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Price Level Risk and Some Long-Run Implications of Alternative Monetary Policy Strategies

James A. Clouse¹
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¹ This paper is dedicated to the memory of Thomas Laubach—a great leader, economist and friend who inspired us all with his intellect, insights, dedication to public service, and courageous leadership. Thomas encouraged work on this set of issues and his remarkable legacy lives on through his work and all who knew him. This draft has also benefitted greatly from insightful comments from Board colleagues Alyssa Anderson, Chris Gust, David Lopez-Salido, and John Roberts. The views expressed in this paper are solely those of the author and do not in any way represent an official view of the Board or its staff.

Abstract

This note focuses on the longer-run implications of alternative monetary policy strategies for the evolution of the price level. The analysis compares the properties of optimal policy in regimes ranging from pure inflation targeting (IT), to a form of weighted-average inflation targeting (WAIT), to pure price level targeting (PLT). Strategies such as WAIT and PLT tend to limit the downward drift in the path of the price level and also mitigate the uncertainty surrounding the expected path of the price level. The influence of alternative monetary policy strategies on the evolution of the price level may have some important long-run implications for entities or groups that rely heavily on long-term nominal debt. Some simple empirical estimates suggest the real value of existing Treasury debt could be boosted significantly in moving from a world in which the ZLB constraint rarely binds to one in which it regularly binds. Similarly, data from the Survey of Consumer Finances indicate that households at lower income levels, and particularly those with mortgage or educational loans outstanding, are exposed to significant price level risk. As a result, such households can experience a significant reduction in their real wealth, on average, in the transition to a world with frequently binding ZLB constraints. The WAIT and PLT regimes significantly mitigate these potential costs for these groups.

Introduction

In an era in which very low equilibrium interest rates are apparently part of the “new normal,” central banks around the world are facing the challenges posed by the heightened risks of episodes with short-term interest rates pinned at the zero lower bound (ZLB) and the associated limitations on the conduct of monetary policy. A large literature has developed focused on designing monetary policy strategies that may allow central banks to more effectively achieve their macroeconomic objectives in a world in which ZLB constraints come into play more frequently.² A key element of the efficacy of such strategies is the credibility of the central bank’s commitment to follow “time inconsistent” policies that may produce near-term benefits but entail significant future costs.

While the design of policy strategies that can enhance the effectiveness of monetary policy in economic stabilization is critically important, there are some longer-run issues in connection with alternative monetary policy strategies that may also warrant attention. This note focuses in particular on the longer-run implications of alternative monetary policy strategies for the longer run evolution of the price level. The analysis below compares the properties of the price level in a stylized model in optimal policy regimes ranging from pure inflation targeting (IT), to a form of weighted-average inflation targeting (WAIT), to pure price level targeting (PLT). The main conclusion is that strategies such as WAIT and PLT tend to limit the downward drift in the expected path of the price level and also mitigate the uncertainty surrounding this anticipated path. These long-run effects of WAIT and PLT regimes could be amplified by any influence that such regimes might have in shaping forward-looking inflation expectations, but the price level effects studied here do not fundamentally depend on this expectational channel.

The influence of alternative monetary policy strategies on the evolution of the price level, in turn, may have some important long-run implications for entities or groups that rely heavily on long-term nominal debt. For example, the federal government relies heavily on nominal debt obligations in financing fiscal deficits. The real value of the federal government’s existing debt obligations then will be higher if the price level follows a lower trajectory than initially anticipated. The tendency for the price level to follow a lower trajectory after repeated encounters with the ZLB can also affect the evolution of real wealth for groups at different points across the wealth distribution. Households with low levels of nominal financial assets relative to nominal debt are far more exposed to lower price level paths than is the case for other groups along the wealth distribution. For these households, a lower projected path for the price level raises the real value of their existing nominal debt obligations by more than the increase in the real value of their nominal assets.

The empirical analysis below attempts to quantify these types of effects. In general, we find that the real value of existing Treasury debt could be boosted significantly, on average, in moving from a world in which the ZLB rarely binds to one in which it may bind fairly frequently. The increase in the real value of government debt may be considerably larger in cases with unusually large or persistent adverse shocks. Similarly, data from the Survey of Consumer Finances (SCF) indicate that households at lower income levels, and particularly those with mortgage or educational loans outstanding, are quite exposed to downside price level risk. As a result, such households can experience a significant reduction in their real wealth, on average, in the

² See Kiley and Roberts (2017), Bernanke, Kiley and Roberts (2019), Eggertson and Woodford (2003), Reifschneider and Williams (2000), Svensson (2003).

transition to a world with frequently binding ZLB constraints. The WAIT regimes and PLT significantly mitigate these potential effects. In the simulations below, even WAIT regimes with relatively short lookback periods can have a sizable effect in reducing the potential costs associated with low trajectories for the price level over time.

The discussion below is organized as follows. Section 1 describes a stylized model of optimal policy that forms the basis for comparisons of alternative monetary policy strategies. Section 2 reports model simulation results for the behavior of the price level under alternative monetary policy strategies. Section 3 considers some implications of the simulation results for the real value of government debt and for the real value of household net worth among different groups. Section 4 concludes.

1. Stylized Model

The model developed below is a much simplified version of the framework employed in the classic paper by Clarida, Gali, and Gertler (1999). The key behavioral relationship is a static version of the Phillips curve that relates the inflation gap in each period to inflation expectations and the output gap. Inflation expectations are assumed here to be fixed and exogenous. As a consequence, none of the results below stem from the influence of alternative policy strategies on private sector expectations regarding the future path of inflation or the future stance of monetary policy.

1.1 Stylized Economic Model

The analysis in this note relies on a highly stylized version of the Phillips curve relationship in which inflation is posited as a function of expected inflation and the output gap.

$$\hat{\pi}_t = \hat{\pi}_t^e + \tau \hat{y}_t + \sigma_v v_t = \hat{\pi}_t^e + \tau \hat{y}_t + \sigma_\varepsilon \varepsilon_t \quad (1)$$

Where

$$\tilde{y}_t = \hat{y}_t + \sigma_n n_t = \text{output gap}$$

$$\hat{y}_t = y_t - y_t^* = \text{portion of output gap controlled by central bank}$$

$$\sigma_\varepsilon \varepsilon_t = \tau \sigma_n n_t + \sigma_v v_t = \text{consolidated shock to Phillips curve}$$

$$\hat{\pi}_t = \pi_t - \pi^* = \text{inflation gap}$$

$$\hat{\pi}_t^e = \pi_t^e - \pi^* = \text{inflation expectations gap}$$

The output gap term \hat{y}_t here is assumed to be controlled by the Federal Reserve's setting of short-term interest rates. The shock term $\sigma_\varepsilon \varepsilon_t$ thus subsumes both direct shocks to the inflation process, $\sigma_v v_t$, as well as the indirect effects of shocks to the output gap on inflation, $\tau \sigma_n n_t$. All underlying shock terms are assumed to be drawn from a normal distribution with unit variance. Inflation expectations here are assumed to be fixed and exogenous. In general, this specification of the Phillips curve has little to recommend it as a realistic representation of the inflation process. But for the purpose of this note, the simple specification helps to isolate and highlight

long run effects of alternative monetary policy strategies that *do not* do not depend on forward looking behavior.³

1.2 Central Bank Objective Function

The central bank in this model chooses the output gap in each period that minimizes its loss function over time. The focus of this paper is largely on the evolution of the price level over the longer-run and not the role of policy in economic stabilization. Consistent with that focus, the central bank objective function here does not include the usual term capturing squared values of the output gap. Absent that usual term, the central bank is free to act aggressively to achieve its objectives for inflation.

With inflation expectations assumed to be exogenous, this problem is a very straightforward exercise in optimal control and, in particular, does not involve issues of discretion versus commitment as in versions of the model that incorporate forward looking behavior by the public. To investigate the long-run implications of alternative monetary policy strategies, we assume the central bank seeks to minimize an objective function of the form⁴:

$$V(Z_{t-1}) = \min_{Z_t} E\left\{\frac{1}{2}Z_t^2 + \beta V(Z_t)\right\} \quad (2)$$

Subject to

$$Z_{t+i} = \rho Z_{t+i-1} + \hat{\pi}_{t+i} \quad (2a)$$

$$\hat{\pi}_{t+i} = \hat{\pi}_{t+i}^e + \tau \hat{y}_{t+i} + \sigma \varepsilon_{t+i} \quad (2b)$$

Here the value of the objective function, $V(Z_{t-1})$, is the minimized discounted value of all future costs conditional on the cumulative inflation gap Z_{t-1} at the end of period (t-1). The variable Z_{t+i} (hereafter the cumulative inflation gap) thus accumulates the current and all past inflation gaps in every period. The coefficient ρ determines how much weight is given to past inflation gaps. When $\rho = 0$, the objective function reduces to the usual one for a central bank pursuing a pure inflation targeting (IT) strategy. When $\rho = 1$, the variable Z_t is a measure of the price level gap or the deviation of the actual price level from a fixed price level path defined by the stated inflation target; the objection function in this case corresponds to that for a pure price level targeting (PLT) regime. For values of ρ between 0 and 1, the variable Z_t is a weighted sum of past inflation gaps. In this case, the objective function captures a type of weighted average inflation targeting (WAIT) where the weights on past inflation gaps decline geometrically over time.

The framework used here to discuss alternative monetary policy regimes differs in some important respects from other work. Here, we are taking the view that a central bank loss function specified as in (2) with any value of ρ between 0 and 1 could be viewed as a reasonable

³ A common concern expressed about average inflation targeting and related strategies for mitigating the risks posed by the ZLB is that their effectiveness depends critically on the extent to which such strategies influence private sector expectations about the future path of inflation and the policy rate. For example, see Kiley and Roberts (2017), Reifschneider and Wilcox (2019), and the summary of the strategic review discussion in the minutes of the October 2019 FOMC meeting.

⁴ The objective function here focuses only on inflation deviations from target and thus may be more suitable as a description of policy for central banks with a single inflation mandate.

representation of a central bank’s “price stability” objective. Given an objective function characterized in this way, the analysis then traces out the implications for optimal policy associated with different parameterizations of the loss function, e.g. different choices of ρ .

A standard framework employed in the literature fixes the form of the central bank loss function with $\rho = 0$ as an approximation to the welfare loss function for households and businesses. Given this loss function, these studies investigate the ability of alternative formulations of monetary policy rules to reduce the value of the loss function.⁵ With forward-looking expectations and the assumption that the central bank can credibly commit to a rule, alternative policy rules such as average inflation targeting or PLT may be able to reduce the value of the loss function by influencing expectations in a way that helps stabilize output and inflation.

Although not the focus of this paper, there is a link between these two approaches. With an objective function as specified above, optimal policy under discretion will result in optimal policy rules similar in some respects to those for optimal policy under commitment with a standard loss function. In effect, the implicit “commitment” to minimize the modified objective function used in the setup here generates some of the features of optimal rules under commitment with the standard objective function.⁶

1.3 Optimal Policy in the Absence of the ZLB Constraint

Before exploring the nature of optimal policy when the zero lower bound (ZLB) constraint is “in the money” or potentially binding, it’s helpful to describe optimal policy in this setup in the absence of the ZLB constraint. We assume that policymakers observe the shocks to output and inflation in each period before determining the appropriate setting of monetary policy. When the central bank faces no constraints on its policy settings and can thus choose any desired level of the ex-post output gap, the optimal policy is simply to set the output gap in each period so that the cumulative inflation gap Z_{t+i} is zero in every period. Thus

$$\hat{y}_{t+i} = -\frac{(\rho Z_{t+i-1} + \hat{\pi}_{t+i}^e + \sigma \varepsilon_{t+i})}{\tau}$$

Starting from a position in which the cumulative inflation gap is zero, the cumulative inflation gap can be maintained at zero in every period by setting the current period inflation gap to zero in every period. The upshot is that, in this model, when the central bank is unconstrained in its policy choices and thus can engineer any desired level of the output gap, the cases with strict inflation targeting ($\rho = 0$), strict price level targeting, ($\rho = 1$), and weighted average inflation targeting, ($0 < \rho < 1$), result in identical outcomes. Inflation in each period is set equal to the stated long-run inflation goal, π^* , the cumulative inflation target Z_t is zero in every period, and the (log) price level then rises along a deterministic path given by:

$$p_{t+i}^* = p_{t+i-1}^* + \pi^*$$

As a result, under all policy strategies, even though the economy is subject to shocks in every period, there is zero uncertainty about the future path of the price level. In what follows, we are

⁵ See Kiley and Roberts (2017), and Bernanke, Kiley, and Roberts (2019), Mertens and Williams (2019), and Reifschneider and Wilcox (2019) for excellent examples of this approach.

⁶ See Gust, Lopez-Salido, and Meyer (2017) for a discussion of this issue.

largely focused on transitions from such a world to one in which the ZLB is periodically binding. We assume that exogenous inflation expectations are consistent with the behavior of inflation in the pre-transition world so that the exogenous inflation expectations gap variable, $\hat{\pi}_{t+i}^e$, is equal to zero.

1.4 Optimal Policy with the Zero Lower Bound Constraint

Even with the very simple structure of the model outlined above, the analysis of alternative policy strategies becomes considerably more complicated when the ZLB constraint is potentially binding. In the context of the model, the ZLB can be represented as an upper bound on the level of the output gap that the central bank can achieve through adjustments in its policy instruments. In what follows, this upper bound on the setting of the output gap will be denoted as \hat{y}_{max} .⁷

The possibility for policy to be constrained by the zero lower bound results in episodes in which the target variable Z_t may depart from zero. That possibility, in turn, provides incentives for the central bank to conduct policy in a way that reduces the likelihood and severity of such episodes.

With the possibility of a binding ZLB constraint, the loss minimization problem for the central bank noted above in equations (2), (2a) and (2b) encompasses the further constraint:

$$\hat{y}_{t+i} < \hat{y}_{max} \quad (2c)$$

We continue to assume that the central bank chooses the level of the output gap in each period after observing the shocks in that period. When the central bank is unconstrained in choosing the desired level of the output gap, the optimal value of the cumulative gap Z_t in the current period is given by:

$$Z_t + \beta V'(Z_t) = 0 \quad (3)$$

The basic intuition for equation (3) is that in choosing the optimal level of the target variable, Z_t , the unconstrained policymaker weighs the marginal the cost of a higher value of Z_t today against the marginal benefit of a higher value of Z_t in reducing the incidence and severity of future episodes in which the policymaker is constrained by the ZLB, $\beta V'(Z_t)$.

The solution for the unconstrained optimal choice of Z_t in each period is just a constant⁸:

$$Z_t = \mu$$

The central bank is thus constrained by the zero lower bound in any period in which it cannot set the output gap so as to achieve this optimal unconstrained level of Z_t . When the central bank is

⁷ Often the ZLB constraint is expressed in term of the policy rate. For example, the output gap may be posited as a function of the policy rate $\hat{y}_t = -\alpha(i - \pi_t^e - r_t^*)$. If inflation expectations and the neutral real rate are constant, the zero lower bound on the nominal short rate implies an upper bound on the level of the output gap that the central bank can achieve given by $\hat{y}_{max} = \alpha(\pi^e + r^*)$.

⁸ The optimal policy in this setup is particularly simple. In variations of the model that include a term capturing squared deviations of output from potential, the optimal policy is more complicated. In that structure, there is still an optimal threshold but it varies over time depending on the past values of the cumulative inflation gap and shocks to the output gap.

constrained, the level of the output gap is set at the maximum level \hat{y}_{max} and the constrained value of the cumulative inflation gap is given by:

$$Z_t^c = \rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t = \varphi_{t-1} + \sigma \varepsilon_t$$

The zero lower bound constraint thus becomes binding at the point:

$$Z_t^c \leq \mu$$

And that condition holds when

$$\sigma \varepsilon_t \leq \mu - (\rho Z_{t-1} + \tau \hat{y}_{max}) = \mu - \varphi_{t-1}$$

The value of μ can be obtained by well-known iterative methods for estimating the value function. In this problem, the setup is simple enough that one can compute the first derivative of the value function and the value of μ directly. See appendix A for technical details.

1.5 Calibrated Buffer Estimates

As noted above, one can readily calculate optimal buffers in this model for any given set of parameter settings. In doing so, it is helpful to have a metric that translates the value of ρ to a more intuitive concept of the “weighted average effective look back period” or WALP for inflation gaps. A simple measure WALP can be defined as:⁹

$$WALP = \rho / (1 - \rho) \tag{4}$$

For convenience, it may be useful to think of a “period” in this model as one year. When $\rho = 0$, the central bank is a pure current period inflation targeter and the lookback period is 0. When $\rho = 1$, the central bank is a pure price level targeter and the effective lookback period is infinite. Values of ρ that correspond to effective average lookback periods of 2, 5, and 10 years are shown in Table 1 below. Many of the results reported below will focus on values of ρ with corresponding lookback period periods in this range.¹⁰

Table 1: Weighted Average Lookback Periods and Corresponding Values of ρ

WALP	Rho
IT	0.00
WAIT 2	0.67
WAIT 5	0.83
WAIT 10	0.91
PLT	1.00

⁹Here the implicit weights for each period are $w(i) = (1 - \rho) \cdot \rho^{i-1}$ and the sum of all these weights $(1 - \rho) \sum_{i=1}^{\infty} \rho^{i-1} = 1$. Applying these weights to the lag lengths i and subtracting 1 to normalize so that the lookback period goes to 0 as ρ converges to 0 yields

$$WALP = \sum_{i=1}^{\infty} w(i) \cdot i - 1 = (1 - \rho) \sum_{i=1}^{\infty} \rho^{i-1} \cdot i - 1 = \rho / (1 - \rho).$$

¹⁰ See Mertens and Williams (2019), Nessen and Vestin (2005), Reifschneider and Wilcox (2019), Vestin (2006), for analysis of average inflation targeting regimes.

To develop estimates of the buffer, we set the time discount factor β at 0.95. The slope of the Phillips curve was set at 0.25 and y_{max} was set at 2. The value for y_{max} was set based on the results for the effect of a 1 percentage point drop in the funds rate on the output gap reported in Laforte (2018) and Chung, Kiley and Laforte (2010) and an assumption that the neutral nominal short rate is about 3 percent.¹¹ With a Phillips curve slope of 0.25, the implied maximum effect of policy on inflation is 0.5 percentage points. The standard deviation of shocks to inflation was set at 1 percentage point based on estimates of inflation forecast uncertainty reported in the Summary of Economic Projections.¹²

Table 2 below reports estimates of the optimal buffer for a range of weighted average lookback periods and value of the variance of shocks to the Phillips curve. As discussed above, generally the estimated buffers increase with the length of the lookback period and the variance of shocks to the Phillips curve. The dependence of the buffer on the length of the lookback period is nonlinear and asymptotes to the levels shown the last column of Table 2.

Table 2: Calibrated Optimal Buffers for Weighted Average Inflation

SIG/WALP	IT	WAIT 2	WAIT 5	WAIT 10	PLT
0.5	0.000	0.033	0.044	0.050	0.059
1	0.000	0.177	0.263	0.326	0.459
2	0.000	0.563	0.896	1.175	1.945

Working down any column, the buffer increases more than proportionately with the increase in the standard deviation of shocks to the Phillips curve. For example, all of the estimated buffers in row 3 of the table are more than twice the level of the buffers estimated in row 2 of the table. This occurs because with the maximum amount of policy stimulus fixed, the ZLB constraint effectively becomes more binding as the variance of Phillips curve shocks increases. Consistent with the general discussion above, the estimated buffers increase as the variance of the shocks increases. The values reported in Table 2 are buffers set for the target variable. The corresponding buffers for the inflation rate in each period are shown in Table 3.

