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Particle Hopping Models, Traffic Flow
Theory, and Traffic Jam Dynamics

Kai Nagel

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Particle hopping models, traffic flow theory, and traffic jam dynamics

Kai Nagel

TSA-DO/SA, MS M997, Los Alamos National Laboratory, NM 87545, U.S.A., kai@lanl.gov

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Although particle hopping models have been introduced into traffic science in the 1950s, their systematic use has only started recently. Two reasons for this are, that they are advantageous on modern computers, and that recent theoretical developments allow analytical understanding of their properties and therefore more confidence for their use. In principle, particle hopping models fit between microscopic models for driving and fluid-dynamical models for traffic flow. In this sense, they also help closing the conceptual gap between these two. This paper starts out with building connections between particle hopping models and traffic flow theory.

I. INTRODUCTION

Traffic jams have always been annoying. At least in the industrialized countries, the standard reaction has been to expand the transportation infrastructure to match demand. During this phase of fast growth, relatively rough planning tools were sufficient. However, in the last years most industrialized societies started to see the limits of such growth. In densely populated areas, there is only limited space available for extensions of the transportation system; and we face increasing pollution and growing accident frequencies as the downsides of mobility. In consequence, planning is now turning to a fine-tuning of the existing systems, without major extensions of facilities. This is for example reflected in the United States by the Clean Air Act and by the ISTEA (Intermodal Surface Transportation and Efficiency Act) legislation. The former sets standards of air quality for urban areas, whereas the latter forces planning authorities to evaluate land use policies, intermodal connectivity, and enhanced transit service when planning transportation.

In consequence, planning and prediction tools with a much higher reliability than in the past are necessary. Due to the high complexity of the problems, analytical approaches are infeasible. Current approaches are simulation-based (e.g. [1-4]), which is driven by necessity, but largely enhanced by the widespread availability of computing power nowadays. Yet, also for computers one needs good simplified models of the phenomena of interest: Just coding a perfect representation of reality into the computer is not possible because of limits of knowledge, limits of human resources for coding all these details, and limits of computational resources.

Practical simulation has to observe trade-offs between *resolution*, *fidelity*, and *scale* [5]. Resolution refers to the smallest entities (objects, particles, processes) resolved in a simulation, whereas fidelity means the degree of realism in modeling each of these entities, and scale means the (spatial, temporal, ...) size of the problem. It is empirically well known, for example from fluid dynamics, that to a certain extent a low fidelity high resolution model (lattice gas automata [6,7]) can do as well as a high fidelity low resolution model (discretization of the Navier-Stokes-equations), or in short: Resolution can replace fidelity.

Current state-of-the-art traffic modeling has a fixed unit of (minimal) resolution, and that is the individual traveler. Since one is aiming for rather large scales (for example the Los Angeles area consists of approx. 10 million potential travelers), it is rather obvious that one has to sacrifice fidelity to achieve reasonable computing times.

One important part of transportation modeling is road traffic. For example in Germany, road traffic currently contributes more than 81% of all passenger and 52.7% of all freight transportation [8]. And despite widespread efforts, the share of road transportation is still increasing. For that reason, it makes sense to start with road traffic when dealing with transportation systems.

Putting these arguments together, one thing which is needed for large scale transportation simulations is a *minimal* representation of road traffic. Particle hopping models clearly are candidates for this, and even if not, building a *minimal* theory of road traffic is certainly the right starting point.

This paper shows how particle hopping models fit into the context of traffic flow theory. It starts out with a historical overview of traffic flow theory (Section II), followed by a short review of fluid-dynamical models for traffic flow (Section III) starting from the Navier-Stokes-equations. Section IV defines different particle hopping models which are of interest in the context of traffic flow. The paper continues by showing connections between the fluid-dynamical traffic flow models and particle hopping models. In some cases, these connections are exact and have long been established, but have never been viewed in the context of traffic theory. These cases are shown in Section V. In other cases, critical behavior of traffic jam clusters can be compared to instabilities in the partial differential equations. This description is only precise for the so-called cruise control limit of the particle hopping models, where jams cannot start spontaneously but have to be started by some external disturbance. This is described in Section VI. Section VII explains in how far these results carry over to other models, in which jams initiate spontaneously due to fluctuations of driver's behavior. Summary and discussion conclude the paper.

This paper derives from [9], which discusses many of the issues of this paper on a more technical level.

II. HISTORICAL OVERVIEW

Vehicular traffic has been a widely and thoroughly researched area in the 1950s and 60s. For a review of traffic theory, see, for example, one of [10–12].

Vehicular traffic theory can be broadly separated into two branches: Traffic *flow* theory, and *car-following* theory.

Traffic flow theory is concerned with finding relations between the three fundamental variables of traffic flow, which are velocity v , density ρ , and current or throughput or flow q . Only two of these variables are independent since they are related through $q = \rho v$.

The first task of traffic flow theory historically was to search for time-independent relations between q , ρ and v , the so-called fundamental diagrams. The form of such a relation is, though, still debated in the traffic flow literature [13,14]. The problem stems mainly from the fact that reality measurements are done in non-stationary conditions. There, only short time averages make sense, and they usually show large fluctuations. Near the end of this paper I will sketch out how a dynamic, particle based description of traffic can account for these difficulties.

The second step of traffic flow theory was to introduce a dynamic, i.e. time-dependent description. This was achieved in 1955 by Lighthill and Whitham [15]. That paper introduces a description based on the equation of continuity plus the assumption that flow (or velocity) depend on the density only, i.e. there is no relaxation time (instantaneous adaption).

Prigogine, Herman, and coworkers developed a kinetic theory for traffic flow [16]. They derived the Lighthill-Whitham situation as a limiting case of the kinetic theory. The kinetic theory anticipates many of the phenomena which come up in later work, but probably because the mathematics of working in this framework is fairly laborious, this theory

has not been developed any further until recently [17].

Instead, in 1979, Payne replaced the assumption of instantaneous adaption in the Lighthill-Whitham theory by an equation for inertia, which is similar to a Navier-Stokes-equation [18]. Kühne, in 1984, added a viscosity term and initiated using the methods of nonlinear dynamics for analyzing the equations [19-22].

In a parallel development, Musha and Higuchi proposed the noisy Burgers equation as a model for traffic and backed that up by measurements of the power spectrum of traffic count data [23].

Ref. [9] puts these fluid-dynamical models into a common perspective.

Car-following theory regards traffic from a more microscopic point of view: The behavior of each vehicle is modeled in relation to the vehicle ahead. As the definition indicates, this theory concentrates on single lane situations where a driver reacts to the movements of the vehicle ahead of him. Many car-following models are of the form

$$a(t + T) \propto \frac{v(t)^m}{[\Delta x(t)]^l} \cdot \Delta v(t), \quad (1)$$

where “ \propto ” means “proportional to”, and a constant with the appropriate units has to be added. a and v are the acceleration and velocity, respectively, of the car under consideration, Δx is the distance to the car ahead, Δv is the velocity difference to that car, and m and l are constants. T is a delay time between stimulus and response, which summarizes all delay effects such as human reaction time or time the car mechanics needs to react to input.

Other examples for car-following equations are $v(t + T) \propto \Delta x$ [24,25] or $a(t) \propto V[\Delta x(t)] - v(t)$ [26], where $V[\Delta x]$ gives a preferred velocity as a function of distance headway.

Mathematically, parts of this theory are very similar to the treatment of atomic movements in crystals, and give results about the stability of chains of cars (“platoons”) in follow-the-leader situations.

One of the achievements of traffic theory of that period was that relations between car-following models and *static* flow-density-relations could be derived.

Car-following theory will not be treated any further in this paper.

A more recent addition to the development of vehicular traffic flow theory are particle hopping models. Imagine a one-dimensional chain of boxes, each box either empty, or occupied by exactly one particle. Movement of particles is achieved by particles jumping from one box to another according to specific movement rules.

In the context of vehicular traffic, one can imagine a road represented by boxes which can fit exactly one car. A rough representation of car movements then is given by moving cars from one box to another. Actually, the first proposition of such a model for vehicular traffic is from Gerlough in 1956 [27] and has been further extended by Cremer and coworkers [28,29].

These models are sometimes also called cellular automata (CA) models [30]. CA models use discrete representations of space (cells) and time, and have a low number of allowed states per cell. For each cell, an updated state at time $t + 1$ is calculated using only local information from time t . Local means that only a small number of neighboring sites is considered. Note that this makes the update completely parallel, that is, once the complete state at time t is known, the states of all cells for time $t + 1$ can be calculated independently from each other.

Particle hopping models and CA are not exactly the same, although the definitions are overlapping. Both model classes use the discrete representation of space and time and the low number of allowed states per cell. But the CA always has a parallel update, whereas for particle hopping models, other updates are possible (see below). In contrast, particle hopping models are defined by the dynamics they are supposed to describe. For example, at least the simpler

of the particle hopping models observe mass conservation.

All CA models in this paper are particle hopping models; the inverse is true except for the ASEP (see below).

