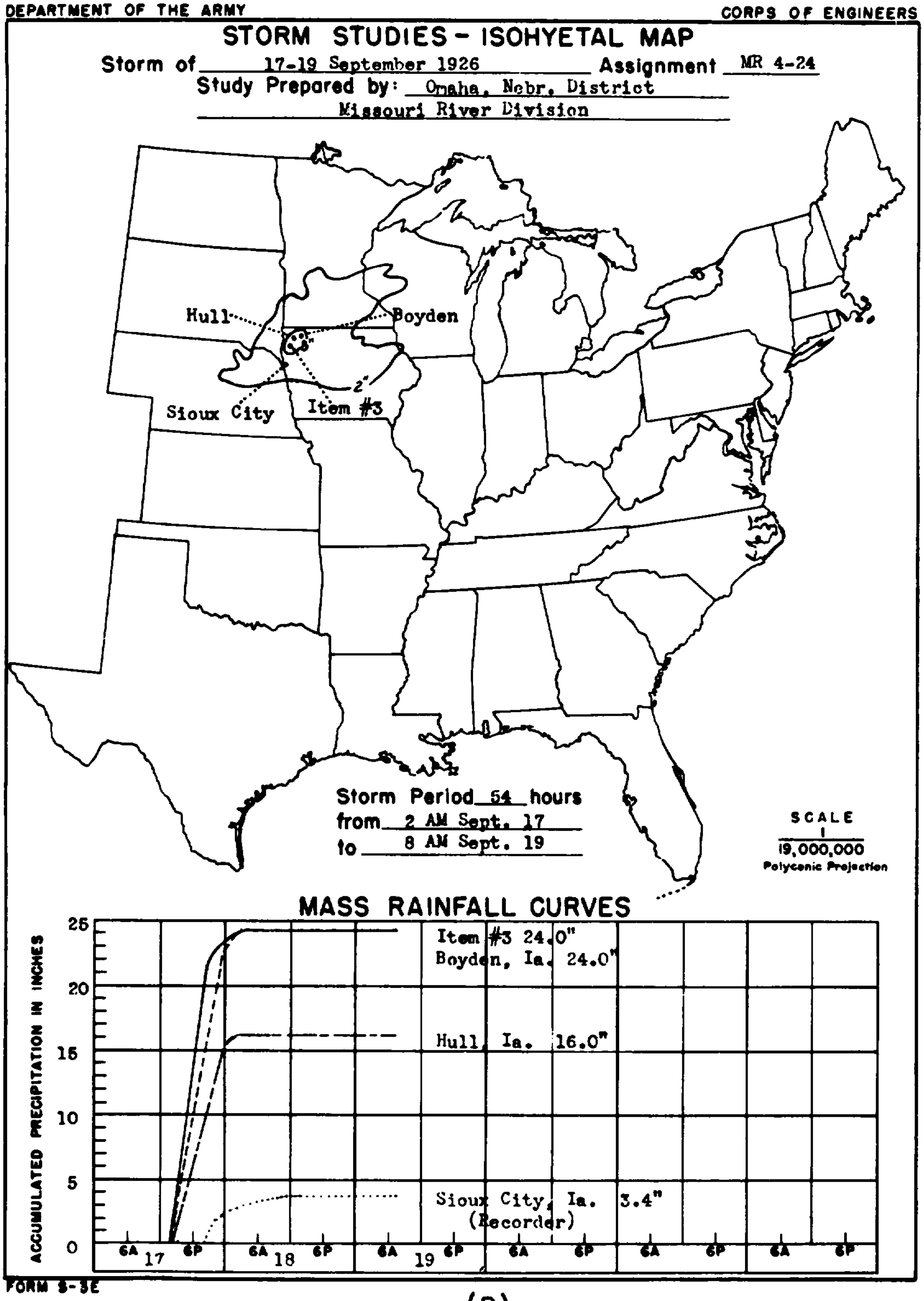
(1) *Transposition limits of major storms.*—The broad limits within which major observed storms can be transposed should be established first. This will require consultation with an experienced hydrometeorologist. However, for the United States east of the 105° meridian, guidelines have been established in Hydrometeorological Report 33 [20].

(2) *Inventory of data of major storms.*—Referring to "Storm Rainfall in the United States" [21], rainfall depth-duration values can be obtained for an area of 200 square miles for all major storms that have been analyzed in the region for which transposition is applicable. Analysis may be required for recent major storms in the region in order to complete the inventory.

(3) *Selection of storms for further study.*—Several of the larger storms are

---

[6] Includes 23 separate reports.



DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

STORM STUDIES - ISOHYETAL MAP

Storm of 17-19 September 1926 Assignment MR 4-24

Study Prepared by: Omaha, Nebr. District

Missouri River Division

Storm Period 54 hours

from 2 AM Sept. 17

to 8 AM Sept. 19

SCALE 1/19,000,000

Polyconic Projection

MASS RAINFALL CURVES

Item #3 24.0"

Boyden, Ia. 24.0"

Hull, Ia. 16.0"

Sioux City, Ia. 3.4" (Recorder)

FORM 5-3E

(B)

*Figure L-8.* Example of summary sheet, "Storm Rainfall in the U.S." (sheet 2 of 2).–288-D-3184(2/2) assumed transposed and the depth-duration values for 200 square miles maximized for maximum moisture charge to identify those storms that give the greatest values. Any individual storm may not yield maximum values for all durations. It may be necessary, therefore, to consider a number of storms in the final analysis.

(4) *Transposition of isohyetal patterns.*—The isohyetal patterns of the storms which yield large values should be obtained, and these patterns then overlaid individually on a map of the subject watershed. The position, within limits, that gives the greatest total basin average rainfall depth should be used. In positioning a transposed storm isohyetal pattern, the orientation of the observed storm pattern is maintained generally within limits of plus or minus 20°.

(5) *Average watershed precipitation of transposed storm.*—The average storm rainfall *within* the watershed boundaries of each transposed storm isohyetal pattern is obtained by planimetry. The depth of precipitation for a given area for the total storm was obtained from a DAD tabulation similar to that shown on figure L-8(A). These values were, of course, measured from the isohyetal pattern in the original storm without regard to any watershed boundaries. Obviously, only an assumption of a *perfect fit* of the transposed isohyetal pattern to the basin configuration would give the same total basin rainfall for the transposed storm as that listed in the DAD tables.

(6) *Fit-factor.*—A fit-factor, $F_f$, that is, the ratio of the watershed average rainfall depth to the storm pattern rainfall depth, for equal areas, is computed for each transposed storm. The importance of the fit-factor to PMS estimates varies depending on the size, shape, and orientation with respect to major storm patterns of each individual watershed. In the example region, watersheds are typically long and narrow with their major axis oriented generally east-west, so that a fit-factor in this region is quite important, except for extremely large drainage basins.

If $\overline{P}_o$ represents the average rainfall depth for the total observed storm for a given area and $\overline{P}_{tr}$ represents the average rainfall depth measured from the isohyetal pattern of the observed storm, as transposed, then

$$F_f = \frac{\overline{P}_{tr}}{\overline{P}_o} \tag{11}$$

It should be obvious that $F_f \leq 1$.

(7) *Total maximization adjustment factor, $Ad_f$.*—The total maximization adjustment factor, $Ad_f$, for a storm, as transposed to a watershed, is the product of the storm moisture maximization factor, $M_f$, and the fit-factor, $F_f$, or,

$$Ad_f = (M_f)(F_f) \tag{12}$$

(8) *Design storm values, depth-duration curve.*—The maximized depth-duration values for each storm, as transposed to a watershed, are computed by multiplying the observed storm depth-duration values by the respective maximization adjustment factor, $Ad_f$. The computed values for each storm should be plotted with accumulative time in hours as the abscissa versus the accumulative rainfall depths in inches as the ordinate.

A design storm depth-duration curve is obtained by drawing a smooth curve. An enveloping curve will give design storm values approaching PMP for a watershed. A curve drawn through the data for one storm only will give selected PMS values.

Since the depth-duration curve is ordered in such a manner as to show only the maximum values of rainfall for various durations, the curve does not indicate a realistic sequence of rainfall increments which might occur during the actual design storm. Incremental design storm values obtained from the smooth depth-duration curve should be arranged in realistic sequence for flood computation.

For storms of long duration (several days), the design storm depth-duration curve may not be smooth throughout but have two or more periods of intense rainfall separated by periods of little or no rainfall. Such storms are frequently critical for very large basins or basins in tropical regions. In these instances, incremental design storm values may be arranged in any realistic sequence, within the limitation that the separate periods will not be so combined as to produce a rainfall sequence that would have exceeded the recommended design storm depth-duration curve at any point.

(9) *Numerical computations.*—Table L-9 presents numerical values for procedures described in the subsections above. Maps showing the transposed storm isohyetal patterns as fitted to the watershed and the planimetry notes for determination of average basin rainfall for each transposed storm are not included. A plot of depth-duration values of the transposed storms, as maximized, and the recommended depth-duration curve of the design storm are shown on figure L-9. In this instance, the design storm duration is 17 hours and rainfall values approach PMP. The enveloping curve on figure L-9 was drawn "by-eye" as adequate for a preliminary PMS estimate. Design storm values read from the curve at 1-hour intervals are listed in table L-10 because a flood hydrologist may wish to use a 1-hour unitgraph to compute an inflow design flood hydrograph for this size watershed.

(b) *Generalized Precipitation Charts.*—Maps showing smoothed isohyets of PMP for the United States east of the 105° meridian and PMS values for the United States west of the 105° meridian are presented here to provide a means of quickly obtaining *preliminary design storm values* for selected watersheds above proposed damsites. It is impossible to show on the generalized charts all of the refinements and variations that can influence the magnitude of design storm values for individual watersheds. Design storm values obtained from the generalized charts represent a reasonable upper limit and, in most instances, will exceed the values obtained for a specific watershed by a detailed hydrometeorological study, as previously discussed.

(1) *Generalized chart for the United States east of the 105° meridian.*—Figure L-10 shows probable maximum 6-hour precipitation values for any area of 10 square miles for the United States east of the 105° meridian. This chart is based on one presented in Hydrometeorological Report No. 33, prepared by the Hydrometeorological Section of the National Weather Service in collaboration with the U.S. Army Corps of Engineers [20]. These 6-hour values for 10-square-mile areas can be modified for durations in excess of 6 hours and for larger areas up to 1,000 square miles by use of figure L-11. No variation is assumed between point and 10-square-mile precipitation. For durations shorter than 6 hours, the time distribution of precipitation can be obtained from curve C, figure L-12. Subsequent to the publication of Hydrometeorological Report No. 33, the Corps of Engineers have recommended[7] that the following adjustment percentages be applied to the depth-duration values obtained from figure L-10 in order to provide for the imperfect *fit* of the isohyetal patterns of observed storms to the shape of a particular basin.

| *Drainage area, square miles* | *Adjustment factor applicable to H.R. 33 rainfall values, percent* |
|---|---|
| 1,000 | 90 |
| 500 | 90 |
| 200 | 89 |
| 100 | 87 |
| 50 | 85 |
| 10 | 80 |

(2) *Generalized chart for the United States west of the 105° meridian.*—Figure L-13 shows probable maximum 6-hour point general-type storm values for areas of the United States west of the 105° meridian. This chart is based on the results of approximately 330 design storm analyses prepared by the Bureau of Reclamation for specific drainage basins west of the 105° meridian, as well as consideration of numerous design storm analyses made by the Special Studies and Hydrometeorological Branches of the National Weather Service.

The variable topography of this part of the United States greatly influences the storm potential and permits only limited transposition of storms. These point storm values can be applied to areas up to 1,000 square miles by use of the curves presented on figure L-14. The 6-hour general-type storm values can be extended for longer duration periods by multiplying the 6-hour value by the

---

[7] Engineer Circular No. 1110-2-27, dated August 1, 1966, "Policies and Procedures Pertaining to Determination of Spillway Capacities and Freeboard Allowances for Dams."

Table L-9.–*Example of design storm derivation for area east of 105° meridian.*

BASIC DATA:
Watershed location: 99°30′ W, 41°00′ N
Drainage area: 200 sq. mi.
Inflow barrier: 2,500 feet

### (A) MAJOR STORMS SELECTED FOR TRANSPOSITION

| Designation No. | Approximate geographic location–name | Date of storm | Inflow barrier, feet | Observed storm dewpoint °F.[1] | Observed storm dewpoint Ref. pt. | Total storm Duration, hrs. | Total storm [2]$\bar{P}_o$ | Reference |
|---|---|---|---|---|---|---|---|---|
| MR4-24 | Boyden, Iowa | 9/17-19/26 | 1,200 | 70 | 175 mi. SSE | 54 | 16.6 | Fig. L-8A |
| MR4-5 | Grant Township, Nebr. | 6/3-4/40 | 1,200 | 66[3] | 120 mi. S | 20 | 11.2 | [21] |
| MR6-15 | Stanton, Nebr. | 6/10-13/44 | 1,500 | 70 | 125 mi. SSE | 78 | 14.4 | [21] |
| R10-1-1[4] | Greeley, Nebr. | 8/12-13/66 | 2,000 | 71 | 80 mi. SSE | 17 | 13.3 | [4] |

[1] 1,000 millibars, or mean sea level.
[2] Average rainfall depth, 200 sq. mi.
[3] Revised value in lieu of 63° F. [21]
[4] Recent storm analysis, preliminary, Bureau of Reclamation, Engineering and Research Center, Denver, Colo.

### (B) STORM TRANSPOSITION AND MAXIMIZATION

(Column heading symbols as previously defined in text.)

| Storm No. | Observed storm Dwpt., °F. | Observed storm Barrier, feet | $W_{p\text{-}1}$ | $W_{p\text{-}2}$ | $W_s$ | $\bar{P}_o$ | Transposed storms Dwpt.,[1] °F. | Transposed storms Barrier, feet | $W_{r\text{-}1}$ | $W_{r\text{-}3}$ | $W_x$ | $\bar{P}_{tr}$ | Maximizing factors $M_f$ | $F_f$ | $Ad_f$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MR4-24 | 70 | 1,200 | 2.27 | 0.25 | 2.02 | 16.6 | 76 | 2,500 | 3.04 | 0.62 | 2.42 | 12.3 | 1.20 | 0.74 | 0.89 |
| MR4-5 | 66 | 1,200 | 1.86 | .22 | 1.64 | 11.2 | 76 | 2,500 | 3.04 | .62 | 2.42 | 9.6 | 1.48 | .86 | 1.27 |
| MR6-15 | 70 | 1,500 | 2.27 | .31 | 1.96 | 14.4 | 76 | 2,500 | 3.04 | .62 | 2.42 | 13.0 | 1.23 | .90 | 1.11 |
| R10-1-1 | 71 | 2,000 | 2.38 | .42 | 1.96 | 13.4 | 77 | 2,500 | 3.19 | .64 | 2.55 | 12.4 | 1.30 | .93 | 1.21 |

[1] From Climatic Atlas of United States [19].

### (C) MAXIMUM OBSERVED DEPTHS, INCHES

| Storm | Duration in hours 3 | 6 | 9 | 12 | 15 | 18 | 24 | 30 | 36 | 48 | 60 | 72 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MR4-24 | | 11.7 | | 15.8 | | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | [1]16.6 | |
| MR4-5 | 5.5 | 9.6 | 11.1 | 11.2 | 11.2 | 11.2 | [2]11.2 | | | | | |
| MR6-15 | | 11.1 | | 12.9 | | 12.9 | 12.9 | 12.9 | 13.1 | 14.1 | 14.3 | [3]14.4 |
| R10-1-1 | 6.7 | 9.4 | 12.5 | 13.1 | 13.2 | [4]13.4 | | | | | | |

[1] Storm ended at 54 hrs.
[2] Storm ended at 20 hrs.
[3] Storm ended at 78 hrs., depth = 14.4 in.
[4] Storm ended at 17 hrs.

### (D) MAXIMUM TRANSPOSED DEPTHS, INCHES

| Storm No. | Storm $Ad_f$ | Duration in hours 3 | 6 | 9 | 12 | 15 | 18 | 24 | 30 | 36 | 48 | 60 | 72 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MR4-24 | 0.89 | | 10.4 | | 14.1 | | 14.8 | 14.8 | 14.8 | 14.8 | 14.8 | [1]14.8 | |
| MR4-5 | 1.27 | 7.0 | 12.2 | 14.1 | 14.2 | 14.2 | 14.2 | [2]14.2 | | | | | |
| MR6-15 | 1.11 | | 12.3 | | 14.3 | | 14.3 | 14.3 | 14.3 | 14.5 | 15.7 | 15.9 | [3]16.0 |
| R10-1-1 | 1.21 | 8.1 | 11.4 | 15.1 | 15.9 | 16.0 | [4]16.2 | | | | | | |

[1] At 54 hrs.
[2] At 20 hrs.
[3] Also at 78 hrs.
[4] At 17 hrs.

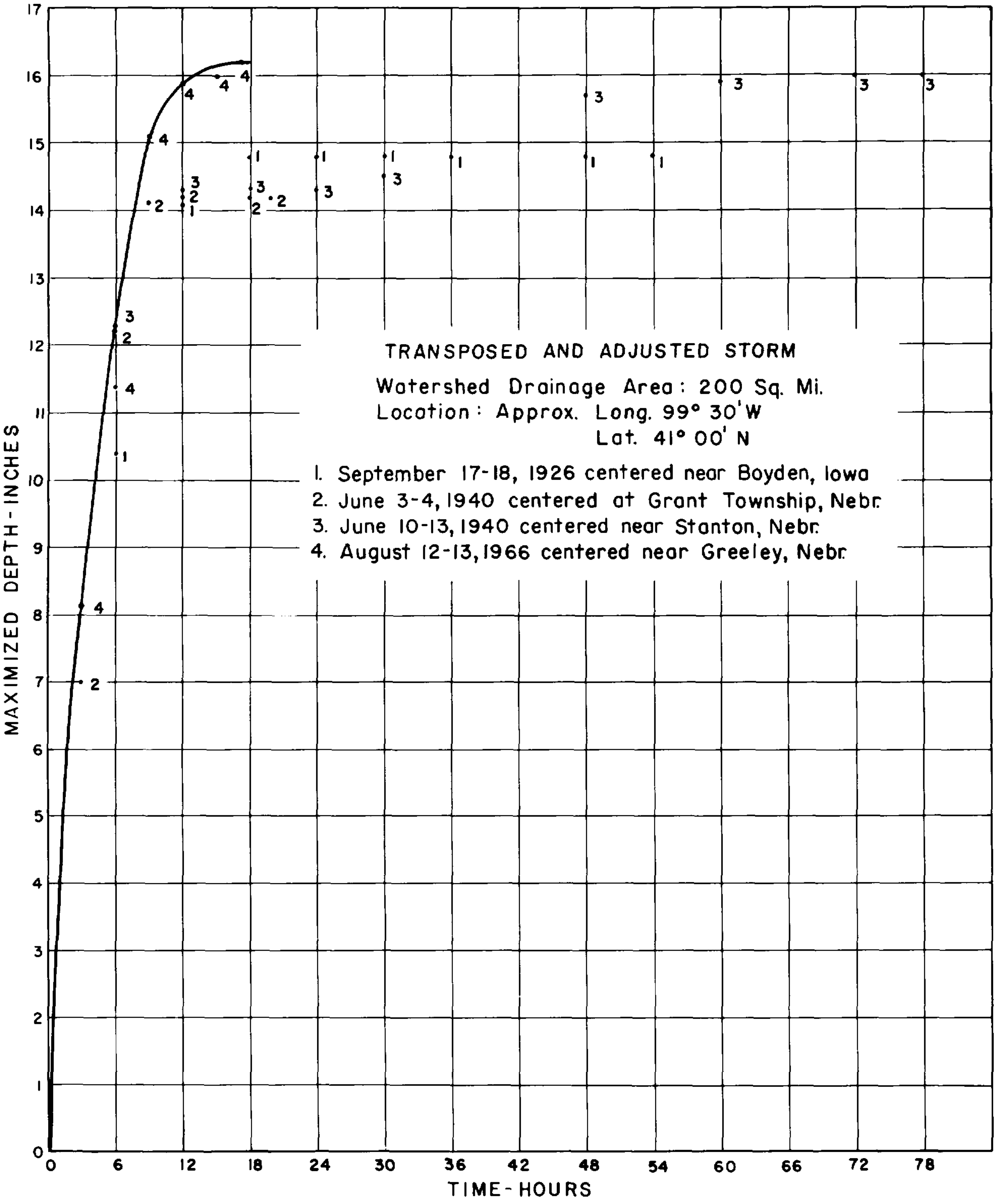


*Figure L-9.* Design storm—depth-duration values.—288-D-3185

Table L-10.—*Design storm depth-duration values, inches.*

BASIC DATA: Hypothetical example.
Watershed area = 200 sq. mi.
Location = approximately 99°30′ W, 41°00′ N

| Time, ending at hour | Accumulated depth, inches | Incremental depth, inches |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 4.20 | 4.20 |
| 2 | 6.40 | 2.20 |
| 3 | 8.10 | 1.70 |
| 4 | 9.70 | 1.60 |
| 5 | 11.10 | 1.40 |
| 6 | 12.30 | 1.20 |
| 7 | 13.30 | 1.00 |
| 8 | 14.30 | 1.00 |
| 9 | 15.10 | .80 |
| 10 | 15.45 | .35 |
| 11 | 15.70 | .25 |
| 12 | 15.90 | .20 |
| 13 | 16.00 | .10 |
| 14 | 16.10 | .10 |
| 15 | 16.15 | .05 |
| 16 | 16.20 | .05 |
| 17 | 16.20 | 0 |
| 18 | 16.20 | 0 |

appropriate factor shown in table L-11. Values for duration of less than 6 hours can be obtained from the appropriate curve of figure L-12.

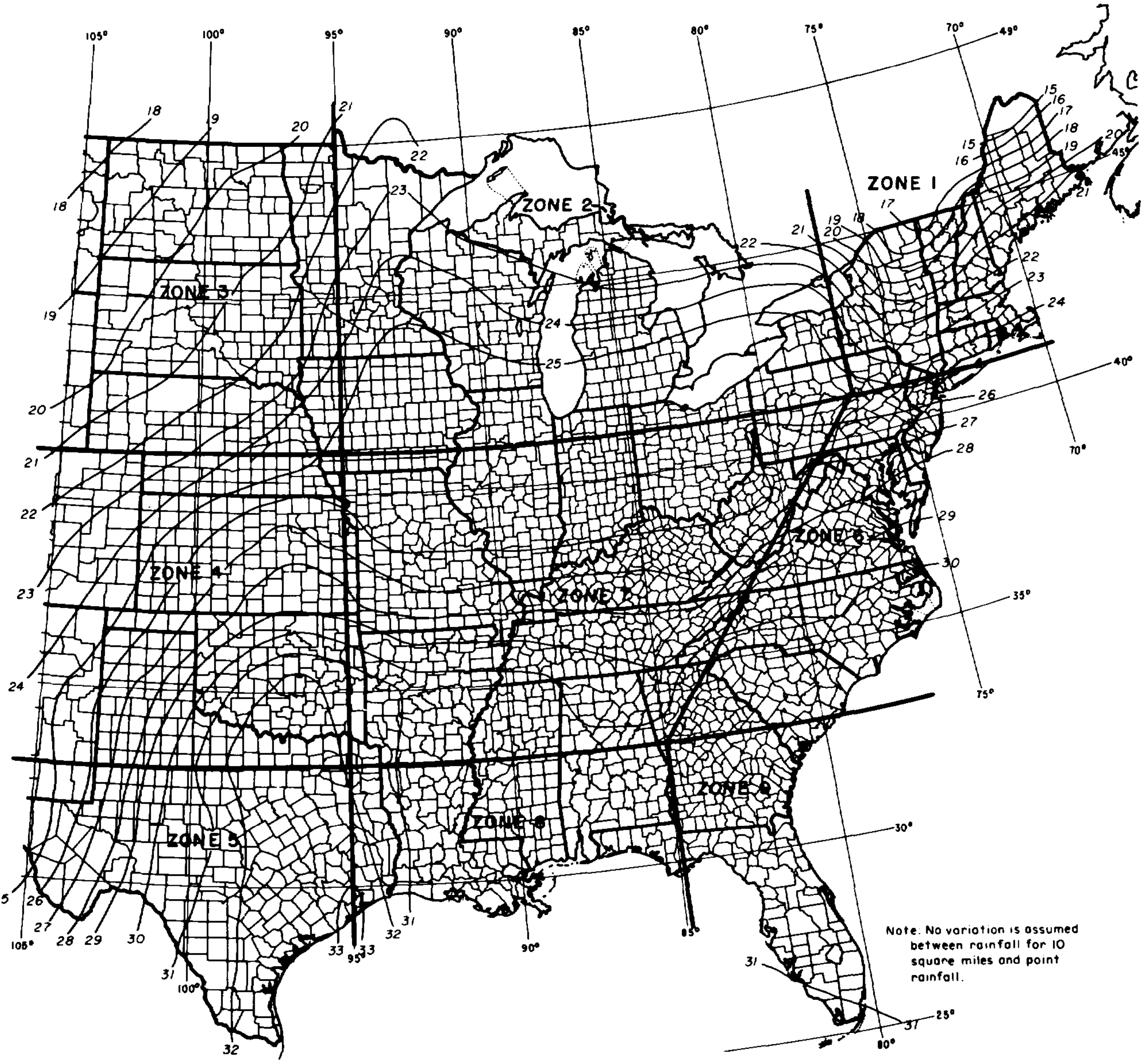
(3) *Use of generalized charts.*—Design storm values for any watershed of a 1,000-square-mile area or less in the conterminous 48 United States may be obtained from the generalized charts, but it must be noted that such design storm values should be considered as only preliminary estimates for watersheds controlled by large dams. Design storm values obtained from figures L-10 and L-13 show considerable difference at their common boundary along the 105° meridian. This is due to the techniques used in determining the values shown on the charts.

Preliminary design storm values for a particular watershed obtained from either generalized chart should be plotted on coordinate paper and an enveloping depth-duration curve drawn. Plotting offers a method of checking the computations, as a smooth curve should be indicated, and also provides the means of obtaining hourly design storm values for the total storm period if needed. Incremental values from the depth-duration curve may be arranged in any sequence desired by a flood hydrologist for computation of a preliminary inflow design flood.

The generalization charts for estimating preliminary design storm values have been limited to an area of 1,000 square miles because generalizations of criteria become more difficult as the size of the area increases. Preliminary design storm estimates can be made for areas greater than 1,000 square miles in regions of nonorographic rainfall by the procedure described in section L-17. The step of determining a fit-factor is omitted. A depth-duration curve is drawn on the basis of information compiled in a tabulation such as table L-9(D), using the moisture maximization factor, $M_f$, instead of the total adjustment factor, $Ad_f$, to compute values for the table. Preliminary design storm estimates for large mountainous basins (with predominately orographic rainfall) should be obtained from a hydrometeorologist.

## F. PRELIMINARY INFLOW DESIGN FLOOD, RAINFALL ONLY

L-18. *General.*—This subchapter outlines procedures for estimating preliminary inflow design flood (IDF) hydrographs using: (1) design storm values from the generalized precipitation charts, figures L-10 and L-13; (2) an estimation of incremental rainfall excesses from runoff curves, section L-7(b)(6); and (3) the lag-time dimensionless-graph method of obtaining unitgraphs, section L-9. An example is given of computation of preliminary inflow design flood hydrographs for a watershed east of the 105° meridian, with accompanying



Note: No variation is assumed between rainfall for 10 square miles and point rainfall.

*Figure L-10.* **Probable maximum precipitation (inches) east of the 105° meridian for an area of 10 square miles and 6 hours' duration.–288-D-3191**

discussions directed toward considerations applicable to all inflow design flood studies. Procedures applicable to watersheds west of the 105° are outlined. A discussion of preparing recommendations for routing preliminary inflow design flood hydrographs through proposed reservoirs concludes this presentation.

L-19. ***Example–Preliminary Inflow Design Flood Hydrographs, Watersheds East of 105° Meridian.***–A hypothetical watershed in a general location east of the 105° meridian has been assumed in order to illustrate several of the problems encountered in IDF computations, all of which would not likely be presented by a specifically located watershed.

