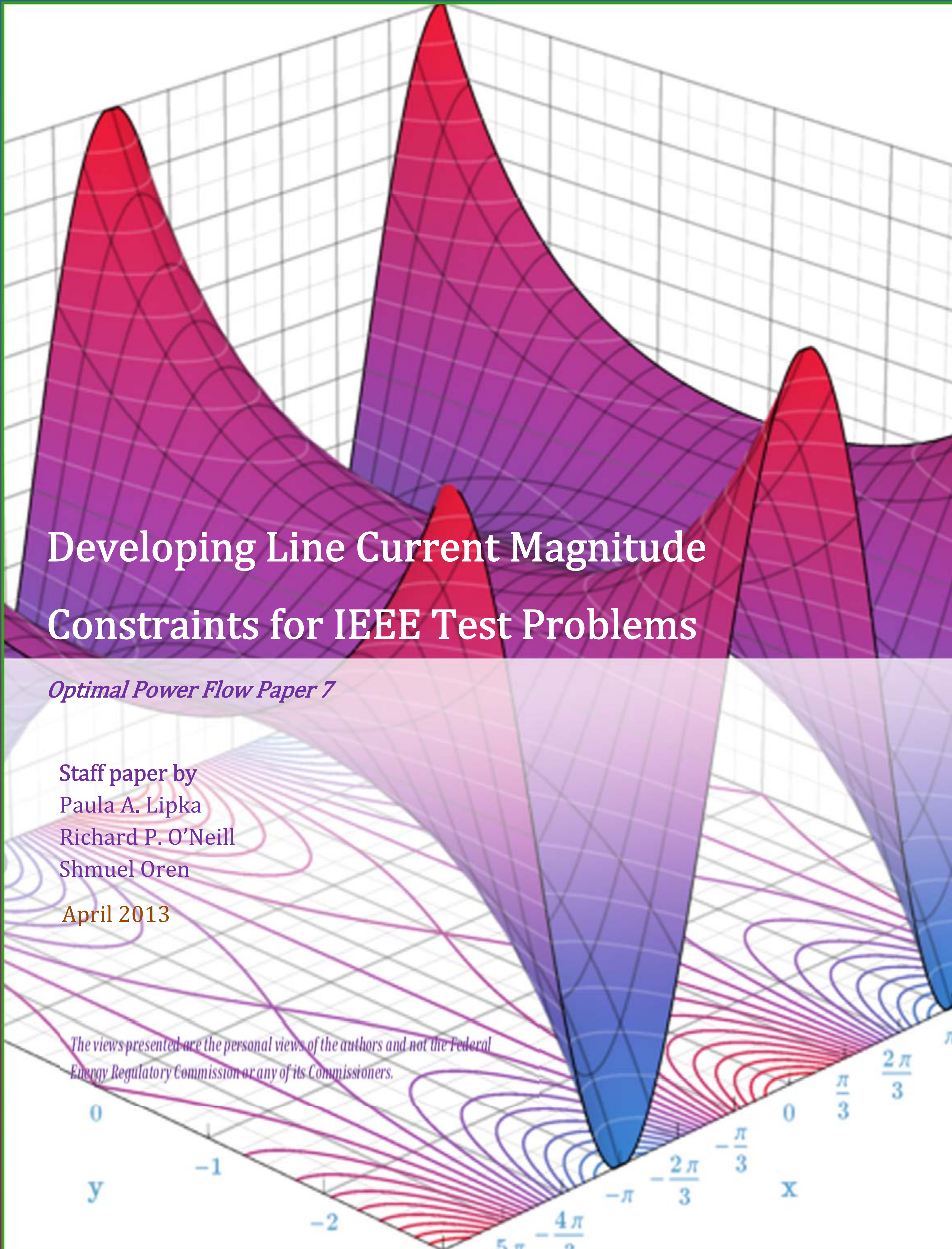

Developing Line Current Magnitude Constraints for IEEE Test Problems

Optimal Power Flow Paper 7

Staff paper by
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April 2013

Abstract and Executive Summary

All power system operators ensure their systems adhere to thermal limits on transmission lines in order to avoid line deformation. Also, thermal limits are used as surrogates for voltage stability. The IEEE test problems do not include data on these limits. The purpose of this paper is to present a simple method for constructing current magnitude constraints and to report on the computational properties in solving the resulting problems. This paper finds limits on the maximum allowable current magnitude that result in a feasible solution for the 14-bus, 30-bus, 57-bus, and 118-bus IEEE test problems. For each test problem, one single limit is applied to all lines that makes the optimal solution without these limits infeasible. For each problem we develop a 'tight' and a 'loose' constraint. We solve the resulting problem using the current voltage formulation. Different test problems exhibit different characteristics. Including these constraints in the ACOPF increases the solution time between 2 to 20 times and costs (objective function) up to 25 percent.

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Table of Contents

1. Introduction

The amount of current that can flow through power system transmission assets (referred to here as lines) is limited by thermal restrictions. The thermal ratings of the transmission are functions of the materials that compose the assets. The excessive heat caused by the line current can deform and degrade transmission lines and cause them to sag. Heat losses are proportional to current magnitude squared. In addition, current magnitude constraints are often used as surrogates for other constraints such as voltage stability. However, most IEEE test problems do not include current magnitude limits on the transmission lines even though they are an important aspect in model testing.

In the absence of these constraints, one approach is to put in constraints based on physical characteristics. Often, there is little information about the lines. It takes considerable time to develop constraints based on physical characteristics, and the result may not be binding constraints.

The purpose of this paper is to develop a methodology for creating line current magnitude constraints using a set of the IEEE test problems. We are interested in creating binding constraints based on maximum current magnitude rather than on apparent power on the lines or on voltage angle differences. The approach we employ is to create constraints from the optimal solution without these constraints. With these constraints, the resulting power flow problem has a feasible solution. Subsequent testing helps to understand how constraining the line flows affects the resulting power flow solution and solution time.

2. Notation

Variables and parameters are indexed over buses denoted by subscripts n and m. Transmission lines are indexed by terminal buses n and m and k. For a complex variable or parameter, the superscript r denotes the real portion and the superscript j denotes the imaginary portion. For example, if $x = a+jb$, $x^r = a$, $x^j = b$ *where* $j = (-1)^{1/2}$.

