UNSIGNALIZED INTERSECTION THEORY

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Chapter 8 - Frequently used Symbols

b_i = proportion of volume of movement *i* of the total volume on the shared lane
C_w = coefficient of variation of service times
D = total delay of minor street vehicles
D_q = average delay of vehicles in the queue at higher positions than the first
$E(h)$ = mean headway
$E(t_c)$ = the mean of the critical gap, t_c
$f(t)$ = density function for the distribution of gaps in the major stream
$g(t)$ = number of minor stream vehicles which can enter into a major stream gap of size, *t*
L = logarithm
m = number of movements on the shared lane
n = number of vehicles
η_c = increment, which tends to 0, when $Var(t_c)$ approaches 0
η_f = increment, which tends to 0, when $Var(t_f)$ approaches 0
q = flow in veh/sec
q_s = capacity of the shared lane in veh/h
$q_{m,i}$ = capacity of movement *i*, if it operates on a separate lane in veh/h
q_m = the entry capacity
q_m = maximum traffic volume departing from the stop line in the minor stream in veh/sec
q_p = major stream volume in veh/sec
t = time
t_c = critical gap time
t_f = follow-up times
t_m = the shift in the curve
$Var(t_c)$ = variance of critical gaps
$Var(t_f)$ = variance of follow-up-times
$Var\ (W)$ = variance of service times
W = average service time. It is the average time a minor street vehicle spends in the first position of the queue near the intersection
W_1 = service time for vehicles entering the empty system, i.e no vehicle is queuing on the vehicle's arrival
W_2 = service time for vehicles joining the queue when other vehicles are already queuing

8.
UNSIGNALIZED INTERSECTION THEORY

8.1 Introduction

Unsignalized intersections are the most common intersection type. Although their capacities may be lower than other intersection types, they do play an important part in the control of traffic in a network. A poorly operating unsignalized intersection may affect a signalized network or the operation of an Intelligent Transportation System.

The theory of the operation of unsignalized intersections is fundamental to many elements of the theory used for other intersections. For instance, queuing theory in traffic engineering used to analyze unsignalized intersections is also used to analyze other intersection types.

8.1.1 The Attributes of a Gap Acceptance Analysis Procedure

Unsignalized intersections give no positive indication or control to the driver. He or she is not told when to leave the intersection. The driver alone must decide when it is safe to enter the intersection. The driver looks for a safe opportunity or "gap" in the traffic to enter the intersection. This technique has been described as gap acceptance. Gaps are measured in time and are equal to headways. At unsignalized intersections a driver must also respect the priority of other drivers. There may be other vehicles that will have priority over the driver trying to enter the traffic stream and the driver must yield to these drivers.

All analysis procedures have relied on gap acceptance theory to some extent or they have understood that the theory is the basis for the operation even if they have not used the theory explicitly.

Although gap acceptance is generally well understood, it is useful to consider the gap acceptance process as one that has two basic elements.

- First is the extent drivers find the gaps or opportunities of a particular size useful when attempting to enter the intersection.
- Second is the manner in which gaps of a particular size are made available to the driver. Consequently, the proportion of gaps of a particular size that are offered to the entering driver and the pattern of the inter-arrival times are important.

This chapter describes both of these aspects when there are two streams. The theory is then extended to intersections with more than two streams.

8.1.2 Interaction of Streams at Unsignalized Intersections

A third requirement at unsignalized intersections is that the interaction between streams be recognized and respected. At all unsignalized intersections there is a hierarchy of streams. Some streams have absolute priority, while others have to yield to higher order streams. In some cases, streams have to yield to some streams which in turn have to yield to others. It is useful to consider the streams as having different levels of priority or ranking. For instance:

Rank 1 stream - has absolute priority and does not need to yield right of way to another stream,

Rank 2 stream - has to yield to a rank 1 stream,

Rank 3 stream - has to yield to a rank 2 stream and in turn to a rank 1 stream, and

Rank 4 stream - has to yield to a rank 3 stream and in turn to a rank 2 stream and to a rank 1 stream.

8.1.3 Chapter Outline

Sections 8.2 discusses gap acceptance theory and this leads to Section 8.3 which discusses some of the common headway distributions used in the theory of unsignalized intersections.

Most unsignalized intersections have more than two interacting streams. Roundabouts and some merges are the only examples of two interacting streams. Nevertheless, an understanding of the operation of two streams provides a basis to extend the knowledge to intersections with more than two streams. Section

8.4 discusses the performance of intersections with two interacting streams.

Section 8.5 to 8.8 discuss the operation of more complex intersections. Section 8.9 covers other theoretical treatments of unsignalized intersections. In many cases, empirical approaches have been used. For instance the relationships for AWSC (All Way Stop Controlled) intersections are empirical. The time between successive departures of vehicles on the subject roadway are related to the traffic conditions on the other roadway elements.

The theory described in this chapter is influenced by the human factors and characteristics as described in Chapter 3, and in particular, Sections 3.13 and 3.15. The reader will also note similarities between the material in this chapter and Chapter 9 dealing with signalized intersections. Finally, unsignalized intersections can quickly become very complicated and often the subject of simulation programs. The comments in Chapter 10 are particularly relevant here.

8.2 Gap Acceptance Theory

8.2.1 Usefulness of Gaps

The gap acceptance theory commonly used in the analysis of unsignalized intersections is based on the concept of defining the extent drivers will be able to utilize a gap of particular size or duration. For instance, will drivers be able to leave the stop line at a minor road if the time between successive vehicles from the left is 10 seconds; and, perhaps how many drivers will be able to depart in this 10 second interval ?

The minimum gap that all drivers in the minor stream are assumed to accept at all similar locations is the critical gap. According to the driver behavior model usually assumed, no driver will enter the intersection unless the gap between vehicles in a higher priority stream (with a lower rank number) is at least equal to the critical gap, t_c. For example, if the critical gap was 4 seconds, a driver would require a 4 second gap between Rank 1 stream vehicles before departing. He or she will require the same 4 seconds at all other times he or she approaches the same intersection and so will all other drivers at that intersection.

Within gap acceptance theory, it is further assumed that a number of drivers will be able to enter the intersection from a minor road in very long gaps. Usually, the minor stream vehicles (those yielding right of way) enter in the long gaps at headways often referred to as the "follow-up time", t_f.

Note that other researchers have used a different concept for the critical gap and the follow-up time. McDonald and Armitage (1978) and Siegloch (1973) independently described a concept where a lost time is subtracted from each major stream gap and the remaining time is considered 'useable.' This 'useable' time divided by the saturation flow gives an estimate of the absorption capacity of the minor stream. As shown below, the effect of this different concept is negligible.

In the theory used in most guides for unsignalized intersections around the world, it is assumed that drivers are both consistent and homogeneous. A consistent driver is expected to behave the same way every time at all similar situations. He or she is not expected to reject a gap and then subsequently accept a smaller gap. For a homogeneous population, all drivers are expected to behave in exactly the same way. It is, of course, unreasonable to expect drivers to be consistent and homogeneous.

The assumptions of drivers being both consistent and homogeneous for either approach are clearly not realistic. Catchpole and Plank (1986), Plank and Catchpole (1984), Troutbeck (1988), and Wegmann (1991) have indicated that if drivers were heterogeneous, then the entry capacity would be decreased. However, if drivers are inconsistent then the capacity would be increased. If drivers are assumed to be both consistent and homogeneous, rather than more realistically inconsistent and heterogeneous, then the difference in the predictions is only a few percent. That is, the overall effect of assuming that drivers are consistent and homogeneous is minimal and, for simplicity, consistent and homogeneous driver behavior is assumed.

It has been found that the gap acceptance parameters t_c and t_f may be affected by the speed of the major stream traffic (Harders 1976 and Troutbeck 1988). It also expected that drivers are influenced by the difficulty of the maneuver. The more difficult

a maneuver is, the longer are the critical gap and follow-up time parameters. There has also been a suggestion that drivers require a different critical gap when crossing different streams within the one maneuver. For instance a turn movement across a number of different streams may require a driver having a different critical gap or time period between vehicles in each stream (Fisk 1989). This is seen as a unnecessary complication given the other variables to be considered.

8.2.2 Estimation of the Critical Gap Parameters

The two critical gap parameters that need to be estimated are the critical gap t_c and the follow-up time t_f. The techniques used to estimate these parameters fit into essentially two different groups. The first group of techniques are based on a regression analysis of the number of drivers that accept a gap against the gap size. The other group of techniques estimates the distribution of follow-up times and the critical gap distribution independently. Each group is discussed below.

Regression techniques.
If there is a continuous queue on the minor street, then the technique proposed by Siegloch (1973) produces acceptable results because the output matches the assumptions used in a critical gap analysis. For this technique, the queue must have at least one vehicle in it over the observation period. The process is then:

- Record the size of each gap, *t*, and the number of vehicles, *n*, that enter during this gap;
- For each of the gaps that were accepted by only *n* drivers, calculate the average gap size, *E(t)* (See Figure 8.1);
- Use linear regression on the average gap size values (as the dependent variable) against the number of vehicles that enter during this average gap size, *n*; and

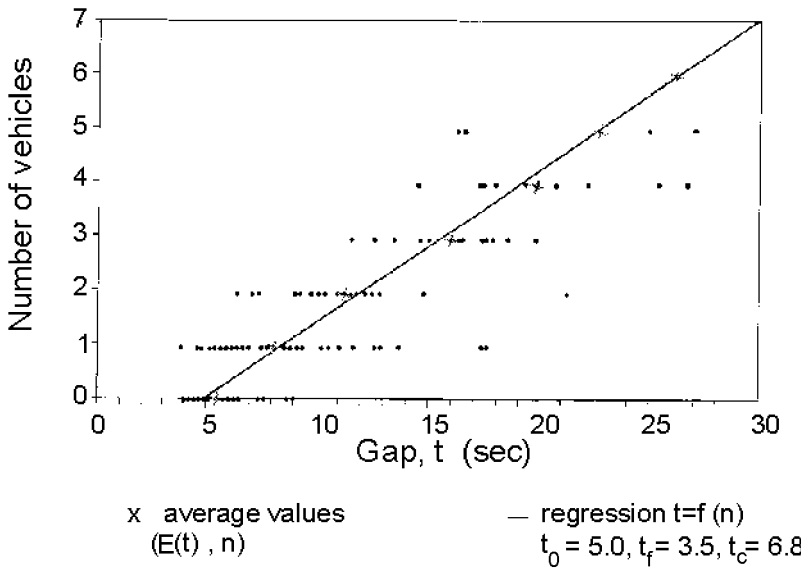


Figure 8.1
Data Used to Evaluate Critical Gaps and Move-Up Times
(Brilon and Grossmann 1991).

- Given the slope is t_f and the intercept of the gap size axis is t_o, then the critical gap t_c is given by

$$t_c = t_o + t_f/2 \qquad (8.1)$$

The regression line is very similar to the stepped line as shown in Figure 8.2. The stepped line reflects the assumptions made by Tanner (1962), Harders (1976), Troutbeck (1986), and others. The sloped line reflects the assumptions made by Siegloch (1973), and McDonald and Armitage (1978).

Independent assessment of the critical gap and follow-up time
If the minor stream does not continuously queue, then the regression approach cannot be used. A probabilistic approach must be used instead.

The follow-up time is the mean headway between queued vehicles which move through the intersection during the longer gaps in the major stream. Consider the example of two major stream vehicles passing by an unsignalized intersection at times 2.0 and 42.0 seconds. If there is a queue of say 20 vehicles wishing to make a right turn from the side street, and if 17 of these minor street vehicles depart at 3.99, 6.22, 8.29, 11.13, 13.14, and so on, then the headways between the minor street vehicles are 6.22-3.99, 8.29-6.22, 11.13-8.29 and so on. The average headway between this group of minor stream vehicles is 2.33 sec. This process is repeated for a number of larger major stream gaps and an overall average headway between the queued minor stream vehicles is estimated. This average headway is the follow-up time, t_f . If a minor stream vehicle was not in a queue then the preceding headway would not be included. This quantity is similar to the saturation headway at signalized intersections.

The estimation of the critical gap is more difficult. There have been numerous techniques proposed (Miller 1972; Ramsey and Routledge 1973; Troutbeck 1975; Hewitt 1983; Hewitt 1985). The difficulty with the estimation of the critical gap is that it cannot be directly measured. All that is known is that a driver's individual critical gap is greater than the largest gap rejected and shorter than the accepted gap for that driver. If the accepted gap was shorter than the largest rejected gap then the driver is considered to be inattentive. This data is changed to a value just below the accepted gap. Miller (1972) gives an alternative method of handling this inconsistent data which uses the data as recorded. The difference in outcomes is generally marginal.

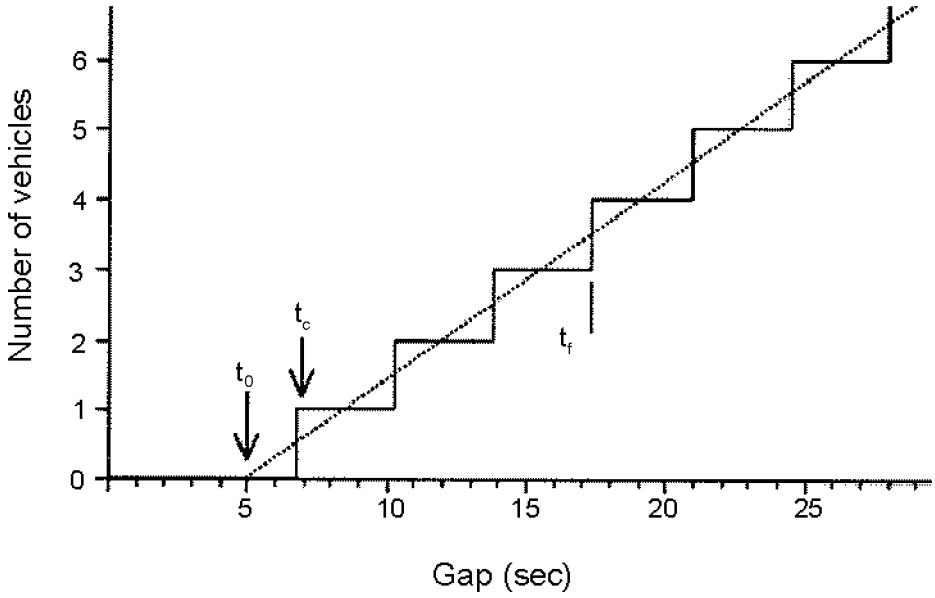


Figure 8.2
Regression Line Types.

Miller (1972), and later Troutbeck (1975) in a more limited study, used a simulation technique to evaluate a total of ten different methods to estimate the critical gap distribution of drivers. In this study the critical gaps for 100 drivers were defined from a known distribution. The arrival times of priority traffic were simulated and the appropriate actions of the "simulated" drivers were noted. This process was repeated for 100 different sets of priority road headways, but with the same set of 100 drivers. The information recorded included the size of any rejected gaps and the size of the accepted gap and would be similar to the information able to be collected by an engineer at the road side. The gap information was then analyzed using each of the ten different methods to give an estimate of the average of the mean of the drivers' critical gaps, the variance of the mean of the drivers' critical gaps, mean of the standard deviation of the drivers' critical gaps and the variance of the standard deviation of the drivers' critical gaps. These statistics enabled the possible bias in predicting the mean and standard deviation of the critical gaps to be estimated. Techniques which gave large variances of the estimates of the mean and the standard deviation of the critical gaps were considered to be less reliable and these techniques were identified. This procedure found that one of the better methods is the Maximum Likelihood Method and the simple Ashworth (1968) correction to the prohibit analysis being a strong alternative. Both methods are documented here. The Probit or Logit techniques are also acceptable, particularly for estimating the probability that a gap will be accepted (Abou-Henaidy et al. 1994), but more care needs to be taken to properly account for flows. Kyte et al (1996) has extended the analysis and has found that the Maximum Likelihood Method and the Hewitt (1983) models gave the best performance for a wide range of minor stream and major stream flows.

The maximum likelihood method of estimating the critical gap requires that the user assumes a probabilistic distribution of the critical gap values for the population of drivers. A log-normal is a convenient distribution. It is skewed to the right and does not have non-negative values. Using the notation:

a_i = the logarithm of the gap accepted by the *i*th driver,
a_i = ∞ if no gap was accepted,
r_i = the logarithm of the largest gap rejected by the ith driver,
r_i = 0 if no gap was rejected,

μ and σ^2 are the mean and variance of the logarithm of the individual drivers critical gaps (assuming a log-normal distribution), and

$f(\,)$ and $F(\,)$ are the probability density function and the cumulative distribution function respectively for the normal distribution.

