

Factoring 101

Factoring is a very useful skill when you want to either solve for the roots of an equation, or just solve for x . In most math classes, factoring is used on quadratics, which have the form $ax^2 + bx + c$ where a, b, c are constants. Let's take a look at the different kinds of factoring.

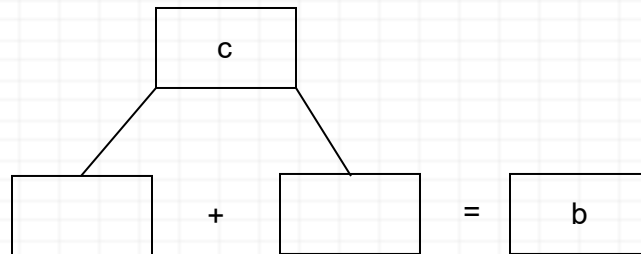
Tree Method

To explain the tree method, I think it's best that we take a look at an example.

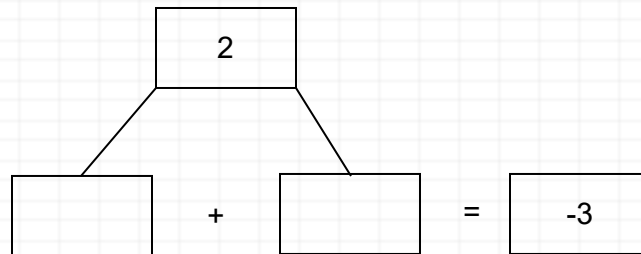
Example:

$$x^2 - 3x + 2$$

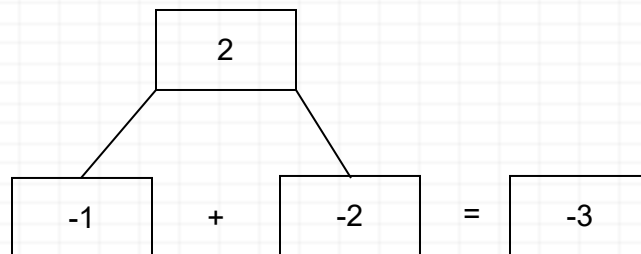
Now, let's set up our tree. Generally, it is going to look like this:



In our case, $b = -3$ and $c = 2$. So, we need something that will multiply to give us 2, and add to give us -3 .



Well, the only factors (numbers that multiply) to give us 2 are 1 and 2. However, $1 + 2$ gives us 3, not -3 . To fix this, let's make both numbers negative, since $-1 * -2 = 2$. Now $-1 + (-2) = -3$, which is what we want.



Therefore, our factored form of $x^2 - 3x + 2$ is $(x - 1)(x - 2)$. To check if this is correct, we could multiply the two factors back together by distributing (more commonly referred to as FOILing). Multiplying back out gives

us $x^2 - 2x - x + 2$ which simplifies to $x^2 - 3x + 2$.

Practice:

1. $x^2 + 9x + 18$

2. $x^2 + 3x - 10$

3. $x^2 - 6x + 8$

Answers:

1. $(x + 6)(x + 3)$

2. $(x - 2)(x + 5)$

3. $(x - 4)(x - 2)$

Important Note: To use the Tree Method, your a value needs to be equal to 1 ($a = 1$). Recall that the a value is the number in front of the x^2 .

Greatest Common Factor (GCF)

Now let's look at another method, Greatest Common Factor, or GCF for short. To factor by the GCF, you need to take out the biggest term that all the terms have.

Example:

$$2x^3 + 8x^2 - 42x.$$

Let's start by taking a look at just the numbers, 2, 8, and -42. What is the greatest factor they have in common? If you said 2, that is correct! So now we know that 2 is part of our GCF. Now let's look at the variables, x^3 , x^2 , x . The GCF here would be x , since it goes into both x^2 and x^3 .

So, our GCF in this case is $2x$. By factoring out $2x$ from each term, we get:

$$2x(x^2 + 4x - 21).$$
 Now we can use the Tree Method to simplify even further, and we get

$$2x(x - 3)(x + 7)$$
 as our fully factored answer.

