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DEVELOPMENT OF GENERALIZED FREE SURFACE FLOW MODELS USING FINITE ELEMENT TECHNIQUES

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INTRODUCTION

The Corps of Engineers' Hydrologic Engineering Center is involved in the development, evaluation, and application of mathematical models. Two finite element hydrodynamic models, one for two-dimensional free surface flow in the horizontal plane and one for the vertical plane are being evaluated. Although the models are formulated to solve dynamic flow problems, all work to date has been with steady state solutions. Recent research has focused on mass continuity performance of the models, proper boundary condition specification, and comparison with finite difference techniques. The objective of this research is to develop generalized mathematical models for routine use by the engineering community. This paper presents recent results of evaluation and application of the models.

THE MODEL FOR TWO-DIMENSIONAL FREE SURFACE FLOW IN THE HORIZONTAL PLANE

The model for two-dimensional free surface flow in the horizontal plane solves the governing equations in the following form:

Continuity

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0 \qquad (1)$$

Momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} + g\frac{\partial a_o}{\partial x} - \frac{\varepsilon_{xx}}{\rho}\frac{\partial^2 u}{\partial x^2} - \frac{\varepsilon_{xy}}{\rho}\frac{\partial^2 u}{\partial y^2} - 2\omega v \sin\phi$$

$$+ \frac{gu}{C^2 h}\left(u^2 + v^2\right)^{1/2} - \frac{\zeta}{h}V_a^2 \cos\psi = 0 \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} + g\frac{\partial a_o}{\partial y} - \frac{\varepsilon_{yx}}{\rho}\frac{\partial^2 v}{\partial x^2} - \frac{\varepsilon_{yy}}{\rho}\frac{\partial^2 v}{\partial y^2} + 2\omega u \sin\phi$$

$$+ \frac{gv}{C^2 h}\left(u^2 + v^2\right)^{1/2} - \frac{\zeta}{h}V_a^2 \sin\psi = 0 \qquad (3)$$

where

u, v = x and y velocity components respectively
t = time
h = depth
a_o = bed elevation
ε = turbulent exchange coefficients
g = gravitational acceleration
ω = rate of earth's angular rotation
ϕ = latitude
C = Chezy roughness coefficient
ζ = empirical wind stress coefficient
V_a = wind speed
ψ = angle between wind direction and x - axis
ρ = fluid density

Before solution, the equations are recast with flow (velocity times depth) and depth as the dependent variables. A linear shape function is used for depth and a quadratic function for flow. The Galerkin method of weighted residuals is used and the resulting non-linear system of equations solved with the Newton-Rapheson scheme. Details of the solution have been published previously by Norton, et al (1973) and King, et al (1975). General discussions of finite element techniques have been published by Zienkiewicz (1971), Hubner (1975), and Strang & Fix (1973).

Evaluation of Continuity Errors

The finite element method yields a solution which approximates the true solution to the governing partial differential equations. The approximate nature of this solution becomes evident when mass continuity is checked at various locations in the solution domain for a steady state simulation. Although overall continuity is maintained (inflow equals outflow over the boundary), calculated flows across internal sections deviate somewhat from the inflow/outflow values. A study was made to evaluate errors in continuity as a function of network density. Poor continuity approximation is important of itself if water quality simulation is the goal. In the present applications, however, water surface elevations and velocities are the variables of interest. Therefore, the impact of continuity errors on these parameters was also investigated.

Flows on the Rio Grande de Loiza flood plain were simulated using several networks. This flood plain was selected because of its complex flow field and a prior study by the U.S. Army Corps of Engineers (1976) had made the data readily available. Model performance had previously been evaluated for simple hypothetical and laboratory flows by Norton et al (1973) and King et al (1975). The Loiza flood plain is about 10 by 10 km (6 by 6 miles) in extent and is characterized by variable bottom topography, one inlet and two outlets, and several islands. Three of the networks used in the study are shown in Figs. 1 to 3 illustrating progressive increase in network detail.

The solution was considered acceptable if flow at all continuity check lines deviated from inflow by less than $\pm$ 5%. Continuity is checked by integrating the normal component of velocity times depth along lines specified by the modeler. The continuity check lines used in this study are indicated by dark lines on Figs. 1 to 3. Note that, because the flow divides around the islands, in some cases the sum of flows across two check lines (such as 5 and 6) should be compared with inflow. Various parameters of the problem are summarized in Table 1. No attempt was made to calibrate the coefficients used.

The continuity approximation improved with increasing network detail, as expected. Flow at the worst check line in the coarsest network (7 + 8) improved from 79.3% to 98.2% of inflow as network detail was increased. Network characteristics, computer execution times, and results of the simulations with these three networks are summarized in Table 2. Average depths and velocities along the continuity check lines are given in Table 3. The check line numbers in Tables 2 and 3 refer to the lines indicated on Figs. 1-3.

