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14. ABSTRACT
The optimal (best) flood-damage-mitigation plan; from the national economic development standpoint, is the plan that yields the maximum net economic benefit consistent with environmental, institutional, social and financial requirements. This document described, in detail, the branch-and-bound enumeration procedure for selecting the optimal combination of flood-damage-mitigation measures and illustrates how the Hydrologic Engineering Center computer programs can be used in this analysis. An example is presented in which the optimal plan is determined for a hypothetical catchment.

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TD-23

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

FLOOD-DAMAGE-MITIGATION PLAN SELECTION

SUMMARY

The optimal (best) flood-damage-mitigation plan, from the national economic development standpoint, is the plan that yields the maximum net economic benefit consistent with environmental, institutional, social, and financial requirements. To identify this optimal plan, candidate measures for various locations in a catchment are proposed, and plans that include combinations of those measures are evaluated and compared. Hydrologic Engineering Center (HEC) computer programs can be used for this plan evaluation. All possible combinations of the measures can be evaluated and compared, but such total enumeration is time-consumming and costly. A branch-and-bound procedure is a more efficient approach. With this procedure, the entire set of plans is divided repeatedly into subsets which are evaluated in the detail necessary to determine that the optimal plan is not in the subset or to identify it if it is.

This document describes, in detail, the procedure for selecting the optimal combination of flood-damage-mitigation measures and illustrates how the HEC programs can be used in the analysis. An example is presented in which the optimal plan is determined for a hypothetical catchment.

FLOOD-DAMAGE-MITIGATION MEASURES

A flood-damage-mitigation plan is a set of measures which are intended to function as a system to mitigate flood damages at one or more locations in a catchment. Depending on the complexity of the channel system and the spatial distribution of damageable property, the plan may include only a single measure at a single site, or it may consist of many measures distributed throughout a catchment.

The measures included in a damage-mitigation plan may be categorized as (1) those that reduce the negative effects of flooding by modifying the flood, (2) those that reduce the effects by modifying the damage susceptibility, or (3) those that reduce the effects by modifying the loss burden. Table 1 identifies specific measures in each of these categories. Measures in the first category modify the motion of flood waters; these are the so-called flood-control measures. Generally measures in this category are implemented with significant first cost and corresponding damage reduction. Measures in the second category, those that modify the damage susceptibility, do not alter the flood, but instead mitigate damage by reducing the potential for flood damage. These measures are the so-called nonstructural measures and typically are implemented as local protection measures. Finally, measures in the third category do not reduce damage at all, but redistribute the burden of the damage through relief or insurance programs.

Table 1. - Flood-damage-mitigation Measures

Measures that modify the flood (1)	Measures that modify the damage susceptibility (2)	Measures that modify the loss burden (3)

Flood protection
    Dikes
    Floodwalls
    Channel improvements
    Floodways
    Diversions
    Reservoirs
	Flood-plain management
	    Land-use regulation
	    Urban renewal
	    Government purchase
	      of property
	    Subsidized relocation
		Redistribute losses
		    Disaster relief
		    Flood insurance
		    Reconstruction grants
		    Tax write-offs

Watershed management
    Terracing
    Gully control
    Bank stabilization
    Forest-fire control
    Revegetation
	Flood proofing
	    Impervious-material
	      construction
	    Land elevation
	    Construction on stilts
	    Installation of flood-
	      shields
	    Closure of backflow
	      valves	

Weather modification
    Storm seeding
	Emergency Measures
	    Flood fighting
	    Flood warning
	    Evacuation	

CRITERIA FOR PLAN SELECTION

From the national economic development standpoint, the best flood-damage-mitigation plan is the plan that yields the maximum difference between total benefit realized due to the measures included in the plan and the total cost of the measures. The total cost is the sum of capital cost, operation, maintenance, power, replacement, and any other plan-related cost. The total benefit is the sum of inundation-reduction benefit and other benefits due to the measures.

<u>Inundation-reduction Benefit Estimation</u>. - The inundation-reduction benefit for any flood-damage-mitigation plan is the difference between flood damage with base conditions and flood damage under the same hydrologic conditions with the plan implemented as proposed (modified-condition). These damages might be estimated for a single historical flood event, without and with the proposed combination of measures in place. However this analysis does not provide information adequate for judging the long-term economic efficiency of the plan. Response to additional floods must an be analyzed to assess this long-term performance. In the extreme, all floods of record could be analyzed, and an average inundation-damage reduction could be computed. However, if the record of observed flood events is short (and it usually is), an analysis based on historical events alone may yield misleading results. The peak discharge of a flood is considered to be a random event. Thus the discharge magnitudes and frequency in the historical record may not adequately represent the magnitudes and frequency of future floods. Consequently, the combination of damage-mitigation measures selected from analysis of only historical events might not truly be the optimal combination in the long-run.

To account for the risk of floods larger and smaller than those observed, statistical analysis techniques, in the form of expected value analysis, can be used. The expected value of a random variable is a probability-weighted average value of that variable. Each possible magnitude of the variable is multiplied by the estimated probability of its occurrence, and the resulting products are accumulated. The result, the expected value, is the estimated long-term average of the random variable. Rare events that have a small probability of exceedance have little impact on the magnitude of the expected value, while more-common events with a larger probability of exceedance have a more significant effect on the magnitude. When modified to employ expected-value analysis, the net-benefit of any specified damage-mitigation plan can be written as

Net benefit = E[DB] -E[DP(P)] + E[OB(P)] - E[C(P)] ....(1)

in which E[ ] denotes the expected value of the argument;
DB = base-condition total-catchment inundation damages;
DP(P) = total-catchment inundation damages with plan P

implemented; OB(P) = other benefits of plan P; and C(P) = total cost of plan P. In systems analysis terminology, Eq. 1 is known as the objective function of the mathematical representation of the planning problem. The goal of plan formulation is to identify the plan, P, which yields the maximum value of the objective function.