The probability that an individual driver's critical gap will be between r_i and a_i is $F(a_i) - F(r_i)$. Summing over all drivers, the likelihood of a sample of n drivers having accepted and largest rejected gaps of (a_i, r_i) is

$$\prod_{i=1}^{n} [F(a_i) - F(r_i)] \qquad (8.2)$$

The logarithm, L, of this likelihood is then

$$L = \sum_{i=1}^{n} \ln[F(a_i) - F(r_i)] \qquad (8.3)$$

The maximum likelihood estimators, μ and σ^2, that maximize L, are given by the solution to the following equations.

$$\frac{\partial L}{\partial \mu} = 0 \qquad (8.4)$$

and

$$\frac{\partial L}{\partial \sigma^2} = 0 \qquad (8.5)$$

Using a little algebra,

$$\frac{\partial F(x)}{\partial \mu} = -f(x) \qquad (8.6)$$

$$\frac{\partial F(x)}{\partial \sigma^2} = -\frac{x-\mu}{2\sigma^2} f(x) \qquad (8.7)$$

This then leads to the following two equations which must be solved iteratively. It is recommended that the equation

$$\sum_{i=1}^{n} \frac{f(r_i) - f(a_i)}{F(a_i) - F(r_i)} = 0 \qquad (8.8)$$

should be used to estimate μ given a value of σ^2. An initial value of σ^2 is the variance of all the a_i and r_i values. Using this estimate of μ from Equation 8.8, a better estimate of σ^2 can be obtained from the equation,

$$\sum_{i=1}^{n} \frac{(r_i - \hat{\mu})\, f(r_i) - (a_i - \hat{\mu})\, f(a_i)}{F(a_i) - F(r_i)} = 0 \qquad (8.9)$$

where $\hat{\mu}$ is an estimate of μ.

A better estimate of the μ can then be obtained from the Equation 8.8 and the process continued until successive estimates of μ and σ^2 do not change appreciably.

The mean, $E(t_c)$, and the variance, $Var(t_c)$, of the critical gap distribution is a function of the log normal distribution parameters, viz:

$$E(t_c) = e^{\mu + 0.5\sigma^2} \qquad (8.10)$$

and

$$Var(t_c) = E(t_c)^2\ (e^{\sigma^2} - 1) \qquad (8.11)$$

The critical gap used in the gap acceptance calculations is then equal to $E(t_c)$. The value should be less than the mean of the accepted gaps.

This technique is a complicated one, but it does produce acceptable results. It uses the maximum amount of information, without biasing the result, by including the effects of a large number of rejected gaps. It also accounts for the effects due to the major stream headway distribution. If traffic flows were light, then many drivers would accept longer gaps without rejecting gaps. On the other hand, if the flow were heavy, all minor stream drivers would accept shorter gaps. The distribution of accepted gaps is then dependent on the major stream flow. The maximum likelihood technique can account for these different conditions. Unfortunately, if all drivers accept the first gap offered without rejecting any gaps, then Equations 8.8 and 8.9 give trivial results. The user should then look at alternative methods or preferably collect more data.

Another very useful technique for estimating the critical gap is Ashworth's (1968) procedure. This requires that the user identify the characteristics of the probability distribution that relates the proportion of gaps of a particular size that were accepted to the gap size. This is usually done using a Probit analysis applied to the recorded proportions of accepted gaps. A plot of the proportions against the gap size on probability paper would also be acceptable. Again a log normal distribution may be used and this would require the proportions to be plotted against the natural logarithm of the gap size. If the mean and variance of this distribution are $E(t_a)$ and $Var_a(t)$, then Ashworth's technique gives the critical gap as

$$E(t_c) = E(t_a) - q_p\ Var(t_a) \qquad (8.12)$$

where q_p is the major stream flow in units of veh/sec. If the log normal function is used, then $E(t_a)$ and $Var(t_a)$ are values given by the generic Equations 8.10 and 8.11. This is a very practical solution and one which can be used to give acceptable results in the office or the field.

8.2.3 Distribution of Gap Sizes

The distribution of gaps between the vehicles in the different streams has a major effect on the performance of the unsignalized intersection. However, it is important only to look at the distribution of the larger gaps; those that are likely to be accepted. As the shorter gaps are expected to be rejected, there is little point in modeling these gaps in great detail.

A common model uses a random vehicle arrival pattern, that is, the inter-arrival times follow an exponential distribution. This distribution will predict a large number of headways less than 1 sec. This is known to be unrealistic, but it is used because these small gaps will all be rejected.

This exponential distribution is known to be deficient at high flows and a displaced exponential distribution is often recommended. This model assumes that vehicle headways are at least t_m sec.

Better models use a dichotomized distribution. These models assume that there is a proportion of vehicles that are free of interactions and travel at headways greater than t_m. These vehicles are termed "free" and the proportion of free vehicles is α. There is a probability function for the headways of free vehicles. The remaining vehicles travel in platoons and again there is a headway distribution for these bunched vehicles. One such dichotomized headway model is Cowan's (1975) M3 model which assumes that a proportion, α, of all vehicles are free and have an displaced exponential headway distribution and the 1-α bunched vehicles have the same headway of only t_m.

In this chapter, the word "queues" is used to refer to a line of stopped vehicles. On the other hand, a platoon is a group of traveling vehicles which are separated by a short headway of t_m. When describing the length of a platoon, it is usual to include a platoon leader which will have a longer headway in front of him or her. A platoon of length one is a single vehicle travelling without any vehicles close-by. It is often useful to distinguish between free vehicles (or platoon leaders) and those vehicles in the platoon but behind the leader. This latter group are called bunched vehicles. The benefits of a number of different headway models will be discussed later.

8.3 Headway Distributions Used in Gap Acceptance Calculations

8.3.1 Exponential Headways

The most common distribution is the negative exponential distribution which is sometimes referred to as simply the "exponential distribution". This distribution is based on the assumption that vehicles arrive at random without any dependence on the time the previous vehicle arrived. The distribution can be derived from assuming that the probability of a vehicle arriving in a small time interval $(t, t+\delta t)$ is a constant. It can also be derived from the Poisson distribution which gives the probability of n vehicles arriving in time t, that is:

$$P(n) = (qt)^n \frac{e^{-qt}}{n!} \qquad (8.13)$$

where q is the flow in veh/sec. For n = 0 this equation gives the probability that no vehicle arrives in time t. The headway, h, must be then greater than t and the probability, from Equation 8.13 is

$$P(h>t) = e^{-qt} \qquad (8.14)$$

The cumulative probability function of headways is then

$$P(h \le t) = 1 - e^{-qt} \qquad (8.15)$$

The probability distribution function is then

$$f(t) = \frac{d[P(h \le t)]}{dt} = q\ e^{-qt} \qquad (8.16)$$

This is the equation for the negative exponential distribution. The parameter q can be estimated from the flow or the reciprocal of the average headway. As an example, if there were 228 headways observed in half an hour, then the flow is 228/1800 i.e. q = 0.127 veh/sec. The proportion of headways expected to be greater than 5 seconds is then

$$\begin{aligned} P(h>5) &= e^{-q\,t} \\ &= e^{-5*0.127} \\ &= 0.531 \end{aligned}$$

The expected number of headways greater than 5 seconds observed in half an hour is then 0.531•228 or 116.

If the flow was 1440 veh/h or 0.4 veh/sec then the number of headways less than 0.1 seconds is then $q * [P(h>0.1)] * 3600$ or 56 per hour. This over-estimation of the number of very short headways is considered to be unrealistic and the displaced exponential distribution is often used instead of the negative exponential distribution.

8.3.2 Displaced Exponential Distribution

The shifted or displaced exponential distribution assumes that there is a minimum headway between vehicles, t_m. This time can be considered to be the space around a vehicle that no other

vehicle can intrude divided by the traffic speed. If the flow is q veh/h then in one hour q vehicles will pass and there are $t_m \bullet q$ seconds lost while these vehicles pass. The remaining time must then be distributed randomly after each vehicle and the average random component is $(1\text{-}t_m \bullet q)/q$ seconds. The cumulative probability distribution of headways is then:

$$F(h) = 1 - e^{-\lambda(h-t_m)} \qquad (8.17)$$

where,

$$\lambda = \frac{q}{1-t_m q} \qquad (8.18)$$

There, the terms, λ and t_m need to be evaluated. These can be estimated from the mean and the variance of the distribution. The mean headway, *E(h)*, is given by:

$$E(h) = 1/q = t_m + \frac{1}{\lambda} \qquad (8.19)$$

The variance of headways is $1/\lambda^2$. These two relationships can then be used to estimate λ and t_m.

This distribution is conceptually better than the negative exponential distribution but it does not account for the platooning that can occur in a stream with higher flows. A dichotomized headway distribution provides a better fit.

8.3.3 Dichotomized Headway Distributions

In most traffic streams there are two types of vehicles, the first are bunched vehicles; these are closely following preceding vehicles. The second group are free vehicles that are travelling without interacting with the vehicles ahead. There have been a number of dichotomized headway distributions developed over time. For instance, Schuhl (1955) proposed a distribution

$$p(h \le t) = 1 - \alpha e^{-t/\bar{h}_f} + (1-\alpha)e^{-(t-t_m)/(\bar{h}_b - t_m)} \qquad (8.20)$$

where there are α vehicles that are free (not in platoons);

there are $(1-\alpha)$ bunched vehicles;

$\bar{h}_f$ is the average headway for free vehicles;

$\bar{h}_b$ is the average headway for bunched or constrained vehicles;

t_m is the shift in the curve.

Other composite headway models have been proposed by Buckley (1962; 1968). However, a better headway model for gap acceptance is the M3 model proposed by Cowan (1975). This model does not attempt to model the headways between the bunched vehicles as these are usually not accepted but rather models the larger gaps. This headway model has a cumulative probability distribution:

$$p(h \le t) = 1 - \alpha e^{-\lambda(t-t_m)} \qquad for\ t > t_m \quad (8.21)$$

and

$$p(h \le t) = 0 \qquad \text{otherwise.}$$

Where λ is a decay constant given by the equation

$$\lambda = \frac{\alpha q}{(1-t_m q)} \qquad (8.22)$$

Cowan's headway model is rather general. To obtain the displaced exponential distribution set α to 1.0. For the negative exponential distribution, set α to 1.0 and t_m to 0. Cowan's model can also give the headway distribution used by Tanner (1962) by setting α to $1-t_m q$, however the distribution of the number of vehicles in platoons is not the same. This is documented below.

Brilon (1988) indicated that the proportion of free vehicles could be estimated using the equation,

$$\alpha = e^{-Aq_p} \qquad (8.23)$$

where A values ranged from 6 to 9. Sullivan and Troutbeck (1993) found that this equation gave a good fit to data, from more than 600 of hours of data giving in excess of 400,000 vehicle headways, on arterial roads in Australia. They also found that the A values were different for different lanes and for different lane widths. These values are listed in Table 8.1.

Table 8.1
"A" Values for Equation 8.23 from Sullivan and Troutbeck (1993).

	Median Lane	All other lanes
Lane width < 3.0 meters	7.5	6.5
3.0 ≤ Lane width ≤ 3.5 meters	7.5	5.25
Lane width> 3.5 meters	7.5	3.7

Typical values of the proportion of free vehicles are given in Figure 8.3.

The hyper-Erlang distribution is also a dichotomized headway distribution that provides an excellent fit to headway data. It is useful in simulation programs but has not been used in traffic theory when predicting capacity or delays. The hyper-Erlang distribution given by Dawson (1969) is:

$$p(h \le t) = 1 - \alpha e^{-(t - t_{mf}/\bar{h}_f - t_{mf})} + (1-\alpha) e^{-(t - t_{mb}/\bar{h}_b - t_{mb})} \sum_{x=0}^{k-1} \frac{k\left[\frac{t - t_{mb}}{\bar{h}_b - t_{mb}}\right]^x}{x!} \quad (8.24)$$

8.3.4 Fitting the Different Headway Models to Data

If the mean headway is 21.5 seconds and standard deviation is 19.55 seconds, then the flow is 1/21.5 or 0.0465 veh/seconds (167 veh/hour). A negative exponential curve that would fit this data is then,

$$p(h \le t) = 1 - e^{-0.0465t}$$

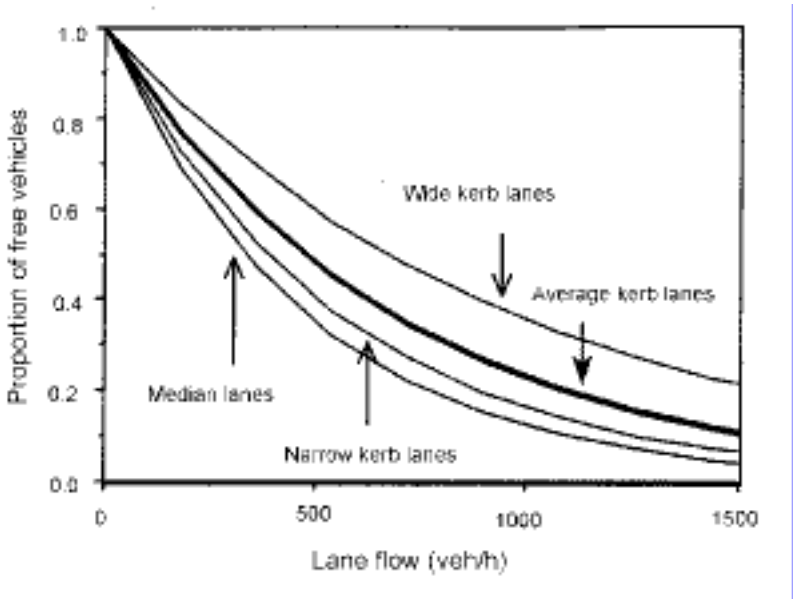


Figure 8.3
Typical Values for the Proportion of Free Vehicles.

To estimate the parameters for the displaced exponential distribution, the difference between the mean and the standard deviation is the displacement, that is t_m is equal to 21.49 – 19.55 or 1.94 seconds. The constant λ used in Equation 8.21 is the reciprocal of the standard deviation. In this case, λ is equal to 1/19.55 or 0.0512 veh/sec. The appropriate equation is then:

$$p(h \le t) = 1 - e^{-0.0512(t-1.94)}$$

The data and these equations are shown in Figure 8.4 which indicates the form of these distributions. The reader should not make any conclusions about the suitability of a distribution from this figure but should rather test the appropriateness of the model to the data collected.

In many cases there are a substantial number of very short headways and a dichotomized headway distribution performs better. As only the larger gaps are likely to be accepted by drivers, there is no point in modeling the shorter gaps in great detail. An example of Cowan's M3 model and headway data from an arterial road is shown in Figure 8.5. Figure 8.6 gives the same data and the hyper-Erlang distribution.

$$P(n) = (1-\alpha)^{n-1}\alpha \qquad (8.25)$$

Under these conditions the mean platoon size is

$$\bar{n} = \frac{1}{\alpha} \qquad (8.26)$$

and the variance by

$$Var(n) = \frac{1-\alpha}{\alpha^2} \qquad (8.27)$$

Another distribution of platoons used in the analysis of unsignalized intersections is the Borel-Tanner distribution. This platooning distribution comes from Tanner's (1962) assumptions where the major stream gaps are the outcome of a queuing process with random arrivals and a minimum inter-departure time of t_m. Although the distribution of these 'revised' major stream gaps is given by Equation 8.21 with α equal to $1-t_m q$,

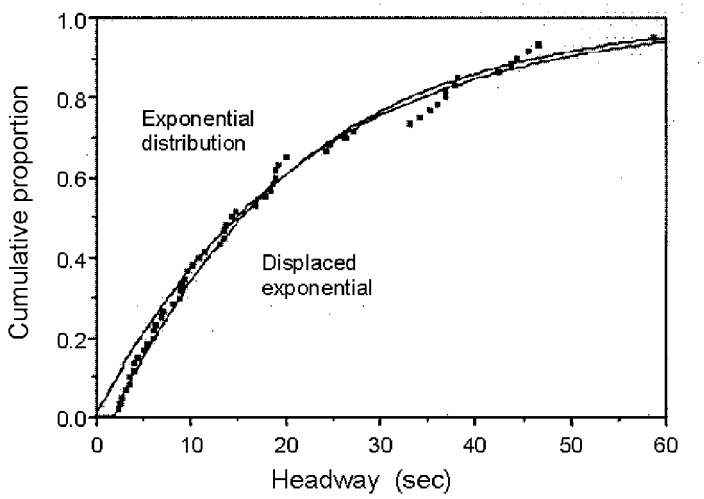


Figure 8.4
Exponential and Displaced Exponential Curves
(Low flows example).

