NATIONAL CENTER For HEALTH STATISTICS | Series 2 Number 4

**VITAL and HEALTH STATISTICS**
DATA EVALUATION AND METHODS RESEARCH

## comparison of two methods of

# Constructing Abridged Life Tables

## by reference to a "standard" table

Comparison of the revised and the prior method of constructing the abridged life tables for the United States.

Washington, D.C.

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U.S. DEPARTMENT OF
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# COMPARISON OF TWO METHODS OF

# CONSTRUCTING ABRIDGED LIFE TABLES

## INTRODUCTION

The publication of an annual series of abridged life tables for the United States was started in 1945. After small biases were detected in the values of 1950 U.S. abridged life tables, studies were undertaken which led to the development of a revised method for constructing the U.S. abridged life tables. This report outlines the revised method used in constructing the abridged life tables since 1954. The construction of the life table for the total population for 1959 is shown in appendix table A. An earlier report[1] outlined the method used in preparing the abridged life tables for the years 1946 to 1953 inclusive, which henceforth will be referred to as the original method.

A test of the accuracy of the revised method of constructing the U.S. abridged life tables is presented which involves a comparison of the 1949-51 abridged life tables constructed by the revised method with the complete decennial 1949-51 life tables which were constructed by elaborate and laborious methods[2]. The 1949-51 abridged life tables constructed by the original method are also compared with those derived from the 1949-51 life tables. Comparing the abridged life tables, constructed by original and revised methods, with the decennial life tables provides a test of the relative accuracy of these methods of constructing the U.S. abridged life tables.

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*This report was prepared by Monroe G. Sirken, of the Division of Health Records Statistics.*

## METHOD OF CONSTRUCTION

The original and the revised methods of constructing the U.S. abridged life tables have in common the fact that each involves reference to a standard life table. According to this method of constructing abridged life tables, certain relationships among the functions of the life table under construction are assumed to be the same as those of another life table already existing (referred to as the "standard" table). In the calculation of the annual abridged life tables since 1954, the decennial U.S. life tables 1949-51[3] have been used as standard tables. When the 1959-61 decennial life tables are constructed, they will become the standard life tables in constructing the U.S. abridged life tables.

The method presented here is based on an observed relationship between the probability of death $({}_nq_x)$ and the age-specific death rate $({}_n\mu_x)$. The function ${}_nq_x$ is the proportion $\frac{{}_nd_x}{l_x}$ where $l_x$ is the number of survivors to exact age $x$ in the hypothetical life table cohort and ${}_nd_x$ is the number of the group who die before reaching exact age $x+n$. The function ${}_n\mu_x$ is the quotient of the number of deaths between exact ages $x$ and $x+n$ during the year and the size of the living population between these exact ages. The age-specific death rate may be defined either in terms of observed population data $({}_nM_x)$ or in terms of the stationary population of the life table $({}_nm_x)$. The former $({}_nM_x)$ is the quotient of the number of deaths in a given calendar year between exact ages x and

$x+n$ and the midyear population between those exact ages. The latter (${}_nm_x$) is the number of deaths (${}_nd_x$) in the life table divided by the number of persons (${}_nL_x$) in the stationary population of the life table between ages $x$ to $x+n$.

According to the revised method of constructing the abridged life table, the relationship between ${}_nq_x$ and ${}_n\mu_x$ is given by the formula

$$(1) \qquad {}_nq_x = \frac{n \; {}_n\mu_x}{1 + (\alpha_{{}_n\mu_x})\,{}_n\mu_x},$$

$$(2) \qquad \alpha_{{}_n\mu_x} = \frac{n}{{}_nq_x} - \frac{1}{{}_n\mu_x}.$$

It will be observed that formula (2) generates 2 sets of conversion constants according to whether ${}_n\mu_x$ is defined as ${}_nM_x$ the observed age-specific mortality rate, or as ${}_nm_x$, the age-specific mortality rate of the stationary population of the life table. The constants $\alpha_{{}_nM_x}$ are used as adjustment factors to convert the observed population age-specific mortality rates into the values on ${}_nq_x$ of the abridged life table. The constants $\alpha_{{}_nm_x}$ are used to calculate the values of ${}_nL_x$ from the values of $l_x$ and ${}_nd_x$ in the abridged life table.
Thus,

$$(3) \qquad \alpha_{{}_nm_x} = \frac{nl_x - {}_nL_x}{{}_nd_x},$$

or

$$(4) \qquad {}_nL_x = nl_x - \alpha_{{}_nm_x}\,{}_nd_x.$$

Greville[4] has also suggested the use of formula (4) to calculate the L-function in the construction of the abridged life table by reference to a standard table.

