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REPORT OF THE INTERNATIONAL ASTRONOMICAL UNION DIVISION I WORKING GROUP ON PRECESSION AND THE ECLIPTIC

J. L. HILTON[1,12], N. CAPITAINE[2], J. CHAPRONT[2], J. M. FERRANDIZ[3], A. FIENGA[4], T. FUKUSHIMA[5], J. GETINO[6], P. MATHEWS[7], J.-L. SIMON[4], M. SOFFEL[8], J. VONDRAK[9], P. WALLACE[10] and J. WILLIAMS[11]

[1]*U.S. Naval Observatory, Washington, DC, USA*
[2]*Observatoire de Paris, Paris, France*
[3]*U. de Alicante, Alicante, Spain*
[4]*Institut de Mécanique Céleste, Paris, France*
[5]*National Astronomical Observatory, Japan, Tokyo, Japan*
[6]*U. de Valladolid, Valladolid, Spain*
[7]*U. of Madras, Chennai, India*
[8]*T. U. Dresden, Dresden, Germany*
[9]*Czech Acad. Sci., Prague, Czech Republic*
[10]*Her Majesty's Nautical Almanac Office, Chilton, Didcot, UK*
[11]*Jet Propulsion Laboratory, Pasadena, CA, USA*
[12]*1274 Hwy 238, Jacksonville, OR 97530, USA, e-mail: jhilton@aa.usno.navy.mil*

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Abstract. The IAU Working Group on Precession and the Equinox looked at several solutions for replacing the precession part of the IAU 2000A precession–nutation model, which is not consistent with dynamical theory. These comparisons show that the (Capitaine et al., Astron. Astrophys., **412**, 2003a) precession theory, P03, is both consistent with dynamical theory and the solution most compatible with the IAU 2000A nutation model. Thus, the working group recommends the adoption of the P03 precession theory for use with the IAU 2000A nutation. The two greatest sources of uncertainty in the precession theory are the rate of change of the Earth's dynamical flattening, ΔJ_2, and the precession rates (i.e. the constants of integration used in deriving the precession). The combined uncertainties limit the accuracy in the precession theory to approximately $2\,\mathrm{mas\,cent}^{-2}$.

Given that there are difficulties with the traditional angles used to parameterize the precession, z_A, ζ_A, and θ_A, the working group has decided that the choice of parameters should be left to the user. We provide a consistent set of parameters that may be used with either the traditional rotation matrix, or those rotation matrices described in (Capitaine et al., Astron. Astrophys., **412**, 2003a) and (Fukushima Astron. J., **126**, 2003).

We recommend that the ecliptic pole be explicitly defined by the mean orbital angular momentum vector of the Earth–Moon barycenter in the Barycentric Celestial Reference System (BCRS), and explicitly state that this definition is being used to avoid confusion with previous definitions of the ecliptic.

Finally, we recommend that the terms *precession of the equator* and *precession of the ecliptic* replace the terms *lunisolar precession* and *planetary precession*, respectively.

Key words: precession and the ecliptic, reference systems

1. Introduction

Precession or, more precisely, precession of the equinox is the result of the motions of two planes. The first plane is that of the Earth's equator. The second is the ecliptic, the mean plane of the orbit of the Earth–Moon barycenter[1] about the Sun. These two planes have been chosen because the equinox has historically provided a convenient fiducial point in the observation of the heavens and the passage of time. For example, the calendar year is tuned to follow the tropical year from equinox to equinox rather than another definition of the year such as perihelion passage or the revolution of the Earth about the Sun in inertial space. These planes are also both dynamically involved in the motion of the Earth's pole.

For the purposes of this report, an inertial space or an inertial coordinate system is one in which the space coordinate grids show no global rotation with respect to a set of distant extragalactic objects. This report is specifically concerned with two coordinate systems. The first coordinate system is the Barycentric Celestial Reference System (BCRS) with its origin at the solar system barycenter and its axes aligned oriented to match the International Celestial Reference System (ICRS). The second coordinate system is the Geocentric Celestial Reference System (GCRS) with its origin at the center of mass of the Earth.

In the past, the motion of the Earth's equator has been called *lunisolar precession*, while the motion of the ecliptic has been called *planetary precession*. The names of these components are based on the dominant forces for each of these motions. However, the accuracy with which the precession can be measured has reached the point where the contribution of the planets to the motion of the Earth's equator is significant. Thus, the name *lunisolar precession* has become misleading. Fukushima (2003) proposed renaming lunisolar precession *equator precession* and planetary precession *ecliptic precession* to describe more accurately these two components of precession.

[1]The ecliptic is defined as the mean plane of the *Earth's* orbit around the Sun (*The Astronomical Almanac* 2006). However, for reasons described in Hilton (2006) it is preferable to use the mean orbital angular momentum vector of the Earth–Moon barycenter. This change of definition is included in recommendation 5 of this report.

Similarly, Capitaine et al. (2003a) proposed the terms *precession of the equator* and *precession of the ecliptic*. Considering that the term equator precession does not differentiate between motion *of* the equator and motion *along* the equator, this report will adopt the terms *precession of the equator* and *precession of the ecliptic*. Further, we recommend that these terms be adopted for general use.

