REPORT

OF THE

SENIOR ASSESSMENT PANEL

FOR THE

INTERNATIONAL ASSESSMENT

OF THE

U.S. MATHEMATICAL SCIENCES

March 1998

In 1993, the Committee on Science, Engineering and Public Policy (COSEPUP) of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine issued the report *Science, Technology and the Federal Government's National Goals For a New Era*. In that report, COSEPUP suggested that the United States adopt the principle of being among the world leaders in all major fields of science so that it can quickly apply and extend advances in science wherever they occur. In addition, the report recommended that the United States maintain clear leadership in fields that are tied to national objectives, capture the imagination of society, or have a multiplicative effect on other scientific advances. To measure international leadership, the report recommended the establishment of independent panels that would conduct comparative international assessments of scientific accomplishments in particular research fields.

Since 1995, the National Science Foundation has been examining various modes of response to the Government Performance and Results Act (GPRA). This act requires an evaluation of how well the Foundation has met its strategic goals, which are:

- To enable the United States to uphold a position of world leadership in all aspects of science, mathematics, and engineering;
- To promote the discovery, integration, dissemination, and employment of new knowledge in the service of society; and
- To achieve excellence in U.S. science, mathematics, engineering and technology education at all levels.

Given the recommendations by COSEPUP, it was decided to conduct an international assessment of the mathematical sciences, as a demonstration project in response to the GPRA requirement. Hence, in March 1997, a Panel was assembled to conduct such an assessment. The Panel consisted of leading mathematicians drawn largely from outside of the United States and of individuals from important U.S. stakeholder communities that are strongly dependent on the mathematical sciences. None had received recent NSF funding for their research in the mathematics sciences. This report is the result of the Panel's deliberations.

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PREFACE

Why should anybody but mathematicians care about the standing of U.S. mathematics in the world? To laymen and to policy-makers and legislators, the answer is far from obvious. A major virtue of this report is that it provides answers they can understand. It explains why virtually all aspects of our society, economy, and national security are increasingly dependent on mathematics, and not just the mathematics already discovered. The dependency also extends to continuing breakthroughs by the "pure" mathematics community.

The report carries a mixed message. The U.S. mathematics community holds a dominant position in the world, but several adverse trends are undermining it. A few are beyond the immediate control of the United States or any other country, affecting mathematics adversely everywhere. Others are peculiar to the United States. Some, such as weak K-12 mathematics education in the United States, are problem areas beyond the scope of this assessment, primarily because they are not very sensitive to National Science Foundation resource allocation policies. Others, however, can be significantly affected by NSF policies.

If we wake up to discover that we have allowed the dominant position of U.S. mathematics to erode, we will pay a heavy price in foregone progress in technology, science, and economic productivity. Only if policy-makers, legislators, and the mathematics community understand this danger alike can they act effectively to avoid it. At least two actions are clearly urgent: a) providing more resources for mathematics and b) using resources in more effective ways.

How objective is a report that makes such a straight forward demand for more resources? The exceptional quality of the panel members alone should remove all concern about parochialism, but it should also be noted that none is supported by the NSF, although one receives support from the Department of Defense. Most are internationally distinguished foreign mathematicians with no potential claim on NSF funding. The panel also includes some "stake holders," i.e., mathematicians and a scientist working in industry, finance, and another university science discipline. They have helped widen the panel's concerns beyond the narrow interests of the pure mathematics community. Notwithstanding the inherently ambiguous nature of the task given the panel, the report has a strong claim to objectivity.

As the chairman and the most thoroughly non-mathematician on the panel, I want to express a word of gratitude to Dr. Donald Lewis, director of the Division of Mathematical Sciences, NSF, for his intellectual guidance and assistance throughout the assessment process.

William E. Odom
Lieutenant General, USA, Retired

EXECUTIVE SUMMARY

The modern world increasingly depends on the mathematical sciences in areas ranging from national security and medical technology to computer software, telecommunications, and investment policy. More and more American workers, from the boardroom to the assembly line, cannot do their jobs without mathematical skills. Without strong resources in the mathematical sciences, America will not retain its pre-eminence in industry and commerce.

At this moment, the U.S. enjoys a position of world leadership in the mathematical sciences. But this position is fragile. It depends very substantially on immigrants who had their mathematical training elsewhere and in particular on the massive flow of experts from the former Communist bloc. The latter, at least, will not continue because there is little talent left to drain and even less new talent being trained.

Young Americans do not see careers in the mathematical sciences as attractive. Funding for graduate study is scarce and ungenerous, especially when compared to funding for other sciences and with what happens in Western Europe. Further, it takes too long to obtain a doctorate because of the distractions of excessive teaching. Students wrongly believe that jobs that call for mathematical training are scarce and poorly paid. Weaknesses in K-12 mathematics education undermine the capabilities of the U.S. workforce.

Based on present trends, it is unlikely that the U.S. will be able to maintain its world leadership in the mathematical sciences. It is, however, essential for the U.S. to remain the world leader in critical subfields, and to maintain enough strength in all subfields to be able to take full advantage of mathematics developed elsewhere. Without remedial action by the universities and National Science Foundation (NSF), the U.S. will not remain strong in mathematics: there will not be enough excellent U.S.-trained mathematicians, nor will it be practicable to import enough experts from elsewhere, to fill the Nation’s needs.

Since the time of Pythagoras, mathematics has been one of the intellectual pinnacles of civilization. Although many mathematicians develop their subject as a purely logical structure, with no reference to the outside world, every area of mathematics, however pure it appears, has important applications: good pure mathematicians will always deserve support. For the benefit of the Nation and of U.S. mathematics, however, there must be more effective interaction between mathematicians and the users of mathematics. All participants in mathematics must share the responsibility for improving this interaction.

Since the National Science Foundation's role is to support scientific activities within universities, we recommend that it encourage programs that:

- Broaden graduate and undergraduate education in the mathematical sciences.

- Provide support for full time graduate students in the mathematical sciences comparable with the other sciences.

- Provide increased opportunity for postdoctoral study for those who wish to become academic researchers as a means to broaden and strengthen their training as professional mathematicians.

- Encourage and foster interactions between university-based mathematical scientists and users of mathematics in industry, government, and other disciplines in universities.

- Maintain and enhance the historical strength of the mathematical sciences in its academic setting as an intellectual endeavor and as a foundation for applications, sustaining the United States' position of world leadership.

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I. INTRODUCTION

Mathematics in a Time of Rapid Change

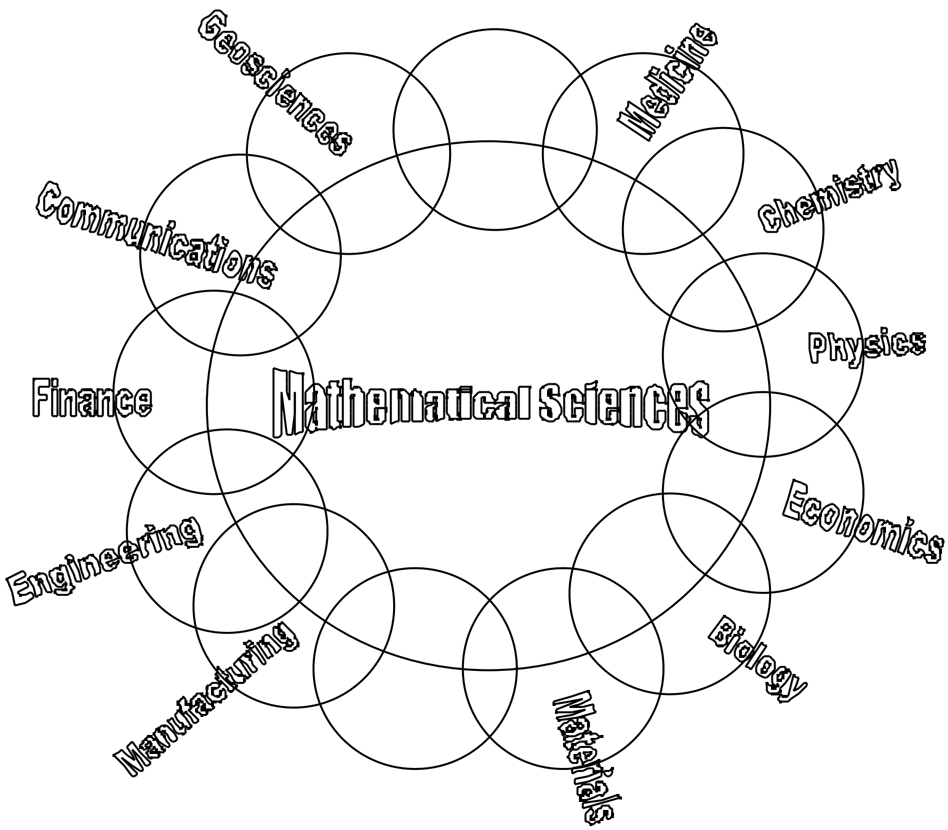
For the mathematical sciences, the past fifty years have been a golden era of great discoveries and new developments spanning the entire spectrum from basic theory to real-world applications. Accomplishments in basic theory have been wide ranging, including the development of sympletic geometry, mirror symmetry, and quantum groups; the discovery of solitons; the proof of Fermat's theorem some 350 years after it was stated; the classification of all simple finite groups (the building blocks for groups generally). Subfields, once viewed as quite disjoint, are now seen as part of a whole. Striking examples of applications also occurred, such as; the designing of the Boeing 777 airliner, which relied on mathematical theory, computation modeling, and powerful simulation techniques to replace physical testing and to speed the design process; the use of wavelets as a fundamental tool for fingerprint analysis; the multipole algorithm in electromagnetic computation; and neural network algorithms in pattern matching applications. U.S. mathematicians have been at the forefront of these and other world developments in mathematics.

The mathematical sciences -- and all other sciences -- are performed in a world that is changing rapidly. The exploding importance of information to all sectors of society and the pervasive role of technology in maintaining the security and prosperity of the nation have placed the mathematical sciences in a position of central importance. Mathematics provides the context for communication and discovery in many other disciplines. Competitive pressures throughout business and government, coupled with the broad expansion of computer analysis and data management, have extended the applications of mathematics to every domain of human activity.

The fundamental changes taking place in many areas of science and technology—especially biology, communications, and computation — are accompanied by important new problems that cannot be solved without new mathematics. The modern desires to improve decision-making (for example, to make real-time stock market or hedging decisions) and to understand very complex problems (for example, to model the impact of human activities on the environment) will require original mathematical techniques. These deep challenges, which are vitally important to the nation, offer novel opportunities for research in mathematics. Without increased openness by mathematicians to problems of other disciplines, mathematics may miss opportunities to contribute to and gain from these developments.

The missed opportunities would extend beyond mathematics. Excellent mathematical ideas developed in biology would probably not be helpful to financial modeling, say, because the jargon used in the two fields is very different. Mathematics can help standardize developments in one field for use by others. Two examples include finite element methods, developed by structural engineers, and sparse matrix methods, developed by power systems engineers and economists. In both cases, standardization and generalization of the results by mathematicians have led to applications in many other fields. The following graphics further illustrate this point.