The assumption underlying the abbreviated method of life table construction used here is that in each age interval $x$ to $x+n$, the constants $\alpha_{{}_n\mu_x}$ ($\mu = M, m$) may be regarded as having the same value in the life table under construction as in the standard table. The constants $\alpha_{{}_nM_x}$ and $\alpha_{{}_nm_x}$ that have been used in the construction of the abridged life tables since 1954 are presented in table 1. They were derived by formula (2) according to relationships observed between ${}_nq_x$ and ${}_n\mu_x$ in the complete U.S. life table for the decennial period 1949-51. Until more current standard tables (U.S. life tables for the decennial period 1959-61) are constructed, these constants will be used each year to construct the U.S. abridged life tables.

## CONSTRUCTING THE 1959 ABRIDGED LIFE TABLES

Basic sources of data used in the preparation of the U.S. life tables for 1959 were the annual mortality tabulations of the National Vital Statistics Division and estimates of the population on July 1, 1959, by age, color, and sex prepared by the U.S. Bureau of the Census.

Values of ${}_nM_x$, the observed population age-specific mortality rates were obtained from the basic mortality and population data. The values of ${}_nq_x$ were calculated by formula (1) using the set of constants $\alpha_{{}_nM_x}$ presented in table 1. (The method of calculating the values of the probability of death during the first year of life and of the final age group 85 years and over is described below.) After the values of ${}_nq_x$ had been obtained, the $l_x$ and ${}_nd_x$ functions were computed in the conventional manner, according to the formula

$${}_nd_x = (l_x)\;({}_nq_x)\;;\; l_{x+n} = l_x - {}_nd_x.$$

Thereafter, the values of ${}_nL_x$ were calculated by formula (4) using the set of constants, $\alpha_{{}_nm_x}$, presented in table 1. The values of $T_x$ were obtained by summing the ${}_nL_x$ column, starting with the oldest age group. In other words,

$$T_x = T_{x+n} + {}_nL_x.$$

Table 1. Conversion factors based on decennial life tables for the United States, 1949-51

| Age interval (years) | Total population | Male | | | Female | | |
|---|---|---|---|---|---|---|---|
| | | Total | White | Nonwhite | Total | White | Nonwhite |
| | $^{\alpha}_{n}M_x = \frac{n}{_nq_x} - \frac{1}{_nM_x}$ | | | | | | |
| 1-5 | 18.7253 | 19.0755 | 18.5164 | 16.7398 | 17.8984 | 18.6698 | 15.7642 |
| 5-10 | 17.1188 | 16.1574 | 21.3402 | 9.5389 | 24.9787 | 22.5984 | 21.2327 |
| 10-15 | -27.7680 | -28.3119 | -31.7154 | -15.5642 | -22.0673 | -18.0455 | -44.7131 |
| 15-20 | 10.2732 | 7.1700 | 7.8382 | .1092 | 18.0825 | 19.9746 | 8.9045 |
| 20-25 | 7.5326 | 7.2706 | 8.1040 | 4.9690 | 5.8762 | 6.3873 | 3.5398 |
| 25-30 | .1806 | -.2167 | -1.0433 | 1.7692 | .3806 | -1.3368 | 2.2291 |
| 30-35 | -1.0910 | 1.2348 | 2.6955 | 5.9535 | -3.9384 | 3.0035 | 2.7942 |
| 35-40 | .9558 | 1.1954 | 1.6008 | -1.8298 | .7362 | 1.1471 | 3.3718 |
| 40-45 | 2.2822 | 2.1367 | 1.7854 | 4.0718 | 2.3874 | 1.0406 | 5.9329 |
| 45-50 | 1.6621 | 1.8081 | 2.1524 | .4126 | 1.4901 | 2.1793 | -.0091 |
| 50-55 | 2.2507 | 2.2277 | 2.1262 | 2.9371 | 2.3393 | 2.0834 | 2.9772 |
| 55-60 | 2.2598 | 2.3750 | 2.3389 | 2.5563 | 2.0481 | 1.9335 | 2.3614 |
| 60-65 | 2.4041 | 2.3848 | 2.3681 | 2.4971 | 2.4313 | 2.3863 | 2.6988 |
| 65-70 | 2.2343 | 2.3584 | 2.3431 | 2.4228 | 2.0201 | 2.0204 | 1.9844 |
| 70-75 | 2.3399 | 2.3872 | 2.3807 | 2.4814 | 2.2793 | 2.2374 | 2.7313 |
| 75-80 | 2.4376 | 2.5014 | 2.5026 | 2.5300 | 2.3645 | 2.3595 | 2.4964 |
| 80-85 | 2.5307 | 2.5607 | 2.5621 | 2.5478 | 2.4998 | 2.4987 | 2.5590 |
| | $^{\alpha}_{n}m_x = \frac{n l_x - {}_nL_x}{_nd_x}$ | | | | | | |
| 1-5 | 2.4152 | 2.3990 | 2.3664 | 2.5044 | 2.4354 | 2.4212 | 2.4671 |
| 5-10 | 2.6834 | 2.6602 | 2.6537 | 2.7082 | 2.7176 | 2.6996 | 2.8127 |
| 10-15 | 2.3174 | 2.2879 | 2.2954 | 2.2475 | 2.3634 | 2.4046 | 2.1910 |
| 15-20 | 2.3205 | 2.3074 | 2.3189 | 2.2390 | 2.3373 | 2.3663 | 2.2543 |
| 20-25 | 2.4423 | 2.4542 | 2.4687 | 2.3895 | 2.4178 | 2.4252 | 2.3851 |
| 25-30 | 2.4468 | 2.4757 | 2.4863 | 2.4377 | 2.4035 | 2.4006 | 2.4064 |
| 30-35 | 2.3839 | 2.3920 | 2.3866 | 2.4039 | 2.3735 | 2.3678 | 2.3792 |
| 35-40 | 2.3421 | 2.3375 | 2.3244 | 2.3909 | 2.3507 | 2.3427 | 2.3711 |
| 40-45 | 2.3293 | 2.3233 | 2.7740 | 2.3698 | 2.3411 | 2.3318 | 2.3658 |
| 45-50 | 2.3437 | 2.3370 | 2.3287 | 2.3703 | 2.3563 | 2.3436 | 2.3834 |
| 50-55 | 2.3536 | 2.3521 | 2.3427 | 2.3946 | 2.3609 | 2.3436 | 2.4024 |
| 55-60 | 2.3700 | 2.3820 | 2.3725 | 2.4442 | 2.3569 | 2.3455 | 2.4361 |
| 60-65 | 2.3980 | 2.4145 | 2.4054 | 2.4912 | 2.3731 | 2.3392 | 2.4729 |
| 65-70 | 2.4055 | 2.4356 | 2.4272 | 2.5175 | 2.3640 | 2.3569 | 2.4974 |
| 70-75 | 2.4280 | 2.4624 | 2.4569 | 2.5313 | 2.3876 | 2.3488 | 2.5064 |
| 75-80 | 2.4879 | 2.5242 | 2.5223 | 2.5607 | 2.4499 | 2.3780 | 2.5321 |
| 80-85 | 2.5747 | 2.6051 | 2.6061 | 2.5985 | 2.5449 | 2.4453 | 2.5693 |

