

Deriving Local Trend Factors for Fair Market Rent Estimation

Multidisciplinary Research Team



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Disclaimer

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EXECUTIVE SUMMARY

The U.S. Department of Housing and Urban Development (HUD) estimates Fair Market Rents (FMRs) for most geographic areas in the United States to set reimbursement values for public programs that address the housing needs of low-income families and households. Because reliable data on rents lag behind the dates that FMRs are needed by, HUD must forecast the FMRs based on trends in the available data. HUD's current methodology includes using a "trend factor" to forecast values several quarters ahead of the most current data from the Bureau of Labor Statistics (BLS). The trend factor is the same for all locations, meaning that areas with relatively high (low) rent inflation will be assigned FMRs that are too low (high). This study examines two ways that HUD could add geographic resolution to the trend factors and, perhaps, reduce concern about the accuracy of FMRs.

The first method relies on BLS data on 22 geographic areas and Autoregressive Integrated Moving Average (ARIMA) modeling to estimate different trend factors that can be compared to HUD's current estimates. Using standard time-series modeling metrics, most of the area-specific estimates appear to be more accurate than the current trend factor. However, the overall impact of using this method is relatively small. In other words, FMRs calculated from HUD's current method are reasonably close to the FMRs that would be calculated from the 22 areas.

The second method uses commercial data from Axiometrics—which are shown to be more appropriate than data from Zillow—to estimate ARIMA models for up to 254 Core-Based Statistical Areas (CBSAs). The findings from this approach suggest that FMRs in several areas may be understated by using HUD's single national trend factor.

The study also highlights the limitations of available data sources, which will need to be considered if HUD migrates toward localization of trend factors. In particular, standardized public data are less available for smaller metropolitan areas, forcing more reliance on commercial data sources.

1 INTRODUCTION

1.1 Understanding of the Problem

The U.S. Department of Housing and Urban Development (HUD) estimates Fair Market Rents (FMRs) for more than 600 metropolitan areas and almost 2,000 non-metropolitan county areas. FMRs are used to determine reimbursement rates in several federal housing programs and must be published at least 30 days before the start of each fiscal year (FY), making FMR *estimates* of the gross rent (the sum of shelter rent and necessary utilities) for the upcoming FY.

Within HUD’s current methodology, FMRs reflect three components: (1) the base rent, as estimated with data from the American Community Survey (ACS), (2) an inflation factor that is applied to the base rent, and (3) a trend factor. The inflation factor uses *actual values* on inflation to bring the base rent to current values, while the trend factor uses *forecasted values* to bring the current values of the base rent to the future values, which will be published as the FMRs. HUD currently uses components of the Consumer Price Index (CPI) to determine inflation factors in the 22 geographic areas where the components are produced and to determine regional values for the remaining areas of the country. Therefore, the inflation factors experience some “localization.” In contrast, HUD currently uses a *national* estimate of the trend factor for all geographic areas.

The national trend factor is used to adjust local area rents applied over seven quarters. In areas where rent growth varies significantly from the national trend, FMRs may not accurately explain current local market conditions and may not capture market rent difference across regions. To address this limitation, HUD is considering the consequences of implementing localized trend factors. This consideration explains why “the goal of this research is to explore and provide a set of alternative FMR trend factor methodologies, analyze the strengths and weaknesses of each, and recommend a methodology that is most in line with current local market conditions and can improve accuracy of trend forecasting for FMR calculations” (MDRT Task Order: Local Trend Factor Research Project: 5). Both the current inflation factor and the current trend factor methodologies face similar constraints, mainly that reliable, relevant public data—such as those produced by the Bureau of Labor Statistics (BLS)—are not available for all geographic areas of interest to HUD. This means that research on localizing the trend factor may be useful for further localizing the inflation factor. For example, assume that the study team finds that trends from a readily available series from Zillow closely match trends in BLS’s Rent of Primary Residence series in the geographic areas where both series exist. In that case, HUD may wish to consider using Zillow trends to further localize the inflation factor in the geographic areas covered by Zillow but not BLS. Therefore, while the scope of this Task Order (TO) is primarily confined to the trend factor, the research may spill over to issues related to the inflation factor.

1.2 HUD’s Recent Efforts to Improve Trend Factor Calculations

To address the limitations with the national trend factor, HUD has explored options to improve calculations of the trend factor. HUD has explored alternative methodologies, such as (1) using national

input variables to forecast local rent and utilities of primary residence from CPI and (2) adding measures of local supply and demand into the rent component of the national model. To account for delayed changes in residential rent due to changes in housing supply and demand, variables like local and national construction permits, oil and gas prices, and a utility index are also included in the model by differencing them across 10 years and adding their lags by two quarters. The two components of the gross rent index (GRI) are forecasted at the smallest geographic area available from CPI data. Local rent and utility costs are used as dependent variables in the model’s run to capture differences in market rents and current market conditions across FMRs.

The phrase “gross rent” is used to describe the theoretically correct value for the FMR. As noted earlier, HUD uses a base value for gross rent from ACS, which is usually at least 2 years old, and then applies inflation and trend factors. In HUD’s work-to-date, the trend factors were derived from the GRI, which was the weighted average of the Rent of Primary Residence Index and the Fuels and Utilities Index (both components of CPI), for which the weights are based on the rent and utility shares of gross rent. Rent and utility shares are obtained from ACS and applied to the forecasted rent and utility estimates. With actual and forecast values of the GRI, HUD then computed some hypothetical local trend factors as the ratio of the forecast (next year) values to the actual (current year) values.¹

HUD’s initial analyses dealing with localizing the trend factor provide the point of departure for this TO. To thoroughly understand and assess the prior work conducted by HUD, the study team received HUD’s background information and the data used to conduct the estimations. Exhibit 1-1 presents a description of the received files.

Exhibit 1-1 | Documents Received from HUD

#	File	Description
1	Approach FY19	HUD’s recent work on local trend factor
2	PUC Forecast	Documentation on voucher per-unit cost (PUC) forecast, from which the national model is derived
3	Report Example	Example of desired template of the final report
4	Subscriptions Folder	Information on data to which HUD’s Office of Policy Development and Research has a subscription
5	Proposals to Update the FMR Formula	Summary of the main issues in estimating FMRs https://www.huduser.gov/portal/sites/default/files/pdf/Proposals-To-Update-the-Fair-Market-Rent-Formula.pdf
6	Gross_Rent_All	A table of all national, regional, and local parameters used in models, by primary sampling unit (PSU) and quarter
7	Gross Rent Component Shares	A table of FY19 gross rent component weights
8	Cpi_psu2	Crosswalk of CPI PSU geographies to 2010 Federal Information Processing Standards (FIPS) codes
9	Local Trend Factor FY19	SAS code documenting processes used in calculating local trend factor

Note: CPI denotes “Consumer Price Index,” FMR denotes “Fair Market Rent,” and FY denotes “fiscal year.”

¹ From the HUD background paper, “Forecasting Regional and Metro FMR Trend Factor,” September 2018.

1.3 Study Objectives

The purpose of this report is to present the results and rationale for a set of models that can be used to compute locally based FMRs. More specifically, this report presents alternative approaches to augmenting local market conditions into calculations of FMRs, starting with extensions of HUD's existing methodology and followed by consideration of models that use alternative sources of data and empirical frameworks. Consequently, this report presents the results of a two-phase research approach with multiple research questions in each phase. The objective of the first phase was to derive a local trend factor using a model similar to the model HUD used in forecasting gross rents nationally. The study team identified six research questions that address the objective of the first phase—

1. What is the best model form for forecasting gross rents locally?
2. What lags are deemed appropriate for illustrating delayed responses in gross rent changes to changes in variable inputs?
3. How do the errors of local forecast estimates compare to the errors of the national forecast estimates?
4. What is the maximum range of error considered acceptable for a local estimate before reverting to the use of a national trend or regional factor?
5. What is the best approach for measuring the accuracy of local trend factors?
6. What is the best approach for calculating estimates for non-metropolitan areas?

In the second phase, the objective was to identify alternative approaches (other than forecasting gross rents) that derive a local trend factor that result in a more accurate FMR calculation. There are six research questions associated with the Phase II objective—

1. What variables best describe supply and demand factors that can be used as predictors of changing gross rents for metropolitan areas?
2. What is the possibility of utilizing forecasted data series from state or metropolitan entities?
3. How are local trend factors calculated using the variables to be identified in question 1 of Phase II?
4. What is the best approach for measuring the accuracy of estimates?
5. What is the accuracy of these local trend factors compared with the accuracy of trend factors derived from local and national gross rent forecasting?
6. Are there metropolitan areas where using a local trend factor consistently works better than using a national trend factor?

The findings in this report address the development of a locally based trend factor that may enhance HUD's recent modeling on forecasting gross rents with BLS CPI components on rent and utilities in Phase I. The findings also address the determination of an ideal dataset that is not confined by the geographies produced by BLS, which may provide opportunities to expand the localization of trend factors and explore alternative geographic definitions in Phase II.

1.4 Contents of This Report

The outline of this report is as follows: following this introduction, section two provides a summary of the data used in the study. Section three provides an overview of the methodologies and model specification used in the study. Section four presents a set of selected models for computing a local trend factor. Section four also describes the rationale for the models' selection and compares their forecasting accuracy against that of HUD's hypothetical model and presents results for the selected models in computing a local trend factor for 22 geographic areas.² Using commercial data, an expanded set of geographies is analyzed in section five, as well as a comparison of data from Axiometrics and Zillow. Section six includes comparisons of the accuracy of various models with comparisons that show the implications for FY19 FMRs of the various models. Finally, section seven offers our conclusions.

² The areas are defined by the availability of quarterly rent indices from BLS.

2 DATA SOURCES AND ISSUES

Data on supply and demand factors of gross rents are available from a wide variety of sources, including government agencies and private research entities, as identified by HUD. The study team used the data obtained from HUD, including the Rent of Primary Residence Index and the Fuels and Utilities Index, residential fixed investment, local permit data, and local and national employment data, among others. We also evaluated additional local supply and demand variables to augment the analysis in Phase I, as well as to address the research questions in Phase II. This section describes the data the study team used to test the different model specifications that we evaluated in deriving local trend factors. Specifically, in this section, we summarize the data on the response variables that we used as dependent variables, and we summarize data on exogenous variables that we used as independent variables in the model specifications. This section also discusses the considerations that were relevant in choosing additional variables (other than the variables that were used in the HUD model) to augment the specified models.

2.1 Dependent Variables

The dependent variables in the study were the Rent of Primary Residence Consumer Price Index (CPI) and the Fuels and Utilities CPI. These two components of CPI for the time interval of January 1995 through January 2018 were obtained from HUD for (1) the United States as a whole, (2) four regions (regional data were used for areas where metropolitan data were not available in the CPI, examined at size class levels—All Classes and Class A³), and (3) 13 metropolitan areas of the United States. Given that these indices are available in the CPI only on a monthly basis for some areas and bimonthly for other areas, HUD converted them to quarterly averages for regions and metropolitan areas. For regions and the three metropolitan areas (New York, Chicago, and Los Angeles) where monthly CPI data are available, HUD averaged the monthly data for each quarter. For the remaining 10 metropolitan areas (Philadelphia; Detroit; Washington, DC; Miami; Atlanta; Boston; Baltimore; Dallas; San Francisco; and Seattle) where CPI data are available bimonthly, HUD used the PROC EXPAND function in Statistical Analysis Software (SAS) to interpolate quarterly estimates for these series. The Rent of Primary Residence CPI is based on repeat surveys of a sample of rental housing units over 6-month periods, with adjustments for aging of the units and vacancies. For this reason, the Rent of Primary Residence CPI is considered a strongly reliable measure of changes in rents, particularly in those areas for which the Bureau of Labor Statistics (BLS) produces local estimates.

2.2 Exogenous Variables

As part of our efforts to improve the forecast accuracy of the estimated models, the study team evaluated and incorporated additional variables that were found to have relationships with the dependent variables. Exogenous variables were selected based on a priori analysis to discern the

³ Based on the BLS definition, “All Classes” represent all metropolitan and non-metropolitan areas in each region; “Class A” represents all metropolitan areas with populations of more than 2.5 million.

response and a stepwise regression approach. First, variables identified to enhance the rent and utility series forecast models were evaluated based on their impact on the response series, mathematically termed as correlation. Using the Pearson's correlation coefficient, the study team listed the identified variables that had a significant impact on the response series. Second, we evaluated the correlation between the chosen independent variables. If one independent variable is excessively correlated with another (multicollinearity), determining their separate influences would be difficult and could result in regression coefficients being sensitive to model specification when both variables are included. For a forecast using exogenous variables, future values of the input variables are estimated using its own past values (such as an Autoregressive Integrated Moving Approach process). To reduce the sensitivity of models, individual variables are added to each model, and the accuracy statistics are calculated based on the coefficient estimation set or training and calibration sets. If the inclusion of a variable enhances model accuracy, the variable is retained. In the ensuing sections, we have described the exogenous variables used for the development of the multivariate models. A summary of the description of these variables is provided in exhibit 2-1.

2.2.1 RENT MODEL VARIABLES

To improve the forecast model for rent, the study team explored additional variables. These variables included seasonally adjusted and unadjusted values of occupancy rate and employment data for metros and U.S. Census Bureau (Census) regions. To capture changes in rent with fluctuations in the labor market, we explored employment data like total employment, unemployment rate, manufacturing employment, labor force participation, and weekly wages. Since these variables are highly correlated to each other and explain similar mechanisms in the labor market, the study team individually included weekly wages and the unemployment rate to forecast rent. The employment data span 21 years (or 84 quarters), while occupancy rates span more than 40 quarters. The model specifications included occupancy rate, unemployment rate, and weekly wages as individual inputs in the forecast model. Using a correlation matrix, the study team based these variables on their impact on the response series. The resulting model specifications were built on the principle explained below and did not contribute to explaining the underlying patterns in the response series; hence, they were not included in further analysis.

2.2.2 UTILITY MODEL VARIABLES

Residential prices for electricity at the state level were used to forecast utilities. The study team seasonally adjusted the series and assessed its impact on the response series. Since the data for electricity are only available at the state level, our forecast models are built for the 13 metro areas that could be mapped to states. CPI indices for energy services and commodities in metros and Census regions were explored and their correlation was examined. Since these indices were significantly⁴ correlated with the dependent variable and were derived from the dependent variable, these indices were dropped from further analysis.

⁴ They are considered significant at 95-percent confidence level.

Exhibit 2-1 | Exogenous Variables Considered in the Development of Models

Data	Source	Geographic Unit	Data Frequency	Data Span
Phase I				
Total Employment	BLS	State; Core-Based Statistical Area (CBSA)	Monthly	1997 to 2018
Manufacturing Employment	BLS	State; CBSA	Monthly	1997 to 2008*
Unemployment Rate	BLS	State; CBSA	Monthly	1997 to 2018**
Occupancy Rate	Axiometrics (from HUD)	County; Tract	Monthly	2008 to 2018
Vacancy Index	U.S. Postal Service (from HUD)	State; County; Tract	Quarterly	2008 to 2018
Weekly Wages	BLS	State; Metropolitan Statistical Area (MSA)	Quarterly	1995 to 2016
Energy CPI	BLS	State; MSA	Quarterly	1997 to 2018
Electricity CPI	BLS	State; MSA	Quarterly	1997 to 2018
Utility (Piped) Gas Service CPI	BLS	State; MSA	Quarterly	1997 to 2018
Gasoline (All Types) CPI	BLS	State; MSA	Quarterly	1997 to 2018
Residential Electricity Prices	Energy Information Administration	State	Quarterly	2001 to 2018
Phase II				
Asking Rent and Concessions	Axiometrics (from HUD)	County; Tract	Monthly	2008 to 2018
Median Rental Price (All Home Types)	Zillow	MSA	Monthly	2010 to 2018
Median Home Values	Zillow	MSA	Monthly	1996 to 2018
Housing Price Index	Federal Housing Finance Agency	State***; MSA; CBSA	Quarterly	1975 to 2018

* Non-adjusted “all employees” data available back to 1939, at the state level.

** Metro data begins in 1990.

*** For non-metropolitan areas, data are available by state and between 1995 and 2008.

Note: BLS denotes “Bureau of Labor Statistics” and CPI denotes “Consumer Price Index.”

2.3 Alternate Rent Series Using Commercial Data

The objective of Phase II (research question 2) is to determine alternative approaches for adding more localization to Fair Market Rent (FMR) estimates. To this end, the study team focused primarily on

expanding the number of metro areas (core-based statistical areas, CBSAs) by using extant data from alternative sources to create quarterly rent series for several geographic areas.

3 METHODOLOGY

As stated previously, the primary objective of this study is to evaluate methodologies that improve trend forecasting for Fair Market Rent (FMR) calculations. Given that the trend factor component of the FMR uses *forecast values* to bring the current values of the base rent to the future values, it is important to use a procedure that produces reliable local rent forecasts to derive the needed improvements in the trend factor. In this study, we used the Autoregressive Integrated Moving Average (ARIMA) model to estimate the future values of local rent and utilities, based on which trend factors for each area were calculated. ARIMA models apply to stationary time series and assume that a time series is a linear combination of its own past values and current and past values of an error term (Box and Jenkins, 1976). Consequently, ARIMA models are generally specified in terms of three different parameters (p , d , and q) where p is the order of the autoregressive term (AR term), d denotes the degree of integration to achieve stationarity, and q is the order of the moving average term (MA term).

The remainder of this section outlines and discusses the types of ARIMA models that were specified and the rationale for choosing the best model used for forecasting. As a prelude to this discussion, section 3.1 presents the structure of the ARIMA model that HUD used in recent efforts to improve trend factor calculations. We use the HUD model as a baseline with which we compared other models under consideration. By making this comparison, we intend to identify the areas where HUD's model is working or not.

3.1 The HUD Model

HUD used an ARIMA (0,1,1) model for the rent series in all areas and an ARIMA (1,1,0) model for all the utility series. Therefore, the current rent depends on the previous error term (shocks), not the previous period rent. To be specific, an MA (1) model is estimated for the rent growth rates or the first-differencing of rent series. Similarly, an AR (1) model is estimated for the utility growth rate. HUD estimated the rent model using two approaches. The first approach was to estimate the model based on national exogenous variables.⁵ In the second approach, HUD estimated the rent model based on local exogenous variables.⁶ However, the utility model was estimated using only national exogenous variables.⁷ To reflect the time delay of the response to the identified exogenous variables, each of these variables was lagged two quarters based on theoretical considerations. In this study, we focused on extending HUD's recent work to find alternative time series models to the current national model.

⁵ The national exogenous variables used in HUD's rent model are the Bureau of Economic Analysis (BEA) National Residential Fixed Investment and the Bureau of Labor Statistics' (BLS) National Civilian Employment data.

⁶ The local exogenous variables for the rent model comprised the U.S. Census Bureau's Local Permit data and BLS Local Employment data.

⁷ The exogenous variables used in HUD's utility model are the national quarterly average spot price in dollars per barrel of West Texas Intermediate crude oil, the quarterly national average price in dollars per short ton of bituminous coal, the quarterly average Henry Hub price of natural gas in dollars per million BTUs, and the Consumer Price Index for All Urban Consumers.

3.2 Model Building Strategy

Building on HUD's recent work, the study team estimated a set of models for forecasting gross rents for all 22 geographic areas. We extended HUD's specification in two different directions: first, we estimated different ARIMA models for the gross rent of each area in the belief that local rent series have a different dynamic (trend) than the national counterpart. Second, we also estimated rent growth in the framework of a multivariate time series model by augmenting exogenous variables. The selection of a proper model is extremely important because it reflects the underlying structure of the series. This fitted model, in turn, is used for future forecasting. We followed the three-stage iterative model-building procedure developed by Box and Jenkins (1976): identification, estimation, and diagnostic checking. The identification stage involves transforming the data (if necessary) to improve the normality and the stationarity of the time series and to determine the general form of the model to be estimated. During the estimation stage, the model parameters are estimated using the method of moments, least square methods, or maximum likelihood methods. Finally, diagnostic checks of the model are performed to reveal possible model inadequacies and to assist in selecting the best model. These stages are discussed in more detail below.

3.2.1 MODEL IDENTIFICATION

The study team began the modeling process by identifying the appropriate model for the rent and utility series, as well as the series for the exogenous variables for each geographic area. Identification of the general form of an ARIMA model involves two steps. First, the data series is analyzed for stationarity and normality. If necessary, to achieve stationarity and normality, the study team performs appropriate differencing of the series. This step is also necessary to determine the integration d order of the time series. All variables used in this study were tested for stationarity (against unit roots) so that the mean, variance, and autocovariances were independent of time. We employed two formal testing tools, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and the Dickey-Fuller Generalized Least Squares (DF-GLS) test, to examine stationarity of time series under study. The KPSS test is adopted mainly as a confirmatory tool to the more popular tests under the opposite null hypothesis. The KPSS test is constructed under the null of stationarity, in contrast to the null hypothesis of unit root for the DF-GLS test. Both tests suggest that the original series of all the variables under study are non-stationary, regardless of the geographic areas. This conclusion led us to transform the original series by taking a first-difference—that is, the growth rates of each series.

We then apply the autocorrelation function (ACF) and partial autocorrelation function (PACF) in the spirit of Box and Jenkins (1976) to the growth rate of each series to identify their underlying dynamics. The ACF and PACF are helpful in confirming the stationarity of the series and identifying whether the underlying model follows an AR, MA, or ARIMA model. Once the series is differenced and the underlying model is identified, we use an information criterion method to determine the appropriate lag length of the input series used in the forecast model. The results of lag selection for the exogenous variables are presented in appendix A. We compute the information criteria using various ARIMA model orders, with an automated function used to determine the range of the autoregressive model orders. To be specific, we use the Schwarz Bayesian information criterion (SBC) for the lag length selection in each model. The basic idea of the SBC rule is to find an optimal model specification based on the trade-off between performance in fitting available data and minimum model complexity. Hence, lower SBC implies either

fewer explanatory variables, better fit, or both. Given any two estimated models, the model with the lowest value of SBC is preferred. Exhibit 3-1 reports the results from this exercise for the rent and utility series at the national level. As shown in the exhibit, the SBC rule suggests that the AR (1) model is best fitting for the national rent growth series and the MA (1) model best fits national utility growth series. Given that the growth series are the first-difference of the original series, this outcome implies that the national rent is best approximated by an ARIMA (1,1,0) model and the national utility series is best captured by an ARIMA (0,1,1) model. Note that the order “d” in ARIMA (p,d,q) denotes the order of integration; hence ARIMA (p,1,q) model for the original series is equivalent to the ARMA (p,q) model for its first-difference. These parameters were then used at the next stage of estimation.

Exhibit 3-1 | Model Selection for the United States Using Minimum Schwarz Bayesian Information Criterion

Rent Model (NIM)			Utility Model (PTS)		
AR Order	MA Order	SBC	AR Order	MA Order	SBC
0	1	- 881.75	0	1	- 486.52
0	2	- 890.62	0	2	- 484.10
1	0	- 924.27	1	0	- 486.19
1	1	- 920.04	1	1	- 483.55
1	2	- 915.70	1	2	- 479.66
2	0	- 920.03	2	0	- 484.70
2	1	- 917.28	2	1	- 482.36

Notes: AR denotes “autoregressive,” MA denotes “moving average,” SBC denotes “Schwarz Bayesian information criterion,” NIM denotes “National Input Model,” a multivariate model with national exogenous variables, and PTS denotes “Pure Time Series” model, a univariate time series model without exogenous variables. Details of the various model specifications are discussed in section 4.

3.2.2 PARAMETER ESTIMATION

After candidate models have been determined from the identification stage, the study team used the parameters (p and q) of these models in the estimate statement of PROC ARIMA in SAS to obtain estimates using maximum likelihood estimation.

3.2.3 MODEL DIAGNOSTIC CHECK

Having identified and estimated the parameters of the model in the previous stages, our next step was to verify the adequacy of the fitted models. The study team performed validation tests on the residual series to determine if any patterns remained unaccounted for. A model is said to be adequate if the residuals are white noise—that is, if no significant correlation is observed among the residuals of the fitted model. We tested for the adequacy of our estimated models by graphically inspecting the ACF and the PACF of the models’ residuals. The residuals of the ACF more importantly provide information about the independence of the residuals; ACF residuals that are significantly different from zero indicate that the model does not adequately represent the data. The residual correlation for the first-differenced rent and utility series for the nation are shown in exhibits 3-2 and 3-3. The plots for the remaining 22 geographic areas are presented in appendix B. The plots indicate no significant correlation between residuals. We conclude that the rent and utility models selected in the parameter estimation stage for

the entire nation appear to be adequate and representative of the data. Consequently, we propose using the national model, especially in cases—such as Boston, Chicago, and Seattle—where there are still significant lags.

Exhibit 3-2 | Residual Correlation Diagnostic for First-Differenced National Rent Series

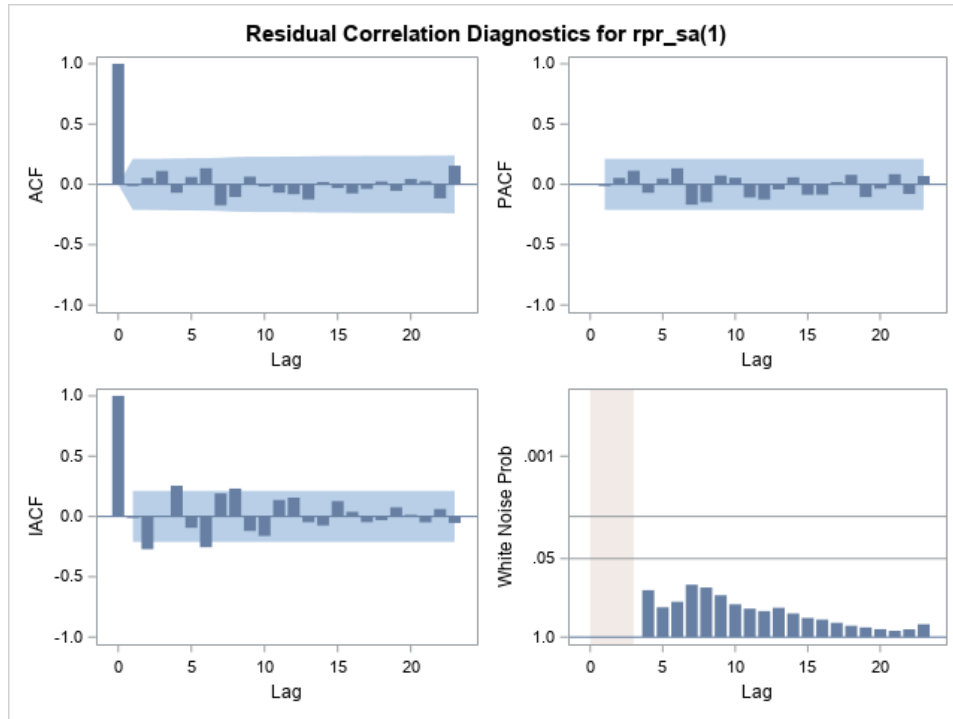
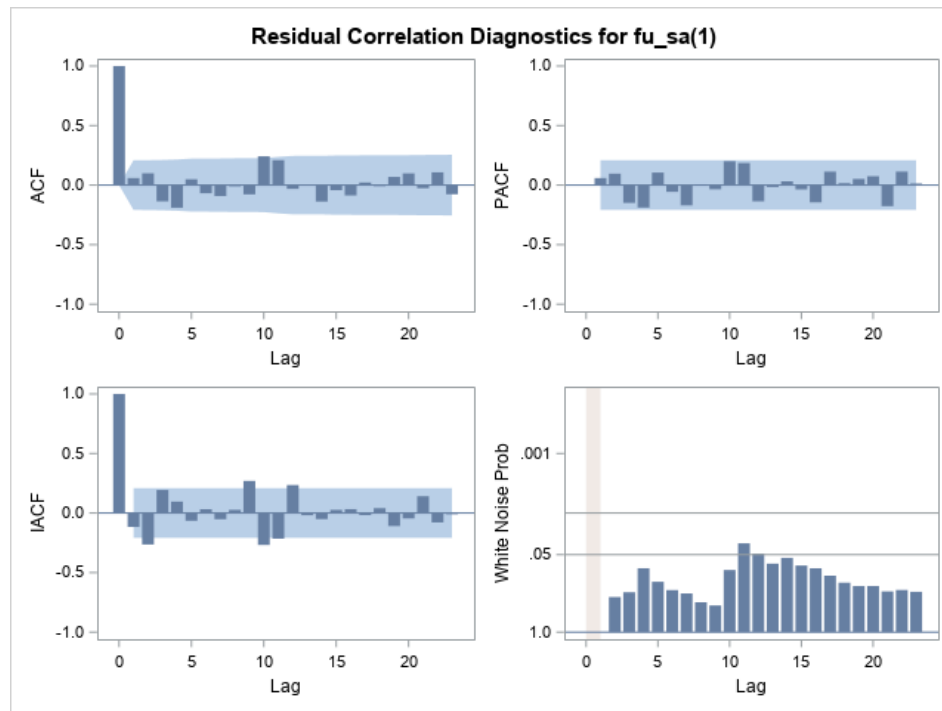


Exhibit 3-3 | Residual Correlation Diagnostic for First-Differenced National Utility Series



3.3 Testing and Measuring Model Performance

From the foregoing procedures, the study team developed three competing models for both rent and utility series for all 22 geographic areas including the nation as a whole. The first model is a univariate time series model using its own past values and error terms. The second is a multivariate time series model that includes other national variables in addition to its own lagged terms. The third is a multivariate model that includes just local explanatory variables in addition to its own lagged terms. Prior to forecasting, the estimation results of univariate models were compared to those of the multivariate approaches, as well as to HUD's model to evaluate each model's forecast accuracy. The response series of each model was divided into two sets: the sample period and the validation period. The study team estimated each specification over seven overlapping sample periods, where each period included 15 years' worth of data. The seven sample periods are: 1996 to 2011, 1997 to 2012, 1998 to 2013, 1999 to 2014, 2000 to 2015, 2001 to 2016, and 2002 to 2017. Estimation over multiple time periods provides a better basis for comparing the model's predictive power, as the results from a particular time period are determined by the specific economic conditions in effect during that period, which may not represent the long-term conditions. To compare the different models, we developed statistics reflecting each model's post-sample forecast accuracy for the validation period subsequent to the sample periods. The length of the validation period varied depending on the sample period that allowed us to develop forecasts that could be compared to the actual data points. For example, for the estimation period 1996 to 2011, we used a longer period 2012:1 to 2018:1 (21 quarters) as the validation, while for the estimation period 2002 to 2017, the validation period comprised just one quarter. The forecasts of the validation period were compared with the actual data for each of the three approaches and were also compared with HUD's forecasts. We conducted this comparison to reveal

how closely the rent and utility predictions of the validation period tracked the corresponding historical data. We used the Root Mean Square Error (RMSE) statistic to compare the forecast performance of each model. RMSE, a widely used criterion for evaluating forecasting performance (Armstrong and Collopy, 1992), provides a measure of the deviation of the true from the forecasted values. Another measure that can be used to evaluate model performance is the Mean Absolute Percentage Error (MAPE). MAPE provides the difference between true and forecast values divided by the true value. We compute MAPE and RMSE to validate the forecast models. However, RMSE is more suited for our analysis, since the series are comparable (Chai and Draxler, 2014). Hence, further analysis is restricted to using RMSE as the measure of forecast model performance. We computed the errors for each time path, from which we calculated average RMSE across the validation period for each model and for each geographical area. Lower RMSE values denote a better forecasting performance. Based on this information, ex-ante forecasts from April 2018 to October 2020 for rents and utilities for each geographic area were made for the validation period.

