Logic, Dynamics, and Their Interactions II

University of North Texas
June 2 - 6, 2014

Abstracts of Plenary Talks

Matthew Foreman, *From odometers to rotations.* University of California, Irvine. `mforeman@math.uci.edu`

An ergodic measure preserving transformation is *odometer based* if its Kronecker factor is an odometer map. It is *circular* if its Kronecker factor is an irrational rotation. This talk presents a canonical functor from odometer based to circular systems and proves that the resulting circular systems can all be realized as ergodic measure preserving transformations of the disk or the torus. The work is joint with B. Weiss.

Alexander S. Kechris, *Amenability, unique ergodicity and random orderings.* California Institute of Technology. `kechris@caltech.edu`

I will discuss some aspects of the topological dynamics and ergodic theory of automorphism groups of countable first-order structures and their connections with logic, finite combinatorics, and probability theory. This is joint work with Omer Angel and Russell Lyons.

Dmitry Kleinbock, *Hausdorff dimension estimates for bounded orbits on homogeneous spaces of Lie groups.* Brandeis University. `kleinboc@brandeis.edu`

This work is motivated by studying badly approximable vectors, that is, $\mathbf{x} \in \mathbf{R}^n$ such that $\|q\mathbf{x} - \mathbf{p}\| \geq cq^{-1/n}$ for all $\mathbf{p} \in \mathbf{Z}^n$, $q \in \mathbf{N}$. Computing the Hausdorff dimension of the set of such $\mathbf{x}$ for fixed c is an open problem. I will present some estimates, based on the interpretation of a badly approximable vector via a trajectory on the space of lattices, and then use exponential mixing to estimate from above the dimension of points whose trajectories stay in a fixed compact set.

Krystyna Kuperberg, *Non-compact minimal sets.* Auburn University. `kuperkm@auburn.edu`

In 1976, Paul A. Schweitzer [3] posed the following question: Does there exist a flow with no minimal set? A nonempty, closed, invariant set is *minimal* if it contains no proper, nonempty, closed, invariant subset. The usual assumption of compactness is omitted in this definition.

The question led to a variety of examples of flows with no minimal sets. In 1999, Takashi Inaba [2] considered the existence of a flow on a surface with infinitely many handles. Note that a flow on a surface with finitely many handles always possesses a minimal set [1].

We will show a basic construction of a flow on a non-compact closed subset of $\mathbb{R}^3$, called a generalized plug, that yields examples of flows with no minimal set on the following spaces:

The Euclidean space of dimension three and higher: The flow is C^∞.

A matchbox manifold M: A matchbox manifold is a separable metric space that is locally homeomorphic to the Cartesian product of the interval (0,1) and a one-dimensional space.

A two-dimensional surface: The non-compact surface without boundary has infinitely many handles and infinitely many holes. The flow is C^∞.

References

[1] G. Hector and U. Hirsch *Introduction to the geometry of foliations, Part A*, Friedr. Vieweg & Sohn,1981.

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[3] Symposium on Differential and Algebraic Topology, Rio de Janeiro, Proceedings: *Differential topology, foliations and Gelfand-Fuks cohomology*, ed. by Paul A. Schweitzer (1976).

Justin Moore, *A new solution to the von Neumann-Day problem for finitely presented groups.* Cornell University. `justin@math.cornell.edu`

I will present a new example of a finitely presented group which is nonamenable but which does not contain a free group on two generators. While such an example has already been given by Olshanski and Sapir, the new example is much easier to analyze. It is a subgroup of Monod's group of piecewise projective homeomorphims of the real projective line and bears a number of similarities to Richard Thompson's group F. This is joint work with Yash Lodha.

Christian Rosendal, *Large scale geometry of metrisable groups.* University of Illinois at Chicago. `rosendal.math@gmail.com`

While there is a quite developed theory for the topological structure of Polish groups covering, e.g., unitary representations and topological dynamics, little progress has been made in the direction of their geometry. We present a general framework that allows one to define and calculate the large scale geometry of a significant class of Polish groups and use this to introduce new invariants for Polish groups.