(a) *Basin Description.*–A map of the assumed watershed above a proposed damsite is shown on figure L-15. The center of the basin is assumed to be located in zone 4 somewhere along the 30-inch, 6-hour PMP for 10-square-mile isohyet, figure L-10. An outline

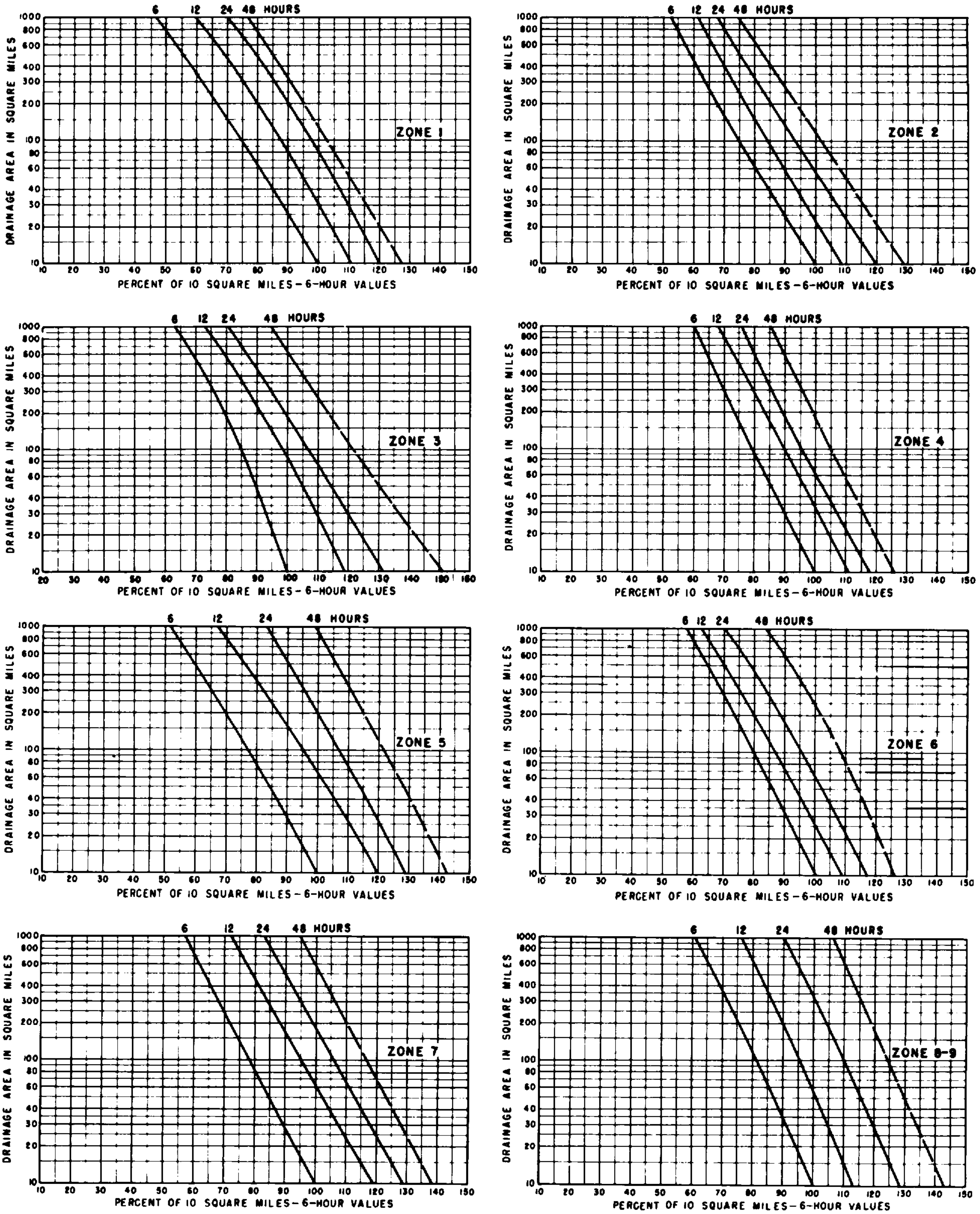


*Figure L-11.* **Depth-area-duration relationships—percentage to be applied to 10 square miles, 6-hour probable maximum precipitation values.—288-D-2450**

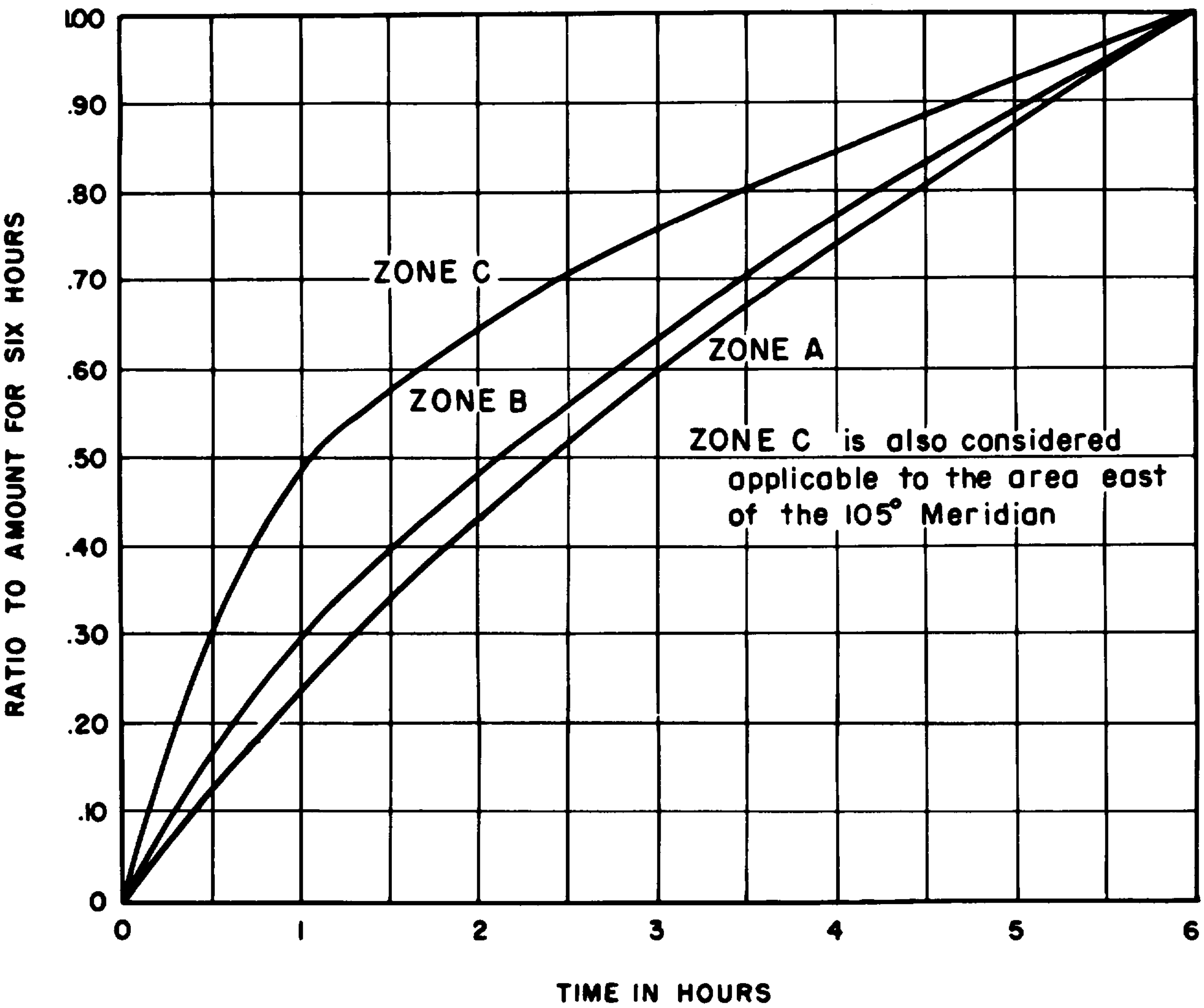


*Figure L-12.* Distribution of 6-hour rainfall for area west of 105° meridian (see fig. L-13 for area included in each zone).–288-D-2758

of the proposed reservoir surface at normal water storage capacity is shown, because the length of natural stream channels to be submerged influences lag-time calculations. It is assumed that runoff characteristics of the areas drained by the two main tributaries differ enough to warrant consideration of dividing the watershed into two subareas, A and B, as there is information available indicating that subarea A definitely has rapid runoff characteristics. All of the area enclosed by the natural divides contributes runoff.

(1) *Drainage areas* are:

| | |
|---|---|
| Total basin | 800 square miles |
| Subarea A | 240 square miles |
| Subarea B | 560 square miles |
| Reservoir surface | 26 square miles |

As the reservoir surface area is about 3 percent of the total basin area in this example, reservoir surface may be considered as land area, except for lag-time computations. Whenever there is found a reservoir surface area of about 10 percent or more of total contributing drainage area, computations should be made separately of the runoff originating from the land area, to which

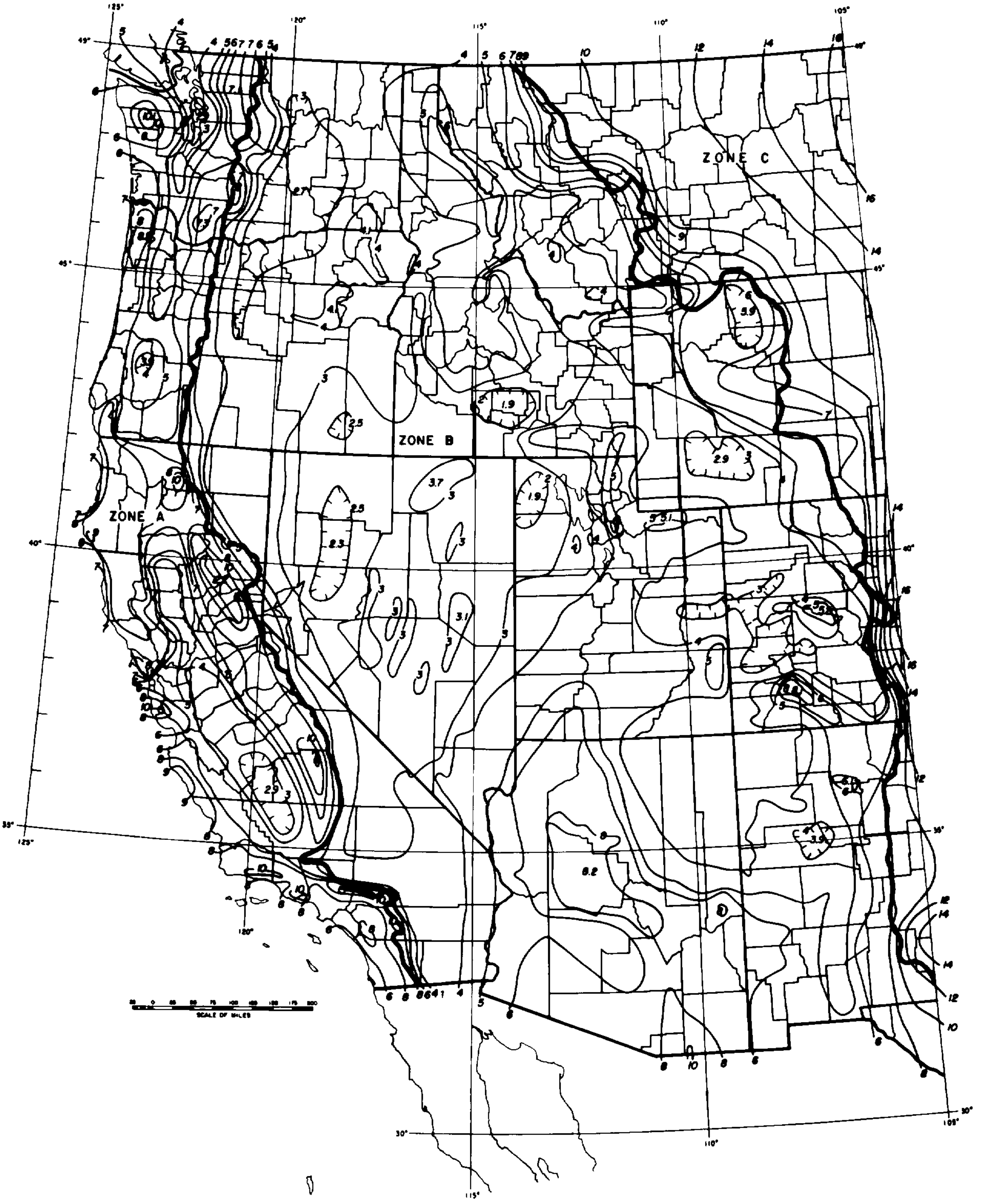


*Figure L-13.* Probable maximum 6-hour point precipitation values in inches for general-type storms west of the 105° meridian.–288-D-3192

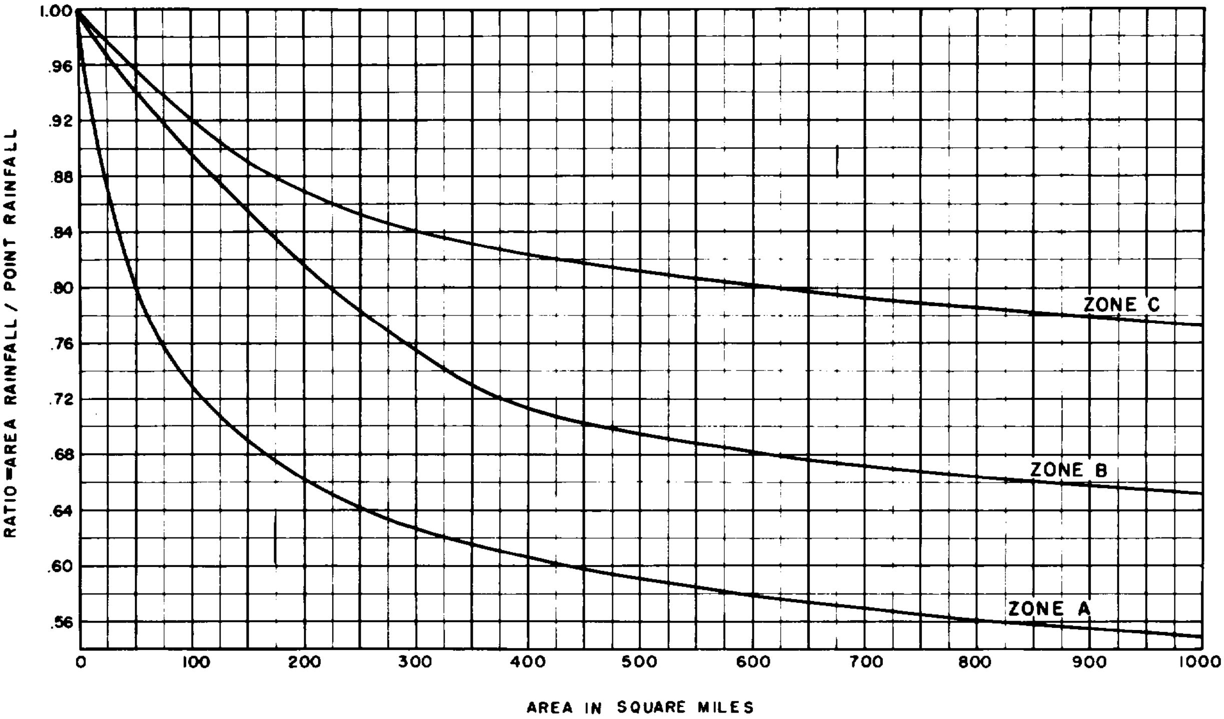


*Figure L-14.* General-type storm–conversion ratio from 6-hour point rainfall to area rainfall for area west of 105° meridian.–288-D-2759

Table L-11.–*Constants for extending 6-hour general-type design-storm values west of 105° meridian to longer duration periods*[1]

| Duration, hours[2] | Constants | | |
|---|---|---|---|
| | Zone A | Zone B | Zone C |
| 8 | 1.20 | 1.18 | 1.14 |
| 10 | 1.39 | 1.36 | 1.26 |
| 12 | 1.58 | 1.53 | 1.36 |
| 14 | 1.76 | 1.66 | 1.43 |
| 16 | 1.93 | 1.77 | 1.50 |
| 18 | 2.10 | 1.87 | 1.57 |
| 20 | 2.26 | 1.95 | 1.64 |
| 22 | 2.42 | 2.03 | 1.71 |
| 24 | 2.57 | 2.10 | 1.78 |
| 30 | 2.95 | 2.28 | 1.97 |
| 36 | 3.26 | 2.38 | 2.15 |
| 42 | 3.55 | 2.40 | 2.25 |
| 48 | 3.79 | 2.41 | 2.28 |
| 60 | 4.14 | | |
| 72 | 4.34 | | |

[1] Multiply 6-hour point rainfall from figure L-13 by indicated constant.

[2] For durations shorter than 6 hours, the time distribution of storm values can be obtained from the appropriate curve presented on figure L-12.

retention losses are applicable to design storm rainfall, and the increased inflow to the reservoir due to design rainfall on the reservoir surface area where retention losses are zero. There are instances where rain falling on reservoir surfaces supplies the major portion of inflow. When rain falling on a reservoir surface must be considered, rainfall increments in inches are converted to equivalent incremental flow in cubic feet per second and combined with respectively timed increments of inflow from the land area. Watersheds in which a reservoir will submerge miles of mainstream channel, and numerous side tributaries flow directly into the reservoir, the watershed should be divided into at least two subareas, the subarea above the head of the reservoir and the area directly tributary to the reservoir. Subarea B, figure L-15, approaches this situation. If a final-type IDF study were made for the example watershed, a better evaluation of a final-type IDF would be obtained by dividing subarea B into two subbasins and

1. Measure stream length $E_1$ to $E_2$; L, miles
2. Measure stream length $E_1$ to x; $L_{ca}$, miles
   Note: Do not include "a," stream length that will be submerged.
3. $S = \dfrac{\text{Elevation } E_2 \text{ minus elevation } E_1.}{L\text{, miles}}$

In the above, x = center of area projected.

Damsites with 2(or more) markedly different tributaries require 2 (or more) unitgraphs.

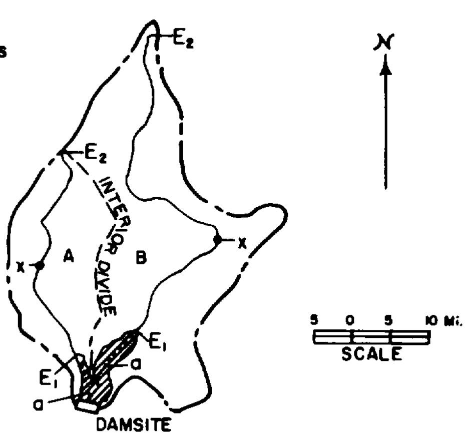


*Figure L-15.* Basin map–example of preliminary inflow design flood computation.–288-D-3186

deriving a unitgraph for each; the subbasins would be above and below the head of the reservoir, point $E_1$, figure L-15.

(2) *Streamflow records.*–Two assumptions are made for lag-time illustrating purposes: first, that there are no streamflow records available for anlaysis; second, that tributary B has been gaged at the mouth near the damsite, and hydrograph analyses have indicated a lag-time of 22 hours for subarea B.

(3) *Soils and cover.*–Use of runoff curves requires hydrologic classification of watershed soils and cover, discussed in section L-7(b)(6), for selection of applicable runoff curve number. These classifications are made by field inspections, examination of soils maps, etc. For this example, it is assumed available information indicates:

Subarea A:

Soils, hydrologic group C
Land use, mostly poor pasture
Runoff curve, AMC-II CN86 (table L-3(A))

Subarea B:

Soils, hydrologic group B
Land use, mostly small grain, contour terraced
Runoff curve, AMC-II CN70 (table L-3(A))

(b) *Dimensionless-Graph Selection.*–As hydrograph analyses cannot be made in the first instance because of lack of streamflow records, a dimensionless-graph must be selected from other sources. The dimensionless-graph shown as (C), figure L-6, which was derived from a flood hydrograph in the general region of the assumed location of the watershed, has been selected as applicable to both subareas of the watershed. It is also used in the second example, where streamflow records are available.

(c) *Lag-Times.*–A cutout of each subarea, including the respective reservoir portion in each, was made, the center of area of each determined and projected to the main streams at the points marked *X* on the stream channels as shown on figure L-15 (see sec. L-9(2)). Longest watercourse lengths listed below were measured from the map. Slope values for this example, *S* in feet per mile, were selected from general data. In the usual study, elevations for computing slope values for a given watershed are obtained from topographic maps.

Subarea A:

$L$ = 29.0 miles from head of reservoir to divide, $E_1$ to $E_2$, figure L-15.
$L_{ca}$ = 12.7 miles from head of reservoir to center of area projected, $E_1$ to $x$, figure L-15.
$S$ = 23.2 feet per mile (assumed in this example).

Subarea B: (Assumption of no streamflow records.)

$L$ = 48.9 miles from head of reservoir to divide, $E_1$ to $E_2$, figure L-15.

$L_{ca}$ = 15.4 miles from head of reservoir to center of area (projected), $E_1$ to $x$, figure L-15.

$S$ = 12.6 feet per mile (assumed for this example).

For use in assumption that streamflow records have indicated a lag-time of 22 hours for tributary B:

$L$ = 59.8 miles from mouth (gage) to divide.

$L_{ca}$ = 26.3 miles from mouth (gage) to center of area, $x$.

$S$ = 16.5 feet per mile (assumed for this example).

Two sets of lag-times are estimated for this example on the basis of the two assumptions regarding available streamflow records. Under the assumption that no streamflow records are available, the generalized lag-time equation is considered applicable.

$$\text{Lag-time, hours} = 1.6 \left[\frac{LL_{ca}}{\sqrt{S}}\right]^{0.33} \quad \text{(Sec. L-8(e)(2).)}$$

Estimated lag-times are:

Subarea A:

$$\frac{LL_{ca}}{\sqrt{S}} = \frac{(29.4)(12.7)}{\sqrt{23.2}} = 77.5$$

Lag-time = 6.7 hours.

Subarea B:

$$\frac{LL_{ca}}{\sqrt{S}} = \frac{(48.9)(15.4)}{\sqrt{12.6}} = 212.2$$

Lag-time = 9.4 hours.

Under the assumption that hydrograph analyses for streamflow gaged near the mouth of tributary B indicates a lag-time of 22 hours for subarea B, the following lag-times are estimated:

Subarea A:

No change, lag-time = 6.7 hours.

Subarea B:

Referring to section L-8(e)(2), if a reliable lag-time for a basin is found by hydrograph analyses at a gaging station, a lag-time for an ungaged portion of the basin may be obtained by passing a curve with slope 0.33 through the point plotted on log-log paper, $\frac{LL_{ca}}{\sqrt{S}}$ versus lag hours. An $\frac{LL_{ca}}{\sqrt{S}}$ value for subarea B above the assumed gaging station is:

$$\frac{(59.8)(26.3)}{\sqrt{16.5}} = 386.7$$

If the generalized lag-time curve has been plotted on log-log paper, plot 387 versus the lag-time of 22 hours and draw a line through the plotted point parallel to the generalized lag-time curve. Read a lag-time of 18 hours for the $\frac{LL_{ca}}{\sqrt{S}}$ value of 212 from the constructed curve. In this example, the proposed reservoir has the effect of reducing the lag-time for subarea B from 22 hours for natural conditions to 18 hours after the dam is built. The effect of a proposed reservoir on natural lag-times should not be overlooked in the preparation of inflow design flood hydrographs.

Of course, the lag-time of 18.0 hours can also be obtained without plotting the curves, by solving the equation,

$$\text{Lag-time} = C \left[\frac{LL_{ca}}{\sqrt{S}}\right]^{0.33}$$

for $C$, substituting 22 hours for lag-time and 386.7 for $\frac{LL_{ca}}{\sqrt{S}}$; this gives $C = 3.08$. Then, using this computed value for $C$, and 212.2 for $\frac{LL_{ca}}{\sqrt{S}}$, lag-time in hours equals 18.0.

(d) *Preliminary Design Storm Values.*—A specific watershed location is identified on the generalized charts, figures L-10 and L-13, by county boundaries within the States and reading the zone and 6-hour PMP values applicable to the watershed. A specific location for the watershed for this example has not been designated other than it is assumed to be in zone 4 where 6-hour probable maximum precipitation (PMP) for 10 square miles is 30 inches (figure L-10). Computation of preliminary design storm values are shown in table L-12. The design storm is assumed to cover the entire watershed area of 800 square miles. Percentages of the 6-hour PMP for 10 square miles applicable to 800 square miles were read from the depth-area-duration relationships on the chart for zone 4, figure L-11, and PMP values for 6, 12, 24, and 48 hours for 800 square miles computed. These values were adjusted to 90 percent of the computed values in accordance with the fit adjustment factors given in section L-17(b)(1). Hourly depth-duration values for the maximum 6-hour period of the storm were computed by percentages read from curve $C$ on figure L-12. Depth-duration values, line 5 of table L-12, were plotted and a preliminary design storm depth-duration curve drawn as shown on figure L-16.

(e) *Arrangement of Design Storm Rainfall Increments and Computation of Increments of Rainfall Excess.*—Arrangement of increments of rainfall of a preliminary design storm estimated from figure L-10 is illustrated in table L-13, along with the computation of respective increments of excess rainfall. Computation of table L-13 is explained in the following paragraphs. General comments on design storm arrangements are included.

(1) *Selection of design storm unit time interval.*—Design storm increments and respective rainfall excesses obtained therefrom must be for the same unit time interval as the unitgraph to which the excesses will be applied to compute an inflow design flood (IDF) hydrograph. Unit time of a unitgraph is related to the lag-time of a basin, being one-fourth or less of the lag-time (sec. L-9(6)). In this example, a 1-hour unitgraph is required for subarea A because a lag-time of 6.7 hours has been estimated for that subarea. A 2-hour unitgraph could be used for subarea B, lag-time 9.4 hours. However, the computed hydrographs for the two subareas must be combined to give the preliminary inflow design flood hydrograph. A better definition of the IDF hydrograph will be obtained if the unitgraphs for the two subareas have the same unit time interval. A 1-hour unitgraph for each subarea was used in this example. Hourly values of preliminary design storm rainfall were read to the nearest tenth inch from the depth-duration curve, figure L-16, from 1 to 24 hours and tabulated in column 2 of table L-13. Hourly increments of rainfall are listed in column 3 of table L-13.

Table L-12.—*Preliminary design storm estimate for hypothetical watershed, east of 105° meridian.*

BASIC DATA:
Location: Hypothetical
Reference: Figure L-10, zone 4, 6-hr. PMP[1], 10 sq. mi.: 30 inches
Areas: Total basin, 800 sq. mi.; subarea A, 240 sq. mi.; subarea B, 560 sq. mi.

| | Item | Time in hours | | | | | | | | | Text reference |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 12 | 24 | 48 | |
| 1. | Percent of 6-hr. PMP[1] for 800 sq. mi. | | | | | | 62 | 70 | 77 | 87 | Fig. L-11 |
| 2. | Computed PMP, 800 sq. mi., inches | | | | | | 18.6 | 21.0 | 23.1 | 26.1 | |
| 3. | PMP, adjusted to 90 percent | | | | | | 16.7 | 18.9 | 20.8 | 23.5 | Sec. L-17(b)(1) |
| 4. | Ratios to 6-hr. rainfall | 0.49 | 0.64 | 0.75 | 0.85 | 0.93 | 1.00 | | | | Fig. L-12, zone C |
| 5. | Design PMP, 800 sq. mi., inches | 8.2 | 10.7 | 12.5 | 14.2 | 15.5 | 16.7 | 18.9 | 20.8 | 23.5 | Fig. L-16 |

[1] PMP = probable maximum precipitation.

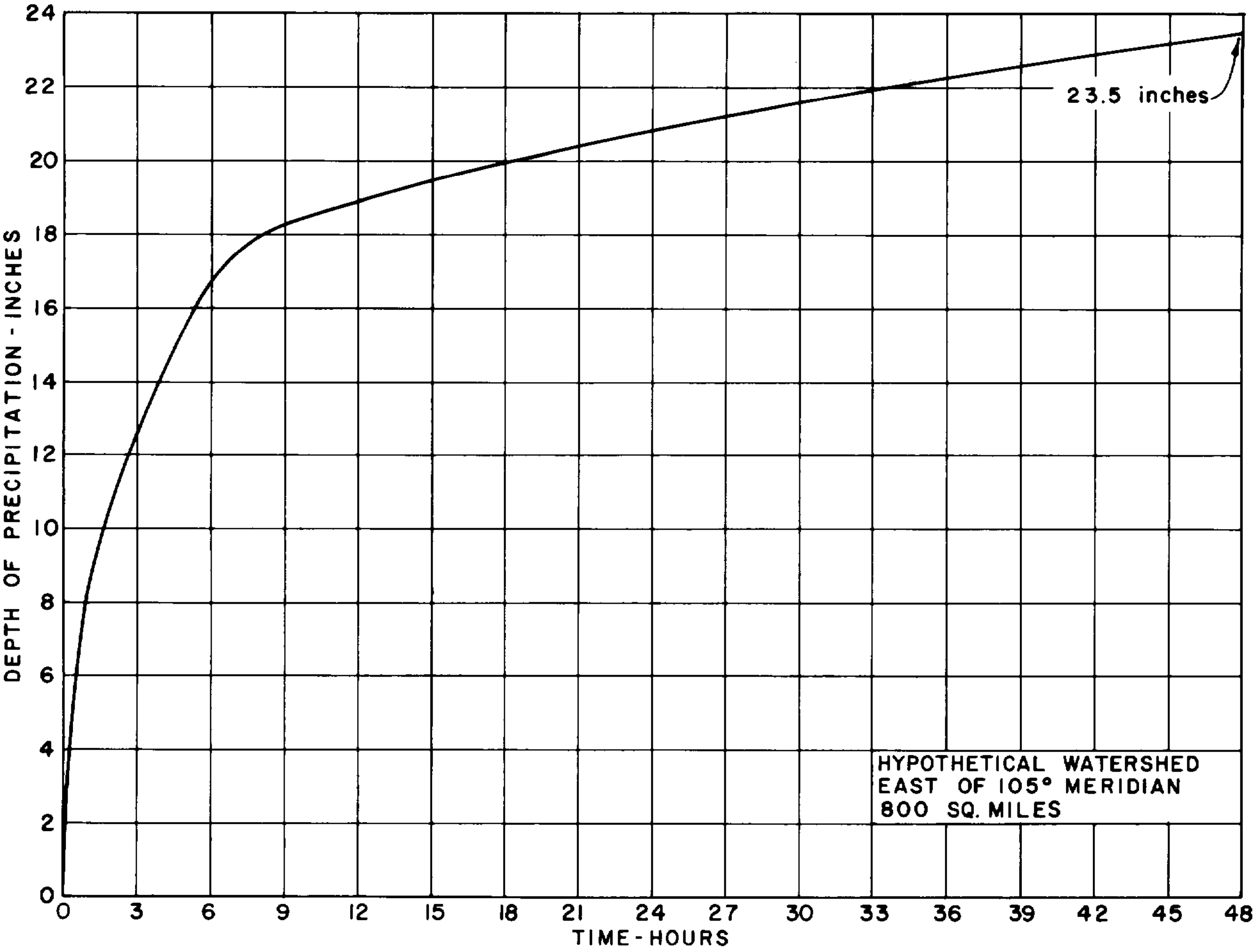


*Figure L-16.* **Preliminary design storm–depth-duration curve.–288-D-3187**

(2) *Arrangement of design storm incremental rainfall.*—Normally, the arrangement with respect to time of increments of design storm rainfall is not established in a design storm study (sec. L-17(a)(8)). Flood hydrologists arrange design storm increments to give rainfall excesses that produce the most critical inflow design flood hydrograph. Except for basins having several thousands of square miles of drainage area, design storm rainfall is assumed to occur with the same time sequence over the total watershed area. If a constant retention loss rate is used to compute rainfall excesses, a critical arrangement may be easily found by arranging design storm increments opposite the ordinates of the unitgraph for the basin, so that the largest rainfall increment (which would give the largest excess increment) is opposite the largest ordinate; and the second largest rainfall increment is opposite the second largest ordinate, etc.