Variables

- p_n real power injected at bus n
- q_n reactive power injected at bus n
- $v^r{}_n$ real part of the voltage at bus n
- $v^j{}_n$ imaginary part of the voltage at bus n
- v_n the voltage magnitude at bus n
- $i^r{}_n$ real part of the current injected at bus n
- $i^j{}_n$ imaginary part of the current injected at bus n
- i_n the current magnitude of injection at bus n
- $i^r{}_{nmk}$ real part of the current on line k at bus n to bus m

i^j_{nmk} imaginary part of the current on line k at bus n to bus m

i_{nmk} the current magnitude on line k at bus n to bus m

Parameters

$c^p{}_n(p_n)$ cost function of real power injected by a generator at bus n

$c^q{}_n(q_n)$ cost function of reactive power injected by a generator at bus n

$c^{pl}{}_n(p_n)$ linear cost function of real power by a generator at bus n

$c^{ql}{}_n(q_n)$ linear cost function of reactive power by a generator at bus n

b_{nmk} imaginary part of the admittance matrix for line k between n and m

g_{nmk} real part of the admittance matrix for line k between n and m

$p^{min}{}_n$ minimum required real power at bus n

$p^{max}{}_n$ maximum allowed real power at bus n

$q^{min}{}_n$ minimum required reactive power at bus n

$q^{max}{}_n$ maximum allowed reactive power at bus n

$v^{min}{}_n$ minimum required voltage magnitude at bus n

$v^{max}{}_n$ maximum allowed voltage magnitude at bus n

$i^{max}{}_{nmk}$ maximum allowed current magnitude on line k from bus n to bus m

3. Current-Voltage ACOPF Model

The current-voltage (IV) ACOPF formulation is used to find a set of voltages and currents at each bus and currents on each transmission line that minimize the objective function in terms of real and reactive power. More detail can be found in Cain et al (2012) and O'Neill et al (2012). The IV-ACOPF model is:

Minimize $\sum_n c^p{}_n(p_n) + c^q{}_n(q_n)$

Subject to

$$i^r{}_{nmk} = g_{nmk}(v^r{}_n - v^r{}_m) - b_{nmk}(v^j{}_n - v^j{}_m)$$

$$i^j{}_{nmk} = b_{nmk}(v^r{}_n - v^r{}_m) + g_{nmk}(v^j{}_n - v^j{}_m)$$

$$i^r{}_n = \sum_{mk} i^r{}_{nmk}$$

$$i^j{}_n = \sum_{mk} i^j{}_{nmk}$$

$$p_n = v^r{}_n i^r{}_n + v^j{}_n i^j{}_n$$

$$p^{min}{}_n \leq p_n \leq p^{max}{}_n$$

$$q_n = v^j{}_n i^r{}_n - v^r{}_n i^j{}_n$$

$$q^{min}{}_n \leq q_n \leq q^{max}{}_n$$

$$(v^r{}_n)^2 + (v^j{}_n)^2 \leq (v^{max}{}_n)^2$$

$$(v^{min}{}_n)^2 \leq (v^r{}_n)^2 + (v^j{}_n)^2$$

$$(i^r{}_{nmk})^2 + (i^j{}_{nmk})^2 \leq (i^{max}{}_{nmk})^2$$

The objective function for test problems is generally presented as a quadratic function, but many applications require linear bid functions. The IV-ACOPF with linearized objective function program has the same constraints as the nonlinear model, but the objective function is a step function approximation of the nonlinear objective function. In solving the ACOPF models, we allow for an infeasible answer that is penalized by the amount the system is infeasible. We add $c^{Penalty}(v^r, v^j, i^r, i^j, p, q)$ to the linearized objective function, where the quantity x^+ is equal to max(x, 0), that is, if x is positive, $x^+ = x$; if x is negative, then $x^+ = 0$. For example, if the real power is greater than the max, the objective function is penalized by that quantity times some cost; if it is less than or equal to the maximum, there is no penalty.

$$c^{Penalty}(v^r, v^j, i^r, i^j, p, q) = \sum_{nmk} cpen_n(v_n - v^{min}{}_n)^+ + cpen_n(v^{max}{}_n - v)^+ +$$

$$cpen_n(i_{nmk} - i^{max}{}_{nmk})^+ + cpen_n(p_n - p^{max}{}_n)^+ + cpen_n(p^{min}{}_n - p_n)^+ +$$

$$cpen_n(q^{min}{}_n - q_n)^+ + cpen_n(q_n - q^{max}{}_n)^+$$

The system infeasibilities could possibly occur because the voltage is above the maximum or below the minimum levels, because the current is above the maximum level, or because real or reactive power violate maximum or minimum limits, as detailed in the penalty cost. Here we set $cpen_n = 10^5$. A problem is declared infeasible if the solution sets $c^{Penalty}$ to be greater than 10^{-3}.

4. Methods

We considered two methods for creating line current magnitude constraints. In both these methods, we solved the alternating current optimal power flow model without line constraints and extracted the optimal current magnitudes of each line from the optimal solution.

In the first method, we constrained the current magnitude on each line to be a fraction (less than 1) of its optimal current magnitude in the unconstrained problem. However, this method did not always return feasible solutions. This is likely because the unconstrained optimization minimizes losses by lowering current magnitudes subject to all other constraints in the model. Therefore, it may not be feasible for all line currents to be restricted to something lower than in the unconstrained solution.

In the second method, we constrained the current magnitude on each transmission line to some fraction of the highest optimal current magnitude over all lines in the unconstrained problem. This method returned local optimal solutions (since the problem is non-convex) for some limits, but, if the current magnitude limit was too low, the solver could not find a feasible point. This current magnitude limit level varied widely depending on the test problem.

Procedure:

1. Solve the ACOPF without any thermal line constraints
2. Extract the optimal line current magnitudes i_{nmk}^* for current magnitude for line k at bus n to m.
3. Let i^{max*} = maximum $\{i_{nmk}^*\}$ over all n, m, and k.
4. Solve the thermally constrained problem by including the constraint $i_{nmk} \le f^* i^{max*}$ for all combinations of n, m, and k where f is a parameter with $0 < f < 1$.

5. Computational Analysis

The input model data used is from the 14, 30, 57, and 118-bus IEEE test system data at http://www.ee.washington.edu/research/pstca/index.html. The generator costs come from MATPOWER (see Zimmerman et. al. 2011). The quadratic cost parameters are shown in the appendix. Where there are multiple transmission lines between two nodes, the lines are aggregated into an equivalent single line between the two nodes. The current magnitude measurement is per unit. Table 1 summarizes the test problem characteristics.

Table 1: IEEE Test System Data

Buses	Lines	Generators		Demand	V^{max}	V^{min}	Best Known Value*
		Number	Total Capacity				
14	20	5	7.72	2.59	1.06	0.94	80.81
30	41	6	326.80	42.42	1.10	0.94	5.745
57	80	7	326.78	235.26	1.06	0.94	417.4
118	186	54	99.66	42.42	1.06	0.94	1297

*Quadratic Objective, Nonlinear Constraints

Hardware. The problems were solved on an Intel Xeon E7458 server with 8 64-bit 2.4GHz processors and 64 GB memory. Minor differences in solution times were recorded when the same problems were run at different times, but the differences were small enough to be considered background noise.

Software. The procedures were formulated in GAMS 23.6.2. The nonlinear solver used was IPOPT version 3.8.

Approach. The IV-ACOPF formulation, as described above, was used to solve the test cases shown below. For each test problem, a figure shows the impact on the line current magnitudes on test systems were examined with the current magnitude level at 75%, 80%, 90%, and 100% of the maximum current magnitude level in the test problem without line current magnitude constraints. The optimal solution may be infeasible, that is, have a penalty function greater than zero. A second figure shows the line index versus the line current magnitude where the line index is sorted from lowest to highest current magnitude level. The red solid line in each graph is the chosen tight current magnitude limit and the dashed green line is the chosen loose current magnitude limit. A cross walk from line number to the buses the line is connecting is in the appendix.

Testing Current Magnitude Constraints. From the testing described in the previous sections, the uniform rule restricting current magnitude to the same percentage of maximum current magnitude has a different impact on the different test cases. Therefore, we picked different percentages of current magnitudes for each case and created two sets of constrained problems: tight and loose. The tight current magnitude level is near the current magnitude level where the problem becomes infeasible. The loose current magnitude level is a level that constrains the problem but is farther away from the point of infeasibility. These levels are shown in Table 2.

