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Nutation Series Evaluation in NOVAS 3.0

by

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Introduction

This circular describes the computation of nutation in the Naval Observatory Vector Astrometry Software (NOVAS),[1] version 3.0. A detailed description of NOVAS 3.0 may be found in USNO Circular 180 (Kaplan et al. 2009),[2] which consists of user's guides for both the Fortran and C versions. Nutation is a small periodic oscillation of the Earth's axis with respect to a space-fixed reference system ("inertial space") caused by the torque of the Sun and Moon—and to a lesser extent, the other planets—on the Earth's equatorial bulge. Nutation is usually modeled by lengthy trigonometric series that, when evaluated for a specific date and time, yield the values of two ecliptic angles: the nutation in longitude, $\Delta\psi$, and the nutation in obliquity, $\Delta\varepsilon$. The evaluation of the nutation series is generally the most computationally intensive part of calculating the positions of celestial objects with respect to the traditional celestial coordinate system defined by the true equator and equinox of date. NOVAS provides several nutation series with different numbers of terms, which the user can select depending on the accuracy requirements.

Background

In the summer of 2000, the IAU General Assembly, meeting in Manchester, England, adopted Resolution B1.6 that called for new models for precession and nutation, to be known collectively as the IAU 2000 precession-nutation model. The new nutation theory was published by Mathews et al. (2002) and the new precession theory is the P03 development of Capitaine et al. (2003). The celestial pole that is predicted by IAU 2000 precession-nutation is referred to as the Celestial Intermediate Pole (CIP).[3] NOVAS 3.0 implements these models.

These models replaced the precession model by Lieske et al. (1977) and the IAU 1980 Theory of Nutation (Seidelmann 1982), the latter developed by Wahr (1981). The pole defined by these older theories was referred to as the Celestial Ephemeris Pole (CEP). The Lieske et al. precession and the Wahr nutation are implemented in NOVAS version 2[4] and earlier (Kaplan 1990).

A general description of precession and nutation, the observational results that motivated the change in theories, and a comparison of the major components of the old and new developments are given in Chapter 5 of USNO Circular 179 (Kaplan 2005).[5] A recent

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[1] http://www.usno.navy.mil/USNO/astronomical-applications/software-products/novas

[2] http://www.usno.navy.mil/USNO/astronomical-applications/publications/circ-180

[3] Strictly, the CIP is defined by both the theoretical predictions and observational corrections.

[4] The previous releases of NOVAS were NOVAS F2.0 (Fortran) and NOVAS C2.0.1 (C).

[5] http://www.usno.navy.mil/USNO/astronomical-applications/publications/circ-179

comparison of observation and theory is given by Capitaine et al. (2009). The agreement between the new theories and observations made over the past decade in the position of the CIP is at the level of a few tenths of a milliarcsecond (that is, about 10^{-9} radian). The main remaining periodic difference is due to the so-called "free core nutation" (FCN) with a period of about 430 days, which has an amplitude and phase that slowly change in unpredictable ways. Unlike true nutation components, the FCN is unforced and is due to a misalignment of the rotation axes of the Earth's liquid core and solid mantle. The International Earth Rotation and Reference System Service (IERS) has provided an empirical model for the FCN but it uses a table of parameter values that change from year to year, based on observations.

Nutation Series in NOVAS

The IAU 2000A nutation series (the new standard) has 1365 terms, an order of magnitude more than the IAU 1980 series. For many NOVAS tasks, the evaluation of the nutation series dominates the computation time, so in many cases NOVAS 3.0 computations are significantly slower than those of those of NOVAS 2.0. Ever-increasing processing speeds may make this issue unimportant for many applications, but not for all. The IAU 2000A series is meant to be precise at about the microarcsecond (5×10^{-12} radian) level, yet the match of the theory with observations is two orders of magnitude poorer than that. For most practical applications, therefore, much of the increased computational burden is pointless. For NOVAS 3.0, an optional, shorter nutation series was, therefore, devised that provides angular output in both $\Delta\psi$ and $\Delta\varepsilon$ that is precise at the 0.1-milliarcsecond (5×10^{-10} radian) level, comparable to the external accuracy of the theory, i.e., as measured against observations.

