Topological and Impedance Element Ranking (TIER) of the Bulk-Power System

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Preliminary Report

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Acknowledgment

Table of Contents

Executive Summary

On August 8, 2005, the Electricity Modernization Act of 2005 of the Energy Policy Act of 2005 (EPAct 2005), was enacted into law.[1] EPAct 2005 adds a new section 215 to the Federal Power Act (FPA) which requires the Federal Energy Regulatory Commission (FERC) to certify an Electric Reliability Organization (ERO) to develop mandatory and enforceable Reliability Standards, which are subject to Commission review and approval. Once approved, the Reliability Standards are enforced by the ERO, subject to Commission oversight or the Commission can independently enforce Reliability Standards.[2] The Reliability Standards are applicable to Users, Owners and Operators of the "Bulk-Power System" (BPS)[3]. The definition for the BPS provided in the statute includes (1) facilities and control systems necessary for operating an interconnected electric energy transmission network (or any portion thereof), (2) electric energy from generating facilities needed to maintain transmission system reliability and it explicitly excludes facilities used in local distribution of electric energy from the definition. The boundary between local distribution and the BPS is not uniquely defined. Different regions in the North American Electric Reliability Council[4] have in the past proposed and used different criteria. Presently FERC is using the NERC definition for "Bulk Electric System"[5], to define the facilities that comprise the BPS, but stated that it would address this definition in a future proceeding.

In this report we introduce a method for ranking branch elements in the electric grid[6] (typically lines and transformers), with the purpose of 1) developing a process to distinguish those facilities that should not be considered part of the Bulk-Power System from those facilities that should be considered part of the Bulk-Power System, 2) identifying the elements needed to operate each of the electric interconnections, and 3) ranking the importance of those elements. A metric, referred to as Topological and Impedance Element Ranking (TIER), is derived that relates the impact of controlling the power flow along a branch to variation in an optimal solution for dispatchable resources.[7] Ideally, with no other restrictions, a topological characteristic of a non-BPS element is that variation of power flow in a non-BPS element should have no impact on the marginal cost profile of optimal dispatch. Therefore, one topological

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[1] Energy Policy Act of 2005, Pub. L. No 109-58, Title XII, Subtitle A, 119 Stat. 594, 941 (2005), to be codified at 16 U.S.C. 824o
[2] 16 U.S.C. 824o(e)(3).
[3] Energy Policy Act of 2005, Pub. L. No 109-58, available at http://www.epa.gov/oust/fedlaws/publ_109-058.pdf
[4] The North American Electric Reliability Council no longer exists and many of its functions have been assumed by the successor entity, North American Electric Reliability Corporation.
[5] FERC Order No. 693, Mandatory Reliability Standards for the Bulk-Power System, March 15, 2007.
[6] Shunt elements such as capacitors and electrical equivalents are not included in this analysis.
[7] The range of value for the metric is one to zero.

characteristic of a non-BPS element is that its TIER would be absolutely zero if it is supplying radial load. Calculation of TIER relies on information about system topology (i.e., interconnection structure), branch element electrical characteristics, and location of relevant dispatchable resources. It does not require information about resource costs. In the terminology of optimization, the method proposed here uses only information associated with network constraints, and is independent of the cost functions or offer curves associated with generators or other dispatchable resources.   For ease of presentation, we will discuss the method using patterns of LMP's.

TIER has been applied to several system models, but only the PJM model has been independently reviewed.   Commission staffs who are familiar with PJM and WECC reviewed the results and their findings suggest that the rankings are understandable and the identified non-BES facilities generally align with existing operations and planning practices. In particular, this expert review process indicated that seeming anomalies in results from these system studies (i.e., elements ranked higher or lower than might be suggested by their nominal voltage) were justified by specific topological considerations. A distribution of TIER values for the PJM system is shown below in Figure EX.1. To interpret this plot, the reader should understand that the horizontal axis represents each of the nearly 9000 individual branch elements that are included in the system model, numbered in decreasing order of their ranking; the vertical axis is the TIER numeric value. On an expanded horizontal scale, this figure would be a scatter plot, with each element appearing as a single "dot;" the density of data points here compresses to a continuous curve.   A notable characteristic of the plot is a sharp transition involving relatively few elements. Prior to this transition, to the left, one observes a gradual decrease in TIER values among the roughly 6000 highest ranked elements each with a TIER value at or exceeding 0.0001. At the transition band with TIER values ranging from 0 to 0.0001, there are 77 elements identified. Beyond the transition, to the right, approximately 2000 lines and transformers connect to radial loads and have identical TIER values equal to zero (as is expected intuitively and hence are off the bottom of the vertical scale in this logarithmic plot). The plot is both informative and suggestive. Connections to radial loads are one topological characteristic of distribution system elements. On the other side, prior to the sharp transition, it is difficult to identify a clear demarcation between elements. The transition identifies a relatively small subset that may require individual consideration of elements.

A table summarizing the relationship between TIER values and rated voltage levels for components is given below in Table EX-1. The average TIER value decreases with voltage level, but there is a wide range of TIER values at each voltage level. The overlap in TIER values, between voltage levels, weighs against exclusive use of voltage level as the distinguishing metric in identifying BPS. For example some 115/138 kV elements can have a TIER value as high as 0.082 which is higher than some 765 kV, 500 kV and 230 kV elements whose TIER values can

be as low as 0.022, 0.0021 and 0.00013 respectively. It is expected that this report will be the subject of a Staff Technical Conference with opportunity for public comment at the conference and written comments to a specific docket after the conference.
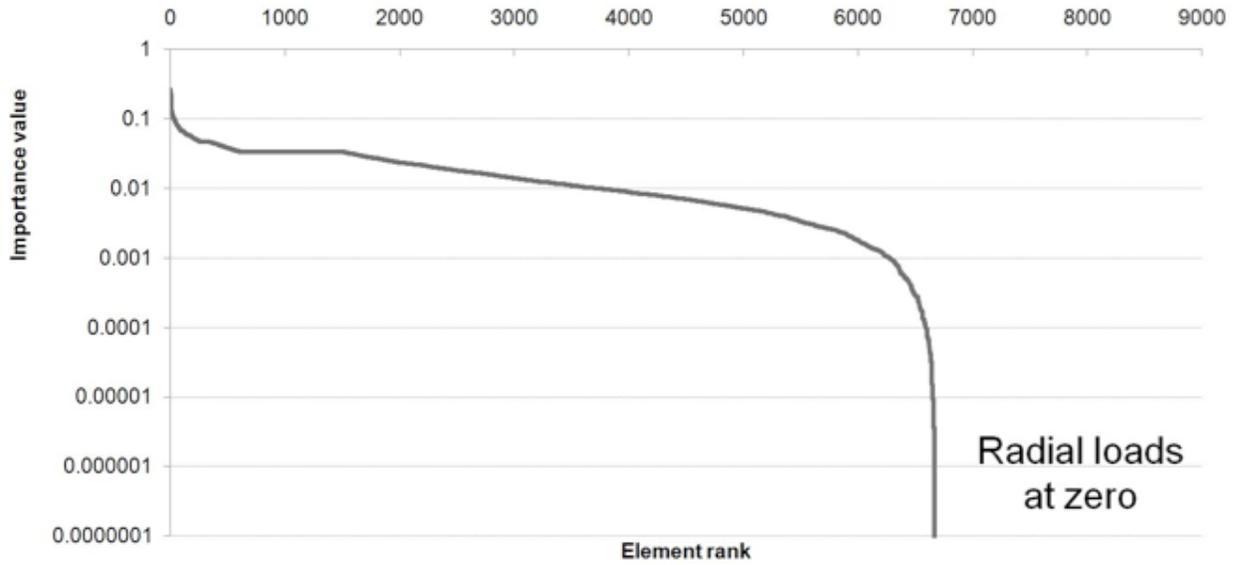
Figure EX- 1 Plot of TIER values for PJM System model. The plot exhibits a sharp transition involving relatively few elements. Approximately 2000 lines and transformers leading to radial load have importance values equal to zero, and are off the scale of this logarithmic plot.

Table EX-1 TIER values for various voltage levels. The average value decreases with voltage level. There is considerable overlap between levels.

Voltage	Low	Average	High
765 kV	0.0220	0.072	0.263
500 kV	0.0021	0.059	0.217
345 kV	0.00013	0.023	0.093
230 kV	3.5×10^{-6}	0.021	0.095
138/115 kV	9.4×10^{-6}	0.010	0.082
69 kV	4.2×10^{-5}	0.0072	0.050
<69 kV	9.1×10^{-7}	0.0045	0.026

1. Introduction

On August 8, 2005, the Electricity Modernization Act of 2005 of the Energy Policy Act of 2005 (EPAct 2005), was enacted into law.[8] EPAct 2005 adds a new section 215 to the Federal Power Act (FPA) which requires the Federal Energy Regulatory Commission (FERC)to certify an Electric Reliability Organization (ERO) to develop mandatory and enforceable Reliability Standards, which are subject to Commission review and approval. Once approved, the Reliability Standards are enforced by the ERO, subject to Commission oversight or the Commission can independently enforce Reliability Standards.[9] The Reliability Standards would be applicable to Users, Owners and Operators of the "Bulk-Power System"[10]. The definition for the Bulk-Power System provided in the statute states

> *The term `bulk-power system' means-- (A) facilities and control systems necessary for operating an interconnected electric energy transmission network (or any portion thereof); and (B) electric energy from generation facilities needed to maintain transmission system reliability. The term does not include facilities used in the local distribution of electric energy.*

This definition includes any elements of the transmission system that are necessary for operating an interconnected electric energy network to achieve Reliable Operation, and specifically excludes local distribution facilities. However, this definition does not directly yield an objective test to classify an element as part of the BPS or not.

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[8] Energy Policy Act of 2005, Pub. L. No 109-58, Title XII, Subtitle A, 119 Stat. 594, 941 (2005), to be codified at 16 U.S.C. 824o

[9] 16 U.S.C. 824o(e)(3).