Table 2: Tight and loose testing levels of current magnitude constraints

	14-bus	30-bus	57-bus	118-bus
Low Level (Tight Constraint): % of Maximum Optimal Current magnitude:	18.5	93.3	76.2	24.3
High Level (Loose Constraint): % of Maximum Optimal Current magnitude:	51.1	95.4	77.0	71.9
Low (Tight Constraint) Current magnitude Level	0.2264	0.3092	1.4027	0.9294
High (Loose Constraint) Current magnitude Level	0.6246	0.3162	1.4168	2.7536

14-Bus Problem. In the 14-bus system when current is not constrained, one line has nearly twice the current magnitude as the next highest current line. As the current magnitude constraint is decreased, the effect on other lines is shown in figures 1 and 2. The current magnitudes that change the least under restriction are the lines with comparatively lower current magnitudes. For the loose constraint, only one current has a binding constraint. For the tight constraint, four constraints are binding or near binding.

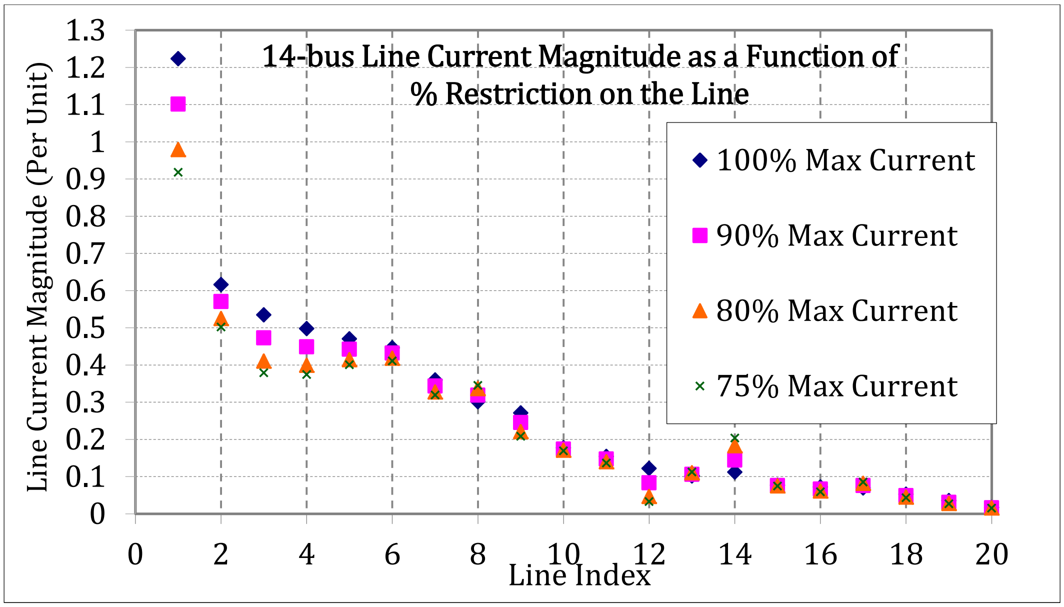

Figure 1. 'Optimal' 14-Bus Current Magnitudes per Line under Percent Restrictions

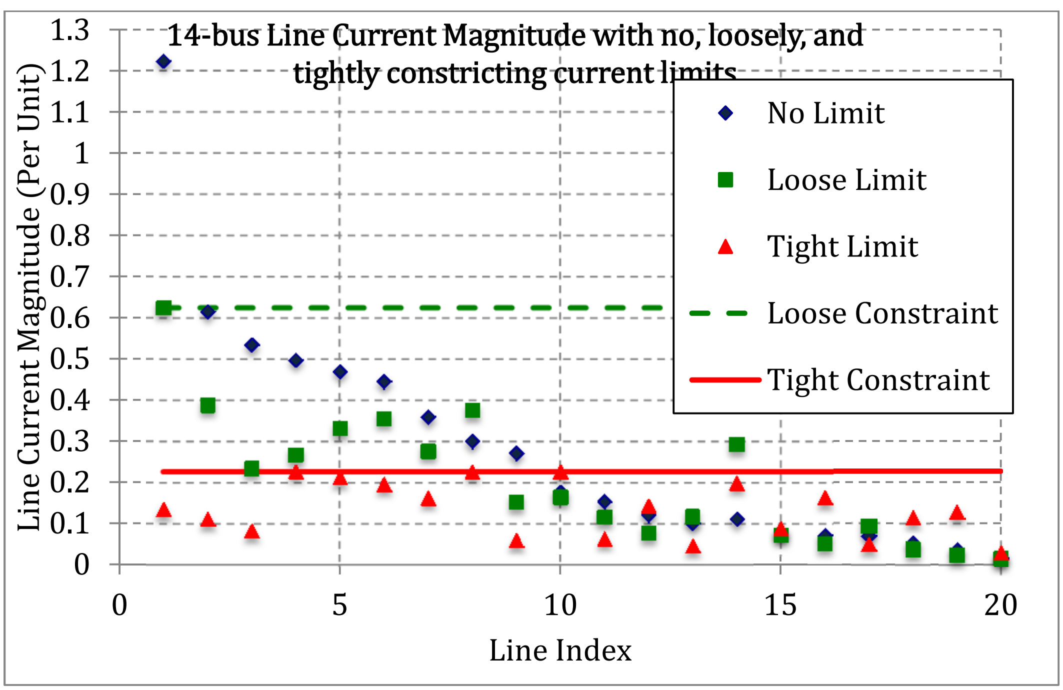

Figure 2. 14-Bus Optimal Current Magnitudes under the Loose and Tight Constraints.

30-Bus Problem. For the 30-bus system, the current level could not be restricted much lower than the maximum optimal current level (0.3314) before the system became infeasible. Restricting the current shifts the line current magnitudes more than in the 14-bus problem; there are many lines where current actually increases. While the difference between the tight (0.3092) and loose (0.3162) constraints is small, these restrictions do have a large impact on the resulting current through some of the lines. In both cases, only two lines have binding constraints

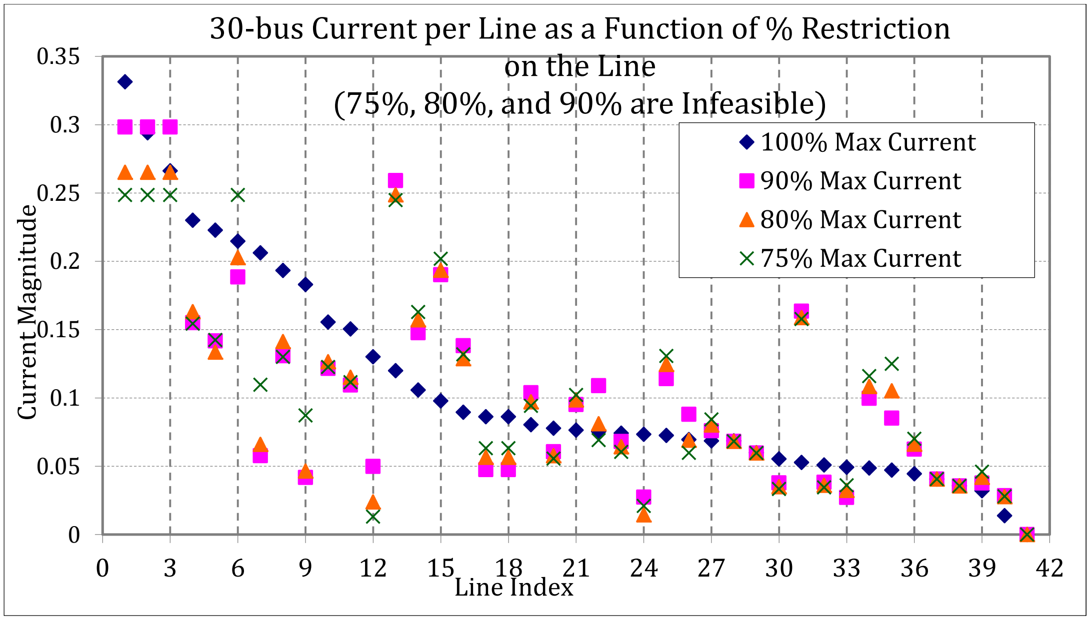

Figure 3. 'Optimal' 30-Bus Current Magnitudes per Line under Percent Restrictions