This shorter nutation series is referred to as the 2000K series, and it exists only in NOVAS 3.0. It has no "official" standing and has no counterpart in either the Standards of Fundamental Astronomy (SOFA)[6] library or the IERS[7] software library. For applications not requiring the highest accuracy, the IAU recommended an abridged nutation series called IAU 2000B, with either 77 or 78 terms (two versions exist). The IAU 2000B series matches IAU 2000A to about 1 milliarcsecond over the period 1995–2050 (McCarthy & Luzum 2003). A 77-term version is available as a function in the C edition of NOVAS 3.0, but there is currently no corresponding subroutine in the Fortran edition. On the other hand, the Fortran edition provides a "quick and dirty" 13-term nutation series as an alternative to 2000K for low-accuracy applications, which is good to better than 50 milliarcseconds.

The names of the code elements that implement these series in NOVAS 3.0 are:

Series Name	No. Terms	Fortran Subroutine	C Function
IAU 2000A	1365	NU2000A	iau2000a
IAU 2000B	77	(none)	iau2000b
2000K	488	NU2000K	nu2000k
NOD 2	13	NOD, version 2	(none)

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[6] http://www.iausofa.org/

[7] http://www.iers.org/MainDisp.csl?pid=38-15

Fortran subroutine NU2000A was written by Patrick Wallace of the Rutherford Appleton Laboratory in the UK and taken from the IERS Conventions web site. It is the same code as in the SOFA subroutine iau_NUT00A but is not restricted by the SOFA Software License.

The 2000K series was created by simply omitting the smallest terms in IAU 2000A until the required output accuracy in both coordinates was obtained. The resulting series includes 488 terms—36% of the number of terms in IAU 2000A. In addition, as a "clean-up" measure, all of the expressions for the fundamental arguments in the series were taken from a single source, Simon et al. (1994). This source is the same as that is used for the arguments of the lunisolar part of IAU 2000A; however, a different source, Souchay et al. (1999), is used for the arguments of the planetary part of that series.

Choosing a Nutation Series

In the Fortran edition of NOVAS 3.0, IAU 2000A is the default nutation series. To use 2000K instead, simply call subroutine LOACC (it has no arguments) once, before any important computations. For applications where the accuracy requirement for celestial coordinates is no better than 0.1 arcsecond, the 13-term NOD 2 series can be used as the low-accuracy choice instead of 2000K, which would save considerable CPU time. Simply substitute version 2 of subroutine NOD (which is in the file of alternative NOVAS subroutines) for the standard version of NOD. A call to LOACC is still required.

In the C edition of NOVAS 3.0, IAU 2000A is invoked when the value of the input variable "accuracy" is 0 (full accuracy); 2000K is invoked if "accuracy" is set to 1 (reduced accuracy). In C, using IAU 2000B instead of 2000K for the reduced-accuracy option is possible by making a small coding change to function nutation_angles. Instructions for making the change are given in the prologue to nutation_angles.

Comparison of Nutation Results

The two output angles of nutation series, $\Delta\psi$ and $\Delta\varepsilon$, are essentially differential Euler angles (in ecliptic longitude and latitude), but their effect on the position of the celestial pole is not equal. The shift in the position of the celestial pole due to nutation is described by $\Delta\psi \sin \varepsilon$ and $\Delta\varepsilon$ (ignoring signs), where ε is the obliquity and $\sin \varepsilon \approx 0.4$. In obtaining the short nutation series, 2000K, that is accurate to 0.1 milliarcsecond, the result is actually a series that is accurate to about 0.1 milliarcsecond in $\Delta\psi$ and 0.04 milliarcsecond in $\Delta\varepsilon$ (because each term contains coefficients for both $\Delta\psi$ and $\Delta\varepsilon$ and no attempt was made to separate them). Therefore, when 2000K is used, the resulting position of the celestial pole is accurate to about 0.04 milliarcsecond in each coordinate direction. On the other hand, the accuracy of apparent sidereal time, which depends on $\Delta\psi \cos \varepsilon$ (where $\cos \varepsilon \approx 0.9$), is about 0.1 milliarcsecond (7 μs).