[10] Energy Policy Act of 2005, Pub. L. No 109-58, available at http://www.epa.gov/oust/fedlaws/publ_109-058.pdfFederal Power Act, Section 215; http://homeland.house.gov/SiteDocuments/20080521141621-50243.pdf

At the present, FERC has adopted the following NERC definition for "Bulk Electric System"[11] that employs a specific voltage level for generation, lines, interconnections, and associated equipment as a generally applicable distinguishing metric:

> *Bulk Electric System: As defined by the Regional Reliability Organization, the electrical generation resources, transmission lines, interconnections with neighboring systems, and associated equipment, generally operated at voltages of 100 kV or higher. Radial transmission facilities serving only load with one transmission source are generally not included in this definition.[12]*

The heart of this definition, unless modified by the regions, is a voltage-level threshold: generation, lines, interconnections, and associated equipment operated or connected at voltages above 100 kilovolts (kV) are considered part of the Bulk Electric System, and elements operated at voltages below 100 kV are generally not included, with the exception of interconnection lines. This definition has the clear advantage of being simple to apply; there is no question as to which elements are included. (Any elements excluded based on the last sentence can be easily identified from a detailed transmission system diagram.) A potential disadvantage of this definition comes from its disregard for the function of the transmission elements. Some interconnected electric energy transmission networks are built with strong underlying networks at voltages below 100 kV (69 kV being a common voltage), while others will build networks that serve the same function at 115 or 138 kV instead. As a result, much larger portions of the electric system may be included in the bulk electric system in some areas, and others may have a fairly small fraction of their transmission system included even if both are necessary for the reliable operation of the network.

While many of the regions do not modify the definition, the largest modification of the definition of the bulk electric system comes from the Northeast Power Coordinating Council (NPCC), a Regional Entity (RE) which oversees the New York ISO and ISO New England areas in the USA as well as parts of the Canadian power systems that are interconnected with NYISO and ISONE and forming part of the Eastern Interconnection. The NPCC's definition of the bulk

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[11] FERC Order 693, Mandatory Reliability Standards for the Bulk Power System, March 16, 2007.
[12] NERC Glossary; http://www.nerc.com/files/Glossary_12Feb08.pdf

electric system can be found in their criteria document, and involves several different tests.[13] The basic premise, however, states that if the failure of an element of the transmission system causes a significant adverse impact outside of a local area, that element should be included in the bulk electric system:

> *Bulk power system: The interconnected electrical systems within northeastern North America comprised of system elements on which faults or disturbances can have a significant adverse impact outside of the local area.*

The NPCC definition of the bulk power system, hereinafter the term bulk electric system is used to align with NERC's term, involves an impact test: if a sustained fault on a bus has a widespread adverse impact outside of a local area that is defined by the entity, then elements connected to that bus are included in the definition.[14]

The application of NPCC's approach in using the impact test as the primary means to define bulk electric system leads to the exclusion of most facilities below 230 kV and even some facilities at 230 kV and above.

The Western Electricity Coordinating Council (WECC) is an RE whose territory covers the entire Western Interconnection of the United States, portions of Canada, and a small part of Mexico. Until recently when WECC stated it would use the NERC definition without any modifications, WECC offered yet another more-inclusive definition[15] that outlines a list of circumstances under which an element should be included as part of the bulk electric system :

1. The system element is listed in the definition of a Transfer Path.
2. An (N-1) outage of the system element necessitates a reduction in a Transfer Path's limit on actual power flow.
3. Measurements of the system element's electrical parameters (e.g. MW, MVAr, amperes, frequency or volts) are included in either a System Operating Limit or an Interconnection Reliability Operating Limit being monitored by the Reliability Coordinator.
4. An (N-1) outage of the system element is included in the list of outages used by a Reliability Coordinator in real-time contingency analysis.

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[13] Northeast Power Coordinating Council, Inc. Document A-10: "Classification of Bulk Power System Elements." April 28, 2007. Retrieved from http://www.npcc.org/viewDoc.aspx?name=A-10.pdf&cat=regStandCriteria
[14] The specifics of both adverse impact and local area are at the selection of the individual Balancing Authorities.
[15] Western Electricity Coordinating Council. "Bulk Electric System." Retrieved from http://www.wecc.biz/ committees/BOD/RPIC/91108/Lists/Agendas/1/4_Bulk%20Electric%20System%20Definition_WECC.doc

5. Planned outages of the system element are coordinated with neighboring transmission providers. As examples, the elements identified in the Northwest Power Pool Coordinated Outage System list of Significant Facilities for Outage Coordination in Section H Appendix B.
6. The system element is either directly involved in supplying off-site station service to nuclear power plants, or its loss causes station service problems that require corrective actions.
7. The system element is listed in the "WECC-Wide Key Facility List - Transmission" table in Appendix A of the WECC Regional Reliability Plan.
8. The system element's status or electrical parameters are incorporated into a remedial action scheme described in the WECC Operating Procedures.
9. The system element is identified by that region's Reliability Coordinator as being part of the "Bulk Electric System".

It is not the purpose of this report to compare and contrast all possible definitions for Bulk Electric System, or Bulk-Power System. Rather, the sampling of currently existing definitions is to show the range of definitions and the possible merit in developing a practical, computable numeric ranking that may be used to provide structure in 1) developing a process to distinguish those facilities that should not be considered part of the Bulk-Power System from those facilities that should be considered part of the Bulk-Power System, 2) identifying the elements needed to operate each of the electric interconnections, and 3) ranking the importance of those elements. The approach we develop uses a sensitivity analysis to classify elements. Generally speaking, we seek to characterize the potential of an individual element to modify or impose network constraints, and in turn, how those constraints impact dispatchable resources in achieving optimal operation. Contingencies are the basis for most constraints and are monitored and controlled in all portions of the grid in order to achieve Reliable Operation.[16] The relative magnitude and additional network locations that are impacted by a contingency on an element provides a practical, objective approach to understanding if that element is needed to enable Reliable Operation of the bulk electric system. In this report we consider as contingencies, the limitation of power flow on each element in the network, individually, and rank the elements by their magnitude and spread of impact.

Sensitivity analysis is a standard tool in most technical fields, including mathematics, engineering, economics, and the sciences. In Chapter 2 of this report we develop a sensitivity

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[16] Reliable Operation means operating the elements of the Bulk-Power System within equipment and electric system thermal, voltage, and stability limits so that instability, uncontrolled separation, or cascading failures of such systems will not occur as a result of sudden disturbance, including a Cybersecurity Incident, or unanticipated failure of system elements as contained in section 215(a)(4) of the FPA.

measure broadly relating network elements to the optimal profile for dispatch. While the material to follow is most easily understood in the context of a traditional optimization of generator operating cost (or market offer price) in \$'s/hour, it is important to stress that the characteristics to be used in this analysis are purely those of the network elements, and are wholly independent of any dollar-valued cost function. However, the approach here does assume that the system is being operated in an optimal fashion with respect to *some* objective function, and the commonly used terminology for such an objective is "cost" function. Hence, for ease of understanding, the exposition to follow will use the terminology of minimizing "cost" in a market, so that sensitivities are then characterized in the familiar units of Locational Marginal Prices (LMPs in \$/MWhr). However, at the risk of repetition, we emphasize again that the method does not depend in any way on knowledge of any generator's \$/hr operating cost or market offer or the existence of a market in a particular portion of the bulk electric system. Indeed, in the discussion to follow, the reader should note that it will be the pattern of LMPs that are used to compute rankings, rather than specific numeric values of these prices.

With this background, we pose the mathematical problem of relating the marginal cost of curtailing flow along an element to variation in the marginal cost profile of dispatchable resources in optimal dispatch. While we have not seen this particular problem presented in the literature in this context, there are related works that use similar sensitivity analyses that influenced the choice for this approach. Researchers have developed a market sensitivity approach to identify load pockets and market participants who may have market power potential[17],[18]. Matrices of revenue/price and dispatch/price sensitivities are calculated and examined. Participants who are able to adjust prices to increase revenues may be able to increase profits. Because production costs are not known, increased revenues would not necessarily indicate increased profits. However, the dispatch/price sensitivities allow the identification of market participants who can adjust prices without changing dispatch.

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[17] B. C. Lesieutre, R. J. Thomas, and T. D. Mount, "Identification of Load Pockets and Market Power in Electric Power Systems," *Journal on Decision Support Systems*, vol. 20, pp. 517-528, November 2005.

[18] M. B. Cain, and F. L. Alvarado, "Metric for Application of Revenues Sensitivity Analysis to Predict Market Power Coalitions in Electricity Markets," Proceedings of the 36th Annual North American Power Symposium, Idaho, pp. 1-8, August 2004.

Unchanged dispatch indicated unchanged production cost, so these works showed that ability to adjust price in such a scenario would be an indicator of market power.

In the above cited market power monitoring work, the known inputs include network information, knowledge of constrained components, and market locational marginal prices (LMPs). The constrained elements require that flows along particular elements be controlled through a pattern of incremental dispatches that in turn admit a non-uniform pattern of LMPs. In certain cases, a small number of participants can exploit such constraints to manipulate prices in a load pocket.

In the research of Cheverez and DeMarco, this relation between line/transformer constraints and LMPs is formally studied[19]. Locational marginal prices must be uniform (i.e., equal at every generator or dispatchable resource) in the absence of constraints.[20] When deviating from this uniform cost situation, the incremental dispatch profile required to curtail flow along an element imposes a pattern of what are termed "admissible" LMP changes. The exact amount of change realized along this new degree of freedom in the optimal power flow solution depends upon the cost functions for dispatchable resources; however, the *pattern* of LMP changes (i.e., the relative amount of change at each location) does not depend on cost functions.

Using the relation between line elements and admissible LMPs, we perform a sensitivity analysis similar to that used in the market monitoring work cited above. This analysis is then used to rank elements by their relative ability to impact LMPs. To explain how this is accomplished it is useful to stress again a fundamental property of economic dispatch in power systems:

> **In the absence of any imposed limit or controls on facilities, optimal economic dispatch results in generators operating at equal marginal costs.**

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[19] D. Chéverez-González; C.L. DeMarco; "Admissible Locational Marginal Prices via Laplacian Structure in Network Constraints," IEEE Transactions on Power Systems, vol. 24, no. 1, pp. 125 - 133, Feb. 2009.

[20] Here we neglect losses. Including losses results in a small perturbation to an optimal power flow; it does not involve a fundamental structural change in the manner of a constraint. It is a second-order effect that is not important for purpose of classifying elements.

