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PROSPECTS FOR DETERMINING ASTEROID MASSES

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ABSTRACT

The orbits of 4583 main belt asteroids are integrated orbits for 57 years and searched for asteroid-asteroid encounters from which it may be possible to determine the masses of 23 of the largest asteroids (diameter $\geqslant$200 km) and 11 smaller asteroids. The search is conducted using a scattering formula which serves as a useful filter for identifying encounters that can lead to a mass determination. A total of 460 such encounters were found. The ten most useful of these encounters are examined in detail. The results show that, to make a reliable mass determination, the mean distance of the perturbed asteroid must be known to within a few times 10^{-8} AU. An observing program targeting the asteroids listed here would have a substantial impact on our knowledge of asteroid masses and densities.

1. INTRODUCTION

The masses of only three asteroids are known with an uncertainty of 10% or less, 1 Ceres (e.g., Viateau & Rapaport 1995), 2 Pallas (e.g., Standish & Hellings 1989), and 4 Vesta (e.g., Standish & Hellings 1989). The current standard planetary ephemerides of the solar system, DE200, was generated including the masses of these three asteroids, along with masses for 7 Iris and 324 Bamberga based on educated guesses, Standish (1990). More recent development ephemerides such as DE403 (Standish *et al.* 1995) use perturbations from 400 asteroids; however, aside from Ceres, Pallas, and Vesta, the masses used are based on guesses derived from their spectra and estimates of their diameters and densities.

10 Hygiea (Scholl *et al.* 1987) and 704 Interamnia (Landgraf 1992) are the only other asteroids for which mass determinations have been published. Both of these asteroids have uncertainties in their masses of approximately 50%. Williams (1984) shows that radar transponder data, such as those obtained from the Viking landers, are potentially sensitive to perturbations from no less than 36 asteroids. Unfortunately, the Viking data set is long enough that only the masses of Ceres, Pallas, and Vesta, can be determined (Standish & Hellings 1989).

The object of this paper is to determine which asteroid-asteroid interactions occurring between the years 1950 and 2017 could be used for determining asteroid masses. The filter used to determine those encounters which could be useful and a preliminary estimate as to how useful a given encounter may be is described in Sec. 2. The results obtained and the ten best interactions found by the filter are analyzed in Sec. 3. Section 4 describes other phenomena found within the data during the search for useful asteroid-asteroid encounters.

2. INTEGRATION AND FILTER

A numerical integration of main belt asteroid orbits was performed to find asteroid encounters likely to provide a good observational basis for determining masses. This integration of 4583 asteroids was made backwards in time, with a 1 day step size, over the interval 1992 June 27 (JD 2448800.5) to 1950 Jan 1 (JD 2433282.5) and then forward from 1992 June 27 to 2017 July 16 (JD 2457950.5). The initial osculating elements were taken from STAMP 1992. Only asteroids with semimajor axes between 1.52 and 5.20 AU were integrated. All asteroids were given zero mass in the integration. Planetary perturbations were provided by integration of the planets at the same time. The initial positions and masses were taken from the DE200 ephemerides (Standish (1990)). The integrator used was the Adams-Pece multi-step integrator (Shampine & Gordon 1975). Once the integration was complete, the distances between all asteroid pair combinations were computed for each day of the integration. Those encounters of the largest asteroids (Table 1) that were less than 0.05 AU in relative distance, and those encounters of other asteroids that were less than 0.01 AU were analyzed. These limits are arbitrary, and resulted in a list of approximately 10,000 possibly interesting encounters.

A simple numerical filter was required to provide a more quantitative estimate of which encounters might result in an orbital change significant enough to have observable consequences. A crude, easy-to-implement discriminatory filter was constructed from the two-body scattering scenario (Fig. 1). In the center of mass frame, the scattering angle is

$$\tan\frac{1}{2}\theta=\frac{GM(m+M)}{v^2bM}, \qquad (1)$$

where θ is the angle through which the asteroids are scattered by the encounter, M and m are the masses of the larger

TABLE 1. The largest asteroids.

No.	Name	Diam.[a] (km)	Tholen[b] Class
1	Ceres	913	G
2	Pallas	523	B
3	Juno	244	S
4	Vesta	501	V
7	Iris	203	S
10	Hygiea	429	C
13	Egeria	215	G
15	Eunomia	272	S
16	Psyche	264	M
19	Fortuna	226	G
24	Themis	249	C
31	Euphrosyne	248	C
45	Eugenia	214	FC
52	Europa	312	CF
65	Cybele	245	P
87	Sylvia	271	P
107	Camilla	237	C
165	Loreley	160	CD
216	Kleopatra	140	M
324	Bamberga	242	CP
451	Patientia	230	CU
511	Davida	337	C
624	Hektor	234	D
704	Interamnia	333	F

[a] Diameters for 19 Fortuna, 24 Themis, and 624 Hektor are from Bowell *et al.* (1979), all others are from Tedesco (1989).
[b] Asteroid classifications are from Tholen (1989).

and smaller asteroids, respectively, v is the relative speed of the two asteroids, b is the impact parameter (the least distance between the two asteroids if no scattering occurs), and G is the gravitational constant.

Values for the masses of the two asteroids involved are required to obtain a scattering angle from equation (1). Measurements of asteroid masses are rare, but measurements of asteroid radii are more common. Equation (1) is re-cast in terms of the radius of the larger asteroid assuming that $M \gg m$ and the larger asteroid is homogeneous and spherically symmetric:

$$\tan \frac{1}{2}\theta = \frac{4}{3}\pi\rho G \frac{r^3}{v^2 b}, \tag{2}$$

where ρ is the density of the larger asteroid and r is its radius. The denominator on the right is quadratic in relative speed but only linear in impact parameter. Hence, a distant encounter at low relative speed may produce a greater perturbation than a closer, faster encounter. Assuming a density of 3 g/cm^3, and expressing the radius in km, the impact parameter in AU, and the relative speed in AU/day, the scattering angle becomes

$$\tan \frac{1}{2}\theta = 2 \times 10^{-21} \frac{r^3}{v^2 b}. \tag{3}$$

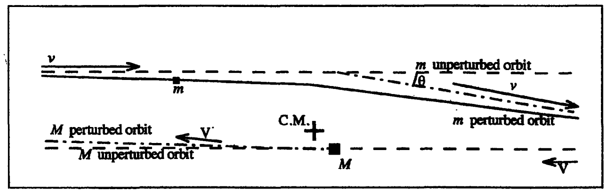

FIG. 1. The scattering of a small asteroid (m) by a more massive asteroid (M) in the center of mass frame of reference.