To compute the expected value of damage, the probability of various damage magnitudes must be estimated. For most flood-damage analyses, inundation damage is assummed to be a unique function of water-surface elevation, so the damage-probability estimates may be derived by manipulation and combination of a discharge-probability function, an elevation-discharge function, and an elevation-damage function, as illustrated by Fig. 1. The discharge-probability and elevation-discharge functions are developed with hydrologic engineering procedures, and the elevation-damage function is developed by survey of flood-plain property. The Expected Annual Flood Damage Computation (EAD) program (10), developed in The Hydrologic Engineering Center, performs the manipulation required, with user-specified functions, and computes expected annual damage. With this program, flood damage can be computed for various damage categories for any number of sites in a catchment.

The discharge-probability, elevation-discharge, and elevation-damage functions required for computation of expected annual damage can be determined with hydrologic and hydraulic analysis and data management programs developed and supported by the HEC. Program HEC-1 (14) can be used to define the discharge-probability function at points in a catchment for existing or modified conditions. This program is a generalized catchment runoff model. It simulates the rainfall-runoff processes and accounts for the motion of a flood wave through catchment channels. With a set of hypothetical storm hyetographs of specified probability as input, HEC-1 can be used to determine the corresponding discharge hydrographs at any point in a catchment. From these hydrographs, a discharge-probability function can be derived, with probabilities assigned based on rainfall frequency or by calibration to runoff frequencies. Program HEC-5 (12) is a reservoir-system simulation program. It can be used to compute the discharge hydrographs throughout a catchment in which reservoirs modify the flood, and thus will yield information required to develop a discharge-probability function for existing or modified conditions. Program HEC-2 (9) computes steady-flow water-surface elevations for natural channels using the standard-step method. This program can be used to derive the elevation-discharge function at location on a stream channel. Program SID (11) can be used to manage elevation-damage data in a catchment. This program manipulates elevation-damage functions for damageable property in a catchment and computes an aggregated elevation-damage function for each stream reach. This elevation-damage function will represent existing conditions, or, if measures that modify damage susceptibility are
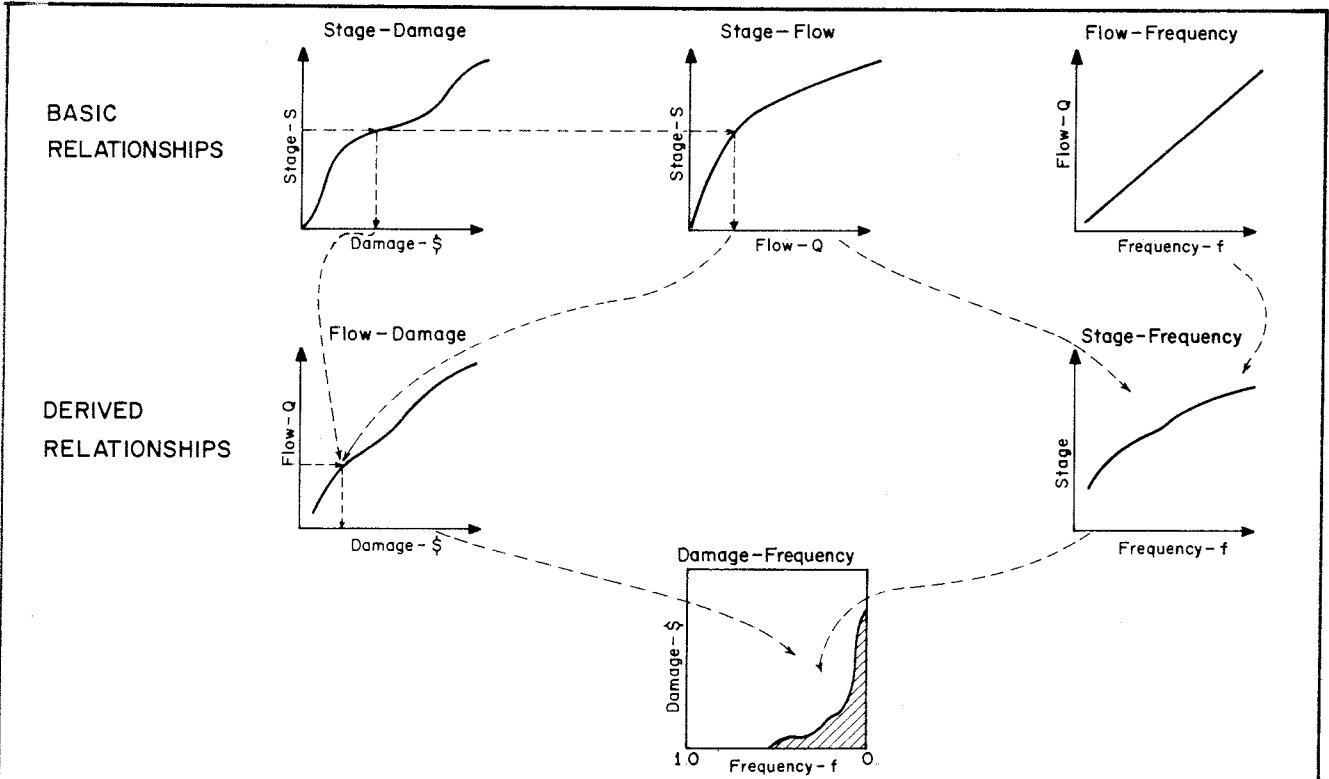
The basic and derived evaluation relationships are shown above. Concepts important to their construction are described herein.

Stage-Flow Relationship: This is a basic hydraulic function that shows for a specific location, the relationship between flow rate and stage. It is frequently referred to as a 'rating curve' and is normally derived from water surface profile computations.

Stage-Damage Relationship: This is the economic counterpart to the stage-flow function and represents the damage which will occur for various river stages. Usually the damage represents an aggregate of the damage which could occur some distance upstream and downstream from the specified location. It is usually developed from field damage surveys.

Flow-Frequency Relationship: This defines the relationship between exceedance frequency and flow at a location. It is the basic function describing the probability nature of stream flow and is commonly determined from either statistical analysis of gaged flow data or through watershed model calculations.

