

Guidelines for the Structural Evaluation of Cargo Tank Rollover Damage Protection Devices

Criteria for DOT406, DOT407, DOT412 cargo tank motor vehicles are included in §178.345. This section requires that guards "...must be designed and installed to withstand loads equal to twice the weight of the loaded cargo tank motor vehicle applied as follows: normal to the tank shell (perpendicular to the tank surface); and tangential (perpendicular to the normal load) from any direction. The stresses shall not exceed the ultimate strength of the material of construction. These design loads may be considered to be uniformly distributed and independently applied. If more than one rollover protection device is used, each device must be capable of carrying its proportionate share of the required loads and in each case at least one-fourth the total tangential load. The design must be proven capable of carrying the required loads by calculations, test and a combination of test and calculations."

Analysis Procedures

One common type of rollover damage protection device is an inverted "U" shaped member made of tubular elements as shown in Figure 1. The devices are frequently installed at a stiffened cross section of the tank. In some designs, gusset plates are used between the tank wall and the rollover device to increase capacity to carry horizontal longitudinal load. Some designs have used a third leg to increase capacity.

The two-legged inverted "U" devices behave as a frame when subjected to vertical load or horizontal transverse load. Such frames can be analyzed using moment distribution procedures, frame analysis from Roark and Young, approximate methods of frames, or finite element methods. For horizontal longitudinal load, these two legged devices can be idealized as cantilever beams.

A difficulty, common to all of the suggested methods of analysis, is idealization of the strength and stiffness of the tank at its juncture with the rollover device.

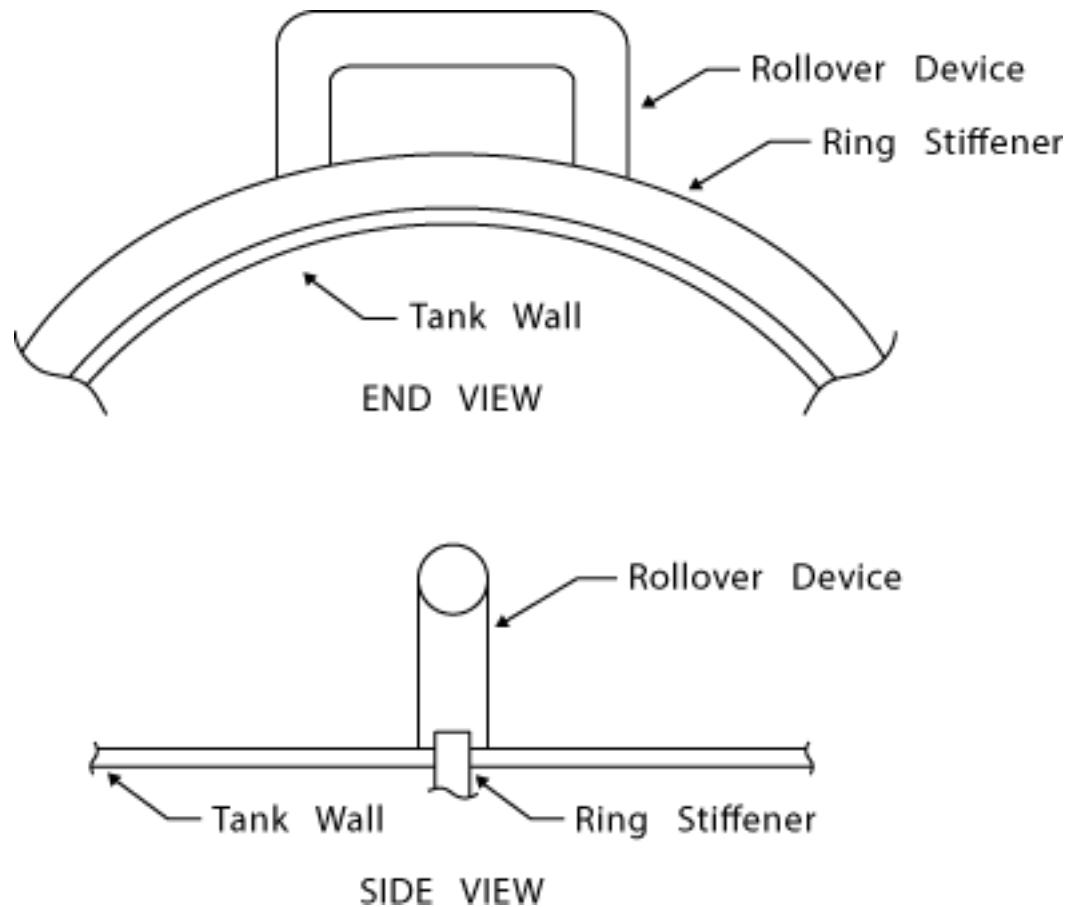
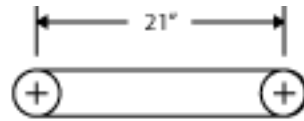


Figure 1. Inverted "U" rollover protection device.

Example 1. Analysis of inverted "U" using moment distribution.

The two-legged inverted "U" device shown in Figure 2 is one of three devices used on a DOT 400 Series cargo tank motor vehicle whose total weight is 49,700 lbs. The device is made of 3-inch standard pipe with an ultimate strength of 65.2 ksi. Properties of the pipe cross section are given in Figure 2. Each device must withstand a load of $2W/n$ in which $2(49,700)/3 = 33.13$ kips.



3-in. PIPE
 $I_y = 3.017 \text{ in}^4$
 $A = 2.229 \text{ in}^2$
 OD = 3.50 in.

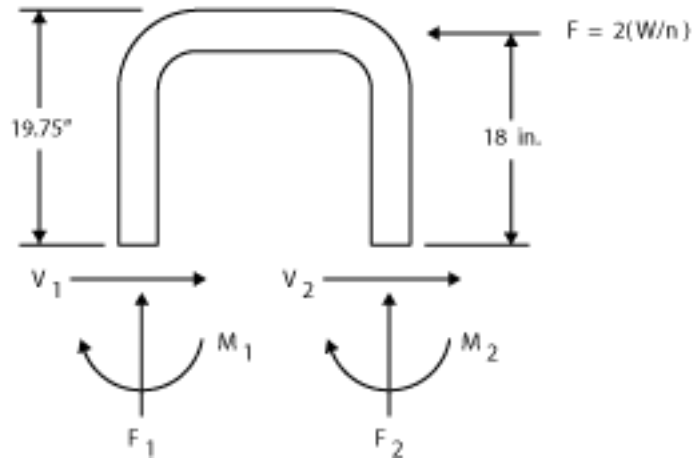


Figure 2. Two-legged, inverted "U" rollover device for DOT 400 Series cargo tank.

For the first analysis, it is assumed that the tank wall provides rigid support for the overturn device at the points of attachment. This is a reasonable assumption but is not accurate because the tank walls are flexible.

A moment distribution analysis table for "side sway" with assumed fixed-end moments of +100 in-k is presented in Figure 3. The results show final joint moments of +77.9 in-k at joints A and D and 56.4 in-k at joints B and C. Further analysis to impose static equilibrium for the horizontal force of 33.13 kips is required.

For member AB:

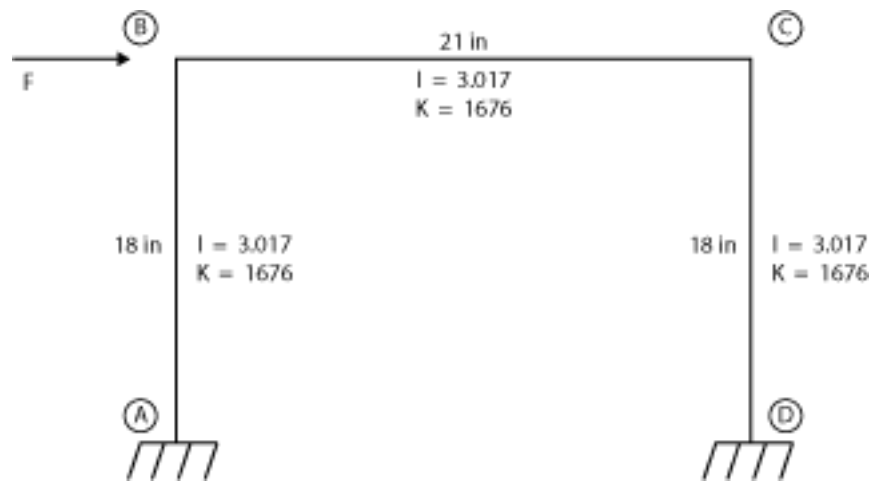
$$\sum M_B = 0: -H_A (18) + 77.9 + 56.4 = 0$$

$$H_A = \frac{77.9 + 56.40}{18}$$

$$H_A = 7.46k$$

also

$$H_D = 7.46k$$



Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	.1676	.1676	.1437	.1437	.1676	.1676
Dist. F.	0	.538	.462	.462	.538	0
F.E.M	+100	+100	0	0	+100	+100
1st Dist	0	-53.8	-46.2	-46.2	-53.8	0
CO	-26.9	0	-23.1	-23.1	0	-26.9
2nd Dist	0	-12.4	+10.7	+10.7	+12.4	0
CO	+6.2	0	+5.3	+5.3	0	+6.2
3rd Dist	0	-2.8	-2.5	-2.5	-2.8	0
CO	-1.4	0	-1.2	-1.2	0	-1.4
4th Dist	0	+0.6	+0.6	+0.6	+0.6	0
Σ	+77.9	+56.4	-56.4	-56.4	+56.4	+77.9

Figure 3. Moment distribution analysis of rollover device with horizontal transverse load and bottom ends of vertical members fixed.