This arrangement is then reversed to give the design storm arrangement in correct time sequence, because rainfall excesses are reversed in sequence of natural occurrence when being applied to unitgraph ordinates by calculators. Otherwise, much additional work must be done: (1) computing discharges for each ordinate of the unitgraph for each excess increment; (2) tabulating the individual discharges in correct time sequence; and (3) adding respectively timed incremental discharges to get the total flood hydrograph. If a retention loss rate which varies with time is used, a critical design storm arrangement is found by trial.

Table L-13.—*Preliminary design storm east of 105° meridian—arrangement of incremental rainfall; computation of incremental excesses, $\Delta P_e$, for subareas A and B.*

BASIC DATA:

Total area (for design storm estimate)—800 sq. mi.

Subarea size and retention data:

Subarea A: 240 sq. mi.; CN 86, selected minimum loss rate, 0.12 in./hr.

Subarea B: 560 sq. mi.; CN 70, selected minimum loss rate, 0.24 in./hr.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| Time, ending hour | Design rainfall depth-duration | | Arrangement of design rainfall | | Rainfall excesses, $P_e$ | | | | | |
| | | | | | Subarea A | | | Subarea B | | |
| | $\Sigma P$, inches | $\Delta P$, inches | $\Delta P$, inches | $\Sigma P$, inches | [2]$\Sigma P_e$, inches | $\Delta P_e$, inches | $\Delta$loss, inches | [3]$\Sigma P_e$, inches | $\Delta P_e$, inches | $\Delta$loss, inches |
| 1 | 8.2 | 8.2 | 1.2 | 1.2 | 0.30 | 0.30 | 0.90 | 0.02 | 0.02 | 1.18 |
| 2 | 10.7 | 2.5 | 1.7 | 2.9 | 1.57 | 1.27 | .43 | .66 | .64 | 1.06 |
| 3 | 12.5 | 1.8 | 1.8 | 4.7 | 3.18 | 1.61 | .19 | 1.82 | 1.16 | .64 |
| 4 | 14.2 | 1.7 | 8.2 | 12.9 | 11.13 | 7.95 | .25 | 8.88 | 7.06 | 1.14 |
| 5 | 15.5 | 1.3 | 2.5 | 15.4 | 13.51 | 2.38 | [4].12 | 11.14 | 2.26 | [5].24 |
| 6 | 16.7 | 1.2 | 1.3 | 16.7 | 14.69 | 1.18 | .12 | 12.20 | 1.06 | .24 |
| 7 | 17.4 | .7 | .7 | 17.4 | 15.27 | .58 | .12 | 12.66 | .46 | .24 |
| 8 | 17.9 | .5 | .5 | 17.9 | 15.65 | .38 | .12 | 12.92 | .26 | .24 |
| 9 | 18.2 | .3 | .3 | 18.2 | 15.83 | .18 | .12 | 12.98 | .06 | .24 |
| 10 | 18.5 | .3 | .3 | 18.5 | 16.01 | .18 | .12 | 13.04 | .06 | .24 |
| 11 | 18.7 | .2 | .2 | 18.7 | 16.09 | .08 | .12 | 13.04 | 0 | .24 |
| 12 | 18.9 | .2 | .2 | 18.9 | 16.17 | .08 | .12 | | | |
| 13 | 19.1 | .2 | .2 | 19.1 | 16.25 | .08 | .12 | | | |
| 14 | 19.3 | .2 | .2 | 19.3 | 16.33 | .08 | .12 | | | |
| 15 | 19.5 | .2 | .2 | 19.5 | 16.41 | .08 | .10 | | | |
| 16 | 19.6 | .1 | .1 | 19.6 | 16.41 | 0 | .12 | | | |
| 17 | 19.8 | .2 | .2 | 19.8 | 16.49 | .08 | .12 | | | |
| 18 | 20.0 | .2 | .2 | 20.0 | 16.57 | .08 | .12 | | | |
| 19 | 20.1 | .1 | .1 | 20.1 | [6] | [6] | | | | |
| 20 | 20.2 | .1 | .1 | 20.2 | | | | | | |
| 21 | 20.4 | .2 | .2 | 20.4 | | | | | | |
| 22 | 20.6 | .2 | .2 | 20.6 | | | | | | |
| 23 | 20.7 | .1 | .1 | 20.7 | | | | | | |
| 24 | [1]20.8 | .1 | .1 | 20.8 | | | | | | |

[1] Balance of design rainfall considered lost to retention.

[2] By equation $\Sigma P_e = \frac{(P - 0.2S)^2}{(P + 0.8S)}$ for CN 86, $S = 1.63$; $0.2S = 0.33$, $0.8S = 1.30$ (table L-4).

[3] By above equation, for CN 70, $S = 4.28$; $0.2S = 0.86$, $0.8S = 3.42$ (table L-4).

[4] $\Delta P_e$ by CN 86 indicates $\Delta$loss = 0.03 in., which is less than 0.12 in. Use 0.12 in. loss/hr.

[5] $\Delta P_e$ by CN 70 indicates $\Delta$loss = 0.15 in., which is less than 0.24 in. Use 0.24 in. loss/hr.

[6] Total of remaining excess not significant for preliminary IDF.

A definite arrangement of design storm increments has been specified for preliminary design storm values obtained from each generalized precipitation chart, figures L-10 and L-13, because the selected general method of computing rainfall excesses using rainfall runoff curves has "built-in" varying retention loss rates. The arrangement specified for preliminary design storm values east of the 105° meridian is illustrated by the arrangement of rainfall increments in column 4, table L-13. The maximum 6-hour period of design rainfall is assumed to occur during the first 6-hour period of the design storm. Hourly precipitation amounts within the maximum 6-hour period are arranged in the following order of magnitude: 6, 4, 3, 1, 2, 5. Increments of design storm rainfall after the sixth hour decrease and are taken directly from the design storm depth-duration curve.

(3) *Computation of increments of rainfall excess.*—The method of estimating excess rainfall increments given in section L-7(b)(6) has been taken from the SCS National Engineering Handbook [3] with the following modifications introduced to give a procedure applicable to preliminary design storm rainfall obtained from generalized precipitation charts.

The rainfall-runoff relationships shown by the curves of figure L-2 were developed by Soil Conservation Service hydrologists from analyses of rainfall and respective runoff records at numerous small area experimental watersheds. The relationships were developed for use with daily nonrecording rainfall data, which are more plentiful in the United States than are recording rainfall data. Data used in the development are totals for one or more storms occurring in a calendar day and nothing is known about their time distributions. The relationships developed, therefore, exclude time as an explicit variable which means that rainfall intensity is ignored.

Strict adherence to use of the runoff curves on figure L-2 results in hourly runoff increments almost equal to hourly precipitation increments after a few hours for many of the design storm values obtained from generalized precipitation charts. Infiltrometer studies indicate that all but impervious clay soils have minimum constant infiltration rates after saturation that may range from 0.05 inch per hour to greater than 1.00 inch per hour, depending on the type of soil. Therefore, to utilize the rainfall-runoff relationships in the computational procedures given in this text, time-sequences of incremental rainfall for a design storm are specified and precipitation excesses are then computed using the runoff curve relationships, with the provision that hourly retention rates indicated by use of the runoff curves be tabulated for each hourly rainfall increment. Progressively through the arranged precipitation sequence, these hourly retention rates are compared with the tabulated minimum retention rates assigned to the four hydrologic soil groups (see table L-14). When the retention rate given by use of a runoff curve becomes less than an assigned minimum retention rate, the minimum rate is used to compute excesses thereafter for the remainder of the storm.

For this example, determination of applicable runoff curve numbers, AMC-II, for subareas A and B has been assumed as described earlier in section L-19(a)(3) on soils and cover. East of the 105° meridian, soil moisture within a watershed which has similar to average conditions present before occurrence of the maximum annual flood (AMC-II) is considered a reasonable assumption for occurrence of a design storm. Therefore, the curve numbers referred to above were obtained from table L-3(A), which lists curve numbers for AMC-II; CN 86 was selected for subarea A and CN 70 for subarea B, to compute rainfall excesses. Minimum retention rates selected are those for general cases, table L-14: 0.12 inch per hour for subarea A, hydrologic soil group C; and 0.24 inch per hour for subarea B, hydrologic soil group B.

Computations of rainfall excesses are made to hundredths of an inch, as shown in table L-13. Runoff curves, figure L-2, cannot be accurately read to hundredths unless plotted to a large scale, so it is recommended that rainfall excesses be computed by the equation shown on figure L-2. The symbol $P_e$ is used in this text to designate direct runoff values, rainfall excesses, in lieu of $Q$ shown on figure L-2. Values of $S$ and $0.2S$ in inches for each curve number are listed in table L-4. Referring to table L-13, computations of hourly rainfall excesses for subarea A are described. This procedure applies to all such computations.

(1) Obtain $S$ and $0.2S$ values from table L-4 for CN 86. Compute $0.8S$ value.

(2) Fill in column 5, $\Sigma P$, by summing the arranged design storm increments.

Table L-14.—*Minimum retention rates for hydrologic soil groups.*

| Hydrologic soil group | Range of minimum retention rates, inches per hour | Recommended rate for use in general case, inches per hour |
|---|---|---|
| A | 0.30-0.45 | 0.40 |
| B | 0.15-0.30 | 0.24 |
| C | 0.08-0.15 | 0.12 |
| D | 0.02-0.08 | 0.04 |

(3) To obtain column 6, begin with the first $\Sigma P$ value that exceeds the applicable $0.2S$ value and, successively by hours, compute $\Sigma P_e$ by the equation:

$$\Sigma P_e = \frac{(P - 0.2S)^2}{(P + 0.8S)} \qquad (13)$$

Each successive $\Sigma P$ value in column 5 of table L-13 becomes the $P$ for the equation, and the values of $0.2S$ and $0.8S$ are those obtained as in (1) above.

(4) Determine increment of excess rain, $\Delta P_e$ for each hour, and tabulate in column 7, then substract $\Delta P_e$ from respective $\Delta P$, column 4, and enter $\Delta$ loss value thus obtained in column 8.

(5) As successively computed, compare $\Delta$ loss value with assigned minimum retention rate: 0.12 inch per hour for subarea A. If loss is greater than 0.12 inch per hour, proceed to next hour and repeat procedure; if loss is less than 0.12 inch, do not use the computed $\Delta P_e$ value. Drop use of runoff equation and use the constant hourly loss rate of 0.12 inch per hour to compute that hour's excess and the rest of the hourly increments of excess rainfall. This change occurred at hour 5 in the example in table L-13.

The hourly increments of excess rainfall listed in column 7 will be applied to a 1-hour unitgraph for subarea A.

In all cases when the generalized precipitation charts are used to estimate preliminary design storm values for a watershed, hourly increments of excess rainfall should be obtained by the above procedure. If a 2-, 3-, or 4-hour unitgraph is to be used for the watershed, the computed hourly rainfall excesses are grouped into respective 2-, 3-, or 4-hour sums and applied to the chosen unitgraph.

(f) *Computation of Preliminary Inflow Design Flood Hydrographs.*—Computation of an inflow design flood (IDF) hydrograph is a routine mathematical process after decisions are made regarding selection of dimensionless-graph, lag-time, retention rate, and design storm values and arrangement. Procedural steps for obtaining a synthetic unitgraph for a watershed have been given in section L-9(7). The principle of obtaining a total flood hydrograph resulting from successive increments of excess rainfall is illustrated on figure L-3. Therefore, detailed tables showing computation of unitgraphs for subareas A and B and the application of respective sets of rainfall excesses to respective unitgraphs are omitted. In lieu thereof, copies of the printouts from the Bureau's Automatic Data Processing (ADP) program for application of the dimensionless-graph lag-time method of computing flood hydrographs are included as tables L-15 and L-16. Table L-15 is a simulated printout of the computed preliminary design flood contribution from subarea A resulting from the incremental rainfall excesses listed in column 7 of table L-13. The program is designed to compute discharges to the nearest cubic foot per second (c.f.s.) so the ordinates of the 1-hour unitgraph for a lag-time of 6.7 hours, listed in the third column of table L-15, are more exact than warranted by the basic data. (The same comment applies also to the computed flood hydrograph discharges.) Table L-16 is a similar printout for subarea B.

(1) *Preliminary inflow design flood hydrograph using generalized lag-time curve for both subareas.*—Design flood contributions for each subarea are tabulated, combined, and total preliminary IDF discharges listed in table L-17. Subarea hydrographs and the total hydrograph are shown on figure L-17. (In usual practice, only the total flood hydrograph is plotted.) A base flow has not been added to computed flood discharges, because base flow discharges are insignificant in relation to the computed flood discharges in this example. A method of obtaining the volume of the IDF hydrograph is detailed in table L-17.

(2) *Preliminary inflow design flood hydrograph, watershed not divided into subareas.*—Under the assumption that no streamflow records are available within the watershed and that the same dimensionless-graph, lag-time curve, and preliminary design storm values are to be used for both subareas, a preliminary inflow design flood hydrograph may be computed using one unitgraph for the total watershed area. Estimating a total basin

Table L-15.—*Simulated automatic data processing printout—preliminary inflow design flood (IDF) contribution, subarea A.*

```
EXAMPLE PRELIMINARY IDF SUBAREA A

UNIT GRAPH DEVELOPED FROM DIMENSIONLESS GRAPH

DIMENSIONLESS FIGURE G-5 DESIGN OF GRAVITY DAMS                TUN = 1.00

LAG DATA GENERAL CURVE SUBAREA A                               LAG = 6.70

AREA = 240.000 SQ MI UNITGRAPH RECES COEF = 0.828781 AT 18.00 HRS

EXCESS OR STORM VOLUME = 16.570 INCHES

HYDROGRAPH VOLUME IN INCHES = 16.573 AND IN AC FT = 212132.7

HOURS        EXCESSES             UNITGRAPH           HYDROGRAPH
              INCHES                 CFS                  CFS

  .00          .000                    0                    0
 1.00          .300                  174                   52
 2.00         1.270                 1247                  595
 3.00         1.610                 4988                 3361
 4.00         7.950                12887                13591
 5.00         2.380                20571                40900

 6.00         1.180                23131                96644
 7.00          .580                20431               184518
 8.00          .380                15042               268585
 9.00          .180                10787               306688
10.00          .180                 7795               291631

11.00          .080                 6238               242607
12.00          .080                 5107               192229
13.00          .080                 4217               150359
14.00          .080                 3575               121417
15.00          .080                 2980                99867

16.00          .000                 2502                83365
17.00          .080                 2140                70906
18.00          .080                 1900                60650
19.00          .000                 1575                52146
20.00          .000                 1305                45261

21.00          .000                 1082                39936
22.00          .000                  897                34392
23.00          .000                  743                29058
24.00          .000                  616                24031
25.00          .000                  510                19680

26.00          .000                  423                16123
27.00          .000                  351                13290
28.00          .000                  291                11018
29.00          .000                  241                 9144
30.00          .000                  200                 7602

31.00          .000                  165                 6324
32.00          .000                  137                 5255
33.00          .000                  114                 4363
34.00          .000                   94                 3631
35.00          .000                   78                 3020

36.00          .000                   65                 2503
37.00          .000                   54                 2074
38.00          .000                   44                 1719
39.00          .000                   37                 1425
40.00          .000                   31                 1181

41.00          .000                   25                  979
42.00          .000                   21                  811
43.00          .000                   17                  672
44.00          .000                   14                  557
45.00          .000                   12                  462

46.00          .000                   10                  383
47.00          .000                    8                  317
48.00          .000                    7                  263
49.00          .000                    6                  218
50.00          .000                    5                  181
51.00          .000                    4                  150
```

Table L-16.—*Simulated automatic data processing printout—preliminary inflow design flood (IDF) contribution, subarea B.*

```
EXAMPLE PRELIMINARY IDF SUBAREA B

UNIT GRAPH DEVELOPED FROM DIMENSIONLESS GRAPH

DIMENSIONLESS FIGURE G-5 DESIGN OF GRAVITY DAMS                TUN = 1.00

LAG DATA GENERAL CURVE SUBAREA B                               LAG = 9.40

AREA = 560.000 SQ MI UNITGRAPH RECES COEF = 0.872335 AT 24.00 HRS

EXCESS OR STORM VOLUME = 13.040 INCHES

HYDROGRAPH VOLUME IN INCHES = 13.041 AND IN AC FT = 389484.2

HOURS        EXCESSES             UNITGRAPH           HYDROGRAPH
              INCHES                 CFS                  CFS

  .00          .000                    0                    0
 1.00          .020                  140                    3
 2.00          .640                  842                  106
 3.00         1.160                 2824                  757
 4.00         7.060                 7558                 3921
 5.00         2.260                16848                14710

 6.00         1.060                26830                42072
 7.00          .460                35618                98125
 8.00          .260                38938               194141
 9.00          .060                38289               304039
10.00          .060                32412               404552

11.00          .000                25421               459442
12.00          .000                19932               467687
13.00          .000                15560               424020
14.00          .000                12603               356437
15.00          .000                10754               290340

16.00          .000                 9518               232873
17.00          .000                 8093               188750
18.00          .000                 6930               157275
19.00          .000                 6232               134614
20.00          .000                 5464               114743

21.00          .000                 4806                98479
22.00          .000                 4229                86598
23.00          .000                 3721                75960
24.00          .000                 3558                66750
25.00          .000                 3103                58891

26.00          .000                 2707                52122
27.00          .000                 2362                47810
28.00          .000                 2060                42485
29.00          .000                 1797                37420
30.00          .000                 1568                32799

31.00          .000                 1368                28697
32.00          .000                 1193                25054
33.00          .000                 1041                21874
34.00          .000                  908                19081
35.00          .000                  792                16645

36.00          .000                  691                14520
37.00          .000                  603                12667
38.00          .000                  526                11050
39.00          .000                  459                 9639
40.00          .000                  400                 8408

41.00          .000                  349                 7335
42.00          .000                  304                 6399
43.00          .000                  266                 5582
44.00          .000                  232                 4869
45.00          .000                  202                 4247

46.00          .000                  176                 3705
47.00          .000                  154                 3232
48.00          .000                  134                 2820
49.00          .000                  117                 2460
50.00          .000                  102                 2146

51.00          .000                   89                 1872
52.00          .000                   78                 1633
53.00          .000                   68                 1424
54.00          .000                   59                 1242
55.00          .000                   52                 1084

56.00          .000                   45                  945
57.00          .000                   39                  825
58.00          .000                   34                  719
59.00          .000                   30                  628
60.00          .000                   26                  547

61.00          .000                   23                  478
62.00          .000                   20                  417
63.00          .000                   17                  363
64.00          .000                   15                  317
65.00          .000                   13                  277

66.00          .000                   11                  241
67.00          .000                   10                  210
```

Table L-17.–*Preliminary inflow design flood hydrograph, east of 105° meridian–same lag-time curve for both subareas.*

| Time, ending at hour | Discharges, 1,000 c.f.s.[1] | | |
|---|---|---|---|
| | Subarea A | Subarea B | Prelim. IDF |
| 0 | 0.00 | 0.0 | 0.0 |
| 1 | .05 | .0 | .1 |
| 2 | .6 | .1 | .7 |
| 3 | 3.4 | .8 | 4.2 |
| 4 | 13.6 | 3.9 | 17.5 |
| 5 | 40.9 | 14.7 | 55.6 |
| 6 | 96.6 | 42.1 | 138.7 |
| 7 | 184.5 | 98.1 | 282.6 |
| 8 | 268.6 | 194.1 | 462.7 |
| 9 | 306.7 | 304.0 | 610.7 |
| 10 | 291.6 | 404.6 | 696.2 |
| 11 | 242.6 | 459.4 | 702.0 |
| 12 | 192.2 | 467.7 | 659.9 |
| 13 | 150.4 | 424.0 | 574.4 |
| 14 | 121.4 | 356.4 | 477.8 |
| 15 | 99.9 | 290.3 | 390.2 |
| 16 | 83.4 | 232.9 | 316.3 |
| 17 | 70.9 | 188.8 | 259.7 |
| 18 | 60.7 | 157.3 | 218.0 |
| 19 | 52.1 | 134.6 | 186.7 |
| 20 | 45.3 | 114.7 | 160.0 |
| 21 | 39.9 | 98.5 | 138.4 |
| 22 | 34.4 | 86.6 | 121.0 |
| 23 | 29.1 | 76.0 | 105.1 |
| 24 | 24.0 | 66.8 | 90.8 |
| 25 | 19.7 | 58.9 | 78.6 |
| 26 | 16.1 | 52.1 | 68.2 |
| 27 | 13.3 | 47.8 | 61.1 |
| 28 | 11.0 | 42.5 | 53.5 |
| 29 | 9.1 | 37.4 | 46.5 |
| 30 | 7.6 | 32.8 | 40.4 |
| [2]33 | 4.4 | 21.9 | 26.3 |
| 36 | 2.5 | 14.5 | 17.0 |
| 39 | 1.4 | 9.6 | 11.0 |
| 42 | .8 | 6.4 | 7.2 |
| 45 | .5 | 4.2 | 4.7 |
| 48 | .3 | 2.8 | 3.1 |
| 51 | .2 | 1.9 | 2.1 |
| 54 | [3].1 | 1.2 | 1.3 |
| 57 | <.1 | .8 | .8 |
| 60 | | .5 | .5 |
| 63 | | .4 | .4 |
| 66 | | .2 | .2 |

Computation of IDF volume:

| | |
|---|---|
| Sum, discharges, 0-29 hrs. | 6,977,200 |
| ½ discharge, hr. 30 | 20,200 |
| Volume, 0-30 hrs. | 6,997,400 c.f.s.-hrs. |
| ½ discharge, hr. 30 | 20,200 |
| Sum, discharges, 33-63 hrs. | 74,400 |
| ½ discharge, hr. 66 | 100 |
| Sum | 94,700 |
| Volume, 30-66 hrs., (3 times 94,700) | 284,100 c.f.s.-hrs. |
| Total IDF volume | 7,281,500 c.f.s.-hrs. |
| Equivalent to | 303,400 c.f.s.-24 hrs. |
| Equivalent to | 600,800 ac.-ft. |

For a check, compare with the sum of volumes in tables L-15 and L-16, or 601,600 ac.-ft.

[1] Instantaneous at designated hour.

[2] Larger time intervals may be used for lower portions of hydrograph recessions.

[3] If needed, discharges "cut off" to shorten computations (see table L-15) may be extended using the hydrograph's recession coefficient.

lag-time by weighting subarea lag-time proportional to the areas of 240 and 560 square miles gives a lag-time of 8.6 hours. A weighted runoff curve number, CN 75, and weighted minimum retention rate, 0.20 inch per hour, are obtained as shown in table L-18. The calculations are shown because this method of weighting curve numbers is used to obtain a weighted CN for a basin (or subbasin) which contains various areas of different soil and cover complexes. Table L-18 shows the computation of incremental rainfall excesses which were applied to a 1-hour unitgraph for the watershed, lag-time 8.6 hours, area 800 square miles. Ordinates of the computed preliminary IDF hydrograph, peak discharge 768,000 c.f.s., volume 597,700 acre-feet, are plotted on figure L-17.

Either of the preliminary IDF hydrographs shown on figure L-17 could be recommended for use for preliminary designs. Under the assumptions made for computing these hydrographs, an acceptable result is obtained by considering the basin as a whole or by dividing the basin into two subareas.

(3) *Preliminary inflow design flood*

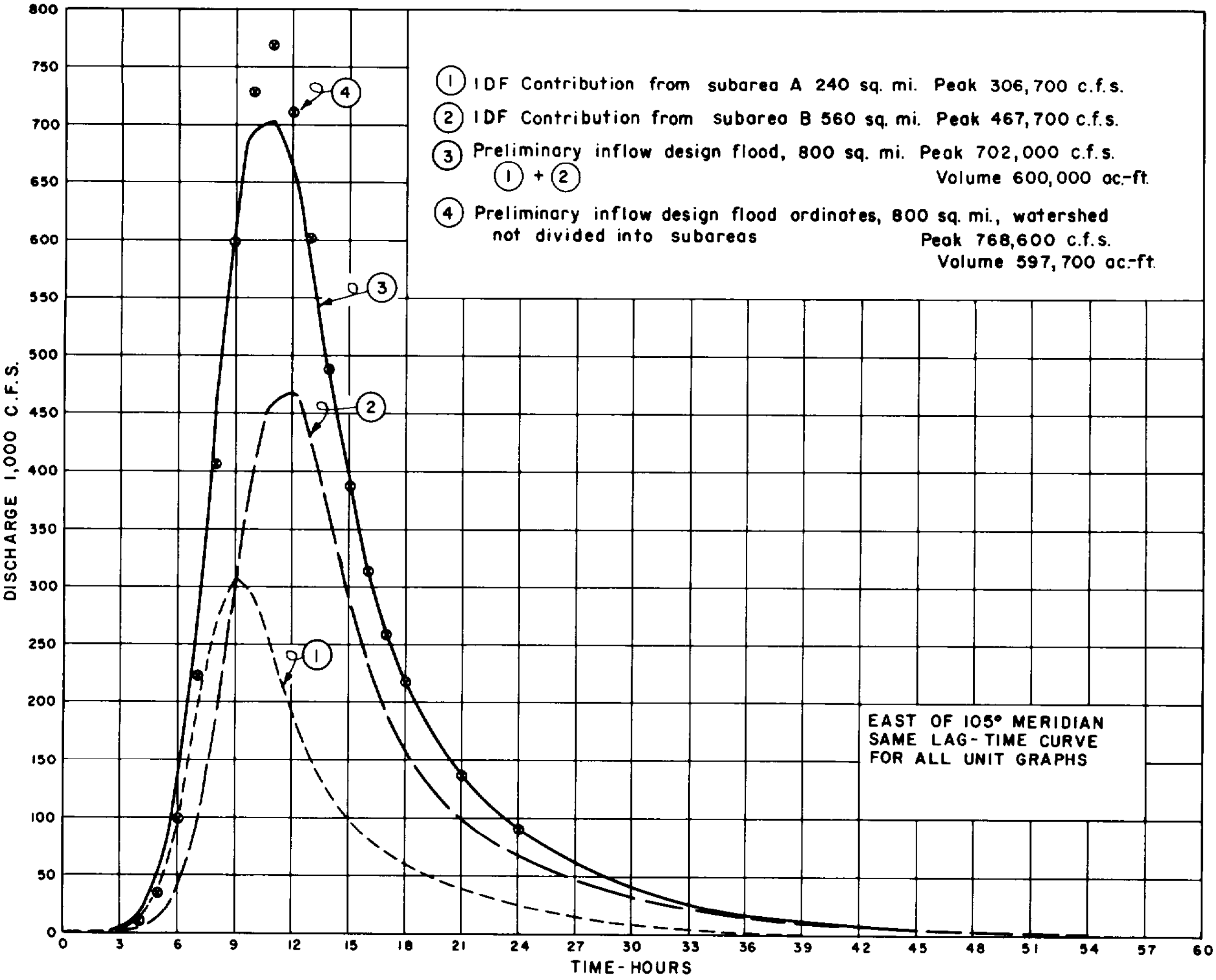


*Figure L-17.* **Example of preliminary inflow design flood hydrographs—same lag-time curve for all unitgraphs.—288-D-3188**

*hydrograph using a different lag-time curve for each subarea.*—As lag-time differences between subarea drainage systems within a basin increase, added consideration needs to be given to dividing the basin into subareas and obtaining the design flood contribution from each subarea for combination to form the inflow design flood. This is demonstrated by the hydrographs shown on figure L-18. Using the assumption given in section L-19(a)(2) that tributary B had streamflow records giving a lag-time of 22.0 hours from which a lag-time of 18.0 hours is obtained for subarea B for inflow to the proposed reservoir (sec. L-19(c)), a 1-hour unitgraph for subarea B was computed. The design flood contribution from subarea A shown on figure L-17 ① is not changed and is replotted on figure L-18 ①.

The increment of rainfall excesses for subarea B, table L-13, column 10, applied to the new unitgraph for subarea B gives the flood contribution shown on figure L-18 ②. Combining the hydrographs from the two subareas, table L-19, gives a preliminary inflow design flood hydrograph, figure L-18 ③, having two peaks, the maximum of which is a peak discharge of 332,500 c.f.s. (as estimated when plotting the graphs) and a 72-hour volume of 597,000 acre-feet. Ordinates of a flood hydrograph

Table L-18.–*Preliminary inflow design flood, east of 105° meridian–computation of incremental excesses, $\Delta P_e$, considering basin as a whole, and using an areal weighted CN and minimum loss rate.*

BASIC DATA:
Subarea A: AMC-II CN 86; min. loss, 0.12 in./hr.; area, 240 sq. mi.
Subarea B: AMC-II CN 70; min. loss, 0.24 in./hr.; area, 560 sq. mi.
WEIGHTED VALUES FOR USE:

$$\frac{(86)(240) + (70)(560)}{800} = 74.8\text{; use AMC-II CN 75}$$

$$\frac{(0.12)(240) + (0.24)(560)}{800} = 0.204\text{; use 0.20 in./hr.}$$

| Time, ending at hour | $\Delta P$,[1] inches | $\Sigma P$, inches | Rainfall excesses, $P_e$ | | |
|---|---|---|---|---|---|
| | | | $\Sigma P_e$,[3] inches | $\Delta P_e$, inches | $\Delta$loss, inches |
| 1 | 1.2 | 1.2 | 0.07 | 0.07 | 1.13 |
| 2 | 1.7 | 2.9 | .89 | .82 | .88 |
| 3 | 1.8 | 4.7 | 2.21 | 1.32 | .48 |
| 4 | 8.2 | 12.9 | 9.61 | 7.40 | .80 |
| 5 | 2.5 | 15.4 | 11.91 | 2.30 | [4].20 |
| 6 | 1.3 | 16.7 | 13.01 | 1.10 | .20 |
| 7 | .7 | 17.4 | 13.51 | .50 | .20 |
| 8 | .5 | 17.9 | 13.81 | .30 | .20 |
| 9 | .3 | 18.2 | 13.91 | .10 | .20 |
| 10 | .3 | 18.5 | 14.01 | .10 | .20 |
| 11 | .2 | 18.7 | 14.01 | 0 | .20 |
| 12 | .2[2] | 18.9 | | | |

[1] Arranged design rainfall, see column 4, table L-13.

[2] Balance of rainfall less than retention loss in this approach.