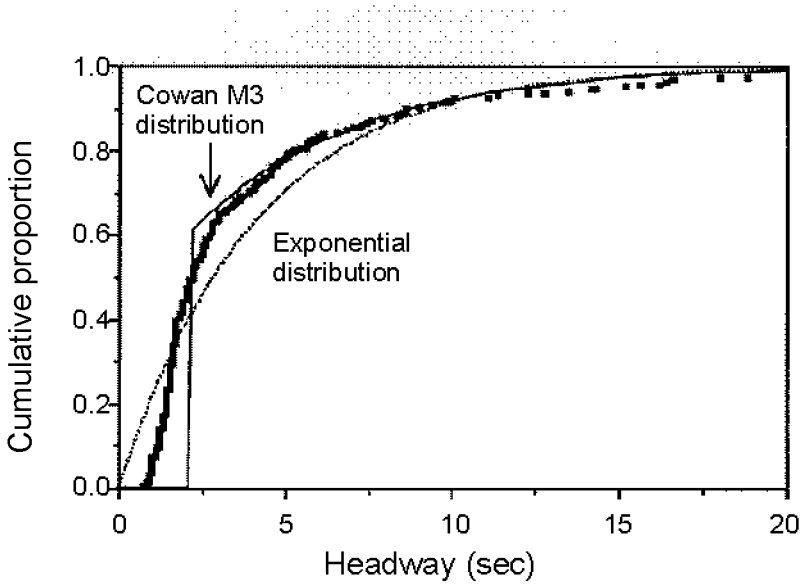


Figure 8.5
Arterial Road Data and a Cowan (1975) Dichotomized Headway Distribution (Higher flows example).

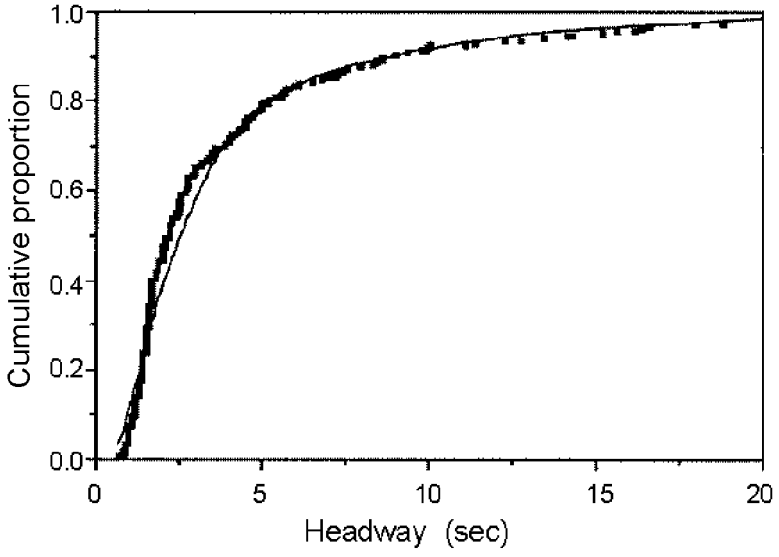


Figure 8.6
Arterial Road Data and a Hyper-Erlang Dichotomized Headway Distribution (Higher Flow Example).

the distribution of the platoon length is Borel-Tanner (Borel 1942; Tanner 1953; 1961; and Haight and Breuer 1960). Again, q is the flow in veh/sec. The Borel-Tanner distribution of platoons gives the probability of a platoon of size n as

$$P(n) = \frac{e^{-nt_m q}(nt_m q)^{n-1}}{n!} \qquad (8.28)$$

where n is an integer.

Haight and Breuer (1960) found the mean platoon size to be $1 / (1 - t_m q)$ or $1/\alpha$ and the variance to be $t_m q / (1 - t_m q)^3$ or $(1- \alpha) /\alpha^3$. For the same mean platoon size, the Borel-Tanner distribution has a larger variance and predicts a greater number of longer platoons than does the geometric distribution. Differences in the platoon size distribution does not affect an estimate of capacity but it does affect the average delay per vehicle as shown in Figure 8.13.

8.4 Interaction of Two Streams

For an easy understanding of traffic operations at an unsignalized intersection it is useful to concentrate on the simplest case first (Figure 8.7).

All methods of traffic analysis for unsignalized intersections are derived from a simple queuing model in which the crossing of two traffic streams is considered. A priority traffic stream (major stream) of the volume q_p (veh/h) and a non-priority traffic stream (minor stream) of the volume q_n (veh/h) are involved in this queuing model. Vehicles from the major stream can cross the conflict area without any delay. Vehicles from the minor stream are only allowed to enter the conflict area, if the next vehicle from the major stream is still t_c seconds away (t_c is the critical gap), otherwise they have to wait. Moreover, vehicles from the minor stream can only enter the intersection t_f seconds after the departure of the previous vehicle (t_f is the follow-up time).

8.4.1 Capacity

The mathematical derivation of the capacity q_m for the minor stream is as follows. Let $g(t)$ be the number of minor stream vehicles which can enter into a major stream gap of duration t. The expected number of these t-gaps per hour is $3600 q_p f(t)$ where,

- $f(t)$ = statistical density function of the gaps in the major stream and
- q_p = volume of the major stream.

Therefore, the amount of capacity which is provided by t-gaps per hour is $3600\ q_p\ f(t)\ g(t)$.

To get the total capacity, expressed in veh/second, we have to integrate over the whole range of major stream gaps:

$$q_m = q_p \int_0^\infty f(t) \cdot g(t) dt \qquad (8.29)$$

where,

- q_m = maximum traffic volume departing from the stop line in the minor stream in veh/sec,
- q_p = major stream volume in veh/sec,
- $f(t)$ = density function for the distribution of gaps in the major stream, and
- $g(t)$ = number of minor stream vehicles which can enter into a major stream gap of size, t .

Based on the gap acceptance model, the capacity of the simple 2-stream situation (Figure 8.7) can be evaluated by elementary probability theory methods if we assume:

(a) constant t_c and t_f values,

(b) exponential distribution for priority stream headways (cf. Equation 8.15), and

(c) constant traffic volumes for each traffic stream.

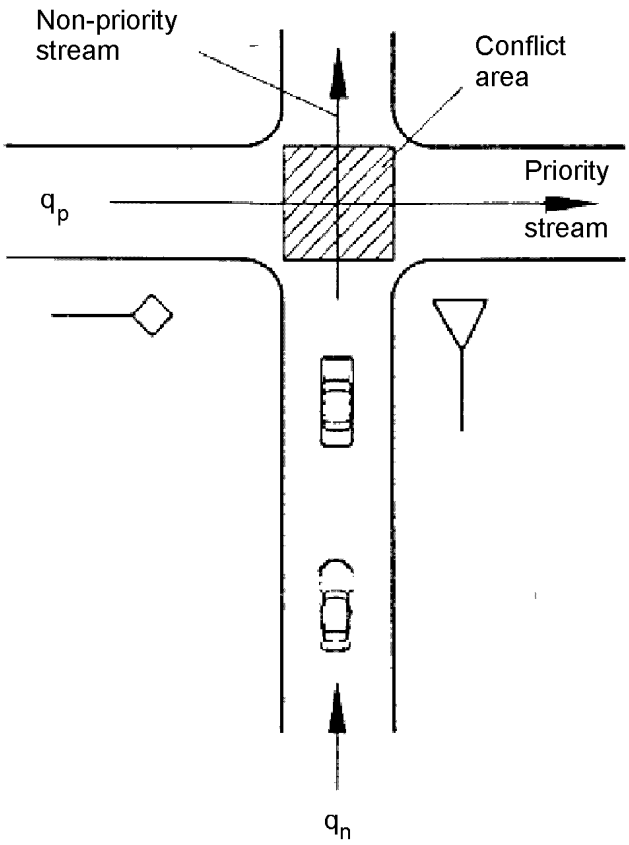


Figure 8.7
Illustration of the Basic Queuing System.

Within assumption (a), we have to distinguish between two different formulations for the term *g(t)*. These are the reason for two different families of capacity equations. The first family assumes a stepwise constant function for *g(t)* (Figure 8.2):

$$g(t) = \sum_{n=0}^{\infty} n \cdot p_n(t) \qquad (8.30)$$

where,

$p_n(t)$= probability that n minor stream vehicles enter a gap in the major stream of duration t,

$$p_n(t) = \begin{cases} 1 & for \quad t_c + (n-1)\cdot t_f \le t < t_c + n \cdot t_f \\ 0 & elsewhere \end{cases}$$

The second family of capacity equations assumes a continuous linear function for *g(t)*. This is an approach which has first been used by Siegloch (1973) and later also by McDonald and Armitage (1978).

$$g(t) = \begin{cases} 0 \ for \ t < t_0 \\ \dfrac{t - t_0}{t_f} \ for \ t \ge t_0 \end{cases} \qquad (8.31)$$

where,

$$t_0 = t_c - \frac{t_f}{2}$$

Once again it has to be emphasized that both in Equations 8.30 and 8.31, t_c and t_f are assumed to be constant values for all drivers.

Both approaches for *g(t)* produce useful capacity formulae where the resulting differences are rather small and can normally be ignored for practical applications (cf. Figure 8.8).

If we combine Equations 8.29 and 8.30, we get the capacity equation used by Drew (1968), Major and Buckley (1962), and by Harders (1968), which these authors however, derived in a different manner:

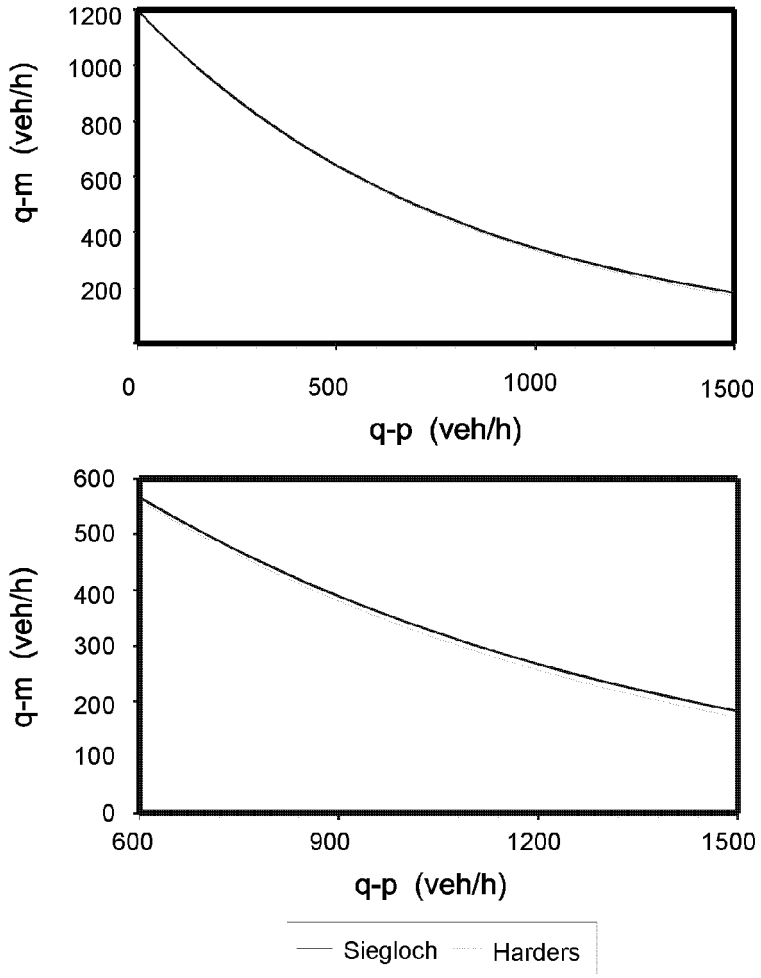


Figure 8.8
Comparison Relation Between Capacity (q-m) and Priority Street Volume (q-p).

$$q_m = q_p \frac{e^{-q_p \cdot t_c}}{1 - e^{-q_p \cdot t_f}} \qquad (8.32)$$

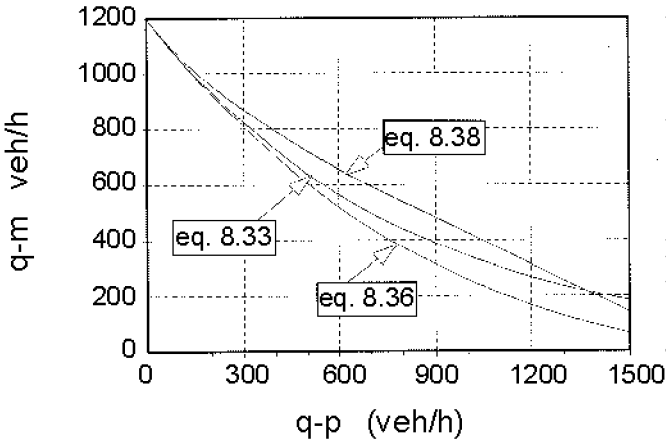
If we combine Equations 8.29 and 8.31 we get Siegloch's (1973) formula,

$$q_m = \frac{1}{t_f} e^{-q_p \cdot t_c} \qquad (8.33)$$

These formulae result in a relation of capacity versus conflicting flow illustrated by the curves shown in Figure 8.8.

The idealized assumptions, mentioned above as (a), (b), (c), however, are not realistic. Therefore, different attempts to drop one or the other assumption have been made. Siegloch (1973) studied different types of gap distributions for the priority stream (cf. Figure 8.9) based on analytical methods. Similar studies have also been performed by Catchpole and Plank (1986) and Troutbeck (1986). Grossmann (1991) investigated these effects by simulations. These studies showed

- If the constant t_c and t_f values are replaced by realistic distributions (cf. Grossmann 1988) we get a decrease in capacity.
- Drivers may be inconsistent; i.e. one driver can have different critical gaps at different times; A driver might reject a gap that he may otherwise find acceptable. This effect results in an increase of capacity.
- If the exponential distribution of major stream gaps is replaced by more realistic headway distributions, we get an increase in capacity of about the same order of magnitude as the effect of using a distribution for t_c and t_f values (Grossmann 1991 and Troutbeck 1986).
- Many unsignalized intersections have complicated driver behavior patterns, and there is often little to be gained from using a distribution for the variables t_c and t_f or complicated headway distributions. Moreover, Grossmann could show by simulation techniques that these effects compensate each other so that the simple capacity equations, 8.32 and 8.33, also give quite realistic results in practice.



Note: Comparison of capacities for different types of headway distributions in the main street traffic flow for t_c = 6 seconds and t_f = 3 seconds. For this example, t_m has been set to 2 seconds.

Figure 8.9
Comparison of Capacities for Different Types of Headway Distributions in the Main Street Traffic Flow.

More general solutions have been obtained by replacing the exponential headway distribution used in assumption (b) with a more realistic one e. g. a dichotomized distribution (cf. Section 8.3.3). This more general equation is:

$$q_m = \frac{\alpha q_p e^{-\lambda(t_c - t_m)}}{1 - e^{-\lambda t_f}} \qquad (8.34)$$

where

$$\lambda = \frac{\alpha q_f}{(1 - t_m q_f)} \qquad (8.35)$$

This equation is illustrated in Figure 8.10. This is also similar to equations reported by Tanner (1967), Gipps (1982), Troutbeck (1986), Cowan (1987), and others. If α is set to 1 and t_m to 0, then Harders' equation is obtained. If α is set to $1 - q_p * t_m$, then this equation reduces to Tanner's (1962) equation:

$$q_m = (1 - q_p \cdot t_m) \cdot \frac{q_p \cdot e^{-q_p \cdot (t_c - t_m)}}{1 - e^{-q_p \cdot t_f}} \qquad (8.36)$$

If the linear relationship for *g(t)* according to Equation 8.37 is used, then the associated capacity equation is

$$q_m = \frac{\alpha \cdot q_p \cdot e^{-\lambda(t_0 - t_m)}}{\lambda \cdot t_f} \qquad (8.37)$$

or

$$q_m = \frac{(1 - q_p t_m) \cdot e^{-\lambda(t_0 - t_m)}}{t_f} \qquad (8.38)$$

This was proposed by Jacobs (1979) .

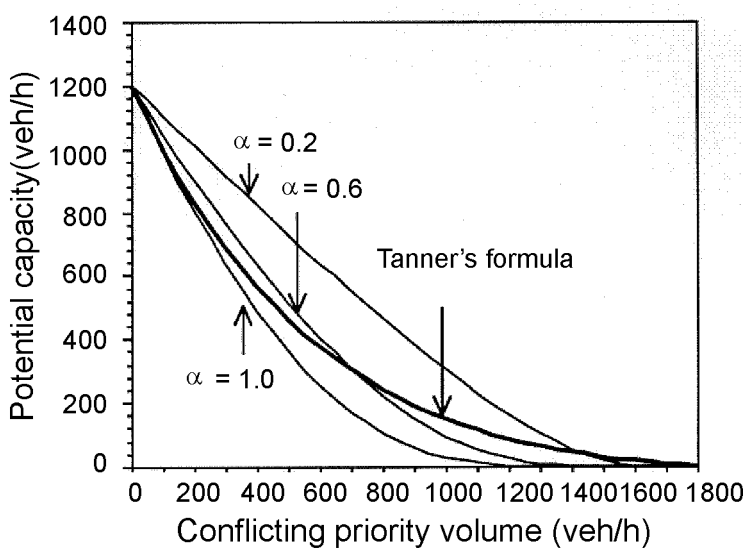


Figure 8.10
The Effect of Changing α in Equation 8.31 and Tanner's Equation 8.36.