Since its adoption, it has become apparent that the IAU 1976 theory of general precession (Lieske et al., 1977, henceforth Lieske) is in error by approximately $300\,\mathrm{mas\,cent}^{-1}$, where $1\,\mathrm{mas} = 0.''001$ and the century (cent[2]) consists of 36,525 Julian days Terrestrial Time (TT). Williams (1994) showed that in addition to the precession in longitude there should also be a secular motion in the obliquity of the Earth which he estimated to be about $-24\,\mathrm{mas\,cent}^{-1}$. This motion in latitude is caused by the slight inclination of the lunar orbit to the ecliptic when averaged over the period of its node. More recent estimates of the motion in obliquity are: Bretagnon et al. (2003), $-25\,\mathrm{mas\,cent}^{-1}$; Capitaine et al. (2003a), $-26\,\mathrm{mas\,cent}^{-1}$; and Fukushima (2003), $-23\,\mathrm{mas\,cent}^{-1}$.

The precession and nutation of the Earth are most accurately observed using Very Long Baseline Interferometer (VLBI) observations. These observations are only sensitive to the linear portion of the precession and insensitive to the ecliptic. Thus, the higher order coefficients of the precession theory along with the orientation of the ecliptic must be derived from dynamical theory. When the IAU 2000 precession–nutation theory (IERS, 2004) was adopted (IAU, 2001) the emphasis of the analysis was on the periodic nutations and correcting the linear portion of the precession VLBI observations. The effect of these changes on the higher-order terms in the precession theory was ignored. Ignoring the higher-order terms results in an error in the precession of about $6.4\,\mathrm{mas\,cent}^{-2}$ in longitude and $0.01\,\mathrm{mas\,cent}^{-2}$ in obliquity. Thus, the precession theory was not consistent with dynamical theory.

Fukushima (2003) showed that the values of ζ_A and z_A, two of the traditional angles for parameterizing the precession, are complementary and highly dependent on the precise values that are adopted for the offset between the dynamical frame and the GCRS at J2000.0. Thus, they are unsuited to polynomial representation.

The ecliptic in use was defined by Lieske using a method similar to that of Newcomb (1894). These yield an ecliptic which follows the geometrical

[2]The IAU style book (Wilkins, 1987) discourages the use of century as a unit; however, it is frequently used for slowly varying angles in celestial mechanics applications. Nor is there a consensus for its abbreviation. The abbreviation used here, *cent*, was chosen as the one found most often in a sample of both British and American English dictionaries.

path of the Earth–Moon barycenter. However, more recent work has used instead an ecliptic that follows the Earth–Moon barycenter's orbital angular-momentum vector in the BCRS, yielding a slightly different result due to the small contribution from the rotation of the ecliptic itself that is included in the former method (see Standish, 1981). Thus both the geometrical, often referred to as "rotating", and inertial definitions of the ecliptic have been used, creating confusion.

The IAU Working Group on Precession and the Ecliptic was formed at the XXVth General Assembly of the IAU in Sydney, Australia to address these topics and make recommendations regarding them to the IAU. This report constitutes the findings of that working group.

2. Choice of Precession Theory

Four recent, high precision theories for the precession and the definition of the ecliptic (Bretagnon et al., 2003; Capitaine et al., 2003a; Fukushima, 2003; Harada and Fukushima, 2004) were considered. After examination of the four theories, the working group agreed that precession of the equator in the Capitaine et al. (2003a) precession theory, P03, is consistent with dynamical theory and recommends it as the high precision precession theory most suited for use with the IAU 2000A nutation. The details of the comparison of the four theories can be found in Capitaine et al. (2004). The two greatest sources of uncertainty in the precession theory are the rate of change of the Earth's dynamical flattening, ΔJ_2, and the precession rates. The portion of the uncertainty from the uncertainty in ΔJ_2 limits the accuracy in the precession theory to approximately 1.5 mas cent^{-2} (Bourda and Capitaine 2004). And the uncertainty in the precession rates is approximately the same. Thus, the combined uncertainties limit the accuracy in the precession theory to about 2 mas cent^{-2}.

While it serves a useful fiducial purpose, dynamical uses for the modern ecliptic are limited. Thus, certain arbitrary decisions required to realize the ecliptic do not impair it. The details of these considerations are given in Hilton (2006). Thus, the working group recommends the precession of the ecliptic included in the P03 precession theory. More specifically, we recommend that the ecliptic pole should be explicitly defined by the mean orbital angular momentum vector of the Earth–Moon barycenter in the BCRS to simplify the dynamics. We also recommend that both the definition used and the process by which the ecliptic has been determined be made explicit when any future definition is adopted, to avoid confusion.

3. Parameterization

There is no unique method of parameterizing the precession. Williams (1994), for example, gives three methods to rotate from the equator and equinox of J2000.0 to the equator and equinox of date in addition to the method used by Lieske (1979). The particular parameterization desired may depend on extraneous details such as whether old computer code needs to be maintained or replaced. Thus, we have chosen to provide the data necessary for four different parameterizations:

- The traditional parameterization used by Newcomb and Lieske.
- The parameterization recommended by Capitaine et al. (2003a) which cleanly separates precession of the equator from precession of the ecliptic.
- The parameterization developed by Fukushima (2003) which allows for flexible switching between the mean equator and equinox of J2000.0 and the GCRS as its initial reference system.
- The parameterization to compute the precession–nutation in the new paradigm adopted by the IAU with respect to the Celestial Intermediate Origin (CIO)[3].