This equation is used to filter the encounters to find those with large scattering angles. The larger the scattering angle, the more likely that a series of observations that spans the encounter will yield a reasonable mass determination for the larger asteroid. Some skepticism is appropriate in interpreting the scattering angles, for the following reasons.

1. The original integration used zero masses for all asteroids. The initial osculating elements of the asteroids used in the integration also refer to orbits determined from observations without asteroid perturbations factored in. Thus the effect of asteroid-asteroid interactions is not present in the integration output, even though such interactions are exactly what is of interest. The integration yielded only a first-order model of the asteroid orbits.
2. The filter described above requires an estimate for the mass or radius of the larger asteroid. A mass estimate based on a radius is subject to large error because of the uncertainty in density, the assumption that the asteroid is spherical and homogeneous, and the large magnification factor that the radius uncertainty projects.
3. Several low-speed encounters span more than 50 days, a period sufficiently long that the center of mass system used for the scattering equation can no longer be considered inertial. In these cases the relative speed is no longer constant. For example, in an encounter between 1 Ceres and 348 May the mean relative speed was 1×10^{-4} AU/day, while the relative speed at closest approach was only 8×10^{-6} AU/day, a factor of 13 smaller.
4. The scattering angle is measured in the center of mass frame, which (for $M \gg m$) is effectively co-moving with the larger asteroid. This will not be the observed geocentric perturbation.

The exact value of the scattering angle is not very meaningful because the scattering angle computed by the filter is subject to these problems in interpretation. However, a "quality factor," which runs from 1 (low probability of significant deflection) to 5 (high probability) is computed based on the scattering angle. The quality factor is the logarithm of the computed deflection angle in arcseconds, rounded to the nearest integer. The computation of the deflection angle was done using the relative speed at closest approach because it gives a sense of the maximum strength of the encounter. In the cases in which the encounter takes place over a long period of time, the calculated value is apt to be orders of magnitude greater than the actual deflection. This is not a weakness of the filter because it is used to establish a scale of relative probabilities of interesting encounters; the most interesting then must be investigated using a more quantitative method. The encounters with a high quality factor are the

TABLE 2. Significant encounters involving large asteroids.

Date of Middle	Larger Asteroid	Type	Smaller Asteroid	Type	Distance (AU)	Time (days)	Quality
12 Jul 1954	1 Ceres	G	1646 Rosseland		0.0491	11	3
4 Apr 1958	1 Ceres	G	255 Oppavia	X	0.0416	26	3
23 Nov 1971	1 Ceres	G	454 Mathesis	C	0.0216	52	3
13 Sep 1973	1 Ceres	G	91 Aegina	C	0.0331	39	3
24 Dec 1975	1 Ceres	G	534 Nassovia	S	0.0226	55	3
13 Jul 1982	1 Ceres	G	548 Kressida	S	0.0488	12	3
6 Sep 1982	1 Ceres	G	2775 Odishaw		0.0465	15	3
10 Apr 1983	1 Ceres	G	786 Bredichina	C	0.0281	54	3
2 Sep 1984	1 Ceres	G	348 May		0.0424	114	5
13 Mar 1994	1 Ceres	G	2475 Semenov		0.0313	20	3
21 Jun 1994	1 Ceres	G	2377 Shcheglov		0.0464	15	3
14 Jan 1996	1 Ceres	G	2933 Amber		0.0196	52	3
17 Jan 2006	1 Ceres	G	2930 Euripides		0.0424	26	3
23 May 2009	1 Ceres	G	1847 Stobbe		0.0255	75	3
11 Aug 2012	1 Ceres	G	308 Polyxo	T	0.0447	17	3
8 Jan 2013	1 Ceres	G	3857 Cellino		0.0281	46	3
17 Jun 2017	1 Ceres	G	2541 1973 *DE*		0.0381	34	3
3 May 1968	2 Pallas	B	2204 Lyyli		0.0232	21	1
1 Jan 1991	2 Pallas	B	2495 Noviomagum		0.0352	17	1
14 Jul 2012	2 Pallas	B	1095 Tulipa		0.0392	12	1
14 May 2014	2 Pallas	B	2995 Taratuta		0.0493	3	1
17 Jun 1967	3 Juno	S	1346 Gotha		0.0446	56	3
5 Feb 1974	3 Juno	S	920 Rogeria	D	0.0444	20	1
31 Dec 1982	3 Juno	S	1767 Lampland	X	0.0056	37	2
4 Feb 1986	3 Juno	S	547 Praxedis	X	0.0464	21	1
30 Dec 1993	3 Juno	S	3389 Sinzot		0.0242	54	1
19 Jun 1951	4 Vesta	V	2066 Palala		0.0201	95	3
10 Dec 1957	4 Vesta	V	197 Arete	S	0.0352	54	3
8 Jan 1960	4 Vesta	V	4297 1938 *HE*		0.0495	8	4
27 Jan 1976	4 Vesta	V	197 Arete	S	0.0346	57	3
19 Jan 1983	4 Vesta	V	3057 Mälaren		0.0244	90	3
3 Sep 1983	4 Vesta	V	2714 Matti		0.0389	43	3
29 Dec 1983	4 Vesta	V	486 Cremona		0.0378	51	3
31 Jan 1991	4 Vesta	V	3802 Dornburg		0.0349	60	3
27 Sep 1991	4 Vesta	V	2873 Binzel		0.0377	108	4
19 Mar 1994	4 Vesta	V	197 Arete	S	0.0420	42	3
12 Jul 1994	4 Vesta	V	113 Amalthea	S	0.0402	54	3
16 Jul 1994	4 Vesta	V	3002 Delasalle		0.0391	151	4
16 Jun 1996	4 Vesta	V	17 Thetis	S	0.0194	138	3
6 May 2012	4 Vesta	V	197 Arete	S	0.0396	49	3
13 Jul 2014	4 Vesta	V	113 Amalthea	S	0.0455	40	4
1 Aug 1962	7 Iris	S	2825 1938 SD_1		0.0225	63	1
2 Feb 1979	7 Iris	S	1825 Klare		0.0114	103	1
12 Nov 1982	7 Iris	S	571 Dulcinea	S	0.0421	37	2
14 Feb 1989	7 Iris	S	836 Jole		0.0477	19	2
20 Oct 1997	7 Iris	S	1007 Pawlowia		0.0376	66	2
11 Feb 1984	10 Hygiea	C	1259 Ógyalla		0.0345	60	3
14 May 1984	10 Hygiea	C	1780Kippes		0.0431	45	3
11 Dec 1989	10 Hygiea	C	2619 Skalnaté Pleso		0.0224	89	3
26 Dec 1995	10 Hygiea	C	465 Alekto		0.0380	74	3
30 Mar 1998	10 Hygiea	C	3946 Shor		0.0144	181	4
25 Dec 2005	13 Egeria	G	757 Portlandia		0.0411	31	1
22 Mar 2014	13 Egeria	G	3489 Lottie		0.0402	28	1
30 Aug 1951	15 Eunomia	S	1284 Latvia	T	0.0330	63	2
11 May 1955	15 Eunomia	S	1313 Berna		0.0310	145	3
17 Oct 1965	15 Eunomia	S	1313 Berna		0.0500	19	5
9 Nov 1968	15 Eunomia	S	1284 Latvia	T	0.0407	70	3
30 Mar 1988	15 Eunomia	S	2613 Plzeň		0.0497	7	2
25 Jun 1956	16 Psyche	M	263 Dresda		0.0341	93	3
6 May 1972	16 Psyche	M	2819 Ensor		0.0449	47	3
13 Sep 1981	16 Psyche	M	2589 Daniel		0.0428	93	3
19 Sep 2001	16 Psyche	M	1442 Corvina		0.0281	90	3
29 Dec 2004	16 Psyche	M	468 Lina	C	0.0493	16	3
16 Jan 1972	19 Fortuna	G	2972 Niilo		0.0350	79	2
11 Jun 1986	19 Fortuna	G	46 Hestia	P	0.0350	67	2
25 Oct 2007	19 Fortuna	G	3289 Mitani		0.0221	150	3

