

**Design of a Suitable Auction Format for Competitive Sale
of Alternative Energy Leases on the OCS
(AE Auction Design Study, Paper 1 of 3)**

Auction Design for Wind Rights



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Abstract

The best sites for offshore wind farms on the US Outer Continental Shelf are scarce. To make the best use of this scarce resource, it is necessary to implement a fair and efficient mechanism to assign wind rights to companies that are most likely to develop offshore wind energy projects. Coastal states, particularly along the eastern seaboard, are taking aggressive actions to spur the growth of an offshore wind sector in their states to help meet their renewable portfolio targets while nurturing the supporting on-shore infrastructure. This paper discusses the design of auctions for wind rights in which price is the sole factor of competition. A second paper, Ausubel and Cramton (2011), extends the analysis to auctions in which multiple factors are used in bid evaluation. This may be especially useful in settings where states (and potential bidders) have already taken actions to foster offshore wind development.

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Terminology

The following terms are used throughout this document.

Term	Description
Activity rule	The rule that limits what bids a bidder can make in subsequent rounds of a multiple round auction based on the bidder's bids in earlier rounds. The activity rule is intended to avoid bid sniping. A bidder with large demands late in the auction must express large demands in the earlier rounds when prices are lower.
Assignment stage	A stage of the auction in which bidders who have won generic lots are assigned specific lots, either based on an additional round of bidding or another mechanism for allocation.
Bid amount	The value or values that the bidder specifies for its bid. This can be a price or a quantity depending on the auction format.
Bid sniping	The tendency to wait until the last instant to place a serious bid as in an eBay auction. Auctions often have activity rules in place to prevent bid sniping.
Bid variable	For the purposes of this paper, the bid variable is the pricing variable, and can be a payment, an operating fee, etc.
Bidder discount	A bidder-specific percentage discount that is applied after winners and gross payments are determined. A bidder's gross payment is reduced by the bidder discount.
Block	An approved subdivision of the Outer Continental Shelf (OCS) intended for leasing purposes. In the Atlantic, an OCS block is 4800 meters square containing 2304 hectares (5693.3 acres) or about 9 square statute miles.
Call for Information and Nominations (Call)	A <i>Federal Register</i> notice that BOEMRE publishes during its renewable energy leasing process. The publication of a Call indicates that (1) BOEMRE has determined competitive interest exists in an area based upon the results of an RFI, or (2) BOEMRE anticipates that there will be competitive interest in an area. The notice solicits nominations of competitive interest in developing a project in the area described, as well as comments from the public. Following the comment period described in the notice, BOEMRE may proceed with its competitive lease issuance process, its non-competitive lease issuance process, both, or neither, depending on the information received in response to the Call and whether an RFI for the area was published previously.
Cap	A competition constraint rule that would prohibit a single bidder from winning more than some percentage (e.g. 45%) of the available lease area. Caps often take prior holdings into account when determining how to apply the rules to a specific bidder.
Clearing Price	The price at which the demand for a lot (or set of lots) is no longer above its supply, and thus is the price when the lot (or set of lots) "clears." This is typical of ascending clock auctions. This clearing price may be less than the winner's bid price.

Clock auction	A multiple round auction in which in each round the auctioneer announces prices and the bidders respond with demands at the specified prices. Prices then increase on products with excess demand and the process repeats. Three common types of clock auctions are a Simultaneous Clock Auction, an Independent Clock Auction and a Clock Auction for a Single Lot.
Clock Price	A price for a lot in a round of a clock auction.
Collusion	Two or more bidders working together to manipulate the auction outcome.
Common value	Model of bidder values in which packages of items have similar values to all bidders. Typically, the bidders do not know the exact common value but rather have an estimate, in which case each bidder is said to face common value uncertainty.
Competition constraint	A rule designed to achieve social goals such as encouraging competition in a given area. Competition constraints may be implemented in a number of ways, such as setting caps.
Competitive lease	A lease that has been issued using BOEMRE's competitive lease issuance procedures. For more information, see BOEMRE's regulations at 30 CFR § 285.211 and 285.220-225.
Complementary goods	X and Y are complementary goods (opposite of substitute goods) if when the price of Y increases demand for X decreases. Complementary goods are typically purchased together and are more valuable together than they are apart (the sum is greater than the parts). The complementarity may be strong or weak. The value of a package of goods with strong complementarities is much higher if sold to one buyer as a package than the sum of values when broken up and sold to multiple buyers. The goods have weak complementarities if the value of the package is only slightly higher when sold together. The level of complementarity between goods is important in auction design.
Demand reduction	A bid for fewer lots at a given price (compared to the demand a price just above), either for an individual bidder or in aggregate.
Discrete increments	This phrase is typically used when describing a clock auction. The price in a clock auction rises in discrete increments if it increases in discrete steps. For example, the price may rise from \$1.00 to \$1.20.
Dynamic auction	Any auction format that involves multiple opportunities to bid and where some information about the bidding is revealed to the bidders during the course of the auction. An English auction is the most common form of dynamic auction.
Eligibility Points	A bidder's eligibility points define the upper limit of lots that the bidder can bid for (based on the sum of bidding points associated with the lots in its bid). In the first round, the number of eligibility points is set by the upfront deposit amount for the bidder. In subsequent rounds, the number of eligibility points is set by the bids placed by the bidder in the previous round (and the activity percentage for that round).

English auction	A format for auctioning a single item. Bidders submit successively higher bids for the item, until no bidder is willing to bid higher. The final bidder wins the item, and pays the amount of his final bid.
Euclidean distance	The Euclidean distance between two points is the length of the straight line connecting those points.
Exit bid	An offer to pay less than the sum of the clock prices for a package of lots in a given round. An exit bid allows a bidder to specify the highest price he is willing to bid for a package of lots instead of only having the “in or out” option of bidding at the clock price for the round. When exit bids are used, the system is better able to assign the lots to the bidder who values them the most, and as it reduces the chance of a tie.
Exit price	The highest price that a bidder wishes to pay for a Lot. Therefore, the price at which he wishes to <i>exit</i> the auction for that Lot.
Exposure	The risk of winning only some lots in a collection of complementary lots and thereby not reaping the complementarities. This occurs when bids are treated independently (such as in an SMRA auction) instead of being treated as a package.
Final Rule	BOEMRE’s offshore renewable energy regulations, found at 30 CFR Part 285. The regulations can be downloaded from BOEMRE’s web site at: http://www.boemre.gov/offshore/RenewableEnergy/PDF/FinalRenewableEnergyRule.pdf .
Final Sale Notice	A <i>Federal Register</i> notice published at least 30 days before the date of the sale describing the final terms and conditions that will be used in the sale. A list of items that will be included with the final sale notice can be found in BOEMRE’s regulations at 30 CFR § 285.216.
First-price auction	An auction in which bidders specify the price they are willing to pay for an item, and if they win that item, they pay this price.
Gaming	Bidding in an auction in a way that does not truthfully represent the bidder’s true value, but may increase the bidder’s chances of a favorable outcome. A good auction design should minimize the possibility of gaming.
Generic lots	Lots that are sufficiently similar that they may be bid as one category and have one price. Bidders may then express a demand for the number of generic lots at a particular price.
Gross payment amount	The amount a winner pays, before the deduction of the bidder-specific discount
Hold up	The strategy of a speculator insisting on getting something from a large bidder as quid pro quo for not pushing prices high on key lots desired by the large bidder.
Independent Clock Auction	A clock auction for many products in which each product closes independently. An activity rule requires that demands for each product cannot increase as prices rise. This format is suitable for settings where values are roughly additive—the value of the package is the sum of the values of its individual products (i.e. the lots are not

	complementary).
Indication of Interest	An applicant's response to a Request for Interest sent to BOEMRE. The applicant must include items listed in 30 CFR § 285.213.
Information policy	The policy that determines the information that is revealed to bidders during the course of a dynamic auction. The information revealed might include bid-specific information such as the price of the bid and the identity of the bidder, or aggregate information such as the total number of bids made on a certain product (demand for that product).
Lease	A legal document that gives the lease holder a reservation with respect to other developers. Before a lessee may develop a tract, BOEMRE needs to approve a Site Assessment Plan and/or a Construction and Operations Plan.
Lease area	The tract that is leased. It is comprised of one or more lots.
Lot	A contiguous set of one or more blocks or sixteenths of blocks that is the basic product that a bidder bids on.
Multiple factor auction	An auction in which the winning bidder is selected following consideration of (1) both monetary and non-monetary factors or (2) solely non-monetary factors.
Outer Continental Shelf (OCS)	All submerged lands lying seaward and outside of the area of lands beneath navigable waters, as defined in section 2 of the Submerged Lands Act (43 U.S.C. 1301), whose subsoil and seabed appertain to the United States and are subject to its jurisdiction and control.
Package auction	An auction that allows package bids.
Package bid	A package bid is a bid on a set of items. In auctions that do not allow package bids, a bidder interested in a set of items must submit multiple bids for each of the items, which exposes the bidder to the possibility that only part of the package is won.
Package clock auction	A clock auction with an additional supplemental round. During the clock auction, bidders specify the packages they wish to purchase at various prices. After the clock auction ends, an additional round is held during which bidders may bid on new packages and improve their bids on packages from the clock auction.
Parking	A strategy in which bidders bid on lots they do not expect to win simply to maintain greater eligibility for later in the auction. This often occurs in an SMRA auction.
Payment amount	The amount a winning bidder pays for the lease. This is the gross payment amount less the bidder-specific discount if any.
Point-Based Activity Rule	An activity rule based on eligibility points. Bidders initially qualify for eligibility points at the beginning of the auction; the number of eligibility points is adjusted based on the bidding history. Each lot is assigned a certain number of "bidding points," and a bidder cannot bid for package bids where the sum of the bidding points for these lots exceeds the bidder's eligibility points.
Power purchase agreement (PPA)	A legal contract between an electricity generator (provider) and a power purchaser (buyer). The contract will specify the duration and the terms of sale, including the

	pricing, quantities, and delivery requirements for the products to be provided such as energy, capacity, ancillary services, and renewable energy credits.
Price discovery	A feature of dynamic auctions in which information about bidder demands is reported to bidders, giving bidders the opportunity to adjust subsequent bids based on the information.
Pricing rule	The rule that determines the price paid by the bidder for each lot that it has won.
Prior holding	Product such as offshore wind leasing rights that a bidder already has that is related to what is being auctioned. Prior holdings are factored in when there are competition constraints.
Proposed Sale Notice	A <i>Federal Register</i> notice with a public comment period of 60 days describing the proposed terms and conditions to be used in the sale. A list of items that will be included with the proposed sale notice can be found in BOEMRE's regulations at 30 CFR § 285.216.
Proxy Bid	A mechanism by which a bidder may submit a bid ahead of time before the auction reaches a given price. The proxy bid is automatically entered into the system when certain conditions are met.
Request for Interest (RFI)	A <i>Federal Register</i> notice in which BOEMRE requests indications of interest and comments relevant to the leasing and potential development of a designated area. BOEMRE uses the information received in response to RFIs to determine whether there is competitive interest in obtaining a lease in the area described in the notice.
Reserve price	The minimum price at which the seller will sell an item.
Revealed Preference Activity Rule	A mechanism by which past bids affect what can be bid for in future rounds. A Revealed Preference Activity Rule can be incorporated into the clock rounds and, for package clock auctions, the supplemental round.
Sealed-bid auction	An auction in which bidders submit bids without receiving any information relating to the bids placed by other bidders.
Second-price auction	An auction in which the highest bid wins and the winner pays the second price. A useful interpretation of this auction is that the bidder pays the smallest price that enables the bidder to win. This encourages the bidder to bid its true value.
Set-aside	A competition constraint rule that sets aside specific lots for bidders meeting certain criteria. A set-aside is sometimes used for new entrants in a market where new entry is desirable to increase competition. However, this is likely not an option for BOEMRE's auctions, due to the absence of any provision for it in BOEMRE's regulations.
Simultaneous ascending auction with package bids (SAAPB)	An SMRA with package bids. In each round each bidder places a bid for a package of lots. The system solves the combinatorial winner determination problem for that round, and determines provisional winner(s) for the lots based on the packages submitted. A bidder can only be a provisional winner for all of the lots in his package or none of the lots in his package; the bidder cannot be a provisional winner for some of the lots in the package bid.

Simultaneous clock auction	A clock auction similar in design to an SMRA. The key difference is that provisional winners are not determined at the end of each round, only the aggregate demands for each product. The auction ends when there is no excess demand for any product. In each round, the auctioneer announces prices and each bidder bids for the package of lots desired at the announced prices. Bids are package bids. An activity rule requires bidders to maintain a level of activity throughout the auction that is commensurate with their desired winnings.
Simultaneous multiple round auction (SMRA)	A format for auctioning multiple items, commonly used for auctioning spectrum licenses. The auction is a natural generalization of the English auction, especially useful when selling many related items. The items are auctioned simultaneously in a sequence of rounds. In each round, each bidder can submit bids on any of the items, raising the provisionally winning bid by at least the bid increment. The auction ends when no bidder is willing to bid higher on any item. An activity rule requires bidders to maintain a level of activity throughout the auction that is commensurate with their desired winnings. Note that this format suffers from the “exposure problem,” as bidders often face significant withdrawal penalties if they attempt to withdraw from any lot in which they are the provisional winner because the combination of lots is not the package they desire.
Sixteenth of a block	BOEMRE’s renewable energy program uses the sixteenth of an OCS block as the smallest unit of leasing. Each sixteenth contains approximately 355.83 acres.
Solver	The software that determines the winners and winning prices. For clock auctions, the solver algorithm is quite simple. For combinatorial situations (such as package clock auctions), standard off-the-shelf optimization software is used to determine which combination of packages yields the best value, given the defined constraints.
Specific lot	Lots that are treated individually, each with its own characteristics, allowing the bidder to specify during the auction the particular lots desired. Specific lots are appropriate when each lot has unique characteristics that determine its value.
Spectrum auction	An auction for radio spectrum (bandwidth at particular frequencies in specified regions).
Substitute goods	X and Y are substitute goods (opposite of complementary goods) if the demand for X (weakly) increases when the price of Y increases.
Substitution	The act of shifting demands across products (lots) in response to price changes, increasing the demand of the product that has become relatively more attractive as a result of the price change.
Supplementary round Or Supplemental round	A special round that occurs at the end of the clock stage in a package clock auction. Bidders bid on new packages and improve their bids on packages from the clock stage.
Tacit collusion	Cooperative behavior among bidders whereby the bidders do not engage in any explicit communication and do not enter into any explicit agreement but they are still able to coordinate on a better joint outcome than would be attained by purely

	competitive bidders. Explicit collusion is usually banned by antitrust law but tacit collusion may be legal. See, for example, “The Economics of Tacit Collusion” (Ivaldi, Julien, Rey, Seabright and Tirole 2003).
Tract	The set of lots that a bidder is interested in.
Vickrey auction	An auction format for multiple identical items. Bidders simultaneously submit demand curves. Each bidder wins the quantity demanded at the clearing price, and pays the opportunity cost of its winnings (the valuations of those bidders that are prevented from winning). For a single-item auction, the Vickrey auction is a second-price auction. When the approach is applied to the auction of non-identical items, the Vickrey auction is often referred to as the generalized Vickrey auction or the Vickrey-Clarke-Groves mechanism.
Winner's curse	The insight that winning an item in an auction is bad news about the item’s value, because winning implies that no other bidder was willing to bid as much for the item. Hence, it is likely that the winner’s estimate of value is an overestimate. Since a bidder’s bid is only relevant in the event that the bidder wins, the bidder should condition the bid on the negative information winning conveys about value. Bidders that fail to condition their bids on the bad news winning conveys suffer from the winner’s curse in the sense that they often pay more for an item than it is worth.
Winner determination	The process of determining winners and winning prices using the solver.

1 Summary

This paper examines auction design for wind rights on the US Outer Continental Shelf (OCS).

1.1 Introduction

Wind energy is growing rapidly in importance as the world moves toward renewable energy sources. In the US, much of the developable wind energy, especially near major population centers, is located offshore. Even in waters less than 30 meters deep, the wind energy potential in New England and the Mid-Atlantic States is estimated to be 56 GW (Musial and Butterfield 2004). Potential wind farm sites differ substantially in value based on such factors as: average wind speed and variance, water depth, and proximity to population centers. Although there is an abundance of potential sites, the best sites are scarce.

The Bureau of Ocean Energy Management, Regulation and Enforcement (BOEMRE) is responsible for assigning wind rights for sites among competing operators on the OCS, much as it does today for oil drilling rights. The experience of the past two decades—particularly the experience worldwide in allocating scarce telecommunication spectrum—has yielded strong support for the notion of allocating scarce public assets by auction. Moreover, it has become abundantly clear that different auction formats have various advantages and disadvantages for accomplishing the objectives of policymakers, and that considerable effort is justified in getting the auction design right.

This paper examines alternative auction designs and provides guidance on what auction designs are best for this application and under what circumstances. More specifically this paper focuses on developing and describing best practice within the class of price-only auctions. The auctions studied determine, in an open competitive process, the assignment and pricing of offshore renewable energy leases on the US Outer Continental Shelf. In a subsequent paper, we address multiple factor auctions, which allow the interaction of price with other factors to determine winners and prices paid for these offshore wind leases.