The first two parameterizations use the position of the mean equator and equinox of J2000.0[4] as their initial reference frame. However, many applications nowadays use the GCRS as the initial reference frame. To start from the GCRS, the coordinates used in the first two parameterizations must first be rotated using a constant bias matrix (top line of Expression 4 in Capitaine et al., 2003a). The third parameterization may start from either the mean equator and equinox of J2000.0 or the GCRS. The description provides the details on how the parameters change between the mean equator and equinox of J2000.0 and the GCRS. The fourth parameterization is meant only to be used with the GCRS as the initial reference frame.

The polynomial coefficients for all the angles for these four parameterizations derived from the P03 precession are given in Table I. The first 15 expressions are from Capitaine et al. (2003a). The remaining six expressions are the result of applying the P03 precession to the angles defined by Fukushima (2003).

[3]The IAU Division I Working Group on Nomenclature for Fundamental Astronomy is recommending the Celestial Ephemeris Origin (CEO) be renamed the Celestial Intermediate Origin (CIO).

[4]The position of the CIP, not including nutation, at J2000.0.

TABLE I

The polynomial coefficients for the precession angles.

Angle	(arcsec.)	Coefficients* $\left(\frac{\text{arcsec.}}{\text{cent.}}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^2}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^3}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^4}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^5}\right)$
ψ_A		5038.481507	−1.0790069	−0.00114045	0.000132851	-9.51×10^{-8}
ω_A	84381.406000	−0.025754	0.0512623	−0.00772503	-4.67×10^{-7}	3.337×10^{-7}
P_A		4.199094	0.1939873	−0.00022466	-9.12×10^{-7}	1.20×10^{-8}
Q_A		−46.811015	0.0510283	0.00052413	-6.46×10^{-7}	-1.72×10^{-8}
π_A		46.998973	−0.0334926	−0.00012559	1.13×10^{-7}	-2.2×10^{-9}
Π_A	629546.7936	−867.95758	0.157992	−0.0005371	−0.00004797	7.2×10^{-8}
ϵ_A†	84381.406000	−46.836769	−0.0001831	0.00200340	-5.76×10^{-7}	-4.34×10^{-8}
χ_A		10.556403	−2.3814292	−0.00121197	0.000170663	-5.60×10^{-8}
z_A	−2.650545	2306.077181	1.0927348	0.01826837	−0.000028596	-2.904×10^{-7}
ζ_A	2.650545	2306.083227	0.2988499	0.01801828	-5.971×10^{-6}	-3.173×10^{-7}
θ_A		2004.191903	−0.4294934	−0.04182264	-7.089×10^{-6}	-1.274×10^{-7}
p_A		5028.796195	1.1054348	0.00007964	−0.000023857	3.83×10^{-8}
X	−0.016617	2004.191898	−0.4297829	−0.19861834	7.578×10^{-6}	5.9285×10^{-6}
Y	−0.006951	−0.025896	−22.4072747	0.00190059	0.001112526	1.358×10^{-7}
$s + \frac{1}{2}XY$	0.0000940	0.00380865	−0.00012268	−0.07257411	0.00002798	0.00001562
γ_{J2000}		10.556403	0.4932044	−0.00031238	-2.788×10^{-6}	2.60×10^{-8}
ϕ_{J2000}	84381.406000	−46.811015	0.0511269	0.00053289	-4.40×10^{-7}	-1.76×10^{-8}
ψ_{J2000}		5038.481507	1.5584176	−0.00018522	−0.000026452	-1.48×10^{-8}
γ_{GCRS}	−0.052928	10.556378	0.4932044	−0.00031238	-2.788×10^{-6}	2.60×10^{-8}
ϕ_{GCRS}	84381.412819	−46.811016	0.0511268	0.00053289	-4.40×10^{-7}	-1.76×10^{-8}
ψ_{GCRS}	−0.041775	5038.481484	1.5584175	−0.00018522	−0.000026452	-1.48×10^{-8}

*Centuries (cent.) are Julian centuries of 36,525 days TT.

†The angle $\epsilon_0 \equiv \epsilon_A(t=0)$.

3.1. LIESKE PARAMETERIZATION

The Lieske (1979) precession matrix is:

$$\begin{aligned}
&P(z_A,\theta_A,\zeta_A)\\
&=R_3(-90^\circ-z_A)R_1(\theta_A)R_3(90^\circ-\zeta_A)\\
&=R_3(-z_A)R_2(\theta_A)R_3(-\zeta_A)\\
&=\begin{pmatrix}\cos z_A\cos\theta_A\cos\zeta_A-\sin z_A\sin\zeta_A & -\cos z_A\cos\theta_A\sin\zeta_A-\sin z_A\cos\zeta_A & -\cos z_A\sin\theta_A\\ \sin z_A\cos\theta_A\cos\zeta_A+\cos z_A\sin\zeta_A & -\sin z_A\cos\theta_A\sin\zeta_A+\cos z_A\cos\zeta_A & -\sin z_A\sin\theta_A\\ \sin\theta_A\cos\zeta_A & -\sin\theta_A\sin\zeta_A & \cos\theta_A\end{pmatrix}
\end{aligned} \tag{1}$$

where R_1, R_2, and R_3 are the basic rotation matrices around the axes of a right handed coordinate system,

$$R_1(\tau)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\tau & \sin\tau\\ 0 & -\sin\tau & \cos\tau\end{pmatrix},\ R_2(\tau)=\begin{pmatrix}\cos\tau & 0 & -\sin\tau\\ 0 & 1 & 0\\ \sin\tau & 0 & \cos\tau\end{pmatrix},\ \text{and}\ R_3(\tau)=\begin{pmatrix}\cos\tau & \sin\tau & 0\\ -\sin\tau & \cos\tau & 0\\ 0 & 0 & 1\end{pmatrix} \tag{2}$$