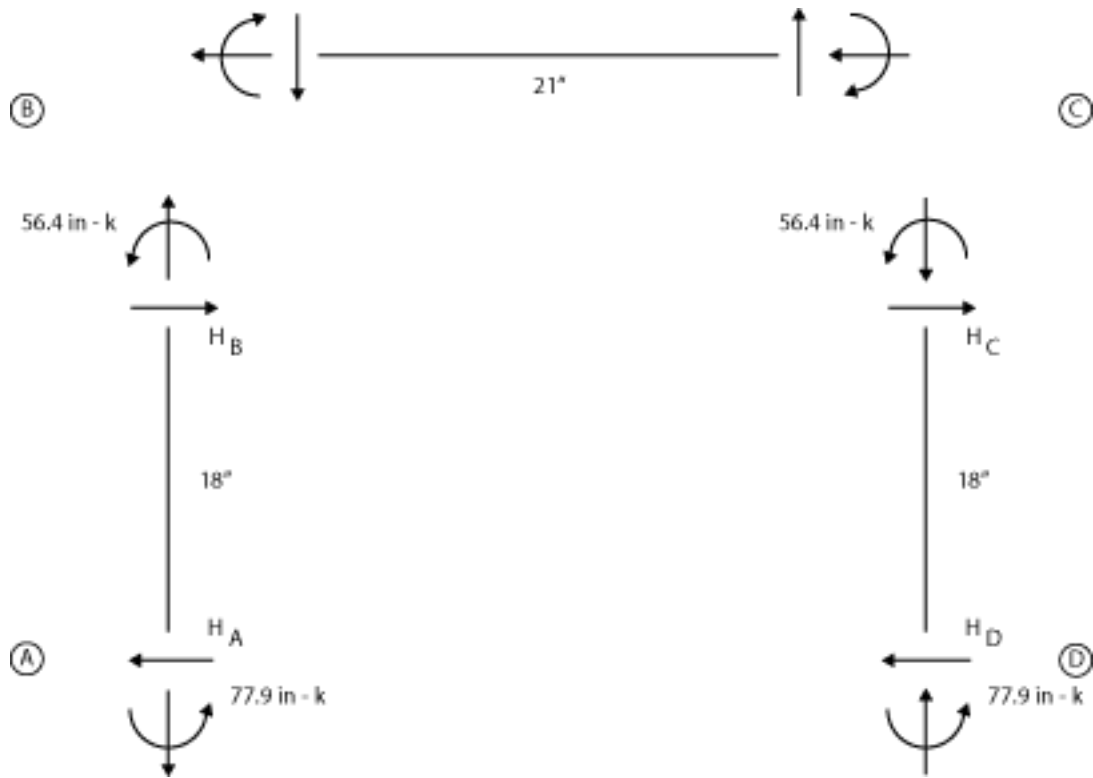


Figure 4. Preliminary results of moment distribution analysis with assumed fixed end moments.

For entire structure:

$$\sum F_x = 0; F - 7.46 - 7.46 = 0$$

$$F = 14.92k$$

The adjustment factor that must be applied to the results in Figure 3 is:

$$\frac{33.13}{14.92} = 2.22$$

All moments (and forces) for the solution presented in Figure 3 must be multiplied by 2.22 for an actual load of 33.13 kips. The final adjusted moments at joint A and D are each 173 in-k as shown in Figure 5.

Further, static analysis results in axial forces of 11.9 kips and shear forces of 16.56 kips in each of the vertical members.

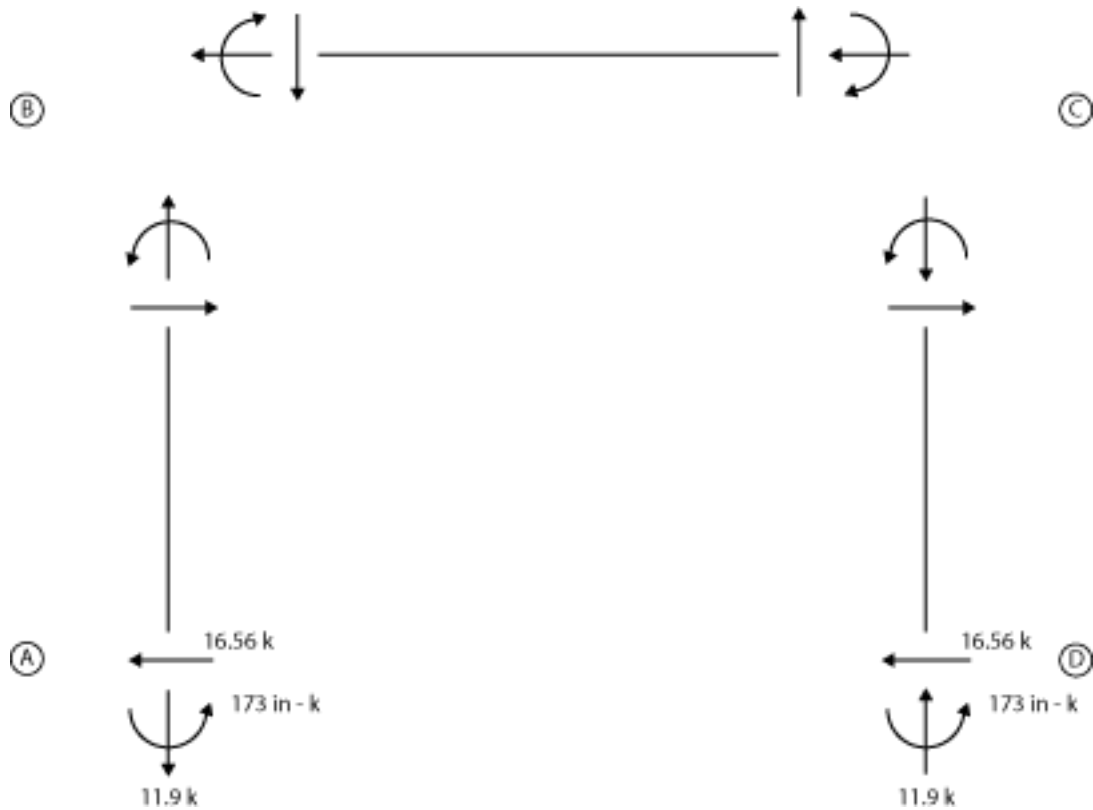


Figure 5. Final results of the moment distribution analysis of structure in figure 31.

The maximum axial stress in member A-B, near joint A, is:

$$Max S = \frac{P_A}{A} + \frac{M}{Z_e}$$

$$Max S = \frac{11.9}{2.229} + \frac{173(1.75)}{3.017}$$

$$Max S = 5.3 + 100.3$$

$$Max S = 105 \text{ ksi} > 65.2 \text{ ksi (No good)}$$

The axial stress in the horizontal member near joints B and C would be:

$$S = \frac{F}{A} + \frac{M}{Z_e}$$

$$M = 56.4(2.22)$$

$$M = 125.2$$

$$S = \frac{16.56}{2.229} + \frac{125.2(1.75)}{3.017}$$

$$S = 5.3 + 100.3$$

$$S = 80.0 \text{ ksi}$$

The axial stress vertical members near joints B and C would be:

$$S = \frac{F}{A} + \frac{M}{Z_e}$$

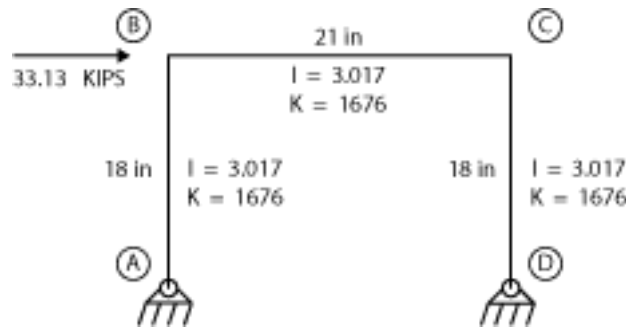
$$S = \frac{11.9}{2.229} + \frac{125.2(1.75)}{3.017}$$

$$S = 5.3 + 72.6$$

$$S = 77.9 \text{ ksi} > 65.2 \text{ ksi (No good)}$$

An analysis of the two leg-devices for horizontal transverse load using the Moment Distribution Method and the assumption that the tank shell provides no stiffness in bending in the legs of the device is presented in Figure 6. The final adjusted moments are shown in Figure 7. The computed maximum tensile or compressive stresses in the vertical legs of the device occur near joints B and C, and is 186 ksi.

