# A proposed assessment and decision-making framework to inform scaup harvest management 

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## 1. Introduction

The continental scaup (greater Aythya marila and lesser Aythya affinis combined) population has experienced a long-term decline (Austin et al. 2000, Afton and Anderson 2001, Austin et al. 2006) in conjunction with 12 consecutive years of liberal harvest frameworks. As a result, waterfowl managers are challenged with the issue of how to manage the harvest of this declining population in the absence of an objective harvest strategy. In response to this dilemma, the SRC requested that a scaup harvest strategy be developed for the 2007 regulations cycle. Here, we report on the development of a proposed decision-making framework to guide scaup harvest management.

We began this work with the intent to develop a framework based on a formal derived harvest strategy that would enable managers to make an informed decision in relation to scaup harvest management. Our intent for this report is to propose a modeling framework from which a harvest strategy can be derived, provide a context for describing the role that scaup harvest management objectives have in structuring the harvest policy, and to propose a decision-making framework for implementation during the 2007 regulations cycle.

## 2. Assessment Framework

The lack of scaup demographic information over a sufficient timeframe and at a continental scale precludes the use of a traditional balance equation to represent scaup population and harvest dynamics. As a result, we continue to use a discrete-time, stochastic, implementation of a logistic growth population model with a harvest process to represent changes in scaup according to

$$
\begin{equation*}
N_{\mathrm{t}}=\left(N_{t-1}+r N_{t-1}\left(1-N_{t-1} / K\right)-H_{t-1}\right) e^{\varepsilon_{t}} . \tag{2.1}
\end{equation*}
$$

With this formulation, annual changes in population size $(N)$ are governed by the intrinsic rate of increase $(r)$, and the carrying capacity $(K)$, while accounting for losses through the harvest $(H)$ and process error ( $\varepsilon$ ). We use a Bayesian approach (Meyer and Millar 1999, Millar and Meyer 2000) to estimate the population parameters, characterize the uncertainty associated with the monitoring programs (observation error), and the ability of our model to predict actual changes in the system (process error).

[^0]In response to feedback from the waterfowl management and research communities, we have incorporated several changes to our original assessment framework (Boomer et al. 2004, Boomer and Johnson 2005). Recent analyses exploring the reliability of the scaup breeding population estimates (Koneff et al. unpublished data) have highlighted some significant differences between prairie and bush survey protocols used by observers in the late 1960's that we believe limit the use of the scaup breeding population estimates prior to 1974. As a result, we have truncated the time series of breeding population estimates used in our assessment to only include years from 1974 through 2005. This change also allowed us to represent total scaup harvest levels based on information from both the United States and Canada.

Our initial assessment relied on the critical assumption that the data used to estimate the population parameters were measured on the same absolute scale. Research conducted to model waterfowl populations from different sources of information has provided evidence of bias in waterfowl survey programs (Martin et al. 1979, Runge et al. 2002). While the source(s) of this bias are not yet known, it is possible to estimate correction factors to reconcile predictions based on disparate sources of information (e.g., USFWS 2006). To address this issue, we chose to include an additional parameter in our assessment to function as a scaling factor that enables us to combine breeding population and harvest estimates in an expression of population change. It is important to note that this term represents the combined limitations and uncertainty of all the monitoring data and functional relationships used in our assessment framework. Although, our initial attempts to estimate a scaling parameter from population and harvest data yielded reasonable estimates, the variance estimates were large. We found that the inclusion of a limited amount of scaup banding and recovery data provided enough information to structure the harvest process and reduce the uncertainty in the scaling parameter estimate. More details regarding this parameterization and the estimation framework can be found in Appendix 1.

## 3. Assessment Results

The state space formulation and Bayesian analysis framework provided reasonable fits to the observed breeding population and total harvest estimates with realistic measures of variation (Figure 1 A and C). The posterior mean harvest rate estimates ranged from 0.03 to 0.08 (Figure $1 \mathrm{~B})$. In general, harvest rates fluctuated over the first decade and then tracked the declining population trend until the early 1990's, when harvest rate estimates increased significantly before dropping in 1999 (Figure 1 D).