Table 3: Implied Buffers for Inflation in “Normal” Periods

SIG/WALP	IT	WAIT 2	WAIT 5	WAIT 10	PLT
0.5	0.000	0.011	0.007	0.005	0.000
1	0.000	0.059	0.045	0.030	0.000
2	0.000	0.186	0.152	0.107	0.000

¹¹ Laforte (2018) reports impulse responses for a 100 basis point change in the funds rate in the FRB/US model with VAR-based expectations. The results point to an impulse response of 50 basis points in the output gap for a 100 basis point change in the funds rate. Chung, Kiley and Laforte (2010) report similar impulse responses for the Estimated Dynamic Optimization model that show a response of about 90 basis points in real GDP for a 100 basis point shock to the funds rate. Here we use a coefficient of 2/3—roughly the average of these two impulse response coefficients—as a reasonable slope of the IS curve. With a 3 percent neutral funds rate and an IS curve coefficient of 2/3, y_{max} is then set at 2.

¹² See the standard reference table on forecast uncertainty in the Summary of Economic Projections.

These are the values of the inflation gap that, if maintained indefinitely, would produce the buffer values for the target variables shown in Table 2. So, for example, with a 2-year weighted average lookback period and with the standard deviation of shocks to the Phillips curve set at 1 percentage point, the central bank would shoot for inflation to be about 6 basis points above target in every period in order to bring the value of the target variable up to the desired level of about 18 basis points over time.

Working down the first column, the inflation buffer for a pure inflation targeter is zero. For weighted average inflation targeters—the WAIT 2, WAIT 5, and WAIT 10 regimes—the inflation buffer corresponding to the buffers for the cumulative average inflation target are smaller than the target buffers and decline with the length of the look back period. In the limit with the infinite lookback period corresponding to price level targeting, the buffer for the inflation rate in each period is zero. As described above, to arrive at this position, the central bank under a PLT regime would allow inflation to move above the target for a while to achieve the price level buffer shown in the last column of Table 2. Once that price level buffer is achieved, the price level targeting central bank would conduct policy to maintain that constant price level buffer which, in turn, implies an inflation rate equal to the long-run target rate in each period.

1.6 Estimated Value Functions

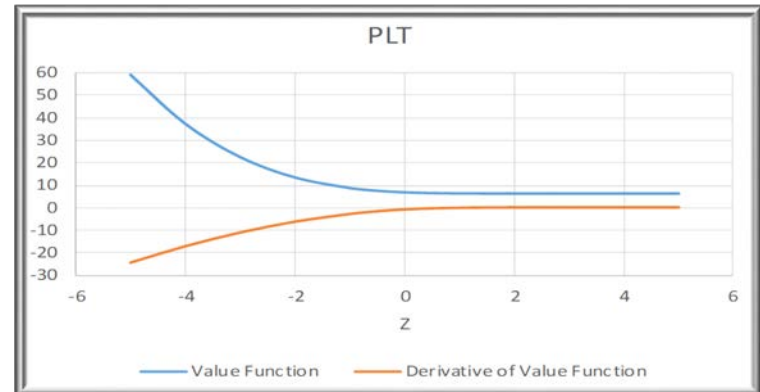
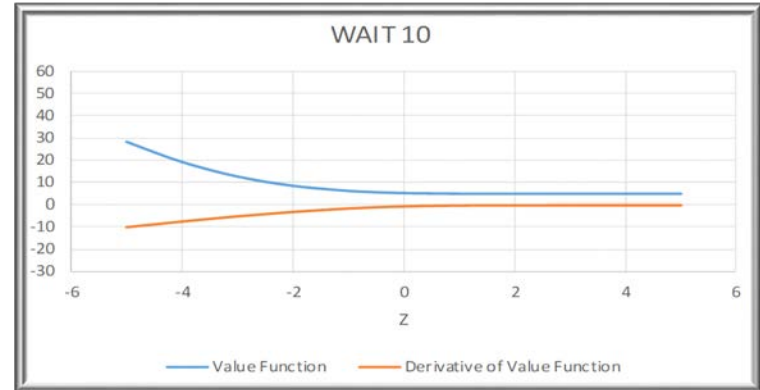
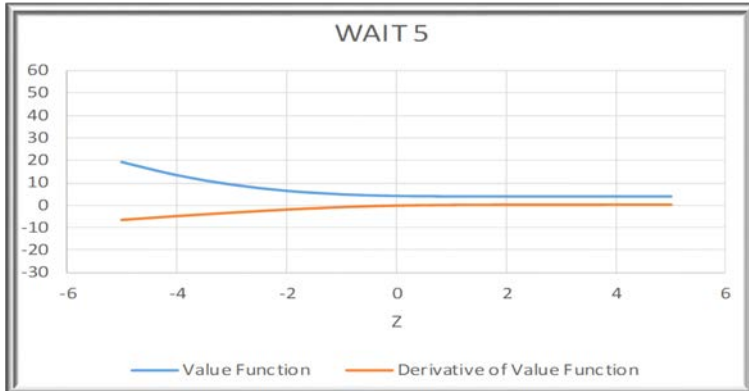
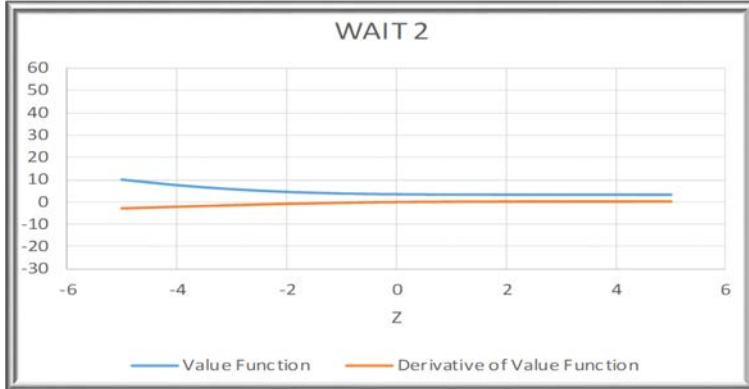
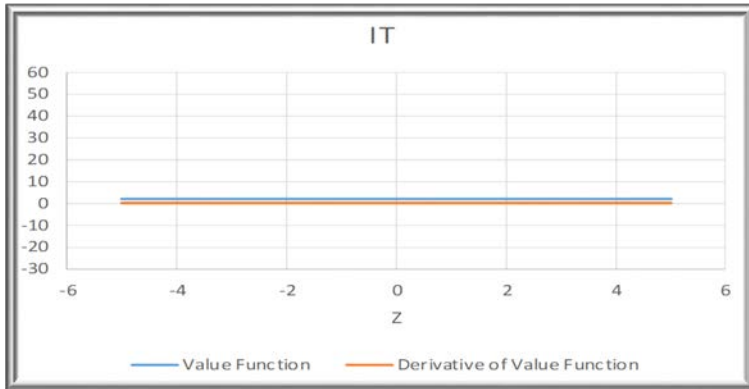
The level and the first derivative of the value function for each of the parameter settings in the second row of Table 2 are shown graphically in Figure 1. For the IT regime, the value function is simply a constant so the first derivative of the function is zero. In moving from the IT regime to the WAIT regimes and ultimately to the PLT regime, the value function moves up sharply as the levels of the cumulative inflation gap turn negative. Conversely, in these regimes, the value function asymptotes to a constant as the level of the cumulative inflation gap becomes large and positive. That result stems from the assumption that the central bank chooses the stance of policy after observing shocks in the current period. With no constraint on the degree of policy tightening, the central bank can (and will) immediately offset any undesired increase in the cumulative inflation gap. When the cumulative inflation gap is negative (or positive and small), the corresponding derivative of the value function turns negative reflecting the marginal value of a higher cumulative inflation gap coming out of the current period in avoiding the ZLB constraint in the future. As the length of the lookback period increases, the first derivative of the value function increases in absolute magnitude; that is, as the lookback period increases, the potential marginal benefit of higher cumulative inflation increases and that leads the central bank to bear the cost of establishing a higher buffer.

1.7 Properties of the Optimal Buffer

The optimal policy in this model thus involves establishing a “buffer” level μ for the cumulative inflation gap Z_t that represents the desired level of cumulative average inflation in periods in which the zero lower bound constraint is not binding. The buffer level, in turn, is a function of the key parameters of the model. In particular, the optimal buffer level is given by a function:

$$\mu = \mu(\beta, \sigma, \rho, \tau \cdot ymax) \text{ with signs associated with the arguments } \mu(+, +, +, -)$$

Figure 1: Value Function



In effect, the optimal buffer maintained when the ZLB is not binding represents a form of “insurance.”¹³ The central bank ideally would like to keep its target variable Z_t at zero as in the unconstrained case but recognizes that there will be some cases in which shocks to inflation bring the zero lower bound into play. The central bank intentionally moves away from its first best outcome in the current period when it is unconstrained in order to reduce the risk that the ZLB will become binding in the future and to be in a better position to exit the ZLB when the ZLB does become binding. The amount of insurance the central bank is willing to take out is directly related to the variance of the underlying shocks that can push the economy to the point at which the ZLB is binding.

Turning to the comparative statics for the buffer as a function of the model parameters, the buffer depends positively on the discount factor because an increase in the discount factor implies that the central bank will place more weight on potential future episodes in which it is constrained by the zero lower bound.

An increase in the standard deviation of shocks to inflation pushes up the optimal inflation buffer by increasing the likelihood, at the margin, of shocks that can push the economy to the zero lower bound.

The optimal buffer is increasing in the parameter ρ as well. As noted above, when the parameter $\rho = 0$, the central bank sets the buffer at zero because there is nothing to be gained by maintaining higher inflation in the current period. As the value of ρ increases, shocks that drive the economy to the ZLB become more costly at the margin because the effects of the shocks on the cumulative average inflation gap are more persistent and thus more likely to lead to longer periods in which the ZLB constraint is binding.

A change in τ —the slope of the Phillips curve—has both direct and indirect effects on the size of the optimal buffer. The direct effect of an increase in τ boosts the extent to which the central bank can offset economic shocks. All else equal, by increasing the efficacy of monetary policy, an increase in the slope of the Phillips curve reduces the optimal buffer. On the other hand, an increase in τ also increases the magnitude of shocks to inflation in each period by allowing a larger portion of the shocks to output to show through to inflation. That effect tends to boost the variance of the shock term in the Phillips curve. (Recall that the variance of the shock term embeds the variance of pure cost push shocks to the Phillips curve as well as the variance of the shocks to the output gap.) The net effect of an increase in the slope of the Phillips curve in this model is to reduce the size of the optimal buffer.

Finally, the buffer is declining in the level of y_{max} . If the amount of stimulus the central bank can provide is quite high, the chances that it will be forced to the ZLB are quite small. As a result, the desired buffer falls as the level of y_{max} increases.

¹³ See Clouse (2018) for a pedagogic discussion of various types of insurance motivations in optimal policy strategy.

1.8 The Buffer and the Behavior of the Price Level in “Normal Times”

It’s useful to note some implications of the results above in “normal times” when the ZLB constraint is not binding. During those episodes, the accumulated inflation gap will be set equal to μ in every period. As a result, the inflation gap during normal times is given by:

$$\hat{\pi} = (1 - \rho)\mu$$

Thus, during normal times, a central bank with $\rho < 1$ conducts policy so as to maintain inflation at a fixed level above the stated long-run goal.¹⁴ The extent of this deviation is a function of the averaging parameter ρ . With a positive inflation gap in each period, the price level gap is increasing over periods when the ZLB is not binding.

$$\hat{p}_{t+i} = \hat{p}_{t+i-1} + \hat{\pi} = \hat{p}_{t+i-1} + (1 - \rho)\mu$$

When the central bank is a price level targeter, $\rho = 1$, the steady state inflation gap is zero and the price level gap is constant at the level determined by the buffer.

$$\hat{p}_{t+i} = \mu$$

So a price level targeting bank will allow inflation to exceed the stated long-run goal for a brief period to reach the desired “normal level” of the price level gap. Thereafter, the inflation gap is set equal to zero in every period so that the price level gap is maintained at the desired level.

Figure 2 provides a graphical representation of the behavior of inflation and the price level in normal times beginning from a position in which all inflation gaps are zero. For a pure inflation targeter, the inflation gap is zero in every period and the price level rises along the baseline path. For a weighted average inflation targeting bank, the inflation gap is positive in normal times and the price level gap rises over time. For a price level targeting bank, the price level gap is positive and the inflation gap is zero. The price level rises at the same rate as the baseline path.

2. Times Series Properties of Inflation and the Price Level

This section analyzes the time-series properties of the price level and inflation under optimal policy. The characteristics of the statistical distributions of the key economic variables can be readily obtained through simulations and the results of simulation exercises are reported below. Some basic properties of the distributions can be gleaned based on an approximation of the time-series process for the cumulative average inflation.

2.1 Approximate Time-Series Behavior of Economic Variables

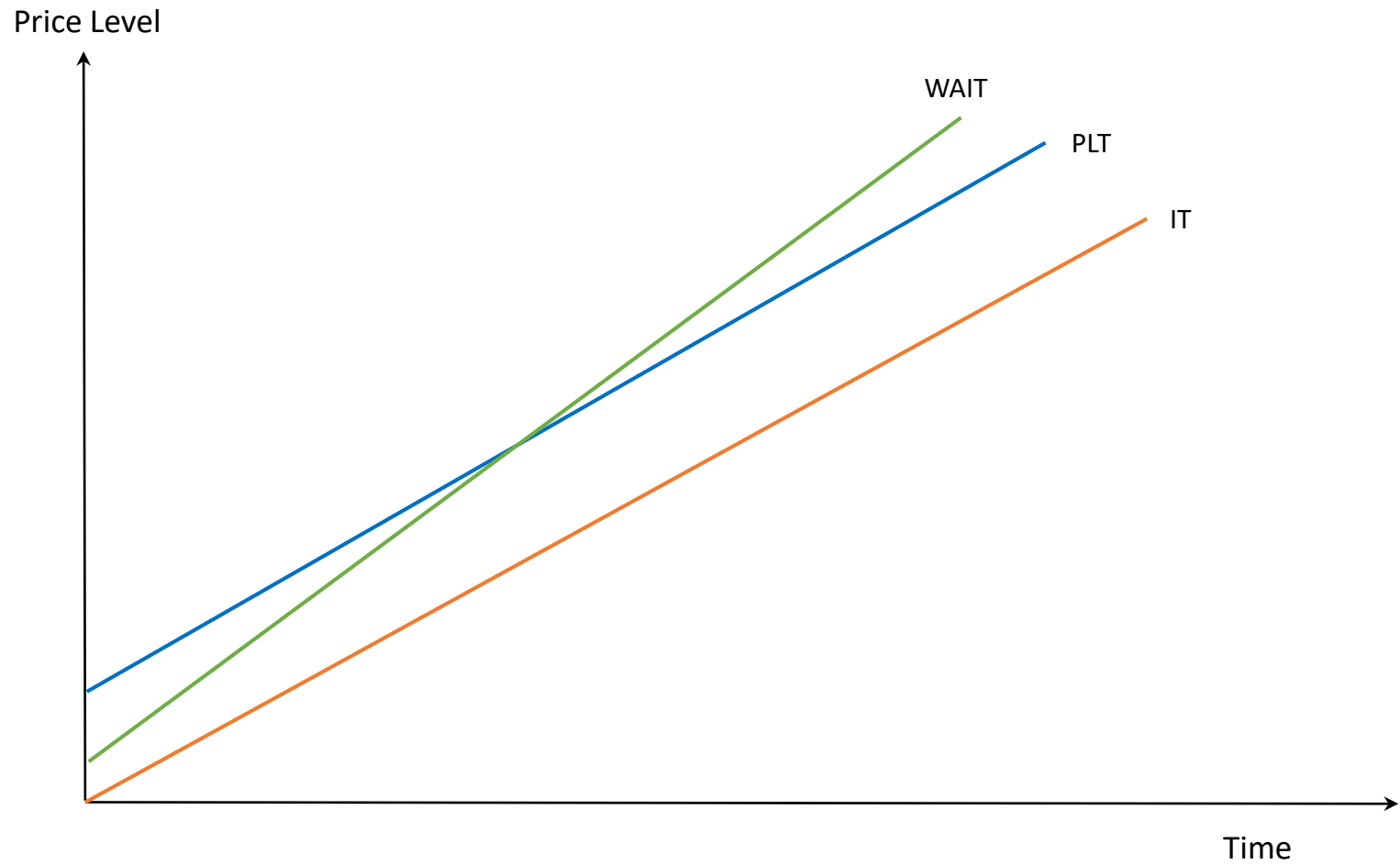
The key relation governing the time series behavior of economic variables in the model under optimal policy is given by¹⁵:

$$Z_t = \min(\rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t, \mu) \quad (5)$$

¹⁴ This result is similar to that in Mertens and Williams (2019) and other papers in which the central bank seeks to minimize a standard objective function by committing to a rule and that shapes expectations about future inflation.

¹⁵ This is an example of the SETAR “threshold” model developed and studied extensively by Tong (1983).

Figure 2: Evolution of Price Level in “Normal Times”



The expected value of Z_t conditional on Z_{t-1} is then:

$$E\{Z_t|Z_{t-1}\} = \varphi_{t-1}G\left(\frac{\mu-\varphi_{t-1}}{\sigma}\right) - \sigma g\left(\frac{\mu-\varphi_{t-1}}{\sigma}\right) + \mu(1 - G\left(\frac{\mu-\varphi_{t-1}}{\sigma}\right)) \quad (6)$$

As discussed in more detail in appendix B, one can linearize this expression around the value of the “risky steady” state Z^* to approximate the time series behavior of the accumulated inflation gap. The risky steady state is the constant value of Z that satisfies equation (6) above.

$$Z_t - Z^* = \rho\gamma(Z_{t-1} - Z^*) + \omega_t \quad (7)$$

Where

$$\gamma = G\left(\frac{\mu - \varphi^*}{\sigma}\right)$$

So, to a first order approximation, the behavior of cumulative average inflation evolves over time as an AR(1) process in this model with the autoregressive coefficient is given by $\rho\gamma$. The parameter γ is the probability that policy remains constrained by the ZLB given that the economy was at the risky steady state in the prior period. And ρ of course is the parameter governing the weighted average lookback period. Based on the calculations above, the autoregressive coefficient $\rho\gamma$ increases with an increase in the lookback period. As a result, the mean rate of reversion to the risky steady state is slower in regimes with a longer lookback period.¹⁶

The shock term here is defined by:

$$\omega_t = \mu - \theta(Z_{t-1}) \quad \text{when } \sigma\varepsilon_t > \mu - \varphi_{t-1}$$

$$\omega_t = \varphi_{t-1} + \sigma\varepsilon_t - \theta(Z_{t-1}) \quad \text{when } \sigma\varepsilon_t \leq \mu - \varphi_{t-1}$$

Combining equations (10) and (3a), the time-series process for inflation is:

$$(1 - \rho\gamma L)/(1 - \rho L) \hat{\pi}_t = (1 - \rho\gamma) Z^* + \omega_t$$

$$(1 - \rho\gamma L) \hat{\pi}_t = (1 - \rho\gamma)(1 - \rho)Z^* + (1 - \rho L)\omega_t$$

So inflation in the model follows an ARMA(1,1) process. Finally, we can use the definition of the price level gap to infer that it follows an ARIMA(1,1,1) process given by:

$$(1 - \rho\gamma L)(1 - L) \hat{p}_t = (1 - \rho\gamma)(1 - \rho)Z^* + (1 - \rho L)\omega_t \quad (8)$$

2.11 Drift and Forecast Variance

Two polar cases of equation (8) help to illustrate the core elements of the long-run behavior of inflation and the price level.

