

**NOAA NESDIS  
CENTER for SATELLITE APPLICATIONS and  
RESEARCH**

**ALGORITHM THEORETICAL BASIS DOCUMENT**

**ABI and VIIRS Ice Thickness and Age**

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Version 1.0

December 17, 2015

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## LIST OF ACRONYMS

ABI - Advanced Baseline Imager  
AIT - Algorithm Integration Team  
APP-x – Extended AVHRR Polar Pathfinder  
ATBD - algorithm theoretical basis document  
AVHRR - Advanced Very High Resolution Radiometer  
AVITA - ABI and VIIRS Ice Thickness and Age  
AWG - Algorithm Working Group  
CIMSS - Cooperative Institute for Meteorological Satellite Studies  
CIS - Canadian Ice Service  
CSIM - CSM Sea Ice Model  
CSM – Climate System Model  
ECMWF - European Centre for Medium-Range Weather Forecasts  
F&PS - Function and Performance Specification  
GCM – Global Circulation Model  
GOCC - Generalized Orthogonal Curvilinear Coordinate  
JPSS - Joint Polar Satellite System  
MODIS - Moderate Resolution Imaging Spectroradiometer  
MRD - Mission Requirements Document  
MSG - Meteosat Second Generation  
NASA - National Aeronautics and Space Administration  
NCAR - National Center for Atmospheric Research  
NCEP - National Centers for Environmental Prediction  
NOAA - National Oceanic and Atmospheric Administration  
NSIDC - National Snow and Ice Data Center  
NWP - Numerical Weather Prediction  
OTIM - One-dimensional Thermodynamic Ice Model  
PIOMAS - Pan-Arctic Ice-Ocean Modeling and Assimilation System  
POP - Parallel Ocean Program  
RTM - Radiative Transfer Model  
S-NPP - Suomi National Polar Orbiting Partnership  
SCICEX - SCientific ICe EXpeditions  
SEVIRI - Spinning Enhanced Visible and Infrared Imager  
SHEBA - Surface HEat Balance of the Arctic Ocean  
SIT – Sea Ice Temperature  
SIC – Sea Ice Concentration  
SSEC – Space Science and Engineering Center  
TED - Thickness and Enthalpy Distribution  
TOA – Top of Atmosphere  
ULS – Upward Looking Sonar  
UW – University of Wisconsin

## ABSTRACT

This document provides a high level description of the physical basis for the determination of sea and lake ice thickness and age information, of each clear pixel data acquired by the Advanced Baseline Imager (ABI) on the GOES-R series of NOAA geostationary meteorological satellites and the Visible Infrared Imaging Radiometer Suite (VIIRS) on the Suomi National Polar Orbiting Partnership (S-NPP) satellite and the Joint Polar Satellite System (JPSS) series of NOAA polar-orbiting meteorological satellites. Ice age is determined from the ice thickness information within the same algorithm module. Thus, the ice thickness and age are both described in this ATBD.

The core of the ABI and VIIRS Ice Thickness and Age (AVITA) algorithm is a One-dimensional Thermodynamic Ice Model (OTIM). Based on the surface energy budget, OTIM contains all components of the surface energy budget that are needed for the estimation of sea and lake ice thickness. Based on ice thickness, ice is classified into open water, new/fresh ice, grey ice, grey-white ice, thin first year ice, medium first year ice, thick first year ice, and multi-year or old ice. Inevitably, OTIM contains parameterizations and/or assumptions of the sea and lake ice and associated snow characteristics of their physical properties, such as ice and snow conductivities, densities, and transmittances, if that information is not available. The validation analysis indicates that the algorithm can meet the accuracy requirements of the Functional and Performance Specification (F&PS).

# 1 INTRODUCTION

## 1.1 *Purpose of This Document*

The ice thickness and age algorithm theoretical basis document (ATBD) provides a high level description of the physical basis for the estimation of ice thickness and age for clear and cloudy pixels identified as ice covered with supplementary information from parameterization schemes and other products retrieved from the Advanced Baseline Imager (ABI) and Visible Infrared Imaging Radiometer Suite (VIIRS) flown on the GOES-R and JPSS series of NOAA geostationary and polar-orbital meteorological satellites, respectively. That supplementary information includes a cloud mask, snow depth, ice surface temperature, ice concentration, surface air temperature, surface air humidity, surface wind, and surface solar and thermal radiation fluxes. The ice thickness and age algorithm provides primary estimates of the ice thickness and age for each ABI or VIIRS pixel covered with ice. The ice thickness and age products are made available to all subsequent algorithms that require knowledge of ice information.

## 1.2 *Who Should Use This Document*

The intended users of this document are those interested in understanding the physical basis of the algorithm and how to use the output of this algorithm to estimate ice thickness and age for a particular application. This document also provides information useful to anyone maintaining or modifying the original algorithm.

## 1.3 *Inside Each Section*

This document is broken down into the following main sections.

- **System Overview:** Provides relevant details of the ABI and VIIRS and a brief description of the products generated by this algorithm.
- **Algorithm Description:** Provides all the detailed description of the algorithm including its physical basis, its input and its output.
- **Assumptions and Limitations:** Provides an overview of the current limitations of the approach and gives the plan for overcoming these limitations with further algorithm development.

## 1.4 *Related Documents*



This ATBD extends the related GOES-R ATBDs by adding information specific to VIIRS.

### ***1.5 Revision History***

Version 1.0 of this document was created by Xuanji Wang of UW/CIMSS and Jeff Key of NOAA/NESDIS/STAR, and is intended to accompany the delivery of the version 1 algorithms to both the GOES-R AWG and the JPSS Algorithm Integration Teams (AIT).

## 2 OBSERVING SYSTEM OVERVIEW

This section will describe the products generated by the ABI and VIIRS Ice Thickness and Age (AVITA) algorithm and the requirements it places on the sensor and other products.

### 2.1 *Products Generated*

The ice thickness and age algorithm is responsible for the estimation of sea and lake ice thickness and age for all ABI or VIIRS pixels covered with ice. In terms of the ABI and JPSS MRDs, at the Threshold level, the ice-free areas are distinguished from first-year ice and older ice areas. The Goal requirement is to distinguish not only ice-free from first-year ice areas, but also to distinguish between the following types of ice: nilas, grey white, first-year thin, first-year medium, first-year thick, second-year, multiyear smooth and multiyear deformed ice. These categories are defined in terms of ice age. The ice thickness and age products will be used by other ABI and VIIRS algorithms that require knowledge of the ice information. The current ice thickness and age design has the ability to estimate sea and lake ice thickness up to 3 meters under both clear and cloudy conditions at night (no sunlight). It would also work during daytime, though the estimation is more uncertain due to the complexities of ice and snow optical properties in the solar spectrum.

### 2.2 *Instrument Characteristics*

The ice thickness and age will be produced for each pixel observed by the ABI or VIIRS and covered with ice. There are no direct ABI or VIIRS channels related to the algorithm which actually relies on some other retrieved products from ABI or VIIRS and parameterization schemes such as cloud mask and ice surface temperature that would use some or all ABI or VIIRS channels for their retrievals.

The algorithm relies on the accuracy of the other products and parameterization schemes such as the cloud mask, ice surface temperature, ice surface albedo, and radiation fluxes. The performance of the ice thickness and age algorithm is therefore sensitive to the accuracy of other ABI or VIIRS retrieved products. We will detail the required input parameters and current validations in the following sections, and the algorithm sensitivity to input uncertainties.

**Table 1.1** Summary of the Current ABI Channel Numbers and Wavelengths.

<i>Channel Number</i>	<i>Wavelength (<math>\mu\text{m}</math>)</i>	<i>Direct Use in AVITA</i>
1	0.47	No
2	0.64	No
3	0.86	No
4	1.38	No
5	1.61	No
6	2.26	No
7	3.9	No
8	6.15	No
9	7.0	No
10	7.4	No
11	8.5	No
12	9.7	No
13	10.35	No
14	11.2	No
15	12.3	No
16	13.3	No

**Table 1.2** Summary of the Current VIIRS Band Numbers and Spectral Ranges.

<i>Band Number</i>	<i>Wavelength (<math>\mu\text{m}</math>)</i>	<i>Direct Use in AVITA</i>
M1	0.402~0.422	No
M2	0.436~0.454	No
M3	0.478~0.498	No
M4	0.545~0.565	No
M5	0.662~0.682	No
M6	0.739~0.754	No
M7	0.846~0.885	No
M8	1.230~1.250	No
M9	1.371~1.386	No
M10	1.580~1.640	No
M11	2.225~2.275	No
M12	3.660~3.840	No
M13	3.973~4.128	No
M14	8.400~8.700	No
M15	10.263~11.263	No
M16	11.538~12.488	No
DNB	0.5~0.9	No
I1	0.600~0.680	No
I2	0.846~0.885	No
I3	1.580~1.640	No
I4	3.550~3.930	No
I5	10.500~12.400	No

### 3 ALGORITHM DESCRIPTION

Complete description of the algorithm at the current level of maturity (which will improve with each revision).

#### 3.1 *Algorithm Overview*

The ice thickness and age algorithm will use a One-dimensional Thermodynamic Ice Model (OTIM) developed by the investigators. OTIM is based on the surface energy balance and contains all components of the surface energy budget to estimate sea and lake ice thickness up to 3 meters. Ice age is based on ice thickness as follows:

- Free or Open water: thickness = 0
- New:  $0.00 < \text{thickness} \leq 0.10$
- Grey:  $0.10 < \text{thickness} \leq 0.15$
- Grey-white:  $0.15 < \text{thickness} \leq 0.30$
- First year Thin:  $0.30 < \text{thickness} \leq 0.70$
- First year Medium:  $0.70 < \text{thickness} \leq 1.20$
- First year Thick:  $1.20 < \text{thickness} < 1.80$
- Older: thickness  $\geq 1.80$

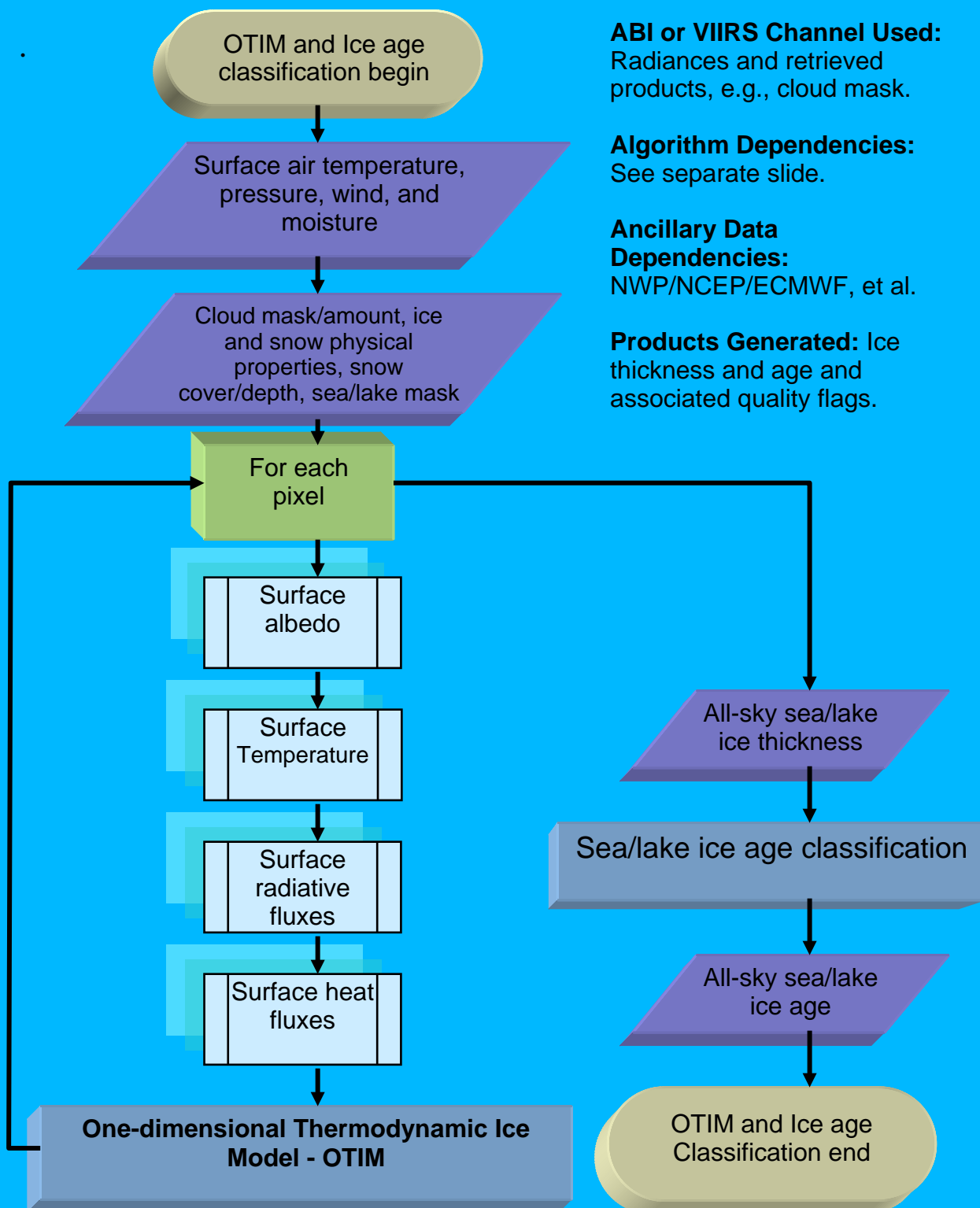
**Pros:** Solid physical foundation with all components of surface energy budget considered. Capable of retrieving daytime and nighttime sea and lake ice thickness under both clear and cloudy sky conditions. Very computationally efficient compared to more complex models such as the Climate System Model (CSM) Sea Ice Model (CSIM). Its sole objective of retrieving ice thickness and age makes it easy to implement with the application of satellite products, flexible, fast and easy to maintain and improve later with more and accurate satellite derived products like radiative fluxes, ice surface temperature and snow depth over the ice.

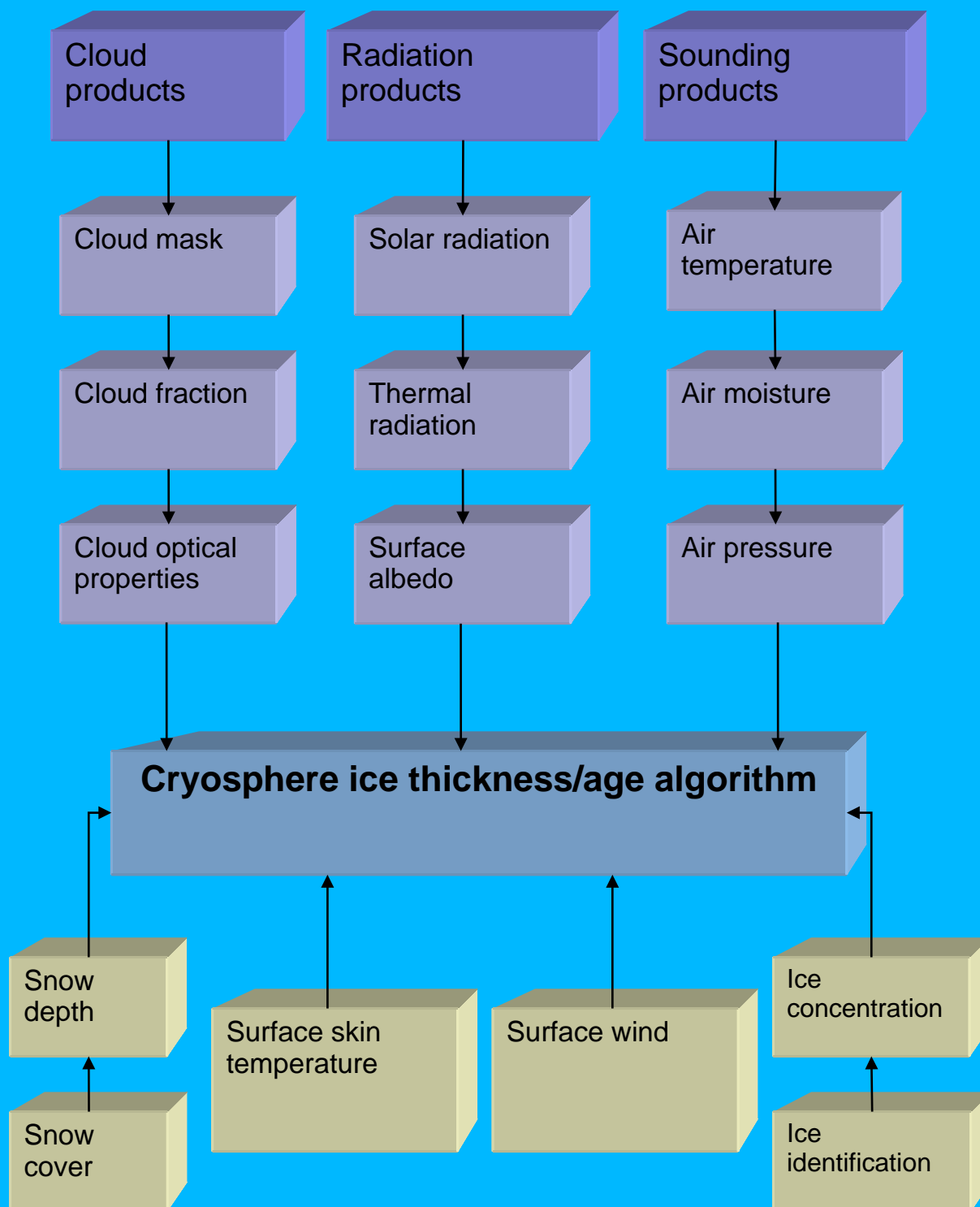
**Cons:** The accuracy of input parameters, e.g., snow depth, surface air humidity, temperature, and wind, will impact the accuracy of ice thickness estimates. Daytime retrieval is sensitive to ice optical properties associated with ice type and thickness, and is less reliable than nighttime retrievals.

#### 3.2 *Processing Outline*

The processing outline of the AVITA is summarized in the following chart. The AVITA is designed to run on segments of data. A segment is comprised of multiple scan lines.

### High Level Flowchart of the AVITA Illustrating the Main Processing Sections.



**The AVITA Algorithm Dependency on Other ABI or VIIRS Products and Data Sources.**

### ***3.3 Algorithm Input***

This section describes the input needed to process the AVITA. While the AVITA is derived for each pixel covered with ice, it does require knowledge of the surrounding atmosphere. In this version, the daytime retrieval was run and improved, but it will still be investigated in the next version due to the fact that complex solar radiation interactions result in a larger uncertainty than with nighttime applications.

#### ***3.3.1 Primary Sensor Data***

The list below contains the primary parameters data used by the AVITA, information that is derived mainly from the ABI or VIIRS observations and geolocation information.

- **Latitude**
- **Longitude**
- **Sensor viewing zenith angle**
- **Solar zenith angle**

#### ***3.3.2 Ancillary Data***

The following lists and briefly describes the ancillary data required to run the AVITA, information that is not included in the ABI or VIIRS observations or geolocation data.

- **Land mask**
- **Ice and Snow Thermal Emissivity**
- **Ice and Snow Optical Properties (Albedo, Transmittance, Absorptivity)**
- **Ice and Snow Physical Properties (Density, Salinity, Conductivity, Contaminant)**
- **Surface Air Temperature**
- **Surface Humidity**
- **Surface Wind**
- **Surface broadband albedo**

#### ***3.3.3 Derived Data***

The following briefly describes the products from other ABI or VIIRS algorithms that the AVITA algorithm uses as input. These data are necessary in order to run the software that calculates ice thickness and sequentially ice age. These data are required information that is not included in the ABI or VIIRS observations or geo-location data.

