Hydro planning by stochastic programming with forward scenario agregation

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Abstract

One advantage of stochastic linear programming (SLP) models for solving hydro planning problems is that their computational complexity is linear in the number of reservoirs (polynomial, actually). But the computational complexity of multi-period SLP is exponential in the number of time stages in the scenario tree. Therefore many authors are effectively using only two or three stages for modeling uncertainties in future inflows or other unknown parameters. We will review certain SLP approaches used in the reservoir optimization literature and we will present some work-in-progress where we experiment with solution strategies based on the scenario aggregation approach, in which nonanticipativity constraints are introduced selectively.

Philosophical introduction

According to Ferris (2012):

- "While the demonstrated value of optimization in solving standard models of increased complexity is of critical importance, we firmly believe that the real value of optimization lies not in solving a single problem, but rather in providing insight and advice on the management of complex systems."
- "Planning and operating the next-generation electric grid involves decisions at time scales ranging from perhaps 15 years, for major grid expansion, through 5-minute markets, and must also account for phenomena occuring in fractions of a second."
- "We argue here against building a single "monster model" that tries to capture all these scales, and propose rather that a collection of coupled or layered models be used for both planning and operation, interfacing via information/solution sharing over multiple time scales and layers of decision making."

My understanding is that many decision support systems were designed along those lines, although there are advocates of the "monster model" approach now.

Slightly more specific introduction

Pereira and Pinto (1985): "Because it is impossible to have perfect forecasts of the future inflow sequences and, in a certain measure, of the future load itself, the operation problem is essentially *stochastic*."

Higle and Sen (1999): "Linear programming is a fundamental planning tool. It is often difficult to precisely estimate or forecast certain critical data elements of the linear program. In such cases, it is necessary to address the impact of uncertainty during the planning process. [...] When one or more of the data elements in a linear program is represented by a random variable, a *stochastic linear program* (SLP) results."

Shapiro and Philpott (2007): "There are many situations where one is faced with problems where decisions should be made sequentially at certain periods of time based on information available at each time period. Such *multi-stage stochastic programming* problems can be viewed as an extension of two-stage programming to a multi-stage setting."

Example from Pereira and Pinto (1985): stages, nodes, and scenarios

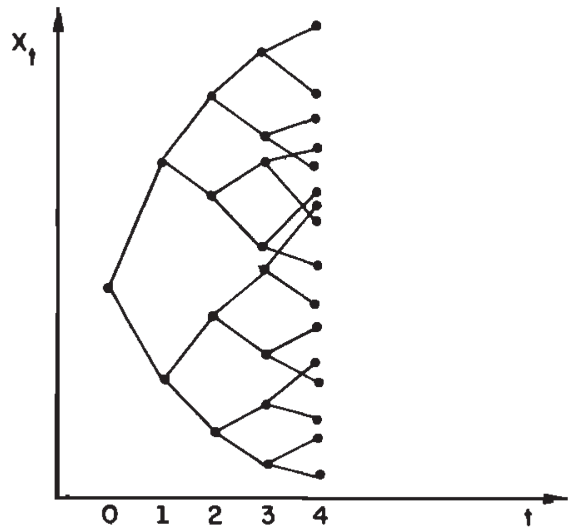

Fig. 2. Multiple inflow scenarios.

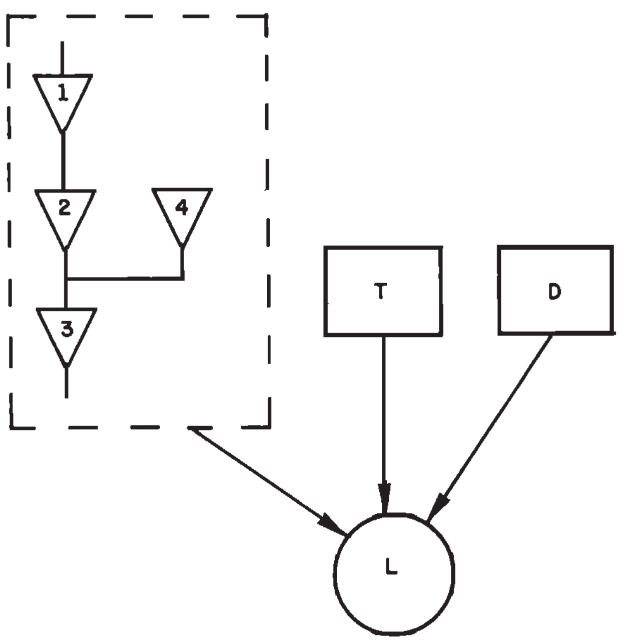

Fig. 5. Four-reservoir hydrothermal system.