Sławomir Solecki, *Algebra and dynamics behind Galvin-Glazer-type results.* University of Illinois at Urbana-Champaign. `ssolecki@illinois.edu`

Galvin-Glazer-type results are an important part of infinite Ramsey theory. A number of them are known. I will describe an algebraic setting that makes it possible to state a general such theorem, which I will state. It leads to a new kind of problems in dynamics of monoids, with monoids acting on compact semigroups via continuous endomorphisms.

Simon Thomas, *Invariant random subgroups of locally finite groups.* Rutgers University. `sthomas@math.rutgers.edu`

Let G be a countable discrete group and let Sub_G be the compact space of subgroups $H \leqslant G$. Then an *invariant random subgroup* of G is a Borel probability measure ν on Sub_G which is invariant under the conjugation action of G. In this talk, I will present some recent results on the invariant random subgroups of locally finite groups and discuss some of the many open problems.

Abstracts of Invited Talks

Vaughn Climenhaga, *Specification, hyperbolicity, and towers.* University of Houston.
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Given a dynamical system with some hyperbolicity, the equilibrium states associated to sufficiently regular potentials often display stochastic behaviour. Two important tools for studying these equilibrium states are specification properties and tower constructions. I will describe how both uniform and non-uniform specification properties can be used to deduce existence of a tower with exponential tails, and hence to establish various statistical properties.

Clinton Conley, *Baire measurable chromatic numbers of locally finite Borel graphs.* Cornell University. clintonc@math.cornell.edu

In stark contrast to the behavior of Borel chromatic numbers, it turns out that the Baire measurable chromatic number of any locally finite Borel graph on a Polish space is strictly less than twice its ordinary chromatic number, provided this ordinary chromatic number is finite. We discuss this result and, time permitting, analogs in the measure-theoretic context when the connectedness equivalence relation is hyperfinite. This is joint work with Ben Miller.

Aleksandra Kwiatkowska, *Uniqueness of an invariant probability measure concentrated on an orbit.* University of California, Los Angeles. akwiatk2@math.ucla.edu

In a recent paper, Ackerman, Freer, and Patel characterized countable models for which there exists an invariant (with respect to the logic action) probability measure concentrated on its orbit as those that have a trivial definable closure. We want to understand when, if there is an invariant probability measure concentrated on the orbit of a countable model M, there is a unique such measure. We show that when M is a Fraïssé limit that has a trivial definable closure, such a measure is unique if and only if for each finite n there is exactly one model of cardinality n in the age of M. This work will be in a joint paper with Ackerman, Freer, and Patel.

Andrew Marks, *Brooks' theorem for measurable colorings.* California Institute of Technology.
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Given a finite graph of degree at most d, a greedy algorithm can be used to give a $(d+1)$-coloring of the graph. A classical theorem of Brooks classifies those graphs where this upper bound on the chromatic number is optimal: the complete graphs on d vertices, and odd cycles when $d = 2$.

We study analogous questions for Borel graphs and colorings with various desirable measurability properties. Kechris, Solecki and Todorcevic have shown every Borel graph of degree at most d has a Borel $(d+1)$-coloring. We classifiy the graphs for which this upper bound is optimal for Baire measurable colorings and μ-measurable colorings. We also give some partial results for Borel colorings.

This is joint work with Clinton Conley and Robin Tucker-Drob.

Volker Mayer, *Random dynamics of transcendental functions.* Université de Lille 1.
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We present joint work with M. Urbański. It concerns random dynamics of transcendental functions $f : \mathbb{C} \to \hat{\mathbb{C}}$ and, more precisely, the spectral gap property in this setting. In classical situations there is a natural and powerful proof of this property whose origin is Birkhoff's contraction principle for operators preserving a positive cone. This method however does not work in our non-compact situation. I will explain how a lemma by Bowen can be used to overcome this difficulty.

Eugen Mihailescu, *Dimension and smooth ergodic theory for systems with overlaps.* Institute of Mathematics of the Romanian Academy. `Eugen.Mihailescu@imar.ro`

In this talk I will give several recent results about the relationships between dimension and smooth ergodic theory, in the context of invariant probability measures supported on certain limit sets of folded saddle-type dynamical systems or iterated function systems.