If the effective stiffness of the tank wall were known and could be included in the analysis, the results would be expected to be somewhere between the two Moment Distribution solutions presented herein.



Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	.1676	.1676	.1437	.1437	.1676	.1676
Dist. F.	1	.538	.462	.462	.538	1
F.E.M	+100	+100	0	0	+100	+100
1st Dist	-100	-53.8	-46.2	-46.2	-53.8	-100
CO	-26.9	-50	-23.1	-23.1	-50	-26.9
2nd Dist	+26.9	+39.3	+33.8	+33.8	+36.3	+26.9
CO	+19.6	+13.4	+16.9	+16.9	+13.4	+19.6
3rd Dist	-19.6	-16.3	-14	-14	-16.3	-19.6
CO	-8.1	-9.8	-7.0	-7.0	-9.8	-8.1
4th Dist	+8.1	-9	+7.8	+7.8	+9.0	+8.1
CO	4.5	4.0	3.9	3.9	4.0	4.5
5th Dist	-4.5	-4.2	-3.6	-3.6	-4.2	-4.5
CO	-2.1	-2.2	-1.8	-1.8	-2.2	-2.1
6th Dist	+2.1	+2.1	+1.8	+1.8	+2.1	+2.1
Σ	0	31.5	-31.5	-31.5	31.5	0

Figure 6. Moment distribution analysis of rollover device with horizontal transverse load and bottom ends of vertical members hinged.

It is noticed that for each of the two solutions by Moment Distribution presented on the previous pages, the sum of the moments for the two ends of a vertical member of the device is 298 in-k at the bottom and 125 in-k at the top. For the second solution (with the ends of the legs pinned) the moments are zero at the bottom and 298 in-k at the top.

For a more accurate solution wherein partial fixity of the tank wall would be accurately included, the moments would also total 298 in-k. The degree of fixity that would result in the lowest maximum stresses in the legs of the device would be one that caused the moments to be equal at each end of the leg (i.e., $298 \div 2 = 149$ in-k). In this case, the stresses in each end of the legs would be:

$$S = \frac{F}{A} + \frac{M}{Z_e}$$

$$S = \frac{F}{A} + \frac{149(1.75)}{3.017}$$

$$S = \frac{F}{A} + 86 \text{ ksi}$$

F/A would be somewhere between 5.3 and 12.7 ksi which would make the maximum stress over 90 ksi.

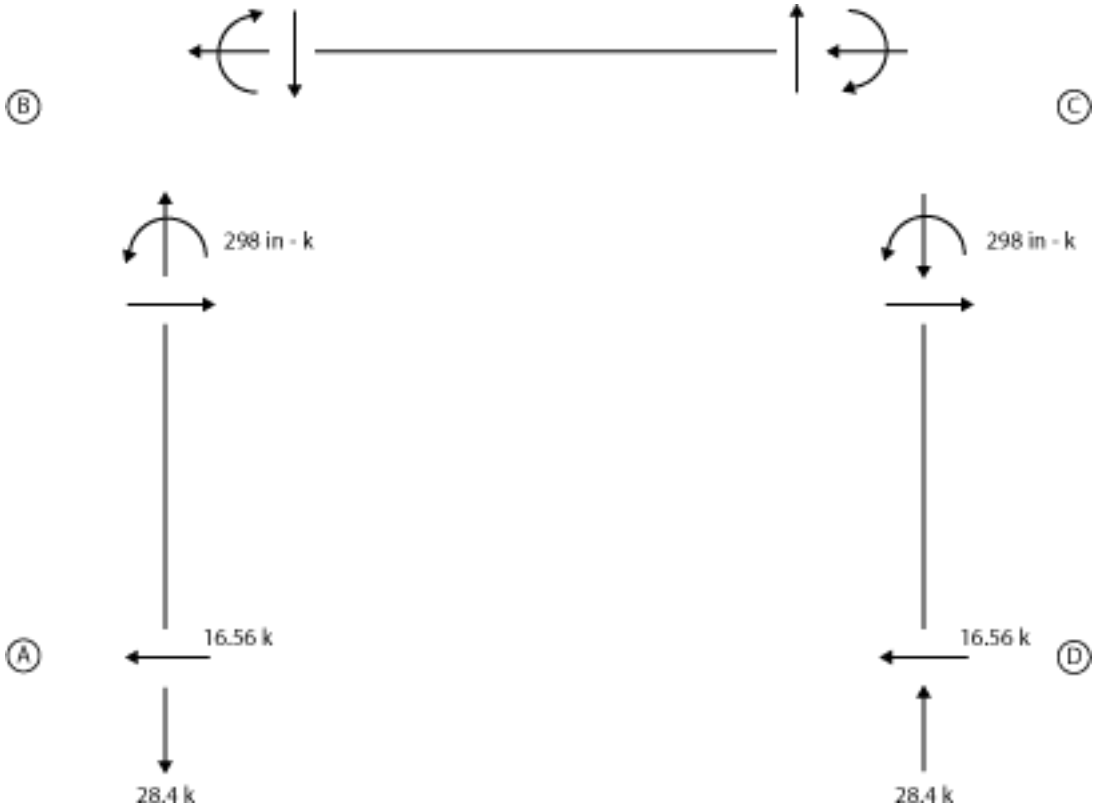


Figure 7. Results of moment distribution analysis of structure in Figure 6.

Roark and Young Analysis

Equations for structural analysis of single bent frames are given in Roark and Young for various combinations of member sizes, member lengths and support conditions. For a frame subjected to a concentrated side load and with fixed supports as shown in Figure 8, the following six general equations for coefficients are given:

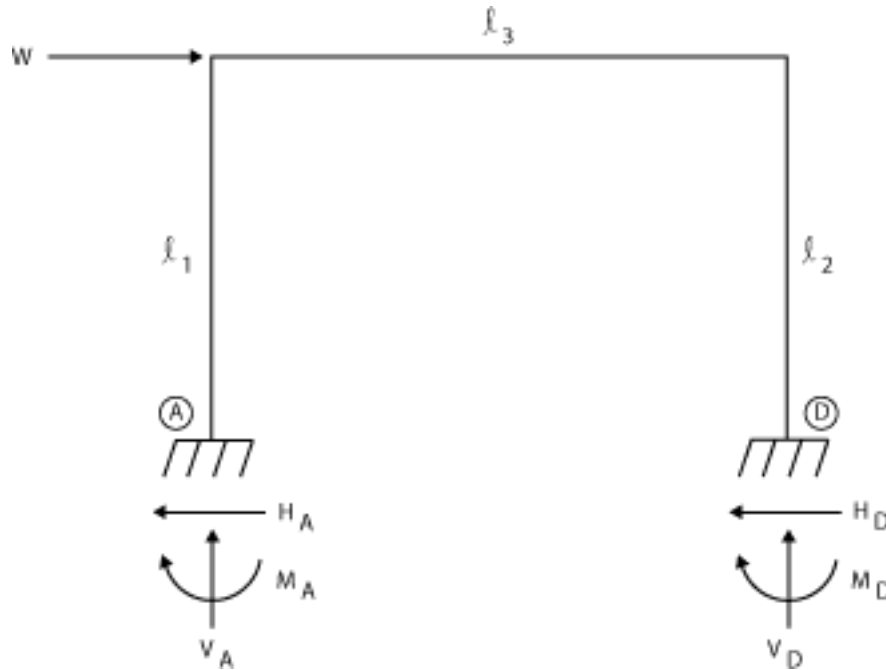


Figure 8: Idealization of rollover protection device for analysis using formulas from Roark and Young.

$$C_{HH} = \frac{l_1^3}{3E_1I_1} + [l_1^3 - (l_1 - l_2)^3]3E_2I_2 + \frac{l_1^2l_3}{E_3I_3}$$

$$C_{HV} = C_{VH} = \frac{l_2l_3(2l_1 - l_2)}{2E_2I_2} + \frac{l_1l_3^2}{2E_3I_3}$$

$$C_{HM} = C_{MH} = \frac{l_1^2}{2E_1I_1} + \frac{l_2(2l_1 - l_2)}{2E_2I_2} + \frac{l_1l_3}{E_3I_3}$$

$$C_{VV} = \frac{l_2 l_3^2}{E_2 I_2} + \frac{l_3^3}{3E_3 I_3}$$

$$C_{VM} = C_{MV} = \frac{l_2 l_3}{E_2 I_2} + \frac{l_3^2}{E_3 I_3}$$

$$C_{MM} = \frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2} + \frac{l_3}{E_3 I_3}$$

For the frame shown in Figure 8, the equations reduce to:

$$C_{HH} = \frac{2l_1^3}{3EI} + \frac{l_1^2 l_3}{EI}$$

$$C_{HV} = \frac{l_1^2 l_3}{2EI} + \frac{l_1 l_3^2}{EI}$$

$$C_{HM} = \frac{l_1^2}{E_2 I_2} + \frac{l_1 l_3}{EI}$$

$$C_{VV} = \frac{l_2 l_3^2}{EI} + \frac{l_3^3}{3EI}$$

$$C_{VM} = \frac{l_2 l_3}{EI} + \frac{l_3^2}{2EI}$$

$$C_{MM} = \frac{l_1}{EI} + \frac{l_2}{EI} + \frac{l_3}{EI}$$

Factors for loads are computed using the three following equations:

$$LF_H = W \left(C_{HM} - l_1 C_{MH} + \frac{l_1^3}{6E_1 I_1} \right)$$

$$LF_V = W (C_{VH} - l_1 C_{VM})$$

$$LF_M = W \left(C_{MH} - l_1 C_{MM} + \frac{l_1^2}{2E_1 I_1} \right)$$

Coefficients and factors for loads can then be used in the following equations to solve for reactions at the left support. Reactions at the right support can then be evaluated using equations of static equilibrium.

$$C_{HH}H_A + C_{HV}V_A + C_{HM}M_A = LF_H$$

$$C_{VH}H_A + C_{VV}V_A + C_{VM}M_A = LF_V$$

$$C_{MH}H_A + C_{MV}V_A + C_{MM}M_A = LF_M$$

Example 2. Structural analysis of frame using equations from Roark and Young.