Our objective is to provide expert guidance on how best to auction wind rights on the OCS. Given the importance of offshore wind to the energy future and energy security of the US, the stakes are high. Getting the auction design right the first time is especially important given the high level of inertia in almost all government programs. The design that is used for the first auction could realistically be anticipated to be the design that is used for the subsequent 20 years. Good auction design and implementation will be essential to the allocation of tracts and to the ultimate success of wind energy policy.

For purposes of this paper, the unit being auctioned in an offshore wind auction is called a “lot,” and the aggregation of lots is called a “tract.” A winning bidder will be awarded a lease for a given tract (the set of lots that it has won) at a given price.

We are not aware of any direct conflicts with BOEMRE’s regulations or statutory authority, and have endeavored to suggest formats and rules the bureau can use under its existing regulatory framework. However, subsequent determinations by BOEMRE could find that some of the formats or rules discussed in this paper may not be implemented as stated without changes or departures from the regulations.

1.2 Desirable properties of auctions for wind rights

Over the last fifteen years, we have researched auction problems in many markets, including spectrum auctions and capacity markets; and we have brought that research to practice in many

markets around the world. Our experience suggests three main principles of effective auction design in the offshore wind setting.

Enhance substitution. First in terms of the auction design, it is important to enhance the possibilities for substitution across the lots that are being sold. This enables the bidders to adapt their tract configurations by substituting among lots during the auction to maximize value. Enhanced substitution is accomplished through both the product design—what is auctioned—and the auction format. In the offshore wind setting, the product design can be almost as important as the auction format. Relevant tools include varying the size of lots and bundling lots together that all bidders would likely view as complementary. In some cases, it may be possible to use generic lots (groups of similar lots) within a lease area, rather than specific lots, to simplify the auction and enhance substitution. Whenever generic lots are used, the auction concludes with an assignment stage that converts the generic assignments into specific assignments. Finally, the auction format can impact greatly the ease of substitution among lots.

Encourage price discovery. Second, encouraging price discovery is extremely important. This requires a dynamic auction process, because unlike some other auction situations (in more liquid markets), there is considerable uncertainty about how much the desired wind tracts are worth. The bidders need to do a lot of analysis to develop a crude valuation model, and their decision-making will benefit further from the collective market insights, which can be aggregated and revealed via a dynamic auction process.

One especially helpful aspect of a dynamic auction is that, through this auction process, the bidders gradually improve the sense of where prices will end up and what packages of lots are most relevant to them. Focusing bidder decisions on what is relevant is a major source of benefit from the dynamic process. In practice, bidders almost never have a fully-specified valuation model. Despite considerable valuation work, there remains much uncertainty about how much the lots are worth and how they should be valued.

A dynamic process, by reducing common value uncertainty, also reduces the winner's curse, the tendency for bidders to overbid and overpay in competitive auctions. Given that offshore wind auctions are new, we can anticipate large uncertainty about values.

Induce truthful bidding. A third principle worth emphasizing is the importance of inducing truthful bidding. Ideally, bids are based on developers' intrinsic valuations. This is accomplished in the auction design through an effective pricing rule and an activity rule. If these are designed well, such that there is little benefit for a bidder to bid untruthfully, the two rules work together to encourage bidders to truthfully express preferences throughout the entire auction. This truthful expression of preferences is what leads to efficiency and price discovery.

A variety of different pricing rules are used in auctions in practice. The two most common rules are "first price" or pay-as-bid pricing, where winners pay the amounts they bid, and "second price," where winners pay an amount that is just sufficient to top the bids of the others.

1.3 A consistent family of auctions

A regulator anticipating many future auctions would do well to identify a consistent family of auctions that are apt to perform well across the range of scenarios that are likely to occur. In this paper, we identify such a consistent family of auctions—a menu of auction formats, with similar underlying principles—that can easily be adapted to a wide range of circumstances.

There are many advantages to using a consistent family of auctions. The consistent family

- provides the regulator with a choice of auction instruments,
- enhances bidder understanding, as all formats are conceptually similar,
- reduces transaction costs, as the same approaches can be reused, and
- minimizes the risks of auction failure.

There are many types of auctions that have been designed and used over time. This paper will focus on a number of variations, which are categorized as follows:

1. *Sealed-bid auction*: Bidders submit bids for the items in a concealed fashion. This process can be as simple as submitting the bids in a sealed envelope (hence its name) or can be done via an online system in a single-round clock auction. The sealed-bid auction may be for a single lot, many lots, or package of lots. We believe that sealed-bid auctions are only appropriate for single item auctions. With multiple items, the valuation problem is sufficiently complex that a dynamic auction with good price discovery is desirable.
2. *Simultaneous multiple round auction (SMRA)*: The auction is a natural generalization of the English auction. It is especially useful when auctioning many related items. The items are auctioned simultaneously in a sequence of rounds. In each round, each bidder can submit bids on any of the items, raising the provisionally winning bid by at least the bid increment. The auction ends when no bidder is willing to bid higher on any item. An activity rule requires bidders to maintain a level of activity throughout the auction that is commensurate with their desired winnings. A variation that allows package bids is also considered.
3. *Clock auction*: A multiple round auction in which in each round the auctioneer announces prices and the bidders respond with demands at the specified prices. Prices then increase on products with excess demand and the process repeats. The key difference between a clock auction and an SMRA is that provisional winners are not determined at the end of each round, only the aggregate demands for each product. The auction ends when there is no excess demand for any product. There are many variations of clock auctions. The simplest variation is a *Clock auction for single item*. In a more complicated variation, *Simultaneous Clock Auction*, bids are package bids. An activity rule requires bidders to maintain a level of activity throughout the auction that is commensurate with their desired winnings. Depending on the product design, a clock auction may be followed by an assignment phase. This would be true for a *Clock auction of multiple generic lots*.
4. *Package clock auction*: A two-stage package auction. The first stage is a clock stage in which bidders specify the packages they wish to purchase at various prices; the second stage is a supplementary round in which bidders bid on new packages and improve their bids on packages from the clock stage. The auction system then takes all the bids from the clock stage and the supplementary round and finds the assignment of lots that maximizes total value. Payments are set using a second price rule. An activity rule based on revealed preference motivates bidders to bid consistently throughout the auction process.

1.4 As simple as possible, but not too simple

Albert Einstein's advice that we should "make things as simple as possible, but not simpler" is an important principle of market design. While many subscribe to Occam's razor, which advocates the simplest solution, Einstein's razor is an important refinement. In our experience, there is a tendency for regulators to adopt auction approaches that are overly simple for the setting. Worse yet, the regulator

mistakenly evaluates simplicity with respect to the complexity of the auction rules, rather than the complexity of participating in the auction and formulating sensible bids. Simple auctions can work well in simple environments, but more complex auction formats are needed in more complex settings. A good example of this is a sealed-bid first-price auction. While such an auction is easy to conduct and explain, it is extremely difficult for bidders to bid in, as the bidder is bidding “in the dark” and needs to guess what bids are going to be placed by competitors to determine what bid to place to win a particular lot.

In making our overall recommendations, we adopt Einstein’s razor. BOEMRE faces a variety of likely auction scenarios. Some are simple, such as the auctioning of a single lot; others are complex, such as the auctioning of multiple interrelated lease areas, each containing numerous lots with strong and varied complementarities among lots.

Based on our “family of auctions,” there are five different auction designs that are well-suited for the offshore wind environment, depending on the specifics of the lease area that will be auctioned:

- sealed-bid second-price auction,¹
- clock auction for a single item
- clock auction for multiple generic units of a single item (followed by an assignment phase),
- simultaneous clock auction, and
- package clock auction.

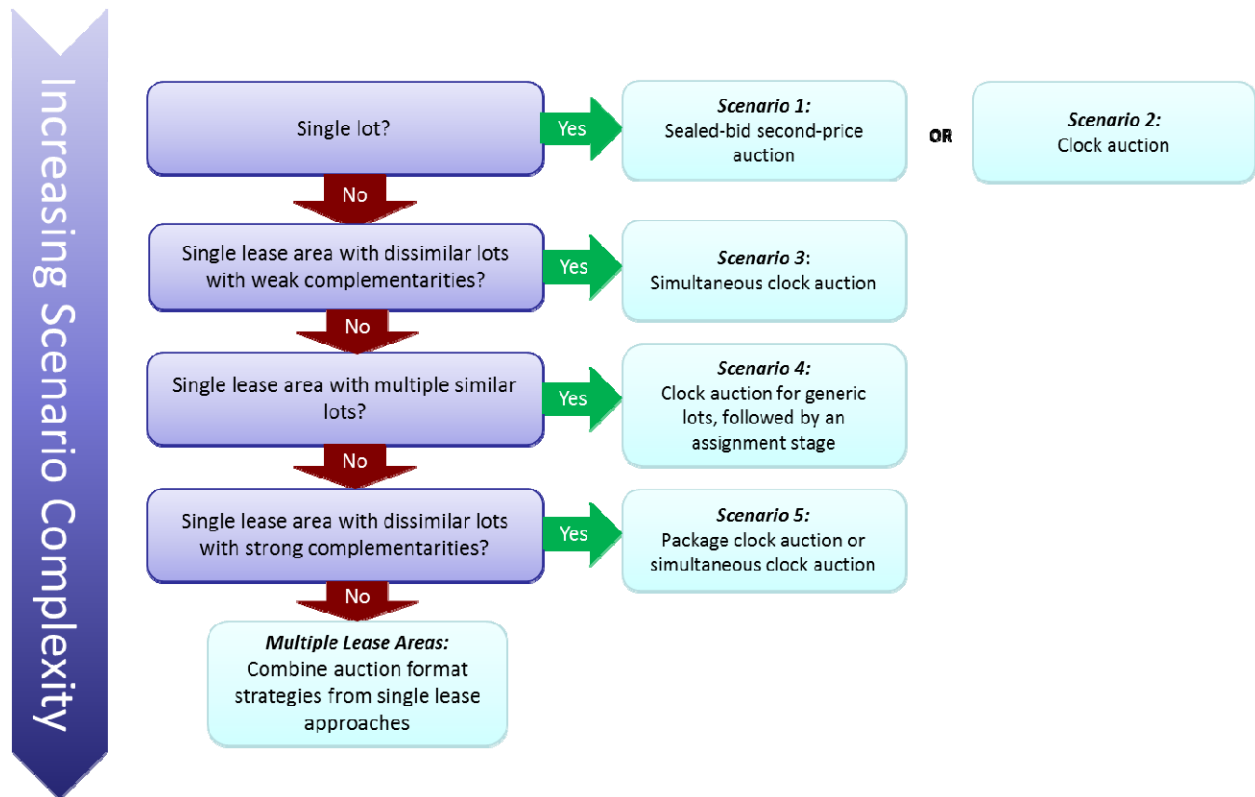
Table 1 provides a hypothetical example of when BOEMRE might want to use each type of auction. Figure 1 summarizes the auction format decision as a flowchart.

¹Due to the speculative nature of offshore wind auctions, we do not recommend a sealed-bid auction at this time. Bidders will want the price discovery features that other auction formats provide. The sealed-bid second-price auction is included in the list for completeness, should it be desired in the future when the common value uncertainty will be lower.

Table 1: Hypothetical situations for offshore wind auction formats

	Hypothetical situation	Scenario (Economic terms)	Suggested Auction Format
1	Suppose the offshore wind auctions have been running for a few years and the bidders have a good sense as to how much each lot is worth. In this case, BOEMRE could auction a single lot using a sealed-bid second-price auction.	Single lease area, one lot	Sealed-bid second-price auction
2	Suppose BOEMRE decides only one lease could realistic fit in a lease area. In this case, BOEMRE would auction it off as a single unit with one winner.	Single lease area, one lot	Clock auction (single lot)
3	Suppose BOEMRE decides on lease areas ahead of time. This might be the case if there is only one logical way to break up a lease area into desired tracts.	Single lease area for dissimilar lots; weak complementarities	Simultaneous clock auction
4	Suppose BOEMRE is able to pre-select, for example, three areas in a lease area as having similar value.	Single lease area for multiple similar contested lots	Clock auction for multiple generic lots, followed by an assignment stage
5	Suppose BOEMRE offers a lease area in which bidders have different overlapping areas of interest, both size and in region. The RFI packages for New Jersey illustrate this situation.	Single lease area for dissimilar lots; strong complementarities	Simultaneous clock auction or package clock auction. This allows bidders to aggregate different combinations of lots to form viable tracts. A lot could consist of a block, or a combination of blocks and/or sixteenths of a block.

Figure 1: Suitable auction formats for various scenarios



1.5 When multiple lease areas should be auctioned together

The decision as to whether to auction off each lease area separately, or whether to auction off two or more lease areas together in a single auction depends on a number of variables.

First, BOEMRE should consider whether lots in adjoining lease areas have strong complementarities. If they do, BOEMRE should consider auctioning off the lease areas together in a single auction, if practical. For example, there might be a shared power line that a wind farm might be able to make use of. Or, the ocean topography might be such that one of the areas might be less valuable to Company B if Company A has a lease for the adjoining lots, as, depending on where the wind turbines are placed, they might be reducing the amount of wind that Company B’s area is exposed to.

Second, BOEMRE needs to consider political, legal and state-specific technical issues when auctioning off multiple lease areas at the same time. It is possible that the state programs such as power purchase agreements (PPAs) are not in place to properly implement a wind farm in federal waters. Combining lease areas from different state waters with different state-specific multiple factor conditions should not be an issue, as these factors can easily be factored into the bids for just those items.

Third, BOEMRE should consider pacing the auctions so that bidders develop a sense of how much the lots are worth. This will reduce the uncertainties regarding how much the lots are worth that will be present during the first few auctions. Once bidders have a greater sense of how much the lots are worth, BOEMRE could consider using a sealed-bid second-price auction in auctions for a single lot. However, sealed-bid auctions are generally poor when multiple lots are auctioned.

Finally, BOEMRE should consider bidding practicalities when grouping multiple lease areas together in a single auction. If there are many lots to bid on, it may be impractical for bidders to think through the

various package combinations to implement their business solution. However, if the number of lots is small, then it might make a lot of sense to combine the lease areas in a single auction, as it is much more practical for both bidders and BOEMRE.

When multiple lease areas are auctioned together, each lease area's lots would probably be grouped together to make it obvious what area the bidder is bidding for. Also, the bidding software would need to identify which lease areas a bidder is eligible to bid for and only allow bidders to bid for those lots. Both of these concerns are easy to mitigate with well-designed auction software.

1.6 Outline

The remainder of the paper is organized as follows. We begin with a discussion of the basic ingredients of market design. Then we discuss each of the basic auction formats that may be considered for the auctioning of wind rights: sealed-bid auction, simultaneous multiple round auction (SMRA), clock auction (especially the simultaneous clock auction), and package clock auction. We will discuss why sealed-bid auctions, SMRA, and independent clock auctions are not appropriate for auctioning offshore wind rights. We will discuss a number of topics that pertain to all auction formats. We then conclude with a description of what our analysis indicates would work well for the renewable energy auction process.

2 Market design

Market design is a relatively new field of study involving several disciplines, especially economics, computer science, and operations research. It extends mechanism design, a theoretical field in economics, into the practical design of real markets.

The process of market design involves a number of initial steps:

- deciding on the objectives to be accomplished in the market;
- an examination of the setting, especially its economic characteristics;
- the development of the product design—what is being auctioned; and
- the selection of the auction format.

We say initial steps, since once a draft market design is complete, it needs to be tested and refined before implementation. The testing step may use theory, simulation, experimentation, and often field pilots to further examine the properties of the market design and search for ways to improve it. Our focus here is only on the initial pre-testing steps.

In addition, the market design process involves a collaboration of market design experts, the regulator, and market participants. Here the focus is on our views as experts. The development of these views has benefited from many discussions with BOEMRE staff and market participants, especially in enhancing of our understanding of the setting.

2.1 Objectives and performance metrics

As we mentioned, the primary objective of the BOEMRE offshore wind auction program is

- *Efficiency*. Awarding wind rights to the companies that can obtain the greatest value and at the lowest cost, and developing them in a timely manner.

In addition there are a number of secondary objectives, including:

- *Competition*. Encouraging competition in the auction;
- *Consistency*. Consistent framework across circumstances.
- *Neutrality*. All companies are treated equally;
- *Revenues*. Receipt of a fair return to taxpayers for wind rights;
- *Simplicity*. A simple process for bidders and the regulator; and
- *Transparency*. An open process in which bids are comparable and it is clear why the winners won.

Fortunately, most of the objectives are standard and broadly consistent with one another. Our practice has been to favor simple and transparent auction designs that promote efficient allocation of the scarce resource through competitive pricing. Such auctions tend to maximize the social value of the wind rights, which is BOEMRE's chief objective.

2.2 Setting

In the summary, we described the salient characteristics of wind rights as we understand them today. Our understanding of the setting has evolved throughout this project, and this has enabled us to further refine the design choices. The relevant engineering and economic issues of potential offshore wind facilities is relatively straightforward. From this we glean that most auctions will involve the auctioning of many related but heterogeneous lots. The lots will be viewed as both substitutes and complements, and the structure of these preferences may be complex, especially across bidders.