Matthew Nicol, *Dynamical Borel-Cantelli Lemmas.* University of Houston. `nicol@math.uh.edu`

Borel-Cantelli Lemmas are useful for establishing the almost sure behavior of stochastic processes. Suppose T is a measure preserving transformation of a probability space (X, μ) and $\{B_i\}$ is a sequence of sets in X such that $\sum_i \mu(B_i)$ diverges.

We may ask: does $T^i(x) \in B_i$ for infinitely many i for μ a.e. $x \in X$? This is an analog of the Borel-Cantelli lemma from probability theory. If B_i is a sequence of nested balls in a metric space this question is sometimes called the shrinking target problem. We will discuss results progress in establishing Borel-Cantelli Lemmas for dynamical systems with some degree of hyperbolicity. In particular, we will discuss limit laws such as the strong law of large numbers, the central limit theoren and the almost sure invariance principle for Birkhoff sums of form $\sum_i 1_{B_i}(T^i x)$.

Brandon Seward, *Krieger's finite generator theorem for ergodic actions of countable groups.* University of Michigan. `b.m.seward@gmail.com`

The classical Krieger finite generator theorem states that if a free ergodic probability-measure-preserving action of $\mathbb{Z}$ has entropy less than $\log(k)$, then the action admits a generating partition consisting of k sets. This was extended to actions of amenable groups independently by A. Rosenthal and Danilenko–Park. We consider a natural analogue of entropy which is defined for actions of arbitrary countable groups and which moreover is identical to entropy when the acting group is amenable. Using this analogue of entropy, we prove Krieger's finite generator theorem for actions of arbitrary countable groups.

Cesar Silva, *Notions of weak mixing and examples in infinite measure.* Williams College. `csilva@williams.edu`

Many of the interesting conditions that are equivalent to the weak mixing property for finite measure-preserving transformations do not remain equivalent in infinite measure. We will discuss several of these conditions, such as double ergodicity, power weak mixing, R-set weak mixing, and rational weak mixing, and also present rank-one constructions that give examples and counterexamples for these properties.

David Simmons, *Diophantine approximation and the geometry of limit sets in Gromov hyperbolic metric spaces.* Ohio State University. `david9550@gmail.com`

Let (X, d) be a Gromov hyperbolic metric space, and let ∂X be the Gromov boundary of X. Fix a group $G \leq \mathrm{Isom}(X)$ and a point $\xi \in \partial X$. We consider the Diophantine approximation of a point $\eta \in \partial X$ by points in the set $G(\xi)$. Our results generalize the work of many authors, in particular Patterson ('76) who proved most of our results in the case that G is a geometrically finite Fuchsian group of the first kind and ξ is a parabolic fixed point of G.

Anush Tserunyan, *Recurrence in probability groups.* University of Illinois at Urbana-Champaign. anush.tserunyan@gmail.com

We consider a class of groups equipped with an invariant probability measure (call them probability groups), which includes all compact groups and is closed under taking ultraproducts with the induced Loeb measure. We prove a triple recurrence result for mixing probability groups, which generalizes a recent theorem of Bergelson and Tao proved for ultra quasirandom groups, nevertheless having a considerably shorter proof. Moreover, the quantitative version of this proof yields a quantitative triple recurrence theorem for probability groups that are mixing up to a small error, such as quasirandom groups introduced by Gowers. In our proofs the amplification of mixing to multiple recurrence is done using a suitable van der Corput difference lemma, and if time permits we will discuss a generalization of this lemma to a general class of filters on semigroups.

Robin Tucker-Drob, *Inner amenability and stability in linear groups.* Rutgers University. rtuckerd@gmail.com

A countable discrete group G is said to be inner amenable if it admits an atomless mean which is invariant under conjugation. Jones and Schmidt introduced a strengthening of inner amenability called stability: G is stable if there exists a free ergodic probability measure preserving action which is orbit equivalent to its independent direct product with a free ergodic action of $\mathbb{Z}$. In this talk I will provide complete characterizations of both inner amenability and stability for linear groups over an arbitrary field. The analysis of stability leads to many new examples of (non-linear) stable groups; notably, all nontrivial countable subgroups of Monod's group $H(\mathbb{R})$ are stable. This includes nonamenable groups constructed by Monod and by Lodha and Moore, as well as Thompsons group F.