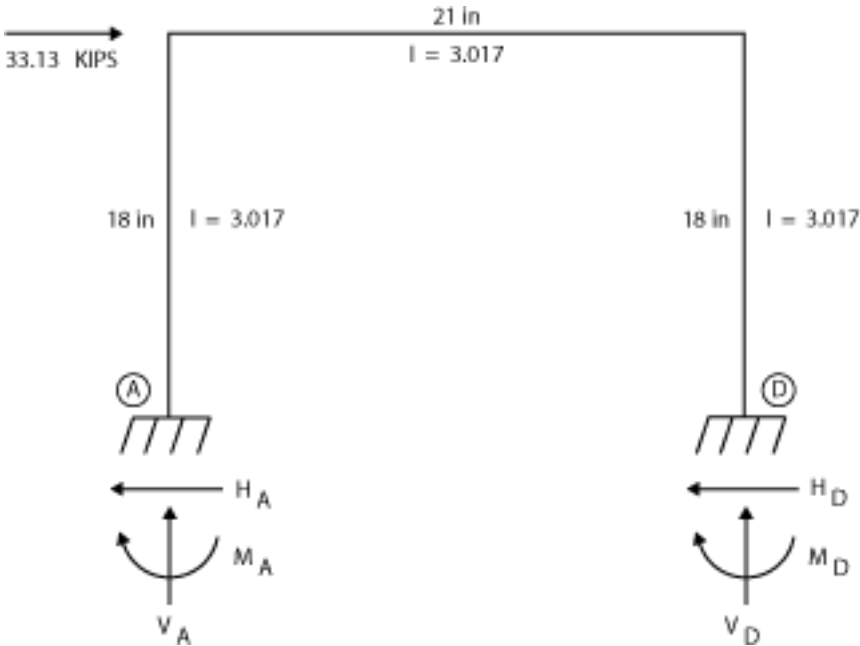


Figure 9. Idealization of frame for analysis using formulas from Roark and Young.

$$\text{Use } E = 29 \times 10^6$$

$$\text{Then } EI = 29 \times 10^6(3.017)$$

$$EI = 87.5 \times 10^6$$

$$C_{HH} = \frac{2l_1^3}{3EI} + \frac{l_1^2 l_3}{EI}$$

$$C_{HH} = \frac{2(18)^3}{3(87.5 \times 10^6)} + \frac{(18)^2(21)}{87.5 \times 10^6}$$

$$C_{HH} = 122.2 \times 10^6$$

$$C_{HV} = \frac{l_1^2 l_3}{2EI} + \frac{l_1 l_3^2}{EI}$$

$$C_{HV} = \frac{(18)^2(21)}{2EI} + \frac{18(21)^2}{EI}$$

$$C_{HV} = 84.24 \times 10^{-6}$$

$$C_{HM} = \frac{l_1^2}{E_2 I_2} + \frac{l_1 l_3}{EI}$$

$$C_{HM} = \frac{(18)^2}{87.5 \times 10^6} + \frac{18(21)}{87.5 \times 10^6}$$

$$C_{HM} = 8.023 \times 10^{-6}$$

$$C_{VV} = \frac{l_2 l_3^2}{EI} + \frac{l_3^3}{3EI}$$

$$C_{VV} = \frac{18(21)}{87.5 \times 10^6} + \frac{21^3}{3(87.5 \times 10^6)}$$

$$C_{VV} = 126.0 \times 10^{-6}$$

$$C_{VM} = \frac{l_2 l_3}{EI} + \frac{l_3^2}{2EI}$$

$$C_{VM} = \frac{18(21)}{87.5 \times 10^6} + \frac{21^2}{2(87.5 \times 10^6)}$$

$$C_{VM} = 6.84 \times 10^{-6}$$

$$C_{MM} = \frac{l_1}{EI} + \frac{l_2}{EI} + \frac{l_3}{EI}$$

$$C_{MM} = \frac{18}{87.5 \times 10^6} + \frac{18}{87.5 \times 10^6} + \frac{21}{87.5 \times 10^6}$$

$$C_{MM} = 0.651 \times 10^{-6}$$

$$LF_H = W \left(C_{HM} - l_1 C_{HM} + \frac{l_1^3}{6EI} \right)$$

$$LF_H = W \left(C_{HM} - l_1 C_{HM} + \frac{l_1^3}{6EI} \right)$$

$$LF_H = W(-11.09 \times 10^{-6})$$

$$LF_V = W(C_{VH} - l_1 C_{VM})$$

$$LF_V = W(84.24 \times 10^{-6} - 18(6.84 \times 10^{-6}))$$

$$LF_V = W(-38.88 \times 10^{-6})$$

$$LF_M = W \left(C_{MH} - l_1 C_{MM} + \frac{l_1^2}{2E_1 I_1} \right)$$

$$LF_M = W \left(8.023 \times 10^{-6} - 18(0.651 \times 10^{-6}) + \frac{(18^2)}{2(87.5 \times 10^6)} \right)$$

$$LF_M = W(-1.847 \times 10^{-6})$$

$$C_{HH}H_A + C_{HV}V_A + C_{HM}M_A = LF_H$$

$$122.2 \times 10^{-6}H_A + 84.24 \times 10^{-6}V_A + 8.023 \times 10^{-6}M_A = -11.09 \times 10^{-6}W$$

$$C_{VH}H_A + C_{VV}V_A + C_{VM}M_A = LF_V$$

$$84.24 \times 10^{-6}H_A + 126 \times 10^{-6}V_A + 6.84 \times 10^{-6}M_A = -38.88 \times 10^{-6}W$$

$$C_{MH}H_A + C_{MV}V_A + C_{MM}M_A = LF_M$$

$$8.023 \times 10^{-6}H_A + 6.84 \times 10^{-6}V_A + 0651 \times 10^{-6}M_A = -1.847 \times 10^{-6}W$$

These equations are solved simultaneously to result in:

$$H_A = +16.6k$$

$$V_A = -11.9k$$

$$M_A = -173 \text{ in } k$$

The combined stress adjacent to joint A due to axial force and bending moment is:

$$\text{Max } S = \frac{V_A}{A} + \frac{M_A}{Z_e}$$

$$\text{Max } S = \frac{11.9}{2.229} + \frac{173}{1.724}$$

$$\text{Max } S = 5.34 + 100.34$$

$$\text{Max } S = 105.68 \text{ ksi}$$

Approximate Methods

Approximate methods of structural analysis can be used to analyze two legged inverted “U” rollover protection devices. One such method is the portal method. In the portal method, it is assumed that points of inflection (zero moment) occur at mid-height of the vertical members. The horizontal shear will be the same value in each of the two vertical members will be the same.

The assumptions stated above make the structure statically determinate and the analysis can be completed using statics.

Example 3. Analysis of inverted “U” using portal (approximate) method.

$$V_1 = V_2$$

$$F_1 = F_2$$

$$M_1 = M_2$$

$$F_1 = \left(\frac{9}{21}\right)F$$

$$V_1 = \frac{F}{2}$$

$$M_1 = 9V_1 = 9\left(\frac{F}{2}\right)$$

$$\max S = \frac{F_1}{A} + \frac{M_1}{Z_e}$$

For the values given in Figure 4, the solution should be:

$$V_1 = \frac{F}{2} = \frac{33.13}{2} = 16.57k$$

$$F_1 = \left(\frac{9}{21}\right)33.13 = 14.2k$$

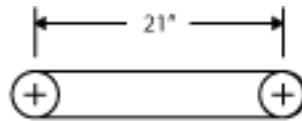
$$M_1 = 9V_1 = 9(16.57) = 149in k$$

$$\max S = \frac{F_1}{A} + \frac{M_1}{Z_e}$$

$$\max S = \frac{14.2}{2.229} + \frac{149}{1.724}$$

$$\max S = 6.4 + 86.4$$

$$\max S = 92.8 ksi$$



3-in. PIPE
 $I_y = 3.017 \text{ in}^4$
 $A = 2.229 \text{ in}^2$
 OD = 3.50 in.