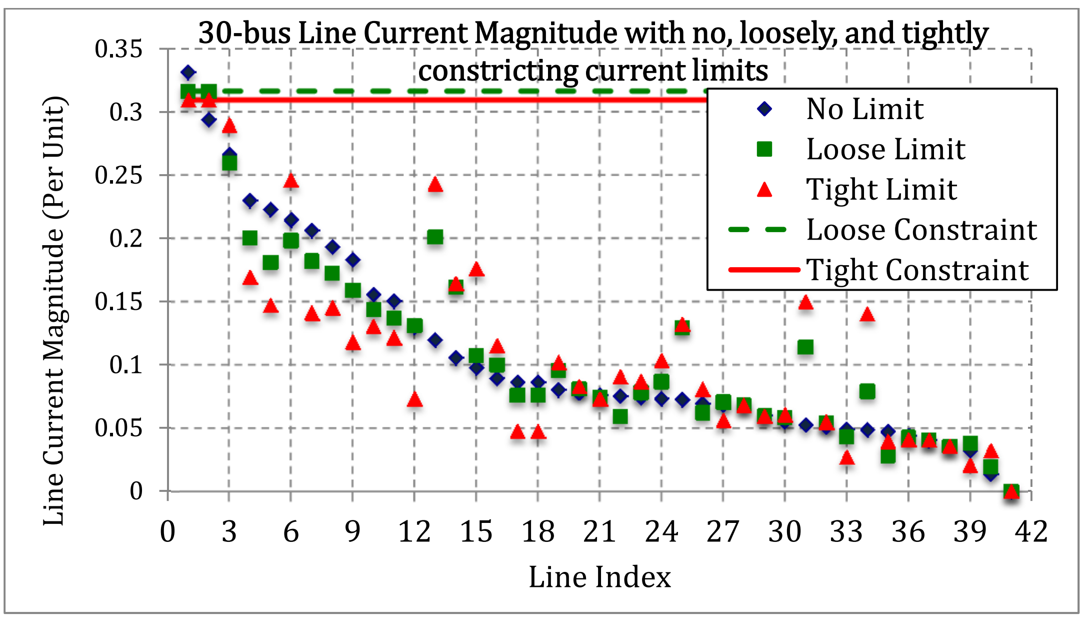

Figure 4. 30-Bus Optimal Current Magnitudes under the Loose And Tight Constraints.

57-Bus Problem. For the 57-bus system, the optimal solution has most of current magnitudes of half or less the highest current magnitude, similar to the 14-bus problem. While about half of the lines exhibit minimal change, the other half have currents that change significantly with the restriction. Like the 30-bus problem, the two current restrictions are close together, with 1.4027 for the loose limit and 1.4168 for the tight limit. In both cases, only two lines have binding constraints. However, even this small difference in limits greatly impacts the current on some lines.

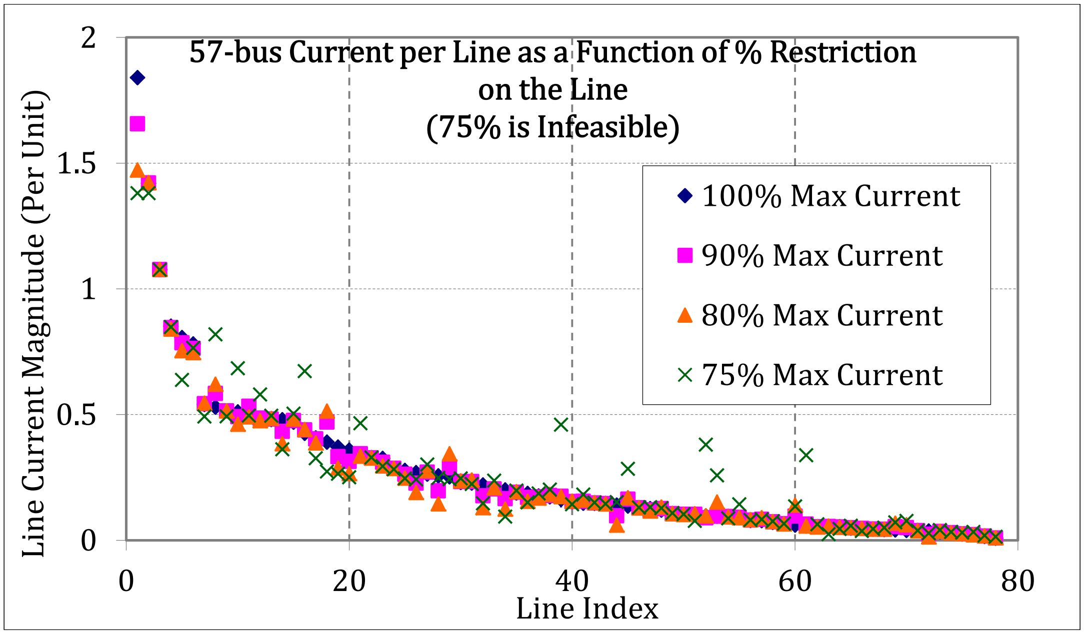

Figure 5. 'Optimal' 57-Bus Current Magnitudes per Line under Percent Restrictions

