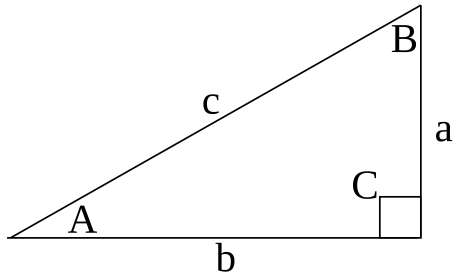
Standard Trigonometric Functions



For angle A:

sine A =	$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$	$a = c \bullet \text{Sin } A$	$A = \text{Sin}^{-1} (a \div c)$
cosine A =	$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$	$b = c \bullet \text{Cos } A$	$A = \text{Cos}^{-1} (b \div c)$
tangent A =	$\frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$	$a = b \bullet \text{Tan } A$	$A = \text{Tan}^{-1} (a \div b)$

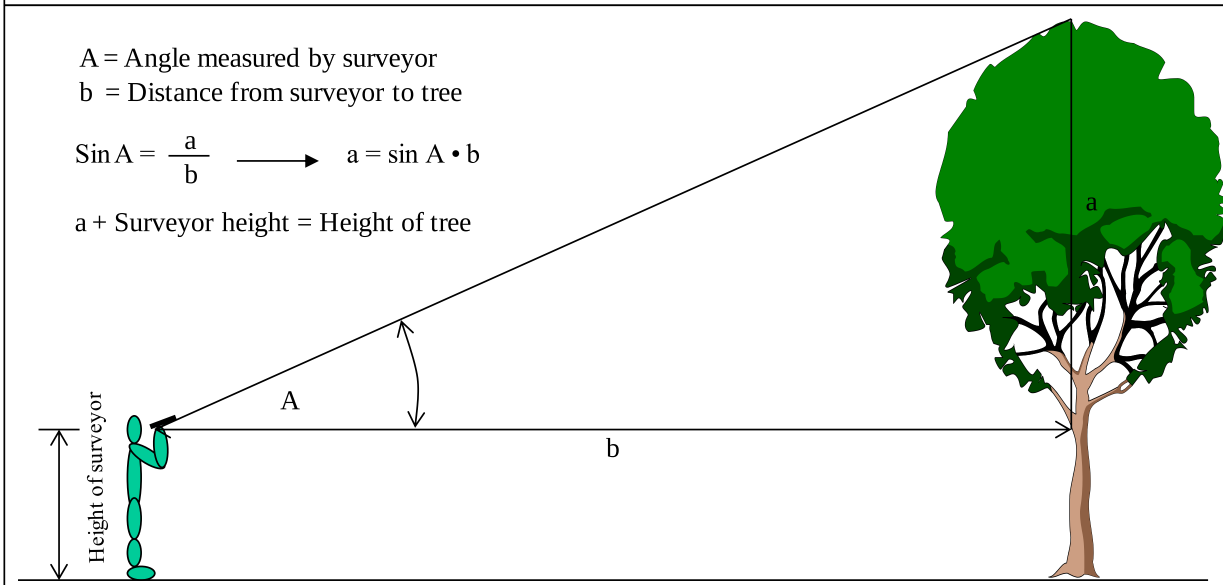
For angle B:

sine B =	$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c}$	$b = c \bullet \text{Sin } B$	$B = \text{Sin}^{-1} (b \div c)$
cosine B =	$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c}$	$a = c \bullet \text{Cos } B$	$B = \text{Cos}^{-1} (a \div c)$
tangent B =	$\frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$	$b = a \bullet \text{Sin } B$	$B = \text{Tan}^{-1} (b \div a)$

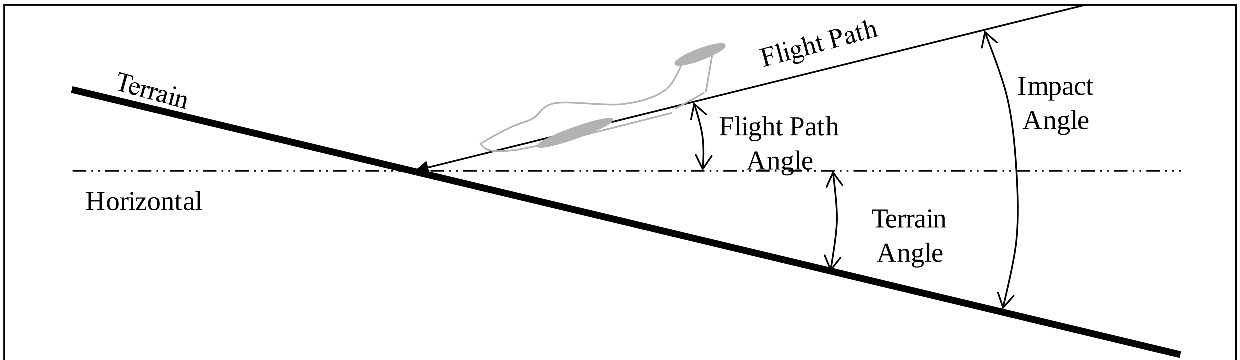
A = Angle measured by surveyor
b = Distance from surveyor to tree