- **Cloud mask & Cloud Fraction**
- **Ice surface skin temperature**
- **Ice mask and concentration**
- **Shadow Mask from Cloud height**
- **Sun glint flag from Cloud Mask**

### 3.4 *Theoretical Description*

Physical and statistical approaches are employed to estimate sea and lake ice thickness and age. In this document, a One-dimensional Thermodynamic Ice Model (OTIM) based on the surface energy budget, containing all components of the surface energy budget has been developed to estimate sea and lake ice thickness. Then based on the knowledge of ice thickness, ice is classified into open water, new/fresh ice, grey ice, grey-white ice, thin first year ice, medium first year ice, thick first year ice, and multi-year or older ice. It inevitably involves parameterizations and/or assumptions of the sea and lake ice and associated snow characteristics, such as ice and snow conductivities, densities, and transmittances.

#### 3.4.1 *Physics of the Problem*

The difficult task for any ice thickness and age estimation based on the thermodynamic theory is to exploit not only ice and snow micro-physical properties which are closely related to ice and snow types and contents but also its environmental conditions such as humidity, air temperature, wind, cloud cover, water salinity and current. In the testing stage of the AVITA, we have extensively used information from the extended AVHRR Polar Pathfinder (APP-x) product, which consists of AVHRR retrievals of surface and cloud properties with our retrieval tool CASPR (Key, 2002) for the period 1982-2004 over the Arctic, the NCAR/NCEP Reanalysis data, parameterization schemes for ice and snow micro-physical properties, and solar and thermal radiation flux parameterizations at the surface developed by other researchers. In addition the AVITA has been tested with the MODIS and SEVIRI data as well. While the current NCAR/NCEP Reanalysis fields often have errors in some critical fields, such as the surface air temperature and humidity over ice and snow, they provide needed and useful information. Nevertheless, parameterization schemes are often needed to estimate environmental conditions, especially for ice and snow areas.

The following sections describe the physical model details of the OTIM. The AVITA uses the OTIM to estimate sea and lake ice thickness. The OTIM treats day and night retrievals differently. When the sun is above the horizon, the solar radiation must be considered in the OTIM which makes the ice thickness estimation very complicated due to the unknown ice and snow types resulting in inaccurate ice and snow optical property estimates in the solar spectrum, as well as unstable numerical solutions for the OTIM. Nighttime retrievals of ice thickness are easier and more accurate, with an analytical solution in the OTIM. The OTIM has been tested with AVHRR, MODIS, and SEVIRI data and validated with submarine and moored Upward Looking Sonar data, meteorological station measurements, and numerical model simulations.

#### 3.4.2 *One-dimensional Thermodynamic Ice Model (OTIM)*

A slab model proposed by Maykut and Untersteiner (1971) is used here as a prototype model, the equation for energy conservation at the top surface (ice or snow) is

$$(1-\alpha_s)F_r - I_0 - F_l^{up} + F_l^{dn} + F_s + F_e + F_c = F_a$$



or

$$(1-\alpha_s)(1-i_0)F_r - F_l^{up} + F_l^{dn} + F_s + F_e + F_c = F_a \quad (1)$$

where  $\alpha_s$  is ice surface broadband albedo where ice may be covered with a layer of snow,  $F_r$  is downward solar radiation flux at the surface,  $I_0$  is the solar radiation flux passing through the ice interior and  $i_0$  is ice slab transmittance,  $F_l^{up}$  is upward longwave radiation flux from the surface,  $F_l^{dn}$  is downward longwave radiation flux from the atmosphere towards the surface,  $F_s$  is turbulent sensible heat flux at the surface,  $F_e$  is turbulent latent heat flux at the surface,  $F_c$  is conductive heat flux within the ice slab,  $F_a$  is residual heat flux that could be caused by ice melting and/or heat advection. By the definitions of the terms in the equation (1),  $\alpha_s$ ,  $F_r$ ,  $I_0$ ,  $F_l^{up}$ ,  $F_l^{dn}$  should be always positive, and  $F_s$ ,  $F_e$ , and  $F_c$  would be positive or negative in terms of the operational symbols used in the equation (1), and  $F_a$  is zero in the absence of a phase change. The details of each term will be addressed in the following subsections.

### 3.4.2.1 Solar Radiation at the Surface

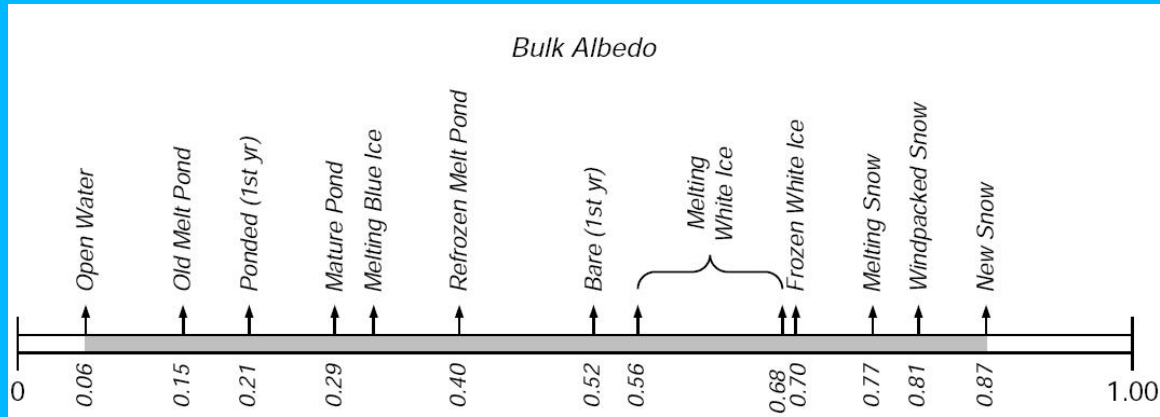
The first term on the left-hand side of the equation (1),  $(1-\alpha_s) F_r$ , is net solar radiation flux at the surface. The surface broadband albedo over entire solar spectrum,  $\alpha_s$ , can be input or estimated by Thomas C. Grenfell (1979) method as described below:

$$\alpha_s = 1 - A \exp(-Bh) - C \exp(-Dh) \quad (2)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are the coefficients of which values can be determined from below Table 2, and  $h$  is ice ( $h_i$ ) or snow ( $h_s$ ) thickness in meter. The other relatively simple approach to determine ice and snow surface albedo include model simulated constant values based on the ice and snow types as discussed by Tuomo M. Saloranta (2000), and the experimental and observational values for a variety of snow and ice surface conditions (Grenfell and Perovich, 2004) as shown in Figure 1 from Donald K. Perovich (1996).

**Table 2.** The coefficient values of A, B, C, and D in the Eq. (2) (from Thomas C. Grenfell, 1979)

<i>Ice type</i>	<i>Cloudiness</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Error</i>
Blue ice 0.8m >= h <sub>i</sub> >= 0.02m	Clear	0.130	15.46	0.820	0.1216	< 1%
	Cloudy	0.150	12.02	0.800	0.2161	< 1%
White ice 0.8m >= h <sub>i</sub> >= 0.02m	Clear	0.419	12.40	0.531	0.1958	< 2.5%
	Cloudy	0.540	10.11	0.410	0.2827	< 3%
Dry packed snow over blue ice 0.4m >= h <sub>s</sub> >= 0.01m	Clear	0.2213	77.48	0.1980	0	< 5%
	Cloudy	0.3181	77.81	0.100	0.050	< 7%



**Figure 1.** Range of observed values of total albedo for sea ice. The albedos are from Burt (1954), Chernigovskiy (1963), Langleben (1971), Grenfell and Maykut (1977), and Grenfell and Perovich (1984).

The downward shortwave radiation flux towards the surface,  $F_r$ , could also be input parameter for the OTIM or parameterized with model built-in parameterization schemes as described below. There are a number of parameterization schemes for the  $F_r$  estimation under both clear and cloudy sky conditions that are described in the following subsections. Key (1996) compared these schemes and discussed the use of which scheme would be better regarding the surface type, location, and atmospheric conditions.

### 3.4.2.1.1 Clear-sky Parameterizations of Solar Radiation

1. Shine and Henderson-Sellers (1985) scheme for the Arctic:

$$F_r^{clr} = 1368\mu^2 [1.2\mu + (1.0 + \mu) e_a 10^{-3} + 0.046]^{-1} \quad (3)$$

where  $\mu$  = cosine of the solar zenith angle,  $e_a = f e_{sa}$ ,  $e_a$  is surface water vapor pressure (hPa),  $e_{sa}$  is surface saturation water vapor pressure (hPa),  $f$  is relative humidity (%). So, knowing surface air temperature  $T_a$  (C) and relative humidity  $f$ , we can calculate  $e_a$  by

$$e_{sa} = 6.11 * 10.0^{(7.5 * T_a / (237.7 + T_a))} \quad (4)$$

where  $e_a = f e_{sa}$ .

2. Moritz (1978) scheme for Baffin Bay, Canada:

$$F_r^{clr} = S_0 \mu (0.47 + 0.47 \mu) \quad (5)$$

where  $S_0$  is solar constant,  $\mu$  is cosine of solar zenith angle.

3. Bennett (1982) scheme for the Arctic:

$$F_r^{clr} = 0.72 S_0 \mu \quad (6)$$

where  $S_0$  and  $\mu$  are the same as the above. This one is recommended for its simplicity and acceptable accuracy.

### 3.4.2.1.2 Cloudy-sky Parameterizations of Solar Radiation

1. Berliand (1960) scheme:

$$F_r^{all} = F_r^{clr} (1 - x c - y c^2) \quad (7)$$

where  $c$  is cloud fraction between 0~1. For land and ocean,  $y=0.38$ ,  $x=0.14$  at  $85^\circ$ ,  $0.41$  at  $55^\circ$ ,  $0.38$  at  $45^\circ$  respectively; and  $x=0.45$  and  $y=0$  at  $75^\circ$ N/S. Its performance is poor over oceans as tested.

2. Laevastu (1960) scheme for the mid-latitude ocean:

$$F_r^{all} = F_r^{clr} (1 - 0.6 c^3) \quad (8)$$

3. Jacobs (1978) scheme for Baffin Island, Canada over the period June to October:

$$F_r^{all} = F_r^{clr} (1 - 0.33 c) \quad (9)$$

4. Bennett (1982) scheme for Arctic sea ice:

$$F_r^{all} = F_r^{clr} (1 - 0.52 c) \quad (10)$$

This one is recommended for its simplicity and acceptable accuracy for this study.

5. Shine (1984) scheme for high albedo surfaces such as ice and snow:

$$F_r^{cld} = (53.5 + 1274.5 \mu) \mu^{0.5} / [1 + 0.139 (1 - 0.9345 \alpha_s) \tau] \quad (11)$$

$$F_r^{all} = [(1 - c) F_r^{clr} + c F_r^{cld}] \quad (12)$$

where  $\alpha_s$  is surface broadband albedo (0~1), and  $\tau$  is cloud optical depth.

### 3.4.2.2 Solar Radiation Passing through Ice Interior

The second term on the left-hand side of the equation (1),  $I_0 = i_0 (1 - \alpha_s) F_r$ , is the solar radiation flux passing through the ice interior.  $i_0$  is ice slab transmittance, i.e. the percentage of the net solar radiation flux that penetrates the ice, which can be estimated by the following parameterization scheme developed by Thomas G. Grenfell (1979):

$$i_o = A \exp(-Bh) + C \exp(-Dh) \quad (13)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are coefficients given in the Table 3, and  $h$  is ice slab thickness in meter.

**Table 3.** The coefficient values of  $A$ ,  $B$ ,  $C$ , and  $D$  in the Eq. (13) (from Thomas C. Grenfell, 1979)

<i>Ice Type</i>	<i>Cloudiness</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Error</i>
Blue ice 0.8m $\geq$ $h_i \geq$ 0.02m	Clear	0.1925	12.96	0.515	1.227	< 4%
	Cloudy	0.1553	12.84	0.755	1.081	< 2%
White ice 0.8m $\geq$ $h_i \geq$ 0.02m	Clear	0.3894	12.39	0.350	1.578	< 4%
	Cloudy	0.3456	10.30	0.590	1.315	< 2.5%
Dry packed snow over blue ice 0.4m $\geq$ $h_s \geq$ 0.01m 0.8m $\leq$ $h_i \geq$ 0.01m	Clear	A = 0.2257 exp(-16.73 $h_s$ ) + 0.4174 exp(-43.89 $h_s$ ) B = 0.7280 exp(-0.1862 $h_s$ ) + 0.3532 exp(-13.04 $h_s$ ) C = 0.1561 exp(-92.79 $h_s$ ) D = [0.06 + 0.0995 exp(-94.20 $h_s$ )] <sup>-1</sup>				< 6%
	Cloudy	A = 0.980 exp(-17.81 $h_s$ ) B = 0.6945 exp(-0.1048 $h_s$ ) + 0.303 exp(-54.92 $h_s$ ) <sup>1.42</sup> C = D = 0.0				< 6%

In the first approximation, the parameter  $i_o$  for the percentage of penetrating shortwave radiation into snow is kept zero, and  $i_o$  for ice is calculated linearly as a function of cloudiness as given in Grenfell and Maykut's 1977 paper listed below:

$$i_o = 0.18(1-c) + 0.35c \text{ for white ice thickness less than 0.1 m,}$$

$$i_o = 0.43(1-c) + 0.63c \text{ for blue ice thickness less than 0.1 m,}$$

where  $c$  is cloud fraction.

### 3.4.2.3 Upward Longwave Radiation from the Surface

The third term on the left-hand side of the equation (1),  $F_l^{up}$ , is upward longwave radiation flux from surface that can be easily estimated with following formulae:

$$F_l^{up} = \varepsilon \sigma T_s^4 \quad (14)$$

where  $\varepsilon$  is longwave emissivity of the ice or snow surface,  $\sigma = 5.6696 \cdot 10^{-8} \text{ W m}^{-2} \text{ deg}^{-4}$  is the Stefan-Boltzman constant, and  $T_s$  is surface skin temperature in K. For simplicity, here ice emissivity of 0.988 is used. Even though some pixels contain a small portion of open water or snow surface, the error in emissivity from improperly defining the surface type is small because snow emissivity at  $0^\circ$  look angle is 0.995, very close to the value of 0.987 for ice and 0.988 for water (Rees, 1993). Due to the substance heterogeneity, both reflectance and emissivity are geometrically considered, e.g. bidirectional reflecting property of a plant.

### 3.4.2.4 Downward Longwave Radiation towards the Surface

The fourth term on the left-hand side of the equation (1),  $F_l^{dn}$ , is downward longwave radiation flux from the atmosphere towards the surface that can be parameterized by following schemes.

#### 3.4.2.4.1 Clear-sky Parameterizations of Longwave Radiation

1. Yu and Rothrock (1996) scheme:

$$F_l^{dn} = \varepsilon^* \sigma T_a^4 \quad (15)$$

where  $\varepsilon^* = 0.7855 (1 + 0.2232 c^{2.75})$  is an effective emissivity for the atmosphere and  $T_a$  is the near-surface air temperature at 2 m above the surface, and  $c$  is fractional cloud cover.

2. Efimova (1961) scheme:

$$F_{l,clr}^{dn} = \sigma T_a^4 (0.746 + 0.0066 e_a) \quad (16)$$

where  $e_a$  is water vapor pressure (hPa).

3. Ohmura (1981) scheme for the temperature range 243-289K:

$$F_{l,clr}^{dn} = \sigma T_a^4 (8.733 \cdot 10^{-3} T_a^{0.788}) \quad (17)$$

where  $\sigma$ ,  $T_a$  are the same as above. This one is recommended for the simplicity and acceptable accuracy in this work.

4. Maykut and Church (1973) scheme for the temperature range of 244-277K:

$$F_{l,clr}^{dn} = 0.7855 \sigma T_a^4 \quad (18)$$

where  $\sigma$ ,  $T_a$  are the same as above.

5. Andreas and Ackley (1982) scheme for the Arctic and Antarctica regions:

$$F_{l,clr}^{dn} = \sigma T_a^4 (0.601 + 5.95 \cdot 10^{-5} e_a^{1500/T_a}) \quad (19)$$

where  $\sigma$ ,  $T_a$  are the same as above,  $e_a$  is the near-surface water vapor pressure (hPa).

#### 3.4.2.4.2 Cloudy-sky Parameterizations of Longwave Radiation

1. Yu and Rothrock (1996) scheme:

$$F_l^{dn} = \varepsilon^* \sigma T_a^4 \quad (20)$$

where  $\varepsilon^* = 0.7855 (1 + 0.2232 c^{2.75})$  is an effective emissivity for the atmosphere and  $T_a$  is the near-surface air temperature at 2 m above the surface, and  $c$  is fractional cloud cover.

2. Jacobs (1978) scheme for Arctic summer and winter:

$$F_l^{dn} = F_{l,clr}^{dn} (1 + 0.26 c) \quad (21)$$

where  $c$  is fractional cloud cover. This one is recommended for the simplicity and acceptable accuracy in this work.

3. Maykut and Church (1973) scheme over 244-277K:

$$F_l^{dn} = F_{l,clr}^{dn} (1 + 0.22 c^{2.75}) \quad (22)$$

where  $c$  is fractional cloud cover.

4. Zillman (1972) scheme:

$$F_l^{dn} = F_{l,clr}^{dn} + \sigma T_a^4 0.96 (1 - 9.2 \cdot 10^{-6} T_a^2) c \quad (23)$$

5. Schmetz et al (1986) scheme:

$$F_l^{dn} = F_{l,clr}^{dn} + (1 - \varepsilon_0) C \varepsilon_c \sigma T_0^4 \exp[(T_B + T_0) / 46] \quad (24)$$

where  $\varepsilon_0$  is the effective sky emittance,  $\varepsilon_c$  is cloud emissivity,  $T_0$  is the near-surface air temperature, and  $T_B$  is cloud base temperature.

### 3.4.2.5 Turbulent Sensible Heat Flux

The fifth term on the left-hand side of the equation (1),  $F_s$ , is the turbulent sensible heat flux at the surface that can be calculated by following formulae if it is an unknown variable in the OTIM.

$$F_s = \rho_a c_p C_s u (T_a - T_s) \quad (25)$$

where  $\rho_a$  is the air density (standard value of  $1.275 \text{ kg}\cdot\text{m}^{-3}$  at  $0^\circ\text{C}$  and 1000 hPa),  $c_p$  is the specific heat of wet air that should be calculated from Eq. (25.1) with wet air specific humidity  $q$ ,  $C_s$  is the bulk transfer coefficients for turbulent sensible heat flux between air and ice surface (Yu chose  $C_s = 0.003$  for very thin ice, and  $0.00175$  for thick ice,  $0.0023$  for neutral stratification as suggested by Lindsay (1998) in his energy balance model for thick arctic pack ice),  $u$  is surface wind speed,  $T_a$  is near surface air temperature at 2 m above the surface, and  $T_s$  is surface skin temperature.

$$C_p = C_{pd} \left(1 - q + \frac{C_{pv}}{C_{pd}} q\right) \quad (25.1)$$

where  $C_{pv}$  is the specific heat of water vapor at constant pressure ( $1952 \text{ J K}^{-1} \text{ kg}^{-1}$ ), and  $C_{pd}$  is the specific heat of dry air at constant pressure ( $1004.5 \text{ J K}^{-1} \text{ kg}^{-1}$ ), so  $C_p$  can simply be written as  $C_p = 1004.5 \cdot (1 + 0.9433 \cdot q)$ .

The wet air density  $\rho_a$  can be calculated using gas law with given surface air pressure  $P_a$  (hPa), surface air virtual temperature  $T_v$  (K), and gas constant  $R_{gas}$  ( $287.1 \text{ J kg}^{-1} \text{ K}^{-1}$ ) by Eq. 25.2.

$$\rho_a = \frac{100P_a}{R_{gas}T_v} \quad (25.2)$$

where  $T_v = (1 + 0.608 \cdot q) \cdot T_a$ ,  $q$  is wet air specific humidity (kg/kg).