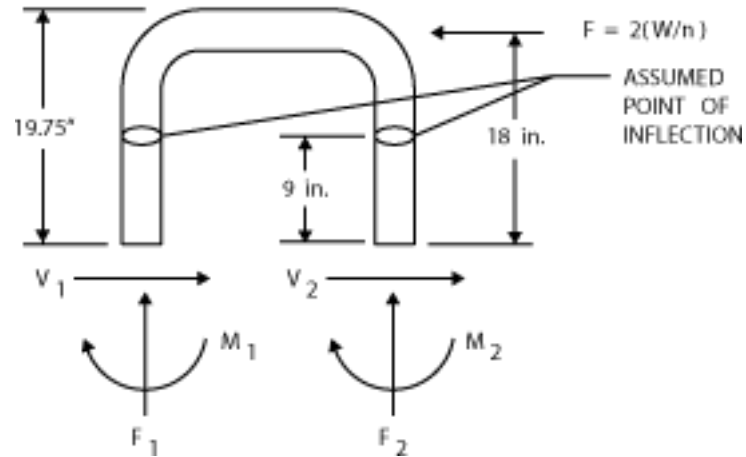


Figure 10. Idealization of rollover protection device for approximate analysis.

Example 4. Analysis of inverted “U” using finite elements.

Stresses in the rollover devices were further evaluated using a finite element analysis procedure. For these analyses, the ends of the legs of the devices were assumed fixed against rotation at the points of attachment to the tank wall.

For the two-leg device subjected to a horizontal transverse load of 33.13 kips the maximum stress is:

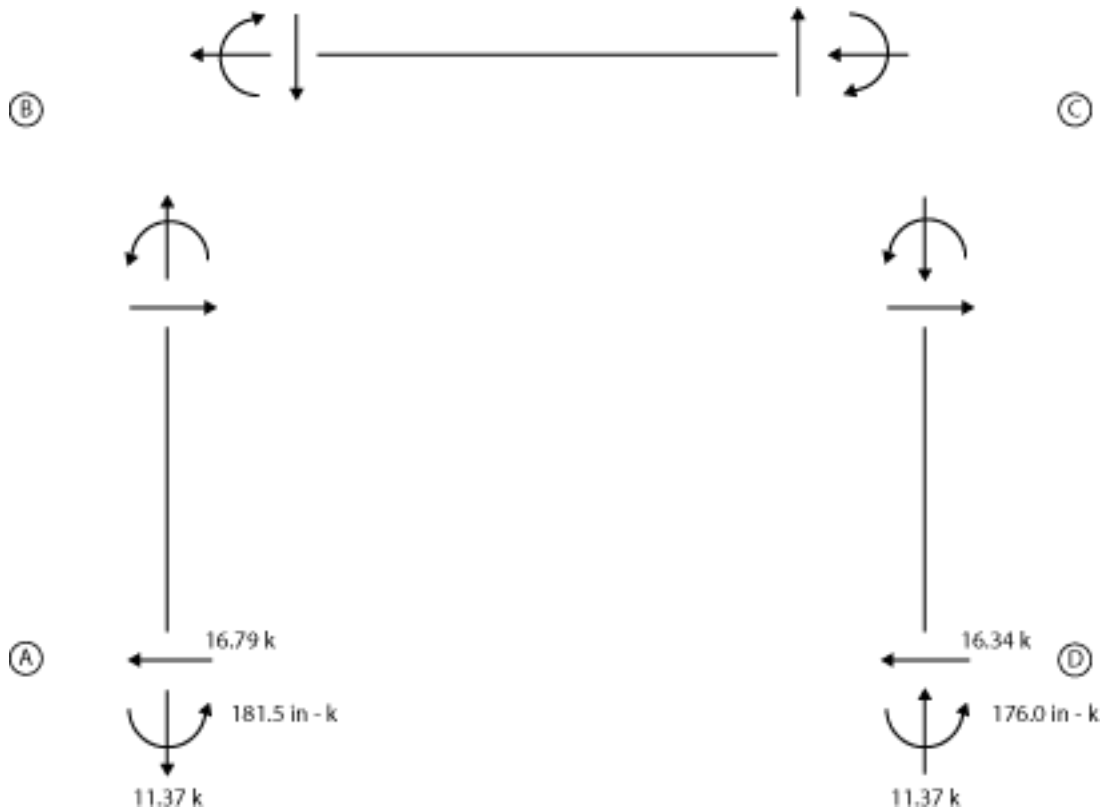
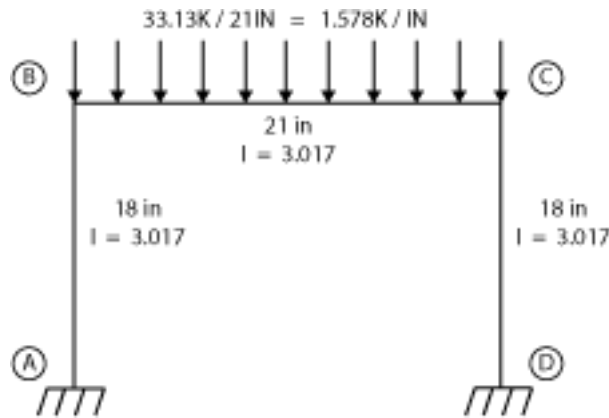


Figure 11. Results of finite elements analysis of device subjected to horizontal transverse load of 33.13 kips.

Method of Analysis	Maximum Normal Stress, ksi	Location of Maximum Normal Stress
Moment Distribution w/ ends fixed	105	Bottom end of vertical members
Moment Distribution w/ ends hinged	186	Top end of vertical members
Roark and Young w/ ends fixed	105	Bottom ends of vertical members
Portal Method	92.8	Both ends at vertical members
Finite Element method w/ ends fixed	110	Bottom end of vertical members

Table 1. Comparison of results of various analysis procedures for inverted “U” rollover protection device subjected to horizontal transverse load.

A moment distribution analysis can be performed for vertical load on the device. Idealization of the structure and load, and the moment distribution table are shown in Figure 12. Further static analysis results are shown in Figure 12. Further static analysis gives the results shown in Figure 13.



Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	.1676	.1676	.1437	.1437	.1676	.1676
Dist. F.	0	.538	.462	.462	.538	0
F.E.M	0	0	+58	-58	0	0
1st Dist	0	-31.2	-26.8	+26.8	+31.2	0
CO	-15.6	0	+13.4	-13.4	0	+15.6
2nd Dist	0	-7.2	-6.2	+6.2	+7.2	0
CO	-3.6	0	+3.1	-3.1	0	03.6
3rd Dist	0	-1.7	-1.4	+1.4	+1.7	0
CO	-9	-0	+7	-7	0	+9
4th Dist	0	-.4	-.3	+.3	+.4	0
Σ	-20.1	-40.5	+40.5	-40.5	+40.5	+20.1

Figure 12. Moment distribution analysis of rollover device with vertical load and bottom ends of vertical members fixed.

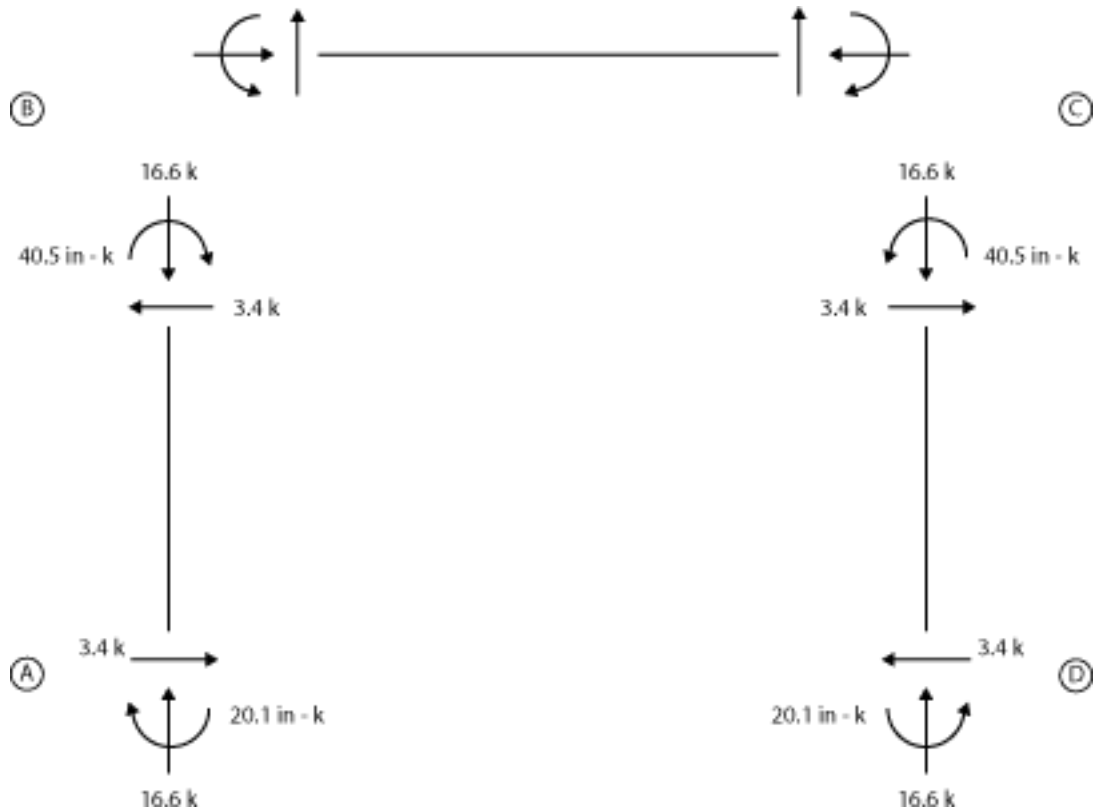


Figure 13. Results of moment distribution analysis for vertical load.