There are three important considerations that are not yet known.

The first and most important is the level of competition. Competition is highly dependent on the location and quality of the lots—higher quality lots in better locations (i.e. with optimal wind conditions or easy access to the transmission grid) typically will result in more competition. Fortunately, the auction methods we outlined in the summary are highly robust to the level of competition, especially when the main focus is making the best use of the wind rights, rather than auction revenues.

The second consideration involves various regional or State factors and political constraints. These are hard to predict in advance and can change both quickly and dramatically. For example, many states have already selected companies to develop offshore wind farms in state waters and this may effectively reduce competition in the federal waters to only those providers. This presents another reason to focus on a consistent family of auctions. This allows a more rapid response to changing regional and political constraints. We anticipate that our recommended family of auctions is sufficiently broad that it includes enough flexibility to changing regional and political circumstances.

State choices can have a large impact on the setting. For example, a state award of a PPA to a particular developer may limit competition and may limit whether and in what way there is conflict for lots.

Third and finally is valuation. Given political uncertainties regarding subsidization, it is likely that valuation of lots in many areas will be uncertain, especially in the early stage of the project. The family of auctions is flexible in terms of pricing, allowing efficient discovery of competitive prices. The clock auction is particularly effective in this regard. Even with large uncertainty about values the auction can be completed with a limited number of rounds.

One thing is clear. There will not be a single setting, but rather a range of settings, which we have organized into a number of scenarios in the summary. The consistent family of auctions we examine is

broad enough to accommodate this wide range of scenarios. In many settings the developers may differ in important ways, for example one may have been selected as a preferred vendor by the state. These differences can be accommodated with multiple factor approaches, which are discussed in our sequel paper, Ausubel and Cramton (2011).

2.3 Product design

The third step is product design—determining how the offshore wind rights are defined and what factors are bid. This is relevant for both the standard auctions considered here and the multiple factor auctions we consider in a separate paper.

Detailed product design is beyond the scope of this project. Moreover, most of the product design work must wait until particular auction opportunities arise. At that point, the lots to be offered can be finalized. Also determined are any pre-auction bundling of lots, the treatment of lots as generic or specific, the size and number of lease areas, the lease duration, and other important details, such as bidder qualification and deposits.

These product design details will follow from at least a rough understanding of prospective bidders' alternative business plans. To what extent are economies of scale important? Under what circumstances are lots (or groups of lots) complements? Under what circumstances are lots (or groups of lots) substitutes? What is the demand for the lease areas and how does this demand change over time? It will not be possible to get precise answers to these questions, but a reasonable understanding is needed to inform the product design.

For auctions with multiple winners, the product design would need to address interference between wind farms. This could be done with mandatory buffers between wind farms. This is analogous to the guard bands that are used in spectrum auctions to assure sufficient spacing between operators to limit interference. Buffer requirements would be part of the definition of offshore wind rights.

For the purposes of this paper, it will suffice to establish what is being bid. In most lease auctions, the bid amounts determine the lease winners as well as the price or payment amount for the lease. The payment amount is a one-time payment made shortly after the auction and in advance of acquiring the lease. We recommend that this approach be adopted for offshore wind rights.

Although there are variations to this lump-sum or “bonus bid” approach, as it is referred to in oil-lease auctions, all of the alternatives are problematic in our mind given the objectives of the auction. Nonetheless, we briefly discuss two of the common alternatives.

The first alternative is to accept payments in installments. This approach was adopted for small businesses in the early Federal Communications Commission (FCC) spectrum auctions. The rationale was to facilitate entry of small businesses that may have more limited access to capital than larger companies. This rationale was a poor one—there are more efficient methods of promoting small business. The use of installment payments encouraged speculative bidding by poorly funded entities. The result was a high frequency of default, which resulted in some of the spectrum resource sitting unused for a period of up to ten years. The basic flaw with the approach was that the installment payments shifted the banking burden of due diligence from the capital markets to the FCC. Unfortunately, it became quite clear that the FCC had no expertise in banking and indeed made basic mistakes in setting up the installment payments program. BOEMRE should not consider using installment payments.

A second approach, which BOEMRE does have experience with from oil lease auctions, is royalties. Royalties and variations such as profit shares can be extremely useful in oil lease auctions. Royalties can

greatly reduce company risk and loosen budget constraints. They also can lessen the possibility of expropriation, which is a common fear in oil lease auctions. These factors imply that companies are apt to be willing to pay substantially more if royalties or profit-sharing agreements are used instead of or in addition to bonus bids.

However, the setting here is quite different from that of oil lease auctions. First, the primary goal is efficiency not revenues. In the case of offshore wind rights, perhaps the greatest government risk is that the auction winner, despite good intentions, will fail to make efficient use of the offshore wind resource. This risk is best addressed with the winner's payment taking the form of a lump-sum shortly after the auction. This approach puts the burden of assessing the capabilities of the bidder where it belongs—with the capital markets. Furthermore, a significant deposit is required of the bidder in order to gain bidding eligibility. This deposit is forfeited in the event the bidder defaults on its payment, which takes place shortly after the auction, not after years of delay. (Some auctions use bid bonds or surety bonds to guarantee performance, but these are much weaker and inferior instruments than cash deposits. They are used in circumstances where performance is difficult to assess. Cash deposits are much preferred in situations like this where performance is crystal clear.)

2.3.1 Lease area definition

Any competitive auction process starts with the step of defining and carefully characterizing the product space of the auction. In some applications it is a straightforward task while in others the process requires a major effort to understand and design the lot structure that will promote the most efficient use of the scarce resource. In case of wind rights on the US Outer Continental Shelf, the product space design is exceptionally important. Value interdependencies and strong complementarities among different locations for wind farms create a difficult environment.

The physical product space is defined in terms of areas of fixed sizes. The predetermined OCS area of interest is subdivided in a number of OCS blocks which are further subdivided into sixteen smaller units. The division of blocks is required to account for different oddly shaped areas on the boundaries of the OCS area. For example, the New Jersey OCS is assembled from 43 full blocks (each can be subdivided into 16 sixteenths of a block) and 33 partial blocks each containing less than 16 sixteenths of a block.

For the purpose of defining the lot structure for the competitive process, it is important to understand that any physical boundaries created by these predetermined OCS blocks or sixteenths of a block have little to do with the bidders' economic valuations. Therefore, a typical auction lot, consisting of OCS blocks or sixteenths of a block, has to be structured in a way that will, most importantly, fit well with bidders' business plans without unnecessarily complicating the auction process. This way, the maximum economic value will be created in terms of both efficiently leasing areas for future electricity production and limiting the costs of the auction process for all parties involved.

It is instructive to think about two extreme examples. One can imagine conducting a package auction with lots defined on the sixteenth of a block level. This would allow bidders to specify their bids precisely. There are over 900 sixteenths of a block in New Jersey. This means there are 2^{900} combinations of packages. Not only would this be impossible for bidders to think through and specify, but solving for the value-maximizing assignment would be impossible.

On the other hand, one can also imagine BOEMRE specifying a few pre-determined lots that would correspond to the tract that a bidder might ultimately want to lease. This greatly reduces the number of lots to a handful of lots per auction, but it does not provide any flexibility for bidders to obtain precisely what they want and does not adequately respond to the patterns of interest elicited in RFIs and Calls.

A middle-ground is to hold an auction at the OCS block-level, or better yet, for lots that correspond to small groups of contiguous blocks. BOEMRE could apply a few simple rules to determine these lots such as the following:

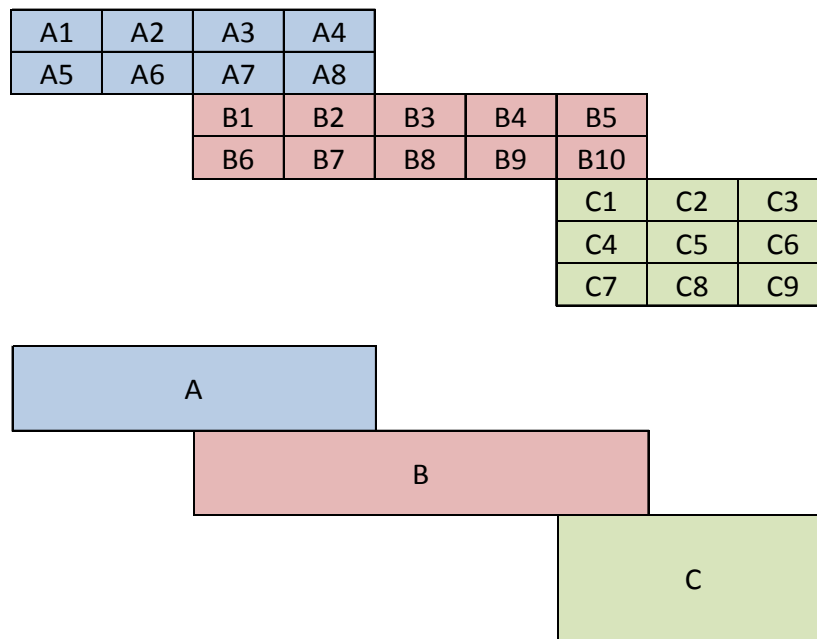
- For partial blocks, group the partial block with one of its neighbors, preferably a full-block. Many of the blocks for New Jersey only contain 1 or 3 sixteenths of a block, and these can easily be combined with a full block next to it. These blocks may otherwise go unsold, as they cannot easily be used by others.
- For blocks that do not have any contiguous blocks on three of its four sides, group it with its neighboring block. Otherwise, this block may go unsold.
- Try to group contiguous blocks that have similar wind conditions and require similar building/operating costs.
- Based on the RFI and Call responses, look for groupings that any potential bidder considers part of a set (a given bidder either wants them as a set, or doesn't want them at all).

So, for 76 blocks in New Jersey, the worst case example is the bidders would need to bid for—and the system would need to process—up to 76 lots. Ideally, some of the 33 partial blocks can be merged with a neighboring block for purposes of bidding. And, based on other considerations, some of the 43 full blocks can also be merged.

2.3.2 Bundling of Complementary Blocks

As stated above, one simplifying step that should be performed during the product design is the pre-bundling of lots for which the bidders agree that the blocks/sixteenths of a block belong together. An example is shown in Figure 2. Suppose the bidders agree that the blocks are logically grouped into three lots. Then the original blocks can be bundled to yield three lots, {A, B, C}, which can be auctioned, for example, using a clock auction with just three prices.

Figure 2. Bundled complementary blocks



2.4 Auction design

The final step, and our central focus here, is auction design. This defines the auction format. Is it sealed-bid auction, clock auction for a single lot, clock auction for generic lots with sealed-bid assignment, simultaneous clock auction, or a package clock auction? Are lots auctioned simultaneously or sequentially? Do bidders bid on individual lots or packages of lots? How are winners determined? What is the pricing rule? What is the activity rule? What is the information policy (what are the bidders told at each point in the bidding process)? All of these questions are answered at least at a high-level in this paper. Sample details will be provided in our final paper.

As mentioned in the summary, rather than considering all possible auction formats at length, we will focus on the set of alternatives that appear most desirable for the range of circumstances BOEMRE is likely to encounter. Also we will defer all discussion of multiple factor auction concerns to our sequel paper.

Beyond the basic auction format, another consideration is whether to use certain instruments to favor one or more classes of bidders. For example, in the US spectrum auctions, the FCC used bidding credits, set asides, and installment payments to favor small businesses and rural operators. We have studied the impact of such policies in our research. Some approaches have been successful, while others have not. Our conclusion is that great care is needed to employ these instruments successfully. These approaches belong to the class of multiple factor auctions, which are treated in detail in our sequel paper.

In some cases, a state-run competition for an offshore wind Power Purchase Agreement (PPA) may adversely impact competition for the auction of wind rights. For example, if there is a single large winner for the PPA, competition in the auction for sites in the adjacent state may be compromised.

To a large extent, the state competition for the PPA is resolving the competition among wind farms in advance of the federal auction. This is not necessarily bad if the state process is sound. In the extreme case, there would be no competition at all in the federal auction and the rights would be assigned to the PPA winner at the reserve price without an auction process, though it is unclear how BOEMRE could reconcile this result with its mandate to offer leases, easements and rights-of-way competitively, absent a finding of no competitive interest.

Most auction formats have a number of auction parameters that must be adjusted based on the particular circumstances. The reserve price (or starting price in an ascending auction) is one such parameter. Other implementation issues of the alternative auction designs are discussed in Sections 3 – Section 7.

2.5 Relevant characteristics of offshore wind auctions and comparison with spectrum auctions

It is worth comparing the offshore wind auctions to that of the Federal Communications Commission (FCC) spectrum auctions. The auction settings are quite similar and therefore much can be learned from the spectrum auctions both in the US and elsewhere.

The offshore wind auction setting is reasonably similar to that of auctions for radio spectrum that have been conducted by the US Federal Communications Commission (FCC) beginning in 1994, as well as by many other countries. Similar to spectrum, wind rights will be auctioned on a geographic basis; that is, the total area to be leased is subdivided into a number of smaller units that will be auctioned off. The bidders will bid for these units, and aggregate them as they see fit to build a package that suits their business needs.

The challenge for any auction is to determine how to define the units being auctioned in a way that enables the auction to produce an efficient outcome. For an offshore wind auction, this unit is defined to be a lot. A lot may be differentiated by water depth, wind quality, bottom conditions, and location. Similarly, in spectrum auctions, the many spectrum blocks are each uniquely distinguished by geography, frequency, propagation characteristics, technology constraints, and adjacent spectrum users/users that could affect signal quality and harmful interference.

The lots themselves are a complex blend of substitutes and complements—lots are substitutes when increasing the price of one does not reduce demand for the other; complements are lots such that the value of the lots combined is greater than the sum of individual values. Bidders typically require areas consisting of a number of complementary, adjacent lots. However, bidders also have flexibility to substitute lots from one area to another or to reconfigure or relocate their project within the area being auctioned. Thus, lots are both substitutes and complements and the structure of preferences may depend on prices. This is nearly identical to the case in spectrum auctions, where adjacent geographic areas are often complementary but bidders would likely consider different blocks in the same spectrum band as substitutes.

Developers of offshore wind facilities are considering using buffer zones between turbines to control for downwind turbulence impacts. Similarly, regulators and telecommunications networks often implement spectrum guard bands (small slices of spectrum that are not used to transmit or receive radio frequencies between two networks) to eliminate harmful interference. In early auctions, spectrum regulators created guard bands and did not include the guard band spectrum in the auction. In more recent auctions, spectrum regulators are leaving the spectrum whole and allowing the auction mechanism to define the optimal guard band frequencies. We recommend that BOEMRE allow the auction mechanism to define the optimal buffer zones for offshore wind facilities.

There are potentially many competing projects of various sizes in an offshore wind auction; the same is true for spectrum auctions. In a spectrum auction, one bidder may be interested in obtaining national spectrum to provide telecommunications services on a nationwide basis while another bidder may be interested in obtaining spectrum in the Northeast for a regional business. In an offshore wind auction, one bidder may be willing to build one large wind farm, whereas other bidders may only want to build a wind farm in a portion of this space. Each bidder likely has its own view on the optimum offshore wind project output requirements and corresponding configuration and space requirements within the lease area. Different bidders also bring different project skills and financial capabilities, which would impact project configuration, valuation, and timing.

The market for wind rights is a long-term market. Winning bidders must make substantial specific investments in building and operating the wind farms. As a result, the leases extend 25 years or longer. In addition, the time between lease sales is uncertain. In spectrum auctions, winners must make substantial investments in the form of radio towers, transmitters and supporting network equipment in order to make use of the spectrum. Typical license terms in a spectrum auction range from between 10 and 20 years and in the US, licenses are generally renewed in perpetuity.

BOEMRE's primary objective in implementing the auctions is efficiency. The goal is to make the most effective use of the scarce offshore wind space in the Outer Continental Shelf. Similar to spectrum auctions—but unlike many other government auctions, such as Treasury auctions, where the goal is to auction the debt at least cost—here the goal is to put the wind energy rights into the hands of those that can put this natural resource to its best use.

Thus, the most relevant characteristics of offshore wind auctions are nearly identical to those of government spectrum auctions. For this reason, this enables us to draw heavily from the worldwide

experience with spectrum auctions over the last 17 years to put forward recommendations for offshore wind auctions, which are still in their infancy in comparison. Indeed, spectrum auction design has been an extremely active area of research, with respect to both theory and practice. We have learned much and it will be important for BOEMRE to take advantage of the key insights from the existing research and experience on spectrum auctions.

Although the offshore wind auctions are similar to the FCC auctions in many ways, we are not recommending BOEMRE use the SMRA format that has been adopted by the FCC in the past. The simultaneous clock auction and package clock auction are variants of the SMRA that address problems observed in spectrum auctions, as detailed later in this document, and these clock auction formats have recently replaced the SMRA format in many recent spectrum auctions world-wide. This is described in greater detail in the sections that follow.