The values of the average remaining lifetime was then obtained by division $\mathring{e}_x = T_x \div l_x$ .

Formulas (1) and (3) respectively were not used to compute the $q_x$ and $L_x$ functions for the first year of life and the final age group 85 years and over. Rather, the special treatment of these age groups used in the construction of U.S. abridged life tables for the years 1945 to 1953 inclusive was continued. The following explanation has been adapted and extracted from a report that describes the method used to construct these earlier tables.[1]

For the age group 85 years and over formula (2) shows that $\alpha_{\,n}M_{85}$ is infinite since $n=\infty$. Hence the assumption that the value of $\alpha_{\,\infty}M_{85}$ is the same in the life table under construction as in the standard table is not useful, and some other assumption must be made. Instead, the ratio $\kappa$, defined as the quotient of the value of $_{\infty}M_{85}$ based on the actual data by the corresponding value $_{\infty}m_{85}$ for the stationary population of the life table was assumed to be the same in the table under construction as in the standard table. But $_{\infty}m_{85}$ is the reciprocal of $\mathring{e}_{85}$, the average remaining lifetime. Thus, the value of $\mathring{e}_{85}$ can be computed by the formula

$$\mathring{e}_{85} = \frac{\kappa}{_{\infty}M_{85}} .$$

According to the standard tables (1949-51), $\kappa$=.9487119 for the total population. The values of $\kappa$ for the 4 subdivisions of the population by color and sex are shown below:

| *Subdivision of the population* | $\kappa$ |
|---|---|
| White males | .9610759 |
| White females | .9554947 |
| Nonwhite males | .8534401 |
| Nonwhite females | .8072982 |

The abridged life table for 1959 can then be computed since

$$T_{85} = l_{85} \times \mathring{e}_{85}.$$

The value of $q_0$ the proportion of liveborn infants dying before reaching age 1, is computed from birth and death statistics, being taken as equal to the adjusted infant death rate. A method of adjusting the infant death rate for the changing number of births is described in a previous publication.[5] The adjustment is made by allocating the deaths of infants occurring during a given year to the year in which the infants were born. The infant deaths so allocated are then related to the births occurring in the respective year of birth. The expression for computing the adjusted infant mortality rate per 1,000 live births may be written:

$$\text{Adjusted rate} = \left[\frac{D\ (1-f)}{E} + \frac{Df}{E'}\right] \times 1{,}000$$

where

- $D$ = number of infant deaths occurring in the given year.
- $f$ = ratio of deaths occurring in the given year among infants born in the preceding year to the total infant deaths of the given year. This is referred to as the "separation factor."
- $E$ = number of births occurring in the given year.
- $E'$ = number of births occurring in the preceding year.