In 1992, CA models for traffic were brought into the statistical physics community. Biham and coworkers used a model with maximum velocity one for one- and for two-dimensional traffic [31]. One-dimensional here refers to roads etc., and includes multi-lane traffic. Two-dimensional traffic in the CA context usually means traffic on a 2-d grid, as a model for traffic in urban areas. Nagel and Schreckenberg introduced a model with maximum velocity five for one-dimensional traffic, which compared favorably with real world data [32]. Both approaches were further analyzed and extended in a series of subsequent papers, both for the one-dimensional [9,33-58] (see also [59]) and the two-dimensional (see, e.g., [60-62]) investigations.

The motivation here was twofold. The first was the aiming for computational speed, to make statistical analysis possible. The second motivation is that the model is still simple enough to be treated analytically, but distinctively different from other particle-hopping models. In addition, CA-methodology is planned to be used as a high-speed option in traffic projects in Germany [2] and in the United States [1].

From a theoretical point of view, the methodology of CA is placed between fluid-dynamical and car-following theories, and is helpful to further clarify the connections between these approaches. This paper aims at contributing to the first part, i.e. understanding and clarifying the relations between particle-hopping models and fluid-dynamical models for traffic flow.

III. FLUID-DYNAMICAL MODELS FOR TRAFFIC FLOW

In traffic flow theory, models can be roughly distinguished into two different classes: the ones which assume instantaneous adaption of velocity to density, and the ones where this adaption needs some time because it has to overcome momentum. The first class is the simpler one; the basic equation here is

$$\partial_t \rho + \partial_x q = 0, \quad (2)$$

where ρ is the density, q is the flow or current or throughput, and ∂_t and ∂_x are partial derivatives with respect to time and space, respectively. The equation is just the equation of continuity, and it simply expresses mass conservation.

In order to make this work, one has to give the flow as a function of density, $q = f(\rho)$, for example $q \propto \rho(1 - \rho)$ (which would be the Greenshield relation, see [11]), or $q \propto \frac{1}{2} - |\rho - \frac{1}{2}|$.

Physically, one finds behind these equations that velocity adapts instantaneously to the surrounding density, i.e.

$$v = q/\rho = f(\rho)/\rho = F(\rho \text{ only}). \quad (3)$$

This is exactly the well-known theory of Lighthill and Whitham [15], and a great deal is known how to handle these equations.

Note that, following a theoretical physics tradition, some variables are made free of units before they are used. For example, density here is renormalized so that it is between zero and one; in traffic one would achieve this by dividing it by the jam density:

$$\rho [\text{no unit}] := \frac{\rho_{\text{real}} [\text{number of cars per km}]}{\rho_{\text{jam}} [\text{number of cars per km}]},$$

where possible units are indicated in the brackets.

Extensions of the Lighthill-Whitham-equations (2, 3) are to add diffusion or noise on the right hand side of the equation, for example

$$\partial_t \rho + \partial_x q = D \partial_x^2 \rho + \eta ,$$

where D is the diffusion coefficient and η is a noise term.

Models including momentum consist of a second equation describing the fact that velocity does in reality *not* adapt instantaneously to the density. An often used form of such a non-instantaneous velocity adaption term is [18,19]

$$\frac{dv}{dt} = \frac{1}{\tau} [V(\rho) - v] + \frac{c_0^2}{\rho} \partial_x \rho + \nu \partial_x^2 v . \quad (4)$$

The equation says that individual acceleration (left hand side) is proportional to the following three effects:

- difference between desired speed $V(\rho)$ and actual speed v ;
- gradient of the density: If traffic gets denser in the driving direction, one slows down; and
- a spatial smoothing effect. This effect can better be derived as a space-averaging effect [9] and is thus not really related to individual accelerations.

$V(\rho)$ still has to be externally given, but the adaption to this value is now (neglecting the other RHS terms) exponentially delayed, i.e. $v(t) = V(\rho) - [V(\rho) - v_0] e^{-t/\tau}$. That means that actual velocity approaches the desired velocity exponentially, with a time constant given by τ . Note that the limit $\tau \rightarrow 0$ makes this adaption infinitely fast, i.e. returning to the instantaneous adaption case. Mathematically, one sees this by the fact that the $\tau \rightarrow 0$ limit makes the relaxation term much bigger than all the other terms of the velocity adaption equation (4).

Usually, for a fluid-dynamical picture, one concentrates on a description on fixed spatial coordinates instead of following the vehicles; then, one replaces the Lagrangian (= individual) derivative dv/dt by the Eulerian (= local) equivalent $\partial_t v + v \partial_x v$, leading to

$$\partial_t v + v \partial_x v = \frac{1}{\tau} [V(\rho) - v] + \frac{c_0^2}{\rho} \partial_x \rho + \nu \partial_x^2 v .$$

One uses this equation in conjunction with the equation of continuity and $q = \rho v$.

A more comprehensive review of the fluid-dynamical equations needed here can be found, e.g., in [9].

IV. DEFINITIONS OF PARTICLE HOPPING MODELS

This section defines several particle hopping models which are candidate models for traffic flow. They all have in common that they are defined on a lattice of, say, length L , where L is the number of sites, and that each site can be either empty, or occupied by exactly one particle. Also, in all models particles can only move in one direction. The number of particles, N , is conserved except at the boundaries.

The section starts out with the Stochastic Traffic Cellular Automaton (STCA), which has been proposed for traffic flow by Nagel and Schreckenberg [32], and which is used as the basis for large scale traffic simulation projects both in the United States [1] and in Germany [2]. The STCA includes strong randomness in the rules. Setting this randomness to zero reduces the STCA to a much simpler, deterministic model, which, when restricting oneself to maximum velocity $v_{max} = 1$, turns out to a well known cellular automaton model. In the third model of this section, randomness is re-introduced, but in this case by changing the update algorithm: Whereas in the first two models all particles are updated synchronously based in "old" information, in this third model, particles are selected in random sequence for individual updates.

A. The Stochastic Traffic Cellular Automaton (STCA)

The Stochastic Traffic Cellular Automaton (STCA), which has been treated in a series of papers [9,40-53], is defined as follows. Each particle (= car) can have an integer velocity between 0 and v_{max} . The complete configuration at time-step t is stored, and the configuration at time-step $t + 1$ is computed from that (parallel or synchronous update). For each particle, the following steps are done in parallel:

- Find number of empty sites ahead (= gap) at time t .
- If $v > gap$ (too fast), then slow down to $v := gap$. [rule 1]
- Else if $v < gap$ (enough headway) and $v < v_{max}$, then accelerate by one: $v := v + 1$. [rule 2]
- **Randomization:** If after the above steps the velocity is larger than zero ($v > 0$), then, with probability p , reduce v by one. [rule 3]
- **Particle propagation:** Each particle moves v sites ahead. [rule 4]

The randomization condenses three different properties of human driving into one computational operation: Fluctuations at maximum speed, over-reactions at braking, and retarded (noisy) acceleration.

Note that, because of integer arithmetic, conditions like $v > gap$ and $v \geq gap + 1$ are equivalent.

Despite its simplicity, this model is astonishingly successful in reproducing realistic behavior such as start-stop-waves and realistic fundamental diagrams [32].

When the maximum velocity of this model is set to one ($v_{max} = 1$), then the model becomes much simpler: For each particle, do in parallel:

- If site ahead was free at time t , move, with probability $1 - p$, to that site.

Since the STCA shows different behavior for $v_{max} \geq 2$ than for $v_{max} = 1$, I will distinguish them using STCA/1 and STCA/2, respectively.

Due to the given discretization of space and time, proper units are often omitted in the context of particle hopping or cellular automata models. Proper units here would be: $[gap]$ = number of cells, $[v]$ = number of cells per time step, $[t]$ = number of time steps, etc. For that reason, it is possible to write something like $v < gap$, which properly would have to be $v < gap/(\text{time step})$. Note that one still needs conversion factors to convert, say, velocity from the

particle hopping model to a real world velocity, e.g. given in kilometers per hour. One should note, though, that every computer program does such a thing. Numbers in computer programs are always unitless, and a proper conversion to real world numbers has to be put in by the program designer.

B. The deterministic limit of the STCA (CA-184)

One can take the deterministic limit of the STCA by setting the randomization probability p equal to zero, which just amounts to skipping the randomization step. It turns out that, when using maximum velocity $v_{max} = 1$, this is equivalent [63] to the cellular automaton rule 184 in Wolfram's notation [30], which is why I use the notation CA-184/1 and CA-184/2.

Much work using CA models for traffic is based on this model. Biham and coworkers [31] have introduced it for traffic flow, with $v_{max} = 1$. Other authors base further results on it [33-35,38-40]. Others [36,37] use it with higher v_{max} . It is also the basis of the two-dimensional CA models for traffic (e.g. [31,60-62]).