3 Sealed-bid auctions

Sealed-bid first-price auctions have been used since 1954 for offshore oil and gas leases. In these auctions, bidders submit a sealed-bid for the lease and the highest bidder is awarded the lease and pays the price he bid. Such an approach is problematic in a situation where the lots are interrelated as is the case here. It may be suitable in a situation where only a single lot is auctioned, but it is a poor design in settings with multiple related items, since there is no way for a bidder to express preferences for alternative packages of items. Bidders must engage in extensive guess work and bad guesses lead to inefficient outcomes.

In 1961, well after the first offshore oil and gas leases were first auctioned, economist William Vickrey published a seminal paper that introduced the second-price auction in which bidders have a dominant strategy to bid their true values.

In a second-price auction, the submitted bids are compared, the highest bid wins the item, but the winner pays the second-highest bid. With this approach the incentive to bid below one's value vanishes. The reason is that the pricing rule automatically does the reduction for the bidder. Indeed, the outcome is equivalent to letting each bidder see all the bids of the others and then optimally reduce his bid to just beat the others whenever winning is profitable or accept losing when winning is unprofitable (one of the others has bid more than the bidder's value). Bidding true value guarantees that the bidder will win whenever it is profitable and pay a price that is set by the best competing bid.

A second-price auction is the better approach when auctioning a single item, since it greatly simplifies bidding strategies and encourages truthful bids.

3.1 Assessment of sealed-bid auctions

Sealed-bid auctions are especially easy to implement. They also perform reasonably well when auctioning a single item. And bidding strategies are simple in the one-item case if a second-price auction is used.

Traditional sealed-bid auctions perform poorly when many related items are auctioned. In such a case, an ascending process is needed to let the bidders express preferences for packages of items.²

² Sealed-bid auctions that allow bids on packages may perform well, but the number of items needs to be small in order to keep the number of packages manageable.

In addition, the sealed-bid auction cannot reveal any information from the market about values and therefore exposes the bidders to a greater possibility of the winner's curse. An ascending process enables the bidder to learn about the values of others and this helps resolve the uncertainty about values.

Thus, even though the second-price sealed-bid auction is closely related to an ascending price auction—in both the winning bidder pays just enough to beat the next-best competitor—an ascending auction reveals additional information about market values, which reduces the winner's uncertainty about value. For this reason we favor ascending auctions in almost all circumstances, but especially those involving many related items.

4 Simultaneous multiple round auctions (SMRA)

Over the last 15 years great advances have been made in our ability to effectively auction many related items. Here we describe the advances that are especially relevant to offshore wind auctions. We begin with the simultaneous multiple round auction. Later sections examine important extensions to the SMRA.

The SMRA is a simple price discovery process for auctioning many related items. This format was pioneered by the Federal Communications Commission for US spectrum auctions. Based on its success in the US, the approach has been utilized worldwide for spectrum auctions since 1994. Although they have been largely effective, many issues were discovered and addressed through various modifications of the auction rules. Today, the SMRAs have largely been replaced by clock auction variants.

4.1 History and overview

In 1994, the first high-stakes SMRA was implemented at the FCC for assigning radio spectrum licenses. At the time of its introduction, it was a major advance in how governments auction many related items. Following its successful use in the US, the approach was adopted by most countries worldwide for spectrum auctions. As described in the summary, spectrum auctions and offshore wind auctions share many salient characteristics. For this reason and given the depth of experience with this auction format, it makes sense to begin with an analysis of the simultaneous multiple round ascending auction as a method for auctioning offshore wind leases.

The SMRA is a simple method of price discovery. It is a natural generalization of the standard English auction to a setting with multiple items. In fact, this is the chief insight of the approach: when auctioning many related items it makes sense to auction them simultaneously so that the bidders can shift their bidding based on the prices that are gradually discovered through the auction process. In this way, they can arrive at a portfolio of items that achieves the bidder's objectives given the prices.

We do not recommend using an SMRA for offshore wind leases because in practice the clock auction variations are apt to perform better. However, we describe the procedures that the FCC used to implement the format and the various problems they encountered using the format in practice. We also describe a variation of the SMRA that includes package bidding that was developed internally by BOEMRE staff and explain why either a simultaneous clock auction or a package clock auction is preferred to assign leases for offshore wind rights.

4.2 Object of bidding

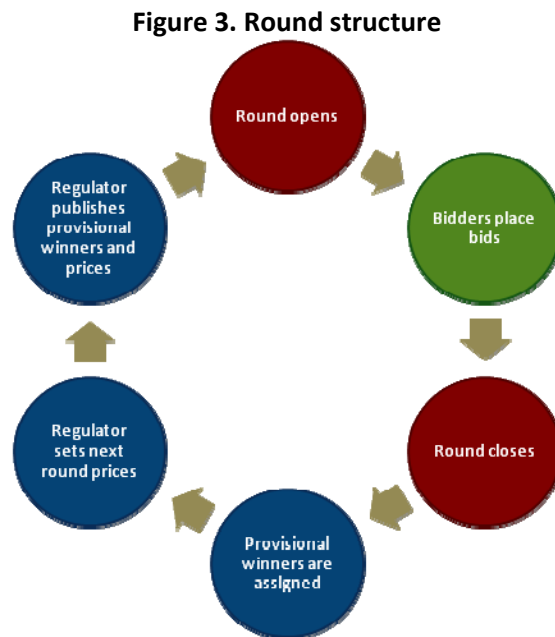
SMRAs generally include many related items that are either substitutes or complements or both.

4.3 Upfront deposits

Before the auction, bidders are required to submit a cash deposit or bank guarantee equal to the aggregate starting bid amounts for the largest set of lots the bidder wishes to win in the auction. We recommend cash deposits. They are a simple and reliable way to guarantee that bids are binding.

4.4 Bidding procedures

The SMRA proceeds in a number of rounds. Figure 3 shows the round structure. All lots are auctioned at the same time. In each round the bidder can raise the high bid on any lot by the bid increment specified for each lot. Bidders are subject to an activity rule (described below). At the end of each round, provisional winners and prices are announced for the next round. All lots remain open until no bids are received in a round on any of the lots. Below is a diagram of the typical round structure in an SMRA.



Bidders have an initial eligibility based on their upfront deposit, which assures that bids are binding commitments. Bidders who want to bid on a large area must submit a proportionately large upfront deposit to help assure their interest is genuine. A bidder may not bid for an area larger than its eligibility.

4.5 Initial round prices and bid increments

Reserve prices for each lot (or starting prices) are announced before the auction and are often proposed in a consultation document in which interested parties may provide comments as to the appropriateness of the proposed reserve price amounts. In Round 1 of the auction, each of the lots is priced at the reserve price. In early FCC auctions, bidders were permitted to bid any dollar amount above the round price. Early in the auction program, certain bidders strategically used the last three digits of their bid amount to signal other bidders (a practice known as “signaling”). After that auction, the FCC modified its process to only allow bidders to select from among a predetermined set of bid amounts for each lot.

If a lot receives a valid bid in the round, the price is incremented by some amount or percentage.³ In SMRAs, the setting of increments is more important than in the clock auctions (described later) since bidders may find themselves in a position where their exit price is between the current provisional winning price and the incremented price in the current round. Without exit bids or a supplemental round, bidders are not able to indicate the exact amount that they are willing to bid up to for a lot. Generally, bid increments should be higher earlier in the auction and should reduce as prices increase.

4.6 Point-based activity rule

In order to ensure that the auction provides valuable price discovery information, it is critical to establish a rule that requires bidders to be active throughout the auction. In eBay auctions that have no activity rules, bidders have a strong incentive to hold back and wait until the last possible second to place their bid. This practice, known as bid sniping, limits price discovery in ascending auctions.

The activity rule requires bidders to remain active in each round to maintain eligibility to continue to bid in subsequent rounds. The absence of an activity rule in a multiple round auction can lead to a poor process where the auction continues for much longer than is necessary as bidders have little incentive to reveal their true bidding preferences.

The activity rule for FCC auctions works as follows. Each lot has a number of eligibility points assigned to it based on the required deposit for that lot (which is typically the starting price for that lot) and therefore the eligibility points for a lot are roughly related to the value of the lot. Activity in a given round must be at least x percent of the bidder's eligibility. As outlined in the round structure, at the end of each round, provisional winners are assigned and announced going into the next round. Bidders who are provisionally winning do not have to increase their bid on the lot in the next round. Bidders are active on a lot if they are either the provisional winner or if they place a bid on a lot that they are not currently provisional winner. The sum of the eligibility points associated with provisionally winning lots and the lots in which he placed a valid bid comprises the bidder's total activity in the current round. Suppose that the rule states that the bidder must be active on 80% of its eligibility in order to maintain his eligibility in the next round. The system would compute the bidders' activity at the end of the round and if the bidder failed to be active on at least 80% of his eligibility points, his eligibility would be permanently reduced commensurately. Thus, for example, bidders seeking large areas must bid for large areas in all rounds of the auction.

This activity rule eliminates the practice of bid sniping or otherwise holding back since bidders are required to bid in all rounds to continue to bid in subsequent rounds.

4.7 Bid withdrawals

Because provisional winners are assigned after each round in the SMRA, a bidder may find himself in the position of winning some of the lots he desires but not others. As prices increase a bidder may be stranded on an unwanted lot or lots because as prices increase, he could no longer afford to continue to bid on all of the lots that he hoped to aggregate to meet his business objectives. The SMRA format therefore includes a rule that allows bidders to withdraw their provisionally winning bid (subject to a bid withdrawal penalty) for these situations. Bid withdrawals are an imperfect solution to a difficult problem created by the SMRA procedure that assigns provisional winners to individual lots. This troubling feature

³ The FCC establishes percentage increments from 5% to 20% of the previous price based on the excess demand for each lot.

of the SMRA rules has prompted many to prefer an auction that allows package bids, such as a simultaneous clock auction or a package clock auction.

4.8 Determination of provisional winners

When the round closes, the auctioneer determines the provisionally winning bidder(s). This is a simple process in an SMRA. The highest bidder for each lot is assigned as the provisional winner for the next round. In the event of a tie bid,⁴ the highest bidder is selected randomly.

4.9 Closing rule and determination of final winners

Final winners are the provisional winners from a round in which no new bids are placed on any of the lots.⁵ In other words, the auction does not close on any lot until it closes on all of the lots. This is an important design feature of the SMRA since it allows bidders the flexibility to switch their bidding to other more attractively priced lots during the course of the auction.

4.10 Defaults

If a bidder defaults by not paying the balance of his winning bid by the due date, the lots remain unsold and are generally included in a subsequent auction. Defaulting bidders are assessed a penalty, which is taken from the bid deposit. (In an FCC auction, the defaulter is assessed a penalty of the difference between his winning bid amount and the amount of the winning bid in the subsequent auction plus a 3% transaction fee).

4.11 Information policy

The initial SMR auctions that were implemented by the FCC were fully transparent and disclosed all of the bidding information after each round of the auction including the amounts of the bids and the identities of the bidders. Over the years, the information policy has changed because bidders sometimes used the bidding information strategically to manipulate the results of the auction, for example to enforce a particular split of the lots using retaliatory bids. For these reasons, the FCC modified its practice and now only discloses aggregate demand information. Bidder identities are concealed until the end of the auction. In most recent auctions, bidder identities are kept secret to minimize gaming opportunities.

Spectrum bidders argued that knowing the identity of neighboring spectrum holders was valuable information that had a material impact the value of the lots. While this was true, the government needed to weigh the benefits and costs in determining the appropriate information policy to implement for spectrum licenses.

4.12 Simultaneous ascending auctions with package bidding

The Federal Communications Commission (FCC) auctions that began in 1994 did allow package bids—bids were on individual lots only. In 1997-98, the FCC commissioned three reports from Charles River Associates Inc. and Market Design Inc. on improving the SMR auction design. Two of these reports (Cramton, McMillan, Milgrom, Miller, Mitchell, Vincent and Wilson, 1997, 1998) focused on augmenting

⁴ Tie bids are common in FCC SMRAs because bidders are limited to a set of pre-defined prices from which they must choose.

⁵ The rule also requires that no bidder has placed a withdrawal or proactive activity rule waiver in the round in which the auction closes.

the SMR format by having bidders submit package bids, rather than independent-item bids, in every round, but otherwise adhering closely to the SMR design. In the CRAI-MDI reports, these augmented SMR auctions are referred to as simultaneous ascending auctions with package bidding (SAAPB).

The basic rules of the SAAPB are as follows. The auction is conducted in multiple rounds. In each round, each bidder submits one or more bids comprising a subset of the items being offered and a price. After each round, the winner determination problem is solved with respect to the standing high bids and the new bids that were submitted, and the provisionally-winning bids are announced. Bidders who were not provisional winners are required to submit new bids in the following round in order to retain their eligibility in the auction. The rounds continue until a round elapses in which no new bids are submitted by any bidder; when such a round occurs, the auction concludes and each provisionally-winning bid is deemed to be a winning bid. Details insofar as the minimum bid increments, eligibility and activity rules, stopping rules, withdrawal rules and bid waivers are discussed in detail in Section 4 of Cramton, McMillan, Milgrom, Miller, Mitchell, Vincent and Wilson (1998).

The FCC commissioned an experimental study of this proposed auction format by Cybernomics, Inc. in 2000. While the experiments suggested reasonably high efficiencies in stylized environments, they also identified several major shortcomings. First, even in stylized environments with only 10 items, the SAAPB lasted three times as many rounds as standard SMR auctions, which themselves are known often to be lengthy.⁶ Second, in all environments tested, the SAAPB generated lower revenues than the SMR auction. Third, academics became increasingly skeptical of the efficiency results that were reported.

Interest in the SAAPB waned and all but disappeared following the publication of Ausubel and Milgrom (2002) and subsequent academic articles. Ausubel and Milgrom viewed the SAAPB as a set of rules appropriate only for “virtual auctions” that are played by “proxy agents” for the bidders. (Each bidder reports its valuations for the various packages to a proxy agent that bids on its behalf in the SAAPBs. In that way, even if the SAAPBs take millions of rounds to conclude, the bidding occurs only in “virtual” time and can be completed almost instantaneously.) The direct mechanism which simply consists of bidders entering their valuations into the proxy agents and the implied result being realized became the basis for “core-selecting auctions,” which are used as the final round of the newer package clock auction (or combinatorial clock auction) design. To the best of our knowledge, the SAAPB has never been used outside the experimental lab.

4.13 Strengths and weaknesses of the SMRA

The simultaneous multiple round auction is an effective and simple price discovery process. It allows arbitrage across substitutes. It lets bidders piece together desirable packages of items. And, because of the dynamic process, it reduces the winner’s curse by revealing common value information during the auction (Kagel and Levin 1986, Kagel et al. 1996).

However the simultaneous multiple round auction has been observed to have many challenges. These challenges are summarized in the following table:

⁶ For example, the Canadian AWS spectrum auction of 2008, a standard SMR auction, lasted 331 rounds and extended over 12 weeks. Major US spectrum auctions have had similar lengths.

Table 2. Challenges of the simultaneous ascending auction

Challenge	Description	Remedy to mitigate problem
Demand reduction	Large bidders reducing demands in order to keep prices low; especially a problem in settings with a few large bidders	Allow package bids and adopt an improved pricing rule
Tacit collusion	Bidders signalling demands and engaging in retaliation in order to get the auction to end sooner at lower prices for their desired set of lots	Limit release of bid information, such as specific bids and bidders until the end of the auction; limit flexibility in making bids
Parking	Bidders bidding on lots they do not expect to win simply to maintain greater eligibility for later in the auction	Adopt a better activity rule, such as the activity rule outlined in Section 5.6
Exposure	The risk of winning only some lots in a collection of complementary lots and thereby not reaping the complementarities	Allow bids on collections of lots (packages)
Hold up	The strategy of a speculator insisting on getting something from a large bidder as quid pro quo from not pushing prices high on key lots desired by the large bidder	Limit release of bid information, such as specific bids and bidders until the end of the auction
Limited substitution	The difficulties a bidder may face in switching from one collection of lots to another	Use generic lots in the initial auction stage
Complex bidding strategies	Incentives for gaming result in highly complex bidding strategies	Adopt rules that encourage truthful bidding throughout the auction, especially a good activity rule (see Section 5.6)

Demand reduction. As a result of the pricing rule, there is a strong incentive for large bidders to engage in demand reduction—reducing the quantity demanded before the bidder’s marginal value is reached in order to win at lower prices.

Tacit collusion. Especially if there is weak competition, bidders have an incentive to engage in tacit collusion. The bidders employ various signaling strategies where they attempt to work out deals through the language of the bids. The goal of the strategies is to divvy-up the items among the bidders at low prices.

Parking. As a result of the activity rule, there are parking strategies. A bidder maintains eligibility by parking its eligibility in particular spots that the bidder is not interested in and then moves to its true interest later. This undermines the activity rule and distorts prices.

Exposure. The simultaneous ascending auction is typically done without package bids. The bidders are bidding on individual lots and there is the possibility that a bidder will win some of the lots that it needs for its business plan, but not all. This exposure to winning less than what the bidder needs has adverse consequences on efficiency. Essentially the bidder has to guess. Either the bidder goes for it or not. When there are complementarities, this is a tough decision for the bidder to make. The bidder may make the wrong decision and win something it actually does not want. For example, the bidder could win an area that is too small or oddly shaped to support their business plan. In this case, the bidder would not have many good options available. The bidder could either withdraw its provisionally winning bid on the lot (and pay a penalty), find a buyer to transfer the lot to after the auction, or default on their bidding obligation (and pay a penalty).