¹⁶ While equation (8) is helpful in understanding some of the time-series dynamics in the model, the approximate time-series representation suggests that the expected value of Z is equal to the risky steady state Z^* . However, the risky steady state Z^* in this model roughly corresponds to the median of the long-run distribution for Z . The mean of the steady state distribution for Z is lower than the risky steady state Z^* .

In the special case when $\rho = 1$, the price level gap process is stationary and given by the AR(1) process:

$$\hat{p}_t = \gamma \hat{p}_{t-1} + \omega_t$$

The inflation gap follows an ARMA(1,1) process with zero mean and the k-step ahead forecast variance for this process is:

$$\sigma_k^2 = \sigma_\omega^2 (1 - \alpha^{k+1}) / (1 - \alpha) \quad \text{where } \alpha = \gamma^2$$

In the polar case when $\rho = 0$, the price level gap process follows a random walk with drift given by:

$$\hat{p}_t = \hat{p}_{t-1} + Z^* + \omega_t$$

The drift is determined by the risky steady state value Z^* . Steady state inflation in this case is equal to Z^* .

The k-step ahead forecast variance for the price level gap thus increases in proportion to the forecast horizon:

$$\sigma_k^2 = k \cdot \sigma_\omega^2$$

In the intermediate cases with $0 < \rho < 1$, the price level gap remains nonstationary and with the given by:

$$\hat{p}_t = \hat{p}_{t-1} + (1 - \rho)Z^* + \left(\frac{1 - \rho L}{1 - \rho \gamma L}\right)\omega_t$$

The magnitude of the drift term $(1 - \rho)Z^*$ is downweighted by the factor $(1 - \rho)$. So the longer the effective lookback period, the smaller the drift in the unit root process for the price level gap. Moreover, the k-step ahead forecast variance is also reduced by the factor $(1 - \rho)$. The k-step ahead forecast variance in this case is complicated but declines monotonically from $k \cdot \sigma_\omega^2$ to $\sigma_k^2 = \sigma_\omega^2 (1 - \alpha^{k+1}) / (1 - \alpha)$ as ρ increase from 0 to 1.

2.2 Simulated Behavior of the Prices Level Under Alternative Policy Regimes

The analytical results above provide some insight into the behavior of inflation and the price level in the model, but they are only approximate. The characteristics of the exact statistical distributions of the key variables can be readily obtained through numerical simulations. Below, we report results for the behavior the price level and the duration of ZLB episodes under alternative monetary policy strategies. The results are based on simulating 40-year periods 5000 times to assess the key characteristics of the variables including mean, variance, and the shape of the resulting distributions at horizons from 1 to 40 years.

As noted above, the analysis above suggests that inflation runs below the announced target on average. This effect is attenuated under the WAIT and PLT regimes. Uncertainty about the price level is also shown to be diminished under the WAIT and PLT regimes relative to the IT regime.

These approximate results hold up for exact distributions for the key economic variables, which can be obtained through simulations of the basic relationship in equation (5):

$$Z_t = \min(\rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t, \mu)$$

Given this time-series process for the target variable, the implied processes for inflation and the price level are then:

$$\hat{\pi}_{t+i} = Z_{t+i} - \rho Z_{t+i-1}$$

And

$$\hat{p}_{t+i} = \hat{p}_{t+i-1} + Z_{t+i} - \rho Z_{t+i-1} \quad (9)$$

A key focus of this paper is the evolution of the price level. It's apparent from equation (9) that the price level gap will generally evolve as a unit root process with a drift determined by the mean value of the target variable.

Figure 3 displays the some of the important characteristics of the evolution of the price level under alternative assumptions about the length of the lookback period. For a pure inflation targeting central bank (orange lines in each panel), bygones are bygones with respect to inflation misses and the corresponding changes in the price level gap. Moreover, as discussed above, the central bank has no incentive to establish a buffer for inflation in normal times. As a result, the average level of the inflation gap is negative, implying a downward mean drift in the price level gap. In addition, uncertainty around the future price level gap as measured by the cross sectional standard deviation in the simulations increases with the square root of the forecast horizon.

The lines labelled WAIT 2, WAIT 5, and WAIT 10 show how the results change in moving from a pure IT regime to one involving weighted average inflation targeting (WAIT). The price level gap under WAIT regimes still embeds a unit root. However, the central bank does establish an inflation buffer that tends to mitigate the downward bias in the average rate of inflation. In general, weighted average inflation targeting substantially mitigates both the downward drift in the price level gap and the uncertainty surrounding the future price level. At least in the context of the model employed here, even WAIT regimes with relatively short lookback periods generate a substantial reduction in price level gap drift and price level gap uncertainty. For example, the WAIT 2 regime with a two-year lookback period has effects on the evolution of the price level gap that are nearly as large as those for the WAIT 5 and WAIT 10 regimes.

The behavior of the price level gap under PLT (yellow lines) is qualitatively different than under the IT and WAIT regimes. The PLT regime reduces the drift in the price level gap to zero. In addition, the price level gap is trend stationary so the uncertainty surrounding future values of the price level gap asymptotes to a fixed level independent of the forecast horizon.

Figure 4 provides another perspective on the evolution of the price level gap over time by focusing on the distribution of the price level gap at fixed horizons. Under the IT regime, the distribution of the price level gap shifts to the left over time reflecting the effects of repeated brushes with the ZLB and the absence of any policy response by the central bank to reverse past inflation misses. In addition, the distribution becomes increasingly disperse with a noticeable

Figure 3: Properties of Price Level Gap

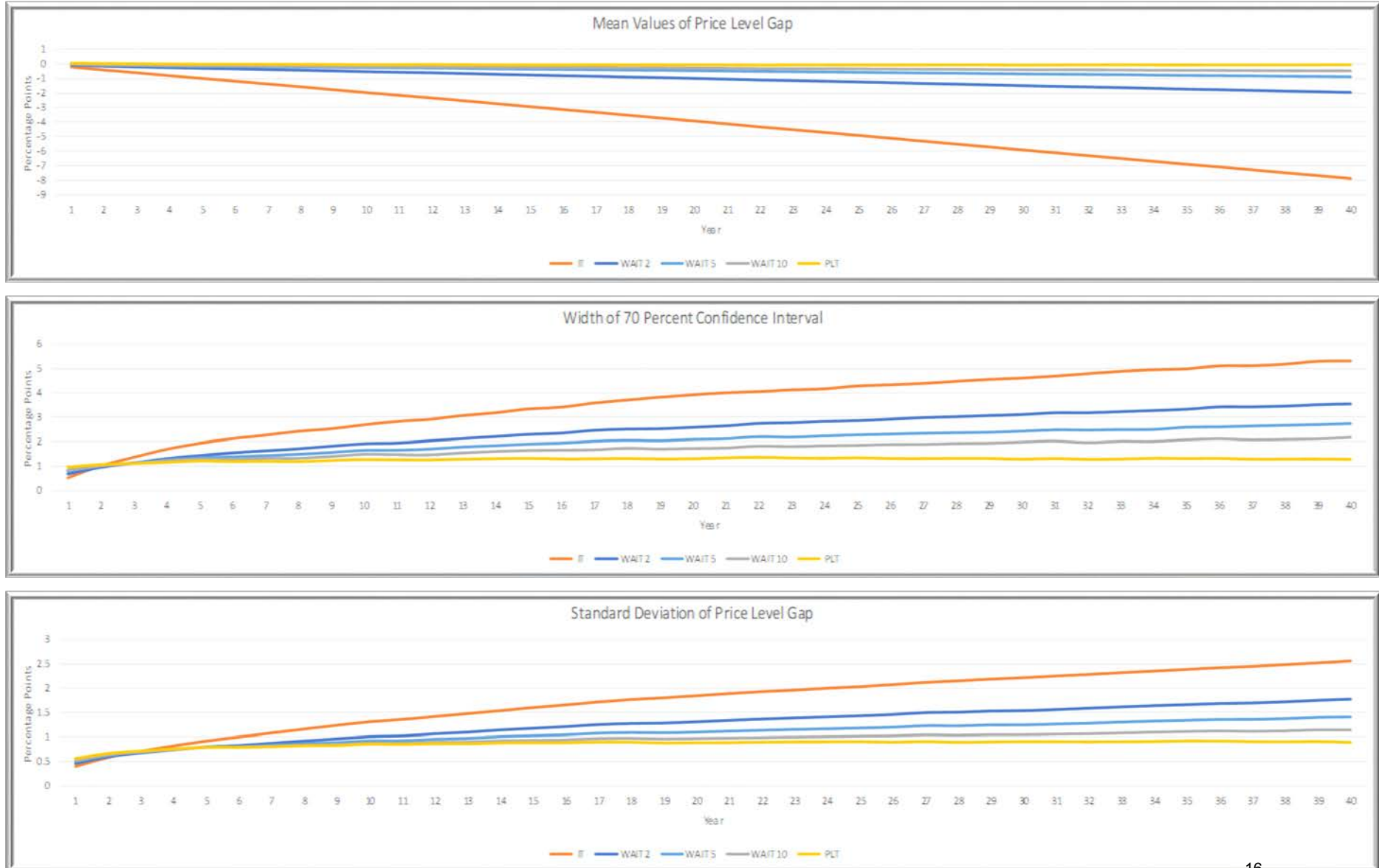
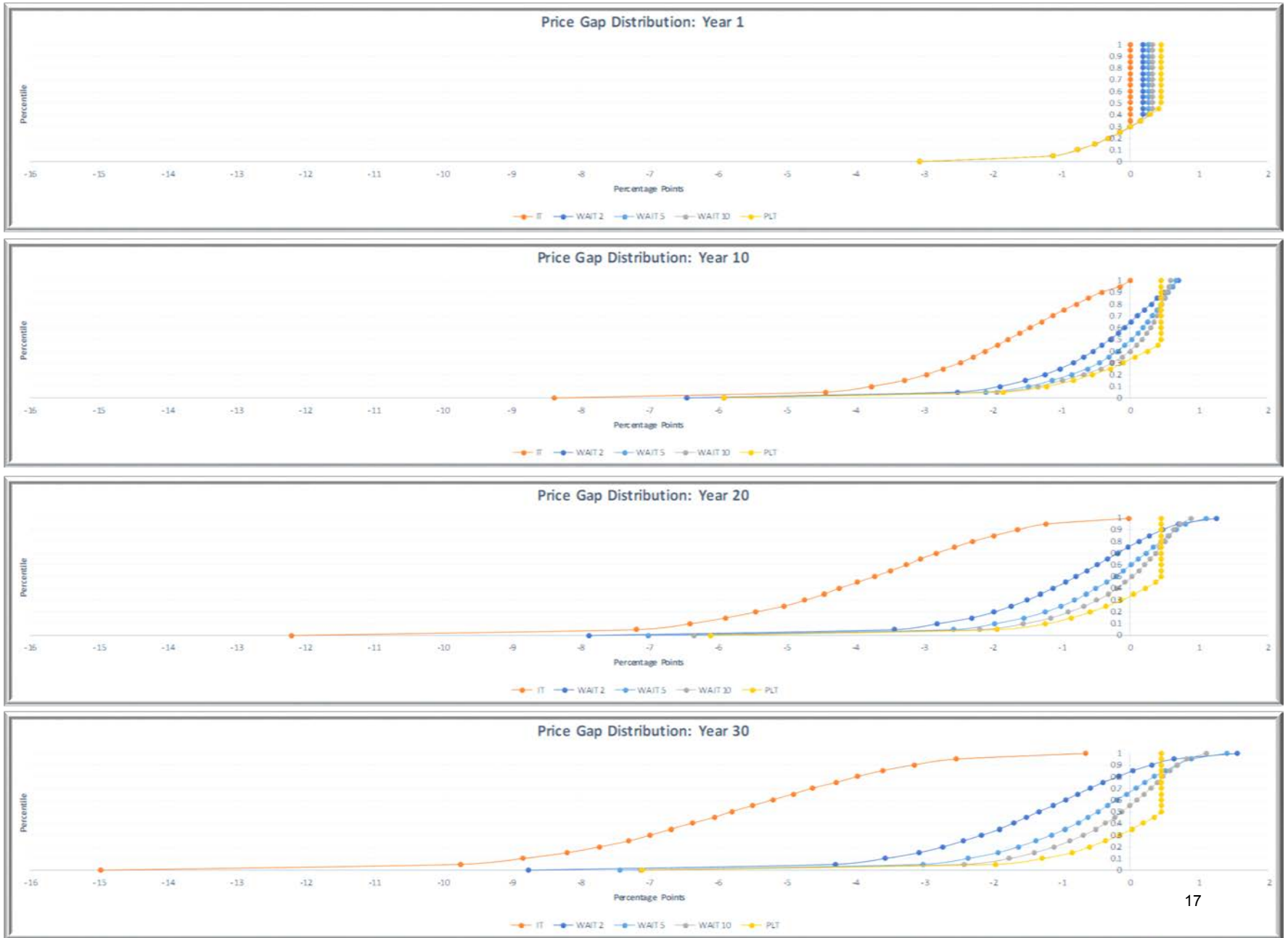


Figure 4: Distribution of Price Level Gap



left tail at longer horizons. These same qualitative features are present for the WAIT 2, 5 and 10 regimes, but the dispersion in the price level gap over time is much attenuated relative to the IT regime. In the case of PLT, the left tail is even more attenuated and a substantial amount of the distribution falls on exactly the desired buffer for the price level gap (the vertical part of the yellow line). In the case of the WAIT 2, 5 and 10 year regimes, the price level gap exhibits some dispersion into larger positive gaps as well. This occurs because under the WAIT regimes, the central bank is shooting for a target buffer for weighted average inflation and that generally requires a positive inflation gap in each period to achieve. So in cases with relatively infrequent adverse shocks and thus infrequent trips to the ZLB, the price level gap for the WAIT regimes tends to increase over time.

2.2 Time and Duration of ZLB Episodes

Differences in the IT, WAIT, and PLT regimes also show up in the frequency and duration of ZLB episodes. Figure 5 reports the number years during an assumed 40-year year period when the ZLB is binding. The chart shows these numbers by percentile so the grouping corresponding to 0.5 represents the median number of years within a 40-year window in which the ZLB constraint is binding.

A notable feature of these estimates is that the frequency of ZLB episodes under the PLT regime and long-lookback WAIT regimes is *higher* than for the IT regime or short-lookback WAIT regimes. That is, the bars for the IT and WAIT 2 regimes are noticeably lower than the bars for WAIT 5, WAIT 10 and PLT regimes. That result stems from the fact that the model employed here involves no forward looking behavior on the part of households and businesses. As a result, unlike the situation in more sophisticated models that incorporate forward looking behavior, the long-lookback WAIT regimes and the PLT regime do not induce changes in expectations about future policy or the price level that help to offset some of the effects of shocks. So if the economy experiences a series of adverse shocks that drive the central bank to the ZLB, those shocks may continue to influence policy settings for a long time as the central bank seeks to overcome the effects of those shocks on its target variable. In contrast, under IT and short-lookback WAIT regimes, the persistence of shocks in affecting the central bank's objective function is far lower with a "bygones are bygones" under a IT regime or a "bygones are mostly bygones" approach under the short lookback WAIT regimes. In effect, under the long lookback WAIT regimes and PLT regime, the central bank works harder and longer to offset prior adverse shocks to inflation. As a result, the central bank ends up at the ZLB more frequently in its efforts to boost inflation and return to the desired target level of the average inflation gap under WAIT or to the desired level of the price level gap under PLT.

This same type of effect is evident in the average duration of ZLB periods shown in Figure 6. As shown by the light blue line, for any given 40-year period, there are an average of six ZLB episodes with a duration of exactly 1 year under the IT regime. The average number of longer duration ZLB episodes falls off quickly under the IT regime. The average number of short-duration ZLB episodes under alternative regimes with longer lookback periods is lower than for the IT regime and there is a corresponding increase in the average number of ZLB episodes with durations of two years or more. The increase in the duration of ZLB episodes again reflects the

Figure 5: Number of Years with Binding ZLB (In 40 Year Period)