The stationary population in the first year of life was obtained by the formula $L_0 = l_0 - (1-f)d_0$.

## EVALUATION OF THE ABRIDGED LIFE TABLE METHODS

A set of U.S. abridged life tables, 1949-51 for subdivisions of the population by color and sex was constructed by the revised method of construction by reference to a standard table. Values of the constants ${}^{\alpha}_{n}M_x$ and ${}^{\alpha}_{n}m_x$ needed in the construction of these tables were derived from the complete U.S. life tables, 1939-41, which served as the standard tables. The decennial U.S. life tables 1949-51 were the criterion tables for the evaluation of the precision of the abridged life tables.

The basic data used in the preparation of the U.S. abridged life tables 1949-51 were essentially the same as those which had been used in the preparation of the complete U.S. life tables 1949-51. These included mortality data by age, sex, and color for the 3-year period 1949-51, extracted from the annual issues of the *Vital Statistics of the United States* published by the National Vital Statistics Division, and population data by age, sex, and color enumerated in the 1950 Census and published by the Bureau of the Census in U.S. *Census of Population,* Volume II, "Characteristics of the Population."

There is close agreement (table 2) between the values of the expectation of life based on the complete life tables and those based on the revised abridged life table method. The abridged life table values exceed the decennial life table values at virtually all ages but the differences are small. For example, the difference between the values of the expectation of life at birth was only .01 years for the total population; it was less than .03 years for white males, white females, and nonwhite males; and .15 years for nonwhite females. For each of these population groups, there is a tendency for the differences between the values of the expectation of life to increase with advancing age. At virtually all ages the differences are greater for nonwhite than for white persons, and within each color group, the differences are greater for females than for males.

Using the same basic data, that is the population data from the 1950 Census and the mortality data for the 3-year period 1949-51, another set of abridged U.S. life tables 1949-51 were prepared by the original abridged life table method. This is the method of construction by reference to a standard table, that had been used to construct the annual abridged U.S. life tables, 1945-53.

The assumptions underlying the original method are that in each age interval $x$ to $x+n$, ${}_nh_x$ defined as the ratio ${}_nq_x \div {}_nM_x$ and the values of the ratio ${}_nj_x = {}_nL_x \div (l_x + l_{x+n})$ were assumed to have the same value in the life table under constructions as in the standard table. Values of the constants ${}_nh_x$ and ${}_nj_x$ needed in the construction of the abridged U.S. life tables 1949-51 by the original method were available[1] for the decennial U.S. life tables 1939-41 which served as the standard tables.

The values of expectation of life based on the original method exceed those of the decennial life table at every age (table 2). At virtually every age, these differences are greater than the amounts by which values of expectation of life based on the revised method exceed those based on the decennial life table. Thus, for the total population the value of expectation of life at birth according to the decennial life table is exceeded by .01 years according to the revised method and it is exceeded by .15 years according to the original method. It is noteworthy that both methods of constructing life tables by reference to a standard table slightly overstate the values of the expectation of life at every age, although the overstatement is consistently less for the revised than for the original life table method.

The absolute values of difference between ${}_nq_x$ values based on the decennial life tables and on the abridged life table are virtually always smaller for the revised than for the original abridged life table method (table 3). Furthermore, the original method in most age groups understates the values of ${}_nq_x$, a tendency which is not evident for the revised method.

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[2]Monroe G. Sirken and Mortimer Spiegelman, "Method of Constructing the 1949-51 National, Divisional, and State Life Tables," *Vital Statistics--Special Reports,* Vol. 41, No. 5, July 31, 1959.

[3]Monroe G. Sirken, "United States Life Tables, 1949-51," *Vital Statistics--Special Reports,* Vol. 41, No. 1, November 23, 1954.

[4]Thomas N. E. Greville, "On the Formula for the L-Function in a Special Mortality Table Eliminating a Given Cause of Death," *Transactions of the Society of Actuaries,* Vol. VI, Meeting No. 14, April 1954.

[5]Iwao M. Moriyama and Thomas N. E. Greville, "Effect of Changing Birth Rates Upon Infant Mortality Rates," *Vital Statistics--Special Reports,* Vol. 19, No. 21, November 10, 1944.

[6]Thomas N. E. Greville, "United States Life Tables and Actuarial Tables, 1939-41," U.S. Government Printing Office, 1947.