### 3.4.2.6 Turbulent Latent Heat Flux

The sixth term on the left-hand side of the equation (1),  $F_e$ , is turbulent latent heat flux at the surface that can be calculated by following formulae if it is an unknown variable in the OTIM.

$$F_e = \rho_a L C_e u (w_a - w_{sa}) \quad (26)$$

where  $\rho_a$  is air density,  $L$  is latent heat of vaporization ( $2.5 \cdot 10^6 \text{ J} \cdot \text{kg}^{-1}$ ) which should include the latent heat fusion/melting ( $3.34 \cdot 10^5 \text{ J} \cdot \text{kg}^{-1}$ ) if the surface is below freezing,  $C_e$  is the bulk transfer coefficients for latent heat flux of evaporation,  $u$  is surface wind speed,  $w_a$  is the air mixing ratio at 2 m,  $w_{sa}$  is the mixing ratio at the surface. Mixing ratio is very close to specific humidity in magnitude,  $w = q / (1 - q) \cong q$ , where  $q$  is the specific humidity.

The bulk transfer coefficients  $C_e$  for latent heat flux is a function of wind speed and air-sea ice temperature difference which can be parameterized as described by Bentamy et al (2003),

$$C_e = \{ a \exp[b(u + c)] + d/u + 1 \} \times 10^{-3}, \quad (26.1)$$

where  $a = -0.146785$ ,  $b = -0.292400$ ,  $c = -2.206648$ , and  $d = 1.6112292$ . The  $C_e$  values range between 0.0015 and 0.0011 for wind speeds between 2 and 20  $\text{m s}^{-1}$ . Schroder et al's study (2003) indicated that  $C_e$  values are always around  $1.0 \times 10^{-3}$  except for rough multi-year ice which has  $C_e$  value of  $1.3 \times 10^{-3}$ , and  $C_s$  value of  $1.5 \times 10^{-3}$  from six ice categories that are gray young ice, mixture of gray and white ice and leads, rough multi-year ice, step change between ice and water, loose ice fields, and grease ice (Table 2 in their paper). Another parameterization scheme of  $C_e$  was developed by Kara et al (2000) for use in general circulation model. They related  $C_e$  to both surface wind speed and air-sea temperature difference, the fitted expression are as follows:

$$C_e = C_{e0} + C_{e1} (T_s - T_a) \quad (26.2)$$

$$C_{e0} = [0.994 + 0.061 \cdot \hat{u} - 0.001 \cdot \hat{u}^2] \cdot 10^{-3} \quad (26.3)$$

$$C_{e1} = [-0.020 + 0.691 \cdot (1/\hat{u}) - 0.871 \cdot (1/\hat{u})^2] \cdot 10^{-3} \quad (26.4)$$

where the wind speed is limited to the interval  $\hat{u} = \max[3.0, \min(27.5, u)]$  to suppress the underestimation of the quadratic fit when  $u > 27.5 \text{ m s}^{-1}$ .

Because the  $C_s$  values are so close in value to the  $C_e$  for sea water, a linear relationship between  $C_e$  and  $C_s$  rather than determine independent  $C_{s0}$  and  $C_{s1}$  coefficients. This also helps to reduce the cost of computing the sensible heat flux in GCMs, the simplest representative linear formulation is found to be  $C_s=0.96 \cdot C_e$  with a negligible intercept ( $3.6 \cdot 10^{-6}$ ) as reported by Kara et al. (2000), we use  $C_s=0.98 \cdot C_e$  in our model for air-sea ice interface turbulent heat transfer.

### 3.4.2.7 Conductive Heat Flux

The seventh term on the left-hand side of the equation (1),  $F_c$ , is conductive heat flux within the ice slab that can be calculated by following formulae as used by Yu and Rothrock (1996).

$$F_c = \gamma (T_f - T_s) \quad (27)$$

where  $\gamma = (k_i k_s) / (k_s h_i + k_i h_s)$ ,  $T_f$  is water freezing temperature and can be derived from a simplified relationship of  $T_f = -0.055 S_w$ , where  $S_w$  is the salinity of seawater and assumed to be 31.0 parts per thousand (ppt) for the Beaufort Sea and 32.5 ppt for the Greenland Sea, which means  $T_f$  is in the unit of degree Celsius.  $h_s$  is snow depth, and  $h_i$  is ice thickness.  $k_s$  is the conductivity of snow which can be formulated by  $k_s = 2.845 \cdot 10^{-6} \rho_{snow}^2 + 2.7 \cdot 10^{-4} \cdot 2.0^{(T_{snow}-233)/5}$  (Ebert and Curry, 1993),  $\rho_{snow}$  is snow density ranging from 225 kg·m<sup>-3</sup> (new snow) ~ 450 kg·m<sup>-3</sup> (water-soaked snow),  $T_{snow}$  is snow temperature in Kelvin. The  $k_s$  can be further simplified as  $k_s = 2.22362 \cdot 10^{-5.655} (\rho_{snow})^{1.885}$  (Yen, Y.-C., 1981). Usually, the  $k_s$  is assumed 0.31 in some applications. The  $k_i$  is the conductivity of ice that can be estimated by  $k_i = k_0 + \beta \cdot S_i / (T_i - 273)$  (Untersteiner's, 1964), where  $\beta = 0.13$  W·m<sup>-2</sup>·kg<sup>-1</sup>,  $k_0 = 2.22 \cdot (1 - 0.00159 \cdot T_i)$  W·m<sup>-1</sup>·K<sup>-1</sup> is the conductivity of pure ice (Curry and Webster, 1999).  $S_i$  is sea ice salinity,  $T_i$  is the temperature within the ice slab. Some experimental relative relationships between  $h_s$  and  $h_i$ ,  $T_i$  and  $T_s$ ,  $S_i$  and  $h_i$  exist as described in the following subsections. See Appendix A for the derivation of the Equation 27 for a two-layer system with snow over ice.

#### 3.4.2.7.1 Relationship between Snow Depth and Ice Thickness

Doronin (1971) used the following relationship to estimate snow depth in terms of ice thickness, which was also used in Yu's paper (1996):

$$\begin{aligned} h_s &= 0, & \text{for } h_i < 5 \text{ cm;} \\ h_s &= 0.05 h_i, & \text{for } 5 \text{ cm} \leq h_i \leq 20 \text{ cm;} \\ h_s &= 0.1 h_i, & \text{for } h_i > 20 \text{ cm.} \end{aligned}$$

In reality, snow accumulation over the ice may not obey the relationship above, and most likely not be that simple. So we set snow depth as one input variable in the OTIM once climate data or measurements are available.

#### 3.4.2.7.2 Relationship between Surface Temperature and Ice Temperature



The ice temperature  $T_i$  is one important factor affecting ice conductivity calculation, which may be significantly different from the surface skin temperature that would be measured or retrieved with remote sensing data. In general, we can obtain surface skin temperature  $T_s$  through satellite retrieval techniques more or less directly, but not  $T_i$  if the surface is covered with thick snow. Yu and Rothrock (1996) suggested that assuming  $T_i$  equal to  $T_s$  can cause 5% and 1% errors when ice is 5 cm thick and 100 cm thick, respectively. That assumption may be valid when it is dark because when it is daytime most of solar radiation is reflected back to the atmosphere from the snow layer; very little of the solar radiation can actually reach the interface of the ice and snow, especially for new snow. Thus the surface skin temperature  $T_s$  is the surface snow temperature, which may differ from the ice temperature significantly. This can result in a large uncertainty in the ice conductivity calculation, resulting in a large error in the calculated ice thickness. This is one of major error sources for the daytime retrieval of ice thickness with the OTIM. More work should be done to correct the solar radiation treatment in the OTIM for sunlit conditions.

### 3.4.2.7.3 Relationship between Sea Ice Thickness and Sea Ice Salinity

There are some experimental relationships between sea ice thickness  $h_i$  and sea ice salinity  $S_i$  as listed below.

1. Cox and Weeks (1974) scheme:

$$\begin{aligned} S_i &= 14.24 - 19.39 h_i, & \text{for } h_i \leq 0.4 \text{ m,} \\ S_i &= 7.88 - 1.59 h_i, & \text{for } h_i > 0.4 \text{ m.} \end{aligned}$$

2. Jin, Stamnes, and Weeks (1994) scheme:

$$\begin{aligned} S_i &= 17.0 - 31.63 h_i, & \text{for } h_i \leq 0.3 \text{ m,} \\ S_i &= 8.0 - 1.63 h_i, & \text{for } h_i > 0.3 \text{ m.} \end{aligned}$$

3. Kovacs (1996) scheme:

$$S_i = 4.606 + 0.91603/h_i, \quad \text{for } 0.10 \text{ m} \leq h_i \leq 2.0 \text{ m.}$$

We modify above schemes from Jin et al (1994) and Kovacs (1996) in the OTIM for the ice thickness greater than 2.0m as listed below:

$$S_i = 2.619 + 1.472/h_i, \quad \text{for } 0.10 \text{ m} \leq h_i \leq 6.0 \text{ m.}$$

### 3.4.2.7.4 Direct Solution from Conductive Heat Flux

If the conductive heat flux  $F_c$  is known with other known parameters like  $k_s$  and  $S$ , the ice thickness can then be retrieved from the equation (27) by simply solving the equation, and the analytical solution can be obtained. Below are the two cases used to solve the equation (27) for ice thickness.

## 1. Fresh Water Ice

For fresh water or lake ice,  $S_w=0$ ,  $S_i=0$ ,  $T_f=273.15K$ ,  $k_i=k_0$ , therefore it is easy to reorganize equation (27) into the following (28).

Let  $T_f - T_s = T_r$ , then we have

$$F_c = \frac{k_0 k_s}{k_s h_i + k_0 h_s} T_r \quad (28)$$

After a series of derivations, we can get the following solution:

$$h_i = \frac{k_0}{F_c} \cdot T_r - \frac{k_0}{k_s} \cdot h_s \quad (29)$$

## 2. Sea Ice

From the above discussion, the ice conductivity can be expressed as

$$k_i = \frac{(k_0 T_i - T_0 k_0 + \beta S_0) h_i + \beta S_1}{(T_i - T_0) h_i} \quad (30)$$

For  $S_i = S_0 + S_1/h_i$ ,  $S_0=2.619$ ,  $S_1=1.472$ , and  $T_0=273.15$ .

Let  $T_i - T_0 = T_k$ , then from the Eq. (27) and (30) we have

$F_c = \frac{k_i k_s}{k_s h_i + k_i h_s} \cdot T_r$ , and let  $a = F_c k_s T_k$ ,  $b = (k_0 T_k + \beta S_0)(F_c h_s - k_s T_r)$ ,  $c = \beta S_1 (F_c h_s - k_s T_r)$ ,  $h = h_i$ , we

have ice thickness monadic quadratic equation as  $ah^2 + bh + c = 0$ , therefore when  $b^2 - 4ac \geq 0$ , two real solutions exist as

$$h_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \text{ and } h_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

When  $b^2 - 4ac = 0$ ,  $h_1 = h_2$ ; and when  $b^2 - 4ac < 0$ , no real solution for ice thickness.

For  $S_i = S_0 + S_1 * h_i$ ,  $S_0=9.0$ ,  $S_1=-1.5$ , and  $T_0=273.15$ .

$$h = \frac{g_2 F_c - K_s g T_r}{K_s K_2 T_r - K_s T_k F_c - K_2 h_s F_c}$$

where  $g = k_0 T_k + \beta S_0$ , and  $g_2 = h_s g$ .

### 3.4.2.8 Solving the OTIM for Ice Thickness

The OTIM can be solved for ice thickness analytically or numerically in terms of input options and variable status as described in the following subsections. First let's rewrite the Eq. (1) into the following form

$$(1 - \alpha)(1 - i)F_r + F_c + F = 0 \quad (31)$$

where  $F = -F_l^{up} + F_l^{dn} + F_s + F_e - F_a$ ,  $\alpha = \alpha_s$ ,

$\alpha = 1 - A_s e^{-B_s h} - C_s e^{-D_s h}$ , where  $h$  is ice or snow thickness, and  $A_s, B_s, C_s, D_s$  are *coefficients* to be determined from Table 2.

$i = A_i e^{-B_i h} + C_i e^{-D_i h}$ , where  $h$  is ice slab thickness,  $i$  is ice slab *transmittance*, and  $A_i, B_i, C_i, D_i$  are *coefficients* to be determined from Table 3.

$$F_c = r(T_f - T_s).$$

For  $S_i = S_0 + S_1 / h$ , we have  $S_i = S_0 + S_1 / h$ , let  $k_0 T_k + \beta S_0 = g$ ,

$$k_i = \frac{(k_0 T_k + \beta S_0)h + \beta S_1}{T_k h} = \frac{gh + \beta S_1}{T_k h}, \quad r = \frac{k_s [(k_0 T_k + \beta S_0)h + \beta S_1]}{k_s T_k h^2 + gh_s h + \beta S_1 h_s} = \frac{k_s (gh + \beta S_1)}{k_s T_k h^2 + gh_s h + \beta S_1 h_s}.$$

$$F_c = \frac{k_s T_r (gh + k_2)}{k_s T_k h^2 + g_2 h + k_2 h_s}, \quad \text{where } k_2 = \beta S_1.$$

For  $S_i = S_0 + S_1 \cdot h$ , we have

$$F_c = \frac{k_s k_2 T_r h + k_s T_r g}{(k_s T_k + k_2 h_s)h + g_2}.$$

#### 3.4.2.8.1 Known Surface Albedo and Known Ice Transmittance

If the values of ice/snow surface albedo  $\alpha$  and the ice slab transmittance  $i$  are both known, let  $(1-\alpha)(1-i)F_r + F = F_i$ , then we have  $F_1 + F_c = 0$ ,

For  $S_i = S_0 + S_1 / h$ , we have  $F_1 + \frac{k_s T_r (gh + \beta S_1)}{k_s T_k h^2 + gh_s h + \beta S_1 h_s} = 0$

$F_1 k_s T_k h^2 + (F_1 h_s g + k_s T_r g)h + F_1 \beta S_1 h_s + k_s \beta S_1 T_r = 0$ , let  $a = F_1 k_s T_k$ ,  $b = F_1 h_s g + k_s T_r g$ , and

$c = F_1 \beta S_1 h_s + k_s \beta S_1 T_r$ , then  $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , when  $b^2 - 4ac \geq 0$ , there are real solutions.

For  $S_i = S_0 + S_1 \cdot h$ , we have  $h = \frac{(h_s F_1 + k_s T_r)g}{k_s T_k F_1 + (h_s F_1 + k_s T_r)k_2}$ .

### 3.4.2.8.2 Known Surface Albedo and Unknown Ice Transmittance

If the value of ice surface albedo  $\alpha$  is known or snow is present over the ice with known depth, but the ice slab transmittance  $i$  is unknown, let  $(1-\alpha)F_r = F_{ra}$ , then  $(1-i)F_{ra} + F_c + F = 0$ .

Let  $F + F_{ra} = F_2$ , then  $F_c - iF_{ra} + F_2 = 0$ ,

For  $S_i = S_0 + S_1 / h$ , we have

$\frac{k_s T_r (gh + \beta S_1)}{k_s T_k h^2 + \beta S_1 h_s + h_s gh} - F_{ra} (A_i e^{-B_i h} + C_i e^{-D_i h}) + F_2 = 0$ , after a series of derivation, we finally have

$F_{ra} (A_i e^{-B_i h} + C_i e^{-D_i h}) (k_s T_k h^2 + g_2 h + k_2 h_s) - [F_2 k_s T_k h^2 + (g k_s T_r + F_2 g_2)h + (k_s T_r + F_2 h_s)k_2] = 0$ ,

where  $k_2 = \beta S_1$ ,  $g_2 = h_s g$ . There is no analytical solution for this nonlinear equation, numerical approach must be applied to solve it for the ice thickness  $h$ .

For  $S_i = S_0 + S_1 \cdot h$ , we have

$\frac{k_s T_r k_2 h + k_s T_r g}{(k_s T_k + k_2 h_s)h + h_s g} - F_{ra} (A_i e^{-B_i h} + C_i e^{-D_i h}) + F_2 = 0$

$F_{ra} (A_i e^{-B_i h} + C_i e^{-D_i h}) [(k_s T_k + k_2 h_s)h + g_2] - [(F_2 k_s T_k + F_2 k_2 h_s + k_2 k_s T_r)h + k_s T_r g + F_2 g_2] = 0$

### 3.4.2.8.3 Unknown Surface Albedo and Known Ice Transmittance

If the value of ice surface albedo  $\alpha$  is unknown, but the ice slab transmittance  $i$  is known, let  $(1-i)F_r = F_{r\tau}$ , then  $(1-\alpha)F_{ri} + F_c + F = 0$ . We know  $\alpha = 1 - (A_s e^{-B_s h} + C_s e^{-D_s h})$ , therefore  $F_{ri}(A_s e^{-B_s h} + C_s e^{-D_s h}) + F_c + F = 0$ .

For  $S_i = S_0 + S_1 / h$ ,  $F_c = \frac{k_s T_r (gh + k_2)}{k_s T_k h^2 + k_2 h_s + g_2 h}$ , we have

$$F_{ri}(A_s e^{-B_s h} + C_s e^{-D_s h})(k_s T_k h^2 + g_2 h + k_2 h_s) + F k_s T_k h^2 + (k_s T_r g + F g_2)h + (k_s T_r + F h_s)k_2 = 0,$$

where  $k_2 = \beta S_1$ ,  $g_2 = h_s g$ . There is no analytical solution for this nonlinear equation, numerical approach must be applied to solve it for the ice thickness  $h$ .

For  $S_i = S_0 + S_1 \cdot h$ , we have

$$F_{ri}(A_s e^{-B_s h} + C_s e^{-D_s h})[(k_s T_k + k_2 h_s)h + g_2] + (F k_s T_k + F k_2 h_s + k_2 k_s T_r)h + k_s T_r g + F g_2 = 0$$

#### 3.4.2.8.4 Unknown Surface Albedo and Unknown Ice Transmittance

If the values of both ice surface albedo  $\alpha$  and ice slab transmittance  $i$  are known, we have  $(1-\alpha)(1-i)F_r + F_c + F = 0$ ,  $\alpha = 1 - (A_s e^{-B_s h} + C_s e^{-D_s h})$ ,  $i = A_i e^{-B_i h} + C_i e^{-D_i h}$ .

For  $S_i = S_0 + S_1 / h$ , we have

$$F_c = \frac{k_s T_r (gh + \beta S_1)}{k_s T_k h^2 + k_2 h_s + g_2 h}, \text{ and } F = -F_i^{up} + F_i^{dn} + F_s + F_e - F_a. \text{ After a series of derivation,}$$

$$F_r(A_s e^{-B_s h} + C_s e^{-D_s h})(1 - A_i e^{-B_i h} - C_i e^{-D_i h})(k_s T_k h^2 + g_2 h + k_2 h_s) + F k_s T_k h^2 + (k_s T_r g + F g_2)h + (k_s T_r + F h_s)k_2 = 0, \text{ where } k_2 = \beta S_1, g_2 = h_s g. \text{ There is no analytical solution for this nonlinear equation, numerical approach must be applied to solve it for the ice thickness } h.$$

For  $S_i = S_0 + S_1 \cdot h$ , we have

$$F_r(A_s e^{-B_s h} + C_s e^{-D_s h})(1 - A_i e^{-B_i h} - C_i e^{-D_i h})[(k_s T_k + k_2 h_s)h + g_2] + (F k_s T_k + F k_2 h_s + k_s k_2 T_r)h + F g_2 + k_s T_r g = 0$$

### 3.4.2.8.5 Nighttime Solution

At night, there is no need to consider solar radiation in the OTIM, so basically we can set  $F_r = 0$ , therefore from  $(1-\alpha)(1-i)F_r + F_c + F = 0$ , we have  $F_c + F = 0$ .