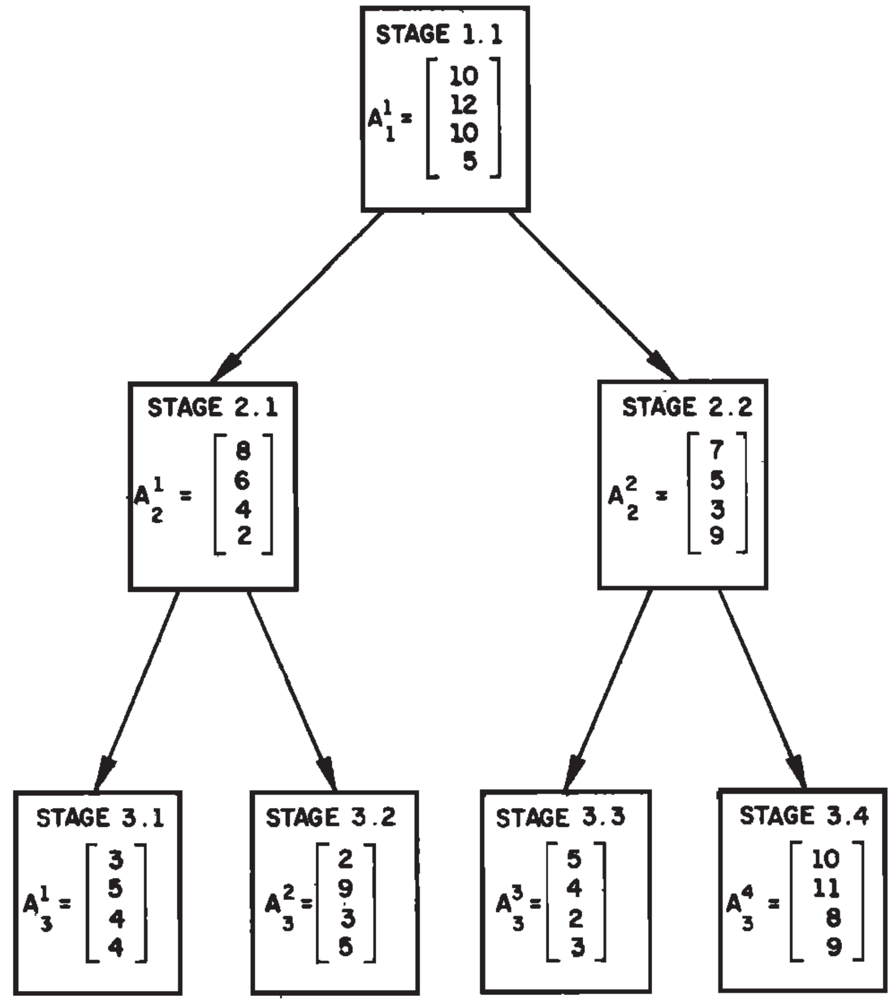

Fig. 6. Inflow vectors for the four-reservoir system.

Stochastic linear programming model

If the objective is to maximize *expected total (or discounted) rewards*, and if the continuous multivariate stochastic process has been discretized into a scenario tree with a *finite number of nodes*, then the SLP model can be written as:

Maximize

$$\sum_{\text{all nodes } n} \text{marginal probability}(n) \times \text{reward of decision variables } (X_n)$$

subject to

$$\text{water conservation equations } \left(X_n, X_{\text{pred}(n)}\right) : \text{ all nodes } n$$

$$\text{storage and flow limits, load, etc. } (X_n) : \text{ all nodes } n$$

$$\text{initial conditions } (X_0)$$

$$\text{terminal conditions } (X_\ell) : \text{ terminal nodes } \ell \text{ (leaves)}$$

end.