$\text{Sin } A = \frac{a}{b} \longrightarrow a = \sin A \bullet b$

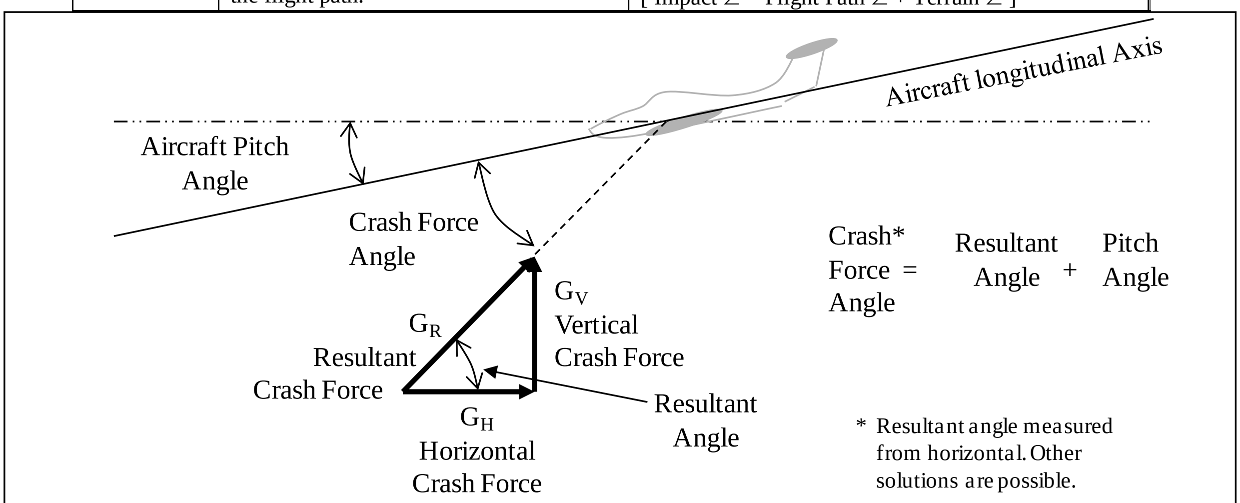
a + Surveyor height = Height of tree



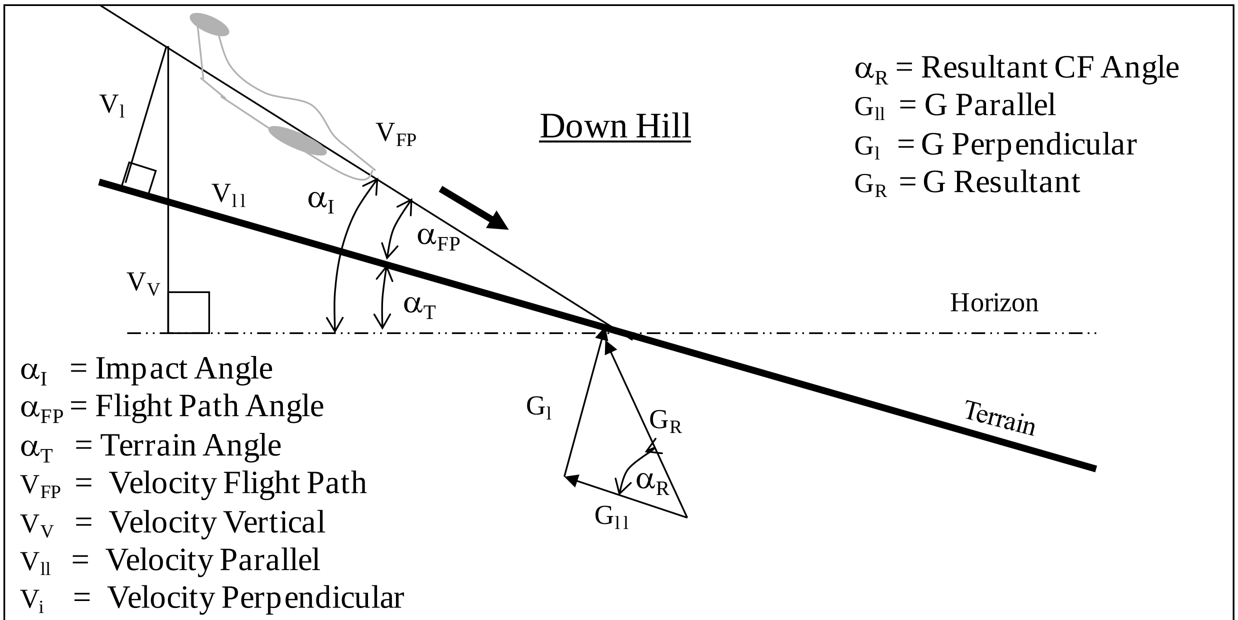
Crash Force Computation Terminology



Term	Description	Notes
Terrain Angle	The angle between the impact surface and the horizontal, measured in the vertical plane of the flight path.	The algebraic sign of the terrain angle is positive when the direction of flight is uphill & negative when the direction of the flight is downhill.
Flight Path Angle	The angle between the aircraft's flight path and the horizontal at the moment of impact.	The algebraic sign of the flight path angle is positive if the aircraft is moving downward immediately prior to impact. The sign is negative if impact occurs while the aircraft is moving upward.
Impact Angle	The angle between the flight path and the terrain, measured in the vertical plane of the flight path.	The impact angle is the algebraic sum of the flight path angle plus the terrain angle. [Impact ∠ = Flight Path ∠ + Terrain ∠]