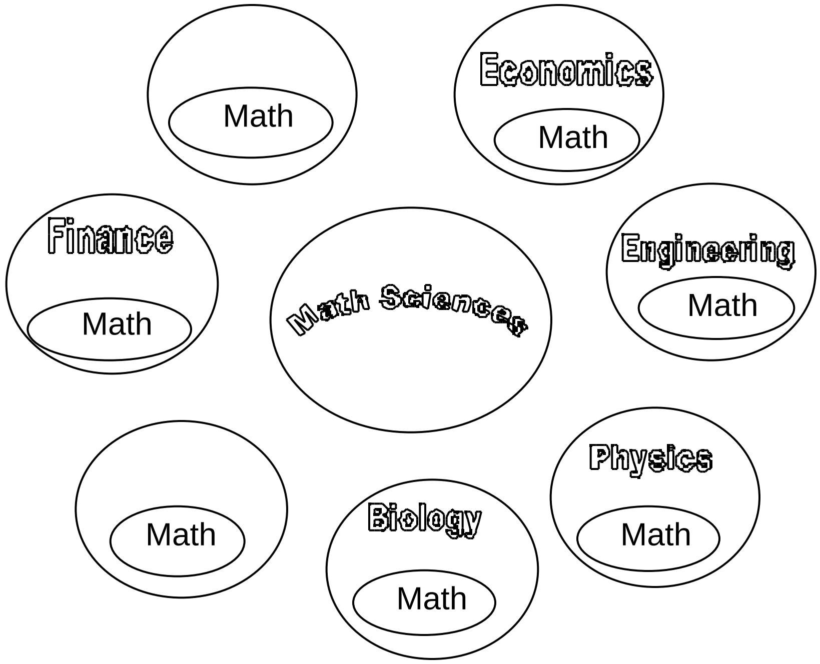
In an ideal world, mathematics has a clean flow from its core out to applications and from applications back to the core. This flow facilitates the adaptation of mathematical concepts from one field, such as physics, to economics, and vice versa.



When the mathematical sciences retreat from such multidisciplinary involvements, mathematics suffers from lack of enrichment by the ideas and challenges of other disciplines. The other disciplines also suffer, for two reasons:

- The disciplines lose the expertise and easy access to the vast knowledge base developed by the mathematical sciences; and,

- The disciplines develop overly specialized mathematical languages and tools that inhibit multidisciplinary communication.



In 1993, there were 22,820 doctoral mathematical scientists employed in the United States. It was estimated[1] that of these mathematical scientists, 14,670 (64.3%) were employed at universities and 4-year colleges (6,427 at doctorate granting universities), 5,160 (22.6%) in industry, and 960 (4.2%) by the Federal government. Of those active in research, 35% reported receiving Federal funding.[2] It is worthwhile to compare with other sciences?

1993 Fulltime Academic Doctorate Faculty

	Total Number	Active in Research	Receiving Fed Support	% Active with Fed Support
Biological Sciences	68194	51767	36143	69%
Physical Sciences	28644	20029	13463	67%
Mathematical Sciences	15475	9517	3250	35%

The mathematical sciences are divided into two largely independent groups: 1) academic mathematicians, and 2) users of mathematics, both inside and outside the university community. The weak coupling between these two groups is a central problem for mathematics worldwide (as it is for some other areas of science and technology). To

[1] **Characteristics of Doctoral Scientists and Engineers in the United States: 1995**, (NSF 97-319), Table 20, page 34. Other employment groups included self-employed, other educational institutions, private not for profit, and state and local government.
[2] **Science and Engineering Indicators**, 1996, Table 5-26, page 203. Table 5-27, page 203.

enhance the vigor of both groups, it is vital that the creators and users of mathematics be more strongly connected. Excellent mathematics, however abstract, leads to practical applications. In turn, hard problems in nature stimulate the invention of new mathematics.

Traditionally, abstract mathematicians follow natural paths of inquiry toward the development of new concepts and new theories. They are often influenced by problems arising outside of mathematics, but as often, perhaps more often, they are driven by the inherent beauty and inner consistency of the results. It might be years or decades before such concepts find application – if they ever do. The physicist Eugene Wigner marveled as to "the unreasonable effectiveness of [abstract] mathematics in the natural sciences;" nowadays, one would add finance and management. Arthur Jaffe (David I [3], p. 120) explains this as follows: "Mathematical ideas do not spring full grown from the minds of researchers. Mathematics often takes its inspirations from patterns in nature. Lessons distilled from one encounter with nature continue to serve as well when we explore other natural phenomena." Even if one values mathematics only for its role in applications, one must value these basic abstract investigations because they provide the foundation upon which applied and computational mathematics, as well as statistics and computer science, are based.

The United States excels in abstract mathematical research. To make the best use of this strength, there is a need for a faster flow of knowledge between the creators and users of mathematics. The progress of science deteriorates when mathematicians or the users of mathematics must develop new knowledge "on demand." Only when the doors of communication are open wide can the mathematical enterprise function at full potential. Users benefit from quick access to known mathematics, and mathematicians are challenged by new formulations and questions from users.

Strengthening the connections between the creators and the users of mathematics, while maintaining historical proficiency in pure mathematics, is the most important opportunity now open to the National Science Foundation in its support of the field. It is imperative to find new ways to speed the flow of mathematical discoveries between academic mathematical scientists and those who use their results, and between different fields of the mathematical sciences.

The third community of the mathematical enterprise consists of the students. During this time of rapid change, the way students are educated must keep pace with quickly shifting realities of vocation and employment. There is a need to broaden and extend the curriculum for future mathematical scientists, as well as for the users of that science, to make them more flexible and ensure that the two groups are equipped to interact

[3] **Renewing U.S. Mathematics,** 1984, National Academy Press, Washington.

effectively in the interests of their disciplines and of society. It will require ingenuity on the part of the mathematical scientists and of other disciplinarians to do so, without losing depth.

The importance of the mathematical sciences to society dictates that we adapt the way we prepare the next generation of mathematical scientists to face new realities, which include increasingly multidisciplinary work and the extension of the mathematical sciences into other fields. **Tomorrow's mathematical scientists must be educated in new ways if they are to contribute to the mathematical enterprise and to society across the full range of employment opportunities and professional challenges.**[4]

The Purpose of This Report

This report is part of the National Science Foundation's response to comply with the **Government Performance and Results Act** (GPRA). The act requires an evaluation of how well the Foundation has met its strategic goals, which are:

- to enable the United States to uphold a position of world leadership in all aspects of science, mathematics, and engineering;

- to promote the discovery, integration, dissemination, and employment of new knowledge in the service of society; and

- to achieve excellence in U.S. science, mathematics, engineering, and technology education at all levels.

In March 1997, the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) convened a Senior Assessment Panel and charged it to undertake an assessment of the Mathematical Sciences in the United States. The Panel was asked to undertake the following tasks: to assess the health and position of leadership of the United States mathematical sciences; to evaluate the connections of mathematics with the other sciences, technology, education, commerce, and industry; to appraise the performance of mathematics in the education and training of professional mathematical scientists; and to make recommendations for action. This report describes the results of the Panel's work.

The principal strategy used by NSF to achieve its objectives is to support research and education in universities. The report therefore focuses on NSF's performance in

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[4] The crucial role that the mathematical sciences must play in the development of technology and in economics competitiveness, and the need to educate students regarding this role is well documented in the National Research Council's Board of Mathematical Sciences report**: Mathematical Sciences, Technology, and Economic Competitiveness**, 1991, National Academy Press, Washington.

supporting the performance and teaching of academic mathematical sciences and in encouraging interactions between academic mathematicians and the users of mathematics.

The Process Used by the Panel

The Panel consisted of leading mathematicians drawn largely from outside the United States and individuals from important U.S. stakeholder communities that are strongly dependent on mathematics (science, technology, education, government, and finance). None had received recent NSF funding in mathematics. Members who were mathematicians brought to the Panel their expertise in the various subdisciplines, the progress of international research, and the means used by other nations to support mathematical research. Members of the stakeholder communities provided judgments on their mathematical needs, on opportunities for mathematicians in these communities, and on the effectiveness of mathematical knowledge in service to society.

The Panel enjoyed staff support from the Division of Mathematical Sciences, which provided data, analysis of data, and a wealth of reports. Members of the Panel met four times (March 20-22, June 5-7, September 5, and September 22, 1997) at the National Science Foundation Headquarters in Arlington, Virginia to study data and reports, to discuss appropriate criteria for making assessment, and to formulate recommendations of a qualitative nature.

The Panel also discussed, at considerable length, the vital importance of mathematics in K-12 education in the United States. Not without regret, the Panel concluded that, given its composition and expertise, it should refrain from making assessments or recommendations in this area. However, the Panel wishes to emphasize its sense of the essential importance of K-12 mathematics to the well-being of the United States, to underscore the assessments and recommendations made in previous reports, and to affirm that much needs to be done in this area. The education of present, and, more importantly, future, teachers, will be the key to the improvement of K-12 mathematics. Because the education of teachers is the task of current and future university and college mathematicians, the quality of graduate students in mathematics and the education they receive will be crucial to the improvement of K-12 mathematics.

The Structure of the Report

This report includes a benchmarking comparison of U.S. mathematics with mathematics in Western Europe and the Pacific Rim. The report combines detailed benchmarking of the individual fields of the mathematical sciences with a strategic analysis of the role of mathematics in building and maintaining U.S. strength across the range of fields that comprise and use mathematics. This range extends from fundamental discoveries in mathematics to the application of mathematics in other scientific and engineering disciplines and in such “user” areas as government, finance, and manufacturing.

The methodological sections of the report describe the procedure followed by the Panel (Chapter III) and the means and data used for making benchmarking comparisons (Chapter IV and Appendix 2).

The substantive results of the Panel's work are presented as follows:

- The mathematical sciences, their contributions to the nation, and their relationship to other disciplines and professions that use mathematics (Chapter II);

- The findings of the panel (Chapter V);

- Objectives for NSF, recommendations for achieving those objectives, and suggested milestones against which to measure progress (Chapter VII).

The report also includes (in Appendices) data which support the findings and materials that address many elements of the report in greater detail.

II. THE MATHEMATICAL SCIENCES: THEIR STRUCTURE AND CONTRIBUTIONS

The Mathematical Sciences

The mathematical sciences are the most abstract of the sciences, as suggested in Table 1.

Table 1: The Intellectual Foci of the Sciences

Field	The Study of
Mathematical sciences	Patterns, structures, the modeling of reality
Physics	Energy, matter, time
Chemistry	Molecules
Biology	Life
Materials science	Materials, structures
Earth sciences	The earth: continents, oceans, the atmosphere
Astronomy	Origin and evolution of planets, stars, and the universe

The mathematical sciences have two major aspects. The first and more abstract aspect can be described as the study of structures, patterns, and the structural harmony of patterns. The search for symmetries and regularities in the structure of abstract patterns lies at the core of pure mathematics. These searches usually have the objective of understanding abstract concepts, but frequently they have significant practical and theoretical impact on other fields as well. For example, integral geometry underlies the development of x-ray tomography (the CAT scan), the arithmetic over prime numbers leads to generation of perfect codes for secure transmission of data on the Internet, and infinite dimensional representations of groups enable the design of large, economically efficient networks of high connectivity in telecommunications.

The second aspect of mathematical science is motivated by the desire to model events or systems which occur in the world – usually the physical, biological, and business worlds. This aspect involves three steps:

- Creating a well-defined model of a real situation, which itself is frequently not well defined. Such modeling involves compromises between the need for the model to be faithful to the real situation and the need for it to be mathematically tractable. An appropriate compromise usually requires the collaboration of an expert on the subject area and an expert on the mathematics.

- Solving the model, through analytic or computational means or a mixture of both.

- Developing general tools, which are likely to be repeatedly useful in solving particular models.

Examples of mathematical modeling include the quantum computer project, DNA-based molecular design, pattern formation in biology, and the fast Fourier transform and multiple algorithms used daily by engineers for numerical computation.

The mathematical sciences are disciplines in themselves, with their own internal vitality and need for nourishment. But they also serve as the fundamental tools and language for science, engineering, industry, management, and finance. They are inextricably linked to these “user” fields and they frequently draw inspiration from them. The mathematical sciences represent a mode of thought based on abstraction that sustains precision and permits careful analysis and explicit calculation. Thus mathematics has a dual nature: it is both an independent discipline valued for precision and intrinsic beauty, and it is a rich source of tools for the world of applications. Mathematics might be described as having abstractness internally and effectiveness externally.