Tanner (1962) analyzed the capacity and delay at an intersection where both the major and minor stream vehicles arrived at random; that is, their headways had a negative exponential distribution. He then assumed that the major stream vehicles were restrained such that they passed through the intersection at intervals not less than t_m sec after the preceding major stream vehicle. This allowed vehicles to have a finite length into which other vehicles could not intrude. Tanner did not apply the same constraint to headways in the minor stream. He assumed the same gap acceptability assumptions that are outlined above. Tanner considered the major stream as imposing 'blocks' and 'anti-blocks' on the minor stream. A block contains one or more consecutive gaps less than t_c sec; the block starts at the first vehicle with a gap of more than t_c sec in-front of it and ends t_c sec after the last consecutive gap less than t_c sec. Tanner's equation for the entry capacity is a particular case of a more general equation.

An analytical solution for a realistic replacement of assumptions (a) and (b) within the same set of formulae is given by Plank and Catchpole (1984):

$$q_m = \beta \cdot \frac{q_p \cdot e^{-q_p \cdot t_c}}{1 - e^{-q_p \cdot t_f}} \qquad (8.39)$$

where

$$\beta = 1 + \frac{1}{2} \cdot q_p^2 \left[Var(t_c) + \frac{Var(t_f)}{(e^{q_p \cdot t_f} - 1)} \right] + \eta_c + \eta_f \qquad (8.40)$$

$Var(t_c)$	=	variance of critical gaps,
$Var(t_f)$	=	variance of follow-up-times,
η_c	=	increment, which tends to 0, when $Var(t_c)$ approaches 0, and
η_f	=	increment, which tends to 0, when $Var(t_f)$ approaches 0.

Wegmann (1991) developed a universal capacity formula which could be used for each type of distribution for the critical gap, for the follow-up time and for each type of the major stream headway distribution.

$$q_m = \frac{1 + E\left[z(G - t_c)/\tau\right]}{E(C) \cdot E(1/\tau)} \qquad (8.41)$$

where,

$E(C)$	=	mean length of a "major road cycle" C,
C	=	G + B,
G	=	gap,
B	=	block,
τ	=	probability ($G > t_c$), and
$z(t)$	=	expected number of departures within the time interval of duration t.

Since these types of solutions are complicated many researchers have tried to find realistic capacity estimations by simulation studies. This applies especially for the German method (FGSV 1991) and the Polish method.

8.4.2 Quality of Traffic Operations

In general, the performance of traffic operations at an intersection can be represented by these variables (measures of effectiveness, MOE):

(a) average delay,
(b) average queue lengths,
(c) distribution of delays,
(d) distribution of queue lengths (i.e number of vehicles queuing on the minor road),
(e) number of stopped vehicles and number of accelerations from stop to normal velocity, and
(f) probability of the empty system (p_o).

Distributions can be represented by:

- standard deviations,
- percentiles, and
- the whole distribution.

To evaluate these measures, two tools can be used to solve the problems of gap acceptance:

- queuing theory and
- simulation.

Each of these MOEs are a function of q_p and q_n; the proportion of "free" vehicles and the distribution of platoon size length in both the minor and major streams. Solutions from queuing theory in the first step concentrate on average delays.

A general form of the equation for the average delay per vehicle is

$$D = D_{\min}\left(1+\frac{\gamma+\varepsilon x}{1-x}\right) \qquad (8.42)$$

where γ and ε are constants
x is the degree of saturation = q_n/q_m

and D_{min} has been termed Adams' delay after Adams (1936). Adams' delay is the average delay to minor stream vehicles when minor stream flow is very low. It is also the minimum average delay experienced by minor stream vehicles.

Troutbeck (1990) gives equations for γ, ε and D_{min} based on the formulations by Cowan (1987). If stream 2 vehicles are assumed to arrive at random, then γ is equal to 0. On the other hand, if there is platooning in the minor stream, then γ is greater than 0.

For random stream 2 arrivals, ε is given by

$$\varepsilon = \frac{e^{q_p t_f} - q_p t_f - 1 + q_p(e^{q_p t_f}-1)D_{\min}}{q_p(e^{q_p t_f}-1)D_{\min}} \qquad (8.43)$$

Note that ε is approximately equal to 1.0. D_{min} depends on the platooning characteristics in stream 1. If the platoon size distribution is geometric, then

$$D_{\min} = \frac{e^{\lambda(t_c-t_m)}}{\alpha q_p} - t_c - \frac{1}{\lambda} + \frac{\lambda t_m^2 - 2t_m + 2t_m\alpha}{2(t_m\lambda+\alpha)} \qquad (8.44)$$

(Troutbeck 1986).

Tanner's (1962) model has a different equation for Adams' delay, because the platoon size distribution in stream 1 has a Borel-Tanner distribution. This equation is

$$D_{\min} = \frac{e^{q_p(t_c-t_m)}}{(1-t_m q_p)q_p} - t_c - \frac{1}{q_p} + \frac{q_p t_m^2(2t_m q_p - 1)}{2(1-t_m q_p)^2} \qquad (8.45)$$

Another solution for average delay has been given by Harders (1968). It is not based on a completely sophisticated queuing theory. However, as a first approximation, the following equation for the average delay to non-priority vehicles is quite useful.

$$D = \frac{1-e^{-(q_p t_c + q_n t_f)}}{q_m/3600 - q_n} + t_f \qquad (8.46)$$

with q_m calculated using Equation. 8.34 or similar.

M/G/1 Queuing System - A more sophisticated queuing theory model can be developed by the assumption that the simple two-streams system (Figure 8.7) can be represented by a M/G/1 queue. The service counter is the first queuing position on the minor street. The input into the system is formed by the vehicles approaching from the minor street which are assumed to arrive at random, i.e. exponentially distributed arrival headways (i.e. "M"). The time spent in the first position of the queue is the service time. This service time is controlled by the priority stream, with an unknown service time distribution. The "G" is for a general service time. Finally, the "1" in M/G/1 stands for one service channel, i.e. one lane in the minor street.

For the M/G/1 queuing system, in general, the Pollaczek-Khintchine formula is valid for the average delay of customers in the queue

$$D_q = \frac{xW(1+C_w^2)}{2(1-x)} \qquad (8.47)$$

where

W = average service time. It is the average time a minor street vehicle spends in the first position of the queue near the intersection

C_w = coefficient of variation of service times

$$C_w = \frac{\sqrt{Var(W)}}{W}$$

Var (W) = variance of service times

The total average delay of minor street vehicles is then

$$D = D_q + W.$$

In general, the average service time for a single-channel queuing system is: l/capacity. If we derive capacity from Equations 8.32 and following and if we include the service time *W* in the total delay, we get

$$D = \frac{1}{q_m}\left(1 + \frac{x}{1-x}C\right) \qquad (8.48)$$

where

$$C = \frac{1 + C_w^2}{2}$$

Up to this point, the derivations are of general validity. The real problem now is to evaluate *C*. Only the extremes can be defined which are:

- Regular service: Each vehicle spends the same time in the first position. This gives $Var(W) = 0$, $C_w^2 = 0$, and $C = 0.5$

This is the solution for the M/D/l queue.

- Random service: The times vehicles spend in the first position are exponentially distributed. This gives $Var(W) = E(W)$, $C_w^2 = 1$, and $C = 1.0$

This gives the solution for the M/M/1 queue.

Unfortunately, neither of these simple solutions applies exactly to the unsignalized intersection problem. However, as an approximation, some authors recommend the application of Equation 8.48 with $C = 1$.

Equation 8.42 can be further transformed to

$$D = D_{\min}\,(1+\gamma)\left(1 + \frac{\gamma+\varepsilon}{1+\gamma} * \frac{x}{1-x}\right) \qquad (8.49)$$

where ϵ and γ are documented in Troutbeck (1990).

This is similar to the Pollaczek-Khintchine formula (Equation 8.48). The randomness constant *C* is given by $(\gamma+\epsilon)/(1+\gamma)$ and the term $1/D_{min}*(1+\gamma)$ can be considered to be an equivalent 'capacity' or 'service rate.' Both terms are a function of the critical gap parameters t_c and t_f and the headway distributions. However, *C*, γ, and ϵ values are not available for all conditions.

For the M/G/1 system as a general property, the probability p_o of the empty queue is given by

$$p_o = 1 - x \qquad (8.50)$$

This formula is of sufficient reality for practical use at unsignalized intersections.

M/G2/1 queuing system **-** Different authors found that the service time distribution in the queuing system is better described by two types of service times, each of which has a specific distribution:

W_1 = service time for vehicles entering the empty system, i.e no vehicle is queuing on the vehicle's arrival

W_2 = service time for vehicles joining the queue when other vehicles are already queuing.

Again, in both cases, the service time is the time the vehicle spends waiting in the first position near the stop line. The first ideas for this solution have been introduced by Kremser (1962; 1964) and in a comparable way by Tanner (1962), as well as by Yeo and Weesakul (1964).

The average time which a customer spends in the queue of such a system is given by Yeo's (1962) formula:

$$D_q = \frac{q_n}{2} * \left(\frac{E(W_1^2) - E(W_2^2)}{v} + \frac{E(W_2^2)}{y}\right) \qquad (8.51)$$

where,

D_q = average delay of vehicles in the queue at higher positions than the first,
$E(W_1)$ = expectation of W_1,
$E(W_1^2)$ = expectation of $(W_1 * W_1)$
$E(W_2)$ = expectation of W_2,
$E(W_2^2)$ = expectation of $(W_2 * W_2)$,
v = $y + z$,
y = $1 - q_n E(W_2)$, and
z = $q_n E(W_1)$.

The probability p_o of the empty queue is

$$p_o = y/v \qquad (8.52)$$

The application of this formula shows that the differences against Equation 8.50 are quite small (< 0.03). Refer to Figure 8.11.

If we also include the service time (= time of minor street vehicles spent in the first position) in the total delay, we get

$$D = \frac{E(W_1^2)}{v} + \frac{q_n}{2} * \left(\frac{y * E(W_1^2) + z * E(W_2^2)}{v * y} \right) \qquad (8.53)$$

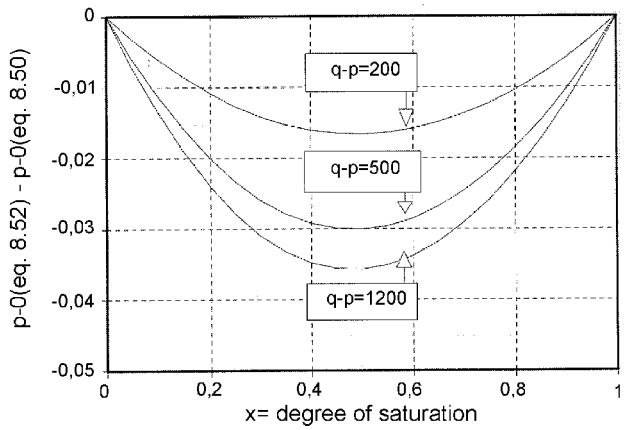
(Brilon 1988):Formulae for the expectations of W_1 and W_2 respectively have been developed by Kremser (1962):

$$E(W_1) = \frac{1}{q_p}(e^{q_p t} \qquad (8.54)$$

$$E(W_2) = \frac{e^{q_p t_c}}{q_p}(1 - e^{-q_p t_f})$$

$$E(W_1^2) = \frac{2}{q_p}(e^{q_p t_c} - 1 - q_p t_c)(\frac{e^{q_p t_c}}{q_p} + t_f - t_c) + t_f^2 - t_c^2$$

$$E(W_2^2) = \frac{2 \cdot e^{q_p t_c}}{q_p^2}(e^{q_p t_c} - q_p t_c)(1 - e^{-q_p t_f}) - q_p t_f \cdot e^{-}$$



q-p = priority flow (veh/h)

Figure 8.11
Probability of an Empty Queue: Comparison of Equations 8.50 and 8.52.

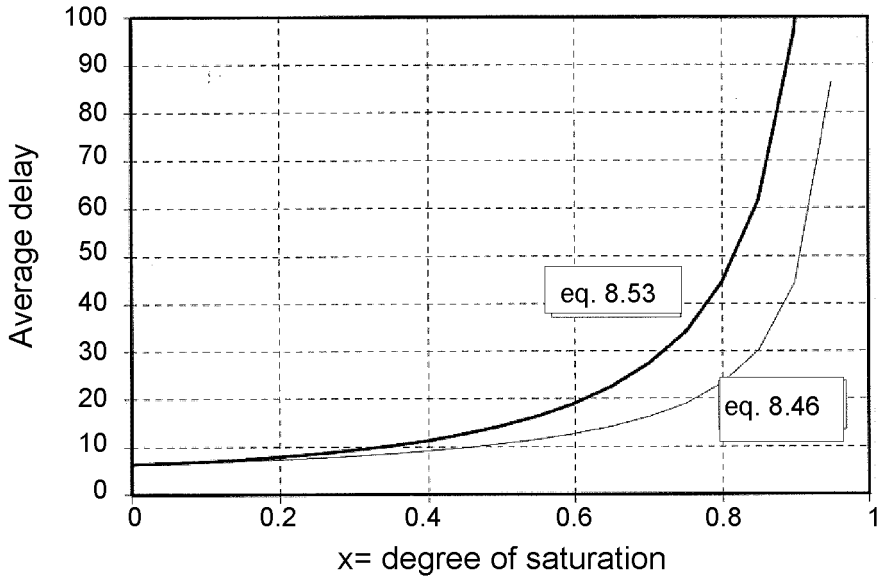
Kremser (1964), however, showed that the validity of these equations is restricted to the special case of $t_c = t_f$, which is rather unrealistic for two-way-stop-control unsignalized intersections. Daganzo (1977) gave an improved solution for $E(W_2)$ and $E(W_2^2)$ which again was extended by Poeschl (1983). These new formulae were able to overcome Kremer's (1964) restrictions. It can, however be shown that Kremer's first approach (Equation 8.56) also gives quite reliable approximate results for t_c and t_f values which apply to realistic unsignalized intersections. The following comments can also be made about the newer equations.

- The formulae are so complicated that they are far from being suitable for practice. The only imaginable application is the use in computer programs.

- Moreover, these formulae are only valid under assumptions (a), (b), and (c) in Section 8.4.1 of the paper. That means that for practical purposes, the equations can only be regarded as approximations and only apply for undersaturated conditions and steady state conditions.

Figure 8.12 gives a graphical comparison for some of the delay formulae mentioned.

Differences in the platoon size distribution affects the average delay per vehicle as shown in Figure 8.13. Here, the critical gap was 4 seconds, the follow-up time was 2 seconds, and the priority stream flow was 1000 veh/h. To emphasize the point, the average delay for a displaced exponential priority stream is 4120 seconds, when the minor stream flow was 400 veh/h. This is much greater than the values for the Tanner and exponential headway examples which were around 11.5 seconds for the same major stream flow. The average delay is also dependent on the average platoon size as shown in Figure 8.14. The differences in delays are dramatically different when the platoon size is changed.



Note: For this example; q_p = 600 veh/h, t_c = 6 sec , and t_f = 3 sec.

Figure 8.12
Comparison of Some Delay Formulae.

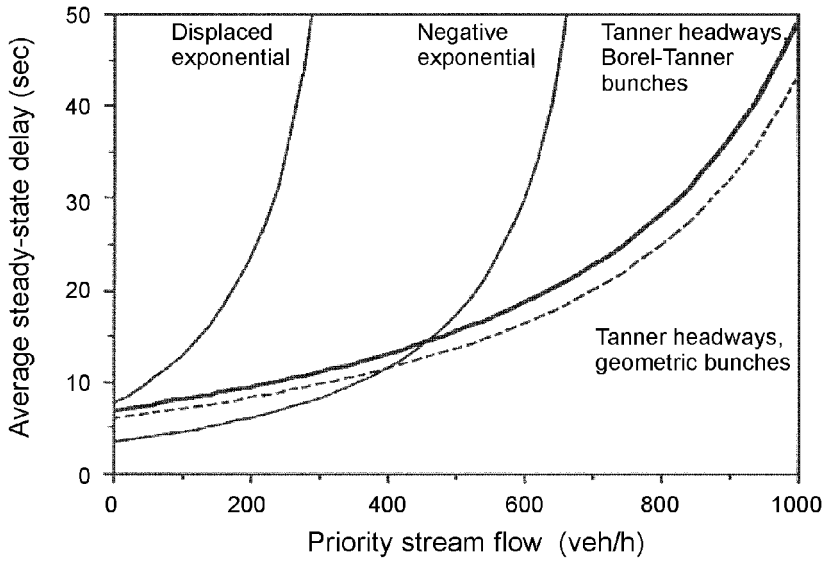


Figure 8.13
Average Steady State Delay per Vehicle
Calculated Using Different Headway Distributions.