Jay Williams, *Finitely generated groups and Borel equivalence relations.* California Institute of Technology. jay.will.math@gmail.com

Thomas and Velickovic proved that isomorphism of finitely generated groups is a universal countable Borel equivalence relation. The proof made heavy use of free products with amalgamation, which are group-theoretically "large" and combinatorial in nature. The question remains if more natural classes of finitely generated groups still have a complex isomorphism relation. We will discuss the best known lower bounds for several classes, including Kazhdan groups, solvable groups, and simple groups.

Christian Wolf, *Localized pressure and equilibrium states.* City College of New York. cwolf@ccny.cuny.edu

In this talk we discuss a notion of localized topological pressure for continuous maps on compact metric spaces. The localized pressure of a continuous potential φ is computed by considering only those (n, ϵ)-separated sets whose statistical sums with respect to an m-dimensional potential Φ are "close" to a given value $w \in \mathbb{R}^m$. We then establish for several classes of systems and potentials φ and Φ a local version of the variational principle. We also construct examples showing that the assumptions in the localized variational principle are fairly sharp. Next, we study localized equilibrium states and show that even in the case of subshifts of finite type and Hölder continuous potentials, there are several new phenomena that do not occur in the theory of classical equilibrium states. In particular, ergodic localized equilibrium states for Hölder continuous potentials are in general not unique.

Anna Zdunik, *Hausdorff and harmonic measures on non-homogeneous Cantor sets.* University of Warsaw. aniazd@mimuw.edu.pl

We consider (not self-similar) Cantor sets defined by a sequence of piecewise linear functions. We prove that the dimension of the harmonic measure on such a set is strictly smaller than its Hausdorff dimension. Some Hausdorff measure estimates for these sets are also provided. This is joint work with Athanasios Batakis.

Abstracts of Special Session Talks

Dylan Airey, *Number theoretic applications of a class of Cantor series fractal functions.* University of Texas at Austin. dylan.airey@utexas.edu

We will introduce a new class of Cantor series fractal functions and discuss their applications to problems involving normal numbers.

Dana Bartošová, *Towards the universal minimal flow of the group of homeomorphisms of the Lelek fan.* University of São Paulo. dana@ime.usp.br

Having constructed the Lelek fan as a natural quotient of a projective Fraïssé limit, we were faced with a problem of generalizing Gowers' combinatorial result of Ramsey type in order to compute the universal minimal flow of the group of automorphisms of the projective Fraïssé limit. We explain this link, give the generalizations and discuss how to proceed towards the universal minimal flow of the Lelek fan itself. This is a joint work with Aleksandra Kwiatkowska.

Kostantinos A. Beros, *Group homomorphism reductions.* University of North Texas.
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We consider a Wadge-like pre-order on the class of K_σ subgroups of a given Polish group, G. This pre-order arises from considering the continuous endomorphisms of G as reducing maps. We describe conditions on the group G which guarantee the existence of maximal subgroups with respect to this pre-order and mention examples of groups in which there are no maximal elements in this pre-order.

Allison Birdsong, *The Banach-Mazur-Schmidt Game.* University of North Texas.
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The Banach-Mazur Game and Schmidt's Game are very similar in the way they are played, yet have very different results. We will introduce a new game, the Banach-Mazur-Schmidt Game, and discuss winning sets for both players. This is a joint project with Rebekah Hansen.

Christopher Caruvana, *Transfinite dimension theory.* University of North Texas.
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We will introduce a notion of transfinite dimension and mention its advantages over more classical notions of transfinite dimension. We will also discuss axioms of finite dimension and how they can be extended in a natural way to the transfinite case.