Table 1 Data for Loiza Flood Plain Simulation

1. Boundary conditions:
   a. Inflow (line 1) = 8200 cms (290,000 cfs)
   b. Outlets (lines 11 & 12), water surface elevation = 2.5 m (8 ft) MSL
   c. All other boundaries; either tangential flow or stagnation points
2. Bed roughness: Chezy C spatially varied from 5.5 to 22 $m^{1/2}$/sec (10 to 40 $ft^{1/2}$/sec)
3. Turbulent exchange coefficients: varied with element size from 24 to 48 m^2/sec (260 to 500 ft^2/sec)

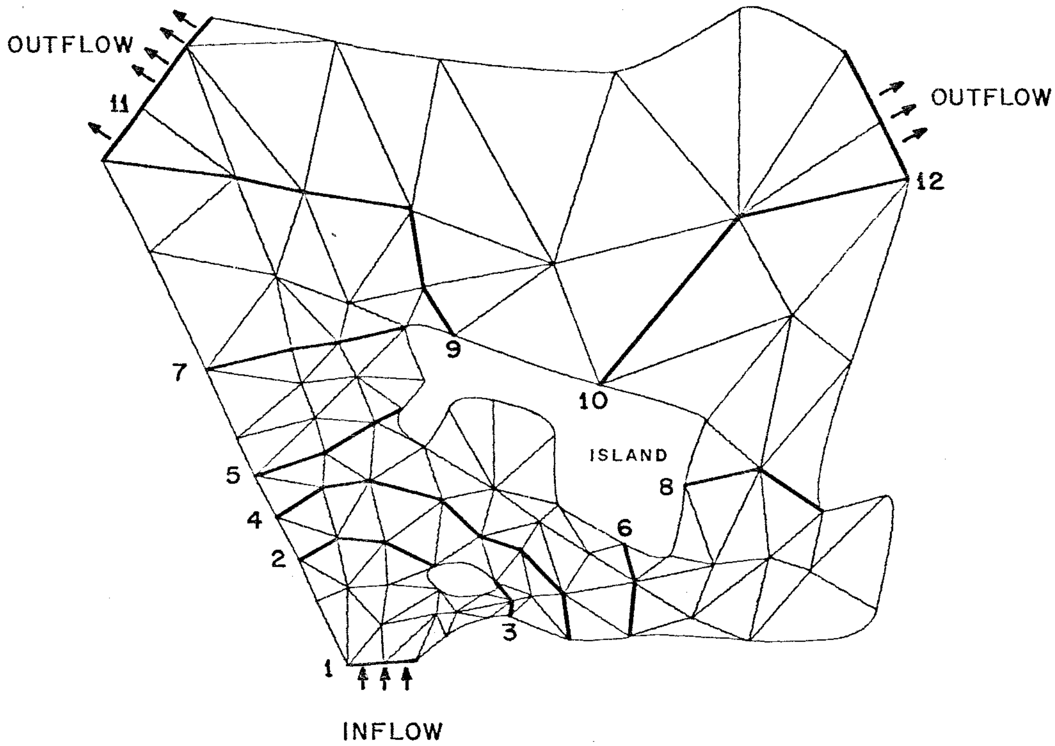

Figure 1 Continuity Check Network 3.1
(Dark Lines Indicate Continuity Check Lines)

Table 2 Continuity Performance of the Networks

Network	3.1	3.3	3.5
No. of Nodes	310	375	432
No. of Elements	131	162	189
CDC 7600 Execution Time (sec)	22	31	45
Check Line	Percent of Inflow		
1 (inflow)	100.0	100.0	100.0
2 + 3	89.2	90.8	96.2
4	114.9	106.8	104.9
5 + 6	87.5	92.0	96.4
7 + 8	79.3	90.1	98.2
9 + 10	99.8	99.4	98.7
11 + 12 (outflow)	100.0	100.0	100.0