Table 2. Differences between values of expectation of life in the complete life table and in abridged life tables, by color, sex and age: United States, 1949-51

| Sex and age | White: $\overset{o}{e}_x$ based on complete life tables | White: Abridged life table values minus complete life table values: Original abridged method | White: Abridged life table values minus complete life table values: Revised abridged method | Nonwhite: $\overset{o}{e}_x$ based on complete life tables | Nonwhite: Abridged life table values minus complete life table values: Original abridged method | Nonwhite: Abridged life table values minus complete life table values: Revised abridged method |
|---|---|---|---|---|---|---|
| MALE | | | | | | |
| 0-1-------- | 66.31 | .10 | .00 | 58.91 | .16 | .02 |
| 1-5-------- | 67.41 | .10 | -.01 | 61.06 | .17 | .02 |
| 5-10------- | 63.77 | .11 | .00 | 57.69 | .22 | .06 |
| 10-15------ | 58.98 | .12 | .00 | 52.96 | .23 | .07 |
| 15-20------ | 54.18 | .12 | .01 | 48.23 | .22 | .06 |
| 20-25------ | 49.52 | .12 | .00 | 43.73 | .23 | .07 |
| 25-30------ | 44.93 | .11 | .00 | 39.49 | .23 | .07 |
| 30-35------ | 40.29 | .12 | .00 | 35.31 | .23 | .07 |
| 35-40------ | 35.68 | .11 | .00 | 31.21 | .24 | .09 |
| 40-45------ | 31.17 | .12 | .00 | 27.29 | .22 | .07 |
| 45-50------ | 26.87 | .11 | .00 | 23.59 | .24 | .10 |
| 50-55------ | 22.83 | .11 | .00 | 20.25 | .22 | .09 |
| 55-60------ | 19.11 | .12 | .00 | 17.36 | .19 | .08 |
| 60-65------ | 15.76 | .12 | .00 | 14.91 | .21 | .11 |
| 65-70------ | 12.75 | .14 | .01 | 12.75 | .27 | .15 |
| 70-75------ | 10.07 | .15 | .01 | 10.74 | .35 | .20 |
| 75-80------ | 7.77 | .15 | .01 | 8.83 | .44 | .27 |
| 80-85------ | 5.88 | .21 | .02 | 7.07 | .54 | .40 |
| 85+-------- | 4.35 | .05 | .05 | 5.38 | .61 | .61 |
| FEMALE | | | | | | |
| 0-1-------- | 72.03 | .20 | .02 | 62.70 | .30 | .15 |
| 1-5-------- | 72.77 | .20 | .01 | 64.37 | .31 | .15 |
| 5-10------- | 69.09 | .22 | .02 | 60.93 | .37 | .19 |
| 10-15------ | 64.26 | .21 | .02 | 56.17 | .36 | .18 |
| 15-20------ | 59.39 | .21 | .02 | 51.36 | .37 | .18 |
| 20-25------ | 54.56 | .22 | .03 | 46.77 | .37 | .19 |
| 25-30------ | 49.77 | .22 | .03 | 42.35 | .37 | .19 |
| 30-35------ | 45.00 | .22 | .03 | 38.02 | .37 | .20 |
| 35-40------ | 40.28 | .22 | .03 | 33.82 | .36 | .20 |
| 40-45------ | 35.64 | .22 | .03 | 29.82 | .35 | .19 |
| 45-50------ | 31.12 | .22 | .03 | 26.07 | .38 | .22 |
| 50-55------ | 26.76 | .22 | .03 | 22.67 | .40 | .24 |
| 55-60------ | 22.58 | .23 | .03 | 19.62 | .37 | .24 |
| 60-65------ | 18.64 | .23 | .03 | 16.95 | .39 | .28 |
| 65-70------ | 15.00 | .22 | .03 | 14.54 | .45 | .35 |
| 70-75------ | 11.68 | .22 | .03 | 12.29 | .51 | .41 |
| 75-80------ | 8.87 | .22 | .05 | 10.15 | .36 | .57 |
| 80-85------ | 6.59 | .19 | .06 | 8.15 | .74 | .77 |
| 85+-------- | 4.83 | .11 | .11 | 6.15 | 1.18 | 1.18 |

Table 3. Differences between values of the probability of dying in the complete life table and in abridged life tables, by color, sex, and age: United States, 1949-51