Damage-Frequency Relationship: This relationship is derived by combining the basic relationships using the common parameters stage and flow. For example, the damage for a specific exceedance frequency is determined by ascertaining the corresponding flow rate from the flow-frequency function, the corresponding stage from the stage-flow function and finally the corresponding damage from the stage-damage relationship. Any changes which occur in the basic relationships because of watershed development or flood plain management measure implementation will change the damage-frequency function and therefore the expected annual damage that is computed as the integral of the function (area underneath).

Other Functional Relationship: The flow-damage relationship is developed by combining the stage-damage with the stage-flow relationship using stage as the common parameter. The stage-frequency relationship is developed by combining the stage-flow with the flow-frequency relationship using flow as the common parameter. The damage-frequency relationship could then be developed as a further combination of these derived relationships.

FIG. 1. - Function Manipulation for Expected Damage
Computation (from ref. 10)

5

to be evaluated, the function will represent modified conditions. Application of the programs HEC-1, HEC-5, HEC-2, and SID with the EAD program for complex analyses is expedited by data-exchange linkages, such as the HEC data storage system (13).

Evaluation of Plan Accomplishments. - The economic effects of flood-damage mitigation measures are determined by evaluating the modifications to the discharge-probability function, to the elevation-damage relationship, or to the elevation-damage relationship, and recomputing the expected damage, expected inundation reduction benefit, and expected net benefit. This may be accomplished with the following procedure:

1.   Derive the damage-probability function for base-conditions based on the discharge-probability, elevation-discharge, and elevation-damage functions, as illustrated by Fig. 1.

2.   Integrate the damage-probability functions to compute expected annual flood damage for base conditions.

3.   Select the flood-damage mitigation plan for evaluation.

4.   Perform the analyses necessary to define modifications (if any) to the discharge-probability, elevation-discharge, and elevation-damage functions.

5.   Derive the modified damage-probability function based on the modified discharge-probability elevation-discharge, and elevation-damage functions, as illustrated by Fig. 1.

6.   Integrate the damage-probability functions to compute expected annual flood damage with the proposed plan.

7.   Compute the inundation-reduction benefit = expected annual damage with base condition - expected annual damage with the proposed plan.

8.   Determine the total economic benefit of the plan by adding to the results of step 7 all other monetary benefits of the proposed plan.

9.   Compute the total cost of the plan.

10.   Compute the net benefit of the plan by subtracting the total cost from the total benefit.

Depending on the damage-mitigation measures considered, the evaluation of step 4 may require application of program HEC-1 to evaluate results of changes to catchment processes, of HEC-2 to evaluate changes to channel processes, of HEC-5 to evaluate the effects of reservoirs, of SID to evaluate modifications to damageable property, or of other analytical plan evaluation tools

(3). Table 2 identifies flood-damage-mitigation measures and identifies programs for evaluation of the modifications due to each.

Again, the EAD program may be used for evaluation. The program allows specification of the required functions for computation of base condition damage and simultaneous specification of modified functions reflecting the proposed plan. The program computes expected annual damage for both conditions and computes the inundation-reduction benefit.

The results of execution of the EAD program are reported in a format illustrated by Table 3. This table is the report of expected annual damage, in thousands of dollars, in the hypothetical Loucks Creek catchment, shown in Fig. 2. The first column of this summary table identifies the damage reaches defined by the analyst. For each of these reaches, data are provided to permit derivation of a damage-probability function for computation of expected annual damage. Col. 2 of the table shows the total expected value of annual damage, by reach, with the base condition. Col. 3 shows the expected damage values with the damage-mitigation plan. These values are computed with user-specified modified discharge-probability, elevation-discharge, or elevation-damage functions. Col. 4 indicates, by reach, the damage reduced by the damage-mitigation plan. For the example shown, the total reduction for all damage categories for all reaches is $1,050,000. This is the inundation-reduction benefit. The net economic benefit can be determined by adding to this value all other economic benefits and subtracting the total cost of the damage-mitigation plan.

Constraints on Plan Selection. - Selection of the optimal combination of measures for a flood-damage mitigation plan is influenced by environmental, social, institutional, and financial considerations. These considerations are expressed as constraints in a mathematical representation of the plan-formulation problem. Typical constraints might require that the total expenditure for the selected plan be limited to available funds (a financial constraint) or that the plan should provide a 100-yr level of protection at some catchment location (an institutional constraint). The mathematical form of the constraints may be simple or extremely complex. For example, if total expenditure is to be limited, the constraint is a simple linear function which computes the sum of the costs of the measures and limits the sum to the amount available. In the case of the requirement for 100-year level of protection, complex hydrologic, hydraulic, and economic models may be required to determine if a given plan satisfies the constraint.

Two categories of constraints influence flood-damage-mitigation plan selection: inviolable and violable. Inviolable constraints must be satisfied at all costs, and a plan which does not satisfy constraints of this category is unacceptable (or infeasible in systems-analysis terminology). The second category

Table 2. – Computer Programs for Evaluation of
Flood-damage-mitigation Measures

Measure	Function modified		
(1)	Stage-discharge (2)	Stage-damage (3)	Discharge-probability (4)
Reservoir	no change	no change	HEC-1, HEC-5
Floodwall	HEC-2	SID	HEC-1[1]
Channel modification	HEC-2	no change	HEC-1[1]
Diversion	no change	no change	HEC-1, HEC-5
Flood forecasting	no change	no change	HEC-5[2]
Flood proofing	no change	SID	no change
Relocation	no change	SID	no change
Flood warning	no change	SID[3]	no change
Land-use control	no change	SID	HEC-1

[1]If floodwall or channel modification significantly alter channel storage

[2]Modifications due to improved reservoir operation with forecast

[3]Evaluation requires subjective analysis

Table 3. - Expected Annual Damage Computation Program
Summary Output

** EXPECTED ANNUAL DAMAGE SUMMARY BY REACH **

** INPUT DATA YEARS   = 1985

** FLOOD PLAIN MANAGEMENT PLANS
        1 - BASE CONDITIONS
        2 - SITE 1 FLOODWALL, SITE 2 STATUS QUO

GRAND SUMMARY - ALL DAMAGE CATEGORIES
   FOR INPUT DATA YEAR 1985
```
----------------------------------------
             . EXPECTED ANNUAL DAMAGE  .
    REACH       BASE      .... PLAN  2....
 NO    ID    CONDITION  DAMAGE    DAMAGE
             (PLAN  1)  W/PLAN   REDUCED
----------------------------------------
    (1)         (2)       (3)       (4)
  1 CRONEL   1200.00    150.00   1050.00
  2 ICATHA    500.00    500.00      0.00
----------------------------------------

    TOTAL    1700.00    485.00   1050.00

----------------------------------------
```