For  $S_i = S_0 + S_1/h$ , we have

$$F_c = \frac{k_s T_r (gh + k_2)}{k_s T_k h^2 + k_2 h_s + g_2 h}, \text{ so } \frac{k_s T_r (gh + \beta S_1)}{k_s T_k h^2 + k_2 h_s + g_2 h} + F = 0. \text{ Finally we have}$$

$$F k_s T_k h^2 + (k_s T_r g + F g_2) h + (k_s T_r + F h_s) k_2 = 0$$

Let  $a = F k_s T_k$ ,  $b = k_s T_r g + F g_2$ ,  $c = (k_s T_r + F h_s) k_2$ , then we have  $ah^2 + bh + c = 0$ , so the solution

for the monadic quadratic equation is  $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , and when  $b^2 - 4ac \geq 0$ , there are

real solutions.

$$\text{For } S_i = S_0 + S_1 \cdot h, \text{ we have } h = -\frac{F g_2 + k_s T_r g}{k_s T_k F + k_2 h_s F + k_s k_2 T_r}.$$

### 3.4.2.9 Physical-Statistical Hybrid Model

The largest uncertainty in the above model for daytime retrieval is the unknown residual heat flux  $F_a$  not being zero, which is large enough to be considered in the OTIM as an important term. Physically,  $F_a$  should be related to the surface and atmospheric conditions such as surface skin temperature, air temperature, air humidity, surface wind speed, and horizontal heat advection within ice and snow. Here we set the  $F_a$  be a function of 17 surface and atmospheric variables that are cosine of Julian day (jcos), cosine and sine of ice melting and freezing days (cosmel, sinmel, cosfre, sinfre), snow depth ( $h_s$ ), surface skin temperature ( $T_s$ ), surface air temperature ( $T_a$ ), surface wind speed ( $U_s$ ), surface air relative humidity ( $h_w$ ), surface air pressure ( $P_r$ ), cloud amount ( $C$ ), surface broadband albedo ( $\alpha$ ), cosine of Solar zenith angle (scos), surface downward shortwave radiative flux ( $F_r$ ), surface downward longwave radiative flux ( $F_l^{\text{dn}}$ ), and surface upward longwave radiative flux ( $F_l^{\text{up}}$ ). For nighttime condition, the  $F_a$  turns to be the function of 14 variables without  $\alpha$ , scos, and  $F_r$ .

A regression equation for the calculation of the residual heat flux was build up by using in-situ measurements of ice thickness from surface, submarine, buoy, mooring, and field campaigns data to calculate  $F_a$  first using this OTIM, and then stepwise regress technique was applied to build up a multi-regress equation for the  $F_a$  predictand calculation with above mentioned variables as predictors. As tested, usually Julian day and ice melting and freezing dates information are not required (not considered in the regression) for the prediction of the  $F_a$ .

### 3.4.2.10 Ice Age

The ABI and JPSS Mission Requirements Documents (MRD) require, at the Threshold level, that ice-free areas be distinguished from first-year ice and older ice areas. The Goal requirement is to distinguish not only ice-free from first-year ice areas, but also to distinguish between the following types of ice: nilas, grey white, first-year medium, first-year thick, second-year, multiyear smooth, and multiyear deformed, commonly called ice age. Generally speaking, older ice is thicker than younger ice. In essence, this assumption is valid as tested and verified by many other researchers (e.g., Tucker et al., 2001; Yu et al., 2004; Maslanik et al., 2007). So ice thickness is used as a proxy for ice age.

There is an internationally accepted terminology for ice form and conditions, coordinated by the WMO. This terminology is used by the Canadian Ice Service as the basis for reporting ice conditions, and adopted by this work, with minor modifications, for classifying ice into different categories. Refer to the *Manual of Standard Procedures for Observing and Reporting Ice Conditions* by the Canadian Ice Service, available at <http://ice-glaces.ec.gc.ca/App/WsvPageDsp.cfm?Lang=eng&lnid=23&ScndLvl=no&ID=172>.

#### Sea-ice types

- **New:** A general term for recently formed ice which includes frazil ice, grease ice, slush and shuga. These types of ice are composed of ice crystals which are only weakly frozen together (if at all) and have a definite form only while they are afloat.
- **Nilas:** A thin elastic crust of ice, easily bending on waves and swell and under pressure growing in a pattern of interlocking “fingers” (finger rafting). Nilas has a matte surface and is *up to 10 cm* in thickness and may be subdivided into dark nilas and light nilas.
- **Grey Ice:** Young ice *10-15 cm* thick. Less elastic than nilas and breaks on swell. Usually rafts under pressure.
- **Grey-white Ice:** Young ice *15-30 cm* thick. Under pressure it is more likely to ridge than to raft.
- **Thin First-year Ice:** First-year ice of not more than one winter's growth, *30-70 cm* thick.
- **Medium First-year Ice:** First-year, ice *70-120 cm* thick.
- **Thick First-year Ice:** First-year ice *120-170 cm* thick.
- **Old Ice:** Sea ice which has survived at least one summer's melt. Topographic features generally are smoother than first-year ice, and *more than 170 cm* thick. May be subdivided into second-year ice and multi-year ice.
- **Second-year Ice:** Old ice which has survived only one summer's melt.
- **Multi-year Ice:** Old ice which has survived at least two summer's melt.

#### Lake-ice types

- **New:** Recently formed ice *less than 5 cm* thick.
- **Thin:** Ice of varying colors, *5-15 cm* thick.
- **Medium:** A further development of floes or fast ice, *15-30 cm* thick.
- **Thick:** Ice *30-70 cm* thick.
- **Very Thick:** Floes or fast ice developed to *more than 70 cm* thickness.

### 3.4.3 Algorithm Output

The final outputs of this algorithm are:

- Ice thickness
- Ice age
- Optional geophysical variables
- Quality control (QC) flags
- Product quality information (PQI)
- Metadata

These are described in Tables 4-8. The ice thickness values are in the range 0 ~ 3.0 m for both sea ice and lake ice. Ice age categories and descriptions of their meanings are given in Table 4.

**Table 4.** AVITA output parameters and their definitions.

<b>Definition</b>	<b>Description</b>
<b>Ice Thickness</b>	Ice thickness is defined as the total vertical length of the ice under and above water surface. The reliable ice thickness retrieved from this algorithm ranges between <i>0 ~ 3.0 m</i> .
<b>Ice age</b>	
1: New	Recently formed ice which includes frazil ice, grease ice, slush and shuga. These types of ice are composed of ice crystals which are only weakly frozen together (if at all) and have a definite form only while they are afloat, usually less than 2 cm.
2: Nilas	A thin elastic crust of ice, easily bending on waves and swell and under pressure growing in a pattern of interlocking “fingers” (finger rafting). Nilas has a matte surface and is <i>up to 10 cm</i> in thickness and may be subdivided into dark nilas and light nilas.
3: Grey	Young ice <i>10-15 cm</i> thick. Less elastic than nilas and breaks on swell. Usually rafts under pressure.
4: Grey-white	Young ice <i>15-30 cm</i> thick. Under pressure it is more likely to ridge than to raft.
5: First-year Thin	First-year ice of not more than one winter's growth, <i>30-70 cm</i> thick.
6: First-year Medium	First-year, ice <i>70-120 cm</i> thick.
7: First-year Thick	First-year ice <i>120-180 cm</i> thick.



8: Older Ice	<p>Sea ice which has survived at least one summer's melt. Topographic features generally are smoother than first-year ice, and <i>more than 180 cm</i> thick. May be subdivided into second-year ice and multi-year ice.</p> <p><b>Second-year Ice:</b> Old ice which has survived only one summer's melt.</p> <p><b>Multi-year Ice:</b> Old ice which has survived at least two summer's melt.</p>
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The algorithm can also output other optional parameters that are closely related to the surface energy budget and ice thickness retrieval. These optional outputs from OTIM are listed and described in Table 5. These optional output parameters will be implemented in the next version of the algorithm.

**Table 5.** AVITA optional output parameters and their definitions.

Parameter	Unit	Description
Cloud mask	0 or 1	Clear or Cloudy over the ice surface, observed.
Surface broadband albedo	0 ~ 1	Ice/snow surface broadband albedo, modeled or in-situ measured, daytime only.
Ice Transmittance	0 ~ 1	Ice slab transmittance for solar radiation, modeled or measured, daytime only.
Surface incoming solar radiation flux	$W \cdot m^{-2}$	Incoming solar radiation flux at the surface, modeled or observed, daytime only.
Surface outgoing thermal radiation flux	$W \cdot m^{-2}$	Outgoing thermal radiation flux at the surface, modeled or observed.
Surface incoming thermal radiation flux	$W \cdot m^{-2}$	Incoming thermal radiation flux at the surface, modeled or observed.
Surface turbulent sensible heat flux	$W \cdot m^{-2}$	The turbulent sensible heat flux at the interface of ice and the above atmosphere modeled or observed.
Surface turbulent latent heat flux	$W \cdot m^{-2}$	The turbulent latent heat flux at the interface of ice and the above atmosphere modeled or observed.
Conductive heat flux	$W \cdot m^{-2}$	Conductive heat flux within the ice slab.
<i>Other potential optional output parameters:</i>		
Surface skin temperature	K	Ice/snow surface skin temperature, observed.
Surface air temperature	K	Surface air temperature at 2 m above the ground, modeled or observed.
Surface air humidity	0%~100%	Surface air humidity, relative or mixing ratio, modeled or observed.
Surface wind	$m \cdot s^{-1}$	Surface wind speed at 2 m above the ground, observed.
Sea water salinity	PPT	Sea water salinity, modeled or observed.
Sea ice salinity	PPT	Sea ice salinity, modeled or observed.

Snow depth	m	Snow accumulation over the ice in meter, modeled or observed.
Water freezing point	K	The temperature at which water freezes, modeled or observed.
Snow conductivity	$W \cdot m^{-1} \cdot K^{-1}$	Snow conductivity, modeled or observed.
Ice conductivity	$W \cdot m^{-1} \cdot K^{-1}$	Ice conductivity, modeled or observed.

Ice thickness and age quality control flags and product quality information are given in Tables 6 and 7.

**Table 6.** AVITA Quality Control (QC, 2 bytes).

Byte	Bit	Quality Flag Name	Description	Meaning
0	0	QC_OUTPUT	Output product quality	00 - good/optimal retrieval
	1			01 - uncertain/suboptimal retrieval
	2		10 - bad/missing retrieval	
	3		11 - non-retrieval	
	4			
	5			
	6			
	7			

**Table 7.** AVITA Products Quality Information (PQI, 4 bytes)\*.

Byte	Bit	Quality Flag Name	Description	Meaning
0	0	QC_INPUT_CLD	Cloud mask	00 - clear
	1			01 - probably clear
	2	QC_INPUT_DAY	Day/Night	0-Day 1-Night
	3	QC_INPUT_SUNGLINT	Sunglint or not	0-Yes 1-No
	4	QC_INPUT_CLDSHADOW	Cloud shadow or not	0-Yes 1-No
	5	QC_INPUT_ICEIDEN	Ice identification	0-Yes 1-No
	6	QC_INPUT_ICECONC	Ice concentration	0-Yes 1-No
	7	QC_INPUT_ICETRAN	Ice transmittance	0-Yes 1-No
1	0	QC_INPUT_SOLZEN	Valid solar zenith angle	0-Yes 1-No
	1	QC_INPUT_SATZEN	Valid satellite zenith angle	0-Yes 1-No
	2	QC_INPUT_ALBEDO	Surface broadband albedo	0-Yes 1-No
	3	QC_INPUT_TSURF	Surface skin temperature	0-Yes 1-No
	4	QC_INPUT_SNOW	Surface snow depth	0-Yes 1-No

	5	QC_INPUT_WIND	Surface wind speed	0-Yes 1-No
	6	QC_INPUT_SURFACE	Surface background flag	00 - in-land water 01 - sea water 10 - land 11 - others
	7			
2	0	QC_INPUT_TAIR	Surface air temperature	0-Yes 1-No
	1	QC_INPUT_PRESSURE	Surface air pressure	0-Yes 1-No
	2	QC_INPUT_HUMIDITY	Surface air relative humidity	0-Yes 1-No
	3	QC_INPUT_SSWD	Surface shortwave downward radiative flux	0-Yes 1-No
	4	QC_INPUT_SLWD	Surface longwave downward radiative flux	0-Yes 1-No
	5	QC_INPUT_SLWU	Surface longwave upward radiative flux	0-Yes 1-No
	6	QC_INPUT_SSHF	Surface turbulent sensible heat flux	0-Yes 1-No
	7	QC_INPUT_SLHF	Surface turbulent latent heat flux	0-Yes 1-No
3	0	QC_INPUT_SCHF	Surface conductive heat flux	0-Yes 1-No
	1	QC_INPUT_SRHF	Surface residual heat flux	0-Yes 1-No
	2	QC_RET_ALGO	Day/Night algorithm selection	0-Day 1-Night
	3	QC_RET_METH	Math method for solution	0-Analytical 1-Numerical
	4	QC_RET_RESU	Retrieval success or fail	0-Success 1-Fail
	5			
	6			
	7			

\* The "Yes/No" flag indicates whether or not that input parameter is available. Some input parameters must be given in order to do ice thickness/age retrieval; these critical input parameters are date and time, cloud mask, land mask, solar zenith angle, surface skin temperature, ice mask and concentration. Other input parameters can be missing or unavailable for input, so-called optional able-to-missing parameters, that will result in the algorithm default values of those parameters being used or calculated by OTIM built-in parameterization schemes, those parameters include surface air temperature, humidity, pressure, and wind speed, surface broadband albedo, ice slab transmittance, cloud fraction, ice and snow thermal and optical physical properties, snow mask and depth, day/night indicator, shortwave and thermal radiation fluxes, and turbulent sensible and latent heat fluxes. Cloud shadow mask, sunglint mask, and satellite view angle are currently not required by the algorithm.

Metadata are listed in Table 8.

**Table 8.** Metadata included with the product.

<b>Variable Name</b>	<b>Definition</b>
Tot_QACat01	Number of QA flag values with Good or Optimal retrievals
Tot_QACat02	Number of QA flag values with Uncertain or Suboptimal retrievals
Tot_QACat03	Number of QA flag values with Bad or missing retrievals
Tot_QACat04	Number of QA flag values with Non-retrievable retrievals
TotWaterPixs	Number of pixels with water surface
TotRetrPixs	Number of valid ice thickness and age retrievals (good + uncertain)
TermntPixPct	Percentage of terminator pixels (Non-retrievable and Bad)
TotDaytimePixs	Number of valid daytime ice thickness and age valid retrievals
TotNighttimePixs	Number of valid nighttime ice thickness and age valid retrievals
MeanIceThk	Mean valid ice thickness retrievals
MaxIceThk	Maximum of valid ice thickness retrievals
MinIceThk	Minimum of valid ice thickness retrievals
STDIceThk	Standard Deviation of valid ice thickness retrievals

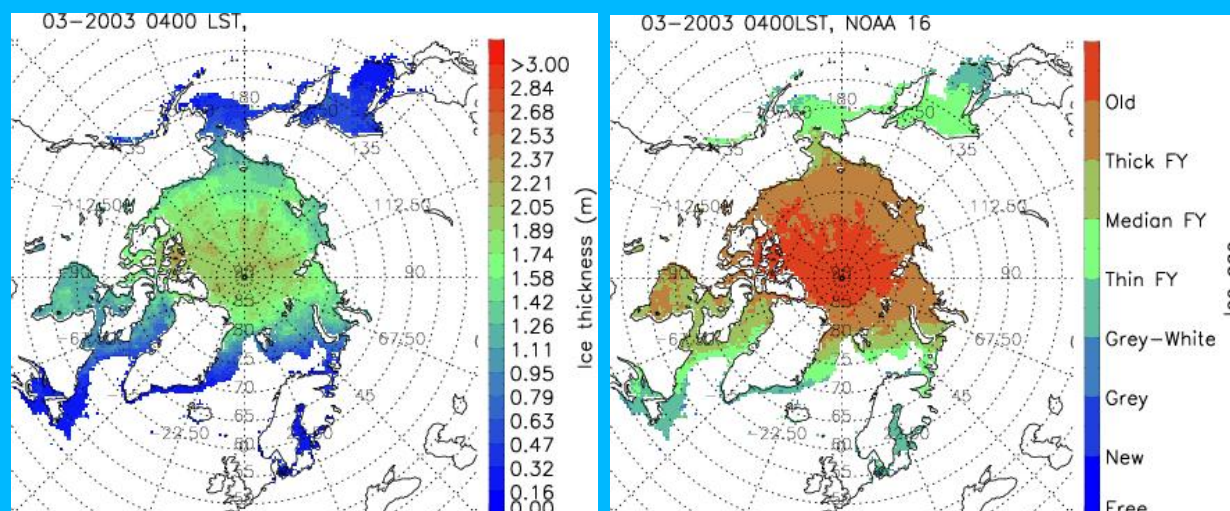
## 4 TEST DATA SETS AND VALIDATEION

### 4.1 Simulated/Proxy Input Data Sets

The simulated/proxy input data sets used to test the AVITA included APP-x, MODIS, SEVIRI observations as detailed in the following subsections.

#### 4.1.1 APP-x Data

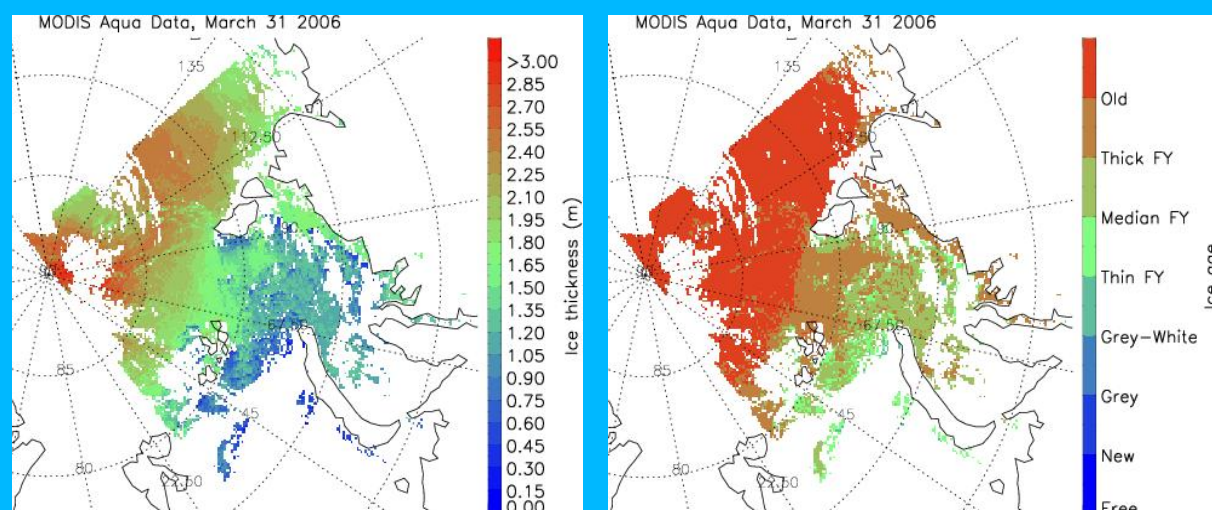
The Advanced Very High Resolution Radiometer (AVHRR) Polar Pathfinder (APP) project (Fowler, et al, 2002) recently produced 23 years of twice-daily, bi-polar surface temperature, surface albedo, and cloud information products. The APP data is being extended to include cloud properties and surface radiative fluxes (Wang and Key, 2003). The extended AVHRR Polar Pathfinder data set, called APP-x data set, covers entire Arctic and Antarctica area and spans 1982-2004 at a spatial resolution of 25 km. The specifically interested data in this work are its cloud information, surface skin temperature, surface broadband albedo, and surface radiation fluxes retrieved from satellite observations as inputs to the OTIM for estimating ice thickness and age along with other ancillary profile data and wind data from NCAR/NCEP. Figure 2 is an example of AVITA retrieved monthly ice thickness and ice age with APP-x data.



**Figure 2.** AVITA retrieved monthly mean ice thickness (left) and ice age (right) with APP-x data for March 2003 under all sky condition.