TABLE 2. (continued)

Date of Middle	Larger Asteroid	Type	Smaller Asteroid	Type	Distance (AU)	Time (days)	Quality
11 Jun 2010	19 Fortuna	G	827 Wolfiana		0.0493	33	4
4 Aug 2013	19 Fortuna	G	2198 Ceplecha		0.0122	167	2
23 Dec 1974	24 Themis	C	2169 Taiwan		0.0370	64	2
23 Dec 1975	24 Themis	C	2296 Kugultinov		0.0157	332	4
20 Nov 1986	24 Themis	C	1768 Appenzella	F	0.0466	21	1
30 Jun 1989	24 Themis	C	1340 Yvette		0.0421	37	1
5 Dec 1995	24 Themis	C	494 Virtus	C	0.0321	40	1
3 May 1969	31 Euphrosyne	C	109 Felicitas	G	0.0429	16	1
15 Aug 1959	45 Eugenia	F	1055 Tynka	S	0.0427	37	2
29 May 1968	45 Eugenia	F	2560 Siegma		0.0331	56	2
5 Nov 1983	45 Eugenia	F	2814 Vieira		0.0280	60	1
27 Nov 1985	45 Eugenia	F	308 Polyxo	T	0.0134	93	2
15 Nov 2014	45 Eugenia	F	4374 1987 *BJ*		0.0469	28	2
8 Aug 1962	52 Europa	C	1605 Milankovitch		0.0385	47	2
22 Jun 1983	52 Europa	C	2837 Griboedov		0.0480	20	2
18 Nov 1988	52 Europa	C	3019 Kulin		0.0481	28	3
8 Jul 1990	52 Europa	C	1558 Järnefelt		0.0399	60	2
10 Feb 1994	52 Europa	C	2405 Welch		0.0239	85	2
27 Aug 1964	65 Cybele	P	147 Protogeneia	C	0.0453	29	1
17 Jul 1965	65 Cybele	P	1624 Rabe		0.0288	41	1
3 May 1968	65 Cybele	P	1082 Pirola	C	0.0485	19	2
17 Dec 1987	65 Cybele	P	1668 Hanna		0.0146	69	1
3 Apr 2016	65 Cybele	P	3071 Nesterov		0.0479	30	2
24 May 1952	87 Sylvia	P	1461 Jean-Jacques	M	0.0224	59	2
8 Aug 1964	87 Sylvia	P	1081 Reseda		0.0099	62	1
18 Aug 1989	87 Sylvia	P	2246 Bowell	D	0.0136	41	1
22 Mar 1991	87 Sylvia	P	1534 Nasi		0.0479	15	2
19 Sep 1996	87 Sylvia	P	3898 1981 SF_9		0.0363	36	1
21 Nov 1955	107 Camilla	C	515 Athalia	I	0.0221	50	1
8 Feb 1974	107 Camilla	C	1882 Rauma		0.0365	55	2
1 Apr 2000	107 Camilla	C	1882 Rauma		0.0493	11	2
15 Jan 2014	107 Camilla	C	1555 Dejan		0.0384	20	1
6 May 2014	107 Camilla	C	670 Ottegebe		0.0412	48	2
18 May 1959	165 Loreley	C	1298 Nocturna		0.0363	38	1
12 Oct 1969	165 Loreley	C	1737 Severny		0.0383	50	1
1 Jul 1981	165 Loreley	C	1913 Sekanina		0.0416	26	1
28 Sep 1985	165 Loreley	C	2964 Jaschek		0.0441	25	1
5 Nov 1986	216 Kleopatra	M	3976 1983 *JM*		0.0419	22	1
30 Jan 1952	324 Bamberga	C	916 America		0.0223	115	2
23 May 1971	324 Bamberga	C	1240 Centenaria		0.0284	44	1
19 Jul 1992	324 Bamberga	C	829 Academia		0.0220	30	1
5 Sep 2004	324 Bamberga	C	1066 Lobelia		0.0284	40	1
2 Jun 2006	324 Bamberga	C	4499 1989 AO_3		0.0347	41	1
7 Mar 1960	451 Patientia	C	977 Philippa	C	0.0285	86	2
9 Nov 1994	451 Patientia	C	698 Ernestina		0.0429	25	1
6 Nov 1995	451 Patientia	C	3286 Anatoliya		0.0308	68	2
20 May 2004	451 Patientia	C	159 Aemilia	C	0.0394	30	1
13 Apr 2017	451 Patientia	C	3286 Anatoliya		0.0150	82	1
20 Sep 1974	511 Davida	C	1847 Stobbe		0.0486	16	2
28 Dec 1980	511 Davida	C	1801 Titicaca		0.0445	23	2
25 Nov 1995	511 Davida	C	4624 Stefani		0.0395	29	1
30 Jul 2003	511 Davida	C	1464 Armisticia		0.0482	11	2
26 Jan 2006	511 Davida	C	3823 Yorii		0.0392	29	2
18 Sep 1986	704 Interamnia	F	881 Athene		0.0470	31	2
29 Nov 1995	704 Interamnia	F	445 Edna	C	0.0385	54	2
28 Feb 2006	704 Interamnia	F	3335 1966 *AA*		0.0326	52	2
10 May 2016	704 Interamnia	F	1971 Hagihara		0.0490	10	2
8 Jan 2017	704 Interamnia	F	3751 Kiang		0.0294	44	2