Longyun Ding, *Borel reduciblity between equivalence relations generated by sequence spaces.* School of Mathematical Sciences, Nankai University. dinglongyun@gmail.com

We investigate the Borel reducibility between equivalence relations of the form as $\mathbb{R}^\omega/G$ where G is a Borel subgroup of $\mathbb{R}^\omega$. We are mainly interested on special cases in which G is a Borel linear subspace. This kind of research begin from Dougherty and Hjorth, while G is one of the classical sequence spaces: $\ell_p\,(p \geq 1)$ or c_0.

In this talk, we will recall some old results and introduce some new results on this topic.

James Freitag, *Complexity for countable differentially closed fields.* University of California, Berkeley. `freitag@berkeley.edu`

For nearly a decade after Vaught's conjecture was proved for differentially closed fields, the number of countable differentially closed fields was not known. Hrushovski and Sokolovic resolved the problem by using and understanding Manin kernels, differential equations which cut out the torsion points on Abelian varieties. A well-known conjecture in differential algebra essentially amounted to saying that these Manin kernels were the only source of complexity for the isomorphism problem for countable differential fields. My recent joint work with T. Scanlon on effective bounds around Andre-Oort has refuted this conjecture by understanding the differential equations associated with isogeny classes of elliptic curves.

Daniel Glasscock, *Marstrand-type theorems for the Counting and Mass Dimensions.* Ohio State University. `glasscock.4@math.ohio-state.edu`

The (upper) mass and counting dimensions of a set $A \subseteq \mathbb{Z}^d$ are, respectively,

$$D_m(A) = \limsup_{n\to\infty} \frac{\log \left|A \cap [-n,n]^d\right|}{\log n} \quad \text{and} \quad D(A) = \limsup_{\|C\|\to\infty} \frac{\log \left|A \cap C\right|}{\log \|C\|},$$

where the limit supremum in $D(A)$ is taken over cubes $C \subseteq \mathbb{Z}^d$ with side length $\|C\|$ tending to infinity. They capture the (maximal) polynomial rate of growth of the set A on larger and larger cubes with and without a fixed center. They may be used to understand "fractal" subsets of the integer lattice; for example, the set of integers expressible in base 3 using only the digits 0 and 2 has mass and counting dimension equal to $\log 2 \big/ \log 3$.

Both dimensions satisfy an analogue of Marstrand's theorem from geometric measure theory. For example, if $A \subseteq \mathbb{Z}^d$ is such that $D(A) = D_m(A)$, then for almost all orthogonal projections P on $\mathbb{R}^d$ with range of dimension k,

$$D_m\big(\lfloor PA \rfloor\big) = \min\big(k, D_m(A)\big).$$

Utilizing the connection between projections and sumsets, we recover typicality results for the mass and counting dimensions of sumsets of scaled sets; for example, if $f_i \in \mathbb{Z}[x]$ has degree $d_i \geq 1$, $1 \leq i \leq m$, then for Lebesgue-almost every $\lambda \in \mathbb{R}^m$,

$$D_m\big(\lfloor \lambda_1 f_1(\mathbb{Z}) + \cdots + \lambda_m f_m(\mathbb{Z}) \rfloor\big) = \min\left(1, \frac{1}{d_1} + \cdots + \frac{1}{d_m}\right).$$

In this talk, I will sketch the proof of these Marstrand-type theorems and discuss their applications. This work builds on recent work of Lima and Moreira; they introduced the counting dimension for subsets of $\mathbb{Z}$ and proved a Marstrand-type theorem for it.

Evgeny I. Gordon, *On nonstandard analysis proofs of G. Birkhoff Ergodic Theorem.* Eastern Illinois University. `yigordon@eiu.edu`

The trivial proof of ergodic theorem for a finite set X and a permutation $\sigma : X \to X$ shows that for an arbitrary function $f : X \to \mathbb{R}$ the sequence of ergodic means $A_n(f,\sigma)$ stabilizes for $n \gg |X|$. We show that if $|X|$ is very big and $|f(x)| \ll |X|$ for almost all x, then $A_n(f,\sigma)$ stabilizes for significantly long segment of very big numbers n that are, however, $\ll |X|$. This statement has a natural rigorous formulation in terms of nonstandard analysis, which is, in fact, equivalent to classical ergodic theorem on hyperfinite Loeb spaces. We discuss some other properties of the sequence $A_n(f,\sigma)$ for a very big finite probability space X and their applications to standard dynamical systems on Lebesgue spaces. Though these statements have quite clear intuitive sense and even can be watched in some computer experiments, some their formulations in terms of sequences of finite probability spaces are very complicated, if not to say unreadable.