The angles, shown in Figure 1, parameterizing the rotation are: θ_A, the arc connecting the mean pole of J2000.0, P_0, with the mean pole of date, P_t; ζ_A, the angle with its vertex at P_0 from the equinoctial colure of J2000.0 to θ_A; and z_A, the angle with its vertex at P_t from θ_A to the equinoctial colure of date. This might be considered the traditional way of parameterizing the precession and is imbedded in many software applications that implement the precession. The angles ζ_A, θ_A, and z_A can be determined from ω_A, the arc from the ecliptic pole of J2000.0 to the mean pole of date; ψ_A, the angle between ϵ_0, the arc from the ecliptic pole of J2000.0 to the mean pole of J2000.0,[5] and ω_A; π_A, the obliquity of the ecliptic of date on the ecliptic of J2000.0; and Π_A, the longitude of the ascending node of the ecliptic of date on the ecliptic of J2000.0 (see Figure 1). The last two quantities, π_A and Π_A, are given in terms of the canonical variables $P_A=\sin\pi_A\sin\Pi_A$ and $Q_A=\sin\pi_A\cos\Pi_A$. The relations between the known quantities and ζ_A, θ_A, and z_A are

$$\begin{aligned}
\cos\theta_A&=\cos\epsilon_0\cos\omega_A+\sin\epsilon_0\sin\omega_A\cos\psi_A,\\
\cos\zeta_A&=\sin\omega_A\sin\psi_A/\sin\theta_A,\ \text{and}\\
\cos(z_A-\chi_A)&=\sin\epsilon_0\sin\psi_A/\sin\theta_A
\end{aligned} \tag{3}$$

where the length of $\epsilon_0\equiv\omega_A(t=0)$, and χ_A, the angle between ω_A and ϵ_A,[6] is

$$\sin\chi_A=\sin(180^\circ-\Pi_A-\psi_A)\sin\pi/\sin\epsilon_A \tag{4}$$

[5]The angular length of the arc ϵ_0 is the obliquity of the ecliptic at J2000.0.

[6]The arc from the ecliptic pole of date to the mean pole of date. The angular length of this arc is the obliquity of the ecliptic of date.

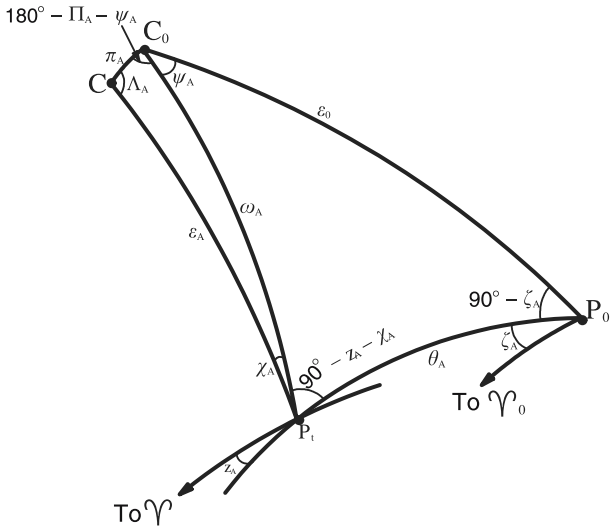

Figure 1. The angles associating the mean pole of J2000.0 (JD 2451545.0), P_0; the mean pole of date, P_t; the ecliptic pole of J2000.0, C_0; the ecliptic pole of date, C; the mean equinox of J2000.0, Υ_0; and the mean equinox of date, Υ, used in the Lieske and Capitaine et al. (2003) parameterizations for the precession.

where

$$\cos\epsilon_A = \cos\omega_A \cos\pi_A + \sin\omega_A \sin\pi_A \cos(180^\circ - \Pi_A - \psi_A) \tag{5}$$

A difficulty arises because the starting point for many applications is now the GCRS rather than mean equator and equinox of J2000.0. Although this can be accommodated in the traditional angles, Fukushima (2003) and Capitaine et al., (2003a) have shown that values of the angles z_A and ζ_A near J2000 are highly sensitive, in a complementary way, to the values adopted for the offset between the mean equator and equinox of J2000.0 and the GCRS and hence cannot satisfactorily be represented by polynomials in t. Thus, unless the accuracy goals are sufficient to neglect the frame bias (about 23 mas overall), either the frame bias has to be introduced as an additional rotation, or a different representation that does not suffer from these difficulties should be adopted.