ones most likely to yield reasonable determinations of the larger asteroid's mass, provided observations of the smaller asteroid are available that are of sufficient quality and quantity and suitably distributed in time.

In practice what is important is not the scattering angle, but the change in the orbital elements of the smaller (perturbed) asteroid and the change in the observed position, due to the encounter. An algebraic manipulator was used to compute the derivative of the scattering angle as a function of the orbital elements. The changes in the elements vary widely

TABLE 3. Significant encounters involving small asteroids.

Date of Middle	Larger Asteroid	Type	Smaller Asteroid	Type	Distance (AU)	Time (days)	Quality
3 Nov 2013	12 Victoria	S	1110 Jaroslawa		0.0047	22	1
19 Sep 2013	14 Irene	S	1078 Mentha	S	0.0062	18	1
10 Jul 1983	20 Massalia	S	356 Liguria	C	0.0095	2	1
1 Jan 2002	28 Bellona	S	4056 Timwarner		0.0052	17	1
17 Sep 2011	70 Panopaea	C	4410 1989 YA		0.0053	18	1
22 Nov 2003	111 Ate	C	2455 Somville		0.0060	15	1
29 Mar 1962	720 Bohlinia	S	1029 La Plata	S	0.0064	101	3
25 Feb 1989	720 Bohlinia	S	1029 La Plata	S	0.0066	100	2
23 Mar 1982	804 Hispania	P	1002 Olbersia		0.0047	23	1
4 Nov 1993	1669 Dagmar	G	2248 Kanda		0.0061	40	1
10 Oct 2005	1686 De Sitter		2918 Salazar		0.0076	32	1

with the aspect of the encounter, of course, but the derivative values showed that the overwhelming majority of the encounters change the mean distance and/or eccentricity while leaving the other orbital elements relatively unchanged. This means that radar observations are potentially very important in asteroid mass determinations and a long term run-off in the position of the perturbed asteroid on the sky is expected.

3. RESULTS

The encounters likely to produce mass determinations are given in Tables 2 and 3. The format for both tables is the same. The first column is the date of the least separation. The second column is the number and name of the larger (perturbing) asteroid in the encounter. The third column is the first letter of the Tholen (1989) classification for the larger asteroid. The fourth column is the number and name of the smaller (perturbed) asteroid. The fifth column is the first letter in the Tholen classification of the smaller asteroid, if known. The sixth column is the distance between the two asteroids at the tabulated time. The seventh column contains the integer number of days for which the distance was less than 0.05 AU for the larger asteroid encounters (Table 2), and less than 0.01 AU for the smaller asteroid encounters (Table 3). The eighth column gives the quality factor, an estimate of the likelihood of producing a useful mass from the encounter.

Table 2 gives a selection of the 449 encounters of the larger asteroids with significant scatterings. Encounters with asteroids before the discovery of the perturbed asteroid have been removed. In the interest of space, only those encounters with high probability (quality factor 3 or greater) or the five best encounters for a given massive asteroid, whichever is more, are given in Table 2. The full list is available from the first author.

Table 2 shows a lack of opportunities for determining the masses of several of the larger asteroids such as 2 Pallas, 3 Juno, and 31 Euphrosyne. These asteroids generally have orbits that are either at high inclination, so the number of encounters are small, or have high eccentricities, so that encounters occur at high speeds. This does not mean that the masses of these large asteroids cannot be determined, but rather that a useful encounter did not take place within the time and distance constraints of the filter. The mass of Pallas, for example, has been determined from its effect on Ceres as a result of encounters during the nineteenth century (Schubart 1975).

Table 3 shows the 11 encounters between smaller aster-

TABLE 4. A comparison of encounters found by the filter and encounters used for making asteroid mass determinations.

Massive Asteroid	Test Asteroid	Reference	Found? (Quality Factor)
1 Ceres	2 Pallas	Schubart (1970)	too early
1 Ceres	4 Vesta	Schubart (1972)	too distant
1 Ceres	32 Pomona	Bowell *et al.* (1994)	yes (2)
1 Ceres	91 Aegina	Bowell *et al.* (1994)	yes (3)
1 Ceres	203 Pompeja	Goffin (1991)	too early
1 Ceres	325 Bamberga	Bowell *et al.* (1994)	too early
1 Ceres	348 May	Williams (1992)	yes (5)
1 Ceres	534 Nassovia	Bowell *et al.* (1994)	yes (3)
1 Ceres	2572 Gregory	Carpino & Kneževič (1995)	yes (2)
1 Ceres	2660 Wasserman	Carpino & Kneževič (1995)	yes (1)
1 Ceres	3643 1978 UN_2	Carpino & Kneževič (1995)	yes (2)
2 Pallas	1 Ceres	Schubart (1973)	too early
4 Vesta	197 Arete	Hertz (1966)	yes (2)
10 Hygiea	829 Academia	Scholl *et al.* (1987)	too early
704 Interamnia	993 Moultona	Landgraff (1992)	rejected

TABLE 5. Difference in the mean distance and eccentricity of perturbed asteroid for the ten best encounters found.