4 PHASE I RESULTS

This section presents results of analyses related to research question 1. The analyses were conducted with HUD’s master data file and can be thought of as testing model specifications against HUD’s initial work. First, we estimated univariate time series models in which the dynamics of a series are solely driven by its own lagged terms. Second, we estimated multivariate time series models by augmenting other housing market fundamental variables. The utility component of rent is forecasted using as exogenous variables the national quarterly average spot price in dollars per barrel of West Texas Intermediate crude oil, the quarterly national average price in dollars per short ton of bituminous coal, the quarterly average Henry Hub price of natural gas in dollars per million BTUs, and the Consumer Price Index (CPI) for All Urban Consumers (CPI-U).

4.1 Selecting Competing Rent and Utility Models

As stated in the previous section, the first step in the specification of each Autoregressive Integrated Moving Average (ARIMA) model is to assess the stationarity of each series. Unit root tests were estimated for each rent and utility series, with the appropriate degree of differencing then adopted in the estimation and the assessment of the best-fitting model. We run two statistical unit root tests, the Dickey-Fuller Generalized Least Squares (DF-GLS) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, to increase the accuracy of the results about the stationarity of the series. All tests were performed with an added trend. Notably, all rent and utility series under consideration were non-stationary in their levels. Exhibits 4-1 and 4-2 show the summary of the DF-GLS and KPSS test results for first-differenced rent series and utility series, respectively.

Exhibit 4-1 | Test for Unit Root and Stationarity for Rent Series

Geographic Code	Area	DF-GLS Test		KPSS Test	
		Test Statistic	p -value	Test Statistic	p -value
0000	National	- 2.7992	0.0000	0.2155	0.2396
0100	North	- 1.7622	0.0000	0.8162	0.0066
0200	Midwest	- 1.9350	0.0000	0.6275	0.0186
0300	South	- 2.8169	0.0000	0.1227	0.4849
0400	West	- 1.8456	0.0000	0.2108	0.2478
S100	North Class A	- 2.0872	0.0000	0.8935	0.0040
S200	Midwest Class A	- 1.7502	0.0000	0.7187	0.0113
S300	South Class A	- 2.5890	0.0000	0.1793	0.3122
S400	West Class A	- 1.7448	0.0000	0.2128	0.2443
S11A	Boston	- 1.7935	0.0000	0.7836	0.0081
S12A	New York	- 2.7245	0.0000	0.6746	0.0142
S12B	Philadelphia	- 1.6747	0.0000	0.5399	0.0313
S23A	Chicago	- 3.2610	0.0000	0.7802	0.0082
S23B	Detroit	- 2.6555	0.0000	0.2327	0.2124

Geographic Code	Area	DF-GLS Test		KPSS Test	
		Test Statistic	p-value	Test Statistic	p-value
S35A	Washington, DC	- 2.0489	0.0000	1.1058	0.0014
S35B	Miami	- 2.1736	0.0000	0.2375	0.2055
S35C	Atlanta	- 2.4856	0.0000	0.3269	0.1135
S35E	Baltimore	- 3.5300	0.0000	0.3936	0.0747
S37A	Dallas	- 2.4950	0.0000	0.4457	0.0547
S49A	Los Angeles	- 1.2421	0.0000	0.3685	0.0870
S49B	San Francisco	- 2.1647	0.0000	0.3396	0.1046
S49D	Seattle	- 2.1044	0.0000	0.3621	0.0906

Notes: DF-GLS denotes “Dickey-Fuller Generalized Least Squares,” and KPSS denotes “Kwiatkowski-Phillips-Schmidt-Shin.” The null hypothesis for the DF-GLS test is that there is a unit root present in the series (series is non-stationary). The null hypothesis for the KPSS test is that there is no unit root present in the series (series is stationary). A coefficient is statistically significant if its p-value is less than or equal to 5 percent.

Exhibit 4-2 | Test for Unit Root and Stationarity for Utility Series

Geographic Code	Area	DF-GLS Test		KPSS Test	
		Test Statistic	p-value	Test Statistic	p-value
0000	National	- 4.6698	0.0000	0.1202	0.4950
0100	North	- 4.1135	0.0000	0.2152	0.2402
0200	Midwest	- 5.3648	0.0000	0.0672	0.7689
0300	South	- 4.9885	0.0000	0.0749	0.7223
0400	West	- 3.5299	0.0000	0.0921	0.6257
S100	North Class A	- 4.0968	0.0000	0.2041	0.2601
S200	Midwest Class A	- 3.9723	0.0000	0.0502	0.8753
S300	South Class A	- 4.3826	0.0000	0.0922	0.6252
S400	West Class A	- 3.7105	0.0000	0.0742	0.7266
S11A	Boston	- 1.7261	0.0000	0.0457	0.9023
S12A	New York	- 3.5986	0.0000	0.1759	0.3204
S12B	Philadelphia	- 2.8424	0.0000	0.2244	0.2251
S23A	Chicago	- 5.5570	0.0000	0.0207	0.9961
S23B	Detroit	- 4.4371	0.0000	0.2466	0.1930
S35A	Washington, DC	- 2.3577	0.0000	0.2624	0.1731
S35B	Miami	- 4.8792	0.0000	0.0875	0.6504
S35C	Atlanta	- 2.7632	0.0000	0.0593	0.8189
S35E	Baltimore	- 2.3565	0.0000	0.0799	0.6929
S37A	Dallas	- 3.0921	0.0000	0.0937	0.6175
S49A	Los Angeles	- 4.4497	0.0000	0.0604	0.8116
S49B	San Francisco	- 4.7253	0.0000	0.0613	0.8060
S49D	Seattle	- 2.7826	0.0000	0.1050	0.5614

Notes: DF-GLS denotes “Dickey-Fuller Generalized Least Squares,” and KPSS denotes “Kwiatkowski-Phillips-Schmidt-Shin.” The null hypothesis for the DF-GLS test is that there is a unit root present in the series (series is non-stationary). The null hypothesis for the KPSS test is that there is no unit root present in the series (series is stationary). A coefficient is statistically significant if its p-value is less than or equal to 5 percent.

Once we achieved stationarity for the rent and utility series by first-order differencing, the next step was to determine the general form of the models to be estimated. For each geographic area, the study team

formulated initial model specifications by examining the autocorrelation and partial autocorrelation functions. For each metro area, we estimated several rent and utility models with varying ARIMA terms. Guided by the Schwarz Bayesian information criterion (SBC), we settled on final ARIMA specifications. Exhibits 4-3 and 4-4 present the structure for three different ARIMA rent and utility models, respectively. We estimated the different models by varying the type of ARIMA specifications and the set of exogenous variables included. The study team developed the first model, Pure Time Series (PTS), without exogenous variables; the second, National Input Model (NIM), with national exogenous variables (National Residential Fixed Investment and Civilian Employment Data for rent and spot price, in dollars per barrel of West Texas intermediate crude oil, price in dollars per short ton of bituminous coal; Henry Hub price of natural gas, in dollars per million BTUs; and CPI-U for utilities); and the third, Local Input Model (LIM), based on local exogenous variables (Local Permit Data and Employment Data for rent, and electricity for utilities).

Exhibit 4-3 | ARIMA Model for Rent by Geographic Area

Geographic Code	Area	PTS	NIM	LIM
0000	National	1,1,0	1,1,0	NA
0100	North	1,1,2	5,1,0	2,1,1
0200	Midwest	0,1,1	3,1,0	3,1,0
0300	South	1,1,0	1,1,0	1,1,0
0400	West	0,1,2	3,1,0	3,1,0
S100	North Class A	0,1,1	3,1,0	0,1,1
S200	Midwest Class A	1,1,2	4,1,0	4,1,0
S300	South Class A	1,1,0	1,1,0	1,1,0
S400	West Class A	1,1,2	3,1,0	3,1,0
S11A	Boston	0,1,1	0,1,1	0,1,1
S12A	New York	0,1,1	0,1,1	1,1,1
S12B	Philadelphia	0,1,1	0,1,1	0,1,1
S23A	Chicago	0,1,1	0,1,1	0,1,1
S23B	Detroit	0,1,1	0,1,2	0,1,2
S35A	Washington, DC	1,1,0	1,1,0	1,1,0
S35B	Miami	1,1,0	1,1,0	1,1,0
S35C	Atlanta	1,1,0	0,1,1	0,1,1
S35E	Baltimore	0,1,1	0,1,1	0,1,1
S37A	Dallas	1,1,0	1,1,0	1,1,0
S49A	Los Angeles	1,1,0	1,1,1	3,1,0
S49B	San Francisco	1,1,0	1,1,0	1,1,0
S49D	Seattle	3,1,0	0,1,1	3,1,0

Note: ARIMA denotes “Autoregressive Integrated Moving Average,” LIM denotes “Local Input Model,” NIM denotes “National Input Model,” and PTS denotes “Pure Time Series” model.

In exhibit 4-3, the NIM for rent produced more consistent and generally smaller SBC results for the 22 areas than either the PTS or the LIM. Based on the SBC rule, we find that the NIM model dominates the other two competing models in most cases.

Exhibit 4-4 | ARIMA Model Utility by Geographic Area

Geographic Code	Area	PTS	NIM	LIM
0000	National	0,1,1	0,1,1	NA
0100	North	2,1,1	0,1,1	NA
0200	Midwest	4,1,0	4,1,0	NA
0300	South	4,1,0	4,1,0	NA
0400	West	2,1,0	2,1,0	NA
S100	North Class A	1,1,0	0,1,1	NA
S200	Midwest Class A	4,1,0	3,1,0	NA
S300	South Class A	4,1,0	3,1,0	NA
S400	West Class A	2,1,2	2,1,0	NA
S11A	Boston	4,1,0	4,1,0	2,1,0
S12A	New York	4,1,0	4,1,0	2,1,1
S12B	Philadelphia	4,1,1	4,1,0	1,1,2
S23A	Chicago	0,1,1	0,1,1	3,1,0
S23B	Detroit	0,1,1	0,1,1	0,1,1
S35A	Washington, DC	4,1,0	4,1,0	4,1,0
S35B	Miami	0,1,1	0,1,1	0,1,1
S35C	Atlanta	4,1,0	4,1,0	0,1,1
S35E	Baltimore	4,1,0	4,1,0	4,1,0
S37A	Dallas	4,1,0	4,1,0	4,1,0
S49A	Los Angeles	0,1,1	0,1,1	0,1,1
S49B	San Francisco	0,1,2	0,1,2	0,1,2
S49D	Seattle	0,1,1	0,1,1	0,1,2

Note: ARIMA denotes “Autoregressive Integrated Moving Average,” LIM denotes “Local Input Model,” NIM denotes “National Input Model,” and PTS denotes “Pure Time Series” model.

In exhibit 4-4, the PTS for utility produced generally smaller SBC results for the geographic areas compared to the alternative NIM. Although the NIM for utility has comparable performance to the PTS for utility in 16 cases, the overall performance of PTS is better than NIM. For cases in which the PTS and NIM ARIMA models differed, the PTS areas had slightly more parsimonious models compared to their NIM counterparts. As in the rent specification, in the minority of cases for utility in which the SBC for a PTS area was not superior to NIM, the results do not outweigh the benefits of the most parsimonious model possible (PTS) across areas.

4.2 Forecast Error Comparison

Using the model with lowest SBC in exhibits 4-3 and 4-4, the study team conducted a test of the model’s forecast performance by splitting each series (dataset for each metro area) into an out-of-sample period. We compared the difference between the forecast estimates and the actual values to determine the forecast accuracy, using the Root Mean Square Error (RMSE). For comparison, we duplicated HUD’s hypothetical model with the national inputs (for rent and utility models) and local inputs (for the rent model) and calculated the RMSE for the same validation period. The lower the value of the measure of

accuracy, the better the performance of the forecast model. The results are presented in exhibits 4-5 and 4-6.

Exhibit 4-5 | Average Root Mean Square Errors Across Validation Period by ARIMA Specification and Geographic Area, Rent Model

Geographic Code	Area	Our Model			HUD's Hypothetical Model	
		PTS	NIM	LIM	NIM	LIM
0000	National	1.629	1.486	NA	2.013	NA
0100	North	1.088	1.562	1.921	2.742	2.761
0200	Midwest	1.324	0.726	0.726	1.159	1.139
0300	South	2.409	2.085	2.116	1.828	1.858
0400	West	5.785	2.814	2.813	5.050	5.115
S100	North Class A	2.957	3.027	3.177	3.716	3.747
S200	Midwest Class A	0.832	0.607	0.598	0.751	0.734
S300	South Class A	4.672	4.488	4.490	4.431	4.428
S400	West Class A	2.918	2.092	2.101	4.928	4.995
S11A	Boston	1.796	1.259	2.962	1.120	1.143
S12A	New York	4.684	4.726	4.733	5.633	5.671
S12B	Philadelphia	3.194	3.450	3.451	3.818	3.776
S23A	Chicago	0.909	1.565	1.646	1.570	1.631
S23B	Detroit	1.756	1.465	1.091	1.382	1.236
S35A	Washington, DC	2.844	3.331	3.178	4.245	4.131
S35B	Miami	4.370	4.402	4.382	4.164	4.227
S35C	Atlanta	9.426	8.929	9.013	9.642	9.607
S35E	Baltimore	4.216	5.442	5.157	3.661	3.647
S37A	Dallas	9.716	9.605	9.640	10.076	10.033
S49A	Los Angeles	2.321	2.062	2.111	3.278	3.252
S49B	San Francisco	2.113	1.921	1.908	6.467	6.450
S49D	Seattle	6.710	11.901	7.204	11.568	11.871

Notes: ARIMA denotes “Autoregressive Integrated Moving Average,” LIM denotes “Local Input Model,” NA denotes “data not available,” NIM denotes “National Input Model,” PTS denotes “Pure Time Series” model, and RMSE denotes “Root Mean Square Error.” The best RMSE for each geographical region is highlighted.

On comparing the RMSEs across models, 2M’s NIM outperforms the PTS and LIM specifications for 10 of the 22 areas. The exogenous variables included in NIM explain the underlying dynamics of the response series better than those in the LIM. Of the remaining 12 areas, the PTS specification performed better than the LIMs in 9 areas. For these regions, we observe that the PTS specification captures the trend and seasonality embedded in the data, without additional input variables. In exhibit 4-5, with a few exceptions, two of the study team’s rent models, NIM and LIM, produced lower-average RMSEs across local and regional areas compared to their HUD rent model counterparts. Compared to HUD’s hypothetical model (NIM), the study team’s model (NIM) improved RMSEs in 15 areas and fared worse in seven others. These 15 areas had an aggregate reduction in RMSE of 17.185 compared to a total reduction of 2.888 in the seven areas where HUD’s hypothetical model performed better than the study model. For the LIMs, the total reduction in RMSEs between the study model (LIM) and HUD’s

hypothetical model (LIM) was 20.853 (in 15 areas) compared to a total reduction of 3.819 (in six areas) where HUD’s hypothetical model fared better than the study model.

Exhibit 4-6 | Average Root Mean Square Errors Across Validation Period by ARIMA Specification and Geographic Area, Utility Model

Geographic Code	Area	Our Model			HUD’s Hypothetical Model	
		PTS	NIM	LIM*	NIM	LIM
0000	National	2.567	1.008	NA	1.639	NA
0100	North	5.534	7.871	NA	4.561	NA
0200	Midwest	4.175	3.831	NA	4.950	NA
0300	South	4.088	1.555	NA	3.811	NA
0400	West	3.007	6.648	NA	4.314	NA
S100	North Class A	4.477	9.139	NA	2.427	NA
S200	Midwest Class A	4.132	4.537	NA	4.712	NA
S300	South Class A	5.205	2.589	NA	3.356	NA
S400	West Class A	3.846	5.464	NA	5.888	NA
S11A	Boston	36.464	28.714	15.059	10.858	NA
S12A	New York	13.489	9.284	10.283	2.113	NA
S12B	Philadelphia	1.496	2.299	2.154	5.845	NA
S23A	Chicago	7.713	4.201	3.125	8.201	NA
S23B	Detroit	4.401	9.012	6.416	4.665	NA
S35A	Washington, DC	7.806	3.934	8.150	5.547	NA
S35B	Miami	5.096	4.024	2.589	5.276	NA
S35C	Atlanta	10.370	14.276	5.347	18.296	NA
S35E	Baltimore	11.820	13.329	14.926	17.338	NA
S37A	Dallas	3.562	7.992	3.092	4.118	NA
S49A	Los Angeles	2.706	6.054	3.525	3.958	NA
S49B	San Francisco	8.864	10.931	8.470	12.693	NA
S49D	Seattle	4.082	6.035	3.247	5.930	NA

* Data were only available for the 13 metro areas.

Notes: ARIMA denotes “Autoregressive Integrated Moving Average,” LIM denotes “Local Input Model,” NA denotes “data not available,” NIM denotes “National Input Model,” PTS denotes “Pure Time Series” model, and RMSE denotes “Root Mean Square Error.” The best RMSE for each geographical region is highlighted.

For the utility models, the PTS specification performs better than the competing NIM and LIM specifications for 9 of the 22 areas. The exogenous variables included in NIM and LIM do not contribute to explaining the underlying patterns in the response series for utilities (exhibit 4-6). Of the remaining 13 areas, the LIM specification performed better than the NIMs in 7 areas. For these metro areas, we observe that the exogenous variables included in the LIM explain the underlying dynamics of the response series better than those in the NIM. Compared to HUD’s hypothetical model (NIM) for utility, the study team’s utility model (PTS) improved RMSEs in 14 areas and fared worse in 8 areas.

The RMSEs are comparable across the three model specifications since they are all measured with the same units. One way to think about choosing a model is to compare the models in percentage terms.

Suppose one model’s RMSE is 30 percent less than another—that may be a significant difference. However, if the RMSE is about 5 percent different, the tradeoff would be model complexity to error measures. While there is no absolute criterion for a “good value” of RMSE, which would make one model preferable over another, the objective of the study team is to choose the model with the lowest RMSE.

4.3 Coefficient Significance of Selected Model

Based on the results in the previous section, the study team selected the model determined to provide the most accurate estimate for rent and utility in-sample among the three competing models. While we deemed NIM to be the best fit for the rent series, we selected PTS for estimating and forecasting utility. In this section, we present results of the parameter estimate of the selected models, as well as their statistical significance. Exhibit 4-7 presents the parameter estimates of NIM for rent and PTS for utility, as well as the exogenous variables used to augment the specified model. For each parameter, the exhibit shows the estimated value, standard error, and *t*-value for the estimate. The exhibit also indicates the lag at which the parameter appears in the model. For the United States, there are four parameters in the rent model and two parameters in the utility model. The mean term is labeled MU. The autoregressive (AR) parameter is labeled AR1,1; this is the coefficient of the lagged value of the change in rent. The moving average (MA) parameter is labeled MA1,1; this is the coefficient of the previous error term in the utility series. The *p*-value provides significance tests for the parameter estimates and indicates whether some terms in the model might be unnecessary. We show only the results for the nation in exhibit 4-7, and present the results for the remaining geographic areas in appendix C. However, we summarize the statistical significance of the parameters across all 22 areas in exhibit 4-8.

Exhibit 4-7 | Parameter Estimates for the United States

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0076	0.0000	0
AR1,1	rpr_sa	0.8488	0.0000	1
NUM1	RES_FIXED_INV	– 0.0074	0.0980	2
NUM2	Civilian_Employment	0.0780	0.0203	3
Utility Model				
MU	fu_sa	0.0073	0.0038	0
MA1,1	fu_sa	– 0.4653	0.0000	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameter; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value is less than or equal to 5 percent.

Exhibit 4-8 | Percentage of Significant Coefficients

Parameter	Number of Areas Using Corresponding Parameters	Percent
Rent Model		
MU	22	100
AR1,1	14	100
AR1,2	6	50
AR1,3	6	100
AR1,4	2	50
AR1,5	1	100
MA1,1	9	100
MA1,2	1	100
RES_FIXED_INV	22	6.5
Civilian_Employment	22	68.2
Utility Model		
MU	22	63.6
AR1,1	15	40
AR1,2	14	64.3
AR1,3	11	63.6
AR1,4	11	90.9
MA1,1	10	80.0
MA1,2	2	100

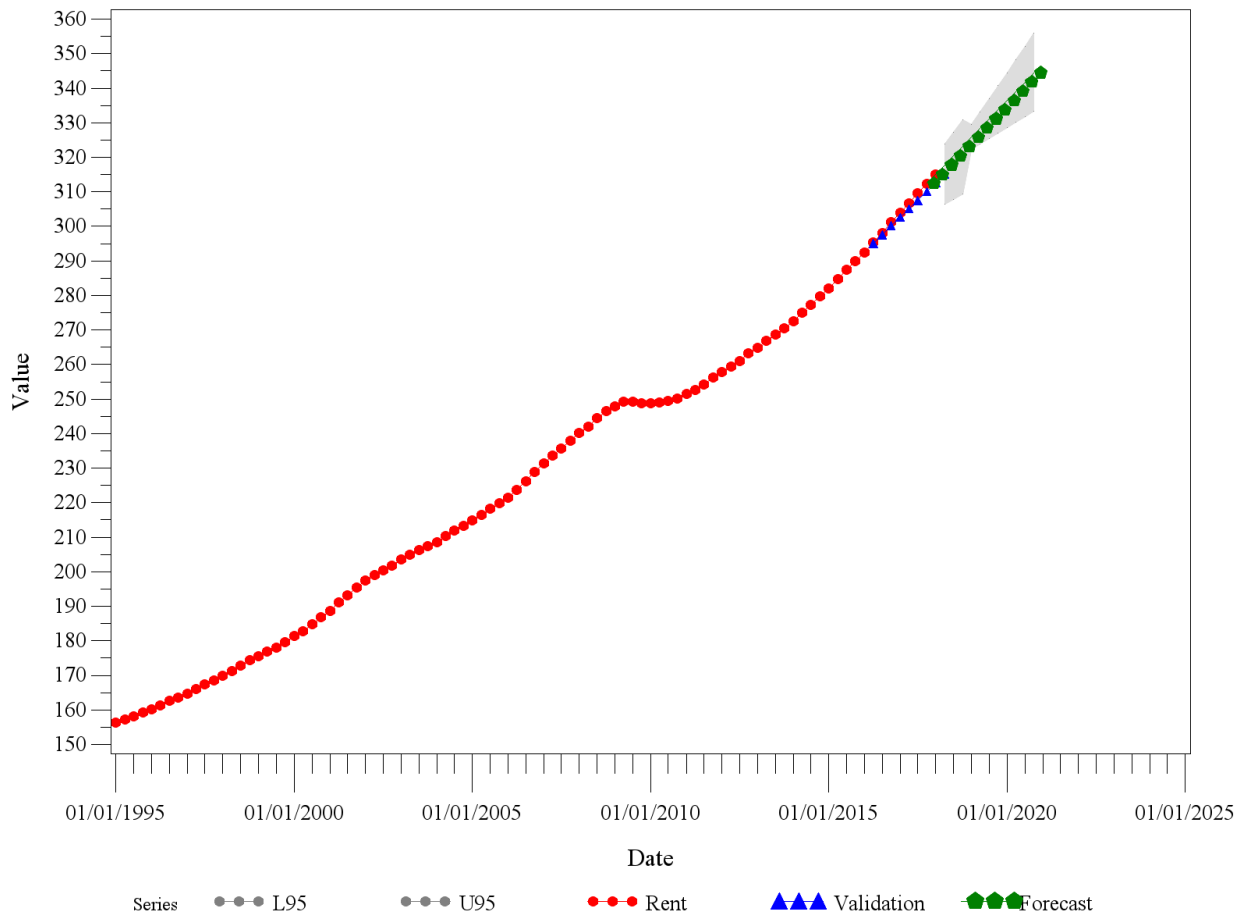
Note: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model.

4.4 Forecasting Rent and Utilities

The study team used the model with the best forecasting measure, based on the model giving lowest RMSE, to forecast rents and utilities for all geographic areas. We forecast the two series for eight quarters ahead using the best models selected in the previous section. We estimated the models for each series using the observations up to 2016 (Q1) and then obtained the forecast values for the following eight quarters until 2018 (Q1). We then computed the forecast errors for the period 2016 (Q2) through 2018 (Q1) by comparing the forecast values with actual observations for the corresponding period. To illustrate how closely the observed data follow the predicted data, we show in exhibits 4-9 and 4-10 the forecast values of rent and utilities respectively for the United States. The results for the remaining geographic areas are presented in appendix D.

Exhibit 4-9 | Forecasts of National Quarterly Rent of Primary Residence

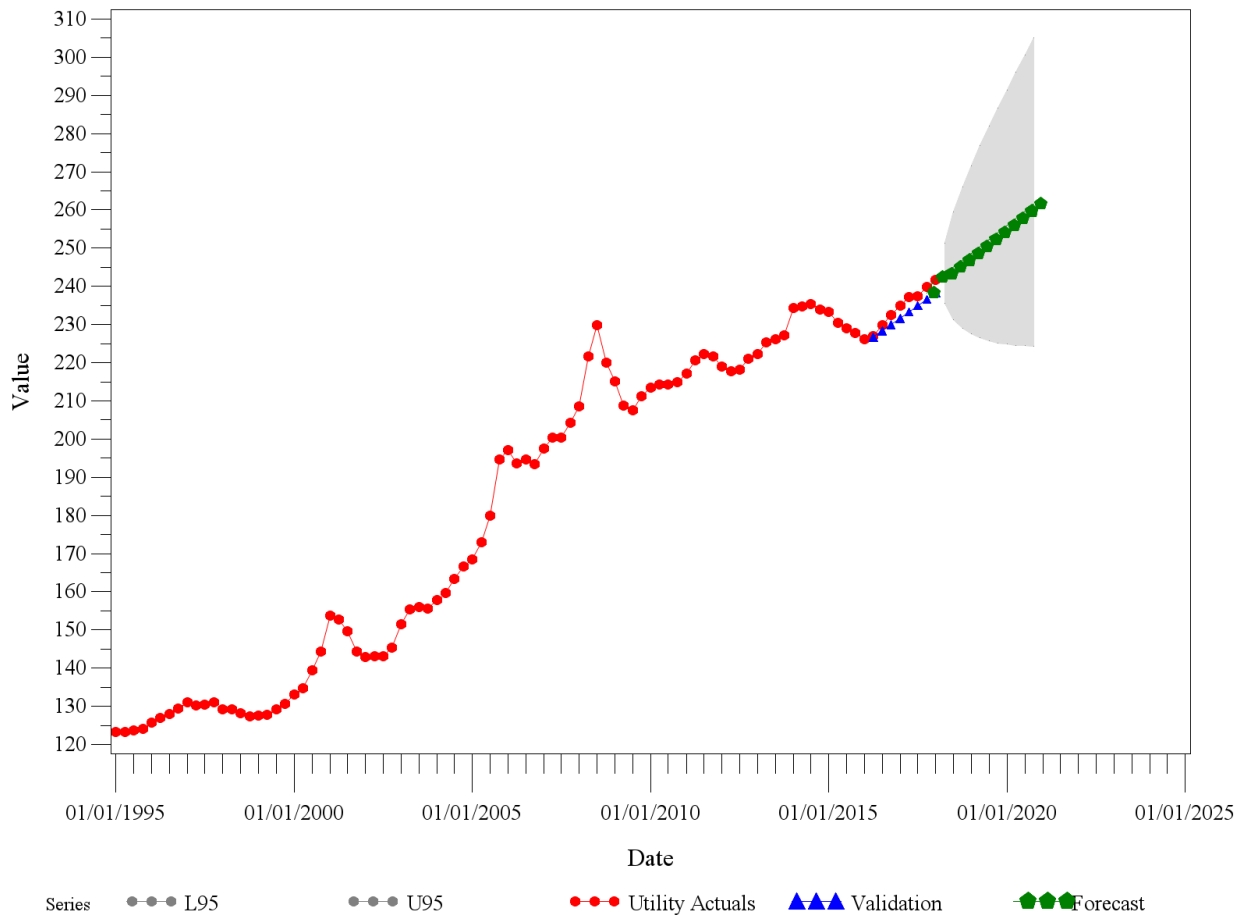
Rent Validation and Forecast for United States



The NIM rent forecast for 2018 (Q2) through 2020 (Q4) closely mimic the national trend for CPI-U Rent of Primary Residence series. The validation series shows slight departures late in the forecast period from both the Consumer Price Index (CPI) Rent series around 2016 (Q2) and from the forecast at 2018 (Q2).

Exhibit 4-10 | Forecasts of National Quarterly Housing Fuels and Utilities

Utilities Validation and Forecast for United States



In exhibit 4-10, the PTS validation series for utility had noticeable deviations between the CPI-U series in several time periods where it exhibits large fluctuations, indicating the presence of more volatile movements in the underlying CPI-U. Other than those time periods, however, the validation values were more consistent with the CPI-U trend. In later quarters, starting in 2016 (Q1), the validation series tracked quite closely with CPI because PTS provided a very precise forecast for CPI-U. For the forecast period 2018 (Q2) through 2020 (Q4), the selected model produces stable forecast values that are consistent with the overall trend direction of CPI-U.

4.5 Estimating a Local Trend Factor

Once the forecast model is estimated and validated, the rent and utility forecasts are used to construct a locally based trend factor for the geographic areas, which could be used in computing Fair Market Rents (FMRs). These trend factors help to account for current market conditions, especially markets where rent prices are escalating rapidly.

The trend factor is used to adjust rents from 2 years prior to the current fiscal year (FY) and is calculated using a gross rent index (GRI). The GRI is the sum of two weighted, independently forecasted components of the CPI-U (for example, Rent of Primary Residence and Fuels and Utilities).