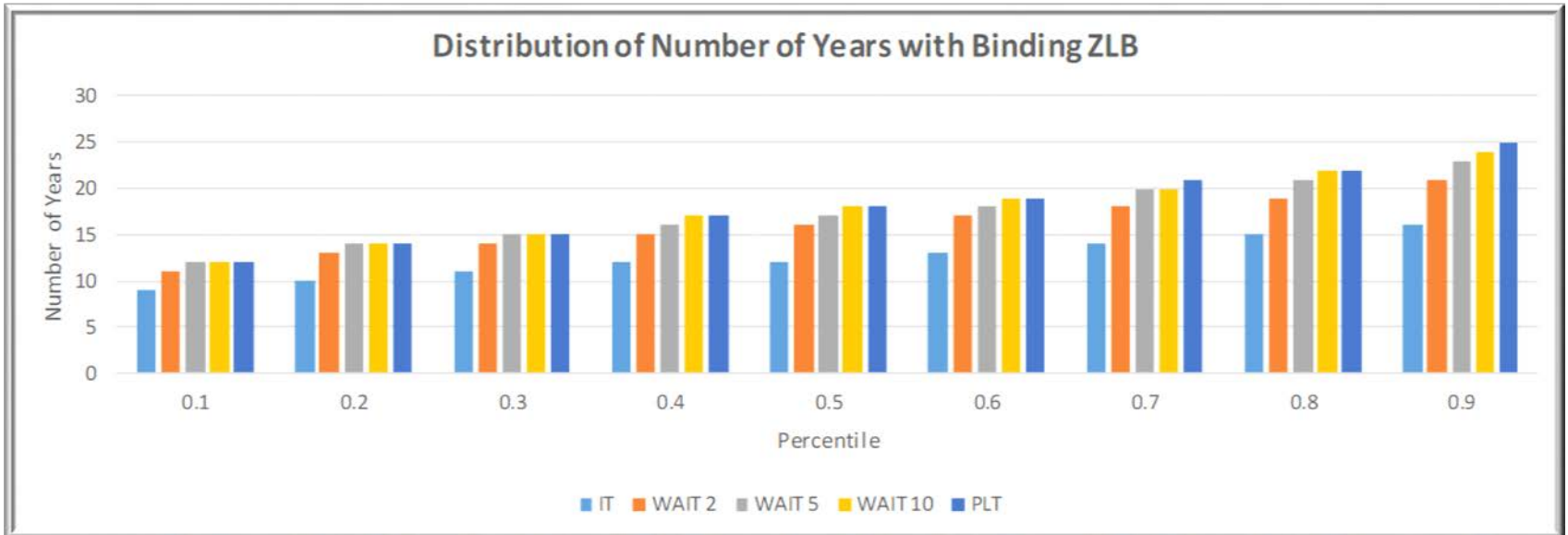
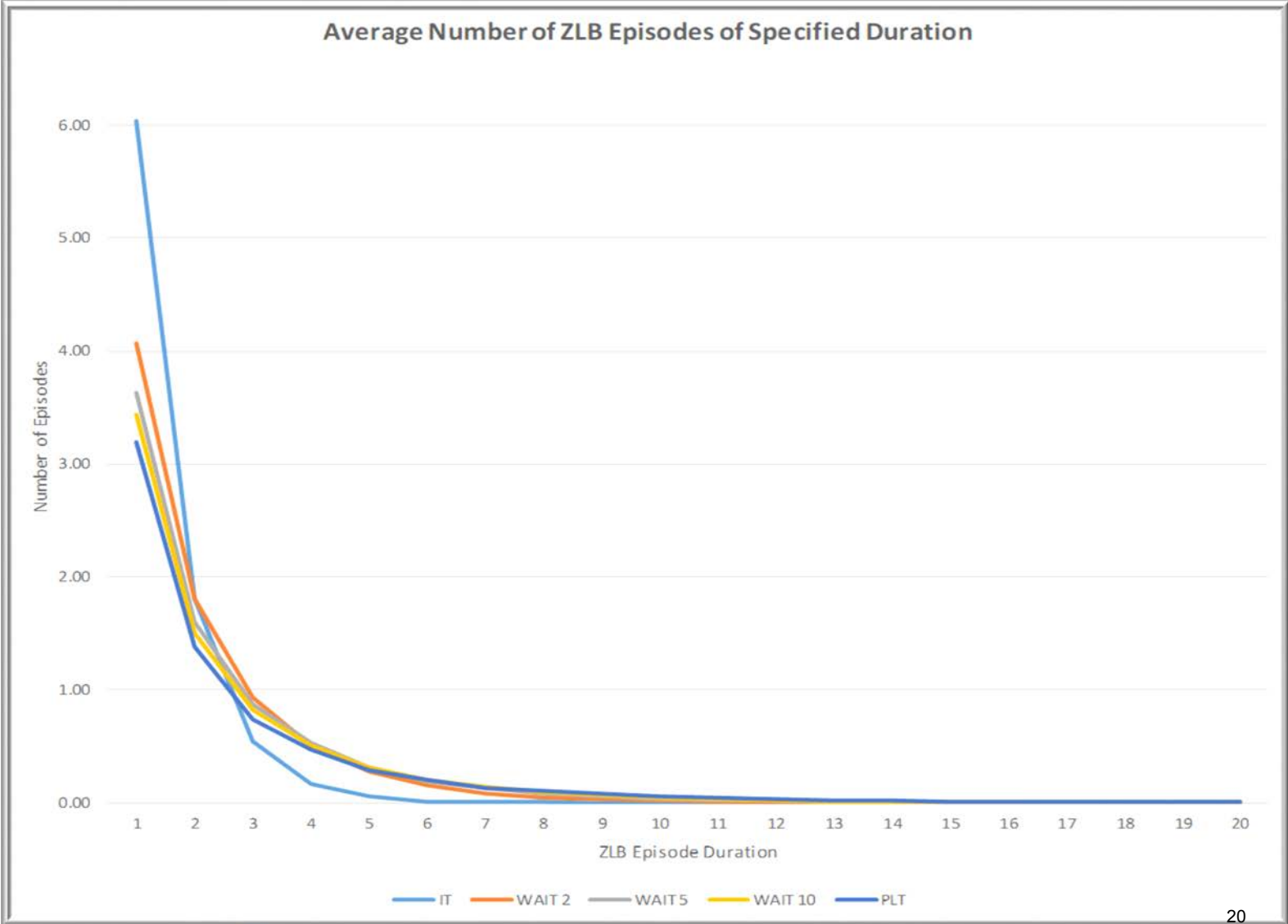


Figure 6: Number of ZLB Episodes of Specified Duration



absence of forward looking behavior in the model and the incentives for the central bank to act aggressively to counter past adverse inflation shocks. In this model, under PLT or a longer-lookback WAIT regime, the central bank will find itself in situations fairly frequently in which it must apply as much stimulus as possible for multiple periods in order to return average inflation or the price level gap to the desired level.

2.3 Comparisons with Alternative Models

As noted above, the model developed here is highly stylized. The simplicity of the model has some important advantages in being able to describe exactly how the economy behaves under optimal policy. However, it is useful to compare the results presented here with comparable results from more fully developed models. In the baseline model developed above, under the IT regime there is about a 30 percent chance of hitting the ZLB and that outcome is associated with inflation, on average, running about 20 basis points below target. Kiley and Roberts (2017) present a range of results based on simulations using both the FRB/US model and the Linde, Smets, Wouters DSGE model.¹⁷ With a standard policy rule and a real neutral short rate of 1 percent, their FRB/US simulations suggest a frequency of ZLB episodes of about 30 percent and with inflation, on average, running about 80 basis points below target. The DSGE simulations suggest that ZLB episodes would have a frequency of about 20 percent with inflation, on average, running about 1 percentage point below target. Mertens and Williams (2019) present a baseline model in which optimal discretionary policy results in a probability of hitting the ZLB of about 30 percent and that is associated with inflation, on average, running about 25 basis points below target. In broad outline, the core simulation results reported here are similar to those reported in other studies, although the effects of the ZLB on inflation in other studies are somewhat larger. It seems likely that the larger price effects in other studies stems from the more sophisticated treatment of the inflation process in those models. Here, for example, we assumed that inflation expectations are constant at the announced target rate. Moreover, the inflation process in the model does not include any element of inertia stemming from the influence of lagged inflation. These assumptions imply a considerable degree of “built in” stability in the inflation process. In contrast, in models that allow inflation expectations to vary over time, initial adverse shocks to output can put downward pressure on inflation expectations that, in turn, boosts real interest rates. The rise in real interest rates can put additional downward pressure on output and inflation. Moreover, these shocks can have lasting effects on inflation if there is a significant backward-looking aspect of the inflation process.

3. Long Term Implications of Alternative Strategies

While highly stylized, the analysis above sheds some light on the potential long-run implications of alternative monetary policy strategies when the ZLB constraint is periodically binding. In this case, inflation can remain below the central bank’s stated inflation goal on average. Moreover, periodic episodes in which the ZLB is binding can impart significant downside risk to the evolution of the price level gap over time. The latter possibility, in turn, highlights one avenue through which alternative monetary policies may have long-run implications—the exposure of

¹⁷ The DSGE model employed in these simulations is that developed by Linde, Smets, and Wouters (2016).

various groups to price level risk. In particular, entities or groups that are most exposed to price level risk are those that hold predominantly real assets financed by longer term nominal debt. Such entities are “long” the price level in the sense that an increase in the price level would be expected to leave the real value their real assets unchanged while reducing the real value of their liabilities. A corollary is that a decline in the price level would boost the real value of the nominal liabilities of these entities while again leaving the real value of their assets unaffected. This is just a variation on the well known “debt deflation” dynamic that has long been a focus of attention among macroeconomists.¹⁸

The calculations below present one way of quantifying this type of price level risk. In each example considered below, the baseline counterfactual is one in which the ZLB rarely if ever binds. As noted above, in the framework employed in this paper, the IT, WAIT and PLT regimes in this case all generate a deterministic path for the price level that increases at the rate of the announced inflation target. In that baseline, it seems reasonable to expect that the interest rates on all nominal financial assets would incorporate this known inflation rate. As a result, all nominal assets then would tend to preserve the real value of an investor’s initial investment over time. The examples below then focus on the price level “risk” that arises from an unexpected transition from this hypothetical baseline in which the ZLB rarely if ever binds to one in which it binds with some frequency.

3.1 The Real Cost of Existing Treasury Debt

One prominent example of an entity that holds predominantly real assets and largely nominal liabilities is the U.S. Treasury. The U.S. Treasury of course has large quantities of nominal Treasury debt outstanding. The real “asset” for the Treasury is the value of its stream of current and future tax revenues. It seems reasonable to view the stream of current and future tax revenues as a real asset in the sense that nominal revenues would be expected to grow over time at the rate of the nominal tax base and thus would automatically incorporate increases associated with a rising overall price level. As a result, the real net position of the federal government might be expected to decline during a transition to a world of regularly binding ZLB constraints and a downward drift in the price level.¹⁹

Of course, the deterioration of the real net position of the federal government could be viewed as a feature rather than a bug. For example, in the spirit of Patinkin (1965), the increase in the real value of Treasury debt could be viewed as a factor that boosts the perceived real wealth of the

¹⁸ See, for example, classic papers by Minsky (1992), Bernanke (1985), and Fisher (1933). See Persson and Svensson (1998) for a study of the effects of changes in the steady state inflation rate on government finances. See Doepke and Schneider (2006) and Doepke, Schneider, and Selezneva (2015) for analysis of the distributional effects of inflation and monetary policy.

¹⁹ The analysis here abstracts from other assets held by the federal government. The Financial Statements of the United States Government (<https://fiscal.treasury.gov/reports-statements/financial-report/balance-sheets.html>) report significant U.S. government holdings of financial assets, much of which is accounted for by about \$1.2 trillion of student loans. All else equal, the real value of such loans would be expected to increase with a downward drift in the price level, offsetting to some extent the increase in the real value of the federal government’s nominal liabilities. However, the increase in the real value of student loans could also be associated with higher default rates on such loans to the extent that they become more burdensome to the borrowers. So the net effect of a decline in the price level on the real value of this nominal asset is difficult to assess.

private sector holders of Treasury debt. That real wealth effect could act as an automatic stabilizer to some extent in the sense that the downward movement in the price level would likely occur in the presence of adverse shocks that drive the economy and the policy rate to the ZLB.²⁰ Based on the experience in Japan and Europe, this potential stabilizing effect appears to be fairly weak. Alternatively, in the tradition the tax smoothing literature, one could view the increase in the real value of existing Treasury debt along with the government budget constraint as necessitating costly future adjustments in marginal tax rates.²¹ The discussion below focuses on quantifying the effects of the transition to a world with a frequently binding ZLB constraint on the real value of Treasury debt, and does not offer any judgment on whether this a desirable or undesirable feature of the economy and government finances.

3.11 Total Treasury Debt

One way to gauge this effect is to measure the increase in the real value of the existing stock of Treasury debt relative to the baseline counterfactual in which the price level moves up at the rate equal to the announced long-run target rate of inflation. There are two distinct potential costs associated with the evolution of the price level in the transition to a world with a periodically binding ZLB constraint. One is the increase in the real value of existing federal debt associated with the ZLB-induced downward bias in the average rate of inflation relative to the long-run target rate. And a second additional potential cost stems from uncertainty about the evolution of the price level and the possibility that realized shocks could push inflation and the price level even lower than expected under the modal outlook.

Table 4 below reports the share of nominal Treasury debt with remaining maturities ranging from 1 to 30 years.²² We estimate the percentage increase in the real value of this existing stock of Treasury debt associated with any price level path using a weighted average of the simulated price level gaps at different horizons under alternative policy regimes with the weights corresponding to the proportion of outstanding Treasury debt in each maturity bucket. That is, the percentage increase in the real value of Treasury debt is constructed as:

$$C = - \sum_1^n \omega_i \hat{p}_i$$

Where ω_i is the share of debt with remaining maturity of “i” years and \hat{p}_i is the simulated price level gap corresponding to each maturity under each regime. To derive a measure of the average increase in cost for existing debt, we use the estimated mean price-level gap at each horizon from

²⁰ See Patinkin (1965) for a discussion of the equilibrating role of changes in the price level on the real value of government debt.

²¹ See Barro (1974) and (1979) for an analysis of government bonds as net worth and the potential costs of increases in government debt associated with the government budget constraint. See Elmendorf and Mankiw (1999) for a comprehensive and critical review of the literature on the economic effects of government debt and deficits.

²² Data on the outstanding stock of marketable Treasury debt are taken from the Monthly Statement of the Public Debt (MSPD). The statistics shown in Table 4 are based on shares held by the public excluding the Federal Reserve’s holdings. In computing these values, the Federal Reserve’s holdings of Treasury securities are netted from the figures reported in the MSPD. Detailed data on the Federal Reserve’s holdings of Treasury securities are available at https://www.newyorkfed.org/markets/soma/sysopen_accholdings.

the simulations reported above. Table 4 reports this measure as the “mean” cost. To derive a measure of the incremental cost that could be associated with inflation that turns out even lower than the expected average rate, we use the simulated price level gap at each horizon corresponding to the lowest 10th percentile in the price level gap simulations. The table reports this measure as the “cost at risk.”

Table 4: Increase in Real Cost of Federal Debt (Percent)

	IT	WAIT 2	WAIT 5	WAIT 10	PLT
Mean	1.16	0.33	0.16	-0.08	-0.01
Cost at Risk	2.35	1.38	1.18	1.08	0.99

As noted in Table 4, both of these measures—the mean cost and the cost at risk—are largest under the IT regime. This simply reflects the outcomes of the price level gap simulations reported in section 4 above; those results show that the average downward drift in the price level gap and the uncertainty surrounding that path to be highest for the IT regime. Both measures of costs decline substantially in moving to the WAIT 2 regime and then fall somewhat further in moving to the longer-lookback WAIT regimes and ultimately to PLT.

The magnitudes of these cost estimates are meaningful. For example, under the IT regime, the estimated real cost of federal debt, on average, increases by a little over 1 percent relative to the counterfactual. And as noted by the cost at risk measure, there is a 10 percent chance that the real cost of existing federal debt could increase by 2.35 percent or more. With the existing stock of marketable Treasury debt held by the public standing at about 75 percent of nominal GDP as of the third quarter of 2019, these estimates suggest that the increase in the real cost of federal debt associated with the IT regime could be $\frac{3}{4}$ percentage point or higher as a share of GDP.

Not surprisingly, the PLT regime exhibits the smallest change in the real cost of federal debt and, in fact, shows a slight decline. The difference between the mean increase in the cost of federal debt and the cost at risk measure is about 1 percentage point under all of the WAIT regimes and PLT. Even though the WAIT regimes still embed a unit root for the price level and thus involve uncertainty about the price level that increases with the length of the forecast horizon, the results here suggest that over a period of 30 years, the WAIT regimes do about as well as PLT in reducing the cost at risk metric.

3.12 Foreign Holdings of Treasury Debt

Another dimension of the effect of the price level on the real cost of Treasury debt stems from the fact that foreign ownership of longer-term Treasury debt is significant. Periods with low inflation then could be viewed, in effect, as transferring real resources from U.S. taxpayers to foreign investors.

The Treasury International Capital (TIC) data provide some sense of the potential scope of such wealth transfers. According to the annual TIC survey from June of 2018, about 35 percent of foreign holdings of longer-term U.S. Treasury securities at that time were concentrated in securities with remaining maturities of more than 5 years. Total foreign holdings of long-term

U.S. Treasury securities amount to \$5.5 trillion, implying about \$1.9 trillion of holdings with maturities beyond 5 years. Table 5 below provides the breakdown by maturity reported in the 2018 Annual TIC survey.

Table 5: Maturity Distribution of Long Term Treasury Debt Held by Foreigners

Years	Percent
<1	11.7
1-2	17.9
2-3	14.4
3-4	10.9
4-5	10.4
5-6	7.0
6-7	6.9
7-8	3.8
8-9	3.7
9-10	3.4
10-15	0.8
15-20	0.5
20-25	3.3
25-30	5.1

Applying the same methodology as above for total federal debt, the corresponding increase in the real cost of debt associated with obligations owned by foreign investors is shown in Table 6. On the whole, the numbers are similar to those for all Treasury debt outstanding shown in Table 4. With about \$5.5 trillion in longer-term Treasury debt held by foreign investors, these percentage increases again represent a significant effective transfer of U.S. resources to the rest of the world.

Table 6: Increase in Real Cost of Treasury Debt Held by Foreigners (Percent)

	IT	WAIT 2	WAIT 5	WAIT 10	PLT
Mean	1.18	0.34	0.17	0.10	0.00
Cost at Risk	2.45	1.42	1.22	1.13	1.04

3.2 Effects on the Distribution of Household Real Net Worth

The evolution of the price level may also have significant implications for the distribution of household wealth.²³ In particular, some households have relatively large amounts of nominal debt that finances real assets such as homes, autos or education/human capital. Such households are “long” the price level in the sense that the real value of their nominal debt falls and the real

²³ The ZLB and alternative monetary policy strategies can have important distributional effects through a variety of channels. See Doepcke et. al. (2006) and (2016) and the recent work by Feiveson et. al. (2020) for excellent discussions of distributional implications of monetary policy.

value of their net worth increases when the price level moves higher. Conversely, when the price level falls during an extended period at the ZLB, such households would see the real value of their net worth fall as the real value of their debt moves up relative to the real value of their assets. Below we explore aspects of this distributional effect across various groups.

3.21 Effects Alternative Strategies Net Worth and the Concentration of Wealth

Data from the 2016 Survey of Consumer Finance (SCF) provide some insight into the potential differential effects of changes in the price level across households. Without detailed information on the nature of the assets and liabilities in each group, it is very difficult to develop precise estimates of the net effects of alternative trajectories for the price level gap on the financial position of various households. However, one way of roughly gauging the effect is to focus on a measure of the response of real net worth with respect to a 1 percent increase in the price level. Real net worth w is defined here as the total nominal value of assets, A , less nominal liabilities, L , divided by the price level. The total nominal value of assets is the sum of the nominal value of “real” assets A_{Real} —those assets with prices that can be expected to reflect changes in the price level over time—and nominal assets $A_{Nominal}$ with maturity values that are fixed in nominal terms. For example, real assets might include stocks, real estate, consumer durables, homes and human capital. Nominal assets would include most types of bonds, for example. The real value of net worth is given by:

$$w = \left(\frac{W}{P}\right) = \left(\frac{A_{Real} + A_{Nominal} - L}{P}\right) = \left(\frac{P \cdot a_{real} + A_{Nominal} - L}{P}\right) = a_{real} + \left(\frac{A_{Nominal} - L}{P}\right)$$

Here a_{real} is the real value of the real assets which is assumed to be independent of the price level. To facilitate comparisons, it’s helpful to scale by the level of real income as:

$$\frac{w}{y} = \left(a_{real} + \left(\frac{A_{Nominal} - L}{P}\right)\right)/y$$

Computing the semi-elasticity of this ratio with respect to the price level provides a measure of household exposure to price level risk given by:

$$d\left(\frac{w}{y}\right) = -\left(\frac{A_{Nominal} - L}{Py}\right) \cdot \left(\frac{dP}{P}\right) = \{exposure\} \cdot \left(\frac{dP}{P}\right) \quad (10)$$

By this exposure metric, households with substantial debts and little in the way of financial assets will have positive exposures to price level risk. Indeed, when the value of nominal assets is zero in this expression, the exposure measure is simply the household’s debt to income ratio. Households with high debt relative to their nominal financial assets are likely to suffer losses in real net worth when the price level falls (or rises less rapidly than expected) because the real value of their debt obligations increases by more than the real value of their nominal assets. The reverse holds for households with substantial nominal financial assets relative to nominal debts. These households tend to suffer losses when the price level increases more than expected because the real value of their nominal assets declines by more than the real value of their debt obligations.