Massive Asteroid	Small Asteroid	Δa (AU)	Δe
1 Ceres	348 May	0.00000057	0.0000004
4 Vesta	4297 1938 *HE*	0.00000001	0.0000006
4 Vesta	2873 Binzel	−0.00000054	−0.0000011
4 Vesta	3002 Delasalle	0.00000008	0.0000002
4 Vesta	113 Amalthea	0.000000442	0.0000000
10 Hygiea	3946 Shor	0.00000024	0.0000000
15 Eunomia	1313 Berna	0.00000042	0.0000008
15 Eunomia	1284 Latvia	0.0000001	0.0000001
19 Fortuna	827 Wolfiana	0.00000001	0.0000000
24 Themis	2296 Kugultinov	0.0000003	0.0000000

oids that could produce masses. These encounters all take place at very small distances and very low speeds. The encounters between 720 Bohlinia and 1029 La Plata in 1962 and 1989 occurred at such low relative speeds at closest approach (6.1 m/s in 1962 and 8.0 m/s in 1989) that mass determinations for both asteroids are apparently possible even though neither of these asteroids is greater than 40 km in diameter (Tedesco 1989). However, encounters useful in determining masses between small asteroids are rare enough that it will be impossible to determine the masses of the majority of them directly based on optical observations.

Two tests have been made to determine how reliable the filter results are. First, the candidates for asteroid mass determination found here are compared with those asteroids used for mass determination in previous studies. Second, the ten encounters most likely to yield asteroid masses are examined in greater detail to determine what the observational signature of these encounters is expected to be.

3.1 Comparison to Other Studies

Table 4 shows all of the small asteroids used for mass determinations found in the literature along with whether or not the filter found the encounter. There are 15 perturbed asteroids that have been used to make asteroid mass determinations of five large asteroids. The filter found eight of the small asteroids. Five of the seven encounters not found were missed because the encounter took place before the start of the integration. One encounter was missed because it occurred at greater than 0.05 AU. The other encounter missed was rejected by the filter as being too weak to result in a reliable mass. The rejected encounter was used by Landgraf 1992 to make a determination of the mass of 704 Interamnia. Inspection of this encounter found it just missed being categorized with a quality factor of 1, that is the filter judged the encounter as being too weak to have a reasonable chance of making a mass determination. Hence, the filter may be a little too conservative in determining whether or not an encounter might be useful. Overall, the filter missed those encounters which were outside its arbitrary time and distance limits and could have missed some weak but useful encounters.

3.2 Examination of the Ten Encounters with the Greatest Potential for Mass Determination

Table 2 contains two quality factor 5 encounters and seven quality factor 4 encounters. These nine encounters, along with the quality factor 3 encounter between Eunomia and 1284 Latvia are examined to determine the observable effect of the massive asteroid on the perturbed asteroid.

Two solutions are made fitting the orbit of the perturbed asteroid to observations The first solution includes the massive asteroid perturbation while the second solution is made without the massive asteroid. The integration of the orbit of the perturbed asteroid covers the time from its discovery to present day or until 25 years after the encounter with the massive asteroid, whichever is later. These two integrations are compared to find the difference in the observables caused by the perturbation of the massive asteroid.

The perturbed asteroid must be fit to the observations to establish the difference between the elements of the perturbed asteroid with and without the perturbation by the massive asteroid. Because the observations contain random errors it is possible to produce a satisfactory orbit for the perturbed asteroid that does not require the perturbation by the massive asteroid, but fits the observations with increased residuals. This is an additional reason why the filter in the previous section is not an error-free source for usable encounters.

Except for Ceres and Vesta, the orbits of both asteroids for each massive asteroid-perturbed asteroid pair are fit to the data currently available from the Minor Planet Center. The observations used to determine the orbits of Ceres and Vesta, were taken from six USNO transit circle catalogs, the W25 (Watts & Adams 1949) covering 1928 through 1935, the $W3_{50}$ (Adams *et al.* 1964) covering 1949 through 1956, the $W4_{50}$ (Adams & Scott 1968) covering 1956 through 1962, the $W5_{50}$ (Hughes & Scott 1982) covering 1963 through 1971, the WL_{50} (Hughes *et al.* 1992), and the $W1_{J00}$ (in preparation) covering 1977 through 1982. All of the catalogs except the WL_{50} were observed using the 6″ transit circle in Washington, DC. The WL_{50} was observed using the USNO 7″ transit circle in El Leoncito, Argentina. Transit circle observations from the Royal Greenwich Observatory between 1897 and 1940 were added to the USNO data. There are 1442 observations of Ceres and 1641 observations of Vesta

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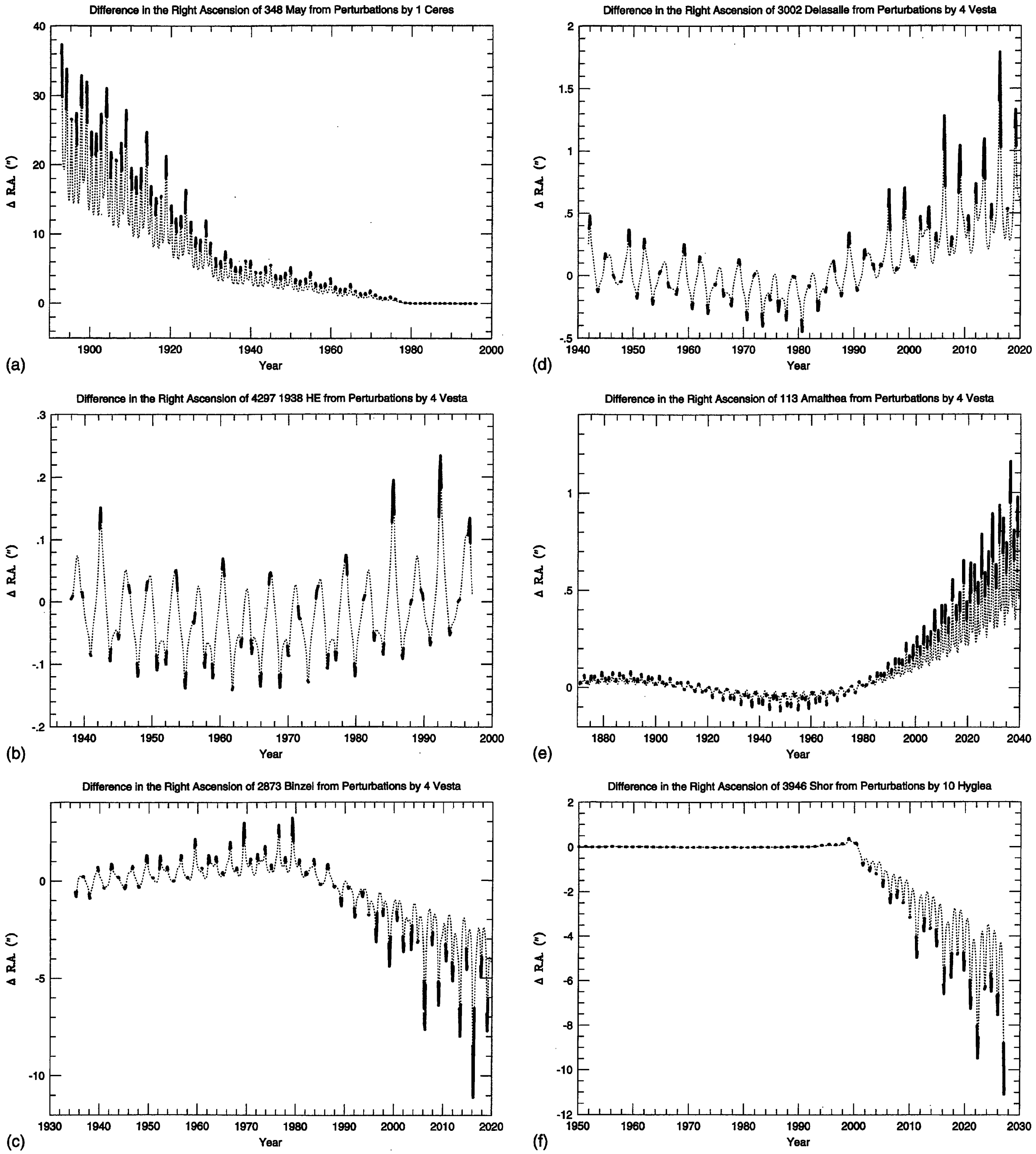