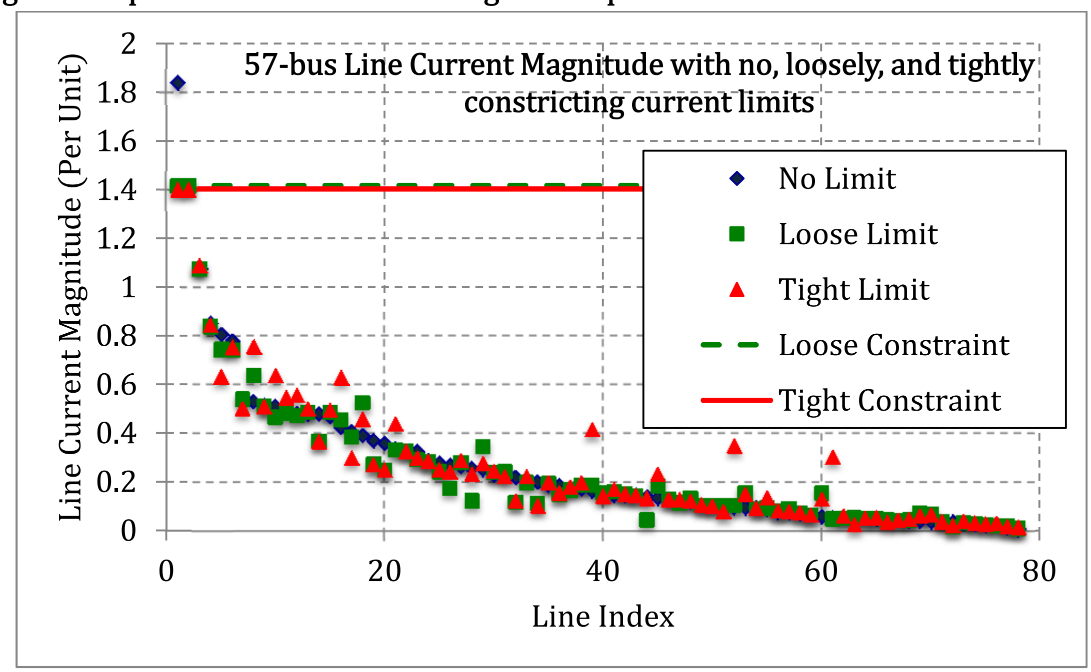

Figure 6. 57-Bus Optimal Current Magnitudes under the Loose and Tight Constraints.

118-Bus Problem. The 118-bus system has several lines with higher currents, similar to the 30-bus system. The current can be restricted greatly before the problem becomes infeasible. Reducing current by up to 75% of its original optimal maximum does not have much impact on line currents except the highest ones, as seen in Figure 7. Constraining the current with the loose limit impacts the currents slightly; however, constraining the current with the tight limit has a major impact on the current magnitudes, as in Figure 8. Under the loose constraint, only one line constraint is binding. Under the tight current magnitudes, 22 of the lines have the highest permissible current under this restriction, and even lines with lower currents exhibit big differences from the unrestricted and loose current cases, both higher and lower than before.

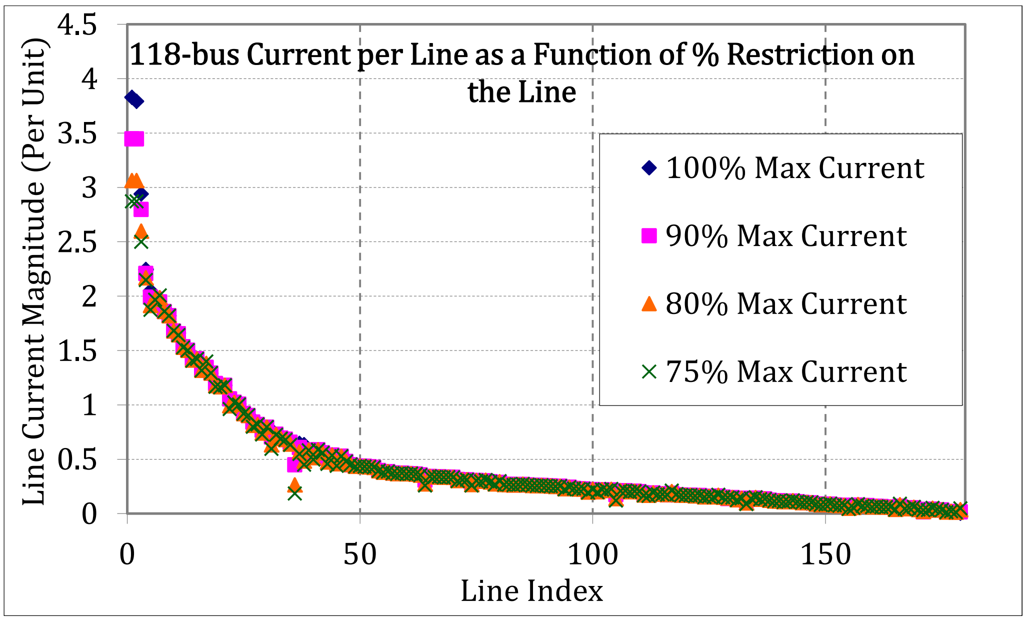

Figure 7. 'Optimal' 118-Bus Current Magnitudes per Line under Percent Restrictions

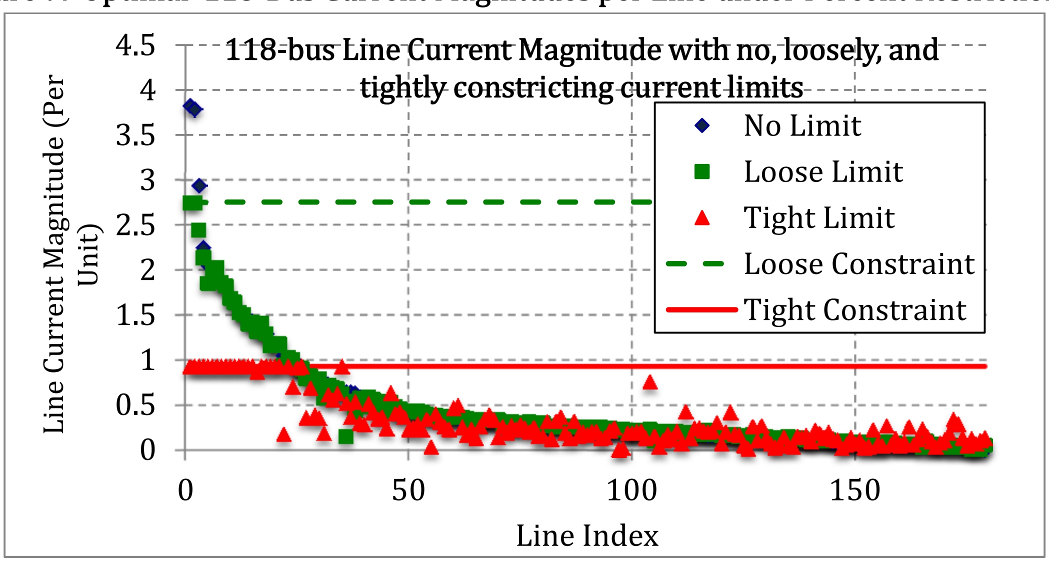

Figure 8. 118-Bus Optimal Current Magnitudes under Loose and Tight Constraints.

Normalized Objective Function Value as a Function of the Current Magnitude Limit

When the current magnitude is restricted, this limit is binding on at least one line. As shown in Figure 9, the different IEEE test cases exhibit infeasibility according to the solver at different percentages of their maximum current magnitude. The denominator of the normalized objective function is the objective function without constraints.