Local trend factors for each region are calculated using the change in average quarterly GRI in the previous calendar year to the average quarterly forecasted GRI index of the respective FY. Specifically, for 2019, the local trend factor is a ratio of the average quarterly GRI for 2018 (Q4) through 2019 (Q3) to 2017 (Q1) through 2017 (Q4).

Exhibit 4-11 presents estimated trend factors for each region using the aforementioned forecasts for rent and utility. Column 3 presents the trend factor using the new model specifications and can be compared to the trend factors constructed by HUD in columns 4 and 5. The new forecasts provide a higher trend factor in 17 of the 22 areas, while five areas show no change.

Exhibit 4-11 | Estimated Trend Factors by ARIMA Specification and Geographic Area

Geographic Code	Area	The Study Team's Selected Model	HUD's Hypothetical Model	
			NIM	LIM
0000	National	1.061	1.057	NA
0100	North	1.048	1.056	1.056
0200	Midwest	1.055	1.050	1.050
0300	South	1.064	1.059	1.059
0400	West	1.077	1.067	1.067
S100	North Class A	1.052	1.058	1.058
S200	Midwest Class A	1.058	1.051	1.051
S300	South Class A	1.069	1.062	1.062
S400	West Class A	1.082	1.072	1.071
S11A	Boston	1.084	NA*	NA*
S12A	New York	1.053	1.052	1.052
S12B	Philadelphia	1.041	1.041	1.041
S23A	Chicago	1.053	1.049	1.050
S23B	Detroit	1.053	1.052	1.053
S35A	Washington, DC	1.058	1.056	1.055
S35B	Miami	1.050	1.058	1.058
S35C	Atlanta	1.061	1.042	1.042
S35E	Baltimore	1.046	1.047	1.046
S37A	Dallas	1.071	1.069	1.069
S49A	Los Angeles	1.081	1.073	1.073
S49B	San Francisco	1.110	1.089	1.089
S49D	Seattle	1.088	1.084	1.082

Note: *Data not available for Boston in HUD's hypothetical model. ARIMA denotes "Autoregressive Integrated Moving Average," LIM denotes "Local Input Model," NIM denotes "National Input Model."

HUD's NIM trend factor was slightly below (0.01) the study team's NIM. The differences between the trend factors produced by the study team's selected model and HUD's hypothetical model ranged from -0.01 to 0.021. A few area results for the study model were smaller than in the HUD model (North, North

Class A, Miami, and Baltimore). Generally, the forecast based on HUD's hypothetical model tends to be smaller than the study team's model in many cities and areas, particularly in the West. The study team's method appears to underestimate the trend only in Miami and Baltimore, although the extent of underestimation looks mild.

In order to determine if the use of a local trend factor is warranted, as opposed to the current methodology of using a national estimate of the trend factor, we compared our local forecast estimates with our national forecast estimate. Specifically, we used the forecast period between 2018 (Q2) and 2019 (Q4) to examine whether the errors of estimates for each geographic area significantly differ from the errors of the estimate for the United States.⁸ The results of this test revealed that all 13 local geographic areas have forecast estimates significantly different from the U.S. estimate, implying that a local trend factor could prove to be beneficial because it may address the varying patterns that exist in each geographical area. To understand the magnitude of this difference and to examine whether using local estimate could improve the accuracy of the trend factor, we compare the FMRs for each area using the local trend factors and the national trend factor in section 6.

⁸ A paired t-test with equal variance for rent and unequal variance for utilities was conducted to address the significant difference between the estimates of local and national estimates at the 95-percent confidence interval.

5 PHASE II RESULTS

The main concern with the analysis in the previous section is that some accuracy may be lost by limiting the number of metro areas. In this section, we look at ways to introduce additional geographic areas with data from Axiometrics and Zillow.⁹ The Axiometrics data are from a monthly survey of rental properties starting in April 2008 and include values for asking rent, effective rent, occupancy rate, number of units, average size of the units, and quality grades of the property and local market areas. The Axiometrics data facilitate the creation of summary measures at various levels of geography.

Zillow offers summary measures, such as their Zillow Rent Indices (ZRI), at various levels of geography. Zillow measures are created by proprietary algorithms/methods and generally began in 2010 although some indices based on home values are available for longer periods of time. Thus, an important consideration in evaluating the usefulness of these data is the extent to which HUD finds these methods acceptable.

Our general approach was to first evaluate these sources of data by comparing rates of change to rates of change calculated from Bureau of Labor Statistics (BLS) and American Community Survey (ACS) data. These comparisons provide “face-validity” assessments of the usability of the series. Next, we looked at a case of forecasting values for 277 Core-Based Statistical Areas (CBSAs) and compared them to actual values from ACS.

Exhibit 5-1 | Cases and Geographic Coverage of Axiometrics Data

Year	Properties in Survey	Number of States	Number of CBSAs
2008	22,681	47	322
2009	25,233	47	350
2010	27,244	48	344
2011	30,530	48	388
2012	33,403	50	416
2013	35,335	51	451
2014	37,358	51	446
2015	40,047	51	495
2016	46,334	51	544
2017	49,542	51	604
2018	51,279	51	610

Notes: CBSAs denotes “Core-Based Statistical Areas.” Unique properties = 51,279. The Number of States column includes the 50 states plus Washington, DC.

⁹ We have looked at the data from the Federal Housing Finance Agency, who produces house price indices (HPIs), starting in 1975, that are based on transactions on the same properties. Indices are produced monthly and quarterly for various geographic regions. This series is of interest to the extent that the relationship between single-family home values is reasonably stable and can be reliably estimated; however, Gallin (2008) is not very encouraging about finding a stable relationship. The main advantage of the HPI series is its lengths but the lack of comparable rent series (in terms of length) hinders the use of the HPIs. Thus, to date we have not identified a relationship between rents and HPI that could be used to forecast trend factors.

Exhibit 5-2 | Analysis Variables Created from the Axiometrics Data

Variable	Description	Notes
<i>Geoid</i>	Location of property	Time invariant; includes lat and lon
<i>Submarket</i>	Submarket name	Time invariant; Axiometrics defined name
<i>Year Built</i>	Year of construction	Time invariant
<i>Rehab</i>	Year of last rehabilitation	Time invariant
<i>Units</i>	Number of units	Time invariant
<i>Area per Unit</i>	Average square feet per unit	Time invariant
<i>Level</i>	Number of stories	Time invariant
<i>Status</i>	Property status code ("S" = stable, "LU" = leased up, and so on)	Monthly
<i>Prop Submarket_Grade</i>	Classification of the property submarket by letter grade	Monthly; lots of missing values
<i>Propmarket_Grade</i>	Classification of the property market by letter grade	Monthly; lots of missing values
<i>Submarket_Grade</i>	Classification of the submarket by letter grade	Monthly
<i>Grent</i>	Asking rent	Monthly
<i>Erent</i>	Asking rent minus concessions	Monthly
<i>Erent per sqft</i>	<i>erent</i> per square foot	Monthly
<i>Occ Rate</i>	Percent of units occupied	Monthly
<i>Con Value</i>	Dollar value of concessions	Monthly
<i>Con Percent</i>	Concessions as a percent of <i>grent</i>	Monthly

5.1 Axiometrics Data

The Axiometrics data are based on a monthly survey of more than 50,000 properties. Some, but not all, properties have been surveyed since 2008 (exhibit 5-1). The main variables available in the Axiometrics data are shown in exhibit 5-2. The Axiometrics data include the asking rent and concessions. The “effective rent” (*erent*) is the asking rent minus the concessions and is the main variable analyzed by the study team.¹⁰ The data also include the precise location of each property, making it possible to create summary measures (for example, mean or 40th percentile *erent*) for various geographic definitions. Additionally, the data include some codes that classify the property (for example, “A,” “B”); indicate dates of rehabs; and convey overall property and market assessment (for example, “stable”). (Full data details will be outlined in an appendix.)

The Axiometrics data offer numerous (almost too many) possibilities for creating time series that are based on summary measures of the individual properties by CBSA (or other geographies). As a result, the study team took an evolutionary approach by starting with the most obvious series. Specifically, we created a set of time series using the properties that responded to the survey in every period. The

¹⁰ Concessions would likely increase (decrease) with decreases (increases) in demand for apartment units. Thus, the effective rent should provide a more accurate measure of the current rent when compared to the asking rent.

advantage of this approach is that property specific factors that are time-invariant will be eliminated by differencing the data. With this set of responses, the study team then created quarterly and annual series for primary sampling unit (PSU) ($n = 22$) and CBSAs ($n = 277$).

5.2 Methods to Assess the Data

To assess the Axiometrics data's usefulness for Fair Market Rent (FMR) trend adjustments, the study team compared the Axiometrics measure of effective rent (*erent*) to the following:

1. The measure of the seasonally unadjusted "rent of primary residence" (*SEHA*) from the Consumer Price Index for All Urban Consumers (CPI-U) data file produced by BLS.
2. The estimates of median gross rent for all bedroom sizes from ACS (*rentACS*).¹¹

We detail our approach to both BLS and ACS data below.

5.2.1 COMPARISON TO BUREAU OF LABOR STATISTICS DATA

To compare Axiometrics data to BLS data, the study team compared the quarterly and yearly percentage change in *erent* and *SEHA* for each of the 22 PSUs available in both datasets.¹² The steps to create a quarterly and yearly series of *erent* for each PSU included the following:

1. Removing projects in the Axiometrics data that did not have complete data (in other words, any that had missing values of *erent* for any month in any year).¹³
2. Using the county Federal Information Processing Standards (FIPS) code to merge the Axiometrics data with the crosswalk provided by HUD that links counties to PSUs and creating a PSU variable that indicates the PSU each project is in (any project not in a PSU was removed from the data).
3. Creating a quarterly estimate of *erent* at the PSU level by averaging the monthly project-level estimates for each quarter (42 in total from 2008 to 2018) in each PSU (23 in total).
4. Creating a yearly estimate of *erent* at the PSU level by averaging the monthly project-level estimates for each year (11 in total) in each PSU (23 in total).

The steps to create a quarterly and yearly series of *SEHA* for each PSU included the following:

1. Creating a quarterly estimate of *SEHA* at the PSU level by averaging the monthly PSU-level estimates for each quarter (the same 42 quarters as the Axiometrics data).
2. Creating a yearly estimate of *SEHA* at the PSU level by averaging the monthly PSU-level estimates for each year (the same 11 as the Axiometrics data).

The study team calculated the percentage change in *erent* and *SEHA* for each PSU (i) in each quarter for year (t) from the preceding quarter or year (t_{-1}) with the following formula:

¹¹ This is variable B25064—median gross rent.

¹² We also did comparisons for each of the four Census regions and can provide these results as an appendix at HUD's request.

¹³ We also did comparisons that included projects with incomplete data, as well as comparisons that only used data from 2010 or later for projects that had complete data. The study team found that the results were best for comparisons that use complete data for all years of the Axiometrics data (2008 to 2018).

$$Change_{i,t} = \frac{X_{i,t} - X_{i,t-1}}{X_{i,t-1}} \times 100.$$

Thus, the study team created PSU-level percentage changes for *erent* and *SEHA* for each quarter and year available in the data. Importantly, nine PSUs only have data from BLS for the fourth quarter of 2017 and later; therefore, the percentage change estimates for these PSUs represent fewer timepoints. We indicate which PSUs have fewer data points in all results below and suggest focusing on the results from the PSUs with complete quarterly series.

Finally, the study team developed several metrics for each PSU to determine the comparability of the percentage change estimates of *erent* and *SEHA* including the following:

- **NOVER/POVER:** the number and percentage of timepoints (quarters or years) in which the *erent* percentage change is higher than the *SEHA* percentage change. One would expect roughly half the timepoints to have a higher *erent* percentage change if *erent* and *SEHA* are comparable measures. If not, then there is evidence that *erent* is consistently either higher or lower than *SEHA* and that the measures are not very comparable.
- **PWITHIN10:** the number and percentage of timepoints in which the *erent* percentage change is within 10-percentage points (either above or below) of the *SEHA* percentage change. Since rents tend to be relatively stable, one would expect virtually all timepoints to show the *erent* and *SEHA* percentage changes are within 10-percentage points of each other if the measures are comparable.
- **PWITHIN5:** the number and percentage of timepoints in which the *erent* percentage change is within 5 percentage points (either above or below) of the *SEHA* percentage change. This measure is more stringent than *PWITHIN10*, but one would still expect many of the timepoints to show the *erent* and *SEHA* percentage changes are within 5-percentage points of each other if the measures are comparable.
- **PWITHIN1:** the number and percentage of timepoints in which the *erent* percentage change is within 1 percentage point (either above or below) of the *SEHA* percentage change. This measure is very stringent and if timepoints show the *erent* and *SEHA* percentage changes are within 1 percentage point of each other, there is evidence that the measures are comparable.
- **PER10:** an indication (either “yes” or “no”) that shows whether an estimated rent value that accounts for the trend in *erent* is within 10 percent of an estimated rent value that accounts for the trend in *SEHA*. The study team calculated estimated rent values for each measure for each PSU by adding the percentage change for each measure from the earliest timepoint in the data to the latest timepoint to \$100. These calculated values are referred to as “simulated” FMRs. For example, if the percentage change was 3.4 percent for *erent* and 5.4 percent for *SEHA*, then the simulated FMRs would be \$103.40 and \$105.40, respectively. *PER10* indicates whether the simulated FMR from *erent* is within 10 percent of the value from *SEHA*.

We present the results for each of the metrics for each PSU below in exhibit 5-3.

5.2.2 COMPARISON TO AMERICAN COMMUNITY SURVEY DATA

To compare Axiometrics data to ACS data, the study team compared the yearly percentage change in *erent* and *rentACS* for each of the 249 CBSAs available in both datasets. The steps to create a yearly series of *erent* for each CBSA included the following:

1. Removing projects in the Axiometrics data that did not have complete data (that is, any that had missing values of *erent* for any month in any year).
2. Using the county FIPS code to merge the Axiometrics data with the crosswalk provided by HUD that links counties to CBSAs and creating a CBSA variable that indicates the CBSA each project is located in (any project not in a CBSA was removed from the data).
3. Creating a yearly estimate of *erent* at the CBSA level by averaging the monthly project-level estimates for each year (10 in total, 2008 to 2017) in each CBSA (249 in total).

The study team downloaded the yearly estimates of *rentACS* for each CBSA from American Factfinder using table B25064. The study team then created the yearly percentage change in *erent* and *rentACS* and the metrics comparing the percentage change estimates using the same process that we describe earlier in section 5.2.1.

5.2.3 RESULTS

Exhibit 5-3 shows the results of each of the metrics comparing quarterly estimates of *erent* from Axiometrics and *SEHA* from BLS. The results indicate a high level of comparability between the two datasets. For many PSUs, *NOVER* shows that *erent* is larger than *SEHA* about half (close to 50 percent) of the time. For some PSUs, *erent* is larger more often (75 percent of the time) or less often (25 percent of the time), but in all cases these are PSUs that have limited data in BLS. The New York-Newark-Jersey City, NY-NJ-PA PSU and Baltimore-Columbia-Towson, MD PSU are slightly more likely to have lower values of *erent* than *SEHA* (*erent* was only larger than *SEHA* 38 and 36 percent of the time, respectively).

Another finding that indicates the comparability of the measures is that the difference in the quarterly percentage change in *erent* and the percentage change in *SEHA* was within 5 percent for all timepoints in all PSUs with the exception of the Seattle-Tacoma-Bellevue, WA PSU, where the difference was still within 5 percent for 93 percent of the time. In addition, the difference was within 1 percent for several quarters across all PSUs, which indicates the measures are highly comparable.

Finally, in all PSUs except Atlanta-Sandy Springs-Roswell, GA, the artificial FMR value (based on an initial value of 100) using the trend from *erent* was within 10 percent of the estimated value using *SEHA*. Again, this shows the two measures are highly comparable.

The comparison of the Axiometrics data to the ACS measure of median gross rent (*rentACS*) also produced results that indicate the Axiometrics measure *erent* is a viable option for estimating trends in FMR. Exhibit 5-4 provides a summary of the results comparing yearly series of the two measures for the 254 CBSAs represented in both datasets. The accompanying Excel workbook provides the full results for each CBSA. Exhibit 5-4 shows that *erent* was higher than *rentACS* about half the time (between 4 to 6 of the 10 years in the analysis) in 196 (79 percent) of the CBSAs. Moreover, the difference in the percentage change in *erent* and the change in *rentACS* was within 10 percent for all years in 156 (63

percent) of the CBSAs. Finally, in 182 (73 percent) of the CBSAs, the simulated FMR estimate using the trend in *erent* was within 10 percent of estimate using the trend in *rentACS*.

Exhibit 5-3 | Number (%) of Quarters in Each Primary Sampling Unit for the Metrics Comparing the Axiometrics Rent Measure to the Bureau of Labor Statistics Measure

PSU NAME	NOVER (POVER)	PWITHIN10	PWITHIN5	PWITHIN1	PER10
Boston-Cambridge-Newton, MA-NH	20 (48%)	41 (100%)	41 (100%)	21 (51%)	Yes
New York-Newark-Jersey City, NY-NJ-PA	16 (38%)	41 (100%)	41 (100%)	22 (54%)	Yes
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	20 (48%)	41 (100%)	41 (100%)	22 (54%)	Yes
Chicago-Naperville-Elgin, IL-IN-WI	19 (45%)	41 (100%)	41 (100%)	19 (46%)	Yes
Detroit-Warren-Dearborn, MI	20 (48%)	41 (100%)	41 (100%)	21 (51%)	Yes
Minneapolis-St. Paul-Bloomington, MN-WI*	2 (50%)	3 (100%)	3 (100%)	2 (67%)	Yes
St. Louis, MO-IL*	1 (25%)	3 (100%)	3 (100%)	3 (100%)	Yes
Washington-Arlington-Alexandria, DC-VA-MD-WV	20 (48%)	41 (100%)	41 (100%)	26 (63%)	Yes
Miami-Fort Lauderdale-West Palm Beach, FL	23 (55%)	41 (100%)	41 (100%)	30 (73%)	Yes
Atlanta-Sandy Springs-Roswell, GA	22 (52%)	41 (100%)	41 (100%)	18 (44%)	No
Tampa-St. Petersburg-Clearwater, FL*	3 (75%)	3 (100%)	3 (100%)	2 (67%)	Yes
Baltimore-Columbia-Towson, MD	15 (36%)	41 (100%)	41 (100%)	24 (59%)	Yes
Dallas-Fort Worth-Arlington, TX	20 (48%)	41 (100%)	41 (100%)	23 (56%)	Yes
Houston-The Woodlands-Sugar Land, TX	19 (45%)	41 (100%)	41 (100%)	26 (63%)	Yes
Phoenix-Mesa-Scottsdale, AZ*	2 (50%)	3 (100%)	3 (100%)	3 (100%)	Yes
Denver-Aurora-Lakewood, CO*	2 (50%)	3 (100%)	3 (100%)	1 (33%)	Yes
Los Angeles-Long Beach-Anaheim, CA	23 (55%)	41 (100%)	41 (100%)	31 (76%)	Yes
San Francisco-Oakland-Hayward, CA	21 (50%)	41 (100%)	41 (100%)	14 (34%)	Yes
Riverside-San Bernardino-Ontario, CA*	3 (75%)	3 (100%)	3 (100%)	3 (100%)	Yes
Seattle-Tacoma-Bellevue WA	21 (50%)	41 (100%)	38 (93%)	12 (29%)	Yes
San Diego-Carlsbad, CA*	3 (75%)	3 (100%)	3 (100%)	3 (100%)	Yes
Urban Hawaii*	3 (75%)	3 (100%)	3 (100%)	1 (33%)	Yes
Urban Alaska*	2 (50%)	3 (100%)	3 (100%)	1 (33%)	Yes

*These areas were not included in the Phase I analysis data due to lack of published monthly data used to construct quarterly averages of Consumer Price Index data prior to 2018. At the time of this report, published monthly data needed to construct quarterly averages for these primary sampling units (PSUs) are only available for four quarters (the fourth quarter of 2017 to the third quarter of 2018).

Notes: PSU denotes “primary sampling unit.” *NOVER (POVER)* is the number (percentage) of quarters that percentage change in *erent* exceeds the percentage change in *SEHA*. *PWITHIN10(5)(1)* is the number of quarters that the absolute value of the difference between the percentage change in *erent* and the percentage change in *SEHA* is within 10(5)(1) percent. *PER10* is “Yes” if the simulated Fair Market Rent (FMR) calculated from *erent* is within 10-percentage points of the simulated FMR calculated from *SEHA*.

Exhibit 5-4 | Summary of Metrics Comparing the Axiometrics Rent Measure to the American Community Survey Measure for Core-Based Statistical Areas

Summary Metrics	Number (%) of CBSAs
<i>POVER</i> is between 40% and 60%	201 (79%)
<i>WITHIN10</i> is satisfied in all timepoints	160 (63%)
<i>PER10</i> is satisfied in all timepoints	186 (73%)
Met all three metrics above	109 (43%)

Notes: CBSAs denotes “Core-Based Statistical Areas.” *POVER* is the percentage of years that percentage change in *erent* exceeds the percentage change in *rentACS*. *PWITHIN10* is the number of years that the absolute value of the difference between the percentage change in *erent* and the percentage change in *rentACS* is within 10 percent. *PER10* is “Yes” if the simulated Fair Market Rent (FMR) calculated from *erent* is within 10 percentage points of the simulated FMR calculated from *rentACS*.

5.1.3 ASSESSMENT OF AXIOMETRICS DATA

The results of our analysis indicate that the rent estimate from the Axiometrics data is often comparable to the rent estimate from both BLS and ACS in metro areas. Therefore, we recommend that HUD further examine the potential of the Axiometrics data in improving predictions of FMR in metro areas.¹⁴

5.3 Zillow Data

The study team examined downloads available from Zillow (<https://www.zillow.com/research/data/>) and decided the best series for capturing changes in rents was ZRI Time Series: Multifamily, SFR, Condo/Co-op (downloaded as: Metro_Zri_AllHomesPlusMultifamily.csv). This is a dollar value monthly index of rents for 661 metropolitan areas (Zillow also provides a crosswalk to CBSAs, which we used to make the data comparable). The only variable in the data is the rent value (*ZRI*). The Zillow methodology seeks to control for other factors, such as living area and market conditions.

5.3.1 METHODS TO ASSESS THE DATA

To assess the Zillow data’s usefulness for FMR trend adjustments, the study team compared the metro region Zillow rent index (*ZRI*) to the following:

1. The measure of the *SEHA* from the CPI-U data file produced by BLS.
2. The estimate of median gross rent from the ACS (*rentACS*).

We detail our approach to both the BLS and ACS data below.

¹⁴ Several CBSAs have fewer than 30 properties (or projects) with rent values in every period—a limitation of the Axiometrics data (see column “*NOBS*” in tab “Yearly AXIO to ACS” in the accompanying Excel workbook). The study team found that by limiting the analysis to the 69 CBSAs with more than 30 Axiometrics projects, the metrics improved substantially. Compared with exhibit 5-4, the percentages are 72, 88, 86, and 62, respectively. Therefore, in section 6, the analysis is confined to these 69 CBSAs.

5.3.2 COMPARISON TO BUREAU OF LABOR STATISTICS DATA

To compare Zillow data to BLS data, the study team compared the quarterly and yearly percentage change in *ZRI* and *SEHA* for each of the 22 PSUs available in both datasets. The steps to create a quarterly and yearly series of *ZRI* for each PSU included the following:

1. Using a crosswalk provided by Zillow to match the metro region IDs in the Zillow data to CBSAs and creating a variable that indicates the CBSA of each Zillow metro region.
2. Using a crosswalk provided by HUD to match the CBSAs in the Zillow data to PSUs and creating a variable that indicates the PSU of each metro region (any metro region not in a PSU was removed from the data).
3. Creating a quarterly estimate of *ZRI* at the PSU level by averaging the monthly estimates for each quarter (33 in total from 2010 to 2018) in each PSU (23 in total).
4. Creating a yearly estimate of *ZRI* at the PSU level by averaging the monthly project-level estimates for each year (nine in total) in each PSU (23 in total).

The steps to create a quarterly and yearly series of *SEHA* for each PSU included the following:

1. Creating a quarterly estimate of *SEHA* at the PSU level by averaging the monthly PSU-level estimates for each quarter (the same 33 quarters as the Zillow data).
2. Creating a yearly estimate of *SEHA* at the PSU level by averaging the monthly PSU-level estimates for each year (the same nine as the Zillow data).

The study team calculated the percentage change in *ZRI* and *SEHA* for each PSU i in each quarter or year t from the preceding quarter or year (t_{-1}) with the following formula:

$$Change_{i,t} = \frac{X_{i,t} - X_{i,t-1}}{X_{i,t-1}} \times 100.$$

Thus, the study team created PSU-level percentage changes for *ZRI* and *SEHA* for each quarter and year available in the data. As above, nine PSUs only have data from BLS for the fourth quarter of 2017 and later; therefore, the percentage change estimates for these PSUs represent fewer timepoints. In addition, Zillow data does not have any data for timepoints earlier than November 2011 for the New York-Newark-Jersey City, NY-NJ-PA PSU. We indicate which PSUs have fewer data points in all results below.

Finally, the study team used the same metrics as above for each PSU to determine the comparability of the percentage change estimates of *ZRI* and *SEHA*. For reference, these metrics are the following—

- **NOVER/POVER**: the number and percentage of timepoints (quarters or years) for which the *ZRI* percentage change is higher than the *SEHA* percentage change. One would expect roughly half the timepoints to have a higher *ZRI* percentage change if *ZRI* and *SEHA* are comparable measures. If not, then there is evidence that *ZRI* is consistently either higher or lower than *SEHA* and that the measures are not very comparable.
- **PWITHIN10**: the number and percentage of timepoints for which the *ZRI* percentage change is within 10-percentage points (either above or below) of the *SEHA* percentage change. Since rents tend to be relatively stable, one would expect virtually all timepoints to show the *ZRI* and

SEHA percentage changes are within 10-percentage points of each other if the measures are comparable.

- **PWITHIN5**: the number and percentage of timepoints for which the *ZRI* percentage change is within 5 percentage points (either above or below) of the *SEHA* percentage change. This measure is more stringent than *PWITHIN10*, but one would still expect many of the timepoints to show the *ZRI* and *SEHA* percentage changes are within 5 percentage points of each other if the measures are comparable.
- **PWITHIN1**: the number and percentage of timepoints for which the *erent* percentage change is within 1 percentage point (either above or below) of the *SEHA* percentage change. This measure is very stringent and if timepoints show the *ZRI* and *SEHA* percentage changes are within 1 percentage point of each other, there is evidence that the measures are comparable.
- **PER10**: an indication (either “yes” or “no”) that shows whether an estimated rent value that accounts for the trend in *ZRI* is within 10 percent of an estimated rent value that accounts for the trend in *SEHA*. The study team calculated estimated rent values for each measure for each PSU by adding the percentage change for each measure from the earliest timepoint in the data to the latest timepoint to \$100. These calculated values are referred to as “simulated” FMRs. For example, if the percentage change was 3.4 percent for *ZRI* and 5.4 percent for *SEHA*, then the simulated FMRs would be \$103.40 and \$105.40, respectively. *PER10* indicates whether the simulated FMR from *ZRI* is within 10 percent of the value from *SEHA*.

We present the results for each of the metrics for each PSU below.

5.3.3 COMPARISON TO AMERICAN COMMUNITY SURVEY DATA

To compare Zillow data to ACS data, the study team compared the yearly percentage change in *ZRI* and *rentACS* for each of the 422 CBSAs available in both datasets. The steps to create a yearly series of *ZRI* for each CBSA included the following:

1. Using a crosswalk provided by Zillow to match the metro region IDs in the Zillow data to CBSAs and creating a variable that indicates the CBSA of each Zillow metro region.
2. Creating a yearly estimate of *ZRI* at the CBSA level by averaging the monthly project-level estimates for each year (eight in total, 2010 to 2017) in each CBSA (420 in total).

The study team downloaded the yearly estimates of *rentACS* for each CBSA from American Factfinder using table B25064. The study team then created the yearly percentage change in *erent* and *rentACS* and the metrics comparing the percentage change estimates using the same process that we describe above in 5.2.2. Importantly, ACS data did not contain data for all years for two CBSAs, and Zillow data did not contain data for all years for 77 CBSAs. We provide a list of these CBSAs and the years for which both ACS and Zillow have data for these CBSAs in the accompanying Excel spreadsheet.

5.3.4 RESULTS

Exhibit 5-5 shows the results of each of the metrics comparing quarterly estimates of *ZRI* from Zillow and *SEHA* from BLS. Unlike the results for the rent measure from the Axiometrics data, the evidence for the comparability of *ZRI* to *SEHA* is more mixed. The difference in the percentage change in *ZRI* and the percentage change in *SEHA* was within 5 percent for all quarters, and it was within 1 percent for

many of the quarters, which indicates a high level of comparability between the two measures. On the other hand, the *POVER* metric shows that *ZRI* was often lower than *SEHA*; in many of the PSUs, *ZRI* was higher than *SEHA* less than 40 percent of the time. Moreover, in 6 of the 23 PSUs, the simulated FMR using the trend in *ZRI* was not within 10 percent of the simulated FMR using the trend in *SEHA*.

Exhibit 5-6 shows a summary of the results of the metrics comparing yearly estimates of *ZRI* to the estimate of median gross rent, *rentACS*, from the ACS data. The full results are in the Excel spreadsheet. These results also show that the Zillow measure is less comparable than the Axiometrics measure. The Zillow rent estimate *ZRI* was higher than the ACS measure *rentACS* about half of the time in 65 percent of CBSAs (a majority), but this percentage is smaller than what the study team found when comparing the Axiometrics data (76 percent). Moreover, the difference in the percentage change in *ZRI* and the change in *rentACS* was within 10 percent for all years in only 50 percent of CBSAs (compared with 64 percent in the comparison of using *erent* from the Axiometrics data).

Exhibit 5-6 also shows that simulated FMR estimate using the rent in *ZRI* was within 10 percent of the estimate using the trend in *rentACS* in only a little more than half of the CBSAs (58 percent). Finally, the percent of CBSAs that met all three of the summary metrics was only 25 percent.

