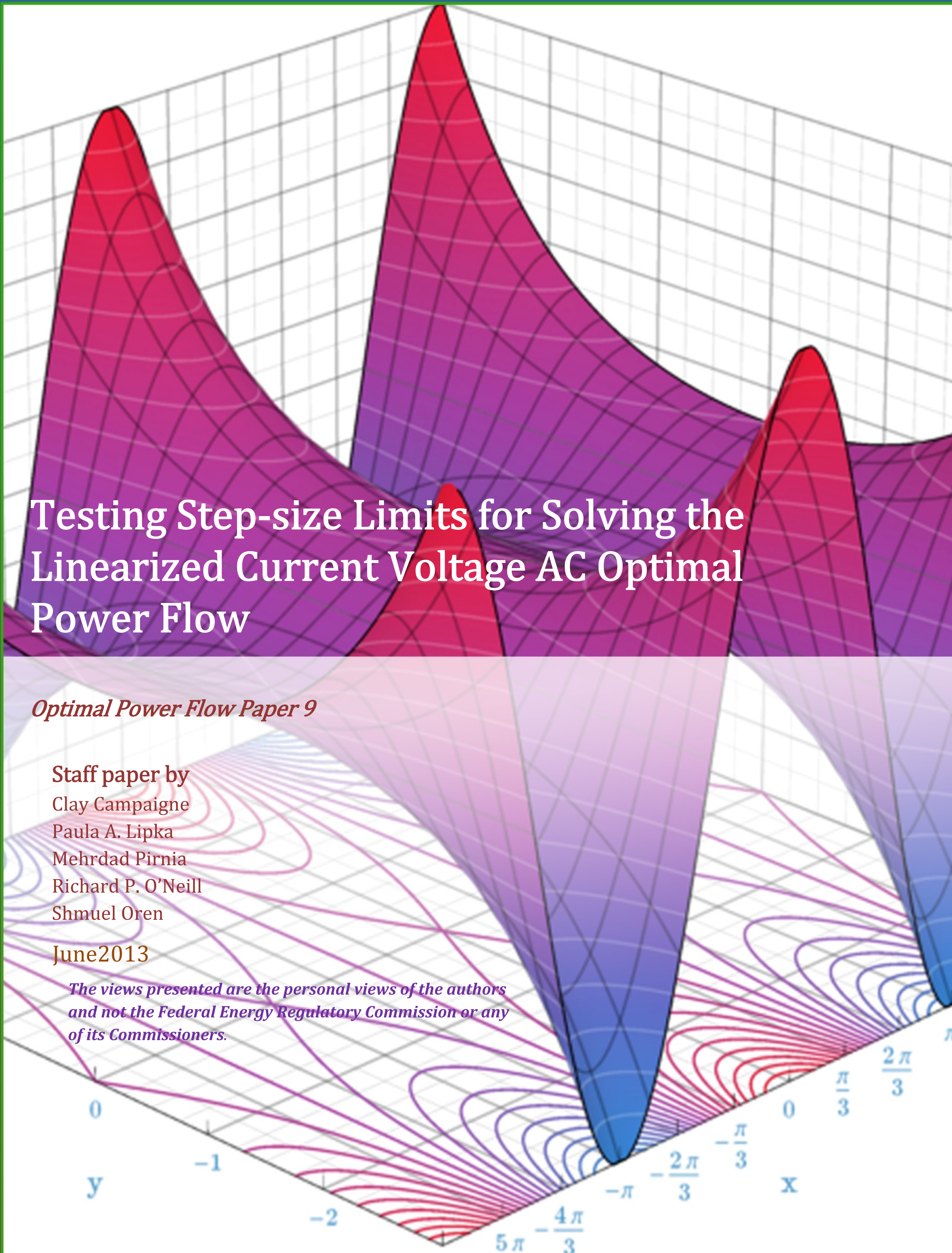

Testing Step-size Limits for Solving the Linearized Current Voltage AC Optimal Power Flow

Optimal Power Flow Paper 9

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Abstract and Executive Summary

In this paper, we seek to improve the performance of the iterative linear program approximation to the current voltage AC optimal power flow (ILIV-ACOPF). By adding a set of constraints that limit the differences between the real and imaginary voltages of successive major iteration solutions, we limit the error in the linear approximation, and we seek to decrease the time to solve and increase the robustness of the procedure. The primary motivation is that the iterative linearization procedure sometimes exhibits periodic behavior ("bouncing" between two solutions). This behavior may add to the solution time or result in a failure to converge. Generally, the step-size constraints improve performance of the iterative linear approximation procedure, but the best parameters of the step-size constraint are problem dependent. Although the convergence tests are different, the linear procedure is considerably faster than the nonlinear solver The tradeoff between the iterative linearization and the nonlinear solver was speed compared to greater accuracy. Increasing the preprocessed cuts from 16 to 32 increases the solution time. As the problem size gets bigger, we see diminishing returns to the number of preprocessed constraints. The tighter tolerance for convergence takes longer, but does not seem to have a major impact on the optimal solution value, except when there is no step-size constraint, where a more restrictive tolerance sometimes results in the linear program not converging. We find that step-size constraints decrease the time to solve and increase the robustness of the procedure. Solution times were up to six times faster using step-size limits.