Hold up. The lack of package bids in traditional simultaneous multiple round auctions makes them vulnerable to hold up, which is basically a speculator stepping in and taking advantage of a bidder (Pagnozzi 2007). The speculator can make it clear to large bidders that it would be expensive to push him out of the way. As a result, the large bidders let the speculator win some desirable lots at low

prices, and then the speculator turns around and sells them to the big players after the auction is over. That is the holdup strategy. It is easy to do and effective. Preventing resale would reduce this problem, but resale is desirable in a rapidly changing dynamic industry.

Limited substitution. There may be limited substitution across licenses depending on how the lots are structured. The spectrum auctions give many examples where limited substitution led to strange prices (Cramton 2009). Generic lots are now commonly used in spectrum auctions to address this problem. However, in the context of offshore wind auctions, generic lots may be infeasible.

Complex bidding strategies. As a result of all these factors and others, the bidding strategies in the SMRA are quite complicated. The auction creates incentives for gaming and the bidders exploit these gaming opportunities in an effort to win what they want at lower prices.

Table 2 above also provides remedies to address the challenges. These remedies have been developed over the last seventeen years as we have gained experience with the auction format in the context of spectrum auctions. The conclusion of this experience has been the evolution of two closely related auction formats: simultaneous clock auctions and package clock auctions. These new designs are the focus of the next two sections.

The severity of the problems above varies with the setting. There may be many settings where the SMRA performs reasonably well. However, we do not view this as a reason not to prefer one of the clock auction methods, which are specifically designed to address the problems of the SMRA. The SMRA is best thought of as an early approach to auction many related items. The clock auctions are simply a subsequent generation of auction methods that build on the strengths of the SMRA and avoid some of the weaknesses revealed from experience.

5 Clock auctions

5.1 History and overview

Clock auctions are an enhancement of simultaneous multiple round auctions. Clock auctions simplify bidding and improve the effectiveness of the auctions. Special cases of the clock auction include a single-lot auction (lease area being auctioned off as one lot). Another special case is auctioning off the area as a set of generic lots (one offshore wind lease area divided into multiple lots of similar value) that is followed by an assignment stage in which bidders specify which lot or lots they want.

5.2 Object of bidding

As with SMRAs, clock auctions generally include many related items that are either substitutes or complements or both. However, it is recommended that blocks that are considered complementary are bundled into single lots, to simplify bidding. This is outlined further in Section 2.3.2.

5.3 Upfront deposits

Bidders submit upfront deposits sufficient to allow them to bid on the largest collection of lots they may desire. Submitting this upfront deposit allows the bidder to bid on smaller packages with fewer eligibility points as the auction progresses.

5.4 Bidding procedures

In the clock auction, all lots are auctioned simultaneously. In each round, the auctioneer announces the current price of each lot. Bidders respond by bidding a quantity for each lot (if the lot is indivisible

and available in supply of one, whether the bidder is “in” or “out”). Bidders have an initial eligibility based on their deposits. They must keep active to maintain their eligibility. Prices increase on lots where there is excess demand.

In a clock auction the price of each lot ticks up, like a clock, in response to excess demand. Key differences from a simultaneous multiple round auction is that in each round bidders respond to the prices by expressing their demands—the lots they desire at the specified prices—and that each round yields the aggregate demand for each lot at the current prices, rather than a list of provisional winning bidders.

In a *simultaneous clock auction*, the stopping rule is that the auction does not end on *any* lot until there is no excess demand on *all* lots. In essence, a simultaneous clock auction is an auction that allows a bidder to specify a specific package of lots that he wants.

The focus on excess demand rather than provisionally winning bids makes it easy to accommodate package bids while maintaining a linear price structure, where the price of a package is simply the sum of the individual lots. This is a key insight as it allows simple package auctions that are free from complex winner determination algorithms or pricing rules. In each round bidders see prices for each lot and then select their desired package given the prices. Prices then increase where we have excess demand. Bidders are free to adjust packages, subject to an activity rule, as prices increase.

The drawback of this package treatment is that there is a possibility, as a result of complementarities, that some lots will go unsold. This happens whenever one or more bidders make large reductions in demand toward the end of the auction. Nonetheless, in the context of offshore wind auctions, it would seem that the cost from unsold lots is not too high and the benefit from simplified implementation may be important.

The clock auction can also be conducted without package bids by adopting a more stringent activity rule (no switching among lots) and lot-by-lot closing. This is known as an *independent clock auction*. In this auction, once a lot has closed, the winning bidder must purchase that lot even though he is still bidding on other lots which are open. If these open lots are complementary to the lot he has won, the bidder may be forced to bid above his value for those lots to win the combination of lots that he wants – otherwise he could be left with a single lot which he did not want on its own. Therefore, it works well when bidders have additive values (the value of a package is equal to the sum of the values of the individual lots in the package), but this seems unlikely in the offshore wind setting, where we expect complementarities to be large. The remainder of this section focuses on *simultaneous clock auctions*, which we feel are more appropriate to the offshore wind setting.

Simultaneous clock auctions retain the benefits of simultaneous multiple round auctions, and yet address many of the challenges as shown in Table 3. The challenges in green are fully addressed and the challenge in light green is partly addressed.

The simultaneous clock auction is a simple price discovery process but

- further simplifies bidding by making the choice a simple yes/no by bidders on each lot,
- allows simpler bidding by not identifying provisionally winning bidders at the end of each round,
- simply accommodates package bids,
- reduces collusion risks by mitigating bid signaling,
- simplifies bidding and improves substitution while avoiding the need for withdrawals, and
- encourages product designs that enhance substitution and simplify the auction.

5.5 Initial prices and bid increments

Initial prices are set in a clock auction in the same manner as in an SMRA.

Bid increments may be less important in a clock auction than in an SMRA because the clock auctions easily accommodate exit bids that allow bidders to precisely express their final prices for lots. Bid increments are normally set based on the relative demand where lots with more bids increase by a higher percentage than lots with fewer bids.

5.6 Bidding activity rules

The activity rule in clock auctions is often 100%. Bidders are required to express their demand for all lots that they wish to win at current prices in every round. If a bidder bids below the required activity, their eligibility is reduced commensurately in the next round. In a clock auction there are no activity rule waivers. However, in some clock auctions, a small number of round extensions are allowed in the event that a bidder experiences a technical problem. There are some variations to these rules which have been utilized in the past, and these are described in Section 5.15 below.

5.7 Bid withdrawals

Bid withdrawals are not necessary in a clock auction because provisional winners are not assigned.

5.8 Closing rule and determination of final winners

Final winners of a simultaneous clock auction are the bidders who remained in the auction during the round in which there was no excess demand on all lots. In other words, the auction does not close on any lot until it closes on all of the lots. This is an important design feature of the simultaneous clock auction since it allows bidders the flexibility to switch their bidding to other more attractively priced lots during the course of the auction.

5.9 Information policy

In clock auction, bidder identities are not revealed. Only aggregate demand on each lot is reported.

5.10 Strengths and weaknesses of the Simultaneous Clock Auction

The simultaneous clock auction addresses many of the challenges of the SMRA, as shown in Table 3. Tacit collusion is much reduced when bidder identities are not revealed. The bids are for packages of lots, so the exposure problem is eliminated. This also prevents the hold-up problem. Substitution is improved because bidders have a great deal of flexibility in changing packages as prices change.

Table 3. Remaining challenges for the simultaneous clock auction

Challenge	Severity relative to simultaneous ascending auction	Remedy to mitigate problem
Demand reduction	Large bidders still have an incentive to reduce demands in order to keep prices low	Adopt an improved pricing rule
Tacit collusion	Fully addressed with standard information policy in clock auction (revealing only aggregate demand)	
Parking	Bidders can still bid on lots they do not expect to win simply to maintain greater eligibility for later in the auction	Adopt a better activity rule
Exposure	Fully addressed with rules best suited for offshore wind application	

Hold up	Fully addressed with standard information policy in clock auction (revealing only aggregate demand)	
Limited substitution	Addressed with expression of demand and the use of generic lots where possible in the initial auction stage	
Complex bidding strategies	Incentives for gaming are much reduced with better information policy and use of generic lots where possible	It may be desirable to further simplify strategies by addressing demand reduction and parking

The main cost of the approach is the possibility that some lots will go unsold. As a result of complementarities one or more large bidders may make substantial drops in demand in the final round, leaving some lots unsold.

This auction approach has performed well since 2001 in large electricity auctions. In these auctions having some quantity unsold is a minor problem, since unsold quantity can be placed in the next quarterly auction.

There is a simple variation of the approach to reduce the quantity of unsold lots. As before, run the clock auction until there is no excess demand for any lot. Then take all the bids that were placed in the entire process and find the collection of bids (no more than one from each bidder) that is feasible and maximizes total value. This collection identifies each winner's lot assignment. Each winner pays its bid.

The package clock auction described in the next section builds on this variation to address the remaining challenges.

5.11 Selecting winners

Winners in the simultaneous clock auction are those that remain in when the auction ends. The price paid for each lot is the final lot price. In the simplest implementation, bidders can freely switch packages subject to the activity rule. In the presence of complements, this means that some lots may go unsold, as a result of a large drop in demand from one or more bidders. Unsold lots could later be resold.

5.12 Use of generic lots when lots are similar

Generic lots greatly simplify the bidding in clock auctions by guaranteeing that each bidder gets contiguous lots and that similar lots sell for similar prices. Instead of selling each lot individually at different prices, lots can be treated as a generic commodity that has a single price that applies to all as shown in Figure 4. The generic lots can be used for any group of lots that are highly substitutable and of comparable value.

Figure 4. Specific vs. generic lots

Specific lots											
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">A1</td> <td style="width: 10%;">A2</td> <td style="width: 10%;">A3</td> <td style="width: 10%;">A4</td> <td style="width: 10%;">A5</td> <td style="width: 10%;">A6</td> <td style="width: 10%;">A7</td> <td style="width: 10%;">A8</td> <td style="width: 10%;">A9</td> <td style="width: 10%;">A10</td> </tr> </table>	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
A1	A2	A3	A4	A5	A6	A7	A8	A9	A10		
Price	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">\$1.49</td> <td style="width: 10%;">\$1.42</td> <td style="width: 10%;">\$1.32</td> <td style="width: 10%;">\$1.07</td> <td style="width: 10%;">\$1.39</td> <td style="width: 10%;">\$1.31</td> <td style="width: 10%;">\$1.28</td> <td style="width: 10%;">\$1.09</td> <td style="width: 10%;">\$1.35</td> <td style="width: 10%;">\$1.22</td> </tr> </table>	\$1.49	\$1.42	\$1.32	\$1.07	\$1.39	\$1.31	\$1.28	\$1.09	\$1.35	\$1.22
\$1.49	\$1.42	\$1.32	\$1.07	\$1.39	\$1.31	\$1.28	\$1.09	\$1.35	\$1.22		

Generic lots											
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> <td style="width: 10%;">A</td> </tr> </table>	A	A	A	A	A	A	A	A	A	A
A	A	A	A	A	A	A	A	A	A		
	Price = \$1.25										

The use of generic lots greatly simplifies the bidding, enhances the substitution across lots, and guarantees that each winner’s contiguous lots. There are two potential downsides. The first is that the lots may not be highly substitutable or of comparable value. This would be the case if adjacent lots had substantially different characteristics such as water depth or wind speed. In this case using generic lots exposes the bidder to risk. The second is that the use of generic lots requires a two-stage auction. The clock stage must be followed by an assignment stage.

- The *clock stage* determines the quantity of contiguous lots won by each bidder. In this stage, bidders are to decide the number of lots they want to buy at the clock price.
- The *assignment stage* determines the specific assignment of lots to each winner given the number of lots won in the clock stage. The options available to a winner are limited to be consistent with pre-defined constraints, such as that each winner is assigned contiguous lots. The assignment stage is a refinement of various “bidder choice” auction formats in which the auction determines a priority of choice among the winners.

Consider an example with $n = 10$ lots for sale to three bidders. In the clock stage, the number of possible package bids is only $n + 1 = 11$ (each bidder can bid for one of 0, 1, 2, ..., or 10 tracts) as opposed to $2^n = 1,024$ (each bidder can bid for any combination of the 10 tracts) if specific lots were used. This illustrates the great simplification of using generic lots. In our example, suppose Blue won four lots and Red and Green each won three lots as shown in Figure 5.

Figure 5. Clock stage when bidding for generic lots

Bidder	Number of lots won				Case	Number of package bids
Blue	A	A	A	A	Generic lots	$n + 1 = 11$
Red	A	A	A		Specific lots	$2^n = 1,024$
Green	A	A	A			

$n = \text{number of lots} = 10$

Not only is the clock stage greatly simplified, but the assignment stage, which determines the specific assignment typically is extremely simple, since it only involves the winners and there are a limited number of ways that contiguous lots can be awarded to these winners. This is described further in the following section.

5.13 Assigning leases

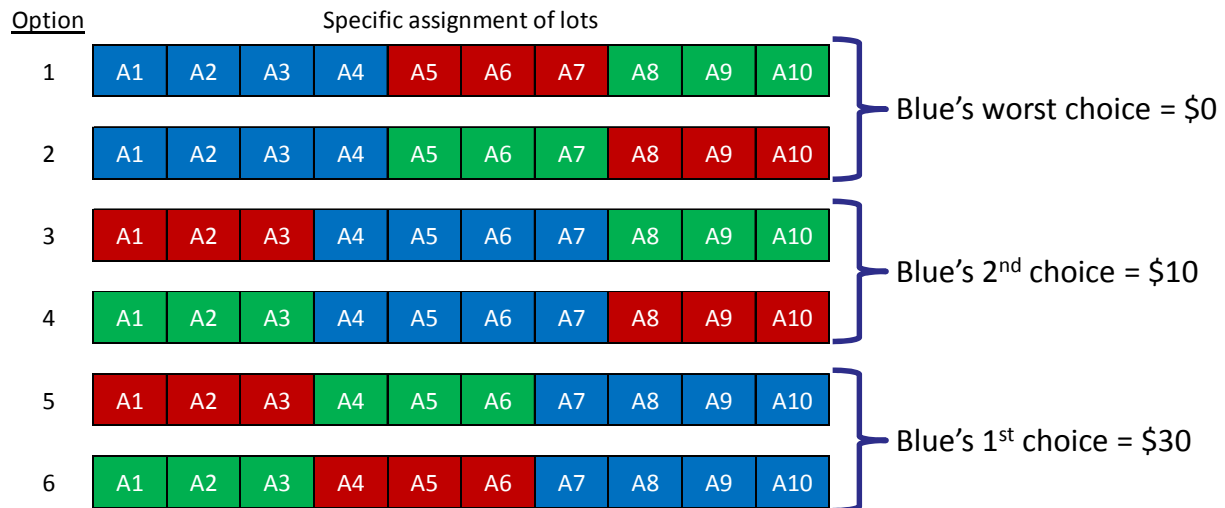
The assignment stage can be thought of as an important extension and improvement of a bidder's choice auction. The assignment approach fixes all quantities before the stage begins, requires that all bidder choices are consistent with these reserved assignments including the need for contiguous lots, and simultaneously gathers information from all bidders about alternative assignments. In this way, the assignment that maximizes total value across all bidders is selected. In contrast, bidder's choice puts too much weight on the preference of the winner with first choice.

The quantities that are fixed in the clock stage are both guaranteed and binding. A bidder cannot later back out, complaining that the outcome of the assignment stage is unsatisfactory. When placing bids in the clock stage the bidder understands the range of possible assignments and places bids knowing there is some possibility that it will get its least favorite assignment (at no additional cost).

In our example above, with $m = 3$ winners, the number of possible assignments is $m! = 3 \times 2 = 6$ if generic lots are used as shown in Figure 6. In contrast if specific lots are used there are $m^n = 3^{10} = 59,049$ different assignments.

Figure 6. Assignment stage in clock auctions with generic lots

Case	Number of specific assignments	
Generic lots	$m! = 6$	$m = \text{number of winners} = 3$
Specific lots	$m^n = 59,049$	$n = \text{number of lots} = 10$



Given that there are so few options for contiguous assignments, eliciting the bidder's preferences in the assignment stage is a simple matter. For example, as shown in the second panel of Figure 6, Blue has to evaluate just three different possibilities: What is Blue's worst choice? What is Blue's second choice? And what is Blue's first choice? Blue would automatically assign an incremental value of \$0 to its worst choice so in fact Blue only has to figure out two numbers: the incremental value of its second choice (\$10) and the incremental value of its first choice (\$30).

The assignment stage is so simple that it can be implemented efficiently with a sealed-bid second-price auction. Indeed, a sealed-bid assignment stage may be required by BOEMRE’s regulations at 30 CFR § 220. Each bidder has to give preference over possible outcomes. Blue has three possible assignments whereas Red and Green have four possible assignments. Note that the options are limited to those that can guarantee *every winner* contiguous lots. This is an important restriction, which is understood by the bidders in the clock stage.