Equivalently, in a market setting, generators operate at equal LMPs. We assert that elements in a local distribution network cannot influence this pattern in the optimal result: generators continue to operate at equal LMPs if a local distribution-only element is curtailed. Conversely, when facilities in the transmission network are curtailed, these elements can impact the pattern of LMPs at an optimal solution.

Using a power system model containing topological information and branch electrical characteristics, we calculate a vector of sensitivities for generator LMPs to the marginal cost of redispatch associated with curtailment of a branch element. The information in the sensitivity vector is condensed to a scalar TIER metric that measures variation in LMPs at optimal dispatch; i.e., this metric characterizes the degree to which the pattern of admissible LMPs departs from the uniform, all equal pattern that must exist at an unconstrained solution. For local radial distribution elements, all their TIER values will equal to zero, identically. For other elements, the TIER value can be used in the ranking and classification of facilities.

We pause to note that this approach, while independent of cost information, is very much dependent on the network model. Accuracy will depend on the inclusion of all transmission lines and transformers of interest. Equivalencing methods are sometimes employed in system studies to approximate multiple physical components by a smaller number of fictitious elements in the model. The motivation can be as simple as a lack of detailed knowledge of part of the system, but more commonly equivalents are used to reduce the size of the system to ease computational requirements. With present tools and computer capabilities, the need for equivalents is greatly reduced if not eliminated. In any case, such equivalencing inherently undermines the objectives of this work. Simply put, a component can be appropriately ranked only if it appears as an element in the power network model. The method presented in this report therefore relies on a suitably complete and detailed network model as input.

It is the purpose of this report to introduce an objective and technical approach to rank branch elements in a power system model. In Chapter 2 we provide the details of the model and derivation of the importance (TIER) metric. That is the main contribution of this work. We also present a small illustrative example carefully worked out in detail. The reader that is less

interested in the mathematics may choose to skip ahead to Chapter 3 in which we present results of our analysis of two large-scale systems: a PJM model and a model for the WECC system. We discuss and summarize this work in Chapter 4.

2. Model and LMP Sensitivity Analysis

In this Chapter we derive a metric, which we will refer to as TIER, for ranking branch elements in a power system model. These typically include power lines and transformers. Shunt elements such as shunt capacitors and reactors, HVDC's, SVC's, STATCOM's, etc. are not included in this analysis.

In the development of this metric we consider three desirable properties for the analysis:

1. The algorithm should be functionally-based and reflect the impact of elements' electrical characteristics and system topology. Both can affect the behavior and results, and in our derivation we are careful to separate the electrical and topological information.

2. Any power line or transformer that only serves radial loads may be considered as having the characteristics of distribution elements. (This is consistent with the NERC definition.) Conversely, any radial connection between generating plants and the rest of the network should be considered among the more important elements in the system. The mathematical output of our algorithm will assign a zero TIER value to radial loads and a high TIER value to radial connections to generators, such as a step-up transformer.

3. It is desirable that the analysis be independent of generator cost functions. As observed above, we focus on the characteristics of the electric grid and do not require data on the generator costs (or offers in a market). The repeated attention to this issue anticipates a conceptual difficulty in presenting the sensitivity-based TIER metric. We base our results on a sensitivity related to LMPs and claim that that this is independent to cost functions. As we explain later, a full LMP solution would depend on cost functions, but the profiles of LMP sensitivities used in our computations do not.

2.1 DC Optimal Power Flow Model and LMP Sensitivities

We base our analysis on the so-called "DC Optimal Power Flow Model" (DCOPF). The DCOPF is a well-known simplified power flow model that has gained increased use in recent years for calculating LMPs in markets and for supplementing long-term production cost models. The DC power flow is a linear approximation of a more detailed nonlinear AC Power flow model[21]. The key differences between the models are that in the DC power flow system losses are neglected (or incorporated using an approximate technique), and reactive power is ignored (neglecting variation in voltage magnitude). Instead, the DC power flow focuses on active power flows in the network. To address these limitations in practice, contingencies that are not based on thermal limitations are simulating by identifying the equivalent thermal limitation on a transmission interface (also known as a flowgate) which would produce either the stability or voltage limitation. Therefore, including all of the contingencies that a DC power flow would not ordinarily consider and allowing for the faster computational capabilities.

To emphasize the topological characteristics of the grid and how they are manifest in the DC power flow, we review some of the steps necessary to derive this well-known model. We begin with the characteristic description of a branch element. The active power flowing through a branch is proportional to the angle differences (of AC voltage waveform) at the connecting terminal buses. The proportionality constant is the electrical susceptance, typically denoted as b:

$$P_l = b_l \sin(\theta_m - \theta_n) \approx b_l(\theta_m - \theta_n)$$

In the equation above, P_l, b_l, θ_m, and θ_n denote the active power flow along the line, the line susceptance (electrical characteristic), and voltage angles at the terminal buses. The more exact nonlinear trigonometric sine function is replaced by its linear "small angle approximation." There are a large number of lines in a typical system, and it is convenient to mathematically represent all the line power flow relations in vector/matrix form

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[21] For a textbook treatment, see Glover, J. Duncan, Mulukutla S. Sarma, and Overbye, Thomas J. "Power System Analysis and Design," Fourth Edition.

$$P_{flow} = diag(b)A^T\theta.$$

Here P_{flow} is the vector of power flows along branch elements, θ is a vector of voltage angles at buses, $diag(b)$ is a diagonal matrix of branch susceptances, and A is a "node-to-branch incidence matrix"[22] Matrix A describes the connections made by transmission elements (transmission lines and transformers) in the system. This matrix has one row for every bus in the system, and one column for each transmission element (transmission line or transformer). Each transmission element is arbitrarily assigned a direction; while this direction does not affect the physical results of the calculations, it will determine which direction of power flow is labeled as positive. In the A matrix, each column has two nonzero entries: a 1 in the row corresponding to the bus where the transmission element begins, and a -1 in the row corresponding to the bus where the element terminates. As a result, A completely describes the location and direction of each transmission line in the system. An example of this matrix will be presented in the small system example discussed later in this chapter. Finally, $diag(b)$ is a diagonal matrix of line susceptances (or the inverse of a diagonal matrix of line reactances), and θ is a vector of phase angles for each bus in the system.

The DC power flow model relates the power "injected" at bus locations to bus voltage angles. The power injected into the network flows along the lines in the network and is described mathematically by

$$\begin{aligned} P_{inj} &= AP_{flow} \\ &= Adiag(b)A^T\theta \end{aligned}$$

In typical DC power flow representations, the matrices in the above relation are combined into a single matrix. Here we retain the separate matrices to explicitly show the dependence on topology, Matrix A, and the on electrical characteristics, $diag(b)$.

To optimize the DC power flow problem, a standard constrained optimization approach is used. The objective function will be denoted as $C(P_g)$, and may most naturally be thought of as the production cost of generation. The exact nature of this function will not influence our

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[22] Leon O. Chua, Charles A Desour, and Ernest S. Kuh, "Linear and Nonlinear Circuits."

result, and hence $C(P_g)$ appears here only as a symbolic "place holder." To minimize the cost of system operation with the constraints of the DC power flow calculation, our problem is

$$\min_{P_g,\theta} \ C(P_g) \quad \text{subject to}$$

$$P_{inj} = A \ diag(b) \ A^T \ \theta \quad \text{and}$$

$$P_{line} = b_{line} A_{line}^T \theta$$

This problem can be solved using the classic method of Lagrange multipliers, a standard technique in constrained optimization problems. The Lagrange function is written for this problem as:

$$L(P_g,\theta) \ = \ C(P_g) \ + \ \lambda^T \left(A \ diag(b) \ A^T \theta - P_{inj} \ \right) + \ \mu_{line} \left(b_{line} A_{line}^T \theta - P_{line} \right)$$

In this equation, both λ and μ_{line} are Lagrange multipliers associated with the constraints. In economic terms, λ represents a vector of "shadow prices" of each bus constraint, characterizing the cost of increasing load at each bus by 1 MW. In power systems terms, this vector λ contains the locational marginal price at each bus in the system. Similarly, μ can be thought of as the shadow price of the line's power, i.e., the incremental cost of curtailing the line's flow by one megawatt. For our sensitivity analysis we also note that μ is the maginal cost of redispatch to control power flow on the line.

Setting the derivatives of this equation equal to zero will yield conditions that must be satisfied at any solution of the constrained optimization problem (in formal terms, these are the Karush-Kuhn-Tucker necessary conditions for optimality). The relationship between λ and μ_{line} comes from a subset of the necessary conditions for an optimal solution, those that are associated purely with the network's behavior. In particular, we employ the condition arising from the derivative with respect to θ being set to zero, which yields:

$$\frac{\partial L(P_g,\theta)}{\partial \theta} \ = \ A^T diag(b) A \ \lambda \ + \ A_{line} b_{line} \mu_{line} = 0$$

From this equation, it is possible to obtain a relationship between λ and μ_{line}. An important note here is that this equation does not take any cost data into account; while knowledge of $C(P_g)$

would be required to fully solve for numeric values of λ and μ_{line}, the relationship between them does not depend on generator cost functions. Regardless of the form of $C(P_g)$, the equation above *must* be satisfied for optimal operation.

This equation forms the basis for all our analysis to follow. It explicitly defines a relation between λ, the bus locational marginal price, and μ, a line marginal cost of redispatch (or incremental cost of curtailment). Solving this equation for λ in terms of μ results in the profile of LMP sensitivities which we use to rank model elements.

We are only interested in variation in admissible LMPs; next we present a mathematical formulation of the equation to solve for the profile of variation while suppressing a common uniform component to the solution.

Rearranging terms yields an equivalent expression, for a standard problem in linear algebra, known as a null space or kernal computation:

$$\begin{bmatrix} A^T diag(b)A & A_{line}b_{line} \end{bmatrix} \begin{bmatrix} \lambda \\ \mu_{line} \end{bmatrix} = 0$$

Finding the null space of the matrix on the left side of this equation will give the relative values of λ and μ_{line}. However, this null space will have two dimensions: one representing the variation in LMPs that can result from the curtailment of flow in the transmission element in question, and another representing a solution of uniform LMPs. We add an extra row to the matrix to restrict the null space to only the variation in LMPs. The following equation will eliminate this uniform component of LMPs:

$$\begin{bmatrix} A^T diag(b)A & A_{line}b_{line} \\ 1^T & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \mu_{line} \end{bmatrix} = 0$$

The solution to this equation is easily obtained using tools to solve linear algebraic equations, and can be effectively applied to very large scale power networks by using "sparse matrix" techniques (simply put, these are methods that save computation by skipping operations

involving zero coefficients). Our work uses the mathematical computation environment of Matlab that employs well-proven sparse matrix algorithms.