The posterior mean estimate of the intrinsic rate of increase $(r)$ is 0.110 while the posterior mean estimate of the carrying capacity $(K)$ is 8.236 million birds (Table 2). The posterior mean estimate of the scaling parameter $(q)$ is 0.541 , ranging between 0.461 and 0.630 with $95 \%$ probability. Based on the estimated population parameters, the estimated average maximum sustainable yield (MSY) on the adjusted scale is 0.211 million scaup ( 0.389 million scaup on the observed scale). One of the benefits of a Bayesian approach is that we can perform an equilibrium analysis during the estimation. This results in a realistic representation of the variation in sustainable harvest levels because all forms of uncertainty are accounted for during the simulation. The resulting yield curve depicts the scaled, sustainable harvest levels with wide credibility intervals that highlights the large amount of uncertainty associated with the estimates of scaup harvest potential (Figure 2). Plotting the current observed harvest levels that have been


Figure 1 A-D. Population assessment results based on a Bayesian analysis and scaup population and harvest data from 1974-2005. A. The posterior mean population estimates and $95 \%$ credibility intervals (gray shading) plotted with the observed breeding population sizes. B. The posterior mean harvest rate estimates and $95 \%$ credibility intervals. C. The posterior mean estimates of the U.S. harvest plotted on the observed scale with $95 \%$ credibility intervals and observed harvest levels. D. The posterior mean scaup population and harvest rate estimates resulting from the Bayesian analysis.

Table 1. Summary statistics (mean, standard deviation, median and $95 \%$ credibility intervals) of the posterior distribution of each population and management parameter resulting from a Bayesian assessment of scaup population and harvest dynamics based on data from 1974 to 2005.

| Parameter | $\underline{\text { mean }}$ | $\underline{\text { Sd }}$ | $\underline{\mathbf{2 . 5 0 \%}}$ | $\underline{\mathbf{5 0 \%}}$ | $\underline{\mathbf{9 7 . 5 0 \%}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $r$ | 0.110 | 0.063 | 0.022 | 0.097 | 0.271 |
| K (millions) | 8.236 | 1.773 | 5.727 | 7.880 | 12.210 |
| MSY (millions) | 0.212 | 0.097 | 0.048 | 0.201 | 0.437 |
| $\sigma_{\text {Process }}^{2}$ | 0.008 | 0.004 | 0.002 | 0.007 | 0.018 |
| PopMSY | 4.118 | 0.886 | 2.863 | 3.940 | 6.105 |
| $q$ | 0.541 | 0.043 | 0.461 | 0.539 | 0.630 |
| MSY | (millions) | 0.389 | 0.171 | 0.093 | 0.372 |
| Deviance | 90.926 | 12.370 | 68.089 | 90.415 | 117.000 |

Observed scale.


Figure 2. Average sustainable scaup harvest levels and $95 \%$ credibility intervals (gray shading) and corresponding equilibrium population sizes estimated with the Bayesian analysis. The years represent the observed breeding population sizes and total harvest levels that have been adjusted with the scaling parameter $(q)$.
scaled by the mean estimate of $q$ on the yield curve suggests that recent levels of exploitation are approaching or have achieved MSY levels.

## 4. Derivation of a harvest strategy

The results from the Bayesian analysis provide a reasonable basis for representing scaup population and harvest dynamics. All major forms of uncertainty were considered in the estimation and the resulting population parameter values and their measures of variation can be used to model scaup population changes and responses to exploitation. Because our goal was to develop an informed decision-making framework to provide an objective basis for scaup harvest management, we believe the decision problem must be structured by a derived, state-dependent policy. With this type of strategy each year's harvest management decision would be based on the current state of the system (e.g., breeding population size). In addition, we also wanted to ensure that the harvest policy would be adjusted each year in relation to past performance and our current understanding of the system. To meet these demands, we used discrete, stochastic dynamic programming to derive an optimal harvest strategy relative to an agreed upon management objective. This procedure is analogous to the analytical methods currently used to derive an optimal state-dependent harvest policy for midcontinent mallards under the annual AHM process (Johnson et al. 1997, USFWS 2006).