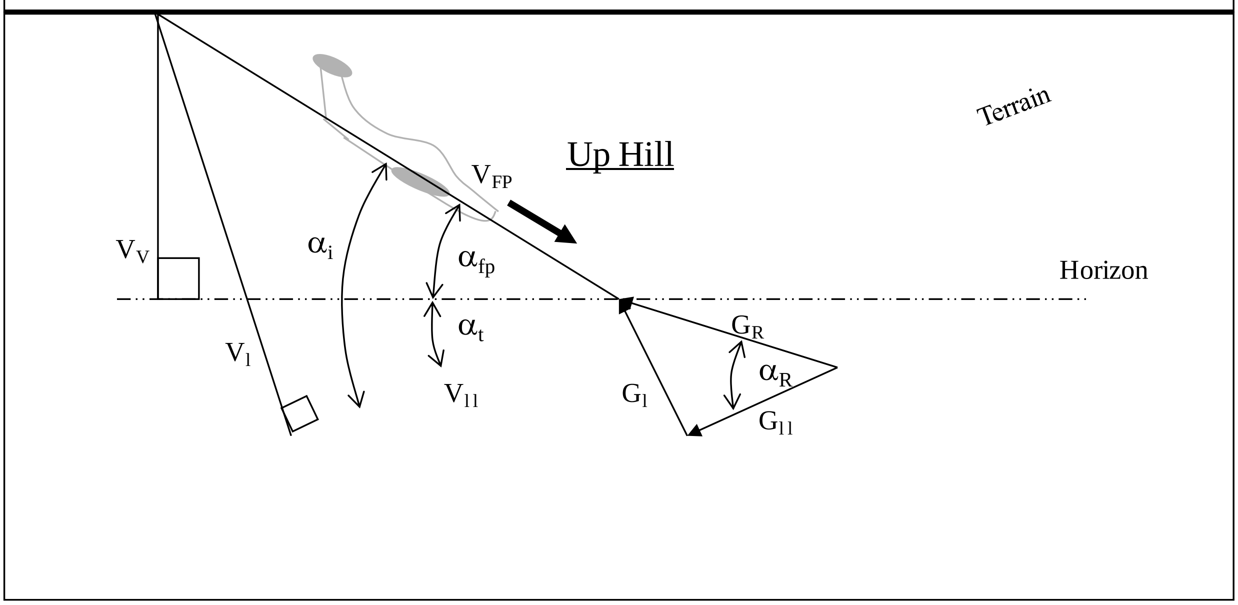


Term	Description	Notes
Crash Force Resultant	The geometric sum of the horizontal and vertical crash forces. Horizontal and vertical crash forces are determined on the basis of horizontal and vertical velocity components at impact and horizontal and vertical stopping distances.	The algebraic sign of the Crash Force Resultant angle is positive when the line of action of the resultant is above the horizontal and is negative if the line of action is below the horizontal.
Attitude at Impact	The aircraft attitude at moment of impact. Pitch – Nose above or below horizon Yaw – Nose right or left Roll – Right or left bank	The algebraic sign of the aircraft pitch angle is negative when the nose of the aircraft points below the horizon and positive when the nose points above the horizon.
Crash Force Angle	The angle between the resultant crash force and the longitudinal axis of the aircraft.	For impacts with little lateral component of force the crash force angle is the algebraic sum of the crash force resultant angle plus the aircraft pitch angle.



