Marginal, Average, and Marginal Average Functions

Hello everybody, and welcome to this video. My name is Haley, and today we will be talking about marginal, average, and marginal average functions.

Notation Notes:
$C(x)$ represents the cost function
$C'(x)$ represents the marginal cost function
$\overline{C}(x)$ represents the average cost function
$\overline{C}'(x)$ represents the marginal average cost function

More generally, the prime (the apostrophe) represents the marginal function, the bar on top represents average, and both together represent the marginal average.

The question is, how do you find the marginal function? Or the average? Or the marginal average? Let's take a look at an example to see.

You are given the cost function $C(x) = 150,000 + 20x - 0.0001x^2$. You are then asked to find the marginal cost function. Luckily, if you are familiar with how to find a derivative, finding the marginal cost function is a similar process. If you want to find the marginal function, you just take the derivative.
So the derivative of $C(x) = 150,000 + 20x - 0.0001x^2$ is $C'(x) = 20 - 0.0002x$, which is the marginal cost function.

What if the question was to find the average cost function? First, let's think about how we normally find the average of something. We just add everything up, then divide by however many things we added. Since we don't know how many things we will be dividing by, let's just say we produce x amount of things. Then, the average cost function will be the cost function divided by x.
In general, the average cost function is $\overline{C}(x) = \dfrac{C(x)}{x}$.
In this case, the average cost function is

$$\overline{C}(x) = \frac{150,000 + 20x - 0.0001x^2}{x}$$
$$\overline{C}(x) = \frac{150,000}{x} + 20 - 0.0001x$$

What if we wanted to find the marginal average cost function? Since marginal indicates you should take the derivative, and average indicates you should divide by x, marginal averages indicates you should "take the derivative of the average function."
We already found the average cost function,

$$\overline{C}(x) = \frac{150,000}{x} + 20 - 0.0001x$$

So we just need to take the derivative.

$$\overline{C}'(x) = \frac{-150,000}{x^2} - 0.0001$$

So that is how you find the marginal average cost function.

It is a similar process if you're trying to find the marginal profit, or marginal revenue, etc.

To summarize,
Marginal: you take the derivative of the original function
Average: you take the original function and divide by x
Marginal Average: you take the derivative of the average function

Practice:

1. Find the marginal cost, average cost, and marginal average cost functions.

$$C(x) = 4000 + 3x^2$$

Pause the video now if you'd like to solve the problem on your own first.

Okay, so the first thing we are going to do is find the marginal cost function. Remember, marginal means to take the derivative. The derivative of $C(x) = 4000 + 3x^2$ is $C'(x) = 6x$. Therefore, the marginal cost function is $C'(x) = 6x$.

Moving on to the average cost function, we take the original cost function ($C(x)$) and we divide that by x. We get $\overline{C}(x) = \dfrac{4000}{x} + \dfrac{3x^2}{x} = \dfrac{4000}{x} + 3x$. So our average cost function is $\overline{C}(x) = \dfrac{4000}{x} + 3x$.

Finally, let's find the marginal average cost function. To do that, we take the average function that we have already found and take the derivative of it. To find the derivative of $\dfrac{4000}{x}$, it would be helpful to rewrite it as $4000x^{-1}$. Then, we can use the Power Rule which should be familiar if you remember derivatives. Therefore, the derivative of $\overline{C}(x) = 4000x^{-1} + 3x$ is $\overline{C}'(x) = -4000x^{-2} + 3 = \dfrac{-4000}{x^2} + 3$, which is our marginal average function.

2. Find the marginal revenue, average revenue, and marginal average revenue functions.

$$R(x) = 2x^2 + 20x - 30$$

Pause the video now if you'd like to solve the problem on your own first.

Okay, so the first thing we are going to do is find the marginal revenue function. Again, marginal means to take the derivative. The derivative of $R(x) = 2x^2 + 20x - 30$ is $R'(x) = 4x + 20$. Therefore, the marginal revenue function is $R'(x) = 4x + 20$.

Moving on to the average revenue function, we take the original revenue function ($R(x)$) and we divide that by x. We get $\overline{R}(x) = \dfrac{2x^2}{x} + \dfrac{20x}{x} - \dfrac{30}{x} = 2x + 20 - \dfrac{30}{x}$. So our average revenue function is

$\overline{R}(x) = 2x + 20 - \dfrac{30}{x}$.

Finally, let's find the marginal average revenue function. To do that, we take the average function that we have already found and take the derivative of it. To find the derivative of $\dfrac{30}{x}$, let's rewrite it as $30x^{-1}$. Therefore, the derivative of $\overline{R}(x) = 2x + 20 - 30x^{-1}$ is $\overline{R}'(x) = 2 + 30x^{-2} = 2 + \dfrac{30}{x^2}$, which is our marginal average function.

Practice:

1. Find the marginal, average, and marginal average functions of $C(x) = 5x^3 - 10x + 2$.
2. Find the marginal, average, and marginal average functions of $R(x) = -x^4 + 4x^2 + 7$

Answers:

1. Marginal: $C'(x) = 18x^2 - 10$

   Average: $\overline{C}(x) = 5x^2 - 10 + \dfrac{2}{x}$

   Marginal Average: $\overline{C}'(x) = 10x - \dfrac{2}{x^2}$

2. Marginal: $R'(x) = -4x^3 + 8x$

   Average: $\overline{R}(x) = -x^3 + 4x + \dfrac{7}{x}$

   Marginal Average: $\overline{R}'(x) = -3x^2 + 4 - \dfrac{7}{x^2}$