We modeled the state dynamics with the discrete logistic population growth model (equation 3.1), but with the harvest scaled by $q$, and we included multiplicative random process error that was assumed to be normally distributed with a mean equal to 0 and variance specified with the posterior mean estimate of $\sigma_{P r o c e s s}^{2}$. The population parameters $r, K$, and the scaling factor $q$ were allowed to vary but not independently. To include a correlation structure for these parameters within the optimization, we defined a joint distribution representing 3 discrete outcomes for each of the 3 parameters. This distribution was specified by first calculating the frequency of occurrence of each of the 27 possible combinations of each parameter value based on the simulation results from the Bayesian analysis and then assigning each parameter a discrete value for each event. We used the 30 and 70 percent quantiles for each parameter as the cutoff ranges from which to define the three values of each parameter. The midpoint of each of these ranges ( $15,50,85$ percent quantiles) were then used as the actual parameter values in the optimization.

We used ASDP software (Lubow 1995) to derive a state-dependent harvest policy under an objective to maximize long-term cumulative harvest (MSY) and an objective to attain a shoulder point (calculated as percentage of MSY) on the yield curve. We evaluated harvest levels from 0 to 5 million (in increments of 50,000 ) for population sizes of 1 to 10 million (in increments of 50,000 ) and harvest objectives ranging from 90 to $100 \%$ MSY (in $2 \%$ increments). For each optimization we assumed perfect control over the harvest decision variable. We then simulated each policy for 5000 iterations to characterize the management performance expected if the harvest policy was followed and the system remained static.

Under an objective to maximize long-term cumulative harvest (MSY) the resulting strategy is extremely knife-edged (Figure 3). This policy prescribes zero harvests for population sizes less than 3.2 million and seeks to hold the population size at maximum productivity (one half the carrying capacity). In contrast to the MSY policy, the harvest strategies necessary to
achieve a shoulder point are considerably less knife-edged and would allow for harvest at lower population sizes (see Figure 3). However, current scaup harvest levels $(317,000)$ exceed the prescribed harvests resulting from optimizations with each of the objective functions we evaluated.

The simulated management performance of each harvest policy demonstrates the tradeoffs that arise when a shoulder point objective is used to derive an optimal policy. As the desired shoulder point moves away from MSY, average harvest levels decrease while the average population increases (Table 2).

## 5. Proposed Implementation

The implementation of this framework requires a clearly articulated objective for scaup harvest management. We propose that this objective should seek to achieve a shoulder point somewhere on the right hand shoulder of the yield curve. Ultimately, input from the waterfowl management community will be necessary to determine the actual percentage of MSY that would serve as the shoulder point specified in the objective function.

We suggest that the proposed estimation and optimization framework be used to derive a state-dependent harvest policy to determine the optimal harvest level for scaup for the 2007 regulations cycle. We believe that a harvest policy should be derived on an annual basis with updated parameter estimates (Figure 4). This annual process will ensure that the harvest policy is consistent in relation to current observations from the monitoring programs and that all available information has been used to update the model of the system.


Figure 3. Optimal scaup harvest levels (observed scale) as a function of the observed breeding population size derived under objective functions ranging from 100 to 90 percent of MSY.