To examine the effect that the curtailment of flow on line has on marginal prices, first consider the case with no constraints, where the flow on any element can change freely. In this case, all locational marginal prices in the system will be equal. Since the flow on any transmission element can increase, an additional megawatt can be supplied to any point in the network for an equal cost. There is no variation in the LMPs in the system.

When the flow on a transmission element is curtailed, variation in LMPs across the network will arise. Because the flow on this element cannot be increased further, the cheapest source of an additional megawatt may not be able to supply every point in the system. As a result, LMPs in one part of the system will tend to increase, while LMPs in the other part will decrease. Although the full solution for LMPs cannot be determined without cost data for generators, the profile of this LMP variation can be determined from the previous equation. The effect on generation dispatch can be seen by examining the effect on LMPs at generator buses. If a flow curtailment on a certain transmission element affects all generator LMPs equally, then the curtailment has no effect on the profile of generators' dispatch (their LMPs may only increase or decrease uniformly). On the other hand, a curtailment that affects generators unequally will have an impact on the generation dispatch in the system. Most importantly, this means that the system dispatch loses a degree of freedom, and correspondingly, the possible LMPs gain a degree of freedom. In the terminology of mathematical optimization, the primal variables (P_g and θ) loose one degree of freedom, while the dual variables (LMPs) gain a degree of freedom.

The solution to the previous equation provides a vector of LMP sensitivities associated with each branch element in the system. Since even a moderately sized system will have hundreds of generators, and a complete Interconnect model will have thousands, the full vector of LMP sensitivities is an impractically large amount of data to use for ranking. Our goal is to characterize in a single numeric quantity how far the system has deviated from the unconstrained case, in which all LMPs are equal. To this end, our method calculates the

standard deviation of values in the vector of LMPs associated only with dispatchable resources (i.e., typically generators, though controllable, dispatchable loads can also be incorporated if desired). This provides a practical, easily computable scalar metric that is used compare and rank elements.

This method of ranking elements leads to a few important characteristics, which will be outlined both in a small example in the following section and also in the results of testing a larger system. As the example in the next section will illustrate, network elements that only serve radial loads will receive a TIER value of zero. The results in the following sections will also show that the lines traditionally considered the most important – the extra-high-voltage lines that form the transmission backbone of a power network – will be ranked highly by this analysis.

2.2 Illustrative Example

To illustrate the analysis given above, we apply the LMP sensitivity analysis to a small system with nine buses, ten network branch elements, three generators, and three loads. The following Figure 2.1 shows a one-line diagram of the system. In this diagram, buses are the thick horizontal lines, and transmission elements are represented by the thinner lines connecting them. Generators are indicated by the large circles at buses 7, 8, and 9, while loads are the downward arrows at buses 3, 4, and 6.
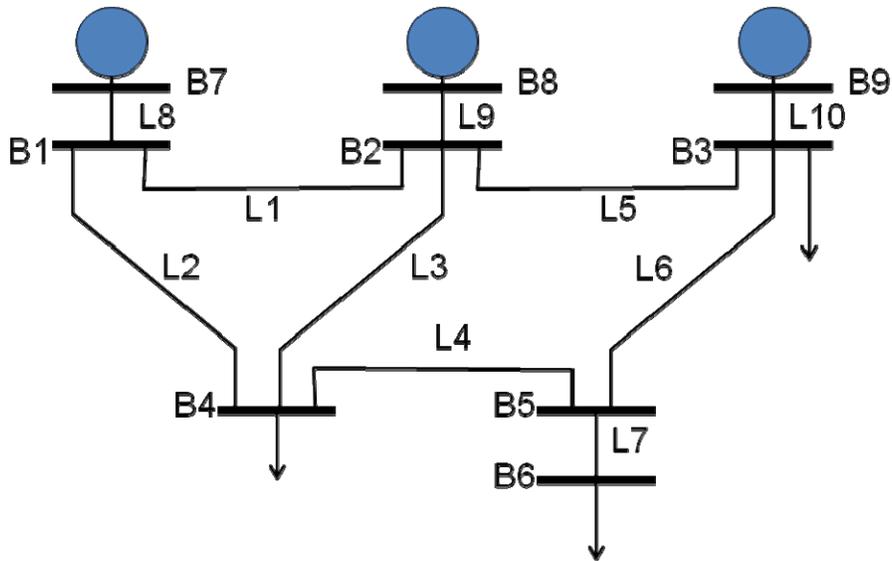
Figure 2.1 A small 9 bus, 10 line example system

Following the description in the previous section, this analysis uses the method of Lagrange multipliers to solve a constrained optimization problem. Mathematically, the problem is stated as

$$\min_{P_g, \theta} \ C(P_g) \quad \text{subject to}$$

$$P_{inj} = A \ diag(b) \ A^T \ \theta \quad \text{and}$$

$$P_{line} = b_{line} A_{line}^T \theta$$

Here we take the opportunity to describe the node-to-branch incidence matrix, A, which represents the system topological information. Each column in A corresponds to a branch and each row corresponds to a bus. In each column there are two nonzero entries in rows associated with the terminals of the line element. For example the first column (L1) has a '1' and '-1' in the first two rows. This indicates that the branch element connects buses B1 and B2 in the network (See Figure 2.1.). The entire Matrix A for this example is system is

26

$$A = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{vmatrix}$$

In this example we set all susceptance values equal to 10: $diag(b)$ is a matrix with all diagonal entries equal to 10. (The other entries are 0.) With uniform electrical characteristics, all differences in results may be attributed to system topology.

Next we step through the sensitivity calculations for a single element, L1. Combining the matrices for the power flow constraints yields

$$P_{inj} = A \; diag(b) \; A^T \; \theta$$

$$= \begin{bmatrix} 30 & -10 & 0 & -10 & 0 & 0 & -10 & 0 & 0 \\ -10 & 40 & -10 & -10 & 0 & 0 & 0 & -10 & 0 \\ 0 & -10 & 30 & 0 & -10 & 0 & 0 & 0 & -10 \\ -10 & -10 & 0 & 30 & -10 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & -10 & 30 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 10 & 0 & 0 & 0 \\ -10 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & -10 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & -10 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \end{bmatrix}$$

To compute the importance value for line L1 in the system (connecting buses 1 and 2), the following equation describes the flow on the line:

$$P_{L1} = b_{L1} A_{L1} \theta$$

$$= 10 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \end{bmatrix}$$

Next we solve the following equation for λ in terms of μ_{L1}:

$$\frac{\partial L(P_g, \theta)}{\partial \theta} = A^T diag(b) A \lambda + A_{L1} b_{L1} \mu_{L1} = 0$$

The solution is given by

$$
\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \end{bmatrix} = \begin{bmatrix} -0.4040 \\ 0.2323 \\ 0.1414 \\ -0.0404 \\ 0.0505 \\ 0.0505 \\ -0.4040 \\ 0.2323 \\ 0.1414 \end{bmatrix} \mu_{L1}
$$

The LMP sensitivities are the vector of numbers on the right hand side of the equal sign; these describe the admissible profile of LMPs that are possible when flow on line 1 is controlled. Before calculating the importance value, we pause to offer an interpretation for the sensitivities. When the flow on line L1 is controlled, typically curtailed to enforce an engineering constraint, the network LMPs will have profile equal to a scaled sensitivity vector plus a uniform contribution.[23] The numeric value of the uniform contribution and the value of the scaling, μ_{L1}, would be computed in a full LMP solution, and would depend on cost functions. However, the profile of the admissible deviation away from the uniform contribution is solely dependent on network structure, i.e., the electrical characteristics, topology, and controlled line. When costs functions are known the optimization problem is solved, the value of μ_{L1} may be interpreted as the marginal cost of redispatch to control the flow on line L1. The LMP sensitivities, as we refer to them in this report, are the sensitivity of locational marginal price to the marginal cost of redispatch for a line element.

The TIER value is calculated as the standard deviation of three entries of this vector, those that correspond to generator buses. These are the entries for λ_7, λ_8, and λ_9. The TIER value is

$$
\sigma_1 = \sqrt{\frac{(-0.4040+0.0101)^2+(0.2323+0.0101)^2+(0.1414+0.0101)^2}{3-1}} = 0.344
$$

---

[23] In a market setting, the constant component of the LMP could be referred to as the Energy Component of the LMP, and the scaled sensitivity would be associated with the Congestion Component.

Repeating the calculations for the remaining lines in the system, the following results are obtained:

Line number	TIER value	Rank
1	0.344	5
2	0.241	6
3	0.139	9
4	0.189	7 (tied)
5	0.396	4
6	0.189	7 (tied)
7	0.000	10
8	0.577	1 (tied)
9	0.577	1 (tied)
10	0.577	1 (tied)

We observe that

- The radial transmission element L7 that connects loads to the rest of the transmission system has an TIER value of zero. This is entirely a result of topology. This is consistent with our understanding of distribution elements' characteristics and several existing definitions of the bulk electric system reviewed earlier in this report, that require radially connected network elements serving only loads to be classified as distribution elements regardless of the size of the load or operating voltage of the element.

- The TIER values for radial transmission elements that connect generators to the rest of the transmission system are all equal. (Lines 8, 9, and 10 show this property in this example.) This too, is a result of topology. Generator step-up transformers are the most common example of this situation. In this example, these elements are the most important elements in the system, which is typical in relatively small systems. As the system grows larger, a radial connection to an individual generator will tend to become

less important than the high-voltage backbone of the transmission system. Conceptually, if there are more generators in the system, the curtailment of one of their radial connections (which is functionally equivalent to the curtailment of the generator's output) has less of an effect on the system as a whole than seen in cases for which there are few other generators available as alternatives.