FIG. 2. - Loucks Creek Catchment

CRONELL

Site I
detention structure
or
levee

ICATHA

Site 2
Channel improvement
or
floodproofing
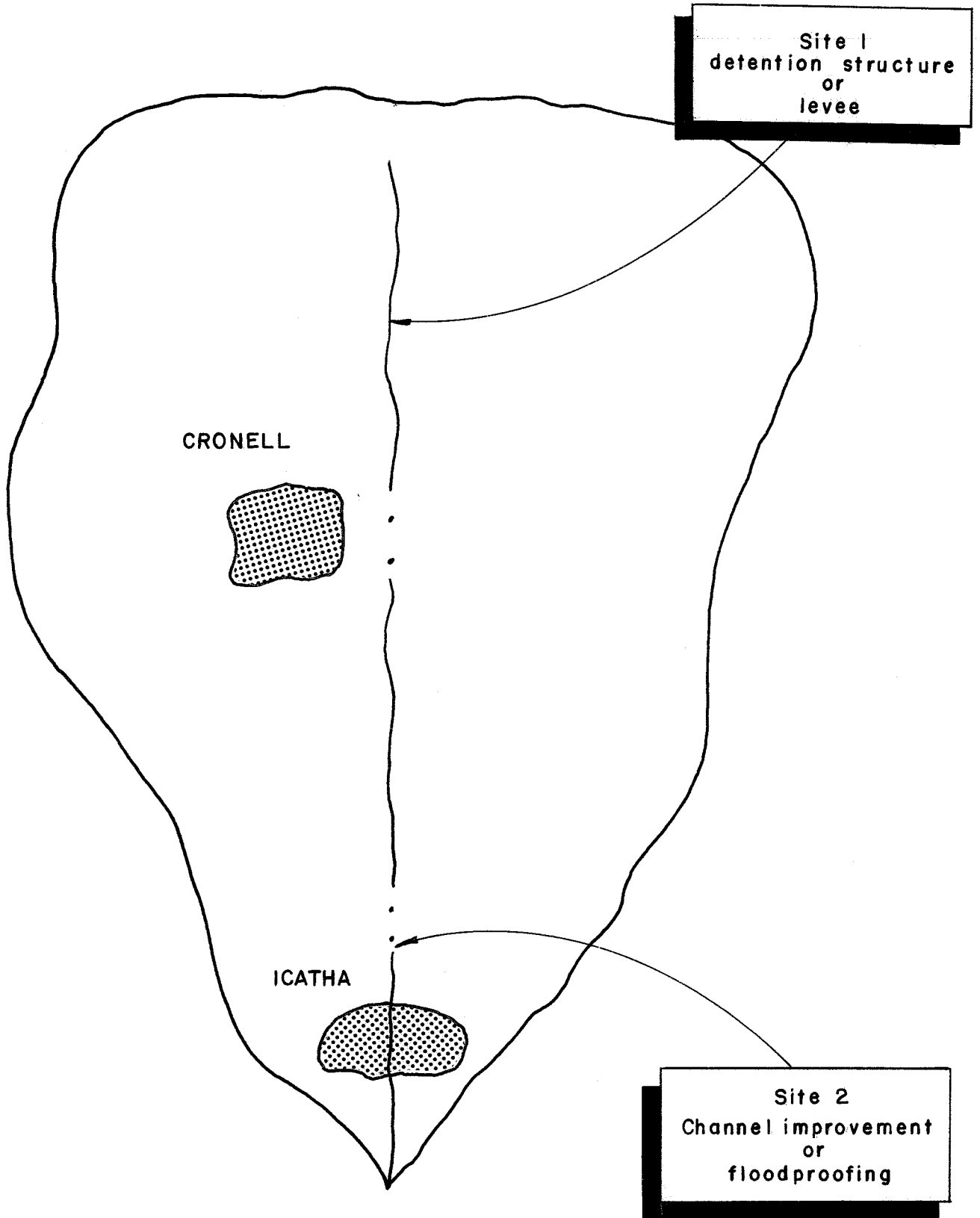
of constraints, the violable constraints, may be violated at some cost, but a plan which does not violate the constraints is preferred to one which does. Constraints of this type may be treated analytically by imposing a penalty on the net benefit computed with Eq. 1, if the plan fails to meet the specified target.

BRANCH-AND-BOUND ENUMERATION

General Properties. - Branch-and-bound enumeration is a general-purpose technique for identifying the optimal solution to an optimization problem without explicitly enumerating all solutions. In a complex situation with many alternatives, enumeration of all solutions is impractical because it is resource-expensive. With branch-and-bound enumeration, the need to evaluate each solution individually is eliminated. This is accomplished by dividing (branching) the entire set of solutions into subsets for which the objective function and constraints can be evaluated. After subdividing the possible solutions, an upper bound is estimated for the objective function achievable with each subset (bounding). This bound is compared with the performance of the best solution thus far identified, and inferior solutions are eliminated. The entire process is repeated to identify the optimal solution to the plan selection problem.

References. - The general characteristics of branch-and-bound methods and applications of the methods have been presented in the management-science and operations-research literature. Lawler and Wood (5) present a survey of the essential features of branch-and-bound methods for constrained optimization problems and describe application to integer and nonlinear programming problems, to the traveling-salesman problem, to the quadratic assignment problem, and to non-mathematical programming problems. Mitten (7) describes the general properties of branch-and-bound methods and presents, in general terms, the conditions for branching and for bounding the results of optimization problems. Garfinkel and Nemhauser (4) describe branch-and-bound methods applicable to integer-programming problems.