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1. Introduction

Since the AC optimal power flow (ACOPF) problem was introduced in 1962 (Carpentier), it has received considerable attention. The ACOPF optimizes the steady state performance of an AC power system by minimizing an objective function such as generation cost, or maximizing market surplus, while satisfying system constraints including nodal real power balance, nodal reactive power balance, bounds on bus voltages, flows on transmission lines, real and reactive power injections, and contingencies. Since the ACOPF's introduction, different objective functions and formulations have been tried (see Cain et al). The canonical ACOPF problem uses the polar representation of voltage, real and reactive power. ACOPF problems have nonconvex continuous functions and can be large. More difficult variations include binary variables for topology control and unit commitment (see, for example, Potluri and Hedman).

Solving an ACOPF problem that meets a power system's realistic physical criteria has continued to be a challenge in power system operations. While most NLP solvers find local optimal solutions most of the time, their lengthy solution times and poor convergence (especially with the introduction of binary variables) have focused attention on linear approximations. Linear and mixed integer programming methods play a significant role in solving these problems with more robust solutions and better execution times. Currently, one of the most common approaches is the DC-OPF, in which the real part of the admittance matrix is considered negligible, reactive power and voltage magnitude variables are fixed in the formulation, and bus voltage angle differences are assumed to be near-zero (Stott, Alsac, and Monticelli, 2009).

A vast body of literature proposes different optimization methods to solve the ACOPF including Lagrangian approaches, sequential quadratic programming, sequential linear programming, interior point methods, and heuristics. Literature reviews appear periodically (see, for example, Dommel and Tinney, Huneault and Galiana, Momoh, et al, Frank and Steponavice and Castillo and O'Neill, 2013a). The literature reviews present an evolution of approaches to solve the ACOPF. Capitanescu et al (2011) reviews the state of the art and challenges to the optimal power flow computations including corrective post-contingency actions, voltage and transient stability constraints, problem size reduction, discrete variables and uncertainty.

O'Neill el al (2012a) formulate the ACOPF in several ways, compare each formulation's properties, and argue that the rectangular current-voltage or "IV" formulation and its linear approximations may be easier to solve than the traditional quadratic power flow "PQV" formulation. The IV-ACOPF isolates nonconvexities to each bus and transmission element rather than allowing nonconvexities in the flow equations as in the canonical polar PQV formulation.

O'Neill et al (2012b) compare solving the IV linear approximation of the ACOPF to solving the ACOPF with several nonlinear solvers. In general, the linear approximation approach is more robust and faster than several of the commercial nonlinear solvers. On several starting points, the nonlinear solvers failed to converge or contained positive relaxation (penalty) variables above the threshold. The iterative linear program approach finds a near-feasible near-optimal solution in almost all problems and starting points.

Lipka et al (2013) added current constraints to the problems and examined the resulting effects on the solution times. Pirnia et al (2013) tested pre-processed circumscribing polygon approximations in combination with and without iterative cuts to the procedure finding the 'sweet spot' for the number of preprocessed cuts in the ILIV-ACOPF. In testing to date, the results have favored the ILIV-ACOPF.

Castillo and O'Neill (2013b) present an experimental framework, statistical methods and numerical results from testing commercial nonlinear solvers with several ACOPF formulations and initializations. The experiments indicate a clear advantage to employing a rectangular formulation over a polar formulation.

In spite of all the work that has been done, the ACOPF remains 'very much a work in progress' (Stott and Alsaç, 2012). Further, they state that solutions to the problems encountered in 'real-life' are 'not easy to obtain' and still require significant individual intervention and tuning.

In this paper, we seek to improve the performance of the ILIV-ACOPF in previous studies by adding a set of constraints that limit the differences between the real and imaginary voltages of successive solutions. The primary motivation for this is that the iterative linearization procedure sometimes converges slowly or fails to converge exhibiting periodic behavior ("bouncing" between two solutions). We find that step-size constraints decrease the time to solve and increase the robustness of the procedure.

2. Notation.

Variables and parameters are indexed over buses using subscripts n and m. Here we refer to transmission assets as lines. Transmission lines are indexed by terminal buses n and m and line number k. For a complex variable or parameter, the superscript r denotes the real portion and the superscript j denotes the imaginary portion. For example, if $x = a + jb$, $x^r = a$, $x^j = b$ *where* $\boldsymbol{j} = (-1)^{1/2}$. The index of a major iteration is h.

Decision Variables

p_n	real power injected at bus n
q_n	reactive power injected at bus n
v^r_n	real part of the voltage at bus n

v^j_n	imaginary part of the voltage at bus n
v_n	voltage magnitude at bus n; $v_n = [(v^r_n)^2 + (v^j_n)^2]^{1/2}$
i^r_n	real part of the current at bus n
i^j_n	imaginary part of the current at bus n
i_n	current magnitude at bus n; $i_n = [(i^r_n)^2 + (i^j_n)^2]^{1/2}$
i^r_{nmk}	real part of the current on line k at bus n connecting to bus m
i^j_{nmk}	imaginary part of the current on line k at bus n connecting to bus m
i_{nmk}	current magnitude on k at n connecting to m ; $i_{nmk} = [(i^r_{nmk})^2 + (i^j_{nmk})^2]^{1/2}$