- Transmission elements that primarily connect generators are ranked fairly highly (for example, lines 1 and 5 here). Pieces of the system that are closer to only loads, such as line 4, tend to be ranked with a lower TIER value. .

3. Sample Results for PJM and WECC Models

The method outlined in the previous sections of this report was utilized to test on models of two different areas of the United States power transmission grid. The first was a detailed model of the PJM Interconnection system, stretching from New Jersey to the Chicago area. The second set of results came from a model of the Western Electricity Coordinating Council (WECC) system, consisting of the entire Western Interconnection of the transmission grid.[24] More details on the structure of these models, as well as characteristics of the results of ranking their transmission elements, will be given in this chapter.

3.1 Model Properties

The PJM model studied here is a very detailed representation of the entire PJM system, including all elements that PJM uses in their real time model for operation of their system. This includes facilities of various voltages and transformers supplying local distribution and all generation that are dispatched to supply either firm load or transactions. Transmission and generation facilities that have not been turned over to PJM for operation (networks at voltages such as 69 or 35 kV) but are in the PJM system footprint are also included. The model contained approximately 8000 buses and 9000 network branch elements inside the PJM area. The following table enumerates buses by voltage level, confirming the model's detailed representation of lower voltage levels.

Table 3. 1 Bus Count by Voltage Level in PJM System Model

Voltage level	Number of buses in PJM area
765 kV	32
500 kV	100
345 kV	213
230 kV	792
138 kV	2547

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[24] While TIER values are automatically calculated for all elements in the model, we only discuss those elements within the USA.

115 kV	752
69 kV	929
46 kV	45
34-35 kV	720
Below 34 kV	1472

One point that should be made here is that many of these voltage levels do not occur in the same geographic regions of the PJM system. For example, there is virtually no overlap between 765 kV and 500 kV systems in the PJM area; 765 kV buses are generally located in the western part of the system, especially in the American Electric Power (AEP) area, while 500 kV buses are primarily located in the eastern part of the system, primarily in Pennsylvania, Maryland, Virginia, and New Jersey. Similarly, the majority of the 345 kV buses are located in the AEP or Commonwealth Edison (ComEd) control areas, which contain almost no 230 kV elements.

One issue that arose with this PJM study model occurred as a result of the interfaces between the PJM system and the surrounding pieces of the transmission grid, which are not modeled in detaile. Areas outside of PJM were modeled less precisely than the areas inside PJM; the remainder of the model, which covered almost the entire Eastern Interconnection, contained fewer buses than the PJM system despite being far larger in area. The real time model contains simplified equivalents in order to reduce the overall size of the model without eliminating detail in the PJM area. As noted previously in this report, it would be inappropriate, if not impossible, to utilize this proposed method to analyze these areas outside PJM. Many of the equivalenced network elements in the model are fictitious, representing attempts to approximate the electrical impact of larger numbers of physical facilities. This being the case, in the study reported here we chose to ignore the pieces of the system beyond PJM's borders. Inevitably, the approximation and errors associated with such equivalancing propogates, and can perturb rankings computed for "non-equivalenced" elements located near the edge of the PJM system. These elements tended to be ranked lower than they otherwise would have been, because the possibility of loop flows (also known as parallel path flows) through other areas (for example, power flowing from PJM into the New York ISO's control area and then back into

PJM) was eliminated. A few different methods for resolving this problem are currently being investigated; most direct would be to simply analyze the Eastern Interconnection in its entirety, rather than analyzing each region separately. This will substantially increase the memory and computing power requirements for the analysis. However, the success of and low computing time requirements for studies to date, using only very modest computer hardware, indicates that computation for the full synchronous interconnect will be perfectly feasible as long as the detailed system modeling is available for the entire Eastern Interconnection.

The WECC system model was similar to the PJM model, but substantially larger. There was a similar degree of detailed modeling of lower-voltage buses, although many generator step-up transformers and distribution step-down transformers were not included. The WECC model included approximately 16,500 buses. Their voltages were broken down as follows:

Table 3. 2 Bus Counted by Voltage Level in WECC System Model (USA Only)

Voltage	Number of buses
500 kV	339
300-400 kV	183
200-299 kV	1504
100-199 kV	6263
51-99 kV	2758
50 kV and below	3364

Because the WECC system model covers the entire Western Interconnection, issues associated with equivalencing the boundary of the modeled system (as described above for the PJM system) were of much less concern. This is an advantage of analyzing the entire interconnection at one time, rather than analyzing different regions separately.

3.2 The Rankings

The calculation of elements' TIER values was carried out using MATLAB, a commonly used scientific computing tool. The original data files were provided in the Siemens PSSE format

for PJM and GE PSLF format for WECC. After importing the data into MATLAB and extracting the previously described *A* matrix and *b* values required for this analysis, the program calculated the TIER values for each line in the system. The computation for the PJM model took about six minutes on a laptop machine of modest computing power. The vast majority of the time was taken up by calculating the null space of a large matrix (roughly 8000-by-8000 for the PJM system, and 16,500-by-16,500 for the WECC system); this large dimensional problem precluded a naive approach, that of just directly invoking MATLAB's built-in null space routine. Instead, the problem was reformulated slightly to allow use of a more efficient method with regards to computing time and memory. This formulation relies on the fact that the null space of the matrix $[A^T \text{diag}(b) A]$ is already known, and is a uniform vector of all equal entries.[25] Adding one column and one row to this matrix creates a square matrix, whose one-dimensional null space is the vector of LMP variations sought in this calculation.

The results of the analysis for the PJM and WECC systems yielded rankings that appeared intuitively reasonable. In particular, the PJM results were examined by PJM staff and staff at FERC who had experience with this system, and were found to be credible representations of the importance of elements. Qualitative trends expected of the rankings were observed; for example, higher-voltage elements were (in general) ranked more highly than lower-voltage elements. However, for instances in which the rank assigned a network element did not follow the expected trend based on voltage level, further scrutiny revealed logical reasons grounded in the network topology that justified the high or low TIER value computed by this method.

One of the qualitative features most expected, and confirmed by the analysis results, was that higher-voltage lines should typically have higher importance values. Clearly, the extra-high voltage (EHV) backbone of any system is generally considered one of the most important parts of the system. Also, when the impedance of these EHV elements is converted to a per-unit value, it becomes substantially lower than the impedance of a lower-voltage element of a similar length and the EHV elements have significantly higher thermal ratings. As a result,

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[25] Another manifestation of the fact that the unconstrained optimal solution yields all equal LMPs.

power flowing over long distances through the network will tend to predominantly use these elements. The following table demonstrates this trend for the PJM system. (For this table, any transformers are listed under the highest voltage level to which they are connected; for example, a 345/115 kV transformer will be included in the 345 kV category. All connections to radial loads and radial generators are excluded.

Table 3. 3 TIER Value Range vs. Voltage Level in PJM System Model

Voltage	Lowest value	Average value	Highest value
765 kV	0.0220	0.072	0.263
500 kV	0.0021	0.059	0.217
345 kV	0.00013	0.023	0.093
230 kV	3.5×10^{-6}	0.021	0.095
138/115 kV	9.4×10^{-6}	0.010	0.082
69 kV	4.2×10^{-5}	0.0072	0.050
Below 69 kV	9.1×10^{-7}	0.0045	0.026

While the average TIER value increases as the operating voltage increases, confirming the "typically" expected trend, the range of importance values among elements within a given voltage level is relatively large. This leads to substantial overlap between TIER values assigned to elements across different voltage levels; for example, a significant fraction of the elements operated at 115 or 138 kV are ranked above a number of the elements operated at 230 kV. As shown in the above table, some 115/138 kV elements can have a TIER value as high as 0.082 which is higher than some 765 kV, 500 kV and 230 kV elements whose TIER values can be as low as 0.022, 0.0021 and 0.00013 respectively. This shows that the topology of a system can have a significant effect on the importance of a network element, regardless of the voltage level at which it is built and operated.

An analogous table is given below for the WECC analysis. Because WECC uses a larger number of distinct voltage levels for its network equipment, the voltage levels are displayed as ranges instead of absolute values. The same general trends are present as in the PJM system;

again, especially for the lower-importance end of each voltage level. A fair amount of overlap in TIER values can be seen from one voltage level to the next.

Table 3. 4 TIER Values vs. Voltages for the WECC System Model (USA only)

Voltage	Lowest value	Average value	Highest value
500 kV	0.0002	0.051	0.285
300-400 kV	0.0027	0.024	0.080
200-299 kV	0.0001	0.017	0.104
100-199 kV	8.63×10^{-8}	0.0075	0.056
51-99 kV	5.25×10^{-6}	0.0071	0.049
50 kV and below	7.82×10^{-6}	0.0054	0.018

It should be noted that in this analysis the numeric TIER value assigned a network element is intended as a relative measure, in comparison with other elements within that same system. Inherent to the algorithm proposed here is the fact that the absolute importance value of an individual element will tend to be smaller if that same element is placed within a larger system. This simply reflects the physical reality that one single element will typically have smaller absolute impact on the overall system, if that system is composed of a very large number of power carrying paths. This property will have no effect on comparing the importance of network elements in the same system. However, with the scaling methods used in this analysis, comparisons in importance values from one interconnection or system to another are not valid. For example, the fact that a 500 kV line in the WECC system has a higher value than a 765 kV line in the PJM system does not mean that one is more important than the other on some absolute, system-independent scale; importance values can only be meaningfully compared within one system.