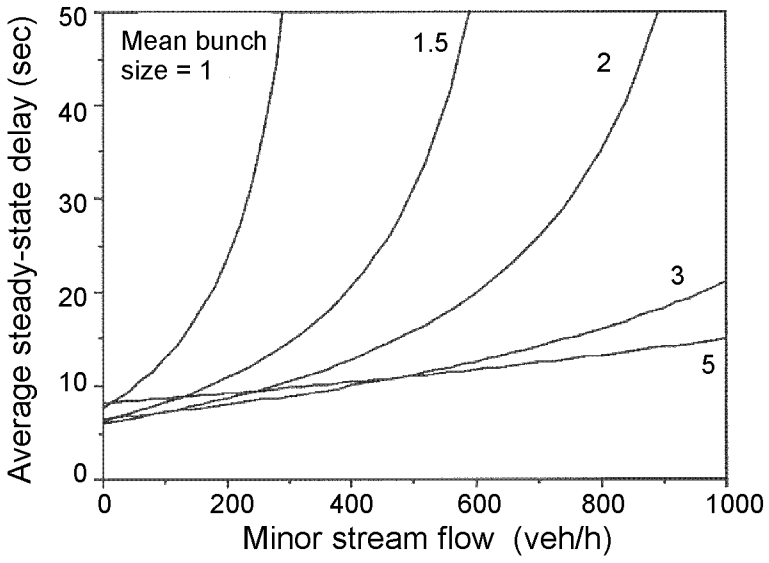


Figure 8.14
Average Steady State Delay per Vehicle by Geometric
Platoon Size Distribution and Different Mean Platoon Sizes.

8.4.3 Queue Length

In each of these queuing theory approaches, the average queue length (L) can be calculated by Little's rule (Little 1961):

$$L = q_n D \tag{8.55}$$

Given that the proportion of time that a queue exists is equal to the degree of saturation, the average queue length when there is a queue is:

$$L_q = q_n D/x = q_m D \tag{8.56}$$

The distribution of queue length then is often assumed to be geometric.

However, a more reliable derivation of the queue length distribution was given by Heidemann (1991). The following version contains a correction of the printing mistakes in the original paper (there: Equations 8.30 and 8.31).

$$p(0) = h_1 \cdot h_3 \cdot (q_p + q_n) \tag{8.57}$$

$$p(1) = p(0) \cdot h_3 \cdot q_n \left[e^{q_n t_f} - (t_c - t_f) \cdot h_2 \right] - q_n \cdot h_1 \cdot h_3$$

$$p(n) = p(n-1) \cdot h_3 \cdot q_n \left[e^{q_n t_f} - (t_c - t_f) \cdot h_2 \right] - h_3 \cdot \sum_{m=0}^{n-2} p(m) \cdot \left[h_2 \frac{(t_c - t_f \cdot q_n)^{n-m}}{(n-m)!} + \frac{(-q_n t_f)^{n-m} \cdot e^{q_n t_f}}{t_f \cdot (n-m-1)!} \right.$$

$p(n)$ = probability that n vehicles are queuing on the minor street

$$h_1 = e^{-q_p t_c} + (e^{-q_p t_f} - 1) \frac{q_n}{q_p}$$

$$h_2 = q_p e^{-q_p t_c - q_n (t_c - t_f)}$$

$$\frac{1}{h_3} = h_2 \cdot q_n \cdot e^{-q_p t_f}$$

These expressions are based on assumptions (a), (b), and (c) in Section 8.4.1. This solution is too complicated for practical use. Moreover, specific percentiles of the queue length is the desired output rather than probabilities. This however, can not be calculated from these equations directly. Therefore, Wu (1994) developed another set of formulae which approximate the above mentioned exact equations very closely:

$$p(0) = 1 - x^{a}$$
$$p(n) = p(0) \cdot x^{a(b(n-1)+1)} \tag{8.58}$$

$p(n)$ = probability that n vehicles are queuing on the minor street

where,

$$x = \frac{q_n}{q_m}$$

x = degree of saturation (q_m according to Equation 8.33).

$$a = \frac{1}{1 + 0.45 \cdot \frac{t_c - t_f}{t_f} \cdot q_p}$$

$$b = \frac{1.51}{1 + 0.68 \cdot \frac{t_c}{t_f} \cdot q_p}$$

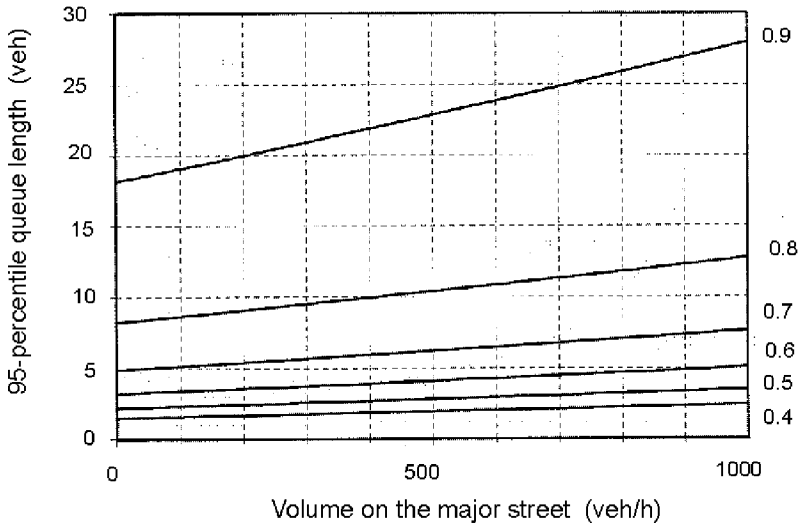
For the rather realistic approximation $t_c \approx 2\, t_f$, we get :

$$a = \frac{1}{1 + 0.45 \cdot q_p} \qquad b = \frac{1.51}{1 + 1.36 \cdot q_p}$$

From Equation 8.58 we get the cumulative distribution function

$$F(n) = p(L \le n) = 1 - x^{a(b \cdot n + 1)} \tag{8.59}$$

For a given percentile, S, (e.g. $S = F(n) = 0.95$) this equation can be solved for n to calculate the queue length which is only exceeded during (1-S)*100 percent of the time (Figure 8.15). For practical purposes, queue length can be calculated with sufficient precision using the approximation of the M/M/1 queuing system and, hence, Wu's equation. The 95-percentile-queue length based on Equation 8.59 is given in Figure 8.15.



The parameter of the curves (indicated on the right side) is the degree of saturation (x).

Figure 8.15
95-Percentile Queue Length Based on Equation 8.59 (Wu 1994).

8.4.4 Stop Rate

The proportion of drivers that are stopped at an unsignalized intersection with two streams was established by Troutbeck (1993). The minor stream vehicles were assumed to arrive at random whereas the major stream headways were assumed to have a Cowan (1975) M3 distribution. Changes of speed are assumed to be instantaneous and the predicted number of stopped vehicles will include those drivers who could have adjusted their speed and avoided stopping for very short periods.

The proportion stopped, $P(x,0)$, is dependent upon the degree of saturation, x, the headways between the bunched major stream vehicles, t_m, the critical gap, t_c . and the major stream flow, q_p . The appropriate equation is:

$$P(x,0) = 1-(1-x)(1-t_m q_p)e^{-\lambda(t_c-t_m)} \qquad (8.60)$$

where λ is given by $\alpha q_p/(1-t_m q_p)$. The proportion of drivers stopped for more than a short period of t, where t is less than the follow-up time t_f , increases from some minimum value, $P(0,t)$, to 1 as the degree of saturation increases from 0 to 1.

The proportion of drivers stopped for more than a short period t, $P(x,t)$, is given by the empirical equation:

$$P(x,t) = P(0,t)+A\{1-P(0,t)\}x+(1-A)\{1-P(0,t)\}x^2 +(1-A)(1-B)(1-x)x \qquad (8.61)$$

where

$$B = 1-(1-\frac{t}{t_f})(1-t_m q_p)e^{-\lambda(t_a-t_m)}$$

$$A = 1-a_0 e^{-\lambda(t_a-t_m)}$$

and

$$P(0,t) = P(0,0)-q_p t\alpha e^{-\lambda(t_a-t_m)} \qquad (8.62)$$

or

$$P(0,t) = 1 - (1 - t_m q_p + q_p t\alpha) e^{-\lambda(t_a - t_m)}$$

If the major stream is random then a_0 is equal to 1.25 and for bunched major stream traffic, it is 1.15. The vehicles that are stopped for a short period may be able to adjust their speed and these vehicles have been considered to have a "partial stop." Troutbeck (1993) also developed estimates of the number of times vehicles need to accelerate and move up within the queue.

8.4.5 Time Dependent Solution

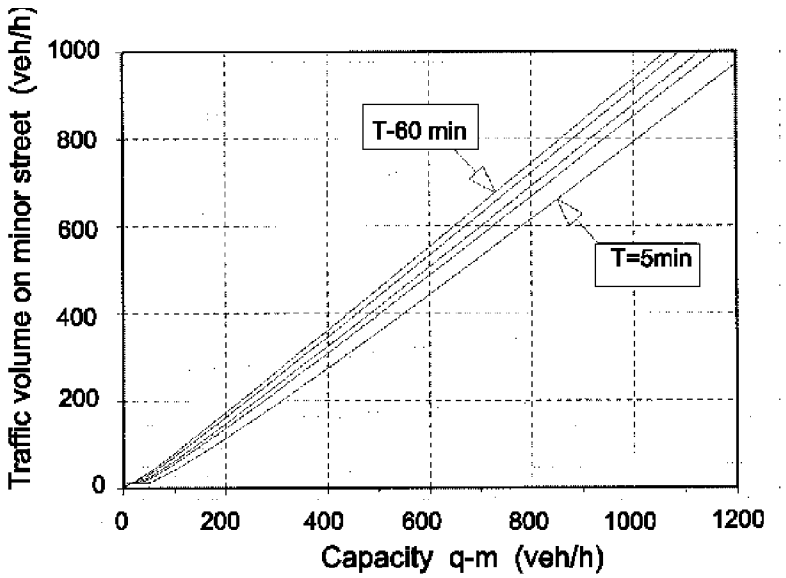
Each of the solutions given by the conventional queuing theory above is a steady state solution. These are the solutions that can be expected for non-time-dependent traffic volumes after an infinitely long time, and they are only applicable when the degree of saturation x is less than 1. In practical terms, this means, the results of steady state queuing theory are only useful approximations if T is considerably greater than the expression on the right side of the following equation.

$$T > \frac{1}{\left(\sqrt{q_m} - \sqrt{q_n}\right)^2} \qquad (8.63)$$

with T = time of observation over which the average delay should be estimated in seconds,

after Morse (1962).

This inequality can only be applied if q_m and q_n are nearly constant during time interval T. The threshold given by Equation 8.63 is illustrated by Figure 8.16. The curves are given for time intervals T of 5, 10, 15, 30, and 60 minutes. Steady state conditions can be assumed if q_n is below the curve for the corresponding T-value. If this condition (Equation 8.63) is not fulfilled, time-dependent solutions should be used. Mathematical solutions for the time dependent problem have been developed by Newell (1982) and now need to be made



Note: The curves are given for time intervals T of 5, 10, 15, 30, and 60 minutes. Steady state conditions can be assumed if q_n is below the curve for the corresponding T-value.

Figure 8.16
Approximate Threshold of the Length of Time Intervals For the Distinction Between Steady-State Conditions and Time Dependent Situations.

more accessible to practicing engineers. There is, however, a heuristic approximate solution for the case of the peak hour effect given by Kimber and Hollis (1979) which are based on the ideas of Whiting, who never published his work.

During the peak period itself, traffic volumes are greater than those before and after that period. They may even exceed capacity. For this situation, the average delay during the peak period can be estimated as:

$$
\begin{aligned}
D &= D_1 + E + \frac{1}{q_m} \\
D_1 &= \frac{1}{2}\left(\sqrt{F^2+G} - F\right) \\
F &= \frac{1}{q_{mo}-q_{no}}\left[\frac{T}{2}(q_m-q_n)y + C(y+\frac{h}{q_m})\right] + E \\
G &= \frac{2Ty}{q_{mo}-q_{no}}\left[C\frac{q_n}{q_m} - (q_m-q_n)E\right] \\
E &= \frac{Cq_{no}}{q_{mo}(q_{mo}-q_{no})} \\
h &= q_m - q_{mo} + q_{no} \\
y &= 1 - \frac{h}{q_n}
\end{aligned}
\qquad (8.64)
$$

q_m = capacity of the intersection entry during the peak period of duration T,
q_{mo} = capacity of the intersection entry before and after the peak period,
q_n = minor street volume during the peak period of duration T, and
q_{no} = minor street volume before and after the peak period

(each of these terms in veh/sec; delay in sec).

C is again similar to the factor C mentioned for the M/G/1 system, where

C = 1 for unsignalized intersections and
C = 0.5 for signalized intersections (Kimber and Hollis 1979).

This delay formula has proven to be quite useful to estimate delays and it has a quite reliable background particularly for temporarily oversaturated conditions.

A simpler equation can be obtained by using the same co-ordinate transfer method. This is a more approximate method. The steady state solution is fine for sites with a low degree of saturation and the deterministic solution is satisfactory for sites with a very high degree of saturation say, greater than three or four. The co-ordinate transfer method is one technique to provide estimates between these two extremes. the reader should also refer to Section 9.4.

The steady state solution for the average delay to the entering vehicle is given by Equation 8.42. The deterministic equation for delay, D_d, on the other hand is

$$
D_d = D_{\min} + \frac{2L_0 + (x_d - 1)q_m T}{2q_m} \qquad x > 1 \qquad (8.65)
$$

and $D_d = 0$ otherwise,

where L_0 is the initial queue,
T is time the system is operating in seconds, and
q_m is the entry capacity.

These equations are diagrammatically illustrated in Figure 8.17. For a given average delay the co-ordinate transformation method gives a new degree of saturation, x_t, which is related to the steady state degree of saturation, x_s, and the deterministic degree of saturation, x_d, such that

$$
x_d - x_t = 1 - x_s = a \qquad (8.66)
$$

Rearranging Equations 8.42 and 8.65 gives two equations for x_s and x_d as a function of the delays D_d and D_s. These two equations are:

$$
x_s = \frac{D_s - D_{\min} - \gamma D_{\min}}{D_s - D_{\min} + \varepsilon D_{\min}} \qquad (8.67)
$$

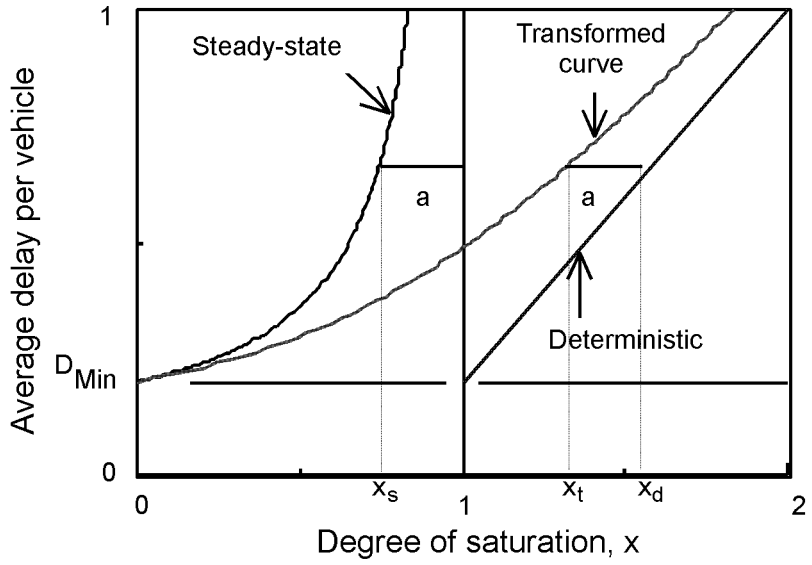


Figure 8.17
The Co-ordinate Transform Technique.

and

$$x_d = \frac{2(D_d - D_{\min}) - 2L_0/q_m}{T} + 1 \qquad (8.68)$$

Using Equation 8.66, x_t is given by:

$$x_t = \frac{2(D_d - D_{\min}) - 2L_0/q_m}{T} - \frac{D_s - D_{\min} - \gamma D_{\min}}{D_s - D_{\min} + \varepsilon D_{\min}} \qquad (8.69)$$

Rearranging Equation 8.69 and setting $D = D_s = D_d$, $x = x_e$ gives:

$$D_t = \frac{1}{2}\left\{\sqrt{A^2 + B} - A\right\} \qquad (8.70)$$

where

$$A = \frac{T(1-x)}{2} - \frac{L_0}{q_m} - D_{\min}(2 - \varepsilon) \qquad (8.71)$$

and

$$B = 4D_{\min}\left\{\frac{T(1-x)(1+\gamma)}{2} + \frac{Tx(\varepsilon+\gamma)}{2} - (1-\varepsilon)\left[\frac{L_0}{q_m} + D_{\min}\right]\right\} \qquad (8.72)$$

Equation 8.66 ensures that the transformed equation will asymptote to the deterministic equation and gives a family of relationships for different degrees of saturation and period of operation from this technique (Figure 8.18).