FIG. 2. The change in the right ascension of the small asteroid caused by the perturbation of the massive asteroid for the ten encounters rated most likely to produce an asteroid mass determination. For each small asteroid, the thick line gives the difference in R.A. when the asteroid is within 45° of opposition with the Earth, while the difference for the rest of the time is given by a dotted line. In all cases, the orbit of the small asteroid was determined by fit to the observational data available from the Minor Planet Center.

used. The masses for Ceres and Vesta are 5.0×10^{-10} $M_\odot$ (Viateau & Rapaport 1995) and 1.38×10^{-10} $M_\odot$ (Schubart & Matson 1979), respectively. The masses of 10 Hygiea and 15 Eunomia are estimated assuming a density of 3 g/cm^3 and using the diameters for these asteroid determined by Tedesco 1989. Similarly, masses for 19 Fortuna and 24 Themis are estimated assuming the same density and using the diameters determined by Bowell *et al.* (1979).

Adjustment of parameters are done using the Planetary Ephemerides Program (PEP) (Ash 1965), a high-accuracy program for generating ephemerides of solar system bodies. The PEP integrator used is an Adams-Morgan integrator. Perturbations from the nine major planets, along with Ceres, Pallas, and Vesta, where appropriate, are included. DE200 (Standish 1990) positions and masses of the planets are used for the perturbations and for the Earth position in fitting

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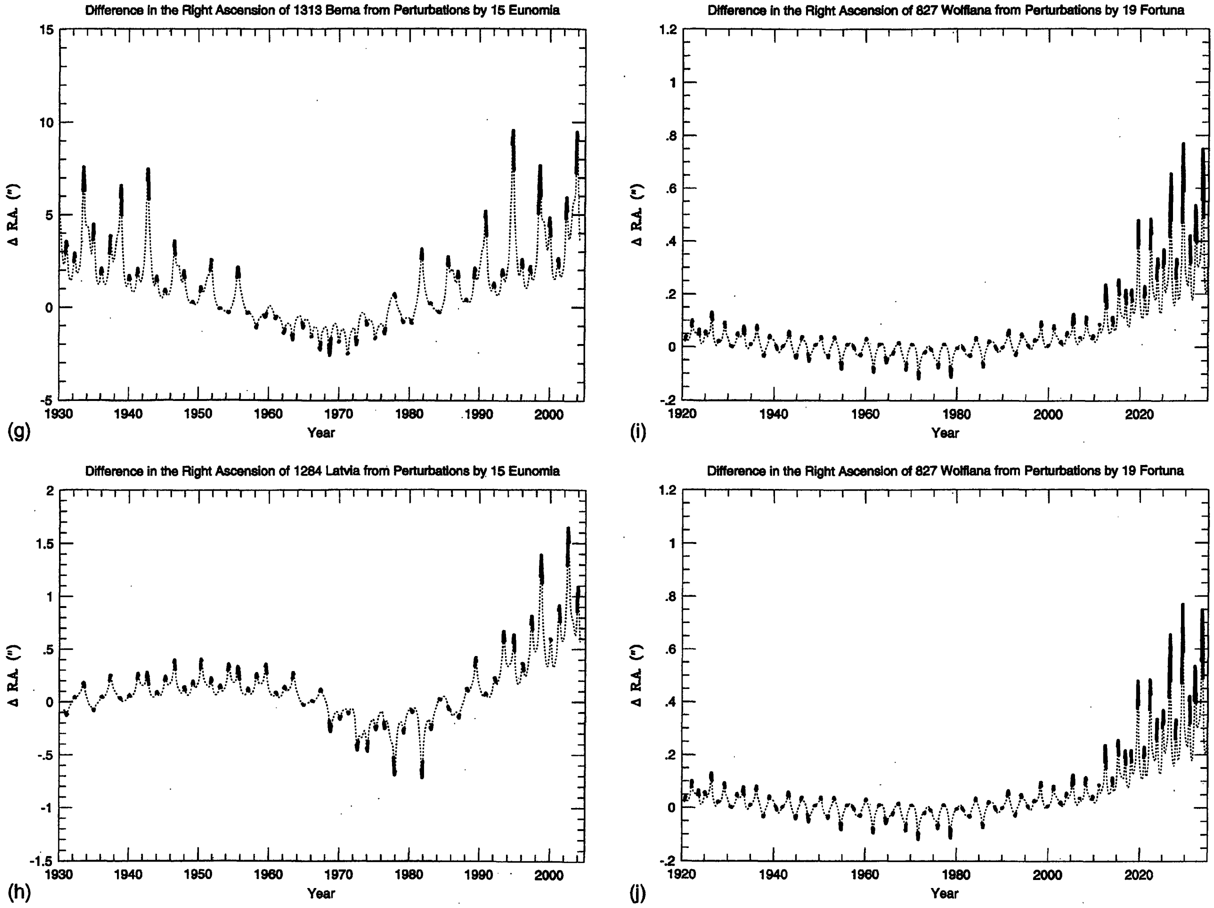

FIG. 2. (continued)

the data.