[3] By equation, $P_e = \frac{(P - 0.2S)^2}{(P + 0.8S)}$, for CN 75, $S = 3.33$, $0.2S = 0.67$, $0.8S = 2.66$ (table L-4).

[4] $\Delta P_e$ by equation indicates $\Delta$ loss of 0.10 in., less than 0.20 in.; use 0.20 in./hr.

computed using a 1-hour unitgraph having a basin weighted lag-time of 14.6 hours and incremental rainfall excesses listed in table L-18 are shown as ④ on figure L-18. This flood hydrograph has a peak of 492,000 c.f.s., excessively high in comparison with the flood hydrograph obtained by combining the two subarea flood hydrographs. The procedure of considering the watershed as a whole does not give an acceptable preliminary IDF hydrograph in this instance.

L-20. ***Preliminary Inflow Design Flood Estimates, Watersheds West of 105° Meridian.***–It is very likely that runoff from snowmelt will contribute a portion of the discharges of an inflow design flood (IDF) hydrograph for large dams at sites west of the 105° meridian. In many instances though, design rainstorm potential is so great that runoff from a design rainstorm gives the major portion of an inflow design flood. Preliminary inflow design flood estimates for many areas west of the 105° meridian can be made using preliminary design storm values obtained from figure L-13 and associated procedures, the methods of arranging design storm incremental rainfall and computing rainfall excesses given in this section, and adding appropriate base flows to the computed rain flood hydrograph. In general, for western mountainous watersheds having seasonal snowmelt runoff which reaches a maximum after mid-May, base flows for addition to the hydrograph computed from a preliminary design rainstorm may be estimated as those discharges likely to occur during the last 5 days of the maximum 15-day period of a 1 percent chance maximum annual 15-day seasonal snowmelt runoff flood. (See secs. L-28

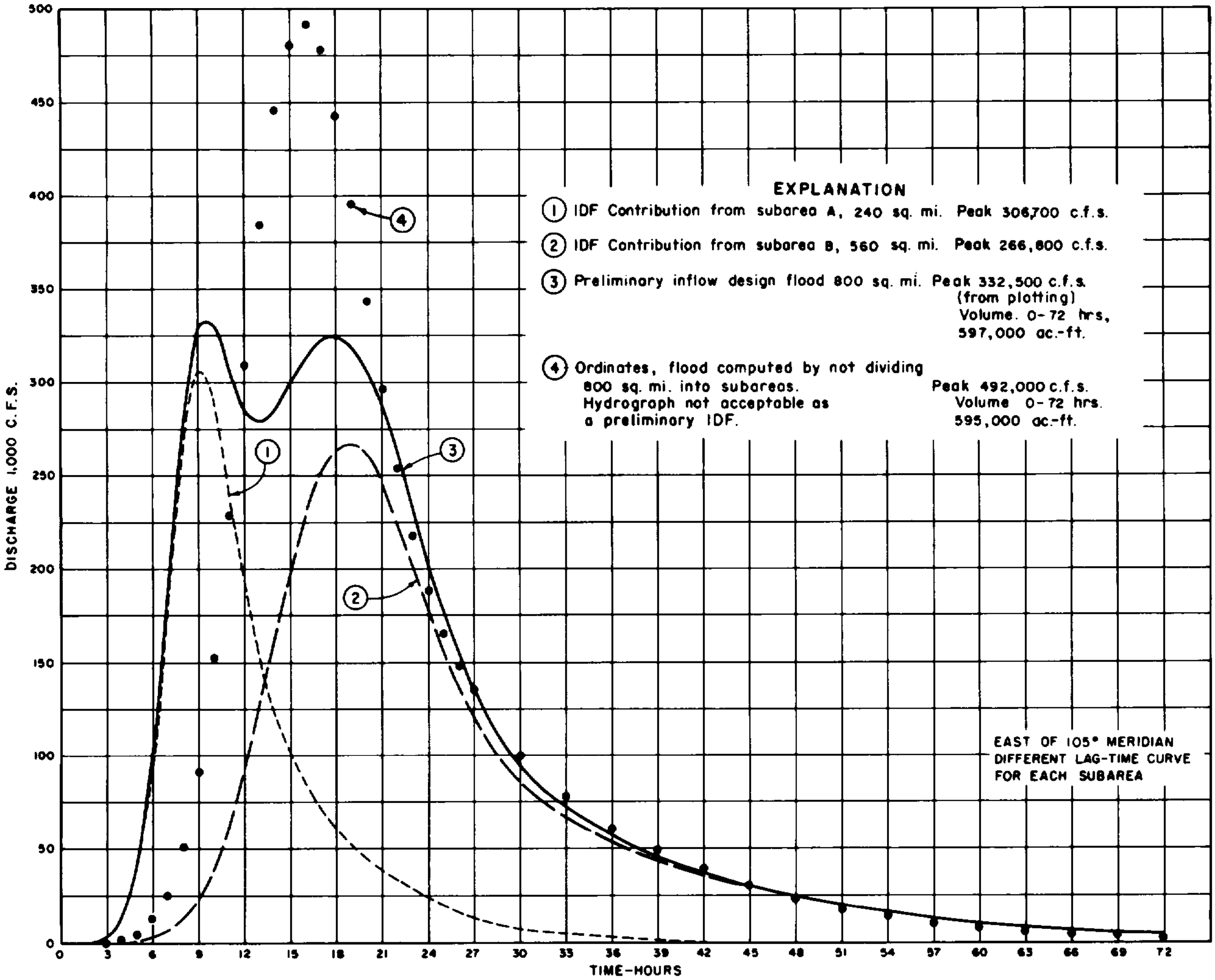


*Figure L-18.* **Example of preliminary inflow design flood hydrograph–different lag-time curve for each subarea.–288-D-3189**

and L-29 for a discussion of statistical analyses–frequency studies.) However, this general approach cannot be used for mountainous watersheds where maximum storm potential occurs during the winter months October through April. Examples are: Sierra Nevada Mountains in California and Nevada, Cascade Range in Oregon and Washington, and Mogollon Rim in Arizona. Extreme floods on streams in these regions result from rain falling on snow-covered watersheds. Estimation of rain-on-snow floods requires special procedures as discussed in sections L-22 through L-26. Exception also must include those watersheds having a large percentage of total basin drainage area at relatively low elevations where the ground may be frozen and winter rain falling on a light snow cover can cause large floods.

Procedures for estimating the rain-flood portion of a preliminary inflow design flood hydrograph from preliminary general-type design storm values for a watershed west of the 105° meridian differ in two respects from the procedures which have been given for watersheds east of the 105° meridian; namely, arrangement of design storm rainfall increments, and assignment of appropriate runoff curve number, CN.

(a) *Preliminary Design Storm Values, Watersheds West of 105° Meridian.*–By geographical location (county) obtain probable

Table L-19.–*Preliminary inflow design flood hydrograph, east of 105° meridian–different lag-time curve for each subarea.*

| Time, ending at hour | Discharges, 1,000 c.f.s.[1] | | |
|---|---|---|---|
| | Subarea [2]A | Subarea [3]B | Prelim. IDF |
| 0 | 0.00 | 0.0 | 0.0 |
| 1 | .05 | <.1 | .1 |
| 2 | .6 | <.1 | .6 |
| 3 | 3.4 | .1 | 3.5 |
| 4 | 13.6 | .5 | 14.1 |
| 5 | 40.9 | 1.3 | 42.2 |
| 6 | 96.6 | 3.2 | 99.8 |
| 7 | 184.5 | 6.9 | 191.4 |
| 8 | 268.6 | 13.1 | 281.7 |
| 9 | 306.7 | 23.5 | 330.2 |
| 10 | 291.6 | 39.5 | 331.1 |
| 11 | 242.6 | 62.8 | 305.4 |
| 12 | 192.2 | 94.1 | 286.3 |
| 13 | 150.4 | 129.1 | 279.5 |
| 14 | 121.4 | 165.3 | 286.7 |
| 15 | 99.9 | 200.7 | 300.6 |
| 16 | 83.4 | 231.0 | 314.4 |
| 17 | 70.9 | 252.5 | 323.4 |
| 18 | 60.7 | 263.5 | 324.2 |
| 19 | 52.1 | 266.8 | 318.9 |
| 20 | 45.3 | 260.5 | 305.8 |
| 21 | 39.9 | 244.4 | 284.3 |
| 22 | 34.4 | 224.4 | 258.8 |
| 23 | 29.1 | 200.8 | 229.9 |
| 24 | 24.0 | 177.5 | 201.5 |
| 25 | 19.7 | 156.4 | 176.1 |
| 26 | 16.1 | 137.6 | 153.7 |
| 27 | 13.3 | 120.8 | 134.1 |
| 28 | 11.0 | 105.9 | 116.9 |
| 29 | 9.1 | 94.9 | 104.0 |
| 30 | 7.6 | 86.0 | 93.6 |
| 33 | 4.4 | 67.6 | 72.0 |
| 36 | 2.5 | 53.2 | 55.7 |
| 39 | 1.4 | 43.2 | 44.6 |
| 42 | .8 | 35.2 | 36.0 |
| 45 | .5 | 28.7 | 29.2 |
| 48 | .3 | 24.8 | 25.1 |
| 51 | .2 | 20.3 | 20.5 |
| 54 | .1 | 16.4 | 16.5 |
| 57 | <.1 | 13.2 | 13.2 |
| 60 | | 10.6 | 10.6 |
| 63 | | 8.5 | 8.5 |
| 66 | | 6.8 | 6.8 |
| 69 | | 5.5 | 5.5 |
| 72 | | 4.4 | 4.4 |
| | | * | |

*Continuing discharges may be computed at 3-hour intervals using recession coefficient of 0.8031. Volume after hour 72:

$$\text{Vol.} = \frac{-q}{\log_e k_3}$$

$$\text{Vol.} = \frac{-4{,}400}{-0.21928}$$

Vol. = 20,060 c.f.s.-3 hrs.
2,508 c.f.s.-24 hrs.
4,970 ac.-ft.

Vol. (0-72 hrs.), 301,050 c.f.s.-hrs.
597,100 ac.-ft.

[1] Instantaneous at designated hour.

[2] Same discharges as for subarea A, table L-17.

[3] 1-hr. unitgraph, lag-time 18.0 hrs., used to compute discharges. Excesses column 10, table L-13.

maximum 6-hour point rainfall value from figure L-13. Note zone designation, A, B, or C, in which watershed is located.

(1) Compute 6-hour basin rainfall by multiplying 6-hour point rainfall by ratio obtained from applicable zone curve, figure L-14, for watershed drainage area, square miles.

(2) Make a tabulation of design storm depth-duration values at 1-hour intervals for a design storm duration extending to the hour beyond which hourly rainfall increments are equal to or less than the minimum hourly retention loss rate for the watershed. Hourly distribution of maximum 6-hour rainfall is obtained from the applicable curve of figure L-12. Design storm values beyond 6 hours are computed at 2-hour intervals by appropriate constants listed in table L-11. From 6 to 24 hours, use average of even-numbered 2-hour accumulative rainfall for the intervening odd-numbered hour. If hourly rainfall increments are needed after 24 hours, draw depth-duration curve for rainfall amounts computed by constants in table L-11 and read hourly values. Compute depth-duration rainfall values to nearest hundredth of inch.

(b) *Arrangement of Design Storm Increments of Rainfall.*—Beginning with the second largest 6-hour design storm rainfall amount, hours 6-12 of depth-duration values, arrange hourly increments of design rainfall in ascending order of magnitude for the first 6 hours of arranged design storm values. For hours 7 through 12, arrange hourly increments of maximum 6-hour rainfall in the following order of magnitude: 6, 4, 3, 1, 2, 5. Hourly rainfall amounts after the 12th hour are arranged in descending order of magnitude.

(c) *Assignment of Runoff Curve Number, CN, and Computation of Increments of Excess Rainfall.*—Watershed soils, cover and land use data are used to estimate an applicable runoff curve number from the information given in section L-7(b)(6). The estimated curve number, CN, is for antecedent moisture condition II, AMC-II. This number is then converted to the respective AMC-III CN listed in table L-4 and the AMC-III CN used to compute hourly rainfall excesses by the method illustrated in table L-13. Antecedent moisture condition III is assumed for watersheds west of the 105° meridian, because late May and June design storm potential is likely to be concurrent with, or immediately after, snowmelt runoff while watershed soil moisture is high.

If a unit time period longer than 1 hour is used for obtaining a unitgraph, the two largest increments of rainfall excesses should be grouped together. If such grouping of hourly excesses results in only 1 hourly excess increment in a unit time period at the beginning and/or end of excess rainfall period, the 1-hour increment of excess is assumed as total excess for the unit time period.

(d) *Floods From Design Thunderstorm Rainfall.*—Data for estimating design thunderstorm rainfall have not been included in this text. If an estimate of a preliminary inflow design flood (IDF) caused by design thunderstorm rainfall is required, preliminary design thunderstorm rainfall estimates for watersheds west of the 105° meridian may be obtained from generalized data in the publication "Design of Small Dams," second edition, [31] along with data for estimating increments of excess rainfall to be applied to a unitgraph. The procedures which have been described in this text for developing a unitgraph can be used to obtain a unitgraph *for that portion of a watershed* over which a design thunderstorm might occur. In the event that this type of preliminary IDF estimate proves critical for design, a hydrometeorologist should be consulted for an estimate of design thunderstorm rainfall for the specific watershed.

L-21. ***Recommendations for Routing Preliminary Inflow Design Floods Through a Proposed Reservoir.***—It is necessary for designers to assume an elevation of the reservoir pool at the start of an inflow design flood for reservoir routing studies to determine required spillway capacity. Normally, the reservoir pool is assumed to be full to the top of planned conservation storage capacity or, when either inviolate or joint use flood control capacity is proposed, full to the top of either type of flood control capacity at the beginning of a preliminary inflow design flood. If large capacities of flood control space are being considered in preliminary planning, criteria for routing a final-type IDF as discussed in sections L-30 and L-31 should be established to the extent possible with information available.

## G. SNOWMELT RUNOFF CONTRIBUTIONS TO INFLOW DESIGN FLOODS

L-22. ***General.***—"Hydraulic engineers responsible for planning and designing multiple-purpose storage reservoirs recognize snow as a form of precipitation possessing certain characteristics which can be evaluated and applied to advantage, both from a hydrologic and an economic viewpoint, in the planning and design of multipurpose storage reservoirs. In northern latitudes and at high elevations, snow falls and accumulates on the earth's surface in frozen crystalline form and usually remains until a proper sequence of meteorologic events provides the thermodynamic conditions essential for either evaporation or melting. Periodic snow surveys provide a reliable index of the relative snow accumulation. With knowledge of the processes of storage, evaporation, and melting, the engineer can predict, with reasonable accuracy (for normal climatic conditions and for known snowpack) the characteristics and amount of streamflow to be expected * * * In the Western United States, the economy of the arid and semiarid lands lying between the mountain ranges is increasingly dependent on development of multiple-purpose storage reservoirs to utilize the streamflow originating in the high mountain snow packs. Engineers of the Western States accept as a blessing the fact that *the predictable characteristics* (italics added) of this streamflow enable economies in planning and designing multiple-purpose reservoirs by the joint use of space allocated to the various functions and by reduction of spillway capacities."

The above extract from Mr. H. S. Riesbol's paper "Snow Hydrology for Multiple-Purpose Reservoirs" [22] is quoted to point out the importance of snow in hydrologic studies and the predictable characteristics of streamflow originating from snowpacks. These predictable characteristics often make possible employment of simple empirical correlations which give acceptable estimates of snowmelt runoff, although this runoff results from a complex thermodynamic process. Discussion of empirical methods of estimating snowmelt runoff as related to inflow design flood estimates is the main objective in these sections. Readers interested in more information about the physical and thermodynamic characteristics of snow and snowmelt processes may consult "Snow Hydrology" [23] and "Handbook of Applied Hydrology" [24].

As previously stated in section L-1, Bureau of Reclamation policy does not provide for combining probable maximum snowmelt runoff with probable maximum rainfall runoff for estimation of an inflow design flood. It is believed that such combinations are unreasonably severe. It is considered more reasonable to combine runoff from a probable maximum rainstorm that could occur during the snowmelt season with a major snowmelt flood, or to combine runoff from a major rainstorm that could occur during the snowmelt season with probable maximum snowmelt runoff. In regions where maximum probable rainstorms can occur during winter months when watersheds may have a large amount of snow on the ground, the amount of snow melted during the design rainstorm must be estimated and runoff calculated from the total combined rain and melted snow water available on the ground surface. Procedures have been developed for computing this type of rain-on-snow floods, utilizing data and analyses described in detail in the report "Snow Hydrology" [23]. One should be mindful that each individual IDF study requires some variations within the framework of a general approach, depending upon watershed characteristics, location, basic data available, and proposed operational capacity of the future reservoir.

L-23. *Major Snowmelt Runoff During Seasonal Melt Period for Combination With Probable Maximum Storm Runoff.*—A method of estimating snowmelt runoff contribution for this type of combination has been described briefly in connection with preliminary IDF estimates for watersheds west of the 105° meridian. Additional items need be considered when making "best possible" preliminary IDF or final-type IDF estimates. Inclusion of flood control capacity and its amount in a proposed reservoir may have a direct bearing on the time duration of flow required in estimation of an inflow design flood hydrograph.

(a) *Damsites for Reservoirs With no Flood Control Capacity Proposed.*—These projects are intended to store seasonal snowmelt runoff as rapidly as possible, allowing only minimum required releases until reservoir capacity becomes full to top of conservation storage. A duration time of 15 days is usually adequate for an inflow design flood hydrograph for this type of structure, as a reservoir may be assumed full to top of conservation capacity at the beginning of the 15-day period. A 1 percent chance (100 year) 15-day volume of snowmelt runoff is usually considered as a major snowmelt flood. It is obtained from a frequency study of maximum annual 15-day snowmelt runoff volumes using runoff records for the contributing watershed, if available, or records for similar nearby watersheds. The 15-day volume indicated by the frequency computations (secs. L-28 and L-29) is adjusted to the specific watershed above a damsite by area relationships.

*Caution*: Occasionally there will be found references or data of an extremely large snowmelt flood exceeding all recently recorded floods and, perhaps, exceeding the 1 percent chance value indicated by frequency analyses of more recent records. These data should not be ignored without making full effort to incorporate the data into the snowmelt flood estimate.

(1) *Assembly of basic streamflow data for frequency analyses.*—Concurrently with tabulation of maximum annual 15-day seasonal snowmelt runoff values from streamflow records, climatological data should be examined to determine if each year's 15-day runoff volume was snowmelt runoff or was increased by rainfall amounts large enough to cause runoff during that period (small rainfall events may be ignored). If a large snowmelt volume is indicated, an estimate of the rain-flood portion can be made and subtracted by plotting the daily discharge values on semilogarithmic paper and sketching an estimated snowmelt recession (due to lower temperatures accompanying rainfall) under the obvious rain-flood portion. This procedure may have to be used in a few regions where almost every year some rainfall runoff is concurrent with snowmelt runoff.

(2) *Daily distribution of 1 percent chance 15-day snowmelt runoff volume.*—Springtime snowmelt runoff coordinates closely with temperature fluctuations. Large areas usually have about the same daily temperature sequence. Usually snow-fed streams in a given vicinity have similar daily distribution patterns of runoff, magnitudes of discharges reflecting individual watershed snowmelt contributing areas. These distribution patterns will also be similar year to year. Therefore, a distribution pattern for one of the larger 15-day volumes recorded for the stream where a damsite is located, or for a nearby similar watershed, can be selected and the 1 percent chance 15-day snowmelt runoff volume for the damsite distributed into daily discharges proportional to the selected recorded flood. An approximately symmetrical 15-day pattern with the maximum daily discharge occurring within the 7th to 10th day of the 15-day period is usually selected. An additional refinement may be included in selecting the distribution pattern, if by chance climatological records show that a small rain event occurred a day or two after the maximum daily discharge of a large recorded 15-day volume and discharges decreased due to lowered temperatures associated with the rain event. This sequence of events agrees with the pattern of natural conditions assumed by the occurrence of a probable maximum rainstorm a day or two after the maximum day of snowmelt runoff.

(3) *Combination of probable maximum rain flood with 1 percent chance 15-day snowmelt flood.*—Selection of an appropriate day within a 15-day period of snowmelt runoff as a beginning time of design rain-flood runoff is a matter of engineering judgment. One reasonable assumption is a 2-day interval between the day of maximum temperature and the beginning of runoff caused by a design storm. Under this assumption, the apparent lag-time in days between maximum temperature and maximum daily snowmelt discharge from a watershed should be considered. The lag-time may be quickly determined by plotting a few of the larger annual maximum 15-day mean daily discharges and respective daily maximum temperatures from an index temperature record. Depending on size and runoff characteristics of a watershed, the time interval between maximum temperature and resulting daily maximum snowmelt discharges at a damsite may vary from zero to 3 or more days. If the time interval is zero days, design rain-flood runoff is added to the snowmelt runoff, beginning on the third day after the peak of the snowmelt flood. As the lag-time interval between

maximum temperature and peak of snowmelt runoff increases, the beginning time for a design rain-flood hydrograph is advanced closer to the peak of the snowmelt flood by 1-day intervals. Thus, for large watersheds, it may be reasonably to combine a design rain flood with the maximum daily discharges of a snowmelt flood.

(b) *Damsites for Reservoirs With Proposed Joint Use Flood Control Capacity.*—A reservoir which has a joint use flood control capacity allocation is intended to control seasonal snowmelt discharges downstream from the dam to a limit of safe channel capacity throughout the entire snowmelt season, and also to store enough water to assure that the reservoir is full to the top of the joint use capacity at the end of each snowmelt season. Forecasts of seasonal snowmelt runoff volumes are a necessary part of this kind of operation.

A seasonal major snowmelt flood as a part of an inflow design flood (IDF) hydrograph usually is required when joint use flood control capacity is proposed. However, if planned joint use capacity is small and there is a likelihood that snowmelt discharges preceding the maximum 15-day period of a 1 percent chance snowmelt flood may fill the joint use pool, a 15-day IDF hydrograph will be adequate. When a seasonal major snowmelt flood hydrograph for combination with a probable maximum rain-flood hydrograph is needed, first consideration is given to the use of streamflow data.

The duration period of a seasonal IDF corresponds with the seasonal duration of the largest snowmelt floods which have occurred in the vicinity. Frequency analyses include annual maximum 30-day, 60-day, and if needed 90-day periods of snowmelt volumes in addition to analysis of the annual maximum 15-day discharge period. A recorded seasonal snowmelt flood is selected as a pattern for runoff distribution. The design rain flood is combined with the estimated snowmelt runoff hydrograph according to the criteria previously discussed.

If available streamflow data are not suitable for satisfactory results using the above approach, one of the methods of temperature-runoff correlations described in the referenced publications may be found adaptable to the situation.

L-24. ***Probable Maximum Snowmelt Floods to be Combined With Major Rain Floods.***—(a) *General.*—An estimate of probable maximum snowmelt runoff may be necessary when making an inflow design flood (IDF) study for a watershed where snowmelt runoff causes the major portion of yearly flow. The degree of refinement needed in making this type of estimate may vary from preliminary comparisons to computation by detailed procedures depending on factors such as the following: storage capacity, space allocations, and operational plans of the proposed reservoir; snowmelt runoff characteristics of the watershed; and difference in magnitudes of probable maximum rainstorm and major rainstorm potentials for the watershed. For some watersheds, a few preliminary computations may show an IDF combination of major snowmelt runoff and probable maximum rain runoff to be definitely critical for design. In other instances detailed computations of each type IDF consisting of combined snowmelt and rain runoff has to be made and both types of IDF hydrographs prepared for use in design of a dam.

Studies prepared by the Bureau of Reclamation show that usually a critical inflow design flood results from a combination of runoff of a major snowmelt flood and a probable maximum rainstorm. In most instances, an approximation of probable maximum snowmelt flood magnitude by simple correlations shows that it will not be critical for design. Development of a *best estimate* of probable maximum snowmelt runoff is a complex procedure and requires special treatment for each site. Therefore, this discussion is limited to general aspects of the problem, with references to publications containing more detailed information.

(b) *Considerations for Estimates of Probable Maximum Snowmelt Floods.*—Estimating probable maximum snowmelt contribution to an inflow design flood can be thought of as requiring three steps: (1) estimating probable maximum seasonal accumulation of snow on a watershed, (2) estimating critical melt rates of the snow pack, and (3) estimating the amount of snowmelt runoff and its timing at the reservoir. The probable maximum seasonal accumulation of snow on a mountainous watershed drained by one main stream can be adequately estimated by a study of winter season precipitation records in and near the watershed, supplemented by snow survey data. Special studies are required for probable maximum seasonal snow accumulation estimates for large multitributary river systems such as the Colorado River above Glen Canyon Dam. One of two basic approaches can be taken to estimate critical snowmelt rates; namely, calculation of snowmelt runoff by means of an air temperature index, or calculation of melt using generalized snowmelt equations based on energy balance considerations. Methods using some form of an air temperature index have given good results for many watersheds. There is some physical basis for using a snowmelt air temperature index. Air temperature is reasonably well correlated, at a particular time and place, with the atmospheric factors which affect melt rates, such as solar radiation and vapor pressure, although it is by no means a perfect index of these factors.

Snowmelt equations which consider energy balance are used to evaluate short-wave radiation melt, long-wave radiation melt, melt due to convective heat transfer from the atmosphere and to latent heat of water vapor condensing into the snow surface, melt due to heat of rain drops, and melt by heat conduction from the ground. The Corps of Engineers report "Snow Hydrology" [23] presents detailed information regarding both approaches. A Corps manual, "Runoff from Snowmelt," EM 1110-2-1406 [25], presents synopses of investigations of melting relationships, generalized basin snowmelt equations and their application in methods of computing maximum snowmelt floods. Selection of an approach to be used depends on the basic data available and the importance of snowmelt runoff contribution to an inflow design flood. Whichever approach is taken, it is necessary to test the snowmelt computation procedures for the basin in question in order to determine basin values of the coefficients involved.

Approximation of a maximum probable snowmelt flood for a period of 10 to 20 days usually is directed toward determination of volume. This volume is then distributed in time by using a large recorded snowmelt runoff hydrograph as a pattern, as previously described in section L-23(a)(2). If a temperature index has been used directly in the computations, the volume may be distributed by a synthetic temperature sequence.

(c) *Springtime Seasonal Probable Maximum Snowmelt Flood Estimates.*—General procedures for estimating total seasonal probable maximum snowmelt runoff are not outlined in detail in this text. Brief statements about some approaches which may be considered for use, and reference to respective specific descriptions, are given below.

(1) *Hydrothermogram approach.*—The paper, "Snow Hydrology for Multiple-Purpose Reservoirs" [22], includes a description of an approach in which during the melting season daily temperatures above a base temperature are directly related to resulting direct runoff by a device referred to as a *hydrothermogram*. A hydrothermogram is a hypothetical discharge hydrograph computed on the assumption that each effective degree of temperature above a base temperature will generate the same amount of runoff volume. This procedure, adjusted to fit individual basin problems, has been found useful in several Bureau of Reclamation IDF studies (unpublished) where probable maximum snowmelt flood estimates were important.

(2) *Generalized melt equations for springtime snowmelt floods.*—The Corps of Engineers Manual, "Runoff from Snowmelt" [25], includes a chapter describing probable maximum snowmelt flood derivation using generalized melt equations. The Salmon River Basin which drains 14,100 square miles of rugged, mountainous regions of central Idaho is cited as an example in the discussion.

(3) *Correlations.*—Correlations between temperature and runoff, snowcover and runoff,

etc., are usually evidenced because of the predictable nature of snowmelt runoff. Hydrologists knowledgeable in the use of correlation studies may find this type of approach useful.

(d) *Major Rain-Flood Estimates for Combination With Probable Maximum Snowmelt Runoff.* –

(1) *Major rainstorm and runoff.*–Design storm studies for watersheds where snowmelt runoff contributes to inflow design floods should also include a hydrometeorological estimate of a major rainstorm that could occur during the snowmelt season. For areas where major rainstorms have often occurred in the vicinity of the watershed during the snowmelt season, the largest rainstorm of record within the area of transposability is fitted to the basin. In areas where major rainstorm occurrences during the spring snowmelt season are infrequent, watershed design storm values without maximization for moisture adjustment may be considered. A hydrograph of runoff from the major rainstorm is computed by the dimensionless-graph lag-time procedures previously discussed, but special attention is given to effects of snowmelt on retention losses applicable to the major rainstorm. The portion of the watershed covered by a melting snowpack will have little or no retention capacity for rainfall, and the portion recently denuded of snow will have high moisture content, hence low retention capacity during rainfall. Guide criteria for combining rain-flood hydrographs and snowmelt flood hydrographs have been discussed in section L-23(a)(3).

(2) *Observed rain floods.*–Occasionally, streamflow data used for snowmelt runoff analyses will include a major rain flood during a snowmelt season. In these instances, special studies are made to separate the rain-flood hydrograph from the snowmelt runoff, and the separated rain-flood hydrograph is used for combination with the estimated probable maximum snowmelt flood hydrograph.

L-25. ***Probable Maximum Rain-On-Snow IDF Estimates.***–There are many watersheds along or near the coasts of the United States where major rainstorms or probable maximum rainstorms can occur during the winter months while the watersheds are partially or completely covered with snow. In many areas, storm systems may consist of precipitation beginning as snow then changing to rain or closely spaced successive storm systems, the first system occurring as snow, the second as rain accompanied by warm temperatures. Devastating floods have resulted from certain rain-on-snow combinations; in other instances, apparently similar conditions have produced only high flows causing little damage. Detailed investigations of differences between rain-on-snow flood magnitudes point toward the following two items as the main contributors to these differences: density conditions of the snowpack at the time of rain occurrence, and convective condensation melt related to wind velocities during the rainstorm. Generalized equations for estimating snowmelt during rainfall, developed as described in "Snow Hydrology" [23], have proved very useful in procedures for estimating runoff due to rainfall on snow.

