iHYCOM documentation

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January 17, 2012

1. Introduction

A 3-dimensional global ocean circulation model, named iHYCOM, is under development at NOAA's Earth System Research Laboratory. The model is destined to become the oceanic counterpart of the finite-volume, flow-following, icosahedral atmospheric model FIM (Bleck et al. 2010). By sharing FIM's icosahedral mesh, iHYCOM can be attached to FIM for coupled ocean-atmosphere simulations without incurring the complexities of an interpolating "flux coupler".

iHYCOM is patterned after the ocean model HYCOM (Bleck 2002). Like HYCOM, it uses an adaptive vertical coordinate representing a combination of fixed-thickness layers in the upper ocean (which become bottom-following in shelf seas) and constant potential-density layers in the deeper ocean. The same vertical grid structure, turned upside down and modified for a gaseous medium, is found in FIM *(ibid.)*.

A distinguishing feature of the vertical grid used in FIM, HYCOM, and iHYCOM is that coordinate layers are not rigidly assigned to either the fixed-depth or the isopycnic class. Any layer, and even regional portions of a layer, can transition from one coordinate mode to the other. The preferred mode for a layer to be in is the isopycnic one, but vertical grid spacing constraints override this assignment in the upper ocean.

The reader should note that the polar singularity found in spherical grids, whose elimination was one of the driving forces behind FIM, is not an issue in ocean modeling because ocean models can take advantage of map projections hiding polar singularities on continents. Hence, the primary motivation for developing iHYCOM was not to circumvent a pole problem but to eliminate spatial interpolation of fluxes at the air-sea interface. While a stand-alone version of iHYCOM driven by prescribed atmospheric fields presently exists and has been put to good use during model development, there is no inherent advantage in configuring a stand-alone ocean model on an icosahedral mesh.

iHYCOM interacts with FIM by receiving surface fluxes of momentum, heat and freshwater, and providing sea surface temperature as well as ice coverage information (including ice surface temperature) in return.

FIM presently treats iHYCOM as a subroutine. This is to say that only one of the two submodels is running at any time and, while active, is making use of all available processors.

iHYCOM is designed to capture the gamut of dynamic processes affecting the global SST on all time scales, such as sea ice formation, thermally and mechanically forced mixed layer entrainment/detrainment, small-scale diapycnal mixing, and wind- as well as thermohaline-forced lateral heat transport. Excluded for the time being are dynamic ice spreading, tidal effects, and a wave submodel for surface roughness prediction.

iHYCOM makes use of the distributed data layout, indirect addressing procedures, and finite-volume numerics developed for FIM and solves a closely related set of prognostic equations. Both models use A-type horizontal staggering of variables. However, their time stepping schemes differ. The 3rd order Adams-Bashforth scheme successfully employed in FIM (Lee et al. 2010) failed in iHYCOM because it does not permit rigorous enforcement of positive-definiteness in the layer thickness tendency equation. For this reason, iHYCOM has inherited from HY-

COM the traditional leapfrog time differencing scheme.

The reason why multistep Adams-Bashforth works in FIM but not in iHYCOM may be threefold:

1. Oceanic orography is steeper than terrestrial orography.
2. The ocean is less stratified and therefore subject to stronger "sloshing" by internal gravity waves.
3. Beyond the shelf break, iHYCOM does not use bottom-following coordinates as FIM does; hence, massless layers are commonplace on the sea floor where they pose a particular numerical challenge due to the steepness of submarine bottom slopes.

(We note in passing that eliminating massless bottom layers by adopting terrain-following coordinates throughout the horizontal domain is not an option in a "blue-water" ocean circulation model because the well-known σ-coordinate pressure gradient error is prohibitively serious near the continental margins and other steep bottom features. Only in shelf seas are bottom slopes sufficiently benign to permit the use of σ coordinates.)

One difference between iHYCOM and HYCOM numerics is worth mentioning. HYCOM, like many other ocean models, gains efficiency by separating barotropic gravity waves from other types of fluid motion and transmitting them using a numerically efficient 2-dimensional shallow-water model (Bleck and Smith 1990). We have been unable to get this split-explicit mode separation scheme to work in iHYCOM. The problem appears to be transverse gravitational sloshing in 2-grid point wide channels, the cause for which we conjecture to be underestimated (or incorrectly coupled) cross-channel pressure gradients on the A grid adopted from FIM.

