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14. ABSTRACT
A reservoir system analysis computer model has been recently developed with the capability to simulate up to ten reservoirs, thirty control points and eight water quality parameters. With this model the user can evaluate a "best" system operation analysis for multipurpose reservoir regulation to obtain target water quality conditions at user specified control points.

The model uses a linear programming algorithm to evaluate the "best" system operation among all the reservoirs and a nonlinear routine for operation of multilevel intakes at any one reservoir in the system. The user may select to operate the system for a balanced reservoir pool operation and its associated water quality or to allow for a modified for distribution between reservoirs to improve the water quality operation.

The water quality routines are capable of analyzing water temperature and up to three conservative and three non-conservative constituents. If at least on of the non-conservative constituents is an oxygen demanding parameter, dissolved oxygen can also be analyzed.

15. SUBJECT TERMS
reservoir system analysis, water quality, optimization, reservoir operation

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US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

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RESERVOIR SYSTEM ANALYSIS FOR WATER QUALITY [*]

By

JAMES H. DUKE[1/], DONALD J. SMITH[2/] AND R. G. WILLEY[3/]

INTRODUCTION

Water managers have been attempting to operate reservoirs in a "best water management strategy" to meet specific project objectives since the first man-made impoundment was constructed. Two things, however, require water managers to occasionally reevaluate a "best water management" strategy. As time goes on, project objectives may change due to either an individual owner/operator's needs and desires or, in the case of large projects, due to public interests changing or finally being recognized. Secondly, state-of-the-art techniques for impoundment regulation change, and may require a reevaluation of operational impacts due to structural or nonstructural changes.

STATE-OF-THE-ART

While water managers are probably performing a best project operation for water quality control for the existing structural facility, studies should be performed to evaluate possible improvements that could be provided by state-of-the-art structural modifications and/or multi-reservoir system operations. Evaluation of the water quality benefits due to possible structural modifications can be performed with several existing state-of-the-art one-dimensional computer programs. [14,26,22,15].

Computer programs to evaluate the impact on water quality due to a specific operation of a large system of reservoirs are not readily available. When it was realized, several years ago, that the U.S. Army Corps of Engineers must have the capability to analyze the operation of large multireservoir systems for water quality, the Hydrologic Engineering Center, HEC, was funded to develop a computer program to meet this need.

PROGRAM DEVELOPMENT

In 1978, various computer programs available within the Corps for evaluating reservoir system operations for water quantity were screened to obtain the best generalized model for adding water quality capability [27,12,16]. The "Simulation of Flood Control and Conservation Systems" Computer Program, HEC-5, was selected due to its generality, documentation, and level of active support in training and maintenance.

The HEC-5 program is designed to simulate the sequential operation of a reservoir-channel system of a branching network configuration. Any time interval from one hour to a month can be used. The model contains the capability to change from one time step to another in order to provide greater temporal resolution during certain periods, such as floods. Channel routing is provided by any of five hydrologic routing techniques. Reservoirs operate to (1) minimize downstream flooding; (2) evacuate flood control storage as quickly as possible; (3) provide for low flow requirements and diversions, and (4) meet hydropower requirements. Hydropower requirements can be defined for individual projects or for a system of projects. Pumped-storage operation can also be simulated. Sizing for conservation demands or storage can be automatically performed, using the safe yield concept, and economic computations can be provided for hydropower benefits and flood damage evaluation.

In 1979, work was initiated to modify HEC-5 to evaluate reservoir operations for water quality control in large reservoir systems. The modifications were identified to be accomplished in three phases as shown in Figure 1. Phase I added the capability to HEC-5 to control water temperature releases at one reservoir to provide for the best combination of downstream needs at up to three control points (i.e, river locations used for controlling flow routing computations, and for controlling quantity and quality target computations). The control of water temperature is accomplished through multilevel intake structure operation. In September 1979, a single reservoir water temperature control program called HEC-5Q [17] was completed.

In 1980, work was initiated to modify the phase I model to add seven more water quality parameters and capability to evaluate either two tandem (i.e., in series) or two parallel (i.e., on two independent tributaries) reservoirs. In September 1980, a two-reservoir model [18] capable of system operation for three conservative and three nonconservative water quality parameters, in addition to dissolved oxygen and water temperature was completed.

Following the phase II development, the model was modified with some small but significant additions and revisions during 1981. These modifications included flow augmentation, improved model efficiency, and recently developed selective withdrawal routines.

The third and last major phase of development involves increasing the HEC-5Q capability to include up to ten reservoirs and thirty control points. The HEC is currently testing the phase III model on a practical application.

MODEL CONCEPTS

Flow Simulation Module

The flow simulation module was developed to assist in planning studies for evaluating proposed reservoirs in a system and to assist in sizing the flood control and conservation storage requirements for each project recommended for the system. The program can be used in studies made immediately after the occurrence of a flood to evaluate preproject conditions and to show the effects of existing and/or proposed reservoirs on flows and damages in the system. The program should also be useful in selecting the