3.2. CAPITAINE ET AL. PARAMETERIZATION

Capitaine et al. (2003a) provide a clean separation between the precession of the equator and precession of the ecliptic providing the precession of the equator with respect to the equator and equinox of J2000.0, ψ_A and ω_A,

and the precession of the ecliptic, P_A and Q_A, with respect to the ecliptic and equinox of J2000.0. The most efficient method of computing the precession matrix using these parameters is first to determine the mean obliquity of date, ϵ_A, and χ_A, the angle between the ecliptic pole of J2000.0 (epoch), C_0, and the ecliptic pole of date, C, with its vertex at P_t (see Figure 1). The angle χ_A is determined using Equation 4 and ϵ_A is determined using Equation 5. The precession matrix is then:

$$P(\chi_A, \omega_A, \psi_A) = R_3(\chi_A)R_1(-\omega_A)R_3(-\psi_A)R_1(\epsilon_0) = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \quad (6)$$

where the components of the rotation matrix are:

$$\begin{aligned}
P_{11} &= \cos\chi_A \cos\psi_A + \sin\chi_A \cos\omega_A \sin\psi_A, \\
P_{12} &= (-\cos\chi_A \sin\psi_A + \sin\chi_A \cos\omega_A \cos\psi_A)\cos\epsilon_0 + \sin\chi_A \sin\omega_A \sin\epsilon_0, \\
P_{13} &= (-\cos\chi_A \sin\psi_A + \sin\chi_A \cos\omega_A \cos\psi_A)\sin\epsilon_0 - \sin\chi_A \sin\omega_A \cos\epsilon_0, \\
P_{21} &= -\sin\chi_A \cos\psi_A + \cos\chi_A \cos\omega_A \sin\psi_A, \\
P_{22} &= (\sin\chi_A \sin\psi_A + \cos\chi_A \cos\omega_A \cos\psi_A)\cos\epsilon_0 + \cos\chi_A \sin\omega_A \sin\epsilon_0, \quad (7) \\
P_{23} &= (\sin\chi_A \sin\psi_A + \cos\chi_A \cos\omega_A \cos\psi_A)\sin\epsilon_0 - \cos\chi_A \sin\omega_A \cos\epsilon_0, \\
P_{31} &= \sin\omega_A \sin\psi_A, \\
P_{32} &= \sin\omega_A \cos\psi_A \cos\epsilon_0 - \cos\omega_A \sin\epsilon_0, \text{ and} \\
P_{33} &= \sin\omega_A \cos\psi_A \sin\epsilon_0 + \cos\omega_A \cos\epsilon_0.
\end{aligned}$$

3.3. FUKUSHIMA PARAMETERIZATION

Fukushima (2003) provides an alternative parameterization that has certain advantages over the parameterization used by Lieske (1979) or the separation into precession of the equator and precession of the ecliptic provided by Capitaine et al. (2003a). This parameterization starts with P_0, and the arc to C_0 on the celestial sphere (see Figure 2). The position of the C is then determined by the arc, ϕ, from P_0 to C, and γ, the angle between ϕ and the arc from P_0 to C_0 with its apex at P_0. The position of the mean pole of date is then determined from ϵ_A and ψ[7], the angle between ϕ and ϵ_A with its apex at C. The angles ϕ, γ, and ψ are related to already known angles by:

[7]The angle ψ is *not* the same as the angle ψ_A. The former is measured from ϕ to ϵ_A with the apex at the ecliptic pole of date, while the latter is measured from ϵ_0 to ω_A with the apex at the ecliptic pole of J2000.0.

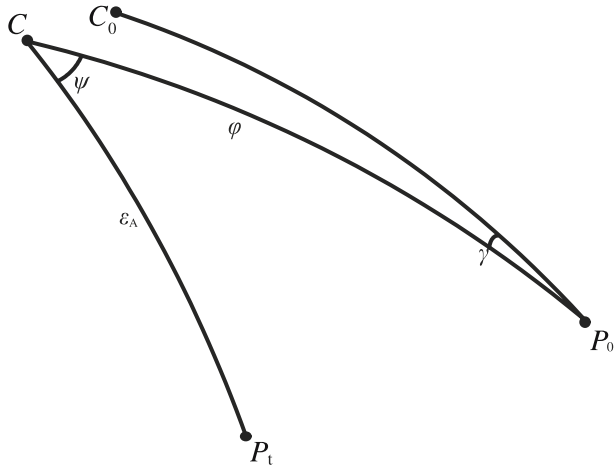

Figure 2. The angles associating the mean pole of date, P_t; and the ecliptic pole of date, C; to the mean pole of J2000.0, P_0; and ecliptic pole of J2000.0, C_0 in the Fukushima (2003) parameterization for the precession.

$$\cos\phi = \cos\pi_A \cos\epsilon_0 - \sin\pi_A \sin\epsilon_0 \cos\Pi_A \tag{8}$$

$$\sin\gamma = \frac{\sin\pi_A \sin\Pi_A}{\sin\phi} = \frac{P_A}{\sin\phi} \tag{9}$$

$$\sin\psi = \frac{\sin\theta_A \cos z_A}{\sin\phi} \tag{10}$$

The precession matrix is then:

$$P(\epsilon_A, \psi, \phi, \gamma) = R_1(-\epsilon_A) R_3(-\psi) R_1(\phi) R_3(\gamma) = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \tag{11}$$

where the components of the rotation matrix are:

$$\begin{aligned}
P_{11} &= \cos\psi \cos\gamma + \sin\psi \cos\phi \sin\gamma, \\
P_{12} &= \cos\psi \sin\gamma - \sin\psi \cos\phi \cos\gamma, \\
P_{13} &= -\sin\psi \sin\phi, \\
P_{21} &= \cos\epsilon_A \sin\psi \cos\gamma - (\cos\epsilon_A \cos\psi \cos\phi + \sin\epsilon_A \sin\phi) \sin\gamma, \\
P_{22} &= \cos\epsilon_A \sin\psi \sin\gamma + (\cos\epsilon_A \cos\psi \cos\phi + \sin\epsilon_A \sin\phi) \cos\gamma, \\
P_{23} &= \cos\epsilon_A \cos\psi \sin\phi - \sin\epsilon_A \cos\phi, \\
P_{31} &= \sin\epsilon_A \sin\psi \cos\gamma - (\sin\epsilon_A \cos\psi \cos\phi - \cos\epsilon_A \sin\phi) \sin\gamma, \\
P_{32} &= \sin\epsilon_A \sin\psi \sin\gamma + (\sin\epsilon_A \cos\psi \cos\phi - \cos\epsilon_A \sin\phi) \cos\gamma, \text{ and} \\
P_{33} &= \sin\epsilon_A \cos\psi \sin\phi + \cos\epsilon_A \cos\phi.
\end{aligned} \tag{12}$$

For some applications, the initial reference frame is the GCRS rather than the mean equator and equinox of J2000.0. Using the GCRS as the initial reference frame requires three changes. The relevant angles are shown in Figure 3.

First, in Equation 8, the arc ϵ_0 is replaced by $\epsilon_0 + \delta\epsilon_0$ where $\delta\epsilon_0$ is the difference between the mean obliquity of the ecliptic at J2000.0 and the arc length from C_0 to the pole of the GCRS, P_{GCRS}. Thus,

$$\cos\phi_{\mathrm{GCRS}} = \cos\pi_{\mathrm{A}}\cos(\epsilon_0 + \delta\epsilon_0) - \sin\pi_{\mathrm{A}}\sin(\epsilon_0 + \delta\epsilon_0)\cos\Pi_{\mathrm{A}}. \tag{13}$$

Second, unlike the mean equator and equinox of J2000.0, the node of the ecliptic of J2000.0 on the GCRS equator is offset 0.″052928 in right ascension. Thus, an additional initial rotation, $\delta\gamma = -0.''052928$, about the z-axis is required to first align the origin of the GCRS with the node of the ecliptic on the equator of the GCRS, and the equation for γ becomes

$$\gamma_{\mathrm{GCRS}} = \sin^{-1}\left(\frac{P_{\mathrm{A}}}{\sin\phi_{\mathrm{GCRS}}}\right) + \delta\gamma. \tag{14}$$

Third, let $\delta\psi$ be the angle between the arc to P_t and the arc to P_{GCRS} with its vertex at C, and $\delta\psi_0$ be the angle between the arc to P_0 and the arc to P_{GCRS} with its vertex at C_0. The angles δ and $\delta\psi$ are very small. Thus, the cosines of these angles are $\sim(1-5\times10^{-17})$. Correct calculation

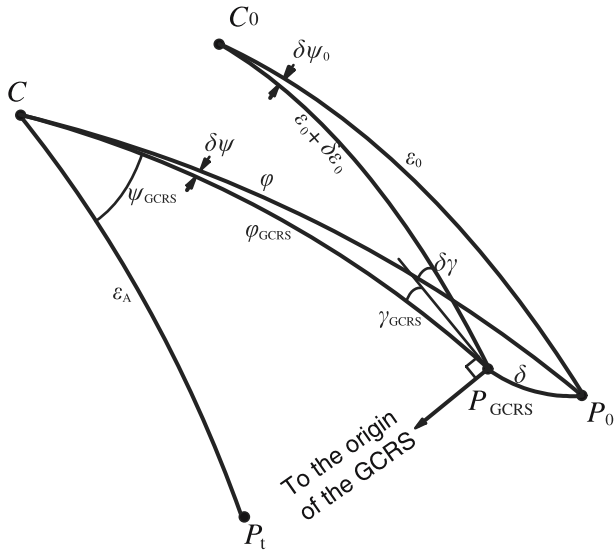

Figure 3. The angles associating the mean pole of date, P_t; and the ecliptic pole of date, C; to the mean pole of J2000.0, P_0; and ecliptic pole of J2000.0, C_0 in the Fukushima (2003) parameterization for the precession.

of these angles using a cosine formulation requires extended precision to retain accuracy. Instead, it is preferable to use the half angle sine formula, Equation C3, of Fukushima (2003). The length of the arc, δ, from P_0 to P_{GCRS} is then

$$\sin^2\left(\frac{\delta}{2}\right) = \sin\epsilon_0 \sin(\epsilon_0 + \delta\epsilon_0)\sin^2\left(\frac{\delta\psi_0}{2}\right) + \sin^2\left(\frac{\delta\epsilon_0}{2}\right). \tag{15}$$

Since the angles $\delta\epsilon_0$ and $\delta\psi_0$ are very small, the sines of these angles may be replaced with their first order approximations without any loss of accuracy. Similarly, $\sin(\epsilon_0 + \delta\epsilon_0)$ may be replaced by $\sin\epsilon_0$ without any loss of accuracy. Thus,

$$\sin^2\left(\frac{\delta}{2}\right) \approx \left(\frac{\delta\psi_0}{2}\right)^2 \sin^2\epsilon_0 + \left(\frac{\delta\epsilon_0}{2}\right)^2. \tag{16}$$