$V_{FP} = \frac{V_V}{\sin \alpha_I}$	$\tan \alpha_{FP} = \frac{V_l}{V_{ll}}$	$G_R = \frac{G_l}{\sin \alpha_R}$	$\tan \alpha_R = \frac{G_l}{G_{ll}}$
$V_l = \sin \alpha_I \bullet V_{FP}$	$V_{ll} = \cos \alpha_I \bullet V_{FP}$	$G_l = \frac{V_l^2}{32.17 \bullet SD}$	$G_{ll} = \frac{V_{ll}^2}{32.17 \bullet SD}$
MPH • 1.467 = Ft/Sec	KTS • 1.69 = Ft/Sec		

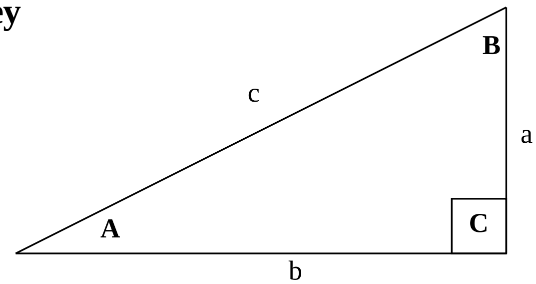
V_V is measured perpendicular to the Horizon, not the Terrain.



Math Key

Solve for:

$\angle$ **A**	Sine A	$= \frac{\text{Opposite}}{\text{Hypotenuse}}$	$= \frac{a}{c}$	
	Cosine A	$= \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$= \frac{b}{c}$	
	Tangent A	$= \frac{\text{Opposite}}{\text{Adjacent}}$	$= \frac{a}{b}$	
$\angle$ **B**	Sine B	$= \frac{\text{Opposite}}{\text{Hypotenuse}}$	$= \frac{b}{c}$	
	Cosine B	$= \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$= \frac{a}{c}$	
	Tangent B	$= \frac{\text{Opposite}}{\text{Adjacent}}$	$= \frac{b}{a}$	



$$\mathbf{a^2 + b^2 = c^2}$$

$$\angle \mathbf{C = 90^o}$$

Example: Solve for length of sides a and b if angle A is 20^o and side c is 10 feet long

Sine of $\angle$A = .3420 Length of side c is 10 feet long Solve for length of side a.	Sine A = a ÷ c 0.3420 = a ÷10 a = (0.3420)(10) **a = 3.420 feet long**	Cosine of $\angle$A = .9397 Length of side c is 10 feet long Solve for length of side b.	Cosine A = b ÷ c 0.9397 = b ÷10 b = (0.9397)(10) **b = 9.397 feet long**

Check accuracy of computations:

$a^2 + b^2 = c^2 \longrightarrow (3.420)^2 + (9.397)^2 = 10^2 \longrightarrow 11.6964 + 88.3036 = 99.6072 \approx 100$

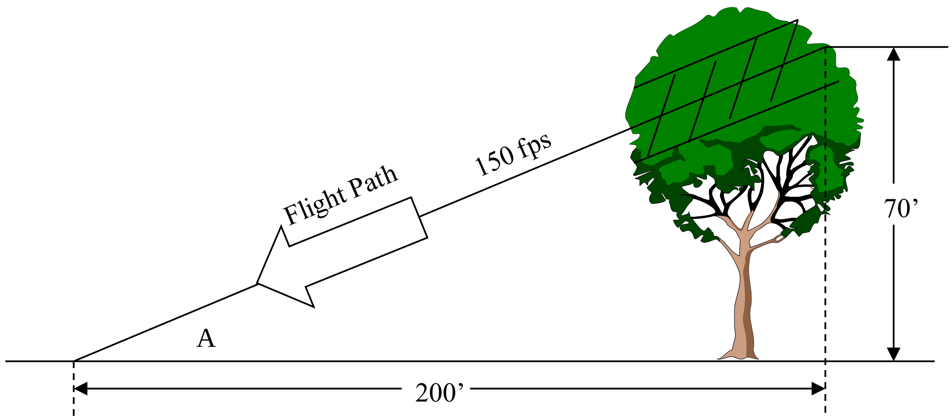
V = Velocity in feet/second (f/s) V_V = Vertical Velocity V_H = Horizontal Velocity V_{FP} = Velocity of flight path V(MPH) • 1.467 = V f/s V(KTS) • 1.69 = V f/s	Gravity (g) = 32.17 GV = G Load Vertical GH = G Load Horizontal SD = Stopping Distance in feet SDV = Stopping Distance Vertical SDH = Stopping Distance Horizontal	KE = Kenetic Energy in foot/pounds (ft/lbs) W = Weight of Object h = Height CF = Centrifugal Force in pounds Radius = 1/2 diameter
Solve for impact angle: Impact $\angle \rightarrow$ $\text{Tan A} = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$	Solve for vertical impact "G" load: $G_V = \frac{(V_V)^2}{g \cdot SD_V}$	Solve for horizontal imbalance: $CF = \frac{W \cdot \text{Radius} \cdot (RPM)^2}{2937}$
Solve for vertical velocity, given flight path velocity (f/s): $\text{Sine A} = \frac{V_V}{V_{FP}}$	Solve for horizontal impact "G" load: $G_H = \frac{(V_V)^2}{g \times SD_H}$	Solve for kinetic energy: $KE = 1/2 \cdot \frac{W}{g} \cdot V^2$ (V in fps, answer in foot/pounds)
Solve for horizontal velocity given flight path velocity (f/s) $\text{Cosine A} = \frac{V_H}{V_{FP}}$	Solve for Velocity: $V = 8\sqrt{h}$	Stopping Distance $SD = \frac{V2}{32.17 \cdot G\text{'s}}$ (Ft needed to survive x amount of G's)
Sovle for velocity: $V^2 = \frac{K.E.}{\frac{1}{2} \cdot \frac{\text{weight}}{32.17}}$	Solve for G's: $G\text{'s} = \frac{V^2}{32.17 \cdot SD}$	MPH • 1.467 = Feet/Second KTS • 1.69 = Feet/Second