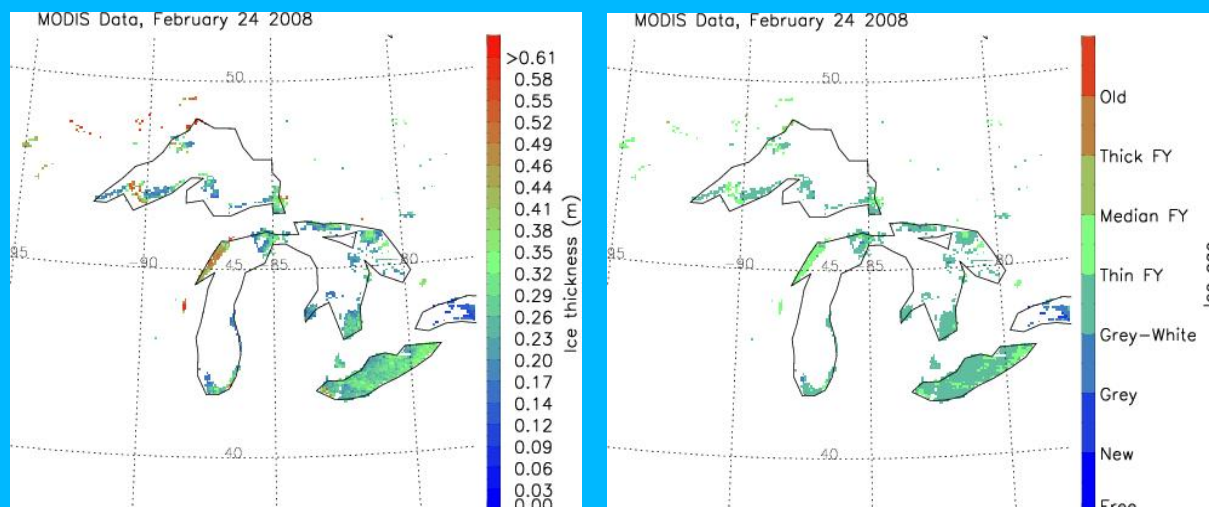
#### 4.1.2 MODIS Data

MODIS (Moderate Resolution Imaging Spectroradiometer) is a key instrument aboard the Terra (*EOS AM*, refer to <http://terra.nasa.gov/>) and Aqua (*EOS PM*, refer to <http://aqua.nasa.gov/>) satellites. Terra's orbit around the Earth is timed so that it passes from north to south across the equator in the morning, while Aqua passes south to north over the equator in the afternoon. The MODIS instrument has a viewing swath width of 2,330 km and views the entire surface of the Earth every one to two days. Its detectors measure 36 spectral bands between 0.405 and 14.385  $\mu\text{m}$ , and it acquires data at three spatial resolutions that are 250m, 500m, and 1,000m. Many data products derived from MODIS observations describe features of the land, oceans and the atmosphere that can be used for studies of processes and trends on local to global scales to improve our understanding of global dynamics and processes occurring on the land, in the oceans, and in the lower atmosphere. MODIS is playing a vital role in the development of validated, global, interactive Earth system models able to predict global change accurately enough to assist policy makers in making sound decisions concerning the protection of our environment. Figure 3 and 4 show the two cases of AVITA retrieved daily ice thickness and ice age with MODIS data.



**Figure 3.** AVITA retrieved ice thickness (left) and ice age (right) with MODIS Aqua data on March 31, 2006 under clear sky condition.

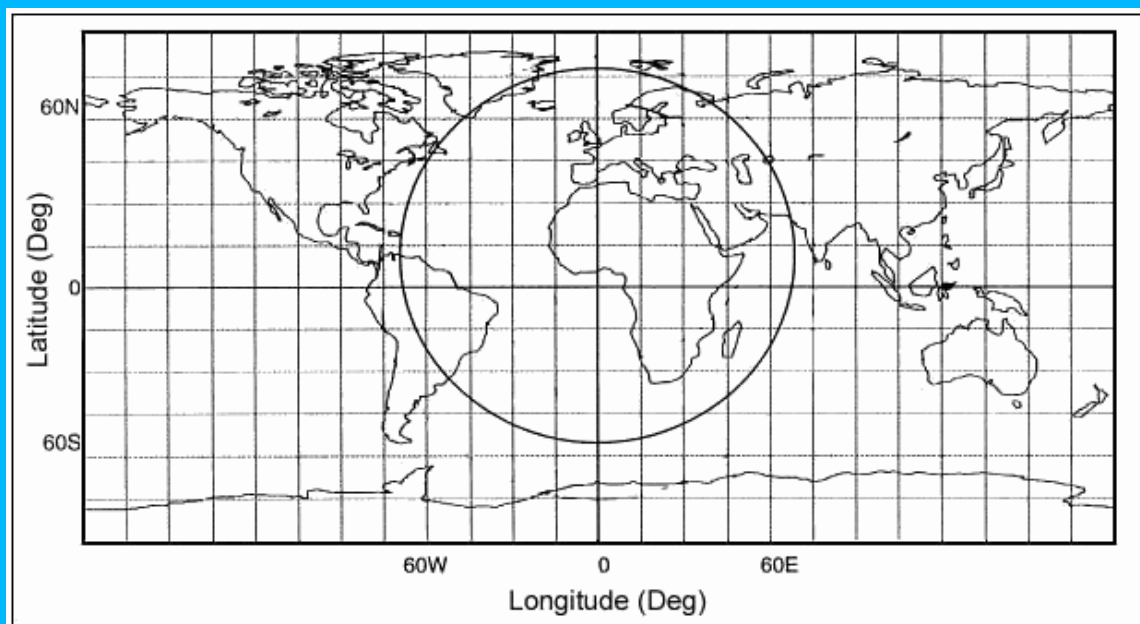




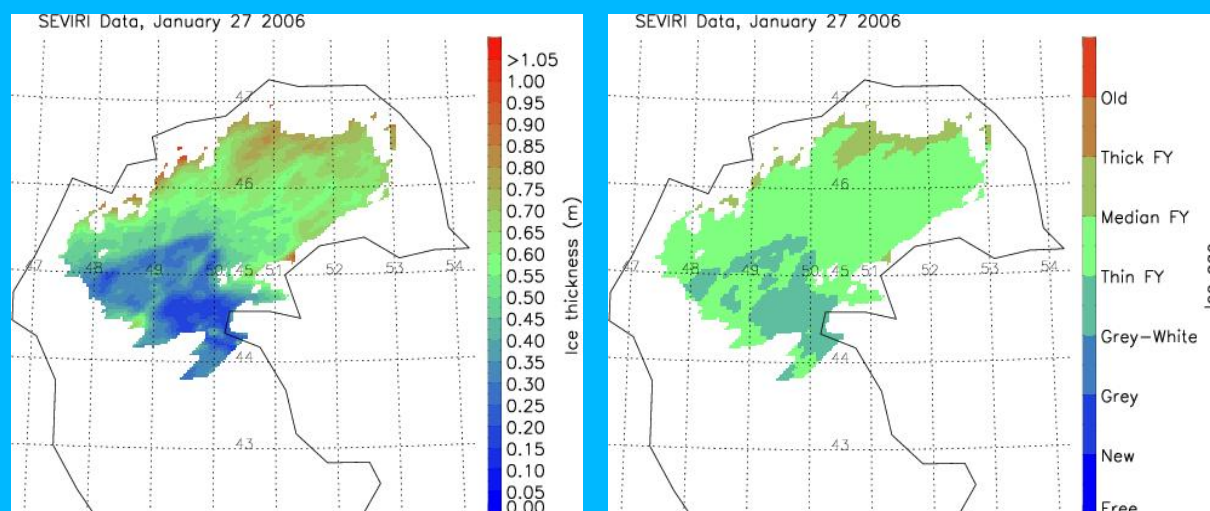
**Figure 4.** AVITA retrieved ice thickness (left) and ice age (right) with MODIS Aqua data on February 24, 2008 under clear sky condition.

### 4.1.3 SEVIRI Data

SEVIRI (Spinning Enhanced Visible & InfraRed Imager) is the primary payload of the MSG (Meteosat Second Generation) satellites which form a joint project between the European Space Agency and Eumetsat, the European Organisation for the Exploitation of Meteorological Satellites since 1977 (*refer to [http://www.eumetsat.int/home/Main/Access\\_to\\_Data/Meteosat\\_Image\\_Services/SP\\_1123237865326](http://www.eumetsat.int/home/Main/Access_to_Data/Meteosat_Image_Services/SP_1123237865326)*). SEVIRI measures reflected and emitted radiance in 11 spectral channels located between 0.6  $\mu\text{m}$  and 14  $\mu\text{m}$  with a nominal spatial resolution of 3 km at the sub-satellite point along with an additional broadband high-resolution visible (HRV, 0.4-1.1  $\mu\text{m}$ ) channel that has a 1 km spatial resolution. The full disc view allows frequent sampling, every 15 minutes, enabling monitoring of rapidly evolving events. The nominal coverage includes the whole of Europe, all of Africa and locations at which the elevation to the satellite is greater than or equal to  $10^\circ$  (Figure 5). Figure 6 shows one case of AVITA retrieved daily ice thickness and ice age with SEVIRI data.



**Figure 5.** MSG Telecommunications coverage area.



**Figure 6.** AVITA retrieved ice thickness (left) and ice age (right) with SEVIRI data on January 27, 2006 under clear sky condition.

#### 4.2 Output from Simulated Input Data Sets

The output results from proxy data sets with AVITA algorithm are given in the Figure 2, 3, 4, and 6 in the previous section.

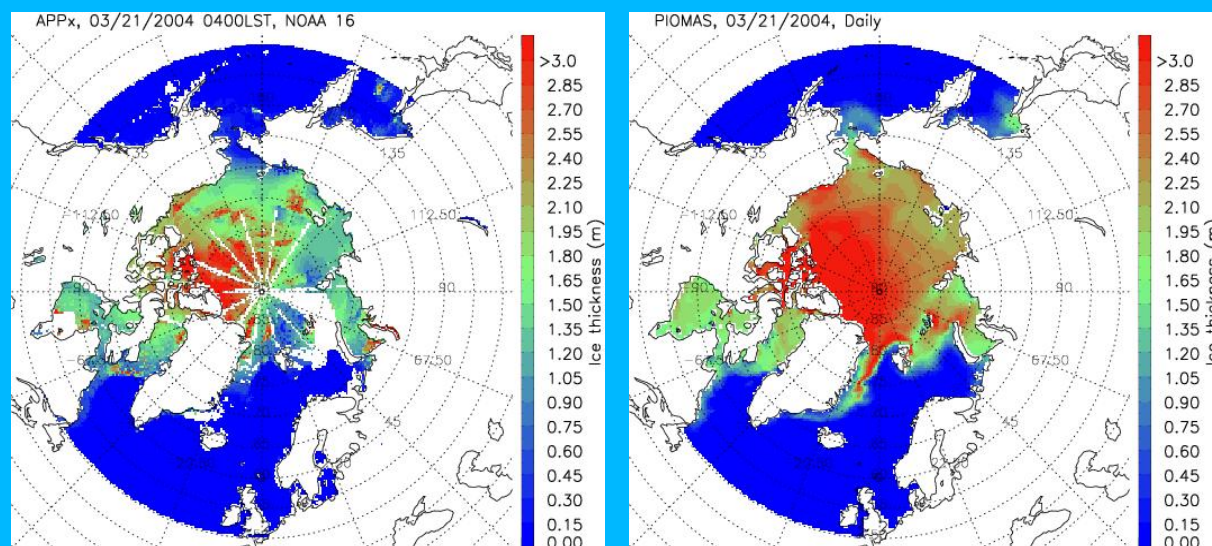


## 4.2.1 Precisions and Accuracy Estimates

To estimate the performance of the AVITA, we have used the comprehensive numerical model simulations, submarine and moored Upward Looking Sonar (ULP) measurements, and meteorological station measurements to assess and validate the AVITA. This section will present our analysis methodology for estimating the precision and accuracy. The next section will provide the quantitative results in terms of the MRD specifications.

### 4.2.1.1 Numerical Model Simulation Analysis

A Pan-Arctic Ice-Ocean Modeling and Assimilation System (PIOMAS) was used for this project for validation purpose. PIOMAS is a coupled Parallel Ocean and sea Ice Model (POIM, Zhang and Rothrock 2003) with capabilities of assimilating ice concentration and velocity data. It is formulated in a generalized orthogonal curvilinear coordinate (GOCC) system and designed to run on computers with a single processor or massively parallel processors. PIOMAS couples the Parallel Ocean Program (POP) with a thickness and enthalpy distribution (TED) sea-ice model. The POP model is developed at the Los Alamos National Laboratory. And the TED sea-ice model is a dynamic thermodynamic model that also explicitly simulates sea-ice ridging. The model originates from the Thorndike et al. (1975) thickness distribution theory and is recently enriched by enthalpy distribution theory (Zhang and Rothrock, 2001).



**Figure 7.** AVITA retrieved ice thickness (left) with APP-x data and PIOMAS simulated ice thickness (right) for March 21, 2004 under all sky condition.

The PIOMAS data sets from the retrospective investigation include model output for 1978-2005. The outputs from the Pan-Arctic Ice-Ocean Modeling and Assimilation System (PIOMAS) provide estimates of some key ice and ocean variables. The data sets only include results for the period of 1978-2005 when satellite ice concentration data are available for assimilation. These data sets include Arctic sea ice thickness and concentration, snow depth, ice growth rate, ocean

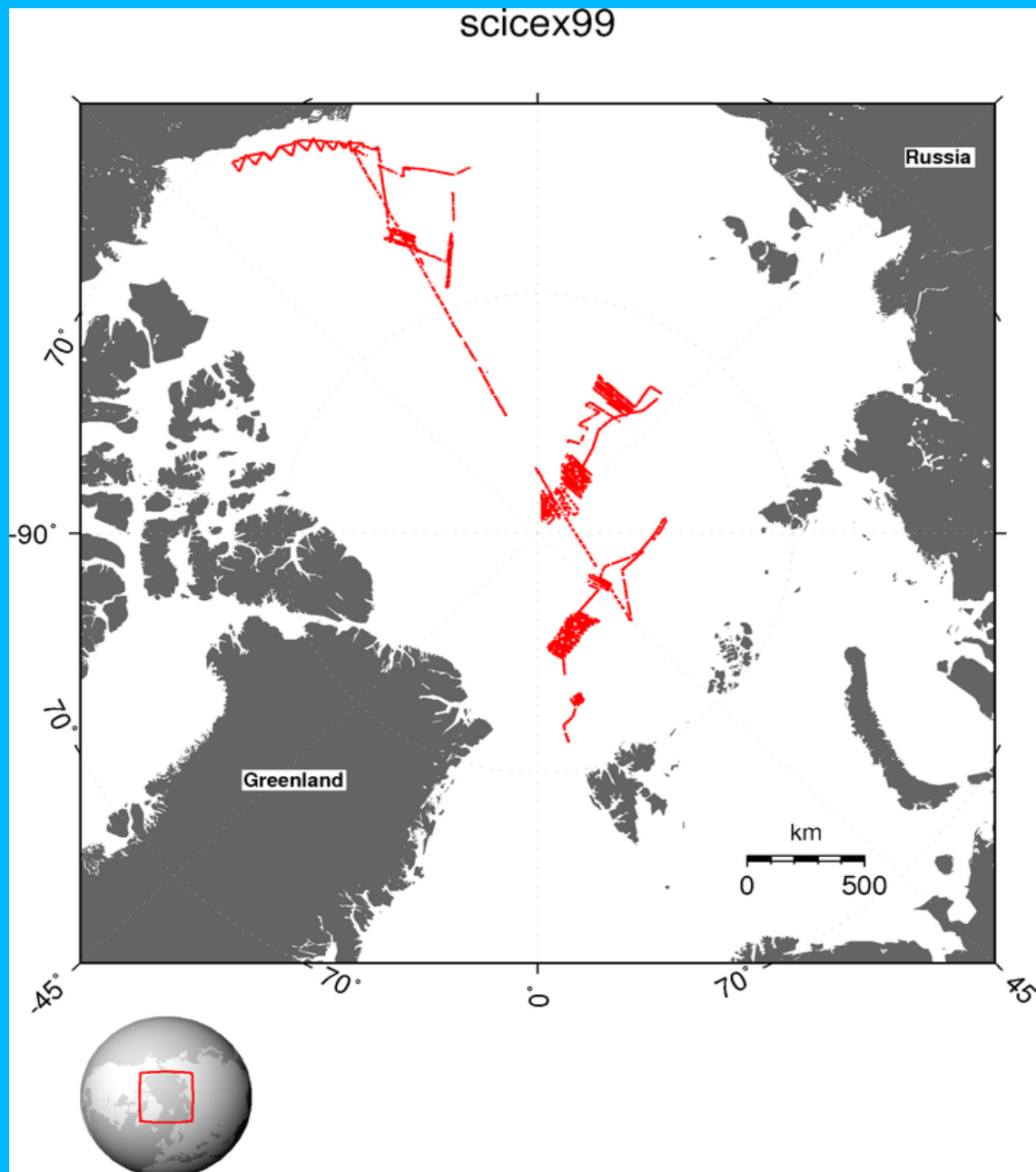
surface salinity, and others. Of special interest of this work is the PIOMAS estimated sea ice thickness. Figure 7 shows the AVITA estimated Arctic sea ice thickness with APP-x data and the PIOMAS estimation on the same day March 21 2004. Overall AVITA estimated sea ice is thinner than PIOMAS, the inconsistency or mismatch will be investigated in the next section with submarine cruise measurements and in-situ station measurements.

#### ***4.2.1.2 Submarine Cruise Measurement Analysis***

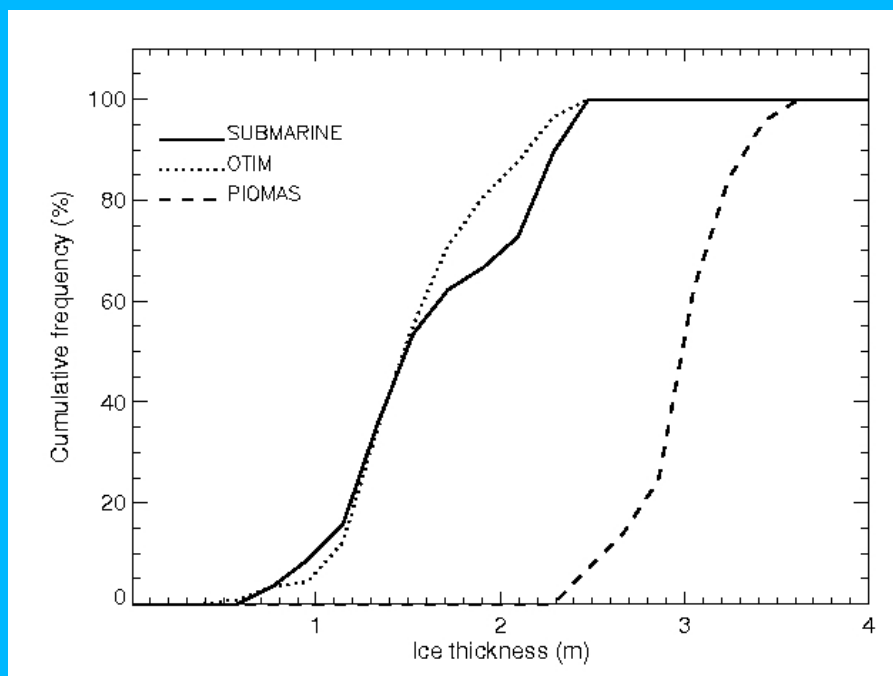
National Snow and Ice Data Center (NSIDC) archives data come from Submarine Upward Looking Sonar Ice Draft Profile Data and Statistics over past decades. This data set consists of upward looking sonar draft data collected by submarines in the Arctic Ocean. It includes data from both U.S. Navy and Royal Navy submarines. Data are provided as ice draft profiles and as statistics derived from the profile data. Statistics files include information concerning ice draft characteristics, keels, level ice, leads, undeformed and deformed ice (refer to <http://nsidc.org/data/g01360.html>). This data set includes submarine data collected in the Arctic Ocean by U.S. Navy and Royal Navy submarines. U.S. Navy guidance has stated that previously classified, submarine-collected ice draft data may be declassified and released according to set guidelines. Those guidelines include restrictions stating that positions of the data must be rounded to the nearest 5 minutes of latitude and longitude, and date is to be rounded to the nearest third of a month. Due to the limitations enforced by those guidelines for temporal and spatial information, not all of the data are suitable to be used by scientific study.