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PHASE I - 1979

SINGLE RESERVOIR SIMULATION
FOR WATER TEMPERATURE

PHASE II - 1980

TWO RESERVOIR SIMULATION
FOR WATER TEMPERATURE
AND SEVEN CONSTITUENTS

1981

FIELD TESTING AND MINOR
MODIFICATIONS

PHASE III - 1982

TEN RESERVOIR SIMULATION
FOR WATER TEMPERATURE AND
SEVEN CONSTITUENTS

1983

FIELD TESTING AND FINALIZE MODEL
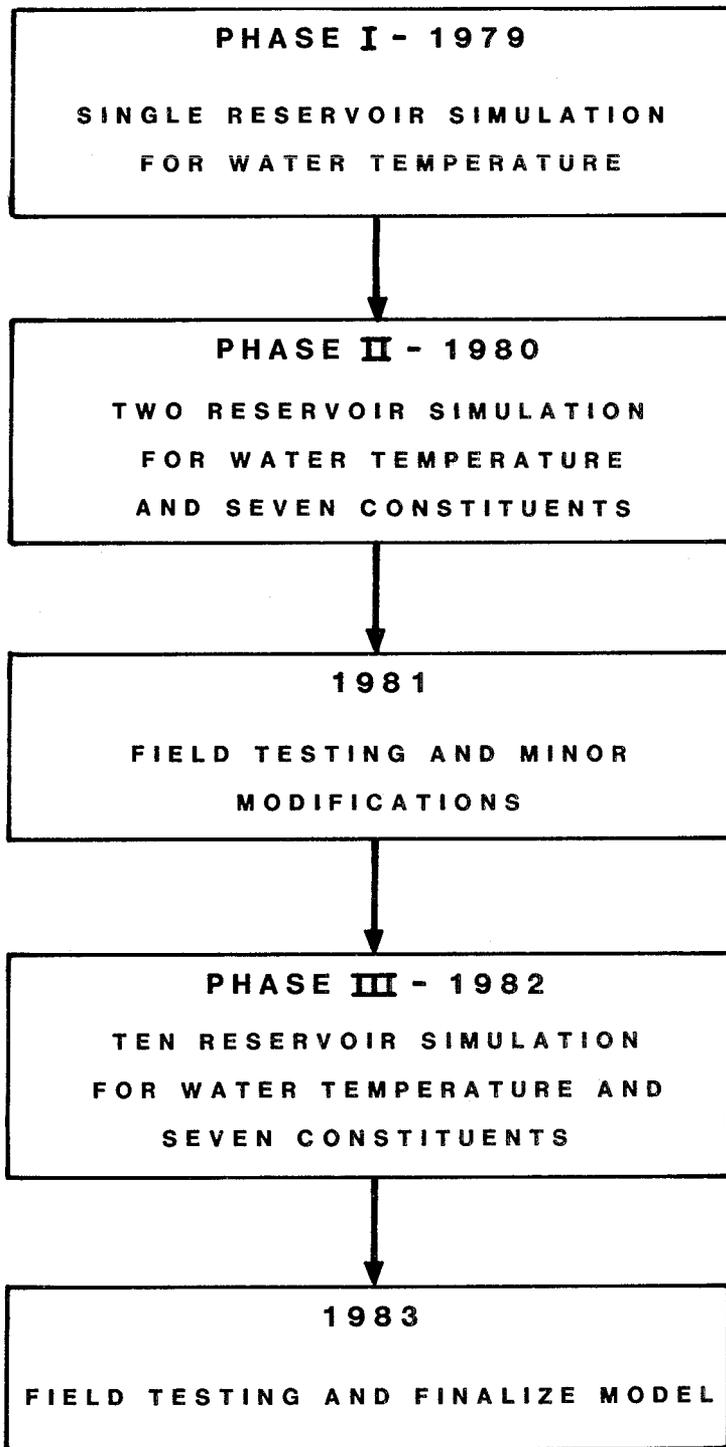
FIGURE 1

Phased Development of HEC-5Q

proper reservoir releases throughout the system during flood emergencies in order to minimize flooding as much as possible and yet empty the system as quickly as possible while maintaining a balance of flood control storage among the reservoirs.

The above purposes are accomplished by simulating the sequential operation of a system of reservoirs of any configuration for short interval historical or synthetic floods or for long duration nonflood periods or for combinations of the two. Specifically the program may be used to determine:

a. Flood control and conservation storage requirements for each reservoir in the system.

b. The influence of a system of reservoirs on the spatial and temporal distribution of runoff in a basin.

c. The evaluation of operational criteria for both flood control and conservation (including hydropower) for a system of reservoirs.

d. The expected annual flood damages, system costs, and system net benefits for flood damage reduction.

e. The system of existing and proposed reservoirs or other alternatives including nonstructural alternatives that result in the maximum net flood control benefits for the system by making simulation runs for selected alternative systems.

Since many papers [19,4,8,9,7,11,23,10] have already been written regarding the detailed technical concepts of the flow simulation module, the remainder of this paper will emphasize the concepts of the water quality simulation module.

Water Quality Simulation Module

The water quality simulation module was developed so that temperature, three user selected conservative and three user selected non-conservative constituents can be simulated. The model allows dissolved oxygen to also be simulated if the user selects either carbonaceous or nitrogeneous oxygen demanding constituents, or both, as one or two of the non-conservative constituents.

The water quality simulation module accepts system flows generated by the flow simulation module and computes the distribution of all the water quality constituents in up to ten reservoirs and their associated downstream reaches. The ten reservoirs may be in any arbitrary parallel and tandem configuration.

The water quality simulation module also selects the gate openings for reservoir selective withdrawal structures to meet user-specified water quality objectives at downstream control points. If the objectives cannot be satisfied, the model will compute the increase in flow (if any) necessary to satisfy the downstream objectives. With these capabilities, the planner may evaluate the effects on water quality of proposed reservoir-stream system modifications and determine how a reservoir intake structure should be operated to achieve desired water quality objectives within the system.

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Each reservoir is assumed to be a control point, in keeping with the concepts used in the development of the flow simulation module. The water quality module will allow for up to thirty control points, including the reservoir control points. The additional control points may be placed in the stream system below the reservoirs at any desirable locations provided the following guidelines are followed:

a. The most downstream point in all systems must be a control point.

b. The confluence of the two streams, on which parallel reservoirs are located, must be a control point.

c. The end of the stream reach below the more upstream reservoir of a tandem reservoir system (at the upstream end of the more downstream reservoir) must be a control point.

The water quality simulation module currently uses flow data from the flow simulation module at intervals of one day and uses computational time steps of one day. Shorter simulation time steps can be used but the model has not been tested with the shorter steps. The model is limited to simulations of one calendar year.

The reservoirs are represented conceptually by series of one dimensional horizontal slices such as those shown in Figure 2. Each horizontal slice or layered volume element is characterized by an area, thickness and volume. In the aggregate the assemblage of layered volume elements is a geometric representation in discretized form of the prototype reservoir. This one dimensional representation has been shown to adequately represent water quality conditions in many deep, well stratified reservoirs by Eiker [2], Baca [2] and WRE [28,29,30].

Within each element, the water is assumed to be fully mixed. This implies that only the vertical dimension is retained during the computation. Each horizontal layer is assumed to be completely homogeneous with all isopleths parallel to the water surface both laterally and longitudinally. External inflows and withdrawls occur as sources or sinks within each layer and are instantaneously dispersed and homogeneously mixed throughout each element from the headwaters of the impoundment to the dam. It is not possible, therefore, to look at longitudinal variations in water quality constituents. Module results are most representative of conditions in the main reservoir body.

Vertical advection is governed by the location of inflow to, and outflow from, the reservoir. Thus the computation of the zones of distribution and withdrawal for inflows and outflows are of considerable significance in operation of the model. The WES withdrawal method [3] is used for determining the allocation of outflow. The Debler inflow allocation method [5] is used for the placement of inflows.

Vertical advection is the net interelement flow and is one of two transport mechanisms used in the module to transport water quality constituents between elements. The vertical advection is defined as the interelement flows which result in a continuity of flow in all elements.