Branch-and-bound methods have been applied in resource planning to problems of sizing, selecting, sequencing, and scheduling projects. Marks and Liebman (6) suggest using a branch-and-bound procedure for locating solid-waste management facilities. Brill and Nakamura (2, 8) propose application of a branch-and-bound method for generating systematically alternative plans for regional wastewater-treatment systems and for evaluating these alternative plans. Ball, Bialas, and Loucks (1) propose a procedure for identifying the least-costly flood-damage-mitigation plan from many alternatives by repeatedly subdividing the set of all possible plans and comparing the minimum cost possible with plans in each subdivision. The procedure presented subsequently in this report is based on that procedure, with modifications to employ maximum net-benefit

11

criteria and with provisions for using HEC programs for plan evaluation.

Branching. - A branch-and-bound procedure identifies the optimal flood-damage-mitigation plan by dividing the set of all possible plans into mutually-exclusive subsets for evaluation. Subdivision is made on the basis of project site, beginning at the most-upstream site in the catchment and progressing downstream. In this context, a site is a location at which alternative flood-damage reduction measures have been proposed for implementation. For each site, one and only one of the proposed measures will be selected and included in the optimal combination. (In mathematics terminology, these measures are mutually-exclusive.) At least one damage location must be downstream of each site to permit evaluation of incremental benefit with common computational tools, such as the EAD program.

For the Loucks Creek catchment example, the proposed mitigation measures shown in Fig. 2 may be grouped at two sites for the branch-and-bound enumeration. Site 1 may be defined as the general area upstream of Cronell at which either status quo will be maintained, or the detention structure or the floodwall will be constructed. Site 2 may be defined as the stream reach in which the channel is to be improved, floodproofing is to be implemented, or status quo is to be maintained. Damage reduction due to the measure included at each site can be determined readily by referring to EAD output; damage reduction at Cronell is due to the damage-mitigation measure at site 1, and the damage reduction at Icatha is due to the combined action of the measures at sites 1 and 2.

For evaluation, the set of flood-damage mitigation plans for a catchment is subdivided based on the sites in which the various measures are grouped. For example, the set of all plans for the Loucks Creek catchment initially may be divided into the following subsets:

1.   a subset that includes all plans with the status quo for site 1;

2.   a subset that includes all plans with the detention structure for site 1; and

3.   a subset that includes all plans with the floodwall for site 1.

This is illustrated conceptually in Fig. 3. Each of these subsets may be divided further as needed to identify the optimal plan. For example, the subset that includes plans with status quo for site 1 may be divided into the following additional subsets:

1.   a subset that includes plans with status quo for site 1 and status quo for site 2;

FIG. 3. — Subdivision of Plans for Loucks Creek Catchment

All plans

(a)

Status quo site 1

Detention structure site 1

Floodwall site 1

(b)

Status quo site 2

Channel Improvement site 2

Flood-proofing site 2

Status quo site 2

Channel improvement site 2

Flood-proofing site 2

Status quo site 2

Channel improvement site 2

Flood-proofing site 2

(c)
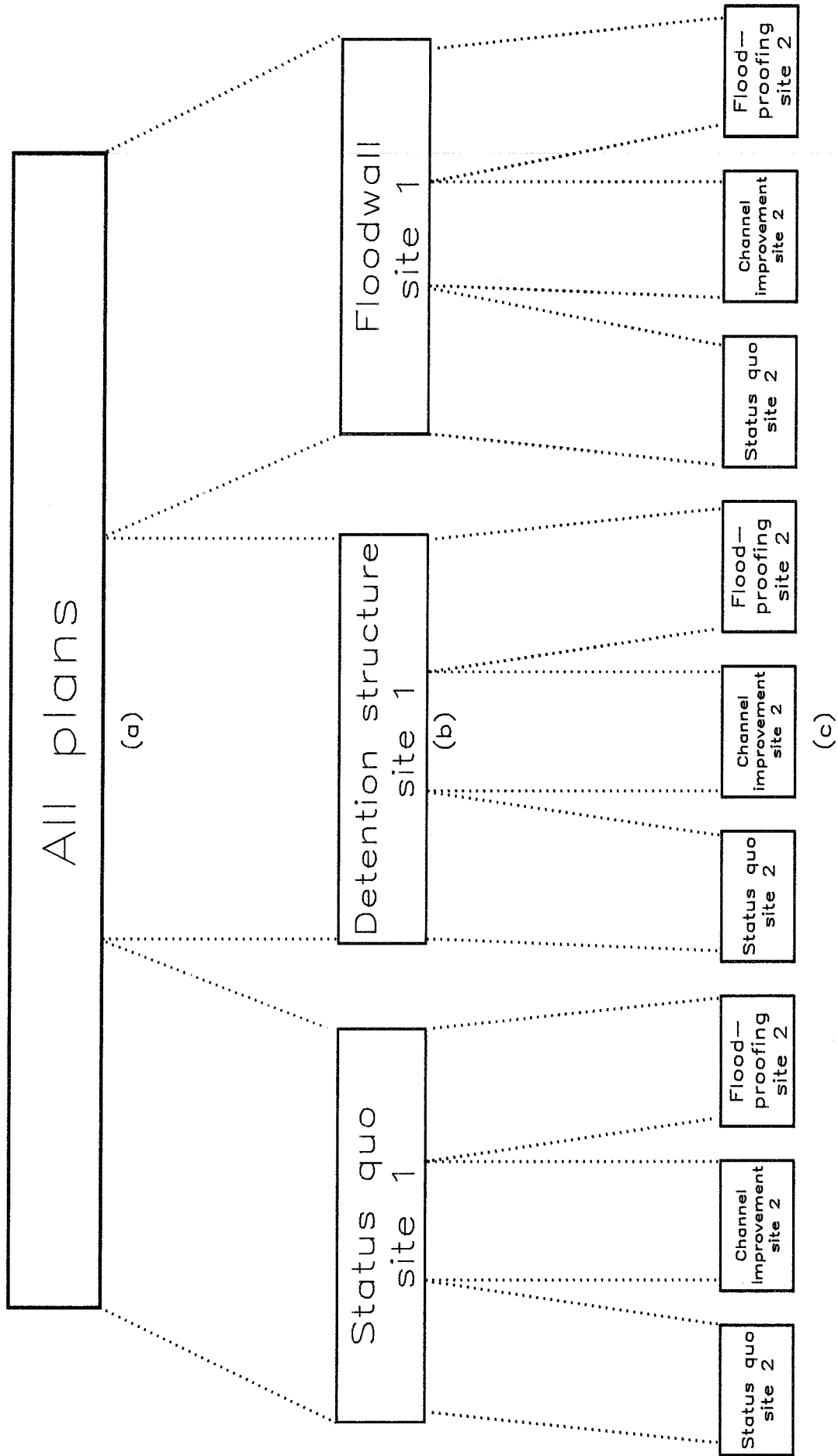
2.   a subset that includes plans with status quo for site 1
     and channel improvement for site 2; and