A program called Scientific Ice Expeditions (SCICEX) used U.S. Navy submarine for research. SCICEX data are not classified and do not have restrictions on reporting the precise location and date for the data; therefore the SCICEX ice draft data in this collection are reported with their date of acquisition, and position is reported to six decimal places, which make the SCICEX data suitable for scientific study (Figure 8). We used SCICEX 1999 ice draft data (hereafter SCICEX-99) in this work because NSIDC obtained permission to release some SCICEX-99 data acquired outside the previously mentioned release box, meaning larger area coverage for the data set. Data are in two types of files, one for ice draft profiles, and the other for statistics derived from the profile data. Ice draft files include a header that gives date and location information followed by a sequential list of drafts spaced at 1.0 m intervals that comprise the bottom-side sea-ice roughness profile. Data in each file fall along a straight-line (great circle) track between the two end points given in the header. The length of the profile in any given file can be up to 50 km, but may be shorter if data dropouts create gaps greater than 0.25 km, or if changes in course cause deviations from a straight-line track. Statistics files include information on ice draft characteristics, keels, level ice, leads, un-deformed, and deformed ice.

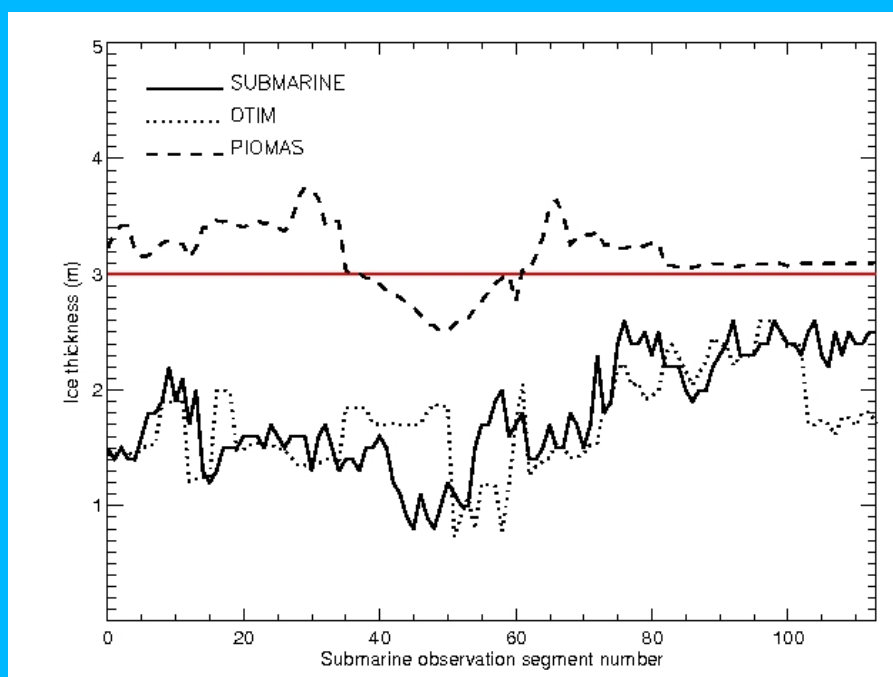
Figure 9 and 10 show the comparisons of the three data sets from APP-x, submarine, and PIOMAS in ice thickness. Results are given in Table 9. Note the submarine actually measures ice draft (ice below the surface), which is roughly 89% of the total ice thickness. Draft can be approximately converted to thickness with an empirical multiplicative factor of 1.1.



**Figure 8.** U.S. Navy submarine track for SCICEX ice draft data collection during April 2 – May 13 in 1999.



**Figure 9.** Comparisons of ice thickness cumulative distribution retrieved by OTIM with APP-x data, measured by submarine, and simulated by numerical model PIOMAS. Submarine ice draft (mean and median only) was already converted to ice thickness by a factor of 1.11.



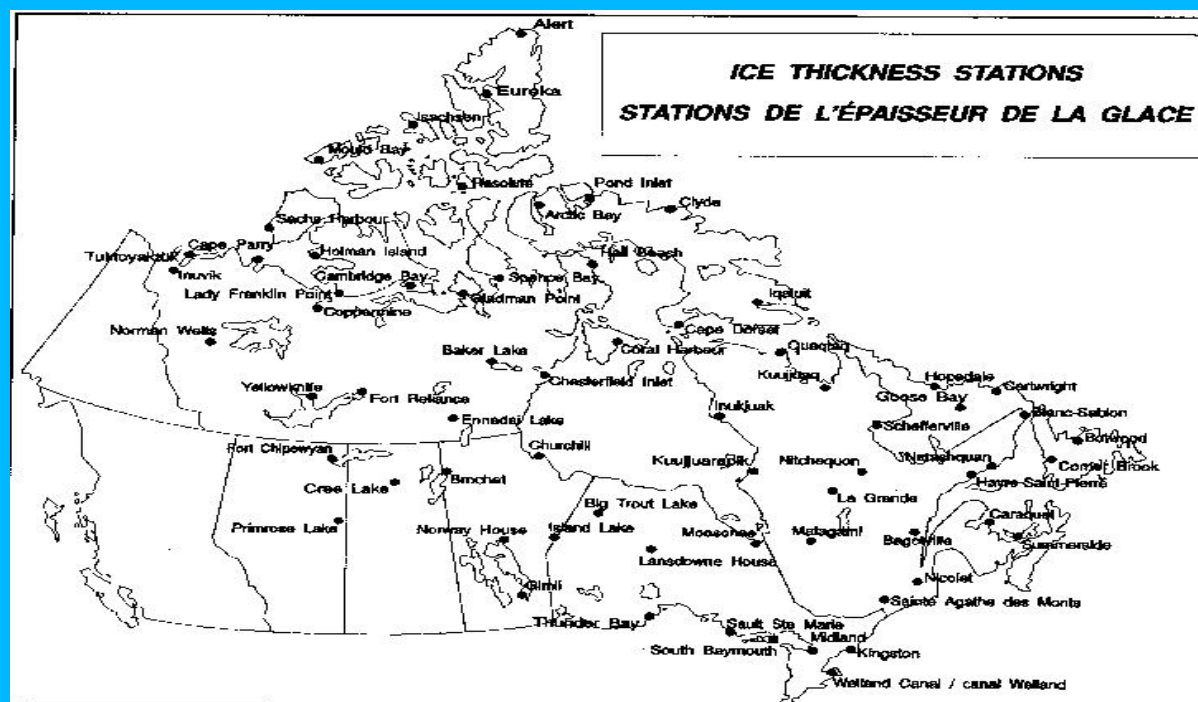
**Figure 10.** Comparisons of ice thickness values retrieved by OTIM with APP-x data, measured by submarine, and simulated by numerical model PIOMAS along the submarine track segments. Submarine ice draft (mean and median only) was already converted to ice thickness by a factor of 1.11.

**Table 9.** The AVITA validation result against submarine measurements.

	<b>AVITA</b>	<b>Submarine</b>
<b>Thickness Mean (m)</b>	1.73	1.80
<b>Bias Mean (m)</b>	-0.07	
<b>Bias Absolute Mean (m)</b>	0.31	
<b>Bias Standard Deviation</b>	0.42	
<b>Accuracy*</b>	83%	
<b>AVITA Ice Age</b>	Ice free water, new/fresh, nilas, grey, grey-white, first year thin, first year medium, first year thick, and multi-year ice.	
<b>ADR Requirements</b>	Distinguish between ice free areas, first-year ice, and older ice.	
<b>CDR Requirements</b>	Distinguish between Ice free, new/fresh ice, nilas, grey white, first Year medium, first Year Thick, second year, and multiyear smooth and deformed ice.	
* $Accuracy = (1.0 - (Bias\ Absolute\ Mean) / (Submarine\ Mean\ Ice\ thickness)) * \%$		

#### 4.2.1.3 Station Measurement Analysis

The Canadian Ice Service (CIS) maintains archived Ice Thickness and On-Ice Snow Depth Measurements for Canadian Stations (Figure 11) back as far as 1947 for the first established stations in the Canadian Arctic (Eureka and Resolute). By the beginning of 2002 most stations from the original Ice Thickness program had stopped taking measurements. Fortunately, due to an increasing interest in updating this historical dataset to support climate change studies a new program was started in the fall of 2002, called New Arctic Program starting from 2002 (refer to <http://ice-glaces.ec.gc.ca/App/WsvPageDsp.cfm?Lang=eng&lnid=5&ScndLvl=no&ID=11703>). Several stations in the Canadian Arctic were re-opened and started taking measurements. These New Arctic Program stations are listed in Table 10. The New Arctic Program Data will be used in this work.



**Figure 11.** Spatial deployment of the Canadian stations for ice thickness and on-ice snow depth measurements.

**Table 10.** Geographic Information of the New Arctic Program Stations (Starting Fall 2002) for Ice Thickness and On-Ice Snow Depth Measurements.

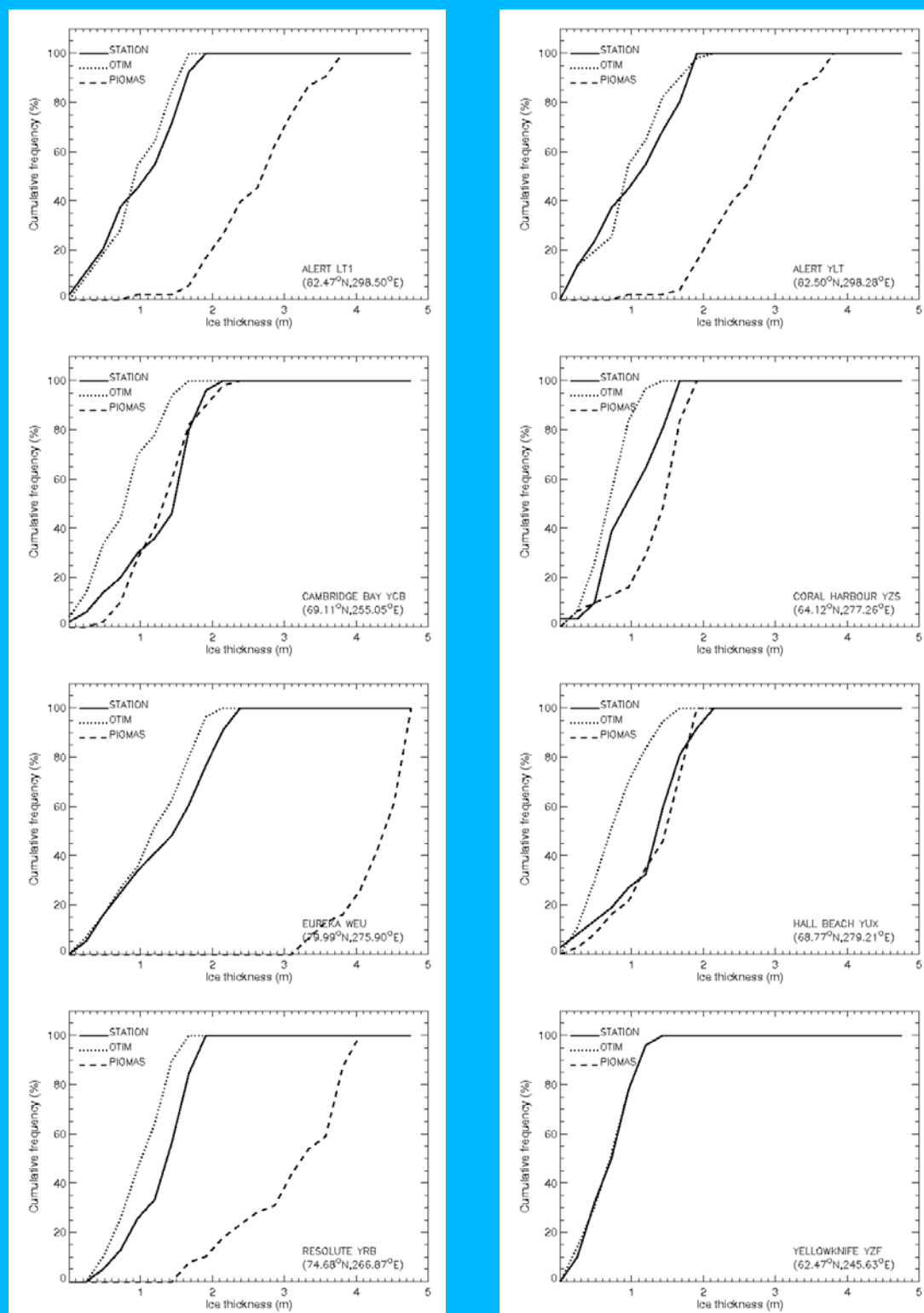
Station ID	Station Name	Start Date	LAT	LON
LT1	ALERT LT1	10/16/2002	82.466667	-61.5
YLT	ALERT YLT	10/16/2002	82.500275	-61.716667
YBK	BAKER LAKE YBK	11/27/2002	64.316666	-95.966667
YCB	CAMBRIDGE BAY YCB	12/07/2002	69.10833	-104.95
YZS	CORAL HARBOUR YZS	11/15/2002	64.119446	-82.741669
WEU	EUREKA WEU	10/11/2002	79.986115	-84.099998
YUX	HALL BEACH YUX	11/10/2002	68.765274	-80.791664
YEV	INUVIK YEV	11/29/2002	68.35833	-132.26138
YFB	IQUALUIT YFB	01/04/2003	63.727779	-67.48333
YRB	RESOLUTE YRB	12/13/2002	74.676941	-93.131668
YZF	YELLOWKNIFE YZF	11/29/2002	62.465556	-114.36556

Most of the data in the current archive at the Canadian Ice Service have been collected by the Atmospheric Environment Program of Environment Canada, but some data are provided by other organizations such as the St-Lawrence Seaway Authority, Trent University and Queen's University. Measurements are taken approximately at the same location every year on a weekly basis starting after freeze-up when the ice is safe to walk on, and continuing until break-up or when the ice becomes unsafe. The location is selected close to shore, but over a depth of water which will exceed the maximum ice thickness. Ice thickness is measured to the nearest centimeter using either a special auger kit or a hot wire ice thickness gauge. The depth of snow on the ice at the location of ice thickness measurement is also measured and reported to the nearest centimeter. Measurements after 1982 include additional information (coded values as per code for additional information at bottom) such as character of ice surface, water features and method of observation. Figures 12 and 13 show the comparisons of the three data sets for ice thickness in cumulative frequency and in absolute magnitude, respectively. Results are given in Table 11.

**Table 11.** The AVITA validation result against in-situ station measurements.

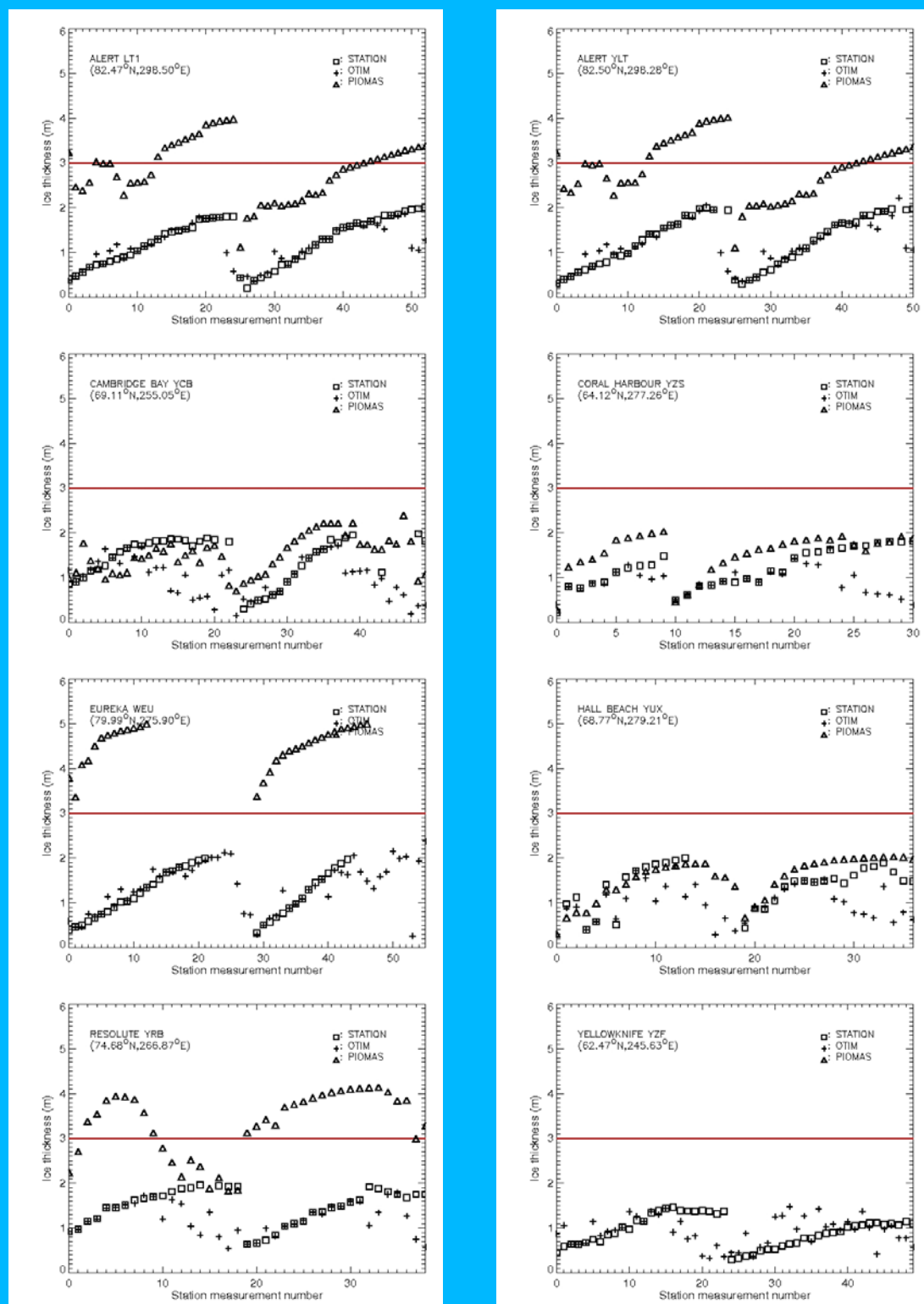
AVITA Station	AVITA ALERT LT1	AVITA ALERT YLT	AVITA CAMBRI DGE BAY YCB	AVITA CORAL HARBOUR R YZS	AVITA EUREKA WEU	AVITA HALL BEACH YUX	AVITA RESOLUTE YRB	AVITA YELLOW KNIFE YZF
<b>Thickness Mean (m)</b>	1.17 1.23	1.21 1.26	1.48 1.51	1.17 1.20	1.36 1.54	1.37 1.46	1.21 1.50	0.91 0.93
<b>Bias Mean (m)</b>	-0.06	-0.06	-0.04	-0.03	-0.18	-0.07	-0.29	-0.01
<b>Bias absolute Mean (m)</b>	0.14	0.16	0.58	0.32	0.27	0.32	0.31	0.31
<b>AVITA Ice Age</b>	Ice free water, new/fresh, nilas, grey, grey-white, first year thin, first year medium, first year thick, and multi-year ice.							
<b>ADR Requirements</b>	Distinguish between ice free areas, first-year ice, and older ice.							
<b>CDR Requirements</b>	Distinguish between Ice free, new/fresh ice, nilas, grey white, first Year medium, first Year Thick, second year, and multiyear smooth and deformed ice.							