In the absence of a mode splitting scheme, we are forced to integrate the 3-dimensional momentum and continuity equations – the set responsible for transmitting gravity waves – using a short time step linked to the phase speed of barotropic gravity waves (referred to as the "barotropic" time step). Tracers can be, and are, advected using a much longer "baroclinic" time step governed by the speed of internal waves and the actual currents.

The lack of a mode splitting scheme has a silver lining. Such schemes – at least the split-explicit ones – are notoriously unstable (Morel et al. 2008) and thus create hazardous working conditions during model development. Furthermore, the particular modal decomposition developed for HYCOM (Bleck and Smith 1990) does not permit changing the bottom pressure in the "baroclinic" set of equations. Hence, the freshwater flux at the surface must be converted in HYCOM into a virtual salt flux, causing potential problems with negative salinities and long-term freshwater conservation, not to mention the omission in the ocean model of the sizeable equatorward return flow of water transported poleward by the atmosphere (Huang 1993). All these difficulties are presently avoided in iHYCOM.

Conservation of mass and tracers is paramount in circulation models used in long-term simulations. For this reason, it is essential for layer models to solve conservation equations in flux form. Even so, conservation of tracers is difficult to enforce in situations where a coordinate layer of finite thickness loses most (but not all) of its mass during a single time step.

The problem is caused by the need to divide tracer *amount* (example: salt in a grid cell), which is the predictand in the conservation equations, by the layer thickness to recover tracer *concentration* (example: salinity) after the transport step. As layer thickness approaches zero, meaning that the operation moves into the vicinity of the zero-over-zero singularity, the resulting concentration value may lie outside the proper bounds, especially if mass export is large in relation to the final layer thickness. As discussed in Bleck et al. (2010), the usual remedy is to substitute a "reasonable" concentration value for the result of the division and to compensate for the implied nonconservation by adding an appropriate global offset to the field. Needless to say, nonconservation ceases to be an issue if a grid cell loses 100% of its mass – rather than, say, 99%.

No such remedies are needed in the case of mass. Mass is rigorously conserved in the model.

2. Equation of State

The vertical coordinate in the isopycnic subdomain of iHYCOM is potential density anomaly referenced to a pressure of 2000 dbar (roughly 2 km), commonly referred to as σ_2[1]. The choice of reference pressure is a compro-

[1]The tradition of denoting density anomaly by σ unfortunately clashes with the meteorological tratition of using σ for terrain-following coordinates.

mise minimizing the deviation of coordinate layer slope from the slope of truly neutral surfaces while at the same time satisfying the monotonicity condition in much of today's global ocean. (Both σ_0 nor σ_4 are vertically non-monotonic in many regions.)

The equation of state (encoded in *sigocn.F90* which is part of *hycom_sigetc.F90*) is taken from Brydon et al. (1999). The thermobaric component of seawater density in the equation of state, whose use would bring the slope of coordinate surfaces closer to that of neutral surfaces over a wide pressure range (Sun et al. 1999), will be added at a future date. While not precisely buoyancy-neutral at pressures other than 2000 dbar, σ_2 surfaces come a long way toward eliminating the false diapycnal component of numerically-induced diffusion of prognostic variables which is an inevitable side effect of solving finite-difference transport equations on constant-depth surfaces. As in the case of FIM, this is the primary motivation for designing a circulation model around an isentropic/isopycnic vertical coordinate.

3. Hydrostatic Equation

Due to its use in a predominantly isopycnic coordinate model, the hydrostatic equation is solved in iHYCOM in the form

$$\frac{\partial M}{\partial \alpha} = p \tag{1}$$

where $M = gz + p\alpha$ is the Montgomery potential, p is pressure, gz is the geopotential, and α is inverse potential density. The integration over α takes place from the bottom up in *hycom_hystat.F90*. Note that M and α are layer variables while p is carried on layer interfaces.

An initial value of M on the sea floor is computed during model initialization in *hycom_init.F90* by a top-down integration of (1), starting with $z = 0$ and $p = 0$ at the surface. Thereafter, the actual sea floor value of M needed for subsequent upward integrations of (1) is obtained by correcting the initially computed M for changes in bottom p and bottom α.