Evaporation

Tributary
Inflow

Tributary
Inflow

**Vertical Advection &
Effective Diffusion**

Volume Element

Outlet

Outflow
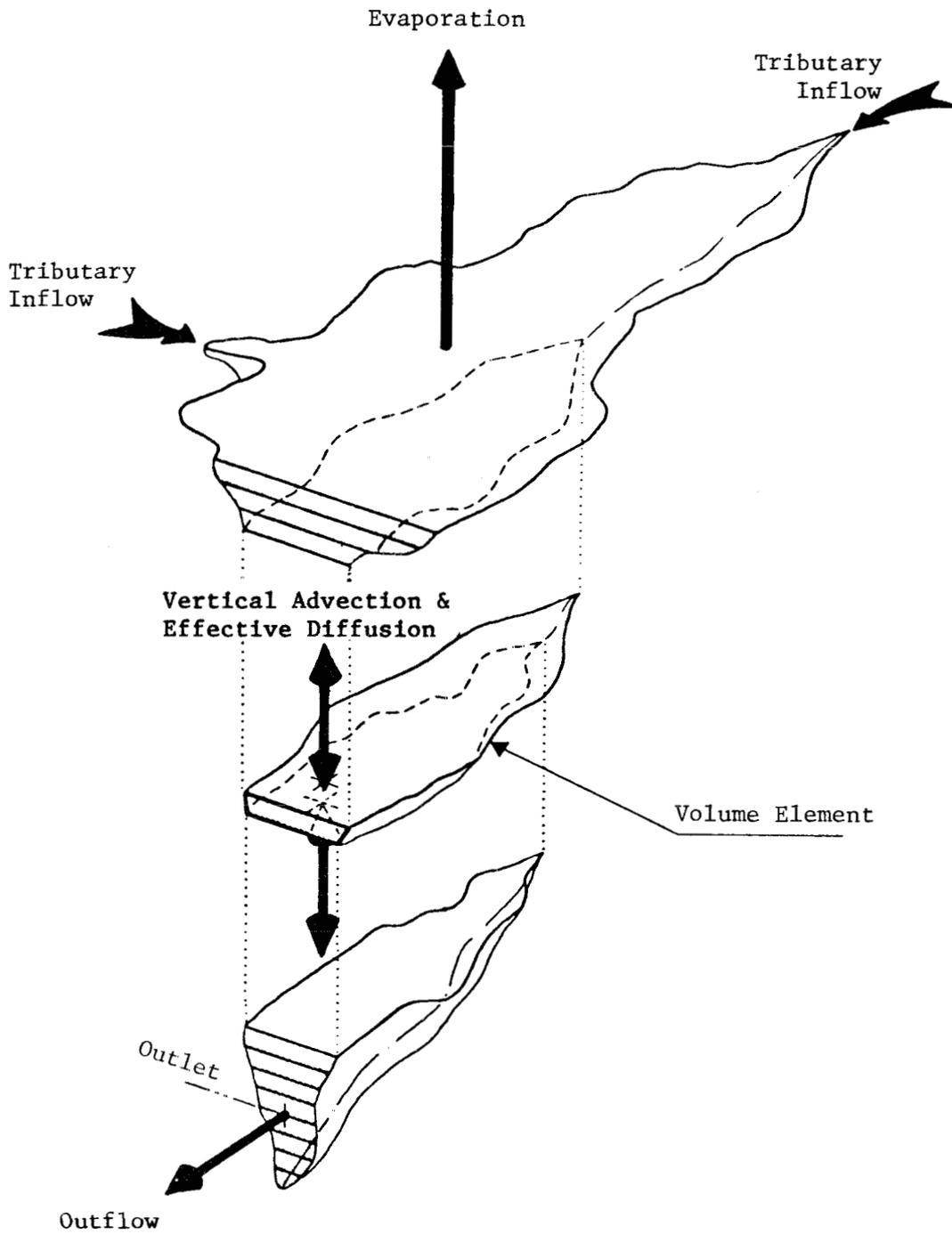
FIGURE 2

Geometric Representation of a Stratified Reservoir
and Mass Transport Mechanisms

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Effective diffusion is the other transport mechanism used in the module to transport water quality constituents between elements. The effective diffusion is composed of molecular and turbulent diffusion and convective mixing.

Wind and flow induced turbulent diffusion and convective mixing are the dominant components of effective diffusion in the epilimnion of most reservoirs. In quiescent well stratified reservoirs, molecular diffusion may be a significant component in the metalimnion and hypolimnion. For deep, well stratified reservoirs with significant inflows to or withdrawals from the hypolimnion, flow induced turbulence in the hypolimnion dominates. For weakly stratified reservoirs, wind induced or wind and flow induced turbulent diffusion will be the dominant component of the effective diffusion throughout the reservoir. One of two methods may be selected by the user to calculate effective diffusion coefficients. For shallow weakly stratified reservoirs, the wind controlled mixing [15] method is appropriate, while the stability method [15] is more appropriate for deeper well stratified reservoirs. Both of these methods have been shown in numerous applications to adequately represent the mixing phenomena for heat and dissolved water quality constituents when properly applied.

The stream system is represented conceptually as a linear network of segments or volume elements. Each element is characterized by length, width, cross-sectional area, hydraulic radius, Manning's n and a flow and depth relationship (see Figure 3). Flow rates at stream control points are calculated within the flow simulation module using any one of the several programmed hydrologic routing methods. Within the flow simulation module, incremental local flows (i.e., inflow between adjacent control points) are assumed deposited at the control point.

Within the water quantity simulation module, the incremental local flow may be divided into components and placed at different locations within the stream reach (i.e., that portion of the stream bounded by the two control points). A flow balance is used to determine the flow rate at element boundaries. Any flow imbalance (i.e., the difference in the flow at the upstream control point plus all tributary inflows and the flow at the downstream control point) is distributed uniformly to the flows at each element boundary. Once interelement flows are established, the depth, surface width, and cross section area are computed at each element boundary assuming normal flow.

Both the streams and reservoirs are represented by a one-dimensional assemblage of fluid elements linked together by interelement flow and diffusion (stream diffusion is assumed to be small). The interelement mass transports and the fundamental principle of conservation of heat can be represented by the following differential equation model of the dynamics of temperature within each fluid element.