3.   a subset that includes plans with the status quo for
     site 1 and floodproofing at site 2.

Fig. 3 (c) illustrates this further subdivision. The subset that
includes the detention structure and the subset that includes the
floodwall for site 1 may be divided in a similar fashion.

Bounding. - The objective function of Eq. 1 is used to
compute the net benefit of any plan in the branch-and-bound
procedure.   In Eq. 1, E[DB] is the expected value of total damage
in the catchment with base conditions. The residual damage term,
E[DP(P)], is the expected value of damage with all measures
implemented, accounting for the damage reduction possible with
the measures acting individually and with the measures acting as
a system (the so-called synergistic benefits). Likewise, OB(P) is
the cumulative benefit throughout the catchment of the individual
measures plus benefit accrued as the measures function together.
The cost, C(P), includes individual measure cost plus any
additional cost to implement the measures in combination (such as
additional wildlife mitigation cost).   Thus, for the Loucks Creek
example, ignoring the other benefits, the value of the objective
function for any is computed with the following equation:

      (a)   expected value of damage for base conditions at
           Cronell

+   (b)   expected value of damage for base conditions at
           Icatha

-    (c)   expected value of damage at Cronell with measure
           for site 1

-    (d)   cost of the measure at site 1

-    (e)   expected value of damage at Icatha with measures
           for sites 1 and 2

-    (f)   cost of measure at site 2

-    (g)   additional cost due to implementing a plan which
           includes combination of measures at sites 1 and 2

$$\dotfill(2)$$

Terms (a) and (b) of this equation are independent of the plan
selected and remain constant throughout the analysis. These may
be determined with the EAD program. Term (c) is a function of the
measure considered for site 1 alone.   Term (d) is a function of
the measure at site 1 only. Term (e) is a function of the
measures selected for sites 1 and 2. This value may be determined

14

directly with the EAD program. Term (f) is a function of the
measure selected for site 2 alone, while term (g) is a function
of the measures at sites 1 and 2. Table 4 shows damage with and
cost of plans for the Loucks Creek catchment. Using these values
the objective function value for a plan that includes the
floodwall for site 1 and status quo for site 2 is

$$
\begin{array}{rl}
\$ \ 1,200,000 & \text{(a)} \\
+ \quad 500,000 & \text{(b)} \\
- \quad 150,000 & \text{(c)} \\
-1,000,000 & \text{(d)} \\
- \quad 500,000 & \text{(e)} \\
- \qquad\quad 0 & \text{(f)} \\
- \qquad\quad 0 & \text{(g)} \\
\hline
\end{array}
$$

Net benefit $ \quad 50,000

Eq. 1 also is used in the branch-and-bound procedure to
estimate the upper bound on net benefit possible with any subset
of plans defined in the branching operation. This subset bound is
computed by evaluating Eq. 1, but including only costs and
benefits of measures that are known with certainty to be in the
subset. For example, the subset bound for site 1 of the Loucks
Creek catchment is computed as follows:

    (a)   expected value of damage for base conditions at
           Cronell

+   (b)   expected value of damage for base conditions at
           Icatha

-    (c)   expected value of damage at Cronell with measure
           for site 1

-    (d)   cost of the measure at site 1

$$\ldots\ldots\ldots\ldots(3)$$

The value computed in this fashion is an upper bound on all plans
that include the specified measure for site 1. Again using the
values shown in Table 4, the bound for the subset of plans that
include the floodwall for site 1 is

$$
\begin{array}{rl}
\$ \ 1,200,000 & \text{(a)} \\
+ \quad 500,000 & \text{(b)} \\
- \quad 150,000 & \text{(c)} \\
-1,000,000 & \text{(d)} \\
\hline
\end{array}
$$

Subset bound $ \quad 550,000

The cost of and regulated damage due to the measure for site 2 is
not included in this computation, because that measure is not

Table 4. - Expected Annual Damage with and Annual
Equivalent Cost of Loucks Creek Catchment Plans

Site 1 measure	Site 2 measure		
	Status quo	Channel improve-ment	Flood-proofing
(1)	(2)	(3)	(4)
Status quo	0	0	0
	0	200	150
	1200	1200	1200
	500	200	335
	1,2,S,E	2,E	S,E
Detention structure	500	500	500
	0	200	150
	459	459	459
	191	0	26
	1,E	2,E	S,E
Flood-wall	1000	1000	1000
	0	200	150
	150	150	150
	500	200	335
	2,E	2,E	S,E

Values in table are arranged as follows:

    Total equivalent annual cost of measure at site 1
    Total equivalent annual cost of measure at site 2
    Expected annual damage at Cronell with plan implemented
    Expected annual damage at Icatha with plan implemented
    HEC programs executed to estimate damage

All damage and cost values are in thousands of dollars.

Abbreviations used for programs are as follows:

    1 = HEC-1 Flood Hydrograph Package Program
    2 = HEC-2 Water Surface Profile Program Program
    S = Structure Inventory for Damage Analysis Program
    E = Expected Annual Flood Damage Computation Program

yet specified. The subset bound, $550,000, is the limit on net benefit that is achievable with plans that include the floodwall at site 1, regardless of the measure included at site 2. This can be verified by comparing Eq. 2 with Eq. 3 and noting that all additional terms in Eq. 2 will always reduce the total. Thus, no matter which measure is included for site 2, the total objective function value will be less than or equal to the subset bound.