Parameters

$c^p_n(p_n)$	quadratic cost of real power at bus n
$c^q_n(q_n)$	quadratic cost of reactive power at bus n
$c^{pl}_n(p_n)$	stepwise linear approximation of $c^p_n(p_n)$
$c^{ql}_n(q_n)$	stepwise linear approximation of $c^q_n(q_n)$
b_{nmk}	susceptance of line k between bus n and m
g_{nmk}	conductance of line k between bus n and m
y_{nmk}	$= g_{nmk} + jb_{nmk}$ admittance of line k between bus n and m
y_{n0}	admittance from bus n to ground
p^{min}_n	minimum required real power at bus n
p^{max}_n	maximum allowed real power at bus n
q^{min}_n	minimum required reactive power at bus n
q^{max}_n	maximum allowed reactive power at bus n
v^{min}_n	minimum required voltage magnitude at bus n
v^{max}_n	maximum allowed voltage magnitude at bus n
$\underline{v}^r_n$	real voltage value at bus n from the previous linear program solution
$\underline{v}^j_n$	imaginary voltage value at bus n from the previous linear program solution
$\underline{i}^r_n$	real current value at bus n from the previous linear program solution
$\underline{i}^j_n$	imaginary current value at bus n from the previous linear program solution
i^{max}_{nmk}	maximum current magnitude on line k connecting bus n to bus m
$\underline{i}^r_{nmk}$	the real current value on line k at bus n to m from the previous linear program solution
$\underline{i}^j_{nmk}$	imaginary current value on line k at bus n to m from the previous linear program solution

3. ACOPF Formulations

The nonlinear IV-ACOPF is used as a benchmark for linear approximation ILIV-ACOPF. The IV-ACOPF formulation is

Minimize $\sum_n c^p{}_n(p_n)+c^q{}_n(q_n)$ (1)

Subj.

To

$$i^r{}_{nmk} = g_{nmk}(v^r{}_n - v^r{}_m) - b_{nmk}(v^j{}_n - v^j{}_m) \qquad \text{for all } n, m, k \qquad (2)$$

$$i^j{}_{nmk} = b_{nmk}(v^r{}_n - v^r{}_m) + g_{nmk}(v^j{}_n - v^j{}_m) \qquad \text{for all } n, m, k \qquad (3)$$

$$i^r{}_n = \sum_{mk} i^r{}_{nmk} \qquad \text{for all } n \qquad (4)$$

$$i^j{}_n = \sum_{mk} i^j{}_{nmk} \qquad \text{for all } n \qquad (5)$$

$$p_n = v^r{}_n i^r{}_n + v^j{}_n i^j{}_n \qquad \text{for all } n \qquad (6)$$

$$p^{min}{}_n \leq p_n \leq p^{max}{}_n \qquad \text{for all } n \qquad (7)$$

$$q_n = v^j{}_n i^r{}_n - v^r{}_n i^j{}_n \qquad \text{for all } n \qquad (8)$$

$$q^{min}{}_n \leq q_n \leq q^{max}{}_n \qquad \text{for all } n \qquad (9)$$

$$(v^r{}_n)^2 + (v^j{}_n)^2 \leq (v^{max}{}_n)^2 \qquad \text{for all } n \qquad (10)$$

$$(v^{min}{}_n)^2 \leq (v^r{}_n)^2 + (v^j{}_n)^2 \qquad \text{for all } n \qquad (11)$$

$$(i^r{}_{nmk})^2 + (i^j{}_{nmk})^2 \leq (i^{max}{}_{nmk})^2 \qquad \text{for all } n, m, k \qquad (12)$$

The line flow and network flow equations, (2)-(5) are linear. The feasible sets for line current magnitudes (12) and voltage magnitudes at buses (10) are discs, and thus can be approximated to any degree of accuracy with circumscribing polygons. The lower bound on voltage magnitude, while non-convex, is seldom binding because the optimization pushes voltages higher to reduce losses.

In rectangular form, the equations for real (6) and reactive power (8) injections and withdrawals in terms of current and voltage are second-order non-convex polynomials. We approximate the quadratic constraint equations (which express p_n and q_n in terms of currents and voltages) with hyperplanes that are tangent to the constraint hypersurfaces using first order Taylor approximations. With the resulting linear approximations, the ILIV-ACOPF(h) at each major iteration *h* is:

Minimize $\sum_n c^{pl}{}_n(p_n)+c^{ql}{}_n(q_n)$ (21)

Subj.