Control
Point

Control Volume Elements

Control Volume Nodes

Advection

Tributary
Inflow

Control Point

Outflow
or Withdrawal

Advection
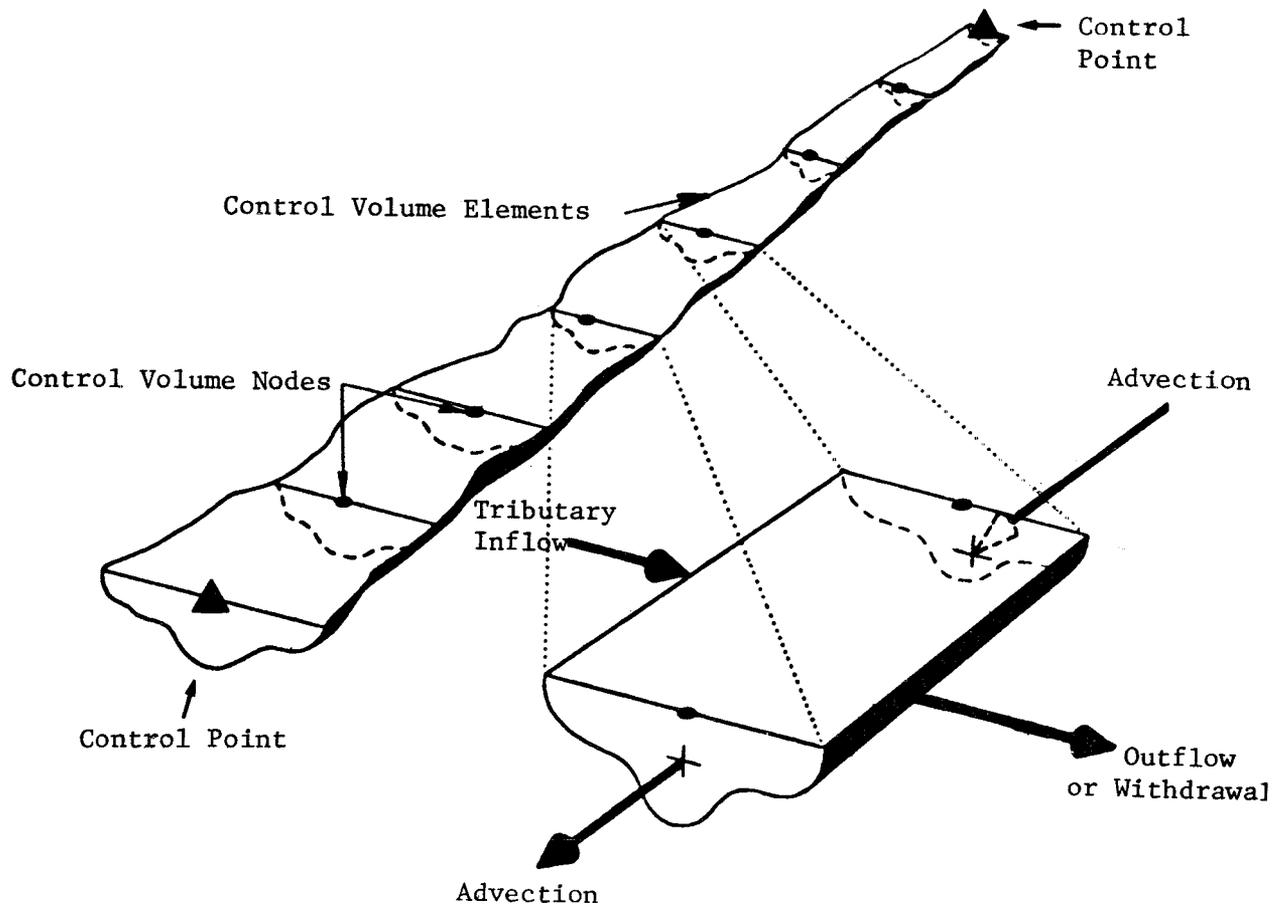
FIGURE 3

Geometric Representation of Stream System and
Mass Transport Mechanisms

$$V \frac{\partial T}{\partial t} = \Delta z \cdot Q_z \frac{\partial T}{\partial z} + \Delta z \cdot A_z \cdot D_z \frac{\partial^2 T}{\partial z^2} + Q_i \cdot T_i - Q_o \cdot T + \frac{A_h H}{\rho \cdot c} - T \frac{\partial V}{\partial t} \qquad (1)$$

where
T = temperature in °C
V = volume of the fluid element in m^3
t = time in seconds
z = space coordinate in meters (vertical for the reservoir and horizontal for the stream)
Q_z = interelement flow in m^3/s
A_z = element surface area normal to the direction of flow in m^2
D_z = effective diffusion coefficient in m^2/s
Q_i = lateral inflow in m^3/s
T_i = inflow water temperature in degrees Celsius
Q_o = lateral outflow in m^3/s
A_h = element surface area in m^2
H = external heat sources and sinks in $kcal/m^2/s$
ρ = water density in kg/m^3
c = specific heat of water in $kcal/kg/°C$

This equation represents the heat conservation equation for the fluid element. The set of equations for all elements within the reservoir or stream system represents the heat conservation within that system. All of the terms on the right side of Equation (1) represent physical heat transfers including the external heat sources and sinks. The external heat sources and sinks that are considered in the module are assumed to occur at the air-water interface. The rate of heat transfer per unit of surface area can be expressed as the sum of the following heat exchange components.

$$H_n = H_s - H_{sr} + H_a - H_{ar} \pm H_c - H_{br} - H_e \qquad (2)$$

where
H_n = net heat transfer
H_s = short wave solar radiation arriving at the water surface
H_{sr} = reflected short wave radiation
H_a = long wave atmospheric radiation
H_{ar} = reflected long wave radiation
H_c = heat transfer due to conduction
H_{br} = radiation from the water surface
H_e = heat loss due to evaporation

All units are in $kcal/m^2/s$.

Complete discussions of the individual terms have been presented by Anderson [1] and in Tennessee Valley Authority Report No. 14 [25].

The method used in the module to evaluate the net rate of heat transfer at the air-water interface was developed by Edinger and Geyer [6]. Their method utilized the concepts of equilibrium temperature and the coefficient of surface heat exchange. The equilibrium temperature is defined as the water temperature at which the net rate of heat exchange between a water surface and the atmosphere is zero. The coefficient of surface heat exchange is the rate at which the heat transfer process proceeds. The equation describing this relationship is:

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$$H_n = K_e (T_e - T_s) \tag{3}$$

where
H_n = net rate of heat transfer in $kcal/m^2/s$
K_e = coefficient of surface heat exchange in $kcal/m^2/s/°C$
T_e = equilibrium temperature in °C
T_s = surface temperature in °C

A Heat Exchange Program which computes these terms is available at the HEC [26].

All heat transfer mechanisms except short wave solar radiation apply at the water surface. Short wave radiation penetrates the water surface and may affect water temperatures several meters below the surface. The depth of penetration is a function of adsorption and scattering properties of the water [13]. This phenomenon is unimportant in the stream routines since elements are assumed vertically mixed.

In the reservoir routines, however, the short wave solar radiation may penetrate several elements. The amount of heat which reaches each element is determined by:

$$I = (1 - \beta) \, I_o e^{-kz} \tag{4}$$

where
I = light energy at any depth in $kcal/m^2/s$
β = fraction of the radiation absorbed in the top foot of depth
I_o = light energy at the water surface in $kcal/m^2/s$
k = light extinction coefficient in $1/m$
z = depth in meters

Combining equations (3) and (4) for the reservoir surface element, the external heat source and sink term becomes:

$$H = K_e(T_e - T_s) - (1 - \beta) \, I_o e^{-k\Delta z} \tag{5}$$

and the external heat source for all remaining reservoir elements becomes:

$$I = I_z(1 - e^{-k\Delta z}) \tag{6}$$

where I_z = light intensity at the top of the element in $kcal/m^2/s$

Water quality constituents other than temperature are represented by Equation (1) with minor modifications:

a. The definition of the variable T is generalized to represent the concentration of any water quality constituent.

b. The distributed heat gain/loss term $A_h H/(\rho c)$ is :

(1) Eliminated for conservative constituents

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(2) Replaced by a first order kinetic decay formulation, $-K_1T$, for non-conservative constituents where K_1 is the decay rate in 1/day.