The same half angle formula is used to find $\delta\psi$

$$\sin^2\left(\frac{\delta}{2}\right) = \sin\phi \sin(\phi_{\mathrm{GCRS}})\sin^2\left(\frac{\delta\psi}{2}\right) + \sin^2\left(\frac{\phi - \phi_{\mathrm{GCRS}}}{2}\right). \tag{17}$$

As before, the angles $\delta\psi$ and $\phi - \phi_{\mathrm{GCRS}}$ are very small. Thus, the sines of these angles may be replaced with their first order approximations without any loss of accuracy. Also, either $\sin\phi_{\mathrm{GCRS}}$ may be replaced by $\sin\phi$, or $\sin\phi$ may be replaced by $\sin\phi_{\mathrm{GCRS}}$ without any loss of accuracy. Thus, making the appropriate substitutions, Equation 17 becomes

$$\sin^2\left(\frac{\delta}{2}\right) \approx \left(\frac{\delta\psi}{2}\right)^2 \sin^2\phi + \left(\frac{\phi - \phi_{\mathrm{GCRS}}}{2}\right)^2. \tag{18}$$

The value of $\delta\psi$ is determined by equating the right hand side of Equation 16 with the right hand side of Equation 18 and solving for $\delta\psi$. The result is

$$\delta\psi \approx \frac{-\sqrt{\sin^2\epsilon_0\,\delta\psi_0^2 + \delta\epsilon_0^2 - (\phi - \phi_{\mathrm{GCRS}})^2}}{\sin\phi}. \tag{19}$$

The negative value of the square root function is specifically chosen since the rotation about the ecliptic pole of date from pole of the GCRS to the Celestial Intermediate Pole (CIP) is clockwise as shown in Figure 3. At J2000.0, $\phi = \epsilon_0$ and $\phi - \phi_{\mathrm{GCRS}} = \delta\epsilon_0$. Thus, $\delta\psi = \delta\psi_0$ at J2000.0, and the rotation matrix reduces to the bias matrix between the GCRS and the mean equator and equinox of J2000.0.

Using these transformations, the pole of the GCRS may be substituted for the mean pole of J2000.0 using $\delta\epsilon_0 = \Delta\epsilon_{\mathrm{GCRS}}$ and $\delta\psi_0 = \Delta\psi_{\mathrm{GCRS}}$

(Mathews et al., 2002). Thus, the bias matrix to rotate from the GCRS to the mean equator and equinox of J2000.0 is incorporated into the precession. The angles $\delta\epsilon_0$ and $\delta\psi_0$ are small enough that for this particular transformation the only significant differences are in the zeroth and first order polynomial coefficients for γ, ϕ, and ψ.

A second advantage of this parameterization is that the intermediate frame resulting after the second rotation, $R_1(\phi)$, is with respect to the ecliptic of date. Thus, nutation may be added simply by adding the nutation in longitude, $\Delta\psi$, to the third rotation and the nutation in obliquity, $\Delta\epsilon$, to the fourth rotation, that is, the rotation to the true pole of date is:

$$NP(\epsilon_A, \Delta\epsilon, \psi, \Delta\psi, \phi, \gamma) = R_1(-\epsilon_A - \Delta\epsilon)R_3(-\psi - \Delta\psi)R_1(\phi)R_3(\gamma). \tag{20}$$

While this parameterization allows the precession and nutation to be applied directly, the two previous parameterizations require nutation to be done in an extra step:

$$NP = N(\epsilon_A, \Delta\epsilon, \Delta\psi)P = R_1(-\epsilon_A - \Delta\epsilon)R_3(\Delta\psi)R_1(\epsilon_A)P. \tag{21}$$

Note that the reason that the nutation matrix has to be separate from the precession matrix in the Lieske and the Capitaine et al. parameterizations is that the precession is referred to the ecliptic pole of J2000.0 while the nutation is referred to the ecliptic pole of date. Were the nutation referred to the ecliptic pole of J2000.0 rather than the ecliptic pole of date, the nutation could be incorporated into the Lieske and Capitaine et al. parameterizations using a method similar to that of the Fukushima parameterization.

3.4. THE CELESTIAL INTERMEDIATE ORIGIN (CIO)

The CIO is intended for use with only a combined bias-precession-nutation matrix (Capitaine et al., 2003b). This parameterization is in terms of the two angles X, in the x-direction (toward the x-origin of the GCRS, and Y, in the y-direction (90° to the east of x), which give the x and y-coordinates of the CIP unit vector in the GCRS (Figure 4). The third parameter is s. Except for a tiny fixed offset, this parameter is the difference between the length of the arcs from Σ_0 to N' and σ to N'. The point Σ_0 is the x-origin of the GCRS, σ is the CIO of date, and N' is the node of the equator of the CIP on the equator of the GCRS. This matrix includes precession, nutation, coupling between precession and nutation, and frame biases.