Exhibit 5-5 | Number (%) of Quarters in Each Primary Sampling Unit for the Metrics Comparing the Zillow Rent Measure to the Bureau of Labor Statistics Measure

PSU NAME	NOVER (POVER)	PWITHIN10	PWITHIN5	PWITHIN1	PER10
Boston-Cambridge-Newton, MA-NH	17 (52%)	32 (100%)	32 (100%)	16 (50%)	Yes
New York-Newark-Jersey City, NY-NJ-PA**	16 (55%)	28 (100%)	28 (100%)	15 (54%)	Yes
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	12 (36%)	32 (100%)	32 (100%)	23 (72%)	Yes
Chicago-Naperville-Elgin, IL-IN-WI	12 (36%)	32 (100%)	32 (100%)	19 (59%)	No
Detroit-Warren-Dearborn, MI	12 (36%)	32 (100%)	32 (100%)	12 (38%)	No
Minneapolis-St. Paul-Bloomington, MN-WI*	1 (20%)	4 (100%)	4 (100%)	4 (100%)	Yes
St. Louis, MO-IL*	0 (0%)	4 (100%)	4 (100%)	2 (50%)	Yes
Washington-Arlington-Alexandria, DC-VA-MD-WV	12 (36%)	32 (100%)	32 (100%)	22 (69%)	Yes
Miami-Fort Lauderdale-West Palm Beach, FL	12 (36%)	32 (100%)	32 (100%)	24 (75%)	Yes
Atlanta-Sandy Springs-Roswell, GA	11 (33%)	32 (100%)	32 (100%)	22 (69%)	No
Tampa-St. Petersburg-Clearwater, FL*	1 (20%)	4 (100%)	4 (100%)	4 (100%)	Yes
Baltimore-Columbia-Towson, MD	9 (27%)	32 (100%)	32 (100%)	17 (53%)	No
Dallas-Fort Worth-Arlington, TX	10 (30%)	32 (100%)	32 (100%)	19 (59%)	No
Houston-The Woodlands-Sugar Land, TX	7 (21%)	32 (100%)	32 (100%)	21 (66%)	No
Phoenix-Mesa-Scottsdale, AZ*	1 (20%)	4 (100%)	4 (100%)	2 (50%)	Yes
Denver-Aurora-Lakewood, CO*	1 (20%)	4 (100%)	4 (100%)	3 (75%)	Yes
Los Angeles-Long Beach-Anaheim, CA	11 (33%)	32 (100%)	32 (100%)	26 (81%)	Yes
San Francisco-Oakland-Hayward, CA	18 (55%)	32 (100%)	32 (100%)	18 (56%)	Yes
Riverside-San Bernardino-Ontario, CA*	3 (60%)	4 (100%)	4 (100%)	4 (100%)	Yes
Seattle-Tacoma-Bellevue, WA	12 (36%)	32 (100%)	32 (100%)	14 (44%)	Yes
San Diego-Carlsbad, CA*	1 (20%)	4 (100%)	4 (100%)	2 (50%)	Yes
Urban Hawaii*	1 (20%)	4 (100%)	4 (100%)	2 (50%)	Yes
Urban Alaska*	2 (40%)	4 (100%)	4 (100%)	3 (75%)	Yes

*These areas were not included in the Phase I analysis data due to lack of published monthly data used to construct quarterly averages of Consumer Price Index data prior to 2018. At the time of this report, published monthly data needed to construct quarterly averages for these primary sampling units (PSUs) are only available for four quarters (the fourth quarter of 2017 to the third quarter of 2018).

**PSU data are only available for 29 quarters (the fourth quarter of 2011 and later).

All other PSUs have data available for 33 quarters.

Notes: PSU denotes “primary sampling unit.” *NOVER (POVER)* is the number (percentage) of quarters that percentage change in *ZRI* exceeds the percentage change in *SEHA*. *PWITHIN10(5)(1)* is the number of quarters that the absolute value of the difference between the percentage change in *ZRI* and the percentage change in *SEHA* is within 10(5)(1) percent. *PER10* is “Yes” if the simulated Fair Market Rent (FMR) calculated from *ZRI* is within 10 percentage points of the simulated FMR calculated from *SEHA*.

Exhibit 5-6 | Summary of Metrics Comparing the Zillow Rent Measure to the American Community Survey Measure for Core-Based Statistical Areas

Summary Metrics	Number (%) of CBSAs
POVER is between 40% and 60%	274 (65%)
WITHIN10 is satisfied in all timepoints	210 (50%)
PER10 is satisfied in all timepoints	243 (58%)
Met all three metrics above	106 (25%)

Notes: CBSAs denotes “Core-Based Statistical Areas.” *POVER* is the percentage of years that percentage change in *ZRI* exceeds the percentage change in *rentACS*. *PWITHIN10* is the number of years that the absolute value of the difference between the percentage change in *ZRI* and the percentage change in *rentACS* is within 10 percent. *PER10* is “Yes” if the simulated Fair Market Rent (FMR) calculated from *ZRI* is within 10 percentage points of the simulated FMR calculated from *rentACS*.

5.3.5 ASSESSMENT OF ZILLOW DATA

The results of our analysis indicate that rent estimate from the Zillow data is less comparable than the Axiometrics estimate to the rent estimates from both BLS and ACS in metro areas. Therefore, we recommend that HUD exclude Zillow data from efforts to improve predictions of the FMR in metro areas.

5.4 Axiometrics Forecasts

As a test of the potential usability of the Axiometrics data, the study team used the quarterly CBSA-level series (through Q4 of 2016) to create an Autoregressive Integrated Moving Average (ARIMA) model for each of the 254 CBSAs and then forecast the four quarters of 2017 in each CBSA.¹⁵ Then, we compared the forecasts to the actual values in ACS in the following way:

1. Compute the annual Axiometrics *erent* in 2016 as the average of the four quarters in 2016 and compute the annual *erent* in 2017 as the average of the four forecasted values.
2. Compute the annual percentage change in *erent* in 2017 for each CBSA, using the values from step 1.
3. Compute the annual percentage change in *rentACS* in 2017 for each CBSA, using the ACS values for 2016 and 2017.
4. Compare the percentage changes expected from the Axiometrics data to the percentage changes in ACS.

We emphasize that we did not necessarily expect to find similar *levels* of rents. However, we do expect (assuming the Axiometrics data are forecastable) to find similar percentage changes in the rents.

¹⁵ Forecasts were made from models estimated with PROC X12 in SAS, using an automatic model selection with seasonality and a second difference to stationarize the growth rate of the series. The statement used to invoke the automatic model selection is based on “time series regression with ARIMA noise, missing values, and outliers” method by Gomez and Maravall (1997a, 1998). This algorithm automatically selects the order of the autoregressive and moving average parameters. The nonseasonal autoregressive (AR) and moving average (MA) orders are given by p , q , while the seasonal AR and MA orders are given by p and q respectively. The model selected using this method is of the form (p,d,q) and (P,D,Q) . The number of differences and seasonal differences are given by d and D . The estimated parameters are presented in the accompanying Excel workbook.

Moreover, to apply the Axiometrics data to the trend factor simply requires accurate percentage changes.

The comparisons are presented in the accompanying Excel spreadsheet. Exhibit 5-7 gives a summary of the findings. The first metric measures whether the percentage differences in the trends are within 10 percent, while the second measures the differences in the trend that are less than 0.1.

The findings from this test suggest that the Axiometrics forecasts are generally within acceptable limits.¹⁶

Exhibit 5-7 | Comparisons of Axiometrics Data Forecasts to American Community Survey Values

Summary Metrics	Number (%) of CBSAs
$\left \frac{(\widehat{erent}_{2017}/\widehat{erent}_{2016}) - (rentACS_{2017}/rentACS_{2016})}{rentACS_{2017}/rentACS_{2016}} \right < 10\%$	238 (94%)
$ (\widehat{erent}_{2017}/\widehat{erent}_{2016}) - (rentACS_{2017}/rentACS_{2016}) < 0.1$	235 (93%)
Met both metrics above	235 (93%)

Notes: ACS denotes “American Community Survey” and CBSAs denotes “Core-Based Statistical Areas.” *erent* is the quarterly average rent calculated from the Axiometrics data. *rentACS* is the annual median gross rent from the 1-Year ACS. A “hat” above a variable denotes a forecasted value, and a subscript denotes the year of the respective value.

¹⁶ Axiometrics produces quarterly forecasts of rents at the project level. The study team compared CBSA-level summaries of Axiometrics’ forecasts to our forecasts of trend factors. Essentially, the forecasts were the same, although Axiometrics did not produce forecasts for as many areas.

6 ACCURACY OF ALTERNATIVE MODELS

The study team’s trend factors for fiscal year 2019 (FY19), based on data from the Bureau of Labor Statistics (BLS), illustrate the potential variability in trend factors among the primary sampling units (PSUs) examined in section 4 (exhibit 6-1). Compared with HUD’s constant value (1.0572), the differences range from a high of +5.3 to -1.6 percentage points. However, the empirical implications of this heterogeneity are small. The largest differences in Fair Market Rents (FMRs) are in San Francisco (\$138) and Boston (\$48). Similarly, compared to HUD’s value, the trend factors for FY19 based on Axiometrics data is different by a maximum of 3.4 percentage points and a minimum of 0.2 percentage points. The largest difference in FMRs is in New York (\$40) and the smallest in Philadelphia (\$2).

Exhibit 6-1 | Comparison of Estimated Fair Market Rents for Select Primary Sampling Units, FY19

PSU	Area	ACS 2016 5-Year Est.	Recent Mover Adj. Factor	Annual 2016 to 2017 CPI Adj.	Trend Factor for FY19		FY19 2-Bedroom FMR			
					HUD	2M	HUD*	2M	2M	
					BLS (Phase I)	Axiometrics (Phase 2)		BLS (Phase 1)	Axiometrics (Phase 2)	
11A	Boston	1,376	1.260	1.0378	1.0572	1.084	1.044	1,902	1,950	1,878
12A	New York	1,324	1.273	1.0272		1.053	1.034	1,831	1,823	1,790
12B	Philadelphia	1,031	1.081	1.0179		1.041	1.055	1,200	1,181	1,197
23A	Chicago	980	1.136	1.0297		1.053	1.042	1,212	1,207	1,194
23B	Detroit	832	1.065	1.0319		1.053	1.076	967	963	984
35A	Washington, DC	1,423	1.078	1.0266		1.058	1.019	1,665	1,666	1,605
35B	Miami	1,114	1.176	1.0495		1.050	1.063	1,454	1,444	1,462
35C	Atlanta	901	1.103	1.0527		1.061	1.091	1,106	1,110	1,141
35E	Baltimore	1,131	1.093	1.0269		1.046	1.034	1,342	1,328	1,313
37A	Dallas	934	1.143	1.0634		1.071	1.055	1,201	1,216	1,198
49A	Los Angeles	1,301	1.242	1.0483		1.081	1.062	1,791	1,831	1,799
49B	San Francisco	1,769	1.415	1.0546		1.110	1.065	2,792	2,930	2,811
49D	Seattle	NA	NA	NA		1.088	1.078	1,899	NA	NA

Note: ACS denotes “American Community Survey,” BLS denotes “Bureau of Labor Statistics,” CPI denotes “Consumer Price Index,” FY denotes “fiscal year,” FMR denotes “Fair Market Rent,” NA denotes “data not available,” and PSU denotes “primary sampling units.” The FY19 2-Bedroom FMR for Boston and San Francisco do not reflect rent surveys.

***Source:** Fiscal Year 2019 Fair Market Rents Documentation System, HUD

https://www.huduser.gov/portal/datasets/fmr.html#2019_query.

To assess the accuracy of trend factors estimated in Phase I with those estimated in Phase II, the study team compared forecasted trend factors retrospectively by assuming a standard for accuracy was the median gross rent from the American Community Survey (ACS) in 2017 (exhibit 6-2). Usually, the study team’s Phase I models outperform the Phase II models, as the Phase II models are better than Phase I in only three PSUs (New York, Baltimore, and Seattle; exhibit 6-2). In New York, for example, the difference

in the Phase I estimate (compared to the ACS value) was 0.011, while the Phase II difference is just 0.006 and was similar for Baltimore and Seattle.

Exhibit 6-2 | Comparison of Change in Rent from 2016 to 2017

PSU	Name	Axiometrics (Phase II)	ACS	BLS (Phase I)	
				Rent	Rent and Utilities
11A	Boston	1.022	1.035	1.031	1.036
12A	New York	1.031	1.025	1.035	1.036
12B	Philadelphia	1.027	1.008	1.020	1.019
23A	Chicago	1.018	1.027	1.029	1.031
23B	Detroit	1.054	1.014	1.023	1.024
35A	Washington, DC	1.015	1.038	1.026	1.028
35B	Miami	1.022	1.054	1.062	1.062
35C	Atlanta	1.053	1.044	1.039	1.035
35E	Baltimore	1.008	1.012	1.024	1.021
37A	Dallas	1.051	1.061	1.052	1.047
49A	Los Angeles	1.069	1.052	1.052	1.050
49B	San Francisco	0.987	1.054	1.062	1.062
49D	Seattle	1.091	1.089	1.048	1.047

Notes: ACS denotes “American Community Survey,” BLS denotes “Bureau of Labor Statistics,” and PSU denotes “primary sampling units.” The 1-year trend for ACS denotes a ratio of 2017 actuals to 2016 actuals. The 1-year trend is calculated as a ratio of 2017 forecast values to 2016 actuals values for Axiometrics and BLS data.

Using the Axiometrics quarterly series through the third quarter of 2018, the study team used the X12 software to build forecasting models for each of the 69 Core-Based Statistical Areas (CBSAs) with more than 30 projects with a full set of monthly rent data. The models were then used to forecast four quarters, starting with the fourth quarter of 2018 through the third quarter of 2019. The ratio of the average of the forecasted values to the average of the actual Axiometrics values from the four quarters of 2017 provides an alternative set of trend factors with additional spatial resolution when compared to the PSU estimates (exhibit 6-3).¹⁷ These values provide some data on the consequences of using a single trend factor for all U.S. Census Bureau (Census) regions. For example, the *aggregate* difference in the South Region is 11 percent, while the West is 50 percent.¹⁸ On average, the Axiometrics series suggests a trend factor of approximately 3-percentage points higher than 1.0572 in the West.

¹⁷ Results for other areas are available at HUD’s request. The difference values are from HUD’s total trend factors: positive (negative) differences would be greater (less) if an adjustment reflecting utilities were made.

¹⁸ These figures are unweighted; hence, Coos Bay completely offsets Albuquerque in the aggregate.

Exhibit 6-3 | FY19 Core-Based Statistical Area Estimated Trend Factors by Region

Region	CBSA	Trend Factor	Difference From 1.0572
Northeast	Boston-Cambridge-Newton, MA-NH	1.044	- 0.013
	Hartford-West Hartford-East Hartford, CT	1.036	- 0.022
	New York-Newark-Jersey City, NY-NJ-PA	1.034	- 0.023
	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	1.055	- 0.002
	Pittsburgh, PA	1.039	- 0.018
	Providence-Warwick, RI-MA	1.082	0.025
South	Boston-Cambridge-Newton, MA-NH	1.044	- 0.013
	Atlanta-Sandy Springs-Roswell, GA	1.091	0.034
	Austin-Round Rock, TX	1.076	0.019
	Baltimore-Columbia-Towson, MD	1.034	- 0.023
	Birmingham-Hoover, AL	1.050	- 0.007
	Charleston-North Charleston, SC	1.050	- 0.007
	Charlotte-Concord-Gastonia, NC-SC	1.039	- 0.018
	Chattanooga, TN-GA	1.055	- 0.002
	Columbia, SC	1.041	- 0.016
	Dallas-Fort Worth-Arlington, TX	1.055	- 0.002
	Durham-Chapel Hill, NC	1.042	- 0.015
	El Paso, TX	1.075	0.017
	Greensboro-High Point, NC	1.082	0.025
	Greenville-Anderson-Mauldin, SC	1.084	0.027
	Gulfport-Biloxi-Pascagoula, MS	1.027	- 0.030
	Houston-The Woodlands-Sugar Land, TX	1.053	- 0.004
	Jackson, MS	1.029	- 0.028
	Jacksonville, FL	1.122	0.065
	Lexington-Fayette, KY	1.030	- 0.027
	Little Rock-North Little Rock-Conway, AR	1.020	- 0.037
	Louisville/Jefferson County, KY-IN	1.044	- 0.013
	Memphis, TN-MS-AR	1.087	0.030
	Miami-Fort Lauderdale-West Palm Beach, FL	1.063	0.006
	Nashville-Davidson-Murfreesboro-Franklin, TN	1.042	- 0.015
	Oklahoma City, OK	1.031	- 0.027
	Orlando-Kissimmee-Sanford, FL	1.129	0.071
	Palm Bay-Melbourne-Titusville, FL	1.188	0.131
	Raleigh, NC	1.059	0.001
	Richmond, VA	1.079	0.022
	San Antonio-New Braunfels, TX	1.034	- 0.023
	Tampa-St. Petersburg-Clearwater, FL	1.103	0.046
	Tulsa, OK	1.012	- 0.046
Virginia Beach-Norfolk-Newport News, VA-NC	1.046	- 0.011	
Washington-Arlington-Alexandria, DC-VA-MD-WV	1.019	- 0.038	
Wilmington, NC	1.043	- 0.014	
Winston-Salem, NC	1.095	0.038	
West	Albuquerque, NM	1.055	- 0.002
	Colorado Springs, CO	1.059	0.002
	Coos Bay, OR Micro Area	1.055	- 0.002

Region	CBSA	Trend Factor	Difference From 1.0572
	Denver-Aurora-Lakewood, CO	1.064	0.007
	Las Vegas-Henderson-Paradise, NV	1.174	0.117
	Los Angeles-Long Beach-Anaheim, CA	1.062	0.005
	Oxnard-Thousand Oaks-Ventura, CA	1.098	0.041
	Phoenix-Mesa-Scottsdale, AZ	1.147	0.090
	Portland-Vancouver-Hillsboro, OR-WA	1.055	- 0.002
	Riverside-San Bernardino-Ontario, CA	1.085	0.028
	Sacramento-Roseville-Arden-Arcade, CA	1.061	0.004
	Salt Lake City, UT	1.101	0.044
	San Diego-Carlsbad, CA	1.105	0.048
	San Francisco-Oakland-Hayward, CA	1.065	0.008
	San Jose-Sunnyvale-Santa Clara, CA	1.109	0.052
	Seattle-Tacoma-Bellevue, WA	1.078	0.021
	Tucson, AZ	1.118	0.061
	Chicago-Naperville-Elgin, IL-IN-WI	1.042	- 0.016
	Cincinnati, OH-KY-IN	1.061	0.004
	Cleveland-Elyria, OH	1.033	- 0.025
	Columbus, OH	1.085	0.027
	Dayton, OH	1.048	- 0.009
Midwest	Detroit-Warren-Dearborn, MI	1.076	0.019
	Indianapolis-Carmel-Anderson, IN	1.084	0.026
	Kansas City, MO-KS	1.064	0.007
	Minneapolis-St. Paul-Bloomington, MN-WI	1.084	0.027
	Omaha-Council Bluffs, NE-IA	1.046	- 0.011
	St. Louis, MO-IL	1.023	- 0.034

Note: CBSA denotes “Core-Based Statistical Area.”

Locations highlighted in exhibit 6-3 indicate areas with trend factors more than 5 percentage points greater than HUD’s current trend factor. In these areas, HUD’s estimated FMRs may be too low. However, the potential gains from using localized trend factors require consideration of a couple of implementation issues. First, HUD’s current methodology inflates ACS values to more current values using elements of the CPI-U—a likely reliable source of information. For most CBSAs, this source of information is not available, meaning that the commercial data will need to be used instead. Also, although the findings in section 5 indicate that this approach is reasonable, it is a significant departure from current policy. Second, HUD will need an expanded pull from the ACS to get baseline values for gross rents and the shares of rent and utilities for the additional areas. Third, numerous geographic areas will not be covered by commercial data. These areas will need FMRs based on other methods—most likely some version of the current method. Finally, the additional complexity of localization may require updating forecasting models on a regular basis, especially in light of the relatively short length of the commercial series, adding costs to implementing localization.

7 CONCLUSION

This research project investigated various avenues to which HUD could add geographic resolution to the trend factors and, perhaps, reduce concern about the accuracy of Fair Market Rents (FMRs). Specifically, the project has investigated alternative approaches to augmenting local market conditions into calculations of FMRs, starting with extensions of HUD’s existing methodology and followed by consideration of models that use alternative sources of data and empirical frameworks. The study team approached this project in two phases.

In Phase I, the study team relied on the Bureau of Labor Statistics’ (BLS) data on 22 distinct geographic areas to estimate different trend factors. This approach is different from HUD’s current methodology, which uses a *national* estimate of the trend factor for all geographic areas. We used the Autoregressive Integrated Moving Average (ARIMA) modeling technique to estimate the forecasts of local rent and utilities, based on which trend factors for each area were calculated. The study team began the modeling process by identifying the best model form for forecasting rent and utility for each geographic area, as well as determining an appropriate lag length for the exogenous variables. In selecting the best model form, we have used the information criterion method—specifically Schwarz Bayesian information criterion (SBC)—to select from a group of established autoregressive (AR) and moving average (MA) orders. We again used SBC to determine the appropriate lag length of each exogenous variable, in order to illustrate the delayed rent and utility responses to changes in these input variables. We have computed SBC using various AR and MA model orders, with an automated function used to determine the range of orders. Our approach produced three competing ARIMA models—a univariate time series model without exogenous variables (Pure Time Series [PTS]), a multivariate model with national exogenous variables (National Input Model [NIM]), and a multivariate model with local exogenous variables (Local Input Model [LIM])—based on which the identified orders of the AR and MA terms were estimated and forecasted rent and utility.

To evaluate each model’s forecast accuracy, we have compared the forecast errors of the models’ estimates using the Root Mean Square Error (RMSE). The results of this comparison showed that the NIM performs better for the rent models, while the PTS performs better than the competing NIM and LIM specifications for the utility models. The study team has selected model specifications based on lower RMSEs in more than 50 percent of the geographical areas. While there is no absolute criterion for a “good value” of RMSE that would make one model preferable over another, it is possible to compare the RMSEs in percentage terms for each geographic area. Suppose the RMSE of one model is over 10 percent lower than another: one could choose the model with the lower RMSE. However, there exists a tradeoff between model complexity and error measure. The study team needs to consider the complexity of a model before choosing one solely based on the error measures. Based on our preferred model, we have used a paired t-test to examine whether the errors of our local estimates significantly differ from the errors of the national estimate. The results of this test showed that the local geographic areas have forecast estimates significantly different from the national estimate. Following the forecasts for each geographical area, the study team was unable to develop forecasts for non-metropolitan regions in Phase I due to the limited availability of data.

In Phase II, the study team first assessed the usability of monthly rent series from Axiometrics and Zillow. The Axiometrics data were found to be more comparable to data from BLS and the American Community Survey (ACS) than with the Zillow data. The study team did not find reliable and consistent data on exogenous variables that may drive rental markets at the Core-Based Statistical Area (CBSA) level. Similarly, there does not appear to be a consistent source of forecasts already produced by states and/or local metropolitan governmental agencies. Subsequently, the Axiometrics data were used to forecast quarterly and annual trends in rents in 254 CBSAs, using PTS models and the X12 software package, to compare the forecasts to actual data from ACS and BLS. Areas with more than 30 sampled properties in the Axiometrics data were used to demonstrate the differences between estimated trend factors from the Axiometrics models to the existing trend factor for fiscal year 2019 (FY19). The Phase II analysis suggests that localization of trend factors is feasible and, in certain areas, may lead to more accurate trend factors.

The findings in this report provide some evidence that localizing the estimates of trend factors will improve the accuracy of FMRs. Carefully constructed time series models for 22 geographic areas generally reduce forecast error, when compared with a single value for all areas. Models of additional geographic areas, constructed with automated algorithms, also seem to offer more accurate trend factors in most areas; however, a perfect standard for determining accuracy is not available. Therefore, one general conclusion is that if accuracy is a primary concern, migrating away from a single trend factor to trend factors for several areas would be sound policy. The overall empirical impact of moving in this direction is relatively small, however. In other words, the empirical payoff to localization may not be obvious when first encountered.

On the other hand, there are clearly geographic areas where localization has relatively large empirical consequences. This suggests a hybrid approach to migrating toward localization by using the results from both phases of this research. This would involve using forecasts from the well-performing models based on BLS data and those based on Axiometrics data to estimate trend factors for numerous CBSAs. Areas without well-performing forecasting models will be assigned trend factors from either national or regional models. This process is not unlike what HUD now does in applying inflation factors.

These observations should be tempered by some enumeration of the limitations of the study. In Phase I of the research, the study team found a dearth of local data that may help forecast the relatively volatile utility series. Residential price for electricity was the only input variable used to forecast the utility series, but since the data were available only at the state level, it failed to capture the varying patterns in the response series. Additionally, exogenous variables related to the rent series, such as labor force participation, weekly wages, and employment in the manufacturing sector, were not available for the North, South, Midwest, and West regions. In Phase II, the analysis was confined primarily to data from Axiometrics once the study team eliminated Zillow sources from further consideration. Additionally, the study team did not find a stable relationship between rents and the Federal Housing Finance Agency House Price Index, eliminating another source of data. The Axiometrics data, while appealing, are based on a limited number of apartment sites and the monthly series is relatively short. In Phase II accuracy assessments, the study team used readily available estimates of median gross rent from ACS. A better standard for comparison would be estimates of 40th percentile rents paid by recent movers.

APPENDIX A. LAG SELECTION RESULTS FOR EXOGENOUS VARIABLES

Exhibit A-1 | Lag Selection for Civilian Employment Using Minimum Schwarz Bayesian Information Criterion (Panel A)

PSU	Area	Lags					
		1	2	3	4	5	6
0000	National	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
0100	North	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
0200	Midwest	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
0300	South	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
0400	West	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
S100	North Class A	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
S11A	Boston	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S12A	New York	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S12B	Philadelphia	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S200	Midwest Class A	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
S23A	Chicago	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S23B	Detroit	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S300	South Class A	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
S35A	Washington, DC	-11.516	-11.563	-11.567	-11.617	-11.584	-11.574
S35B	Miami	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S35C	Atlanta	-11.511	-11.515	-11.555	-11.595	-11.596	-11.592
S35E	Baltimore	-11.516	-11.563	-11.567	-11.617	-11.584	-11.574
S37A	Dallas	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S400	West Class A	-11.267	-11.245	-11.286	-11.264	-11.226	-11.181
S49A	Los Angeles	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S49B	San Francisco	-11.313	-11.290	-11.332	-11.312	-11.275	-11.231
S49D	Seattle	-11.511	-11.515	-11.555	-11.595	-11.596	-11.592

Exhibit A-1 reports the SBC for the optimal lag length of the independent variable (civilian employment) used in estimating the national input rent model.

Exhibit A-2 | Lag Selection for Civilian Employment Using Minimum Schwarz Bayesian Information Criterion (Panel B)

PSU	Area	Lags					
		7	8	9	10	11	12
0000	National	-11.187	-11.147	-11.104	-11.061	-11.065	-11.021
0100	North	-11.137	-11.096	-11.053	-11.011	-11.008	-10.964
0200	Midwest	-11.137	-11.096	-11.053	-11.011	-11.008	-10.964
0300	South	-11.137	-11.096	-11.053	-11.011	-11.008	-10.964
0400	West	-11.137	-11.096	-11.053	-11.011	-11.008	-10.964
S100	North Class A	-11.137	-11.096	-11.053	-11.011	-11.008	-10.964
S11A	Boston	-11.187	-11.147	-11.104	-11.061	-11.065	-11.021
S12A	New York	-11.187	-11.147	-11.104	-11.061	-11.065	-11.021
S12B	Philadelphia	-11.187	-11.147	-11.104	-11.061	-11.065	-11.021

PSU	Area	Lags					
		7	8	9	10	11	12
S200	Midwest Class A	- 11.137	- 11.096	- 11.053	- 11.011	- 11.008	- 10.964
S23A	Chicago	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S23B	Detroit	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S300	South Class A	- 11.137	- 11.096	- 11.053	- 11.011	- 11.008	- 10.964
S35A	Washington, DC	- 11.527	- 11.540	- 11.608	- 11.562	- 11.530	- 11.495
S35B	Miami	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S35C	Atlanta	- 11.553	- 11.567	- 11.559	- 11.512	- 11.510	- 11.532
S35E	Baltimore	- 11.527	- 11.540	- 11.608	- 11.562	- 11.530	- 11.495
S37A	Dallas	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S400	West Class A	- 11.137	- 11.096	- 11.053	- 11.011	- 11.008	- 10.964
S49A	Los Angeles	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S49B	San Francisco	- 11.187	- 11.147	- 11.104	- 11.061	- 11.065	- 11.021
S49D	Seattle	- 11.553	- 11.567	- 11.559	- 11.512	- 11.510	- 11.532

Exhibit A-1 reports the SBC for the optimal lag length of the independent variable (civilian employment) used in estimating the national input rent model.

Exhibit A-3 | Lag Selection for Residential Fixed Investment Using Minimum Schwarz Bayesian Information Criterion (Panel A)

PSU	Area	Lags					
		1	2	3	4	5	6
0000	National	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
0100	North	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
0200	Midwest	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
0300	South	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
0400	West	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
S100	North Class A	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
S11A	Boston	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S12A	New York	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S12B	Philadelphia	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S200	Midwest Class A	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
S23A	Chicago	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S23B	Detroit	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S300	South Class A	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
S35A	Washington, DC	- 7.115	- 7.176	- 7.146	- 7.100	- 7.099	- 7.070
S35B	Miami	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S35C	Atlanta	- 7.156	- 7.213	- 7.185	- 7.138	- 7.136	- 7.115
S35E	Baltimore	- 7.115	- 7.176	- 7.146	- 7.100	- 7.099	- 7.070
S37A	Dallas	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S400	West Class A	- 7.216	- 7.269	- 7.242	- 7.197	- 7.205	- 7.182
S49A	Los Angeles	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S49B	San Francisco	- 7.226	- 7.284	- 7.258	- 7.214	- 7.221	- 7.197
S49D	Seattle	- 7.156	- 7.213	- 7.185	- 7.138	- 7.136	- 7.115

Exhibit A-4 | Lag Selection for Residential Fixed Investment Using Minimum Schwarz Bayesian Information Criterion (Panel B)

PSU	Area	Lags					
		7	8	9	10	11	12
0000	National	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
0100	North	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
0200	Midwest	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
0300	South	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
0400	West	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
S100	North Class A	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
S11A	Boston	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S12A	New York	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S12B	Philadelphia	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S200	Midwest Class A	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
S23A	Chicago	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S23B	Detroit	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S300	South Class A	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
S35A	Washington, DC	-7.028	-7.043	-7.000	-6.961	-6.939	-6.958
S35B	Miami	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S35C	Atlanta	-7.071	-7.079	-7.040	-6.996	-6.971	-6.986
S35E	Baltimore	-7.028	-7.043	-7.000	-6.961	-6.939	-6.958
S37A	Dallas	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S400	West Class A	-7.141	-7.150	-7.107	-7.066	-7.043	-7.041
S49A	Los Angeles	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S49B	San Francisco	-7.155	-7.170	-7.128	-7.087	-7.059	-7.062
S49D	Seattle	-7.071	-7.079	-7.040	-6.996	-6.971	-6.986

Exhibit A-2 reports the SBC for the optimal lag length of the independent variable (residential fixed investment) used in estimating the national input rent model.