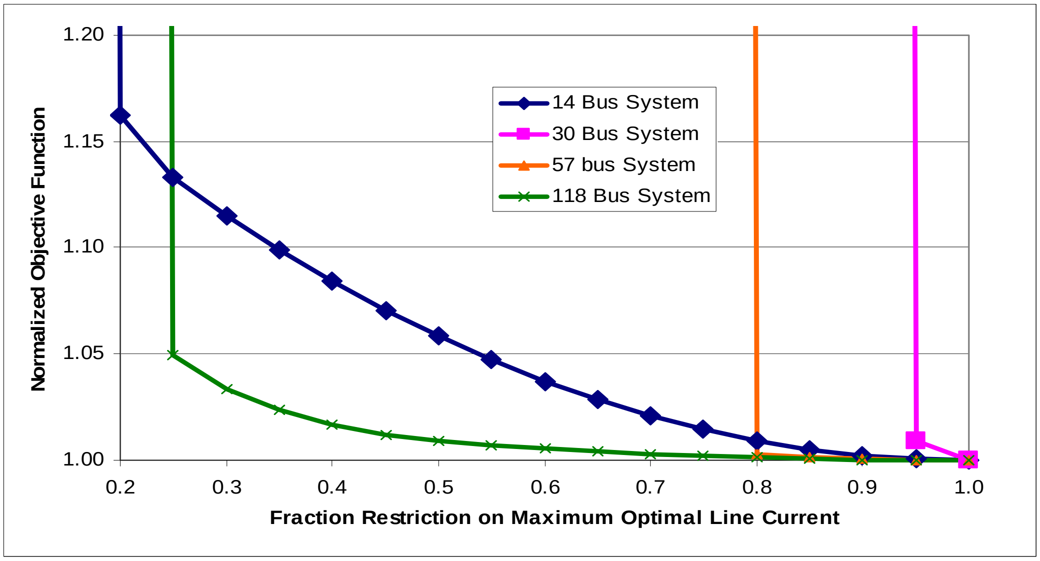

Figure 9. Normalized Objective Function versus Current Magnitude Limit

Normalized Solution Time as a Function of the Current Magnitude Limit

As shown in Figure 10, the computational time to solve (find a local optimal solution or declare the problem infeasible) the constrained problem only seems to grow greatly when the current magnitude is restricted to 5-15% of its original value. When it is restricted to 20% or more of its original value, the added constraints do not seem to increase or decrease the computational time predictably.

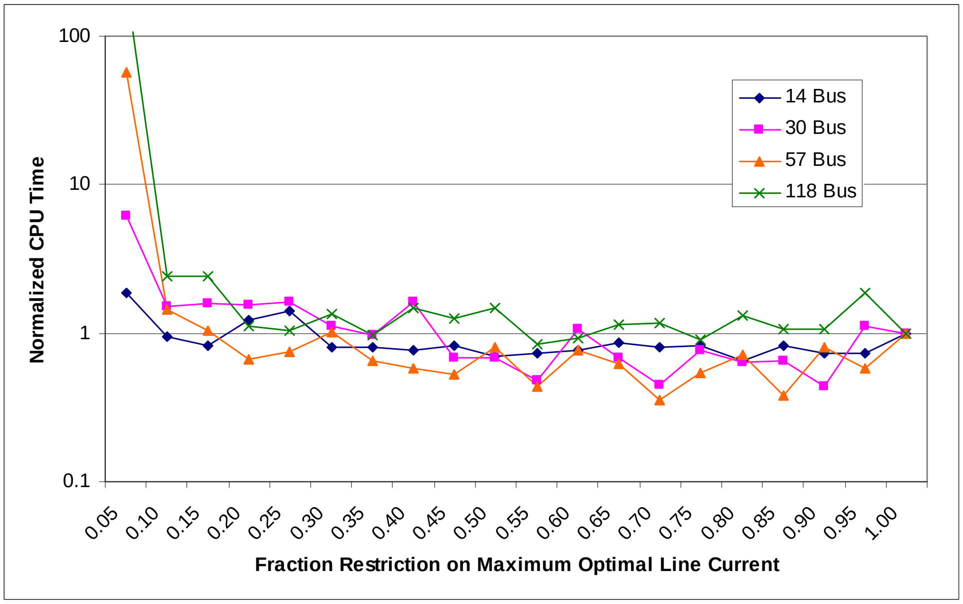

Figure 10: Impact of Current magnitude Constraints on CPU Time

Effect of Current Magnitude Constraints on the Optimal Value. When binding current magnitude constraints are added to the model, the feasible region shrinks and the objective function value increases, as shown in Figures 11 and 12 and Table 3. The objective function value of the 14-bus problem with the tight constraint is 30 percent greater. For other problems, the increases in objective function value with current magnitude constraints were 6 percent or less.

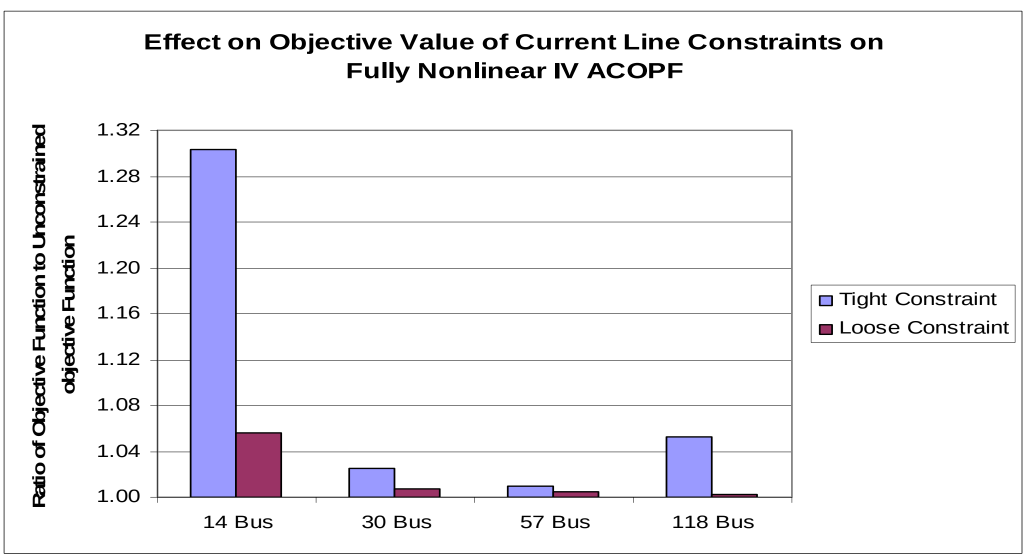

Figure 11. Value of the quadratic objective function with current magnitude constraints

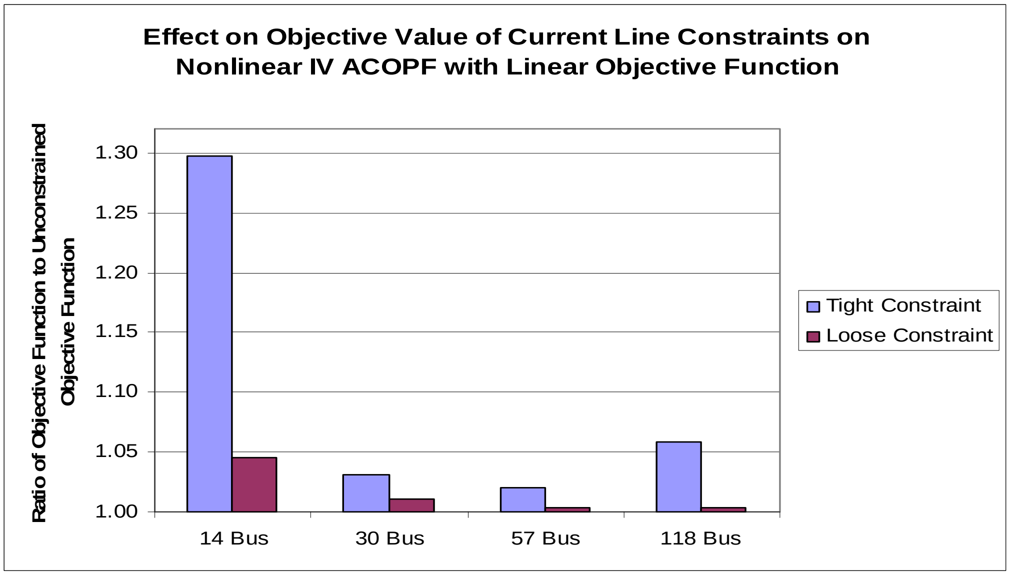

Figure 12. Value of the linear objective function with current magnitude constraints