C. The Asymmetric Stochastic Exclusion Process (ASEP)

The probably most-investigated particle hopping model is the Asymmetric Stochastic Exclusion Process (ASEP) (e.g. [63-68]). It is defined as follows:

- Pick one particle randomly. [rule 1]
- If the site to the right is free, move the particle to that site. [rule 2]

The ASEP is closely related to CA-184/1 and STCA/1 (both with maximum velocity one). The main difference is the update schedule: Instead of doing something with all particles simultaneously, one picks one particle at a time.

In contrast, in CA-184/1, one picks *all* particles synchronously and moves them according to rule 2 of the ASEP. In order to make this work, one has to use "old" information (i.e. from iteration t) to decide if the site to the right is free (i.e. if the particle can be there at time $t + 1$). For the ASEP, this distinction between "old" and "new" information is not necessary because one only picks one particle at a time and all others do not move.

In STCA/1 (with $p = 1/2$), one picks randomly *half of all* particles synchronously and moves them according to rule 2 of the ASEP.

In order to compare the ASEP with the other, synchronously updated models, one has to note that, in the ASEP, *on average* each particle is updated once after N single-particle updates. A time-step (also called update-step or iteration) in the ASEP is therefore completed after N single-particle updates ($= N$ attempted hops).

It was already noted earlier [63] that going from ASEP to CA-184/1, i.e. changing the update from asynchronous to synchronous, produces very different dynamics. In this paper, I will in addition show that re-introducing the randomness via the randomization (rule 3) in the STCA again leads to different results.

V. PARTICLE HOPPING MODELS, FLUID DYNAMICS, AND CRITICAL EXPONENTS

Writing about both particle hopping and fluid-dynamical models for traffic flow does not make much sense as long as one cannot compare them. Fortunately, such a comparison is possible and will turn out to be quite instructive. Actually, for some of the mentioned particle hopping models, fluid-dynamical limits are known exactly. By fluid-dynamical limits one technically means the limit where the grid size Δx goes to zero, both the number of grid points, n , and the number of particles, N , go to infinity, while one keeps the system size, $L = n \cdot \Delta x$ and the density $\rho = N/n$ both constant.

More intuitively, a fluid-dynamical description of a particle hopping model is a description where one averages over enough particles so that the granularity of the original system is no longer visible. As a consequence, phenomena on the level of a few particles cannot be expected to be correctly described, but larger scale phenomena can.

The cases for which the fluid-dynamical limits are known will be presented in this section. In many cases, though, such as for the STCA, these limits are not known. In these cases, the concept of critical exponents still helps to classify the models and to make comparisons to fluid-dynamical models. In order to prepare for this exercise, I will already talk about critical exponents in this section.

The most straightforward way to put the concept of critical exponents into the context of traffic flow is to consider "disturbances" (i.e. jams) of length x and ask for the time t to dissolve them. For example, one would intuitively assume that a queue of length x at a traffic light which just turned green would need a time t proportional to x until everybody is in full motion. By this argument, the dynamic exponent z , defined by $t \sim x^z$, should be one.

Yet, there can be more complicated cases. Imagine again a queue at a traffic light just turned green but this time also some fairly high inflow at the end of the queue. The jam-queue itself will start moving backwards, clearing its initial position in time $t \sim x$. However, the dissolving of the jam itself may be governed by different rules. An example of this will be given below.

Both for the ASEP/1 and for the CA-184/1, fluid-dynamical limits and critical exponents are well known (see, e.g., [63-66]), and this section will therefore describe these two cases plus the generalization which leads to CA-184/5. Note that, compared to the introduction of the particle hopping models in the last section, we start "backwards": Much is known about some of the sub-cases or variations of the STCA, yet much less about the STCA itself.

A. ASEP/1

The classic stochastic asymmetric exclusion process corresponds to the noisy Burgers equation. More precisely, the ASEP particle process corresponds to a diffusion equation $\partial_t \rho + \partial_x q = D \partial_x^2 \rho + \eta$ with a current [63,67] of $q = \rho(1-\rho)$. Interestingly, this is exactly the Lighthill-Whitham case, specialized to the Greenshields flow relation, with terms added for noise and diffusion. *In other words, the ASEP/1 particle hopping process and the Lighthill-Whitham-theory (plus noise plus diffusion), specialized to the case of the Greenshields flow-density relation, describe the same behavior.* In consequence, many phenomena of this particle hopping process can be understood using the Lighthill-Whitham theory.

Inserting yields

$$\partial_t \rho + \partial_x \rho - \partial_x \rho^2 = D \partial_x^2 \rho + \eta . \quad (5)$$

In the steady state, this model shows kinematic waves (= small jams), which are produced by the noise and damped by diffusion (Fig. 1). These non-dispersive waves move forwards (wave velocity $c = dq/d\rho = 1 - 2\rho > 0$) for $\rho < 1/2$ and backwards ($c < 0$) for $\rho > 1/2$. At $\rho = 1/2$, the wave velocity is exactly zero ($c = 0$), and this is the point of maximum throughput [68]. If traffic were modeled by the ASEP, then one could detect maximum traffic flow by standing on a bridge: Jam-waves moving in flow direction indicate too low density (cf. Fig. 1), jam-waves moving against the flow direction indicate too high density.

The ASEP is one of the cases where clearing a site follows a different exponent than dissolving a disturbance.¹ As long as $\rho \neq 1/2$, a disturbance of size x moves with speed $c \neq 0$ and therefore clears the initial site in time $t \sim c \cdot x \sim x^1$, i.e. with dynamical exponent $z = 1$. In order to see how the disturbance itself dissolves, one transforms into the coordinate system of the wave velocity. One conventionally does that by first separating between the average density $\langle \rho \rangle_L$ and the fluctuations ρ' . By inserting $\rho = \langle \rho \rangle_L + \rho'$ one obtains

$$\partial_t \rho' + (1 - 2\langle \rho \rangle_L) \partial_x \rho' - 2\rho' \partial_x \rho' = D \partial_x^2 \rho' + \eta. \quad (6)$$

When transforming this into the moving coordinate system $x' = x + (1 - 2\langle \rho \rangle_L) \cdot t$, one obtains

$$\partial_t \rho' - 2\rho' \partial_x \rho' = D \partial_x^2 \rho' + \eta,$$

which is the classic noisy Burgers equation [63].

For this equation it is well known that the dynamical exponent is $z = 3/2$. In other words, in the original coordinate system a disturbance four times as big as another one, $x' = 4x$, needs $t' \sim x' = 4x \sim 4t$, i.e. four times as much time to clear the site, but $t' \sim x'^{3/2} = (4x)^{3/2} \sim 8t$, i.e. 8 times as much time until the jam-structure itself is no longer visible in the noise. A precise treatment of this uses, e.g., correlations between tagged particles [64].

The drawback of this model with respect to traffic flow is that it does neither have a regime of laminar flow nor "real", big jams (see also Fig. 1). Because of the random sequential update, vehicles with average speed \bar{v} fluctuate severely around their average position given by $\bar{v}t$. As a result, they always "collide" with their neighbors, even at very low densities, leading to "mini-jams" everywhere. This is clearly unrealistic for light traffic.

Actually, this fact is also visible in the speed-density-diagram. Using the Greenshields flow-density relation, one obtains

$$v = \frac{q}{\rho} \propto 1 - \rho. \quad (7)$$

This is in contrast to the observed result that, at low densities, speed is nearly independent of density (practically no interaction between vehicles).

B. CA-184

Using a maximum velocity higher than one does not change the general behavior of CA-184. It therefore makes sense to directly discuss the general case.

¹Note that technically, all these remarks are only valid for *small* disturbances. The problem is that if one is no longer close to the steady state, one sees transient behavior which may be different [63].

As explained above, the CA-184/1 is the deterministic counterpart of the ASEP/1. But taking away the noise from the particle update completely changes the universality class (i.e. the exponent z) [63]. The model now corresponds to the non-diffusive, non-noisy equation of continuity

$$\partial_t \rho + q' \partial_x \rho = 0 \quad (8)$$

with a (except at $\rho = \rho_{qmax}$) linear flow

$$q' = \frac{dq}{d\rho} = \begin{cases} v_{max} & \text{for } \rho < \rho_{qmax} \\ -1 & \text{for } \rho > \rho_{qmax} \end{cases} \quad (9)$$

The intersection point of the fundamental diagram divides two phenomenological regimes: light traffic ($\rho < \rho_{qmax}$) and dense traffic ($\rho > \rho_{qmax}$).

A typical situation for light traffic is shown in Fig. 2 (with $v_{max} = 5$). After starting from a random initial condition, the traffic relaxes to a steady state, where the whole pattern just moves $v_{max} = 5$ positions to the right in each iteration. Cars clearly have a tendency of keeping a *gap* of $\geq v_{max} = 5$ between each other. As a result, the current, q , in this regime is

$$q_{<} = \rho \cdot v_{max} \quad (10)$$

The velocity of the kinematic waves in this regime is $c_{<} = q'_{<} = v_{max}$. This means that disturbances, such as holes, just move with the traffic, as can also be seen in Fig. 2.