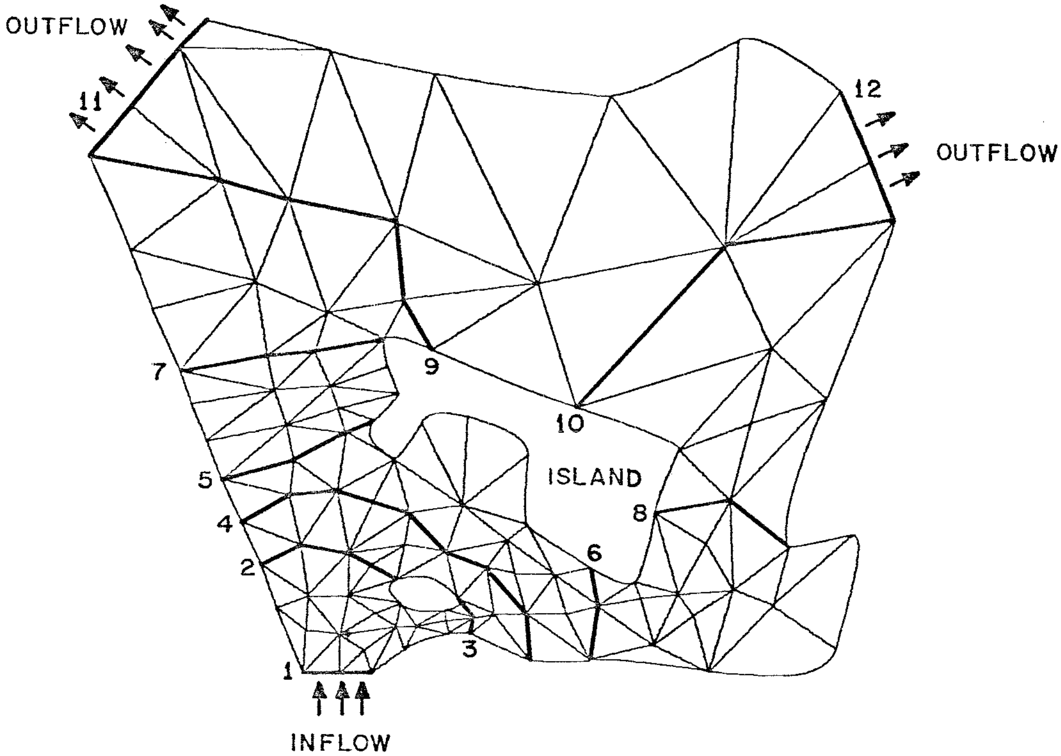

Figure 2 Continuity Check Network 3.3

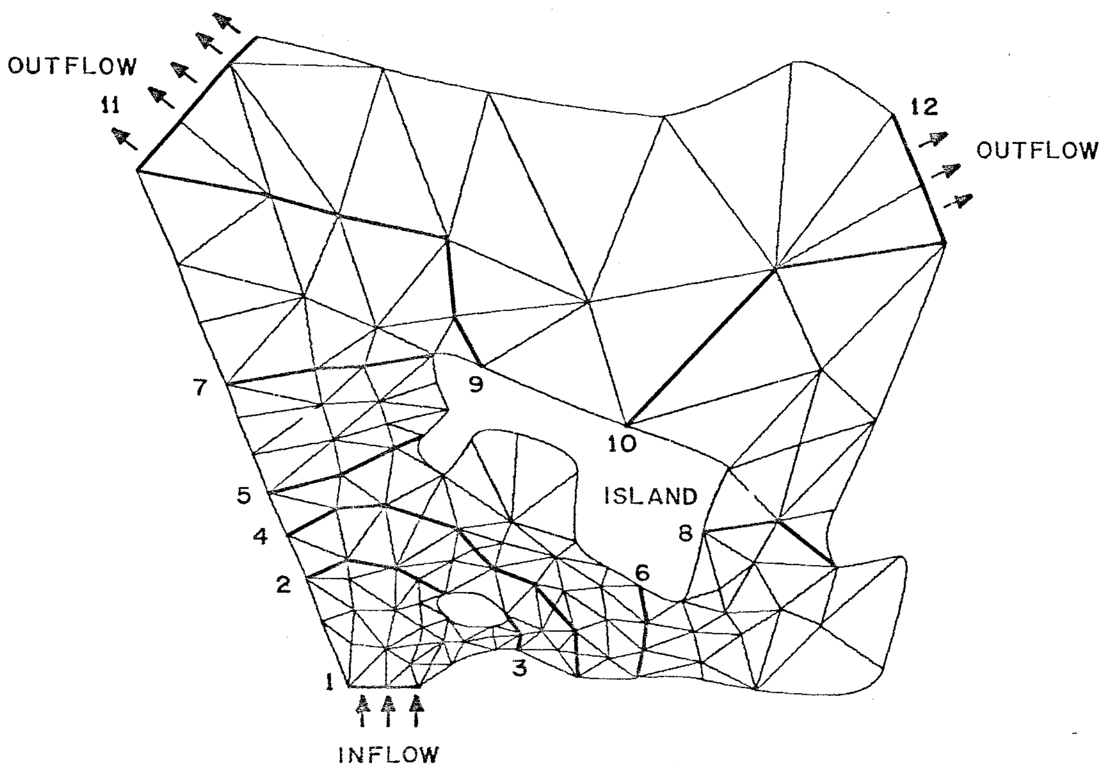

Figure 3 Continuity Check Network 3.5

Table 3 Flows (as percent of inflow), depth, and velocities for the networks

Line	NETWORK	3.1	3.3	3.5
1	%	100	100	100
	Y(m)	5.09	5.34	5.36
	V(mps)	1.92	1.83	1.83
2	%	50.4	50.1	53.7
	Y(m)	3.02	2.84	2.87
	V(mps)	.70	.74	.79
3	%	38.8	40.7	42.5
	Y(m)	2.48	2.51	2.63
	V(mps)	2.52	2.62	2.61
4	%	114.9	106.8	104.9
	Y(m)	2.91	2.78	2.78
	V(mps)	.66	.65	.63
5	%	36.0	37.9	39.7
	Y(m)	1.91	1.98	2.02
	V(mps)	.75	.76	.78
6	%	51.5	54.1	56.7
	Y(m)	3.82	3.90	3.76
	V(mps)	.90	.93	1.01
7	%	36.8	37.9	40.9
	Y(m)	1.86	1.88	1.91
	V(mps)	.62	.63	.67
8	%	42.5	52.2	57.3
	Y(m)	2.26	2.37	2.37
	V(mps)	.80	.94	1.03
9	%	42.1	41.0	42.6
	Y(m)	2.48	2.48	2.50
	V(mps)	.24	.23	.24
10	%	57.7	58.4	56.1
	Y(m)	3.09	3.07	3.05
	V(mps)	.31	.31	.30
11	%	45.8	46.0	47.0
	Y(m)	2.36	2.37	2.37
	V(mps)	.67	.68	.69
12	%	54.2	54.0	53.0
	Y(m)	2.31	2.31	2.30
	V(mps)	1.06	1.06	1.05

For most of the check lines, the improvement in continuity obtained with increasing network detail was associated with changes in both velocity and depth. In two cases, lines 6 & 8, the velocity changes were substantial. This region of the flow field is characterized by a rapid change of direction. The results reinforce the caveat that increased network detail is important in such regions. Furthermore, it appears that depth is somewhat less sensitive to errors in continuity than is velocity. Therefore, if one is interested in water surface elevations only, a less stringent continuity performance criterion could be accepted than if velocities are of interest.