4. Momentum Equations

The horizontal momentum equations are solved in the form

$$\left(\frac{\partial u}{\partial t}\right)_s - fv = -\left(\frac{\partial M}{\partial x}\right)_s + p\left(\frac{\partial \alpha}{\partial x}\right)_s - g\frac{\partial \tau_x}{\partial p} + \frac{1}{\Delta p}\nabla_s \cdot (\nu \Delta p \nabla_s u) \tag{2}$$

$$\left(\frac{\partial v}{\partial t}\right)_s + fu = -\left(\frac{\partial M}{\partial y}\right)_s + p\left(\frac{\partial \alpha}{\partial y}\right)_s - g\frac{\partial \tau_y}{\partial p} + \frac{1}{\Delta p}\nabla_s \cdot (\nu \Delta p \nabla_s v) \tag{3}$$

where ν is an eddy viscosity, Δp is the layer thickness, τ_x,τ_y are wind- or bottom-induced stress components, and s is a vertically monotonic but otherwise arbitrary variable denoting the vertical coodinate. Subscript s indicates that partial derivatives are to be evaluated at $s = const$. These equations are similar to the momentum equations in FIM and are solved in *hycom_momtum.F90* in a very similar way.

Layers in contact with the sea floor, or close to it, are subjected to a bottom stress expressed in traditional bulk form based on the flow speed averaged over the lowest 10 m and incremented by a tidal component of 0.5 m/s. Bottom stress is assumed to decrease linearly to zero over a fixed depth range, also chosen to be 10 m. Note that layer models require an actual stress profile, rather than just a bottom value, to properly partition the stress, because it is not known *a-priori* which layers will be close to the sea floor at a particular time and location.

Wind stress is partitioned among near-surface layers in the same manner. The depth range over which the surface stress decreases lineary to zero is set to 50 m.

Aside from vertical momentum fluxes caused by surface and bottom stress, there are fluxes induced by turbulence in the surface mixed layer.

The reasons for adding explicit horizontal viscous terms on the r.h.s. of (2) and (3) are twofold:

- Lateral sidewall drag is an important element of western boundary current dynamics;
- Some lateral stirring of momentum is deemed beneficial in coarse-mesh, noneddy-resolving applications.

Even though u, v are the regular Cartesian velocity components regardless of the definition of the vertical coordinate, one must remember that all horizontal derivatives on the r.h.s. of (2) and (3) are evaluated along the actual coordinate surfaces. Since α is constant (or nearly so) in interior coordinate layers, the gradient of M dominates the 2-term pressure force expression in those layers. Only in nonisopycnic layers near the model top do both terms play a major role.

Lateral drag is presently communicated only among grid cells belonging to the same coordinate layer. In the interior, this has the effect of rendering momentum exchange via eddy stirring an isopycnal process, as it tends to be in reality.

Sidewall drag is evaluated by viewing the ocean bottom figuratively as an assemblage of variable-height hexagonal "basalt" columns and determining the vertical extent to which a given model layer is in contact with one of those columns. The portion of the layer extending above the column is assumed not to be affected by its presence. Prorating the effect of sidewalls in this fashion avoids temporal discontinuities in sidewall drag in layers subjected to gravity wave sloshing near steep bottom slopes.

5. Continuity Equation

iHYCOM, like FIM, is a stacked shallow-water model. The solution procedure for the continuity equation, which largely controls the vertical spacing of layer interfaces, mimics the procedure used in FIM, but allowances must be made, of course, for the presence of coastal boundaries. The coastline in iHYCOM follows icosahedral cell edges, which is to say that a given grid cell is either totally land or totally water. This allows us to rigorously apply the kinematic boundary condition stipulating zero flow across coastlines.

The layer-integrated continuity equation is identical to the one solved in FIM:

$$\frac{\partial \Delta p}{\partial t} + \nabla_s \cdot (\mathbf{v}\Delta p) + \left(\dot{s}\frac{\partial p}{\partial s}\right)_2 - \left(\dot{s}\frac{\partial p}{\partial s}\right)_1 = 0. \quad (4)$$

Here, indices 1,2 denote the upper and lower interface, respectively, of the layer under consideration. Note that s always appears in combination with $\partial p/\partial s$ to account for the fact that the dimensions of s are arbitrary and may, in fact, be physically meaningless.