A property of the results that is potentially relevant to classification can be seen in the distribution of the TIER values. The following graph, displaying results from the PJM system, shows the TIER values on a logarithmic scale on the vertical axis. The horizontal axis may be interpreted as an indexed numeric label of each network element, with the elements ordered from most important (element #1) to least important (element # 8970). Because the vertical axis of the graph uses a logarithmic scale, elements having zero importance values (associated with lines serving radial loads) are off the scale towards the negative.
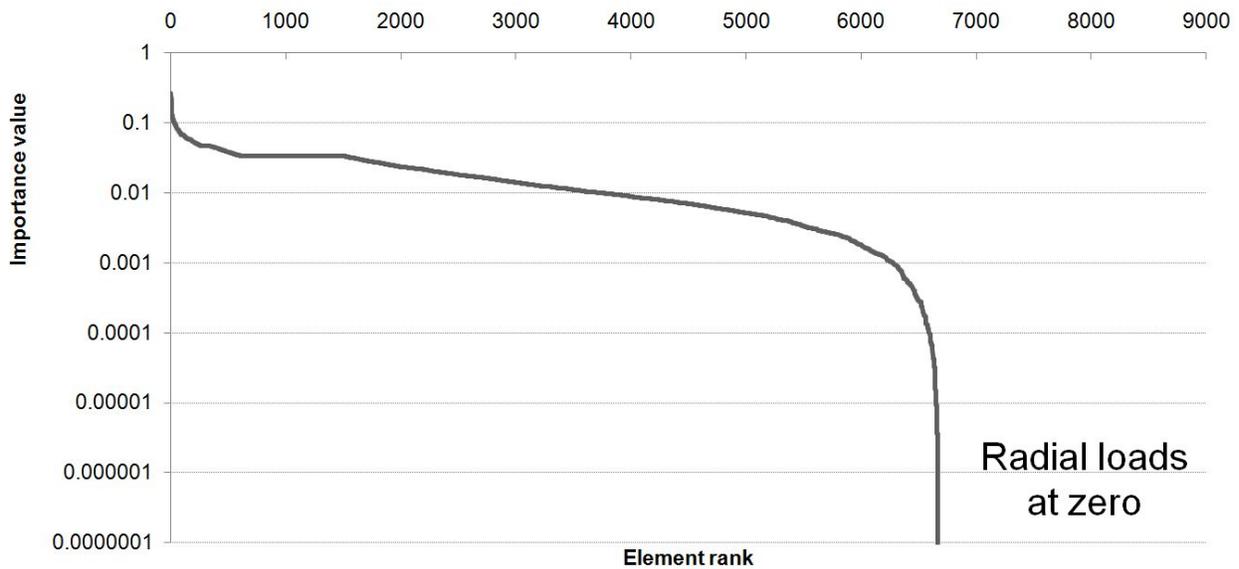
Figure 3. 1 TIER value vs. rank for the PJM System Model

Additional structural features of the method can be observed from this graph. First of all, the very most important elements, on the far left side of the graph, span a fairly wide range of TIER values (in particular, they span roughly one order of magnitude on the logarithmic scale). These are followed by a flat portion in the graph, representing elements ranked roughly 625 to 1500. These elements having all equal importance values are the 875 generator step-up transformers in the system. The most notable feature is the steep drop-off in TIER value for those elements ranked in the range of roughly 6250 to 6750. This indicates that there is a fairly small percentage of elements (fewer than 500 out of the total 8970, or 5.5%) in the system having TIER values below 0.001, but greater than the "hard" zero value that is assigned to radial load

serving elements . And there are fewer than 80 elements (less than 1%) with a TIER value between 0.0001 and zero.

A similar characteristic can be seen in the TIER value plot for the WECC system, as shown below.
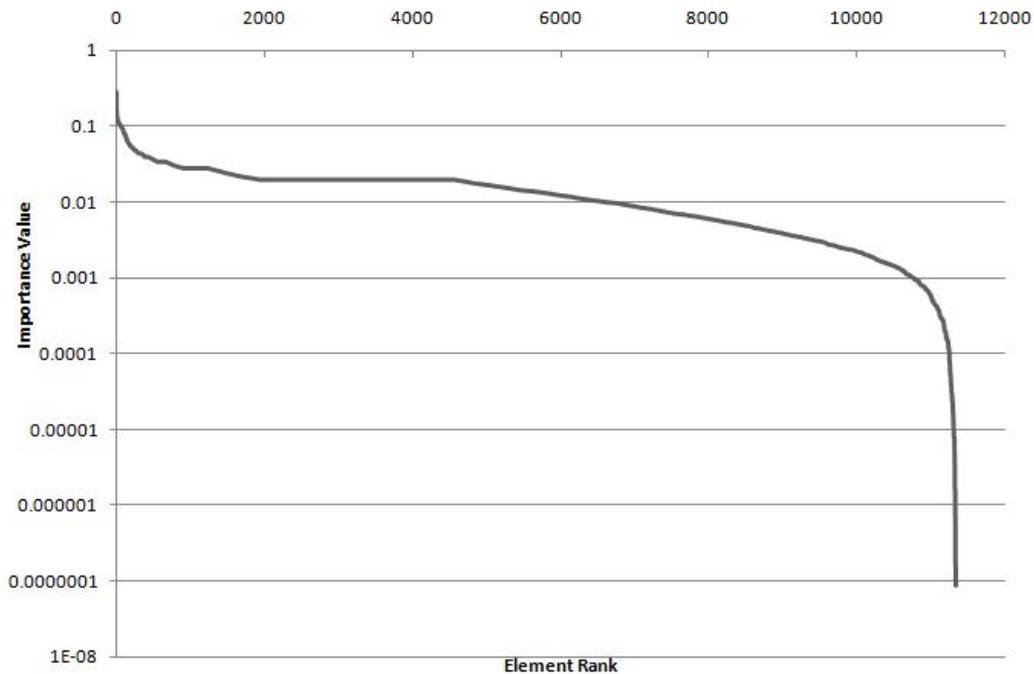
Figure 3. 2 TIER value vs. rank for the WECC System Model (USA only).

The TIER value results provide an opportunity to revisit an issue regarding robustness of the mathematical model, that was noted earlier in the theoretical development of Section 2 of this report. In computing TIER values, we have chosen the simplification of the DC power flow approximation to represent the network electrical behavior. In the terminology reviewed in Section 2 of this report, this corresponds to use of a "small angle" approximation to model incremental power flow on a network branch element. This is a widely used and valid approximation, but neglects potential impact of changes in system operating point. For the purposes of this method, it is very attractive to avoid dependence on the specifics of a single

operating point, but does this expedience compromise accuracy? Specifically, would the use of a (potentially) more accurate linearization about the exact power flow solution significantly change the element TIER values, relative to those calculated using the simpler small angle approximation? The PJM data included the full power flow solution, so this question could be examined in the context of our PJM study, and the resulting answer was clearly "no." As evidence, consider the numeric results displayed in the figure below. This plot is a scatter plot of the importance values for all network elements in the PJM study with TIER values greater than zero. The horizontal axis is the TIER value assigned to the element based on the small angle approximation (i.e., the DC power flow used here), while the vertical axis shows the TIER value calculated without this approximation (i.e., using a linearization of the line flows at the full power flow solution). Both axes use a logarithmic scale consistent with the previous figures. The ideal case of exact agreement between the DC power flow approximation and the linearization about the exact power flow solution would force all points to lie precisely on a 45 degree line down the center of this plot. As may be observed in the figure below, the actual results for the PJM study come extraordinarily close to this ideal of a 45 degree line. This example reinforces the appropriateness and validity of the DC power flow approximation as used in our method, and indicates that the PJM element TIER values were calculated quite accurately without the need for the data of a full power flow solution at a particular operating point[26].

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[26] Both models compared here neglect voltage variations and reactive power injections and constraints.
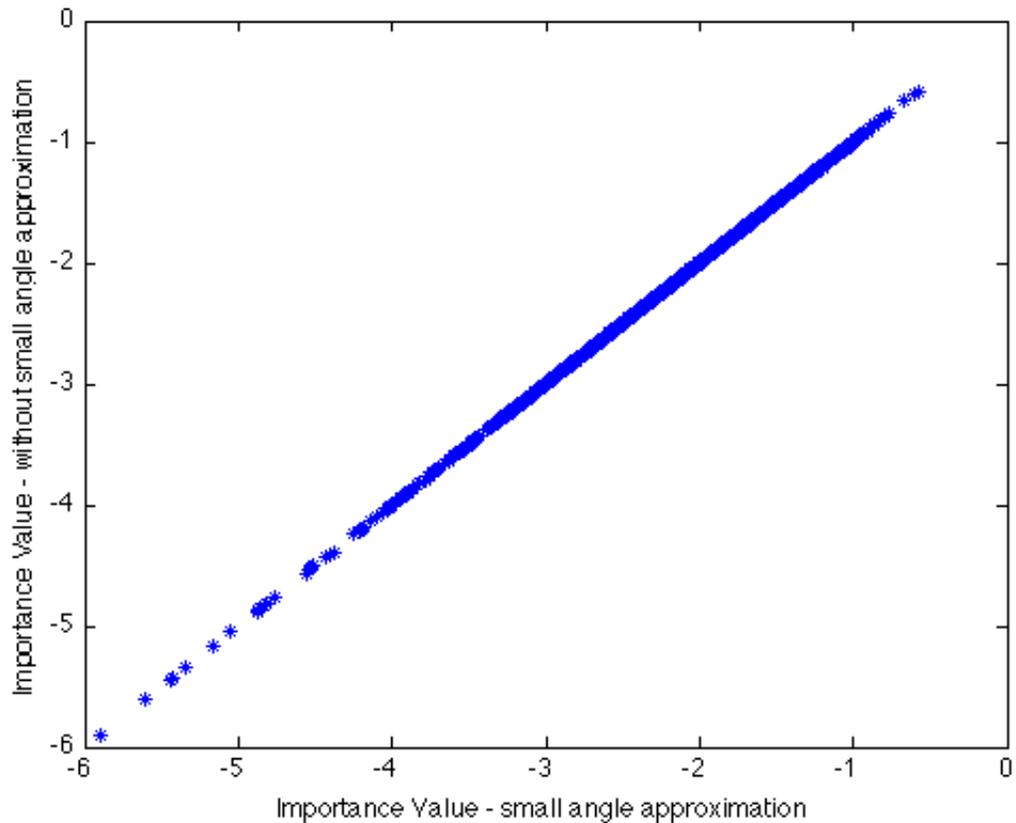
Figure 3. 3 Plot of Importance values, comparing results using a linear DC power flow and a nonlinear power flow model (logarithmic scale: values on both axes indicate power of 10).

An interesting way to interpret the importance value results is via graphical display of variation in the admissible LMP basis vector. The following graphs show plots of the LMP vectors associated with four sample network elements in the PJM system. In each of these, the vertical axis measures the relative impact on LMP experienced at each generator bus in the system. The horizontal axis indexes the generators, from 1 to 875. (The indices correspond to the numbering used in the PJM PSSE data set; as a very rough rule of thumb, lower numbers correspond to generators located in the eastern side of the PJM service area, while higher numbers correspond to those located farther to the west.)