In addition to estimates of snowpack melting rates, procedures for estimating runoff caused by rain-on-snow conditions include evaluations of snowpack release of free water to the ground surface, retention losses, and distribution in time of the runoff at the point of interest. A procedure used by the Corps of Engineers is given in the manual, "Runoff from Snowmelt" [25]. The procedure used in Bureau of Reclamation studies is described in Engineering Monograph No. 35, "Effect of Snow Compaction on Runoff from Rain on Snow" [26]. In both procedures snow melting rates during rainfall are computed by the same melting equations and water released at ground surface is determined. Excesses are computed by subtracting retention losses, and are distributed in time by a basin unitgraph. Differences between the procedures lie in estimations of snowpack free-water holding capacities.

The Corps procedure establishes a limit of liquid water holding capacity of a snowpack as a percentage of snowpack water content. Nearly all data considered when developing the limit of water holding capacity were obtained from spring snowpack of densities above 35 percent. The procedure in Engineering Monograph No. 35 relates snowpack liquid water holding capacity to snowpack densities just preceding the start of rainfall, and to increases in snowpack density due to melting and added rainfall until the pack attains a density of 40 or 45 percent when release of liquid water to the ground surface is assumed to begin. Development of the procedure was directed primarily for use for evaluating wintertime conditions where a rainstorm system closely follows a snowstorm and the newly deposited snowpack has had little time to change in structure. Topics of discussion in Engineering Monograph No. 35 are a development of the procedure and reconstitution of the December 1955 flood on South Yuba River near Cisco, Calif. Estimation of a probable maximum rain-on-snow flood is not discussed in the monograph. Data required for use of the procedure for IDF computations are: (1) estimates of watershed snowcover depth and water content antecedent to a design storm occurrence; and (2) hydrometeorological data of temperatures and wind velocities concurrent with design storm rainfall increments.

L-26. *Special Situations.*—(a) *Frozen Ground.*—Frozen ground conditions seldom occur in well-forested areas or under deep snowpacks. On the other hand, open areas where periods of subfreezing temperatures and light snowfall are normal can develop frozen soil conditions such that retention losses are practically nil. These areas may experience severe winter floods due to combinations of shallow snowcover, rising temperature, and relatively minor rainfall. Frozen ground conditions may also reduce lag-time. Analyses for this type of condition require individual watershed study.

(b) *Snowmelt in the Great Plains Region of the United States.*—Probable maximum precipitation potential is so great in the Great Plains region that snowmelt runoff is not usually considered in inflow design flood studies except for large drainage areas with headwaters in the Rocky Mountains. In the northern Great Plains, major floods have resulted from rapid spring snowmelt and frozen ground conditions. Consideration of this type of flood may be necessary for large drainage areas near the northern border of the United States.

## H. ENVELOPE CURVES

L-27. *General.*—Peak discharge envelope curves and flood volume envelope curves can be prepared by drawing curves enveloping plotted points representing maximum recorded values for various drainage areas. The values plotted should represent similar type floods (rain floods or snowmelt floods) that have occurred within the broad geographical subdivision within which the subject watershed lies, and should not be limited to events of a single small river system. Preparation of envelope curves for a general area provides an engineer with valuable information on past flood history and an indication of the flood of record comparable to the subject area. However, they should not be relied upon as a means of estimating probable maximum flood values. Design flood values purporting to be the probable maximum should be higher than those obtained from envelope curves. Only in specific instances where a watershed has definitely lower flood potential than neighboring watersheds due to soil type, surface storage, etc., would it be good judgment to adopt an inflow design flood of smaller magnitude than that of a flood which has occurred nearby.

A simple method of preparation of envelope curves is to tabulate maximum peak discharges (or volumes of a selected duration) and respective drainage areas prior to plotting points. In most instances, the drainage area above a stream gaging station or the point of a large flood discharge measurement is given in the U.S. Geological Survey water supply paper listing the flood. When it is known that only a portion of the drainage area above a point of measurement contributed to a flood, the size of that contributing portion should be used in the envelope curve analysis. Discharges or volumes are plotted versus respective drainage areas using log-log paper. Data thus plotted usually indicate a curved line envelopment on log-log paper which may be approximated by a straight line for small ranges in areas. High discharges from local thunderstorms may suggest consideration of two curves–one for smaller areas subject to such occurrences and another for larger areas where maximum discharges originate from general storms.

## I. STATISTICAL ANALYSES—ESTIMATES OF FREQUENCY OF OCCURRENCE OF FLOODS

**L-28.** *General.*–Estimates of the magnitude of floods which have frequencies of 1 in 5, 1 in 10, or 1 in 25 years are helpful in estimating requirements for stream diversion during construction. These floods are often termed the "5-, 10-, or 25-year flood." The magnitude of more rare events such as the 50- or 100-year flood may be required for reasons such as to establish sill location of emergency spillways, etc. The usual term of expression, "*x*-year flood," should not lead to the wrong conclusion that the event indicated can happen only once in *x* years, and having occurred, will not happen again for another period of *x* years. It does mean that over a long span of years we can expect as many *x*-year floods (or larger) as there are *x*-year-long periods within that span. Floods occur randomly and may be bunched or spread out unevenly with respect to time. No predictions are possible for determining their distribution; the probable maximum flood *can* occur the first year after the project is built, though of course, the odds are heavily against it.

The frequency of a flood should be considered as the chances of occurrence of a flood of that size (or one larger) in any one year. Stated another way, the chances of the flood in any one year being equaled or exceeded by floods of the magnitudes indicated as the 5-, 10-, 25-, or 100-year floods have ratios of 20:100, 10:100, 4:100, and 1:100, respectively.

Many methods of flood frequency determinations based on streamflow data have been published. Excellent summaries of these methods, along with comments on factors affecting their accuracy and limitations, are contained in the papers entitled "Review of Flood Frequency Methods" [27] and "Methods of Flow Frequency Analysis" [28]. While the many methods of flood frequency determinations made from streamflow data are all based on acceptable statistical procedures, the difference in methodology can give appreciably different results when extensions are made beyond the range of adequate data. To provide for a uniformity in Federal water resources planning, the Water Resources Council has recommended that all Government agencies use the Log-Pearson type III distribution as a base method. The method is described in the publication "A Uniform Technique for Determining Flood Flow Frequencies" [29]. Hazen's method [30] gives results that are comparable to those obtained with the Log-Pearson type III method and is easier to use when computations are made by hand with or without the aid of mechanical calculating machines. A procedural outline for Hazen computations is presented in section 59 of "Design of Small Dams," second edition [31].

If streamflow data for a period of 20 years or more are available for the subject watershed or comparable watersheds, frequency curve computations yield acceptable results for estimates up to the 25-year flood and may be extrapolated to indicate the 100-year flood with a fair assurance of obtaining acceptable values.

**L-29.** ***Hydrographs for Estimating Diversion Requirements During Construction.***–Usually, inflow design flood (IDF) studies include hydrographs of floods for different frequencies of occurrence to be used for estimation diversion requirements during construction of a dam.

The hydrograph of a particular frequency flood is usually sketched to conventional shape using the peak discharge value and corresponding volume value obtained from computed frequency curves. In some instances, a peak discharge and associated volume of a recorded flood will correspond closely with a particular frequency value, in which case the recorded flood hydrograph is used.

## J. FINAL-TYPE INFLOW DESIGN FLOOD STUDIES

L-30. *General.*—Preparations of final-type inflow design flood (IDF) studies differ from preliminary studies only in the degree of refinement used to estimate each variable causing flood runoff. For example, a basin unitgraph may be derived from a single large flood hydrograph in a preliminary study, whereas in a final-type study several flood hydrographs are analyzed and a selected basin unitgraph tested by reproduction of recorded flood hydrographs. Perhaps the most important consideration in the preparation of final-type studies is making certain that all available hydrological and meteorological data available, including historical and recent events, have been considered properly. A hydrometeorologist prepares the design storm study for the basin, including therein design temperatures and wind velocities if rain-on-snow floods are to be considered. Preliminary estimates of each flood-producing variable are reviewed and revised if additional data so indicate. Preliminary dam and reservoir operation plans are examined for certainty that the critical IDF situation for the chosen type of design and operation has been used.

Hydrologists and hydrometeorologists must estimate effects of ever-varying natural phenomena. Studies of these phenomena as related to a particular watershed begin with the inception of a project and continue thereafter, unless the project is determined infeasible and not built.

L-31. ***Flood Routing Criteria.***—Normally, the reservoir pool is assumed to be full to the top of conservation storage at the start of the routing of the inflow design flood (IDF). However, when either inviolate or joint use flood control space is provided, the determination of space available at the beginning of the inflow design flood will depend upon the spacing of preceding storms, the relative magnitude of snowmelt contribution to the design flood, and the operational criteria proposed for the reservoir.

(a) *Preceding Storms.*—In some areas of the west, for example areas for which the Gulf of Mexico is the moisture source, the meteorological situation is such that a major storm could occur a few days prior to the maximum possible storm. In these areas, the flood control pool is assumed to be partially or completely occupied at the start of the inflow design flood. The determination of the portion of flood control pool that is occupied depends upon the distance of the area from the moisture source and a study of historical flood events in the area.

(b) *Seasonal Flood Hydrograph.*—For those areas in which floods occur on a fixed seasonal basis, largely as the result of snowmelt, it is frequently desirable to prepare a flood-season hydrograph including the inflow design flood and maximum antecedent and supervening flows that could reasonably be expected to occur with the inflow design flood. This hydrograph is then routed through the reservoir with the conservation pool full at the beginning of the season inflow, if that assumption can be justified on the basis of carryover storage. Otherwise, the minimum drawdown for the beginning date of seasonal inflow is selected from project operation studies.

(c) *Operational Criteria.*—The assumed reservoir elevation at the start of the inflow design flood will also be dependent upon the type of flood control space, which may be a fixed inviolate amount or a varying amount, normally referred to as joint use storage space.

The varying amount of flood control storage required will be based on operational parameters which show the needed amount of flood control storage based on antecedent precipitation, or the needed amount of storage based on forecasts of the seasonal runoff expected from the snowcover measurements.

## K. BIBLIOGRAPHY

### L-32. *Bibliography.*

[1] "Criteria and Practice Utilized in Determining the Required Capacity of Spillways," USCOLD Committee on "Failures and Accidents to Large Dams, Other than in Connection with the Foundations," United States Committee on Large Dams, C/O Engineers Joint Council, 345 East 47th Street, New York, N.Y. 10017.

[2] "Estimation of Maximum Floods," Technical Note No. 98, Report of a working group of the Commission for Hydrometeorology, World Meteorological Organization, Secretariat of the World Meteorological Organization, Geneva, Switzerland, 1969.

[3] U.S. Department of Agriculture, Soil Conservation Service National Engineering Handbook, Section 4, Hydrology, January 1971 or most recent publication. (For sale by Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.)

[4] Sherman, L. K., "Streamflow from Rainfall by the Unit-Graph Method," Engineering News-Record, vol. 108, pp. 501-505, 1932.

[5] Johnstone, D., and Cross, W. P., "Elements of Applied Hydrology," The Ronald Press Co., New York, N.Y., 1949.

[6] Federal Inter-Agency River Basin Committee, Bulletin No. 1, "Instructions for Compilation of Unit Hydrograph Data," attached to the minutes of the 28th meeting of the Subcommittee on Hydrologic Data, March 1948.

[7] Barnes, B. S., "Discussion of Analysis of Runoff Characteristics," Trans. ASCE, vol. 105, 1940, p. 106.

[8] Langbein, W. B., "Channel Storage and Unit Hydrograph Studies," Trans. American Geophysical Union, 1940, Part II, pp. 620-627.

[9] Snyder, F. F., "Synthetic Unit-Graphs," Trans. American Geophysical Union, vol. 19, 1938, pp. 447-454.

[10] Clark, C. O., "Storage and the Unit Hydrograph," Trans. ASCE, vol. 110, 1945, pp. 1419-1488.

[11] Crawford, N. H., and Linsley, R. K., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Technical Report No. 39, July 1966, Department of Civil Engineering, Stanford University, Stanford, Calif.

[12] Meserve, E. C., "Use of Clark Unit Graphs and Application of Clark Method to Pond Creek Study," April 1952, AWR Joint Study on Pond Creek, Little Rock District, Corps of Engineers.

[13] Linsley, R. K., Kohler, M. A., and Paulhus, J.L.H., "Hydrology for Engineers," McGraw-Hill Book Co., Inc., New York, N.Y., 1958.

[14] Tatum, F. E., "A Simplified Method of Routing Flood Flows through Natural Valley Storage," unpublished memorandum, U.S. Engineers Office, Rock Island, Ill., May 29, 1940.

[15] Wilson, W. T., "A Graphical Flood-Routing Method," Trans. American Geophysical Union, Part III, 1941.

[16] California Division of Water Resources, "Report on Control of Floods, San Joaquin River and Tributaries between Friant Dam and Merced River," July 1954.

[17] "Manual for Depth-Area-Duration Analysis of Storm Precipitation," WMO No. 237, TP. 129, Secretariat of the World Meteorological Organization, Geneva, Switzerland.

[18] "Tables of Precipitable Water and Other Factors for a Saturated Pseudo-Adiabatic Atmosphere," Technical Paper No. 14, 1951. U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service, Washington, D.C.

[19] "Climatic Atlas of the United States," U.S. Department of Commerce, Environmental Science Service Administration, Environmental Data Service, Washington, D.C. 20402, June 1968.

[20] U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service, Hydrometeorological Reports (selected):

Report No. 2, "Maximum Possible Precipitation over the Ohio River Basin above Pittsburgh, Pennsylvania," 1942.

Report No. 3, "Maximum Possible Precipitation over the Sacramento Basin of California," 1943.

Report No. 20, "An Estimate of Maximum Possible Flood-Producing Meteorological Conditions in the Missouri River Basin Above Garrison Dam Site," 1945.

Report No. 21, "A Hydrometeorological Study of the Los Angeles Area," 1939.

Report No. 21A, "Preliminary Report on Maximum Possible Precipitation, Los Angeles Area, California," 1944.

Report No. 21B, "Revised Report on Maximum Possible Precipitation, Los Angeles Area, California," 1945.

Report No. 22, "An Estimate of Maximum Possible Flood-Producing Meteorological Conditions in the Missouri River Basin Between Garrison and Fort Randall," 1946.

Report No. 23, "Generalized Estimates of Maximum Possible Precipitation Over the United States East of the 105th Meridian, for Areas of 10, 200, and 500 Square Miles," 1947.

Report No. 24, "Maximum Possible Precipitation Over the San Joaquin Basin, California," 1947.

Report No. 25, "Representative 12-Hour Dewpoints in Major United States Storms East of the Continental Divide," 1947.

Report No. 25A, "Representative 12-Hour Dewpoints in Major United States Storms East of the Continental Divide," 2d edition, 1949.

Report No. 28, "Generalized Estimate of Maximum Possible Precipitation Over New England and New York," 1952.

Report No. 33, "Seasonal Variation of the Probable Maximum Precipitation East of the 105th Meridian for Areas from 10 to 1,000 Square Miles and Durations of 6, 12, 24, and 48 Hours," 1956.

Report No. 36, "Interim Report–Probable Maximum Precipitation in California," Washington, D.C., 1961.

Report No. 39, "Probable Maximum Precipitation in the Hawaiian Islands," Washington, D.C., 1963.

Report No. 40, "Probable Maximum Precipitation, Susquehanna River Drainage above Harrisburg, Pennsylvania," Washington, D.C., 1965.

Report No. 41, "Probable Maximum and TVA Precipitation over the Tennessee River Basin above Chattanooga," Washington, D.C., 1965.

Report No. 42, "Meteorological Conditions for the Probable Maximum Flood on the Yukon River above Rampart, Alaska," Washington, D.C., 1966.

Report No. 43, "Probable Maximum Precipitation, Northwest States," Washington, D.C., 1966.

Report No. 44, "Probable Maximum Precipitation over the South Platte River, Colorado, and Minnesota River, Minnesota," Washington, D.C., 1969.

*Cooperative Studies Reports, Cooperative Studies Section, Division of Climatological and Hydrologic Services, National Weather Service, in cooperation with the Bureau of Reclamation:*

Report No. 9, "Maximum Possible Flood-Producing Meteorological Conditions." (1) Colorado River Basin above Glen Canyon Damsite, (2) Colorado River Basin above Bridge Canyon Damsite, (3) San Juan River Basin above Bluff Damsite, (4) Little Colorado River Basin above Coconino Damsite. June 1949.

Report No. 11, "Critical Meteorological Conditions for Design Floods in the Snake River Basin," February 1953.

Report No. 12, "Probable Maximum Precipitation on Sierra Slopes of the Central Valley of California," Washington, D.C., March 1954.

[21] "Storm Rainfall in the United States, Depth-Area-Duration Data," Department of the Army Office of the Chief of Engineers, Washington, D.C., 1945.

[22] Riesbol, H. S., "Snow Hydrology for Multiple-Purpose Reservoirs," Trans. ASCE, vol. 119, 1954, pp. 595-627.

[23] "Snow Hydrology," Summary Report of Snow Investigations, U.S. Corps of Engineers, June 1956.

[24] "Handbook of Applied Hydrology," A Compendium of Water-Resources Technology, Ven Te Chow (Editor-in-Chief), McGraw-Hill Book Co., Inc., New York, N.Y., 1964.

[25] "Runoff from Snowmelt," EM 1110-2-1406, U.S. Corps of Engineers, 1960. (For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., price $1.75.)

[26] Bertle, F. A., "Effect of Snow Compaction on Runoff From Rain on Snow," Engineering Monograph No. 35, Bureau of Reclamation, 1966. (For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, or Bureau of Reclamation, Engineering and Research Center, Attention 922, Building 67, Denver Federal Center, Denver, Colorado 80225.)

[27] "Review of Flood Frequency Methods," Final Report of the Subcommittee of the Joint Division Committee on Floods, Trans. ASCE, vol. 118, 1953, pp. 1220-1231.

[28] "Methods of Flow Frequency Analysis," Bulletin No. 13, Subcommittee on Hydrology, Inter-Agency Committee on Water Resources (now the Hydrology Committee, Water Resources Council), Washington, D.C., April 1966.

[29] "A Uniform Technique for Determining Flood Flow Frequencies," Bulletin No. 15, Hydrology Committee, Water Resources Council, Washington, D.C., December 1967.

[30] Hazen, A., "Flood Flows," John Wiley & Sons, Inc., New York, N.Y., 1930.

[31] "Design of Small Dams," second edition, Bureau of Reclamation, 1973.

# Sample Specifications for Concrete

M-1. *Introduction.*—Designs of any structure are based on assumptions regarding the quality of work which will be obtained during construction. It is through the means of specifications that the assumed quality is described, and it is important that conformance to the specifications be obtained for all work.

This appendix includes sample specifications for concrete in the dam and its appurtenances. For the construction of a particular dam, these specifications will be supplemented by local conditions, selected provisions, and special measures required for the construction of the structure.

The sample specifications are written on the basis that the concrete mixes to be used in the work will be designed and controlled by the purchaser (referred to in the specifications as the Contracting Authority or simply as the Authority) within the maximum water to cement or water to cement plus pozzolan ratio and slump limitations specified, the limitations for quality and grading of aggregates, and the limitations for the other materials as specified. Also, the specifications are written on the basis that the quantity of sand and each size of coarse aggregate to be used in the concrete mixes will be determined by the purchaser. The quality limitations shown in the specifications for sand and coarse aggregate are considered as standard limits. These limits may be reduced when only substandard materials are available within economical hauling distance, and provided it has been determined by tests of concrete made with such aggregates that durable concrete meeting the design strength criteria can be produced.

Under these specifications the purchaser's own engineering force or an engineering organization retained by the purchaser would accomplish testing of proposed aggregates and other materials, perform the design of mixes, and handle the inspection and quality testing throughout the contract. If the purchaser will require the contractor to provide such mix design, inspection and control, the specifications should so provide and should include specific design compressive strength(s) at designated age(s) for the concrete. The concrete mixes should be designed to provide compressive strengths of test cylinders such that 80 percent of the cylinders will have compressive strength(s) at the specified age(s) greater than the design compressive strength [1].[1]

References to "designations" in the sample specifications refer to designations in the appendix of the Bureau of Reclamation Concrete Manual, eighth edition [1]. Where materials or other requirements are to conform to Federal specifications, or other standard specifications such as ASTM, the construction specifications for specific work should provide that the specifications for the materials or requirements concerned should be in compliance with the latest editions or revisions thereof in effect on the date bids are received or award of contract is made, whichever is appropriate.

The maximum size of coarse aggregate for concrete arch dams may be smaller than the 6-inch maximum-size aggregate provided for in

---

[1] Numbers in brackets refer to items in the bibliography, sec. M-25.

the sample specifications. Arch dams generally require a high compressive strength, and Bureau of Reclamation studies on the effect of maximum size of aggregate on the compressive strength of concrete have shown that high strength concrete can generally be produced with less cement utilizing smaller maximum-size aggregate. Where low strength concrete is adequate, a larger maximum-size aggregate is used to achieve optimum cement economy. For the double-curvature thin-arch concrete dams being constructed by the Bureau of Reclamation, a maximum-size coarse aggregate in the range of 3 to 4½ inches may be used, depending on the quality of the aggregates and cement efficiency as determined by mix design studies.

M-2. *Contractor's Plants, Equipment, and Construction Procedures.*—Prior to the installation of the contractor's plants and equipment for processing, handling, transporting, storing, and proportioning concrete ingredients, and for mixing, transporting, and placing concrete, the contractor shall submit drawings covering his plans for approval by the Contracting Authority, showing proposed plant arrangement, including plans of locations and description of facilities for sampling of concrete and concrete materials as hereinafter provided. Included with the plans shall be a description of the equipment the contractor proposes to use in sufficient detail that an adequate review can be accomplished. The drawings and description of plant, equipment, and sampling and testing facilities shall be submitted at least 60 days prior to plant erection.

After completion of installation, the operation of the plant and equipment shall be subject to the approval of the Contracting Authority.

Sampling and testing facilities for use by the Authority shall be provided by the contractor and shall include power-driven mechanical sampling devices, satisfactory to the Authority, as may be necessary for procuring and handling representative test samples of aggregates and other concrete materials during batching; and for obtaining samples of concrete as discharged from the mixers, for mixer efficiency, slump, and other tests, except that power-driven mechanical sampling devices will not be required for sampling concrete from truck mixers if and when the use of truck mixers is permitted by these specifications. The concrete sampling device shall be capable of procuring samples of concrete from any point in the discharge stream as the concrete is being discharged from the mixer.

After completion of the plant installation, the operation of the sample taking facilities shall be demonstrated to the satisfaction of the Authority that they are suitable for the purpose intended. If truck mixers are used where permitted by these specifications, the contractor shall provide a stable, level platform with adequate shelter, satisfactory to the Authority, for concrete tests at the point of discharge from the truck mixers. The contractor shall also provide ample and protected working space adjacent to the batching and mixing plants, free from plant vibration; and shall furnish necessary utilities such as compressed air, water, heat, and electrical power for operation of the Authority's testing equipment and for execution of tests by Authority personnel of concrete and concrete materials at the batching and mixing plants.

Where these specifications require specific types of equipment to be used or specific procedures to be followed, such requirements are not to be construed as prohibiting use by the contractor of alternative types of equipment or procedures if it can be demonstrated to the satisfaction of the Authority that equal results will be obtained by the use of such alternatives. Approval of plants and equipment or their operation, or of any construction procedure, shall not operate to waive or modify any provisions or requirement contained in these specifications governing the quality of the materials or of the finished work.

The cost of providing facilities and working space for procuring and handling representative test samples of concrete and concrete materials at the batching and mixing plants shall be included in the prices bid in the schedule for concrete.

The contractor shall keep the Authority advised as to when batching and mixing of concrete, installation of reinforcement and forming, preparations for placing and placing of concrete, finishing, and repair of concrete will be performed. Unless inspection is waived in each specific case, these construction activities shall be performed only in the presence of a duly authorized Authority inspector.

M-3. ***Composition.***–(a) *General.*–Concrete shall be composed of cement, pozzolan, sand, coarse aggregate, water, and admixtures as specified, all well mixed and brought to the proper consistency. It is contemplated that pozzolan will be used in all concrete except for miscellaneous items of concrete where elimination of pozzolan is directed by the Contracting Authority.

(b) *Maximum Size of Aggregate.*–The maximum size of coarse aggregate in concrete for any part of the work shall be the largest of the specified sizes, the use of which is practicable from the standpoint of satisfactory consolidation of the concrete by vibration.

Except where it is determined by the Authority that, owing to closely spaced reinforcement or other reasons, the use of a smaller maximum size of aggregate is necessary to obtain satisfactory placement of the concrete, the maximum size of aggregate shall be as follows:

(1) Six-inch maximum-size aggregate shall, in general, be used in concrete for the dam, stilling basins, gravity walls, and elsewhere in other equally massive portions of structures where concrete containing the 6-inch maximum-size aggregate can be properly placed.

(2) Three-inch maximum-size aggregate shall be used in concrete for walls that are 15 inches or more in thickness and in slabs that are 8 inches or more in thickness, such as in massive floors and walls, and elsewhere where concrete containing 6-inch maximum-size aggregate cannot be placed, except that the requirements of subsection (3) below shall apply for tunnels, and for structures under conditions indicated.

(3) Three-inch maximum-size aggregate shall be used in concrete in tunnels where the concrete is 12 inches or more in thickness and the reinforcement, if any, consists of only one row or will not otherwise prevent satisfactory placement of the concrete, as determined by the Authority: *Provided*, that the contractor may use 2½-inch maximum-size aggregate to facilitate pumping: *Provided further*, that the contractor may use 2½-inch maximum-size aggregate in concrete that would otherwise contain 3-inch maximum-size aggregate whenever concrete containing 2½-inch maximum-size aggregate is being used at that time in work requiring pumping. One and one-half-inch maximum-size aggregate shall be used in concrete in tunnels where the concrete is less than 12 inches in thickness and for greater thicknesses when it is determined by the Authority that concrete containing a larger maximum size of aggregate cannot be properly placed.

(4) One and one-half-inch maximum-size aggregate shall be used in concrete for walls (except tunnel walls) that are less than 15 inches in thickness and in slabs that are less than 8 inches in thickness. However, where the walls or slabs are so heavily reinforced that 1½-inch size aggregate cannot be properly placed, as determined by the Authority, ¾-inch maximum-size aggregate may be permitted.

(5) In locations where concrete is to be placed against excavated surfaces and the thickness of concrete to be placed is greater than that shown on the drawings, correspondingly larger maximum size aggregate from that specified for the thickness of concrete shown on the drawings shall be used: *Provided*, that aggregate with a maximum size greater than that indicated above will not be required.

(c) *Mix Proportions.*–The proportions in which the various ingredients are to be used for different parts of the work and the appropriate water to portland cement plus pozzolan ratio will be determined by the Authority. Adjustments in the mix proportions and water to portland cement plus pozzolan ratio will be made by the Authority from time to time during the progress of the work, as tests are made of samples of the aggregates and the resulting concrete. These adjustments will have the objective of procuring concrete having suitable workability, density, impermeability, durability, and required strength, without the use of an excessive amount of cement.

It is contemplated that the composition of the concrete will be within the ranges given in the tabulation below.

The proportions shown in the referenced tabulation may be modified by the Authority to suit the work or the nature of the materials, or to comply with limitations on the water to portland cement plus pozzolan ratio, and the contractor shall be entitled to no extra compensation by reason of such modification.

The net water to portland cement plus pozzolan ratio of the concrete (exclusive of water absorbed by the aggregates) shall not exceed 0.47, by weight, for concrete in thin sections of structures which will be exposed to frequent alternations of freezing and thawing, such as curbs, gutters, sills, the top 2 feet of walls, piers, and parapets; and walls of structures in the range of fluctuating water levels or subject to spray. The net water to portland cement plus pozzolan ratio shall not exceed 0.53, by weight, for other concrete in structures which will be exposed to freezing and thawing. The net water to portland cement plus pozzolan ratio shall not exceed 0.60, by weight, for mass concrete in the dam, stilling basin, gravity walls, and elsewhere in other equally massive portions of structures; and for concrete in structures that will be covered with fill material or be continually submerged or otherwise protected from freezing and thawing.

(d) *Consistency.*—The amount of water used in the concrete shall be regulated as required to secure concrete of the proper consistency and to adjust for any variation in the moisture content or grading of the aggregates as they enter the mixer. Addition of water to compensate for stiffening of the concrete before placing will not be permitted. Uniformity in concrete consistency from batch to batch will be required.

The slump of the concrete, after the concrete has been deposited but before it has been consolidated, shall not exceed 2 inches for mass concrete; for concrete in the tops of walls, piers, parapets, and curbs; and for concrete in slabs that are horizontal or nearly horizontal. Similarly, the slump shall not exceed 4 inches for concrete in sidewalls and arch of tunnel lining; and 3 inches for all other concrete. The Authority reserves the right to require a lesser slump whenever concrete of such lesser slump can be consolidated readily into place by means of the vibration specified in section M-18(c) (Consolidation). The use of buckets, chutes, hoppers, or other equipment which will not readily handle and place concrete of such lesser slump will not be permitted.

(e) *Tests.*—The compressive strength of the concrete will be determined by the Authority through the medium of tests of 6- by 12-inch cylinders made and tested in accordance with designations 29 to 33, inclusive, of the eighth edition of the Bureau of Reclamation Concrete Manual [1], except that, for all concrete samples from which cylinders are to be cast, the pieces of coarse aggregate larger than 1½ inches will be removed by screening or hand picking. Slump tests will be made by the Authority in accordance with designation 22.

**M-4.** ***Cement.***—(a) *General.*—Cement for concrete, mortar, and grout shall be furnished

| Maximum size of aggregate (inches) | Cementing materials, portland cement plus pozzolan (approximate) | | Sand, percent of total aggregate, by weight | Coarse aggregate, percent of total coarse aggregate only, by weight | | | |
|---|---|---|---|---|---|---|---|
| | Total pounds per cubic yard of concrete | Percent pozzolan (by weight of portland cement plus pozzolan) | | 3/16 to ¾ inch | ¾ to 1½ inches | 1½ to 3 inches | 3 to 6 inches |
| 6<br>3<br>1½<br>¾ | (Values to be determined by laboratory tests and inserted here for specifications.) | | | | | | |

by the contractor. The cement shall be free from lumps, unground clinker, tramp metal, and other foreign material, and shall be otherwise undamaged when used in concrete. If the cement is delivered in paper bags, empty paper bags shall be disposed of as directed. The contractor shall inform the Contracting Authority in writing, at least 60 days before first shipments are required, concerning the mill or mills from which the cement is to be shipped; whether cement will be ordered in bulk or in bags; and the purchase order number, contract number, or other designation that will identify the cement to be used by the contractor.