As in FIM, the continuity equation is solved using Flux Corrected Transport (Zalesak 1979), with high-order fluxes based on centered 2^{nd} order finite difference expressions.

Due to the small size of baroclinic eddies in the ocean, their influence on the large-scale flow has to be parameterized in coarse-mesh ocean models. Aside from adding explicit lateral "eddy" mixing terms in (2) and (3), the effect of baroclinic instability on the resolved-scale buoyancy field needs to be taken into account. A widely adopted parameterization is that of Gent and McWilliams (1990) which captures the instability-induced slumping of tilted isopycnals by invoking a peristaltic or "bolus" flux transferring mass laterally within isopycnic layers such that available potential energy decreases.

In an isopycnic model, the GM parameterization can be implemented elegantly by smoothing the interface pressure field. The important point to note here is that baroclinic instability is an adiabatic process, implying that interface smoothing may not lead to mass transfer between layers. Instead, vertical displacement of interfaces resulting from smoothing must be translated into an *intralayer* mass flux. This is accomplished by viewing the smoothing operator $A\nabla^2 p$ (where A is the product of a diffusivity coefficient and the model time step) as the divergence of an "interface pressure flux" $A\nabla p$, i.e., as $\nabla \cdot (A\nabla p)$. The mass or layer thickness flux in a layer is then simply the difference of $A\nabla p$ at its upper and lower interface.

To make this work in the case of variable bottom topography, the smoothing operation must not allow an interface to descend below the bottom. This is achieved by setting $A\nabla p$ to zero wherever an interface coincides with the sea floor. Furthermore, in situations where an interface impinging on an inclined bottom slope is tilted such that flattening would make it intersect the bottom, this is likewise not permitted. These steps guarantee that in the idealized situation of an ocean at rest with horizontal interfaces intersecting variable topography, interface smoothing has no effect.

Interface smoothing is done in iHYCOM at the end of *hycom_cnuity.F90*. The resulting bolus fluxes are added to the "regular" mass fluxes and hence contribute appropriately to lateral tracer transport.

One word of caution: Interface smoothing captures the GM effect only if a layer interface is isopycnic. In iHYCOM, all interfaces not coinciding with the sea floor are smoothed, but there is no pretense of physical significance in smoothing interfaces in the isobaric or terrain-following coordinate subdomain.

A scheme for extending GM to the nonisopycnic coordinate subdomain in iHYCOM (and its parent model HYCOM) is under development (Bleck 2012). The approach taken is to transform the native grid to a purely isopycnic one, smooth the resulting interfaces, and transform the mass fluxes inferred from the thickness changes on the transformed grid back to the native grid. Note that this back-and-forth transform is a null operation in the isopycnic subdomain.

By transforming tracers to the isopycnic grid and numerically diffusing them there, rather than on the native grid, isoneutral eddy mixing (Redi 1982) can (and will) also be extended to the nonisopycnic subdomain.

6. Tracer Transport

Because vertical mesh size in iHYCOM varies in space and time, solving tracer conservation equations in flux form is mandatory. Temperature and salinity are transported using the long-time step approach developed for tracer transport in FIM. (In iHYCOM, the long time step is the "baroclinic" time step mentioned in the Introduction.) The salient aspects of this method are laid out here for convenience. For additional details see Sun and Bleck (2006) and the FIM documentation (Bao et al. 2011).

Let Δt be the time step appropriate for transmitting gravity waves. Solving tracer transport equations on a longer time step $J\Delta t$ ($J > 1$) commensurate with actual flow rather than gravity wave speed, the conservation equation must be based on a rigorously time-integrated form of the mass continuity equation (4),

$$\frac{\Delta p^{n+J} - \Delta p^n}{J\Delta t} + \nabla_s \cdot \overline{\mathbf{v}\Delta p}^J + \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J\right)_2 - \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J\right)_1 = 0, \qquad (5)$$

where the overbar denotes integration over J time steps. To assure that the equation is exactly satisfied in the model, the dynamically active fields must already have been stepped forward from time level n to $n+J$. At that instant, both the tendency term and the horizontal flux divergence term in (5) can be determined, the latter by summing up the instantaneous fluxes over the past J time steps. The time-integrated vertical flux terms can then be obtained by vertically summing up (5), using $\dot{s} = 0$ (material surface boundary condition) at the top and bottom of the column.