The final assignment is an assignment that yields the highest total incremental value—the sum of the bids across winners for the particular option. This is option 1 as shown in Figure 7. The extra price each winner pays for the resulting specific assignment is the Vickrey price (or second price) calculated as follows. Blue pays \$0, since Blue gets its worst choice. To calculate the price paid by Red, we imagine Red reducing each of its bids as much as possible until Red getting its second choice is tied with Red getting its worst choice. This happens when Red’s bids are reduced by \$2, so that option 3 becomes tied for optimal with an incremental value of \$45. Red therefore pays \$10. For Green we do the same calculation. Green can reduce its *bids* by \$9 before the incremental value of the winning assignment falls to \$38 and becomes tied with the next-best option 6 in which Green gets its worst choice. Thus, Green pays 35 – 9 = \$26. This pricing rule makes bidding your true incremental value a dominant strategy—regardless of what the other bidders do, your best strategy is to truthfully bid your value. (The calculation requires that all of Green’s bids are reduced, not just its bid on the efficient assignment.)

Figure 7. The value-maximizing assignment with second pricing

<u>Option</u>	Sealed-bid second-price auction to determine specific assignment			Incremental value
1	Blue = \$0	Red = \$12	Green = \$35	\$47
	Blue pays \$0	Red pays \$10	Green pays \$26	
2	Blue = \$0	Green = \$15	Red = \$20	\$35
3	Red = \$0	Blue = \$10	Green = \$35	\$45
4	Green = \$0	Blue = \$10	Red = \$20	\$30
5	Red = \$0	Green = \$11	Blue = \$30	\$41
6	Green = \$0	Red = \$8	Blue = \$30	\$38

One important simplification in our example is that the lots are arranged in a line. With this simplification it is always trivial to specify the options for specific assignments that are consistent with awards of contiguous lots and there are always a small number ($m!$ where m is the number of winners) of options.

More generally, lots are arranged in a two-dimensional grid. This poses a challenge in specifying the algorithm that determines the possible options based on the outcome of the clock stage. While the generalization to a two-dimensional grid is far from trivial, this challenge can be overcome. What is critical is that in advance of the clock stage, the bidders fully understand how the clock bidding

translates into a generic outcome and then what options are available in the assignment stage as a function of the generic outcome. The best approach depends on the layout of lots, and any pre-bundling of lots, both of which are established for the particular auction as part of the product design work.

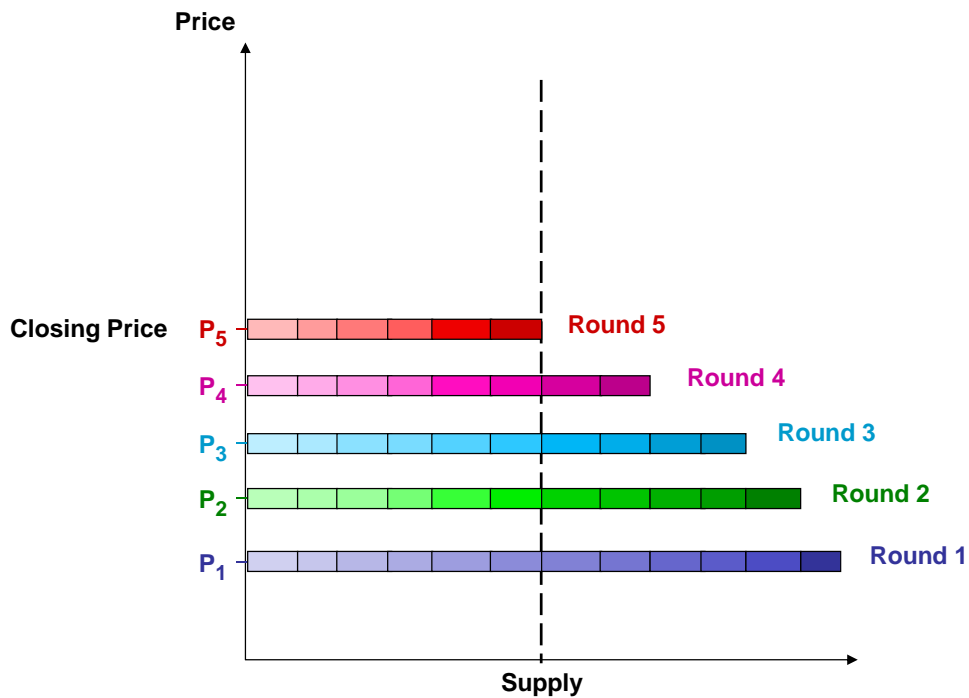
Even so, lots may be too heterogeneous with respect to water depth, bottom type, environmental concerns, distance to shore, view shed concerns, existing uses (navigation, recreation, etc.), and wind speed to accommodate generic lots, and therefore specific lots may be required in many BOEMRE applications.

5.14 Clock auction with exit bids

In practice, clock auctions are conducted using discrete rounds and significant bid increments, typically between 5% and 15% depending on the level of excess demand. Discrete rounds are used because bids must be binding commitments and even brief Internet access issues would create serious problems if a continuous price clock were used. In addition, discrete rounds give the bidders time to reflect and enable the auctioneer to better mitigate tacit collusion, both of which improve price discovery. See Ausubel and Cramton (2004) for a richer discussion.

Figure 8 shows the progress of a clock auction. The auction starts at a low price at which there is substantial excess demand (demand > supply). The price is then increased until there is no more excess demand (demand = supply). This occurs in round 5 in the example.

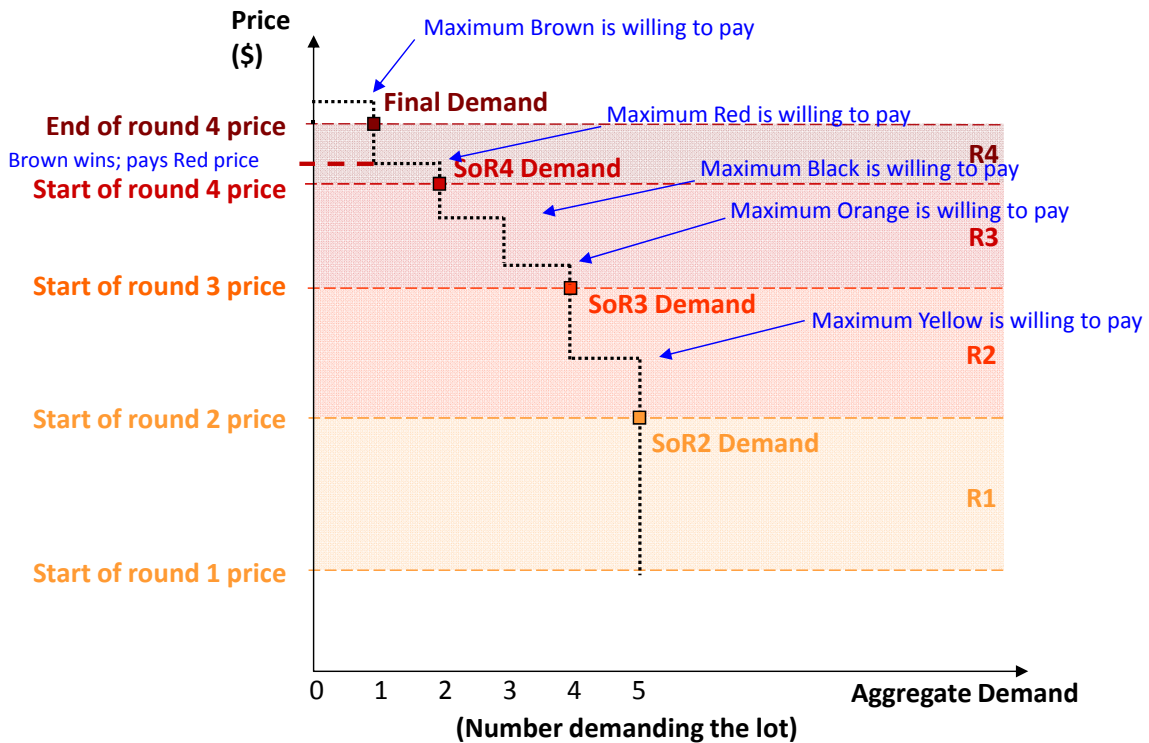
Figure 8. A clock auction



As a result of discrete rounds with significant bid increments, best-practice in clock auctions is to let bidders specify the prices at which they desire to reduce demand, which are called exit bids. Thus, if the price increases from \$10 to \$11 in a round, the bidder may either maintain its \$10 quantity at the \$11 price, or may specify the exact prices between \$10 and \$11 at which it wishes to reduce demand. When specific lots are auctioned, so each lot has a separate price clock, then a bidder wishing to reduce

demand simply names the exit price, such as \$10.75. For multiple generic lots, the bidder names a price for each quantity reduction. An example is shown in Figure 9 for a single lot.

Figure 9. A clock auction with exit bids



This approach is both simple and powerful. It reduces the possibility of ties, since the price of each exit is stated by the bidder. Moreover, the efficiency loss associated with the discrete increments is eliminated. Each bidder’s demand curve is continuous, just as with a continuous clock. As a result, the auctioneer can choose a larger bid increment to speed up the auction process. This can both reduce transaction costs and mitigate the risk of tacit collusion.

5.15 Alternative activity rules

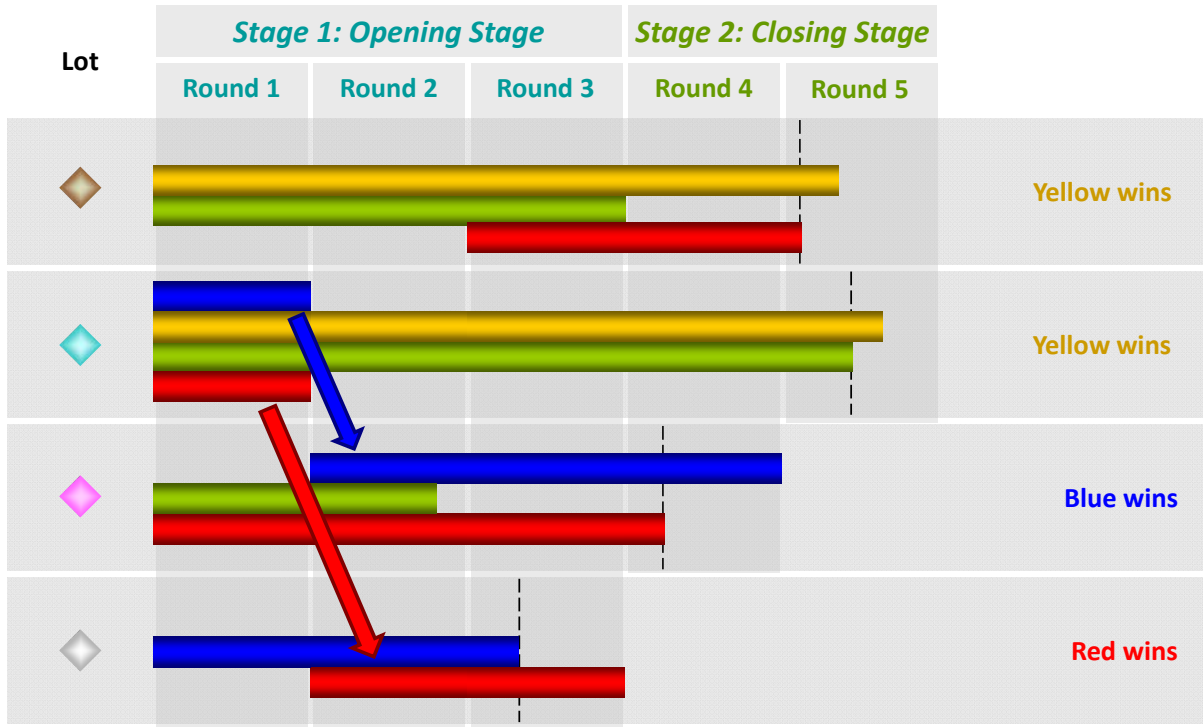
With many specific lots or groups of generic lots, the auction requires many price clocks, one for each specific lot and each group of generic lots. This also requires a more sophisticated activity rule to determine the constraints on switching across the lots. This choice involves a difficult tradeoff: allowing great flexibility in switching encourages substitution but also enables parking strategies that undermine price discovery; whereas, allowing little flexibility in switching encourages price discovery by preventing parking, but limits substitution across lots. The resolution of this tradeoff requires both judgment and good knowledge of the particular application.

We consider two alternative activity rules that attempt to resolve this trade-off. The simplest implementation of a simultaneous clock auction is one that sharply limits switching among lots after the first few rounds, and this is known as the “limited-switching rule.”

Limited-switching rule. There are two stages to the auction: an opening stage and a closing stage. In the opening stage, bidders can switch freely from one lot to another. In the closing stage, bidders can only bid on lots that they bid on in the previous round. In essence, the auction can be considered as a

simultaneous clock auction for the opening stage (i.e. for the first few rounds), and an *independent clock auction* for the closing stage (the remaining rounds). Figure 10 shows an ascending clock auction with a limited-switching activity rule. Blue and Red switch their bids during the opening stage in which bidders are allowed to freely switch across lots.

Figure 10. A clock auction with a limited-switching activity rule



The opening stage addresses the coordination problem that the bidders would face with a “no switching” rule. As a result of budget constraints or limited demands, bidders may be unwilling to bid on all the lots even though the initial prices are low. They must therefore decide which among the many lots to bid on initially. If too few bidders select some lots in the initial round, the opening stage enables bidders to shift or expand their demands on the lots that received too few bids in the initial round.

Then in the closing stage, bidders cannot switch across lots, but only exit if the price gets too high. This allows some substitution, as bidders will tend to exit those lots that are a poorer value, but the substitution is far from perfect. On the plus side, no switching in the closing stage eliminates all parking strategies.

This limited switching activity rule is used without package bids. Once a winner is identified for a lot (there is no excess demand), then the winner is locked in and cannot subsequently withdrawal. The activity rule works best when bidder values are largely additive: the value of the package is the sum of the individual values. This rule has worked well in diamond auctions, since 2008. However, we do not recommend it for offshore wind auctions, since bidder values are not apt to be additive.

Aggregate rule. Bidders can switch freely among lots, subject to the constraint that the bidder’s aggregate bid activity is weakly decreasing over time. Thus, in aggregate a bidder can maintain its overall level of activity or reduce its level of activity, but cannot increase its activity.

To measure activity it is necessary to assign each lot a number of eligibility points that reflects the estimated relative value of the lot. In the simplest case of generic lots, each lot is given the same

number of points, since it is presumed that all generic lots within the same lease area are of about the same value. In contrast, the eligibility points for specific lots may differ. For example, a superior lot likely worth 50% more than an inferior lot, may be assigned 3 points while the inferior lot is 2 points. A bidder's activity in a round is the sum of the eligibility points for the lots the bidder is demanding.

With this activity rule bidders may switch freely among lots subject to the constraint that their total activity never increases as prices rise. Thus, a bidder may at any time switch from 2 superior lots with 3 points each to 3 inferior lots with 2 points each. In both cases, the activity is $2 \times 3 = 6$ points.

This aggregate rule allows greater substitution among lots. However, the rule enables parking strategies, which may distort bidding and discourage price discovery, especially if it is difficult for the regulator to estimate relative values across lots.

The aggregate rule has been used in nearly all spectrum auctions. Based on our knowledge of preferences in the offshore wind context, we believe that an aggregate activity rule would be best in most cases. The synergies among adjacent lots seem most important in the offshore wind application.

5.16 Assessment of clock auctions

Clock auctions have many advantages.

- *Transparency.* Clock auctions are an open, transparent, and fair auction process.
- *Efficient allocation.* Clock auctions have a strong tendency to put the lots in the hands of those who value them the most.
- *Competitive pricing.* Clock auctions, through a simple price discovery process, tend to identify competitive market prices.
- *Effective process.* Clock auctions are a fully dynamic auction, yet can be run from start to finish in one day and can accommodate limitations in Internet access.
- *Activity rule.* Clock auctions have an activity rule, which eliminates bid sniping—the tendency to wait until the last minute to place serious bids as in eBay.

These advantages are seen in theory, in the experimental lab, and in practice. Clock auctions address many of the challenges of the traditional simultaneous multiple round (SMRA) auctions. It is for this reason that clock auctions have largely replaced simultaneous ascending auctions in recent years.

The main challenge for clock auctions is how packages are dealt with. With *Independent clock auctions*, bids are for individual lots. This limits a bidder's ability to express preferences for a complementary package of lots. With *simultaneous clock auctions*, bidders can express preferences for a complementary package of lots, but there is the tendency for unsold lots, demand reduction, and the possibility of too few bids.

The package clock auction addresses these limitations.

6 Package clock auctions

6.1 Background

Package clock auctions are an extension of simultaneous clock auctions. In a package clock auction, all bids are package bids (collections of synergistic lots). The approach becomes important when there are strong complementarities across the lots that differ among the bidders. When bidding on individual lots, a bidder is exposed to the risk of winning only some of a complementary set of lots. Package

bidding eliminates the exposure problem by allowing bidders to bid on packages of lots, which is important when complementarities are strong. The presence of strong complementarities, however, is not sufficient for requiring a package auction. For example, if the structure of complements is the same for all bidders, then the complementary sets of lots can be pre-bundled before the auction starts, just as a shoe store pre-bundles left and right shoes and sells only pairs of shoes. Rather, a package auction is needed in circumstances where the complementarities are both strong and varied across bidders—the bidders disagree how the lots should be bundled.