APPENDIX B. RESIDUAL CORRELATION DIAGNOSTIC RESULTS

Exhibit B-1 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Northeast—All Classes

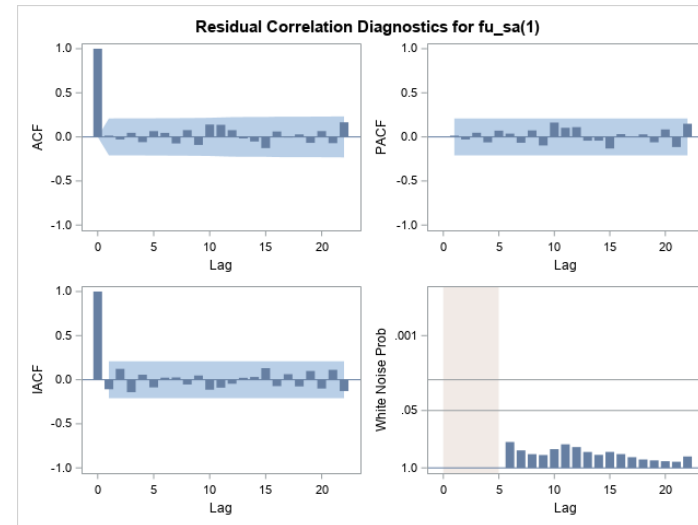
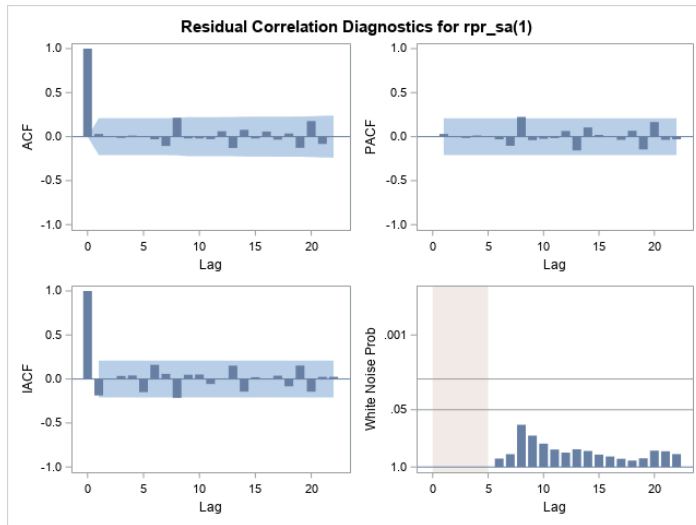


Exhibit B-2 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Midwest—All Classes

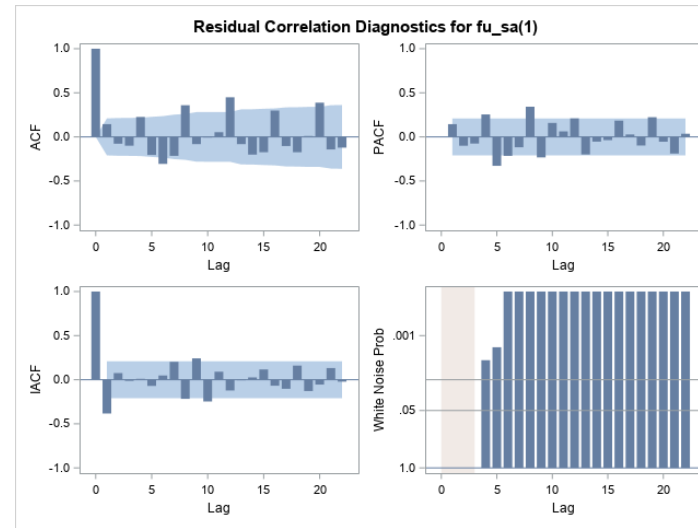
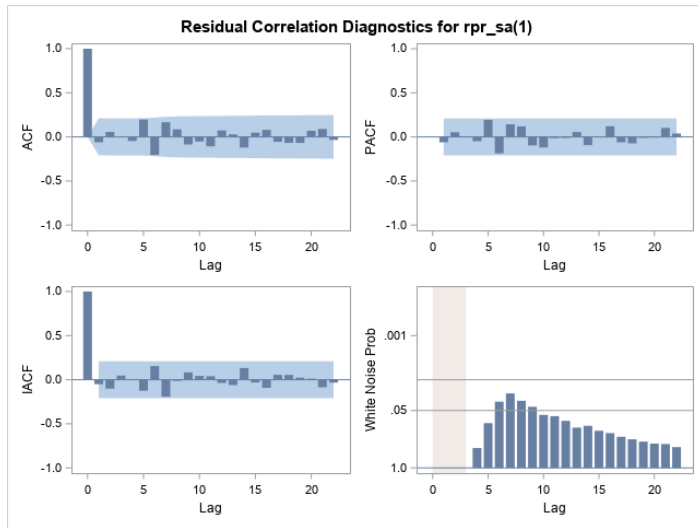


Exhibit B-3 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, South—All Classes

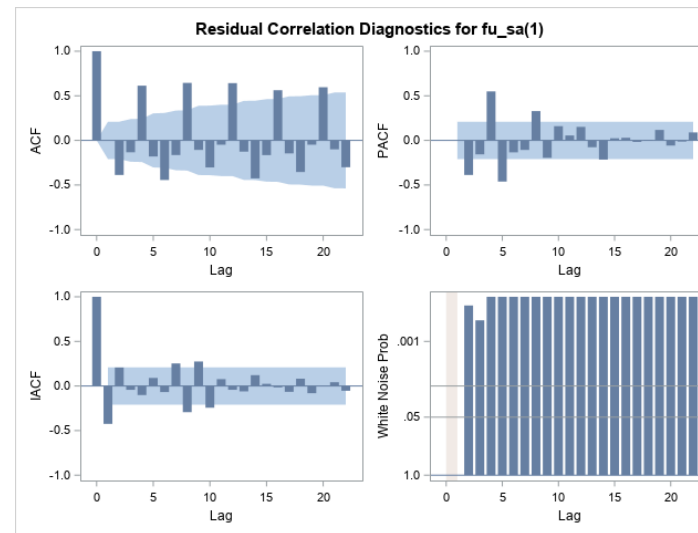
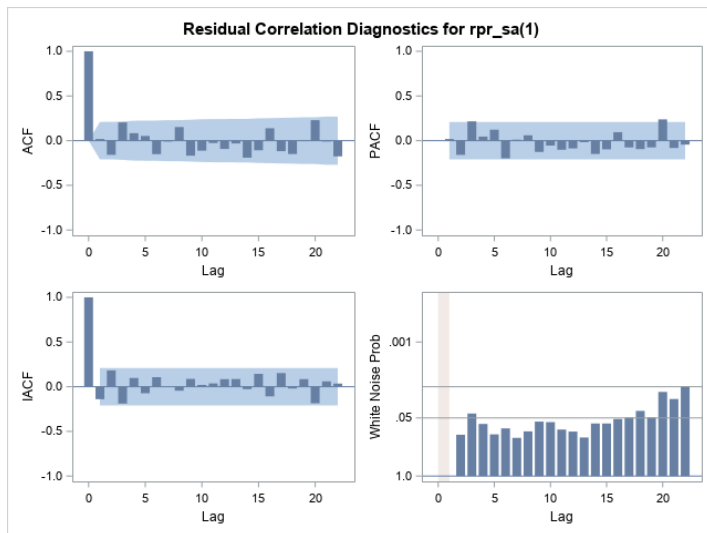


Exhibit B-4 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, West—All Classes

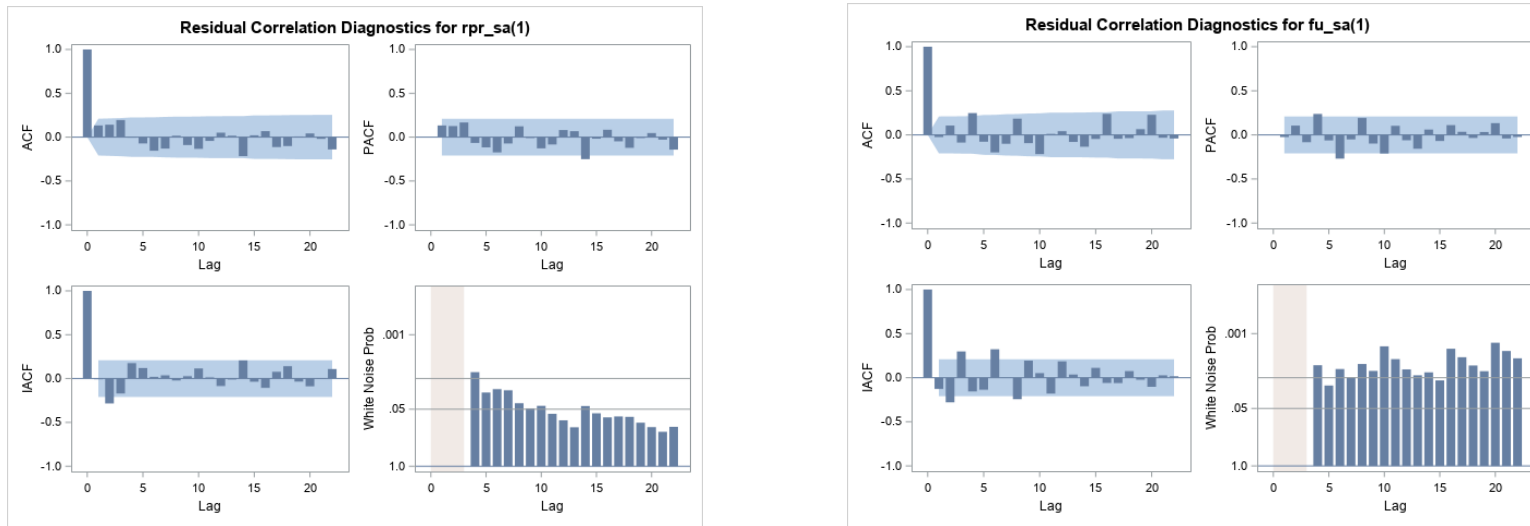


Exhibit B-5 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Northeast—Class A

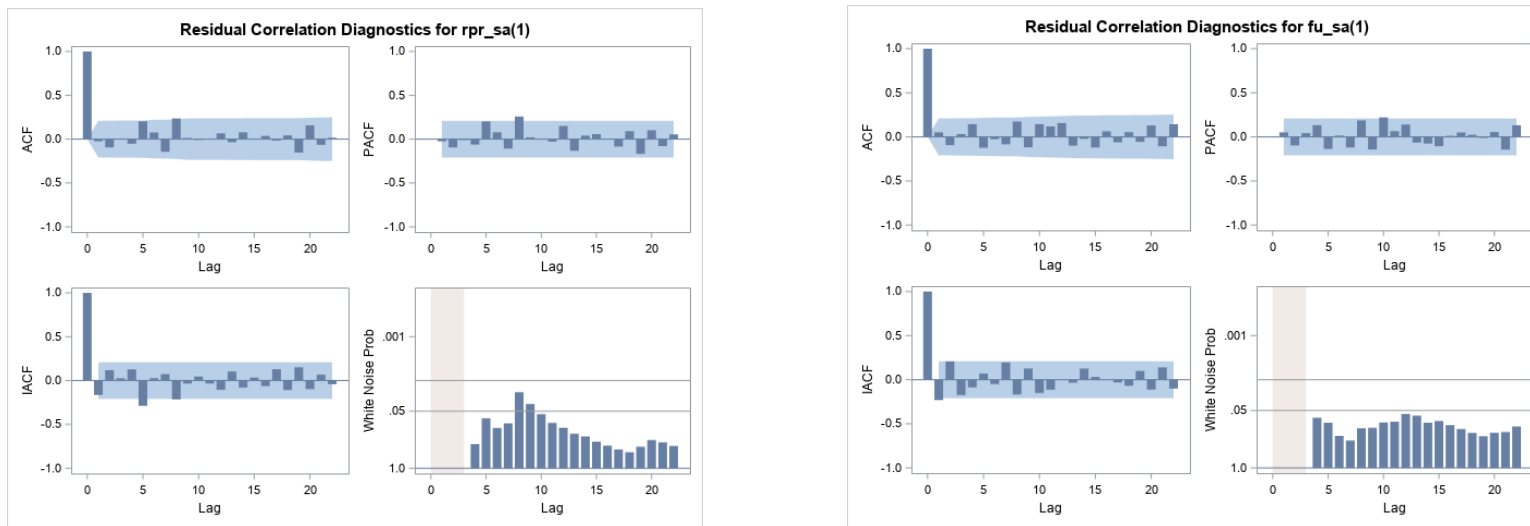


Exhibit B-6 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Boston

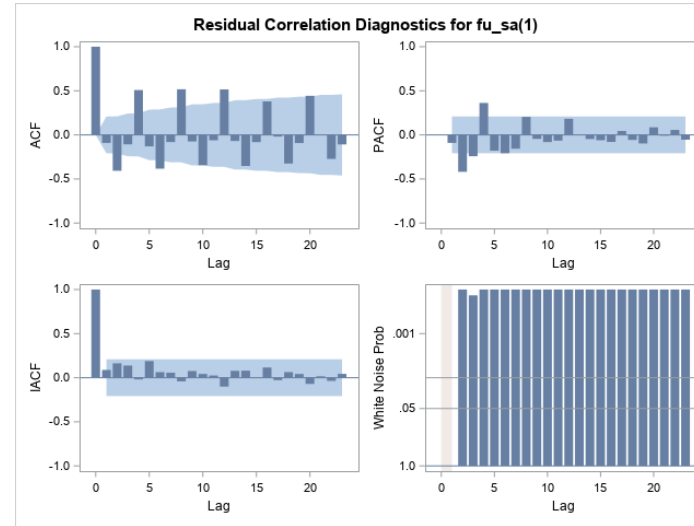
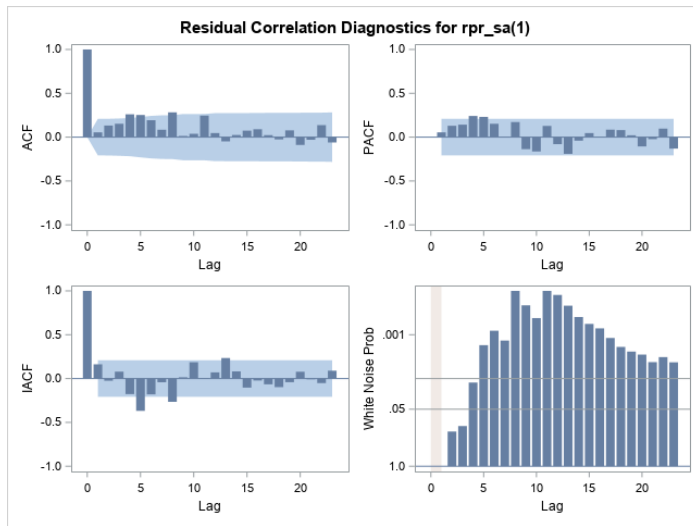


Exhibit B-7 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, New York

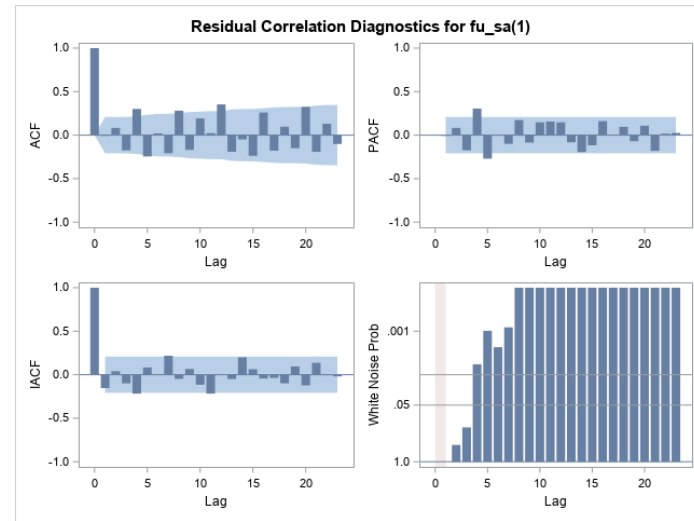
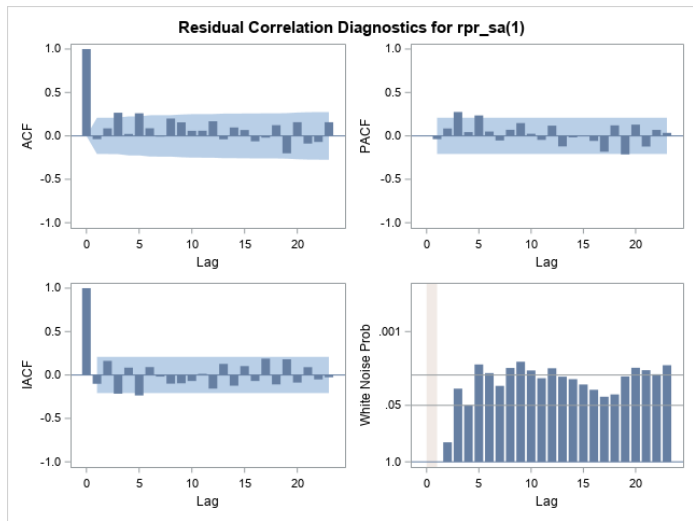


Exhibit B-8 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Philadelphia

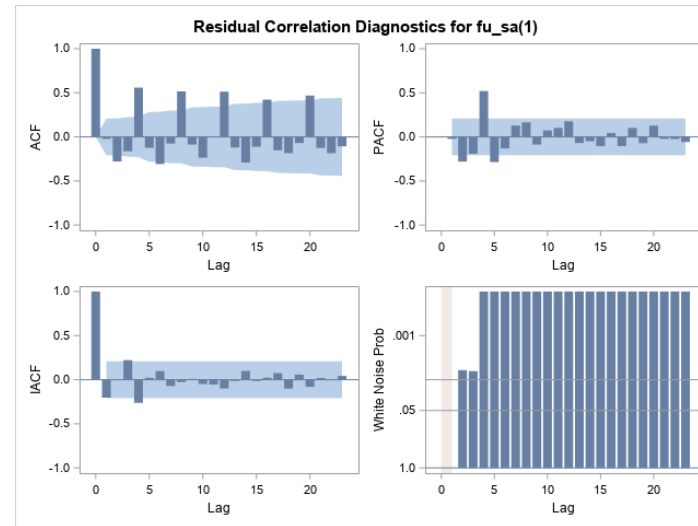
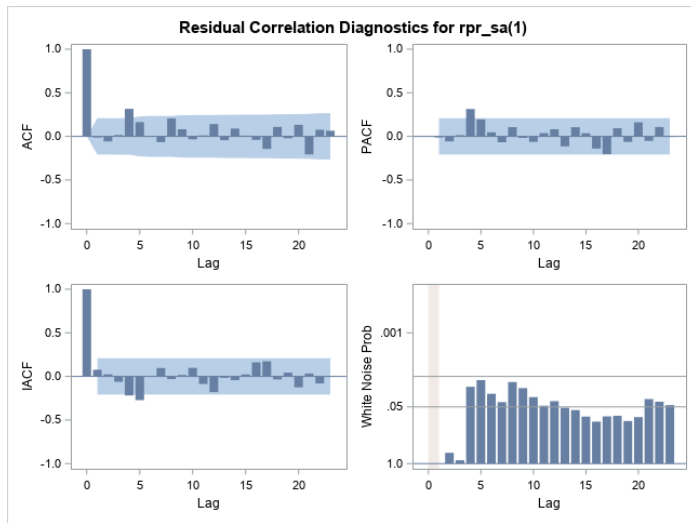


Exhibit B-9 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Midwest—Class A

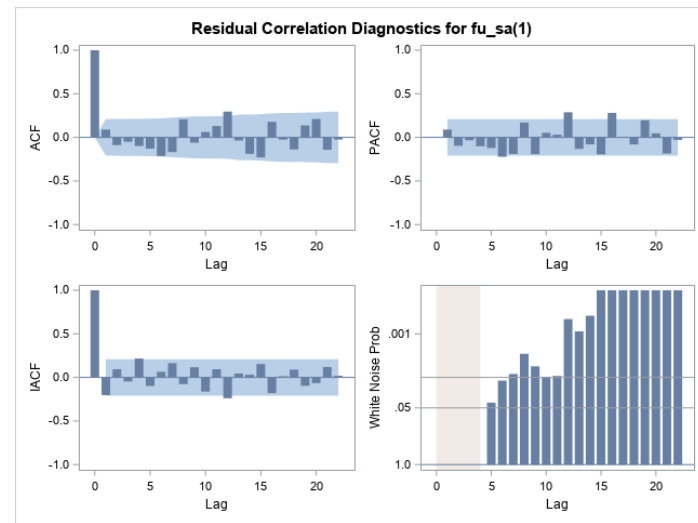
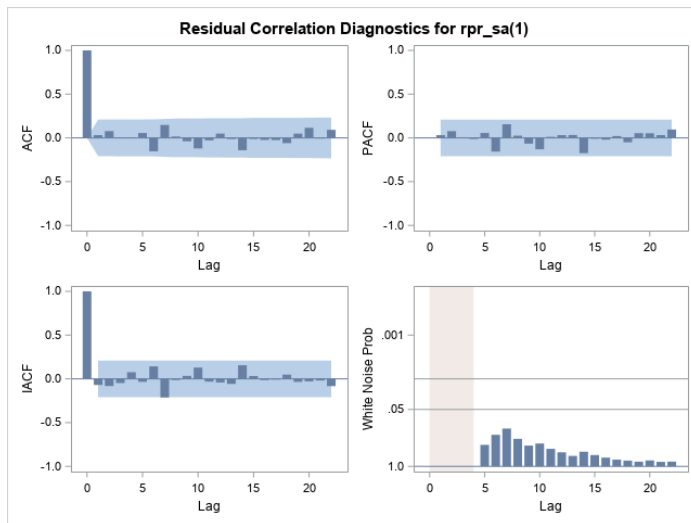


Exhibit B-10 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Chicago

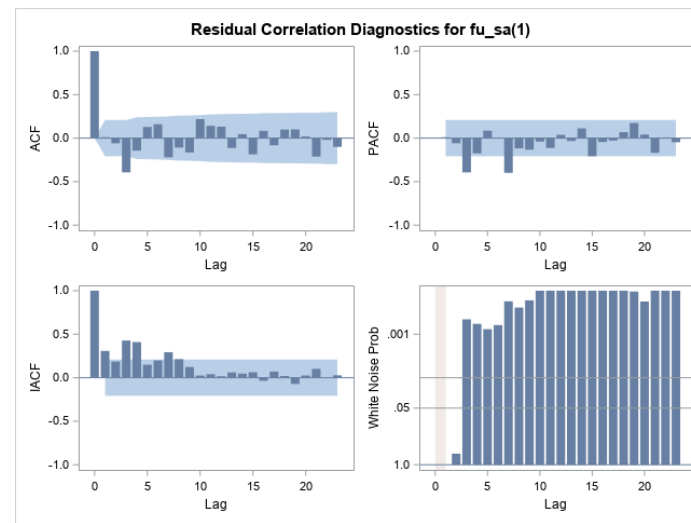
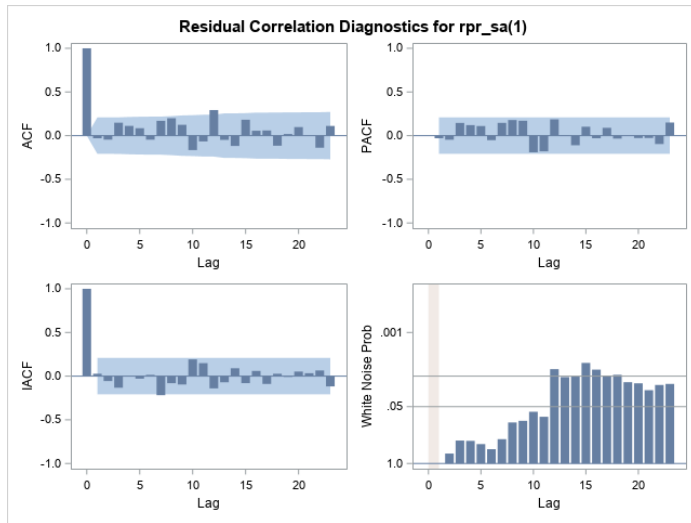


Exhibit B-11 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Detroit

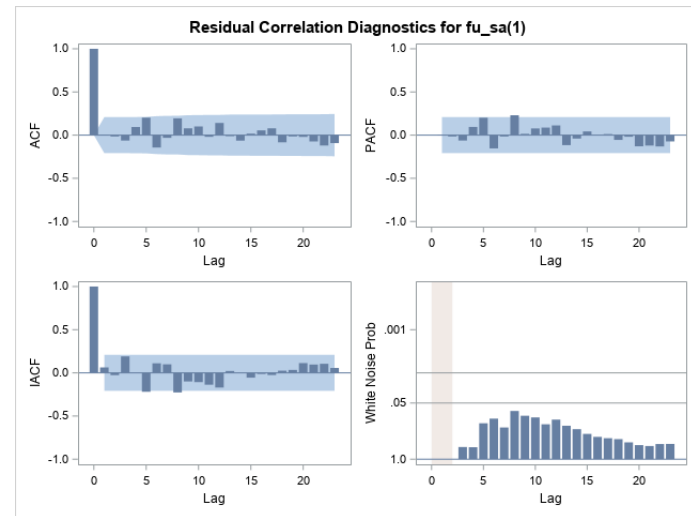
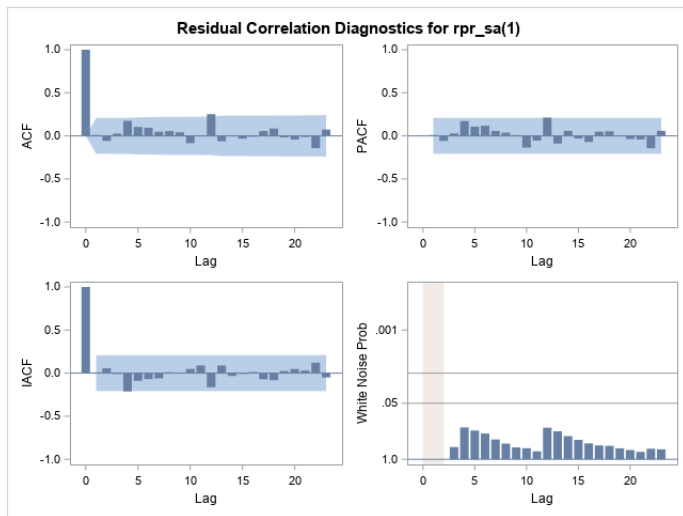


Exhibit B-12 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, South—Class A

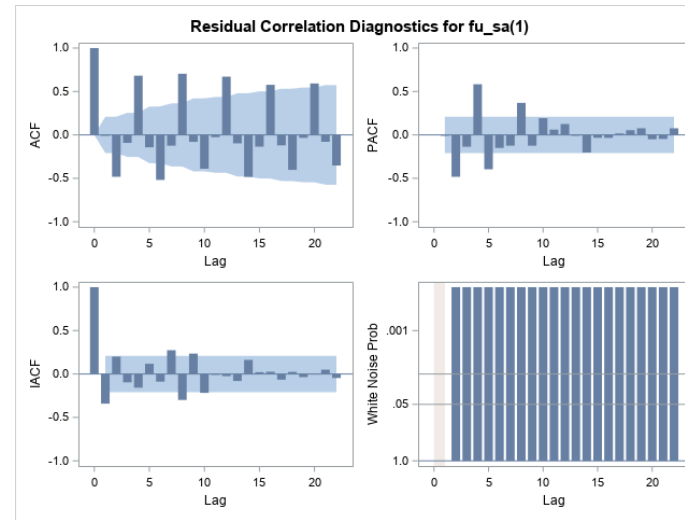
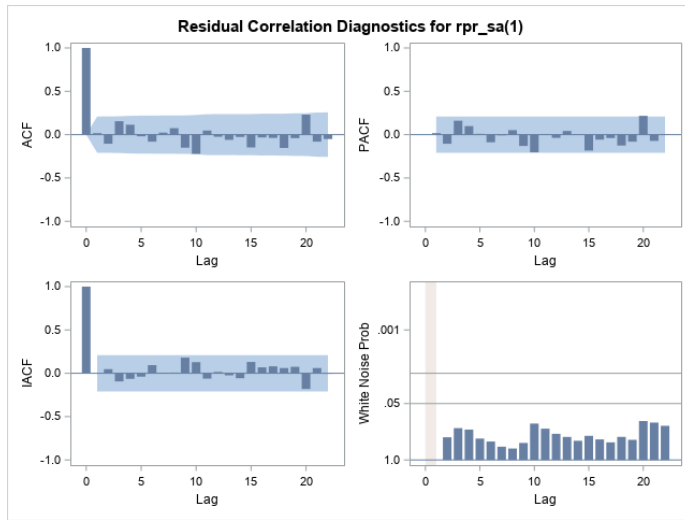