By combining (5) with the equation $dQ/dt = 0$, expressing conservation of a tracer Q during transport (sources and sinks of Q can be evaluated separately), we arrive at the transport equation

$$\frac{(Q\Delta p)^{n+J} - (Q\Delta p)^n}{J\Delta t} + \nabla_s \cdot (Q\,\overline{\mathbf{v}\Delta p}^J) + \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J \hat{Q}\right)_2 - \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J \hat{Q}\right)_1 = 0. \qquad (6)$$

which can be solved for the tracer amount $Q\Delta p$ at time level $n+J$. Here, the caret indicates values interpolated to layer interfaces. For details regarding the conversion of $Q\Delta p$ to Q in massless or near-massless layers, see the FIM documentation *(ibid.)*.

Eqn. (6) is solved using Flux Corrected Transport where high-order fluxes are again based on centered 2^{nd} order finite differencing.

7. Surface Mixed Layer

Turbulent effects in the surface mixed layer are simulated using the *K Profile Parameterization* scheme of Large et al. (1994). The routine performing this task, named *hycom mxkpp.F90*, is a straightforward adaptation of the routine used for this purpose in HYCOM. Following the approach taken in the HYCOM version incorporated into the NASA-GISS climate model, iHYCOM applies the KPP scheme only down to the depth of the surface mixed layer. Diapycnal mixing in the oceanic interior is simulated using the method of McDougall and Dewar (1998) described in a later section.

Use of the turbulence parameterization scheme of Canuto et al. (2001) is planned for the future.

8. Coordinate Maintenance

As pointed out earlier, the vertical grid in iHYCOM is adaptive in the sense that the depth of a coordinate surface is obtained by reconciling two potentially contradictory requirements, namely, maintenance of (a) the isopycnic nature of the coordinate and (b) minimum layer thickness. In case of a conflict, (b) trumps (a). Details of the ”grid generator” are laid out in Appendix C of Bleck (2002).

Vertical advection has traditionally been treated in FIM and HYCOM as part of the coordinate maintenance (grid generation) task. The uneven vertical grid spacing poses special challenges for the advection scheme with regard to conservation, positive-definiteness and monotonicity of the advcted fields. HYCOM and iHYCOM use the *Piecewise Parabolic Method* (PPM) which incorporates the constraints just mentioned without being unduly dissipative. As in FIM, vertical advection is formulated so as to avoid violating the CFL criterion for linear stability when layer thickness goes to zero while vertical velocity remains finite.

9. Diapycnal Mixing

iHYCOM uses the McDougall and Dewar (1998) scheme adapted from HYCOM to model small-scale diapycnal mixing in the water column beneath the surface mixed layer. The scheme was specifically developed for isopycnic coordinate models. It diffuses temperature and salinity vertically, but does so without modifying the density in individual coordinate layers (within the limits of a linearized state equation). What does change in the course of the diffusive process is the thickness of layers. Thin layers in the interior of the column often grow at the expense of thick layers while those at the top and bottom gradually vanish. The scheme predicts an infinite inflation rate for massless layers, requiring an arbitrary bound on mass transfer into a massless layer.

The algorithm used in HYCOM is unable to inflate two or more massless layers sandwiched together because simultaneous mass input from both neighbors of a given layer is required to thicken it. This limitation has been removed in iHYCOM by embedding the HYCOM-based routine *hycom_diapfl.F90* in another one, called *hycom_diamix.F90*, which searches each grid column for sequences of two or more massless layers.

If such a sequence is present, the routine removes these layers from the column but then calls *hycom_diapfl.F90* repeatedly, each time inserting a different one of the eliminated massless layers. That particular layer, by virtue of it’s being solitary, can now be inflated. The time step in this series of calls is reduced appropriately to avoid ”over-diffusing” the remaining, mass-containing layers. This could potentially lead to ”under-inflation” of the massless layers; however, the rate of inflation of such layers is fairly arbitrary, as pointed out earlier.

The requirement to conserve T, S as well as the initially assigned layer densities during the diffusion process means that complete homogenization of a water column is generally not possible. Except under special circumstances, the final state arrived at by the McDougall-Dewar scheme will consist of 2 model layers which, due to their different densities, also differ in their T, S properties.