Each bid in a package clock auction is a binding commitment for the entire auction. There is no need to allow bidders to withdraw from bids in a package clock auction, as the bidder should be happy to win at any price less than the price they bid for a given package. This makes bidding in the package clock auction simpler than the SMRA from a bidder perspective. There is no need to carefully track and plan the use of withdrawals or attempt to keep activity from falling. Instead, bidders are best served by simply bidding what they would most like to win at each price.

In addition, as we will see, package bidding can help mitigate the incentives of large bidders to exercise market power in which they reduce demands in order to keep prices low. The package clock auction uses a pricing rule that encourages truthful bidding.

The package clock auction has been designed to be simple from a bidder's perspective. In each round of the auction the bidder is asked to name its preferred package given the specified prices. The bidder is shown the aggregate demands for each lot. Prices for lots with excess demand are increased. The bidder is asked again to name its most preferred package given the new prices. This process continues until there is no excess demand for any lot. Following this process of price and assignment discovery, the bidder is asked to place any additional bids or any improvements to prior clock bids. Then all the bidder's bids together with all the bids of the others are considered in finding the value-maximizing assignment of lots. The winning bidder pays the competitive "second price" for its winning package. Both the pricing rule and the activity rule are chosen to encourage truthful bidding in each round of the process. This lets the bidder focus on identifying its most preferred package given the prices.

To be clear, the package clock auction consists of two stages, both of which would be implemented in a single stage in the BOEMRE Final Rule. The package clock auction begins with a clock stage for price discovery in which the auctioneer names a price for each lot and bidders specify the package of lots they are willing to bid for at that set of prices. For all lots with excess demand, the price is increased in the next round. This process is repeated until there is no excess demand on all lots.

Following the clock stage, the bidders have a much better sense of what the likely prices are and what they are apt to win. At this point a supplementary round is held. This is a final sealed-bid round. In this, bidders can list all packages that they are interested in winning, and their value of each, subject to a set of activity rules. These rules limit what packages a bidder is permitted to bid for, as well as the maximum and minimum prices the bidder can specify for a package, based on his bids in the clock stage. All bids placed in the clock stage are carried forward into the supplemental round, but these bids can be revised subject to these rules. Without these rules, the bidder would be able to specify any bids it wanted in the sealed-bid round, and the clock stage would have little meaning. Therefore, the rules encourage the bidder to bid truthfully in the clock stage, and prevent bid sniping, as the bidder must reveal its preferences in the clock stage.

Once the clock bids and the supplementary bids are collected, an optimization is run to determine the generic assignment (or specific assignment if generic lots are not used) and prices. The assignment is the one that maximizes the total value based on the bids. This is the classic "winner determination

problem.” Winner determination is a set-packing problem. It is solved efficiently with specialized optimization software, such as CPLEX, designed to solve large integer programming problems.

The assignment stage is the final step of the package clock auction. This is only needed if generic lots are used. It resolves a specific assignment if generic lots are used. The assignment stage is a sealed-bid second-price auction as was described earlier.

Three important features of the package clock auction are

- no exposure problem, since bids are for packages,
- second pricing to encourage truthful bidding, and
- an activity rule to eliminate bid-sniping and promote price discovery.

We now describe the pricing rule and the activity rule. Both may seem complicated at first; however, this complexity of the rule actually simplifies the bidder’s bidding strategy. Bidders are given strong incentives to bid truthfully throughout the entire auction. This enhances price and assignment discovery. Most importantly, it enables the bidder to focus on developing a better understanding of values and relevant tradeoffs.

6.2 Pricing rule

With an objective of efficiency, best practice is to adopt second pricing. Consider an example with a single lot. The highest bidder wins the lot and pays the *second-highest price*. This makes bidding true value a dominant strategy; it is the best strategy regardless of what the other bidders are doing. For instance, if you have the highest bid of \$100 and my bid of \$90 is the second highest, then you win and pay \$90; this gives you an incentive to bid your true value, \$100, since this lets you win in all circumstances that are profitable, regardless of what the others are bidding.

Second pricing generalizes to auctions with many items, either Vickrey pricing or a variant called bidder-optimal core pricing (or minimum-revenue core pricing). Under both variations, the assignment is the same. The winners are determined to maximize the total value, assuming bids to be values.

Vickrey prices can be found for each winner as follows. Gradually reduce each of the winner’s bids until the point where the winner no longer wins. The winner pays the reduced bid that is just sufficient to let the bidder win given the bids of the others. Notice that Vickrey pricing is equivalent to the following: the winner is told all the bids of the others and then sets its price at the lowest level that still enables the bidder to win what it wins in the efficient assignment.

Vickrey pricing makes bidding your true value a dominant strategy. This is an extremely desirable theoretical property. However, when there are complementarities, it is possible that the Vickrey prices will be too low in the sense that losing bidders may complain that a winner is paying too little—that they were willing to pay more. Bidder-optimal core pricing is a variant of Vickrey pricing that includes these competitive constraints.

The core is an assignment of lots and payments by the winners such that

1. the assignment is efficient in that it maximizes total value, and
2. it is unblocked in that no collection of bidders offered the seller a better deal.

Bidder-optimal core pricing finds the smallest payments that are consistent with the core. For the auction of a single item, bidder-optimal core pricing is identical to second pricing, which is the same as Vickrey pricing: the high bidder wins and pays the second-highest price. For auctioning many items that are substitutes, bidder-optimal core pricing again is identical to Vickrey pricing: the winners form the

value-maximizing assignment and each pays the opportunity cost of its winnings, the smallest bid possible that still enables the winner to win, holding fixed the bids of others. For auctioning many related items with both substitutes and complements, then bidder-optimal core prices may be higher than Vickrey prices in order to respect competition constraints.

Nearest-Vickrey core pricing was adopted in each of the UK spectrum auctions, both the two that have already been held as well as the proposed auctions for the 2.6 GHz spectrum and the digital dividend spectrum. Nearest-Vickrey core pricing was also used in the Netherlands 2.6 GHz auction, and proposed in the US auction for takeoff and landing slots at the New York City airports. Erdil and Klemperer (2009) argue that marginal incentives for truthful bidding may be improved by using a reference point other than the Vickrey prices for selecting among bidder-optimal core prices. In particular, they recommend a reference point that is independent of the winners' bids. In instances where the regulator can identify a suitable reference point that is independent of the winners' bids this may be desirable. This would be the case if the regulator had good information about relative prices in advance of the auction. However, in circumstances where the regulator does not have good information about relative prices, it may be better and simpler to use the Vickrey prices as the reference prices.

Bidder-optimal core pricing has several advantages. First, it minimizes the bidders' incentive to distort bids in a Pareto sense: there is no other pricing rule that provides strictly better incentives for truthful bidding. Bidder-optimal core pricing implies Vickrey pricing, whenever Vickrey is in the core. For example, when lots are substitutes, Vickrey is in the core, and the bidders have an incentive to bid truthfully. Since the prices are in the core, it avoids the problem of Vickrey prices being too low as a result of complements.

Further information can be found in Appendix A.

6.3 Activity rule based on revealed preference

In a package clock auction, one can use a quantity-based rule, like the aggregate activity rule in the clock stage, but one also needs to specify how the rule limits bids in the supplementary round. This linkage between the clock bids and the supplementary bids is of critical importance, for otherwise the bidder could bid snipe, submitting all of its bids in the supplementary round.

Together with Paul Milgrom, we proposed an alternative activity rule based on revealed preference for the package clock auction (Ausubel et al. 2006). Revealed preference is the underlying motivation for all activity rules. The intent is to require the bidder to bid in a way throughout the auction that is consistent with the bidder's true preferences. Since we do not know the bidder's true preferences, the best we can hope for is for the bidder to bid in a manner that is consistent with its revealed preferences. In the simplest case of a single-product clock auction, this is equivalent to monotonicity in quantity, just like the aggregate activity rule, but when we have multiple products the two rules differ in important ways.

With a revealed preference rule, bidders are required to bid in a way that is consistent with profit maximization. Behavior inconsistent with profit maximization is not allowed. In particular, bidders can only move toward packages that become better values in the clock stage and can only express supplementary bids that are consistent with bids on the clock packages. These constraints are stated mathematically as follows:

At time $t' > t$, package $q_{t'}$ has become relatively cheaper than q_t

$$(P') \quad q_{t'} \cdot (p_{t'} - p_t) \leq q_t \cdot (p_{t'} - p_t)$$

Supplementary bid $b(q)$ must be less profitable than revised package bid at t
(S') $b(q) \leq b(q_t) + (q - q_t) \cdot p_t$

The first constraint simply says that your shift to a different package appears consistent with your prior bids; that is, your new package is now relatively less expensive than the prior packages you did bid for. This explains why you did not bid on the package earlier.

Similarly, the second constraint says the value being expressed for the package q is not so large that a profit maximizing bidder would have preferred to bid for q at some earlier time. Thus, your bid on q in the supplementary stage is consistent with your earlier bids.

This activity rule may seem complicated but it is nothing more than a collection of linear constraints on a bidder's bids. The auction system can readily manage these constraints for the bidder to facilitate bid entry. From the point of view of the bidder, the strategic implication is clear: understand your values as quickly as possible and bid in a manner that is consistent with profit maximization, or risk undesirable limits on what you can bid on in the later rounds of the auction.

A version of the revealed preference rule has been adopted for a number of recent package clock auctions in Europe and is being considered in Canada and Australia.

6.4 Properties of package clock auctions with the revealed preference activity rule

Although the intent of the package clock auction is to better address complementarities, it actually performs well in settings with substitutes and no complementarities. In the substitute case, bidding on the most profitable package throughout the auction is best for each bidder. When this is done, the clock stage yields the competitive equilibrium with an efficient assignment and supporting prices. As a result, the final assignment at the end of the clock stage is the assignment at the end of the auction. Indeed, the clock stage does all the work and the supplementary round is not necessary, and as a result of the activity rule, cannot alter the assignment from the assignment at the end of the clock stage.

As a result of this property, the package clock auction can be safely used in circumstances where the regulator believes that complementarities may be important, but the regulator is not sure. If complementarities turn out to be minor, then no harm is done. The regulator can rest assured that the Hippocratic Oath is satisfied—first do no harm.

Of course the standard simultaneous ascending auction works well in the substitutes case, especially if second pricing is adopted so that bidders have incentives to bid truthfully and not engage in demand reduction.

With complements, the use of package bids becomes important. In this general case, supplementary bids are needed if the clock stage ends with excess supply (unsold lots) as a result of complementarities. Nonetheless, even in this case, the bidder can guarantee winning the final package it bid for in the last clock round by, in the supplementary round, raising its bid for its final clock package by the price of the unsold lots in the last clock round. This demonstrates that the clock stage can have a large impact in resolving the uncertainty about who will win what and at what prices.

Even with complementarities, if the clock stage ends with no excess supply, then the assignment at the end of the clock stage cannot change. The supplementary round in this case cannot alter the assignment.

Of course if the complementarities are large and there are few bidders, as may well be the case in wind rights auctions, then it is possible that the clock stage ends with large excess supply when a large bidder makes a large exit in the auction involving many lots. In this case, both prices and assignments

can change substantially as a result of the supplementary round and the subsequent optimization to determine the efficient assignment and bidder-optimal core prices.

6.5 *Selecting winners*

Optimization methods are required in package auctions to determine the value-maximizing assignment and price the winning packages. These methods are now commonly used in government auctions, especially in spectrum auctions. The optimization may appear to be a “black box,” but in fact, the algorithms result in a unique determination of both assignment and prices and are readily tested and audited by all stakeholders. Thus, a high level of transparency is still maintained.

6.6 *Assessment of package clock auction*

The package clock auction has many advantages. It

- eliminates exposure by allowing package bids,
- minimizes gaming as a result of package bids, second pricing, and the revealed preference activity rule,
- enhances substitution by using generic lots whenever possible,
- is readily customized to a variety of settings, and
- accommodates settings with strong and varied complementarities among bidders, yet works well in simpler settings.

One of the challenges of the package clock auction is that it tends to favor large bidders as a result of the threshold problem—small bidders have an incentive to hold back in order to pay less for items won. A second challenge for the regulator is the complexity of implementation. However, these auctions are becoming increasingly common and well-understood. Acquiring quality auction services in an RFP should be straightforward.

Package clock auctions may appear complex. However, in situations with strong and varied complementarities across lots or lease areas, the complexity is needed. The complexity of the package auction rules greatly *simplifies* the bidders’ bidding strategies. With appropriate rules, the package clock auction facilitates excellent price and assignment discovery, enabling the bidders to focus valuation efforts on relevant packages, and then to truthfully report their valuations in the auction. Laboratory tests with sophisticated bidders show nearly fully efficient outcomes in complex bidding environments.

Package clock auctions are used for assigning spectrum in several countries in Europe. Many auctions have been successfully conducted with this format. Spectrum regulators in Canada, Australia and the United States are considering using package clock auctions for future auctions since bidders and regulators are both pleased with the results of the package clock auctions conducted in Europe.

7 *Topics that apply to all auction formats*

The following auction topics apply to all auction formats outlined in this paper.

7.1 *Pre-auction procedures*

Once all Expression of Interest (EOI) documents have been received, bidders are qualified for the auction. Qualification takes into consideration factors such as: financial backing and technical viability.

Auction Rules are provided to parties that have had their EOIs approved. In addition, the standard lease contract will be provided to these companies. Note that the contracts should be complete, with the exception of the blocks to be allocated to the company and price paid. Bidders must agree to both the auction rules and contract as part of their submission to participate in the auction. BOEMRE may wish to execute the contracts as soon as possible following the auction, subject to its legal responsibility to provide the Department of Justice (DOJ) at least 30 days following the auction in which to conduct an antitrust review of the sale. Crucially, there must be no possibility of negotiation following the auction. If a bidder feels that its bid is non-binding in any way and could choose not to honor its bid (through failed negotiations, for example), then the bidder will not bid honestly. This would not alter BOEMRE's ability to add mitigations or development restrictions at a later time, as needed for reasons identified in a NEPA analysis. These may be added as conditions of plan approval at a later time.

The Auction Rules should include the following:

- Terms and conditions to participate in the auction
- Definition of terms (glossary)
- Process for submission of bids
- Forms listing individuals who are authorized to bid, non-disclosure forms for these individuals
- Process for submission of any bidder deposits
- Process surrounding the auction itself, including what constitutes a bid, the bid submission and information release processes during/after each round and after the auction itself, and an overview of the process for calculating winners and prices
- Conditions for suspending or rescheduling auction
- Permitted communications during the auction
- Process after the auction has concluded

An Instructions to Bidders manual should be sent to all parties who will participate in the auction. This document contains:

- High level summary of the auction process
- Proposed auction schedule
- IT requirements to participate in the auction
- User guide of the auction system, and how to submit bids electronically, including a troubleshooting guide
- Process of submission of bids using a means of backup if the bidder loses internet connectivity
- Contact details during the auction

Before any online auction, training of the auction system should be provided to bidders. At a minimum, this should include a presentation to bidders and a mock auction, in which bidders can practice taking part in an auction. Additional training, such as on-site training, could also be provided if deemed necessary.

Finally, bidders will need to be sent their credentials to access the auction system in advance of the auction. This could be as simple as a username and password, or could be more sophisticated, such as a token key or client certificate.

7.2 Bidder definition

The regulator must specifically define rules and procedures for qualifying related entities in an auction. Auctions generally have strict rules that prohibit bidders from communicating with one another regarding their bidding interests or strategies. These measures are more fully described below. Related entities with common ownership or management personnel may not be able to abide by these rules. In high stakes auctions, requiring disclosure of the ownership structure of the bidding entity can be important to enforcing rules intended to promote a competitive auction and downstream market.

The FCC has a field tested auction application form that they use for their auction program. The application requires applicants to fully disclose their ownership structures including identifying the shareholders that hold 10% or more of the company, the executives and directors, any related entities, and any agreements with other auction applicants. In the FCC case, related entities are allowed to bid as completely separate entities provided they do not apply for the same licenses. Bidders who have applied for any overlapping licenses are prohibited from communicating with one another regarding bidding strategies once they apply to participate in an auction until after the final auction payment deadline.

In the case of offshore wind, BOEMRE may want to implement a similar rule that allows related entities to enter an auction of multiple lease areas as separate entities only if their block selections do not overlap.

Ownership information is also necessary if BOEMRE decides to restrict the amount of offshore wind lease space that any one entity (or multiple related entities) can hold in a lease area or areas. For example, in many international spectrum auctions, operators are only allowed to hold a percentage of the available spectrum to help ensure a competitive downstream market. In these cases, related entities are normally subject to a single bidder limit.

If BOEMRE wants to increase competition for the lease area, one way to do this is to set limits on how much OCS space can be won by a single company – either overall or per auction.