Exhibit B-13 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Washington, DC

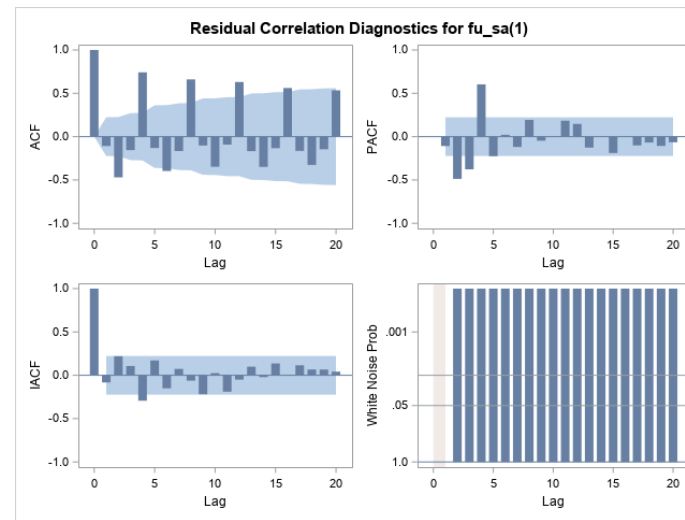
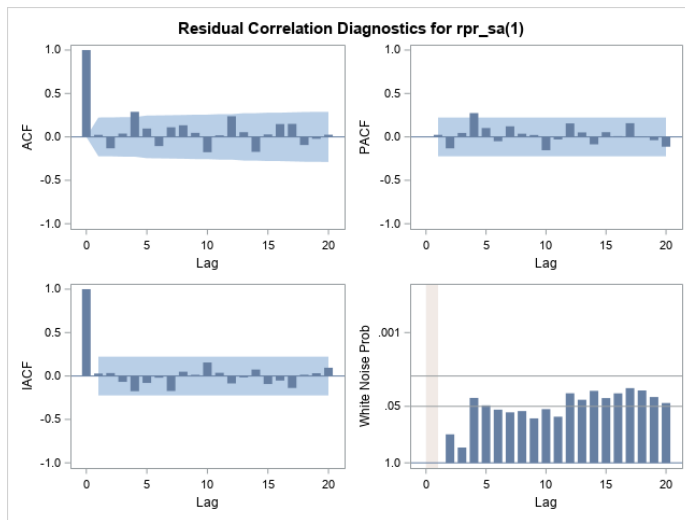


Exhibit B-14 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Miami

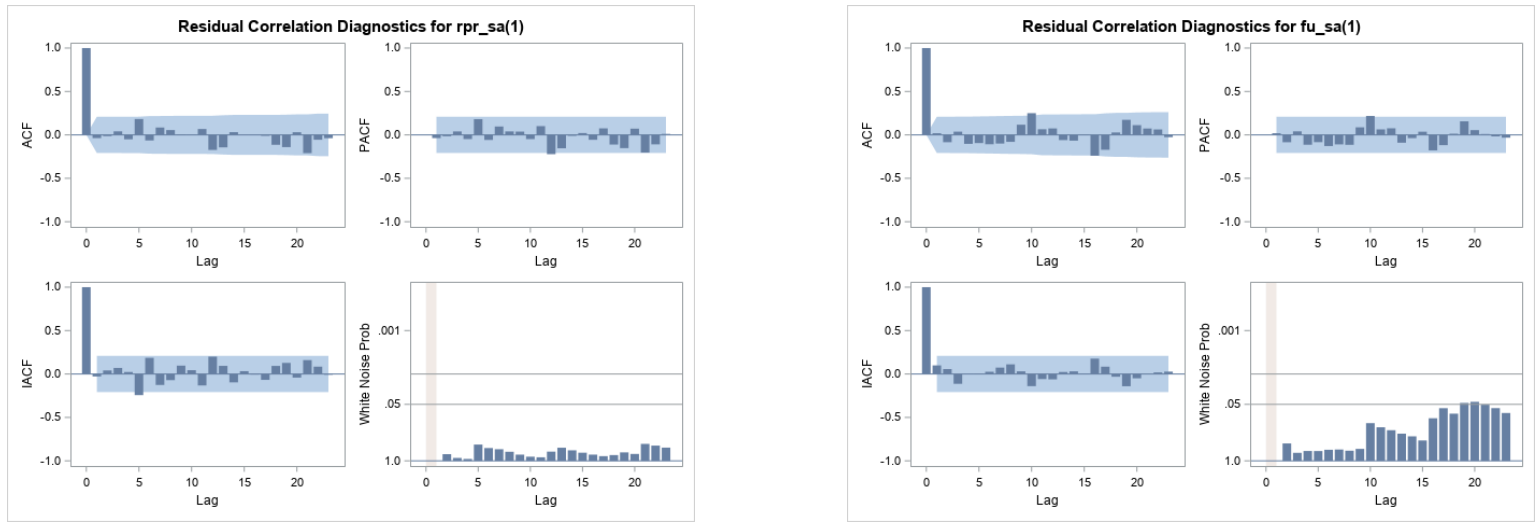


Exhibit B-15 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Atlanta

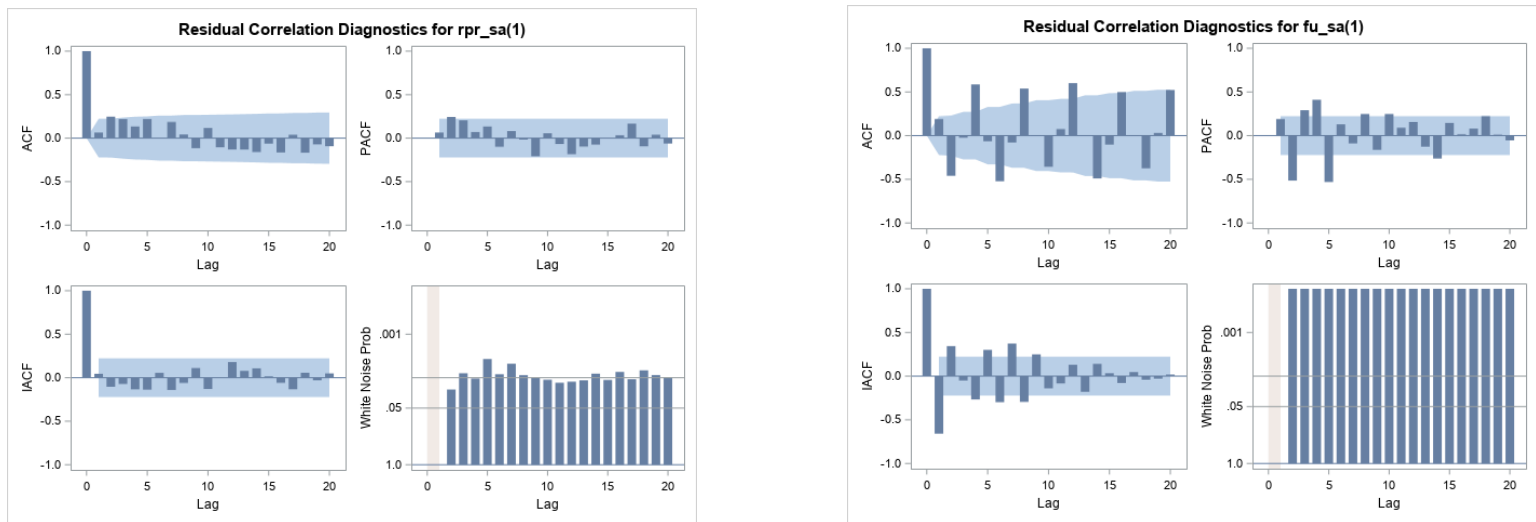


Exhibit B-16 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Baltimore

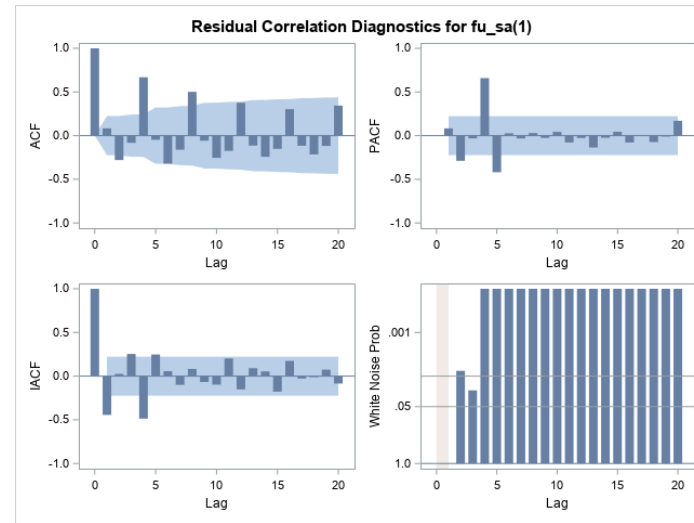
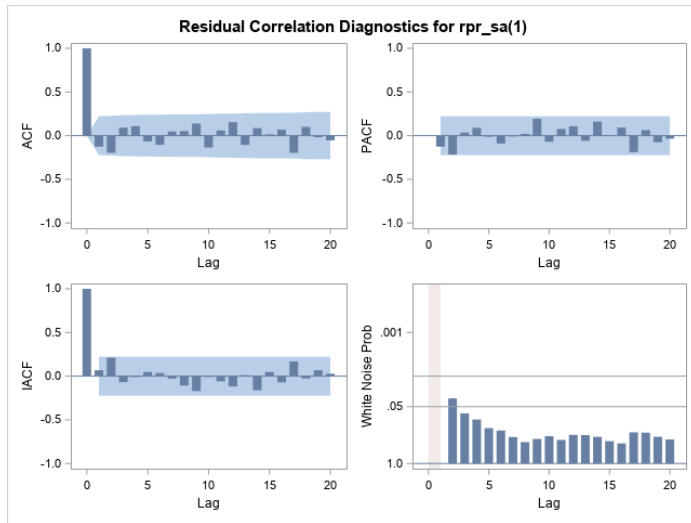


Exhibit B-17 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Dallas

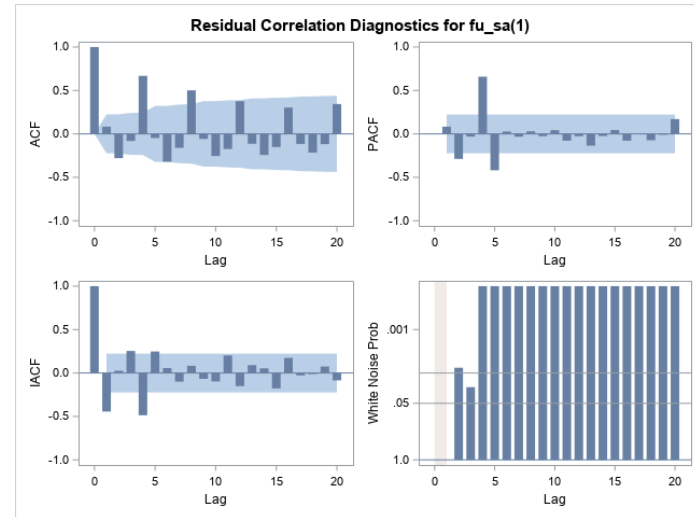
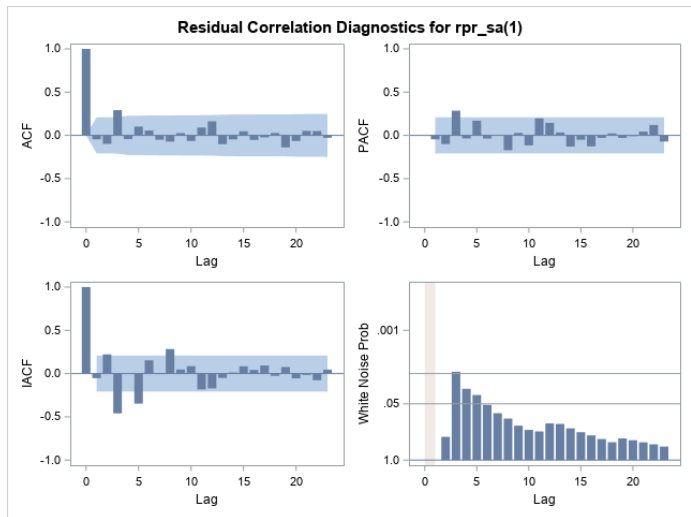


Exhibit B-18 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, West—Class A

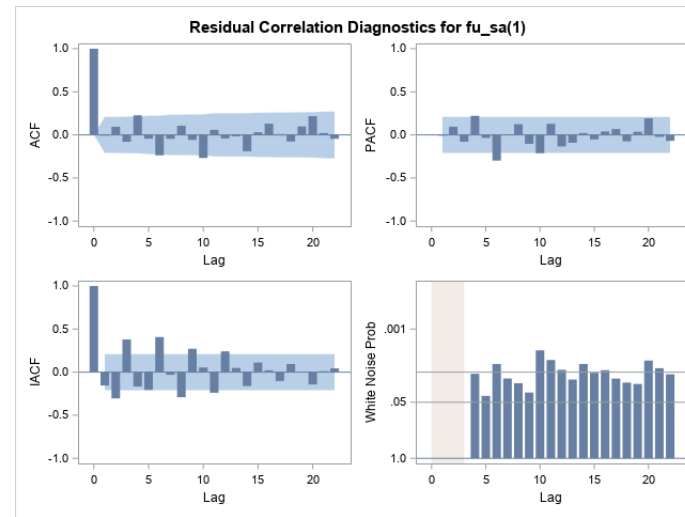
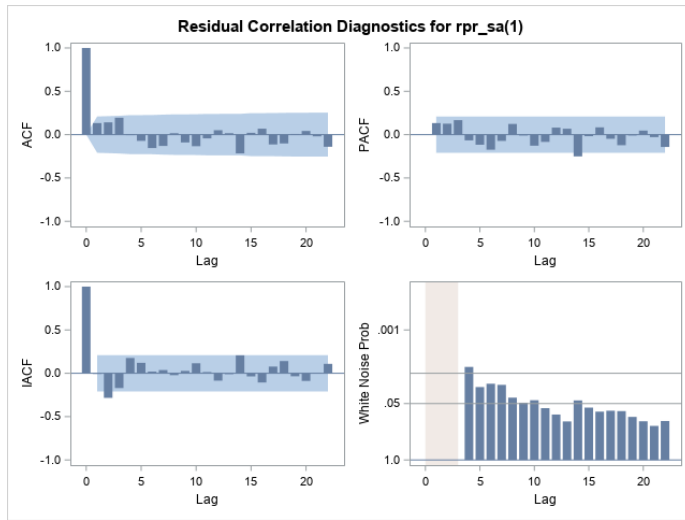


Exhibit B-19 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Los Angeles

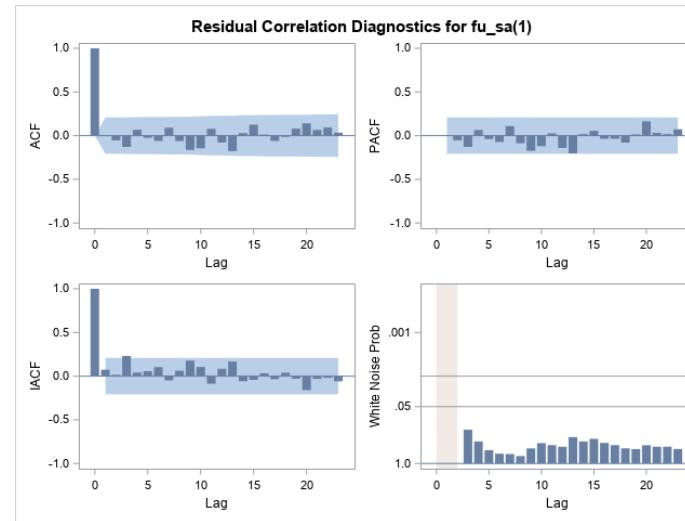
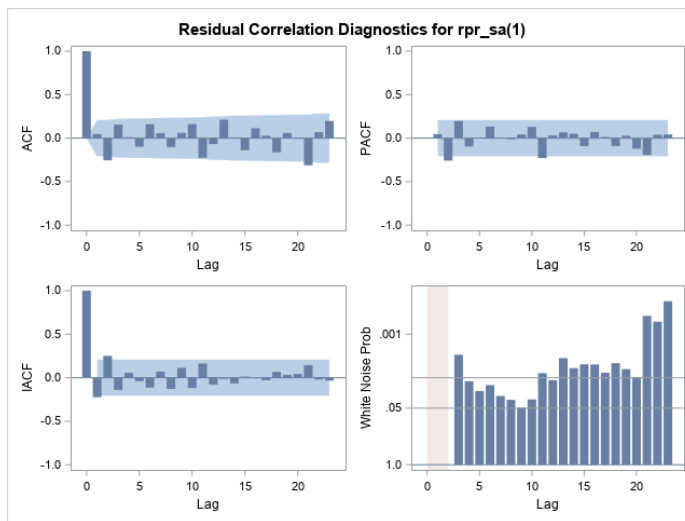


Exhibit B-20 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, San Francisco

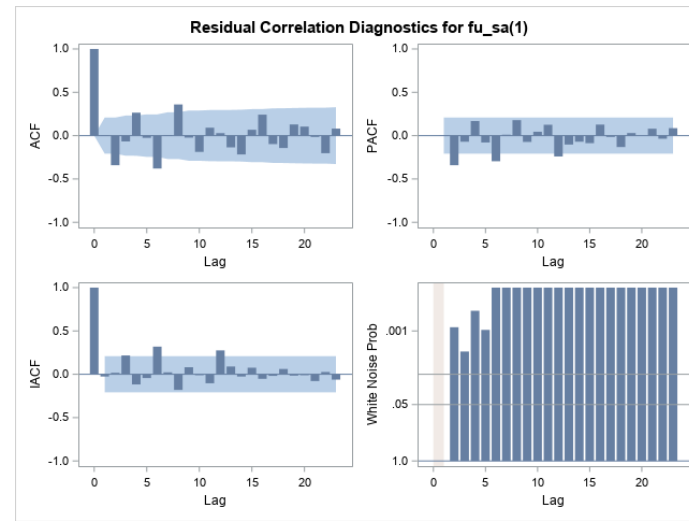
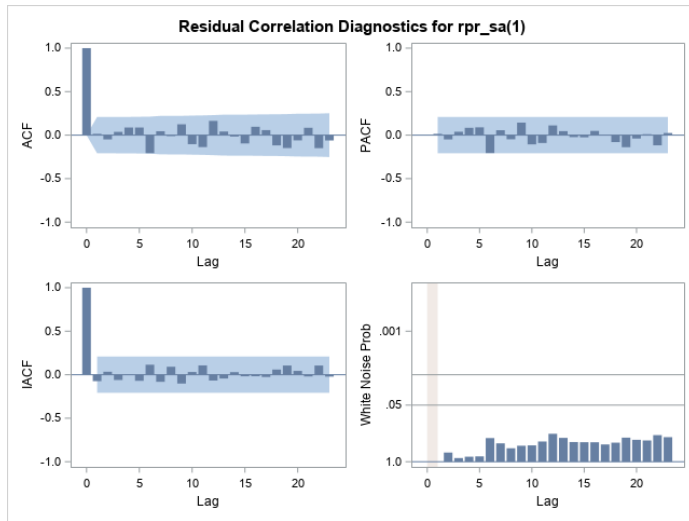
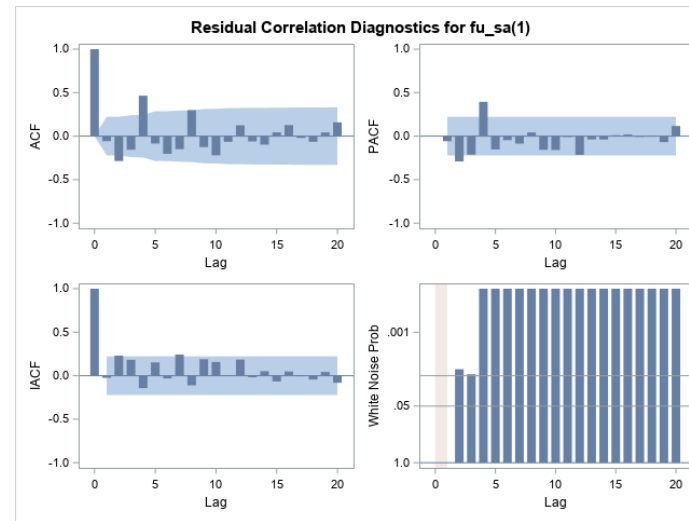
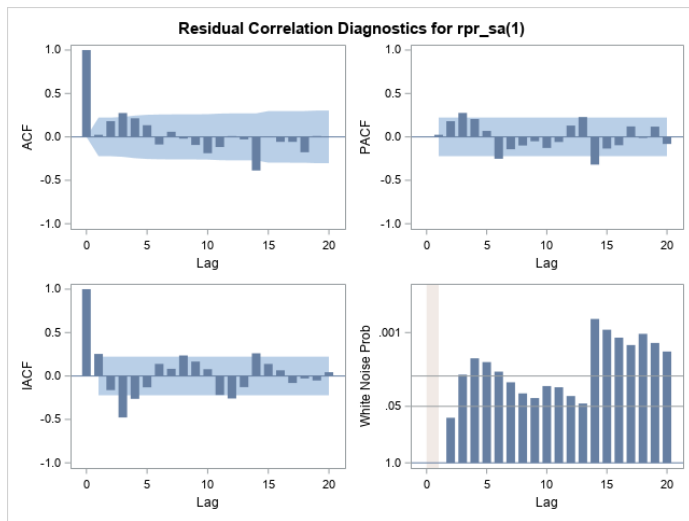


Exhibit B-21 | Residual Correlation Diagnostic for First-Differenced Rent and Utility Series, Seattle



APPENDIX C. PARAMETER ESTIMATE RESULTS FOR THE NATIONAL INPUT RENT MODEL

Exhibit C-1 | Parameter Estimates for Northeast—All Classes

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0084	0.0000	0
AR1,1	rpr_sa	0.5017	0.0000	1
AR1,2	rpr_sa	– 0.1695	0.1615	2
AR1,3	rpr_sa	0.3278	0.0017	3
NUM1	RES_FIXED_INV	0.0054	0.5860	2
NUM2	Civilian_Employment	– 0.0356	0.6581	3
Utility Model				
MU	fu_sa	0.0064	0.0598	0
AR1,1	fu_sa	0.0411	0.6983	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-2 | Parameter Estimates for Northeast—Midwest—All Classes

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0063	0.0000	0
AR1,1	rpr_sa	0.4956	0.0000	1
AR1,2	rpr_sa	– 0.2278	0.0604	2
AR1,3	rpr_sa	0.2572	0.0320	3
AR1,4	rpr_sa	0.2409	0.0286	4
NUM1	RES_FIXED_INV	– 0.0024	0.7442	2
NUM2	Civilian_Employment	0.1409	0.0155	3
Utility Model				
MU	fu_sa	0.0072	0.0381	0
AR1,1	fu_sa	0.0455	0.6596	1
AR1,2	fu_sa	– 0.0923	0.3299	2
AR1,3	fu_sa	– 0.3800	0.0001	3
AR1,4	fu_sa	0.2883	0.0054	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-3 | Parameter Estimates for South—All Classes

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0074	0.0000	0
AR1,1	rpr_sa	0.7648	0.0000	1
NUM1	RES_FIXED_INV	0.0075	0.3769	2
NUM2	Civilian_Employment	0.1770	0.0056	3
Utility Model				
MU	fu_sa	0.0070	0.0844	0
AR1,1	fu_sa	- 0.0149	0.8608	1
AR1,2	fu_sa	- 0.1931	0.0231	2
AR1,3	fu_sa	- 0.0956	0.2586	3
AR1,4	fu_sa	0.5759	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-4 | Parameter Estimates for West—All Classes

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0091	0.0000	0
AR1,1	rpr_sa	0.8904	0.0000	1
AR1,2	rpr_sa	- 0.4612	0.0005	2
AR1,3	rpr_sa	0.4114	0.0000	3
NUM1	RES_FIXED_INV	- 0.0081	0.3186	2
NUM2	Civilian_Employment	0.1777	0.0115	3
Utility Model				
MU	fu_sa	0.0087	0.0001	0
MA1,1	fu_sa	- 0.5383	0.0000	1
MA1,2	fu_sa	- 0.9230	0.0000	2
AR1,1	fu_sa	- 0.2403	0.0001	1
AR1,2	fu_sa	- 0.8861	0.0000	2

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-5 | Parameter Estimates for Northeast—Class A

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0076	0.0000	0
AR1,1	rpr_sa	0.3826	0.0004	1
AR1,2	rpr_sa	- 0.2163	0.0669	2
AR1,3	rpr_sa	0.3262	0.0034	3
AR1,4	rpr_sa	- 0.0285	0.8058	4
AR1,5	rpr_sa	0.3349	0.0031	5
NUM1	RES_FIXED_INV	0.0000	0.9956	2
NUM2	Civilian_Employment	0.0692	0.2895	3
Utility Model				
MU	fu_sa	0.0071	0.0496	0
MA1,1	fu_sa	- 0.9943	0.0013	1
AR1,1	fu_sa	- 0.7226	0.0000	1
AR1,2	fu_sa	0.2773	0.0000	2

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-6 | Parameter Estimates for Midwest—Class A

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0059	0.0000	0
AR1,1	rpr_sa	0.7060	0.0000	1
AR1,2	rpr_sa	- 0.4679	0.0001	2
AR1,3	rpr_sa	0.4772	0.0000	3
NUM1	RES_FIXED_INV	0.0009	0.8556	2
NUM2	Civilian_Employment	0.1212	0.0080	3
Utility Model				
MU	fu_sa	0.0070	0.0404	0
AR1,1	fu_sa	0.0521	0.6032	1
AR1,2	fu_sa	- 0.0814	0.3780	2
AR1,3	fu_sa	- 0.3649	0.0001	3
AR1,4	fu_sa	0.3615	0.0003	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-7 | Parameter Estimates for South—Class A

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0071	0.0000	0
AR1,1	rpr_sa	0.6676	0.0000	1
NUM1	RES_FIXED_INV	– 0.0148	0.0647	2
NUM2	Civilian_Employment	0.2178	0.0004	3
Utility Model				
MU	fu_sa	0.0069	0.0486	0
AR1,1	fu_sa	0.0263	0.7667	1
AR1,2	fu_sa	– 0.1684	0.0550	2
AR1,3	fu_sa	– 0.1260	0.1522	3
AR1,4	fu_sa	0.5450	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-8 | Parameter Estimates for West—Class A

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0084	0.0000	0
AR1,1	rpr_sa	0.8465	0.0000	1
AR1,2	rpr_sa	– 0.6543	0.0000	2
AR1,3	rpr_sa	0.6459	0.0000	3
NUM1	RES_FIXED_INV	– 0.0139	0.0680	2
NUM2	Civilian_Employment	0.1553	0.0218	3
Utility Model				
MU	fu_sa	0.0086	0.0000	0
AR1,1	fu_sa	0.2041	0.0451	1
AR1,2	fu_sa	– 0.3111	0.0023	2

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-9 | Parameter Estimates for Boston

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0080	0.0000	0
MA1,1	rpr_sa	- 0.5871	0.0000	1
NUM1	RES_FIXED_INV	0.0261	0.1490	2
NUM2	Civilian_Employment	0.0651	0.6806	3
Utility Model				
MU	fu_sa	0.0099	0.1013	0
AR1,1	fu_sa	0.4380	0.0000	1
AR1,2	fu_sa	- 0.6193	0.0000	2
AR1,3	fu_sa	0.2576	0.0232	3
AR1,4	fu_sa	0.1051	0.3267	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-10 | Parameter Estimates for New York

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0090	0.0000	0
MA1,1	rpr_sa	- 0.5757	0.0000	1
NUM1	RES_FIXED_INV	0.0057	0.5972	2
NUM2	Civilian_Employment	- 0.0556	0.5555	3
Utility Model				
MU	fu_sa	0.0062	0.1506	0
AR1,1	fu_sa	- 0.0243	0.8115	1
AR1,2	fu_sa	0.0448	0.6547	2
AR1,3	fu_sa	- 0.1686	0.0928	3
AR1,4	fu_sa	0.3094	0.0024	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-11 | Parameter Estimates for Philadelphia

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0062	0.0000	0
MA1,1	rpr_sa	- 0.3314	0.0016	1
NUM1	RES_FIXED_INV	- 0.0065	0.6796	2
NUM2	Civilian_Employment	0.0827	0.5323	3
Utility Model				
MU	fu_sa	0.0058	0.0825	0
MA1,1	fu_sa	- 0.3947	0.0131	1
AR1,1	fu_sa	- 0.1734	0.2139	1
AR1,2	fu_sa	- 0.1236	0.1620	2
AR1,3	fu_sa	- 0.2189	0.0192	3
AR1,4	fu_sa	0.5189	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-12 | Parameter Estimates for Chicago

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0065	0.0000	0
MA1,1	rpr_sa	- 0.3554	0.0005	1
NUM1	RES_FIXED_INV	0.0124	0.3297	2
NUM2	Civilian_Employment	0.2430	0.0249	3
Utility Model				
MU	fu_sa	0.0062	0.4067	0
MA1,1	fu_sa	- 0.2577	0.0116	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-13 | Parameter Estimates for Detroit

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0048	0.0000	0
MA1,1	rpr_sa	- 0.3014	0.0040	1
MA1,2	rpr_sa	0.3293	0.0017	2
NUM1	RES_FIXED_INV	0.0390	0.0045	2
NUM2	Civilian_Employment	0.3002	0.0192	3
Utility Model				
MU	fu_sa	0.0084	0.0066	0
MA1,1	fu_sa	- 0.0992	0.3467	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-14 | Parameter Estimates for Washington, DC

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0087	0.0000	0
AR1,1	rpr_sa	0.4967	0.0000	1
NUM1	RES_FIXED_INV	- 0.0206	0.2064	2
NUM2	Civilian_Employment	0.2896	0.0263	3
Utility Model				
MU	fu_sa	0.0071	0.0366	0
AR1,1	fu_sa	- 0.2097	0.0115	1
AR1,2	fu_sa	- 0.2106	0.0079	2
AR1,3	fu_sa	- 0.2314	0.0040	3
AR1,4	fu_sa	0.6900	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-15 | Parameter Estimates for Baltimore

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0081	0.0000	0
MA1,1	rpr_sa	- 0.6654	0.0000	1
NUM1	RES_FIXED_INV	- 0.0117	0.5978	2
NUM2	Civilian_Employment	0.5928	0.0021	3
Utility Model				
MU	fu_sa	0.0073	0.3891	0
AR1,1	fu_sa	- 0.0641	0.3948	1
AR1,2	fu_sa	- 0.1163	0.1046	2
AR1,3	fu_sa	- 0.1653	0.0229	3
AR1,4	fu_sa	0.7301	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-16 | Parameter Estimates for Miami

Parameter	Variable	Estimate	p-value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0082	0.0000	0
AR1,1	rpr_sa	0.8011	0.0000	1
NUM1	RES_FIXED_INV	0.0211	0.1407	2
NUM2	Civilian_Employment	- 0.0015	0.9891	3
Utility Model				
MU	fu_sa	0.0050	0.1202	0
MA1,1	fu_sa	- 0.2410	0.0186	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its p-value was less than or equal to 5 percent.