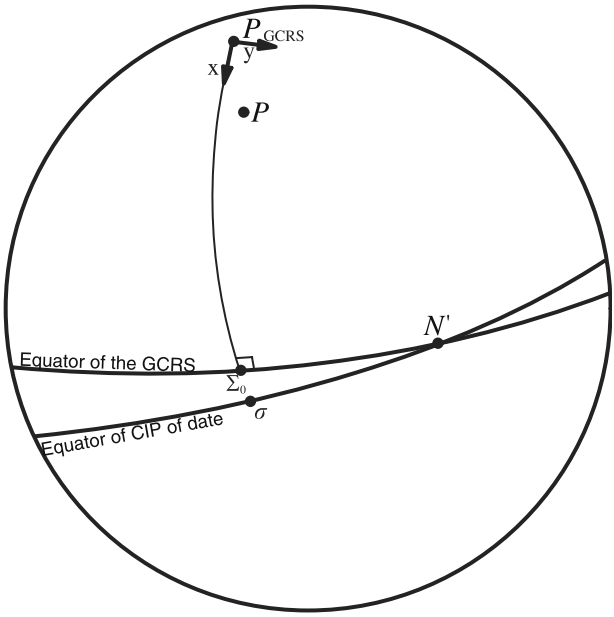

Figure 4. The Celestial Intermediate Origin (CIO), σ, and the position of the Celestial Intermediate Pole (CIP), P, with respect to the pole, P_{GCRS}, and equator of the Geocentric Celestial Reference System (GCRS) and its origin, Σ_0. The parameter X is the component of the arc from P_{GCRS} to P in the x direction and Y is the component in the y direction. Thus the CIP is located at $[X, Y, 1-(X^2+Y^2)^{1/2}]$ with respect to the GCRS located at $[0, 0, 1]$. The third parameter, s, is the difference between the arcs $\sigma N'$ and $\Sigma_0 N'$.

The bias-precession-nutation matrix is:

$$\mathrm{NPB} = R_3(-s) \cdot \begin{pmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{pmatrix} \tag{22}$$

where

$$\begin{aligned} a &\approx \frac{1}{2} + \frac{(X^2+Y^2)}{8} \\ X &= \sum_{i=0}^{5} x_i t^i + \sum_j \sum_{k=0}^{3} t^k (a_{sjk} \sin b_k + a_{cjk} \cos b_k), \quad \text{and} \\ Y &= \sum_{i=0}^{5} y_i t^i + \sum_j \sum_{k=0}^{3} t^k (d_{sjk} \sin f_k + d_{cjk} \cos f_k) \end{aligned} \tag{23}$$

t is the time in Julian centuries from J2000.0 TT, x_i and y_i are the coefficients for the frame bias and precession in X and Y, respectively, a_{sjk}, a_{cjk},

d_{sjk}, and d_{cjk} are the coefficients for the nutation and coupling between precession and nutation, and b_k and f_k are the fundamental angular arguments for the nutation and coupling terms.

The values for the coefficients for the secular part of the relations for X and Y are almost entirely due to precession plus the bias between BCRS and the J2000 mean pole. These coefficients are given in Table I. The parameter s is also represented by a series, but it is more efficient to derive s from the series for $s + XY/2$, also in Table I. The equations for the secular portions of X, Y, and $s + XY/2$ are also given in Equations 49–51 of Capitaine et al. (2003a).

Determination of the coefficients for the periodic parts of the series for X and Y require both the precession *and* the nutation portion of IAU 2000A (Mathews et al. 2002). Thus, developing coefficients for the periodic parts of the series was deemed beyond the scope of the Working Group. However, only very slight adjustments are required to the IAU 2000 nutation, and likewise to the periodic part of the X and Y series, in order to become consistent with P03, and the periodic part of $s + XY/2$ is insensitive to this change. An explanation of the theory and the sources for machine readable tabulations of the coefficients compatible with the IAU 2000A precession–nutation model are on pg. 44 of IERS (2004).

The elements of the matrix in Equation 22 are dimensionless. Thus, the values of X, Y, s, and the coefficients for the nutation and coupling between precession and nutation should be in radians. However, these values are normally given in arcseconds, as in Table I, and have to be converted.

4. Recommendations

The Working Group on Precession and the Ecliptic, recognizing:

1. the need for a precession theory consistent with dynamical theory and compatible with the IAU 2000A nutation theory,
2. the gravitational attraction of the planets make a significant contribution to the motion of the Earth's equator, making the terms *lunisolar precession* and *planetary precession* are misleading,
3. the need for a definition of the ecliptic for both astronomical and civil purposes, and
4. recognizing the ecliptic has been defined both with respect to an observer situated in inertial space (inertial definition) and an observer comoving with the ecliptic (rotating definition),

recommends:

1. The terms *lunisolar precession* and *planetary precession* be replaced by *precession of the equator* and *precession of the ecliptic*, respectively.
2. The IAU adopt the P03 precession theory, of Capitaine et al., (2003a, Astron. Astrophys., 412, 567–586) for the precession of the equator (Equations 37) and the precession of the ecliptic (Equations 38); the same paper provides the polynomial developments for the P03 primary angles and a number of derived quantities for use in both the equinox based and CIO based paradigms.
3. The choice of precession parameters be left to the user.
4. The recommended polynomial coefficients for a number of precession angles are given in Table I of this report, including the P03 expressions set out in Tables 3–5 of Capitaine et al. (2005, Astron. Astrophys., 432, 355–367), and those of the alternative Fukushima (2003, Astron. J., 126, 494) parameterization; the corresponding matrix representations are given in Equations 1, 6, 11, and 22.
5. The ecliptic pole should be explicitly defined by the mean orbital angular momentum vector of the Earth–Moon barycenter in the BCRS, and this definition should be explicitly stated to avoid confusion with older definitions.

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