Practice:

1. $3x^2 - x$

2. $9x^4 + 3x^2 + 12x^3$

3. $9x^2 + 3x + 5$

Answers:

1. $x(3x - 1)$, GCF of x

2. $3x^2(x + 1)(3x + 1)$, GCF of $3x^2$

3. No GCF, already in simplest form

Note: GCF is usually used in combination with one of the other methods for factoring.

Difference Of Two Squares (DOTS)

Difference of Two Squares (DOTS) is more of a shortcut. You can still factor without knowing DOTS using the Tree Method, this is just a quicker way to do it. You'll know you can use DOTS if it has the form $a^2x^2 - c^2$.

If it has this form, the factors will be $(ax - c)(ax + c)$.

Example:

$$49x^2 - 25$$

This can be rewritten as $7^2x^2 - 5^2$. Therefore, our factors are $(7x - 5)(7x + 5)$. We can verify this is true by distributing back out, but I'll leave that up to you if you wish to see.

Practice:

1. $x^2 - 1$

2. $9x^2 - 16$

3. $x^2 + 1$

Answers:

1. $(x - 1)(x + 1)$

2. $(3x - 4)(3x + 4)$

3. DOTS does not apply here since we are not subtracting (Difference of Two Squares, not Addition of Two Squares). Therefore, this is actually not factorable in the real numbers.

By Grouping

To factor by grouping, we are also going to need GCF. We want to group terms together that have the GCF.

Example:

$$6x^3 - 3x^2 - 4x + 2$$

Let's go ahead and group as follows: $\underbrace{6x^3 - 3x^2}_{\text{GCF: } 3x^2} \underbrace{-4x + 2}_{\text{GCF: } -2}$. It'll be clear why I took -2 to be the GCF in a

minute.

After factoring out the GCFs, we have

$$3x^2(2x - 1) - 2(2x - 1)$$

You might notice that there is a $2x - 1$ next to the GCFs. That actually becomes the new GCF of $3x^2$ and -2 . If it helps, think of $2x - 1$ as a variable, let's say a . Then we would have $3x^2(a) - 2(a)$. Now, it's a little more clear that we have a as the GCF. So, if we factor out the new GCF, we have

$$a(3x^2 - 2)$$

And since we said $a = 2x - 1$, we can plug it back in to get

$$(2x - 1)(3x^2 - 2)$$

which is the final factored form of $6x^3 - 3x^2 - 4x + 2$.

Now, as for why I took out -2 as opposed to $+2$. If you take out a $+2$, you end up with

$$3x^2(2x - 1) + 2(-2x + 1)$$

and you'll get stuck since unless you see you're also supposed to take out the negative. However, if you forget, it's okay, since you can just take the negative out at this step in order to make them the same.

Practice:

1. $9x^3 - 5x^2 + 18x - 10$

2. $2x^3 + 3x - 10x^2 - 15$

3. $12x^3 + 24x^2 + 3x + 6$

Answers:

1. $(x^2 + 2)(9x - 5)$

2. $(x - 5)(2x^2 + 3)$

3. $3(4x^2 + 1)(x + 2)$. Note that sometimes the GCF is 1, which is fine.

Note: Sometimes, you'll have to move terms around in order to get the best combination of GCFs. It's alright to try a couple different ones! Typically, the best ones to group are the ones that have the most powers of x in common.