Exhibit C-17 | Parameter Estimates for Atlanta

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0036	0.0138	0
MA1,1	rpr_sa	- 0.6989	0.0000	1
NUM1	RES_FIXED_INV	0.0628	0.0318	2
NUM2	Civilian_Employment	0.5303	0.0153	3
Utility Model				
MU	fu_sa	0.0091	0.0164	0
AR1,1	fu_sa	- 0.1811	0.0519	1
AR1,2	fu_sa	- 0.3307	0.0004	2
AR1,3	fu_sa	- 0.2162	0.0211	3
AR1,4	fu_sa	0.5591	0.0000	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-18 | Parameter Estimates for Dallas

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0075	0.0000	0
AR1,1	rpr_sa	0.6139	0.0000	1
NUM1	RES_FIXED_INV	0.0051	0.8198	2
NUM2	Civilian_Employment	- 0.0328	0.8516	3
Utility Model				
MU	fu_sa	0.0060	0.2004	0
AR1,1	fu_sa	0.0189	0.8486	1
AR1,2	fu_sa	- 0.2122	0.0293	2
AR1,3	fu_sa	- 0.1436	0.1407	3
AR1,4	fu_sa	0.3700	0.0002	4

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-19 | Parameter Estimates for Los Angeles

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0082	0.0001	0
MA1,1	rpr_sa	0.3979	0.0013	1
AR1,1	rpr_sa	0.9244	0.0000	1
NUM1	RES_FIXED_INV	0.0161	0.1526	2
NUM2	Civilian_Employment	0.2424	0.0033	3
Utility Model				
MU	fu_sa	0.0081	0.0158	0
MA1,1	fu_sa	- 0.3324	0.0008	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-20 | Parameter Estimates for San Francisco

Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0098	0.0000	0
AR1,1	rpr_sa	0.8090	0.0000	1
NUM1	RES_FIXED_INV	0.0028	0.8581	2
NUM2	Civilian_Employment	0.3128	0.0087	3
Utility Model				
MU	fu_sa	0.0108	0.0000	0
MA1,1	fu_sa	0.0341	0.7382	1
MA1,2	fu_sa	0.2889	0.0046	2

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

Exhibit C-21 | Parameter Estimates for Seattle

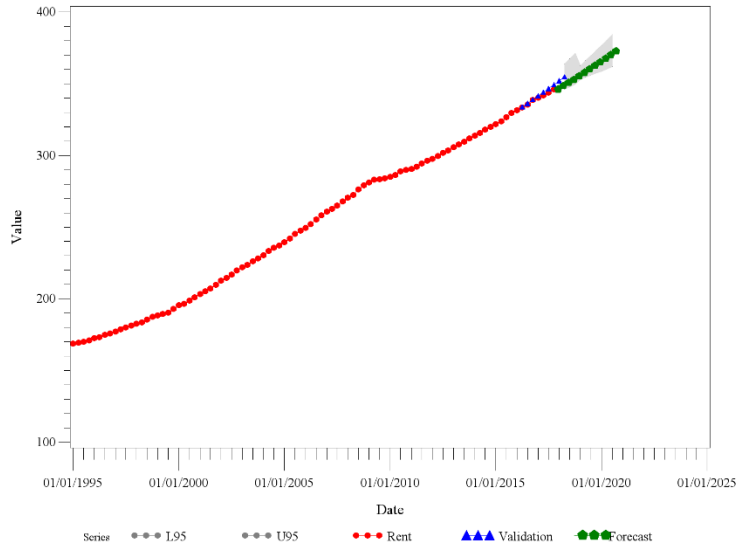
Parameter	Variable	Estimate	<i>p</i> -value	Lag/Shift
Rent Model				
MU	rpr_sa	0.0090	0.0000	0
MA1,1	rpr_sa	- 0.7814	0.0000	1
NUM1	RES_FIXED_INV	- 0.0752	0.0005	2
NUM2	Civilian_Employment	0.4563	0.0180	3
Utility Model				
MU	fu_sa	0.0097	0.0017	0
MA1,1	fu_sa	- 0.3813	0.0003	1

Notes: MU denotes the mean term; AR denotes the autoregressive parameters; MA is the moving average parameter; rpr_sa is the dependent variable on which the rent model was run; fu_sa is the dependent variable on which the utility model was run; RES_FIXED_INV and Civilian_Employment are used as independent variables in the national input rent model. An estimate is statistically significant if its *p*-value was less than or equal to 5 percent.

APPENDIX D. FORECAST RESULTS

Exhibit D-1 | Forecasts of Quarterly Rent and Utility, Northeast—All Classes

Rent Validation and Forecast for North



Utilities Validation and Forecast for North

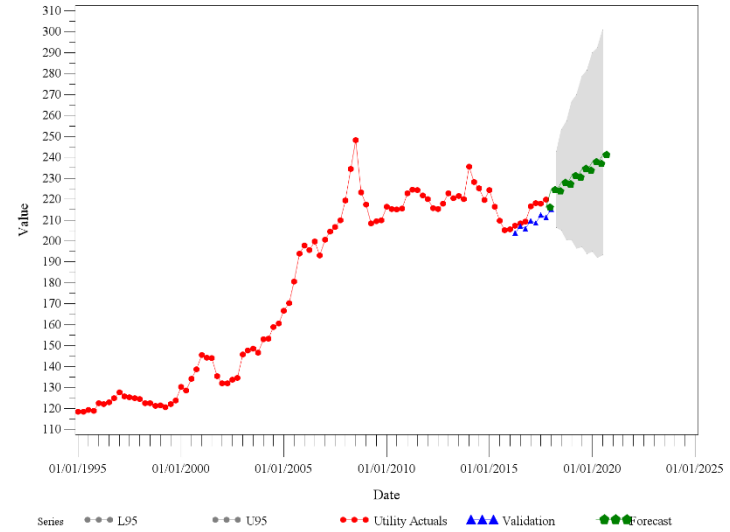
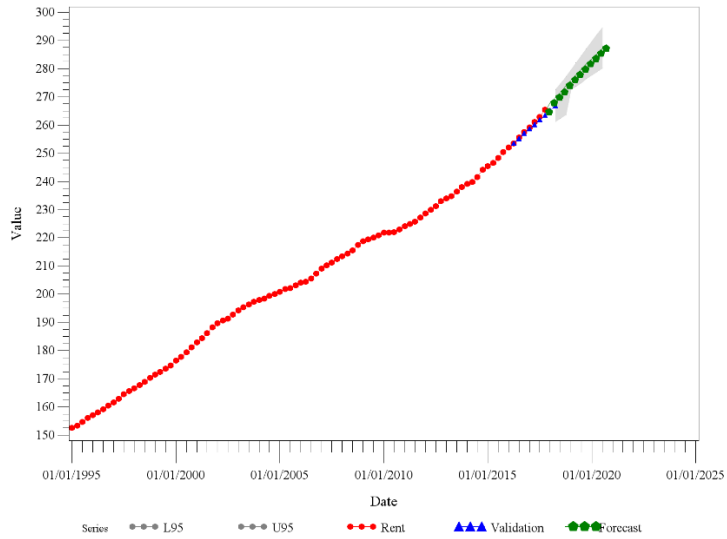


Exhibit D-2 | Forecasts of Quarterly Rent and Utility, Midwest—All Classes

Rent Validation and Forecast for Midwest



Utilities Validation and Forecast for Midwest

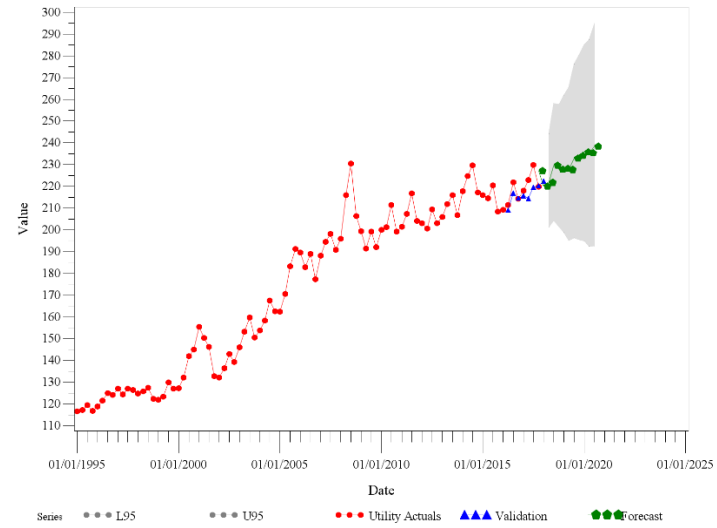
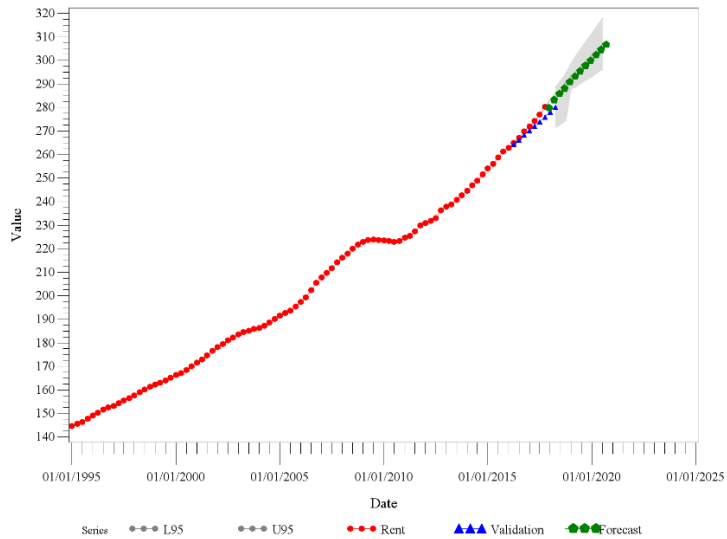


Exhibit D-3 | Forecasts of Quarterly Rent and Utility, South—All Classes

Rent Validation and Forecast for South



Utilities Validation and Forecast for South

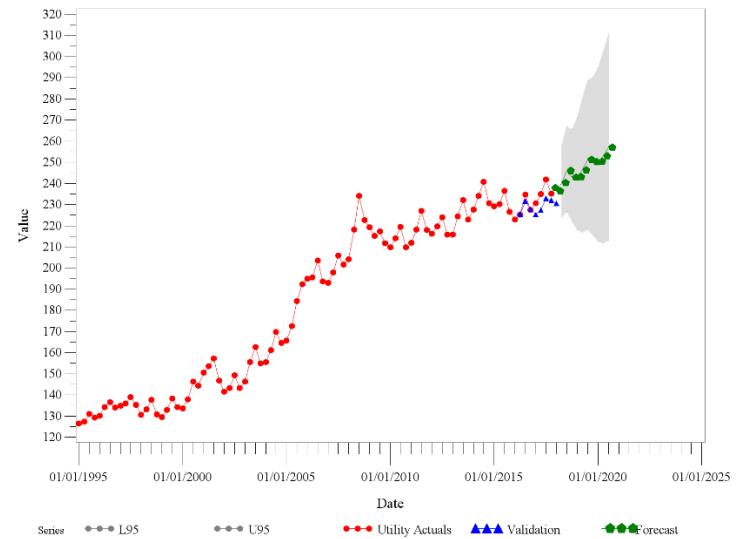


Exhibit D-4 | Forecasts of Quarterly Rent and Utility, West—All Classes

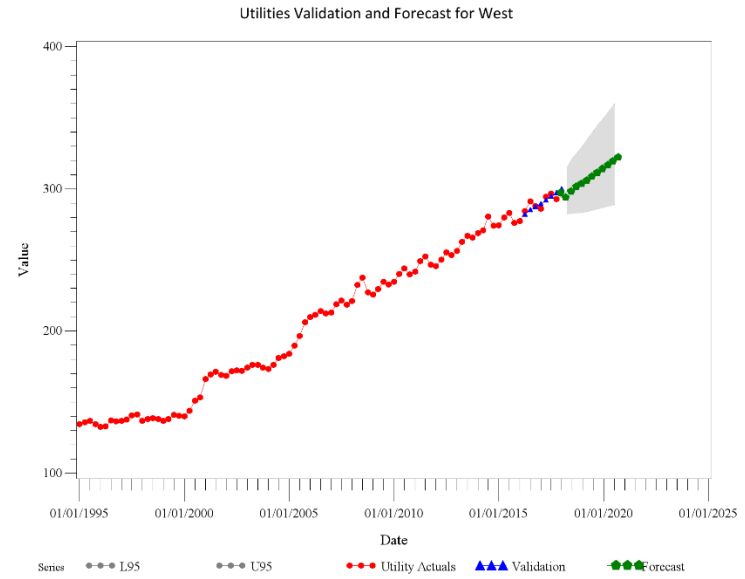
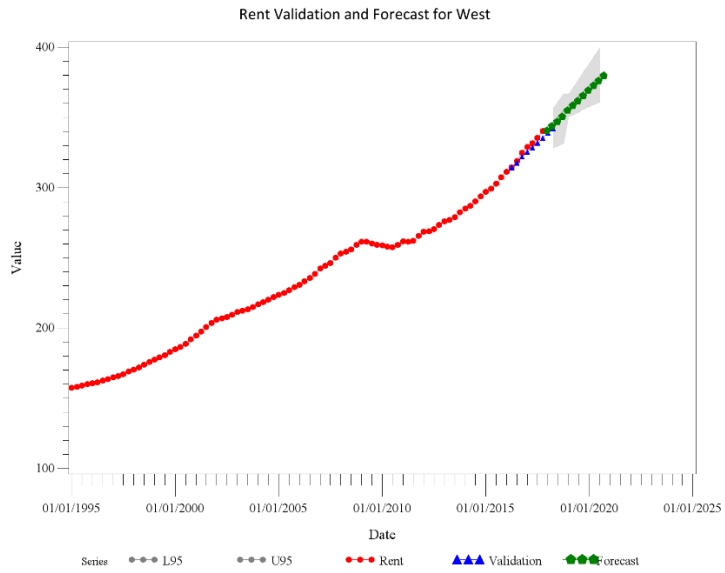


Exhibit D-5 | Forecasts of Quarterly Rent and Utility, Northeast—Class A

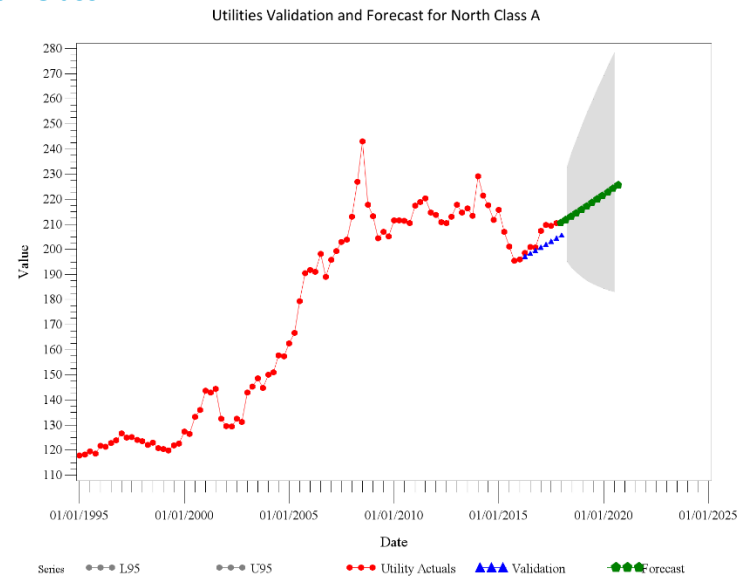
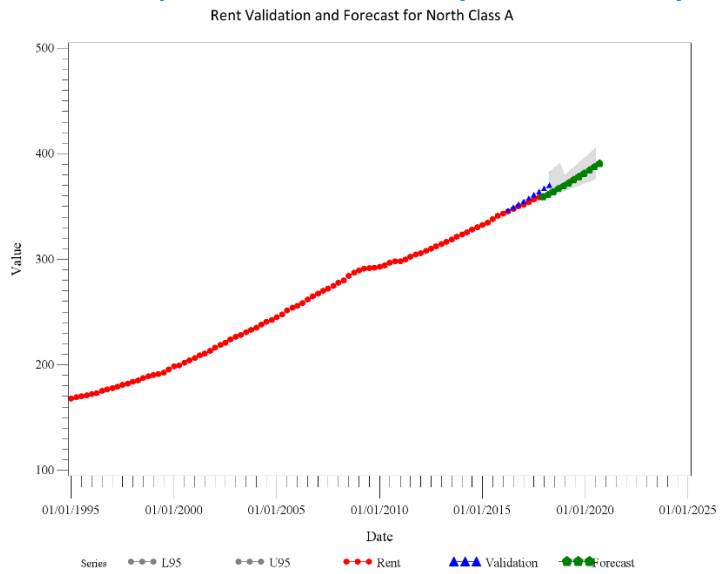
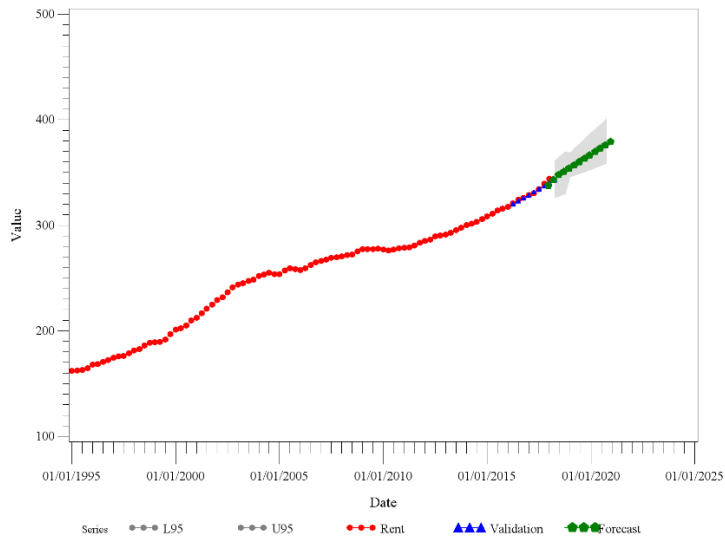


Exhibit D-6 | Forecasts of Quarterly Rent and Utility, Boston

Rent Validation and Forecast for Boston



Utilities Validation and Forecast for Boston

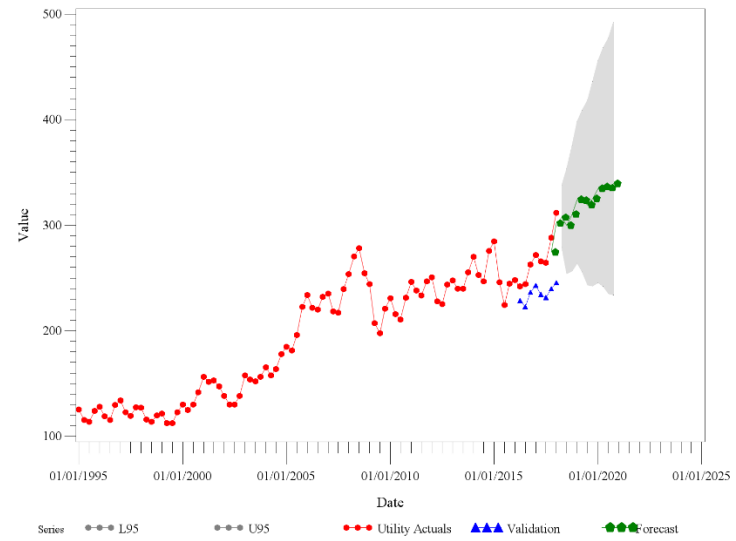
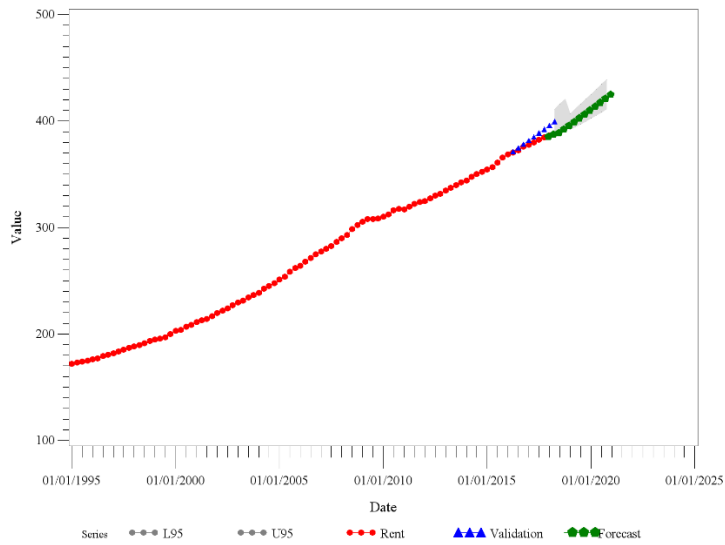


Exhibit D-7 | Forecasts of Quarterly Rent and Utility, New York

Rent Validation and Forecast for New York



Utilities Validation and Forecast for New York

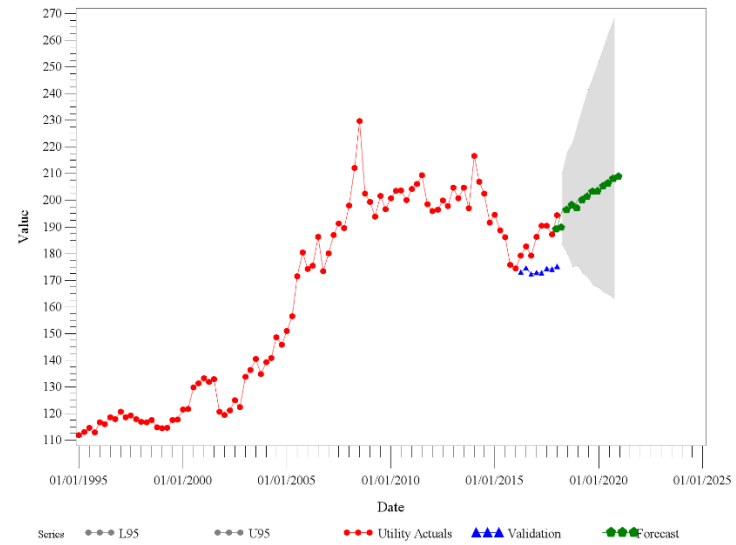
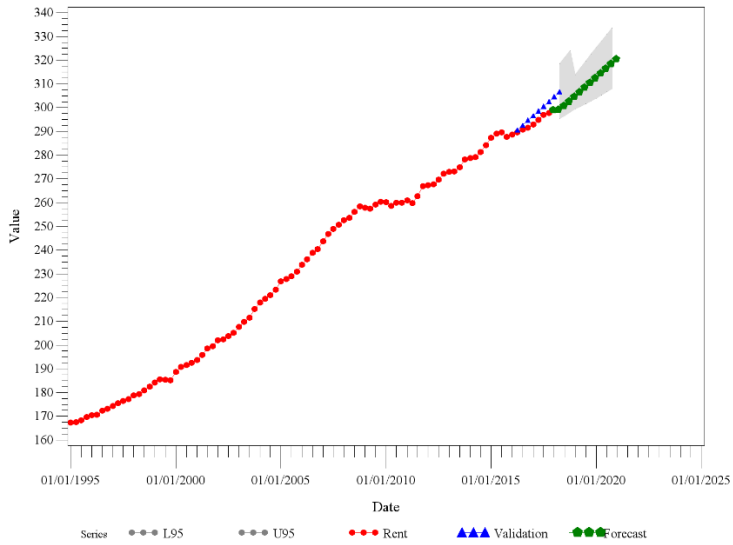


Exhibit D-8 | Forecasts of Quarterly Rent and Utility, Philadelphia

Rent Validation and Forecast for Philadelphia



Utilities Validation and Forecast for Philadelphia

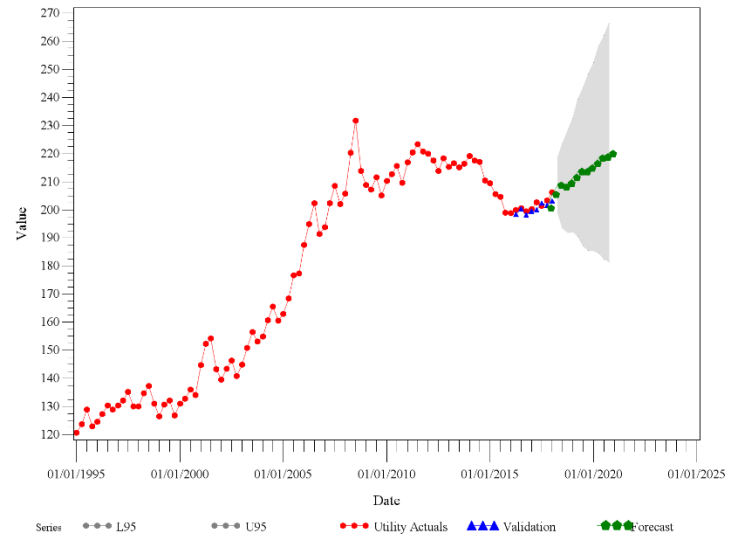
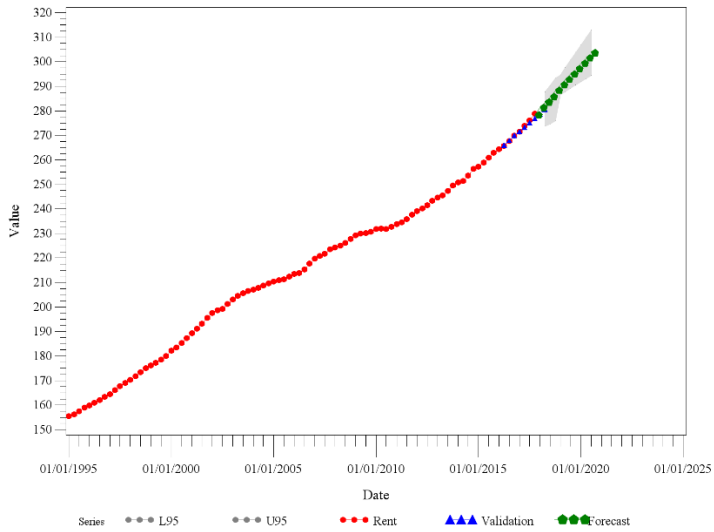


Exhibit D-9 | Forecasts of Quarterly Rent and Utility, Midwest—Class A

Rent Validation and Forecast for Midwest Class A



Utilities Validation and Forecast for Midwest Class A

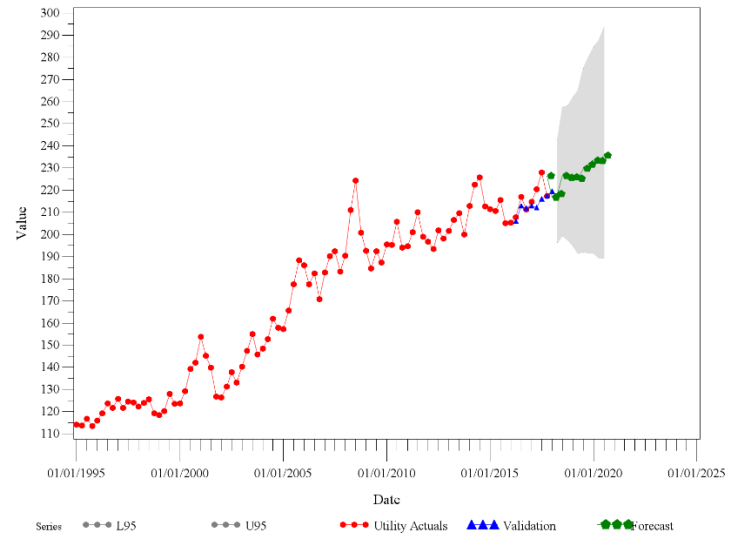


Exhibit D-10 | Forecasts of Quarterly Rent and Utility, Chicago

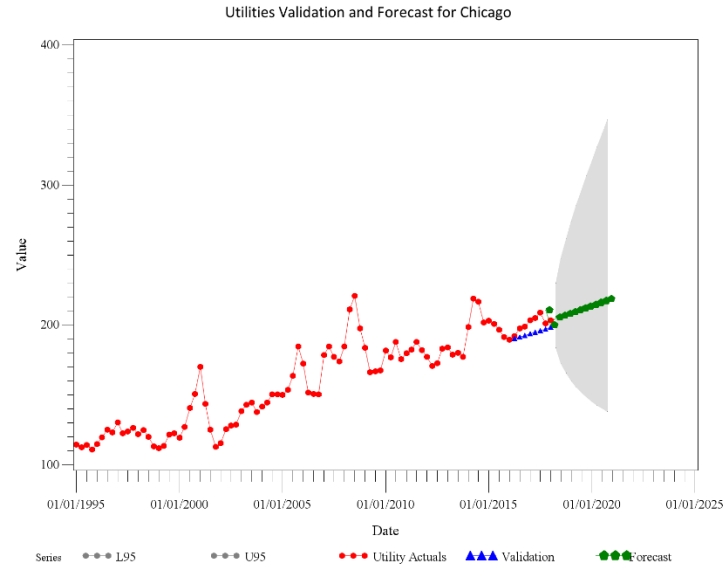
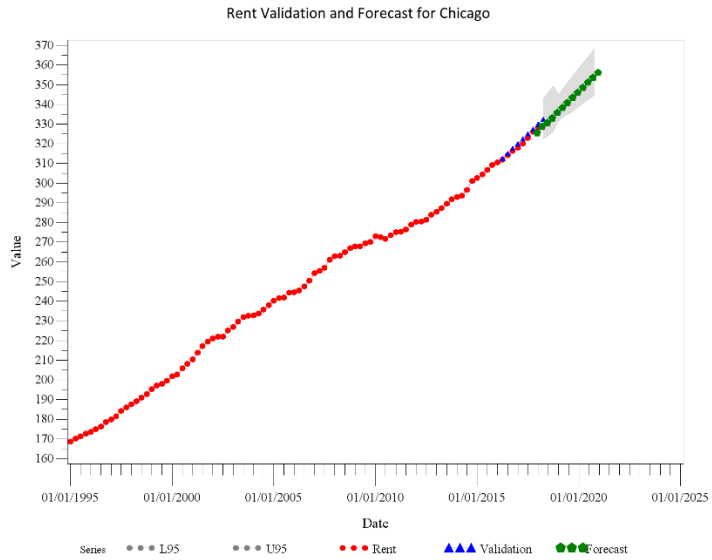