Application to McNary Dam Second Powerhouse Study

An example of a "production" type application of the horizontal flow model is the second powerhouse site selection study for McNary lock and dam on the Columbia River. Flow fields downstream of the dam were simulated for several possible locations of the second powerhouse. Of interest were velocities, both magnitudes and directions, in the vicinity of the approach channel to the navigation lock. The study area and several of the possible second powerhouse locations are shown on Fig. 4. Finite element networks for the existing condition and for the south shore powerhouse with excavated discharge channel are shown in Figs. 5 and 6. Data are summarized in Table 4. The roughness coefficient was calibrated to reproduce an observed condition.

This study was greatly facilitated by an automatic reordering algorithm (Collins (1973)) which has been incorporated into the model. This algorithm makes modification of a network (compare Figs. 5 and 6) straightforward in that the entire network need not be re-numbered. The existing numbering scheme is utilized for input/output and the system of equations internally re-ordered to reduce storage.

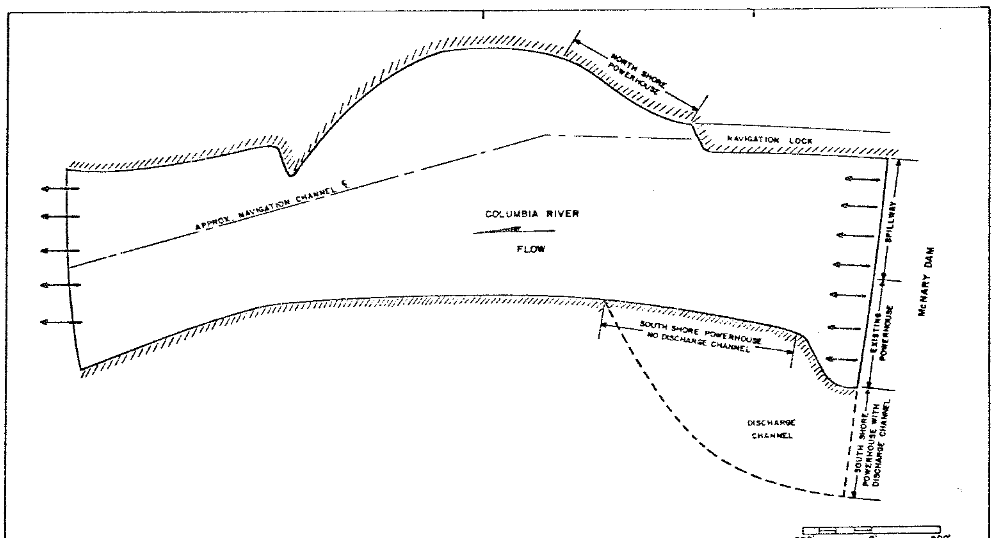

FIG. 4
STUDY AREA SHOWING POSSIBLE SECOND POWERHOUSE LOCATIONS

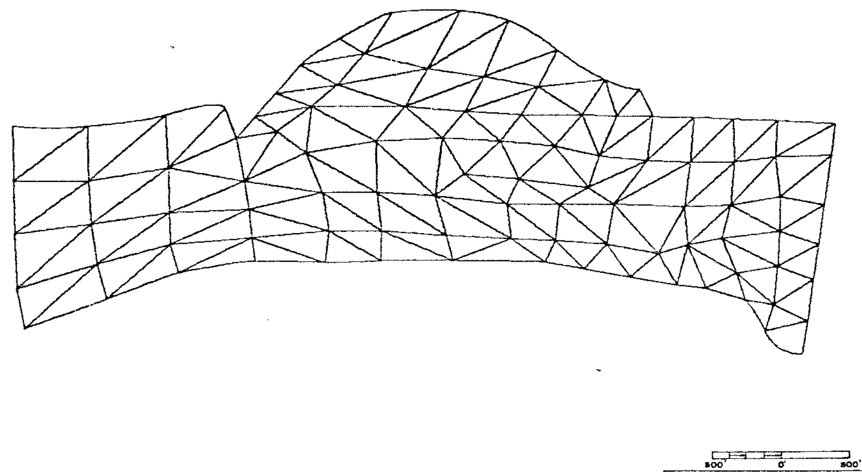

FIG 5
FINITE ELEMENT NETWORK
FOR RUNS 1-4

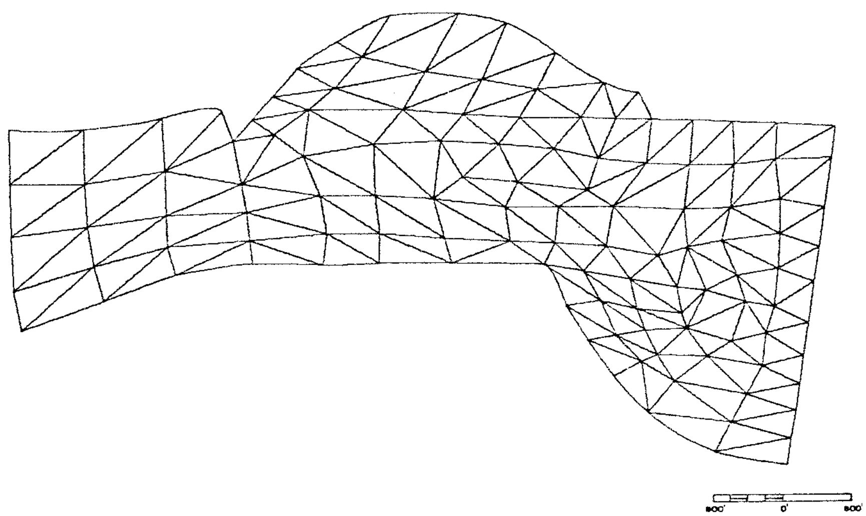

FIG 6
FINITE ELEMENT NETWORK
FOR RUN 5

Table 4 Data for McNary Second Powerhouse Study

1. Upstream boundary condition:
   a. Spillway: Q = 7000 cms (250,000 cfs) for calibration runs
      Q = 0 for production runs
   b. Existing powerhouse: Q = 6500 cms (230,000 cfs)
   c. Second powerhouse: Q = 7000 cms (250,000 cfs)
2. Downstream boundary condition: Water surface elevation = 82.4 m (270.3 ft) MSL*
3. All other boundaries: Either tangential flow or stagnation points
4. Roughness: Chezy C = 55 $m^{1/2}$/sec (100 $ft^{1/2}$/sec)
5. Turbulent exchange coefficients: Varied with element size from 4.8 to 14.4 m^2/sec (50 to 150 ft^2/sec)