A simpler equation was developed by Akçelik in Akcelik and Troutbeck (1991). The approach here is to rearrange Equation 8.42 to give:

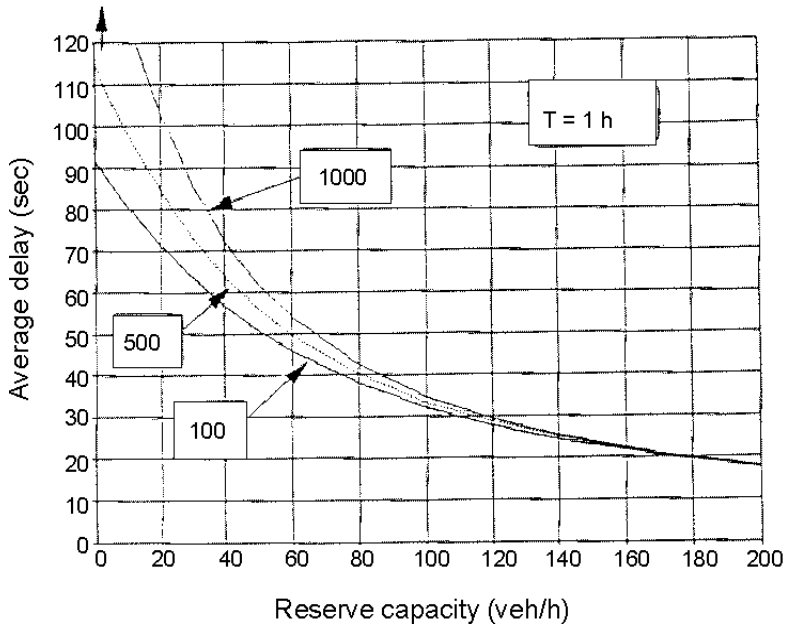


Figure 8.18
A Family of Curves Produced from the Co-Ordinate Transform Technique.

$$a = 1 - x_s = \frac{D_{\min}(\gamma + \varepsilon x_s)}{D_s - D_{\min}}$$

and this is approximately equal to:

$$a \approx \frac{D_{\min}(\gamma + \varepsilon x_t)}{D_s - D_{\min}} \qquad (8.73)$$

If this is used in Equation 8.66 and then rearranged then the resulting equation of the non-steady state delay is:

$$D - D_{\min} = \frac{1}{2}\left[\frac{L_0}{q_m} + \frac{(x-1)T}{4} + \sqrt{\left[\frac{L_0}{2q_m} + \frac{(x-1)T}{4}\right]^2 + \frac{TD_{\min}(\varepsilon x + \gamma)}{2}}\right] \qquad (8.74)$$

A similar equation for M/M/1 queuing system can be obtained if ε is set to 1, γ is set to zero, and D_{min} is set to $1/q_m$; the result is:

$$D = \frac{1}{q_m} + \frac{T}{4}\left\{(x-1) + \sqrt{(x-1)^2 + \frac{8x}{q_m T}}\right\} \qquad (8.75)$$

The average delay predicted by Equation 8.74 is dependent on the initial queue length, the time of operation, the degree of saturation, and the steady state equation coefficients. This equation can then be used to estimate the average delay under oversaturated conditions and for different initial queues. The use of these and other equations are discussed below.

8.4.6 Reserve Capacity

Independent of the model used to estimate average delays, the reserve capacity (*R*) plays an important role

$$R = q_{emax} - q_n \qquad (8.76)$$

In the 1985 edition of the HCM but not the 1994 HCM, it is used as the measure of effectiveness. This is based on the fact that average delays are closely linked to reserve capacity. This close relationship is shown in Figure 8.19. In Figure 8.19, the average delay, *D*, is shown in relation to reserve capacity, *R*. The delay calculations are based on Equation 8.64 with a peak hour interval of duration *T*= 1 hour. The parameters (100, 500, and 1000 veh/hour) indicate the traffic volume, q_p, on the major street. Based on this relationship, a good approximation of the average delay can also be expressed by reserve capacities. What we also see is that - as a practical guide - a reserve capacity

$R > 100$ pcu/h generally ensures an average delay below 35 seconds.

Brilon (1995) has used a coordinate transform technique for the "Reserve Capacity" formulation for average delay with oversaturated conditions. His set of equations can be given by

$$D = -B + \sqrt{B^2 + b} \qquad (8.77)$$

where

$$B = \frac{1}{2}\left(bR - \frac{L_0}{q_m} \right) \qquad (8.78)$$

$$b = \left\{ \frac{1}{q_m - R_f} \left[\frac{L_0 - R_f T}{2} \left(1 - \frac{R_f}{R_0} \right) - \frac{L_0}{q_m} \right] \right\} \frac{1}{|R_f|} \qquad (8.79)$$

$$R_f = \frac{100 \cdot 3600}{T} \qquad (8.80)$$

$$L_0 = \frac{q_{n0}}{R_0} = \frac{q_{m0} - R_0}{R_0} \qquad (8.81)$$

T	=	duration of the peak period
q_m	=	capacity during the peak period
q_n	=	minor street flow during the peak period
R	=	reserve capacity during the peak period $= q_{emax} - q_n$
L_0	=	average queue length in the period before and after the peak period
q_{n0}	=	minor street flow in the period before and after the peak period
q_{m0}	=	capacity in the period before and after the peak period
R_0	=	reserve capacity in the period before and after the peak period

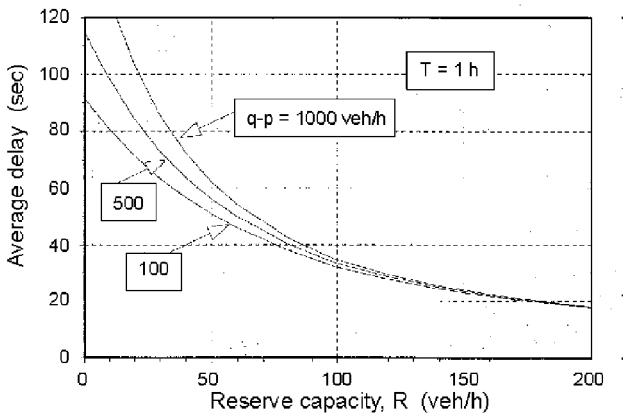


Figure 8.19
Average Delay, D, in Relation to Reserve Capacity R.

All variables in these equations should be used in units of seconds (sec), number of vehicles(veh), and veh/sec. Any capacity formula to estimate q_m and q_{m0} from Section 8.4.1 can be used.

The numerical results of these equations as well as their degree of sophistication are comparable with those of Equation 8.75.

8.4.7 Stochastic Simulation

As mentioned in the previous chapters, analytical methods are not capable of providing a practical solution, given the complexity and the assumptions required to be made to analyze unsignalized intersections in a completely realistic manner. The modern tool of stochastic simulation, however, is able to overcome all the problems very easily. The degree of reality of the model can be increased to any desired level. It is only restricted by the efforts one is willing to undertake and by the available (and tolerable) computer time. Therefore, stochastic simulation models for unsignalized intersections were developed very early (Steierwald 1961a and b; Boehm, 1968). More recent solutions were developed in the U. K. (Salter 1982), Germany (Zhang 1988; Grossmann 1988; Grossmann 1991), Canada (Chan and Teply 1991) and Poland (Tracz 1991).

Speaking about stochastic simulation, we have to distinguish two levels of complexity:

1) ***Point Process Models*** - Here cars are treated like points, i.e. the length is neglected. As well, there is only limited use of deceleration and acceleration. Cars are regarded as if they were "stored" at the stop line. From here they depart according to the gap acceptance mechanism. The effect of limited acceleration and deceleration can, of course, be taken into account using average vehicle performance values (Grossmann 1988). The advantage of this type of simulation model is the rather shorter computer time needed to run the model for realistic applications. One such model is KNOSIMO (Grossmann 1988, 1991). It is capable of being operated by the traffic engineer on his personal computer during the process of intersection design. A recent study (Kyte et al , 1996) pointed out that KNOSIMO provided the most realistic representation of traffic flow at unsignalized intersections among a group of other models.

   KNOSIMO in its present concept is much related to German conditions. One of the specialities is the restriction to single-lane traffic flow for each direction of the main street. Chan and Teply (1991) found some easy modifications to adjust the model to Canadian conditions as well. Moreover, the source code of the model could easily be adjusted to traffic conditions and driver behavior in other countries.

2) ***Car Tracing Models*** - These models give a detailed account of the space which cars occupy on a road together with the car-following process but are time consuming to run. An example of this type of model is described by Zhang (1988).

Both types of models are useful for research purposes. The models can be used to develop relationships which can then be represented by regression lines or other empirical evaluation techniques.

8.5 Interaction of Two or More Streams in the Priority Road

The models discussed above have involved only two streams; one being the priority stream and the second being a minor stream. The minor stream is at a lower rank than the priority stream. In some cases there may be a number of lanes that must be given way to by a minor stream driver. The capacity and the delay incurred at these intersections have been looked at by a number of researchers. A brief summary is given here.

If the headways in the major streams have a negative exponential distribution then the capacity is calculated from the equation for a single lane with the opposing flow being equal to the sum of the lane flows. This results in the following equation for capacity in veh/h:

$$q_{emax} = \frac{3600qe^{-qt_a}}{1-e^{-qt_f}} \qquad (8.82)$$

where q is the total opposing flow.

Tanner (1967) developed an equation for the capacity of an intersection where there were *n* major streams. The traffic in each lane has a dichotomized headway distribution in which there is a proportion of vehicles in bunches and the remaining vehicles free of interaction. All bunched vehicles are assumed to have a headway of t_m and the free vehicles have a headway equal to the t_m plus a negative exponentially distributed (or random) time. This is the same as Cowan's (1975) M3 model. Using the assumption that headways in each lane are independent, Tanner reviewed the distribution of the random time periods and estimated the entry capacity in veh/h as:

$$q_{emax} = \frac{3600\left[\lambda(1-t_{m1}q_1)(1-t_{m2}q_2)\cdots(1-t_{m1}q_1)e^{-\lambda(t_a-t_m)}\right]}{1-e^{-\lambda t_f}} \qquad (8.83)$$

where $\lambda = \lambda_1 + \lambda_2 + \ldots\ldots + \lambda_n$ (8.84)

$$\lambda_i = \alpha_i q_i / (1-t_m q_i) \qquad (8.85)$$

q_i is the flow in the major stream *i* in veh/sec.
α_i is the proportion of free vehicles in the major stream *i*.

This equation by Tanner is more complicated than an earlier equation (Tanner 1962) based on an implied assumption that the proportion of free vehicles, α_i, is a function of the lane flow. That is

$$\alpha_i = (1-t_m q_i)$$

and then λ_i reduces to q_i. Fisk (1989) extended this earlier work of Tanner (1962) by assuming that drivers had a different critical gap when crossing different streams. While this would seem to be an added complication it could be necessary if drivers are crossing some major streams from the left before merging with another stream from the right when making a left turn.

Her equation for capacity is:

$$q_{emax} = \frac{3600q\left[\prod_{i=1}^{n}(1-t_{mi}q_i)e^{-q_i t_{ai}}\right]e^{qt_m}}{1-e^{-qt_f}} \qquad (8.86)$$

where $q = q_1 + q_2 + \ldots\ldots + q_n$

8.5.1 The Benefit of Using a Multi-Lane Stream Model

Troutbeck (1986) calculated the capacity of a minor stream to cross two major streams which both have a Cowan (1975) dichotomized headway distribution. The distribution of opposing headways is:

$$F(t) = \frac{2q_1q_2t}{(q_1+q_2)} \qquad for \quad t < t_m \qquad (8.87)$$

and

$$F(t) = 1-\alpha' e^{-\lambda'(t-t_m)} \qquad for \quad t > t_m \qquad (8.88)$$

where

$$\alpha' = \frac{\alpha_1 q_1(1-q_2t_m)+\alpha_2 q_2(1-q_1t_m)}{(q_1+q_2)} \qquad (8.88a)$$

or after a little algebra,

$$\alpha' q = \lambda' \prod_{i=1}^{n}(1-q_i t_m) \qquad (8.88b)$$

and

$$\lambda' = \lambda_1 + \lambda_2 \qquad (8.89)$$

As an example, if there were two identical streams then the distribution of headways between vehicles in the two streams is given by Equations 8.87 and 8.88. This is also shown in figures from Troutbeck (1991) and reported here as Figure 8.20.

Gap acceptance procedures only require that the longer headways or gaps be accurately represented. The shorter gaps need only be noted.

Consequently the headway distribution from two lanes can be represented by a single Cowan M3 model with the following properties:

$$F(t) = 1-\alpha * e^{-\lambda'(t-t_m^*)} \qquad t > t_m^* \qquad (8.90)$$

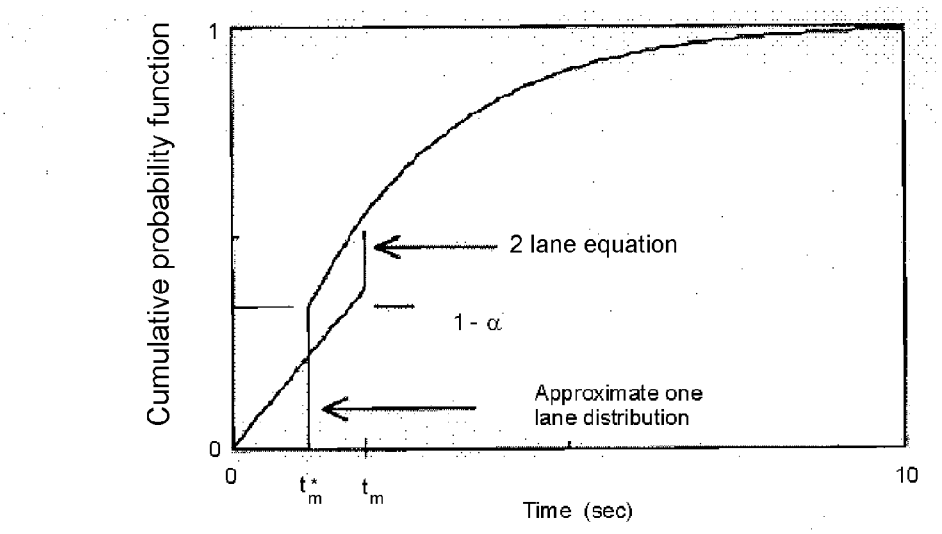


Figure 8.20
Modified 'Single Lane' Distribution of Headways (Troutbeck 1991).

and otherwise *F(t)* is zero. This modified distribution is also illustrated in Figure 8.20. Values of α* and t_m^* must be chosen to ensure the correct proportions and the correct mean headway are obtained. This will ensure that the number of headways greater than *t*, 1–*F(t)*, is identical from either the one lane or the two lane equations when *t* is greater than t_m^*.

Troutbeck (1991) gives the following equations for calculating α* and t_m^* which will allow the capacity to be calculated using a modified single lane model which are identical to the estimate from a multi-lane model.

The equations

$$(1-t_m^* q_1 - t_m^* q_2)e^{\lambda' t_m^*} = (1-t_m q_1)(1-t_m^* q_2)e^{\lambda' t_m} \quad (8.91)$$

and

$$\alpha * e^{\lambda' t_m^*} = \alpha' e^{\lambda' t_m} \quad (8.92)$$

are best solved iteratively for t_m with $t_{m,i}$ being the *i*th estimate. The appropriate equation is

$$t_m^*, i+1 = \frac{1-(1-t_m q_1)(1-t_m q_2)e^{\lambda'(t_m - t_{m,i}^*)}}{q_1 + q_2} \quad (8.93)$$

α* is then found from Equation 8.93.