7.3 Mitigating anti-competitive behavior

It is important that applicants be prohibited from engaging in collusive behavior, including discussing their bids and bidding strategies, once the auction process has begun, which commonly is defined as the due date for the auction application, and especially during the time when these bids can be received. This should be stated in the auction rules. For a sealed-bid auction, the risk of collusion is lower as bidders do not see any information relating to the bids of others. For multiple round auctions, we recommend doing the following to reduce the risk of collusive behavior:

- The auction should be designed in such a way to limit signaling (one bidder using its bids to signal something to another bidder). This is generally done by:
 - Providing information back to bidders that is aggregated (so that it does not show information about a single bid), if possible. This is a common approach in clock auctions and in package clock auctions.
 - If information about single bids is revealed, it should be made anonymous, and the design should minimize the flexibility that the bidder has in specifying its bid. For example, by providing predefined bid amounts that bidders can choose from

instead of allowing the bidder to specify any dollar amount. This is a common approach in SMRAs.

7.4 Operations during auction

The auction rules must be clear so that if a bid is placed during the auction that meets the auction rules, that bid cannot be disputed. Therefore, any acceptance and rejection of bids would be done instantly by the auction system during the auction to allow the bidder to revise their bids accordingly. However, bidders must be permitted to appeal or dispute the results of the auction within a predefined time-period after the conclusion of the auction. To support this process, we would recommend the following processes:

- Having a trustee or a pre-arbitral referee onsite with the auction management team to oversee the auction and to field any disputes raised during the auction, so that a quick decision can be made as to whether to reject the complaint or suspend the auction while it is investigated further
- Recording all actions performed by bidders during the auction on the auction system in an audit log, which can be analyzed in the event of a dispute
- Recording all conversations with bidders during the auction
- Independently verifying the auction results using an independent system and only releasing the results to bidders once the results have been verified

It is difficult to identify collusive behavior, particularly when the procedures to reduce the risk of this behavior are in place. Therefore, while the auction management team should monitor the auction for any behavior that may seem suspicious, given the severity of being found guilty of collusion, the management team would need strong evidence of such activities before any accusations could be made.

It is critical that the bids be sent and received in a secure fashion. Ideally, all bidders should be required to log into a computer-based auction system and transmit their bids via secure means (SSL encryption, etc.) Strict procedures should be established for receiving bids via other means. For example, processing bid data via an open email is not a good idea. Nor is entering bids from an attached unencrypted spreadsheet. First, it is possible for such an email to be intercepted and read, decreasing the security of the bids. More importantly, it would be easy for someone to send in false data. Care must be taken when receiving bids over the phone or via fax for the same reason.

7.5 Information policy – public data

BOEMRE should establish policies as to what auction data should be available to the general public, and when this data is published. Once the auction is over, it is normal practice to detail who were the winners of the auction, what they won, and the price they paid (not the price they bid) for these lots. Other general information about the auction could also be posted, such as the number of participants and number of rounds held. If the auction is a multiple round auction, BOEMRE may also want to consider publishing information publically during the auction. This would obviously be a subset of what is provided to bidders, and would typically include the round number, the prices associated with that round, and perhaps some aggregated bid information.

7.6 Post-auction procedures

As stated in the pre-auction procedures above, ideally there should not be negotiation with bidders following the auction. The auction results should be final. If there are some options that a bidder may

choose to take or refuse following the auction, these can be built into the contract, but there must always be a default answer, such that if a bidder does not respond with an answer, the contract still stands, and the bidder is still bound by the contract.

Therefore, in summary the post-auction procedures would involve:

- a request to the DOJ to evaluate the auction. DOJ has 30 days to conduct this review.
- successful bidders indicating whether they wish to take any options within the contract,
- the transfer of completed signed contracts, which include details of the blocks to be leased and associated price (note that these may have been signed before the auction, or otherwise must be signed shortly after the auction),
- the transfer of outstanding funds from the successful bidders to BOEMRE to fulfill their bids,
- the transfer of guarantees from BOEMRE to unsuccessful bidders, and
- reporting the results to the general public (providing a press release).

7.7 Defaults

If a bidder defaults by not paying the balance of his winning bid by the due date, the lots remain unsold and are generally included in a subsequent auction. Defaulting bidders' upfront payment to participate in the auction is immediately forfeited and a stiff penalty should be assessed.

8 Summary evaluation of auction formats

This paper has provided a brief summary of the modern auction techniques suitable for the auctioning of wind rights. The auction design problem is similar to that of spectrum auctions. Fortunately, we are able to leverage our knowledge of the substantial developments that have occurred in spectrum auctions over the last seventeen years.

There are now a variety of methods that are suitable for wind rights, and indeed, these methods form a consistent family of auctions. Our recommendation depends on the particular auction setting as shown in Table 4.

Table 4. Summary of auction formats

	Scenario (Economic terms)	Suggested Auction Format	Reason
1	Single lease area, one lot	Sealed-bid second-price auction, single lot	Simplest to implement, induces truthful bidding as a dominant strategy, full efficiency
2	Single lease area, one lot	Clock auction (single lot)	Simple to implement; induces truthful bidding as a bidding strategy; full efficiency; reduced winner's curse; privacy of winning bid
3	Single lease area for dissimilar lots; weak complementarities	Simultaneous clock auction	Simple to implement; excellent price discovery; good bidding incentives
4	Single lease area for multiple similar contested lots	Clock auction for multiple generic lots, followed by an assignment stage	Simple to implement; excellent price discovery; good bidding incentives; sealed-bid assignment stage determines specific assignment
5	Single lease area for dissimilar lots; strong complementarities	Either simultaneous clock auction or package clock auction. Minimum lot size should reflect bidders' indication of interest. A lot could consist of a block, or a combination of blocks and/or sixteenths of a block.	Allows bidders the chance to aggregate different combinations of lots to form viable tracts. No exposure problem. Separate price clock needed for each lot since lots are dissimilar. Simultaneous clock auction offers simpler implementation. Package clock auction offers higher efficiency and less unsold lots.

Although the package clock auction performs well in the widest variety of circumstances, it also is the most complicated to implement. BOEMRE likely will face some simpler scenarios in which a simpler auction design can be just as effective as a package clock auction. For example, if the sale involves auctioning a single lot, then a second-price auction performs well, as does a clock auction for a single lot. If the auction is for a number of similar lots, then BOEMRE could use a clock auction for generic lots. Such an auction is highly efficient and easy to implement. However, if the sale involves many dissimilar lots and complementarities among the lots are strong and different among the bidders, then the full power of a package clock auction is likely to be needed.

This paper focused solely on price-only auctions following an initial qualification stage. The interaction of price and non-price factors can be taken further, and in the sequel paper (Ausubel and Cramton 2011) we will examine multiple-factor auctions, which enable the direct interaction between price and factors other than price during the auction to determine winners. A final paper will summarize the main results of the first two papers. It will also apply the ideas presented in these papers to one or more of the potential auctions on the East Coast Outer Continental Shelf.

Appendix A – Pricing rules for package clock auctions

To illustrate bidder-optimal core pricing, we have provided a simple example below in which there are five bidders and two lots. Suppose there are five bidders, 1, 2, 3, 4, 5, bidding for two lots, A and B. The following bids are submitted:

$$b_1\{A\} = 28$$

$$b_2\{B\} = 20$$

$$b_3\{AB\} = 32$$

$$b_4\{A\} = 14$$

$$b_5\{B\} = 12$$

Bidders 1 and 4 are interested in A, bidders 2 and 5 are interested in B, and bidder 3 is interested in the package A and B.

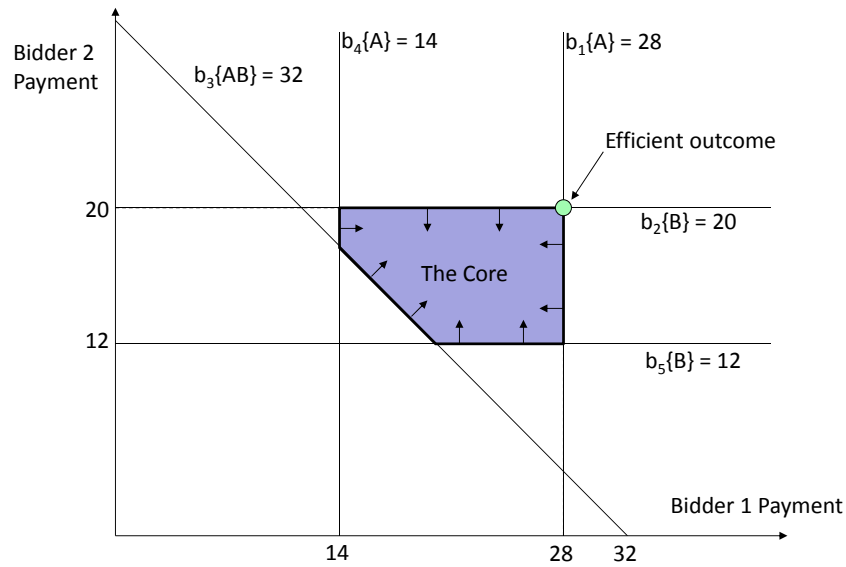
Determining the value maximizing assignment is easy in this example. Bidder 1 gets A and bidder 2 gets B, generating 48 in total value. No other assignment yields as much. Vickrey prices are also easy to calculate. If we remove bidder 1, then the best assignment gives A to bidder 4 and B to bidder 2, resulting in 34, which is better than the alternative of awarding both A and B to bidder 3, which yields 32. Thus, the social opportunity cost of bidder 1 winning A is $34 - 20 = 14$ (the value lost from bidder 4 in this case). Similarly, if we remove bidder 2, then the efficient assignment is for bidder 1 to get A and bidder 5 to get B, resulting in 40. Then the social opportunity cost of bidder 2 winning B is $40 - 28 = 12$ (the value lost from bidder 5). Hence, the Vickrey outcome is for bidder 1 to pay 14 for A and for bidder 2 to pay 12 for B. Total revenues are $14 + 12 = 26$. Notice that bidder 3 has cause for complaint, since bidder 3 offered 32 for both A and B.

Now consider the core for this example. The core is represented in the payment space of the winning bidders—in this case the payments of bidders 1 and 2. Each bid defines a half-space of the payment space:

- Bidder 1's bid of 28 for A implies 1 cannot pay more than 28 for A.
- Bidder 2's bid of 20 for B implies 2 cannot pay more than 20 for B.
- Bidder 3's bid of 32 for AB implies that the sum of the payments for A and B must be at least 32.
- Bidder 4's bid of 14 for A implies that bidder 1 must pay at least 14 for A.
- Bidder 5's bid of 12 for B implies that bidder 2 must pay at least 12 for B.

The core is the intersection of these half-spaces as shown in Figure 11.

Figure 11. The Core

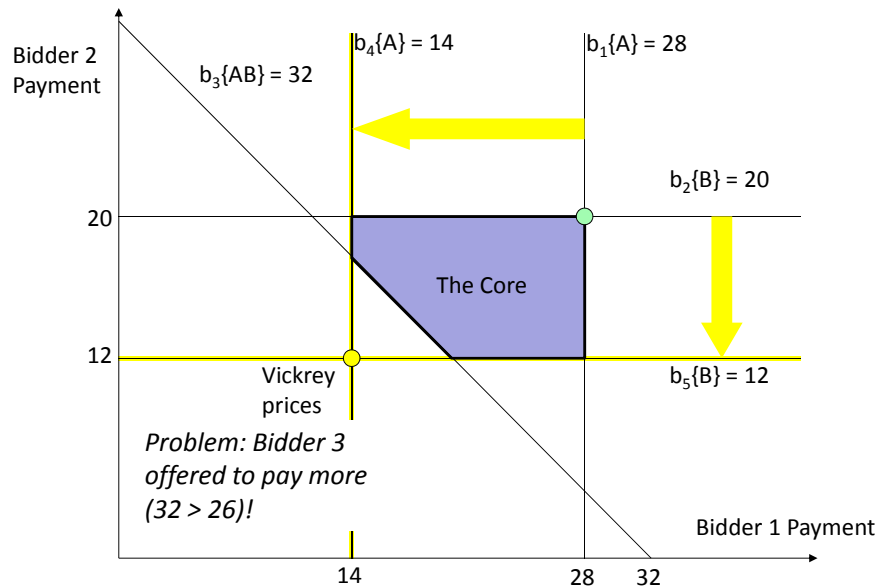


The x-axis of this graph represents the Payment for Bidder 1, and the y-axis represents the Payment for Bidder 2. Thus, each point on the graph represents a unique pair of Bidder 1 and Bidder 2 Payments. For example, the Efficient outcome is represented by a Bidder 1 Payment of 28 and a Bidder 2 Payment of 20.

This example is quite general. First, unlike in some economic settings, in an auction, the core is always nonempty. The reason is that the core always includes the efficient outcome. The reason is that all the constraints are southwest of the efficient point, since the efficient point maximizes total value. Second, the core is always a convex polytope, since it is the intersection of numerous half-spaces. Third, complementarities, like bidder 3's bid for AB, are the source of the constraints that are neither vertical nor horizontal. These are the constraints that can put the Vickrey prices outside the core. Without complementarities, all the constraints will be vertical and horizontal lines, and there will be a unique extreme point to the southwest: the Vickrey prices.

The graphical representation of the core is also a useful way to see the Vickrey prices. Vickrey is asking how much can each winner unilaterally reduce its bids and still remain a winner. As shown in Figure 12, bidder 1 can reduce its bid to 14 before bidder 1 is displaced by bidder 4 as a winner. Similarly, bidder 2 can reduce its bid to 12 before being displaced by bidder 5. Thus, the Vickrey prices are 14 and 12. The problem is that these payments sum to 26, which violates the core constraint coming from bidder 3's bid of 32 for AB.

Figure 12. Vickrey prices: how much can each winner's bid be reduced holding others fixed?



Bidder-optimal core prices can also be thought of as maximal reductions in the bids of winners, but rather than reducing the bids of each winner one at a time, we jointly reduce all the winning bids, as shown in Figure 13, until the southwest face of the core is reached. As can be seen, this does not result in a unique core point, since the particular point on the southwest face depends on the rate at which each winner's bids are reduced. The bidder-optimal core points consist of the entire southwest face of the core. If the southwest face is a unique point, then it is the Vickrey prices; if the southwest face is not unique then the face is a core constraint involving complementarities, and the Vickrey prices lie outside the core.

Figure 13. Bidder-optimal core prices: jointly reduce winning bids as much as possible

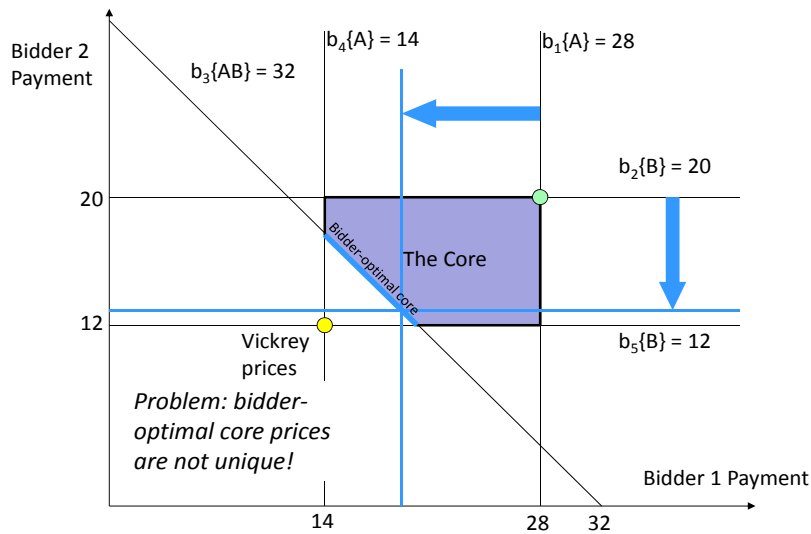
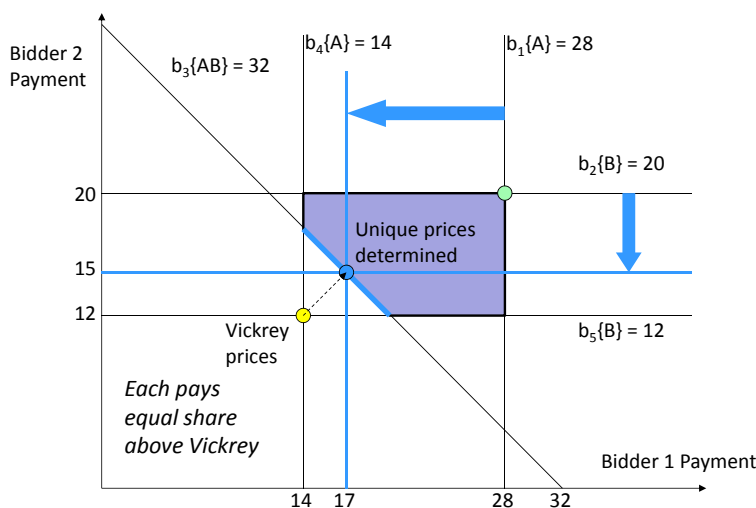


Figure 14. Core point closest to Vickrey prices



Nonetheless, there is always a unique bidder-optimal core point that is closest to the Vickrey prices. This is seen in Figure 14, as the bidder-optimal core point that forms a 90 degree angle with the line that passes through the Vickrey prices. This point minimizes the Euclidean distance from the Vickrey prices.

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