The two parts of this duality are intimately connected. The search for order, symmetries, and regularities in patterns is the heart of research in pure mathematics. Results of this research are very durable, sometimes finding important application in unexpected ways decades after their discovery. A major reason for this is that results in mathematics, once proven, are never disproved -- even though they may be superceded by more powerful results. Other sciences, by contrast, move towards truth by a process of successive approximations.

In the United States, mathematics research, which is carried out principally at universities, may be segmented (somewhat arbitrarily) into nine sub-fields, as described in Table 2.:

Table 2: Major Subfields of Mathematical Sciences

Subfield	**The Study of**
Foundations	Logical underpinnings of mathematics
Algebra and Combinatorics	Structures, discreteness
Number Theory and Algebraic Geometry	Properties of numbers and polynomials
Topology and Geometry	Spatial structures, patterns, shapes
Analysis	Extensions and generalizations of the calculus
Probability	Randomness and indeterminate phenomena
Applied Mathematics	Problems arising in nature
Computational Mathematics	Problems whose solution uses the computer
Statistics	Analysis of data

The boundaries between these subfields are neither fixed not solid, and some of the most interesting and fruitful developments in mathematics come at the interfaces of subfields. Some areas of research appear in more than one of these categories; e.g., for example, Theoretical/Mathematical Physics appears in Topology/Geometry, Analysis, and Applied Mathematics.

The Mathematical Sciences Research Community

The mathematical sciences research community differs from other research communities in several ways. Mathematical research is the epitome of "small" science; that is, much research is done by individuals working alone, with modest equipment needs such as workstations. (Increasingly, however, some mathematicians need access to supercomputers and visualization labs.) Also, mathematical research is long-lasting, and rich in references to older literature, so that mathematicians are more dependent than other scientists on good libraries. Finally, mathematicians are more closely associated with teaching and with educational institutions than other scientists. Most research mathematicians are university based, so that their culture has an academic orientation.

In 1995[5], approximately 16,000 (over 65%) of the doctoral mathematical scientists in the United States were located at institutions of higher education. Of these, 6,427 worked at doctorate-granting institutions and represent the heart of the U.S. academic research community. Less than 25% of doctoral mathematicians were employed in private industry, and 4.2% were employed in government. Of the 1994-95 cohort of U.S. doctorate recipients, more than 50% anticipated faculty positions at educational institutions, with an additional 25% planning U.S. postdoctoral appointments, presumably as a precursor to academic careers.

Mathematical scientists in industry seldom carry the title "mathematician;" they are usually known as "engineers," "systems analysts," or by other titles, (see SIAM report)[6]. Thus they lack the mathematical identity and consciousness of their academic counterparts and in contrast to chemists and engineers, tend to be poorly connected to the university community.

Mathematical Sciences as an International Discipline

Both by its abstract nature and by convention, mathematics knows neither linguistic nor political boundaries. Its language is usually decipherable from equations and relations alone; when words are needed, mathematicians around the world use English by common agreement -- just as scholars once used Latin. In the same spirit, mathematicians have managed to transcend political differences and borders, even during the Cold War. And because mathematicians do not require specialized laboratories to conduct their research, they travel freely between universities and between countries. The result of these customs and agreements is that mathematics is an extraordinarily open and international

[5] **Notices of American Mathematical Sciences**, 1997, vol. 44, page 917.
[6] **Mathematics in Industry**, 1995, SIAM, Philadelphia

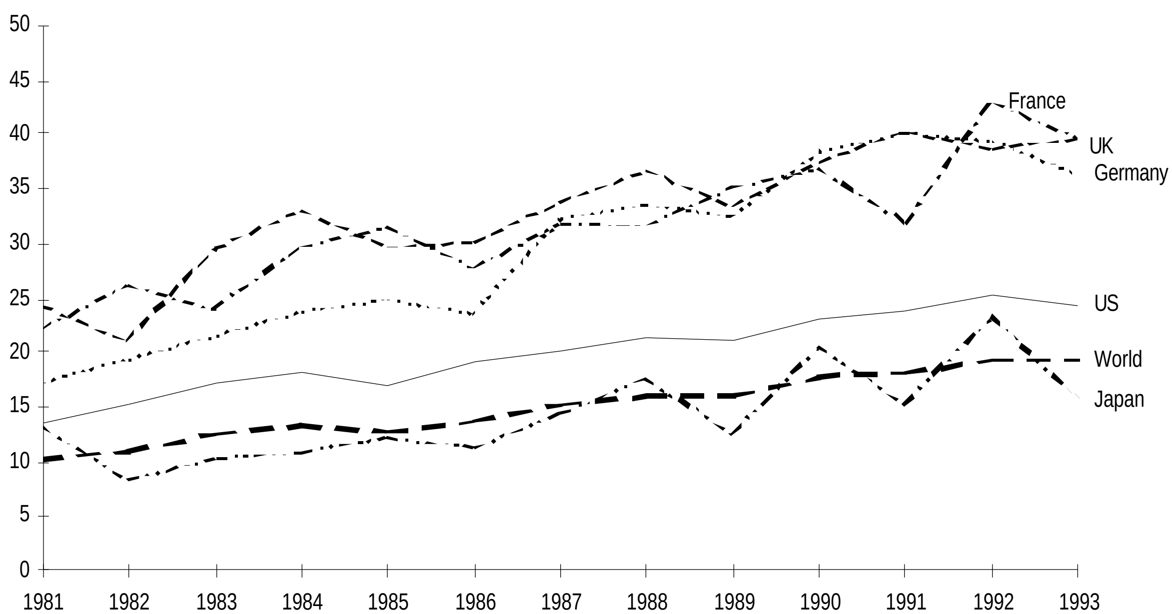
activity.

The number of highly active research mathematical scientists worldwide is small –probably well under 10,000 – so that a given subarea may be populated by only a tiny number of highly specialized individuals. They know each other well regardless of their country of residence; share a common, specialized vocabulary; and collaborate extensively even over long distances. Mathematical science conferences typically host participants from many countries; meeting one's peers is essential for the exchange of ideas which may not appear in published work.

Because of this international culture, mathematicians frequently take up sequential residencies in different countries or alternate between countries. The United States, with its commitment to freedom, a high standard of living, and excellent universities, has benefited enormously from flows of foreign-born mathematicians; in the same spirit, Americans serve on mathematical science faculties in almost every country in Europe. For these reasons, local changes in the support of the mathematical sciences in any country can result in the rapid migration of mathematicians, such as the great emigrations from Europe before World War II and the former Soviet Union at the end of the Cold War.

Mathematicians also collaborate internationally on research, a trend that has been growing consistently for nearly two decades. The number of papers co-authored by mathematicians in the five major mathematical nations[7] with researchers in other countries rose about 50% between 1981 and 1993, and this tendency continues.

Percent of Papers Internationally Coauthored in the Mathematical Sciences



[7] C

The growth in co-authored papers by researchers in the United Kingdom, France, and Germany reflects the growing unification of the countries in the European Union.

Mathematics students tend to gather in the strongest research centers, a tradition that began over a century ago. Before 1940, it was common for the best U.S. students to study in Europe; after World War II, the U.S. reputation in mathematics grew rapidly, and for the past 15 years, a majority of Ph.D. graduates of U.S. institutions have been non-U.S. citizens. In 1996, non-U.S. citizens earned 55% of total doctoral degrees in mathematical and computer sciences[8]. Other strong international research centers are also attracting foreign students. In France, international students now earn one out of three doctoral degrees awarded in all fields of science; in Japan, that proportion is 40%; and in England, 27%, with many students from commonwealth countries and the United States.[9] Germany supports foreign graduate students and postdoctorates on Humboldt Fellowships.

The Role of Mathematics in Society

Although most of the mathematical research community is university-based, the impact of mathematics on society is pervasive. Mathematics underpins most current scientific and technological activities. Whole new areas of mathematics are evolving in response to problems in experimental science (biology, chemistry, geophysics, medical science), in government (defense, security), and in business (industry, technology, manufacturing, services, finance). All of these areas now require the analysis and management of huge amounts of loosely structured data, and all need mathematical models to simulate phenomena and make predictions. Modeling and simulation are essential to fields where observable data are scarce or involve a great deal of uncertainty, such as astronomy, climatology, and public policy analysis. Addressing such complex problems calls for openness to all of mathematics and to the emergence of new mathematics. Progress requires radical theoretical ideas as well as significantly greater collaboration between pure mathematicians, statisticians, computer scientists, and experimental scientists.

The applications of mathematics in the future will require closer partnerships between mathematical scientists and the broader universe of scientists and engineers. Meeting the complexity of tomorrow's challenges will demand insights across the full spectrum of the mathematical sciences. Both the theoretical and the industrial impact of this development will be enormous. Table 3 illustrates some of the present and potential contributions of mathematics to society.

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[8] **Notices of American Mathematical Sciences**, 1997, vol. 4, page 915

[9] **Science and Engineering Indicators,** 1996, (NSB 96-21), Appendix Table 2-33, page 69.

Table 3: Illustrations of Some Uses of Mathematics in Society

Problem/Application	Contribution from Mathematics
MRI and CAT Imaging	Integral geometry
Air traffic control	Control theory
Options valuation	Black-Scholes options model and Monte Carlo simulation
Global reconnaissance	Signal processing, image processing, data mining
Stockpile stewardship	Operations research, optimization theory
Stability of complex networks	Logic, computer science, combinatorics
Confidentiality and integrity	Number theory, cryptology/combinatorics
Modeling of atmospheres and oceans	Wavelets, statistics, numerical analysis
Agile, automated manufacturing	Geometry, visualization, robotics, control theory, in process quality control
Design and training	Simulation, modeling, discrete mathematics
Analysis of the human genome	Data mining, pattern recognition, algorithms
Rational drug design	Data mining, combinatorics, statistics
Seiberg-Witten questions (string theory)	Geometry
Interpreting data on the universe	Data mining, modeling, singularity theory
Design systems for composite materials	Control theory, computation, partial differential equations
Earthquake analysis and prediction	Statistics, dynamical systems/turbulence, modeling, in process control

The sciences have always used mathematics to formulate theory and underpin simulation and statistics to design productive experiments. Wherever numbers or symbols are manipulated, the manipulations rest on mathematical relationships. With the advent of high-speed computers and sensors, some experimental sciences can now generate enormous volumes of data --- the human genome project is an example -- and the new tools needed to organize this data and extract significant information from it will depend on the mathematical sciences. Hence the mathematical sciences are now essential to all three aspects of science: observation, theory, and simulation.

The following examples illustrate ways in which mathematics contribute to areas of broad concern to our nation:

National security. The security of complex communications systems--voice, data, and electronic--rests on mathematically sophisticated tools. Stockpile stewardship--the maintenance of the nuclear arsenal without testing--will be based on mathematical modeling and advanced computation. The operation of national surveillance systems requires extensive use of mathematics for collection and analysis of data. Military systems are being transformed by the application of mathematical-based systems for intelligence, logistics, and warfighting.

Technology. Mathematics is ubiquitous in the design, manufacturing, and use of technology. No complex system--from microprocessors to aircraft engines, from satellite communications networks to home marketing systems, and from the air traffic control system to the laptop computer--could exist without the application of mathematics.

Education. The role of mathematics in educating the work force is crucial for the well-being of the nation. Scientists and engineers depend on the mathematical sciences and need a sound foundation in that discipline to succeed. For the average citizen, a grounding in mathematics, at least through the secondary level, is essential to modern citizenship. Innumeracy is as crippling as illiteracy.

Medicine. The operation of modern medical imaging systems--CAT scanners, nuclear imagers, Magnetic Resonance Imaging (MRI)--depends on the mathematical processing of signals. The success of the human genome project will require the use of mathematics to search for information that correlates genetic sequences to human disease. Elucidating the complex geometry of protein folding is the key to understanding protein functions. The management of hospital patient records will increasingly require the application of mathematics to construct efficient databases.