**Figure 12.** Comparisons of ice thickness cumulative distribution retrieved by OTIM with APP-x data, measured by stations, and simulated by numerical model PIOMAS for some locations as shown in the lower-right corner of the plot.





**Figure 13.** Comparisons of ice thickness values retrieved by OTIM with APP-x data, measured by stations, and simulated by numerical model PIOMAS at the station locations as shown in the lower-right corner of the plot.

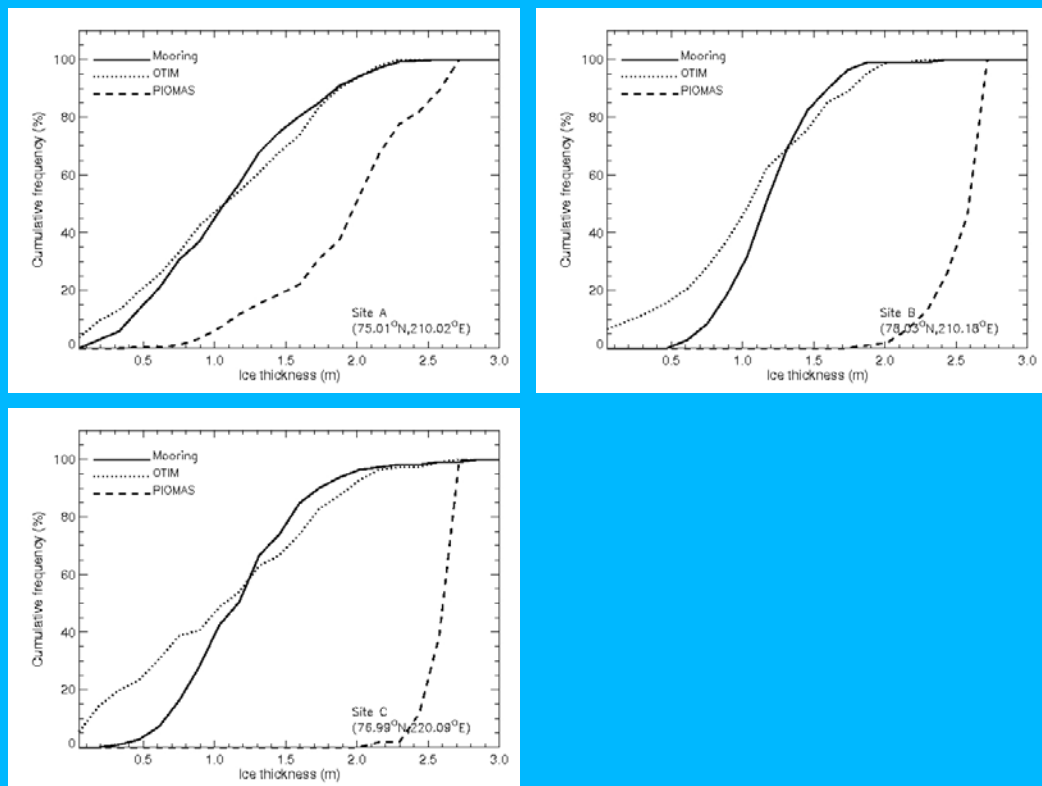
#### 4.2.1.4 Mooring Measurement Analysis

There are ice draft mooring data from Beaufort Gyre Exploration Project (BGEP; <http://www.whoi.edu/beaufortgyre/index.html>) from 2003 up to the present at three mooring sites in the Beaufort Sea area. Since 2003, Upward Looking Sonars (ULS) were deployed beneath the Arctic ice pack on Beaufort Gyre Observing System (BGOS; <http://www.whoi.edu/beaufortgyre>) bottom-tethered moorings (Ostrom et al., 2004; Kemp et al., 2005). Over 15 million observations are acquired for every mooring location in each year. Detailed ULS data processing can be found at <http://www.whoi.edu/beaufortgyre/pdfs/BGOS%20ULS%20Data%20Processing%20Procedure.pdf>. We used ice draft mooring data from 2003 and 2004 from three mooring sites because of no APP-x data available beyond 2004. The mooring ice draft is converted to ice thickness by multiplying a factor of 1.11 as the same process for submarine ice draft. The daily mean ice draft mooring data from 2003 to 2004 were used in the comparison, excluding the data from May through August due to the non-retrievals of OTIM for polar day period.

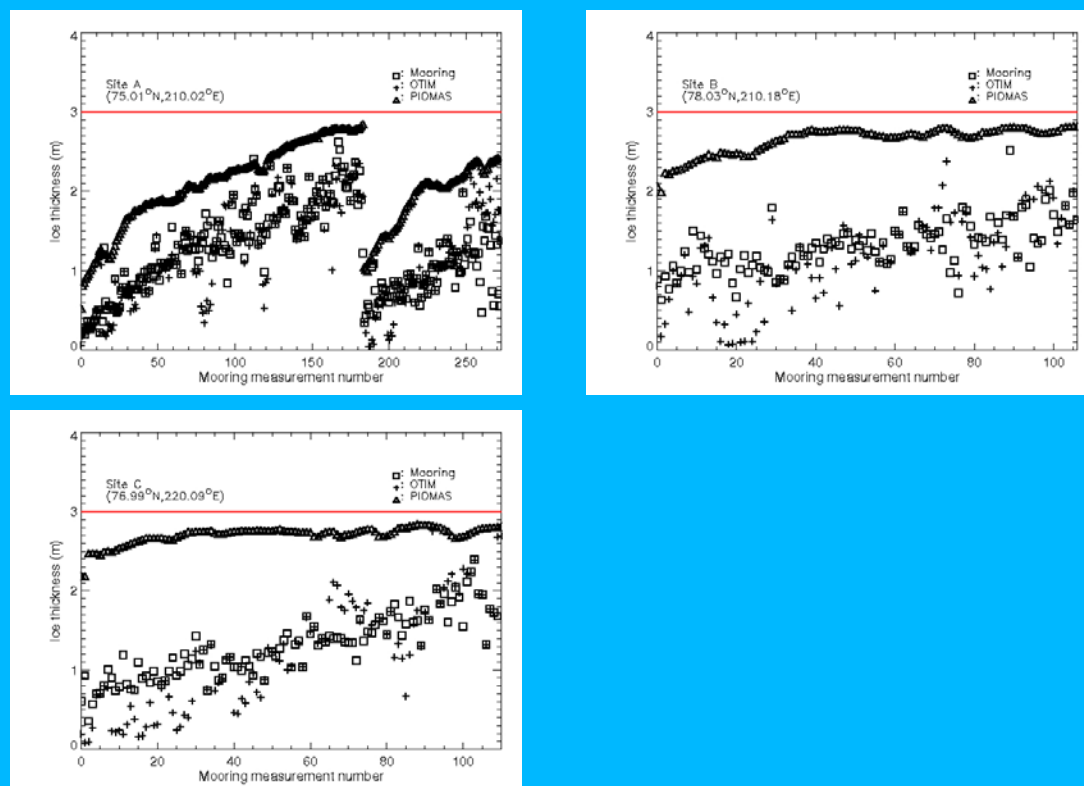
Table 12 lists mooring sites location information, time period, and comparison statistical results. Figure 14 and 15 show the comparisons of the three data sets, i.e., OTIM using APP-x, PIOMAS simulations, and mooring measurements at three sites, as a cumulative frequency ice thickness distribution and as point-to-point comparisons. Table 12 gives the statistical results of ice thickness from OTIM and from mooring measurements for all 3 sites when both of them have valid ice thickness data. The overall error is comparable to the error of OTIM against submarine and station measurements.

**Table 12.** The OTIM validation results against mooring measurements over 2003-2004.

Mooring Location OTIM	Thickness mean (m)	Bias mean (m)	Bias absolute mean (m)
Site A (75°0.499'N, 149°58.660'W) OTIM	1.24 1.22	-0.02 (-1.2%)	0.19 (15.3%)
Site B (78°1.490'N, 149°49.203'W) OTIM	1.32 1.17	-0.15 (-11.4%)	0.29(21.9%)
Site C (76°59.232'N, 139°54.562'W) OTIM	1.32 1.20	-0.12 (-9.1%)	0.28 (21.2%)
ALL MOORING AVERAGE OTIM AVERAGE	1.29 1.20	-0.09 (-6.9%)	0.25 (19.4%)



**Figure 14.** Comparisons of ice thickness cumulative distribution retrieved by OTIM with APP-x data, simulated ice thickness from the PIOMAS model and the ULS measurements at the mooring sites A, B, and C.



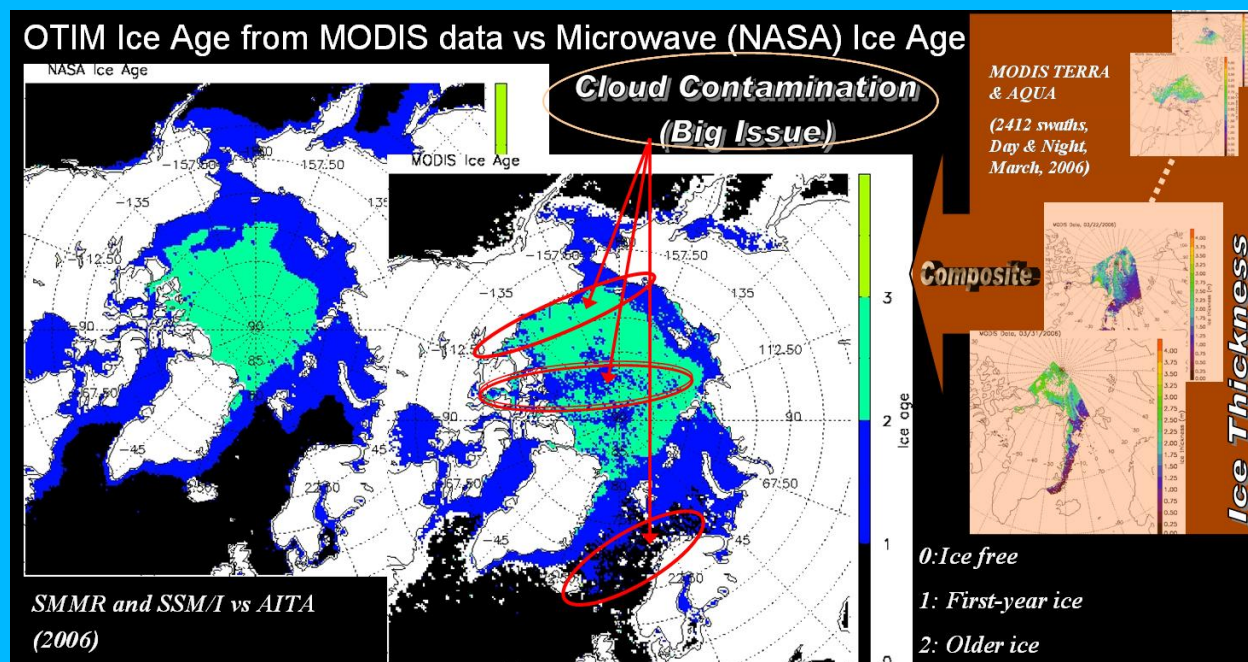
**Figure 15.** Comparisons of ice thickness values retrieved by OTIM with APP-x data, ULS measured ice thickness at the mooring site A, and simulated ice thickness from the PIOMAS model.

#### 4.2.1.5 Microwave Data Derived Ice Age Analysis

There is a data set of sea ice concentrations (the fraction, or percentage, of ocean area covered by sea ice) available at <http://nsidc.org/data/nsidc-0051.html>. This data set is generated from brightness temperature data derived from Nimbus-7 Scanning Multichannel Microwave Radiometer (SMMR) and Defense Meteorological Satellite Program (DMSP) -F8, -F11 and -F13 Special Sensor Microwave/Imager (SSM/I) radiances at a grid cell size of 25 x 25 km. These data include gridded daily (every other day for SMMR data) and monthly averaged sea ice concentrations for both the north and south polar regions. The data are generated using the NASA Team algorithm developed by the Oceans and Ice Branch, Laboratory for Hydrospheric Processes at NASA Goddard Space Flight Center (GSFC), and include data since 26 October 1978. Final data are produced from SMMR brightness temperature data processed at NASA GSFC and SSM/I brightness temperature data processed at the National Snow and Ice Data Center (NSIDC). These sea ice concentration data from passive microwave observations were used as an independent validation data resource for ice age product. We used this microwave sea ice concentration data to derive sea ice age in the Arctic Ocean, i.e., ice free, first-year ice, and older ice by tracking daily sea ice concentration for each pixel over a year-long period. If an areal Sea Ice Concentration (SIC) is less than 15% all the year round on any day, it is identified as ice free area, if only for certain period of time over a year, it is identified as first-year ice area, and if the areal SIC is never less than 15% on any day for a year, it is identified as older ice area.

The ice age product derived from passive microwave data this way is reckoned ice age proxy truth.

We collected 2412 data granules covering the Arctic from MODIS Terra and Aqua for March, 2006, and applied our algorithm to derive ice age product, and then composed all those MODIS granules to form a fully covered Arctic ice age map for the comparison with microwave derived ice age truth. Figure 16 shows MODIS derived ice age (middle) with our algorithm from a composite of Terra & Aqua MODIS data in March, 2006, and the ice age (left) derived from microwave ice concentration (NASA team algorithm) data using a tracking method over the year 2006. Inaccuracies in the MODIS cloud mask can severely affect the accuracy of ice thickness and age products. The performance of our ice age product algorithm is assessed by performance metrics of product accuracy and precision. The product accuracy is defined as the percentage ratio of the OTIM retrieval against truth, and the product precision is defined as the standard deviation of the errors between OTIM retrievals and truth. Tables 13, 14, and 15 list the statistical results between OTIM retrieval and truth in terms of total number of pixels in each category of ice free, first-year ice, and older ice, product accuracy, and product precision. As seen, overall our algorithm derived ice age product fulfills the requirements of 80% product accuracy and less than one category product precision. MODIS daytime data seems to have higher product accuracy and precision. That is due to the very few daytime data (small samples) available from MODIS Terra & Aqua for the Arctic Ocean in March 2006, and the most part of Arctic Ocean are in dark around the clock in winter.



**Figure 16.** Sensitivity of ice thickness to expected uncertainties in the controlling variables for daytime case with reference ice thickness of 0.3 (red), 1 (black), and 1.8 (blue) meters.

**Table 13.** The statistical matrix of the comparison in ice age between OTIM derived ice age with MODIS data and NASA team algorithm derived ice age with passive microwave data.

<i>Note: Number in each cell stands for the number of pixels that belong to the ice age categories corresponding to NASA and OTIM ice age classifications used to do statistics, i.e., accuracy and precision in ice age classification.</i>		NASA Ice Age Truth*			
		Ice Free	First-year Ice	Older Ice	Total
OTIM Ice Age This Study)	Ice Free	(D&N:32278) (N:32288) (D:34681)	(D&N:0) (N:0) (D:0)	(D&N:0) (N:0) (D:0)	(D&N:32278) (N:32288) (D:34681)
	First-year Ice	(D&N:2381) (N:2371) (D:30)	(D&N:12623) (N:12615) (D:93)	(D&N:1141) (N:1141) (D:0)	(D&N:16145) (N:16127) (D:123)
	Older Ice	(D&N:52) (N:52) (D:0)	(D&N:2632) (N:2634) (D:0)	(D&N:5919) (N:5919) (D:0)	(D&N:8603) (N:8605) (D:0)
	Total	(D&N:34711) (N:34711) (D:34711)	(D&N:15255) (N:15249) (D:93)	(D&N:7060) (N:7060) (D:0)	(D&N:57026) (N:57020) (D:34804)

\* D=Day, N=Night, D&N=Day and Night

**Table 14.** The statistical results in terms of product accuracy for the comparison in ice age between OTIM derived ice age with MODIS data and NASA team algorithm derived ice age with passive microwave data.

Ice Age (OTIM vs Microwave)*	
Statistics	Accuracy <sup>1</sup>
Ice Free	D&N:93%, N:93%, D:~100%
First-year Ice	D&N:92%, N:92%, D:~100%
Older Ice	D&N:84%, N:84%, D:~100%
All	D&N:89%, N:89%, D:~100%
Error Sources	<ol style="list-style-type: none"> <li>1. Ice identification algorithm</li> <li>2. Cloud mask/shadow detection</li> <li>3. Relationship between thickness and age</li> <li>4. Ice motion/Dynamic processes</li> </ol>

\* D=Day, N=Night, D&N=Day and Night

**Table 15.** The statistical results in terms of product precision for the comparison in ice age between OTIM derived ice age with MODIS data and NASA team algorithm derived ice age with passive microwave data.

<i>Note: Number in each cell stands for the number of pixels that belong to the ice age category difference corresponding to NASA and OTIM ice age classifications used to do statistics, i.e., accuracy and precision in ice age classification.</i>	<b>Ice Age Difference (OTIM vs Microwave)</b>		
	<b>No Difference</b>	<b>1 Category Difference</b>	<b>2 Category Difference</b>
	(D&N:49820) (N:49822) (D:34774)	(D&N:7154) (N:7146) (D:30)	(D&N:52) (N:52) (D:0)
<b>Precision</b>	<b>(D&amp;N:0.34 Category) (N:0.34 Category) (D:0.03 Category)</b>		

\* D=Day, N=Night, D&N=Day and Night

## 4.2.2 Error Budget

In estimation of ice thickness by using the OTIM, many factors affect the accuracy of ice thickness. The uncertainties from all of the input controlling variables in the OTIM will finally propagate into ice thickness through the ways of parameterizations and model algorithms. Theoretically and mathematically speaking, we can describe the estimate of ice thickness as a function of heat fluxes and surface albedo and transmittance:

$$\hat{h}_i = f(\hat{\alpha}_s, \hat{i}_0, \hat{F}_r, \hat{F}_l^{up}, \hat{F}_l^{dn}, \hat{F}_s, \hat{F}_e, \hat{F}_c, \hat{F}_a) \quad (31)$$

where the variables with carets “^” are the variables defined in Equation (1). In the OTIM model we used parameterization schemes (Key et al, 1996) as described in previous sections to calculate  $\hat{F}_l^{up}$ ,  $\hat{F}_l^{dn}$ ,  $\hat{F}_s$ ,  $\hat{F}_e$ ,  $\hat{F}_c$ , all of which are functions of surface skin and air temperatures ( $T_s$ ,  $T_a$ ), surface air pressure ( $P_a$ ), surface air relative humidity ( $R$ ), ice temperature ( $T_i$ ), wind speed ( $U$ ), cloud amount ( $C$ ), and snow depth ( $h_s$ ), therefore ice thickness is actually the function of those variables expressed in Equation (32):

$$\hat{h}_i = f(\hat{\alpha}_s, \hat{i}_0, \hat{F}_r, \hat{T}_s, \hat{T}_i, \hat{T}_a, \hat{P}_a, \hat{R}, \hat{U}, \hat{C}, \hat{h}_s, \hat{F}_a) \quad (32).$$

Suppose the true ice thickness  $h_i$  is estimated from the true values of all controlling variables in the Equation (32), and let  $x_i$  represent the variables in equation (32) with true values, and  $\hat{x}_i$  represent those variable with estimated values, and  $x$ 's subscript  $i$  is from 1 to 12 representing 12 variables in the Equation (32), thus if the uncertainties in the controlling variables are independent and random, the statistics of the error ( $\hat{h}_i - h_i$ ) can be expressed in terms of the uncertainties in the variables on which it depends:



$$(\hat{h}_i - h_i) = \sum (\hat{x}_i - x_i) \frac{\partial h_i}{\partial x_i} \quad (33)$$

or the variance in the thickness error, as

$$\sigma_{h_i}^2 = \sum \sigma_{x_i}^2 \left( \frac{\partial h_i}{\partial x_i} \right)^2 \quad (34).$$

However, as discussed by Key et al (1997), if the variables are not independent of each other, then the covariances between them must be considered. Unfortunately, data needed to estimate the covariance between all pairs of variables are often not available. If the covariance between pairs of variables is unknown, then it can be shown (Taylor, 1982) that the total uncertainty will never exceed

$$\sigma_{h_i} \leq \sum \sigma_{x_i} \left| \frac{\partial h_i}{\partial x_i} \right| \quad (35)$$

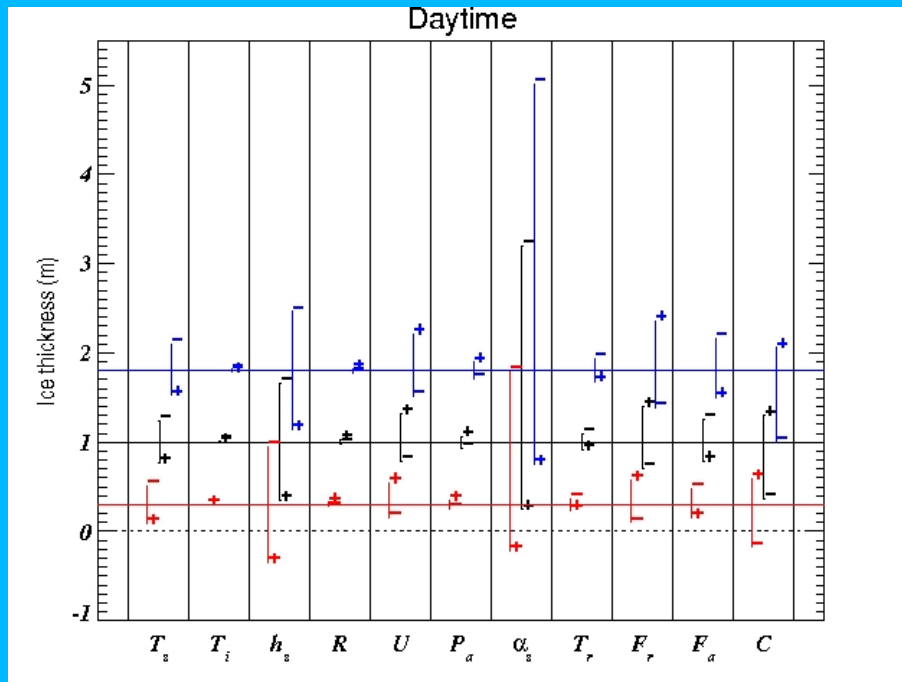
Table 13 and 14 give estimates of the partial derivatives needed in the Equations (33), (34), and (35), computed using differences ( $\Delta h_i / \Delta x_i$ ). These partial derivatives can be used for the calculations of the sensitivity of the ice thickness to errors in the controlling variables.