We can use the SCF data to compute this exposure measure for all households in the survey and then plug in the price level gaps at the five-year horizon associated with alternative monetary policy strategies to gain a sense of the impact of these alternative strategies across households.²⁴

Table 7 reports how the net worth to income ratio for households at each percentile would fare in response to the simulated changes in the price level under the different monetary policy strategies discussed above. The column labelled “exposure” reports the value of the exposure measure at percentiles ranging from 1 percent to 99 percent. The price level gaps across alternative strategies are taken from the simulations results reported above at the five year horizon. As noted in the first column of Table 7, the IT strategy results in the largest downward drift in the price level and thus generates the largest declines in the net worth to income ratio for households with the highest exposure measure. For example, as shown in the last row of Table 7, households at the 99th percentile for the exposure measure would suffer a decline in net worth equal to about 6.6 percent of income under the IT strategy. Conversely, households in the lowest percentiles record large gains in their net worth to income ratio. As noted in the first row, households at the first percentile for exposure would experience an increase in their net worth equal to about 11 percent of their income. These effects are substantially attenuated in moving to the WAIT and PLT regimes; that attenuation stems from the significantly smaller mean decline in the price level under these strategies. Table 8 reports similar results using the 10 percent lower tail of the price gap distributions from the simulations.

The tables suggest that declines in the price level may have the effect of concentrating real wealth among those groups that have accumulated substantial financial assets. Tables 9 and 10 focus on this issue specifically by examining changes in the distribution of real net worth under the alternative monetary policy regimes. As noted in the first column of Table 9, net worth is highly concentrated; households in the upper 10 percent of the net worth distribution hold about 2/3 of total net worth and the top 1 percent account for about 40 percent of total net worth.

As one might expect, the larger price level gaps associated with the IT strategy result in the most noticeable effects on the concentration of real net worth. Based on the mean price level gaps at the five year horizon in the simulations, the concentration of real net worth in the lower net worth percentiles drops by about 1 basis point. This effect results from the decline in the price level under the IT regime (relative to the counterfactual in which prices increase along an assumed constant growth rate path of 2 percent) and the exposure of lower net worth groups to a decline in the price level given the size of their debt relative to financial assets. At the opposite end of the spectrum, the decline in the price level boosts the concentration of real net worth in the top 10 percent of households by a few basis points. Again, this stems from the large holdings of financial assets relative to debt for these households. The WAIT and PLT strategies tend to damp the downward drift in the price level and thus damp the effect of the downward drift in the price level on the distribution of net worth. Table 10 reports similar results for the 10 percent

²⁴ The SCF provides detailed data on the holdings of various assets. To arrive at the measure of nominal financial assets used in equation (10), we use the data reported for total financial assets and subtract the value reported for equity. The equity variable includes direct holdings of stocks as well as estimates of indirect holdings through mutual funds, annuities, trusts, and retirement accounts.

Effect of Alternative Monetary Policy Strategies on Wealth to Income Ratio

Table 7: Mean Effect: Full Sample, Five Year Horizon

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-11.76	0.116	0.035	0.020	0.013	0.002
5	-4.35	0.043	0.013	0.007	0.005	0.001
10	-2.28	0.023	0.007	0.004	0.003	0.000
25	-0.40	0.004	0.001	0.001	0.000	0.000
50	0.06	-0.001	0.000	0.000	0.000	0.000
75	1.10	-0.011	-0.003	-0.002	-0.001	0.000
90	2.42	-0.024	-0.007	-0.004	-0.003	0.000
95	3.33	-0.033	-0.010	-0.006	-0.004	-0.001
99	6.62	-0.066	-0.020	-0.011	-0.007	-0.001

Table 8: 10th Percent Tail Effect: Full Sample, Five Year Horizon

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-11.76	0.263	0.165	0.147	0.141	0.133
5	-4.35	0.097	0.061	0.054	0.052	0.049
10	-2.28	0.051	0.032	0.028	0.027	0.026
25	-0.40	0.009	0.006	0.005	0.005	0.005
50	0.06	-0.001	-0.001	-0.001	-0.001	-0.001
75	1.10	-0.025	-0.015	-0.014	-0.013	-0.012
90	2.42	-0.054	-0.034	-0.030	-0.029	-0.027
95	3.33	-0.075	-0.047	-0.042	-0.040	-0.038
99	6.62	-0.148	-0.093	-0.083	-0.079	-0.075

Table 9: Effects on the Distribution of Wealth (Mean Price Level Gap)

Wealth Percentiles	Levels						Deviations				
	SCF	IT	WAIT 2	WAIT 5	WAIT 10	PLT	IT	WAIT 2	WAIT 5	WAIT 10	PLT
0-9.9	-0.50	-0.51	-0.50	-0.50	-0.50	-0.50	0.00	0.00	0.00	0.00	0.00
10-19.9	0.02	0.02	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00
20-29.9	0.16	0.15	0.15	0.15	0.15	0.16	-0.01	-0.01	-0.01	-0.01	0.00
30-39.9	0.47	0.46	0.47	0.47	0.47	0.47	-0.01	0.00	0.00	0.00	0.00
40-49.9	1.06	1.05	1.06	1.06	1.06	1.06	-0.01	0.00	0.00	0.00	0.00
50-59.9	1.89	1.88	1.89	1.89	1.89	1.89	-0.01	0.00	0.00	0.00	0.00
60-69.9	3.23	3.22	3.23	3.23	3.23	3.23	-0.01	0.00	0.00	0.00	0.00
70-79.9	5.48	5.48	5.48	5.48	5.48	5.48	0.00	0.00	0.00	0.00	0.00
80-89.9	11.33	11.33	11.33	11.33	11.33	11.33	0.00	0.00	0.00	0.00	0.00
90-94.9	12.08	12.08	12.08	12.08	12.08	12.08	0.00	0.00	0.00	0.00	0.00
95-98.9	26.57	26.60	26.58	26.58	26.57	26.57	0.03	0.01	0.01	0.00	0.00
99-100	38.21	38.23	38.21	38.21	38.21	38.21	0.02	0.00	0.00	0.00	0.00

Table 10: Effects on the Distribution of Wealth (10 percentile Price Level Gap)

Wealth Percentiles	Levels						Deviations				
	SCF	IT	WAIT 2	WAIT 5	WAIT 10	PLT	IT	WAIT 2	WAIT 5	WAIT 10	PLT
0-9.9	-0.50	-0.52	-0.51	-0.51	-0.51	-0.51	0.00	0.00	0.00	0.00	0.00
10-19.9	0.02	0.02	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00
20-29.9	0.16	0.15	0.15	0.15	0.15	0.15	-0.01	-0.01	-0.01	-0.01	-0.01
30-39.9	0.47	0.45	0.46	0.46	0.46	0.46	-0.02	-0.01	-0.01	-0.01	-0.01
40-49.9	1.06	1.04	1.05	1.05	1.05	1.05	-0.02	-0.01	-0.01	-0.01	-0.01
50-59.9	1.89	1.87	1.88	1.88	1.88	1.88	-0.02	-0.01	-0.01	-0.01	-0.01
60-69.9	3.23	3.21	3.22	3.22	3.22	3.22	-0.02	-0.01	-0.01	-0.01	-0.01
70-79.9	5.48	5.47	5.47	5.48	5.48	5.48	-0.01	-0.01	0.00	0.00	0.00
80-89.9	11.33	11.33	11.33	11.33	11.33	11.33	0.00	0.00	0.00	0.00	0.00
90-94.9	12.08	12.09	12.08	12.08	12.08	12.08	0.01	0.00	0.00	0.00	0.00
95-98.9	26.57	26.63	26.61	26.61	26.60	26.60	0.06	0.04	0.04	0.03	0.03
99-100	38.21	38.26	38.24	38.24	38.23	38.23	0.05	0.03	0.03	0.02	0.02

lower tail of price level gaps. In this case, the share of wealth in the highest categories rises by about 5 basis points while the wealth shares at the lower percentiles drops by a couple basis points.

The effects of alternative strategies on the concentration of wealth reported here are relatively modest, in part reflecting the relatively small price level gaps in the simulations. For example, the mean price level gap under the IT strategy at the 5 year horizon is only about 1 percentage point. Of course, larger price level gaps would tend to amplify this effect to some degree. As one benchmark for comparison, the price level gap for the total PCE price index over the five years 2015-2019 is about 3 percent—about 3 times the mean price level gap for the IT strategy based on the simulations.

3.22 Factors Affecting the Cross Sectional Distribution of Price Level Exposure

The exposure measure developed above is the key factor determining the cross sectional effects of changes in the price level. Moreover, Tables 7 and 8 suggest that the exposure measure varies considerably across households. Some simple tables help to identify the factors that explain that variation.

Tables 11 through 17 report the percent of households by percentile groups for the exposure variable and by groups for other variables of interest. For example, Table 11 reports the fraction of households across income and exposure groups. Each column of the table reports the fraction of all households in a particular income group that falls in the percentile categories for the exposure variable. The percentages in each column thus sum to 100. The percentages shown in the last column correspond to the full sample and thus map directly to the definitions of the percentile exposure categories. That is, 1 percent of the households are in the lowest 1 percent of households by exposure measure, 4 percent of all households are the 1-5 percent category of households by exposure measure and so on. Looking first at the lower percentiles for the exposure variable, households in the upper income categories are overrepresented in this category; for example, 14.6 percent of households in the highest income category are in the lowest exposure category versus only 1 percent of all households that fall in this exposure category. These higher income households have amassed substantial financial assets. As a result, an increase in the price level depresses their real net worth—their exposure measure is large and negative, placing them in the lowest exposure categories. At the other extreme, households in the lowest income category are overrepresented in the highest exposure category—almost 4 percent of households in this income category fall in the highest exposure category versus the 1 percent of all households that fall in this exposure category. Tables 12-17 report similar statistics for categories defined by other variables including net worth, age, race, gender and whether the household has a mortgage or education loan outstanding. Not surprisingly, the data in Table 12 show a large over representation of high net worth groups in the low exposure categories and a sizable over representation of low net worth groups in the higher exposure categories. Table 13 highlights the high concentration of younger households in the high exposure groups as well as the high concentrations of older households in the low exposure groups. The differences across race reported in Table 14 are noticeable with white households overrepresented in the low exposure categories and nonwhite households over

Table 11: Distribution of Households by Income and Exposure

Exposure	Distribution of Households by Income and Exposure Groups												All
	0-9.9	10-19.9	20-29.9	30-39.9	40-49.9	50-59.9	60-69.9	70-79.9	80-89.9	90-94.9	95-98.9	99-100	
0-1	1.0	1.1	0.8	0.5	0.5	1.1	0.8	0.8	1.1	1.9	2.9	3.1	1
1-5	1.1	2.4	3.2	3.8	3.8	3.3	4.1	4.9	4.9	6.6	11.2	8.4	4
5-10	0.7	2.3	4.3	5.5	3.9	4.7	4.7	5.2	6.2	12.2	12.6	12.8	5
10-25	7.5	9.0	10.5	12.2	12.7	17.0	16.6	16.9	20.2	23.6	29.1	38.6	15
25-50	53.5	43.5	36.3	28.5	22.4	17.2	13.6	11.8	12.6	9.0	12.5	18.4	25
50-75	19.9	23.8	24.2	22.6	28.5	29.5	28.1	28.3	26.9	19.9	17.8	10.4	25
75-90	5.9	8.7	10.6	15.3	16.3	15.1	19.5	23.7	20.6	19.9	8.7	6.7	15
90-95	1.6	3.3	4.7	5.7	5.7	6.3	7.6	5.1	5.7	5.0	4.0	1.2	5
95-99	5.3	4.0	4.0	5.1	5.6	5.5	4.2	3.2	1.7	2.0	1.2	0.2	4
99+	3.7	2.0	1.5	0.8	0.6	0.5	0.6	0.1	0.3	0.1	0.0	0.1	1
All	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 12: Distribution of Households by Net Worth and Exposure

Exposure	Distribution of Households by Net Worth and Exposure Groups												All
	0-9.9	10-19.9	20-29.9	30-39.9	40-49.9	50-59.9	60-69.9	70-79.9	80-89.9	90-94.9	95-98.9	99-100	
0-1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	1.3	2.2	4.1	7.3	14.6	1
1-5	0.0	0.0	0.0	0.1	0.3	0.3	3.3	6.5	11.0	13.5	25.2	16.9	4
5-10	0.0	0.0	0.0	0.3	1.0	1.4	4.3	12.3	14.1	16.5	17.8	14.7	5
10-25	0.0	0.2	3.3	10.0	13.3	15.9	21.0	27.9	30.9	30.0	24.5	29.4	15
25-50	1.5	72.2	55.9	33.3	18.4	19.5	12.8	14.8	10.8	11.5	8.8	11.3	25
50-75	51.1	24.5	26.4	23.1	26.6	27.5	29.1	17.1	13.8	12.8	9.1	7.3	25
75-90	23.8	1.9	7.1	18.7	28.1	23.2	17.9	13.8	10.7	5.5	4.6	4.0	15
90-95	9.0	1.0	4.5	8.5	6.3	5.1	6.3	3.7	3.5	3.3	1.4	0.4	5
95-99	10.8	0.2	2.2	5.3	4.9	5.8	4.1	2.2	2.7	2.4	1.2	1.2	4
99+	3.8	0.0	0.7	0.7	1.0	1.4	1.2	0.5	0.4	0.4	0.2	0.2	1
All	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 13: Distribution of Households by Age and Exposure

Exposure	Distribution of Households by Age and Exposure Groups						All
	<35	35-44	45-54	55-64	65-74	>=75	
0-1	0.2	0.0	0.3	1.0	2.7	3.0	1
1-5	0.5	0.5	1.2	4.8	8.6	13.0	4
5-10	0.4	0.7	2.8	7.7	10.7	11.7	5
10-25	6.7	8.5	14.1	19.6	22.0	24.6	15
25-50	28.6	24.1	23.1	24.2	24.4	25.0	25
50-75	32.8	29.4	30.2	22.7	16.0	11.0	25
75-90	18.8	22.9	17.9	11.5	7.1	7.6	15
90-95	6.0	8.6	5.1	3.5	3.6	1.9	5
95-99	4.5	4.7	4.6	3.8	3.5	1.9	4
99+	1.6	0.5	0.7	1.3	1.3	0.3	1
All	100	100	100	100	100	100	100

Table 14: Distribution of Households by Race and Exposure

Exposure	Distribution of Households by Race and Exposure Groups				All
	White	Black	Hispanic	Other	
0-1	1.4	0.2	0.1	0.6	1
1-5	5.5	0.8	0.7	2.4	4
5-10	6.7	1.5	0.9	3.6	5
10-25	18.1	9.1	7.0	12.0	15
25-50	19.9	32.8	42.8	28.3	25
50-75	23.2	32.1	23.4	27.5	25
75-90	15.7	14.6	13.7	12.6	15
90-95	5.0	4.0	5.1	6.4	5
95-99	3.7	4.3	3.8	5.6	4
99+	0.8	0.7	2.5	0.9	1
All	100	100	100	100	100

Table 15: Distribution of Households by Gender and Exposure

Exposure	Distribution of Households by Gender and Exposure Group		
	male	female	All
0-1	0.9	1.3	1
1-5	3.8	4.7	4
5-10	5.2	4.5	5
10-25	16.0	12.4	15
25-50	22.9	30.6	25
50-75	25.5	23.6	25
75-90	16.1	12.1	15
90-95	5.1	4.6	5
95-99	3.6	5.0	4
99+	0.9	1.2	1
All	100	100	100

Table 16: Distribution of Households by Mortgage Loan Status and Exposure

Exposure	Distribution of Households by Mortgage Loan Status and Exposure Group		
	No	Yes	All
0-1	1.5	0.4	1
1-5	5.3	2.2	4
5-10	5.9	3.8	5
10-25	17.7	11.3	15
25-50	38.6	6.2	25
50-75	24.1	26.2	25
75-90	4.6	29.5	15
90-95	1.0	10.6	5
95-99	1.0	8.1	4
99+	0.5	1.7	1
All	100	100	100

Table 17: Distribution of Households by Education Loan Status and Exposure

Exposure	Distribution of Households by Education Loan Status and Exposure Group		
	No	Yes	All
0-1	1.3	0.0	1
1-5	5.0	0.5	4
5-10	6.1	1.3	5
10-25	17.9	4.9	15
25-50	29.9	7.9	25
50-75	20.6	40.5	25
75-90	11.6	27.0	15
90-95	3.9	8.8	5
95-99	3.0	7.5	4
99+	0.8	1.7	1
All	100	100	100

represented at the higher exposures. The distributions in the exposure measure by gender reported in Table 15 are very similar between men and women although women are slightly over represented at both the upper and lower extremes of the exposure distribution. Tables 16 and 17 show a significant concentration of households with mortgage or education loans outstanding toward the upper ends of the exposure distribution.

The overall picture that emerges from these tables is that households with the lowest exposure measures are also often those with older heads of households with high incomes and high net worth. A disproportionate share of these heads of households are white and tend not to have mortgage or education loans outstanding. Tables 11-17 reported statistics based on categories defined by the unconditional distribution of the exposure measure. The disproportionate share of some groups in the tails of these exposure categories suggests that distribution of the exposure measure *conditional* on various factors could differ significantly from the unconditional distribution.

Tables 18-21 investigate this aspect of the data and report the distribution of the exposure measure for different groups.²⁵ For example, in Table 18, the distribution of the exposure measure for households in the lowest income decile is shown in the first column. Not surprisingly, the value of the exposure measure at every percentile is higher than that reported for the full sample reported in Table 7; the difference is most pronounced for the highest percentiles. These higher exposure levels then result in larger declines in net worth relative to income under the alternative monetary policy strategies. For example, households in the lowest income decile have an upper end of the distribution of the exposure measure of 10.29. Under the price decline for the IT strategy at the five year horizon, that results in a drop in real net worth of about 10 percent relative to real income. Tables 19-21 report similar results conditioning on households that are young (age less than 35), or that have mortgage or education loans outstanding. The results here again point to higher exposures for these groups than for the overall population and thus larger declines in net worth relative to income in response to price declines.

The simple cross tabs reported above are helpful in understanding how price level shifts could affect some types of households more than others. The patterns suggested by the tables are borne out in results based on more formal statistical tools. Figures 7 and 8 report the results of quantile regressions with our measure of price level risk exposure as the dependent variable and a list of characteristics of individual households as explanatory variables.

EDU	=	Dummy Variable for Education Loan Outstanding
MORT	=	Dummy Variable for Mortgage Loan
INCOME_X	=	Dummies for Income Categories
FEMALE	=	Dummy for Gender of Head of Household
BLACK	=	Dummy for Race of Head of Household
HISPANIC	=	Dummy for Race of Head of Household

²⁵ These statistics are weighted using the SCF survey population weights for each household in the survey. The SCF has a system of imputations so that five “implicates” are reported for each household. The statistics reported here use all implicates and divide the population weights by 5.