*For production runs in which total river discharge was 13600 cms (480,000 cfs). This elevation was varied according to a known stage-discharge relationship for other discharges.

A vector plotting routine was used to display simulated flow fields. Two such plots are shown on Figs. 7 and 8. Plots of this type are considered essential for interpreting and analyzing complex flow fields.

Continuity errors were generally less than ±5% with the exception of the constriction near the downstream boundary where errors were on the order of -15%. If future detailed studies are made, and velocities in that area become important, more network detail will be provided.

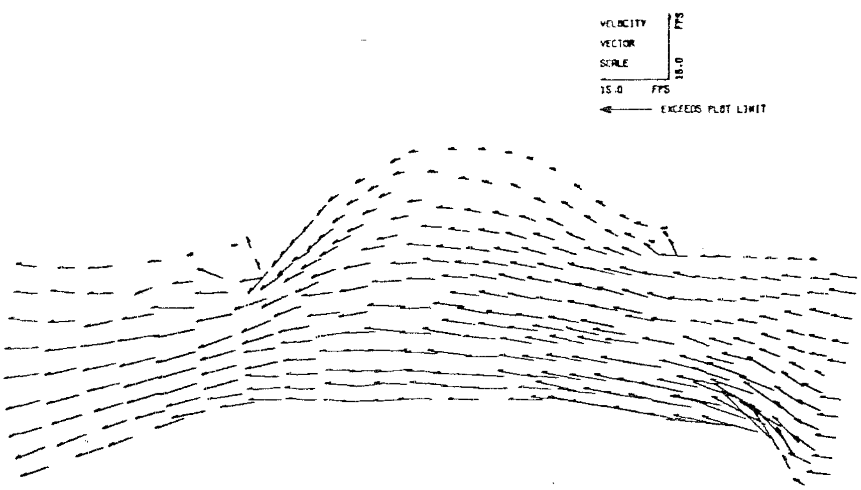

Figure 7 Velocities for Spillway Q = 7000 cms (250,000 cfs), Existing Powerhouse Q = 6500 cms (230,000 cfs), Slip Boundary Conditions

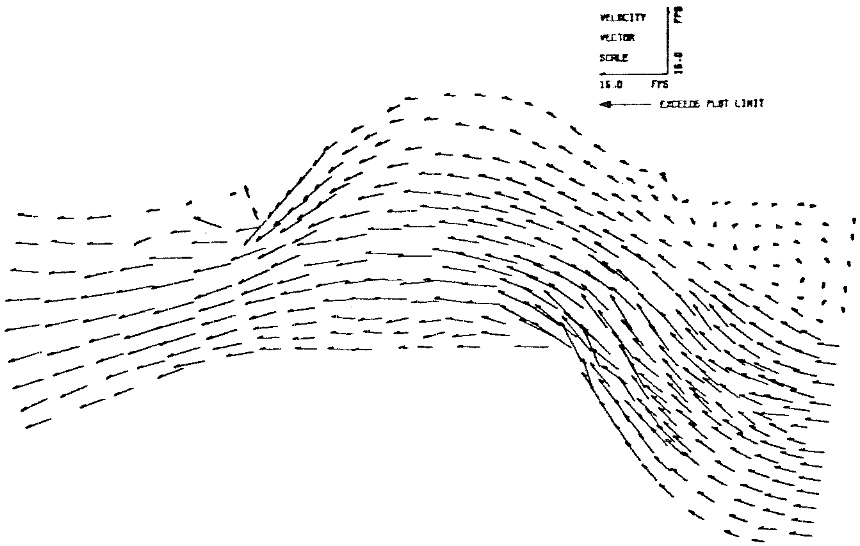

Figure 8 Velocities for Spillway Q = 0, Existing Powerhouse Q = 6500 cms (230,000 cfs), Second Powerhouse Q = 7000 cms (250,000 cfs), Slip Boundary Conditions

The model allows two valid types of boundary conditions at boundaries where no flow enters or leaves the system. One is the stagnation point where both components of velocity are zero; the other is the slip boundary condition where the velocity on the boundary is tangential to the boundary. The slip condition requires use of curved-sided elements on the boundaries. Use of curved boundaries with tangential flow is favored. Use of stagnation points along the boundaries results in a substantially different solution as shown in Fig. 9. Not only is the velocity distribution altered, but calculated head loss in the reach is about 0.21 m (0.7 ft.) greater than with the slip boundary condition. Continuity performance for the two simulations was similar, though in other problems analyzed by Resource Management Associates (1977), the slip condition was superior. Use of different boundary conditions should be investigated in an attempt to identify under what conditions the modeler should choose slip or stagnation point boundaries.