| Sex and age interval | White | | | Nonwhite | | |
|---|---|---|---|---|---|---|
| | $_nq_x$ based on complete life tables | Abridged life table values minus complete life table values | | $_nq_x$ based on complete life tables | Abridged life table values minus complete life table values | |
| | | Original abridged method | Revised abridged method | | Original abridged method | Revised abridged method |
| MALE | | | | | | |
| 1-5------ | .00544 | .00016 | .00015 | .01043 | .00080 | .00067 |
| 5-10----- | .00347 | .00001 | .00002 | .00498 | -.00002 | .00001 |
| 10-15---- | .00354 | -.00005 | -.00006 | .00522 | -.00005 | -.00006 |
| 15-20---- | .00652 | .00002 | .00007 | .01102 | .00000 | .00000 |
| 20-25---- | .00852 | .00002 | .00012 | .01801 | -.00007 | .00001 |
| 25-30---- | .00853 | -.00003 | -.00001 | .02168 | -.00018 | -.00003 |
| 30-35---- | .01013 | -.00002 | .00004 | .02703 | -.00013 | .00028 |
| 35-40---- | .01480 | -.00008 | -.00004 | .03616 | -.00077 | -.00066 |
| 40-45---- | .02381 | -.00009 | -.00003 | .05005 | .00031 | .00086 |
| 45-50---- | .03821 | -.00021 | -.00008 | .07365 | -.00198 | -.00130 |
| 50-55---- | .05963 | -.00009 | .00008 | .10658 | -.00301 | -.00110 |
| 55-60---- | .09098 | -.00032 | .00000 | .14721 | -.00073 | .00031 |
| 60-65---- | .13163 | -.00064 | -.00008 | .18614 | .00028 | .00066 |
| 65-70---- | .18580 | -.00142 | -.00018 | .22524 | .00094 | .00123 |
| 70-75---- | .26348 | -.00287 | .00021 | .27260 | -.00325 | -.00092 |
| 75-80---- | .37002 | -.00838 | .00025 | .33636 | -.00581 | .00149 |
| 80-85---- | .49946 | -.02093 | .00330 | .41444 | -.02013 | -.00279 |
| FEMALE | | | | | | |
| 1-5------ | .00457 | .00011 | .00011 | .00894 | .00059 | .00047 |
| 5-10----- | .00246 | .00013 | .00002 | .00396 | .00002 | .00004 |
| 10-15---- | .00210 | .00000 | -.00001 | .00355 | .00001 | -.00006 |
| 15-20---- | .00312 | .00001 | .00002 | .00846 | -.00008 | .00004 |
| 20-25---- | .00396 | -.00001 | .00001 | .01291 | -.00009 | .00003 |
| 25-30---- | .00485 | -.00001 | .00000 | .01665 | -.00009 | .00001 |
| 30-35---- | .00657 | -.00001 | .00000 | .02196 | -.00042 | -.00012 |
| 35-40---- | .00945 | -.00006 | -.00003 | .03100 | -.00059 | -.00061 |
| 40-45---- | .01440 | -.00005 | -.00002 | .04410 | -.00008 | .00067 |
| 45-50---- | .02200 | -.00012 | -.00003 | .06382 | -.00028 | -.00020 |
| 50-55---- | .03294 | -.00016 | -.00001 | .08845 | -.00289 | -.00101 |
| 55-60---- | .05039 | -.00041 | -.00008 | .12020 | -.00165 | -.00013 |
| 60-65---- | .07812 | -.00080 | -.00001 | .15221 | -.00022 | .00119 |
| 65-70---- | .12021 | -.00219 | -.00050 | .18615 | -.00278 | -.00145 |
| 70-75---- | .19465 | -.00401 | -.00016 | .22601 | -.00008 | .00188 |
| 75-80---- | .30096 | -.01071 | -.00114 | .28105 | -.00445 | .00041 |
| 80-85---- | .43860 | -.02056 | -.00257 | .34418 | -.00583 | .00117 |

# APPENDIX

## EXPLANATION OF THE COLUMNS OF TABLE A

*Column 1—Age interval* ($x$ to $x+n$).—The age interval shown in column 1 is the interval between the two exact ages indicated. For instance, "20-25" means the 5-year interval between the 20th and the 25th birthdays.

*Column 2—Population* (${}_nP_x$).—This column shows the estimated midyear population for the indicated age interval. Births for 1958 and 1959 were used in computing $q_0$.

*Column 3—Deaths* (${}_nD_x$).—This column shows the number of deaths for the age interval during 1959.

*Columns 4 and 5—Death rates* (${}_nM_x$).—The age-specific death rate shown in column 4 is the central death rate for the age interval. In column 5, these rates have been adjusted proportionately for deaths for which age was not reported on the death certificate.

*Column 6—Conversion factor* ($\alpha_{{}_nM_x}$).—This column is derived from a "standard" table, in this instance, the life table for the total population of the United States, 1949-51. These conversion factors are shown in table 1.

*Columns 7 and 8—Proportion dying* (${}_nq_x$).—The number shown in column 7 is the denominator of the proportion of the cohort dying in the age interval according to formula (1), page 3. Column 8 shows the proportion of the cohort who are alive at the beginning of an indicated age interval who will die before reaching the end of that age interval. For example, for the population in the age interval 20-25, the proportion dying is 0.0061—out of every 1,000 persons alive and exactly 20 years old at the beginning of the period, 6.1 will die before reaching their 25th birthday. In other words, the ${}_nq_x$ values represent probabilities that persons who are alive at the beginning of a specific age interval will die before reaching the beginning of the next age interval. The "proportion dying" column forms the basis of the life table; the life table is so constructed that all other columns are derived from it.