When bulk cement is not unloaded from the primary carriers directly into weathertight hoppers at the batching plant, transportation from the mill, railhead, or intermediate storage to the batching plant shall be accomplished in adequately weathertight trucks, conveyors, or other means which will protect the cement completely from exposure to moisture. Separate facilities, other than those provided for pozzolan, shall be provided for unloading, transporting, storing, and handling bulk cement. Locked unloading facilities shall be provided, and unloading of cement shall be performed only in the presence of the Authority or his representative. Immediately upon receipt at the jobsite, bulk cement shall be stored in dry, weathertight, and properly ventilated bins which shall be constructed so that there will be no dead storage. All storage facilities shall be subject to approval and shall be such as to permit easy access for inspection and identification.

The bins shall be emptied and cleaned by the contractor when so directed; however, the intervals between required cleanings will normally be not less than 4 months. If cement is obtained from more than one cement plant, shipments from each plant shall be blended with those from the other plant or plants by placing the cement from the different plants in alternate layers when unloading into silos at the railhead or at the jobsite, or by any other method satisfactory to the Authority. To prevent undue aging of cement furnished in bags, after delivery, the contractor shall use the bagged cement in the chronological order in which it was delivered to the jobsite. Each shipment of cement in bags shall be stored so that it may readily be distinguished from other shipments.

The cement shall meet the requirements of Federal Specification SS-C-192G [9], including Amendment 3 for type II, low-alkali cement, and shall meet the false-set limitation specified therein. In addition, cement for contraction joint grouting shall be air separated, and 100 percent of the finished product, after processing at the cement plant, shall pass a No. 30 United States standard sieve and 97.7 percent shall pass a No. 100 United States standard sieve. Cement for contraction joint grouting shall also be screened at the jobsite through a No. 16 crimped screen which shall be installed by the contractor between the mixer and agitator in the grout plant. The cement for contraction joint grouting shall be furnished in waterproof bags which will prevent hydration of the cement from exposure and also prevent lumping of the cement due to warehouse set for a minimum of 90 days. Cement for foundation grouting shall be furnished in bags: *Provided*, that bulk cement may be used for such grouting if a suitable method, satisfactory to the Authority, is used for weighing and accounting for the cement used.

(b) *Inspection.*–Except for sieve fineness of cement for contraction joint grouting, the cement will be sampled and tested by the Authority in accordance with Federal Test Method Standard No. 158A [11], including Change Notice 1 thereto, except that for initial penetration under method 2501.1 the rod shall be released 20 seconds after completion of mixing, and except that the note at the end of method 2501.1 concerning variations in initial penetration will be disregarded.

Fineness tests of the cement for contraction joint grouting will be made by the Authority in accordance with ASTM Designation C 184 [5], except that the tests will be performed on No. 30 and No. 100 sieves.

Acceptance tests, except for false set but including fineness tests, will be made on samples taken as bins of cement are filled and reserved for exclusive Authority use.

Acceptance tests for false set will be made on samples taken from the cement at the latest time, prior to shipment in cars or trucks, that the cement is still in possession of the cement company. Cement not meeting test requirements will be rejected, and the contractor shall be entitled to no adjustments in price or completion time by reason of any delays occasioned thereby.

The contractor will be charged the cost of testing of all Authority-tested cement which has been ordered in excess of the amount of cement used for the work under these specifications. The charges to be made for the cost of testing excess cement will be at the rate of 3.5 cents per hundredweight (cwt.), which charge includes the Authority overhead, and will be deducted from payments due the contractor.

(c) *Measurement and Payment.*—Measurement, for payment, of cement furnished in bags will be on the basis of the number of bags of cement used at the mixer. Measurement, for payment, of bulk cement will be on the basis of batch weights at the batching plant. Any cement, either bulk or in bags, used for grouting, finishing, or other miscellaneous work will be measured for payment in the most practicable manner. One bag of cement shall be considered as 0.94 hundredweight.

Payment will be made for cement used in concrete placed within the pay lines for concrete; and for cement used in concrete placed outside the concrete pay lines, unless the requirement for such concrete is determined by the Authority to be the result of careless excavation, or excavation intentionally performed by the contractor to facilitate his operations. No payment will be made for cement used as follows: cement used in wasted concrete, mortar, or grout; cement used in the replacement of damaged or defective concrete; cement used in extra concrete required as a result of careless excavation; and cement used in concrete placed by the contractor in excavation intentionally performed by the contractor to facilitate his operations. As determined by the Authority, payment will be made for a reasonable amount of cement used in grout required to keep the pipelines full during the grouting operations.

Payment for furnishing and handling cement will be made at the applicable unit prices per hundredweight or bag bid therefor in the schedule, which unit prices shall include the cost of rail and truck transportation of the cement from the mill to the jobsite and the cost of storing the cement.

M-5. ***Pozzolan.***—(a) *General.*—Pozzolan for concrete shall be furnished by the contractor. The contractor shall use pozzolan concrete as provided in section M-3 (Composition). The pozzolan shall be in accordance with Federal Specification SS-P-570B [10].

When bulk pozzolan is not unloaded from primary carriers directly into weathertight hoppers at the batching plant, transportation from the source railhead or intermediate storage to the batching plant shall be accomplished in adequately designed trucks, conveyors, or other means which will protect the pozzolan completely from exposure to moisture. Separate facilities, other than those for cement, shall be provided for unloading, transporting, storing, and handling bulk pozzolan. Locked unloading facilities shall be provided and unloading of pozzolan shall be performed only in the presence of the Contracting Authority or his representative.

Immediately upon receipt at the jobsite, bulk pozzolan shall be stored in dry, weathertight, and properly ventilated bins. All storage facilities shall be subject to approval and shall be such as to permit easy access for inspection and identification. Sufficient pozzolan shall be in storage at all times to complete any concrete lift or placement started. The bins shall be emptied and cleaned by the contractor when so directed; however, the intervals between required cleanings will normally be not less than 4 months. The pozzolan shall be free from lumps and shall be otherwise undamaged when used in concrete.

The contractor shall inform the Authority in writing, within 60 days after date of notice to proceed, concerning the source or sources from which he proposes to obtain the pozzolan; together with information as to location, shipping point or points, purchase order number, contract number, or other designation and information that will identify the pozzolan to be used by the contractor.

(b) *Inspection.*—The pozzolan will be sampled and tested by the Authority in accordance with Federal Specification SS-P-570B [10]. Acceptance tests will be made on a lot or lots of pozzolan, which lot or lots shall be reserved in bulk storage in sealed bins at the source for exclusive Authority use. Untested lots shall not be intermingled or combined with tested and approved lots until such lots have been tested and approved. Pozzolan will also be sampled at the jobsite when determined necessary. Release for shipment and approval for use will be based on compliance with 7-day lime-pozzolan strength requirements and other physical and chemical and uniformity requirements for which tests can be completed by the time the 7-day lime-pozzolan strength test is completed. Release for shipment and approval for use on the above basis will be contingent on continuing compliance with the other requirements of the specifications. No pozzolan shall be shipped until notice has been given that the test results are satisfactory and all shipments will be made under supervision of the Authority. Any lot or lots of pozzolan not meeting test requirements will be rejected. Rejected pozzolan shall be replaced with acceptable pozzolan, and the contractor shall be entitled to no adjustments in price or completion time by reason of any delays occasioned thereby.

The contractor will be charged the cost of testing of all Authority-tested pozzolan which has been ordered in excess of the amount of pozzolan used for the work under these specifications. The charges to be made for the cost of testing excess pozzolan will be at the testing rate per ton plus overhead cost to the Authority and will be deducted from payments due the contractor.

(c) *Measurement and Payment.*—Measurement, for payment, of pozzolan will be made on the basis of batch weights at the batching plant with deductions made for the percentage of moisture in the pozzolan. The moisture content will be determined by heating a 500-gram sample to constant weight in an oven at 105° C. The percentage of moisture will be 100 times the quantity obtained by dividing the loss in weight, in grams, by the weight in grams of the moist sample. Any pozzolan used for miscellaneous work will be measured in the most practicable manner.

Pozzolan will be paid for on the basis of the number of tons (2,000 pounds net dry weight) used in the work covered by these specifications. No payment will be made for pozzolan used as follows: pozzolan used in wasted concrete; pozzolan used in the replacement of damaged or defective concrete; pozzolan used in extra concrete required as a result of careless excavation; and pozzolan used in concrete placed by the contractor in excavation intentionally performed by the contractor to facilitate his operations.

Payment for furnishing and handling pozzolan will be made at the unit price per ton bid therefor in the schedule, which unit price shall include the cost of rail and truck transportation of the pozzolan from the mill to the jobsite and the cost of storing the pozzolan.

**M-6.** *Admixtures.*—(a) *Accelerator.*—Calcium chloride shall not be used in concrete in which aluminum or galvanized metalwork is to be embedded or in concrete where it may come in contact with prestressed steel. The contractor shall use 1 percent of calcium chloride, by weight of the cement, in all other concrete placed when the mean daily temperature in the vicinity of the worksite is lower than 40° F. Calcium chloride shall not be used otherwise, except upon written approval of the Contracting Authority. Request for such approval shall state the reason for using calcium chloride and the percentage of calcium chloride to be used and the location of the concrete in which the contractor desires to use the calcium chloride. Calcium chloride shall not be used in excess of 2 percent, by weight of the cement. Calcium chloride shall be measured accurately and shall be added to the batch in solution in a portion of the mixing water. Use of calcium chloride in the concrete shall in no way relieve the contractor of responsibility for compliance with the requirements of these specifications governing protection and curing of the concrete.

(b) *Air-Entraining Agents.*—An air-entraining agent shall be used in all concrete. The agent used shall conform to ASTM Designation C 260 [6], except that the limitation and test on bleeding by concrete containing the agent and the requirement relating to time of setting shall not apply. The agent shall be of uniform consistency and quality within each container and from shipment to shipment. Agents will be accepted on manufacturer's certification of compliance with specifications: *Provided*, that the Authority reserves the right to require submission of and to perform tests on samples of the agent prior to shipment and use in the work and to sample and test the agent after delivery at the jobsite.

The amount of air-entraining agent used in each concrete mix shall be such as will effect the entrainment of the percentage of air shown in the following tabulation in the concrete as discharged from the mixer:

| *Maximum size of coarse aggregate in inches* | *Total air, percent by volume of concrete* |
|---|---|
| ¾ | 6.0 plus or minus 1 |
| 1½ | 4.5 plus or minus 1 |
| 3 | 3.5 plus or minus 1 |
| 6 | 3.0 plus or minus 1 |

The agent in solution shall be maintained at uniform strength and shall be added to the batch in a portion of the mixing water. This solution shall be accurately batched by means of a reliable mechanical batcher which shall be so constructed that the full measure of solution added to each batch of concrete can be observed in a sight gage by the plant operator prior to discharge of the solution into the mixer. When calcium chloride is being used in the concrete, the portion of the mixing water containing the air-entraining agent shall be introduced separately into the mixer.

(c) *Water-Reducing, Set-Controlling Admixture.*—The contractor shall, except as hereinafter provided, use a water-reducing, set-controlling admixture, referred to herein as WRA, in all concrete. The WRA used shall be either a suitable lignosulfonic-acid or hydroxylated-carboxylic-acid type.

The WRA shall be of uniform consistency and quality within each container and from shipment to shipment. WRA will be accepted on manufacturer's certification of conformance to Bureau of Reclamation "Specifications and Method of Test for Water-Reducing, Set-Controlling Admixtures for Concrete," dated August 1, 1971: *Provided*, that the Authority reserves the right to require submission of and to perform tests on samples of the agent prior to shipment and use in the work and to sample and test the agent after delivery at the jobsite.

If Authority testing of the WRA is required, the contractor shall submit a sample of the WRA and five bags (94 pounds each) of the cement proposed for use in the work at least 90 days before use is expected. The size of the sample of WRA to be submitted shall be 1 liquid gallon.

The quantity of WRA to be used in each concrete batch shall be determined by the Authority and for the lignosulfonic-acid type shall not exceed 0.40 percent, by weight of cement plus pozzolan, of solid crystalline lignin, and for the hydroxylated-carboxylic-acid type shall not exceed 0.50 percent, by weight of cement plus pozzolan, of liquid.

Since the quantity of WRA required will vary with changing atmospheric conditions, the quantity used shall be commensurate with the prevailing conditions. The Authority reserves the right to use lesser quantities or no WRA in concrete for any part of the work, depending on climatic or other job conditions, and the contractor shall be entitled to no additional compensation by reason of reduction in or elimination of WRA in any concrete to be placed under these specifications.

The WRA solution shall be measured for each batch by means of a reliable visual mechanical dispenser. The WRA, in a suitably dilute form, may be added to water containing air-entraining agent for the batch if the materials are compatible with each other, or shall be introduced separately to the batch in a portion of the mixing water if the two are incompatible.

When requested, the contractor shall submit test data by the manufacturer showing effects of the WRA on mixing water requirements, setting time of concrete, and compressive strength at various ages up to 1 year.

The contractor shall be responsible for any difficulties arising or damages occurring as a result of the selection and use of WRA, such as delay or difficulty in concrete placing or damage to the concrete during form removal. The contractor shall be entitled to no additional compensation above the unit prices bid in the schedule for concrete by reason of such difficulties.

(d) *Furnishing Admixtures.*–Air-entraining agent, accelerator, and WRA, as required, shall be furnished by the contractor, and the cost of the materials and all costs incidental to their use shall be included in the applicable prices bid in the schedule for concrete in which the materials are used.

M-7. ***Water.***–The water used in concrete, mortar, and grout shall be free from objectionable quantities of silt, organic matter, alkali, salts, and other impurities.

M-8. ***Sand.***–(a) *General.*–The term "sand" is used to designate aggregate in which the maximum size of particles is 3/16 of an inch. Sand for concrete, mortar, and grout shall be furnished by the contractor and shall be natural sand, except that crushed sand may be used to make up deficiencies in the natural sand grading. The contractor shall maintain at least three separate stockpiles of processed sand; one to receive wet sand, one in the process of draining, and one that is drained and ready for use. Sand to be used in concrete shall be drawn from the stockpile of drained sand which shall have been allowed to drain for a minimum of 48 hours. Sand, as delivered to the batching plant, shall have a uniform and stable moisture content, which shall be less than 6 percent free moisture.

(b) *Quality.*–The sand shall consist of clean, hard, dense, durable, uncoated rock fragments. The maximum percentages of deleterious substances in the sand, as delivered to the mixer, shall not exceed the following values:

| *Deleterious substance* | *Percent, by weight* |
|---|---|
| Material passing No. 200 screen (designation 16) . . . . . . . . . . . . . . . . . | 3 |
| Lightweight material (designation 17) . . . . . | 2 |
| Clay lumps (designation 13) . . . . . . . . . . . | 1 |
| Total of other deleterious substances (such as alkali, mica, coated grains, soft flaky particles, and loam) . . . . . . . . . | 2 |

The sum of the percentages of all deleterious substances shall not exceed 5 percent, by weight. Sand producing a color darker than the standard in the colorimetric test for organic impurities (designation 14) may be rejected. Sand having a specific gravity (designation 9), saturated surface-dry basis, of less than 2.60 may be rejected. The sand may be rejected if the portion retained on a No. 50 screen, when subjected to 5 cycles of the sodium sulfate test for soundness (designation 19), shows a weighted average loss of more than 8 percent, by weight. The designations in parentheses refer to methods of tests described in the eighth edition of the Bureau of Reclamation Concrete Manual [1].

(c) *Grading.*–The sand as batched shall be well graded, and when tested by means of standard screens (designation 4) shall conform to the following limits:

| *Screen No.* | *Individual percent, by weight, retained on screen* |
|---|---|
| 4 | 0 to 5 |
| 8 | * 5 to 15 |
| 16 | *10 to 25 |
| 30 | 10 to 30 |
| 50 | 15 to 35 |
| 100 | 12 to 20 |
| Pan | 3 to 7 |

*If the individual percent retained on the No. 16 screen is 20 percent or less, the maximum limit for the individual percent retained on the No. 8 screen may be increased to 20 percent.

The grading of the sand shall be controlled so that at any time the fineness moduli (designation 4) of at least 9 out of 10 consecutive test samples of finished sand will not vary more than 0.20 from the average fineness modulus of the 10 test samples.

M-9. *Coarse Aggregate.*–(a) *General.*–The term "coarse aggregate," for the purpose of these specifications, designates aggregate of sizes within the range of 3/16 of an inch to 6 inches or any size or range of sizes within such limits. The coarse aggregate shall be reasonably well graded within the nominal size ranges hereinafter specified. Coarse aggregate for concrete shall be furnished by the contractor and shall consist of natural gravel or crushed rock or a mixture of natural gravel and crushed rock.

Coarse aggregate, as delivered to the batching plant, shall have a uniform and stable moisture content.

(b) *Quality.*–The coarse aggregate shall consist of clean, hard, dense, durable, uncoated rock fragments. The percentages of deleterious substances in any size of coarse aggregate, as delivered to the mixer, shall not exceed the following values:

| | *Percent, by weight* |
|---|---|
| Material passing No. 200 screen (designation 16) . . . . . . . . . . . . . . . . . . | ½ |
| Lightweight material (designation 18) . . . . . | 2 |
| Clay lumps (designation 13) . . . . . . . . . . . | ½ |
| Other deleterious substances . . . . . . . . . . . | 1 |

The sum of the percentages of all deleterious substances in any size, as delivered to the mixer, shall not exceed 3 percent, by weight. Coarse aggregate may be rejected if it fails to meet the following test requirements:

(1) Los Angeles rattler test (designation 21).–If the loss, using grading A, exceeds 10 percent, by weight, at 100 revolutions or 40 percent, by weight, at 500 revolutions.

(2) Sodium sulfate test for soundness (designation 19).–If the weighted average loss after 5 cycles is more than 10 percent by weight.

(3) Specific gravity (designation 10).–If the specific gravity (saturated surface-dry basis) is less than 2.60.

The designations in parentheses refer to methods of test described in the eighth edition of the Bureau of Reclamation Concrete Manual [1].

(c) *Separation.*–The coarse aggregate shall be separated into nominal sizes and shall be graded as follows:

| Designation of size (inches) | Nominal size range (inches) | Minimum percent retained on screens indicated | |
|---|---|---|---|
| | | Percent | Size of screen (inches) |
| ¾ | 3/16 to ¾ | 50 | 3/8 |
| 1½ | ¾ to 1½ | 25 | 1¼ |
| 3 | 1½ to 3 | 20 | 2½ |
| 6 | 3 to 6 | 20 | 5 |

Coarse aggregate shall be finished screened on vibrating screens mounted over the batching plant, or at the option of the contractor, the screens may be mounted on the ground adjacent to the batching plant. The finish screens, if installed over the batching plant, shall be so mounted that the vibration of the screens will not be transmitted to, or affect the accuracy of the batching scales. The sequence of coarse aggregate handling and plant management shall be such that, if final and/or submerged cooling are used, excessive free moisture shall be removed and diverted outside of the plant by dewatering screens prior to finish screening so that a uniform and stable moisture content is maintained in the plant storage and batching bins. The method and rate of feed shall be such that the screens will not be overloaded and will operate properly in a manner that will result in a finished product which consistently meets the grading requirements of these specifications. The finished products shall pass directly to the individual batching bins. Material passing the 3/16-inch screen that is removed from the coarse aggregate as a result of the finished screening operation shall be wasted.

Separation of the coarse aggregate into the specified sizes, after finish screening, shall be such that, when the aggregate, as batched, is tested by screening on the screens designated in the following tabulation, the material passing the undersize test screen (significant undersize) shall not exceed 2 percent, by weight, and all material shall pass the oversize test screen:

| Aggregate size designation (inches) | Size of square opening in screen (inches) | |
|---|---|---|
| | For undersize test | For oversize test |
| ¾ | No. 5 mesh (U.S. standard screen) | 7/8 |
| 1½ | 5/8 | 1¾ |
| 3 | 1¼ | 3½ |
| 6 | 2½ | 7 |

Screens used in making the tests for undersize and oversize will conform to ASTM Designation E 11 [7], with respect to permissible variations in average openings.

**M-10.** ***Production of Sand and Coarse Aggregate.***—(a) *Source of Aggregate.*—Sand and coarse aggregate for concrete, and sand for mortar and grout may be obtained by the contractor from any approved source as hereinafter provided.

If sand and coarse aggregate are to be obtained from a deposit not previously tested and approved by the Contracting Authority, the contractor shall submit representative samples for preconstruction test and approval at least 60 days after date of notice to proceed. The samples shall consist of approximately 200 pounds each of sand and 3/16- to 3/4-inch size of coarse aggregate, and 100 pounds of each of the other sizes of coarse aggregate.

The approval of deposits by the Authority shall not be construed as constituting the approval of all or any specific materials taken from the deposits, and the contractor will be held responsible for the specified quality of all such materials used in the work.

In addition to preconstruction test and approval of the deposit, the Authority will test the sand and coarse aggregate during the progress of the work and the contractor shall provide such facilities as may be necessary for procuring representative samples.

If any deposit used by the contractor is located within an approved area owned or controlled by the Authority, no charge will be made to the contractor for materials taken from such deposit and used in the work covered by these specifications. Any royalties or other charges required to be paid for materials taken from deposits not owned or controlled by the Authority shall be paid by the contractor.

(b) *Developing Aggregate Deposit.*—The contractor shall carefully clear the area of the deposit, from which aggregates are to be produced, of trees, roots, brush, sod, soil, unsuitable sand and gravel, and other objectionable matter. If the deposit is owned or controlled by the Authority, the portion of the deposit used shall be located and operated so as not to detract from the usefulness of the deposit or of any other property of the Authority and so as to preserve, insofar as practicable, the future usefulness or value of the deposit. Materials, including stripping, removed from deposits owned or controlled by the Authority and not used in the work covered by these specifications shall be disposed of as directed.

The contractor's operations in and around aggregate deposits shall be in accordance with the provisions of the specifications sections on environmental protection.

(c) *Processing Raw Materials.*—Processing of the raw materials shall include screening, and washing as necessary, to produce sand and coarse aggregate conforming to the requirements of sections M-8 (Sand) and M-9 (Coarse Aggregate). Processing of aggregates produced from any source owned or controlled by the Authority shall be done at an approved site. Water used for washing aggregates shall be free from objectionable quantities of silt, organic matter, alkali, salts, and other impurities. To utilize the greatest practicable yield of suitable materials in the portion of the deposit being worked, the contractor may crush oversize material and any excess material of the sizes of coarse aggregate to be furnished, until the required quantity of each size has been secured: *Provided*, that crusher fines produced in manufacturing coarse aggregate that will pass a screen having 3/16-inch square openings shall be wasted or rerouted through the sand manufacturing plant. Crushed sand, if used to make up deficiencies in the natural sand grading, shall be produced by a suitable ball or rod mill, disk or cone crusher, or other approved equipment so that the sand particles shall be predominately cubical in shape and free from objectionable quantities of flat or elongated particles.

The crushed sand and coarse aggregate shall be blended uniformly with the uncrushed sand and coarse aggregate, respectively. Crushing and blending operations shall at all times be subject to approval by the Authority. The handling, transporting, and stockpiling of aggregates shall be such that there will be a minimum amount of fines resulting from breakage and abrasion of material caused by free fall and improper handling. Where excesses in any of the sand and coarse aggregate sizes occur, the contractor shall dispose of the excess material as directed by the Authority.

(d) *Furnishing Aggregates.*—The cost of producing aggregates required for work under these specifications and the cost of aggregates not obtained from a source owned or controlled by the Authority shall be included in the unit prices bid in the schedule for concrete in which the aggregates are used, which unit prices shall also include all expenses of the contractor in stripping, transporting, and storing the materials. The contractor shall be entitled to no additional compensation for materials wasted from a deposit, including crusher fines, excess material of any of the sizes into which the aggregates are required to be separated by the contractor, and materials which have been discarded by reason of being above the maximum sizes specified for use.

M-11. ***Batching.***—(a) *General.*—The contractor shall provide equipment and shall maintain and operate the equipment as required to accurately determine and control the prescribed amounts of the various materials, including water, cement, pozzolan, admixtures, sand, and each individual size of coarse aggregate entering the concrete. The amounts of bulk cement, pozzolan, sand, and each size of coarse aggregate entering each batch of concrete shall be determined by separate weighing, and the amounts of water and each admixture shall be determined by separate weighing or volumetric measurement. Where bagged cement is used, the concrete shall be porportioned on the basis of integral bags of cement unless the cement is weighed.

When bulk cement, pozzolan, and aggregates are hauled from a central batching plant to the mixers, the cement and pozzolan for each batch shall either be placed in an individual compartment which during transit will prevent the cement and pozzolan from intermingling with each other and with the aggregates and will prevent loss of cement and pozzolan; or the cement and pozzolan shall be completely enfolded in and covered by the aggregates by loading the cement, pozzolan, and aggregates for each batch simultaneously into the batch compartment. The bins of batch trucks shall be provided with suitable covers to protect the materials therein from wind or wet weather. Each batch compartment shall be of sufficient capacity to prevent loss in transit and to prevent spilling and intermingling of batches as compartments are being emptied. If the cement and pozzolan are enfolded in aggregates containing moisture, and delays occur between filling and emptying the compartments the contractor shall, at his own expense, add extra cement to each batch in accordance with the following schedule:

| **Hours of contact between cement and wet aggregate* | *Additional cement required* |
|---|---|
| 0 to 2 | 0 percent |
| 2 to 3 | 5 percent |
| 3 to 4 | 10 percent |
| 4 to 5 | 15 percent |
| 5 to 6 | 20 percent |
| Over 6 | Batch will be rejected. |

---

*The Contracting Authority reserves the right to require the addition of cement for shorter periods of contact during periods of hot weather and the contractor shall be entitled to no additional compensation by reason of the shortened period of contact.

Batch bins shall be constructed so as to be self-cleaning during drawdown and the bins shall be drawn down until they are practically empty at least three times per week. Materials shall be deposited in the batch bins directly over the discharge gates. The 1½-, 3-, and 6-inch coarse aggregates shall be deposited in the batcher bins through effective rock ladders, or other approved means. To minimize breakage, the method used in transporting the aggregates from one elevation to a lower elevation shall be such that the aggregates will roll and slide with a minimum amount of free fall.

Equipment for conveying batched materials from the batch hopper or hoppers to and into the mixer shall be so constructed, maintained, and operated that there will be no spillage of the batched materials or overlap of batches. Equipment for handling portland cement and pozzolan in the batching plant shall be constructed and operated so as to prevent noticeable increase of dust in the plant during the measuring and discharging of each batch of material. If the batching and mixing plant is enclosed, the contractor shall install exhaust fans or other suitable equipment for removing dust.

(b) *Equipment.*—The weighing and measuring equipment shall conform to the following requirements:

(1) The construction and accuracy of the equipment shall conform to the applicable requirements of Federal Specification AAA-S-121d [8] for such equipment, except that an accuracy of 0.4 percent over the entire range of the equipment will be required.

The contractor shall provide standard test weights and any other equipment required for checking the operating performance of each scale or other measuring device and shall make periodic tests over the ranges of measurements involved in the batching operations. The tests shall be made in the presence of an Authority inspector, and shall be adequate to prove the accuracy of the measuring devices. Unless otherwise directed, tests of weighing equipment in operation shall be made at least once every month. The contractor shall make such adjustments, repairs, or replacements as may be necessary to meet the specified requirements for accuracy of measurement.

(2) Each weighing unit shall include a visible springless dial which will register the scale load at any stage of the weighing operation from zero to full capacity. The minimum clear interval for dial scale graduations shall be not less than 0.03 inch. The scales shall be direct reading to within 5 pounds for cement and 20 pounds for aggregate. The weighing hoppers shall be constructed so as to permit the convenient removal of overweight materials in excess of the prescribed tolerances. The scales shall be interlocked so that a new batch cannot be started until the weighing hoppers have been completely emptied of the last batch and the scales are in balance. Each scale dial shall be in full view of the operator.

(3) The equipment shall be capable of ready adjustment for compensating for the varying weight of any moisture contained in the aggregates and for changing the mix proportions.

(4) The equipment shall be capable of controlling the delivery of material for weighing or volumetric measurement so that the combined inaccuracies in feeding and measuring during normal operation will not exceed 1 percent for water; 1½ percent for cement and pozzolan; 3 percent for admixtures; 2 percent for sand, ¾-inch aggregate, and 1½-inch aggregate; and 3 percent for 3- and 6-inch coarse aggregate.

(5) Convenient facilities shall be provided for readily obtaining representative samples of cement, pozzolan, admixtures, sand, and each size of coarse aggregate from the discharge streams between bins and the batch hoppers or between the batch hoppers and the mixers.

(6) The operating mechanism in the water-measuring device shall be such that leakage will not occur when the valves are closed. The water-measuring device shall be constructed so that the water will be discharged quickly and freely into the mixer without objectionable dribble from the end of the discharge pipe. In addition to the water-measuring device, there shall be supplemental means for measuring and introducing small increments of water into each mixer when required for final tempering of the concrete. This equipment shall introduce the added water well into the batch. Each water-measuring device shall be in full view of the operator.

(7) Dispensers for air-entraining agents, calcium chloride solutions, and WRA shall have sufficient capacity to measure at one time the full quantity of the properly diluted solution required for each batch, and shall be maintained in a clean and freely operating condition. Equipment for measuring shall be designed for convenient confirmation by the plant operator of the accuracy of the measurement for each batch and shall be so constructed that the required quantity can be added only once to each batch.

(8) The mixing plant shall be arranged so that the mixing action in at least one of the mixers can be conveniently observed from its control station. Provisions shall be made so that the mixing action of each of the other mixers can be observed from a safe location which can be easily reached from the control station. Provisions shall also be made so that the operator can observe the concrete in the receiving hopper or buckets after being dumped from the mixers.

(9) Equipment that fails to conform to the requirements of this section shall be effectively repaired or satisfactorily replaced.