Dense traffic is different (Fig. 3). Again starting from a random initial configuration, the simulation relaxes to a steady state where the whole pattern moves one position to the *left* in each iteration. Note that cars still move to the right; if one follows the trajectory of one individual vehicle, for this car regions of relatively free movement are alternating with regions of high density and slow speed. Although in a too static way, this captures some of the features of start-stop-traffic. The average speed in the steady state equals the number of empty sites divided by the number of particles: $\langle v \rangle_L = (L - N)/N$; the current is $q_{>} = \rho \cdot \langle v \rangle_L$, or, with $\rho = N/L$,

$$q_{>} = 1 - \rho \quad (11)$$

This straight line intersects with the one from light traffic at $\rho = 1/(1 + v_{max})$, which is therefore the density corresponding to maximum throughput $q_{max} = v_{max}/(1 + v_{max})$.

The velocity of the kinematic waves in the dense regime is $q'_{>} = -1$, which corresponds to the backwards moving pattern in Fig. 3.

Since the second term of Eq. 8 (with 9) is (except at $\rho = \rho_{qmax}$) linear in the density, these are linear Burgers equations, and the dynamic exponent z is equal to 1 [63].

More precisely, the following happens: The outflow of a jam in this model always operates at flow $q_{out} = q_{max}$ and density $\rho_{out} = \rho_{qmax}$. The time t until a jam of length x dissolves therefore obeys the average relation $t \propto x/(q_{max} - q_{in})$, where q_{in} is the average inflow to the jam. Since $q \propto \rho$ for $\rho \leq \rho_{qmax}$, one can write that as

$$t \propto \frac{x}{\rho_{qmax} - \rho(q_{in})} \quad (12)$$

This means that for $\rho < \rho_{qmax}$, the critical exponent z is indeed one, but at $\rho = \rho_{qmax}$, t diverges. This effect is also visible when disturbing the system from its stationary state: The transient time t_{trans} until the system is again stationary scales as [36]

$$t_{trans} \sim \frac{1}{\rho_{qmax} - \rho} \quad (13)$$

The scaling law (13) is actually also true for $\rho > \rho_{qmax}$, albeit for a different reason with a slightly more complicated phenomenology. See [36] for more details.

Two observations are important at this point:

(i) Many papers in the physics literature [31,33–38,40] use this model for their investigations. Also the 2d-grid models (see, e.g., [60–62]) essentially use this driving model, although the two-dimensional interactions seem to change the flow-density relationship [69]. The CA-184 model lacks at least two features which are, as I will argue later, important with respect to reality: (a) Bi-stability: Laminar flow above a certain density becomes unstable, but can exist for long times. (b) Stochasticity: CA-184 is completely deterministic, i.e. a certain initial condition always leads to the same dynamics. Real traffic, however, is stochastic, that is, even identical initial conditions will lead to different outcomes, and a model should be capable of calculating some distribution of outcomes (by using different random seeds).

(ii) The fluid-dynamical model behind the so-called cell transmission model [70], which is a discretization of the Lighthill-Whitham-theory, is similar to Eq. 8 with 9, especially with respect to the range of physical phenomena which are represented. The only difference is that the q - ρ -relation of Ref. [70] has a flat portion at maximum flow instead of the single peak of Eq. 9. That means that in the cell transmission model low density and high density traffic behave similarly to CA-184, but traffic at capacity has a regime where waves do not move at all.

Using other q - ρ -relations in discretized Lighthill-Whitham-models (e.g. [71,72]), will lead to other relations for the wave speeds, but the range of physical phenomena (backwards or forwards moving waves) which can be represented will always resemble CA-184; especially, neither the bi-stability nor the stochasticity can be represented.

VI. CRUISE CONTROL LIMITS

No fluid-dynamical limits for the other particle hopping models are known. Yet, one can still gain further insight by looking into the traffic jam dynamics of the different models. In order to separate out the traffic jam dynamics from other effects, as a first step one would like to modify the models in such a way that always only one jam at a time exists. This is achieved by introducing the “cruise control limit”. Here, fluctuations at free driving (i.e. when $v = v_{max}$ and $gap \geq v_{max}$) are set to zero. The result is that traffic in these models, once all vehicles are in the free driving regime, remains deterministic and laminar for all times. A single jam can then be initiated by perturbing one single car by, say, stopping it and letting it re-accelerate. In general, many different choices for the local perturbation give rise to the same large scale behavior. The perturbed car eventually re-accelerates back to maximum velocity. In the meantime, though, a following car may have come too close to the disturbed car and has to slow down. This initiates a chain reaction – an emergent traffic jam.

A. CA-184-CC

Takayasu and Takayasu [39] introduced a model which amounts to a deterministic cruise control situation for CA-184/1. This may not be obvious from the rules, but it will become clear from the dynamic behavior. Since they use only maximum velocity $v_{max} = 1$, the rules are short: For all particles do in parallel:

- A particle with velocity one is moved one site ahead when the site ahead is free ($gap \geq 1$).
- A particle at rest ($v = 0$) can only move when $gap \geq 2$.

The important new feature of this model is a bi-stability [39]. This bi-stability occurs for $\rho_{c1} = 1/3 \leq \langle \rho \rangle_L := N/L \leq \rho_{c2} = 1/2$, where some initial conditions lead to laminar flow but others lead to traffic including jams. $\langle \cdot \rangle_L$ means the average over the whole (closed) system of length L . Takayasu and Takayasu found the following:

(i) Starting with maximally spaced particles and initial velocity one, one finds stable configurations with flow $\langle q \rangle_L = \langle \rho \rangle_L \cdot v_{max} = \langle \rho \rangle_L$ for low densities $\langle \rho \rangle_L \leq 1/2 =: \rho_{c2}$. For high densities $\langle \rho \rangle_L > \rho_{c2}$, a jam phase appears for *all* initial conditions since not all particles can keep $gap \geq 1$. Once a jam has been created, all particles in the outflow of this jam have $gap = 2$. For $t \rightarrow \infty$, this dynamics reorganizes the system into jammed regions with density one and zero current, and laminar outflow regions with $\rho_{out} = 1/3$ and $q_{out} = 1/3$. Simple geometric arguments then lead, for the whole system, to $\langle q \rangle_L = (1 - \langle \rho \rangle_L)/2$ and $\langle v \rangle_L = (1/\langle \rho \rangle_L - 1)/2$.

(ii) Starting, however, with an initial condition where all particles are clustered in a jam, this jam is only sorted out up to $\langle \rho \rangle_L \leq 1/3 =: \rho_{c1}$, leading to $\langle q \rangle_L = \langle \rho \rangle_L$ and $\langle v \rangle_L = 1$. For $\langle \rho \rangle_L > \rho_{c1}$, the initial jam survives forever, yielding $\langle q \rangle_L = (1 - \langle \rho \rangle_L)/2$ and $\langle v \rangle_L = (1/\langle \rho \rangle_L - 1)/2$. One observes that, for $\rho_{c1} < \langle \rho \rangle_L < \rho_{c2}$, this initial condition leads to a different final flow state than the initial conditions in (i). — Note that ρ_{c1} is equal to the outflow density ρ_{out} .

(iii) Starting from an arbitrary initial condition, the density-velocity relation converges to one of the above two types.

Note that up to before this section, all relations between q , v , and ρ were also locally correct, which is why averaging brackets were omitted. Now, this is no longer true. For example densities slightly above ρ_{c2} do not really exist on a local level; they are only possible as a global composition of regions with local densities $\rho = \rho_{c1}$ plus others with local densities $\rho = 1$.

Since the model is deterministic, one can calculate the behavior from the initial conditions. For any particle i with initial velocity zero one can determine the influence that particle has on following particles $i+1, i+2, \dots$. For particle $i+k$ to be the first one not to be involved in the jam caused by i , one needs the average gap between i and k to be larger than two. This corresponds to a density between i and k of $\rho_{ik} < 1/(gap+1) = 1/3 = \rho_{c1}$. The sequence $(gap_{i+j})_j$ describes a random walk, which is positively (negatively) biased for $\rho > \rho_{c1}$ ($\rho < \rho_{c1}$), and unbiased at the critical point $\rho = \rho_{c1}$ [39].

B. STCA-CC/1

The cruise control limit of the STCA algorithmically amounts to the following: For all cars do in parallel:

- A vehicle is stationary when it travels at maximum velocity v_{max} and has free headway: $gap \geq v_{max}$. Such a vehicle just maintains its velocity.

- Else (i.e. if a vehicle is not stationary) the standard rules of the STCA are applied.

Both acceleration and braking still have a stochastic component. The stochastic component of braking is realistic, but it is irrelevant for the results presented here.

The cruise control limit of the STCA is in some sense a mixture between the CA-184 and the full STCA. Since the STCA-CC has no fluctuations at free driving, the maximum flow one can reach is with all cars at maximum speed and $gap = v_{max}$. Therefore, one can manually achieve flows which follow, for $\rho \leq \rho_{c2}$, the same q - ρ -relationship as the CA-184, where ρ_{c2} now denotes the density of maximum flow of the deterministic model CA-184, i.e. $\rho_{c2} = 1/(v_{max} + 1)$.