Table 2. Summary statistics of simulated harvest policies derived under objective functions chosen as a percentage of MSY. The average of the simulated population and harvest levels are displayed along with the frequency of prescribed optimal harvest levels (in millions).

|  | Average |  |  | Expected frequency of harvest levels |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\% \mathrm{MSY}$ | N | H | $\mathrm{H}=0$ | $\mathrm{H} \leq 0.1$ | $0.1<\mathrm{H} \leq 0.3$ | $\mathrm{H}>0.3$ |  |
| 100 | 3.893 | 0.388 | 22.98 | 21.55 | 14.82 | 40.65 |  |
| 98 | 4.219 | 0.380 | 0.04 | 2.90 | 40.41 | 56.65 |  |
| 96 | 4.404 | 0.374 | 0.00 | 1.94 | 42.80 | 55.27 |  |
| 94 | 4.648 | 0.363 | 0.00 | 1.35 | 45.67 | 52.98 |  |
| 92 | 4.746 | 0.358 | 0.00 | 1.18 | 44.57 | 54.24 |  |
| 90 | 4.866 | 0.351 | 0.00 | 1.27 | 47.57 | 51.16 |  |



Figure 4. The anticipated sequence of events involved with the implementation of a decisionmaking framework to inform scaup harvest management. Within this framework, a statedependent harvest policy would be derived annually based on the latest monitoring information and updated population parameters.

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## Appendix 1. Modeling and Estimation Framework

We use a state-space formulation of scaup population and harvest dynamics within a Bayesian estimation framework (Meyer and Millar 1999, Millar and Meyer 2000). This analytical framework allows us to represent uncertainty associated with the monitoring programs (observation error) and the ability of our model formulation to predict actual changes in the system (process error).

### 8.1 Process Model

Given a logistic growth population model that includes harvest (Schaefer 1954), scaup population and harvest dynamics are calculated as a function of the intrinsic rate of increase ( $r$ ), the carrying capacity $(K)$, along with the harvest $\left(H_{t}\right)$. Following Meyer and Millar (1999), we scaled population sizes by $K$ (i.e., $\mathrm{P}_{\mathrm{t}}=\mathrm{N}_{t} / K$ ) and assumed that process errors ( $\varepsilon_{\mathrm{t}}$ ) are lognormally distributed with a mean of 0 and variance $\sigma_{\text {Process }}^{2}$. The state dynamics can be expressed as

$$
\begin{gather*}
P_{1974}=P_{0} e^{\varepsilon_{1974}}  \tag{8.1.1}\\
P_{\mathrm{t}}=\left(P_{t-1}+r P_{t-1}\left(1-P_{t-1}\right)-H_{t-1} / K\right) e^{\varepsilon_{t}}, \mathrm{t}=1975, \ldots, 2005 \tag{8.1.2}
\end{gather*}
$$

where $\mathrm{P}_{0}$ is the initial ratio of population size to carrying capacity. To predict total scaup harvest levels, we modeled scaup harvest rates $\left(h_{t}\right)$ as a function of the pooled direct recovery rate $\left(f_{\mathrm{t}}\right)$ observed each year with

$$
\begin{equation*}
h_{t}=f_{t} / \lambda_{t} . \tag{8.1.3}
\end{equation*}
$$

We specified reporting rate $\left(\lambda_{t}\right)$ distributions based on estimates for mallards (Anas platyrhynchos) from large scale historical and existing reward banding studies (Henny and Burham 1972, Nichols et. al. 1995, Garrettson et al. unpublished data). We accounted for increases in reporting rate believed to be associated with changes in band type (e.g., from AVISE and new address bands to 1-800 toll free bands) by specifying year specific reporting rates according to

$$
\begin{array}{ll}
\lambda_{t} \sim \operatorname{Normal}(0.38,0.04) & t=1974, \ldots, 1996 \\
\lambda_{t} \sim \operatorname{Normal}(0.70,0.04) & t=1997, \ldots, 2005 . \tag{8.1.5}
\end{array}
$$

We then predicted total scaup harvest $\left(H_{t}\right)$ with

$$
\begin{equation*}
H_{\mathrm{t}}=h_{t}\left[P_{t}+r P_{t}(1-P)\right] K, \mathrm{t}=1974, \ldots, 2005 . \tag{8.1.6}
\end{equation*}
$$