**M-12.** *Mixing.*–(a) *General.*–The concrete ingredients shall be mixed thoroughly in batch mixers of approved type and size and designed so as to positively ensure uniform distribution of all of the component materials throughout the mass at the end of the mixing period. The adequacy of mixing will be determined by the method of "Variability of Constituents in Concrete" in accordance with the provisions of designation 26 of the eighth edition of the Bureau of Reclamation Concrete Manual [1]. Mixers when tested shall meet the following criteria:

(1) The unit weight of air-free mortar in samples taken from the first and last portions of the batch as discharged from the mixer shall not vary more than 0.8 percent from the average of the two mortar weights.

(2) For any one mix, the average variability for more than one batch shall not exceed the following limits:

| *Number of tests* | *Average variability (percent based on average mortar weight of all tests)* |
|---|---|
| 3 | 0.6 |
| 6 | .5 |
| 20 | .4 |
| 90 | .3 |

(3) The weight of coarse aggregate per cubic foot in samples taken from the first and last portions of the batch as discharged from the mixer shall not vary more than 5.0 percent from the average of the two weights of coarse aggregate.

The Contracting Authority reserves the right to either reduce the size of batch to be mixed or to increase the mixing time when the charging and mixing operations fail to produce a concrete batch which conforms throughout to the above-numbered criteria and in which the ingredients are uniformly distributed and the consistency is uniform. Water shall be added prior to, during, and following the mixer-charging operations. Overmixing, requiring addition of water to preserve the required consistency, will not be permitted. Any concrete retained in mixers so long as to require additional water in excess of 3 percent of the design mix water (net water-cement plus pozzolan ratio water, not including water absorbed by aggregates) to permit satisfactory placing shall be wasted. Any mixer that at any time produces unsatisfactory results shall be repaired promptly and effectively or shall be replaced.

Use of truck mixers in accordance with subsection (c) below will be permitted only for miscellaneous items of concrete work where and as approved by the Authority.

(b) *Central Mixers.*–Mixers shall not be loaded in excess of their rated capacity unless specifically authorized. The concrete ingredients shall be mixed in a batch mixer for not less than the period of time indicated in the following tabulation for various mixer capacities after all of the ingredients except the full amount of water are in the mixer, except that the mixing time may be reduced if, as determined by the Authority, thorough mixing conforming to subsections (a) (1) and (2) above can be obtained in less time.

| *Capacity of mixer* | *Time of mixing* |
|---|---|
| 2 cubic yards or less | 1½ minutes |
| 3 cubic yards | 2 minutes |
| 4 cubic yards | 2½ minutes |
| Larger than 4 cubic yards | To be determined by tests performed by the Authority |

(c) *Truck Mixers.*—Use of truck mixers will be permitted only when the mixers and their operation are such that the concrete throughout the mixed batch and from batch to batch is uniform with respect to consistency and grading. Any concrete retained in truck mixers sufficiently long as to require additional water to permit placing shall be wasted.

Each truck mixer shall be equipped with (1) an accurate watermeter between supply tank and mixer, the meter to have indicating dials and totalizer, and (2) a reliable revolution counter, which can be readily reset to zero for indicating the total number of revolutions of the drum for each batch. Each mixer shall have affixed thereto a metal plate on which the drum capacities for both mixing and agitating are plainly marked in terms of volume of concrete in cubic yards and the maximum and minimum speeds of rotation of the drum in revolutions per minute.

Mixing shall be continued for not less than 50 nor more than 100 revolutions of the drum at the manufacturer's rated mixing speed after all the ingredients, except approximately 5 percent of the water which may be withheld, are in the drum. The mixing speed shall be not less than 5 nor more than 20 revolutions per minute. Thereafter, additional mixing, if any, shall be at the speed designated by the manufacturer of the equipment as agitating speed; except that after the addition of withheld water, mixing shall be continued at the specified mixing speed until the water is dispersed throughout the mix. After a period of agitation a few revolutions of the drum at mixing speed will be required just prior to discharging. In no case shall the specified maximum net water-cement plus pozzolan ratio be exceeded.

When a truck mixer or agitator is used for transporting concrete, the concrete shall be delivered to the site and the discharge completed within 1½ hours after the introduction of the cement into the mixer. Each batch of concrete, when delivered at the jobsite from commercial ready-mix plants, shall be accompanied by a written certificate of batch weights and time of batching.

Mixers shall be examined daily for changes in condition due to accumulation of hard concrete or mortar or to wear of blades. No mixer shall be charged in excess of its rated capacity for mixing or agitating; however, if any mixer cannot produce concrete meeting the requirements heretofore specified when mixing at rated capacity, within the specified limitation on the number of revolutions of the mixing drum at mixing speed, the size of batch mixed in that mixer may be reduced until, upon testing, a uniformly mixed batch, conforming to the mixer performance tests as provided in subsection (a) above, is obtained.

M-13. *Temperature of Concrete.*—The temperature of mass concrete for the dam shall, when concrete is being placed, be not more than 50° F. and not less than 40° F. For all other concrete, the temperature of concrete when it is being placed shall be not more than 90° F. and not less than 40° F. in moderate weather or not less than 50° F. in weather during which the mean daily temperature drops below 40° F. Concrete ingredients shall not be heated to a temperature higher than that necessary to keep the temperature of the mixed concrete, as placed, from falling below the specified minimum temperature. Methods of heating concrete ingredients shall be subject to approval by the Contracting Authority.

If concrete is placed when the weather is such that the temperature of the concrete would exceed the maximum placing temperatures specified, as determined by the Authority, the contractor shall employ effective means as necessary to maintain the temperature of the concrete, as it is placed, below the maximum temperatures specified. These means may include placing at night; precooling the aggregates by cool airblast, immersion in cold water, vacuum processing, or other suitable method; refrigerating the mixing water; adding chip or flake ice to the mixing water; or a combination of these or other approved means. The contractor shall be entitled to no additional compensation on account of the foregoing requirements.

**M-14.** ***Forms.***—(a) *General.*—Forms shall be used, wherever necessary, to confine the concrete and shape it to the required lines. Forms shall have sufficient strength to withstand the pressure resulting from placement and vibration of the concrete, and shall be maintained rigidly in position. Forms shall be sufficiently tight to prevent loss of mortar from the concrete. Chamfer strips shall be placed in the corners of forms so as to produce beveled edges on permanently exposed concrete surfaces. Interior angles on such surfaces and edges at formed joints will not require beveling unless requirement for beveling is indicated on the drawings. Inside forms for nearly horizontal circular tunnels having an inside diameter of 12 feet or more shall be constructed to cover only the arch and sides. The bottom 60° of the inside circumference shall be placed without forming: *Provided*, that the contractor may increase the angle of the inside circumference to be placed without forming on written approval of the Contracting Authority. Request for approval shall be accompanied by complete plans and description of the placing methods proposed to be used.

Forms for tunnel lining shall be provided with openings along each sidewall and in each arch, each opening to be not less than 2 by 2 feet. The openings shall be located in the crown and along each sidewall, as follows:

> (1) Openings in the crown shall be spaced at not more than 8 feet on centers and shall be located alternately on each side of the tunnel centerline.
>
> (2) Openings in sidewall forms for tunnels having an inside diameter less than 12 feet shall be located at midheight of the tunnel in each sidewall and shall be spaced at not more than 8 feet on centers along each sidewall.
>
> (3) Openings in sidewall forms for tunnels having an inside diameter of 12 feet or more shall be located along two longitudinal lines in each sidewall, the locations of which are satisfactory to the Authority. The openings along the two selected longitudinal lines in each sidewall shall be staggered and shall be spaced at not more than 8 feet on centers along each longitudinal line.

The cost of all labor and materials for forms and for any necessary treatment or coating of forms shall be included in the unit prices bid in the schedule for the concrete for which the forms are used.

(b) *Form Sheathing and Lining.*—Wood sheathing or lining shall be of such kind and quality or shall be so treated or coated that there will be no chemical deterioration or discoloration of the formed concrete surfaces. The type and condition of form sheathing and lining, and the fabrication of forms for finishes F2, F3, and F4 shall be such that the form surfaces will be even and uniform. The ability of forms to withstand distortion caused by placement and vibration of concrete shall be such that formed surfaces will conform with applicable requirements of these specifications pertaining to finish of formed surfaces. Where finish F3 is specified, the sheathing or lining shall be placed so that the joint marks on the concrete surfaces will be in general alinement both horizontally and vertically. Where pine is used for form sheathing, the lumber shall be pinus ponderosa in accordance with the Standard Grading Rules of the Western Wood Products Association or shall be other lumber of a grading equivalent to that specified for pine. Plywood used for form sheathing or lining shall be concrete form, class I, grade B-B exterior, mill oiled and edge sealed, in accordance with Product Standard PS 1-66 of the Bureau of Standards [12]. Materials used for form sheathing or lining shall conform with the following requirements, or may be other materials producing equivalent results:

| *Required finish of formed surface* | *Wood sheathing or lining* | *Steel sheathing or lining** |
|---|---|---|
| F1 | Any grade–S2E | Steel sheathing permitted.<br>Steel lining permitted. |
| F2 | No. 2 common or better, pine shiplap, or plywood sheathing or lining. | Steel sheathing permitted.<br>Steel lining permitted if approved. |
| F3 | No. 2 common or better pine tongue-and-groove or plywood sheathing or lining, except where special form material is prescribed. | Steel sheathing not permitted.<br>Steel lining not permitted. |
| F4 | For plane surfaces, No. 1 common or better pine tongue-and-groove or shiplap or plywood. For warped surfaces, lumber which is free from knots and other imperfections and which can be cut and bent accurately to the required curvatures without splintering or splitting. | Steel sheathing permitted.<br>Steel lining not permitted. |

*Steel "sheathing" denotes steel sheets not supported by a backing of wood boards. Steel "lining" denotes thin steel sheets supported by a backing of wood boards.

(c) *Form Ties.*–Embedded ties for holding forms shall remain embedded and, except where F1 finish is permitted, shall terminate not less than two diameters or twice the minimum dimension of the tie in the clear of the formed faces of the concrete. Where F1 finish is permitted, ties may be cut off flush with the formed surfaces. The ties shall be constructed so that removal of the ends or end fasteners can be accomplished without causing appreciable spalling at the faces of the concrete. Recesses resulting from removal of the ends of form ties shall be filled in accordance with section M-19 (Repair of Concrete).

(d) *Cleaning and Oiling of Forms.*–At the time the concrete is placed in the forms, the surfaces of the forms shall be free from encrustations of mortar, grout, or other foreign material. Before concrete is placed, the surfaces of the forms shall be oiled with a commercial form oil that will effectively prevent sticking and will not soften or stain the concrete surfaces, or cause the surfaces to become chalky or dust producing. For wood forms, form oil shall consist of straight, refined, pale, paraffin base mineral oil. For steel forms, form oil shall consist of refined mineral oil suitably compounded with one or more ingredients which are appropriate for the purpose. The contractor shall furnish certification of compliance with these specifications for form oil.

(e) *Removal of Forms.*–To facilitate satisfactory progress with the specified curing and enable earliest practicable repair of surface imperfections, forms shall be removed as soon as the concrete has hardened sufficiently to prevent damage by careful form removal. Forms on upper sloping faces of concrete, such as forms on the watersides of warped transitions, shall be removed as soon as the concrete has attained sufficient stiffness to prevent sagging. Any needed repairs or treatment required on such sloping surfaces shall be performed at once and be followed immediately by the specified curing.

To avoid excessive stresses in the concrete that might result from swelling of the forms, wood forms for wall openings shall be loosened as soon as this can be accomplished without damage to the concrete. Forms for the openings shall be constructed so as to facilitate such loosening. Forms for conduits and tunnel lining shall not be removed until the strength of the concrete is such that form removal will not result in perceptible cracking, spalling, or breaking of edges or surfaces, or other damage to the concrete. Forms shall be removed with care so as to avoid injury to the concrete and any concrete so damaged shall be repaired in accordance with section M-19 (Repair of Concrete).

M-15. *Tolerances for Concrete Construction.*–(a) *General.*–Permissible surface irregularities for the various classes of concrete surface finish as specified in section M-20 (Finishes) are defined as "finishes," and are to be distinguished from tolerances as described herein. The intent of this section is to establish tolerances that are consistent with modern construction practice, yet are governed by the effect that permissible deviations will have upon the structural action or operational function of the structure. Deviations from the established lines, grades, and dimensions will be permitted to the extent set forth herein: *Provided*, that the Contracting Authority reserves the right to diminish the tolerances set forth herein if such tolerances impair the structural action or operational function of a structure or portion thereof.

Where specific tolerances are not stated in these specifications or shown on the drawings for a structure, portion of a structure, or other feature of the work, permissible deviations will be interpreted conformably to the tolerances stated in this section for similar work. Specific maximum or minimum tolerances shown on the drawings in connection with any dimension shall be considered as supplemental to the tolerances specified in this section, and shall govern. The contractor shall be responsible for setting and maintaining concrete forms within the tolerance limits necessary to insure that the completed work will be within the tolerances specified. Concrete work that exceeds the tolerance limits specified in these specifications or shown on the drawings shall be remedied or removed and replaced at the expense of and by the contractor.

(b) *Tolerances for Dam Structures.*—

| | | |
|---|---|---|
| (1) Variation of constructed linear outline from established position in plan | In any length of 20 feet, except in buried construction | ½ inch |
| | Maximum for entire length, except in buried construction | ¾ inch |
| | In buried construction | twice the above amounts |
| (2) Variation of dimensions to individual structure features from established positions | Maximum for overall dimension, except in buried construction | 1¼ inches |
| | In buried construction | 2½ inches |
| (3) Variation from plumb, specified batter, or curved surfaces for all structures, including lines and surfaces of columns, walls, piers, buttresses, arch sections, vertical joint grooves, and visible arrises | In any length of 10 feet, except in buried construction | ½ inch |
| | In any length of 20 feet, except in buried construction | ¾ inch |
| | Maximum for entire length, except in buried construction | 1¼ inches |
| | In buried construction | twice the above amounts |
| (4) Variation from level or from grades indicated on the drawings for slabs, beams, soffits, horizontal joint grooves, and visible arrises | In any length of 10 feet, except in buried construction | ¼ inch |
| | Maximum for entire length, except in buried construction | ½ inch |
| | In buried construction | twice the above amounts |
| (5) Variation in cross-sectional dimensions of columns, beams, buttresses, piers, and similar members | Minus | ¼ inch |
| | Plus | ½ inch |
| (6) Variation in the thickness of slabs, walls, arch sections, and similar members | Minus | ¼ inch |
| | Plus | ½ inch |

| | | |
|---|---|---|
| (7) Footings for columns, piers, walls, buttresses, and similar members: | | |
| (a) Variation of dimensions in plan | Minus | ½ inch |
| | Plus | 2 inches |
| (b) Misplacement or eccentricity | 2 percent of the footing width in the direction of misplacement but not more than | 2 inches |
| (c) Reduction in thickness | | 5 percent of specified thickness |
| (8) Variation from plumb or level for sills and sidewalls for radial gates and similar watertight joints* | | Not greater than a rate of ⅛ inch in 10 feet |

*Dimensions between sidewalls for radial gates shall be not more than shown on the drawings at the sills and not less than shown on the drawings at the top of the walls.

| | | |
|---|---|---|
| (9) Variation in locations of sleeves, floor openings, and wall openings | | ½ inch |
| (10) Variation in sizes of sleeves, floor openings, and wall openings | | ¼ inch |

## (c) *Tolerances for Tunnel Lining.—*

| | | |
|---|---|---|
| (1) Departure from established alinement or from established grade | Free-flow tunnels and conduits | 1 inch |
| | High-velocity tunnels and conduits | ½ inch |
| (2) Variation in thickness, at any point | Tunnel lining | minus 0 |
| | Conduits | minus 2½ percent or ¼ inch, whichever is greater |
| | Conduits | plus 5 percent or ½ inch, whichever is greater |
| (3) Variation from inside dimensions | | ½ of 1 percent |

## (d) *Tolerances for Placing Reinforcing Bars and Fabric.—*

| | | |
|---|---|---|
| (1) Reinforcing steel, except for bridges: | | |
| (a) Variation of protective covering | With cover of 2½ inches or less | ¼ inch |
| | With cover of more than 2½ inches | ½ inch |
| (b) Variation from indicated spacing | | 1 inch |
| (2) Reinforcing steel for bridges: | | |
| (a) Variation of protective covering | With cover of 2½ inches or less | ⅛ inch |
| | With cover of more than 2½ inches | ¼ inch |
| (b) Variation from indicated spacing | | 1 inch |

M-16. ***Reinforcing Bars and Fabric.***—(a) *Furnishing.*—The contractor shall furnish all the reinforcing bars and fabric required for completion of the work. Reinforcing bars shall conform to ASTM Designation A 615, grade 40 or 60, or ASTM Designation A 617, grade 40 or 60. (See reference [3] or [4].) Fabric shall be electrically welded-wire fabric and shall conform to ASTM Designation A 185 [2].

(b) *Placing.*—Reinforcing bars and fabric shall be placed in the concrete where shown on the drawings or where directed. Splices shall be located where shown on the drawings: *Provided*, that the location of splices may be altered subject to the written approval of the Contracting Authority, and *Provided further*, that, subject to the written approval of the Authority, the contractor may splice bars at additional locations other than those shown on the drawings. Reinforcing bars in splices located where shown on the drawings, in relocated splices approved by the Authority, or in additional splices approved by the Authority, will be included in the measurement, for payment, of reinforcing bars.

Unless otherwise prescribed, placement dimensions shall be to the centerlines of the bars. Reinforcement will be inspected for compliance with requirements as to size, shape, length, splicing, position, and amount after it has been placed.

Before the reinforcement is embedded in concrete, the surfaces of the bars and the surfaces of any bar supports shall be cleaned of heavy flaky rust, loose mill scale, dirt, grease, or other foreign substances which, in the opinion of the Authority, are objectionable. Heavy flaky rust that can be removed by firm rubbing with burlap or equivalent treatment is considered objectionable.

Reinforcement shall be accurately placed and secured in position so that it will not be displaced during the placing of the concrete, and special care shall be exercised to prevent any disturbance of the reinforcement in concrete that has already been placed. Welding or tack welding of grade 60 or grade 75 reinforcing bars will not be permitted except at locations shown on the drawings. Chairs, hangers, spacers, and other supports for reinforcement may be of concrete, metal, or other approved material. Where portions of such supports will be exposed on concrete surfaces designated to receive F2 or F3 finish, the exposed portion of the supports shall be of galvanized or other corrosion-resistant material, except that concrete supports will not be permitted. Such supports shall not be exposed on surfaces designated to receive an F4 finish. Unless otherwise shown on the drawings, the reinforcement in structures shall be so placed that there will be a clear distance of at least 1 inch between the reinforcement and any anchor bolts, form ties, or other embedded metalwork.

(c) *Reinforcement Drawings to be Prepared by the Contractor.*—The contractor shall prepare and submit for approval of the Authority reinforcement detail drawings for all structures including bar-placing drawings, bar-bending diagrams, and bar lists.

The contractor's reinforcement detail drawings shall be prepared from reinforcement design drawings included with these specifications and from supplemental reinforcement design drawings to be furnished by the Authority. The position, size, and shape of reinforcing bars are not shown in all cases on the drawings included with these specifications. Supplemental reinforcement design drawings in sufficient detail to permit the contractor to prepare his reinforcement detail drawings will be furnished to the contractor by the Authority after final designs have been completed and after equipment data are received from equipment manufacturers. As the supplemental reinforcement design drawings may not be available in time to enable the contractor to purchase prefabricated reinforcing bars, it may be necessary for the contractor to purchase bars in stock lengths, and to cut and bend the bars in the field.

At least ________ days before scheduled concrete placement, the contractor shall submit to the Authority for approval three prints of each of his reinforcement detail drawings. The contractor's reinforcement detail drawings shall be prepared following the recommendations established by the American

Concrete Institute's "Manual of Standard Practice for Detailing Reinforced Concrete Structures" (ACI 315-65) unless otherwise shown on the reinforcement design drawings. The contractor's drawings shall show necessary details for checking the bars during placement and for use in establishing payment quantities. Reinforcement shall conform to the requirements shown on the reinforcement design drawings.

The contractor's reinforcement detail drawings shall be clear, legible, and accurate and checked by the contractor before submittal. If any reinforcement detail drawing or group of drawings is not of a quality acceptable to the Authority, the entire set or group of drawings will be returned to the contractor, without approval, to be corrected and resubmitted. Acceptable reinforcement detail drawings will be reviewed by the Contracting Authority for adequacy of general design and controlling dimensions. Errors, omissions, or corrections will be marked on the prints, or otherwise relayed to the contractor, and one print of each drawing will be returned to the contractor for correction. The contractor shall make all necessary corrections shown on the returned prints. The corrected drawings need not be resubmitted unless the corrections are extensive enough, as determined by the Authority, to warrant resubmittal. Such Authority review and approval shall not relieve the contractor of his responsibility for the correctness of details or for conformance with the requirements of these specifications.

(d) *Measurement and Payment.*—Measurement, for payment, of reinforcing bars and fabric will be made only of the weight of the bars and fabric placed in the concrete in accordance with the drawings or as directed.

Payment for furnishing and placing reinforcing bars will be made at the applicable unit price per pound bid in the schedule for the various sizes of reinforcing bars and fabric, which unit prices shall include the cost of preparing reinforcement detail drawings, including bar-placing drawings and bar-bending diagrams; of submitting the drawings to the Authority; of preparing all necessary bar lists and cutting lists; of furnishing and attaching wire ties and metal or other approved supports, if used; and of cutting, bending, cleaning, and securing and maintaining in position, all reinforcing bars and fabric as shown on the drawings.

M-17. ***Preparations for Placing.***—(a) *General.*—No concrete shall be placed until all formwork, installation of parts to be embedded, and preparation of surfaces involved in the placing have been approved. No concrete shall be placed in water except with the written permission of the Contracting Authority, and the method of depositing the concrete shall be subject to his approval. Concrete shall not be placed in running water and shall not be subjected to the action of running water until after the concrete has hardened. All surfaces of forms and embedded materials that have become encrusted with dried mortar or grout from concrete previously placed shall be cleaned of all such mortar or grout before the surrounding or adjacent concrete is placed.

(b) *Foundation Surfaces.*—Immediately before placing concrete, all surfaces of foundations upon or against which the concrete is to be placed shall be free from standing water, mud, and debris. All surfaces of rock upon or against which concrete is to be placed shall, in addition to the foregoing requirements, be clean and free from oil, objectionable coatings, and loose, semidetached, or unsound fragments. Earth foundations shall be free from frost or ice when concrete is placed upon or against them. The surfaces of absorptive foundations against which concrete is to be placed shall be moistened thoroughly so that moisture will not be drawn from the freshly placed concrete.

(c) *Surfaces of Construction and Contraction Joints.*—Concrete surfaces upon or against which concrete is to be placed and to which new concrete is to adhere, that have become so rigid that the new concrete cannot be incorporated integrally with that previously placed, are defined as construction joints.

All construction joints shall be cured by water curing or by application of wax base curing compound in accordance with the provisions of section M-22 (Curing). Wax base curing compound, if used on these joints, shall be removed in the process of preparing the joints to receive fresh concrete. The surfaces of the construction joints shall be clean, rough, and surface dry when covered with fresh concrete. Cleaning shall consist of the removal of all laitance, loose or defective concrete, coatings, sand, curing compound if used, and other foreign material. The cleaning and roughening shall be accomplished by wet sandblasting, washing thoroughly with air-water jets, and surface drying prior to placement of adjoining concrete: *Provided*, that high-pressure water blasting utilizing pressures not less than 6,000 pounds per square inch may be used in lieu of wet sandblasting for preparing the joint surfaces if it is demonstrated to the satisfaction of the Authority that the equipment proposed for use will produce equivalent results to those obtainable by wet sandblasting. High-pressure water blasting equipment, if used, shall be equipped with suitable safety devices for controlling pressures, including shutoff switches at the nozzle that will shut off the pressure if the nozzle is dropped. The sandblasting (or high-pressure water blasting if approved), washing, and surface drying shall be performed at the last opportunity prior to placing of concrete. Drying of the surface shall be complete and may be accomplished by air jet. In the process of wet sandblasting construction joints, care shall be taken to prevent undercutting of aggregate in the concrete.

The surfaces of all contraction joints shall be cleaned thoroughly of accretions of concrete or other foreign material by scraping, chipping, or other means approved by the Authority.

M-18. ***Placing***.—(a) *Transporting*.—The methods and equipment used for transporting concrete and the time that elapses during transportation shall be such as will not cause appreciable segregation of coarse aggregate, or slump loss in excess of 1 inch, in the concrete as it is delivered into the work. The use of aluminum pipe for delivery of pumped concrete will not be permitted.

(b) *Placing*.—The contractor shall keep the Contracting Authority advised as to when placing of concrete will be performed. Unless inspection is waived in each specific case, placing of concrete shall be performed only in the presence of a duly authorized Authority inspector.

The surfaces of all rock against which concrete is to be placed shall be cleaned and, except in those cases where seepage or other water precludes drying of the rock face, shall be dampened and brought to a surface-dry condition. Except for tunnels, surfaces of highly porous or absorptive horizontal or nearly horizontal rock foundations to which concrete is to be bonded shall be covered with a layer of mortar approximately three-eighths of an inch thick prior to placement of the concrete. The mortar shall have the same proportions of water, air-entraining agent, cement, pozzolan, and sand as the regular concrete mixture, unless otherwise directed. The water-cement plus pozzolan ratio of the mortar in place shall not exceed that of the concrete to be placed upon it, and the consistency of the mortar shall be suitable for placing and working in the manner hereinafter specified. The mortar shall be spread and shall be worked thoroughly into all irregularities of the surface. Concrete shall be placed immediately upon the fresh mortar.

A mortar layer shall not be used on concrete construction joints. Unless otherwise directed in formed work, structural concrete placements shall be started with an oversanded mix containing ¾-inch maximum-size aggregate; a maximum net water-cement plus pozzolan ratio of 0.47, by weight; 6 percent air, by volume of concrete; and having a maximum slump of 4 inches. This mix shall be placed approximately 3 inches deep on the joint at the bottom of the placement.

Retempering of concrete will not be permitted. Any concrete which has become so stiff that proper placing cannot be assured shall be wasted. Concrete shall be deposited in all cases as nearly as practicable directly in its final position and shall not be caused to flow such that the lateral movement will permit or cause segregation of the coarse aggregate from the concrete mass. Methods and equipment employed in depositing concrete in forms shall be such as will not result in clusters or groups of coarse aggregate particles being separated from the concrete mass, but if clusters do occur they shall be scattered before the concrete is vibrated. Where there are a few scattered individual pieces of coarse aggregate that can be restored into the mass by vibration, this will not be objectionable and should be done.

Concrete in tunnel lining may be placed by pumping or any other approved method. Where the concrete in the invert is placed separately from the concrete in the arch and without inside forms, it shall not be placed by pneumatic placing equipment unless an approved type of discharge box which prevents segregation is provided and used. The equipment used in placing the concrete and the method of its operation shall be such as will permit introduction of the concrete into the forms without high-velocity discharge and resultant separation. After the concrete has been built up over the arch at the start of a placement, the end of the discharge line shall be kept well buried in the concrete during placement of the arch and sidewalls to assure complete filling. The end of the discharge line shall be marked so as to indicate the depth of burial at any time. Special care shall be taken to force concrete into all irregularities in the rock surfaces and to completely fill the tunnel arch. Placing equipment shall be operated by experienced operators only.

Where tunnel lining placements are terminated with sloping joints, the contractor shall thoroughly consolidate the concrete at such joints to a reasonably uniform and stable slope while the concrete is plastic. If thorough consolidation at the sloping joints is not obtained, as determined by the Authority, the Authority reserves the right to require the use of bulkheaded construction joints. The concrete at the surface of such sloping joints shall be clean and surface dry before being covered with fresh concrete. The cleaning of such sloping joints shall consist of the removal of all loose and foreign material.

Except as intercepted by joints, all formed concrete other than concrete in tunnel lining, including mass concrete in the dam, shall be placed in continuous approximately horizontal layers. The depth of layers for mass concrete shall generally not exceed 18 inches, and the depth for all other concrete shall generally not exceed 20 inches. The Authority reserves the right to require lesser depths of layers where concrete in 20-inch layers cannot be placed in accordance with the requirements of these specifications. Except where joints are specified herein or on the drawings, care shall be taken to prevent cold joints when placing concrete in any portion of the work. The concrete placing rate shall be such as to ensure that each layer is placed while the previous layer is soft or plastic, so that the two layers can be made monolithic by penetration of the vibrators. To prevent featheredges, construction joints that are located at the tops of horizontal lifts near sloping exposed concrete surfaces shall be inclined near the exposed surface, so that the angle between such inclined surfaces and the exposed concrete surface will be not less than $50^{\circ}$.

In placing unformed concrete on slopes so steep as to make internal vibration of the concrete impracticable without forming, the concrete shall be placed ahead of a nonvibrated slip-form screed extending approximately 2½ feet back from its leading edge. Concrete ahead of the slip-form screed shall be consolidated by internal vibrators so as to ensure complete filling under the slip-form.

In placing mass concrete in the dam, the contractor shall, when required, maintain the exposed area of fresh concrete at the practical minimum, by first building up the concrete in successive approximately horizontal layers to the full width of the block and to full height of the lift over a restricted area at the downstream end of the block, and then continuing upstream in similar progressive stages to the full area of the block. The slope formed by the unconfined upstream edges of the successive layers of concrete shall be kept as steep as practicable in order to keep its area to a minimum. Concrete along these edges shall not be vibrated until adjacent concrete in the layer is placed, except that it shall be vibrated immediately when weather conditions are such that the concrete will harden to the extent that it is doubtful whether later vibration will fully consolidate and integrate it with more recently placed adjacent concrete. Clusters of large aggregate shall be scattered before new concrete is placed over them. Each deposit of concrete shall be vibrated completely before another deposit of concrete is placed over it.

Concrete shall not be placed during rains sufficiently heavy or prolonged to wash mortar from coarse aggregate on the forward slopes of the placement. Once placement of concrete has commenced in a block, placement shall not be interrupted by diverting the placing equipment to other uses.

Concrete buckets shall be capable of promptly discharging the low slump, 6-inch mass concrete mixes specified, and the dumping mechanism shall be designed to permit the discharge of as little as a ½-cubic-yard portion of the load in one place. Buckets shall be suitable for attachment and use of drop chutes where required in confined locations.