(3) Replaced by a first order reaeration term, $K_2(DO*-DO)$, for dissolved oxygen where K_2 is the reaeration rate, $DO*$ is the dissolved oxygen saturation concentration at the ambient temperature and DO is the existing dissolved oxygen concentration.

The reservoir reaeration rate is computed as follows:

$$K_2 = (a + bW^2)/z \tag{7}$$

where
K_2 = reaeration rate in 1/day at 20°C
a, b = empirical coefficients derived by curve fit from Kanwisher [20] to be 0.641 and 0.128 respectively.
W = wind speed in m/s
z = surface element thickness in meters

The stream reaeration rate is computed using the O'Connor-Dobbins [24] method:

$$K_2 = (D_m U)^{0.5}/D^{1.5} \tag{8}$$

where
K_2 = reaeration rate in 1/day at 20° C
D_m = molecular diffusion coefficient in m^2/day
U = flow velocity in m/s
D = average stream depth in meters

All first order kinetic rates are adjusted for local ambient temperatures using a multiplicative correction factor.

$$\Theta = T_c^{(T-20)} \tag{9}$$

where
Θ = kinetic rate multiplicative correction factor
T_c = empirically determined temperature correction factor
T = local ambient water temperature

Reservoir Solution Technique

Within the reservoirs, a Gaussian reduction scheme is used for solving the differential equations which represent the response of the water quality constituents. Equation (1) is rewritten in a form where a "forward time and central difference" scheme is used to describe all the derivative processes. For element i adjacent to elements i-1 and i+1 (see Figure 4) the general mass balance equation becomes:

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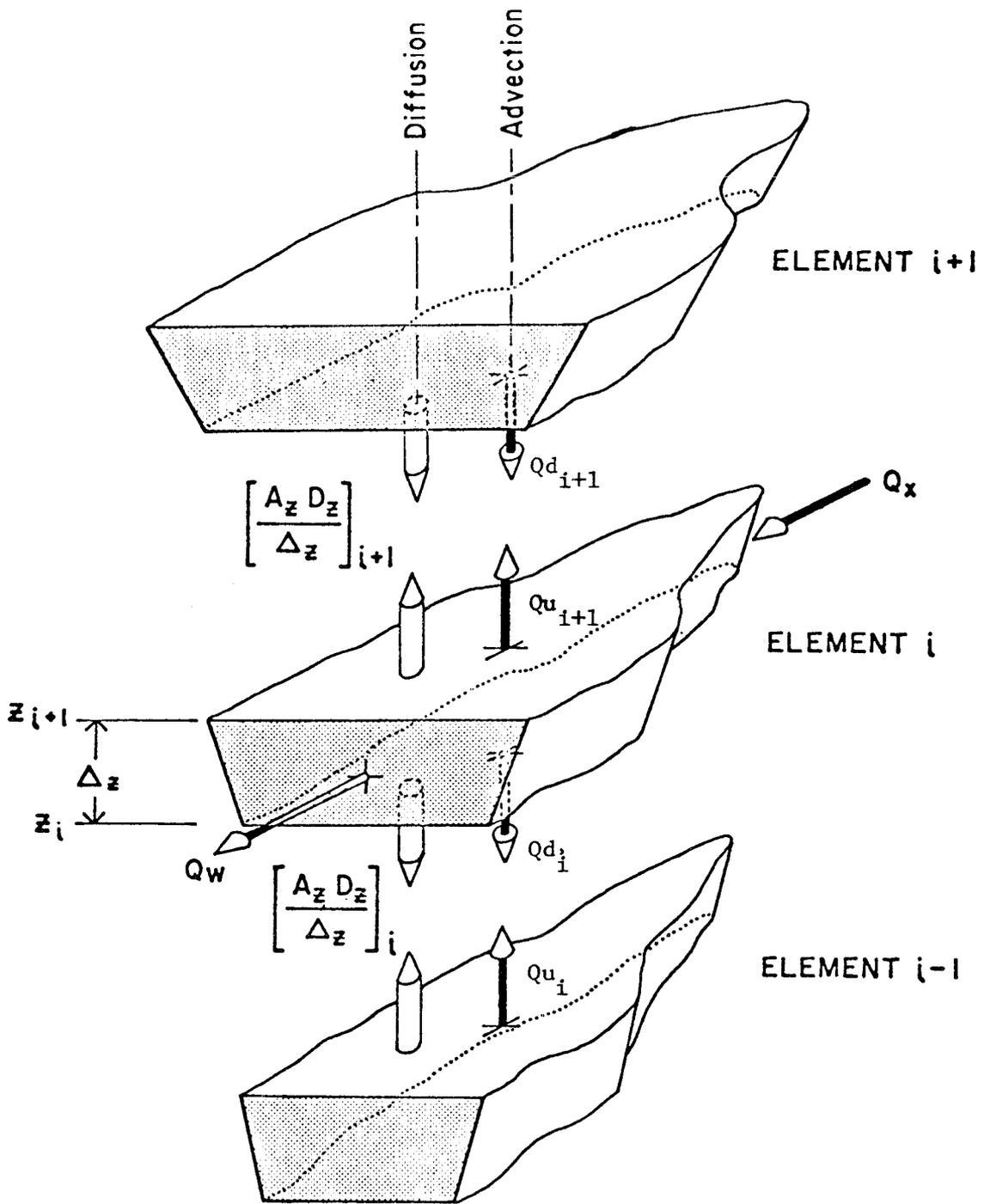
FIGURE 4

Physical Mass Transfers Between Elements

$$V_i \left[\frac{\partial T}{\partial t}\right]_i = T_{i-1}\left\{\left[\frac{A_z D_z}{\Delta z}\right]_i + Qu_i\right\} - T_i\left\{\left[\frac{A_z D_z}{\Delta z}\right]_i + \left[\frac{A_z D_z}{\Delta z}\right]_{i+1} + Qd_i\right.$$

$$\left. + Qu_{i+1} + Qw + \frac{\partial V}{\partial t}\right\} + T_{i+1}\left\{\left[\frac{A_z D_z}{\Delta z}\right]_{i+1} + Qd_{i+1}\right\} + \Sigma Q_x T_x + \frac{H}{\rho c} \tag{10}$$

where   subscripts i, i-1, i+1 denote element numbers, and

V = volume of the fluid element in m^3
T = temperature in °C or water quality
      constituent concentration, mg/l
t = computation time step in seconds
A_z = element area at the fluid element boundary in m^2
D_z = effective diffusion coefficient in m^2/s
Δz = element thickness (length of stream) in meters
Q_u = upward advective flow (stream flow) between elements in
      m^3/s
Q_d = downward advective flow between elements in m^3/s
Q_w = rate of flow removal from the element in m^3/s
Q_x = rate of inflow to the element in m^3/s
T_x = inflow water temperature in °C or constituent
      concentration in mg/l
H = external sources and sinks of heat in kcal/s
ρ = water density in kg/m^3
c = specific heat of water in kcal/kg/°C

Recall that the $H/(\rho c)$ term is replaced by $-K_1 T$ or $K_2(DO*-DO)$ for non-conservative water quality constituents and dissolved oxygen respectively. A finite difference equation of this type is formed for each element and integrated with respect to time. The system of finite difference mass balance equations represents the response of water quality within the entire reservoir and, with the aid of numercial integration technique, the equations are solved with respect to time.