### 8.2 Observation Model

We compared our predictions of population and harvest numbers from our process model to the observations collected by the Waterfowl and Breeding Habitat Survey (WBPHS) and the Harvest Survey programs with the following relationships, assuming that the population and harvest
observation errors were additive and normally distributed. May breeding population estimates were related to model predictions by

$$
\begin{align*}
& N_{t}^{\text {Observed }}-P_{t} K=\varepsilon_{t}^{B P O P}, \text { where } \\
& \varepsilon_{t}^{B P O P} \sim N\left(0, \sigma_{t, B P O P}^{2}\right), t=1974, \ldots, 2005, \tag{8.2.1}
\end{align*}
$$

where $\sigma_{t, B P O P}^{2}$ is specified for each year with the variance estimates resulting from the WBPHS.
We adjusted our harvest predictions to the observed harvest data estimates with a scaling parameter (q) according to

$$
\begin{align*}
& H_{t}^{\text {Observed }}-\left(h_{t}\left[P_{t}+r P_{t}(1-P)\right] K\right) / q=\varepsilon_{t}^{H}, \mathrm{t}=1974, \ldots, 2005, \text { where } \\
& \varepsilon_{t}^{H} \sim N\left(0, \sigma_{t, \text { Harvest }}^{2}\right) \tag{8.2.2}
\end{align*}
$$

We assumed that appropriate measures of the harvest observation error $\sigma_{t, H a r v e s t}^{2}$ could be approximated by assuming a coefficient of variation for each annual harvest estimate equal to 0.15 (Paul Padding pers. comm.). The final component of the likelihood included the year specific direct recovery rates that were represented by the rate parameter $\left(f_{t}\right)$ of a Binomial distribution indexed by the total number of birds banded preseason and estimated with.

$$
\begin{align*}
& f_{t}=m_{t} / M_{t},  \tag{8.2.3}\\
& m_{t} \sim \operatorname{Binomial}\left(M_{t}, f_{t}\right)
\end{align*}
$$

where $m_{t}$ is the total number of scaup banded preseason in year $t$ and recovered during the hunting season in year $t$ and $M_{t}$ is the total number of scaup banded preseason in year $t$.

### 8.3 Bayesian Analysis

Following Meyer and Millar (1999), we developed a fully conditional joint probability model, by first proposing prior distributions for all model parameters and unobserved system states and secondly by developing a fully conditional likelihood for each sampling distribution.

## Prior Distributions

For this analysis, a joint prior distribution is required because the unknown system states P are assumed to be conditionally independent (Meyer and Millar 1999). This leads to the following joint prior distribution for the model parameters and unobserved system states

$$
\begin{align*}
& P\left(r, K, q, f_{t}, \lambda_{t}, \sigma_{p r o c e s s}^{2}, P_{0}, P_{1, \ldots, T}\right) \\
& =p(r) p(K) p(q) p\left(f_{t}\right) p\left(\lambda_{t}\right) p\left(\sigma_{\text {Process }}^{2}\right) p\left(P_{0}\right) p\left(P_{1} \mid P_{0}, \sigma_{\text {Process }}^{2}\right) \times \prod_{t=2}^{n} p\left(P_{t} \mid P_{t-1}, r, K, f_{t-1}, \lambda_{t-1}, \sigma_{\text {Process }}^{2}\right) . \tag{8.3.1}
\end{align*}
$$

In general, we chose non-informative priors to represent the uncertainty we have in specifying the value of the parameters used in our assessment. However, we were required to use existing information to specify informative priors for the initial ratio of population size to carrying capacity $\left(\mathrm{P}_{0}\right)$ as well as the reporting rate values $\left(\lambda_{t}\right)$ specified above that were used to adjust the direct recovery rates estimates to harvest rates.

We specified that the value of $\mathrm{P}_{0}$, ranged from the population size at maximum sustained yield $\left(\mathrm{P}_{0}=\mathrm{N}_{\mathrm{MSY}} / K=(K / 2) / K=0.5\right)$ to the carrying capacity $\left(\mathrm{P}_{0}=\mathrm{N} / K=1\right)$, using a uniform distribution on the log scale to represent this range of values. We assumed that the exploitation experienced at this population state was somewhere on the right-hand shoulder of a sustained yield curve (i.e., between MSY and $K$ ). Given that we have very little evidence to suggest that historical scaup harvest levels were limiting scaup population growth, this seems like a reasonable prior distribution.