Finance. Mathematics has become indispensable in measuring risk and modeling the behavior of financial instruments, financial institutions, and financial systems (individual countries, trading blocs, and global systems, such as international settlements). The combination of probability theory and advanced financial models with increased data, capacity, algorithm efficiency, and computational speed facilitates the sophisticated modeling of interest rates, currencies, commodities, equities, and other financial instruments. Better understanding of areas such as value at risk, portfolio theory for credit exposures, and non-linear instruments depends on the application of mathematics.

Environmental monitoring. Building useful models of oceans and atmospheres to predict the impact of human activities on the environment is essential to the formulation of sound public and regulatory policies. Climate models require the manipulation of massive quantities of data and the study of complex simultaneous interactions (for example, among the many trace chemicals in the atmosphere). All such models are based on uncertainty; to judge their validity will require heavy use of mathematics.

In broad terms, both the techniques of the sciences and the needs of society are dramatically more complex than those of the past. The ability to understand new needs and systems, and to predict and control their behavior, will require two elements: i) new mathematical ideas and methods, and ii) more effective collaboration between all groups actively concerned with them -- disciplinary scientists, engineers, computer scientists, and members of relevant professions, from medicine to public policy.

III. CRITERIA FOR ASSESSMENT

The National Science Foundation is the major non-university source of research support for the academic mathematical sciences in the United States. Assessing the performance of NSF in this area is then equivalent to assessing the performance and health of U.S. academic mathematics and the impact of academic mathematics on users of mathematics in university, industry, and government.

At the same time, assessing the value of mathematics by measuring the return on public investment cannot be done by simply comparing the amount of expenditure on mathematics with the amount of wealth created. The "effects" of mathematics may not be immediate, or direct, or attributable to a single funded program. A more accurate assessment of the field requires one to accept the consensus view of all branches of science and technology -- that support for mathematics is essential -- and to ask whether that support is producing health within mathematics and a valued impact outside it.

The Panel concluded that the most accurate way to measure the impact of academic mathematics is to examine the three primary activities of mathematicians:

1. Generating concepts in fundamental mathematics;
2. Interacting with areas that use mathematics, such as science, engineering, technology, finance, and national security; and
3. Attracting and developing the next generation of mathematicians.

We assessed each of these activities separately, by different criteria.

1. Criteria for Assessing Contributions to Fundamental Mathematics. We believe that it is the mathematicians themselves who are best qualified to assess the intellectual impact of U.S. mathematics. We asked mathematicians to use the following benchmarks in their assessment: (i) academic recognition of the accomplishments of mathematical scientists, as measured by publications, awards, and presentations at major conferences; (ii) judgments of mathematical scientists not resident in the United States; (iii) the attractiveness of the U.S. mathematics community to foreign mathematical scientists; and (iv) the speed with which U.S. mathematical scientists can respond to discoveries occurring both within and outside the United States, as well as to discoveries by other scientists and engineers, and incorporate these discoveries into their own work. We used these measures to benchmark the standing of U.S. academic mathematical sciences in relation to other nations.

2. Criteria for Assessing Interactions Between Mathematicians and Users of Mathematics. Evaluating the effectiveness of interactions between academic mathematics and the users of mathematics is necessarily more subjective. As one measure, we used the levels of employment of mathematicians in different segments of society. As the SIAM report[10] demonstrates, these levels are difficult to ascertain. As a second measure, we used three types of subjective judgments, based on both surveys and personal information: (i) the perceived importance of mathematics to other areas; (ii) the effectiveness of the academic mathematical sciences in solving the problems of these areas; and (iii) the speed with which new mathematical ideas are transmitted to other areas and to the private sector.

3. Criteria for Assessing Undergraduate, Graduate, and Post-Doctoral Education. Four measures are used for this assessment: (i) the flow of students (both U.S. national and foreign) into degree programs in U.S. universities; (ii) the quality of the research performed by university alumni/ae; (iii) the ability of students trained in U.S. departments to find rewarding jobs that make good use of their training; and (iv) the nature of the work that mathematicians perform when employed.

We have not explicitly examined the influence of university research and teaching on education at the K-12 level. U.S. universities teach those who become the teachers of K-12 mathematics, so university instruction in mathematics is essential to the teaching of arithmetic and mathematics in grade school and high school. The challenges of assessing K-12 teaching and of proposing strategies for curriculum reform are, however, beyond the scope of this report and are being addressed by others. We do, however, wish to stress the importance of these challenges. The health of the U.S. mathematical sciences, and the economic well-being of the United States, are both directly related to the quality of K-12 mathematics education.

In addition, we have not examined post-professional or continuing education for mathematical scientists or for the teachers and users of the mathematical sciences. We anticipate that education at these levels will become increasingly important in the future.

[10] **Mathematics in Industry**, 1995, SIAM, Philadelphia.

IV. INTERNATIONAL COMPARISONS

The Panel examined numerous categories of data in trying to benchmark the activity and health of U.S. mathematical sciences in relation to those of other countries. It found a number of indicators (see Appendices) and qualitative observations to be significant, as discussed below.

Bibliometrics

In comparing the United States with Europe, the Panel found the United States mathematics produce more research papers. Pacific Rim nations trail that of the United States and Europe, but have increased their output significantly since the early 1980s (not shown).[11]

Publications in the Mathematical Sciences

	1989	1991	1992
United States	39.9%	42.1%	38.9%
United Kingdom	5.7%	6.3%	5.9%
Germany	6.6%	7.0%	6.5%
France	7.5%	4.6%	8.5%
Other Western Europe	12.7%	12.6%	13.1%
Japan	4.3%	4.6%	3.6%
Other Pacific Rim	3.9%	3.7%	4.4%

In a separate study by CHI Research, Inc. of ISI journals with emphasis on research publications in pure mathematics, the proportion of papers published by various nations remained stable, with U.S. mathematicians authoring 40-50%. The study noted that international coauthorship was increasing.

Data on the number of Ph.D.s in the mathematical sciences is difficult to determine by geographic region. More generally, for the natural sciences, in 1992 there were 6593 doctorates in Asia, 18,951 doctorates in Europe, and 13,344 doctorates in North America (with the United States producing 12,555 doctorates). However, the report noted, "a declining pool of college-age students in Europe has not resulted in declining numbers of natural science and engineering degrees, as has occurred in the United States."[12]

[11] **Science and Engineering Indicators,** 1996, (NSB 96-21), Appendix Table 5-31.
[12] **Human Resources for Science & Technology: The European Region,** (NSF 96-316).

International Congress Participation

Data on invited speakers at international conferences vary somewhat. In 1994, 50% of invited one-hour speakers at the quadrennial International Congress of Mathematicians were from the United States. In the forthcoming 1998 Congress, 38% of the invited one-hour speakers are from the United States and 48% are from Europe; on the other hand, 48% of the 45-minute speakers are from the United States (1/3 of them having non-U.S. origins) and 36% are from Europe. U.S. speakers accounted for 35% of the plenary speakers at the 1995 International Congress of Industrial and Applied Mathematics.

Awards

A tally of the leading awards in mathematics provides another useful benchmark. Of the 16 mathematicians awarded the Fields Medal between 1970 and 1990, eight resided in the United States at the time of the award and 11 currently do. Four of the eight medalists honored in the 1990s reside in the United States, but only one was born in the United States. (The Nobel Prize is not awarded in mathematics; one U.S. mathematician has received the Nobel Prize in economics.) Of the mathematicians who have received the Wolf Prize, an award for distinguished scientists, 14 of 27 are U.S. mathematicians and two others have spent substantial time in the United States.

Subdiscipline comparisons

The Panel, on the basis of its own expertise, undertook a qualitative benchmarking of subdisciplines, which appears in full as Appendix 2. The Panel concluded that the United States has strengths in all subdisciplines, but that it is not the major contributor in some. In several fundamental subdisciplines, including Foundations, Symbolic Computation, and Ordinary Differential Equations, foreign contributions outweigh those of the United States. Overall, however, the U.S. mathematical research enterprise is judged capable of responding to advances occurring anywhere in the world, an ability enhanced by the very high level of interaction among mathematicians worldwide.

Budgetary comparisons

The Panel was unable to provide meaningful country-by-country comparisons of research funding. One reason for this is the wide variations in budgetary and institutional relations between governments and universities. For example, in Canada, Europe, and Japan, faculty are paid on a 12-month basis for both teaching and research. This means that all faculty in these countries have funding for summer research; in the United States, only 35% of active researchers do. This "have" and "have-not" situation is very destructive to the fabric of the U.S. mathematical sciences community and decidedly discouraging to young U.S. researchers. Funding agencies in Europe and Japan play differing roles from that of NSF or the DoD agencies, often using different means to promote the health of science and mathematics within their countries. For example, the Japanese Society for the Promotion of Science spends most of its money to support university science libraries and visits by distinguished foreign scientists.

A few generalizations are possible, however. In Europe, bursaries for students admitted to graduate school meet their full living costs and do not need to be supplemented by money earned through teaching. The docent programs in Germany and Eastern Europe support students completing the doctorate and intending to seek a post in a research university, as do the collegiate fellowships in England. Recently, France has introduced postdoctoral fellowships, as has the European Union. By contrast, the Panel finds that in the United States, there are few research assistantships for the mathematical sciences. The United States is overly dependent on teaching funds to provide support for graduate students in the mathematical sciences, a custom that prolongs the time to degree and makes the field of mathematics less attractive to U.S. students. Also, except for a few NSF funded postdoctoral research fellowships and a few research instructorships funded by universities, postdoctoral research opportunities for mathematical scientists in the United States are exceedingly scarce.

In Asia, overall investment in mathematics is rising. The Asian Tiger countries -- Singapore, Taiwan, Korea, and Hong Kong -- are building strong research universities and research institutes with significant mathematical components. Japan has begun a five-year program to double its funding for basic science. The Chinese National Science Foundation has given the mathematical sciences its highest priority for development.

Research institutes

Institutes and conference centers are important elements in the infrastructure supporting the mathematical sciences – as important to the field as are lab facilities and observatories to physics and astronomers. The Institute for Advanced Study (IAS) in Princeton, N.J. was the first institute to assemble, for short periods (4-12 months), groups of mathematical science researchers. Such institutes have become popular in the mathematical sciences and are viewed as making significant contributions to the advance of the discipline because they enable explorations of new developments, facilitate collaboration among mathematical scientists and assist in the sharing of ideas between mathematical scientists and those of other disciplines. The NSF provides partial funding to IAS primarily for support of young researchers, core funding for three other research institutes (MSRI, IMA, NISS) as well as for DIMACS, a Science and Technology Center that operates much as the other institutes but concentrates on discrete mathematics, algorithms and theoretical computer science. Western Europe has six research institutes plus two conference centers, Canada has three, and the Pacific Rim has several and is planning more. Germany, which is less than a third the size of the United States, has two Max Planck Research Institutes, the Oberwohlfach Conference Center and seven Sonderforschungberich, which are attached to universities and have some aspects of an institute and some of the U.S. Science and Technology Centers. Some institutes have permanent faculty and some, e.g. those whose core funding comes from the NSF, do not. The Centre Nationale des Recherches Scientifiques (CNRS), in France, and the National Academies of Sciences in Russia and Eastern Europe have significant numbers of full-time researchers in the mathematical sciences.

Collaboration with other disciplines

Communication between academic mathematical scientists and other scientists is poor the world over. Many mathematical scientists have a limited vision of their capacity to interact with other scientists. Graduate education is frequently highly specialized. The structure of universities, where decisions on promotions, awards, and salaries are made by disciplinary departments, mitigates against collaboration outside one's discipline. The difficult and time-consuming task of understanding a second discipline is also an inhibitor. A few U.S. programs, such as the University of Illinois' Beckman Institute, have been created to advance multidisciplinary research, but few of these efforts have involved mathematical scientists.