Stream Solution Technique

For the stream, a linear programming algorithm is used to solve a fully implicit backward difference in space, forward difference in time, finite difference approximation of Equation (1). This approximation has the general form

$$a_{i,i-1} C_{i-1}^{t+1} + a_{i,i} C_i^{t+1} + a_{i,i+1} C_{i+1}^{t+1} = b_i \tag{11}$$

where the "a" terms are coefficients formed from the area, dispersion coefficients, flows, lengths of the computational elements and time step for each volume element; the "C" terms are the unknown temperatures and constituent concentrations in each volume element; the "b" terms are constants formed from initial conditions or previously computed conditions, tributary inputs of heat or mass loads and, depending upon the context, the reservoir releases.

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Two matrix formats are used in the stream water quality simulation module. The first is used to solve for temperature and constituent concentrations given all external inputs this format is

$$|A|\,\bar{c} = \bar{b} \tag{12}$$

where $|A|$ is the matrix of coefficients, $\bar{c}$ is the vector of unknown temperatures or constituent concentrations and $\bar{b}$ is the vector of constants. This first format is used in the water quality simulation module to compute the final results after all reservoir operations have been completed. In effect, the linear programming algorithm is used simply as a matrix solver for a simulation model.

The second, and more complex, matrix format used in the water quality simulation module is for determining the temperature and constituent concentrations that must come from the reservoirs to satisfy all water quality targets in the stream system. In effect, the second format is used to (1) determine which control point controls the release for each constituent and (2) determine the reservoir release water quality that most closely satisfies the targets at the controlling point. This decision making capability is achieved by (1) transforming the constituent concentrations at each control point into a specification of the target and the deviation of the simulated concentration above or below the target, and (2) making the concentrations in the reservoir releases unknown so that they can be computed.

The transformation used at control points to specify the target is:

$$C_i^{t+1} = C_{o_i} + C_{+i}^{t+1} - C_{-i}^{t+1} \tag{13}$$

where
C_i^{t+1} = simulated temperature in degrees Celsius or constituent concentration in milligrams/liter

C_{o_i} = target temperature in degrees Celsius or constituent concentration in milligrams/liter for a control point

C_{+i}^{t+1} = deviation of simulated temperature or constituent concentrations above the control point target

C_{-i}^{t+1} = deviation of simulated temperature or constituent concentration below the control point target

This transformation is substituted into Equation (11) to yield the following equation which is applied to those volume elements that are located at control points.

$$a_{i,i-1}C_{i-1}^{t+1} + a_{i,i}C_{+i}^{t+1} - a_{i,i}C_{-i}^{t+1} + a_{i,i+1}C_{i+1}^{t+1} = b_i - a_{i,i}C_{o_i} \tag{14}$$

where the $(a_{i,i}C_{oi})$ term has been moved to the right hand side of the equation since it is known. Thus, the m x m simulation matrix has now been transposed into a m x n rectangular matrix where n = m + NCP and NCP is the number of control points.

Equation (14) is the general form of the equation used for all volume elements in formulating decision making problem. It includes, as variables, the constituent concentrations in the reservoir releases, although the inclusion is not obvious. For those volume elments that are just below reservoirs, the C_{i-1}^{t+1} concentrations represent the constituent concentrations in the reservoir releases. In the simulation model, where the reservoir release constituent concentrations are known, the $a_{i,i-1} C_{i-1}^{t+1}$ terms were included in the $\overline{b}$ vector for those volume elements just below reservoirs. For the decision making model, the $a_{i,i-1} C_{i-1}^{t+1}$ terms are included as unknowns. Thus the m x n simulation matrix has been made even more elongated in variables and n is now m + NCP + NRES where NRES is the number of reservoirs in the system.

One additional set of equations is included in the water quality simulation module to ensure that realistic results are obtained in computing reservoir release water quality. These equations are applied to define the range of constituent concentrations that may be released from the reservoirs. Normally the range is defined by two inequalities:

$$C_r^{t+1} \geq C_{min}^{t+1} \qquad (15)$$

$$C_r^{t+1} \leq C_{max}^{t+1} \qquad (16)$$

where C_{min}^{t+1} = minimum temperature or constituent concentration in reservoir water quality profile

C_{max}^{t+1} = maximum temperature or constituent concentration in reservoir water quality profile

C_r^{t+1} = final computed temperature or constituent concentration in reservoir release

In practice, these inequalities are written as equalities by adding slack and surplus variables.

$$C_r^{t+1} - X_{surplus} = C_{min}^{t+1} \qquad (17)$$

$$C_r^{t+1} + X_{slack} = C_{max}^{t+1} \qquad (18)$$

With the problem so formulated, the $|A|$ matrix of Equation (12) consists of (m + 2 * NRES) rows and (m + NCP + NRES) unknowns and the $\overline{b}$ vector consists of (m + 2 * NRES) constants. The $|A|$ matrix may be conceptually partitioned as shown in Figure 5, where it is assumed that reservoirs are above volume elements 1 and 3, that these reservoirs are in tandem and that volume elements 1, 3, 7 and m are control points.

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	Stream Water Constituent Concentration Variables*	Variables for Negative Deviations at Control Points	Reservoir Release Variables	Slack and Surplus Variables
Element Numbers	a_{11} a_{12}	$-a_{11}$	$a_{0,1}$	
	a_{21} a_{22}	$-a_{21}$		
	a_{33} a_{34}	$-a_{33}$	$a_{2,3}$	
	a_{43} a_{44} a_{45}	$-a_{43}$		
	$\cdot$			
	a_{65} a_{66} a_{67}	$-a_{67}$		
	a_{76} a_{77} a_{78}	$-a_{77}$		
	a_{87} a_{88} a_{89}	$-a_{87}$		
	$\cdot$ $\cdot$ $\cdot$			
	$a_{m-1,m}$ $a_{m,m}$	$-a_{m-1,m}$		
		$-a_{m,m}$		
Inequality Statements			$+1$	-1
			$+1$ $+1$	$+1$
			$+1$	-1
			$+1$	$+1$

* Variables 1, 3, 7 and m are expressed as deviations from target concentrations at the respective control points.