Problem #1

An airplane impacts on level ground after passing through the top branches of a tree. By measurement, you determine that the airplane struck the tree 70 feet above the ground at a point 200 feet horizontally from the point of impact.

Find:

1. The angle of impact.
2. The horizontal and vertical velocities at impact if the flight path velocity is 150 feet per second.

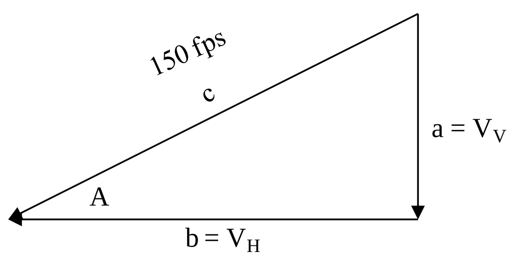


① **Find the angle of impact**

$\text{Tan } A = \dfrac{a}{b} \longrightarrow A = \tan^{-1}(70 \div 200) = 19.29^{\circ}$

② **The horizontal and vertical velocities at impact if the flight path velocity is 150 feet per second.**

a. Draw sketch of airplane impact angle



$A = 19.29^{\circ}$

b. Find V_V and V_H using sine and cosine

$V_V = a, \quad V_H = b$

$V_V = (\sin A = a \div c) \longrightarrow \sin A \cdot c = a \longrightarrow \sin 19.29\ (150) = 49.55 \text{ fps}$

$V_H = (\cos A = b \div c) \longrightarrow \cos A \cdot c = b \longrightarrow \cos 19.29\ (150) = 141.58 \text{ fps}$

Problem #2

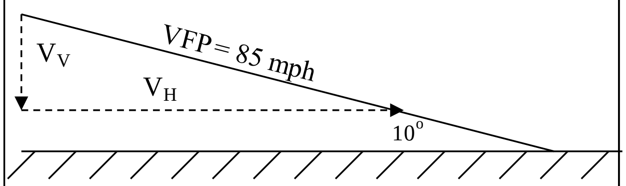
An aircraft crashes in a level open field. Flight path angle is 10 degrees and the true airspeed is 85 mph. Initial impact occurs with the fuselage level (zero pitch angle). The impact causes a 2-foot-deep gouge, and the aircraft comes to rest 25 feet from initial impact. The fuselage is crushed 12 inches vertically and 5 feet horizontally.

Find:

1. The aircraft ground speed (V_H) and vertical velocity (V_V) in feet per second.
2. The mean vertical and horizontal accelerations, in G's.
3. The magnitude and direction of the mean crash resultant.