*Column 9—Number surviving* ($l_x$).—This column shows the number of persons, starting with a cohort of 100,000 live births, who survive to the exact age marking the beginning of each age interval. The $l_x$ values are computed from the ${}_nq_x$ values, which are successively applied to the remainder of the original 100,000 persons still alive at the beginning of each age interval. Thus, out of 100,000 live born babies, 97,357 will complete the first year of life and enter the second; 96,948 will begin the sixth year; 96,051 will reach 20; and 17,877 will live to age 85.

*Column 10—Number dying* (${}_nd_x$).—This column shows the number dying in each successive age interval out of 100,000 live births. Out of 100,000 persons born alive, 2,643 die in the first year of life, 409 in the succeeding 4 years, 584 in the 5-year period between exact ages 20 and 25, and 17,877 die after reaching age 85. Each figure in column 10 is the difference between two successive figures in column 9.

*Column 11—Conversion factor* ($\alpha_{{}_nm_x}$).—This column is derived from a "standard" table, in this instance, the life table for the total population of the United States, 1949-51. These conversion factors are shown in table 1.

*Columns 12 and 13—Stationary population* (${}_nL_x$ *and* $T_x$).—Suppose that a group of 100,000 individuals is born every year and that the proportions dying in each such group in each age interval throughout the lives of the members are exactly those shown in column 8. If there were no migration and if the births were evenly distributed over the calendar year, the survivors of these births would make up what is called a stationary population—stationary because in such a population the number of persons living in any given age group would never change. Thus, a census taken at any time in such a stationary community would always show the same total population and the same numerical distribution of that population among the various age groups. In such a stationary population supported by 100,000 annual births, column 9 shows the number of persons who, each year, reach the birthday which marks the beginning of the age interval indicated in column 1, and column 10 shows the number of persons who die each year in the indicated age interval.

Column 12 shows the number of persons in the stationary population in the indicated age interval. For example, the figure given in the age interval 20-25 is 478,829. This means that in a stationary population supported by 100,000 annual births and with proportions dying in each age group always in accordance with column 8, a census taken on any data would show 478,829 persons between exact ages 20 and 25.

Column 13 shows the number of persons in the stationary population in the indicated age interval (column 12) and all subsequent age intervals. For example, in the stationary population referred to in the last illustration, column 13 shows that there would be at any given moment, a total of 5,030,781 persons who have passed their 20th birthday. The population at all ages 0 and above (in other words, the total population of the stationary community) would be 6,965,532.

*Column 14--Average remaining lifetime* ($\mathring{e}_x$) .—The average remaining lifetime (also called expectation of life) at any given age is the average number of years remaining to be lived by those surviving to that age on the basis of a given set of age-specific rates of dying. In order to arrive at this value, it is first necessary to observe that the figures in column 12 can also be interpreted in terms of a single life table cohort without introducting the concept of the stationary population. From this point of view, each figure in column 13 represents the total time (in years) lived between two indicated birthdays by all those reaching the earlier birthday among the survivors of a cohort of 100,000 live births. Thus, the figure 478,829 in the age interval 20-25 is the total number of years lived between the 20th and 25th birthdays by the 96,051 persons (column 9) who reached the 20th birthday out of 100,000 live born babies. The corresponding figure (5,030,781) in column 13 is the total number of years lived after attaining age 20 by the 96,051 persons reaching that age. This number of years divided by the number of persons (5,030,781 divided by 96,051) gives 52.4 years as the average remaining lifetime at age 20.