Troutbeck (1991) also indicates that the error in calculating Adams' delay when using the modified single lane model instead of the two lane model is small. Adams' delay is the delay to the minor stream vehicles when the minor stream flow is close to zero. This is shown in Figure 8.21. Since the modified distribution gives satisfactory estimates of Adams' delay, it will also give satisfactory estimates of delay.

In summary, there is no practical reason to increase the complexity of the calculations by using multi-lane models and a single lane dichotomized headway model can be used to represent the distribution of headways in either one or two lanes.

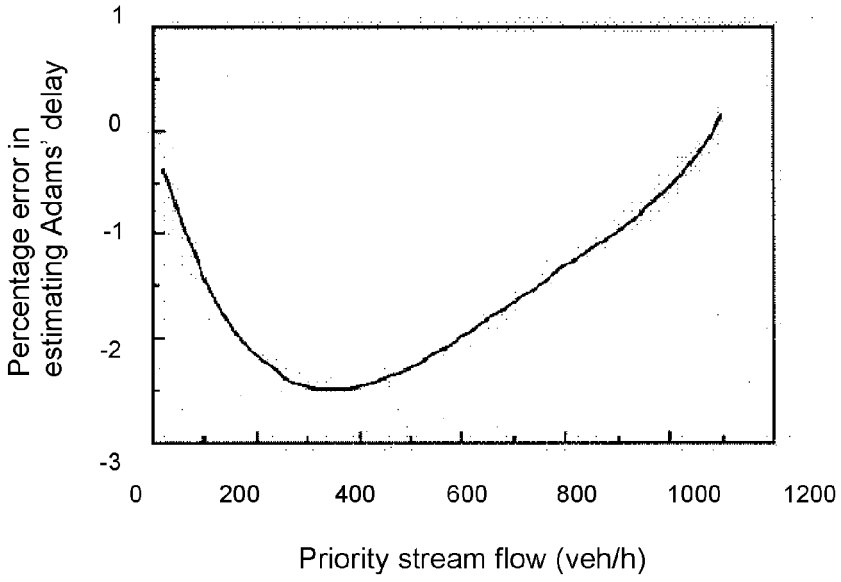


Figure 8.21
Percentage Error in Estimating Adams' Delay Against the Major Stream Flow for a Modified Single Lane Model (Troutbeck 1991).

8.6 Interaction of More than Two Streams of Different Ranking

8.6.1 Hierarchy of Traffic Streams at a Two Way Stop Controlled Intersection

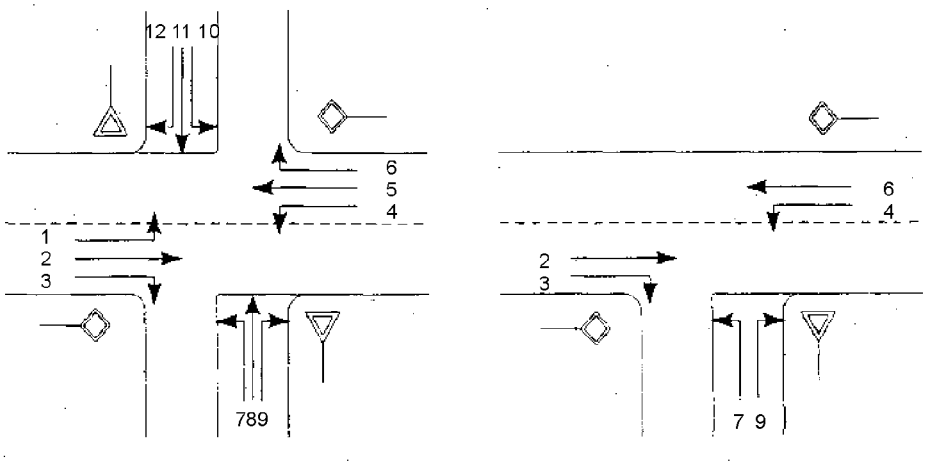
At all unsignalized intersections except roundabouts, there is a hierarchy of streams. Some streams have absolute priority (Rank 1), while others have to yield to higher order streams. In some cases, streams have to yield to some streams which in turn have to yield to others. It is useful to consider the streams as having different levels of priority or ranking. These different levels of priority are established by traffic rules. For instance,

Rank 1 stream	has absolute priority and does not need to yield right of way to another stream,
Rank 2 stream	has to yield to a Rank 1 stream,
Rank 3 stream	has to yield to a Rank 2 stream and in turn to a Rank 1 stream, and
Rank 4 stream	has to yield to a Rank 3 stream and in turn to Rank 2 and Rank 1 streams (left turners from the minor street at a cross-intersection).

This is illustrated in Figure 8.22 produced for traffic on the right side. The figure illustrates that the left turners on the major road have to yield to the through traffic on the major road. The left turning traffic from the minor road has to yield to all other streams but is also affected by the queuing traffic in the Rank 2 stream.

8.6.2 Capacity for Streams of Rank 3 and Rank 4

No rigorous analytical solution is known for the derivation of the capacity of Rank-3-movements like the left-turner from the minor street at a T-junction (movement 7 in Figure 8.22, right side). Here, the gap acceptance theory uses the impedance factors p_0 as an approximation. p_0 for each movement is the probability that no vehicle is queuing at the entry. This is given with sufficient accuracy by Equation 8.50 or better with the two service time Equation 8.52. Only during the part $p_{0,rank\text{-}2}$ of the total time, vehicles of Rank 3 can enter the intersection due to highway code regulations.



Note: The numbers beside the arrows indicate the enumeration of streams given by the Highway Capacity Manual (1994, Chapter 10).

Figure 8.22
Traffic Streams And Their Level Of Ranking.

Therefore, for Rank-3-movements, the basic value q_m for the potential capacity must be reduced to $p_0 \cdot q_m$ to get the real potential capacity q_e:

$$q_{e,rank\text{-}3} = p_{0,rank\text{-}2} \cdot q_{m,rank\text{-}3} \tag{8.94}$$

For a T-junction, this means

$$q_{e,7} = p_{0,4} \cdot q_{m,7}$$

For a cross-junction, this means

$$q_{e,8} = p_x \cdot q_{m,8} \tag{8.95}$$

$$q_{e,11} = p_x \cdot q_{m,11} \tag{8.96}$$

with

$$p_x = p_{0,1} \cdot p_{0,4}$$

Here the index numbers refer to the index of the movements according to Figure 8.22. Now the values of $p_{0,8}$ and $p_{0,11}$ can be calculated according to Equation 8.50.

For Rank-4-movements (left turners at a cross-intersection), the dependency between the p_0 values in Rank-2 and Rank-3-movements must be empirical and can not be calculated from analytical relations. They have been evaluated by numerous simulations by Grossmann (1991; cf. Brilon and Grossmann 1991). Figure 8.23 shows the statistical dependence between queues in streams of Ranks 2 and 3.

In order to calculate the maximum capacity for the Rank-4-movements (numbers 7 and 10), the auxiliary factors, $p_{z,8}$ and $p_{z,11}$, should be calculated first:

$$p_{z,i} = 0.65 p_{y,i} - \frac{p_{y,i}}{p_{v,i}+3} + 0.6\sqrt{p_{y,i}} \tag{8.97}$$

diminished to calculate the actual capacities, q_e. Brilon (1988, cf. Figures 8.7 and 8.8) has discussed arguments which support this double introduction.

The reasons for this are as follows:

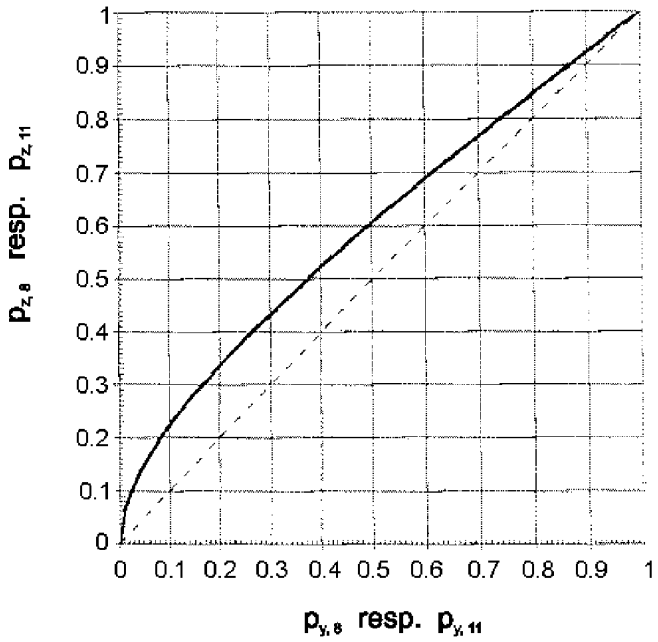


Figure 8.23
Reduction Factor to Account for the Statistical Dependence Between Streams of Ranks 2 and 3.

- During times of queuing in Rank-2 streams (e.g. left turners from the major street), the Rank-3 vehicles (e.g. left turners from the minor street at a T-junction) cannot enter the intersection due to traffic regulations and the highway code. Since the portion of time provided for Rank-3 vehicles is p_0, the basic capacity calculated from Section 8.4.1 for Rank-3 streams has to be diminished by the factor p_0 for the corresponding Rank-2 streams (Equations 8.95 to 8.99).
- Even if no Rank-2 vehicle is queuing, these vehicles influence Rank-3 operations, since a Rank-2 vehicle approaching the intersection within a time of less than t_c prevents a Rank-3 vehicle from entering the intersection.

Grossmann (1991) has proven that among the possibilities considered, the described approach is the easiest and quite realistic in the range of traffic volumes which occur in practical applications.

8.7 Shared Lane Formula

8.7.1 Shared Lanes on the Minor Street

If more than one minor street movement is operating on the same lane, the so-called "shared lane equation" can be applied. It calculates the total capacity q_s of the shared lane, if the capacities of the corresponding movements are known. (Derivation in Harders, 1968 for example.)

$$\frac{1}{q_s} = \sum_{i=1}^{m} \frac{b_i}{q_{m,i}} \qquad (8.100)$$

q_s = capacity of the shared lane in veh/h,
$q_{m,i}$ = capacity of movement i, if it operates on a separate lane in veh/h,

b_i = proportion of volume of movement i of the total volume on the shared lane,
m = number of movements on the shared lane.

The formula is also used by the HCM (1994, Equation 10-9).

This equation is of general validity regardless of the formula for the estimation of q_m and regardless of the rank of priority of the three traffic movements. The formula can also be used if the overall capacity of one traffic stream should be calculated, if this stream is formed by several partial streams with different capacities, e.g. by passenger cars and trucks with different critical gaps. Kyte at al (1996) found that this procedure for accounting for a hierarchy of streams, provided most realistic results.

8.7.2 Shared Lanes on the Major Street

In the case of a single lane on the major street shared by right-turning and through movements (movements no. 2 and 3 or 5 and 6 in Figure 8.22), one can refer to Table 8.2.

If left turns from the major street (movements no. 1 and 4 in Figure 8.22) have no separate turning lanes, vehicles in the priority l movements no. 2 and 3, and no. 5 and 6 respectively in Figure 8.21 may also be obstructed by queuing vehicles in those streams. The factors $p_{0,1}$* and $p_{0,4}$* indicate the probability that there will be no queue in the respective shared lane. They might serve for a rough estimate of the disturbance that can be expected and can be approximated as follows (Harders 1968):

$$p_{o,i}^{*} = 1 - \frac{1 - p_{0,i}}{1 - q_j t_{Bj} - q_k t_{Bk}} \qquad (8.101)$$

where: $i = 1$, $j = 2$ and $k = 3$ (cf. Figure 8.22)

or

$i = 4$, $j = 5$ and $k = 6$ (cf. Figure 8.22)

q_j = volume of stream j in veh/sec,
q_k = volume of stream k in veh/sec, and
t_{Bj} and t_{Bk} = follow-up time required by a vehicle in stream j or k (s).
(1.7 sec < t_B < 2.5 sec, e.g. t_B = 2 sec)

In order to account for the influence of the queues in the major street approach lanes on the minor street streams no. 7, 8, 10, and 11, the values $p_{0,1}$ and $p_{0,4}$, according to Equation 8.47 have to be replaced by the values $p_{0,1}$* and $p_{0,4}$* according to Equation 8.101. This replacement is defined in Equations 8.95 to 8.97.

8.8 Two-Stage Gap Acceptance and Priority

At many unsignalized intersections there is a space in the center of the major street available where several minor street vehicles can be stored between the traffic flows of the two directions of the major street, especially in the case of multi-lane major traffic (Figure 8.24). This storage space within the intersection enables the minor street driver to cross the major streams from each direction at different behavior times. This behavior can contribute to an increased capacity. This situation is called two-stage priority. The additional capacity being provided by these wider intersections can not be evaluated by conventional capacity calculation models.

Brilon et al. (1996) have developed an analytical theory for the estimation of capacities under two-stage priority conditions. It is based on an earlier approach by Harders (1968). In addition to the analytical theory, simulations have been performed and were the basis of an adjustment factor α. The resulting set of equations for the capacity of a two-stage priority situation are:

$$c_T = \frac{\alpha}{y^{k+1} - 1} \left\{ y(y^k - 1) \cdot [c(q_5) - q_1] + (y - 1) \cdot c(q_1 + q_2 + q_5) \right\}$$

for $y \neq 1$

$$c_{T(y=1)} = \frac{\alpha}{k + 1} \left\{ k[c(q_5) - q_1] + (c(q_1 + q_2 + q_5) \right\} \qquad (8.104)$$

for $y = 1$

c_T = total capacity of the intersection for minor through traffic (movement 8)

Table 8.2
Evaluation of Conflicting Traffic Volume q_p
Note: The indices refer to the traffic streams denoted in Figure 8.22.

Subject Movement	No.	Conflicting Traffic Volume q_p
Left Turn from Major Road	1	$q_5 + q_6$ [3]
	7	$q_2 + q_3$ [3]
Right Turn from Minor Road	6	q_2 [2] $+ 0.5\, q_3$ [1]
	12	q_5 [2] $+ 0.5\, q_6$ [1]
Through Movement from Minor Road	5	$q_2 + 0.5\, q_3$ [1] $+ q_5 + q_6$ [3] $+ q_1 + q_4$
	11	$q_2 + q_3$ [3] $+ q_5 + 0.5\, q_6$ [1] $+ q_1 + q_4$
Left Turn from Minor Road	4	$q_2 + 0.5\, q_3$ [1] $+ q_5 + q_1 + q_4 + q_{12}$ [4)5)6)] $+ q_{11}$ [5]
	10	$q_5 + 0.5\, q_6$ [1] $+ q_2 + q_1 + q_4 + q_6$ [4)5)6)] $+ q_8$ [5]

Notes

1) ***If there is a right-turn lane, q_3 or q_6 should not be considered.***
2) ***If there is more than one lane on the major road, q_2 and q_5 are considered as traffic volumes on the right lane.***
3) ***If right-turning traffic from the major road is separated by a triangular island and has to comply with a YIELD- or STOP-Sign, q_6 and q_3 need not be considered.***
4) ***If right-turning traffic from the minor road is separated by a triangular island and has to comply with a YIELD- or STOP-sign, q_9 and q_{12} need not be considered.***
5) ***If movements 11 and 12 are controlled by a STOP-sign, q_{11} and q_{12} should be halved in this equation. Similarly, if movements 8 and 9 are controlled by a STOP-sign, q_8 and q_9 should be halved.***
6) ***It can also be justified to omit q_9 and q_{12} or to halve their values if the minor approach area is wide.***

where

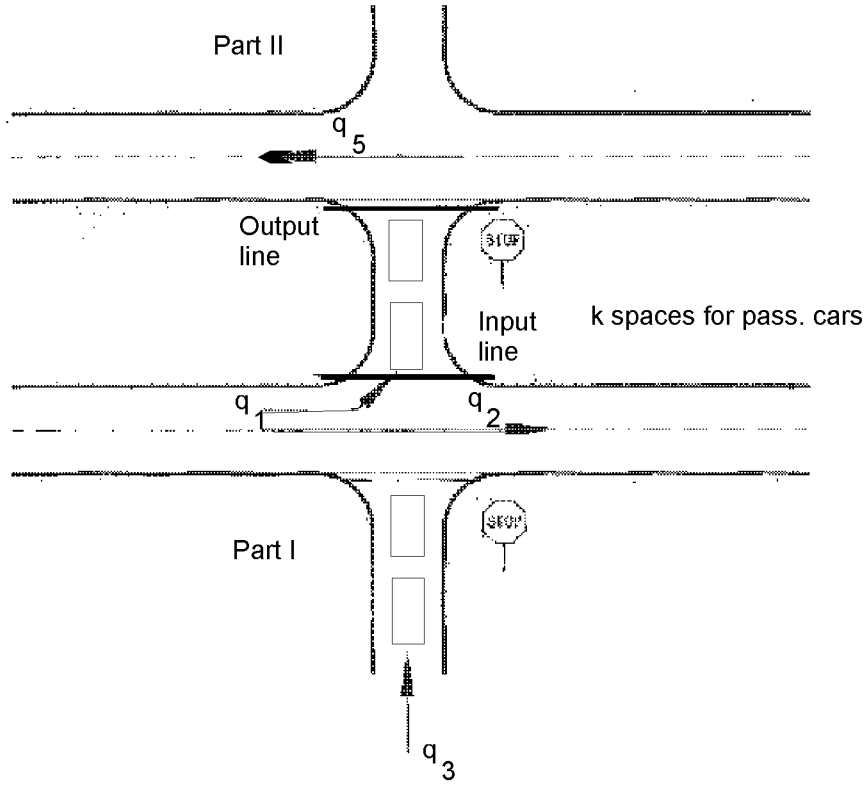
$$y = \frac{c(q_1 + q_2) - c(q_1 + q_2 + q_5)}{c(q_5) - q_1 - c(q_1 + q_2 + q_5)}$$

q_1 = volume of priority street left turning traffic at part I
q_2 = volume of major street through traffic coming from the left at part I
q_5 = volume of the sum of all major street flows coming from the right at part II.

$$\alpha = 1 \qquad \text{for } k=0$$

$$a = 1 - 0.32\exp(-1.3\cdot\sqrt{k}) \qquad \text{for } k > 0 \qquad (8.105)$$

Of course, here the volumes of all priority movements at part II have to be included. These are: major right (6, except if this movement is guided along a triangular island separated from the through traffic) , major through (5), major left (4); numbers of movements according to Figure 8.22.