to

$$i^r{}_{nmk} = g_{nmk}(v^r{}_n - v^r{}_m) - b_{nmk}(v^j{}_n - v^j{}_m) \qquad \text{for all } n, m, k \qquad (22)$$

$$i^j{}_{nmk} = b_{nmk}(v^r{}_n - v^r{}_m) + g_{nmk}(v^j{}_n - v^j{}_m) \qquad \text{for all } n, m, k \qquad (23)$$

$$i^r{}_n = \sum_{mk} i^r{}_{nmk} \qquad \text{for all } n \qquad (24)$$

$$i^j{}_n = \sum_{mk} i^j{}_{nmk} \qquad \text{for all } n \qquad (25)$$

$$p_n = \underline{v}^r{}_n i^r{}_n + \underline{v}^j{}_n i^j{}_n + v^r{}_n \underline{i}^r{}_n + v^j{}_n \underline{i}^j{}_n - (\underline{v}^r{}_n \underline{i}^r{}_n + \underline{v}^j{}_n \underline{i}^j{}_n) \qquad \text{for all } n \qquad (26)$$

$$p^{min}{}_n \leq p_n \leq p^{max}{}_n \qquad \text{for all } n \qquad (27)$$

$$q_n = \underline{v}^j{}_n i^r{}_n - \underline{v}^r{}_n i^j{}_n - v^r{}_n \underline{i}^j{}_n + v^j{}_n \underline{i}^r{}_n - (\underline{v}^j{}_n \underline{i}^r{}_n - \underline{v}^r{}_n \underline{i}^j{}_n) \qquad \text{for all } n \qquad (28)$$

$$q^{min}{}_n \leq q_n \leq q^{min}{}_n \qquad \text{for all } n \qquad (29)$$

$$cos(\underline{\theta}_s)\, v^r{}_n + sin(\underline{\theta}_s)\, v^j{}_n \leq v^{max}{}_n \qquad \text{for } s = 0, 1, \ldots, s^{max}; \text{ for all } n \qquad (30)$$

$$(\underline{v}^r{}_{nd}/\underline{v}_{nd})\, v^r{}_n + (\underline{v}^j{}_{nd}/\underline{v}_{nd})\, v^j{}_n \leq v^{max}{}_n \qquad \text{for } d = 0, \ldots, h\text{-}1; \text{ for all } n \qquad (31)$$

$$cos(\underline{\theta}_s)\, i^r{}_{nmk} + sin(\underline{\theta}_s)\, i^j{}_{nmk} \leq i^{max}{}_{nmk} \qquad \text{for } s = 1, \ldots, s^{max}; \qquad (32)$$

$$(\underline{i}^r_{nmkd}/\underline{i}_{nmkd})\, i^r_{nmk} + (\underline{i}^j_{nmkd}/\underline{i}_{nmkd})\, i^j_{nmk} \leq i^{max}_{nmk} \quad \text{for all } n, m, k \text{ for } d = 0, \ldots, h\text{-}1; \text{ for all } n, m, k \tag{33}$$

where s^{max} is the number of sides of the preprocessed circumscribing polygons, and d indexes the iterative tight cuts.

The line flow and network flow equations, (22)-(25) are linear and unchanged from the IV_ACOPF. For the voltage magnitudes, the preprocessed outer bounds are in (30) and the iterative tight cuts are in (31). For the line current magnitudes, the preprocessed outer bounds are in (32) and the iterative tight cuts are in (33). The lower bound on voltage magnitude, while non-convex, is seldom binding because the optimization pushes voltages higher to reduce losses.

The constraints (30)-(33) are all constraints on magnitude of the component of the complex voltage or current vector in the direction of some unit-norm reference vector. Accordingly, the left-hand-side is the dot product of voltage or current with the unit-norm vector. In (30) and (32), the pre-processed cut constraints that specify a regular polygon are evenly spaced angles $\underline{\theta}_s = 2\pi s/s^{max}$ where s = 1, …, s^{max}. are the tight cuts from the previous candidate solutions, (31) and (33), that violated the nonlinear constraints, correspond to step (4) of the ILIV-ACOPF procedure described in Section 5 below.

In rectangular form, real (26) and reactive power (28) injections and withdrawals in terms of current and voltage are approximated by the first order Taylor series. We treat (11) as active constraints, that is, we only use a Taylor's series approximation when the constraint is violated.

4. Step-size Limits on the Real and Reactive Power Approximations

An unconstrained local linearization can result in relatively large errors. We introduce the step-size constraints, which constrain the voltages to lie within a box centered on the base point, $\underline{v}^r_n, \underline{v}^j_n, \underline{i}^r_n, \underline{i}^j_n$, for the current major iteration to correct this problem. We control the errors in p_n and q_n by controlling the step-size for the voltage as follows. The equation for real power is

$$p_n = v^r_n i^r_n + v_n^j i_n^j,$$

and its first-order Taylor approximation around $\underline{v}^r_n, \underline{v}^j_n, \underline{i}^r_n, \underline{i}^j_n$ is

$$p^{approx}_n = \underline{v}^r_n i^r_n + \underline{v}^j_n i^j_n + v^r_n \underline{i}^r_n + v^j_n \underline{i}^j_n - (\underline{v}^r_n \underline{i}^r_n + \underline{v}_n^j \underline{i}_n^j).$$

The approximation error is

$$p_n - p^{approx}_n = (v^r_n - \underline{v}^r_n)(i^r_n - \underline{i}^r_n) + (v^j_n - \underline{v}^j_n)(i^j_n - \underline{i}^j_n).$$

Similarly, for reactive power,

$$q_n = v^j_n i^r_n - v^r_n i^j_n, \text{ and}$$

$$q^{approx}_n = \underline{v}^j_n i^r_n - \underline{v}^r_n i^j_n - v^r_n \underline{i}^j_n + v^r_n \underline{i}^j_n - (\underline{v}^j_n \underline{i}^r_n - \underline{v}^r_n \underline{i}^j_n).$$

