Manipulating Fractions

Hello everybody, and welcome to this video. My name is Haley, and today we are going to talk about manipulating fractions.

So, you have probably clicked on this video because you, like many other people, dislike fractions and don't like dealing with them. However, you are in a class that requires you to work with them and you think you should probably figure it out. Hopefully, by the end of this video you won't be as afraid of them anymore!

Let's start off by defining some vocabulary. You have probably heard these terms thrown around, but you might not know or remember what they mean.

Vocabulary:
Numerator: top half of the fraction
Denominator: bottom half of the fraction
Improper fraction: a fraction where the numerator is larger than the denominator
Mixed number: a whole number plus a proper fraction
Multiple: a number that can be divided by another number without a remainder
Factor: numbers we can multiply together to get another number
Least common multiple (LCM): the lowest quantity that is a multiple of two or more given numbers
Greatest common factor (GCF): is the largest positive integer that divides each of the numbers

Multiplying Fractions
For multiplying fractions, we just multiply across!

$$\frac{9}{17} * \frac{3}{4} = \frac{27}{68}$$

and since it doesn't simplify, we are done.

Same idea if we have variables:

$$\frac{x}{5} * \frac{2x^2}{3} = \frac{2x^3}{15}$$

Practice:
1. $\frac{1}{14} * \frac{28}{3}$
2. $\frac{2}{x} * \frac{xy}{5}$
3. $\frac{7}{6} * \frac{3}{28}$

Adding/Subtracting Fractions
For both adding and subtracting fractions, we need to find a common denominator. By finding a common denominator, I mean that we want to end up with both fractions having the same denominator. We can do this by either finding the least common multiple (LCM), or multiplying the denominators together. Multiplying the denominators together and using that as your common denominator will always work, you just might have to do more reducing to get simplest form.

Let's start with an example.

Say that we had $\frac{2}{3}+\frac{5}{6}$. Let's go ahead and do the problem using the LCM and by multiplying together.

LCM:

To find the least common multiple, start with the smaller number and start listing out multiples. Basically, count by that number. For 3, we get

$$3, 6, 9, 12...$$

We don't have to count very far before we hit the least common multiple, which in this case would be 6. Our common denominator is going to be 6. Now we have to figure out what to multiply both sides by to get the common denominator. $\frac{5}{6}$ is fine since the denominator is already 6, but we need to multiply $\frac{2}{3}$ by 2 to get 6 on the bottom. We can't just multiply the bottom by 2 since that would change the value of the fraction. Instead we multiply by $\frac{2}{2}$ since that's equal to 1 and multiplying by 1 doesn't change the value of what you're multiplying by. Therefore, we have

$$\frac{5}{6}+\frac{2}{3}\left(\frac{2}{2}\right)=\frac{5}{6}+\frac{4}{6}=\frac{9}{6} \text{ which, when simplified, gives us } \frac{3}{2}$$

Depending on your teacher, they might be okay with having the number be an improper fraction, other times they might want it to be a mixed number. If they are fine with having an improper fraction, you can stop after reducing to simplest form and in this case your answer would be $\frac{3}{2}$. To write as a mixed number, see how many times the denominator goes into the numerator and put the remainder on top of the denominator. 2 goes into 3 one time evenly, and $3-2$ gives us a remainder of 1. Our mixed number will be $1\frac{1}{2}$.

Multiplying together:

If all that seemed complicated, you can just multiply the denominators together to get a common denominator. In this case, the common denominator would be $3(6) = 18$. Multiplying both fractions, we get

$$\frac{5}{6}\left(\frac{3}{3}\right)+\frac{2}{3}\left(\frac{6}{6}\right)=\frac{15}{18}+\frac{12}{18}=\frac{27}{18} \text{ which gives us } \frac{3}{2} \text{ when simplified}$$

Now, what if we aren't sure what the denominator is? For example, if we had

$$\frac{4}{x}-\frac{2}{5}$$

We would do the exact same method as before, we just can't simplify quite as much. Our common denominator is going to be $5x$ since we just multiply the denominators together. Then,

$$\frac{4}{x}\left(\frac{5}{5}\right)-\frac{2}{5}\left(\frac{x}{x}\right)=\frac{20}{5x}-\frac{2x}{5x}=\frac{20-2x}{5x}$$

Our answer ends up being $\frac{20-2x}{5x}$ instead of a fraction with only numbers, but that's okay.

Practice:

4. $\frac{4}{5} - \frac{3}{7}$
5. $\frac{7}{4} + \frac{2}{3}$
6. $\frac{5}{3} + \frac{x}{2}$

Dividing Fractions
Dividing fractions can be a little trickier. You don't need a common denominator, but you do need to be aware of what you are actually doing. You might've heard of the phrase "Keep, Change, Flip," for dividing fractions, but why does that work?

Let's say we are doing $30 \div \frac{2}{5}$. This is saying the same thing as $30 = \frac{2}{5} \times x$ where x is what we are trying to find. So, to solve for x, we multiply by 5 and then divide by 2.

$$30 = \frac{2}{5} \times x$$
$$5 \times 30 = 2 \times x$$
$$150 \div 2 = x$$
$$75 = x$$

This is the same process as $30 \times \frac{5}{2}$, since we get $\frac{30 \times 5}{2} = \frac{150}{2} = 75$. So, instead of having to set up the above process every time, we can simply "Keep, Change the sign, Flip the fraction."

Practice:
7. $\frac{4}{9} \div \frac{8}{x}$
8. $\frac{1}{3} \div \frac{5}{27}$
9. $\frac{2}{15} \div \frac{4}{5}$

Simplifying Expressions with Fractions
Now that we have the basics of fractions down, let's take a look at solving a fractional expression.

$$\frac{9x}{4x+5} + \frac{8x^2}{6x+5}$$

From our basic rules, we know that we need to multiply the denominators together to get the common denominator. Our common denominator is $(4x+5)(6x+5)$.

$$\frac{9x}{4x+5}\left(\frac{6x+5}{6x+5}\right) + \frac{8x^2}{6x+5}\left(\frac{4x+5}{4x+5}\right)$$

$$\frac{9x(6x+5) + 8x^2(4x+5)}{(4x+5)(6x+5)}$$

Now we can simplify by distributing the $9x$ and $8x^2$ respectively.

$$\frac{54x^2+45x+32x^3+40x^2}{24x^2+20x+30x+25}$$

And now we can combine like terms to get our simplified answer.

$$\frac{32x^3+94x^2+45x}{24x^2+50x+25}$$

So we still use the same rules, it just looks a little more complicated when you add variables in.

10. $\frac{3y^4}{3y+2}-\frac{4y^2}{5y+2}$
11. $\frac{\frac{1}{x}}{\frac{x^2}{2}}$
12. $\frac{7x}{3}+\frac{2y}{5}$

Answers to Practice Problems:

1. $\frac{2}{3}$
2. $\frac{2y}{5}$
3. $\frac{1}{8}$
4. $\frac{13}{35}$
5. $\frac{29}{12}$
6. $\frac{10+3x}{6}$
7. $\frac{x}{18}$
8. $\frac{9}{5}$
9. $\frac{1}{6}$
10. $\frac{15y^4+6y^4-12y^3-8y^2}{15y^2+16y+4}$
11. $\frac{2}{x^3}$

12. $\frac{35x+6y}{15}$