G. Tony Jacobs, *Periodicity and cubic numbers - Towards a solution of Hermite's problem.* University of North Texas. `GeorgeJacobs@my.unt.edu`

It has long been known that the continued fraction expansion of a quadratic irrational number is eventually periodic. In 1848, Hermite asked whether a similar algorithm could be developed that would associate periodic sequences with cubic irrational numbers. We take inspiration from Richard Mollin's recent work linking continued fractions with sequences of ideals of algebraic integers. In a limited class of cubic fields, we find that we are able to mimic this process of 'ideal reduction'. In these cases, we have a geometric process which gives us periodic sequences associated to certain cubic numbers in these fields.

Adriane Kaïchouh, *Amenability and convex Ramsey theory.* Université Lyon 1. `kaichouh@math.univ-lyon1.fr`

Moore recently characterized the amenability of automorphism groups of countable ultrahomogeneous structures by a Ramsey-type property (in the vein of the Kechris-Pestov-Todorčević correspondence). We will present this property and give a generalization of Moore's result to automorphism groups of separable metric structures, which encompass all Polish groups. We will also discuss some nice consequences of this characterization.

Ed Krohne, *Existence of continuous graph homomorphisms from* $2^{\mathbb{Z}^2}$. University of North Texas. `EdwardKrohne@my.unt.edu`

Building on our previous work showing that there is no continuous 3-coloring of $2^{\mathbb{Z}^2}$, we investigate the broader question of the existance of a continuous graph homomorphism $f_G : 2^{\mathbb{Z}^2} \to G$ for a finite graph G. Because four-cycles form a cycle basis for the graph $\mathbb{Z}^2$, they are crucially important to homomorphisms from $\mathbb{Z}^2$. In particular, we define a CW-complex G^* by filling in all the four-cycles of G with faces, and then compute the homotopy group $\pi_1(G^*)$. We examine several positive and negative existence conditions for f_G based on $\pi_1(G^*)$ and give examples of the limitations of the homotopy group in determining the existence of f_G. We also showcase SAT-solving based software we developed to help answer these questions, which are applicable to a variety of questions in Borel combinatorics.

Tue Ly, *Diophantine approximation in number fields and Homogeneous Dynamics.* Brandeis University. `lntue@brandeis.edu`

Let K be a number field of degree d, and S be the set of all archimedean absolute values of K, then $K_S \cong \mathbb{R}^d$. A natural extension of Diophantine approximation in our setting is to consider how well you can approximate elements of K_S by the image of K via the diagonal embedding. In this talk, I will discuss about the analogues of the traditional results in Diophantine approximation in this setting, and their connection to the dynamics on certain space of lattices.

Bill Mance, *Unexpected distribution phenomenon resulting from Cantor series expansions.* University of North Texas. `mance@unt.edu`

We explore in depth the number theoretic and statistical properties of certain sets of numbers arising from their Cantor series expansion. As a direct consequence of our main theorem we deduce numerous new results as well as strengthen known ones.

Joel Moreira, *On $\{x+y, xy\}$ patterns in large sets of infinite fields.* Ohio State University. `moreira.6@osu.edu`

Certain problems from combinatorics can be solved employing tools from ergodic theory. Examples include an old lemma of Schur which states that, given any finite partition of the positive integer numbers, there exists a triple of the form $\{x, y, x+y\}$ in the same cell of the partition. Additionally, it can be shown that there also exists a triple of the form $\{x, y, xy\}$ in the same cell.

It has been an open problem ever since whether one can find a quadruple $\{x, y, x+y, xy\}$ in the same cell.

I will explain how one can try to apply ideas from dynamics to attack this problem and I will present recent joint work with V. Bergelson on analogues of this question with the set of positive integers replaced with an infinite field.