Table 3: Objective function value for each test system and constraint type

Problem Type	Constraint type	14-bus	30-bus	57-bus	118-bus
Quadratic Objective Nonlinear Constraints	Tight Constraint	105.4	5.89	421.5	1364.9
	Loose Constraint	85.3	5.79	419.2	1300.1
	Unconstrained	80.8	5.75	417.4	1296.6
Linear Objective Nonlinear Constraints	Tight Constraint	107.4	6.10	432.2	1388.4
	Loose Constraint	86.5	5.98	425.5	1315.5
	Unconstrained	82.8	5.92	423.8	1311.5

Effect of Current Magnitude Constraints on the Solution Time. Current magnitude constraints also impact the solution time of each test system as shown in Figures 13 and 14 and Table 4. Adding constraints at least doubles the solution time of the problem. For the 30-bus and 118-bus problems, the more tightly constrained problem solves faster than the less constrained problem. For the 14-bus and 118-bus problems, the less constrained problem solves faster than the more constrained problem. The 57-bus and 118-bus solution time took significantly longer than the 14-bus and 30-bus systems, but solution time is not solely based on the number of buses. The linear objective function required considerably more time to solve.

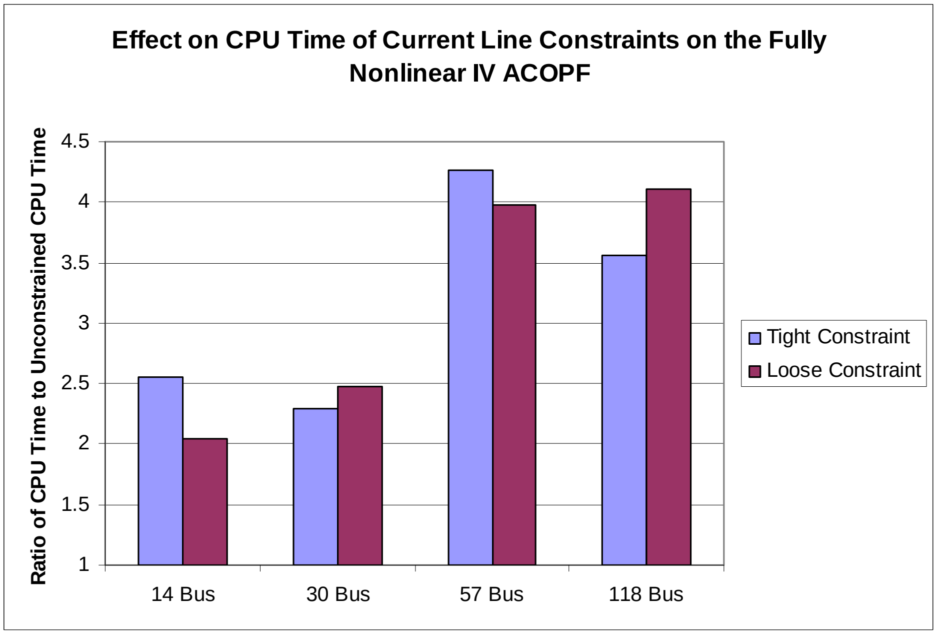

Figure 13. CPU Time with current magnitude constraints of quadratic objective

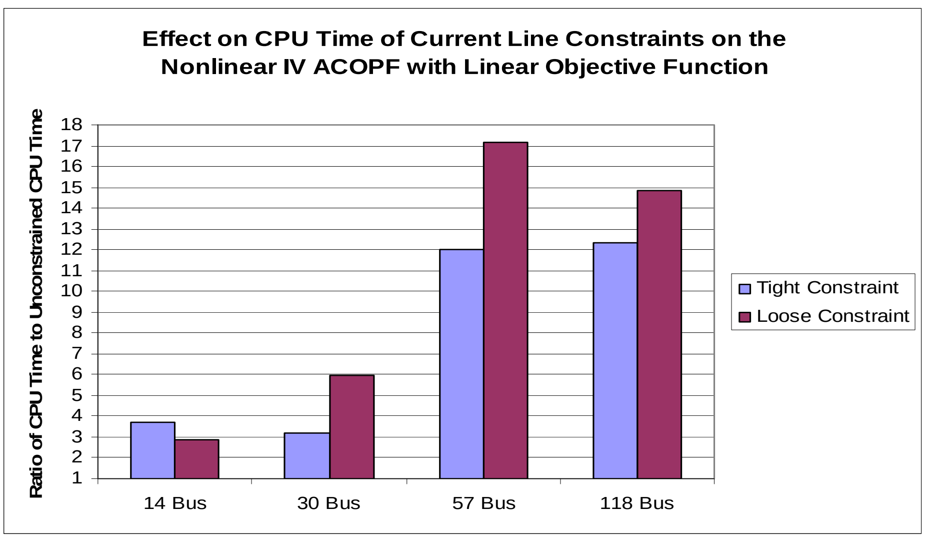

Figure 14. CPU Time of the current magnitude constrained linear objective

Table 4. CPU Time for all bus systems, constraint types, and model types

Problem Type	Constraint type	14-bus	30-bus	57-bus	118-bus
Quadratic Objective Nonlinear Constraints	Tight Constraint	0.72	1.80	8.34	28.95
	Loose Constraint	0.58	1.94	7.77	33.33
	Unconstrained	0.28	0.78	1.95	8.13
Linear Objective Nonlinear Constraints	Tight Constraint	1.16	1.55	8.42	32.20
	Loose Constraint	0.91	2.88	12.05	38.75
	Unconstrained	0.31	0.48	0.70	2.61

6. Conclusions

For each test problem, one single limit is applied to all lines that makes the optimal solution without these limits infeasible. We solve the resulting problem using the IV-ACOPF formulation. For each problem we develop a 'tight' and a 'loose' constraint.

For the 14, 30, 57 and 118-bus problems, creating line current magnitude constraints for the ACOPF problem can result in infeasible problems. As one tightens the current magnitude constraints, the objective function increases gradually at first, then increases exponentially near the point of infeasibility. Different test problems exhibit different characteristics in the line current magnitude distribution and at what current magnitude level constraint the problem becomes infeasible.

The current magnitude constraints also increase the solution time, although stricter constraints do not necessarily increase the solution time more than looser constraints. For problems like this case where the problem becomes infeasible quickly, it may work well to restrict only some of the line current magnitudes rather than all of them.

Including these constraints in the ACOPF increases the solution time between 2 to 20 times and objective function up to 25 percent.

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Appendix

The generator costs take on the form cost = aP+bP²+.0001*|Q|, where P is the real power and |Q| is the magnitude of reactive power q. We list all generator costs used for each test system.