Many nations have begun to promote the collaboration of mathematical scientists in multidisciplinary research. England appears to be doing so aggressively, especially under the aegis of the Isaac Newton Institute. In France, there is significant interaction between mathematical scientists and engineers, in part due to the commonality of their secondary and collegiate education. Collaborative initiatives are becoming prominent in countries emphasizing research that drives the economy (e.g., nations of the European Union and those of the Pacific Rim). Many U.S. mathematicians are becoming more involved in multidisciplinary research, probably more than those in other countries, but science and engineering require much more involvement by U.S. mathematicians. Nonetheless, progress is being made. More than 10% of the NSF/DMS budget is invested in projects cofunded with other Divisions and Directorates. The DMS Group Infrastructure Program did provide funding for collaborative programs, and proposals from U.S. mathematicians have been well received by the MPS Office of Multidisciplinary Activities and by the IGERT program. Other recent NSF initiatives promote multidisciplinary research, often with a strong mathematical component, such as the Knowledge and Distributed Intelligence (KDI) Initiative begun this fiscal year.

Interactions between academia and industry

Another trend, most apparent in England and the Netherlands, is to foster interactions between the very different cultures of academic research and the private sector. England has been especially active on this front with its Smith Institute and the OCIAM at Oxford. Activity in the Isaac Newton Institute has led to significant private investment in academic mathematics. Other European countries have begun such programs, but progress has been slow; mixing the cultures of academia and industry is not easy, and there is concern over intellectual property rights and industrial privacy. In Japan, there is little interaction. In the United States, some mathematics departments and institutes (notably the Institute for Mathematics and Its Applications) interact with industrial and financial entities, but they are very much in the minority. Involvement of the U.S. mathematical sciences in the NSF GOALI (Grant Opportunities for Academic Liaison with Industry) program is small compared to that of other sciences, but is beginning to grow. The Division of Mathematical Sciences (DMS) has recently collaborated with DARPA (the Defense Advanced Research Projects Agency) to fund several initiatives that require collaboration by mathematical scientists with other academic and industrial scientists.

The expectation of an academic career, rather than one in industry, is particularly strong in the United States. Some 75% of new U.S. doctoral mathematical scientists anticipate academic positions. In continental Europe, by contrast, many universities have Diploma programs from which students seek nonacademic positions. The English universities have recently introduced a new degree intermediate between the baccalaureate degree and the Ph.D. which serves that purpose. Many U.S. mathematical science departments have discussed a "professional masters" degree, which would emulate the Diploma programs, but few departments have tested or established them.

Undergraduate Education

Although U.S. doctoral programs in the mathematical sciences are extremely strong, U.S. undergraduate programs offer less exposure to mathematics, at less depth, than do those in Europe and Asia. There are two important reasons for this: (i) U.S. undergraduates arrive at the college level with less knowledge of mathematics: 50% or more are unprepared to begin the calculus; (ii) In other countries, undergraduate mathematical science students concentrate entirely on the mathematical sciences and related subjects, while U.S. students spend at least 50% of their time on unrelated subjects. This has meant that graduates of U.S. undergraduate programs must spend time catching up with their European counterparts, extending time to the doctorate. On the other hand, the U.S. system allows students greater opportunities to explore outside their discipline and to change specialties. For example, of the students with A and A+ in high school who enrolled in U.S. undergraduate schools in the mathematics sciences in 1984, 75% switched to other programs while 61% of those earning a mathematical science degree by 1989 were recruits from other fields.[13]

Graduate Education

Ph.D. recipients from the best universities, whatever the country, are at the same level of

[13] **Best and Brightest: Education and Career Paths of Top Science and Engineering Students**, 1997, Commission on Professionals in Science and Technology, Washington, DC.

achievement and preparedness for research. U.S. graduate departments generally offer a wider range of fields in which to specialize than is the case in Europe or Asia, but recent developments in the European Union mean that students in Europe can easily move to other universities, possibly in another country. This allows them greater variety of specialization. European graduate students also receive funding sufficient to cover their living expenses. In the United States, most mathematical science students need to teach to cover their living expenses.

Anecdotal information suggests that a much larger percentage of students who begin a doctoral degree program in the mathematical sciences in the United States fail to earn that degree than is the case in Western Europe. This is especially true for U.S students. A number of U.S. universities report that by the third year, no U.S. citizen remains in their doctoral program.

Attractiveness of the field to the young

During the last decade, the number of U.S. citizens pursuing degrees in the mathematical sciences has suffered a decided decline. Between 1985 and 1995, U.S. freshmen interested in the mathematical sciences declined by 32%, and by 23% among the top students.[14] This situation is mirrored in other nations. The Netherlands, Germany, France, Russia, and Poland all have reported significant losses in mathematics enrollment during the past five years. In the last three years, there has been a steady decline in the numbers of applications to U.S. graduate schools in mathematics by Chinese students, which is probably a sign of diminishing interest in that country.

European trends toward applications and centralization

In the European Union, the European Commission has shifted emphasis in the direction of mathematical applications which enhance wealth creation or the quality of life. In the current Fourth Framework Programme, almost the only support of pure science is budgeted in the Human Capital and Mobility program. This program does carry benefits for mathematics as a whole in supporting the movement of postgraduates and postdocs between countries of the EU, in funding conferences and networks, and in supporting an agenda that covers a larger proportion of the needs of mathematicians than those of laboratory-based scientists. However, the shifting of funds from local to central control and from fundamental research to applications does not have the universal support of the mathematics community.

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[14] **Best and Brightest: Education and Career Paths of Top Science and Engineering Students**, 1997, Commission on Professionals in Science and Technology, Washington, DC.

Conclusion

On the positive side, the U.S. mathematics community leads other nations in a large number of subdisciplines and is judged overall to be capable of responding to breakthroughs occurring elsewhere in any area of mathematics. Individually, U.S. mathematicians have won more than their share of prestigious awards in the field.

At the same time, the mathematics community in the United States shares with other nations significant disciplinary challenges including a condition of isolation from other fields of science and engineering, a decline in the number of young people entering the field, and a low level of interaction with nonacademic fields, especially in the private sector.

The Panel gained the sense that mathematicians in the United States feel themselves disadvantaged in comparison with mathematicians of other countries, most notably in public support. This low morale is not evident in Western Europe or the Pacific Rim. The European Union is expanding opportunities and funding for graduate students and postdoctorates. U.S. students are overly dependent on teaching income, which extends time to degree, decreases the attractiveness of mathematics to younger students, and contributes substantially to the fragility of the U.S. mathematical enterprise.

V. FINDINGS

Finding 1: Academic Success

U.S. mathematics has been and remains distinguished. Academic mathematicians in the United States have been very successful in creating new fundamental concepts. This excellence has been clearly and repeatedly recognized in the large number of professional awards received by U.S. mathematical scientists. In addition, U.S. mathematical scientists have been quick to develop and extend new concepts created elsewhere. There is no question that the U.S. academic community has been among the strongest in the world since World War II and remains so today.

The success of mathematicians from U.S. graduate programs in the mathematical sciences attracts students from every country, including Western Europe. Generally, U.S. graduate programs are larger and broader than those available elsewhere, which adds to their appeal.

Although the United States is the strongest national community in the mathematical sciences, this strength is somewhat fragile. If one took into account only home-grown experts, the United States would be weaker than Western Europe. Interest by native-born Americans in the mathematical sciences has been steadily declining. Many of the strongest U.S. mathematicians were trained outside the United States and even more are not native born. A very large number of them emigrated from the former Soviet Union following its collapse. (Russia's strength in mathematics has been greatly weakened with the disappearance of research funding and the exodus of most of its leading mathematicians.) Western Europe is nearly as strong in mathematics as the United States, and leads in important areas. It has also benefited by the presence of émigré Soviet mathematical scientists.

It is worth noting that prior to World War II, the United States lagged well behind Europe in mathematical research. After the war, the presence of German refugees, growth of federal investment in science, and expansion of the university system all fueled the powerful growth of U.S. mathematical sciences. But federal funding has not kept pace with the growth in the size of the mathematical science community, and the growth of the university system has stopped in all but two or three states. The impetus that led to U.S. leadership in the mathematical sciences no longer prevails.

The U.S. lead in the mathematical sciences is declining in some subfields, which are further endangered by a lack of young people in several areas where U.S. leaders will soon retire. An example is Foundations, which during the past two decades has failed to attract enough young mathematicians to contribute to or respond to advances in other countries. As a result, the average age of the leaders in mathematical logic in the United States is above 50 years (even higher in proof theory), significantly higher than in other fields of mathematics. In symbolic computation, a subarea where Europe is strong, the United States has considerable commercial presence but little academic depth. The separation of computer science from the mathematical sciences in U.S. universities has had a negative impact on combinatorics, discrete mathematics, symbolic computation, and other areas. It has also resulted in the training of computer scientists who have limited mathematical backgrounds.

U.S. strength in mathematics rests heavily on mathematicians who have come from outside the United States. Many distinguished U.S. mathematicians who have received international awards were neither born nor trained in the United States. An increasing number of all U.S. academic mathematicians received their early training outside the United States. A yet-to-be-published study by COSEPUP reports that 21% of tenured faculty and 58% of tenure-track faculty at 10 highly rated mathematics departments received their undergraduate degree outside the United States. This situation is not confined to the highly rated departments. The citizenship of full-time mathematics faculty with Ph.Ds hired during 1991-92 by U.S. universities and colleges were as follows: 37% were U.S. citizens, 16% were Western Europeans, 13% were Eastern Europeans, 22% were Asiatics and 12% were citizens of other countries.[15] Of these hires, 26% came directly from overseas. U.S. industry constantly seeks to recruit mathematical scientists outside of the United States and sends abroad much work which requires mathematical skills. Although mathematics is a very international field, this trend suggests that U.S. academic mathematics is not as robust as suggested by its high level of academic recognition. Unless the United States can make mathematics more attractive as a career to U.S. citizens, several developments threaten to push the supply of trained mathematicians below that needed by academia, let alone by industry: (i) the collapse of the Soviet Union as a producer of highly trained mathematicians; (ii) the pressure on U.S. graduate students who are Chinese citizens to return to China after completing their studies; (iii) worldwide decline of student interest in mathematics; and (iv) competition by Western Europe to retain first-rank European-trained mathematicians.

Lack of financial support thwarts the careers of many young mathematical scientists. Not only is there a lack of sufficient postdoctoral fellowships for new doctorates in the mathematical sciences, but few young researchers are successful in obtaining research grants. With only 35% of academic research mathematical scientists receiving such grants, it is exceedingly difficult for young researchers to pursue careers in research. This lack of support, especially when compared with support for young researchers in the physical, biological, and engineering sciences, discourages young mathematicians, many of whom have left academia for Wall Street and other nonacademic fields. This loss of young researchers has the potential to undermine future U.S. strength in the mathematical sciences.

[15] **Notices of American Mathematical Society**, 1992, vol. 1939, pages 314-315.

Finding 2: Interactions with Users of Mathematics

Academic mathematics is insufficiently connected to mathematics outside the university. One of the greatest – and most difficult -- opportunities for academic mathematics is to build closer connections to industry. The poor communication between the university and industry cannot be blamed exclusively on either party. Academic mathematics is an intense, focused, and sometimes solitary intellectual activity. By contrast, mathematical scientists in industry tend to work in teams, usually addressing analytical challenges rather than developing new concepts. A further difficulty is that most companies do not have a separate division devoted to mathematics or, indeed, the job classifications of "mathematician" or "statistician." This situation, which evolved in an era when mathematics was much less pervasive in industry and less central to economic competitiveness than it is today, makes it difficult for academic mathematicians to contact their industrial counterparts.