A perturbing body contributes nothing of significance to a model unless it causes a change in at least one other model parameter that is greater than the uncertainty in that parameter. The parameters that are affected the most by a perturbing body, in most cases, are the mean distance and eccentricity. Table 5 shows the change in the mean distance and eccentricity caused by the massive asteroid.

Four of the perturbed asteroids, 3002 Delasalle, 113 Amalthea, 3946 Shor, and 827 Wolfiana, have encounters either in the future or so recent that the effect of the perturbation is not expected to be apparent in the current data. The six encounters that have already taken place have an average change in the mean distance between the perturbed and unperturbed orbits of 2.8×10^{-7} AU and average change in the eccentricity of 5×10^{-7}. The ten perturbed asteroids have an average of 83 observations. The average uncertainty in the mean distance is 6×10^{-8} AU, and the average uncertainty in the eccentricity is 4×10^{-7}. The three asteroids with the most observations have an average of 165 observations, the uncertainty in the mean distance is 2×10^{-8} AU, and the average uncertainty in the eccentricity is 2×10^{-7}. The change in the mean distance is much more detectable than the change in the eccentricity, because the uncertainty in the mean distance is a factor of nearly seven smaller than the uncertainty in eccentricity for the perturbed asteroid. A mass determination with an uncertainty of 10% for these encounters requires the mean distance of the perturbed asteroid have an uncertainty on the order of 10^{-8} AU. Comparing the average number of observations with the average uncertainty in the mean distance, a perturbed asteroid orbit with an uncertainty in the mean distance on the order of 10^{-8} AU can be achieved with about 100 observations with a 1″ scatter in each coordinate. The perturbed asteroid orbit needs to be well determined both before and after the encounter to provide the best possible mass. A mass determination with an uncertainty of 10% or better is reasonable.

Figure 2 shows the change in the right ascension of the perturbed asteroid caused by the perturbation of the massive asteroid for each of the ten highest rated encounters. Each asteroid plot shows the change the massive asteroid causes in the right ascension of the perturbed asteroid, in seconds of arc. The orbit of the perturbed asteroid, both with and without the massive asteroid perturbation, is fit to the observations currently available from the Minor Planet Center for each perturbed asteroid. The thick line gives the difference in R.A. when the asteroid is within 45° of opposition with the Earth. The difference for the rest of the time is given by a dotted line. Each plot covers the period from the dis-

TABLE 6. Possible resonant asteroid pairs.

Larger Asteroid	Smaller Asteroid	Resonance
4 Vesta	113 Amalthea	1:1
4 Vesta	197 Arete	5:4
4 Vesta	1945 Wesselink	9:8
4 Vesta	2676 Aarhus	1:1
4 Vesta	2708 Burns	3:2
4 Vesta	2720 Pyotr Pervyj	1:1
7 Iris	4550 1977 HH_1	3:2
15 Eunomia	1284 Latvia	1:1
15 Eunomia	1313 Berna	1:1
15 Eunomia	1738 Oosterhoff	3:4
19 Fortuna	3543 1964 VA_3	3:2
19 Fortuna	3583 Burdett	1:1
107 Camilla	1882 Rauma	4:5
451 Patientia	159 Aemilia	1:1
451 Patientia	3286 Anatoliya	4:5
720 Bohlinia	1029 La Plata	1:1

TABLE 7. Asteroids that encounter more than one scattering asteroid.

Scattered Asteroid	First Encounter	Second Encounter
46 Hestia	19 Fortuna	24 Themis
77 Frigga	4 Vesta	24 Themis
84 Klio	1 Ceres	52 Europa
197 Arete	1 Ceres	4 Vesta
308 Polyxo	1 Ceres	45 Eugenia
720 Bohlinia	19 Fortuna	1029 La Plata
993 Moultona	45 Eugenia	52 Europa
1259 Ógyalla	10 Hygiea	19 Fortuna
1550 Tito	1 Ceres	10 Hygiea
1825 Klare	7 Iris	10 Hygiea
1847 Stobbe	1 Ceres	511 Davida
1971 Hagihara	10 Hygiea	704 Interamnia
2455 Somville	16 Psyche	111 Ate
2633 Bishop	4 Vesta	15 Eunomia
2775 Odishaw	1 Ceres	7 Iris
3071 Nesterov	52 Europa	65 Cybele
3371 Giacconi	15 Eunomia	24 Themis

TABLE 8. Asteroids encounters which have the largest quality factor for each of the 34 asteroids for which masses may be determined.