It is encouraging to note that the McNary study required no code changes to the model.

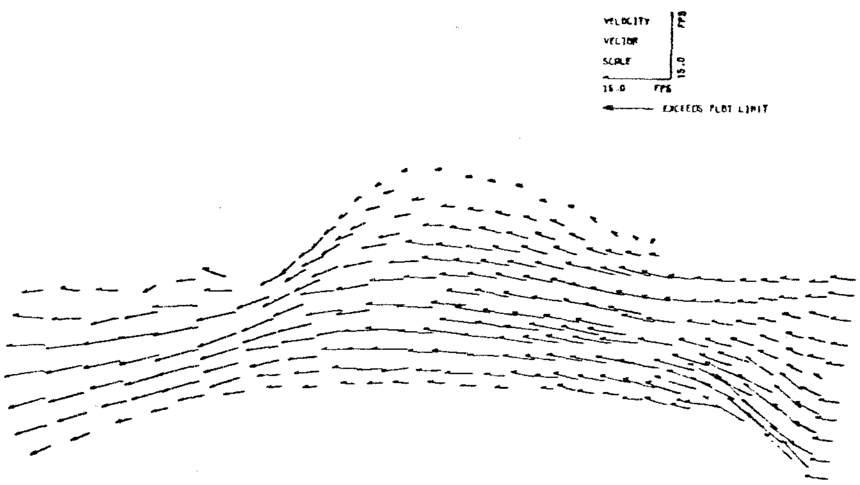

Figure 9 Velocities for Spillway Q = 7000 cms (250,000 cfs), Existing Powerhouse Q = 6500 cms (230,000 cfs), Stagnation Point Boundary Conditions

TWO-DIMENSIONAL MODELS IN THE VERTICAL PLANE

Two-dimensional (longitudinal and vertical) hydrodynamic models have been developed to aid the Corps in the description and analysis of reservoir water quality. The importance of the longitudinal as well as the vertical exchange in long, relatively narrow and deep impoundments has been studied by Pritchard (1971), Anthony and Drummond (1973) and the Tennessee Valley Authority (1969). Investigations such as these have shown that the hydrodynamics of a stratified reservoir influences the water quality and, therefore, the biological productivity of deep impoundments. Additional objectives for the development of multidimensional models are to be able to predict the effects that outlet type and location, degree of stratification, and reservoir operation have on the water quality in downstream rivers and streams.

As well as the general interest in simulating flows in the vertical plane, this research has provided the opportunity to compare the performance of an implicit finite difference method (FDM) model with that of a finite element method (FEM) model. The FDM model was developed by Edinger and Buchak (1977) and is named LARM (Laterally Averaged Reservoir Model). The FEM vertical model was developed by Norton et al (1973) and King et al (1975). Although initial development of the vertical FEM model was accomplished at the same time as that of the horizontal model previously discussed, further refinement and use of the vertical model has lagged considerably.

The primary objectives of the comparison of these two hydrodynamic models were: (1) to compare the relative ease with which the required data and boundary conditions could be prepared and coded; (2) to compare the overall performance of the two different approaches with respect to stability, convergence, accuracy and practicality; and finally, (3) to compare relative run times and simulation costs between the two methods for similar problems. The following paragraphs present the fundamental equations used by the two models.

Governing Equations

Both models incorporate similar forms of the so-called phenomenological equations for momentum, along with the continuity equation and a form of the convective-diffusion equation for thermal or material transport in the vertical. Note, however, that the FEM model retains the vertical momentum equation, which is replaced by the hydrostatic pressure distribution in the FDM model. Both models utilize a Cartesian coordinate system with the longitudinal x dimension positive downstream. The vertical z dimension is referenced positive upward from the x-axis in the FEM model, while it is positive downward from the x-axis in LARM. Both models allow for a variable width in the lateral y direction.

FEM Hydrodynamic Model

Momentum Equation:

$$\rho b \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) + \frac{\partial (pb)}{\partial x} - \varepsilon_{xx}\frac{\partial}{\partial x}\left(b\frac{\partial u}{\partial x}\right) - \varepsilon_{xz}\frac{\partial}{\partial z}\left(b\frac{\partial u}{\partial z}\right)$$

$$+ \rho g C^{-2} u|u|A_b - \zeta V_a^2 \operatorname{Cos} \psi \, A_s = 0 \qquad (4)$$

$$b\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) + \frac{\partial (pb)}{\partial z} + b\rho g - \varepsilon_{xz}\frac{\partial}{\partial x}\left(b\frac{\partial w}{\partial x}\right)$$

$$- \varepsilon_{zz}\frac{\partial}{\partial z}\left(b\frac{\partial w}{\partial z}\right) = 0 \qquad (5)$$