The first graph is for a transmission element which serves only a radial load. If the flow on this element is curtailed, there is no variation in generator LMPs induced – the line if "flat." As a result, this element's importance value (standard deviation) is equal to zero.
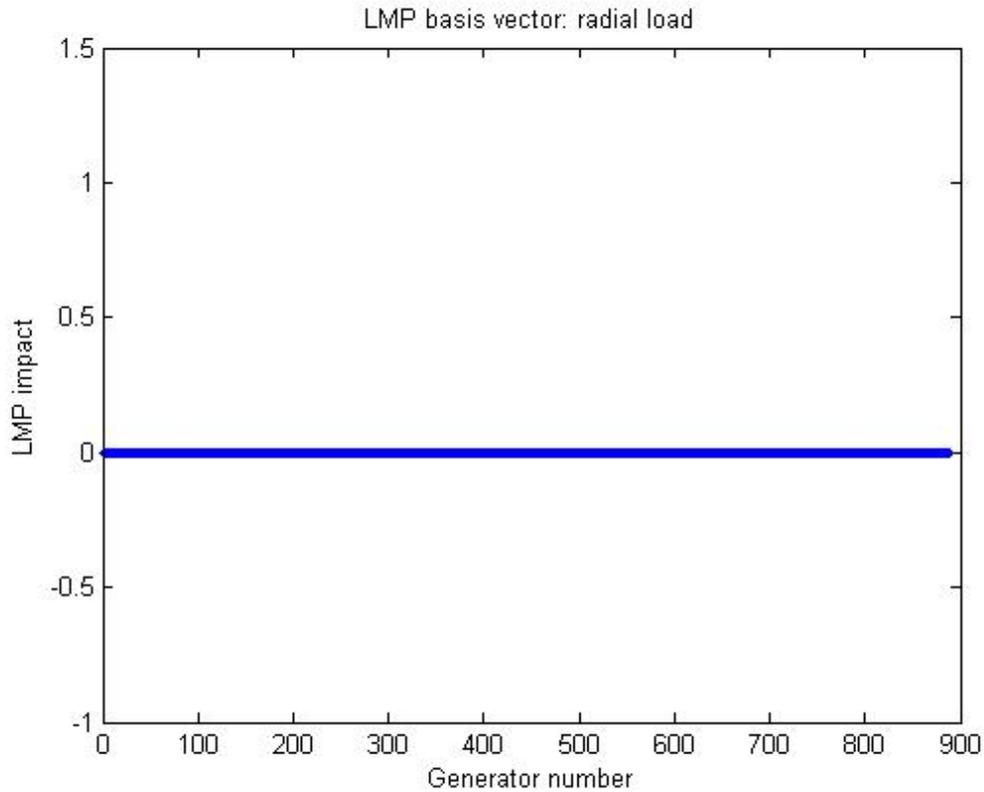
Figure 3. 4 LMP basis vector for an element serving a radial load

The next graph shows the LMP basis vector for a transmission element which serves a radial generator (in this case, the element is a generator step-up transformer). As one would expect, there is a very large impact, but on only the LMP for this one generator in the system – the generator located at the end of the transmission element in question. All of the remaining generators see a very small impact in the opposite direction (so close to zero as to be imperceptible on the scale of the graph). This leads to an importance value of 0.0336, and is equal to the importance value of all other radial elements serving a single generator.
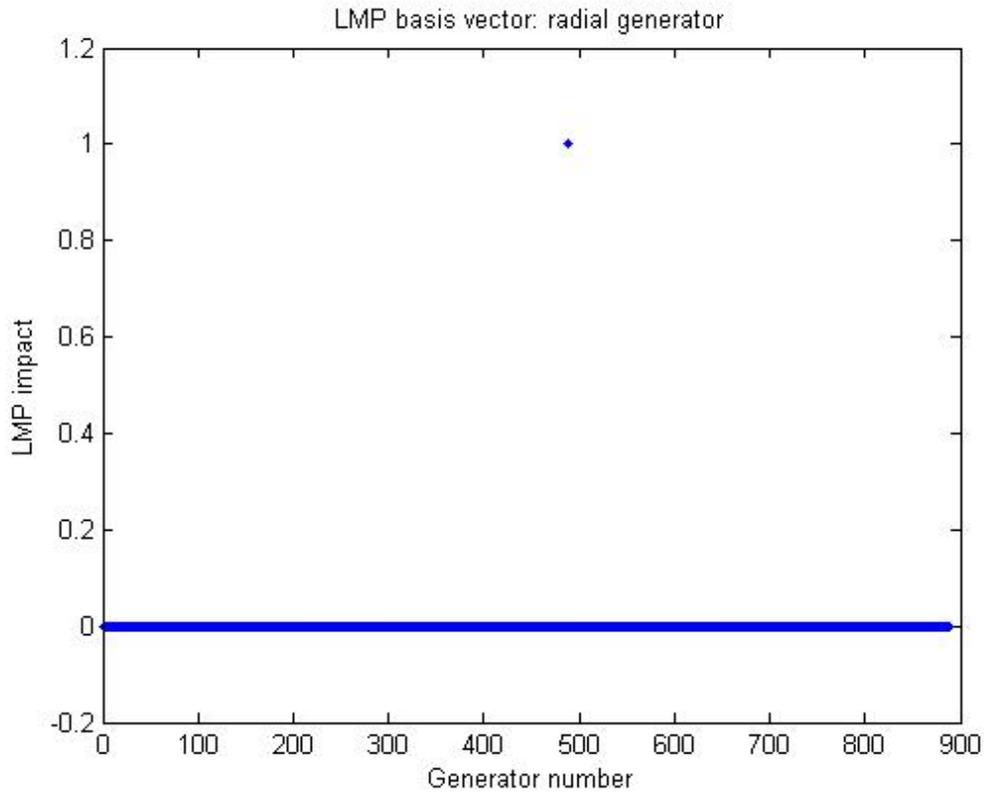
Figure 3. 5 LMP basis vector for an element serving a radial generator

The third graph displays the admissible LMP profile associated with an element in the 35 kV network in Eastern PJM. As the graph shows, there is a small impact on generator LMPs near the line, but virtually no impact on generators located far from this line. This matches intuitive physical reasoning: curtailing the flow on such a low voltage element can be accomplished with changes in generators nearby, but dispatch of generators hundreds of miles away will have negligible effect on the element's flow. The importance value of this line is approximately 3.3 x 10^{-4}.
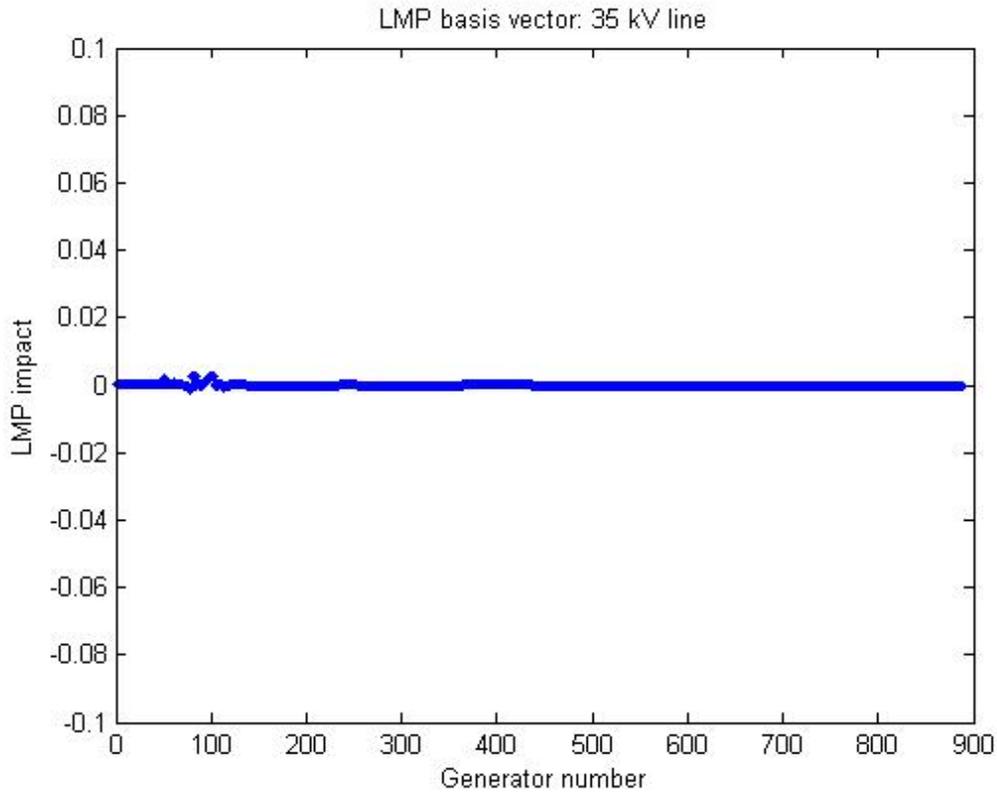
Figure 3. 6 LMP basis vector for a 35 kV subtransmission line