The estimated uncertainties in the controlling variables in Equation (32), e.g. surface skin temperature  $T_s$ , are now used to assess the accuracy with which ice thickness can be estimated using satellite data products. Since ice thickness vary nonlinearly with respect to the controlling variables under investigation, its sensitivity to errors varies over the range of the input controlling variables. Therefore, uncertainty in ice thickness is estimated for a set of reference values that represent the typical values for certain thick ice as listed in Table 13 and 14.

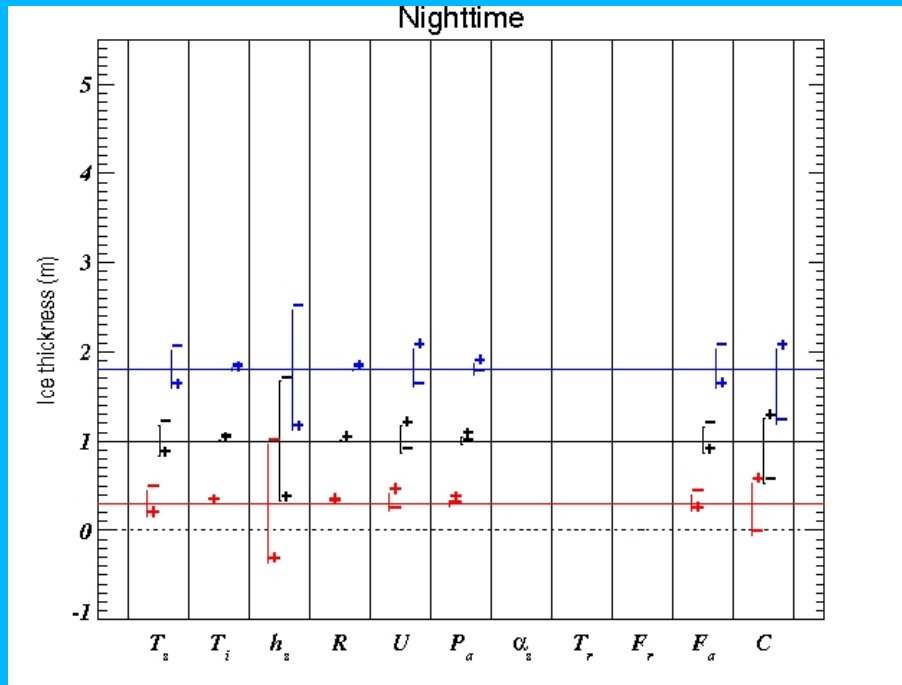
To estimate  $\sigma_{h_i}$ , we need to first estimate the uncertainties of all controlling variables in Equation (32). According to Wang and Key's study (2005), for the satellite retrieved surface broadband albedo  $\alpha_s$ , the uncertainty would be as large as 0.10 in absolute magnitude that is used here. Regarding the ice slab transmittance  $i_0$ , we use absolute uncertainty of 0.05 in this study, which is kind of arbitrary and probably larger than actual value. Satellite retrieved surface downward shortwave radiation flux  $F_r$  can be biased high or low by 20% of the actual value or  $35 \text{ W m}^{-2}$  as compared with in-situ measurements (Wang and Key, 2005). Wang and Key (2005) also estimated the uncertainties in satellite-derived surface skin temperature  $T_s$  and cloud amount  $C$  with respect to the Surface Heat Balance of the Arctic Ocean (SHEBA) ship measurements (Maslanik et al., 2001) that can be as large as 2 K and 0.25 in absolute magnitude, respectively, and we take 2 K as surface air temperature  $T_a$  uncertainty as well. Since surface may be covered with a layer of snow, ice slab temperature  $T_i$  may be different from  $T_s$ , therefore assuming  $T_i = T_s$  may introduce additional error in ice thickness estimation, we elect to assign 5 K uncertainty in  $T_i$  to estimated its impact on the ice thickness since there is no known information about the difference between  $T_i$  and  $T_s$ , and Satellite can only retrieve surface skin temperature  $T_s$ , not  $T_i$ . The uncertainties in surface air pressure and relative humidity together with surface temperature will affect the ice thickness estimation indirectly through the impact of turbulent sensible and latent heat fluxes. A change of 50 hPa surface air pressure may induce changing weather pattern,



we take  $50 \text{ hPa}$  as possible maximum uncertainty of surface air pressure. The uncertainty in geostrophic wind  $U_G$  could be  $2 \text{ m s}^{-1}$  as determined by the buoy pressure field (Thorndike and Colony, 1982), and the relationship  $U = 0.34U_G$  gives the uncertainty in surface wind speed  $U$  of  $0.7 \text{ m s}^{-1}$ , we take  $1 \text{ m s}^{-1}$  as possible actual uncertainty in this study. An uncertainty of 10% in surface air relative humidity is adopted in this work. Snow cover directly affects conductive heat flux, surface albedo, and the radiative fluxes at the interface of the ice-snow. Snow depth  $h_s$  plays a big role, accurate and spatially wide covered measurements are usually not available coincidentally in time and space with satellite observations, and also changes over time with wind and topography. It is hard to know the uncertainty in snow depth estimation, and we think it is reasonable to give 50% of the given snow depth as its uncertainty in general. The last uncertainty source is the surface residual heat flux  $F_a$ , which is associated ice growth, ablation, and possible horizontal heat gain/loss. In the case of no melting and horizontal heat gain/loss,  $F_a$  is zero, which is widely accepted by ice models if the surface temperature is below freezing point. We set uncertainty of  $F_a$   $2 \text{ W m}^{-2}$  as an initial guess. The overall error caused by the uncertainties in those controlling variables for ice thickness estimation may not be equal to the summation of all errors from each individual uncertainty source because the opposite effects may cancel each other among the uncertainty sources resulting in a less error as mathematically described by Equation (35).



**Figure 17.** Sensitivity of ice thickness to expected uncertainties in the controlling variables for daytime case with reference ice thickness of 0.3 (red), 1 (black), and 1.8 (blue) meters.



**Figure 18.** Sensitivity of ice thickness to expected uncertainties in the controlling variables for nighttime case with reference ice thickness of 0.3 (red), 1 (black), and 1.8 (blue) meters.

Tables 16 and 17 list the controlling variables used in ice thickness sensitivity study for daytime and nighttime cases with aforementioned uncertainties in controlling variables and their impacts for typical ice thickness of 1 meter. The results of this sensitivity study are shown graphically in Figures 17 and 18 based on the reference ice thickness values of 0.3, 1.0, 1.8 meters with those expected uncertainties in controlling variables. The bars give the overall range in the ice thickness corresponding to the uncertainties listed in the Tables. Plus signs in Figures 17 and 18 are the ice thickness values for positive uncertainties in the indicated variables; minus signs show the direction of change in ice thickness for a decrease in the controlling variable value.

**Table 16.** Sensitivity of ice thickness estimates to uncertainties in the controlling variables during daytime case with reference ice thickness of 1 meter.

Name	Ref. Value	Error (Dx)	IceThk_Dh	IceThk_Dh/h	IceThk_Dh/Dx
$T_s$ (K)	253.23	+2.000 -2.000	-0.235 +0.245	-0.235 +0.245	-0.117 -0.122
$T_i$ (K)	253.23	+5.000 -5.000	-0.008 +0.008	-0.008 +0.008	-0.002 -0.002
$h_s$ (m)	0.20	+0.100 -0.100	-0.654 +0.654	-0.654 +0.654	-6.544 -6.544
$R$ (%)	90.00	+9.000 -9.000	+0.024 -0.024	+0.024 -0.024	+0.003 +0.003
$U$ (m/s)	5.00	+1.000 -1.000	+0.316 -0.208	+0.316 -0.208	+0.316 +0.208
$P_a$ (hPa)	1000.00	+50.00 -50.00	+0.066 -0.063	+0.066 -0.063	+0.001 +0.001
$\alpha_s$ (0~1)	0.85	+0.100 -0.100	-0.757 +2.195	-0.757 +2.195	-7.566 -21.953
$T_r$ (0~1)	0.05	+0.050 -0.050	-0.086 +0.092	-0.086 +0.092	-1.711 -1.848
$F_r$ ( $w/m^2$ )	101.44	+20.288 -20.288	+0.395 -0.295	+0.395 -0.295	+0.019 +0.015
$F_a$ ( $w/m^2$ )	0.00	+2.000 -2.000	-0.212 +0.260	-0.212 +0.260	-0.106 +0.130
$C$ (0~1)	0.50	+0.250 -0.250	+0.297 -0.639	+0.297 -0.639	+1.189 +2.555

**Table 17.** Sensitivity of ice thickness estimates to uncertainties in the controlling variables during nighttime case with reference ice thickness of 1 meter.

<i>Name</i>	<i>Ref. Value</i>	<i>Error (Dx)</i>		<i>IceThk_Dh</i>		<i>IceThk_Dh/h</i>		<i>IceThk_Dh/Dx</i>	
$T_s$ (K)	241.09	+2.000	-2.000	-0.172	+0.179	-0.172	+0.179	-0.086	-0.090
$T_i$ (K)	241.09	+5.000	-5.000	-0.008	+0.008	-0.008	+0.008	-0.002	-0.002
$h_s$ (m)	0.20	+0.100	-0.100	-0.667	+0.667	-0.667	+0.667	-6.666	-6.666
$R$ (%)	90.00	+9.000	-9.000	+0.006	-0.006	+0.006	-0.006	+0.001	+0.001
$U$ (m/s)	5.00	+1.000	-1.000	+0.166	-0.133	+0.166	-0.133	+0.166	+0.133
$P_a$ (hPa)	1000.00	+50.00	-50.00	+0.043	-0.041	+0.043	-0.041	+0.001	+0.001
$F_a$ (w/m <sup>2</sup> )	0.00	+2.000	-2.000	-0.137	+0.155	-0.137	+0.155	-0.068	0.078
$C$ (0~1)	0.50	+0.250	-0.250	+0.248	-0.476	+0.248	-0.476	+0.992	+1.903

## 5 PRATICAL CONSIDERATIONS

### 5.1 Numerical Computation Considerations

The AVITA is implemented sequentially. Because ice thickness retrieval via OTIM relies on the values of the ancillary data flags, the ancillary data flags need to be computed first. The AVITA will be implemented into the AIT and uses its numerical routines for processing.

### 5.2 Programming and Procedural Considerations

The AVITA requires knowledge of spatial information for accurate pixel geographic locations and land mask information for identifying sea, lake, river, and et al. In addition, the temporal information is required for each pixel regarding the solar radiation in case daytime algorithm is used. Beyond this reliance, the AVITA is purely a pixel by pixel algorithm.

### 5.3 Quality Assessment and Diagnostics

The following procedures are recommended for diagnosing the performance of the AVITA.

- Monitor the percentage of pixels retrieved for ice thickness, and check the value uniformity over the small and smooth area without cracks, melting ponds, and leads.
- Check input ancillary data such as surface skin temperature, air temperature, humidity, wind speed, and snow depth for all pixels of the AVITA. See how those ancillary variables changes affect ice thickness estimation.
- Periodically image the individual test results to look for artifacts or non-physical behaviors.
- Maintain a close collaboration with the other teams using the AVITA in their product generation.

## 5.4 *Exception Handling*

The AVITA includes checking the validity of input data before applying the OTIM and ice age algorithm. The AVITA also expects the main processing system (i.e. AIT) to flag any pixels with missing geolocation or viewing geometry information.

The AVITA does check for conditions where the AVITA can not be performed. These conditions include missing input variables values and unsolvable numerical solutions. In these cases, the appropriate flag is set to indicate that no ice thickness and age are produced for that pixel.

## 5.5 *Algorithm Validation*

As discussed and detailed in section 4.2, the validations were performed with modeled ice thickness data from PIOMAS, submarine and mooring site measurements, and in-situ station measurements. Our testing and validations span the multiple years for every season, and cover both sea ice and lake ice, though most of them are within Arctic Ocean where the submarine, mooring sites, and station measurements were made for years.

The mean absolute error is 0.31 m for samples with a mean ice thickness of 1.80 m, i.e., a 17% mean absolute bias when comparing to the submarine uplooking sonar ice draft measurements in terms of ice thickness. The results of comparisons with mooring sites and in-situ Canadian station measurements are similar. In terms of ice age classifications, the algorithm can easily meet the MRD requirements by classifying ice into ice-free, first-year, and older ice with accuracy greater than 80% and product precision less than one category.

## 6 ASSUMPTIONS AND LIMITATIONS

The following sections describe the current limitations and assumptions in the current version of the AVITA.

### 6.1 Performance

The following list contains the current assumptions and proposed mitigation strategies.

1. Atmospheric profile and wind speed data are available from NWP or other teams' retrieved products. In case no profile data available, it is valid assumption as used by other researchers that surface air temperature generally is about 0.5 ~ 2 degree higher than ice/snow surface temperature depending on the cloud condition, and relative humidity is about 90% over ice/snow, and wind speed of 10~20 m/s at night. But wind speed should be observed or simulated to guarantee to be realistic.
2. Radiation fluxes are available from NWP or other teams' products, otherwise parameterizations will be used and assumed reliable and accurate enough for each pixel. (*Use parameterization schemes over ice and/or snow surface from Bennett (1982), Ohmura (1981), Jacob (1978) as recommended*).
3. Snow maps and climatological depths are available from NWP or other teams' products, or general assumption of 2~50 cm snow depth will be used over ice. (*Use snow information from NWP or elsewhere*).
4. Land mask maps are also available to identify different surface types.
5. All of the static ancillary data is available at the pixel level. (*Reduce the spatial resolution of the surface type, land mask and/or coast mask to pixel size*).

### 6.2 Assumed Sensor Performance

We assume the sensors will meet its current specifications and retrieved products from other teams will be accurate enough for the AVITA, otherwise AVITA built-in parameterization schemes will be used for certain input variables. The AVITA will be critically dependent on the following retrieved products.

- Surface skin and air temperature.
- Surface broadband albedo
- Radiation fluxes at the surface.
- Snow depth.
- Atmospheric moisture and wind.

### ***6.3 Pre-Planned Product Improvements***

The AVITA serves other applications. Its development is closely tied to the development and feedback from the other team algorithms. At this point, it is therefore difficult to predict what the future modifications will be. However, the following discussion contains our current best guess of the future AA modifications.

#### ***6.3.1 Daytime Algorithm Modification***

The solar radiation is inevitably involved in the daytime ice thickness retrieval, making the OTIM extremely difficult to solve analytically for ice thickness due to the complicated ice/snow micro-macro physical properties in solar spectrum, which vary a lot with changes of ice/snow clarity, density, chemicals contained, salinity, particle size and shape, structure, and thickness itself that are hard to know beforehand. We plan to search and/or develop reliable and efficient parameterizations for ice/snow reflectance, transmittance, emissivity, conductivity, and others as well as to develop a parameterization scheme for estimating residual heat flux for taking into account the un-equilibrium state at the interface between ice/snow and the atmosphere, in particular, for daytime condition.

#### ***6.3.2 Optimization***

The OTIM has been optimized to minimize computation time. Other ways to optimize product generation will continue to be investigated.

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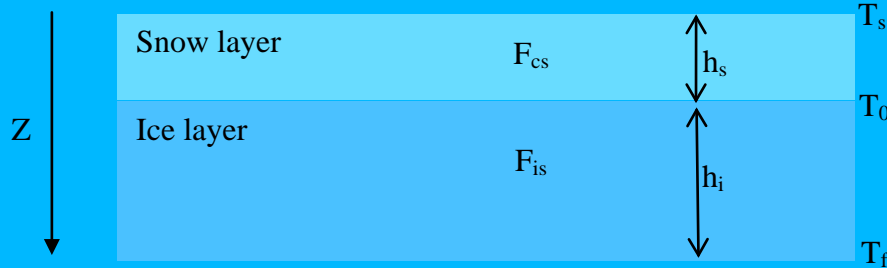


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## Appendix A. Conductive heat flux for two-layer system with snow over ice

Consider a two-layer system, with a slab of ice overlaid by a layer of snow as shown below. We assume the temperature gradients in the snow and ice are each linear and thus conductive heat



flux is constant with depth. At the snow/ice interface, the conductive flux in the snow must equal the conductive flux in the ice, i.e.,  $F_{ci} = F_{cs}$ . As we define the direction to the snow/ice is positive, so we can derive the conductive heat flux for the two-layer system with a snow layer overlaid a slab of ice as shown below. Downward direction is defined positive, so  $F_c = k \cdot dT/dh$ , where  $dT$  is temperature difference, and  $dh$  is the snow/ice thickness.

$F_c = k \frac{dT}{dZ}$ , and so for the snow layer, we have  $F_{cs} = k_s \frac{T_0 - T_s}{h_s}$ , and the same for the ice slab,

$F_{ci} = k_s \frac{T_f - T_0}{h_i}$ , so we have  $k_s \frac{T_f - T_0}{h_i} = k_s \frac{T_0 - T_s}{h_s}$ , after series of derivation, we finally get

$T_0 = \frac{k_s h_i T_s + k_i h_s T_f}{k_i h_s + k_s h_i}$ , therefore  $F_{cs} = \frac{k_i k_s}{k_i h_s + k_s h_i} (T_f - T_s)$ ,  $F_c = F_{cs} = \frac{k_i k_s}{k_i h_s + k_s h_i} (T_f - T_s)$ .