The maximum normal stresses in members A-B and C-D occur near joints B and C and are:

$$\max S = \frac{F}{A} + \frac{M}{Z_e}$$

$$\max S = \frac{16.6}{2.229} + \frac{40.5}{1.724}$$

$$\max S = 7.5 + 23.5$$

$$\max S = 31.0 \text{ ksi}$$

It is noted that an approximate analysis such as $S = \frac{P}{A}$ would result in:

$$S = \frac{33.13}{(2)(2.229)}$$

$$S = 7.4 \text{ ksi}$$

which is much less than 31.0 ksi computed using moment distribution.

A finite element analysis of the rollover device shown in Figure 12 and subjected to vertical load was performed and the results are presented in Figure 14. The maximum normal stress in vertical member A-B occurs near joint B and is:

$$\max S = \frac{F}{A} + \frac{M}{Z_e}$$

$$\max S = \frac{16.565}{2.229} + \frac{38.086}{1.724}$$

$$\max S = 7.43 + 22.09$$

$$\max S = 29.5 \text{ ksi}$$

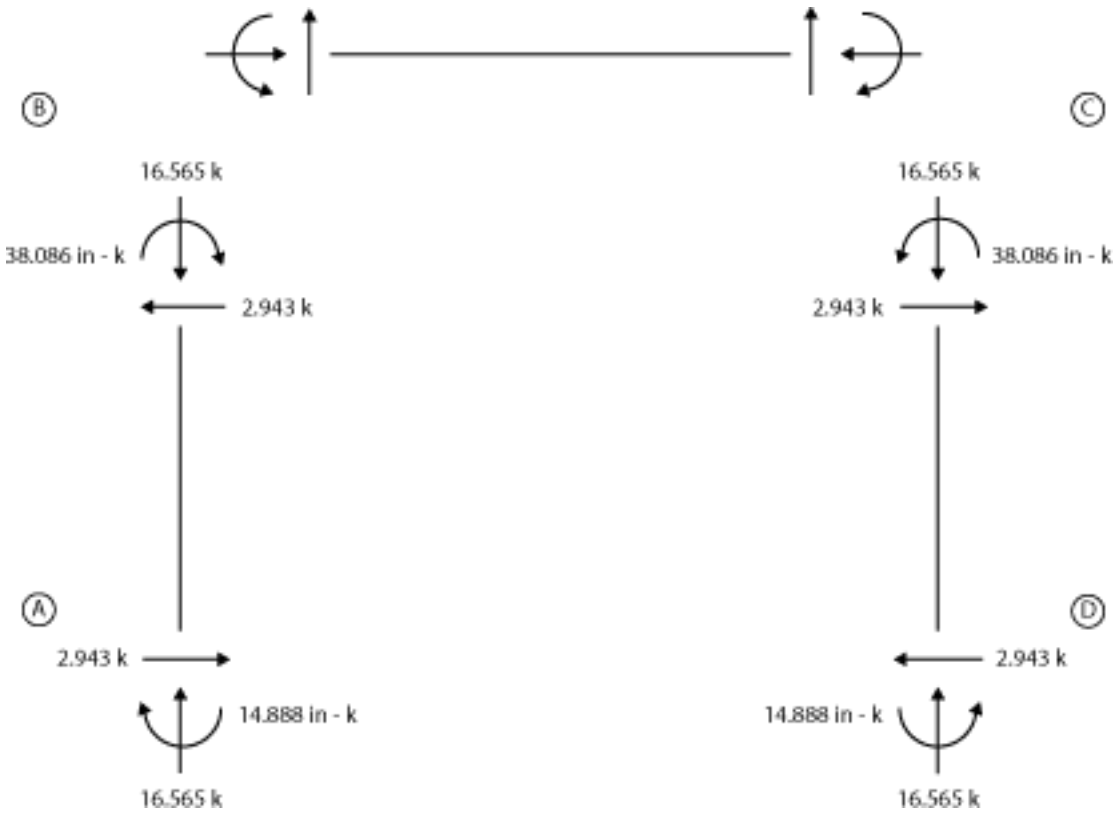


Figure 14. Results of finite element analysis for structure in Figure 12.

Rectangular box-like structures fabricated from flat plates and resembling “tombstones” have been used for rollover protection devices. One such design used on a DOT 400 series trailer is shown in Figure 15.

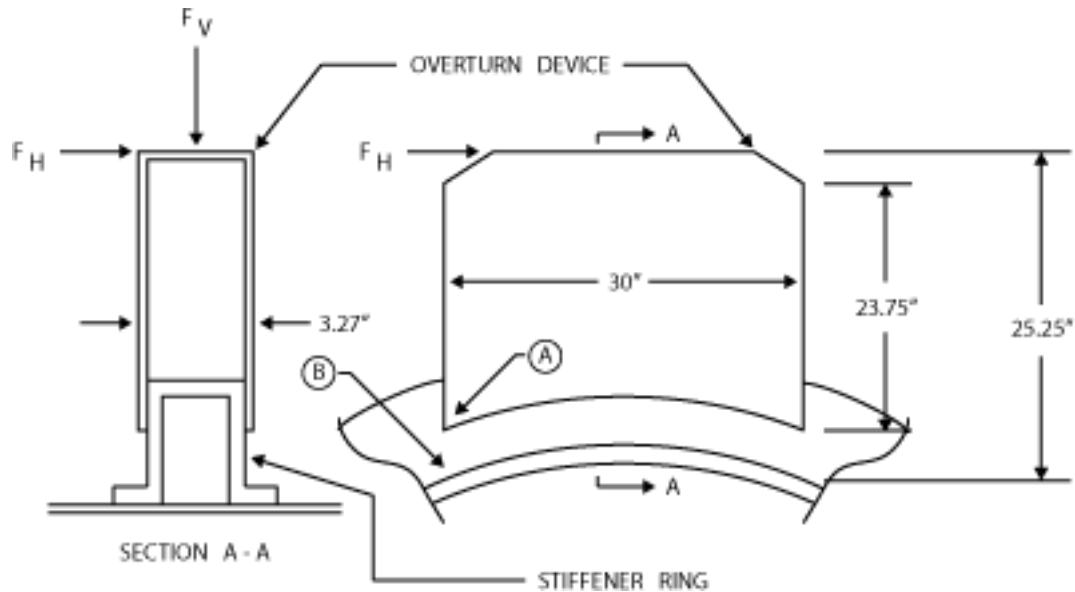


Figure 15. Box-like or “tombstone” rollover protection device.

The box is made from 10 gage (0.135 in.) Type 304 stainless steel. The device is welded to a stiffener ring made from 12 gage (0.105 in.) stainless steel. Two of these devices are used on a trailer having a total weight of 61,500 lbs. The vertical load that must be supported by the two devices is twice the total weight of the loaded (i.e., $2(61,500)=123,000$ lbs.). Each device must support half of this load or 61,500 lbs. The direct compressive stress in the device due to vertical load would be:

$$S = \frac{F}{A} = \frac{61,500}{8.91} = 6,900 \text{ psi}$$

The devices are required to support a horizontal longitudinal load and a horizontal transverse load of twice the total weight. Each device would be required to support 61,500 lbs. For horizontal load, the device will behave as a cantilever beam. The elastic section modulus for bending in the longitudinal direction is 12.55 in.^3 and the bending stress is:

$$S = \frac{M}{Z_e}$$

$$S = \frac{61,500(23.75)}{12.55}$$

$$S = 116,400 \text{ psi}$$

The elastic section modulus for bending in the transverse direction is 52.5 in³ and the bending stress is:

$$S = \frac{M}{Z_e}$$

$$S = \frac{61,500(23.75)}{52.5}$$

$$S = 27,820 \text{ psi}$$

The average shear stress in the walls of the box when subjected to horizontal transverse load would be:

$$S_s = \frac{V}{A}$$

$$S_s = \frac{61,500}{(2)(30)(.135)}$$

$$S_s = 7,593 \text{ psi}$$

The average shear stress in the walls of the box when subjected to horizontal longitudinal load would be:

$$S_s = \frac{V}{A}$$

$$S_s = \frac{61,500}{(2)(3.27)(.135)}$$

$$S_s = 69,657 \text{ psi}$$

Buckling of Plates in Compression (Roark and Young)

Elastic buckling of the thin plates used in “tombstone” devices should be checked. The compression “flange” for horizontal longitudinal bending can be checked using formulas from Roark and Young analysis tables.¹ Case 1 in that table is for a rectangular plate loaded with uniform compression on two opposite edges.