Appendix Table A. Computation of abridged life table

| AGE INTERVAL<br>Period of life between two exact ages stated in years | Estimated population July 1, 1959 within age interval | Deaths in 1959 within age interval | Death rate unadjusted<br>Col. 3 / Col. 2 | Death rate adjusted for age not stated<br>Col. 4 X 1.00054 | Conversion factor<br>(See table 1) | Denominator of formula (1) (See page 2)<br>1+Col.5 X Col. 6 |
|---|---|---|---|---|---|---|
| $x$ to $x+n$ | ${}_nP_x$ | ${}_nD_x$ | | ${}_nM_x$ | ${}^{\alpha}_{n}M_x$ | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 0-1[1] | ... | 112,008 | ... | ... | ... | ... |
| 1-5 | 16,000 | 17,116 | 0.001069 | 0.001070 | 18.7253 | 1.02004 |
| 5-10 | 18,703 | 9,028 | .000483 | .000483 | 17.1188 | 1.00827 |
| 10-15 | 16,435 | 7,402 | .000450 | .000450 | -27.7680 | 0.98749 |
| 15-20 | 12,850 | 11,931 | .000928 | .000929 | 10.2732 | 1.00954 |
| 20-25 | 10,867 | 13,337 | .001227 | .001228 | 7.5326 | 1.00925 |
| 25-30 | 10,922 | 14,084 | .001290 | .001290 | 0.1806 | 1.00023 |
| 30-35 | 11,928 | 19,734 | .001654 | .001655 | -1.0910 | 0.99819 |
| 35-40 | 12,299 | 28,477 | .002315 | .002316 | 0.9558 | 1.00221 |
| 40-45 | 11,382 | 41,569 | .003652 | .003654 | 2.2822 | 1.00834 |
| 45-50 | 10,907 | 62,544 | .005734 | .005737 | 1.6621 | 1.00954 |
| 50-55 | 9,575 | 87,521 | .009141 | .009145 | 2.2507 | 1.02058 |
| 55-60 | 8,228 | 114,895 | .013964 | .013970 | 2.2598 | 1.03157 |
| 60-65 | 7,133 | 148,102 | .020763 | .020773 | 2.4041 | 1.04994 |
| 65-70 | 5,752 | 191,536 | .033299 | .033315 | 2.2343 | 1.07444 |
| 70-75 | 4,284 | 214,256 | .050013 | .050037 | 2.3399 | 1.11708 |
| 75-80 | 2,971 | 210,524 | .070860 | .070893 | 2.4376 | 1.17281 |
| 80-85 | 1,520 | 177,601 | .116843 | .116898 | 2.5307 | 1.29583 |
| 85 and over[1] | 860 | 174,369 | .202753 | .202850 | ... | ... |

[1]For method of computing values at these ages, see text on page 4.

for the total population of the United States, 1959

| Proportion of persons alive at beginning of age interval dying during interval | Number surviving to exact age $x$ out of 100,000 born alive | Number dying in age interval | Conversion factor | STATIONARY POPULATION: In age interval | STATIONARY POPULATION: In this and all subsequent intervals | Average years of life remaining to survivors at age $x$ |
|---|---|---|---|---|---|---|
| $\frac{n \text{ Col. } 5}{\text{Col. } 7}$ | Col. 9 (Line above)- Col. 10 (Line above) | Col. 8 X Col. 9 | (See table 1) | n Col. 9 - (10) X (11) | Sum of Col. 12 for this line and all below | $\frac{\text{Col. } 13}{\text{Col. } 9}$ |
| ${}_nq_x$ | $l_x$ | ${}_nd_x$ | ${}^{\alpha}_{n}m_x$ | ${}_nL_x$ | $T_x$ | $\mathring{e}_x$ |
| (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| 0.0264 | 100,000 | 2,643 | ... | 97,681 | 6,965,532 | 69.7 |
| .0042 | 97,357 | 409 | 2.4152 | 388,440 | 6,867,851 | 70.5 |
| .0024 | 96,948 | 232 | 2.6834 | 484,117 | 6,479,411 | 66.8 |
| .0023 | 96,716 | 221 | 2.3174 | 483,068 | 5,995,294 | 62.0 |
| .0046 | 96,495 | 444 | 2.3205 | 481,445 | 5,512,226 | 57.1 |
| .0061 | 96,051 | 584 | 2.4423 | 478,829 | 5,030,781 | 52.4 |
| .0064 | 95,467 | 616 | 2.4468 | 475,828 | 4,551,952 | 47.7 |
| .0083 | 94,851 | 786 | 2.3839 | 472,381 | 4,076,124 | 43.0 |
| .0116 | 94,065 | 1,087 | 2.3421 | 467,779 | 3,603,743 | 38.3 |
| .0181 | 92,978 | 1,684 | 2.3293 | 460,967 | 3,135,964 | 33.7 |
| .0284 | 91,294 | 2,594 | 2.3437 | 450,390 | 2,674,997 | 29.3 |
| .0448 | 88,700 | 3,974 | 2.3536 | 434,147 | 2,224,607 | 25.1 |
| .0677 | 84,726 | 5,737 | 2.3700 | 410,034 | 1,790,460 | 21.1 |
| .0989 | 78,989 | 7,814 | 2.3980 | 376,207 | 1,380,426 | 17.5 |
| .1550 | 71,175 | 11,034 | 2.4055 | 329,333 | 1,004,219 | 14.1 |
| .2240 | 60,141 | 13,469 | 2.4280 | 268,002 | 674,886 | 11.2 |
| .3022 | 46,672 | 14,106 | 2.4879 | 198,265 | 406,884 | 8.7 |
| .4511 | 32,566 | 14,689 | 2.5747 | 125,010 | 208,619 | 6.4 |
| 1.0000 | 17,877 | 17,877 | ... | 83,609 | 83,609 | 4.7 |

*U.S. National Center for Health Statistics.*

Comparison of two methods of constructing abridged life tables by reference to a "standard" table; comparison of the revised and the prior method of constructing the abridged life tables for the United States. Washington, U.S. Department of Health, Education, and Welfare. Public Health Service, 1964.

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