Donald Robertson, *Straus' example and ergodic theory.* Ohio State University. `robertson.250@osu.edu`

A result due to Hindman states that, no matter how the positive integers are finitely partitioned, one cell of the partition contains a sequence and all its sums without repetition. Straus, answering a question of Erdos, later gave an example showing that a density version of Hindman's result does not hold. He exhibited sets of positive integers with arbitrarily large density, each having the property that no shift contains a sums set of the above kind. In this talk I will present recent joint work with V. Bergelson, C. Christopherson and P. Zorin-Kranich in which we generalize Straus' example to a class of locally compact, second countable, amenable groups and show, using ergodic theory techniques recently developed by Host and Austin, that positive density subsets of groups outside this class must contains sets with strong combinatorial properties.

Youngwhan Son, *Joint ergodicity along generalized linear functions.* Weizmann Institute of Science. `younghwan.son@weizmann.ac.il`

Invertible commuting measure preserving transformations $T_1, \ldots, T_s$ are called *jointly ergodic*, if for any $f_1, \ldots, f_s \in L^\infty(X)$,

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} T_1^n f_1(x) \cdots T_s^n f_s(x) = \prod_{i=1}^{s} \int f_i \, d\mu \quad \text{in } L^2.$$

Berend and Bergelson proved that the necessary and sufficient conditions for joint ergodicity are that $T_1 \times \cdots \times T_s$ is ergodic and $T_i^{-1} T_j (i \neq j)$ are ergodic.

In this talk I will present joint work with Bergelson and Leibman in which we generalize the above result to the sequences of transformations of the form $T_i^{\phi_i(n)} (i = 1, 2, \ldots, s)$, where ϕ_i are integer valued generalized linear functions, namely the functions constructible from the conventional linear functions with the help of the operations of addition, multiplication by constants, and taking the integer part (or the fractional part).

Milan Stehlík, *Topological aggregation: a unified framework for both Quantum and Classical Logic?.* Johannes Kepler University. `mlnstehlik@gmail.com`

The concept of dependence and information aggregation is present in all scientific areas. When working with mathematical concepts of dependence and aggregation, we often use more structural properties than it is needed. Once it gives us the advantage of simplicity but sometimes it is a burden diverting attention from important clues to new discovery. By employing aggregation on topological space (see Stehlík [3]), one can address two imprecision structures at once: imprecision in logic operation by employing the "fuzzy" concept together with imprecision of space irregularities by relaxation of the metrizability condition. The latter one may play a crucial role in a fuzzy logical inference on complex sets. During presentation we will show how topological aggregation can introduce the unified approach for quantum and classical logic and illustrate how sharpen are some paradoxes. In particular, in quantum mechanics, *observables* correspond to self-adjoint operators, see von Neumann [2]. Consequently, some logical operations in quantum logics have different properties from their classical analogies, e.g., OR. The logical disjunction behaves differently in quantum logic (see e.g. Aerts et al. [1]). Thus space $B_{sa}(H)$ (the space of all bounded self-adjoint operators in Hilbert space H) is a good candidate for the construction of an example showing how "infinite-dimensionality" of H-space (basic assumption in quantum physics) can violate the Lipschitz property in the operator norm of an aggregation operator A. Eventually, applications to cancer research modeling can be mentioned and emphasized.

References

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Andrew Zucker, *The Generic Point Problem and Closed Subgroups of S_∞*. Carnegie Mellon University. `zucker.andy@gmail.com`

A topological group G is said to have the *Generic Point Property* if the universal minimal flow $M(G)$ has a *generic point*, a point whose orbit is comeager. This in turn implies that any minimal G-flow has a generic point. Angel, Kechris, and Lyons asked the following question, known as the Generic Point Problem: If G is a Polish group and the universal minimal flow $M(G)$ is metrizable, does G have the Generic Point Property? In this talk, we will discuss the case where G is a closed subgroup of S_∞; in this case, the answer is affirmative. To show this, we give a new, explicit characterization of the greatest G-ambit and introduce some new tools in structural Ramsey theory.