Continuity Equation:

$$\frac{\partial}{\partial x}(bu) + \frac{\partial}{\partial z}(bw) = 0 \qquad (6)$$

Convective-Diffusion Equation for Density:

$$\frac{\partial b\rho}{\partial t} + b\left(u\frac{\partial \rho}{\partial x} + w\frac{\partial \rho}{\partial z}\right) - D_x\frac{\partial}{\partial x}\left(b\frac{\partial \rho}{\partial x}\right) - D_z\frac{\partial}{\partial z}\left(b\frac{\partial \rho}{\partial z}\right) = 0 \qquad (7)$$

where

u, w = fluid velocity in the x and z directions respectively
b = breadth
p = pressure
D_x, D_z = eddy diffusion coefficients in the x and z directions respectively
A_b = area over which bottom stress is effective
A_s = the area over which the wind stress is effective

Other variables have previously been defined.

FDM Hydrodynamic Model LARM

Momentum Equation:

$$\frac{\partial}{\partial t}(ub) + \frac{\partial}{\partial x}(u^2 b) + \frac{\partial}{\partial z}(uwb) + \frac{1}{\rho}\frac{\partial}{\partial x}(pb) - \frac{\partial}{\partial x}\left(b\varepsilon_x \frac{\partial u}{\partial x}\right)$$

$$- \frac{\partial}{\partial z}(\tau_z b) = 0 \qquad (8)$$

Boundary stresses are found using the following expressions:
at the surface:

$$\tau_z = \zeta \frac{\rho_a}{\rho} V_a^2 \cos \psi \qquad (9)$$

at the bottom:

$$\tau_z = \frac{\rho g}{C^2} u|u| \qquad (10)$$

Hydrostatic Pressure distribution:

$$\frac{\partial p}{\partial z} - \rho g = 0 \qquad (11)$$

Continuity Equation:

$$\frac{\partial}{\partial x}(ub) + \frac{\partial}{\partial z}(wb) = qb \qquad (12)$$

Thermal Convective-Diffusion Equation:

$$\frac{\partial (Tb)}{\partial t} + \frac{\partial (uTb)}{\partial x} + \frac{\partial (wTb)}{\partial z} - \frac{\partial}{\partial x}\left(D_x b \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial z}\left(D_z b \frac{\partial T}{\partial z}\right) = \frac{fb}{\rho c_p} \qquad (13)$$

Equation of state:

$$\rho = \rho(T) \qquad (14)$$

where

ρ = fluid density
ρ_a = density of air
τ_z = boundary shear stress
q = lateral inflow per unit volume
T = temperature
D_x, D_z = heat transport dispersion coefficients
f = heat inflow per unit volume
c_p = specific heat

Other variables have been previously defined.

Description of the Test Problem

Data collected by the Tennessee Valley Authority (TVA)(1969) were used to test and compare the two vertical models in a reservoir simulation. These data were for the Fontana Reservoir in North Carolina. The models were applied to the first 23 km (14.5 miles) of the reservoir upstream from the dam. To simplify geometric requirements, a uniform reservoir breadth of 638 m (2095 ft) was used. This breadth was selected to conserve reservoir volume. The bottom profile and elevations were determined from sediment investigation cross sections. Conditions that existed in the reservoir during the last week of March, 1966 were used to provide the boundary conditions for the simulation. Water temperature profiles, water surface elevations, and flows into and out of the test reach were obtained from the TVA (1969) data. The reservoir was stratified and was approximately 108 m (353 ft.) deep at the dam. Surface heat exchange, wind velocity, and tributary inflows were all assumed to be zero for the purposes of this investigation; a steady inflow and outflow of 140 cms (5000 cfs) was used.

Discussion

The time and effort necessary to describe the reservoir geometry for both the FDM and FEM models were comparable. To achieve calculated results at comparable locations in space, optional quadrilateral elements were used so that the finite element network (Fig. 10) was almost identical to FDM grid (not shown). It is recognized that this network does not exploit the capability of the FEM model to allow increased geometric resolution where desired, such as near the reservoir outflow point, but this simplification was useful for comparison of results.

The convergence of the FEM solution was noted to be somewhat more sensitive to the magnitude of the turbulent exchange coefficients than the FDM model. The ranges of values of the coefficients over which convergent solutions can be obtained for the two models have not yet been firmly established. Additional sensitivity investigations shall be undertaken at a later time. Ariathurai, et al (1977) examined similar equations and found that stability and convergence of the solution could be related not only to spatial and temporal step sizes but also to the Peclet number which is the ratio of convective transport to diffusive transport.

The flow fields calculated with the FDM and FEM models are shown in Figs. 11 and 12 respectively. The vertical scale of Figs. 10-12 is exaggerated by a factor of 100. Coefficients used (refer to equations 4-7) were: ε_{XX}=24, ε_{XZ}=4.8x10^{-3}, ε_{ZZ}=240, D_X=23, D_Z=9.3x10^{-7} m^2/sec (260,0.05, 2600, 250, 10^{-5} ft^2/sec). Although the models have numerous detailed differences, particularly in the description of boundary conditions, the calculated flow fields are similar and reasonable. For the test application, the reservoir was thermally stratified, with the incoming fluid cooler and more dense than the fluid in the surface layers. The stable density gradient in the region of the thermocline tends to inhibit vertical momentum and material transport, yet circulation appears in the upper layers. The circulation in the surface layers is driven by internal horizontal shearing between the cool water flowing toward the outlet and the warmer water above. A similar flow pattern is also observed in the bottom region below the main flow in the