Above a certain ρ_{c1} , these flows are unstable to small local perturbations. This density will turn out to be a "critical" density; for that reason I will use $\rho_c \equiv \rho_{c1}$.

Now assume that a single jam has been initiated in an infinite system of density ρ_0 . It is straightforward to see [49] that $n(t)$, the number of cars in the jam, follows a usually biased, absorbing random walk, where $n(t) = 0$ is the absorbing state (jam dissolved): Every time a new car arrives at the end of the jam, $n(t)$ increases by one, and this happens with probability $q_{in} = v_{max} \cdot \rho_0$, which is the inflow rate. Every time a car leaves the jam at the outflow side, $n(t)$ decreases by one, and this happens with probability q_{out} . When $q_{in} = q_{out}$, $n(t)$ follows an unbiased absorbing random walk. $q_{in} \neq q_{out}$ introduces a bias or drift term $\propto (q_{in} - q_{out}) \cdot t$.

This picture is consistent with Takayasu and Takayasu's observations for the CA-184-CC model. The main difference is that now *both* the inflow gaps and the outflow gaps form a random sequence. Another difference is conceptually: Takayasu and Takayasu have looked at the transient time starting from initial conditions, whereas Nagel and Paczuski look at jams starting from a *single disturbance*. The latter leads to a cleaner picture of the traffic jam dynamics because it concentrates on the transition from laminar to start-stop-traffic which is observed in real traffic.

The statistics of such absorbing random walks can be calculated exactly. For the unbiased case one finds that

$$\langle n(t) \rangle \sim t^\eta, \quad P_{surv}(t) \sim t^{-\delta} \text{ and } \langle w(t) \rangle_{surv} \sim t^{\eta+\delta}, \quad (14)$$

where P_{surv} is the survival probability of a jam until time t , $w(t)$ means the width of the jam, i.e. the distance between the leftmost and the rightmost car in the jam. $\langle \cdot \rangle$ means the ensemble average over *all* jams which have been initiated, and $\langle \cdot \rangle_{surv}$ means the ensemble average over *surviving* jams. For the critical exponents, one finds as well from theory as from numerical simulations $\delta = 1/2$ and $\eta = 0$.

$\eta = 0$ re-confirms that, at the critical density ρ_c , jams in the average *barely survive* (unbiased random walk).

If one now uses q_{in} as order parameter, and, say, $P_{surv}(t)$ as control parameter, then we have a second order phase transition, where

$$P_{surv}(t) \begin{cases} = 0 & \text{for } q_{in} < q_{out} \text{ and } t \rightarrow \infty, \\ \sim t^{-\delta} & \text{for } q_{in} = q_{out} \text{ and } t \rightarrow \infty, \text{ and} \\ = const & \text{for } q_{in} > q_{out} \text{ and } t \rightarrow \infty. \end{cases} \quad (15)$$

For that reason, we call $q_c := q_{out}$ the critical flow, and the associated density $\rho_c := \rho(q_c)$ the critical density.

It is important to note that $q_{in} > q_{out}$ as a stable, longtime state is only possible due to the particular definition of the cruise control limit and in an open (or infinite) system. If one would use a closed system, the outflow of the jam would eventually go around the loop and turn into the inflow of the jam (see Fig. 4), leading to the situation $q_{in} = q_{out}$; if one would go away from the cruise control limit, eventually other jams would form upstream of the one

under consideration, and the outflow of these jams would eventually be the inflow of the jam under consideration, again leading to $q_{in} = q_{out}$.

All this is true for an open system, or a system which is large enough and where times are short enough so that the closed boundaries are not felt. In a closed system, the jams ultimately absorb all the excess density $\rho_{excess} = \rho - \rho_c$; as a result, all traffic between jams operates at ρ_c .

Average measurements of the long time behavior of traffic flow can therefore show at most $q_c = q_{out}$.

This picture is consistent with recent results both in fluid-dynamical models and mathematical car-following models for traffic flow.

In Ref. [73], traffic simulations using a fluid-dynamical model starting from nearly homogeneous conditions eventually form stable waves. The fluid-dynamical model one can, by usual linearization, find the parameters for the onset of instability. What is $\langle n(t) \rangle \sim t^\eta$ in the particle hopping model becomes the amplitude $A(t) \sim e^{\lambda t}$ in the fluid-dynamical model, and at the onset of instability $A = const$ is similar to $\langle n(t) \rangle = const$ (since $\eta = 0$). Therefore, the wave in the fluid-dynamical model corresponds to the *average* jam-cluster in the particle-hopping model.

Bando and co-workers [26] also find the separation of traffic into laminar and jammed phases in a deterministic continuous mathematical car-following model.

C. STCA-CC/2

Replacing maximum velocity $v_{max} = 1$ by $v_{max} \geq 2$ does not change the critical behavior, but it adds a complication [49]. Now, jam clusters can branch, with large jam-free holes in between branches of the jam (see Fig. 5). As a result, space-time plots of such jams now appear to show fractal properties, and in simulations at the critical density, $w(t)$ does not follow any longer a clean scaling law, whereas $n(t)$ and P_{surv} still do. For further details, see [49].

VII. RETURNING TO THE STOCHASTIC TRAFFIC CA (STCA)

A. STCA/1

For the STCA at $v_{max} = 1$, from visual inspection (Fig. 6) one finds that distinguishable jams do not exist here. Instead, the space-time plot looks much more like one from the ASEP.

This is confirmed by theoretical analysis. One technical possibility to find q - ρ relations for a given particle hopping model is the n -cluster method. The essential idea here is to derive transition probabilities for transitions from one local system configuration to another. Since a configuration of length l at time $t + 1$ causally derives from a configuration of length $l + 2v_{max}$ at time t , one has to make some approximations to close the upcoming equations.

In the case of the ASEP, it turns out that already the simplest of these approximations, called 1-cluster or mean field approximation, leads to the exact result, that is, all higher order corrections are zero. In the case of the STCA/1, the 1-cluster approximation is not exact, indicating more complicated dynamics than for the ASEP, but the 2-cluster approximation is. The difference between the ASEP and the STCA/1 in this analysis is that in the STCA/1 one finds an effective repulsive force of range one between particles, caused by the parallel update. This helps to keep particles more equidistant than in the ASEP case, thus leading to a higher flow. For further details, see [51,53].

B. STCA/2

For $v_{max} \geq 2$, the n -cluster analysis does no longer lead to an exact solution, indicating that a different dynamical regime has now completely taken over. (In practice, though, the n -cluster analysis is already fairly close to simulation results for $n \simeq 5$.) Visual inspection of space-time plots (Fig. 7) confirms that the dynamics now is much more similar to the cruise control limit, i.e. to STCA-CC/2 (Fig. 5), than to the ASEP (Fig. 1).

The important difference is that jams now start spontaneously and independently of other jams because vehicles fluctuate even at maximum speed. One consequence is that this modifies the scaling regime [49,9]. Another consequence is that the supercritical laminar flow (i.e. $\rho_{c1} < \rho < \rho_{c2}$ in the cruise control limit description) now is meta-stable at best, and eventually decomposes into laminar regions with $\rho = \rho_{c1}$, and jams.

A more precise explanation of this is as follows. Laminar traffic will always and at all densities, due to small fluctuations, have small disturbances which can develop into jams. The inflow to the jam decides if such a jam can become long-lived or not: Since the average outflow q_{out} is fixed by the driving dynamics, $q_{in} > q_{out}$ makes the jam (in the average) long-lived, $q_{in} < q_{out}$ not. $q_{in} = q_{out}$ defines a critical point, i.e. $q_c \equiv q_{out}$ and $\rho_c \equiv \rho_{out}$, where traffic jam clusters in the average barely survive, as e.g. quantified by $\langle n(t) \rangle = t^\eta$ with $\eta = 0$.

Since in supercritical systems $q_{in} > q_c = q_{out}$, a jam, once initiated, grows until the inflow reduces to q_c or lower. This can happen, e.g., because of another jam upstream, or because the rush hour is over.

This dynamical picture explains the high variations in the short time measurements. Measuring at a fixed position in a situation like in Fig. 7, one can measure arbitrary combinations of supercritical laminar traffic, critical laminar traffic, jams, or traffic during acceleration or slowing down. See Fig. 8 for a comparison between short-time (300 time steps) averages and a schematic picture. Data points along the (a) branch belong to stable and laminar traffic. Data points along the (c) branch belong to still laminar, but only meta-stable traffic. Data points along the (d) branch belong to creeping high density traffic.

All other data points are mixtures between regimes, where two or more regimes have been captured during the 300 iterations interval. Essentially, these data points should lie between point (b) and branch (d), yet, due to high fluctuations and due to the effects of acceleration and braking, which are not captured in the steady state arguments, we see huge fluctuations. For example, when a car is just leaving a jam, the density decreases, but the velocity adaption is lagging somewhat behind. Therefore, the car has too low speed for the given density, leading to too low a flow value.