FIGURE **5**

Structure of |A| Matrix for Two Tandem Reservoir, Four Control Point System With m Volume Elements

There are a number of solutions that will satisfy a matrix that is not square (i.e., m x m). The purpose of using a linear programming solution is to select the solution that best satisfies the objectives of the reservoir operation on downstream water quality. However, it is know that one class of solutions will never appear: at no time will the variables that describe the positive and the negative deviations from the control point target constituent concentrations appear simultaneously in the solution. At all times, one or the other deviation will appear but not both. It is also known that the reservoir release constituent concentrations will always appear in the final solution. Thus, selecting the solution that best satisfies the objectives of the reservoir operation on downstream water quality reduces to selecting which control point deviation variable appears in the final solution and the numerical value attached to that variable. Once this numerical value is known, it is known that the deviation of the opposite sign is zero so that the actual control point constituent concentration can be computed using Equation (13).

The objective function is used in a linear programming formulation to quantitatively describe the desirability of any given solution to a formulated problem. In the water quality simulation module, a minimization routine is used with the objective

$$\text{minimize } z = \bar{p}\,\bar{c} \tag{19}$$

The actual value of z is immaterial to the water quality simulation module; it is just an index by which the desirability of the solution is determined. The vector $\bar{c}$ is the same vector $\bar{c}$ as in Equation (12) except that, as previously described, it includes the variables representing

1. The deviations from the control point targets for those volume elements that represent control points,
2. The constituent concentrations in all other volume elements, and
3. The constituent concentrations in the reservoir releases.

The vector $\bar{p}$ represents the penalty associated with the appearance of a given variable in the final solution. Logically, the penalties in $\bar{p}$ are nonzero only at control points and are applied only for the variables that represent deviations from the target.

The water quality simulation module is structured flexibly so that different penalties can be assigned for each control point, for each constituent and for each deviation, above and below. The magnitude of the penalty is unimportant, as long as it is nonzero where necessary and realistically represents the desired policy. For instance, for a temperature target expressed as "the temperature at control point I shall not exceed 20° C", or

$$T_I \leq 20 \tag{20}$$

the penalty for the positive deviation at control point I could be set to 1.0 and the penalty for the negative deviation could be set to 0.0. Obviously, when trying to minimize z, Equation (19), the linear programming algorithm would try to ensure that the variable representing the negative deviation would appear in the final solution since a lower value of the index z would result.

If it was twice as important that the temperature target at control point I be maintained than at another control point, say J, then the penalty associated with a positive deviation from the target at I could be set to 2.0 and the penalty associated with a similar positive deviation at J could be set to 1.0. Of course, the penalties associated with negative deviations at both I and J would be set to 0.0.

Similar logic is used for setting penalties for constituents that must always exceed a target value, such as dissolved oxygen. The nonzero penalties are applied to the variables representing negative deviations, and the variables that represent positive deviations are given penalties of 0.

Gate Selection

Once the desired reservoir release water quality has been computed, using the above procedure, the water quality simulation module proceeds to determine: (1) the reservoir gates from which releases can be made; (2) the gates that should be used, and, (3) the water quality of the releases.

The port selection algorithm serves to determine which ports should be open and what flow rate should pass through each open port in order to maximize a function of the downstream water quality concentrations. Solution of this problem is accomplished by using mathematical optimization techniques. The objective function is related to meeting downstream target qualities subject to various hydraulic constraints on the individual ports.

Kaplan [21] solved a similar, although more difficult, problem by including in the constraint set upper and lower bounds on the relase concentration of each water quality constituent. Kaplan also considered as part of his objective function the reservoir water quality that resulted from any particular operation strategy. A penalty function approach was used to incorporate the many constraints into the objective function which could then be solved as an unconstrained nonlinear problem. For the problem of interest with respect to HEC-5Q, with appropriate transformations it is possible to formulate a quadratic objective function with linear constraints. Mathematical optimization techniques are available to exploit the special structure of this problem and to solve it efficiently.

The hydraulic structure under consideration is composed of two wet wells, containing up to eight ports each, and a flood control outlet. It is assumed that releases through any of these ports (including the flood control outlet) leave the reservoir through a common pipe. At any given time, only one port in either wet well and the flood control outlet may be operated. Hence, the algorithm provides flows through three ports at most.

The HEC-5Q model also provides for releases through an uncontrolled spillway. These releases are not a part of the gate selection algorithm, but the water quality of the spillway releases are considered by the gate selection algorithm.

The algorithm proceeds by considering a sequence of problems, each representing a different combination of open ports. For each combination the optimal allocation of total flow to ports is first determined and then a water quality index is determined for the optimal allocation of flows. The combination of open ports with the highest water quality index and its associated allocation of flows, define the optimal operation strategy for the time period under consideration.

There are four different types of combinations of open ports. For one-port problems, all of the flow is taken from a single port and the water quality index is computed. For two-port problems, combinations of one port in each wet well and combinations of each port with the flood gate are considered. For three-port problems, combinations of one port in each wet well and the floodgate are considered. The total flow to be released downstream is specified external to the port selection module, but if the flow alteration option is selected, then the flow can be treated as an additional decision variable and the flow for which the water quality index is maximized is also determined.

For each combination of open ports, a sequence of flow allocation strategies is generated using a gradient method, a gradient projection method, or a Newton projection method as appropriate. The value of any flow allocation strategy is determined by evaluation of a water quality index subject to the hydraulic constraints of the system. The sequence converges to the optimal allocation strategy for the particular combination of open ports.

To evaluate the water quality index for a feasible flow allocation strategy, first the release concentration for every water quality constituent is computed.

$$R_c = \frac{\displaystyle\sum_{p=1}^{N_p} \left[\Phi_{cp} \, Q_p \right]}{\displaystyle\sum_{p=1}^{N_p} Q_p} \; ; \quad c = 1, \, N_c \tag{21}$$

where:

c = index for constituents
R_c = release concentration for constituent c
p = index for open ports
N_p = number of open ports
Φ_{cp} = concentration of constituent c at port p
Q_p = flow through port p
N_c = number of constituents under consideration

The deviation of release qualities from downstream target qualities can be computed.

$$D_c = |R_c - T_c| \; ; \; c = 1, N_c \tag{22}$$

where:

D_c = deviation of constituent c
t_c = downstream target quality for constituent c

The subindex S_c for each constituent can be determined by:

$$S_c = f (D_c) \quad ; \quad c = 1, N_c \tag{23}$$

Where the function $f(D_c)$ takes the quadratic form:

$$f(D_c) = a + cD_c^2 \tag{24}$$

Suggested coefficients for some water quality constituents are shown in Table 1.

Table 1

Coefficients in Constituent Suboptimization

of Gate Selection Procedure

Quality	a	c
Temperature	100	-4.0
DO	100	-4.0
TDS	100	-0.000625
BOD	100	-0.444
E. Coli	100	-0.025
NH3	100	-0.16
NO3	100	-0.16

Finally, the scalar water quality index can be determined by:

$$Z = \sum_{c=1}^{N_c} W_c S_c \tag{25}$$

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where:

Z = water quality index

W_c = weighting factor for constituent c; the sum of the weighting factors for all constituents must equal one

S_c = subindex for constituent c

In summary, the problem of determining the optimal allocation of flows to ports for a particular combination of open ports and for a specified total flow rate Q, can be expressed as follows:

$$\underset{Q_p}{MAX} \left[\sum_{c=1}^{N_c} W_c S_c \right] \tag{26}$$

Subject to:

$$\sum_{p=1}^{N_p} Q_p = Q \tag{27}$$

$$F_{min,p} = Q_p = F_{max,p} \quad ; \quad p = 1, N_p \tag{28}$$

Where F_{min} and F_{max} are the minimum and maximum acceptable flow rates through a port.