1 To find V_H and V_V

a. Sketch conditions at impact



b. Convert 85 mph to ft/sec

(85 mph)(1.467) = 124.69 ft/sec

Answer rounded to 124.7 ft/sec

c. Determine vertical and horizontal velocities (Round answers to nearest 10th for next step)

$\sin 10^o = V_V \div V_{FP} \longrightarrow V_V = V_{FP} \cdot \sin 10^o \longrightarrow V_V = (124.7)(0.174) = \underline{\underline{21.70 \text{ ft/sec}}}$

$\cos 10^o = V_H \div V_{FP} \longrightarrow V_H = V_{FP} \cdot \cos 10^o \longrightarrow V_H = (124.7)(0.985) = \underline{\underline{122.83 \text{ ft/sec}}}$

2 To find G_V and G_H

a. Determine vertical and horizontal stopping distances

Vertical stopping distance $(S_V) = (2 \text{ ft})_{gouge} + (1 \text{ ft})_{structure} = 3 \text{ ft}$

Horizontal stopping distance $(S_H) = (25 \text{ ft})_{gouge} + (5 \text{ ft})_{structure} = 30 \text{ ft}$

b. Determine vertical and horizontal accelerations (G's)

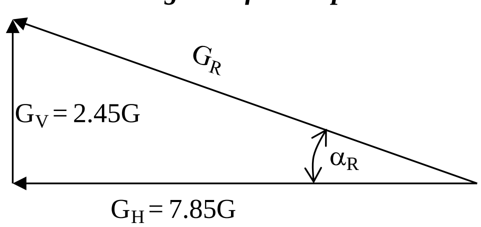
$$G = \frac{V^2}{64S}$$

$G_V = (21.7)^2 \div (64 \cdot 3) = \underline{\underline{2.45 \text{ G}}}$

$G_H = (122.8)^2 \div (64 \cdot 30) = \underline{\underline{7.85 \text{ G}}}$

3 To find α_R and G_R

a. Sketch the vector diagram of the impact accelerations



b. Use tangent trig function to find direction of resultant acceleration

$$\tan \alpha_R = \frac{G_V}{G_H} = \frac{2.45}{7.85} = 0.312$$

$\alpha_R = \arctan 0.312 = \underline{\underline{17.3^o}}$

c. Use the cosine function to find the magnitude of the resultant acceleration

$$\cos 17.3^o = \frac{7.85}{G_R} \longrightarrow G_R = \frac{7.85}{\cos 17.3^o} = 8.22 \text{ G}$$

Problem #3

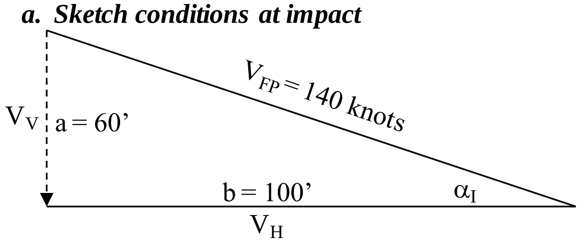
An aircraft crashes on level ground at an airspeed of 140 knots. Accident investigators discover that the airplane struck the top of a tree at a point 60 feet above the ground and crashed 100 feet from the base of the tree. The aircraft came to rest at the end of a gouge 32 feet long and 3 feet deep. Measurements show that the airplane was crushed 60 inches longitudinally and 12 inches vertically.

Find:

1. Horizontal and vertical velocities, in feet per second.
2. Mean vertical and horizontal accelerations, in G's.
3. Magnitude and direction of the crash force resultant.

1 To find V_H and V_V

a. Sketch conditions at impact



b. Convert 140 knots to ft/sec

(140 kts)(1.69) = 236.6 ft/sec

c. Determine impact angle

$$\tan \alpha_I = \frac{a}{b} \longrightarrow \alpha_I = \tan^{-1}\left(\frac{60}{100}\right) = 30.96^o$$

(Answer rounded to 31º for next computation)

d. Determine vertical and horizontal velocities (Round answers to nearest 10th)

$\sin 31^o = V_V \div V_{FP} \longrightarrow V_V = (236.6)(\sin 31^o) \longrightarrow V_V = (236.6)(0.515) = \underline{\underline{121.849 \text{ ft/sec}}}$

$\cos 31^o = V_H \div V_{FP} \longrightarrow V_H = (236.6)(\cos 31^o) \longrightarrow V_H = (236.6)(0.857) = \underline{\underline{202.766 \text{ ft/sec}}}$

2 To find G_V and G_H

a. Determine vertical and horizontal stopping distances

Vertical stopping distance $(S_V) = (3 \text{ ft})_{gouge} + (1 \text{ ft})_{structure} = 4 \text{ ft}$

Horizontal stopping distance $(S_H) = (32 \text{ ft})_{gouge} + (5 \text{ ft})_{structure} = 37 \text{ ft}$

b. Determine vertical and horizontal accelerations (G's) (Round answers to nearest 10th)