Construction joints shall be approximately horizontal unless otherwise shown on the drawings or prescribed by the Authority, and shall be given the prescribed shape by the use of forms, where required, or other means that will ensure suitable joining with subsequent work. All intersections of construction joints with concrete surfaces which will be exposed to view shall be made straight and level or plumb.

If concrete is placed monolithically around openings having vertical dimensions greater than 2 feet, or if concrete in decks, top slabs, beams, or other similar parts of structures is placed monolithically with supporting concrete, the following instructions shall be strictly observed:

(1) Placing of concrete shall be delayed from 1 to 3 hours at the top of openings and at the bottoms of bevels under decks, top slabs, beams, or other similar parts of structures when bevels are specified, and at the bottom of such structure members when bevels are not specified; but in no case shall the placing be delayed so long that the vibrating unit will not readily penetrate of its own weight the concrete placed before the delay. When consolidating concrete placed after the delay, the vibrating unit shall penetrate and revibrate the concrete placed before the delay.

(2) The last 2 feet or more of concrete placed immediately before the delay shall be placed with as low a slump as practicable, and special care shall be exercised to effect thorough consolidation of the concrete.

(3) The surfaces of concrete where delays are made shall be clean and free from loose and foreign material when concrete placing is started after the delay.

(4) Concrete placed over openings and in decks, top slabs, beams, and other similar parts of structures shall be placed with as low a slump as practicable and special care shall be exercised to effect thorough consolidation of the concrete.

(c) *Consolidation.*—Concrete shall be consolidated to the maximum practicable density, so that it is free from pockets of coarse aggregate and entrapped air, and closes snugly against all surfaces of forms and embedded materials. Consolidation of concrete in structures shall be by electric- or pneumatic-drive, immersion-type vibrators. Vibrators having vibrating heads 4 inches or more in diameter shall be operated at speeds of at least 6,000 revolutions per minute when immersed in the concrete. Vibrators having vibrating heads less than 4 inches in diameter shall be operated at speeds of at least 7,000 revolutions per minute when immersed in the concrete. Immersion-type vibrators used in mass concrete shall be heavy duty, two-man vibrators capable of readily consolidating mass concrete of the consistency specified: *Provided*, that heavy-duty, one-man vibrators may be used if they are operated in sufficient number, and in a manner and under conditions as to produce equivalent results to that specified for two-man vibrators: *Provided further*, that where practicable in vibrating mass concrete, the contractor may employ gang vibrators, satisfactory to the Authority, mounted on self-propelled equipment in such a manner that they can be readily raised and lowered to eliminate dragging through the fresh concrete, and provided all other requirements of these specifications with respect to placing and control of concrete are met.

Consolidation of concrete in the sidewalls and arch of tunnel lining shall be by electric- or pneumatic-driven form vibrators supplemented where practicable by immersion-type vibrators. Form vibrators shall be rigidly attached to the forms and shall operate at speeds of at least 8,000 revolutions per minute when vibrating concrete.

In consolidating each layer of concrete the vibrator shall be operated in a near-vertical position and the vibrating head shall be allowed to penetrate and revibrate the concrete in the upper portion of the underlying layer. In the area where newly placed concrete in each layer joins previously placed concrete, particularly in mass concrete, more than usual vibration shall be performed, the vibrator penetrating deeply and at close intervals into the upper portion of the previously placed layer along these contacts. In all vibration of mass concrete, vibration shall continue until bubbles of entrapped air have generally ceased to escape. Additional layers of concrete shall not be superimposed on concrete previously placed until the previously placed concrete has been vibrated thoroughly as specified. Care shall be exercised to avoid contact of the vibrating head with surfaces of the forms.

M-19. ***Repair of Concrete.***—Concrete shall be repaired in accordance with the Bureau of Reclamation "Standard Specifications for Repair of Concrete," dated November 15, 1970. Imperfections and irregularities on concrete surfaces shall be corrected in accordance with section M-20 (Finishes and Finishing).

M-20. ***Finishes and Finishing.***—(a) *General.*—Allowable deviations from plumb or level and from the alinement, profile grades, and dimensions shown on the drawings are specified in section M-15 (Tolerances for Concrete Construction): these are defined as "tolerances" and are to be distinguished from irregularities in finish as described herein. The classes of finish and the requirements for finishing of concrete surfaces shall be as specified in this section or as indicated on the drawings. The contractor shall keep the Contracting Authority advised as to when finishing of concrete will be performed. Unless inspection is waived in each specific case, finishing of concrete shall be performed only in the presence of an Authority inspector. Concrete surfaces will be tested by the Authority where necessary to determine whether surface irregularities are within the limits hereinafter specified.

Surface irregularities are classified as "abrupt" or "gradual." Offsets caused by displaced or misplaced form sheathing or lining or form sections, or by loose knots in forms or otherwise defective form lumber, will be considered as abrupt irregularities and will be tested by direct measurements. All other irregularities will be considered as gradual irregularities and will be tested by use of a template, consisting of a straightedge or the equivalent thereof for curved surfaces. The length of the template will be 5 feet for testing of formed surfaces and 10 feet for testing of unformed surfaces.

(b) *Formed Surfaces.*—The classes of finish for formed concrete surfaces are designated by use of symbols F1, F2, F3, and F4. No sack rubbing or sandblasting will be required on formed surfaces. No grinding will be required on formed surfaces, other than that necessary for repair of surface imperfections. Unless otherwise specified or indicated on the drawings, the classes of finish shall apply as follows:

*F1.*—Finish F1 applies to formed surfaces upon or against which fill material or concrete is to be placed, to formed surfaces of contraction joints, and to the upstream face of the dam below the minimum water pool elevation. The surfaces require no treatment after form removal except for repair of defective concrete and filling of holes left by the removal of fasteners from the ends of tie rods as required in section M-19 (Repair of Concrete), and the specified curing. Correction of surface irregularities will be required for depressions only, and only for those which, when measured as described in subsection (a) above, exceed 1 inch.

*F2.*—Finish F2 applies to all formed surfaces not permanently concealed by fill material or concrete, or not required to receive finishes F1, F3, or F4. Surface irregularities, measured as described in subsection (a) above, shall not exceed one-fourth of an inch for abrupt irregularities and one-half of an inch for gradual irregularities: *Provided*, that surfaces over which radial gate seals will operate without sill or wall plates shall be free from abrupt irregularities.

*F3.*—Finish F3 applies to formed surfaces, the appearance of which is considered by the Authority to be of special importance, such as surfaces of structures prominently exposed to public inspection. Included in this category are superstructures of large powerplants and pumping plants, parapets, railings, and decorative features on dams and bridges and permanent buildings. Surface irregularities, measured as described in subsection (a) above shall not exceed one-fourth of an inch for gradual irregularities and one-eighth of an inch for abrupt irregularities, except that abrupt irregularities will not be permitted at construction joints.

*F4.*—Finish F4 applies to formed surfaces for which accurate alinement and evenness of surface are of paramount importance from the standpoint of eliminating destructive effects of water action. When measured as described in subsection (a) above, abrupt irregularities shall not exceed one-fourth of an inch for irregularities parallel to the direction of flow, and one-eighth of an inch for irregularities not parallel to the direction of flow. Gradual irregularities shall not exceed one-fourth of an inch. (*Note*: When waterflow velocities on formed concrete surfaces of outlet works, spillways, etc., are calculated to exceed 40 feet per second, further limitations should be considered for the allowable irregularities to prevent cavitation.)

(c) *Unformed Surfaces.*—The classes of finish for unformed concrete surfaces are designated by the symbols U1, U2, and U3. Interior surfaces shall be sloped for drainage where shown on the drawings or directed. Surfaces which will be exposed to the weather and which would normally be level shall be sloped for drainage. Unless the use of other slopes or level surfaces is indicated on the drawings or directed, narrow surfaces, such as tops of walls and curbs, shall be sloped approximately three-eighths of an inch per foot of width; broader surfaces such as walks, roadways, platforms, and decks shall be sloped approximately one-fourth of an inch per foot. Unless otherwise specified or indicated on the drawings, these classes of finish shall apply as follows:

*U1.*—Finish U1 (screeded finish) applies to unformed surfaces that will be covered by fill material or by concrete. Finish U1 is also used as the first stage of finishes U2 and U3. Finishing operations shall consist of sufficient leveling and screeding to produce even, uniform surfaces. Surface irregularities measured as described in subsection (a) above, shall not exceed three-eighths of an inch.

*U2.*—Finish U2 (floated finish) applies to unformed surfaces not permanently concealed by fill material or concrete, or not required to receive finish U1 or U3. U2 is also used as the second stage of finish U3. Floating may be performed by use of hand- or power-driven equipment. Floating shall be started as soon as the screeded surface has stiffened sufficiently, and shall be the minimum necessary to produce a surface that is free from screed marks and is uniform in texture. If finish U3 is to be applied, floating shall be continued until a small amount of mortar without excess water is brought to the surface, so as to permit effective troweling. Surface irregularities, measured as described in subsection (a) above, shall not exceed one-fourth of an inch. Joints and edges of gutters, sidewalks, and entrance slabs, and other joints and edges shall be tooled where shown on the drawings or directed.

*U3.*—Finish U3 (troweled finish) applies to the inside floors of buildings, except floors requiring a bonded-concrete finish or a terrazzo finish, and to inverts of draft tubes and tunnel spillways. When the floated surface has hardened sufficiently to prevent an excess of fine material from being drawn to the surface, steel troweling shall be started. Steel troweling shall be performed with firm pressure so as to flatten the sandy texture of the floated surface and produce a dense uniform surface, free from blemishes and trowel marks. Surface irregularities, measured as described in subsection (a) above, shall not exceed one-fourth of an inch.

(*Note*: When waterflow velocities on unformed concrete surfaces of outlet works, spillways, etc., are calculated to exceed 40 feet per second, further limitations on U2 and/or U3 finishes should be considered for the allowable irregularities to prevent cavitation.)

M-21. *Protection.*—The contractor shall protect all concrete against injury until final acceptance by the Contracting Authority. Fresh concrete shall be protected from damage due to rain, hail, sleet, or snow. The contractor shall provide such protection while the concrete is still plastic and whenever such precipitation, either periodic or sustaining, is imminent or occurring, as determined by the Authority.

Immediately following the first frost in the fall the contractor shall be prepared to protect all concrete against freezing. After the first frost, and until the mean daily temperature in the vicinity of the worksite falls below 40° F. for more than 1 day, the concrete shall be protected against freezing temperatures for not less than 48 hours after it is placed.

After the mean daily temperature in the vicinity of the worksite falls below 40° F. for more than 1 day, the following requirements shall apply:

(a) *Mass Concrete.*—Mass concrete shall be maintained at a temperature not lower than 40° F. for at least 96 hours after it is placed. Mass concrete cured by application of curing compound will require no additional protection from freezing if the protection at 40° F. for 96 hours is obtained by means of approved insulation in contact with the forms or concrete surfaces; otherwise, the concrete shall be protected against freezing temperatures for 96 hours immediately following the 96 hours protection at 40° F. Mass concrete cured by water curing shall be protected against freezing temperatures for 96 hours immediately following the 96 hours of protection at 40° F. Discontinuance of protection of mass concrete against freezing temperatures shall be such that the drop in temperature of any portion of the concrete will be gradual and will not exceed 20° F. in 24 hours. After March 15, when the mean daily temperature rises above 40° F. for more than 3 successive days, the specified 96-hour protection at a temperature not lower than 40° F. for mass concrete may be discontinued for as long as the mean daily temperature remains above 40° F.: *Provided*, that the specified drop in temperature limitation is met, and that the concrete is protected against freezing temperatures for not less than 48 hours after placement.

(b) *Concrete Other Than Mass Concrete.*—All concrete other than mass concrete shall be maintained at a temperature not lower than 50° F. for at least 72 hours after it is placed. Such concrete cured by application of curing compound will require no additional protection from freezing if the protection at 50° F. for 72 hours is obtained by means of approved insulation in contact with the forms of concrete surfaces; otherwise, the concrete shall be protected against freezing temperatures for 72 hours immediately following the 72 hours protection at 50° F. Concrete other than mass concrete cured by water curing shall be protected against freezing temperatures for 72 hours immediately following the 72 hours protection at 50° F. Discontinuance of protection of such concrete against freezing temperatures shall be such that the drop in temperature of any portion of the concrete will be gradual and will not exceed 40° F. in 24 hours. After March 15, when the mean daily temperature rises above 40° F. for more than 3 successive days, the specified 72-hour protection at a temperature not lower than 50° F. may be discontinued for as long as the mean daily temperature remains above 40° F.: *Provided*, that the specified drop in temperature limitation is met, and that the concrete is protected against freezing temperatures for not less than 48 hours after placement.

(c) *Use of Unvented Heaters.*—Where artificial heat is employed, special care shall be taken to prevent the concrete from drying. Use of unvented heaters will be permitted only when unformed surfaces of concrete adjacent to the heaters are protected for the first 24 hours from an excessive carbon dioxide atmosphere by application of curing compound: *Provided*, that the use of curing compound on such surfaces for curing of the concrete is permitted by and the compound is applied in accordance with section M-22 (Curing). (Include this proviso only when the use of sealing compound is not permitted on some concrete surfaces.)

M-22. *Curing.*–(a) *General.*–Concrete shall be cured either by water curing in accordance with subsection (b) or by application of wax base curing compound in accordance with subsection (c), except as otherwise hereinafter provided.

The unformed top surfaces of walls and piers shall be moistened by covering with water-saturated material or by other effective means as soon as the concrete has hardened sufficiently to prevent damage by water. These surfaces and steeply sloping and vertical formed surfaces shall be kept completely and continually moist, prior to and during form removal, by water applied on the unformed top surfaces and allowed to pass down between the forms and the formed concrete faces. This procedure shall be followed by the specified water curing or by application of curing compound.

(b) *Water Curing.*–Concrete cured with water shall be kept wet for at least 21 days for concrete containing pozzolan and for at least 14 days for concrete not containing pozzolan. Water curing shall start as soon as the concrete has hardened sufficiently to prevent damage by moistening the surface, and shall continue until completion of the specified curing period or until covered with fresh concrete: *Provided*, that water curing of concrete may be reduced to 6 days during periods when the mean daily temperature in the vicinity of the worksite is less than 40° F.: *Provided further*, that during the prescribed period of water curing, when temperatures are such that concrete surfaces may freeze, water curing shall be temporarily discontinued. The concrete shall be kept wet by covering with water-saturated material or by a system of perforated pipes, mechanical sprinklers, or porous hose, or by any other approved method which will keep all surfaces to be cured continuously (not periodically) wet. Water used for curing shall be furnished by the contractor and shall meet the requirements of these specifications for water used for mixing concrete in accordance with section M-7 (Water).

(c) *Wax Base Curing Compound.*–Wax base curing compound shall be applied to surfaces to form a water-retaining film on exposed surfaces of concrete, on concrete joints, and where specified, to prevent bonding of concrete placed on or against such joints. The curing compound shall be white pigmented and shall conform to Bureau of Reclamation "Specifications for Wax-Base Curing Compound," dated May 1, 1973. The compound shall be of uniform consistency and quality within each container and from shipment to shipment.

Curing compound shall be mixed thoroughly and applied to the concrete surfaces by spraying in one coat to provide a continuous, uniform membrane over all areas. Coverage shall not exceed 150 square feet per gallon, and on rough surfaces coverage shall be decreased as necessary to obtain the required continuous membrane. Mortar encrustations and fins on surfaces designated to receive finish F3 or F4 shall be removed prior to application of curing compound. The repair of all other surface imperfections shall not be made until after application of curing compound.

When curing compound is used on unformed concrete surfaces, application of the compound shall commence immediately after finishing operations are completed. When curing compound is to be used on formed concrete surfaces, the surfaces shall be moistened with a light spray of water immediately after the forms are removed and shall be kept wet until the surfaces will not absorb more moisture. As soon as the surface film of moisture disappears but while the surface still has a damp appearance, the curing compound shall be applied. Special care shall be taken to insure ample coverage with the compound at edges, corners, and rough spots of formed surfaces. After application of the curing compound has been completed and the coating is dry to touch, any required repair of concrete surfaces shall be performed. Each repair, after being finished, shall be moistened and coated with curing compound in accordance with the foregoing requirements.

Equipment for applying curing compound and the method of application shall be in accordance with the provisions of chapter VI of the eighth edition of the Bureau of Reclamation Concrete Manual [1]. Traffic and other operations by the contractor shall be such as to avoid damage to coatings of curing compound for a period of not less than 28 days. Where it is impossible because of construction operations to avoid traffic over surfaces coated with curing compound, the film shall be protected by a covering of sand or earth not less than 1 inch in thickness or by other effective means. The protective covering shall not be placed until the applied compound is completely dry. Before final acceptance of the work, the contractor shall remove all sand or earth covering in an approved manner. Any curing compound that is damaged or that peels from concrete surfaces within 28 days after application, shall be repaired without delay and in an approved manner.

(d) *Costs.*—The costs of furnishing and applying all materials used for curing concrete shall be included in the price bid in the schedule for the concrete on which the curing materials are used.

M-23. ***Measurement of Concrete.***—Measurement, for payment, of concrete required to be placed directly upon or against surfaces of excavation will be made to the lines for which payment for excavation is made. Measurement, for payment, of all other concrete will be made to the neatlines of the structures, unless otherwise specifically shown on the drawings or prescribed in these specifications. In the event cavities resulting from careless excavation, as determined by the Contracting Authority, are required to be filled with concrete, the materials furnished by the Authority and used for such refilling will be charged to the contractor at their cost to the Authority at the point of delivery to the contractor. In measuring concrete for payment, the volume of all openings, recesses, ducts, embedded pipes, woodwork, and metalwork, each of which is larger than 100 square inches in cross section will be deducted.

M-24. ***Payment for Concrete.***—Payment for concrete in the various parts of the work will be made at the unit prices per cubic yard bid therefor in the schedule, which unit prices shall include the cost of all labor and materials required in the concrete construction, except that payment for furnishing and handling cement, and payment for furnishing and placing reinforcing bars will be made at the unit prices bid therefor in the schedule.

M-25. ***Bibliography.***

*Bureau of Reclamation*

[1] "Concrete Manual," eighth edition, 1975.

*American Society for Testing and Materials*

[2] ASTM Designation: A 185, "Welded Steel Wire Fabric for Concrete Reinforcement."
[3] ASTM Designation: A 615, "Deformed Billet-Steel Bars for Concrete Reinforcement."
[4] ASTM Designation: A 617, "Axle-Steel Deformed Bars for Concrete Reinforcement."
[5] ASTM Designation: C 184, "Standard Method of Test for Fineness of Hydraulic Cement by the No. 100 and 200 Sieves."
[6] ASTM Designation: C 260, "Standard Specifications for Air-Entraining Admixtures for Concrete."
[7] ASTM Designation: E-11, "Standard Specifications for Wire-Cloth Sieves for Testing Purposes."

*General Services Administration*
*(Federal Supply Service)*

[8] Federal Specification AAA-S-121d, "Scale (weighing; General Specifications for)."
[9] Federal Specification SS-C-192G (Including Amendment 3), "Portland Cement."
[10] Federal Specification SS-P-570B, "Pozzolan (for Use in Portland Cement Concrete)."
[11] Federal Test Method Standard No. 158A, "Cements, Hydraulic; Sampling, Inspection, and Testing."

*U.S. Department of Commerce, Bureau of Standards*

[12] Product Standard PS 1-66, "Softwood Plywood, Construction and Industrial."

## << Appendix N

# Sample Specifications for Controlling Water and Air Pollution

N-1. *Scope.*—The following sample specifications prescribe water quality controls and preventive measures for discharge of wastes and/or pollution into a river, lake, or estuary due to construction operations; and the prevention and control of air pollution. They are written in the form of mandatory provisions which should be required of the contractor.

## A. PREVENTION OF WATER POLLUTION

N-2. *General.*—The contractor shall comply with applicable Federal and State laws, orders, and regulations concerning the prevention, control, and abatement of water pollution. Permits to discharge wastes into receiving waters shall be obtained by the contractor either from the State water pollution control agency or from the Environmental Protection Agency.

The contractor's construction activities shall be performed by methods that will prevent entrance or accidental spillage of solid matter, contaminants, debris, and other objectionable pollutants and wastes into streams, flowing or dry watercourses, lakes, and underground water sources. Such pollutants and wastes include but are not restricted to refuse, garbage, cement, concrete, sewage effluent, industrial waste, radioactive substances, mercury, oil and other petroleum products, aggregate processing tailings, mineral salts, and thermal pollution. Pollutants and wastes shall be disposed of at sites approved by the Contracting Authority.

The contractor shall control his construction activities so that turbidity resulting from his operations shall not exist in concentrations that will impair natural or developed water supplies, fisheries, or recreational facilities downstream from the construction area.

At least 40 days prior to beginning of construction of each phase of work, the contractor shall submit for approval two copies of his plans for the treatment and disposal of all waste and for control of turbidity in the __________ River which may result from his operations. The plans shall be submitted to the Construction Engineer, Post Office Box _____ , __________ , ____________. The plans shall include complete design and construction details of turbidity control features. Such plans shall also show the methods of handling and disposal of oils or other petroleum products, chemicals, and similar industrial wastes.

Except as otherwise provided in section N-4(a) below, approval of the contractor's plans shall not relieve the contractor of the responsibility for designing, constructing, operating, and maintaining pollution and turbidity control features in a safe and systematic manner, and for repairing at his expense any damage to, or failure of, the pollution and turbidity control structures and equipment caused by floods or storm runoff.

N-3. *Control of Turbidity.*–Turbidity increases above the natural turbidities in the __________ River that are caused by construction activities shall be limited to those increases resulting from performance of required construction work in the river channel and will be permitted only for the shortest practicable period required to complete such work and as approved by the Contracting Authority. This required construction work will include such work as diversion of the river, construction or removal of cofferdams and other specified earthwork in or adjacent to the river channel, pile driving, and construction of turbidity control structures.

The spawning period for trout (or other game fish) in the __________ River is normally during the period ________ through ________. Accordingly, no change in the diversion or channelization of the river will be permitted during this particularly sensitive period.

Mechanized equipment shall not be operated in flowing water except as necessary to construct approved crossings or to perform the required construction, as outlined above.

The contractor's methods of unwatering, of excavating foundations, of operating in the borrow areas, and of stockpiling earth and rock materials shall include preventive measures to control siltation and erosion, and to intercept and settle any runoff of muddy waters. Waste waters from construction of dam and appurtenances, aggregate processing, concrete batching and curing, drilling, grouting, and similar construction operations shall not enter flowing or dry watercourses without the use of special approved turbidity control methods.

N-4. *Turbidity Control Methods.*–(a) *General.*–Turbidity control shall be accomplished through the use of plans approved by the Contracting Authority in accordance with section N-2 above.

The Bureau of Reclamation's methods for control of turbidity during construction at the damsite as set forth in (c) below are acceptable methods. The contractor may adopt these methods or he may submit for approval alternative methods of equivalent adequacy. If the contractor elects to utilize the Bureau's methods and his plans for implementation are approved by the Contracting Authority, and if such approved plans do not effectively control turbidity due to no fault of the contractor, additional work will be directed for which payment will be made in accordance with the "General Provisions" portion of the specifications. If the contractor elects to propose for approval different methods of turbidity control, the contractor shall bear the full responsibility for their satisfactory operation in controlling turbidity. The approval of the contractor's alternate proposals by the Contracting Authority shall not be construed to relieve the contractor from his responsibility.

The contractor's plans, submitted in accordance with section N-2 above, shall show complete design and construction details for implementing either the Bureau's methods or the contractor's alternative methods.

(b) *Requirements for Turbidity Control During Construction at the Damsite.*–The turbidity control method to be used during construction at the damsite shall: (1) Provide for treatment of all turbid water at the damsite resulting from construction of dam and appurtenances; washing of aggregate obtained from approved sources, if such washing is performed at the damsite; drilling; grouting; or similar construction operations: Provided, that the Contracting Authority may direct that clear water removed from foundations be discharged directly to the river without treatment. The treatment plant shall have a capacity to treat 0 to ________ gallons of turbid water per minute so that the turbidity of any effluent discharged to the river does not exceed ______ Jackson turbidity units.

(2) Include bypass and control equipment suitable for blending treated and untreated waste waters and obtaining effluents of varying degrees of turbidity. The decision to discharge to the river completely treated effluent or a

blend of treated and untreated effluent will be the responsibility of the Contracting Authority, and will depend on the natural turbidity existing in the river at any particular time.

(3) Have a capability of adjusting the *pH* and alkalinity values of any effluent discharged to the river.

(4) Use only chemicals which have been approved by the Environmental Protection Agency for use in potable water and which have been proven to be harmless to terrestrial wildlife and aquatic life.

(5) Have provisions for accumulating, transporting, and depositing sludge in disposal areas so that the material will not wash into the river by high flows or storm runoff, as approved by the Contracting Authority.

(6) Provide for removal of the treatment plant, cleanup of the site, and restoration of the site to its original condition as approved by the Contracting Authority. All materials, plant, and appurtenances used for turbidity control shall remain the property of the contractor.

(c) *Bureau's Methods of Turbidity Control at the Damsite.*—The Bureau of Reclamation's methods for controlling turbidity during construction at the damsite are based on collecting turbid waters in sumps, and pumping from the sumps to: (1) A water clarification plant, Dorr-Oliver[1] Pretreater ( -foot diameter by __-foot water depth), or equal, with automatic chemical dosage feeders for hydrated lime, alum, and an acid or coagulant aid if needed; or

(2) A treatment plant consisting of equalizing tanks, sedimentation flumes, settling tanks, and ponds combined with innocuous stabilizing and flocculating chemicals as required. Such a treatment plant shall be the Dow Turbidity Control System, as proposed by Dow Chemical U.S.A.,[1] or equal.

(d) *Sampling and Testing of Water Quality.*—The Contracting Authority will do such water quality sampling and testing in connection with construction operations as is necessary to ensure compliance with the water quality standards of the State of __________ and the Environmental Protection Agency.

Turbidities of all effluents discharged to the river from the contractor's construction operations shall be monitored by continuous recorders such as the HACH 6491 or 7855 strip chart recorder provided with CR Surface Scatter Turbidimeter Model 2411 or 2426,[1] or equal, which shall be furnished, installed, and operated by the contractor. Locations of the recorders shall be as approved by the Contracting Authority.

Copies of the recordings shall be submitted daily to the Contracting Authority and shall include the date, time of day, and name of person or persons responsible for operation of the equipment and recorder.

Sampling and testing by the Contracting Authority in no way relieves the contractor of the responsibility for doing such monitoring as is necessary for the controlling of his operations to prevent violation of the water quality standards.

N-5. *Payment.*—Payment for control of turbidity during construction at the damsite will be made at the applicable lump-sum price bid therefor in the schedule, which lump-sum price shall include the cost of furnishing all labor, equipment, and materials for designing, constructing, operating, maintaining, and removing all features necessary for control of turbidity in accordance with these sections.

Payment of percentages of the lump-sum price for control of turbidity during construction at the damsite will be made as follows:

(1) Fifty percent of the lump sum in the first monthly progress estimate after completion of the initial installation of the approved plant for treatment of the turbid water.

(2) Twenty-five percent of the lump sum in the first monthly progress estimate after completion of all concrete placement in the dam.

(3) Twenty-five percent of the lump sum in the first monthly progress estimate after completion of the turbidity control operation at the damsite, and removal of equipment.

---

[1] Mention of these firms should not be construed as an indication that they are the only suppliers of these or similar products nor as an endorsement by the Bureau of Reclamation.

The costs of all other labor, equipment, and materials necessary for control of turbidity at locations other than the damsite and for prevention of water pollution for compliance with these sections shall be included in the prices bid in the schedule for other items of work.

## B. ABATEMENT OF AIR POLLUTION

N-6. *General.*—The contractor shall comply with applicable Federal, State, and local laws and regulations concerning the prevention and control of air pollution.

In his conduct of construction activities and operation of equipment, the contractor shall utilize such practicable methods and devices as are reasonably available to control, prevent, and otherwise minimize atmospheric emissions or discharges of air contaminants.

The emission of dust into the atmosphere will not be permitted during the manufacture, handling, and storage of concrete aggregates, and the contractor shall use such methods and equipment as are necessary for the collection and disposal, or prevention, of dust during these operations. The contractor's methods of storing and handling cement and pozzolans shall also include means of eliminating atmospheric discharges of dust.

Equipment and vehicles that show excessive emissions of exhaust gases due to poor engine adjustments, or other inefficient operating conditions, shall not be operated until corrective repairs or adjustments are made.

Burning shall be accomplished only at times and at locations approved by the Contracting Authority. Burning of materials resulting from clearing of trees and brush, combustible construction materials, and rubbish will be permitted only when atmospheric conditions for burning are considered favorable by appropriate State or local air pollution or fire authorities. In lieu of burning, such combustible materials may be removed from the site, chipped, or buried as provided in section______.

Where open burning is permitted, the burn piles shall be properly constructed to minimize smoke, and in no case shall unapproved materials such as tires, plastics, rubber products, asphalt products, or other materials that create heavy black smoke or nuisance odors be burned.

Storage and handling of flammable and combustible materials, provisions for fire prevention, and control of dust resulting from drilling operations shall be done in accordance with the applicable provisions of the Department of Labor "Safety and Health Regulation for Construction" and the Bureau of Reclamation Supplement thereto.

Dust nuisance resulting from construction activities shall be prevented in accordance with section______.

The costs of complying with this section shall be included in the prices bid in the schedule for the various items of work.

N-7. ***Dust Abatement.***—During the performance of the work required by these specifications or any operations appurtenant thereto, whether on right-of-way provided by the Contracting Authority or elsewhere, the contractor shall furnish all the labor, equipment, materials, and means required, and shall carry out proper and efficient measures wherever and as often as necessary to reduce the dust nuisance, and to prevent dust which has originated from his operations from damaging crops, orchards, cultivated fields, and dwellings, or causing a nuisance to persons. The contractor will be held liable for any damage resulting from dust originating from his operations under these specifications on Authority right-of-way or elsewhere.

The cost of sprinkling or of other methods of reducing formation of dust shall be included in the prices bid in the schedule for other items of work.

# INDEX