Exhibit D-11 | Forecasts of Quarterly Rent and Utility, Detroit

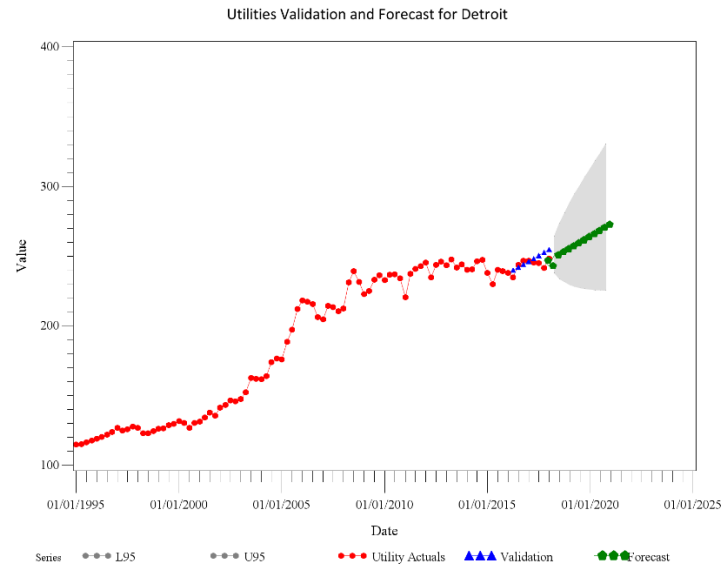
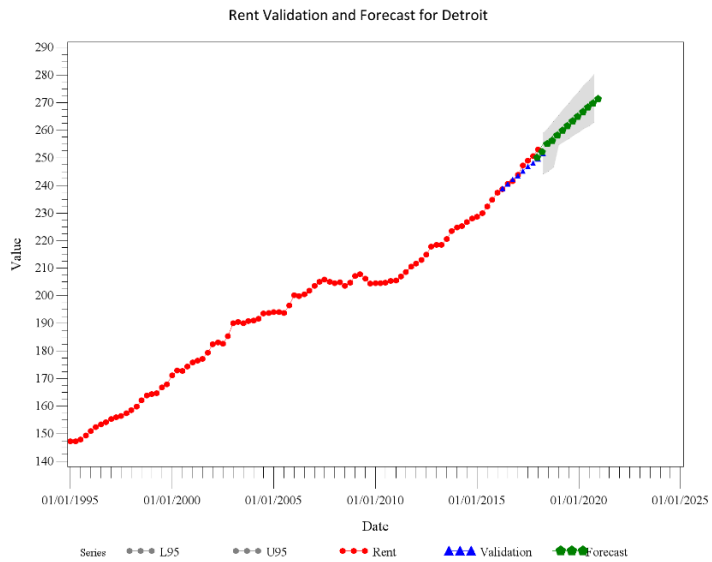
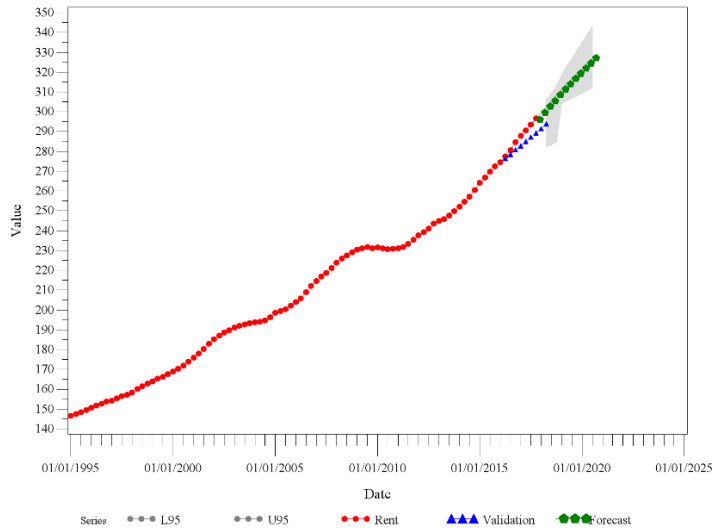


Exhibit D-12 | Forecasts of Quarterly Rent and Utility, South—Class A

Rent Validation and Forecast for South Class A



Utilities Validation and Forecast for South Class A

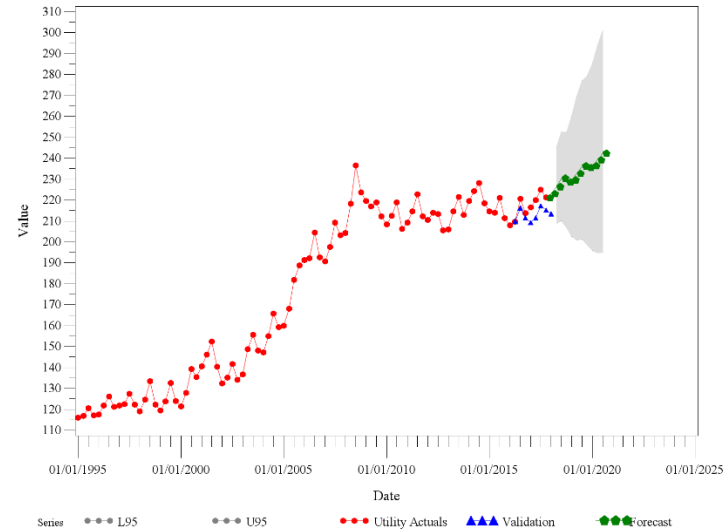
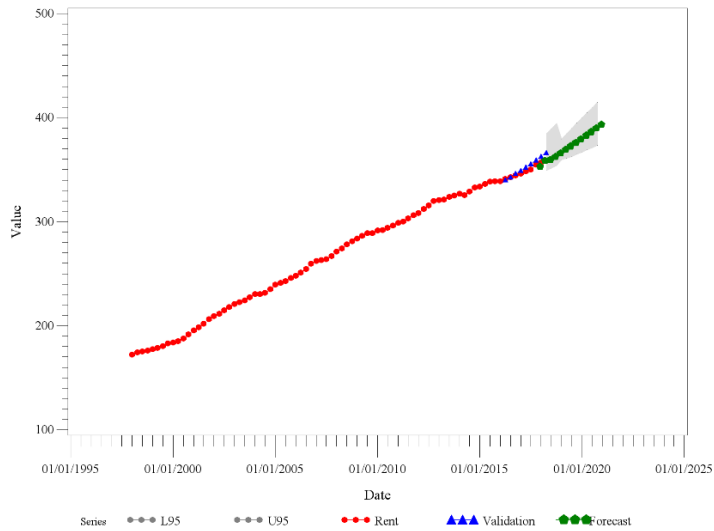


Exhibit D-13 | Forecasts of Quarterly Rent and Utility, Washington, DC

Rent Validation and Forecast for Washington DC



Utilities Validation and Forecast for Washington DC

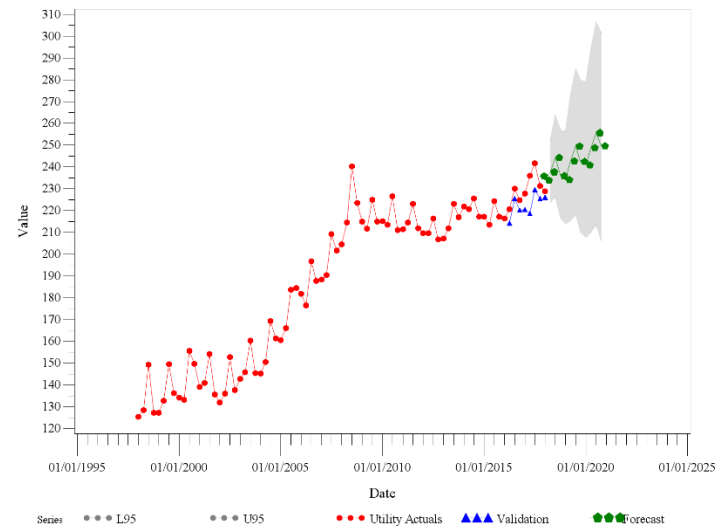
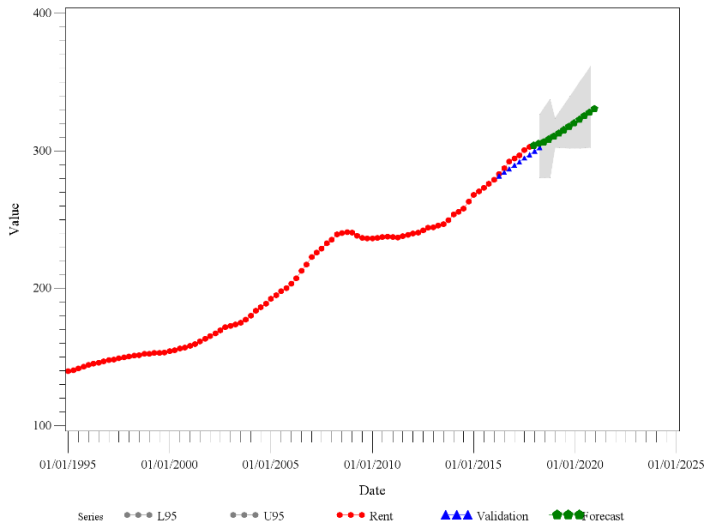


Exhibit D-14 | Forecasts of Quarterly Rent and Utility, Miami

Rent Validation and Forecast for Miami



Utilities Validation and Forecast for Miami

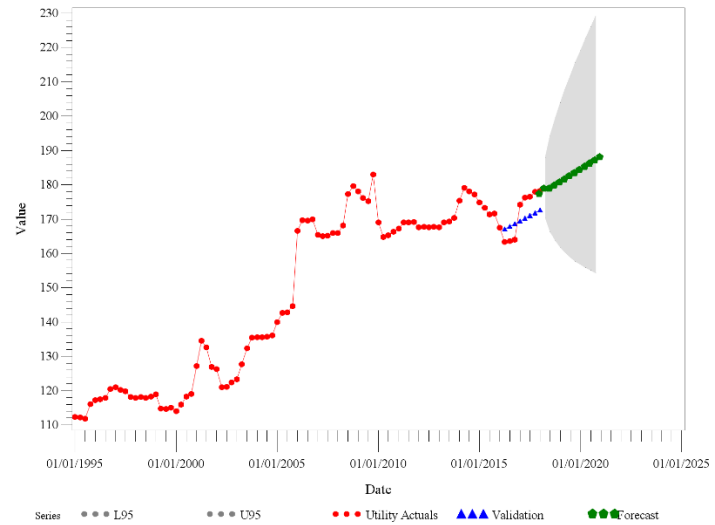
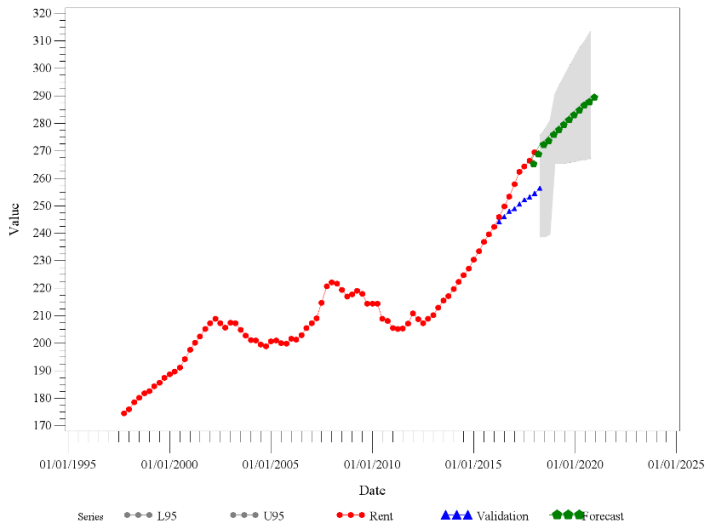


Exhibit D-15 | Forecasts of Quarterly Rent and Utility, Atlanta

Rent Validation and Forecast for Atlanta



Utilities Validation and Forecast for Atlanta

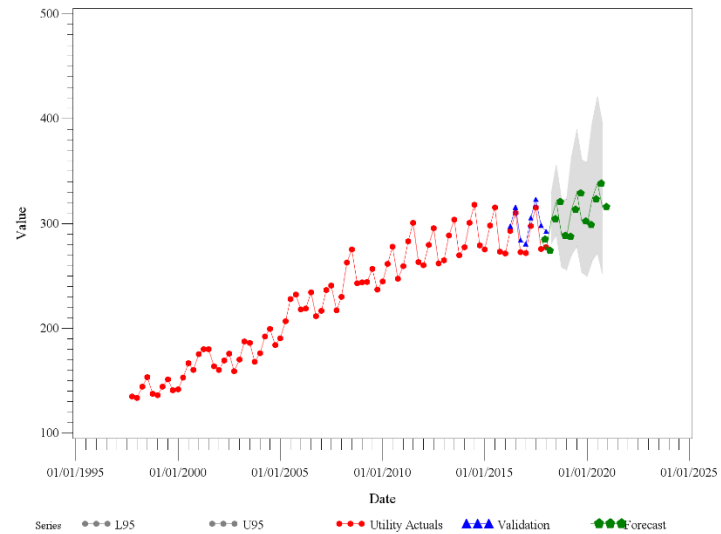
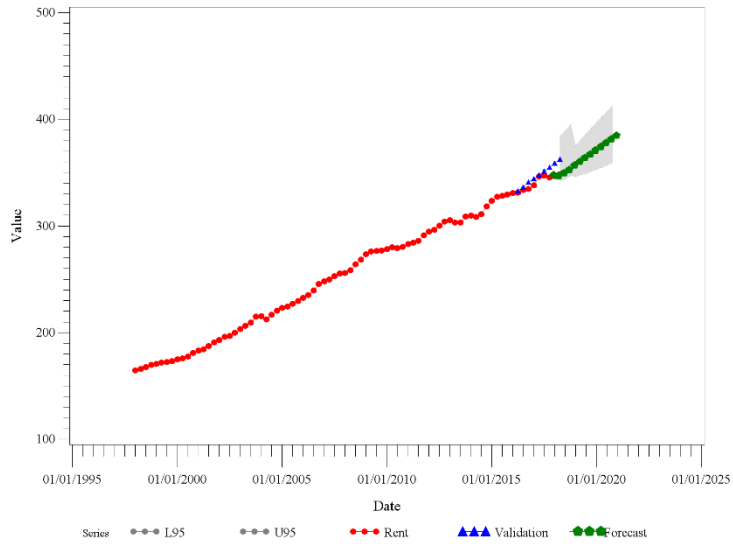


Exhibit D-16 | Forecasts of Quarterly Rent and Utility, Baltimore

Rent Validation and Forecast for Baltimore



Utilities Validation and Forecast for Baltimore

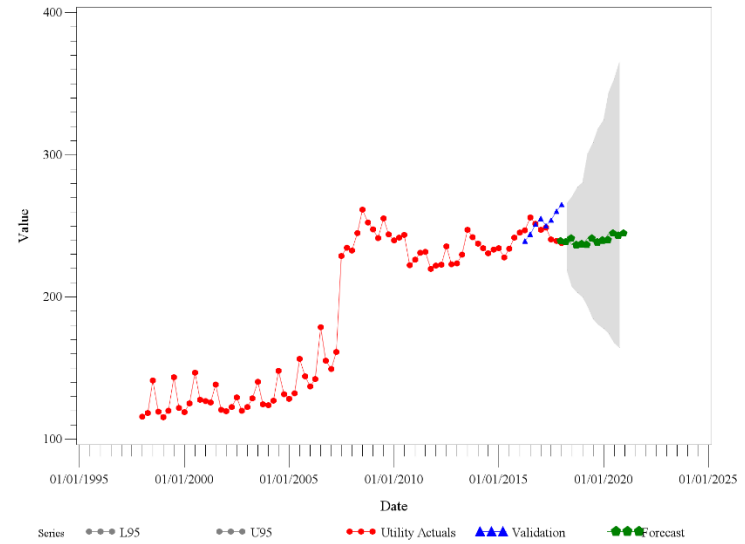
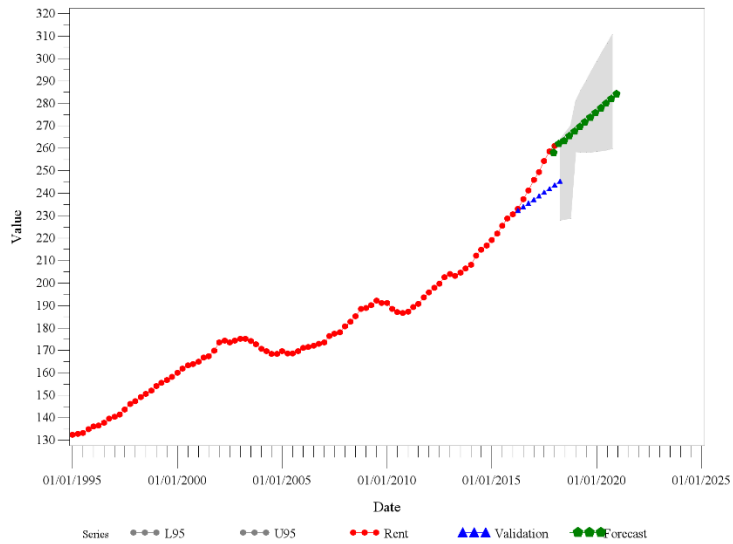


Exhibit D-17 | Forecasts of Quarterly Rent and Utility, Dallas

Rent Validation and Forecast for Dallas



Utilities Validation and Forecast for Dallas

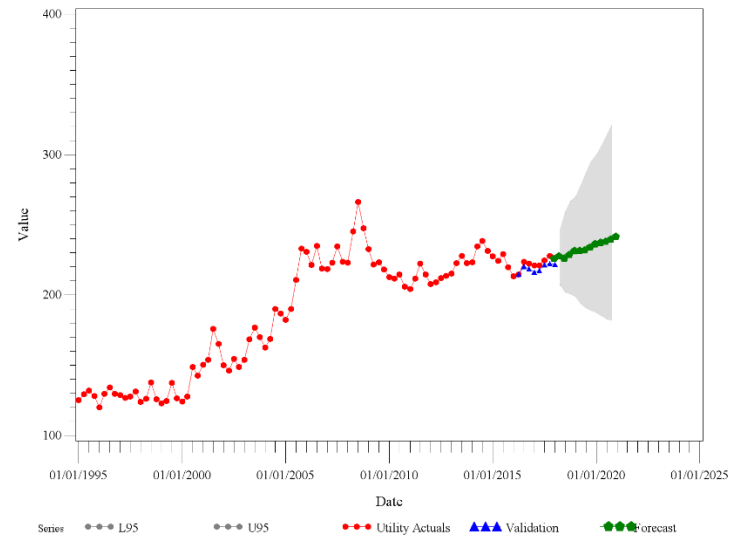
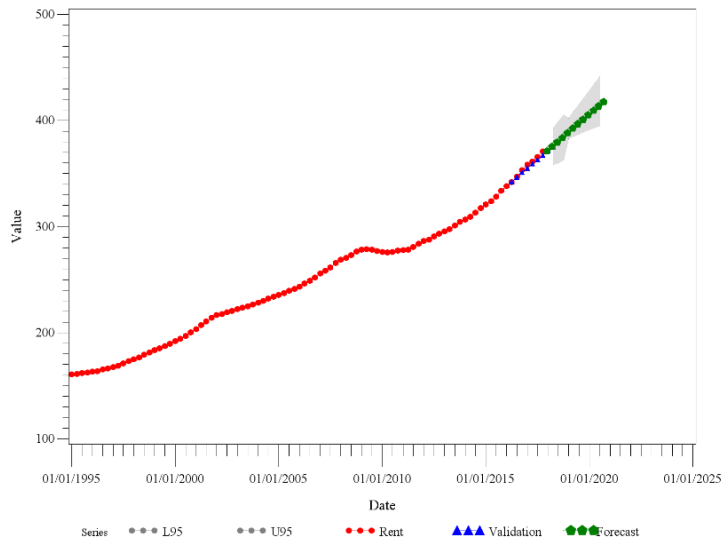


Exhibit D-18 | Forecasts of Quarterly Rent and Utility, West—Class A

Rent Validation and Forecast for West Class A



Utilities Validation and Forecast for West Class A

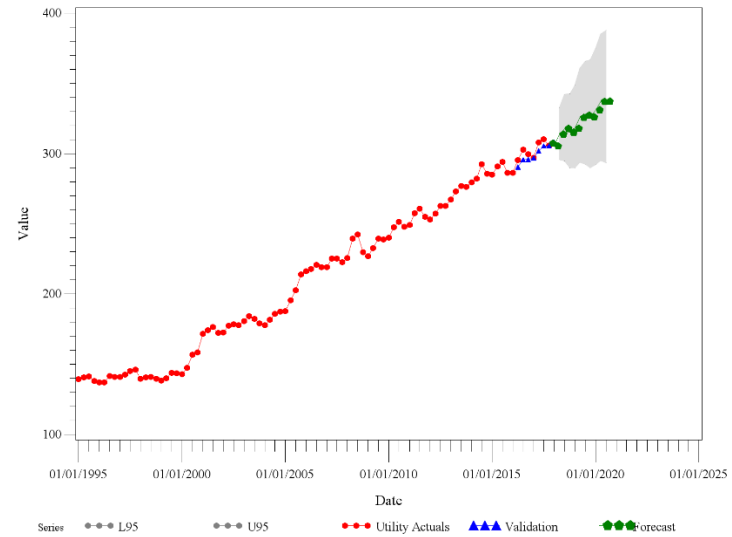
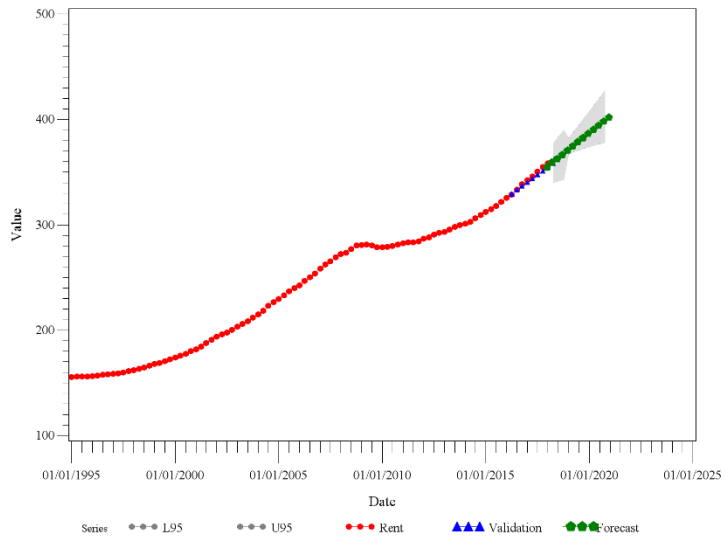


Exhibit D-19 | Forecasts of Quarterly Rent and Utility, Los Angeles

Rent Validation and Forecast for Los Angeles



Utilities Validation and Forecast for Los Angeles

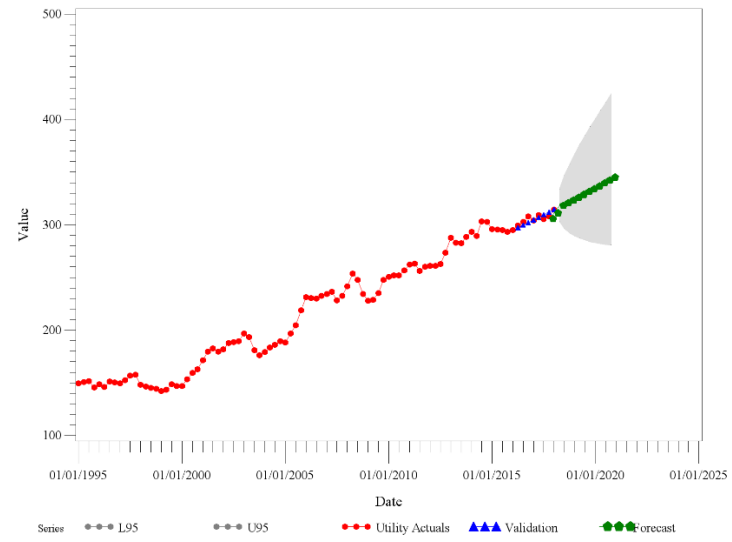
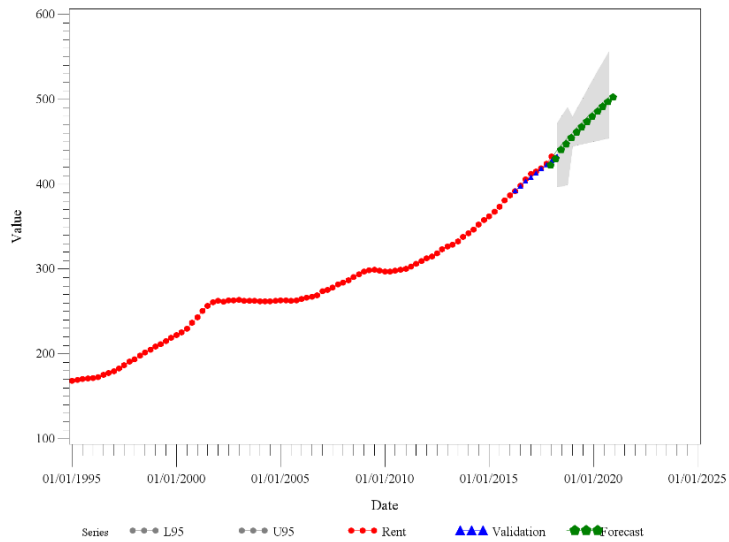


Exhibit D-20 | Forecasts of Quarterly Rent and Utility, San Francisco

Rent Validation and Forecast for San Francisco



Utilities Validation and Forecast for San Francisco

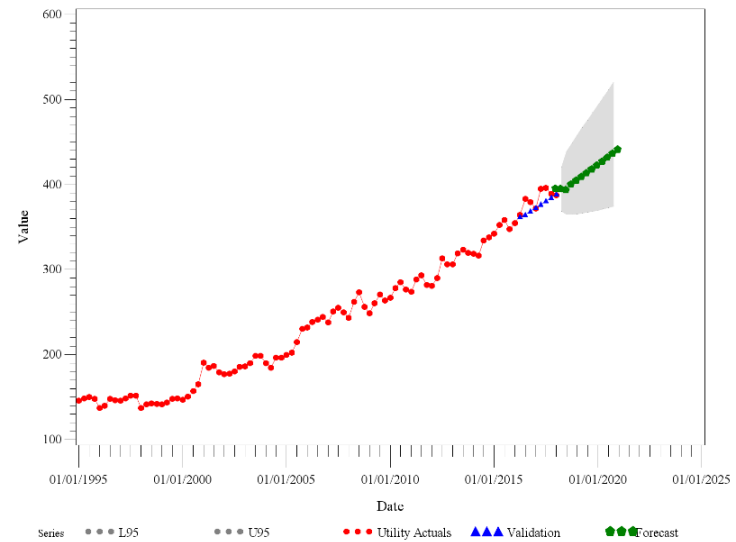
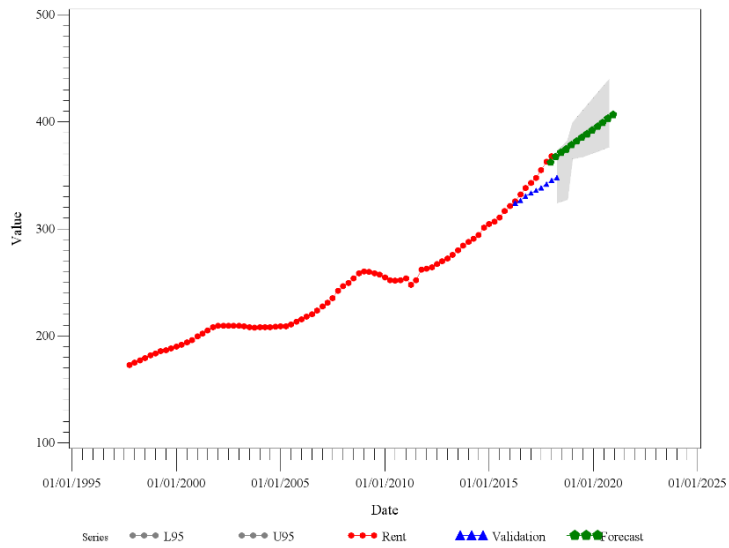
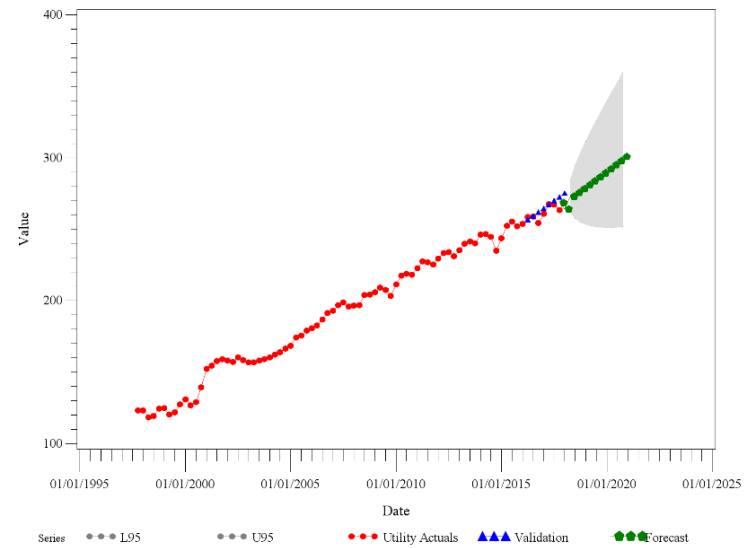


Exhibit D-21 | Forecasts of Quarterly Rent and Utility, Seattle

Rent Validation and Forecast for Seattle



Utilities Validation and Forecast for Seattle



REFERENCES

- Armstrong, J. Scott, and Fred Collopy. 1992. "Error Measures for Generalizing About Forecasting Methods: Empirical Comparisons." *International Journal of Forecasting* 8 (1): 69–80. DOI: [10.1016/0169-2070\(92\)90008-W](https://doi.org/10.1016/0169-2070(92)90008-W).
- Box, George E. P., and Gwilym M. Jenkins. 1976. *Time Series Analysis: Forecasting and Control*. San Francisco: Holden-Day.
- Chai, Tianfeng, and Roland R. Draxler. 2014. "Root Mean Square Error (RMSE) or Mean Absolute Error (MAE)?—Arguments Against Avoiding RMSE in the Literature." *Geoscientific Model Development* 7 (3): 1247–1250. DOI: [10.5194/gmd-7-1247-2014](https://doi.org/10.5194/gmd-7-1247-2014).
- Gallin, Joshua. 2008. "The Long-Run Relationship Between House Prices and Rents." *Real Estate Economics* 36 (4): 635–658. DOI: [10.1111/j.1540-6229.2008.00225.x](https://doi.org/10.1111/j.1540-6229.2008.00225.x).
- Gómez, Víctor, and Agustín Maravall. 1997. Guide for Using the Programs TRAMO and SEATS: Beta Version: June 1998. Madrid: Banco de España.
- Gómez, Víctor, and Agustín Maravall. 1998. Guide for Using the Programs TRAMO and SEATS: Beta Version: December 1997. Working Paper No. 9805. Madrid: Banco de España.

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