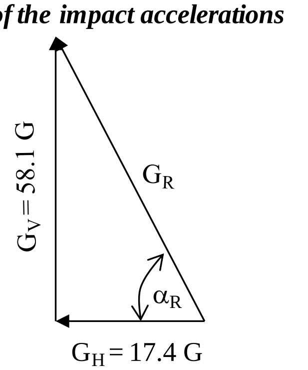
$$G = \frac{V^2}{64S}$$

$G_V = (121.8)^2 \div (64 \bullet 4) = \underline{\underline{57.950 \text{ G}}}$

$G_H = (202.8)^2 \div (64 \bullet 37) = \underline{\underline{17.368 \text{ G}}}$

3 To find a_R and G_R

a. Sketch the vector diagram of the impact accelerations



b. Use tangent trig function to find direction of resultant acceleration

$$\tan \alpha_R = \frac{G_V}{G_H} = \frac{58.0}{17.4} = 3.33$$

$\alpha_R = \tan^{-1}(3.33) = \underline{\underline{73.28^o}}$ (Round answer to nearest 10th)

c. Use the cosine function to find the magnitude of the resultant acceleration

$$\sin 73.3^o = \frac{G_V}{G_R} = \frac{58.0}{G_R} \longrightarrow G_R = \frac{58.0}{\sin 73.3^o} = \underline{\underline{60.55 \text{ G}}}$$

Problem #4

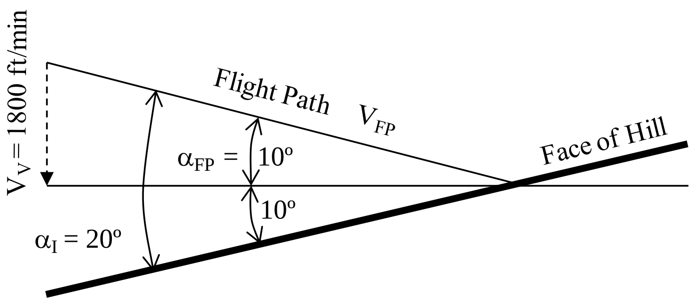
An aircraft crashes against a 10-degree uphill slope. The impact angle is 20 degrees. At the time of the impact, the aircraft vertical velocity was 1,800 feet per minute. The airplane came to rest after sliding 80 feet. Maximum depth of the gouge was 1 foot, and inspection revealed that the airplane structure was crushed 1 foot vertically. There was no horizontal crushing of the structure.

Find:

1. Flight path velocity.
2. Mean longitudinal and vertical acceleration, in G's, with respect to the face of the hill.
3. Mean crash force resultant magnitude and direction..

(1) To find flight path velocity

a. Sketch conditions at impact



b. Convert V_V to feet/sec

$$\frac{1{,}800 \text{ ft/min}}{60} = 30 \text{ ft/sec}$$

c. Determine flight path velocity

$$\sin \alpha_{FP} = \sin 10º = \frac{V_V}{V_{FP}} = \frac{30}{V_{FP}}$$

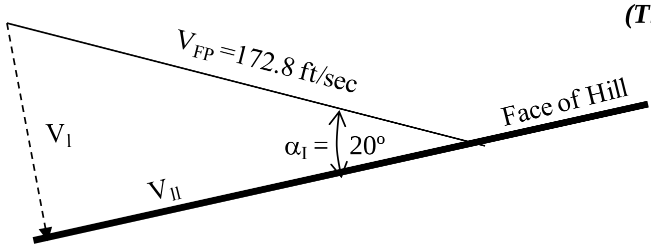
$$V_{FP} = \frac{30}{\sin 10º} = \underline{\underline{172.76 \text{ ft/sec}}}$$

(Round answer to nearest 10th)

(2) To find G_l and G_{ll}

a. Sketch the known conditions, with respect to the face of the hill.

(This may be included in the earlier sketch).



b. Determine velocity parallel to face of hill and velocity perpendicular to face of hill. (Round answers to nearest 10th)

(1) $\cos 20º = V_{ll} \div V_{FP}$ ⟶ $V_{ll} = (V_{FP})(\cos 20º) = (172.8)(\cos 20º) = \underline{\underline{162.378 \text{ ft/sec}}}$

(2) $\sin 20º = V_l \div V_{FP}$ $V_l = (V_{FP})(\sin 20º) = (172.8)(\sin 20º) = \underline{\underline{59.101 \text{ ft/sec}}}$

c. Determine stopping distances parallel to the face of the hill and perpendicular to the face of the hill.