This picture also makes precise the hysteresis argument of Treiterer and coworkers [75], also confirmed later [76,77]. These measurements confirm the idea that the traffic density can go above the critical point while still being laminar, similar to the gas which can be super-cooled by increasing the density. Yet, both for traffic and for super-cooled gases, this state is only meta-stable and eventually leads to a phase separation into jams and laminar flow. Quantitative evidence of this will be given in a separate paper (in preparation).

VIII. SUMMARY AND DISCUSSION

At a first glance, particle hopping models seem a somewhat crude approximation of real world traffic. Yet, they produce surprisingly realistic dynamics, for example with respect to start-stop wave formation and with respect to fundamental diagrams. The reason for this is that even when the microscopic dynamics is only crudely represented, the macroscopic behavior can still be very realistic. This has already been known for some time and in some cases

even been proven for example for the lattice gas methods for Navier-Stokes-equations [6]. More interesting in the context of traffic flow theory are results for one-dimensional systems, and they turn out to be even more instructive than expected.

This paper starts with the definition of a certain particle hopping model (called STCA for Stochastic Traffic Cellular Automaton) where the crude driving rules have been taken directly from reality. After that, sub-cases or variations of this model are discussed, which make it arguably less realistic, but have the advantage that these cases are well understood. It turns out that two of these models are described by certain cases of the Lighthill-Whitham theory, which has been used in the traffic context for about 40 years now. And in addition, the way in which the STCA goes beyond these models (by including momentum) is exactly the same way in which recent work goes beyond Lighthill-Whitham theory. Moreover, one can show that certain important aspects of the traffic jam dynamics of the STCA are phenomenologically the same as in the modern fluid-dynamical models. Yet, the STCA goes even beyond that at least with respect to fluctuations, which the STCA includes but the fluid-dynamical theories do not.

Thus, one learns that particle hopping models, crude as they are on the microscopic level, are "good" enough to lead to reasonable behavior on the fluid-dynamical level. Usually, one expects particle hopping models still to be realistic also for somewhat smaller scales than the fluid-dynamical scale.

In consequence, if one is for example interested in mostly macroscopic quantities which could be obtained in principle by a fluid-dynamical model, but needs microscopic ingredients such as individual trip plans, (some) different vehicles classes, or – most importantly – some information about fluctuations, then particle hopping models seem to be the best way to go, especially if one wants to save computational resources.

And here it is where we come back to the starting point in the introduction: Since current progress in the traffic forecasting process is heading towards the regional scale using microscopic models, this is exactly the problem which was posed: How can we model the individual vehicle as part of the simulation process without using up too many of the computational resources on the one hand and without becoming too unrealistic on the other hand?

It is, though, clear that there are limits to how well particle hopping models will be able to represent microscopic properties of traffic. Sometimes, it will be possible to expand the particle hopping model, for example by choosing a higher resolution [78], but often enough, it will be necessary to resort to a higher fidelity model. Nevertheless, the body of theory which is already available or currently being developed for particle hopping models puts them into a special position here: Understanding what a model does is the best way of knowing what it cannot do.

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FIG. 1. Space-time plot for random sequential update, $v_{max} = 1$ (ASEP/1), and $\rho = 0.3$. Clearly, the kinematic waves are moving forwards. For $\rho > 1/2$, the kinematic waves would be moving backwards.

FIG. 2. Space-time plot for CA-184/5 and subcritical density.

FIG. 3. Space-time plot for CA-184/5 and supercritical density.

FIG. 4. Space-time plot for STCA-CC/1, at supercritical density, with one disturbance. The jam first grows according to $n(t) \sim (j_{in} - j_{out}) \cdot t$. Eventually, via the periodic boundary conditions, the outflow reaches the jam as inflow, and $n(t)$ follows a random walk (apart from finite size effects).

FIG. 5. Space-time plot for parallel update (cruise control limit), $v_{max} = 5$, $\rho = 0.09$, i.e. slightly above critical. The flow is started in a deterministic, supercritical configuration, but from a single disturbance separates into a jam and a region of exactly critical density.—This is phenomenologically the same plot as Fig. 4 except that $v_{max} = 5$.

FIG. 6. Space-time plot for parallel update, $v_{max} = 1$.

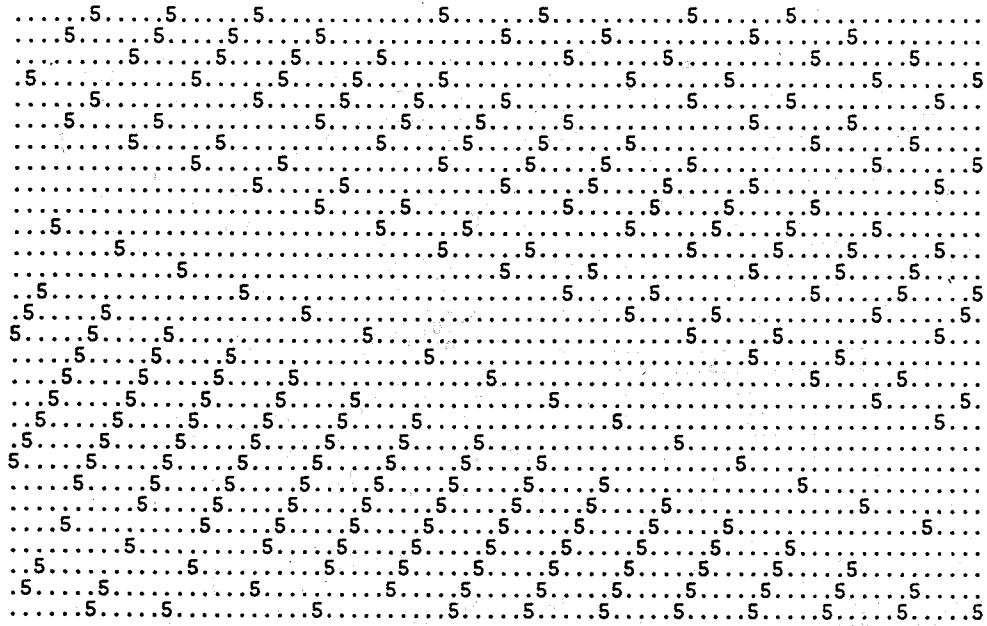
FIG. 7. Space-time plot for parallel update, $v_{max} = 5$, $\rho = 0.09$ (i.e. slightly above $\rho(q_{max})$), starting from ordered initial conditions. The ordered state is meta-stable, i.e. "survives" for about 300 iterations until is spontaneously separates into jammed regions and into regions with $\rho = \rho(q_{max})$.

FIG. 8. Flow-density fundamental diagrams for the STCA. *Top*: Simulation output from the STCA. Short-time averages are taken over 300 simulation steps and thus mimic the 5-minute averages often taken in reality. *Bottom*: Schematical view. (a) is the subcritical branch, (b) is the critical point, (c) is the supercritical branch, and (d) is the branch where traffic only creeps. 5-minute averages at densities between ρ_c at (b) and the creep branch are mixtures between the dynamical regimes.