Remark: the scenario tree indicates the time $t(n)$ and predecessor node pred(n).

Practical considerations

- Many authors use more complicated notations for mathematical generality or just to emphasize a particular point of view (scenarios, time stages, branches).
- The previous formulation shows that all stochastic data can be stored by nodes, with the "navigation" between nodes being part of to the solution algorithm.
- Each scenarion represents one possible future. Careful specification of a representative scenario tree is a critical step in formulating a useful SLP.
- An optimal solution specifies decision variables X_1 for the 1st stage, with a recourse strategy in subsequent stages.
- Marginal value of stored water is also obtained in the neighborhood of the initial condition, useful for spot market decisions, etc. See Tilmant et al (2008).
- The number of nodes increases exponentially with the number of time stages.
- Some popular solution methods include Bender's decomposition and SDDP (stochastic dual dynamic programming). See Pereira and Pinto (1985, 1991).
- Some authors reported experimentation with *scenario aggregation* (also called *progressive hedging*): Dos Santos et al (2008, 2009), Gonçalves et al (2011), Iroumé et al (2010).

Scenario aggregation

- The purpose of this approach is to define a *scenario-oriented* model formulation for combining the solutions of many small deterministic linear programs.
- A scenario is a sequence of successive nodes going from the root to a leaf ℓ of the scenario tree. (You could also start from a leaf ℓ and follow the predecessors until the root node.)
- The decision variables X_n are replaced by $X_{n,\ell}$ so there are many decision variables at intermediate nodes, one for every scenario ℓ where n belongs.
- The formulas for the constraints and objective function are the same but they are enumerated differently:

$$\sum_{\text{leaves } \ell} \; \sum_{\substack{\text{nodes } n \\ \text{in scenario} \\ \text{ending at } \ell}} \text{scenario probability}(\ell) \times \text{reward of decision variables } (X_{n,\ell})$$

- The constraints of nodes n are duplicated for every scenario ℓ where n belongs.
- Nonanticipativity constraints must be added at all intermediate nodes that belong to more than one scenario.

Augmented Lagrangian relaxation

- The scenario oriented model is much larger than the original SLP.
- It was solved using Cplex in a case study by Lee et al (2008)...?!
- The *nonanticipativity* constraints (or *implementability* conditions) for node n can be written as:

$$X_{n,\ell} - \bar{X}_n = 0 : \text{ for all scenarios } \ell \text{ where } n \text{ belongs,}$$

  where $\bar{X}_n$ are new variables (mean values at node n).
- They are relaxed and added to the objective function (augmented Lagrangian):

$$\ldots + \sum_{\text{leaves } \ell} \sum_{\substack{\text{nodes } n \\ \text{in scenario} \\ \text{ending at } \ell}} w_{n,\ell}(X_{n,\ell} - \bar{X}_n) + \frac{\rho}{2}(X_{n,\ell} - \bar{X}_n)^2$$

  where $w_{n,\ell}$ is a Lagrange multiplier and ρ is a penalty factor.
- For given values of $\bar{X}_n$, $w_{n,\ell}$ and ρ, the problem is *separable* into subproblems.
- Can solve each quadratic scenario subproblems using a gradient method.
- The unknown variables are to be solved iteratively.

Solution strategies

- Scenario aggregation was proposed by Rockafellar and Wets (1991) and some solution strategies are discussed in Mulvey and Ruszczynski (1995).
- Some authors are very enthusiastic about the potential of the progressive hedging method for hydrothermal coordination: see Iroumé et al (2010).
- Computational experience reported in Dos Santos et al (2008, 2009) and Gonçalves et al (2011).
- Use of augmented Lagrangian decomposition also reported in Escudero (2001).
- May use parallel processing for solving scenarios.
- May introduce nonanticipaticity constraints selectively.
- Work in progress. . .

Thank you for your attention!

Some references on following slides.

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