The approximation error is

$$q_n - q^{approx}_n = (v^j_n - \underline{v}^j_n)(i^r_n - \underline{i}^r_n) + (v^r_n - \underline{v}^r_n)(\underline{i}^j_n - i^j_n).$$

Since the bus currents, (2)-(5), are a linear function of voltage differences, the difference in bus currents can be expressed as

$$i^r_n - \underline{i}^r_n = \sum_m g_{nm}[(v^r_n - \underline{v}^r_n) - (v^r_m - \underline{v}^r_m)] - \sum_m b_{nm}[(v^j_n - \underline{v}^j_n) - (v^j_m - \underline{v}^j_m)]$$

and

$$i^j_n - \underline{i}^j_n = \sum_m g_{nm}[(v_n^j - \underline{v}^j_n) - (v^j_m - \underline{v}^j_m)] + \sum_m b_{nm}[(v^r_n - \underline{v}^r_n) - (v^r_m - \underline{v}^r_m)].$$

If $|v_n^r - \underline{v}^r_n|, |v_n^j - \underline{v}^j_n|, |v^r_m - \underline{v}^r_m|, |v^j_m - \underline{v}^j_m|$ *are all less than* δ, *then*

$$|i^r_n - \underline{i}^r_n| \le \sum_m 2\delta(|g_{nm}| + |b_{nm}|) = 2\delta[(\sum_m |g_{nm}|) + (\sum_m |b_{nm}|)].$$

Similarly,

$$|i^j_n - \underline{i}^j_n| \le 2\delta[(\sum_m |g_{nm}|) + (\sum_m |b_{nm}|)].$$

Using definition of the admittance matrix, $\sum_{m(\neq n)} g_{nm} = -g_{nn}$ *and* $\sum_{m(\neq n)} b_{nm} = -b_{nn}$,

$$|i^r_n - \underline{i}^r_n| \le 2\delta(|g_{nn}| + |b_{nn}|).$$

and

$$|i^j_n - \underline{i}^j_n| \le 2\delta(|g_{nn}| + |b_{nn}|).$$

Our original worst case bounds are then:

$$|p_n - p^{approx}_n| \le |(v^r_n - \underline{v}^r_n)(i^r_n - \underline{i}^r_n) + (v^j_n - \underline{v}^j_n)(i^j_n - \underline{i}^j_n)| \le 4\delta^2(|g_{nn}| + |b_{nn}|)$$

and

$$|q_n - q^{approx}_n| \le 4\delta^2(|g_{nn}| + |b_{nn}|).$$

In order to control the errors that arise from linearly approximating p_n and q_n, and also to prevent periodic behavior in the iterative procedure that slows convergence and may not terminate with an optimal solution, we introduce new constraints on $|v_n^r - \underline{v}_n^r|$ and $|v_n^j - \underline{v}_n^j|$, so that they are more and more tightly restricted at each major iteration. In the first major iteration, v^r and v^j are allowed to vary as long as they satisfy the preprocessed voltage cuts. In subsequent major iterations, each must stay within a square centered on the value from the previous solution. The size of these squares decreases over successive major iterations. We added the following constraints to the ILIV-ACOPF(h):

$$-av^{max}/h^b \le v^r_n - v^r_n \le av^{max}/h^b \quad (34)$$

$$-av^{max}/h^b \le v^j_n - v^j_n \le av^{max}/h^b \quad (35)$$

In (34) and (35), we tried $b = 1$ (linear stepsize decay) and $b = 2$ (quadratic step-size decay). The results of the experiments are displayed below. In the systems we studied, v^{max}_n is the same for all n, so we drop the n subscript.

5. Linear Iteration Approach

The ILIV-ACOPF method is:

1) Set $h = 0$. Choose a starting point $\underline{v}^r_{n0}$ and $\underline{v}^j_{n0}$. Add a circumscribing polygon for each maximum voltage magnitude and maximum current magnitude constraint. Approximate p_n and q_n with hyperplanes that are tangent to the constraint hypersurfaces.

2) Set $h = h+1$. Solve the resulting ILIV-ACOPF(h) to obtain optimal values for the current and voltage, $\underline{v}^r_{nh}$, $\underline{v}^j_{nh}$, $\underline{i}^r_{nh}$, $\underline{i}^j_{nh}$

3) Check the result for convergence of the optimal values using the actual nonlinear p_n and q_n equations (6) through (12). If within convergence tolerance, stop; otherwise continue.

4) Add another set of tight constraints to the iterative convex approximation cutting off the optimal solution to the linear approximation, but an infeasible solution to the nonlinear formulation. Relinearize the p_n and q_n approximations and adjust the stepsize. Go to step 2.

