Understanding Truth Tables

Definitions:

∧ - "and" or **conjunction**, statement is True only when both values are True.

i.e., **T ∧ T** = **T** , **T ∧ F** = **F**

v - "or" or **disjunction**, statement is True if at least one value is True.

i.e., **T v T** = **T** and **T v F** = **T**

~ - "not" or **negative**, negates or changes the value.

i.e., **~ T** = **F** and **~F** = **T**

→ - "if - then" or **conditional**, statement which determines a value of a scenario playing out given the item occurred, the condition. Note that these are always true if the condition, the first statement, is false.

i.e., **T → T** = **T** and **F → F** = **T**

↔ - "if and only if" or **bi-conditional**, like the conditional statement but goes both ways. Note that these are generally true when the values of the statements on each side are the same.

i.e., **T ↔ T** = **T** and **F ↔ F** = **T**

Statements can **ONLY** be True or False, **T** or **F**, but **NOT** both.

Statements can be put together to make **compound statements** which can get confusing and hard to follow, but if you make a table like in the example below, then it'll make life much easier.

Basic Truth Table for two statements, **p** and **q**.

p	q	~p	~q	p ∧ q	p v q	p → q	p ↔ q
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

Notice there are 4 rows of possible scenarios, this table encompasses all the possible scenarios of **p** and **q** values: **p** is **T** and **q** is **T**, or **p** is **T** and **q** is **F**, or **p** is **F** and **q** is **T**, or **p** is **F** and **q** is **F**. The number of possible scenarios, or rows, can be calculated by $\mathbf{2^k}$ where **k** is the **number of statements you have**. So if you were working with 3 statements: **p**, **q**, and **r**, you'd have **8** rows for whatever problem you were trying to solve.

Example:

Typical problems may have you construct the truth table to find the truth values of **compound statements** like **(p ∧ q) v (r v q)**, so we can construct a truth table for this problem and work through it in steps, doing one column at a time.

Step 1) We set up our table, notice the first three columns are just our basic statements: **p**, **q**, and **r**, these will not change

p	q	r	(p ∧ q)	(r v q)	(p ∧ q) v (r v q)
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Step 2) Now we can fill out the first two **compound statements** in the parenthesis, column 4 and 5, using the **conjunction** and **disjunction** rules from the basic truth table from the previous page.

p	q	r	(p ∧ q)	(r v q)	(p ∧ q) v (r v q)
T	T	T	T	T	
T	T	F	T	T	
T	F	T	F	T	
T	F	F	F	T	
F	T	T	F	T	
F	T	F	F	F	
F	F	T	F	T	
F	F	F	F	F	

Step 3) Now we can use the truth values in columns 4 and 5 and take the **disjunction** of the two to complete column 6, then we're done.

p	q	r	(p ∧ q)	(r v q)	(p ∧ q) v (r v q)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	F	T	T
F	F	F	F	F	F