space (road) -->

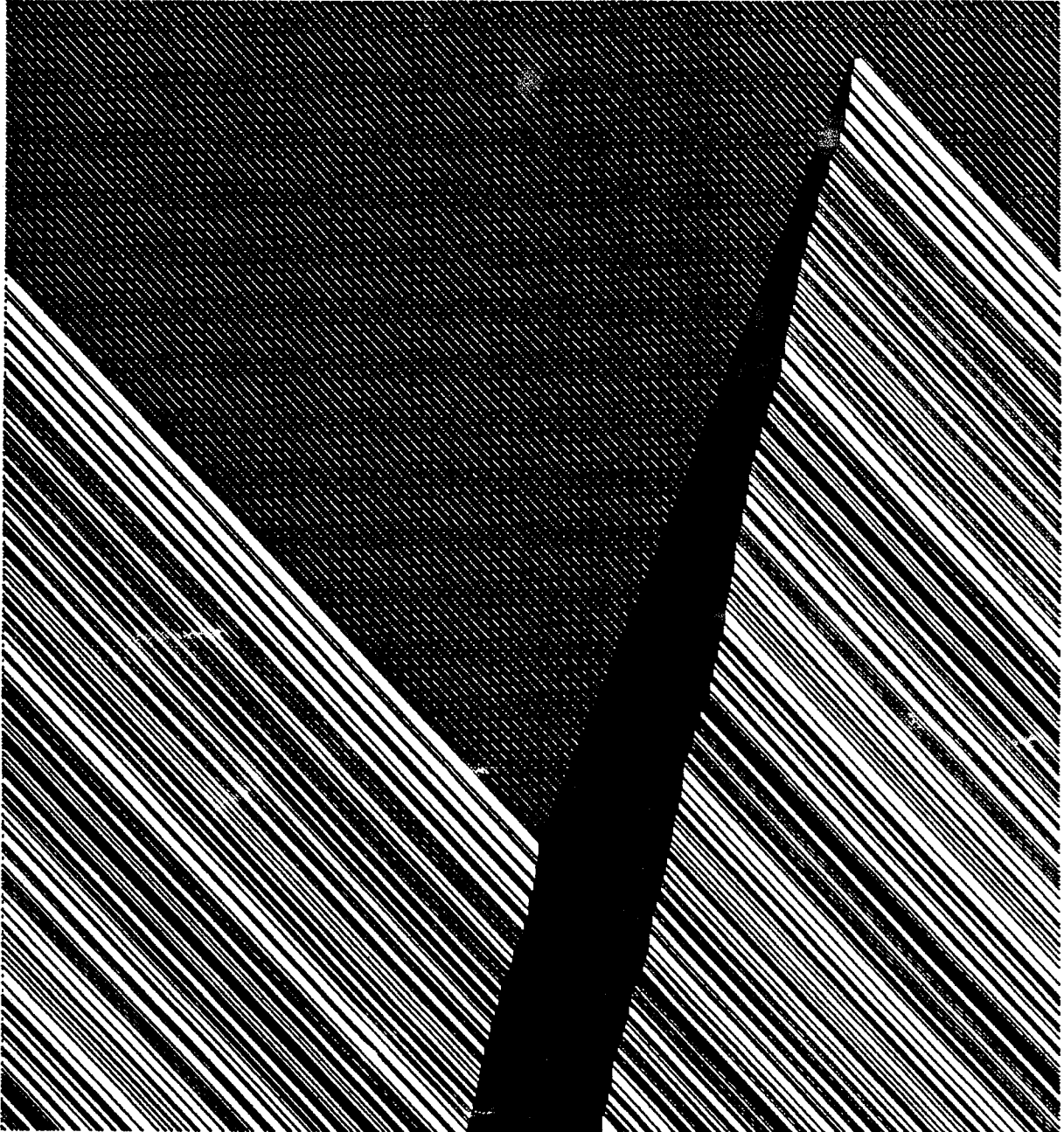
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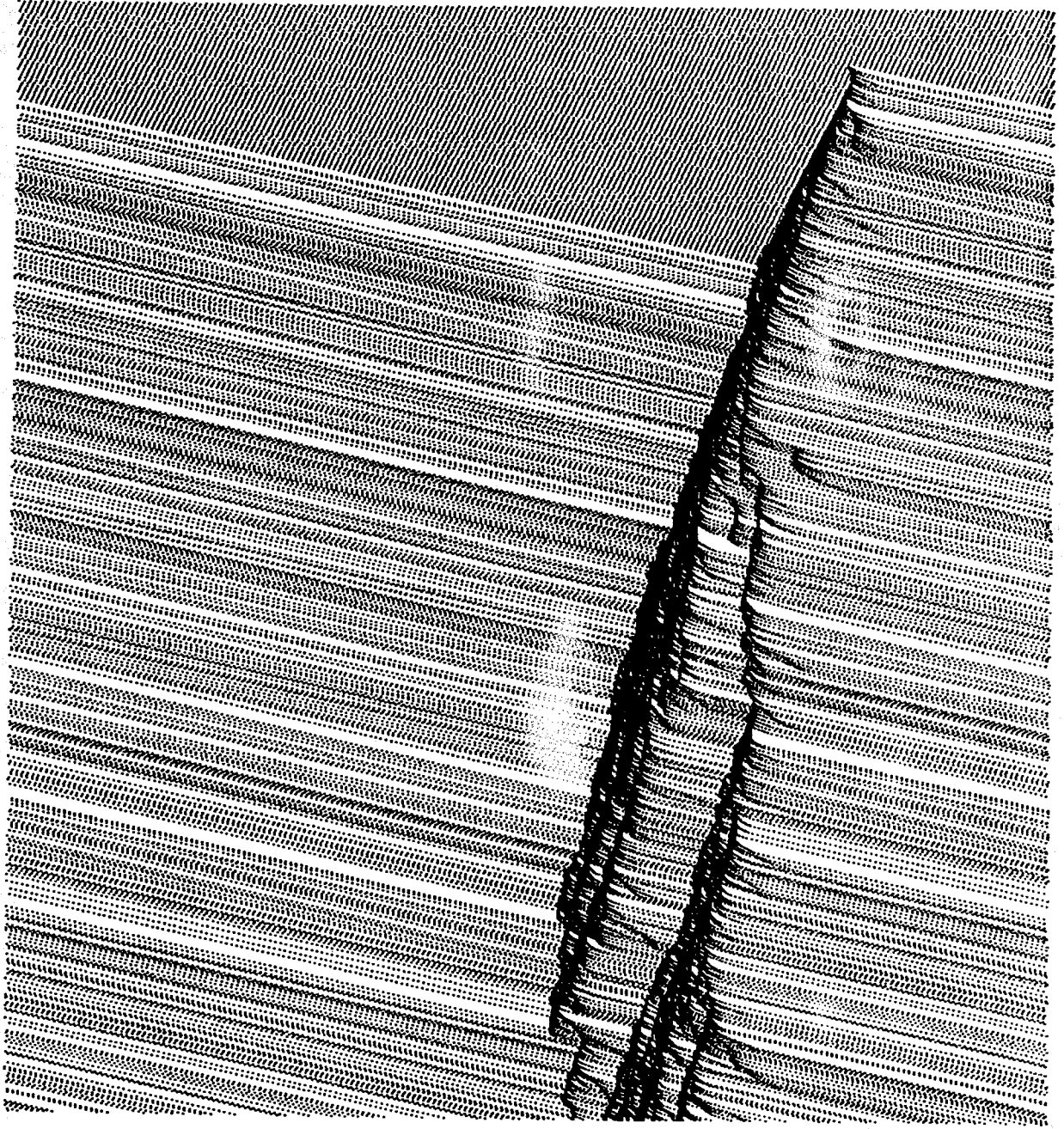
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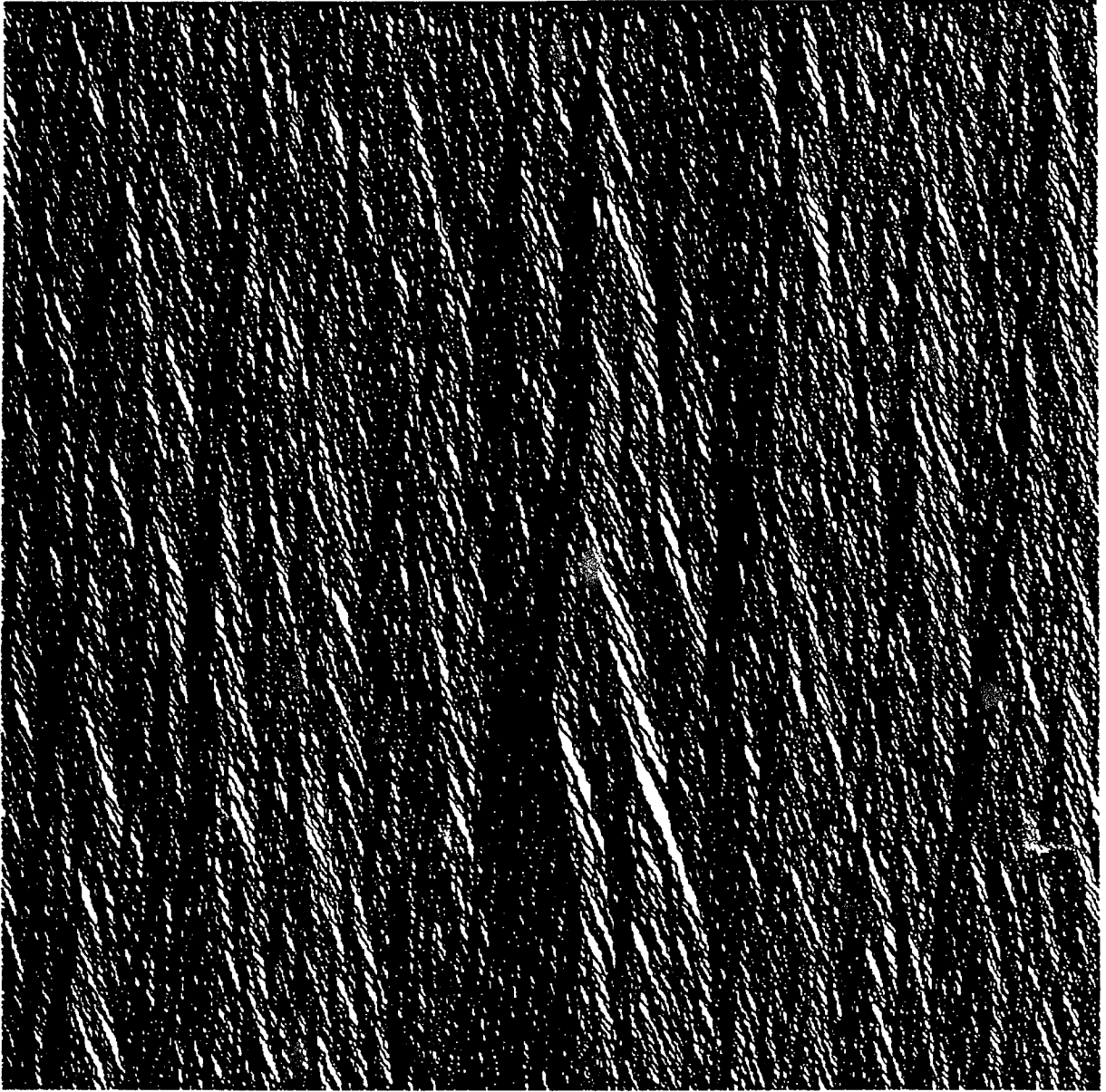
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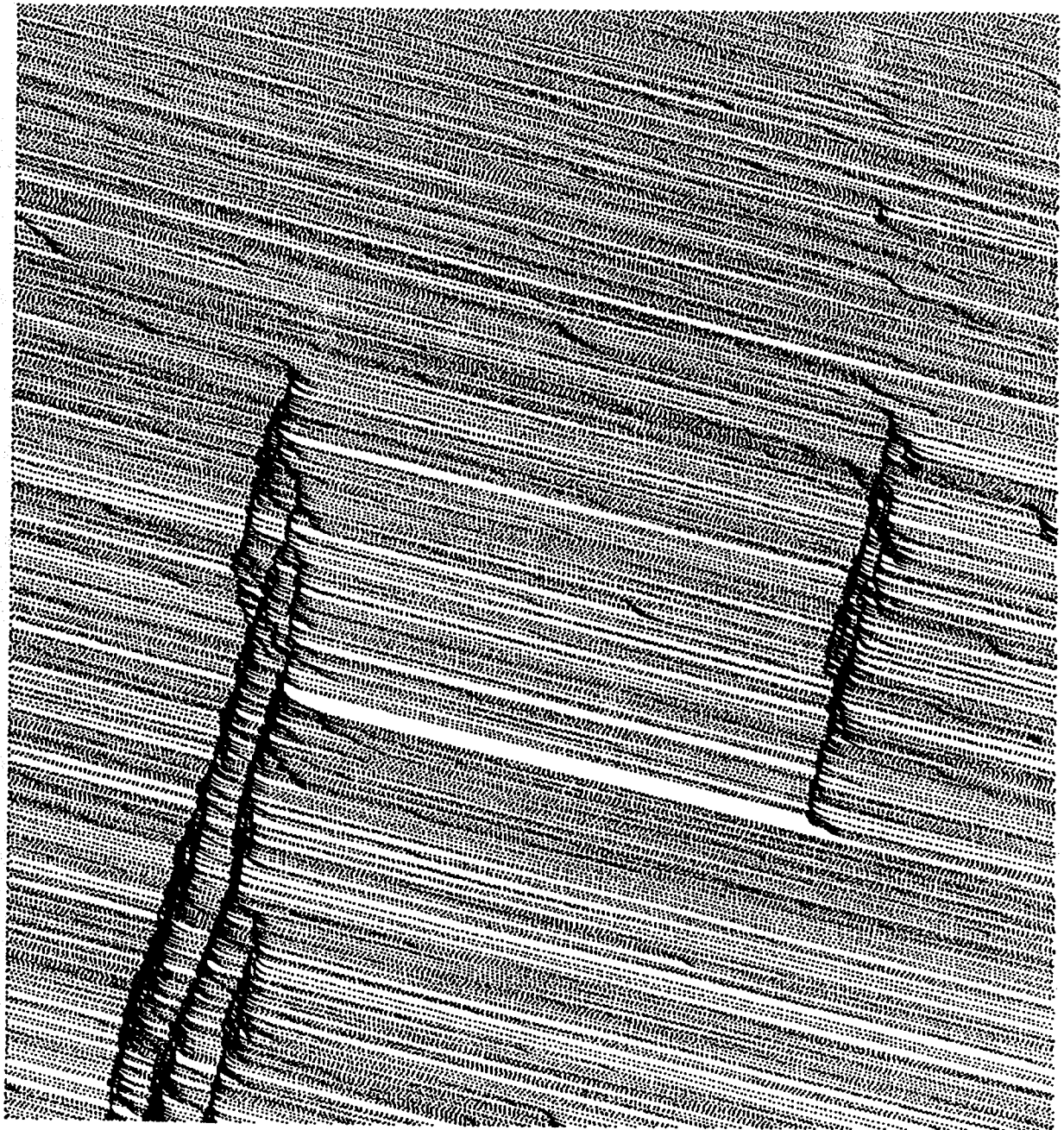