Date of Middle	Larger Asteroid	Type	Smaller Asteroid	Type	Distance (AU)	Time (days)	Quality
2 Sep 1984	1 Ceres	G	348 May		0.0424	114	5
1 Jan 1991	2 Pallas	B	2495 Noviomagum		0.0352	17	1
31 Dec 1982	3 Juno	S	1767 Lampland	X	0.0056	37	2
16 Jul 1994	4 Vesta	V	3002 Delasalle		0 . 0391	151	4
14 Feb 1989	7 Iris	S	836 Jole		0.0477	19	2
30 Mar 1998	10 Hygiea	C	3946 Shor		0.0144	181	3
3 Nov 2013	12 Victoria	S	1110 Jaroslawa		0.0047	22	1
22 Mar 2014	13 Egeria	G	3489 Lottie		0.0402	28	1
19 Sep 2013	14 Irene	S	1078 Mentha	S	0.0062	18	1
17 Oct 1965	15 Eunomia	S	1313 Berna		0.0500	19	5
13 Sep 1981	16 Psyche	M	2589 Daniel		0.0428	93	3
11 Jun 2010	19 Fortuna	G	827 Wolfiana		0.0493	33	3
10 Jul 1983	20 Massalia	S	356 Liguria	C	0.0095	2	1
23 Dec 1975	24 Themis	C	2296 Kugultinov		0.0157	332	4
1 Jan 2002	28 Bellona	S	4056 Timwarner		0.0052	17	1
3 May 1969	31 Euphrosyne	C	109 Felicitas	G	0.0429	16	1
15 Nov 2014	45 Eugenia	T	4374 1987 *BJ*		0.0469	28	2
18 Nov 1988	52 Europa	C	3019 Kulin		0.0481	28	3
3 Apr 2016	65 Cybele	P	3071 Nesterov		0.0479	30	3
17 Sep 2011	70 Panopaea	C	4410 1989 YA		0.0053	18	1
22 Mar 1991	87 Sylvia	P	1534 Nasi		0.0479	15	2
6 May 2014	107 Camilla	C	670 Ottegebe		0.0412	48	2
22 Nov 2003	111 Ate	C	2455 Somville		0.0060	15	1
1 Jul 1981	165 Loreley	C	1913 Sekanina		0.0416	26	2
5 Nov 1986	216 Kleopatra	M	3976 1983 *JM*		0.0419	22	2
30 Jan 1952	324 Bamberga	C	916 America		0.0223	115	2
7 Mar 1960	451 Patientia	C	977 Philippa	C	0.0285	86	2
20 Sep 1974	511 Davida	C	1847 Stobbe		0.0486	16	2
29 Nov 1995	704 Interamnia	F	445 Edna	C	0.0385	54	2
29 Mar 1962	720 Bohlinia	S	1029 La Plata	S	0.0064	101	2
23 Mar 1982	804 Hispania	P	1002 Olbersia		0.0047	23	1
29 Mar 1962	1029 La Plata	S	720 Bohlinia	S	0.0064	101	3
4 Nov 1993	1669 Dagmar	G	2248 Kanda		0.0061	40	1
10 Oct 2005	1686 De Sitter		2918 Salazar		0.0076	32	1

covery of the perturbed asteroid until either present day, or 25 years after the encounter with the massive asteroid, whichever is greater. There is a significant perturbation of the perturbed asteroid by the massive asteroid in 90% of these encounters. The only insignificant perturbation is that of 1938 *HE* which shows a maximum perturbation of 0.″1 over the time period plotted. The perturbation of Latvia, rated with a quality factor of 3 by the filter is found to undergo a larger perturbation than five of the seven quality factor 4 encounters.

The filter is shown to be useful in finding encounters that may be useful in making asteroid mass determinations. Although an encounter occurred in all cases, the filter does not guarantee that the encounter resulted, or will result, in an orbital change large enough that the change in the observations are large enough to allow an accurate asteroid mass determination. It does provide a starting point for looking for asteroid masses since nine of the ten high quality encounters produce large changes. The filter found all but one of the encounters previously used to make mass determinations within the time span and distance limits covered. The encounter missed lies just outside the criterion of a quality factor 1 encounter.

4. ASTEROID-ASTEROID RESONANCES AND MULTIPLE ENCOUNTERS

Thirteen examples of possible resonances between asteroids were found from the encounter data set, in addition to the resonances between Vesta and Arete (Hertz 1968) and Eunomia with Latvia and Berna (Scholl *et al.* 1987). These resonances are given in Table 6.

No formal determination of whether or not these resonances are real, in the sense that the perturbations of the larger asteroid are responsible for determining the orbit of the smaller asteroid, has been made. There are indications that these are resonances, not coincidences. For the non-1:1 resonance asteroids the commensuration of their periods are high, with less than 0.1% difference between the period of the smaller asteroid and that of the resonance period with the larger asteroid. For most of the 1:1 resonances the periods are less close to being commensurate, with differences of a few percent in the worst cases. The differences in the periods of these asteroids appear to indicate that they are coincidences, rather than resonances. However, five of the eight 1:1 pairs, 4 Vesta-113 Amalthea, 4 Vesta-2676 Aarhus, 4 Vesta-2720 Pyotr Pervyj, 19 Fortuna-3583 Burdett, and 451 Patientia-159 Aemilia, have encounters at both nodes of their orbits, that is not only are the periods nearly commensurate, but the orbits must be aligned to to allow close encounters at both of the nodes. It is possible that the alignment of the orbits is due to Jupiter's influence, but this influence has been shown to only affect the positions of the perihelia and not the nodes of the orbits (Watson 1956). The chance that the orbital alignment of all these asteroid pairs is a coincidence is small. A sixth 1:1 resonance pair, Bohlinia-La Plata, is very close to being commensurate. The Williams (1989) proper elements give a synodic period of one with respect to the other of over 3000 yr.

There were 17 different small asteroids that were found to interact with more than one large asteroid. These asteroids are given in Table 7. These 17 asteroids account for 36, or 7.8%, of the asteroid-asteroid encounters. Since the observations of the perturbed asteroid need to be made for several orbital periods before and after the encounter, the multiple asteroid encounters shows that care needs to be taken so that the effects of these interactions are separated out in making asteroid mass determinations.

Not only are encounters with more than one large asteroid at different times found, the integration database found several hundred occasions in which two small asteroids approached a single large asteroid at the same time. There are also several examples of four asteroid, one large and three small, approaching within 0.05 AU of each other at the same time in the database.

5. CONCLUSIONS

A total of 460 asteroid-asteroid encounters were found that could be used to determine the masses of 34 different asteroids. The ability of a given encounter to make a mass determination is rated with a quality factor running from 5 (high probability) to 1 (low probability).

The change in the mean distance of the small asteroids for the 10 highest quality factor encounters was 2.5×10^{-7} AU and the change in the eccentricity was 5×10^{-7}. The average number of astrometric observations for the small asteroids was 83 and produced an average uncertainty of 6×10^{-8} AU in the mean distance and 4×10^{-7} in eccentricity. An asteroid mass determination with an uncertainty of 10% or better is possible for these encounters provided a hundred or so astrometric observations of the small asteroid with an accuracy of 1″ in each coordinate from both before and after the encounter with the massive asteroid.

The majority of the encounters passed by the filter had a large asteroid greater than 200 km in diameter. There were some encounters that passed with the larger asteroid as small as 40 km in diameter. Table 8 gives the asteroids that have the largest quality factor for each of the 34 asteroids for which encounters were found. Observations of these asteroids have a particularly good chance of enhancing knowledge of asteroid masses and densities.

A total of 16 possible resonances between pairs of asteroids that result in close encounters were found. Seventeen occurences of small asteroids interacting with more than one large asteroid were also found. Care must be taken in making mass determinations, because perturbations by multiple asteroids are not rare.

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