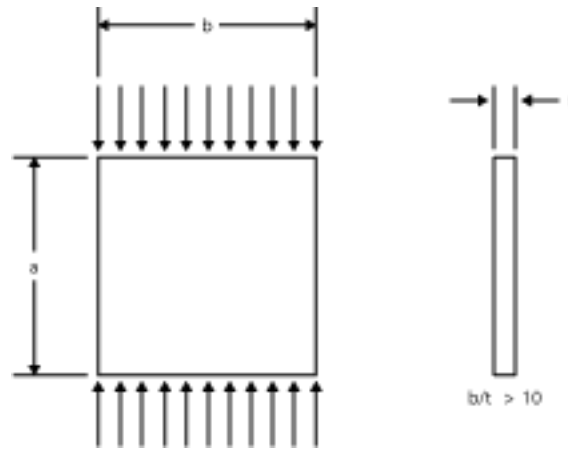


Figure 16. Rectangular plate under equal uniform compression on two opposite edges b with $\frac{b}{t} > 10$.

Case 1a is for all edges of the plate simply supported and the formula for critical buckling stress is:

$$S_{cr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

Where K depends on the ratio of $\frac{a}{b}$ and b is the length of each loaded edge.

$\frac{a}{b}$ =	0.2	0.3	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.7	3	∞
K	22.20	10.90	6.92	4.23	3.45	3.29	3.40	3.68	3.45	3.32	3.29	3.32	3.40	3.32	3.29	3.29

For the example at hand $\frac{a}{b}$ would be $\frac{24.75}{30} = 0.825$ which would give a value of $K = 3.43$.

¹ Specifically, Table 15.2 – Formulas for Elastic Stability of plates and Shells, in *Roark's Formulas for Stress and Strain*, 7th edition.

The critical bulking stress would be:

$$S_{cr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

$$S_{cr} = \left[\frac{(3.43)(28 \times 10^6)}{1 - 0.3^2} \right] \left(\frac{0.135}{30} \right)^2$$

$$S_{cr} = 2,317 \text{ psi}$$

For all edges of the plate clamped, the formula for critical buckling stress is the same as given above and values of K are:

a/b	=	1	2	3	∞
K	=	7.7	6.7	6.4	5.73

If the value of K = 7.7 is used for the device being considered the critical buckling stress would be:

$$S_{cr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

$$S_{cr} = \left[\frac{(7.7)(28 \times 10^6)}{1 - 0.3^2} \right] \left(\frac{0.135}{30} \right)^2$$

$$S_{cr} = 4,798 \text{ psi}$$

It is noted that both of the values computed above are extremely small in comparison with both the yield strength of the material and the computed stress due to load.

Buckling of Plates in Shear (Roark and Young)

Case 4 in Table 35 in Roark and Young is for a rectangular plate under uniform shear on all edges.

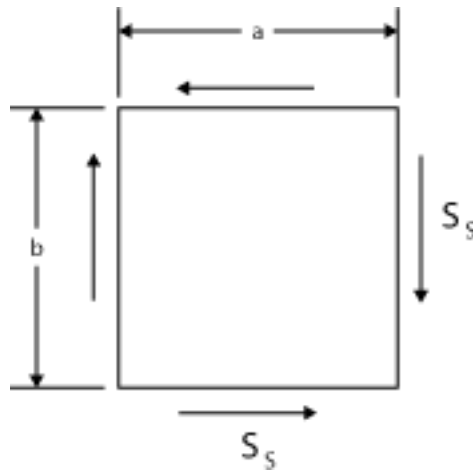


Figure 17. Rectangular plate under uniform shear on all edges.

For all edges of the plate simply supported, the critical shear stress is:

$$S_{scr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

where K depends on the ratio $\frac{a}{b}$ and b is the length of the shorter sides.

a/b	=	1	1.2	1.4	1.6	1.8	2	2.5	3	∞	
K	=	7.75	6.58	6.00	5.84	5.76	5.59	5.43	5.18	5.02	4.40

For an $\frac{a}{b}$ ratio of $\frac{30}{24.75} = 1.21$, K would be 6.55. The critical shear stress would be:

$$S_{scr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

$$S_{scr} = \left[\frac{(6.55)(28 \times 10^6)}{1 - 0.3^2} \right] \left(\frac{0.135}{24.75} \right)^2$$

$$S_{scr} = 5,996 \text{ psi}$$

For all edges of the plate clamped, the formula for critical shear stress is the same as above but the following values of K are given:

a/b	=	1	2	∞
K	=	12.7	9.5	7.38

For an $\frac{a}{b}$ ratio of 1.21, K would be 12.0 and the critical shear stress would be:

$$S_{scr} = \left[\frac{KE}{1 - \nu^2} \right] \left(\frac{t}{b} \right)^2$$

$$S_{scr} = \left[\frac{(12.0)(28 \times 10^6)}{1 - 0.3^2} \right] \left(\frac{0.135}{24.75} \right)^2$$

$$S_{scr} = 10,985 \text{ psi}$$

Buckling of Plates in Shear (Salmon and Johnson)

Salmon and Johnson give the following formula for critical buckling shear stress for a thin rectangular plate simply supported on all edges and subjected to shear:

$$S_{scr} = \frac{[K\pi^2 E]}{\left[[12(1 - \nu^2)] \left(\frac{b}{t} \right)^2 \right]}$$

where:

$$K = 5.34 + 4.0 \left(\frac{b}{a} \right)^2$$

E = modulus of elasticity, psi

E = modulus of elasticity, psi

ν = Poisson's ratio

t = thickness of plate, in.

b = short dimension of plate, in.

a = long dimension of plate, in.

From the previous computations, the ratio of b/a would be 0.825 and the value of K would be:

$$K = 5.34 + 4.0 \left(\frac{b}{a} \right)^2$$

$$K = 5.34 + 4.0(0.825)^2$$

$$K = 8.06$$

The critical buckling shear would be:

$$S_{scr} = \frac{[K\pi^2 E]}{\left[[12(1 - \nu^2)] \left(\frac{b}{t} \right)^2 \right]}$$

$$S_{scr} = \frac{[8.06\pi^2(28 \times 10^6)]}{\left[[12(1 - 0.3^2)] \left(\frac{24.75}{.135} \right)^2 \right]}$$

$$S_{scr} = 6,068 \text{ psi}$$

This compares to 5,996 psi from the Roark and Young formula.

Buckling of Plates in Shear (Guide for Stability Design)

The Structural Stability Research Council, in their Guide to Stability Design-Criteria for Metal Structures, give the following formula for critical buckling shear for a thin rectangular plate subjected to shear with all edges fixed:

$$S_{scr} = \frac{[K\pi^2 E]}{\left[[12(1 - \nu^2)] \left(\frac{b}{t}\right)^2 \right]}$$

where:

$$K = 8.98 + 5.6 \left(\frac{b}{a}\right)^2$$

and all other variables as defined by Salmon and Johnson in the previous section.

From the previous computation, the ratio of b/a would be 0.825 and the value of K would be:

$$K = 8.98 + 5.6(0.825)^2$$

$$K = 12.79$$

The critical buckling shear stress would be:

$$S_{scr} = \frac{[12.79\pi^2 28 \times 10^6]}{\left[[12(1 - 0.3^2)] \left(\frac{24.75}{0.135}\right)^2 \right]}$$

$$S_{scr} = 9,630 \text{ psi}$$

This compares to 10,985 psi from the Roark and Young formula.

Example 5. Analysis of continuous overturn rails.