Eliminating Subsets. - The goal of the branch-and-bound procedure is to eliminate, without explicit evaluation, subsets of plans that are clearly inferior. The characteristics of the subset bound make this possible. If a subset bound is less than the net benefit achievable with any trial optimal plan, the subset cannot possibly contain a plan that is superior to the trial optimal plan. The value of the subset bound cannot increase as the subset is further subdivided, so the bound, and hence, the net benefit, cannot increase. Therefore, the entire subset can be eliminated from further consideration, and other subsets can be considered.

For example, if the maximum net benefit possible with the subset that includes all plans with the floodwall at site 1 is $550,000, and the net benefit is $565,000 for a plan that includes the detention structure for site 1 and status quo for site 2, the entire subset that includes the floodwall can be eliminate from further consideration. Regardless of the measure included for site 2, the net benefit will not exceed $550,000. Thus no plan in the subset is superior to the plan which yields $565,000.

Procedure. - The step-by-step procedure that follows describes the branch-and-bound procedure for identifying the economic optimal flood-damage-mitigation plan:

a.   Initialize. - Assign unique indices to the sites for which flood-damage mitigation measures have been proposed. Each site downstream of a given site must have a larger index. Prepare a list of all measures proposed for each site, including the status quo as the first measure for each. The first plan is the status quo plan, so note that for each site, the measure included in the plan is status quo. Set the initial trial optimum (straw man) as -999. For evaluation of the subset bound, define a site pointer, S, and set S = 1. Go to step i.

b.   Evaluate subset bound. - Compute the subset bound for site S. (Note that this subset bound may have been computed previously. If so, go to step c.) The damage values required for computation of the partial objective function are available from the results of step i. If this subset bound is less than the trial optimum, go to step d. Otherwise, go to step c.

c.   Move downstream. - Consider the next site downstream
     (set S=S+1). If this site is the most-downstream site
     (S=N), go to step e. Otherwise, go to step b.

d.   Eliminate subset. - All plans in this subset can be
     eliminated from further consideration because the
     optimal plan cannot include the combination of measures
     specified at this point. Go to step e.

e.   Modify plan. - If all measures for site S have been
     considered, go to step k. Otherwise, replace the
     measure currently included for site S in the plan with
     the next measure in the list for that site. Go to step
     f.

f.   Check for complete plan. - If a complete plan has been
     formulated with a measure included for each site, set
     S=1, and go to step h. Otherwise, consider the next
     downstream site (set S=S+1), and go to step g.

g.   Add new measure for site S. - Add to the plan the first
     measure included in the list for site S. Go to step f.

h.   Evaluate constraints. - Determine if this plan
     satisfies the system requirements. If it does not, go
     to step e. Otherwise go to step i.

i.   Evaluate objective function. - Compute the value of the
     objective function of Eq. 1 for the complete plan. Go
     to step j.

j.   Compare. - If the trial optimum exceeds the objective
     function value for this plan, go to step b.
     Otherwise a better plan has been identified. In that
     case, the measures included in the plan are now the
     trial optimal plan and the trial optimum is the
     objective function value for this plan. Go to step b.

k.   Backtrack. - Eliminate the measure included for site S
     in the plan. Reconsider the previous site (set S=S-1).
     If no such site exists (if S=0), go to step l.
     Otherwise go to step e.

l.   Terminate. - Does the best plan identified include
     status quo for all sites? If so, none of the plans has
     positive net benefit, and none is economically
     feasible. Otherwise the optimal value of the objective
     function is the current recorded value, and the optimal
     plan includes the measures recorded in step j. Stop.

     Example Problem Solution. - The steps of application of the
branch-and-bound method to identification of the optimal plan for
the Loucks Creek catchment are as follows:

1.   The sites are numbered 1 and 2 for the Loucks Creek
      catchment. The measures are as shown in Table 3. The
      first plan to be evaluated includes status quo for
      sites 1 and 2. The trial optimum is set to $-999
      initially. The site pointer, S, is set to 1 at this
      point.

2.   The objective function, computed in step i, is base-
      condition damage at Cronell + base-condition damage at
      Icatha - damage at Cronell with status quo for site 1 -
      the cost of status quo at site 1 - damage at Icatha
      with status quo for site 2 - the cost of status quo at
      site 2 = $1,200,000 + 500,000 - 1,200,000 - 0 - 500,000
      - 0 = 0.

3.   By comparison in step j, this plan is an improvement,
      so the status quo plan is noted as the trial optimal
      plan, and the trial optimal objective function value is
      now 0.

4.   The subset bound is evaluated in step b for site S.
      S=1, so this corresponds to evaluating the subset bound
      for all plans that include the first measure for site
      1. The subset bound is base-condition damage at Cronell
      + base-condition damage at Icatha - damage at Cronell
      with status quo for site 1 - the cost of status quo
      measure at site 1 = $1,200,000 + 500,000 - 1,200,000 -
      0 = 500,000. These damage values are determined from
      computations previously performed in step i. The subset
      bound exceeds the trial objective function value, so in
      step c, set S = S + 1 = 2. The subset is subdivided
      further.

5.   In step e, status quo for site 2 is replaced with channel
      improvement. This creates another complete plan.

6.   The objective function is evaluated in step i. The
      objective function value is base-condition damage at
      Cronell + base-condition damage at Icatha - damage at
      Cronell with status quo for site 1 - the cost of status
      quo at site 1 - damage at Icatha with channel
      improvement for site 2 - the cost of the channel
      improvement at site 2 = $1,200,000 + 500,000 -
      1,200,000 - 0 - 200,000 - 200,000 = 100,000.

7.   The objective function value computed in step i exceeds
      the trial optimum, so the trial optimal objective
      function value is updated in step j, and the measures
      included in this plan are recorded as the trial optimal
      plan.

8.   The subset bound has been evaluated previously in step
      b, so in step d, the channel improvement measure for

19

site 2 is replaced with flood-proofing.