Finally, the last graph given here shows the LMP profile for one of the most important lines in the PJM system. Specifically, this is the fourth highest ranked line in the system: a 500 kV line in the middle of PJM. The graph confirms that the greatest effect on LMPs occurs at the generators near this line. However, generator LMPs are impacted across the entire PJM system, including generators at the far opposite end. This also corresponds with what would be expected: the flow on this element couples to the dispatch of nearly all of the generators in the system, and curtailing the flow on the line will affect almost any generator. Its importance value is approximately 0.217.
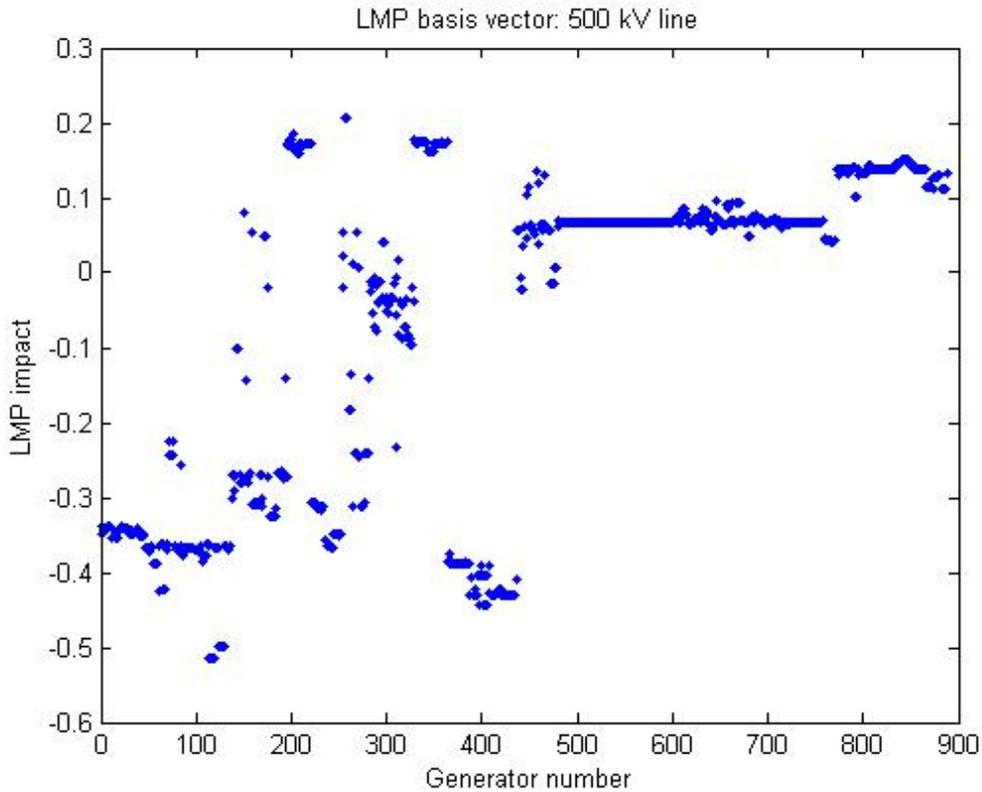
Figure 3. 7 LMP basis vector for a 500 kV EHV transmission line

Lastly, it is instructive to examine a few cases in the PJM system where an element's TIER value is either much higher or much lower than other elements operated at the same nominal voltage. This often occurs as the result of a system's topology, where the connections made by a transmission element cause it to be either much more or much less important than elements with similar voltages and impedances. One simple example, which occurs often in any system, is the case of a transmission element serving a radial load. Curtailing the flow on this element (and simultaneously load), regardless of its voltage level or impedance, will have no effect on the optimal profile for generator dispatch. This can lead to a TIER value of zero for lines operated at 138 or even 230 kV, despite their high voltage and low impedance.

There are other elements in the PJM system which, while they are not strictly radial, mostly exist to serve local load. These elements, as expected, tend to result in relatively low TIER values, regardless of their voltage level. One example of this type of element occurs on a 115 kV line in the southern portion of PJM. There are no generators located along this line; the only connections to it are loads located at each substation, as well as additional radial loads served from one substation. There is also a 230 kV line which offers a much lower-impedance path between two of the substations. While changes in generation dispatch will affect the power flow on this 230 kV line, the high impedance of the 115 kV path (and the 230/115 kV transformers on each end) dictates that the majority of the power flowing there will only be serving the local load located on this line. Accordingly, the 115 kV line receives a modest importance value of 9.0×10^{-4}. This gives it a rank below most of the other 115 kV lines in the PJM system, and below the majority of the 69 kV lines as well.

The reverse situation, a lower-voltage transmission element with a relatively high importance value, is also a common occurrence in the PJM system. One example of this situation occurs in Eastern PJM.   A power plant is located at this substation, along with (in the peak-load situation in the PJM model provided) more load than the generation. This substation is connected to the rest of the transmission grid by two 69 kV lines, each leading to its own 230/69 kV transformer. Because these two lines are in parallel, neither of them is a direct radial connection to a generator. However, a curtailment on either of these lines will have a large effect on the dispatch of generation; the generators at this location will be forced to run at full output to serve the load at the same bus. As a result, the LMPs can basically vary independently of LMPs in the rest of the system if the 69 kV lines must be curtailed. This gives these lines a fairly high importance value of 0.0335, which is above the average importance value for 115 kV and even 230 kV transmission elements.

4. Discussion and Conclusions

We have presented a practical, objective computable numeric method for ranking branch elements in a power system model relative to one set of actions necessary to assure Reliable Operation of the interconnected electric energy transmission network. The chosen action is based on the generation redispatch and is measured in terms of the sensitivity of locational marginal price to a marginal cost of redispatch in controlling flow along an element. A standard deviation to measure variation from uniform marginal cost dispatch (uniform LMP for markets) for dispatchable resource is used for a scalar metric which is defined as Topological and Impedance Element Ranking (TIER). For radial loads, this sensitivity metric will be zero. We emphasize the following points

- The model and calculation combines topological information about the network, and electrical characteristics of the elements.

- The analysis does not require any knowledge of specific cost functions. It relies on properties of the network (topology, susceptance).

- The analysis may be performed without data on a specific operating point of the system (i.e., a full power flow solution is not required).

- The calculations are not complicated and can be performed using ordinary consumer level computers and Matlab software.

We have applied this method to large system models and have achieved understandable results (reviewed by experts). A typical distribution of importance rankings, as presented for the PJM and WECC models in this report, is characterized by a sharp transition region corresponding to relatively few elements, separating the zero-valued connections to radial loads and a region with elements that have intermediate TIER value. We also note that the TIER value for an element declines on average with voltage level, however there is considerable overlap. Below we show again the distribution of TIER values, but color the points by the voltage level of the element (using the highest value for transformers). To make the points easier to study, we vertically offset the points associated with each voltage level. The points associated with EHV lines appear more densely to the left of the plot (higher importance value), and the lowest voltage elements appear more densely to the right. It is clear on this plot that there is much overlap of TIER values for the various voltage levels. Therefore, it is difficult to distinguish between many of the branch elements solely by their voltage level of operation. System topology is important.
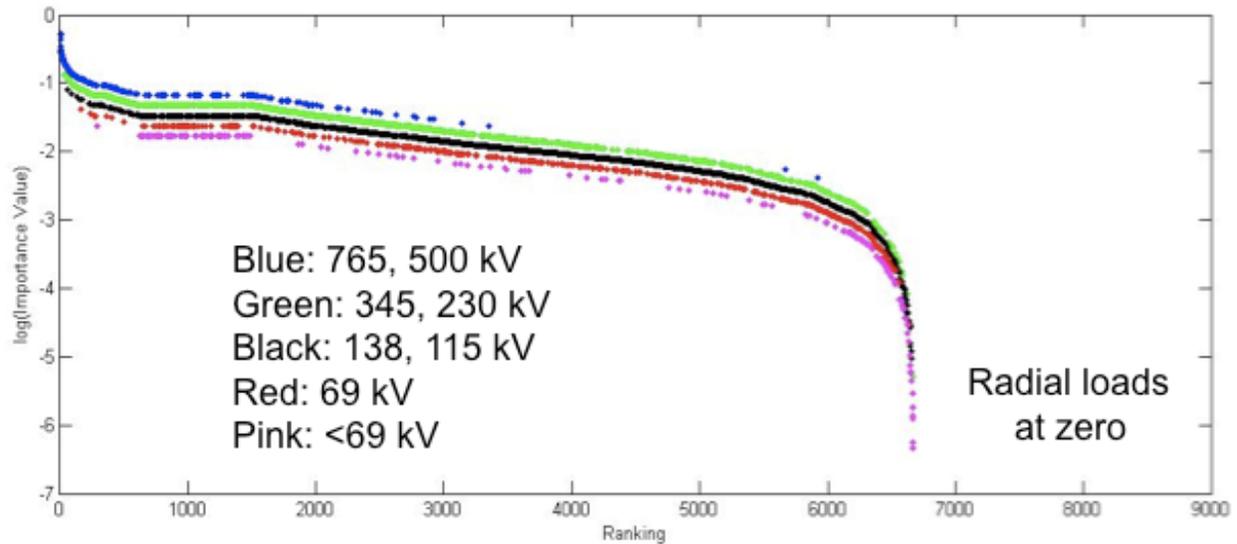
Figure 4. 1 Distribution of TIER values for PJM System Model – offset to show values for different voltage levels.

As it is the purpose of this research to develop a method to 1) develop a process to distinguish those facilities that should not be considered part of the Bulk-Power System from those facilities that should be considered part of the Bulk-Power System, 2) identify the elements needed to operate each of the electric interconnections, and 3) rank the importance of those elements, we offer one technically sound, objective input to policy makers concerning the classification of those elements. We understand that this research was not intended to address control systems and that some elements supplying radial loads (or portions of) may be included in the Bulk-Power System because of their control or Protection Systems.   The elements with a zero-valued TIER, which are single connections to radial loads, could be classified as having characteristics of local distribution elements.

We conclude with a few comments about models.

- This method is model-based, and hence relies on an appropriate model for input. Generally, more detail is better. Elements may only be ranked if they appear in the model. There needs to be some discussion about the specific elements to include for the purpose of classifying elements.

- There is an issue with equivalents – simplified representations for parts of the grid outside a study area of interest. These equivalents may affect the accuracy of these calculations, especially near the edge of the study region. Therefore, we recommend applying this method to a model of the entire Interconnect. There is no computational limitation that should prevent such an analysis.

- Network shunt elements, High Voltage DC, and Static VAR Systems or similar power systems that are series or shunt connected with active control via electronics are not included in this analysis. Future research may expand this sensitivity approach to include such elements if needed. However, it is likely that the classification of many of such elements as bulk power system elements will be obvious, and consistent with the classification of the neighboring connected branch elements.

- Further research is warranted to consider normalizing the metric to allow for absolute comparisons between different networks. Presently the TIER metric uses "raw" sensitivities, and the resulting values should only be used for comparison of the elements within the network used to calculate these values.

- Further research could also focus on application of the TIER metric to study critical system facilities. In the present work, we focus on distinguishing distribution elements, at the low end of the TIER scale. It would be valuable to compare how the metric ranks critical elements at the high end, relative to other impact-based analyses.