The original McDougall-Dewar scheme does not distinguish between diffusion coefficients for T and S, and no attempt has been made to generalize the scheme in this direction. The diffusivity in iHYCOM is set to the larger of two values, $2 \times 10^{-5}\mathrm{m}^2\mathrm{s}^{-1}$ and $2 \times 10^{-7}\mathrm{m}^2\mathrm{s}^{-2}/N$, where N is the buoyancy frequency.

10. Sea Ice Model

Sea-ice related processes are presently modeled in the simplest possible way with an ”energy loan” model developed for HYCOM. It resembles the single-layer model discussed in the Appendix of Semtner (1976). Freezing takes place whenever latent heat is needed to keep the mixed layer temperature from dropping below the freezing level. When the ocean-ice system is being heated, the incoming energy is used to melt the ice before the water temperature is allowed to rise above the freezing level.

One major task of an ice model is to find the ice surface temperature. In the energy loan model this temperature is obtained iteratively by stipulating that the vertical heat flux inside the ice agrees with the prescribed heat flux through the ice-air interface. To improve convergence, the effect of the temperature change on the outgoing radiation, and hence on the energy flux computation at the next time step, is incorporated into the iterative scheme.

Details are as follows. Denoting thermal conductivity by k, ice thickness by h, ice surface temperature by T, and the freezing temperature of seawater by T_{frz}, the downward heat flux through the ice is

$$k\frac{T - T_{frz}}{h}.$$

This flux is supposed to match the atmospheric net surface heat flux F.

If the atmosphere prescribes an F value at variance with the above, the new balance, taking into account changes in long-wave radiaton implied by changing T, is

$$k\frac{T + \Delta T - T_{frz}}{h} = F - \sigma\left[(T + \Delta T)^4 - t^4\right]$$

where σ is the Stefan-Boltzmann constant.

By expanding the right-hand side into terms of up to 2^{nd} order in ΔT,

$$\left[(T + \Delta T)^4 - t^4\right] \approx 4T^3\Delta T + 6T^2\Delta T^2,$$

a quadratic equation in ΔT is obtained:

$$\Delta T^2 + \Delta T\frac{(k/h) + 4\sigma T^3}{6\sigma T^2} = \frac{F - (k/h)(T - T_{frz})}{6\sigma T^2}.$$

To accomodate abrupt changes in atmospheric heat flux F, it is advisable to iterate this equation a few times.

Due to the sensitivity of F to surface temperature fluctuations and the danger of exciting oscillatory behavior in F, rapid temporal changes in T must be avoided. This is done by adding only a fraction of the final value of ΔT to T.

11. Miscellaneous

11a. *Hydrologic Cycle*

A model ocean will gradually become saltier if precipitation falling onto land is not returned to the ocean. Pending acquisition of a data base cataloguing watersheds worldwide, FIM estimates the direction of river runoff from the terrain slope. To close the hydrological cycle, that is, to avoid collecting rain water in interior bowls like the Caspian Sea basin, the algorithm elevates these bowls until an outflow direction can be established.

To avoid draining water into the ocean that in reality would evaporate, precipitation falling into undrained basins will eventually be allowed to flood the land and made available to the atmospheric boundary layer module for evaporation.

No attempt is presently made to properly account for the time spent by runoff water in the various rivers. Precipitation which falls into a grid cell during an atmospheric time step and is not re-evaporated or used to moisten the soil is simply passed to the neighboring downstream cell, together with water that arrived during the previous time step from adjacent upstream cells. Effectively, then, the river flow speed is uniformly set to 1 grid cell per time step.

The river routing scheme is an adaptation of the one used in climate models at the NASA Goddard Institute for Space Studies.

11b. *Land/Sea Boundary Reconciliation*

Even though iHYCOM is on the same horizontal grid as FIM, the land mask used in FIM is not necessarily optimal for iHYCOM where the width of straits and the existence or absence of land bridges is important for the ocean circulation. For this reason, the land mask prepared for iHYCOM trumps the FIM mask in coupled runs, requiring a reconciliation strategy. In grid cells which FIM thinks are ocean but iHYCOM says are land, various land surface parameters (vegetation type etc.) need to be set. This is done by nearest-neighbor extrapolation. Grid cells converted from land to ocean require less work but nevertheless need to be properly identified in FIM as ocean points.