We used non-informative prior distributions to represent the variance and scaling terms, while the priors for the population parameters $r$ and $K$ were chosen to be vague but within biological bounds. These distributions were specified according to

$$
\begin{gather*}
P_{0} \sim \operatorname{Uniform}(\ln (0.5), 0),  \tag{8.3.2}\\
K \sim \operatorname{Lognormal}(2.17,0.667),  \tag{8.3.3}\\
r \sim \operatorname{Uniform}(0.00001,2),  \tag{8.3.4}\\
f_{t} \sim \operatorname{Beta}(0.5,0.5),  \tag{8.3.5}\\
q \sim \operatorname{Uniform}(0.0,2),  \tag{8.3.6}\\
\sigma_{\text {Process }}^{2} \sim \operatorname{Inverse} \operatorname{Gamma}(0.001,0.001) . \tag{8.3.7}
\end{gather*}
$$

## Likelihood

We related the observed population, total harvest estimates, and observed direct recoveries to the model parameters and unobserved system states with the following likelihood function:

$$
\begin{align*}
& P\left(N_{1, \ldots, T}, H_{1, \ldots, T}, m_{1, \ldots, T}, M_{1, \ldots, T} \mid r, K, f_{t}, \lambda_{t}, q, \sigma_{\text {process }}^{2}, \sigma_{\text {Harvest }}^{2}, P_{1, \ldots, T}\right) \\
& =\prod_{t=1}^{T} p\left(N_{t} \mid P_{t}, K, \sigma_{B P O P}^{2}\right) \times \prod_{t=1}^{T} p\left(H_{t} \mid P_{t}, r, K, f_{t}, \lambda_{t}, q, \sigma_{\text {Harvest }}^{2}\right) \times \prod_{t=1}^{T} p\left(m_{t} \mid M_{t}, f_{t}\right) . \tag{8.3.8}
\end{align*}
$$

## Posterior Evaluation

Using Bayes theorem we then specified a posterior distribution for the fully conditional joint probability distribution of the parameters given the observed information according to

$$
\begin{align*}
& P\left(r, K, q, f_{t}, \lambda_{t}, \sigma_{\text {process }}^{2}, P_{0}, P_{1, \ldots, T} \mid N_{1, \ldots, T}, H_{1, \ldots, T}, m_{1, \ldots, T}, M_{1, \ldots, T}\right) \\
& \propto p(r) p(K) p(q) p\left(f_{t}\right) p\left(\lambda_{t}\right) p\left(\sigma_{\text {Process }}^{2}\right) p\left(P_{0}\right) p\left(P_{1} \mid P_{0}, \sigma_{\text {Process }}^{2}\right) \times \prod_{t=2}^{n} p\left(P_{t} \mid P_{t-1}, r, K, f_{t-1}, \lambda_{t-1}, \sigma_{\text {Process }}^{2}\right)  \tag{8.3.9}\\
& \times \prod_{t=1}^{T} p\left(N_{t} \mid P_{t}, K, \sigma_{B P O P}^{2}\right) \times \prod_{t=1}^{T} p\left(H_{,} \mid P_{t}, r, K, q, f_{t}, \lambda_{t}, \sigma_{\text {Harvest }}^{2}\right) \times \prod_{t=1}^{T} p\left(m_{t} \mid M_{t}, f_{t},\right) .
\end{align*}
$$

We used Markov Chain Monte Carlo (MCMC) methods to evaluate the posterior distribution using WinBUGS (Spiegelhalter et al. 2003). We randomly generated initial values and simulated 5 independent chains each with $1,000,000$ iterations. We discarded the first half of the simulation and thinned each chain by 250 , yielding a sample of 10,000 points. We calculated Gelman-Rubin statistics (Brooks and Gelman 1998) to monitor for lack of convergence.

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