Table 7: Mean Effect: Full Sample, Five Year Horizon

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-11.76	0.116	0.035	0.020	0.013	0.002
5	-4.35	0.043	0.013	0.007	0.005	0.001
10	-2.28	0.023	0.007	0.004	0.003	0.000
25	-0.40	0.004	0.001	0.001	0.000	0.000
50	0.06	-0.001	0.000	0.000	0.000	0.000
75	1.10	-0.011	-0.003	-0.002	-0.001	0.000
90	2.42	-0.024	-0.007	-0.004	-0.003	0.000
95	3.33	-0.033	-0.010	-0.006	-0.004	-0.001
99	6.62	-0.066	-0.020	-0.011	-0.007	-0.001

Table 18: Mean Effect: Lowest Income Decile

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-11.19	0.111	0.034	0.019	0.012	0.002
5	-1.14	0.011	0.003	0.002	0.001	0.000
10	-0.40	0.004	0.001	0.001	0.000	0.000
25	-0.07	0.001	0.000	0.000	0.000	0.000
50	0.00	0.000	0.000	0.000	0.000	0.000
75	0.52	-0.005	-0.002	-0.001	-0.001	0.000
90	2.51	-0.025	-0.008	-0.004	-0.003	-0.001
95	4.92	-0.049	-0.015	-0.008	-0.005	-0.001
99	10.29	-0.102	-0.031	-0.017	-0.011	-0.002

Table 19: Mean Effect: Age Less than 35

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-2.63	0.026	0.008	0.004	0.003	0.001
5	-0.66	0.007	0.002	0.001	0.001	0.000
10	-0.27	0.003	0.001	0.000	0.000	0.000
25	-0.02	0.000	0.000	0.000	0.000	0.000
50	0.38	-0.004	-0.001	-0.001	0.000	0.000
75	1.35	-0.013	-0.004	-0.002	-0.001	0.000
90	2.64	-0.026	-0.008	-0.004	-0.003	-0.001
95	3.51	-0.035	-0.011	-0.006	-0.004	-0.001
99	9.26	-0.092	-0.028	-0.016	-0.010	-0.002

Table 20: Mean Effect: Outstanding Mortgage

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-7.31	0.072	0.022	0.012	0.008	0.001
5	-2.82	0.028	0.008	0.005	0.003	0.001
10	-1.30	0.013	0.004	0.002	0.001	0.000
25	0.14	-0.001	0.000	0.000	0.000	0.000
50	1.09	-0.011	-0.003	-0.002	-0.001	0.000
75	2.16	-0.021	-0.006	-0.004	-0.002	0.000
90	3.31	-0.033	-0.010	-0.006	-0.004	-0.001
95	4.38	-0.043	-0.013	-0.007	-0.005	-0.001
99	8.46	-0.084	-0.025	-0.014	-0.009	-0.002

Table 21: Mean Effect: Outstanding Education Loan

Percentile	Exposure	IT	WAIT2	WAIT5	WAIT10	PLT
1	-3.57	0.035	0.011	0.006	0.004	0.001
5	-0.67	0.007	0.002	0.001	0.001	0.000
10	-0.07	0.001	0.000	0.000	0.000	0.000
25	0.27	-0.003	-0.001	0.000	0.000	0.000
50	0.94	-0.009	-0.003	-0.002	-0.001	0.000
75	2.01	-0.020	-0.006	-0.003	-0.002	0.000
90	3.21	-0.032	-0.010	-0.005	-0.004	-0.001
95	4.42	-0.044	-0.013	-0.008	-0.005	-0.001
99	8.91	-0.088	-0.027	-0.015	-0.010	-0.002

ASIAN_OTHER = Dummy for Race of Head of Household
 AGE_X = Dummies for Age Categories

Figures 7 and 8 show the estimated coefficients for each quantile ranging including .01, .05, .10, .25, .50, .75, .90, .95 and .99.²⁶ The various dummy variables are coded so that the reference group is composed of white males that are more than 75 years old that are in the top 1 percent of the income distribution with no mortgage or education loans outstanding. The constant in the quantile regressions thus captures the distribution for the exposure to price level risk for this reference group. As shown in the left panel of figure 7, at the lowest end of the distribution, the constant is large and negative, consistent with idea that individuals in this very select group have sizable holdings of financial assets relative to their debt and thus benefit substantially from a decline in the price level. For example, at the 5 percent quantile, the constant is about -10, implying that a 1 percent drop in the price level tends to boost the real net worth of this group *relative* to their income by 10 percent. Even at the upper end of the distribution—those households in the reference group with a smaller gap between their nominal financial assets and liabilities—the coefficient is still negative implying that they benefit modestly from a decline in the price level.

The panel in the right displays the estimated effects for race and gender. The effect of gender on price level risk exposure is fairly small and tends to be more noticeable at the very low end of the exposure distribution. The coefficients on the racial and ethnic group dummy variables are mostly positive—that is, nonwhite groups tend to have higher price level risk exposures than for whites—but the differences are fairly modest in magnitude for most quantiles. The effect tends to be more noticeable at the lowest quantiles, consistent with the fact that nonwhite groups—even at these lower quantiles—reported lower levels of financial assets relative to their debts.

The upper left panel reports the effects of dummies for various income categories. In general, the estimated coefficients across all quantiles are positive and most noticeable at the lowest quantiles. Again, that reflects the pattern that lower income households also tend to have lower holdings of nominal financial assets relative to their debt. That in turn tends to boost their price level risk exposures relative to the reference group.

The panel at the top right shows the effects of the dummies for mortgage loans and education loans. Not surprisingly, the coefficients on these dummies suggest that they tend to boost the price level risk exposure at every quantile and particularly for the high quantiles. These results are very consistent with the simple tables reported earlier showing that households with outstanding mortgage loans and education loans tend to have much higher exposures than households without such liabilities.

The bottom panel in Figure 8 reports the coefficients on age groups. These again are generally positive and largest at the lower quantiles. In part, this effect likely represents the “life-cycle” of savings. Younger households have often acquired debt to finance homes and their educations, resulting in relatively large price level risk exposures—e.g. large debts and low financial assets. Older households, by contrast, have often paid down many types of debt and accumulated

²⁶ Full regression results for the quantile regressions are reported in appendix C.

Figure 7: Baseline Quantile Regression Results

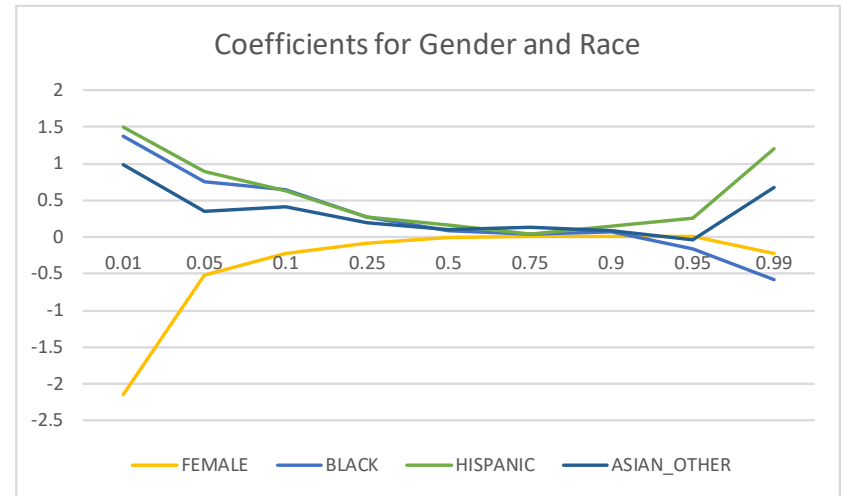
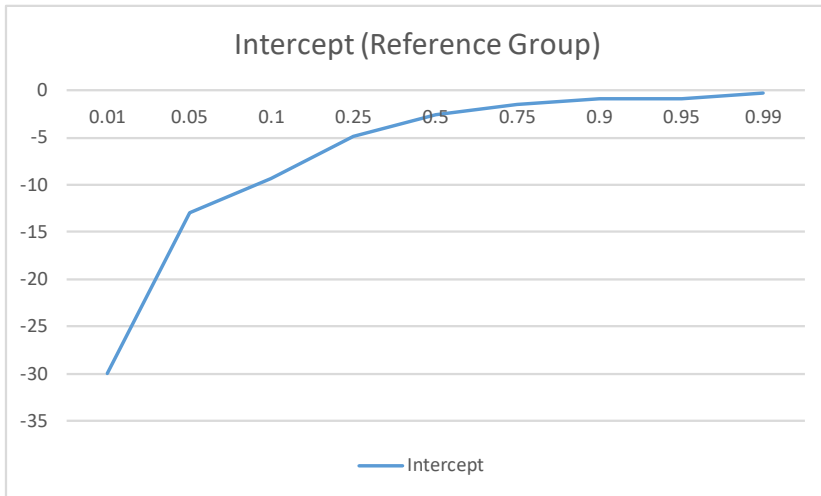
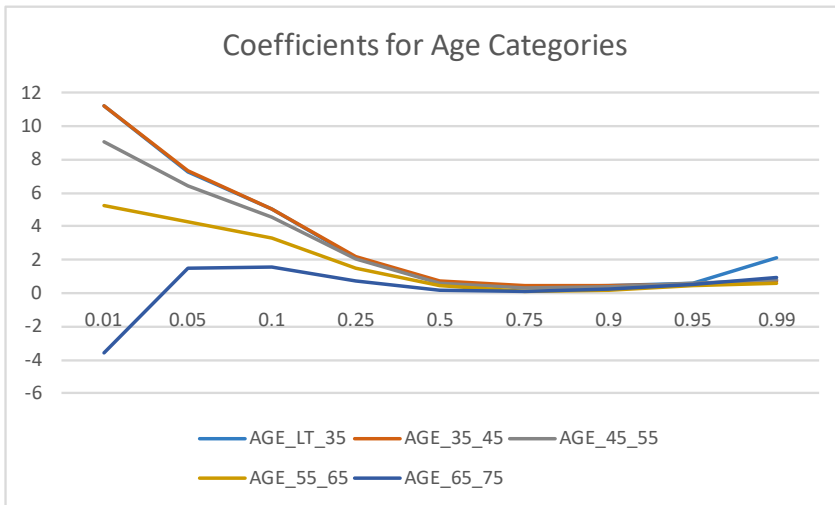
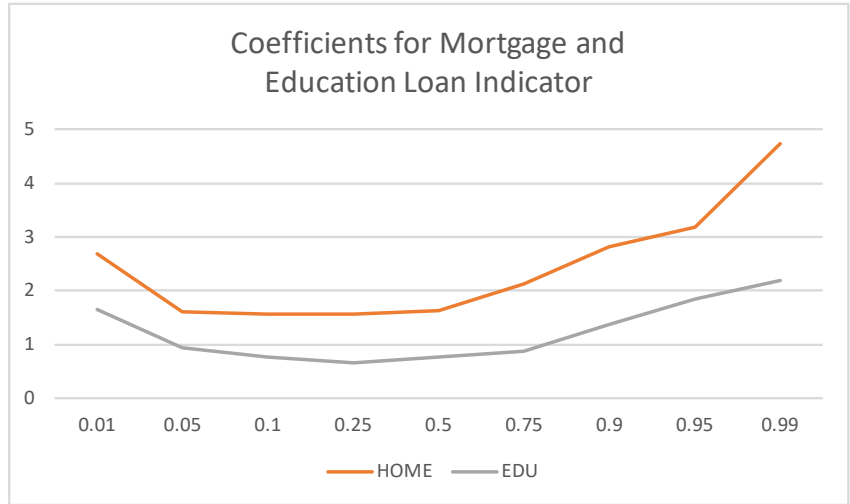
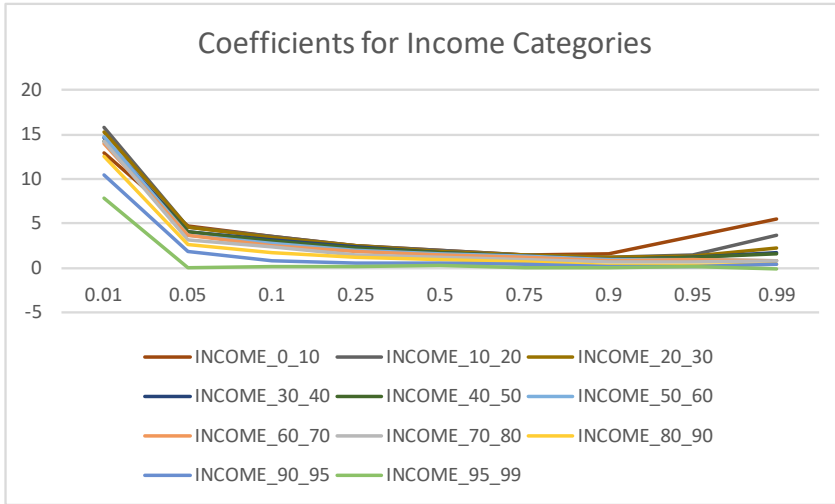


Figure 8: Quantile Regression Results



significant holdings of financial assets for retirement. Such households (at the lower quantiles) tend to have larger volumes of financial assets than younger households and thus lower exposures to downside price level risks.

Although there is a good deal of heterogeneity across households in the SCF, the basic picture that emerges from the statistical analysis is that households most heavily exposed to downside price level risk are those in relatively low income groups and that have mortgage loans or educational loans outstanding. In addition, younger households tend to have substantially larger exposures to downside price level risk than older households. That effect, in turn, importantly reflects the lifecycle effects of younger households taking on debt to finance homes and their educations while maintaining relatively low levels of financial assets. Older households, however, have often paid down mortgage debt and other debt and built up substantial holdings of financial assets to finance their retirements. Other characteristics of households appear to have significant effects. The analysis suggests that—controlling for income and other factors—nonwhites and particularly Hispanics seem to have somewhat higher price level exposures than some other groups.

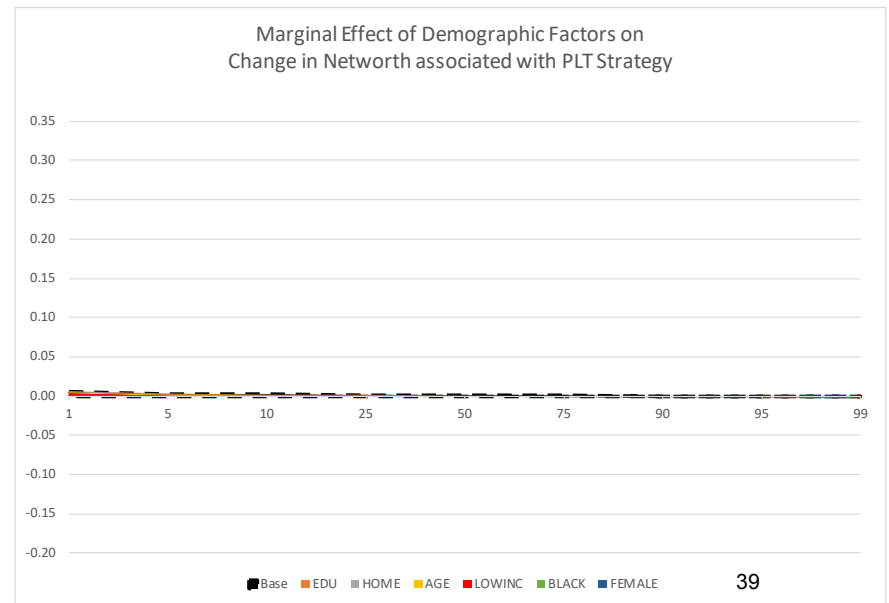
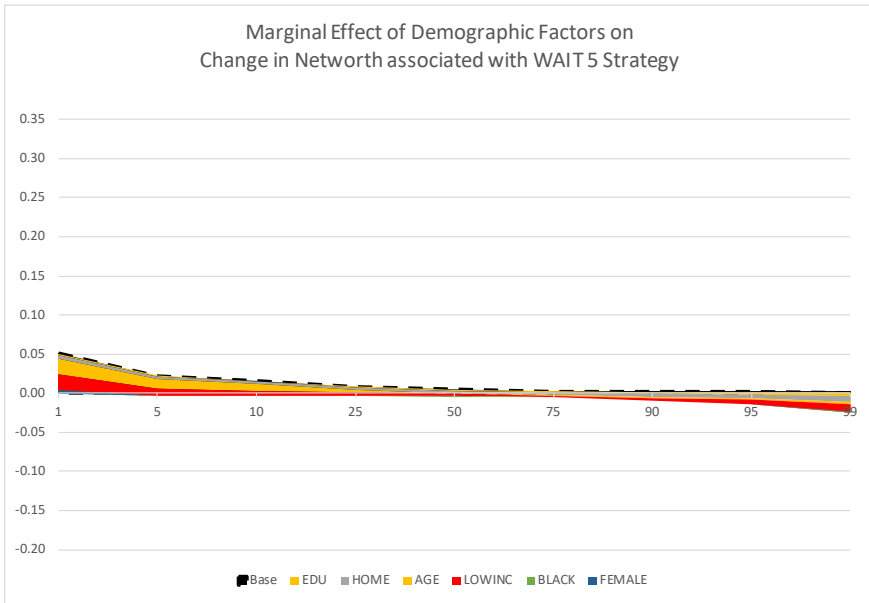
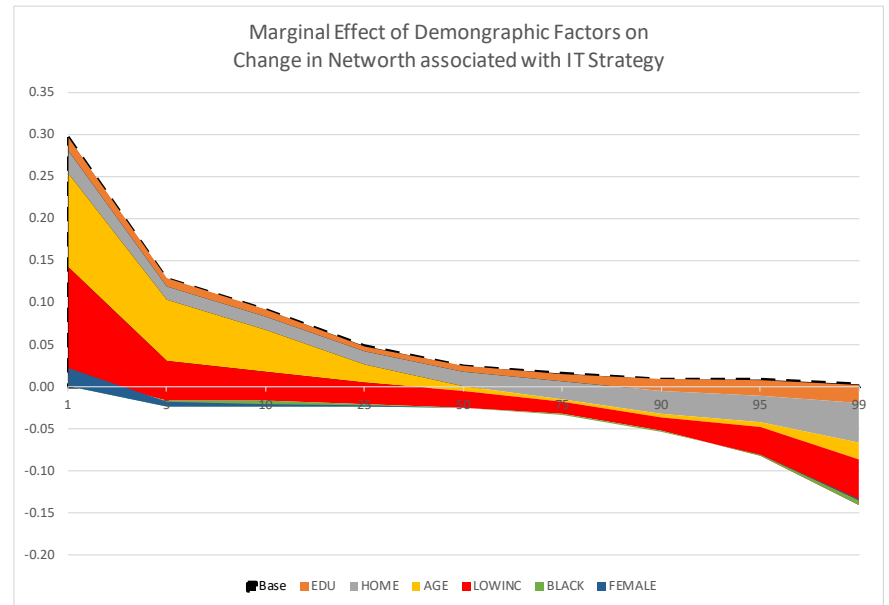
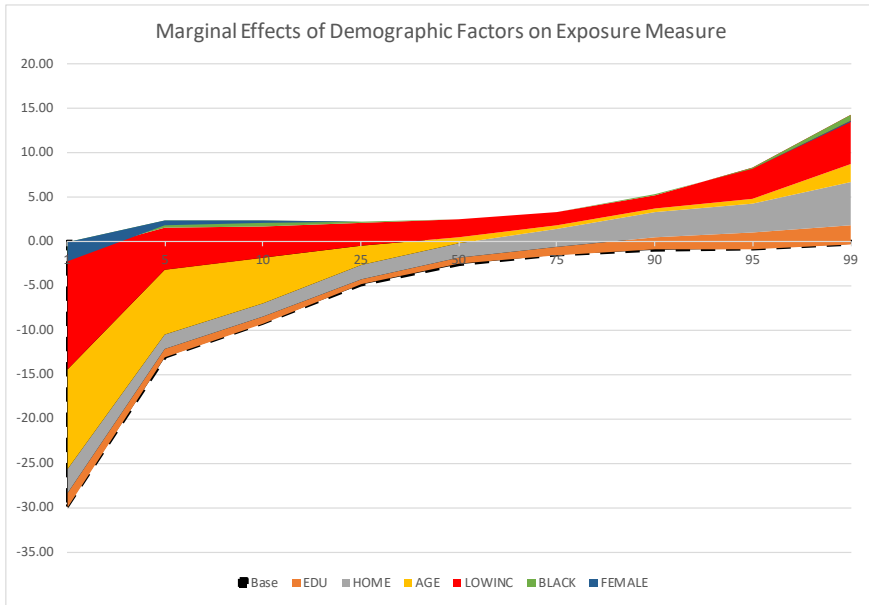
Figure 9 presents a graphical translation of these coefficient estimates on the change in net worth relative to income across policy strategies. The upper left panel reports the exposure measure for the base group—elderly white males in the highest income group with no mortgage or education loans outstanding. The exposure measure for this group is represented by the dashed black line at the bottom edge of the area chart. Each shaded area then reports the marginal effect of the variable shown on the value of the exposure measure at selected points along the conditional distribution. For example, the yellow swath shows the marginal effect of being in the youngest age category. That change substantially boosts the exposure measure at the lowest percentiles. The grey shaded area shows the marginal effect of having a home mortgage loan outstanding. The effect is positive throughout and especially large at the higher percentiles. The red shaded area shows the effect of being in the lowest income decile. This factor substantially boosts the exposure measure at the lowest percentiles and the upper percentiles.