When an acceptable flow range Q_{lower} to Q_{upper} is specified, then the problem is written as:

$$\underset{Q_p}{MAX} \left[\sum_{c=1}^{N_c} W_c S_c \right] \tag{29}$$

Subject to:

$$Q_{lower} \leq \sum_{p = 1}^{N_p} Q_p \leq Q_{upper} \tag{30}$$

$$F_{min,p} \leq Q_p \leq F_{max,p} \; ; \; P = 1, N_p \tag{31}$$

These problems are solved very efficiently by using mathematical optimization techniques that take advantage of the problem structure, namely a quadratic objective function with linear constraints.

Flow Alterations

The flow alteration routine is designed to change the reservoir releases, computed by the flow simulation module, to better satisfy the stream control point water quality objectives. The routine is designed about a mass balance for all reservoir releases and all control points affected by those releases. Tributary inflows and other flow changes are included. Second order effects, such as reaeration and external heating due to increased or decreased stream surface area are not included.

The procedure is as follows:

1. The relative mass that needs to be added in the flow at the control point (for those constituents below the target) or reduced in the flow at the control point (for those constituents above the target) is computed using:

$$\Delta M = Q_{cp} (C_o - C_{cp}) \tag{32}$$

where:

Q_{cp} = flow at the control point as determined by the flow simulation module

C_o = target constituent concentration at the control point

C_{cp} = computed constituent concentration at the control point

2. The average reservoir release concentration is computed for all reservoirs for which the constituent concentration in the releases is greater than the target concentration at the control point of interest (for those constituents below the target) or for which the constituent concentration in the releases is less than the target at the control point of interest (for those constituents above the target). Thus:

$$\overline{C}_R = (\sum_{i=1}^{n} Q_{Ri} C_i) / \sum_{i=1}^{n} Q_{Ri} \tag{33}$$

where:

$\overline{C}_R$ = average constituent concentration in reservoir releases for only those reservoirs releasing flow with constituent concentrations adequate to dilute the control point concentration and bring it to the target

Q_{Ri} = flow release from reservoir i

C_i = constituent concentration in release from reservoir i

n = number of reservoirs

and the sums are taken only over those reservoirs i that are capable of diluting the control point constituent concentration that is worse than the target.

3. The total dilution flow requirement is then computed by the following quotient:

$$Q_A = \frac{\Delta M}{\overline{C}_R} \tag{34}$$

where Q_A is the total flow release needed to bring the constituent concentration at the control point of interest to the target.

4. The flow Q_A is then apportioned to the reservoirs capable of bringing the control point constituent concentration to the target in proportion to the flows originally computed for those reservoirs by the flow simulation module.

Thus the flow augmentation requirement can be computed for each control point and for each constituent. The various computed flow rates are then combined by using the coefficients of the linear programming objective function and the deviation of the respective constituent concentrations from the target concentrations at each respective control point as follows:

$$Q_k = \frac{1}{\sum_{i=1}^{N_{cp}} \sum_{j=1}^{N_{cc}} P_{ij}(C_{ij}-C_{io})} \sum_{i=1}^{N_{cp}} \sum_{j=1}^{N_{cc}} Q_k P_{ij}(C_{ij}-C_{io}) \tag{35}$$

where:

Q_k = flow release from reservoir k

N_{cp} = number of control points affected by both reservoirs

N_{cc} = number of constituents

P_{ij} = linear programming objective function coefficient for constituent j at control point i

C_{ij} = computed concentration of constituent j at control point i

C_{io} = target concentration of constituent i

Once the Q_k is determined, using equation (35), the flow simulation module is recalled and the daily computations for flow and water quality are solved again for the final results.

SUMMARY

HEC-5Q is capable of simulating the effects of the operation of as many as ten reservoirs and the streams of the basin. The reservoirs may be operated to satisfy a number of objectives, including flood control, low flow maintenance, hydropower production, water conservation and water quality control. The water quality portion of the model will simulate temperature and seven water quality constituents including an option for dissolved oxygen. The model will internally determine the water quality needed from all reservoir releases to meet specified downstream water quality objectives and will determine the gate openings in each reservoir that will yield the appropriate reservoir release water quality. Should it be necessary, flows will be altered to ensure that downstream water quality objectives are met. As currently formulated, the model does not use foresight in an attempt to ensure that a "global" optimum solution is found that meets water quality objectives. The model selects the "best" solution for system-wide reservoir operation on a daily basis.

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Appendix — Notation

The following symbols are used in this paper:

A = surface area of fluid element

$|A|$ = matrix of coefficients describing water elements characteristics

a = empirical elements

$\bar{b}$ = vector of constants from initial conditions

C = simulated water quality concentration

C_o = target water quality concentration

c = specified heat of water

$\bar{c}$ = vector of water quality concentration

D = effective diffusion coefficient

D_c = deviation of actual water quality concentration from target value

D_m = molecular diffusion coefficient

d = flow depth

F = acceptable flow rate

H = external heat source or sink

H_a = long wave atmospheric radiation

H_{ar} = reflected long wave radiation

H_{br} = water surface radiation

H_c = heat of conduction

H_e = heat of evaporation

H_n = net heat transfer

H_s = short wave solar radiation

H_{sr} = reflected short wave radiation

I = light energy at specified depth

I_o = light energy at water surface

K = heat exchange coefficient

K_1 = decay rate

K_2 = reaeration rate

k = light extinction coefficient

m = number of rows in matrix

N_{cc} = number of water quality constituents

N_{cp} = number of control points effected by more than one reservoir

N_p = number of open ports on discharge structure

n = number of columns in matrix

O = dissolved oxygen concentration

O^* = saturation dissolved oxygen concentration

p = index for open ports on discharge structure

P_{ij} = linear programming objective functions

Q = interelement flow

Q_d = downward advective flow

Q_i = lateral inflow

Q_k = reservoir release

Q_o = lateral outflow

Q_u = upward advective flow

Q_w = flow withdrawal

Q_x = flow into element

R_c = release concentration

T = water temperature

T_c = water temperature correction factor

T_e = equilibrium temperature

T_s = water surface temperature

T_x = inflow water temperature

t = time

U = flow velocity

V = volume of fluid element

S_c = water quality subindex

W = wind speed

W_c = water quality index weighting factor

Z = water quality index

z = space coordinate (vertical for reservoir/horizontal for stream)

Δz = element thickness

ρ = water density

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