We presently describe the step (3) convergence criteria in detail. At each major iteration h, the optimal solution to the linear approximation produces two sets of values for the nonlinear equations of the voltage magnitude and real and reactive power injection/withdrawal: (i) The first order Taylor series approximations p_{nh}, q_{nh} and v_{nh}, employed in the linearization, and (ii) the actual "nonlinear" values that the decision variables give rise to when plugged in to the true, nonlinear equations. The absolute percentage by which the nonlinear values violate the constraints (7), (9), and (10) is calculated for each bus. If the violated p or q constraint (min or max) level is non-zero, the percentage violation is the absolute value of the difference of the nonlinear value and the violated constraint, divided by the constraint value. If the right-hand-side of the violated constraint is zero (which can occur for p and q, but not v) since the previous calculation would result in division by zero, the percentage violation is taken to be the nonlinear power injection/withdrawal divided by the respective real or imaginary component of the power flowing through the node. Then, for each of v_n, p_n, and q_n, these nodal percentage constraint violations are aggregated across nodes into two metrics of aggregate violation: the maximum percentage violation over nodes, and the sum of percentage violations over nodes. For each of the two aggregate metrics, the three values corresponding to violations of constraints on v, p and q are added together. After some experimentation, we settled on setting the threshold for the sum of summed violations to be five times the threshold for sum of maximum violations (henceforth "max percentage violations"). When these feasibility criteria are met, the solution is determined to be AC feasible and the procedure is terminated. If these violation criteria are not met, the solution is determined to not be AC feasible and we continue the iterative procedure.

6. Computational Testing

Problems. The test problems consistent of the 14, 30, 57, and 118 bus IEEE test cases (see Table 1) at http://www.ee.washington.edu/research/pstca/index.html. The quadratic generator costs come from MATPOWER (see Zimmerman et al). We formulate the 20-step linear approximation to each quadratic function. Where there are multiple transmission lines between two nodes, the lines are aggregated into an equivalent single line between the two nodes. Each test problem has two levels (tight and loose) of line current constraints (see Lipka et al, 2013).

Table 1: IEEE Test Bus System Data

Buses	Lines	Generators		Total Demand	Best Known Value Tight Current Limit		Best Known Value Loose Current Limit	
		No.	Capacity		quadratic	linear	quadratic	linear
14	20	5	7.724	2.590	105.4	107.4	85.3	86.5
30	41	6	326.80	42.42	5.89	6.10	5.79	5.98
57	80	7	326.78	235.26	421.5	432.2	419.2	425.5
118	186	54	99.66	42.42	1364.9	1388.4	1300.1	1315.5

Hardware and Software. The problems were solved on an Intel Xeon E7458 server with 8 64-bit 2.4GHz processors and 64 GB memory. However, no parallelization was used during one problem. Minor differences in solution times were recorded when the problems were run at different times, but the differences were small enough to be considered background noise. The problems were formulated in GAMS 23.6.2. The nonlinear programs used solver IPOPT version 3.8. Linear programs used GUROBI version 4.0.0 with the aggressive presolve option. The implementation was simplistic in that each linear program was solved from scratch at each major iteration. Starting from where the previous linear program terminated was not an option in the GAMS solver. There may be easily gained speedups by not starting each major iteration from scratch.

Optimization Parameters Settings. For IPOPT, we use the default parameters. For step-size constraints, we examine b=1 (linear step-size) and b=2 (quadratic step-size) and $a = 0.5$ and 1. We choose 16- and 32-sided circumscribing polygons based on the testing in Pirnia et al (2012)

Initalization. As the initial starting point, we use a flat start, $\underline{v}^r_n = 1$ and $\underline{v}^i_n = 0$, for all buses n. Other starting points, for example, hot starts, random starts and DC starts, can be used (see Castillo et al 2013). In commercial practice, a hot start may be available from previous solutions, that is, the solution from the last time period or similar topology.

Termination. The maximum number of major iterations was set to 100. We examine two sets of convergence criteria: (i) 0.1% for the max percentage violations and 0.5% for the summed violations (a "tolerance" of .001), as well as (ii) 0.5% for the max violations and 2.5% for the summed violations (a tolerance of .005). The maximum CPU time was never reached.

14-bus problem. The results for the 14-bus problem are shown in Tables 2 and 3. The objective function values were within .5 percent of the best-known value. The linear approximations with step-size limits solve faster than the linear approximation without step-size constraints and the IPOPT solver. For a tolerance of 0.001 with no step-size limits, for both 16 and 32 cuts at the loose current limits and for 32 cuts with tight current limits, the procedure was terminated at the major iteration maximum. However, by restricting the step-size for the voltages, the program converges in less than 8 major iterations. It takes less time to solve with 16 preprocessed cuts than 32. For the loose current limits with step-size restrictions (see Table 2), the quadratic step-size converged to similar objective values in fewer iterations than the linear step-size. The tighter tolerance level takes on average 5.9% more iterations (although sometimes, mysteriously, tightening the requirements for the feasibility check actually reduced the number of major iterations). For the tight current limits problem (see Table 3) and the tighter tolerance with 16 pre-processed cuts, the performance was almost the same with and without step-size limits. Overall, the performance was best with a 16 preprocessed cuts and a quadratic step-size with *a* (the step-size coefficient) = .5.