AC Method

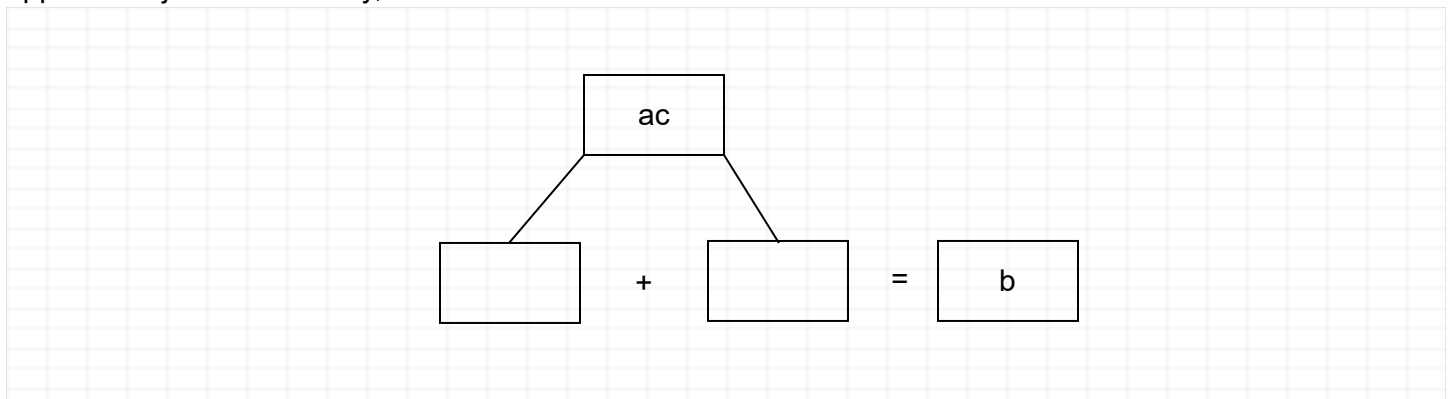
Recall that the standard form of a quadratic is $ax^2 + bx + c$, where a, b, c are constants. If you refer back to the Tree Method, you'll notice I made a note about how it only works if $a = 1$. The AC Method deals with when $a \neq 1$ and combines the Tree Method and Grouping Method.

Example:

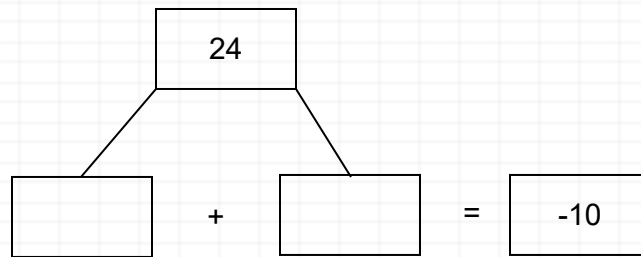
$$8x^2 - 10x + 3$$

In this example, our a value is 8, which means we can't use just the Tree Method. There's not a GCF for all 3 constants, and it's not a Difference of Two Squares. And it's not easy to see how things should be grouped, if they can be at all. So, that leads us to the AC Method.

It's the same idea as the Tree Method to start, except we have to make sure we multiply and use ac as opposed to just c . Generally, we want this:



In this case, $ac = 8 \cdot 3 = 24$ so that goes in the top box. And $b = -10$, so that goes in the far right box.

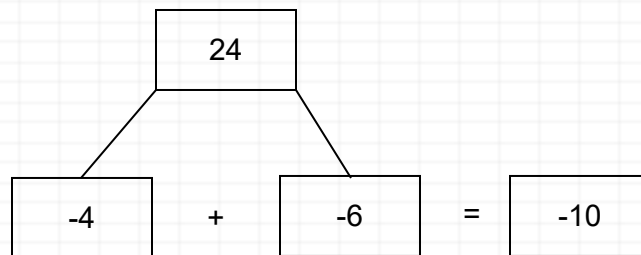


Similar to before, we need factors of 24 that add to give us -10 .

Factors of 24 include:

± 1	± 24
± 2	± 12
± 3	± 8
± 4	± 6

The only factors that give us -10 are -4 and -6 .



So, instead of directly giving us the factors like in the Tree Method, it actually breaks up the b value so we can use grouping.

$$8x^2 - 4x - 6x + 3$$

Now, we can group like we typically would.

$$\begin{aligned} & \underbrace{8x^2 - 4x} \quad \underbrace{-6x + 3} \\ & = 4x(2x - 1) - 3(2x - 1) \\ & = (2x - 1)(4x - 3) \end{aligned}$$

Practice:

1. $6x^2 + 7x + 2$
2. $3x^2 + 14x + 8$
3. $2x^2 + 11x + 12$

Answers:

1. $(3x + 2)(2x + 1)$

2. $(x + 4)(3x + 2)$

3. $(2x + 3)(x + 4)$