The remaining panels on the page show the same type of chart using the price gaps from the simulation under alternative policy regimes to calculate the effect of the decline in the price level on net worth relative to income. The black dashed line at the upper edge of these charts corresponds to the results for the base group. The base group records substantial gains under the IT strategy based on their large (negative) exposure measure in the top left panel. The transition to the low income and young categories substantially boosts the exposure measure and thus pulls down the gains at the lowest percentiles and increases the losses for the highest percentiles. The other panels show the same effects under the WAIT 5 and PLT strategies. As for previous results, the more muted price declines under these strategies results in more muted effects on net worth to income across all groups.

4. Conclusions

The analysis above highlights the effects alternative monetary strategies in shaping the evolution of the price level over long time periods. Of note, alternative strategies that involve an element

Figure 9: Marginal Effects of Explanatory Variables on Conditional Distribution



of “makeup” for past deviations of inflation from target as in the WAIT and PLT regimes reduce both the downward bias in the average rate of inflation that stems from the constraints posed by the ZLB as well as the uncertainty surrounding the price path.

Over the longer-term, variations in the evolution of the price level stemming from alternative monetary policy regimes may have meaningful effects on the financial position of entities or groups that rely heavily on nominal debt to finance “real” assets. Of course, the influence of alternative monetary policy strategies on these groups operates through many different channels, importantly including the effects such strategies may have in stabilizing the economy and employment in response to adverse shocks. The particular channel addressed in this paper focuses on the “behind the scenes” price level effects that play out slowly over time. Entities or groups that are “long” the price level stand to lose when the price level path follows a lower-than-expected trajectory. The analysis presented above suggests that the movement away from a “bygones are bygones” approach under a pure IT regime and toward one that incorporates some element of “makeup” as under the WAIT and PLT regimes could have meaningful beneficial effects for groups exposed to downside price level risks by reducing the negative bias in inflation and the downside risks to evolution of the price level. In the case of the federal government, an IT regime can result in a significant increase in the real cost of existing Treasury debt; that cost falls significantly under the WAIT and PLT regimes. The empirical analysis presented above suggests that when the zero lower bound is a significant risk, the downward drift in the price level associated with an IT regime tends to transfer real wealth from the lower end of the wealth distribution to the upper end. In examining the incidence of price level effects across different groups, we find that households most exposed to downside price level risk are younger with lower incomes and with significant debts outstanding, particularly home and educational loans. Controlling for other factors, differences in exposure to price level risk by race/ethnicity and general are comparatively small. Again, the WAIT and PLT regimes tend to mitigate these distributional effects by damping the downward drift in the price level relative to an IT regime.

A. Technical Details

A.1 Basic Properties of the Value Function

Some properties of the value function are evident from the form of equation (3). In particular, as the level of Z_{t-1} increases to infinity, the value function asymptotes to:

$$\lim_{Z_{t-1} \rightarrow \infty} V(Z_{t-1}) = \frac{1}{2}\mu^2 + \beta V(\mu)$$

The reason the value function asymptotes to a constant is that in this model, a very high value of the target value from the previous period Z_{t-1} implies that there is essentially no chance that the shock to inflation could be large enough that the central bank would be constrained by the zero lower bound. As a result, the expected loss in the current period is just the optimal constrained “buffer” μ plus the expected present discounted value of expected future losses given that the target value taken out of the current period will be equal to μ .

And when the level of the target variable is deeply negative, the value function converges to:

$$\lim_{Z_{t-1} \rightarrow -\infty} V(Z_{t-1}) = \int_{-\infty}^{\infty} \frac{1}{2}(\varphi_{t-1} + \sigma\varepsilon_t)^2 g(\varepsilon_t)/(1 - \beta) = \frac{1}{2}(\varphi_{t-1}^2 + \sigma^2)/(1 - \beta)$$

Where

$$\varphi_{t-1} = (\rho Z_{t-1} + \hat{\pi}_t^e + \tau \hat{y}_{max})$$

The intuition for this expression is that when the target variable from the prior period is deeply negative, there is a very high likelihood that the central bank could be trapped at the zero lower bound for many periods. In the limit, the central bank would be trapped at the zero lower bound forever, in which case the objective function is quadratic.

The first and second derivatives of the value function with respect to Z_{t-1} are given by:

$$V'(Z_{t-1}) = \int_{-\infty}^{(\mu - \varphi_{t-1})/\sigma} \rho(\varphi_{t-1} + \sigma\varepsilon_t)g(\varepsilon_t) + \beta\rho \int_{-\infty}^{(\mu - \varphi_{t-1})/\sigma} V'(\varphi_{t-1} + \sigma\varepsilon_t)g(\varepsilon_t) \quad (A1a)$$

$$V''(Z_{t-1}) = \int_{-\infty}^{(\mu - \varphi_{t-1})/\sigma} \rho^2 g(\varepsilon_t) + \beta\rho^2 \int_{-\infty}^{(\mu - \varphi_{t-1})/\sigma} V''(\varphi_{t-1} + \sigma\varepsilon_t)g(\varepsilon_t) \quad (A1b)$$

The first term on the right hand side of (A1b) is positive. Solving (A1b) forward implies that $V''(Z_{t-1})$ is a discounted, probability weighted sum of positive terms so the function $V''(Z_{t-1})$ must be positive as well. As a result, the function $V'(Z_{t-1})$ is globally convex. Moreover, as noted above, the value of $V'(Z_{t-1})$ asymptotes to a maximum value of zero as the value of Z_{t-1} increases to infinity. That fact and the positive second derivative then implies that $V'(Z_{t-1}) < 0$.

A.2 Optimal Policy

The first order condition that applies in cases when the central bank is unconstrained in its choice of the output gap is:

$$Z_t + \beta V'(Z_t) = 0 \quad (A2)$$

Equation (A1a) can be solved forward as:

$$V'(Z_t) = \rho \int_{-\infty}^{(\mu-\varphi_t)/\sigma} Z_{t+1}^c g(\varepsilon_{t+1}) + \beta \rho^2 \int_{-\infty}^{(\mu-\varphi_t)/\sigma} \int_{-\infty}^{(\mu-\varphi_{t+1})/\sigma} Z_{t+2}^c g(\varepsilon_{t+1})g(\varepsilon_{t+2}) + \beta^2 \rho^3 \int_{-\infty}^{(\mu-\varphi_t)/\sigma} \int_{-\infty}^{(\mu-\varphi_{t+1})/\sigma} \int_{-\infty}^{(\mu-\varphi_{t+2})/\sigma} Z_{t+3}^c g(\varepsilon_{t+1})g(\varepsilon_{t+2})g(\varepsilon_{t+3})+\dots \quad (A3)$$

In this expression, Z_{t+i}^c is the value of cumulative average inflation in period $t+i$ along a path in which the central bank has been constrained by the zero lower bound in consecutive periods beginning in period $t+1$. The values of Z_{t+i}^c along such paths is given by:

$$Z_{t+i}^c = \rho^i Z_t + \frac{\tau \hat{y}_{max}(1 - \rho^i)}{1 - \rho} + \sigma \sum_{j=0}^{i-1} \rho^j \varepsilon_{t+i-j}$$

Thus $V'(Z_t)$ is thus the marginal value of an increase in Z_t along all future paths in which the ZLB constraint is binding in consecutive periods beginning in period $+1$. The buffer value μ defined by equation (A2) is the value that balances the future marginal benefit of a higher value of the Z_t today in reducing the costs along those paths against the marginal cost today associated with a higher value of Z_t .

The solution for the optimal value of the buffer μ can be obtained by well known iterative methods including value function iteration and policy function iteration. The setup here is simple enough that it's possible to calculate the optimal value of the buffer directly from (A3) and (A2). For any given value of μ , the value of $V'(Z_t)$ above can be readily calculated numerically. Here, we have simulated 40 year periods 5000 times. We identify each iteration in which the economy is at the ZLB in consecutive periods and then average over the values of constrained cumulative average inflation in these iterations as defined in (A3). These values can then be used to develop an estimate of $V'(Z_t)$ conditional on the initial choice of buffer. The value of the optimal buffer μ then just involves a simple search that to find the value of μ such that equation (A3) holds and with:

$$\mu + \beta V'(Z_t = \mu) = 0$$

B. Approximate Time Series Behavior around the Risky Steady State

This section investigates the stochastic nature of the long-run time-series behavior of inflation and the price level and the associated uncertainty. The concept of the “risky steady state” discussed in Hill et. al. (2016) is helpful in in characterizing the approximate time-series behavior of inflation and the price level.

B.1 The Risky Steady State

The analysis above shows that under optimal policy, the cumulative inflation gap Z_t follows a type of switching process in which:

$$Z_t = \mu \quad \text{when } \sigma \varepsilon_t > \mu - \varphi_{t-1}$$

And

$$Z_t = \rho Z_{t-1} + \hat{\pi}_t^e + \tau \hat{y}_{max} + \sigma \varepsilon_t \quad \text{when } \sigma \varepsilon_t \leq \mu - \varphi_{t-1}$$

During prolonged periods when the ZLB is not binding, the cumulative inflation gap is then just a constant μ . And using the definition of the cumulative inflation gap, the optimal current period inflation gap during such periods is given by $\hat{\pi}_t = (1 - \rho)\mu$. During periods when the ZLB is binding, the cumulative inflation gap evolves according to:

$$Z_t = \rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t$$

From this expression, the expected value of Z_t conditional on all information in the prior period is given by:

$$E\{Z_t | Z_{t-1}\} = \theta(Z_{t-1}) = \varphi_{t-1} G\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right) - \sigma g\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right) + \mu(1 - G\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right))$$

Where $G(\cdot)$ and $g(\cdot)$ are the cumulative distribution and density, respectively, for the unit normal distribution and again

$$\varphi_{t-1} = \rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t$$

This equation can be used to define the “risky steady state” as in Hill et. al. (2016) which is the value of $Z_t = Z^*$ such that:

$$Z^* = \theta(Z^*) = \varphi^* G\left(\frac{\mu - \varphi^*}{\sigma}\right) - \sigma g\left(\frac{\mu - \varphi^*}{\sigma}\right) + \mu(1 - G\left(\frac{\mu - \varphi^*}{\sigma}\right)) \quad (\text{B1})$$

Where

$$\varphi^* = \rho Z^* + \tau \hat{y}_{max}$$

The risky steady state is the value around with the cumulative inflation gap tends over time if shocks are set to zero. Similar to the optimal buffer, the risky steady state is a function of the key parameters of the model including the variance of the shock terms, the maximum amount of policy stimulus \hat{y}_{max} that can be provided and the parameter ρ determining the degree of averaging. A unique risky steady state solution exists in this model for all cases with $\tau \hat{y}_{max} > 0$.²⁷

The risky steady states corresponding to the cases considered above are shown in Table B1 below.

²⁷ In the case with $\hat{y}_{max} = 0$ and $\rho = 1$, the target variable Z_t behaves as a “truncated” random walk since there is nothing that pulls the value of the target variable back toward zero in cases with a series of negative shocks.

Table B1: Risky Steady State Values for the Average Inflation Gap

SIG	IT	WAIT 2	WAIT 5	WAIT 10	PLT
0.5	-0.042	-0.016	-0.006	0.000	0.009
1	-0.198	-0.109	-0.044	0.013	0.147
2	-0.573	-0.417	-0.230	-0.013	0.755

In general, the risky steady state values in Table B1 are below the corresponding values of the buffer in Table 3 in the main text. The difference reflects the tendency for shocks to push the economy to the ZLB so that inflation over the longer run tends to run below the buffer level set in “normal times” when the ZLB is not binding.

B.2 Approximate Time-Series Behavior of Economic Variables

The key relation governing the time series behavior of economic variables in the model under optimal policy is given by²⁸:

$$Z_t = \min(\rho Z_{t-1} + \tau \hat{y}_{max} + \sigma \varepsilon_t, \mu) \quad (B2)$$

As noted above, the expected value of Z_t conditional on Z_{t-1} is then:

$$E\{Z_t | Z_{t-1}\} = \varphi_{t-1} G\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right) - \sigma g\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right) + \mu(1 - G\left(\frac{\mu - \varphi_{t-1}}{\sigma}\right)) \quad (B3)$$

Given the value of the risky steady state Z^* as defined above, one can approximate the time series behavior of the accumulated inflation gap by linearizing equation (B3) around Z^* :

$$Z_t - Z^* = \rho\gamma(Z_{t-1} - Z^*) + \omega_t \quad (B4)$$

Where

$$\gamma = G\left(\frac{\mu - \varphi^*}{\sigma}\right)$$

So, to a first order approximation, the behavior of cumulative average inflation evolves over time as an AR(1) process in this model with the autoregressive coefficient is given by $\rho\gamma$. The parameter γ is the probability that policy remains constrained by the ZLB given that the economy was at the risky steady state in the prior period. And ρ of course is the parameter governing the weighted average lookback period. Based on the calculations above, the autoregressive coefficient $\rho\gamma$ increases with an increase in the lookback period. As a result, the mean rate of reversion to the risky steady state is slower in regimes with a longer lookback period.

The shock term here is defined by:

$$\omega_t = \mu - \theta(Z_{t-1}) \quad \text{when } \sigma \varepsilon_t > \mu - \varphi_{t-1}$$

²⁸ This is an example of the SETAR “threshold” model developed and studied extensively by Tong (1983).

$$\omega_t = \varphi_{t-1} + \sigma \varepsilon_t - \theta(Z_{t-1}) \quad \text{when } \sigma \varepsilon_t \leq \mu - \varphi_{t-1}$$

Combining equations (10) and (3a), the time-series process for inflation is:

$$(1 - \rho\gamma L)/(1 - \rho L) \hat{\pi}_t = (1 - \rho\gamma) Z^* + \omega_t$$

$$(1 - \rho\gamma L) \hat{\pi}_t = (1 - \rho\gamma)(1 - \rho)Z^* + (1 - \rho L)\omega_t$$

So inflation in the model follows an ARMA(1,1) process. Finally, we can use the definition of the price level gap to infer that it follows an ARIMA(1,1,1) process given by:

$$(1 - \rho\gamma L)(1 - L) \hat{p}_t = (1 - \rho\gamma)(1 - \rho)Z^* + (1 - \rho L)\omega_t \quad (\text{B5})$$

B.3 Drift and Forecast Variance

Two polar cases of equation (12) help to illustrate the core elements of the long-run behavior of inflation and the price level.

In the special case when $\rho = 1$, the price level gap process is stationary and given by the AR(1) process:

$$\hat{p}_t = \gamma \hat{p}_{t-1} + \omega_t$$

The inflation gap follows an ARMA(1,1) process with zero mean and the k-step ahead forecast variance for this process is:

$$\sigma_k^2 = \sigma_\omega^2 (1 - \alpha^{k+1}) / (1 - \alpha) \quad \text{where } \alpha = \gamma^2$$

In the polar case when $\rho = 0$, the price level gap process follows a random walk with drift given by:

$$\hat{p}_t = \hat{p}_{t-1} + Z^* + \omega_t$$

The drift is determined by the risky steady state value Z^* . Steady state inflation in this case is equal to Z^* .

The k-step ahead forecast variance for the price level gap thus increases in proportion to the forecast horizon:

$$\sigma_k^2 = k \cdot \sigma_\omega^2$$

In the intermediate cases with $0 < \rho < 1$, the price level gap remains nonstationary and with the given by:

$$\hat{p}_t = \hat{p}_{t-1} + (1 - \rho)Z^* + \left(\frac{1 - \rho L}{1 - \rho\gamma L}\right)\omega_t$$

The magnitude of the drift term $(1 - \rho)Z^*$ is downweighted by the factor $(1 - \rho)$. So the longer the effective lookback period, the smaller the drift in the unit root process for the price level gap. Moreover, the k-step ahead forecast variance is also reduced by the factor $(1 - \rho)$. The k-step ahead forecast variance in this case is complicated but declines monotonically from $k \cdot \sigma_\omega^2$ to $\sigma_k^2 = \sigma_\omega^2 (1 - \alpha^{k+1}) / (1 - \alpha)$ as ρ increase from 0 to 1.

Appendix C: Quantile Regression Results

Quantile regressions were executed using proc quantreg in SAS. All regressions were executed with observations weighted by the SCF population weights. The full statistical results are reported below.