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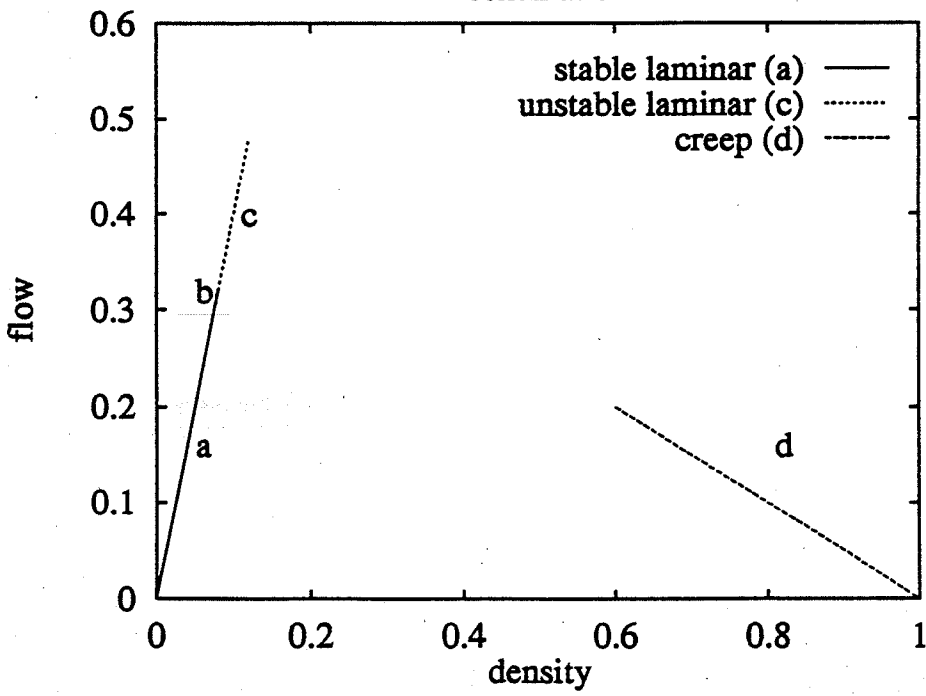
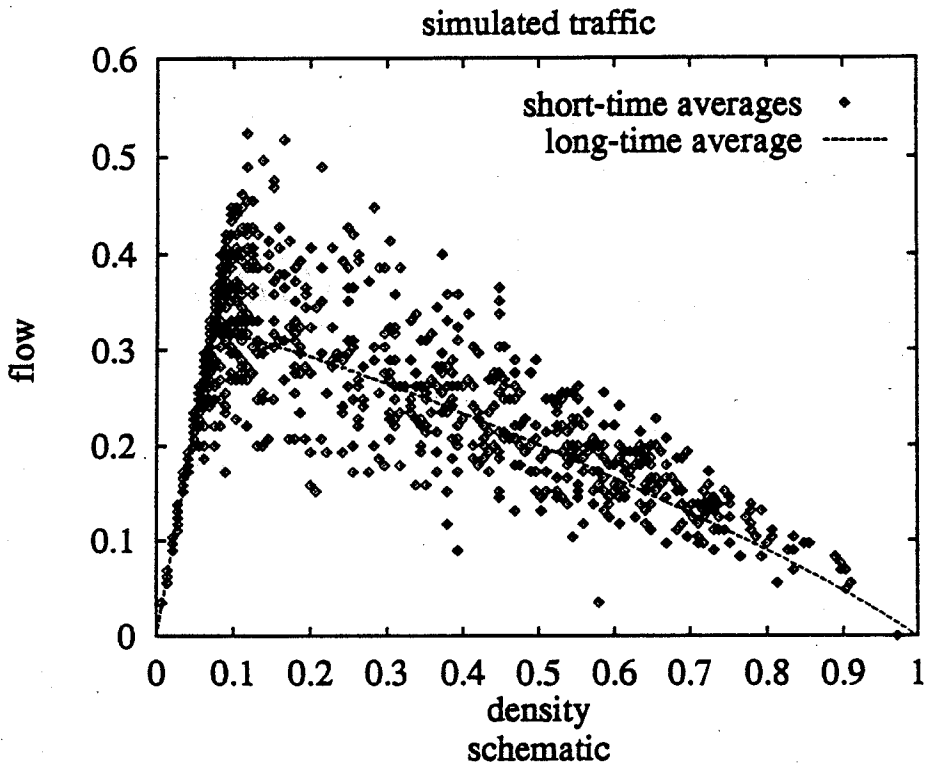
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space (road) -->

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