The following graphs (Figures 1–4, respectively) show the differences between nutation series as evaluated by NOVAS. On each graph, $\Delta\psi$ is shown in red and $\Delta\varepsilon$ is shown in blue. The vertical scale is measured in milliarcseconds. Figure 1 shows the difference between the

IAU 1980 nutation and the IAU 2000A nutation (i.e., old – new) over the 21st century, and Figure 2 shows the detail of the difference over the decade 2005–2015. Figure 3 and Figure 4 are similar, except that the difference shown is between 2000K and IAU 2000A and displayed on a finer vertical scale. Over the 21st century, we have the following statistics, expressed in milliarcseconds:

	IAU 1980 – IAU 2000A		2000K – IAU 2000A	
	$\Delta\psi$	$\Delta\varepsilon$	$\Delta\psi$	$\Delta\varepsilon$
Average difference	5.7	2.8	0.019	0.008
Minimum difference	–11.5	–8.1	–0.126	–0.041
Maximum difference	+18.1	+3.9	+0.089	+0.038

The "average difference" is the average of the absolute values of all the differences. The graphs and statistics are based on differences in $\Delta\psi$ and $\Delta\varepsilon$ computed at a mean interval of one day. However, in order to avoid possible aliasing problems, the actual time interval between any two successive points was a randomly selected value between 0 and 2 days.

About this Circular

An electronic version of this circular is available on the USNO website.[8] Errata and updates will be provided there as well.

Acknowledgements

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[8] http://www.usno.navy.mil/USNO/astronomical-applications/publications/circ-181

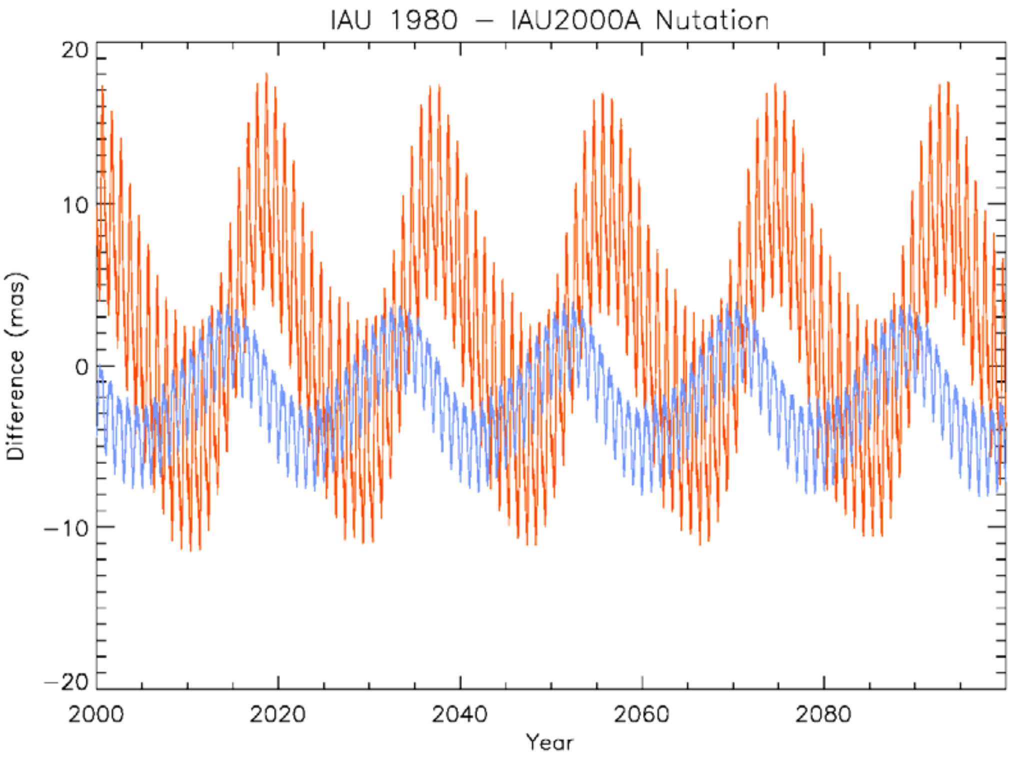

Figure 1. Difference between IAU 1980 nutation and IAU 2000A nutation over the 21st century. Δψ is shown in red and Δε is shown in blue.