Quantile Regression Results

Table 22

The QUANTREG Procedure
Quantile Level = 0.01

Quantile Level and Objective Function	
Quantile Level	0.01
Objective Function	4973536.1541
Predicted Value at Mean	-8.6207

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-29.9509	4.0192	-37.8286 -22.0732	-7.45	<.0001
HOME	1	2.6807	0.3250	2.0436 3.3178	8.25	<.0001
EDU	1	1.6473	0.1932	1.2687 2.0260	8.53	<.0001
FEMALE	1	-2.1485	0.4609	-3.0519 -1.2451	-4.66	<.0001
BLACK	1	1.3804	0.2486	0.8932 1.8676	5.55	<.0001
HISPANIC	1	1.4969	0.3058	0.8976 2.0962	4.90	<.0001
ASIAN_OTHER	1	0.9818	0.4083	0.1815 1.7820	2.40	0.0162
INCOME_0_10	1	12.9747	4.8446	3.4790 22.4703	2.68	0.0074
INCOME_10_20	1	15.7996	3.8911	8.1729 23.4262	4.06	<.0001
INCOME_20_30	1	15.2873	4.0321	7.3842 23.1903	3.79	0.0002
INCOME_30_40	1	14.6352	3.8857	7.0190 22.2514	3.77	0.0002
INCOME_40_50	1	14.2083	3.8718	6.6195 21.7971	3.67	0.0002
INCOME_50_60	1	14.7190	3.8699	7.1339 22.3042	3.80	0.0001
INCOME_60_70	1	13.9246	3.8794	6.3207 21.5285	3.59	0.0003
INCOME_70_80	1	14.1650	3.8574	6.6042 21.7257	3.67	0.0002
INCOME_80_90	1	12.4997	3.8975	4.8604 20.1391	3.21	0.0013
INCOME_90_95	1	10.4267	3.8820	2.6218 18.2316	2.62	0.0088
INCOME_95_99	1	7.8724	4.9132	-1.7577 17.5026	1.60	0.1091
AGE_LT_35	1	11.2147	0.6560	9.9389 12.5005	17.10	<.0001
AGE_35_45	1	11.2045	0.6381	9.9537 12.4552	17.56	<.0001
AGE_45_55	1	9.0442	0.7101	7.6523 10.4360	12.74	<.0001
AGE_55_65	1	5.2470	0.9334	3.4175 7.0766	5.62	<.0001
AGE_65_75	1	-3.5822	1.7937	-7.0979 -0.0665	-2.00	0.0458

Table 23

The QUANTREG Procedure
Quantile Level = 0.05

Quantile Level and Objective Function	
Quantile Level	0.05
Objective Function	10045737.016
Predicted Value at Mean	-3.5177

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-12.9990	0.5713	-14.1188 -11.8793	-22.75	<.0001
HOME	1	1.6082	0.0797	1.4519 1.7645	20.17	<.0001
EDU	1	0.9320	0.0494	0.8351 1.0289	18.86	<.0001
FEMALE	1	-0.5157	0.0633	-0.6398 -0.3917	-8.15	<.0001
BLACK	1	0.7494	0.0683	0.6156 0.8832	10.98	<.0001
HISPANIC	1	0.8907	0.0550	0.7828 0.9985	16.19	<.0001
ASIAN_OTHER	1	0.3464	0.0872	0.1785 0.5174	3.97	<.0001
INCOME_0_10	1	4.7681	0.5140	3.7606 5.7757	9.28	<.0001
INCOME_10_20	1	4.5646	0.5169	3.5514 5.5778	8.83	<.0001
INCOME_20_30	1	4.5892	0.5164	3.5760 5.6004	8.88	<.0001
INCOME_30_40	1	4.0458	0.5421	2.9832 5.1083	7.46	<.0001
INCOME_40_50	1	4.0022	0.5178	2.9874 5.0170	7.73	<.0001
INCOME_50_60	1	3.7138	0.5191	2.6964 4.7311	7.15	<.0001
INCOME_60_70	1	3.6321	0.5248	2.6035 4.6608	6.92	<.0001
INCOME_70_80	1	3.1147	0.5359	2.0644 4.1650	5.81	<.0001
INCOME_80_90	1	2.6712	0.5419	1.6091 3.7334	4.93	<.0001
INCOME_90_95	1	1.7902	0.6325	0.5505 3.0299	2.83	0.0047
INCOME_95_99	1	0.0370	0.7899	-1.5112 1.5851	0.05	0.9627
AGE_LT_35	1	7.2696	0.1480	6.9795 7.5596	49.13	<.0001
AGE_35_45	1	7.3136	0.1488	7.0199 7.6073	48.81	<.0001
AGE_45_55	1	6.4488	0.2020	6.0528 6.8448	31.92	<.0001
AGE_55_65	1	4.2484	0.2166	3.8238 4.6731	19.61	<.0001
AGE_65_75	1	1.4654	0.3560	0.7677 2.1631	4.12	<.0001

Table 24

The SAS System
The QUANTREG Procedure
Quantile Level = 0.1

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Quantile Level and Objective Function	
Quantile Level	0.1
Objective Function	12384647.214
Predicted Value at Mean	-2.0950

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-8.2700	0.4221	-10.0972 -6.4427	-21.96	<.0001
HOME	1	1.5710	0.0568	1.4596 1.6823	27.66	<.0001
EDU	1	0.7590	0.0306	0.6990 0.8190	24.80	<.0001
FEMALE	1	-0.2258	0.0487	-0.3231 -0.1284	-4.55	<.0001
BLACK	1	0.6523	0.0391	0.5757 0.7289	16.69	<.0001
HISPANIC	1	0.6257	0.0344	0.5583 0.6932	18.18	<.0001
ASIAN_OTHER	1	0.4094	0.0592	0.2934 0.5254	6.92	<.0001
INCOME_0_10	1	3.5720	0.3864	2.8146 4.3294	9.24	<.0001
INCOME_10_20	1	3.5076	0.3948	2.7337 4.2815	8.88	<.0001
INCOME_20_30	1	3.3988	0.3911	2.6321 4.1654	8.69	<.0001
INCOME_30_40	1	3.1052	0.4097	2.3062 3.9122	7.59	<.0001
INCOME_40_50	1	3.0503	0.3998	2.2666 3.8340	7.63	<.0001
INCOME_50_60	1	2.8070	0.3899	2.0428 3.5713	7.20	<.0001
INCOME_60_70	1	2.5420	0.4013	1.7555 3.3285	6.33	<.0001
INCOME_70_80	1	2.3619	0.4122	1.5540 3.1697	5.73	<.0001
INCOME_80_90	1	1.7631	0.4199	0.9401 2.5861	4.20	<.0001
INCOME_90_95	1	0.8559	0.4710	-0.2673 1.7791	1.82	0.0692
INCOME_95_99	1	0.0988	0.5481	-0.9755 1.1731	0.18	0.8570
AGE_LT_35	1	5.0689	0.1263	4.8224 5.3175	40.14	<.0001
AGE_35_45	1	5.0578	0.1290	4.8049 5.3107	39.20	<.0001
AGE_45_55	1	4.5175	0.1447	4.2340 4.8010	31.23	<.0001
AGE_55_65	1	3.2779	0.1781	2.9288 3.6270	18.40	<.0001
AGE_65_75	1	1.5417	0.2836	0.9859 2.0975	5.44	<.0001

Quantile Regression Results (continued)

Table 25

The QUANTREG Procedure
Quantile Level = 0.25

Quantile Level and Objective Function	
Quantile Level	0.25
Objective Function	17607381.942
Predicted Value at Mean	-0.5887

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-4.8722	0.1493	-5.1649 -4.5796	-32.63	<.0001
HOME	1	1.5616	0.0308	1.5012 1.6219	50.73	<.0001
EDU	1	0.6593	0.0193	0.6215 0.6971	34.18	<.0001
FEMALE	1	-0.0796	0.0155	-0.1090 -0.0483	-5.08	<.0001
BLACK	1	0.2689	0.0186	0.2294 0.3022	14.32	<.0001
HISPANIC	1	0.2706	0.0204	0.2306 0.3107	13.24	<.0001
ASIAN_OTHER	1	0.1920	0.0208	0.1512 0.2328	9.22	<.0001
INCOME_0_10	1	2.5565	0.1282	2.3053 2.8077	19.95	<.0001
INCOME_10_20	1	2.5193	0.1271	2.2702 2.7684	19.82	<.0001
INCOME_20_30	1	2.4779	0.1282	2.2266 2.7291	19.33	<.0001
INCOME_30_40	1	2.3470	0.1322	2.0879 2.6061	17.75	<.0001
INCOME_40_50	1	2.2040	0.1324	1.9446 2.4634	16.65	<.0001
INCOME_50_60	1	1.9196	0.1361	1.6528 2.1863	14.10	<.0001
INCOME_60_70	1	1.7942	0.1348	1.5301 2.0584	13.31	<.0001
INCOME_70_80	1	1.4714	0.1372	1.2024 1.7404	10.72	<.0001
INCOME_80_90	1	1.1287	0.1390	0.8562 1.4011	8.12	<.0001
INCOME_90_95	1	0.4801	0.1697	0.1476 0.8126	2.83	0.0047
INCOME_95_99	1	0.1462	0.1678	-0.1827 0.4782	0.87	0.3836
AGE_LT_35	1	2.1237	0.0573	2.0115 2.2359	37.09	<.0001
AGE_35_45	1	2.1608	0.0578	2.0475 2.2742	37.38	<.0001
AGE_45_55	1	2.0215	0.0600	1.9038 2.1391	33.68	<.0001
AGE_55_65	1	1.4896	0.0782	1.3164 1.6228	18.80	<.0001
AGE_65_75	1	0.7247	0.0997	0.5292 0.9202	7.27	<.0001

Table 26

The SAS System
The QUANTREG Procedure
Quantile Level = 0.5

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Quantile Level and Objective Function	
Quantile Level	0.5
Objective Function	17827592.078
Predicted Value at Mean	0.3069

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-2.5371	0.0793	-2.6926 -2.3816	-31.98	<.0001
HOME	1	1.6301	0.0174	1.5960 1.6643	93.51	<.0001
EDU	1	0.7631	0.0149	0.7338 0.7923	51.11	<.0001
FEMALE	1	-0.0018	0.0116	-0.0245 0.0208	-0.16	0.8738
BLACK	1	0.0929	0.0152	0.0631 0.1227	6.11	<.0001
HISPANIC	1	0.1710	0.0135	0.1445 0.1975	12.64	<.0001
ASIAN_OTHER	1	0.1021	0.0193	0.0663 0.1380	5.88	<.0001
INCOME_0_10	1	1.9350	0.0741	1.7899 2.0802	26.13	<.0001
INCOME_10_20	1	1.9396	0.0744	1.7937 2.0855	26.06	<.0001
INCOME_20_30	1	1.8876	0.0744	1.7418 2.0334	25.37	<.0001
INCOME_30_40	1	1.7612	0.0730	1.6182 1.9042	24.14	<.0001
INCOME_40_50	1	1.7241	0.0754	1.5762 1.8719	22.86	<.0001
INCOME_50_60	1	1.5895	0.0751	1.4424 1.7366	21.18	<.0001
INCOME_60_70	1	1.4196	0.0797	1.2654 1.5739	18.04	<.0001
INCOME_70_80	1	1.2173	0.0799	1.0606 1.3740	15.23	<.0001
INCOME_80_90	1	0.9118	0.0811	0.7529 1.0708	11.24	<.0001
INCOME_90_95	1	0.5909	0.0967	0.4013 0.7806	6.11	<.0001
INCOME_95_99	1	0.2408	0.0752	0.0933 0.3883	3.20	0.0014
AGE_LT_35	1	0.6416	0.0265	0.5897 0.6936	24.23	<.0001
AGE_35_45	1	0.7090	0.0280	0.6542 0.7638	25.36	<.0001
AGE_45_55	1	0.6089	0.0269	0.5562 0.6616	22.64	<.0001
AGE_55_65	1	0.4419	0.0268	0.3893 0.4944	16.48	<.0001
AGE_65_75	1	0.2071	0.0482	0.1126 0.3017	4.29	<.0001

Table 27

The QUANTREG Procedure
Quantile Level = 0.75

Quantile Level and Objective Function	
Quantile Level	0.75
Objective Function	13653512.936
Predicted Value at Mean	0.9439

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-1.5413	0.1109	-1.7586 -1.3240	-13.90	<.0001
HOME	1	2.1143	0.0203	2.0744 2.1542	103.91	<.0001
EDU	1	0.8613	0.0257	0.8109 0.9117	33.50	<.0001
FEMALE	1	0.0169	0.0127	-0.0081 0.0418	1.33	0.1845
BLACK	1	0.0415	0.0175	0.0072 0.0758	2.37	0.0176
HISPANIC	1	0.0468	0.0152	0.0159 0.0787	3.00	0.0027
ASIAN_OTHER	1	0.1364	0.0282	0.0811 0.1916	4.84	<.0001
INCOME_0_10	1	1.4504	0.1087	1.2374 1.6634	13.35	<.0001
INCOME_10_20	1	1.4799	0.1097	1.2649 1.6949	13.49	<.0001
INCOME_20_30	1	1.4793	0.1115	1.2607 1.6979	13.26	<.0001
INCOME_30_40	1	1.3639	0.1118	1.1448 1.5830	12.20	<.0001
INCOME_40_50	1	1.3100	0.1153	1.0839 1.5361	11.36	<.0001
INCOME_50_60	1	1.2760	0.1110	1.0584 1.4936	11.49	<.0001
INCOME_60_70	1	1.1376	0.1112	0.9198 1.3555	10.23	<.0001
INCOME_70_80	1	0.9293	0.1177	0.6985 1.1600	7.89	<.0001
INCOME_80_90	1	0.6585	0.1173	0.4285 0.8885	5.61	<.0001
INCOME_90_95	1	0.4009	0.1224	0.1610 0.6407	3.28	0.0011
INCOME_95_99	1	0.0808	0.1364	-0.1865 0.3481	0.59	0.5535
AGE_LT_35	1	0.3722	0.0258	0.3217 0.4227	14.44	<.0001
AGE_35_45	1	0.4639	0.0261	0.4129 0.5150	17.81	<.0001
AGE_45_55	1	0.3216	0.0225	0.2774 0.3657	14.27	<.0001
AGE_55_65	1	0.1357	0.0185	0.0996 0.1719	7.35	<.0001
AGE_65_75	1	0.0739	0.0166	0.0414 0.1064	4.46	<.0001

Quantile Regression Results (continued)

Table 28

The QUANTREG Procedure
Quantile Level = 0.9

Quantile Level and Objective Function	
Quantile Level	0.9
Objective Function	8671562.4182
Predicted Value at Mean	1.7520

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-0.9312	0.1417	-1.2089 -0.6536	-6.57	<.0001
HOME	1	2.8256	0.0328	2.7614 2.8899	86.19	<.0001
EDU	1	1.3748	0.0560	1.2649 1.4846	24.53	<.0001
FEMALE	1	0.0067	0.0312	-0.0545 0.0678	0.21	0.8304
BLACK	1	0.0746	0.0358	0.0044 0.1448	2.08	0.0372
HISPANIC	1	0.1560	0.0560	0.0462 0.2658	2.78	0.0054
ASIAN_OTHER	1	0.0891	0.0435	0.0038 0.1745	2.05	0.0406
INCOME_0_10	1	1.5440	0.1706	1.2096 1.8784	9.05	<.0001
INCOME_10_20	1	1.1366	0.1483	0.8459 1.4273	7.66	<.0001
INCOME_20_30	1	1.1900	0.1488	0.8984 1.4817	8.00	<.0001
INCOME_30_40	1	0.9768	0.1416	0.6993 1.2542	6.90	<.0001
INCOME_40_50	1	0.9841	0.1511	0.6879 1.2802	6.51	<.0001
INCOME_50_60	1	0.9277	0.1360	0.6612 1.1943	6.82	<.0001
INCOME_60_70	1	0.7631	0.1355	0.4975 1.0286	5.63	<.0001
INCOME_70_80	1	0.6098	0.1430	0.3295 0.8900	4.26	<.0001
INCOME_80_90	1	0.3117	0.1397	0.0378 0.5857	2.23	0.0257
INCOME_90_95	1	0.0923	0.1435	-0.1890 0.3736	0.64	0.5201
INCOME_95_99	1	-0.0059	0.1554	-0.3106 0.2987	-0.04	0.9697
AGE_LT_35	1	0.4382	0.0476	0.3449 0.5315	9.21	<.0001
AGE_35_45	1	0.4539	0.0440	0.3676 0.5402	10.31	<.0001
AGE_45_55	1	0.4158	0.0490	0.3197 0.5119	8.48	<.0001
AGE_55_65	1	0.1869	0.0452	0.0982 0.2756	4.13	<.0001
AGE_65_75	1	0.2463	0.0486	0.1511 0.3415	5.07	<.0001

Table 29

The SAS System
The QUANTREG Procedure
Quantile Level = 0.95

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Quantile Level and Objective Function	
Quantile Level	0.95
Objective Function	5958717.6526
Predicted Value at Mean	2.5531

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-0.8284	0.0937	-1.0122 -0.6447	-8.84	<.0001
HOME	1	3.1816	0.0468	3.0898 3.2734	67.92	<.0001
EDU	1	1.8381	0.0762	1.6888 1.9875	24.12	<.0001
FEMALE	1	0.0021	0.0431	-0.0823 0.0865	0.05	0.9615
BLACK	1	-0.1564	0.0429	-0.2405 -0.0722	-3.64	0.0003
HISPANIC	1	0.2499	0.1107	0.0330 0.4668	2.26	0.0239
ASIAN_OTHER	1	-0.0445	0.0956	-0.2318 0.1428	-0.47	0.6417
INCOME_0_10	1	3.5531	0.1524	3.2543 3.8519	23.31	<.0001
INCOME_10_20	1	1.5125	0.1083	1.3003 1.7248	13.97	<.0001
INCOME_20_30	1	1.3525	0.0958	1.1646 1.5404	14.11	<.0001
INCOME_30_40	1	1.1516	0.1016	0.9524 1.3507	11.33	<.0001
INCOME_40_50	1	1.1987	0.1066	0.9898 1.4077	11.25	<.0001
INCOME_50_60	1	0.9048	0.0927	0.7232 1.0865	9.76	<.0001
INCOME_60_70	1	0.9052	0.0944	0.7201 1.0902	9.59	<.0001
INCOME_70_80	1	0.7059	0.0885	0.5335 0.8803	7.99	<.0001
INCOME_80_90	1	0.2968	0.0822	0.1357 0.4579	3.61	0.0003
INCOME_90_95	1	0.1810	0.1140	-0.0426 0.4046	1.59	0.1125
INCOME_95_99	1	0.1538	0.0905	-0.0235 0.3311	1.70	0.0891
AGE_LT_35	1	0.5633	0.0770	0.4124 0.7142	7.32	<.0001
AGE_35_45	1	0.5291	0.0634	0.4049 0.6533	8.35	<.0001
AGE_45_55	1	0.5988	0.0632	0.4749 0.7227	9.47	<.0001
AGE_55_65	1	0.4979	0.0599	0.3704 0.6053	8.14	<.0001
AGE_65_75	1	0.5028	0.0731	0.3596 0.6461	6.88	<.0001

Table 30

The QUANTREG Procedure
Quantile Level = 0.99

Quantile Level and Objective Function	
Quantile Level	0.99
Objective Function	2441112.3435
Predicted Value at Mean	4.9320

Parameter Estimates						
Parameter	DF	Estimate	Standard Error	95% Confidence Limits	t Value	Pr > t
Intercept	1	-0.3021	0.5515	-1.3831 0.7790	-0.55	0.5839
HOME	1	4.7483	0.1553	4.4438 5.0527	30.57	<.0001
EDU	1	2.1889	0.2071	1.7830 2.5949	10.57	<.0001
FEMALE	1	-0.2231	0.1220	-0.4622 0.0160	-1.83	0.0675
BLACK	1	-0.5801	0.1083	-0.7924 -0.3678	-5.36	<.0001
HISPANIC	1	1.2036	0.3317	0.5534 1.8537	3.63	0.0003
ASIAN_OTHER	1	0.6818	0.4084	-0.1187 1.4823	1.67	0.0950
INCOME_0_10	1	5.4913	0.6678	4.1824 6.8002	8.22	<.0001
INCOME_10_20	1	3.6747	0.6880	2.3262 5.0232	5.34	<.0001
INCOME_20_30	1	2.2774	0.6216	1.0590 3.4959	3.66	0.0002
INCOME_30_40	1	1.7356	0.5661	0.6261 2.8452	3.07	0.0022
INCOME_40_50	1	1.6038	0.6338	0.3616 2.8461	2.53	0.0114
INCOME_50_60	1	0.8617	0.5505	-0.2173 1.9406	1.57	0.1175
INCOME_60_70	1	0.7402	0.5884	-0.3543 1.8346	1.33	0.1850
INCOME_70_80	1	0.8213	0.5232	-0.2042 1.8469	1.57	0.1165
INCOME_80_90	1	0.4711	0.5703	-0.6467 1.5888	0.83	0.4088
INCOME_90_95	1	0.3485	0.5600	-0.7491 1.4461	0.62	0.5337
INCOME_95_99	1	-0.1417	0.5327	-1.1859 0.9025	-0.27	0.7802
AGE_LT_35	1	2.1082	0.2607	1.5971 2.6192	8.09	<.0001
AGE_35_45	1	0.7124	0.1547	0.4092 1.0157	4.61	<.0001
AGE_45_55	1	0.8040	0.1540	0.5022 1.1058	5.22	<.0001
AGE_55_65	1	0.6112	0.1672	0.2834 0.9389	3.66	0.0003
AGE_65_75	1	0.9485	0.2416	0.4750 1.4221	3.93	<.0001

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