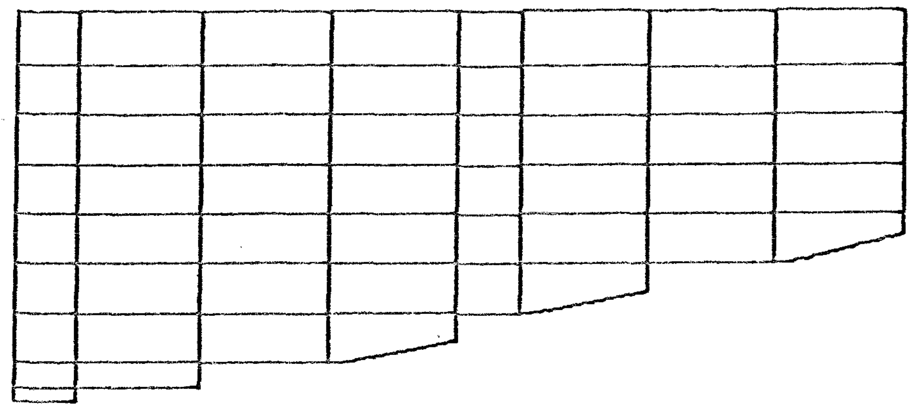

Figure 10 Finite Element Network for Reservoir Simulation

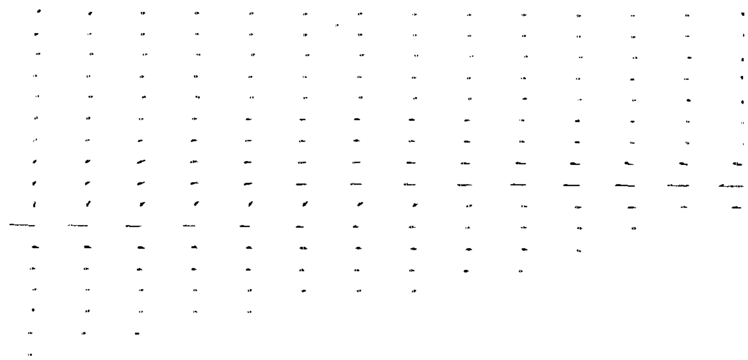

Figure 11 Reservoir Velocities Calculated with the Finite Difference Model

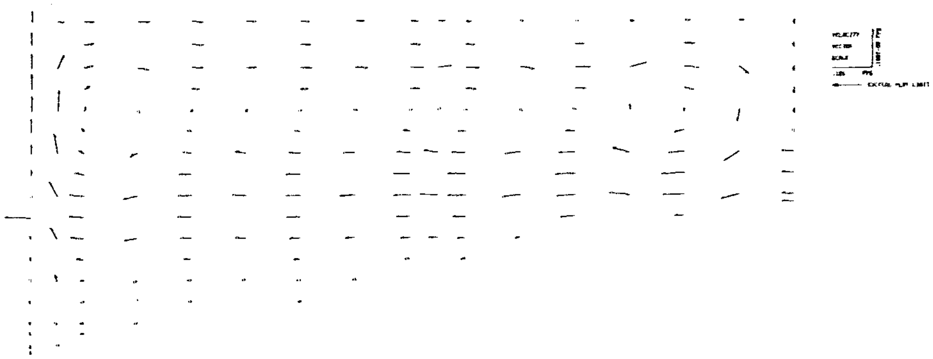

Figure 12 Reservoir Velocities Calculated with the Finite Element Model

FDM model (Fig. 11). Generally, the FEM solution predicts larger vertical velocity components, perhaps due to the retention of the vertical momentum equation. Comparison of the solutions with available field data will be undertaken once general performance characteristics of the two models are further defined.

For these steady state simulations, the FDM took about 6 times more CDC 7600 computer time than the FEM. The primary reason is that, to achieve a steady state solution, the FDM model must be run through pseudo-time with constant boundary values until transients from initial conditions die out (about 75-100 days in this case). The FEM model, however, has the capability of solving the system once with zero time derivatives to arrive at a steady state solution. Comparative costs for dynamic simulations will depend primarily upon length of time step and number of elements used to define the study region.

SUMMARY

The work to date with the horizontal flow model indicates the following:

(1) Internal continuity errors can be reduced to acceptable levels by increasing network detail, particularly in areas of large curvature of the velocity field.

(2) Errors in continuity tend to be reflected more strongly in the velocity than the depth.

(3) General application of the model to steady state simulations is feasible at present.

The preliminary work with the vertical flow models indicates the following:

(1) The finite element method model is less costly than the finite difference model for steady state solutions.

(2) Simulation of flows in which density gradients are important requires careful selection of turbulent exchange and eddy diffusion coefficients.

(3) The finite element model predicts larger vertical velocities than the finite difference model, perhaps due to the retention of the vertical momentum equation.

(4) More experience with, and development of, the vertical models will be required before "production" applications can be easily made.

Indicated areas of further work are:

(1) Verification of models' performance when an adequate data set becomes available.

(2) Development of guidance on selection of turbulent exchange coefficients, relationship to flow properties etc.

(3) Investigate models' behavior for dynamic simulations.

(4) Evaluate use of stagnation vs. slip boundary conditions in the finite element models.

(5) Extend simulations with the vertical models to variable breadth problems.

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