11c. *Finite-difference operations near coastal boundaries*

Despite its regular appearance, the icosahedral grid used in FIM and iHYCOM (Wang and Lee 2011) is, technically speaking, "unstructured". As outlined in Lee and MacDonald (2009) and Bao et al. (2011), finite-volume numerics can be implemented on unstructured grids by converting spatial derivative operations (divergence, curl, gradient) into line integrals around individual grid cells. This requires interpolation of variables from cell centers to cell perimeters. (Recall that FIM and iHYCOM use A grid staggering.)

In FIM, line integral segments along each of the 5 or 6 edges of a grid cell are evaluated using Simpson's rule which requires function values in the center and at each end of the integration interval. Edge *center* values are obtained by averaging the 2 nearest cell center values while *end* values, located at the corners of the pentagons/hexagons defining the grid, are obtained by averaging the 3 nearest cell center values. At present, no allowance is made for distortions in the shape and size of the icosahedral grid cells.

iHYCOM evaluates line integrals in the same manner, but values needed for the interpolation are not always available because an adjacent cell might be on land or have zero thickness due to sharply rising bottom topography. Variables in these "ghost" cells are obtained by extrapolation which is carried out in *hycom_edgvar.F90* as part of the general interpolation task. Details are as follows.

1. *momentum:* Let **v** be the velocity vector in the ocean cell adjacent to the ghost cell. The vector in the ghost cell is then set to either **v** or **-v**, depending on whether free- or no-slip sidewall conditions are specified.

2. *interface pressure:* Pressure in the ghost cell is set to the value in the adjacent ocean cell.

3. *pressure gradient force:* the PGF on the r.h.s. of (2) and (3) is based on line integrals over M and α. Ghost values for these 2 variables are set to the value in the adjacent ocean cell.

If neighboring bottom topography only partially covers the lateral face of a grid cell, the weight of the ghost values of u, v, M, α in the interpolation procedure is reduced accordingly. This partial weighting, already mentioned at the end of the momentum equation section, is a feature adopted from HYCOM. The degree of lateral blocking of an ocean cell extending from p_1 to $p_2 (> p_1)$ by neighboring bottom topography is determined by comparing the difference between the neighboring sea floor pressure and p_1 to the difference $p_2 - p_1$. The ghost value and the actual value in the neighboring cell are weighted linearly based on the ratio of the two pressure differences, provided the ratio lies in the interval (0,1).

11d. *Mass Flux Diagnostics*

Hydrographic measurements have long been used to estimate the mass transport across strategically placed transects in the world ocean. This information is vital for validating ocean models. Hence, diagnostic tools are needed to compute the mass transport across prescribed transects in the model.

The unstructured grid in iHYCOM makes this a a nontrivial task. Unlike the primary prognostic variables, mass fluxes in iHYCOM are carried on cell *edges* and are usually not archived. Diagnosing the transport across a prescribed transect, be it a meridian, parallel, or an oblique line drawn across a Strait, requires locating all cell edge segments that, if connected, form a polygon adhering as closely as possible to the chosen transect line.

The relative complexity of this task explains why such a utility has only recently been developed. We hope soon to be able to make statements about climate-relevant measures of model accuracy, such as meridional heat flux in individual basins, the strength of the Antarctic Circumpolar Circulation (typically measured in the Drake Passage), the amount of water flowing from the Pacific to the Indian Ocean through the Indonesian passage, and the strength of the thermohaline overturning circulation. Since western boundary currents are an important conduit for poleward heat transport, information about their strength must also be extracted from model results and compared to observations.

The algorithm for constructing edge polygons works as follows. To avoid a potentially time-consuming global search for all grid cells located on the chosen transect, a search is conducted for only the starting cell (for example, the westernmost cell in a transect running west-to-east). From there, the algorithm relies on the universal lookup table by which FIM and iHYCOM identify cell neighbors.

As the algorithm advances from cell to cell along the transect, it makes use of the fact that a straight line crossing a grid cell always intersects its perimeter in two places. The edge through which the transect line exits a given cell is also the edge through which it enters the next cell. Given knowledge of the latitude and longitude of each cell vertex, finding the *exit* edge in the new cell is straightforward provided the transect line is a meridian or parallel.