It is clear that both **industrial and academic mathematics must reach out to one another if the two are to interact effectively**. Industry could enhance communication by organizing its mathematicians so they can be easily identified and contacted by their university colleagues. Academic mathematicians will have a larger perspective of their discipline if liaisons can be developed between industry and academics, as exists in chemistry, pharmacology, and engineering. Good models exist, at Boeing, Lucent, IBM, AT&T, the applied mathematics groups in the pharmaceutical companies, and the financial industry, where mathematical scientists are easy to identify, work on well defined and sophisticated mathematical problems, and welcome faculty consultants and student interns. Effective interactions like these are creating new specialties in applied mathematics, such as financial engineering and computational drug design.

Academic mathematics could interact fruitfully with other disciplines in ways which are often obscured by the inward focus of mathematics and science departments. We believe that mathematics is a field of almost unlimited opportunity -- provided that it looks outward toward its interfaces with other fields. The opportunities at disciplinary interfaces -- for example, in bioinformatics, communications networks, and global climate modeling -- are not only important in a practical sense, but they are also intellectually challenging. By tradition, however, academic mathematicians are reluctant to seek such interactions – as are members of other science and engineering disciplines. This reluctance means foregoing much professional stimulation and precludes the solution of problems that require new concepts and techniques in mathematics. This is less the case with statisticians, who have always worked with others.

A narrow vision of mathematics in academic departments translates into a narrow education for graduate students, most of whom are oriented toward careers only in academic mathematics. Although it may be appropriate for some departments to maintain a "pure" academic focus, a higher level of interaction with other disciplines is essential for the mathematical enterprise as a whole as it is for other disciplines.

The structure of universities mitigates against multidisciplinary research. While the above finding criticizes mathematical scientists for not collaborating more actively with other scientists and engineers, part of the fault lies with the organization and culture of universities, here and abroad, which restrains collaboration across scientific boundaries. The academic award system does not encourage collaboration; in fact, individuals who straddle fields reduce their chances of tenure. Given the growing need for multidisciplinary research, forward-looking universities must find ways to break down the disciplinary walls that inhibit collaboration.

Scientific problems of the future will be extremely complex and will require collaborative mathematical modeling, simulation, and visualization. Mathematical modeling and experimental observation go hand in hand. Modeling, which is built on both observation and theory, leads to further experiment and more precise measurements. Good modeling demands the most relevant mathematical theory. It is nearly impossible for a single researcher to maintain sufficient expertise in both mathematics/computational science and a scientific discipline to model complex problems alone. A well defined model requires multidisciplinary teams that include both mathematical and disciplinary scientists. Each member of such teams will need to understand the expertise of the other members well enough to recognize their competencies and limitations. Developing this degree of breadth takes time and commitment from all members**. Funding agencies need to provide financial support that recognizes and rewards multidisciplinary activities and to recognize the long time required to become competent in such work.**

The existence of physically separate departments of "applied mathematics" and "pure mathematics" has often perpetuated a narrow view of what mathematics can or should be applied. Historically "applied mathematics" has meant the application of the subarea "analysis" to problems in the physical sciences and engineering. This view of applied mathematics has greatly limited the application of *all* of mathematics to real world problems. With the burgeoning opportunities now available, the view must be that every area of mathematics can contribute and benefit from interactions with other disciplines and with industry and commerce. The division into "pure" and "applied" has been highly destructive to the discipline and must be healed.

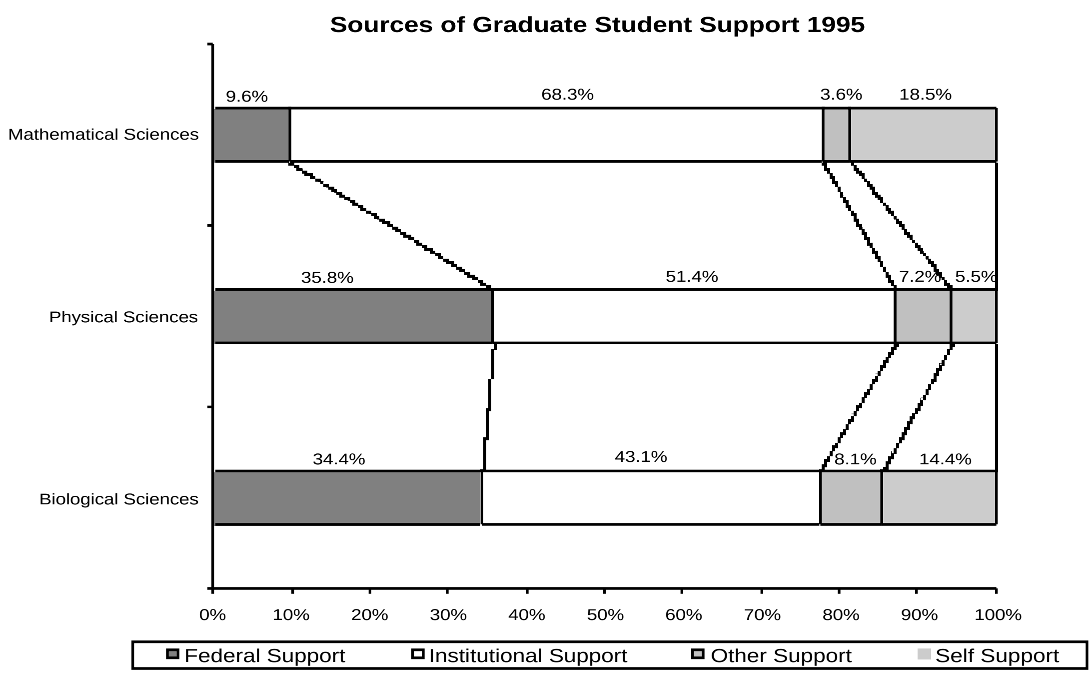
Finding 3: Educating the Next Generation

U.S. graduate programs in the mathematical sciences, especially the top 25, are considered to be among the very best in the world, attracting many students from other nations. For the last decade, more than 50% of Ph.D. degree recipients in the mathematical sciences from U.S. graduate schools received their undergraduate degrees from outside the United States. The graduates of the U.S. graduate problems have excelled at what they have been educated to do. Their publications are deeper and more numerous than those of earlier generations.

Despite the excellence of the U.S. graduate programs in the mathematical sciences, the students of these programs are provided substantially less federal funding than are students of the other sciences. They depend almost entirely on teaching assistants stipends and on their own resources. This treatment sends a clear message that the United States does not place high value on the mathematical sciences. This is certainly not the case in Western Europe.

Numbers of Full-time Graduate Students and Source of Support [16]

	Federal Support	Institutional Support	Other Support	Self Support
Biological Sciences	16593	20805	3923	6962
Physical Sciences	10353	14858	2079	1602
Mathematical Sciences	1291	9169	478	2484



There is a paucity of research assistantships and postdoctoral positions in mathematics**.** The data shows that in 1995, (see next page), new doctoral recipients in the mathematical

[16] **Graduate Students and Postdoctorates in Science and Engineering,** Fall 1995, (NSF 97-312), Table 25.

sciences were much more likely to move directly into teaching appointments than their counterparts in the physical or biological sciences, and much less likely to obtain research assistantships or postdoctoral fellowships. The "bench science" nature of the physical and biological sciences explains only a part of this disparity. The lack of an adequate number of postdoctoral fellowships slows the professional development of young mathematical scientists and reduces the attractiveness of the field. There is a clear need to decrease significantly the use of TAs in departments of mathematics and to increase research assistant positions as well as postdoctoral fellowships.

Percentage Distribution of Definite Post Graduate Plans of Science Doctorates, 1995[17]

All Recipients

	In U.S.					Abroad	Unknown
	Total in U.S.	Post Doc Study	Academic Employment	Industrial Employment	Other		
All Sciences	87.0	43.2	21.2	11.8	10.8	12.8	3.0
Chemistry	97.1	60.0	7.6	20.7	3.8	7.6	3.0
Physics	81.2	53.1	7.8	14.4	5.9	17.9	10.0
Biological Sciences	91.0	74.5	8.0	4.3	4.3	8.7	3.0
Mathematical Sciences	82.6	22.3	42.8	13.3	4.3	16.8	6.0

U.S. Citizens

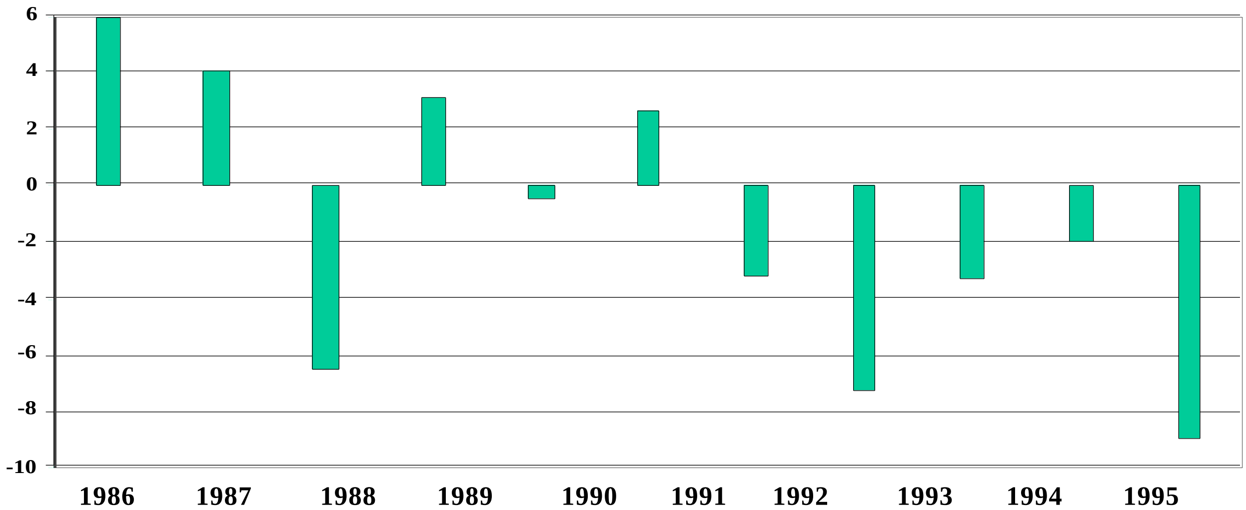
	In U.S.					Abroad	Unknown
	Total in U.S.	Post Doc Study	Academic Employment	Industrial Employment	Other		
All Sciences	95.3	42.6	25.9	12.8	14.0	4.6	1.1
Chemistry	95.8	56.8	10.6	23.4	5.1	4.0	1.0
Physics	89.6	52.6	10.4	17.2	9.5	9.7	6.0
Biological Science	95.3	73.8	10.7	5.2	5.5	4.6	0.1
Mathematical Sciences	95.3	21.9	52.3	13.7	7.4	4.4	0.3

Graduate applications in the mathematical sciences have declined. Since 1985, the number of U.S. undergraduate majors has declined, and since the early 1990s, the percentage of U.S. mathematics majors going on to graduate school in the mathematical sciences has dropped. The American Mathematical Society reports[18] that the number of U.S. citizens enrolling in graduate school for the first time dropped 6.9% in fall 1997 from 1996; it dropped 8.9% in subfields other than statistics, applied mathematics, and operations research. Both the number of U.S. and foreign applicants have declined by 1/6 in each of the last three years. Since 1992, many of the best mathematics departments have actually reduced graduate enrollments (6.3% from 1996 to 1997), in part because of lack of graduate student funding and in part because of a tight job market. Even so, a number of these departments failed to meet enrollment targets for first-year students for

[17] **Selected Data on Science and Engineering Awards**, 1995, (NSF 96-303), page 80.
[18] **Notices of American Mathematical Society**, 1997, v44, page 920.

fall 1997, indicating not only a decline in interest, but also in the quality of the applicant pool.