9. The objective function is evaluated in step i. The objective function value is base-condition damage at Cronell + base-condition damage at Icatha - damage at Cronell with status quo for site 1 - the cost of status quo at site 1 - damage at Icatha with flood-proofing for site 2 - the cost of the flood-proofing at site 2 = $1,200,000 + 500,000 - 1,200,000 - 0 - 335,000 - 150,000 = 15,000.

10. The objective function value is less than the best value found thus far, so no updating is required in step j.

11. In step e, it is noted that all measures proposed for site 2 have been considered with status quo for site 1, so in step k the first measure for site 1 is replaced with the second measure proposed.

12. The first measure for site 2 is added to the plan in step g.

13. The objective function is evaluated in step i. The net benefit is base-condition damage at Cronell + base-condition damage at Icatha - damage at Cronell with the detention structure for site 1 - the cost of the detention structure at site 1 - damage at Icatha with status quo for site 2 - the cost of status quo at site 2 = $1,200,000 + 500,000 - 459,000 - 500,000 - 191,000 - 0 = 550,000.

14. The objective function value exceeds the best value found thus far, so the trial optimum is updated in step j. The best plan found thus far includes the detention structure for site 1 and status quo for site 2.

15. The subset bound is evaluated for all plans that include the second measure for site 1. This value is base-condition damage at Cronell + base-condition damage at Icatha - damage at Cronell with the detention structure for site 1 - the cost of the detention structure at site 1 = $1,200,000 + 500,000 - 459,000 - 500,000 - 0 = 741,000. The subset bound exceeds the trial optimal objective function value ($550,000) so the subset is not eliminated; the optimal plan may be in this subset.

16. Status quo is replaced with channel improvement for site 2 in step e.

17. The objective function is evaluated in step i. The net benefit is base-condition damage at Cronell + base-condition damage at Icatha - damage at Cronell with the

detention structure for site 1 - the cost of the
detention structure at site 1 - damage at Icatha with
channel improvement for site 2 - the cost of channel
improvement at site 2 = $1,200,000 + 500,000 - 459,000
- 500,000 - 0 - 200,000 = 541,000.

18. The objective function value is less than the best
value found thus far, so no updating is required in
step j.

19. The channel improvement for site 2 is replaced with
flood-proofing in step e, and the constraints and the
objective function are evaluated in steps h and i,
respectively. The objective-function value is base-
condition damage at Cronell + base-condition damage at
Icatha - damage at Cronell with the detention structure
for site 1 - the cost of the detention structure at
site 1 - damage at Icatha with flood-proofing for site
2 - the cost of flood-proofing at site 2 = $1,200,000 +
500,000 - 459,000 - 500,000 - 26,000 - 150,000 =
565,000.

20. The objective-function value exceeds the trial optimum,
so the measures included are recorded and the trial
optimum value is updated in step j.

21. All measures proposed for site 2 have been considered
with the detention structure for site 1, so now, via
backtracking in step k, the floodwall is considered
for site 1.

22. The first measure (status quo) is included for site
2 in step f.

23. The plan is evaluated in steps h and i. The net
benefit is base-condition damage at Cronell + base-
condition damage at Icatha - damage at Cronell with the
floodwall for site 1 - the cost of the floodwall at
site 1 - damage at Icatha with status quo for site 2 -
the cost of status quo at site 2 = $1,200,000 + 500,000
- 150,000 - 1,000,000 - 500,000 - 0 = 50,000.

24. The objective function value does not exceed the best
value found thus far, so no updating is required in
step j.

25. The subset bound is evaluated for all plans that
include the floodwall for site 1. This value is base-
condition damage at Cronell + base-condition damage at
Icatha - damage at Cronell with the floodwall for site
1 - the cost of the floodwall at site 1 = $1,200,000 +
500,000 - 150,000 - 1,000,000 = 550,000. In this case
the subset bound is less than the trial optimum, so all
plans that include the floodwall at site 1 can be

eliminated from further consideration.

The steps of the branch-and-bound procedure are summarized in Table 5.

The economic optimal plan for the Loucks Creek catchment includes the detention structure at site 1 and flood-proofing at site 2. The net benefit is $565,000. This plan is identified in the sixth evaluation of the objective function for a complete plan. Two of nine possible plans are eliminated from consideration through comparison of the subset bound. This represents a significant reduction in effort for evaluation of the proposed plans, with this simple example. Using information readily available from evaluation of a limited number of plans, inferior plans are eliminated without evaluation. In this case, approximately 25 percent of the plans are thus eliminated. However, owing to the systematic branch-and-bound procedure, it is certain that the optimal plan is not eliminated. In a more complex catchment, the results may be much more dramatic, as a significant portion of the alternatives may be eliminated without explicit evaluation. Regardless of the complexity of the plans, however, the procedure is guaranteed to identify the optimal plan.

CONCLUSIONS

The optimal flood-damage-mitigation plan, from the national economic development standpoint, is the plan that yields the maximum net economic benefit consistent with all environmental, institutional, social, and financial requirements. Such a mitigation plan typically consists of a combination of measures that modify the flood, measures that modify damage susceptibility, and measures that modify loss burden. Given proposed alternative mitigation measures for various sites in a catchment, a branch-and-bound procedure is an efficient, systematic approach for identifying the optimal combination. With such a procedure, the entire set of plans is divided repeatedly into subsets which are evaluated in the detail necessary to determine that the optimal plan is not in the subset or to identify it if it is. Hydrologic Engineering Center (HEC) computer programs provide technical information required for the evaluation.

ACKNOWLEDGEMENTS

Table 5. – Summary of Branch-and-bound Enumeration Steps for Loucks Creek Catchment

Measure for site 1 (1)	Measure for site 2 (2)	Subset bound or objective function of Eq. 1 (3)	Trial optimum (4)
–	–	–	-999
1	1	0	0
1	–	500	0
1	2	100	100
1	3	15	100
2	1	550	550
2	–	741	550
2	2	541	550
2	3	565	565
3	1	50	565
3	–	540	565

APPENDIX I. - REFERENCES

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