To easily locate cells along a transect that is not a meridian or parallel, we change to a new spherical coordinate system whose equator coincides with the chosen transect line. This reduces the problem to one of constructing an edge polygon along a parallel (which in this case also happens to be a great circle).

While the resulting polygon by design stays close to the transect line, the above algorithm will occasionally create unnecessary "meanders" of the edge polygon around that line. The implied lengthening of the integration path is unlikely to materially affect the accuracy of mass flux integrals. Nevertheless, there will always be meandering, and mass fluxes computed on multiple transect lines will be noisy if the transect lines are spaced close together.

12. Outstandig Issues

12a. *Interface Form Drag*

Form drag $p\nabla\phi$ is a quantity usually reserved for discussions of flow over solid obstacles. However, non-solid coordinate layer interfaces exert form drag on the layers above and below as well. Treating interface form drag correctly is important in both weather and ocean models. For example, Arakawa and Lamb (1977) point out that a pressure gradient formula which, for one reason or another, implies an inconsistent form drag will violate the law that there can be no barotropic spinup over flat bottom. In the ocean, form drag can be used to explain why wind stress is able to accelerate flow beneath the oceanic Ekman layer.

Correct vertical transmission of form drag $p\nabla\phi$ – or of its curl, the pressure torque $\nabla p \times \nabla\phi$ – is important for the following reason. The depth-integrated transport in a western boundary current (WBC) is largely controlled by the cross-basin zonal integral of torques applied at the top and bottom of the water column, i.e., wind stress torque and bottom torque (Sverdrup balance). For the strength of the WBC to be simulated correctly in a model, pressure torques passed across interior layer interfaces must cancel if summed up vertically. To achieve this in a generalized vertical coordinate model is not trivial. The task at hand is to transform the PGF in the flux form of the momentum equation into the sum of a horizontal gradient and a vertical derivative,

$$\frac{\partial p}{\partial s}\left[\alpha\nabla_s p + \nabla_s\phi\right] = \nabla\left(\frac{\partial p}{\partial s}\alpha\phi\right) + \frac{\partial}{\partial s}(p\nabla_s\phi) \qquad (7)$$

where the last term expresses the difference of form drags at the top and bottom of each coordinate layer. The salient step is to convert the *finite-difference* analog of the r.h.s. of (7) back to non-flux form by dividing it by layer thickness $\partial p/\partial s$. (Remember that the momentum equations in HYCOM and iHYCOM are solved in nonflux form.) Finally, an attempt should be made to restructure the resulting 2-term expression so that it reduces to a finite-difference analog of $\alpha\nabla p$, $\nabla\phi$, or ∇M if $s = z$, $s = p$, or $s = \theta$, respectively.

For rectilinear horizontal grids and HYCOM's hybrid-isopycnic vertical coordinate, such a finite-difference reduction is outlined in Appendix A of Bleck (2002). In unstructured horizontal grids, re-tracing these steps may be impossible because gradients are expressed on such grids as line integrals around grid cells, not as finite-difference operators that can be manipulated like true differentials.

Attempts to evaluate the PGF in iHYCOM using the raw 2-term formulation

$$\left(\frac{\partial p}{\partial s}\right)^{-1}\left[\nabla_s\left(\frac{\partial p}{\partial s}\alpha\phi\right) + \frac{\partial}{\partial s}(p\nabla_s\phi)\right]$$

which in principle would render the Sverdrup balance and its implications for WBC transport correctly, have not been successful so far.

The problem encountered here may transcend that of form drag conservation. The underlying issue may be one of accuracy, and it may manifest itself in more than one way. As pointed out earlier, the PGF in HYCOM is formulated as $\nabla M - p\nabla\alpha$. In horizontal nonisopycnic layers (i.e., near the ocean surface) it is not clear to what extent PGF values computed from this 2-term expression will match those based on $\alpha\nabla p$. For this to hold, we must be able to combine, in finite difference form, the $\nabla(\alpha p)$ part of ∇M with $-p\nabla\alpha$ into a single-term expression resembling $\alpha\nabla p$. No tools exist at present to perform this reduction.

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