Table 2. 14-Bus Case with Loose Current Limits

			No Step-size	Linear Step-size		Quadratic Step-size		IPOPT
	Cuts	Tolerance	Constraints	a = .5	a = 1	a = .5	a = 1	results
Objective Value	16	0.001	86.101	86.098	86.098	86.341	86.137	86.5
	16	0.005	85.957	86.098	85.942	86.211	86.153	86.5
	32	0.001	86.101	86.019	86.102	86.202	86.146	86.5
	32	0.005	85.959	86.097	85.959	86.202	86.624	86.5
CPU Time seconds	16	0.001	4.857	0.238	0.367	0.209	0.244	0.91
	16	0.005	0.3	0.216	0.395	0.154	0.166	0.91
	32	0.001	6.86	0.363	0.594	0.246	0.322	0.91
	32	0.005	0.304	0.319	0.383	0.219	0.311	0.91
Number of Major Iterations	16	0.001	100	4	4	4	4	n/a
	16	0.005	7	4	7	3	3	n/a
	32	0.001	100	5	8	3	4	n/a
	32	0.005	5	4	5	3	5	n/a

Table 3 14-Bus Case with Tight Current Limits

	Cuts	Tolerance	No Step-size Constraint	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective. Value	16	0.001	105.69	105.69	105.69	106.50	105.70	107.4
	16	0.005	105.69	105.69	105.69	105.00	105.70	107.4
	32	0.001	86.1	106.53	106.53	106.59	106.53	107.4
	32	0.005	85.96	106.53	106.53	106.59	106.53	107.4
CPU Time seconds	16	0.001	0.299	0.3	0.271	0.301	0.279	1.16
	16	0.005	0.305	0.282	0.262	0.17	0.27	1.16
	32	0.001	8.93	0.322	0.32	0.293	0.325	1.16
	32	0.005	0.907	0.331	0.307	0.29	0.315	1.16
Number of Major Iterations	16	0.001	5	5	5	5	5	n/a
	16	0.005	5	5	5	3	5	n/a
	32	0.001	100	4	4	4	4	n/a
	32	0.005	11	4	4	4	4	n/a

30-bus problem. The results for the 30-bus problem are shown in Tables 4 and 5. The objective function values were within 1 percent of the best-known value. The linear approximations with step-size limits are faster than the linear approximation without step-size constraints and the IPOPT solver. In terms of CPU time, the approximation with 16 preprocessed cuts outperforms the approximation with 32. In all cases, the number of major iterations is 3 or 4. For the .005 and .001 tolerance and the loose current constraint, the quadratic step performs best with 16 preprocessed cuts. For the .005 and .001 tolerance and the tight current constraint, there is no clear step-size winner. Overall, there is not much difference with different step-sizes.

Table 4. 30-Bus Case with Loose Current Limits

	Cuts	Tolerance	No Step-size Constraints	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective Value	16	0.001	5.98	5.98	5.98	5.98	5.98	5.98
	16	0.005	5.98	5.98	5.98	5.98	5.98	5.98
	32	0.001	5.98	5.98	5.98	5.98	5.98	5.98
	32	0.005	5.98	5.98	5.98	5.98	5.98	5.98
CPU Time seconds	16	0.001	0.709	0.664	0.88	0.666	0.738	2.88
	16	0.005	0.751	0.647	0.721	0.508	7.52	2.88
	32	0.001	0.945	0.892	0.84	0.869	0.942	2.88
	32	0.005	0.938	0.882	0.911	0.926	0.904	2.88
Number of Major Iterations	16	0.001	4	4	4	4	4	n/a
	16	0.005	4	4	4	3	4	n/a
	32	0.001	3	3	3	3	3	n/a
	32	0.005	3	3	3	3	3	n/a

Table 5. 30-Bus with Tight Current Limits

	Cuts	Tolerance	No Step-size Constraints	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective Value	16	0.001	6.1	6.09	6.1	6.1	6.1	6.10
	16	0.005	6.06	6.06	6.06	6.06	6.06	6.10
	32	0.001	6.1	6.1	6.1	6.08	6.1	6.10
	32	0.005	6.08	6.08	6.08	6.08	6.08	6.10
CPU Time seconds	16	0.001	0.817	0.67	0.617	0.773	0.697	1.55
	16	0.005	0.572	0.488	0.507	0.535	0.53	1.55
	32	0.001	1.287	1.21	1.161	0.854	1.21	1.55
	32	0.005	1.02	0.891	0.861	0.852	0.992	1.55
Number of Major Iterations	16	0.001	4	4	4	4	4	n/a
	16	0.005	4	3	3	3	3	n/a
	32	0.001	4	4	4	3	4	n/a
	32	0.005	3	3	3	3	3	n/a

57-bus problem. The results for the 57-bus problem are shown in Tables 6 and 7. The objective function values were within 2 percent of the best-known value. With loose current limit, a quadratic step-size function with a = .5 and 16 preprocessed cuts performs best. With the tight current limit, the different step-size constraint parameters do not produce a clear winner. It takes less time to solve with 16 preprocessed cuts than preprocessed 32 cuts.