Annual Percentage Change in Full-time, First year Graduate Students in Mainline Departments[19]



Good mathematics students can and do excel in many disciplines and are heavily recruited by others. The lack of research assistantships for mathematical science students puts the mathematical science departments at a disadvantage when recruiting.

Careers in mathematics have become less attractive to U.S. students. At least seven factors contribute to this change: (i) Students mistakenly believe that the only jobs available are collegiate teaching jobs, a job market which is saturated (more than 1,100 new Ph.D.s compete for approximately 600 academic tenure-track openings each year); (ii) Academic training in the mathematical sciences tends to be narrow and to leave students poorly prepared for careers outside academia; (iii) Neither students nor faculty understand the kinds of positions available outside academia to those trained in the mathematical sciences; (iv) Within the United States, the number of graduate students on fellowships and research assistantships is significantly lower in the mathematical sciences than it is in the other sciences, so students must depend on teaching assistantships for financial support. For this reason, students perceive that other disciplines are more highly valued by society; (v) Universities depend on graduate students to teach undergraduates; too often, graduate students are exploited as cheap labor rather than nurtured and mentored as students. This custom extends the time to degree, which now averages more than 7 years, and places entering freshmen under the tutelage of novice graduate instructors. The uneven quality of this teaching diminishes the appeal of mathematics as an area of study; (vi) The United States offers limited opportunities for postdoctoral work in the mathematical sciences. In most fields of science in the United

[19] **Notices of American Mathematical Society**, 1998, v44, p 920.

States, and in mathematics in Europe, postdoctoral experience is valued as an opportunity to increase one's breadth of training and to develop independent research. The absence of postdoctoral funding makes mathematics less attractive and decreases opportunities for new Ph.D.s to develop their talent; and (vii) Funding for U.S. research mathematical science faculty is substantially below that for other sciences and engineering fields; sending a very negative signal about the status of the discipline in the United States.

The curriculum in U.S. institutions for undergraduates needs to be strengthened, broadened, and designed for more active participation by students in discovery. The faculty needs to be more involved with students to enable them to experience the joy of mathematical discovery. Also, U.S. students motivated by a desire to apply mathematics often find they must study in departments of economics and engineering, where they may not receive a firm foundation in mathematics.

Numbers of U.S. bachelor degree recipients in the mathematical sciences increased by 4% from 1989-1995. In that period the number of students receiving conventional mathematics degrees declined by 6.4% and this decline has continued. Several smaller trends moved against this decrease: from 1994-1995, the number of B.S. degrees in mathematics education and statistics doubled, while the number of actuarial majors tripled.[20]

The mathematical sciences play an essential role in precollege education. As mentioned above, the Panel has chosen to defer to other reports on K-12 education, and to underscore the assertion by these reports that mathematics education is crucially important in preparing the workforce of the future. In the United States, the present situation is not acceptable. The U.S. mathematical research community has the opportunity and the obligation to participate in the education of precollege teachers of mathematics, especially those planning to teach at the high school and community college level.

There are mounting pressures to reduce the time allotted to mathematics in the undergraduate education of scientists, engineers, and business majors. At the same time, there is a desire, for pedagogical reasons, to better integrate mathematical concepts and methods with those of the scientific and engineering disciplines. Unless appropriate steps are taken to respond to these felt needs, there is a distinct possibility that "service teaching" in mathematics will be "absorbed" by other disciplines, and that it will no longer be controlled by or provide employment for the mathematical research community. These possibilities are already manifest in some engineering programs. The academic mathematical community must cooperate with faculty from other disciplines in responding to needs for more effective, collaborative instruction. They must respond to a new trend in pedagogy that makes use of scientific problems to motivate mathematical

[20] **Notices of American Mathematical Sciences,** 1997, vol. 44, page 926.

theory and to increase the use of computers, especially with regard to visualization. Students need a sound understanding of mathematics as a basis for life-long learning. The dual nature of mathematics as a theoretical field of its own and as a discipline intrinsically linked to applications must be better reflected in the undergraduate curriculum.

There are exciting mathematical science career opportunities outside the academy. No hard data on career paths for mathematicians outside the academy were available to the Panel (see SIAM Report (op cit)). To gain insight, however, a member of the Panel undertook a survey of financial firms through the International Association of Financial Engineers to assess the use of mathematics in finance, principally by banks and broker-dealers. The survey, which can be found at http.//:www.cmra.com, indicates the high importance of quantitative and mathematical methods to this industry, with 40% of the respondents reporting heavy dependence and at least 75% moderate dependence. One-third of the respondents reported that more than 10% of their professional staffs were quantitative professionals, and more than 80% of the respondents planned to hire additional quantitative professionals. More than 70% of the respondents stated that the proportion of quantitative professionals on their staffs had increased in the past 15 years. Notably, 63% of the respondents stated that ***academic quantitative research should focus more on industrial applications*** and that ***there is not enough communication between theoretical mathematical research in academia and applied mathematical research in the practitioner world.*** This survey confirmed the Panel's judgments that (i) there are many exciting opportunities for mathematical careers outside academia, and (ii) many of these opportunities are foregone because of poor communication and academia's undervaluation of nonacademic employment. With a reorientation of curriculum and of employment expectations, possibilities for a career outside academia are very bright for mathematically talented and well-trained individuals.

Summary of Findings

There is a danger that academic mathematics will be perceived as solely a scholarly endeavor rather than as a full participant in the explosion of scientific and technological advances that began five decades ago. If this danger materializes, mathematics, irrespective of its great beauty, will not compete successfully for resources with disciplines whose goals are more obviously relevant to the solution of societal problems. Were this to occur, mathematics would be relegated to a more limited role, similar to that of some humanistic disciplines. Many intellectual activities of a mathematical nature would be absorbed by the other sciences and engineering because of their importance to applications. As described in Chapter II, this would result in major losses of synergy between academic mathematics and the users of mathematics -- to the detriment of both. It would also reduce our effectiveness in solving critical problems of society. It would do this by distorting the common sequence of events by which a scientific or technological problem is approached. That is, a problem may be generated by theory or experiment, but at some point it moves into mathematical questions; the refinement of these questions into a mathematical problem and the solution of this problem require substantial effort of an exclusively mathematical nature. Conversely, abstract mathematical ideas, which are developed without an application in mind, often prove to be the key to the formulation and solution of scientific and technological problems.

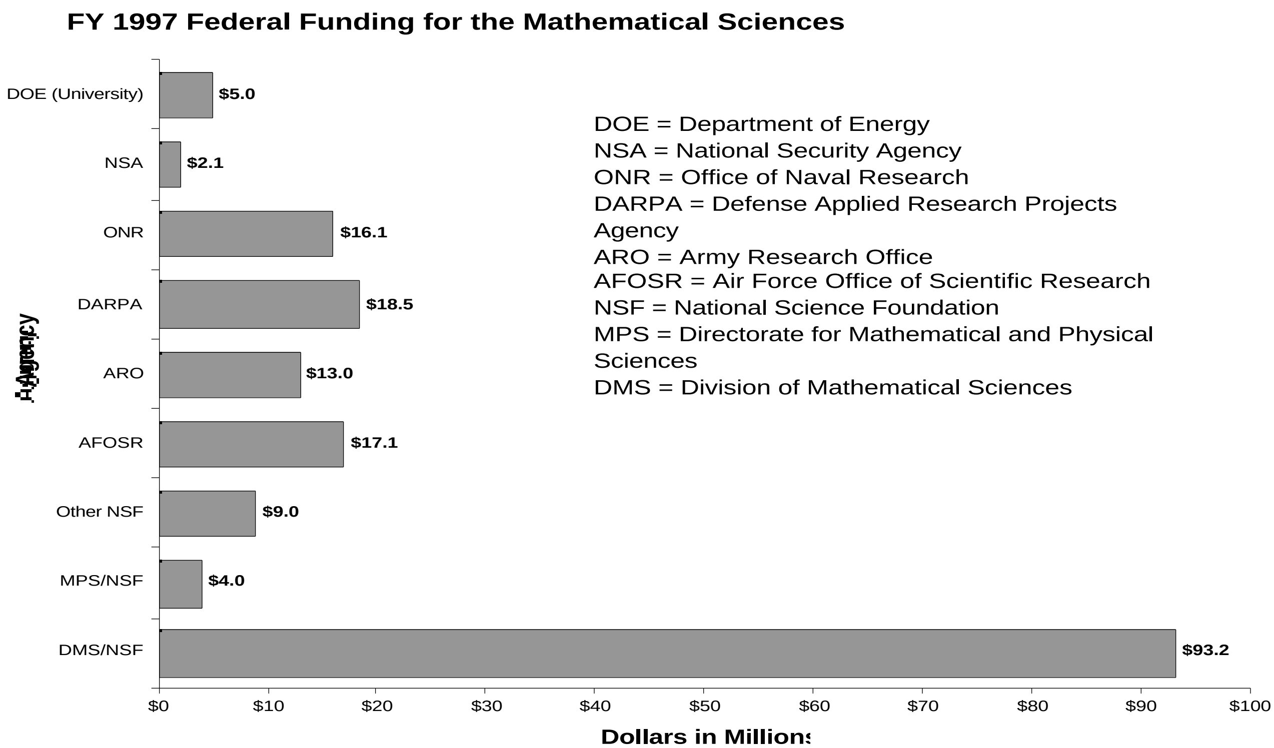
Academic mathematical science must strike a better balance between theory and application. At one extreme, a narrowly inward-looking community will miss both the opportunities that arise outside the mathematical sciences and the opportunities that are part of scientific and technological developments. At the other extreme, an exclusive concern with applications and collaborative research would severely limit the mathematical sciences and deprive the scientific community of the full benefits of mathematical inquiry. At present, the balance is tilted too far toward inwardness.

For U.S. mathematical science to thrive, the discipline must be made more attractive to young Americans with bright and inquisitive minds. The reforms suggested above will enrich the discipline and expand students' abilities to contribute to their profession and to society. In addition, mathematics students must be funded at a level comparable to students in the other sciences and engineering, and they must be made aware of the range of employment opportunities available outside academia. Young academic researchers must be supported to enable them to reach their full potential.

VI. FEDERAL AND NON-FEDERAL SUPPORT FOR RESEARCH IN ACADEMIC MATHEMATICAL SCIENCE

Federal support

The U.S. mathematical sciences rest on a narrow base of support. The NSF provides the majority of support for mathematical research in U.S. universities and institutions.



Source[21]

The percentage of support supplied by NSF has grown in recent years -- not because of significant growth in the NSF/DMS budget, but because of significant cutbacks by other agencies, especially DoD. Traditionally, these agencies have used a larger percentage of their budget for the support of graduate and postdoctoral students than has DMS; hence, these cutbacks have a significant negative impact on student opportunities. Because NSF now provides such a large fraction of the support for research in the mathematical sciences, it has both a high level of responsibility for the stewardship of the mathematical sciences and high leverage in enforcing change.