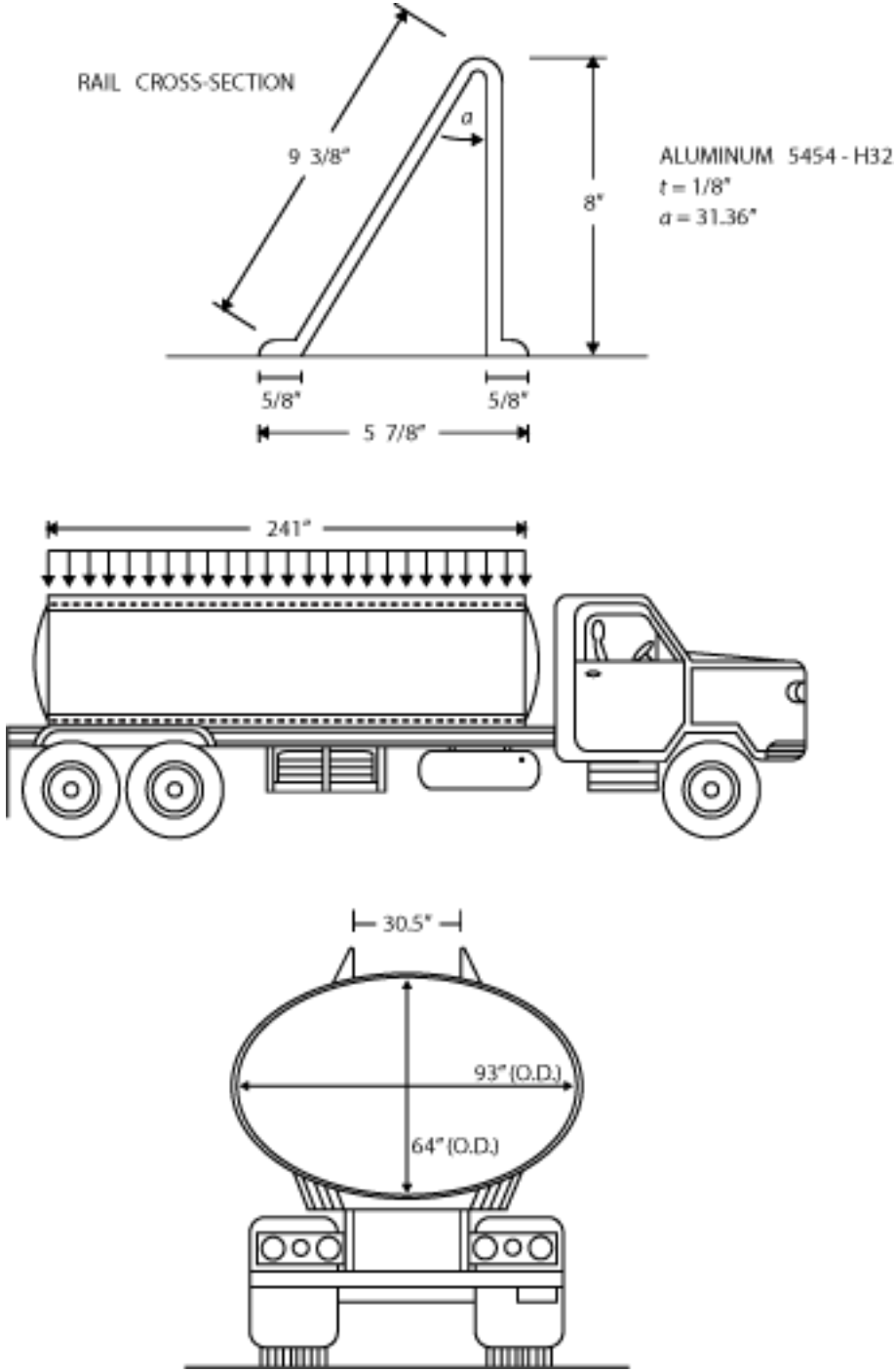


Figure 18. Continuous overturn protection rail on single unit cargo tank truck.

The continuous overturn rails shown on the vehicle in Figure 18 are to be analyzed for the required loads of 2 g's in the vertical, horizontal transverse and horizontal longitudinal directions.

The analysis presented serves to illustrate the manner in which over turn rails have been analyzed. IT IS NOT A RECOMMENDED METHOD. The method is based on assumed uniform support at the rail throughout its length and that assumption has not been substantiated. The authors think that concentrations of force at bulk heads and baffles make that assumption inappropriate.

$$\text{Gross weight of vehicle} = 51,000 \text{ lbs.}$$

$$\text{Vertical load } F = 2G = 2(51,000\text{lbs.}) = 102,000 \text{ lbs.}$$

$$\frac{102,000}{241} = 423.25 \frac{\text{lbs}}{\text{inch}} \text{ (uniformly distributed)}$$

$$\text{Horizontal load: } W = 2G = 2(51,00 \text{ lbs}) = 102,000 \text{ lbs.}$$

$$\frac{102,000}{241} = 423.25 \frac{\text{lbs}}{\text{inch}} \text{ (uniformly distributed)}$$

A free body diagram for the rail for vertical load is shown in Figure 19.

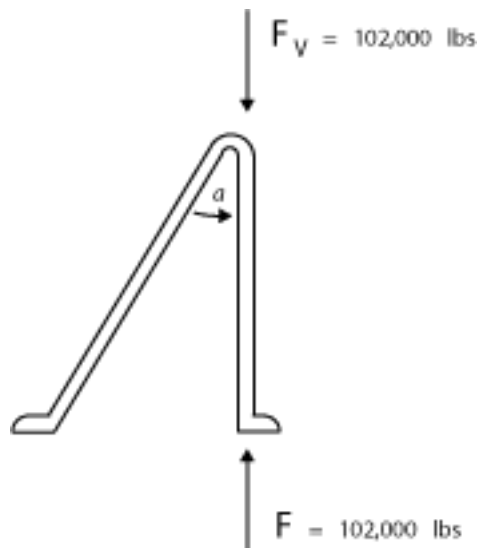


Figure 19. Free body diagram for overturn rail for vertical load.

Strength of the rail element in compression is controlled by either the yield strength or the compressible buckling strength. The compressive buckling strength is computed using Euler's formula.

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

where:

$$E = 10.2 \times 10^6 \text{ psi}$$

$$I = \frac{bh^3}{12} = \frac{(241)(0.125)^3}{12} = 0.0392 \text{ in}^4$$

For the 8" leg:

$$P_{cr} = \frac{\pi^2(10.2 \times 10^6)(0.0392)}{8^2}$$

$$P_{cr} = 61,600 \text{ lbs.}$$

$$S_{cr} = \frac{P_{cr}}{A} = \frac{61,600}{241 \times 0.125}$$

$$S_{cr} = 2,047 \text{ psi}$$

For 9 3/8" leg:

$$P_{cr} = \frac{\pi^2(10.2 \times 10^6)(0.0392)}{8^2}$$

$$S_{cr} = \frac{P_{cr}}{A} = \frac{44,900}{241 \times 0.125}$$

$$S_{cr} = 1,490 \text{ psi}$$

The actual compressive stress in the 8" leg is:

$$S = \frac{F}{A}$$

$$S = \frac{102,000}{[241(0.125)(2)]} \text{ (2 devices)}$$

$$S = 1,693 \text{ psi} < 2,047 \text{ psi} \text{ OK}$$

A free body diagram for the overturn rail subjected to horizontal transverse load is shown in Figure 20.

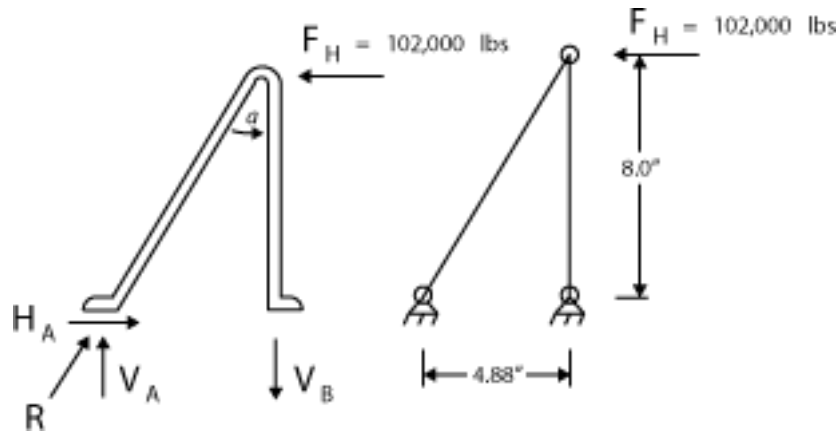


Figure 20. Free body diagram for overturn rail for horizontal transverse load.

Computation of stresses for this loading yields the following:

$$H_A = 102,000 \text{ lbs}$$

$$V_B = \frac{102,000(8)}{4.88}$$

$$V_B = 167,213 \text{ lbs}$$

$$S_B = \frac{167,213(241)}{.125}$$

$$S_B = 5,551 \text{ psi (tension)}$$

$$V_A = 167,213 \text{ lbs}$$

$$R = (167,213^2 + 102,000^2)^{0.5}$$

$$R = 195,868 \text{ lbs}$$

$$S_A = \frac{195,868}{(241)(0.125)}$$

$$S_A = 6,502 \text{ psi (compression)} > 1,490 \text{ psi FAILS}$$

Note this analysis assumes that the overturn rails are fully supported by the shell along their entire length.

Analysis of Local Stresses

Local stresses which occur at pads, cradles, or other supports must be considered in accordance with Appendix G of the ASME Code. Appendix G references several other publications for analyzing different types of local stresses. For local stresses in cylinders due to external loads it references the Welding Research Council (WRC) Bulletin No. 107.

One common loading situation which must be considered is the stresses in the tank wall due to loads on overturn protection devices.



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