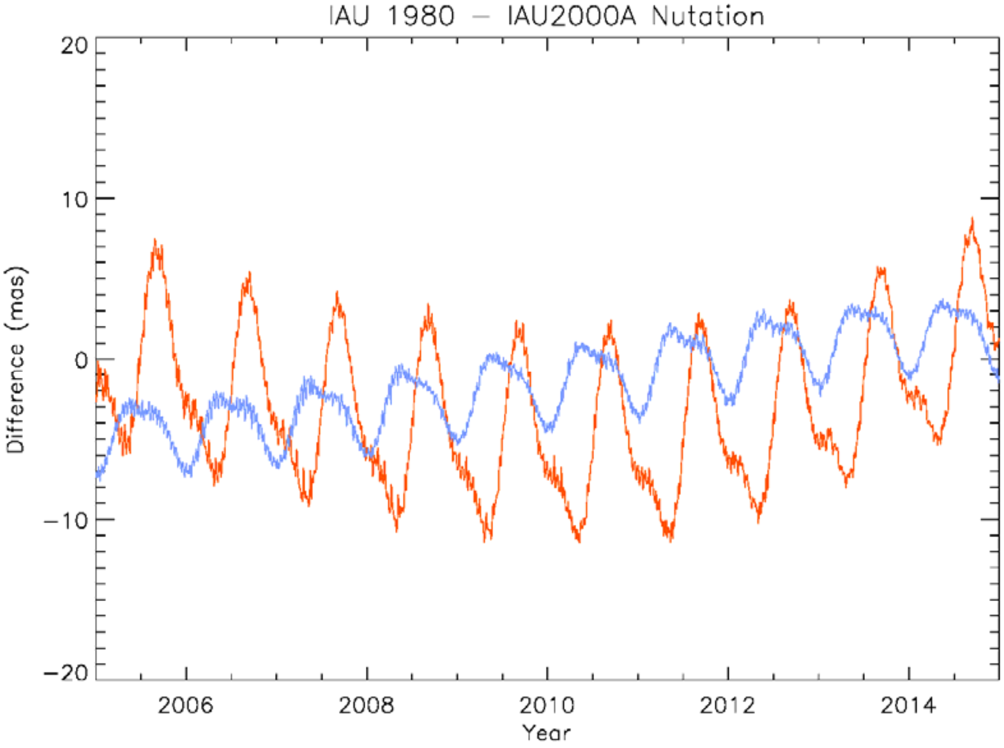

Figure 2. Difference between IAU 1980 nutation and IAU 2000A nutation from 2005 to 2015. Δψ is shown in red and Δε is shown in blue.