(1) S_{ll} = 80 feet

(2) S_l = (1 foot) + (1 foot) = 2 feet

d. Determine accelerations parallel to the slope and perpendicular to the slope.

$$G = \frac{V^2}{64S}$$

(1) $G_{ll} = (162.4)^2 \div (64 \bullet 80) = 5.15$ G's

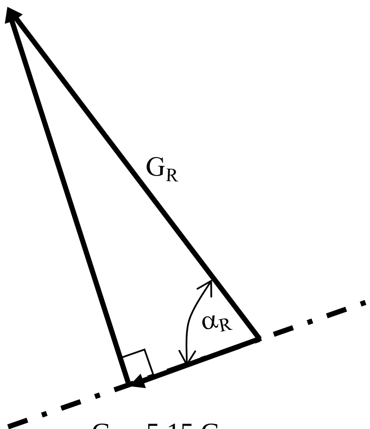
(2) $G_l = (59.1)^2 \div (64 \bullet 2) = 27.28$ G's

Problem #4 continued:

③ <u>To find a_R and G_R</u>

a. Sketch the vector diagram of the mean accelerations.

$G_l = 27.28$ G



$G_{ll} = 5.15$ G

b. Determine α_R.

$$\tan \alpha_R = \frac{G_l}{G_{ll}} = \frac{27.28}{5.15} = 5.29$$

$$\alpha_R = \tan^{-1}(5.29) = \underline{\underline{79.29^o}}$$

(Round answer to nearest 10th)

c. Determine magnitude of crash force resultant.

$$\sin 79.3 = \frac{G_l}{G_R} = \frac{27.28}{G_R}$$

$$G_R = \frac{27.28}{\sin 79.3^o} = 27.76 \text{ G}$$

Problem #5

(Based on Problem #4: An aircraft crashes against a 10-degree uphill slope. The impact angle is 20 degrees. At the time of the impact, the aircraft vertical velocity was 1,800 feet per minute. The airplane came to rest after sliding 80 feet. Maximum depth of the gouge was 1 foot, and inspection revealed that the airplane structure was crushed 1 foot vertically. There was no horizontal crushing of the structure.)

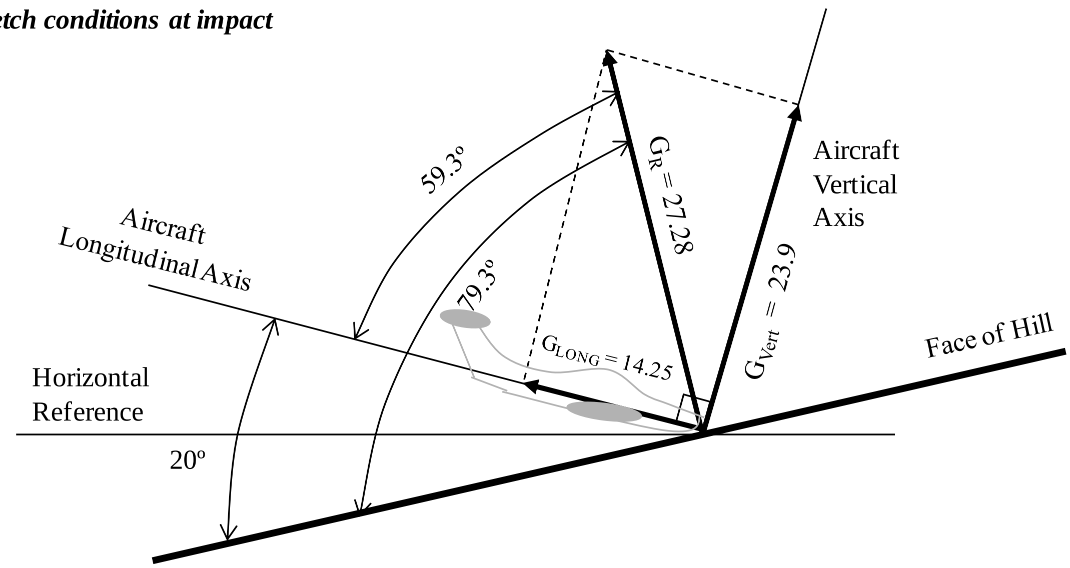
ADD: The longitudinal axis of the airplane is parallel to the flight path.

Find:

Longitudinal and vertical accelerations with respect to the aircraft axes.

1 Determine longitudinal and vertical accelerations with respect to the aircraft axes.

a. Sketch conditions at impact



b. Determine from the sketch that the 27.28 G crash force resultant acts at an angle of 59.3 degrees from the longitudinal axis of the airplane (79.3º - 20º = 59.3º)

Therefore, the mean longitudinal acceleration of the airplane is:

$G_{longitudinal} = (G_R)(\cos 59.3º) = (27.28)(\cos 59.3º) = \underline{\underline{13.927\ ft/sec}}$

And the mean vertical acceleration of the airplane is:

$G_{vertical} = (G_R)(\sin 59.2º) = (27.28)(\sin 59.3º) = \underline{\underline{23.456\ ft/sec}}$