Table A1: Generator Cost Coefficients for the 14, 30 and 57-bus Problems

14-bus	Cost Coefficient		30-bus	Cost Coefficient		57-bus	Cost Coefficient	
Generator	a	b	Generator	a	b	Generator	a	b
1	0.04303	20	1	0.02000	2.00	1	0.07758	20
2	0.25000	20	2	0.01750	1.75	2	0.01000	40
3	0.01000	40	13	0.02500	3.00	3	0.25000	20
6	0.01000	40	22	0.06250	1.00	6	0.01000	40
8	0.01000	40	23	0.02500	3.00	8	0.02222	20
			27	0.00834	3.25	9	0.01000	40
						12	0.03226	20

Table A2: 118-bus Generator Costs

Generator	Cost Coefficient		Generator	Cost Coefficient		Generator	Cost Coefficient	
	a	b		a	b	r	a	b
1	0.01000	40	46	0.52632	20	87	2.50000	20
4	0.01000	40	49	0.04902	20	89	0.01647	20
6	0.01000	40	54	0.20833	20	90	0.01000	40
8	0.01000	40	55	0.01000	40	91	0.01000	40
10	0.02222	20	56	0.01000	40	92	0.01000	40
12	0.11765	20	59	0.06452	20	99	0.01000	40
15	0.01000	40	61	0.06250	20	100	0.03968	20
18	0.01000	40	62	0.01000	40	103	0.25000	20
19	0.01000	40	65	0.02558	20	104	0.01000	40
24	0.01000	40	66	0.02551	20	105	0.01000	40
25	0.04545	20	69	0.01936	20	107	0.01000	40
26	0.03185	20	70	0.01000	40	110	0.01000	40
27	0.01000	40	72	0.01000	40	111	0.27778	20
31	1.42857	20	73	0.01000	40	112	0.01000	40
32	0.01000	40	74	0.01000	40	113	0.01000	40
34	0.01000	40	76	0.01000	40	116	0.01000	40
36	0.01000	40	77	0.01000	40			
40	0.01000	40	80	0.02096	20			
42	0.01000	40	85	0.01000	40			

Table A3: 14 and 30-bus Line Index Mapping

14-bus		
Line Index	Buses Connected	
	To	From
1	1	2
2	1	5
3	2	3
4	4	5
5	2	4
6	5	6
7	2	5
8	7	9
9	4	7
10	6	13
11	4	9
12	3	4
13	9	14
14	7	8
15	6	12
16	6	11
17	9	10
18	13	14
19	10	11
20	12	13

30-bus		
Line Index	Buses Connected	
	To	From
1	6	8
2	12	13
3	21	22
4	2	6
5	4	6
6	1	2
7	1	3
8	2	4
9	3	4
10	5	7
11	2	5
12	4	12
13	27	28
14	25	27
15	15	23
16	6	7
17	9	10
18	6	9
19	8	28
20	10	20
21	15	18
22	10	21
23	10	17
24	12	15
25	24	25
26	10	22
27	12	16
28	27	30
29	27	29
30	19	20
31	6	28
32	12	14
33	6	10
34	22	24
35	23	24
36	18	19
37	25	26
38	29	30
39	16	17
40	14	15
41	9	11
35	23	24
36	18	19
37	25	26
38	29	30
39	16	17
40	14	15
41	9	11

Table A4: 57-bus Line Index Mapping

57-bus			57-bus		
	Buses Connected			Buses Connected	
Line Index	To	From	Line Index	To	From
1	8	9	40	50	51
2	14	46	41	38	44
3	10	51	42	35	36
4	24	26	43	24	25
5	7	8	44	4	5
6	7	29	45	6	7
7	15	45	46	41	43
8	1	15	47	52	53
9	9	55	48	54	55
10	3	15	49	11	41
11	1	2	50	41	42
12	1	17	51	38	49
13	13	49	52	12	16
14	9	11	53	3	4
15	46	47	54	25	30
16	14	15	55	49	50
17	6	8	56	34	35
18	2	3	57	53	54
19	9	10	58	32	34
20	9	13	59	41	56
21	1	16	60	10	12
22	4	18	61	12	17
23	28	29	62	18	19
24	11	43	63	48	49
25	27	28	64	30	31
26	5	6	65	36	40
27	44	45	66	20	21
28	4	6	67	32	33
29	12	13	68	37	39
30	37	38	69	22	38
31	38	48	70	22	23
32	11	13	71	56	57
33	13	14	72	23	24
34	9	12	73	39	57
35	36	37	74	42	56
36	26	27	75	40	56
37	29	52	76	19	20
38	47	48	77	31	32
39	13	15	78	21	22

Table A5: 118-bus Line Index Mapping

118-bus			118-bus			118-bus			118-bus		
Line Index	Buses		Line Index	Buses		Line Index	Buses		Line Index	Buses	
	To	From		To	From		To	From		To	From
1	9	10	46	77	78	91	51	52	136	51	58
2	8	9	47	66	67	92	104	105	137	105	106
3	5	8	48	45	49	93	93	94	138	100	101
4	37	38	49	76	77	94	2	12	139	34	43
5	17	30	50	22	23	95	103	104	140	105	107
6	49	66	51	59	61	96	75	77	141	29	31
7	26	30	52	49	50	97	55	59	142	32	114
8	89	92	53	47	69	98	20	21	143	70	71
9	68	116	54	100	106	99	34	36	144	19	20
10	68	69	55	56	59	100	82	83	145	70	75
11	64	65	56	94	95	101	37	40	146	27	32
12	59	63	57	100	104	102	99	100	147	43	44
13	89	90	58	75	118	103	62	67	148	106	107
14	23	25	59	85	88	104	68	81	149	7	12
15	80	81	60	74	75	105	1	3	150	60	62
16	63	64	61	94	100	106	54	59	151	40	42
17	38	65	62	92	93	107	101	102	152	52	53
18	65	66	63	59	60	108	40	41	153	3	12
19	25	26	64	11	12	109	78	79	154	90	91
20	25	27	65	21	22	110	12	117	155	12	16
21	77	80	66	62	66	111	23	24	156	15	33
22	4	5	67	48	49	112	17	31	157	32	113
23	60	61	68	80	97	113	27	115	158	105	108
24	61	64	69	110	111	114	80	99	159	56	58
25	69	75	70	49	69	115	96	97	160	24	72
26	69	70	71	37	39	116	53	54	161	86	87
27	34	37	72	92	94	117	16	17	162	55	56
28	88	89	73	45	46	118	41	42	163	71	73
29	15	17	74	11	13	119	47	49	164	98	100
30	42	49	75	110	112	120	91	92	165	39	40
31	5	6	76	103	110	121	82	96	166	13	15
32	23	32	77	27	28	122	77	82	167	1	2
33	100	103	78	103	105	123	85	86	168	108	109
34	69	77	79	35	37	124	70	74	169	71	72
35	65	68	80	31	32	125	83	84	170	114	115
36	8	30	81	80	98	126	56	57	171	15	19
37	5	11	82	50	57	127	33	37	172	54	56
38	30	38	83	6	7	128	95	96	173	35	36
39	49	54	84	61	62	129	17	113	174	19	34
40	17	18	85	44	45	130	94	96	175	109	110
41	79	80	86	83	85	131	18	19	176	76	118
42	85	89	87	80	96	132	92	100	177	24	70
43	4	11	88	92	102	133	12	14	178	54	55
44	49	51	89	46	47	134	46	48	179	14	15
45	3	5	90	84	85	135	28	29			