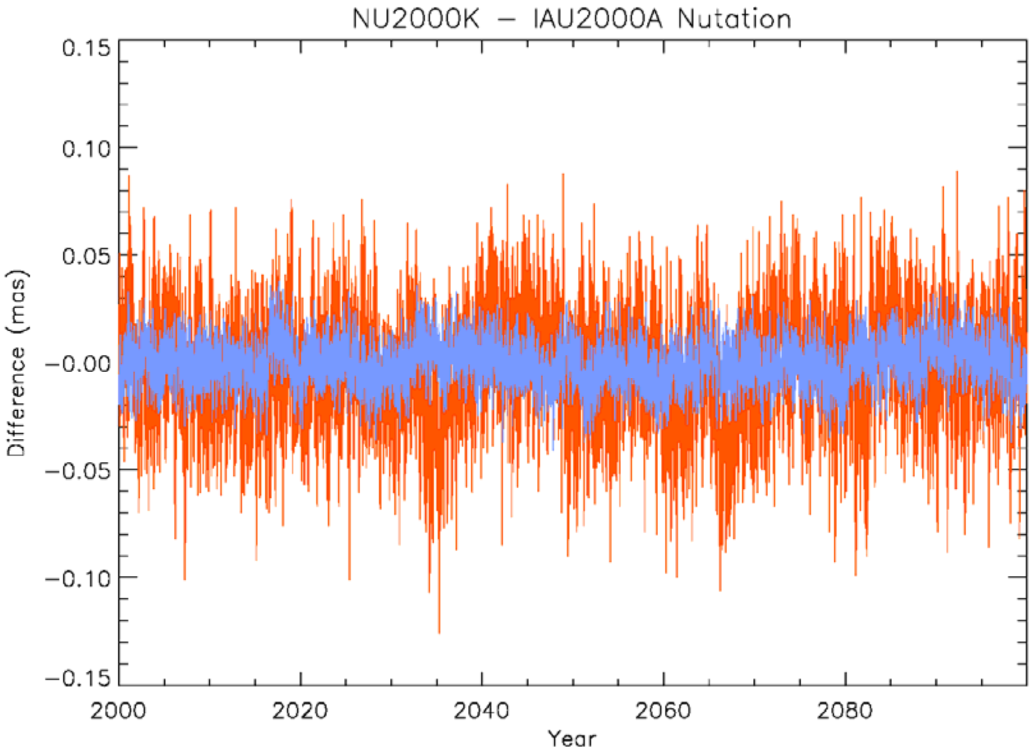

Figure 3. Difference between 2000K nutation and IAU 2000A nutation over the 21st century. Δψ is shown in red and Δε is shown in blue.

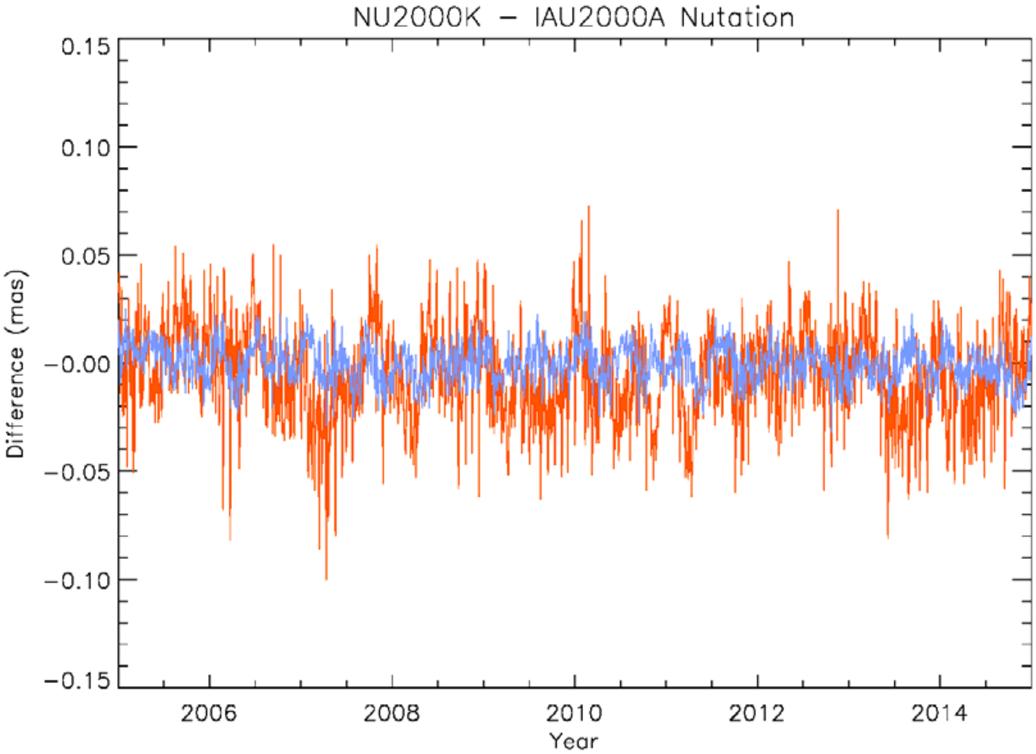

Figure 4. Difference between 2000K nutation and IAU 2000A nutation from 2005 to 2015. Δψ is shown in red and Δε is shown in blue.

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