Table 6. 57-Bus Case with Loose Current Limits

	Cuts	Tolerance	No Step-size Constraints	Linear Step Size Rate a = .5	Linear Step Size Rate a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective Value	16	0.001	422.6	422.6	422.6	424.3	423.27	425.5
	16	0.005	422.6	422.6	422.6	422.84	423.27	425.5
	32	0.001	422.42	422.96	422.42	424.41	423.47	425.5
	32	0.005	422.42	422.8	422.42	423.9	423.47	425.5
CPU Time seconds	16	0.001	2.3	2.14	2.425	1.94	2.12	12.05
	16	0.005	2.317	1.93	2.205	1.53	2.1	12.05
	32	0.001	5.27	6.38	4.3	3.68	3.69	12.05
	32	0.005	4.98	5.3	4.67	2.88	3.73	12.05
Number of Major Iterations	16	0.001	4	4	4	4	4	n/a
	16	0.005	4	4	4	3	4	n/a
	32	0.001	5	7	5	4	4	n/a
	32	0.005	5	6	5	3	4	n/a

Table 7. 57-Bus Case with Tight Current Limits

	Cuts	Tolerance	No Step-size Constraints	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective Value	16	0.001	422.66	422.66	422.66	423.62	424.42	432.2
	16	0.005	423.43	423.43	423.4	423.62	424.4	432.2
	32	0.001	422.55	422.55	422.55	425.13	423.69	432.2
	32	0.005	422.71	422.71	422.71	424.01	423.69	432.2
CPU Time seconds	16	0.001	3.13	2.79	2.74	1.52	2.82	8.42
	16	0.005	1.943	1.59	1.64	1.66	2.16	8.42
	32	0.001	9.07	8.74	9.03	3.44	3.6	8.42
	32	0.005	4.028	3.69	3.63	2.62	3.71	8.42
Number of Major Iterations	16	0.001	5	5	5	3	5	n/a
	16	0.005	3	3	3	3	4	n/a
	32	0.001	9	9	9	4	4	n/a
	32	0.005	4	4	4	3	4	n/a

118-bus problem. The results for the 118-bus problem are shown in Tables 8 and 9. The objective function values were within .5 percent of the best-known value. For the loose current limits problem, the quadratic step-size with $a = .5$ converged much faster than the linear step-size and the IPOPT solver. With a tight current limit, the linear step-size performed best, although it converges to an objective value about .5% below both the IPOPT optimum the quadradic step-size optimum. It takes less time to solve with 16 preprocessed cuts than with preprocessed 32 cuts.

Table 8. 118-Bus Case with Loose Current Limits

	Cuts	Tolerance	No Step-size Constraint	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step Size Rate a = .5	Quadratic Step Size Rate a = 1	IPOPT Results
Objective Value	16	0.001	1315.2	1315.4	1315.2	1313.6	1313.3	1315.5
	16	0.005	1307.7	1309.2	1307.7	1313.6	1310.6	1315.5
	32	0.001	1314.3	1315.4	1314.3	1312	1314.6	1315.5
	32	0.005	1298.1	1308.4	1314.3	1312	1314.6	1315.5
CPU Time seconds	16	0.001	243.27	190.08	348.91	6.83	12.08	38.75
	16	0.005	29.31	11.3	25.84	6.98	9.7	38.75
	32	0.001	167.31	221.8	148.75	12.31	28.16	38.75
	32	0.005	403.95	23.92	148.97	12.23	26.17	38.75
Number of Major Iterations	16	0.001	100	100	100	3	5	n/a
	16	0.005	11	5	11	3	4	n/a
	32	0.001	41	69	41	3	7	n/a
	32	0.005	100	6	41	3	7	n/a

Table 9. 118-Bus Case with Tight Current Limits

	Cuts	Tolerance	No Step-size Constraint	Linear Step-size a = .5	Linear Step-size a = 1	Quadratic Step-size a = .5	Quadratic Step-size a = 1	IPOPT Results
Objective Value	16	0.001	1381.15	1384.47	1384.47	1388.25	1388.31	1388.4
	16	0.005	1381.15	1381.01	1381.01	1388.25	1388.31	1388.4
	32	0.001	1388.36	1384.24	1384.24	1388.65	1388.31	1388.4
	32	0.005	1380.39	1380.22	1380.22	1390.52	1388.31	1388.4
CPU Time seconds	16	0.001	12.3	10.04	10.56	11.77	18.47	32.2
	16	0.005	8.32	7.9	7.94	11.33	19.3	32.2
	32	0.001	415.541	17.14	17.55	19.23	30.66	32.2
	32	0.005	13.26	12.66	12.97	15.48	30.52	32.2
Number Of Major Iterations	16	0.001	3	4	4	5	8	n/a
	16	0.005	3	3	3	5	8	n/a
	32	0.001	100	4	4	5	8	n/a
	32	0.005	3	3	3	4	8	n/a

7. Conclusions

Overall, the step-size constraints improve performance of the linear procedure, but the best parameters of the step-size constraint are problem-dependent. Although the convergence tests are different, the linear procedure with step-size constraints is considerably faster (up to six times) the nonlinear solver and without a step-size constraint. The tradeoff between the iterative linearization and the nonlinear solver was speed compared to greater accuracy. Increasing the preprocessed cuts from 16 to 32 increases the solution time. As the problem size gets bigger, we see diminishing returns to the number of preprocessed constraints. The tighter tolerance for convergence takes longer, but does not seem to have a major impact on the optimal solution value, except when there is no step-size function, where a more restrictive tolerance sometimes results in the linear program not converging. Nearly uniformly, the ILIV-ACOPF objective values are slightly lower than the IPOPT objective values, by an average of .9%. Tightening the tolerances reduced the percentage underestimate of cost from 1.1% to .95%, a consistent but essentially negligible pattern.

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