Note: The theory is independent of the number of lanes in the major street.

Figure 8.24
Minor Street Through Traffic (Movement 8) Crossing the Major Street in Two Phases.

$c(q_{1+}q_2)$	=	capacity at part I
$c(q_5)$	=	capacity at part II
$c(q_1+q_2+q_5)$	=	capacity at a cross intersection for minor through traffic with a major street traffic volume of $q_1+q_2+q_5$

(All of these capacity terms are to be calculated by any useful capacity formula, e.g. the Siegloch-formula, Equation 8.33)

The same set of formulas applies in analogy for movement 7. If both movements 7 and 8 are operated on one lane then the total capacity of this lane has to be evaluated from c_{T7} and c_{T8} using the shared lane formula (Equation 8.95). Brilon et al. (1996) provide also a set of graphs for an easier application of this theory.

8.9 All-Way Stop Controlled Intersections

8.9.1 Richardson's Model

Richardson (1987) developed a model for all-way stop controlled intersections (AWSC) based on M/G/1 queuing theory. He assumed that a driver approaching will either have a service time equal to the follow-up headway for vehicles in this approach if there are no conflicting vehicles on the cross roads (to the left and right). The average service time is the time between successive approach stream vehicles being able to depart. If there were conflicting vehicles then the conflicting

vehicles at the head of their queues will depart before the approach stream being analysed. Consequently, Richardson assumed that if there were conflicting vehicles then the average service time is the sum of the clearance time, T_c, for conflicting vehicles and for the approach stream.

For simplicity, Richardson considered two streams; northbound and westbound. Looking at the northbound drivers, the probability that there will be a conflicting vehicle on the cross road is given by queuing theory as ρ_w. The average service time for northbound drivers is then

$$s_n = t_m (1-\rho_w) + T_c \rho_w \quad (8.106)$$

A similar equation for the average service time for westbound drivers is

$$s_w = t_m (1-\rho_e) + T_c \rho_e \quad (8.107)$$

where,

ρ_i is the utilization ratio and is $q_i s_i$,
q_i is the flow from approach i,
s_i is the service time for approach i
t_m is the minimum headway, and
T_c is the total clearance time.

These equations can be manipulated to give a solution for s_n as

$$s_n = \frac{q_w t_m T_c + t_m - q_w t_m^2}{1 - q_w q_n (T_c^2 - 2 t_m T_c + t_m^2)} \quad (8.108)$$

If there are four approaches then very similar equations are obtained for the average service time involving the probability there are no cars on either conflicting stream. For instance,

$$s_n = t_m (1-\rho_{ew}) + T_c \rho_{ew} \quad (8.109)$$

$$s_s = t_m (1-\rho_{ew}) + T_c \rho_{ew} \quad (8.110)$$

$$s_e = t_m (1-\rho_{ns}) + T_c \rho_{ns} \quad (8.111)$$

$$s_w = t_m (1-\rho_{ns}) + T_c \rho_{ns} \quad (8.112)$$

The probability of no conflicting vehicles being $1-\rho_{ns}$ given by

$$1-\rho_{ns} = (1-\rho_s)(1-\rho_n) \quad (8.113)$$

hence,

$$\rho_{ns} = 1 - (1-q_n s_n)(1-q_s s_s) \quad (8.114)$$

and

$$\rho_{ew} = 1 - (1-q_e s_e)(1-q_w s_w) \quad (8.115)$$

Given the flows, q_n, q_s, q_e, and q_w and using an estimate of service times, p_{ns} and p_{ew} can be estimated using Equations 8.114 and 8.115. The iterative process is continued with Equations 8.109 to 8.112 providing a better estimate of the service times, s_n, s_s, s_e, and s_w.

Richardson used Herbert's (1963) results in which t_m was found to be 4 sec and T_c was a function of the number of cross flow lanes to be crossed. The equation was

$$t_c = 3.6 * 0.1 \; number\ of\ lanes$$

and T_c is the sum of the t_c values for the conflicting and the approach streams.

The steady-state average delay was calculated using the Pollaczek-Khintchine formula with Little's equation as:

$$W_s = \frac{2\rho - \rho^2 + q^2 Var(s)}{2(1-\rho)q} \quad (8.116)$$

or

$$W_s = \frac{\rho}{q}\left[1 + \frac{\rho\left(1 + \frac{q^2}{\rho^2} Var(s)\right)}{2(1-\rho)}\right]$$

This equation requires an estimate of the variance of the service times. Here Richardson has assumed that drivers either had a service time of h_m or T_c. For the northbound traffic, there were $(1-\rho_{ew})$ proportion of drivers with a service time of exactly t_m and ρ_{ew} drivers with a service time of exactly T_c. The variance is then

$$Var(s)_n = t_m^2(1-\rho_{ew}) + T_c^2\rho - s_n^2 \qquad (8.117)$$

and

$$\rho = \frac{s_n - t_m}{T_c - t_m} \qquad (8.118)$$

This then gives

$$Var(s)_n = t_m^2 \frac{T_c - s_n}{T_c - t_m} + T_c^2 \frac{s_n - t_m}{T_c - t_m} - s_n^2 \qquad (8.119)$$

for the northbound traffic. Similar equations can be obtained for the other approaches. An example of this technique applied to a four way stop with single lane approaches is given in Figure 8.25. Here the southbound traffic has been set to 300 veh/h. The east-west traffic varies but with equal flows in both directions. In accordance with the comments above, t_m was 4 sec and T_c was 2*t_c or 7.6 sec.

Richardson's approach is satisfactory for heavy flows where most drivers have to queue before departing. His approach has been extended by Horowitz (1993), who extended the number of maneuver types and then consequently the number of service time values. Horowitz has also related his model to Kyte's (1989) approach and found that his modified Richardson model compared well with Kyte's empirical data.

Figure 8.25 from Richardson's research, gives the performance as the traffic in one set approaches (north-south or east-west)increases. Typically, as traffic flow in one direction increases so does the traffic in the other directions. This will usually result in the level of delays increasing at a more rapid rate than the depicted in this figure.

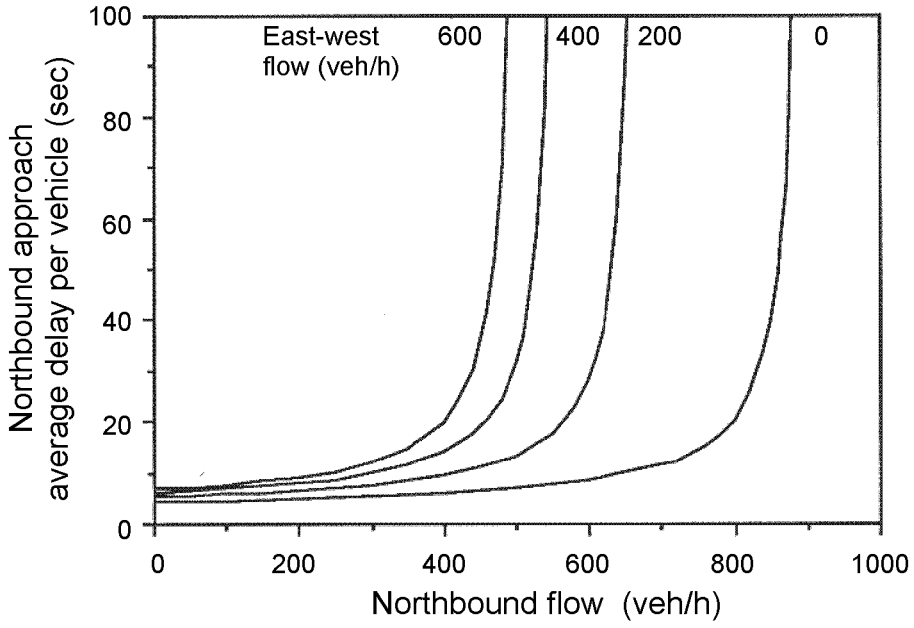


Figure 8.25
Average Delay For Vehicles on the Northbound Approach.

8.10 Empirical Methods

Empirical models often use regression techniques to quantify an element of the performance of the intersection. These models, by their very nature, will provide good predictions. However, at times they are not able to provide a cause and effect relationships.

Kimber and Coombe (1980), of the United Kingdom, have evaluated the capacity of the simple 2-stream problem using empirical methods. The fundamental idea of this solution is as follows: Again, we look at the simple intersection (Figure 8.7) with one priority traffic stream and one non-priority traffic stream during times of a steady queue (i.e. at least one vehicle is queuing on the minor street). During these times, the volume of traffic departing from the stop line is the capacity. This capacity should depend on the priority traffic volume q_p during the same time period. To derive this relationship, observations of traffic operations of the intersection have to be made during periods of oversaturation of the intersection. The total time of observation then is divided into periods of constant duration, e.g. 1 minute. During these 1-minute intervals, the number of vehicles in both the priority flow and the entering minor street traffic are counted. Normally, these data points are scattered over a wide range and are represented by a linear regression line. On average, half of the variation of data points results from the use of one-minute counting intervals. In practice, evaluation intervals of more than 1-minute (e.g. 5-minutes) cannot be used, since this normally leads to only few observations.

As a result, the method would produce linear relations for q_m:

$$q_m = b - c \cdot q_p \qquad (8.120)$$

Instead of a linear function, also other types of regression could be used as well, e.g.

$$q_m = A \cdot e^{-Bx} \quad . \qquad (8.121)$$

Here, the regression parameters A and B could be evaluated out of the data points by adequate regression techniques. This type of equation is of the same form as Siegloch's capacity formula (Equation 8.33). This analogy shows that A=3600/t_f.

In addition to the influence of priority stream traffic volumes on the minor street capacity, the influence of geometric layout of the intersection can be investigated. To do this, the constant values b and c or A and B can be related to road widths or visibility or even other characteristic values of the intersection layout by another set of linear regression analysis (see e.g. Kimber and Coombe 1980).
The advantages of the empirical regression technique compared to gap acceptance theory are:

- there is no need to establish a theoretical model.
- reported empirical capacities are used.
- influence of geometrical design can be taken into account.
- effects of priority reversal and forced priority are taken into account automatically.
- there is no need to describe driver behavior in detail.

The disadvantages are:

- transferability to other countries or other times (driver behavior might change over time) is quite limited: For application under different conditions, a very big sample size must always be evaluated.
- no real understanding of traffic operations at the intersection is achieved by the user.
- the equations for four-legged intersections with 12 movements are too complicated.
- the derivations are based on driver behavior under oversaturated conditions.
- each situation to be described with the capacity formulae must be observed in reality. On one hand, this requires a large effort for data collection. On the other hand, many of the desired situations are found infrequently, since congested intersections have been often already signalized.

8.10.1 Kyte's Method

Kyte (1989) and Kyte et al. (1991) proposed another method for the direct estimation of unsignalized intersection capacity for both AWSC and TWSC intersections. The idea is based on the fact that the capacity of a single-channel queuing system is the inverse of the average service time. The service time, t_w, at the unsignalized intersection is the time which a vehicle spends in the first position of the queue. Therefore, only the average of these times (t_w) has to be evaluated by observations to get the capacity.

Under oversaturated conditions with a steady queue on the minor street approach, each individual value of this time in the first

position can easily be observed as the time between two consecutive vehicles crossing the stop line. In this case, however, the observations and analyses are equivalent to the empirical regression technique .

Assuming undersaturated conditions, however, the time each of the minor street vehicles spends in the first position could be measured as well. Again, the inverse of the average of these times is the capacity. Examples of measured results are given by Kyte et al. (1991).

From a theoretical point of view, this method is correct. The problems relate to the measurement techniques (e.g. by video taping). Here it is quite difficult to define the beginning and the end of the time spent in the first position in a consistent way. If this problem is solved, this method provides an easy procedure for estimating the capacity for a movement from the minor street even if this traffic stream is not operating at capacity.

Following a study of AWSC intersections, Kyte et al. (1996) developed empirical equations for the departure headways from an approach for different levels of conflict.

$$h_i = h_{b\text{-}i} + h_{LT\text{-}adj} P_{LT} + h_{RT\text{-}adj} P_{RT} + h_{HV\text{-}adj} P_{HV} \qquad (8.122)$$

where:

h_i	is the adjusted saturation headway for the degree of conflict case *i*;
$h_{b\text{-}i}$	is the base saturation headway for case *i*;
$h_{LT\text{-}adj}$ and $h_{RT\text{-}adj}$	are the headway adjustment factors for left and right turners respectively;
P_{LT} and P_{RT}	are the proportion of left and right turners;
$h_{HV\text{-}adj}$	is the adjustment factor for heavy vehicles; and
P_{HV}	is the proportion of heavy vehicles.

The average departure headway, $\overline{d}$, is first assumed to be four seconds and the degree of saturation, x , is the product of the flow rate, V and $\overline{d}$. A second iterative value of $\overline{d}$ is given by the equation:

$$\overline{d} = \sum_{i=1}^{5} P(C_i) h_i$$

where $P(C_i)$ is the probability that conflict C_i occurs. These values also depend on estimates of $\overline{d}$ and the h_i values. The service time is given by the departure headway minus the move-up time.

Kyte et al. (1996) recognizes that capacity can be evaluated from two points of view. First, the capacity can be estimated assuming all other flows remain the same. This is the approach that is typically used in Section 8.4.1. Alternatively capacity can be estimated assuming the ratio of flow rates for different movements for all approaches remain constant. All flows are incrementally increased until one approach has a degree of saturation equal to one.

The further evaluation of these measurement results corresponds to the methods of the empirical regression techniques. Again, regression techniques can be employed to relate the capacity estimates to the traffic volumes in those movements with a higher rank of priority.

8.11 Conclusions

This chapter describes the theory of unsignalized intersections which probably have the most complicated intersection control mechanism. The approaches used to evaluate unsignalized intersections fall into three classes.

(a) Gap acceptance theory which assumes a mechanism for drivers departure. This is generally achieved with the notion of a critical gap and a follow on time. This attributes of the conflicting stream and the non priority stream are also required. This approach has been successfully used to predict capacity (Kyte et al. 1996) and has been extended to predict delays in the simpler conditions.

(b) Queuing theory in which the service time attributes are described. This is a more abstract method of describing driver departure patterns. The advantages of using queuing theory is that measures of delay (and queue lengths) can be more easily defined for some of the more complicated cases.

(c) Simulation programs. These are now becoming more popular. However, as a word of caution, the output from these models is only as good as the algorithms used in the model, and some simpler models can give excellent results. Other times, there is a temptation to look at the output from models without relating the results to the existing theory. This chapter describes most of the theories for unsignalised intersections and should assist simulation modelers to indicate useful extension to theory.

Research in these three approaches will undoubtably continue. New theoretical work highlights parameters or issues that should be considered further. At times, there will be a number of counter balancing effects which will only be identified through theory.

The issues that are likely to be debated in the future include the extent that one stream affects another as discussed in Section 8.6; the similarities between signalized and unsignalized intersections; performance of oversaturated intersection and variance associated with the performance measures.

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