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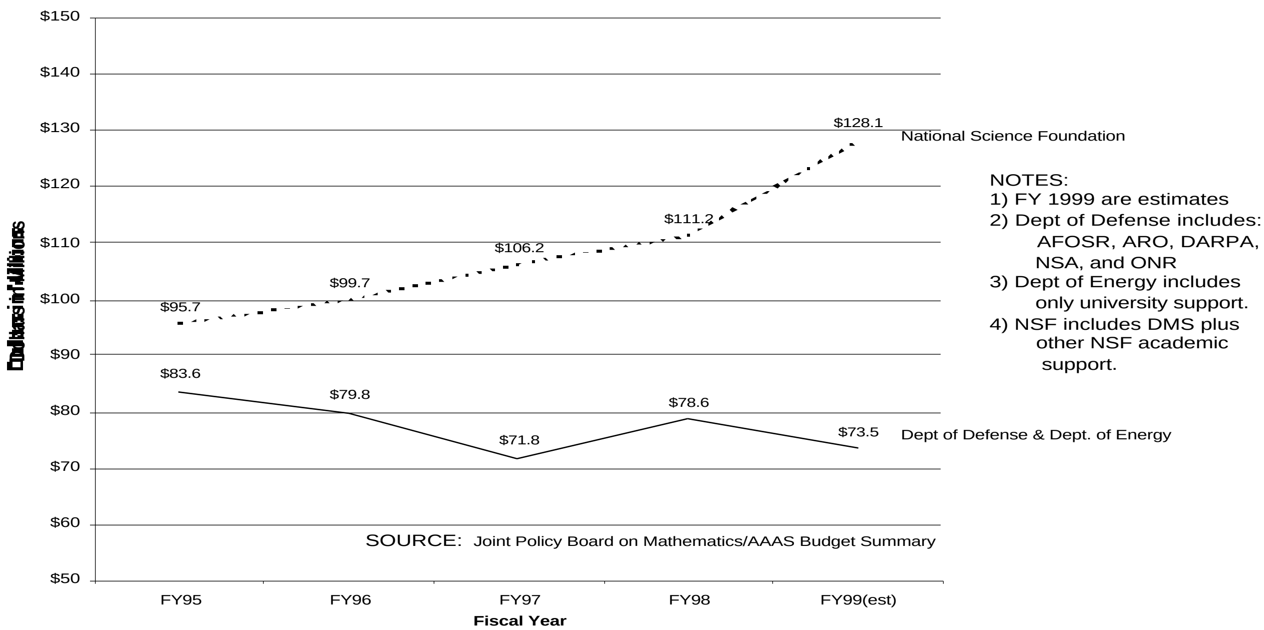
[21] **Research and Development, FY 1997, AAAS Report XXI**. Note: ONR support is listed at 16.1M, however, it is estimated that only $7M is for academic research.

Federal Funding Trends from FY95



As the David I (op. cit.) and David II[22] reports made clear, federal funding for the mathematical sciences is disproportionately low compared to funding for the other sciences and engineering. The discrepancy in funding carries over to support for graduate students and postdoctorates.

Source of Support for Graduate Students[23]

	Number of Students			
	Federal Support	Institutional Support	Other Support	Self Support
Biological Sciences	16593	20805	3923	6962
Physical Sciences	10353	14858	2079	1602
Mathematical Sciences	1291	9169	478	2484

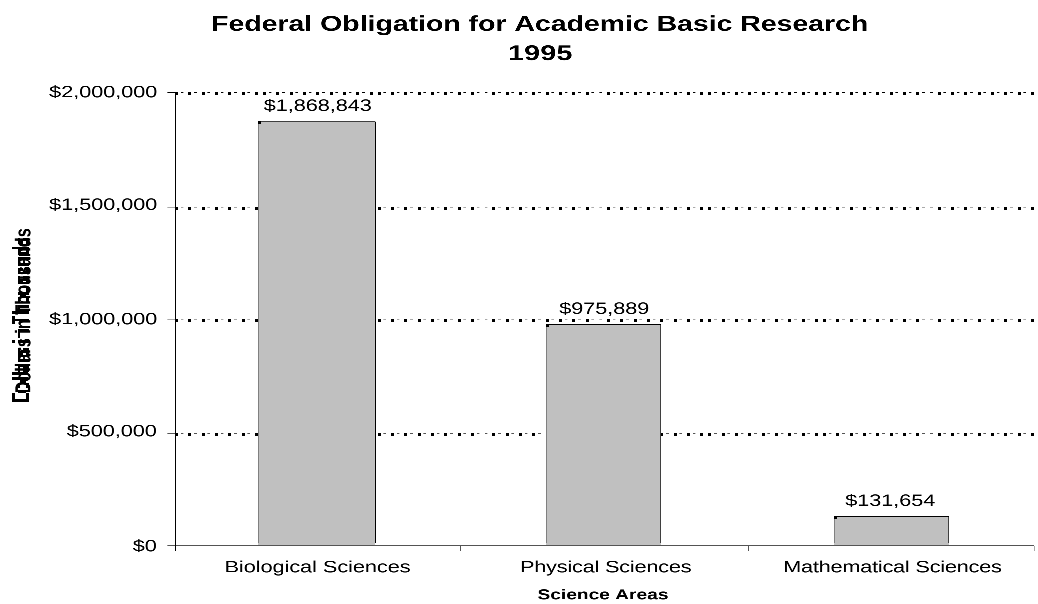
Science Postdoctoral Appointees in Doctorate Grantship Institutions[24]

	1990	1991	1992	1993	1994	1995
Chemistry	3630	3647	3564	3555	3710	3581
Physics	1723	1813	1948	1823	1844	1821
Biological Sciences[25]	5428	5806	6071	6308	6760	7057
Mathematical Sciences	248	206	201	224	239	255

[22] **A plan for the 1990's,** 1990, National Academy Press.
[23] **Renewing U.S. Mathematics: Graduate Students and Postdoctorates in Science and Engineering,** Fall 1995, (NSF 97-312).
[24] **Graduate Students and Postdoctorates in Science and Engineering,** Fall 1995, (NSF 97-312) p.88.
[25] Biology, Biophysics, Botany, Cell Biology, Zoology, Genetics, and Microbiology.

The NSF, because of its dominant role in funding mathematics, provides a much larger share of federal funding for the mathematical sciences than it does for the physical and biological sciences and engineering**. The lack of broader support for the mathematical sciences among the federal agencies significantly contributes to instability within this field and to the danger of losing the U.S. position of leadership.**

This difference in support greatly undermines the mathematical sciences within the universities, especially when the mathematical sciences department sits in a college of science or engineering. Too many universities judge the quality of a department by the indirect costs it generates rather than by its standing relative to other departments in same discipline. Because federal funding for the mathematical sciences is so much lower than for the other sciences and engineering (see Table below), the mathematical sciences are judged within universities not as research departments but as service teaching units and are funded accordingly. One consequence is that the mathematical science faculty teach twice the number of courses as do other scientists and hence have less time for research.



Source[26]

Unfortunately, many U.S. universities use generation of external funding as a significant measure in making tenure decisions. Given the underfunding of the mathematical sciences, many worthy young researchers do not obtain funding and so fail to obtain tenure. This situation reduces the appeal of the mathematical sciences as a career. Given the essential role of the mathematical sciences in future scientific endeavors, the United States cannot

[26] **Graduate Students and Postdoctorates in Science and Engineering,** Fall 1995, (NSF 97-312).

expect to remain in the forefront of science without adequately funding the mathematical sciences.

The current funding imbalance of support of graduate students and postdoctorates between that for the mathematical sciences and that for the other sciences follows several decades of policy changes. Throughout the sixties, Federal support for graduate students in the mathematical sciences was on a par with the other sciences, with most of the funding coming via the Department of Education as authorized by the National Defense Education Act (NDEA). At the end of that decade, there was a surplus of new science doctorates and funding for NDEA ceased. At that time, NSF placed its emphasis on research and the charge to develop the next generation of scientists was not a priority. Between 1969 and 1972, the number of federally funded Predoctoral Fellowships and Traineeships in the mathematical sciences dropped from 1,179 to fewer than 150, and the number of full-time fellowships and research assistantships is not much larger today. When NDEA funding ended, the lab sciences successfully argued that they needed graduate assistants and postdoctorates to carry on their research, and funding for such was included in NSF grants to principal investigators in the lab sciences, but not for those in the mathematical sciences. During the 1970s and early 1980s, the Foundation was funding 1,200 postdoctorates in physics and twice that number in chemistry, but only 56 in the mathematical sciences. The David I report of 1984 made a strong case for correcting this imbalance of support. However, this has not occurred, even at a time when the Foundation has again begun to emphasize both research and education and the integration of both.

Non-Federal support

The major source of non-federal support for the mathematical sciences is the major research universities. They have provided a limited number of research professorships and a limited number of research instructorships for recent doctorates. In addition, some portion of academic appointments are for research. Currently, via teaching assistantships, universities are providing the bulk of support for doctoral students.

Academic mathematics receives relatively little direct support from the private sector. Industry provides substantial industrial support for areas that use mathematics (computer science, communications, bioinformatics, engineering, earth sciences), but this support favors scientists and engineers in allied disciplines rather than trained mathematicians. The U.S. insurance and finance companies, whose employees receive their basic education in the mathematical sciences and who continue to collaborate and consult with academic mathematical sciences faculty, also provide little direct support. This is not the case in England, where similar businesses have a tradition of investing in mathematics departments.

Direct industrial investment in the mathematical sciences is largely restricted to areas of applications. Although there are rare exceptions (e.g., limited unrestricted funding for internal mathematics by Lucent, Microsoft, IBM, AT&T, NSA, and NEC), most industrial mathematics focuses on short-term product applications. Targeted funding also predominates in other arenas where mathematics is used heavily-- finance, government, and corporate laboratories.

The flux of foreign-trained professional mathematicians into U.S. universities and businesses brings both a high level of skill and a hidden form of support: These individuals represent substantial educational investments by their native countries. However, foreign professionals may not have long-term commitments to U.S. interests. Further, changes in emigration policies or economic opportunities could restrict this flow quickly and even reverse it. Rapid changes of this nature could undermine the health of certain U.S. universities and industries.

Because of the academic orientation of U.S. mathematical scientists, there is scant evidence of the entrepreneurism that prevails in the other sciences, particularly in chemistry, materials science, molecular biology, computer science, and engineering. For examples, few of the NSF Science and Technology centers have a strong mathematical orientation. The lack of an entrepreneurial environment in the mathematical sciences restricts potential sources of external funding and reduces the earning power of students who aim primarily for academic employment.

VII. OBJECTIVES, RECOMMENDATIONS, AND MILESTONES FOR NSF

Objectives of NSF Support for Mathematics

NSF's specific objective in the area of the mathematical sciences should be to build and maintain an academic community in mathematics that is both intellectually distinguished and relevant to society. This objective contains an important shift in emphasis, in keeping with the NSF's Strategic Plan, which is the explicit inclusion of societal relevance as both a criterion for performance and an objective in the academic mathematical sciences.

Traditionally, U.S. departments of mathematical sciences have focused on problems in mathematics that are intellectually challenging for their own sake. In the last three to five years, however, shifts in emphasis have begun to appear. Underlying these shifts is the belief that mathematics has enormous benefits to offer to the country, to other areas of science and technology, and to industry, commerce, and government. These areas, in turn, have much to offer the mathematical sciences in providing challenging mathematical problems, jobs for mathematics students, and opportunities for mathematicians to work with professionals of varied disciplines. Therefore, **NSF's broad objective in mathematics should be to build and maintain the mathematical sciences in the United States at the leading edge of the mathematical sciences, and to strongly encourage it to be an active and effective collaborator with other disciplines and with industry.**

NSF should also ensure the production of mathematical students sufficient in number, quality, and breadth to meet the nation's needs in teaching, in research in the mathematical sciences and in other disciplines, and in industry, commerce, and government.

NSF should approach these objectives through the following strategies:

- Show an increase in funding for the mathematical sciences to bring the number of researchers supported in this discipline to a level comparable to those in the physical and biological sciences and engineering;

- Encourage activities that connect mathematics to other areas of science, technology, business, finance, and government;

- Strengthen the connections between "pure" and "applied" mathematics;

- Broaden the exposure of professional and student mathematicians to problems in other fields;

- Make the mathematical sciences more attractive to U.S. students so the mathematical sciences can better complete for the brightest young people; and

- Maintain and strengthen abstract mathematics, which is fundamental to the health of the mathematical sciences.

Recommendations for NSF

Encourage broader education for graduate and undergraduate students in the mathematical sciences. NSF should focus its support on Ph.D. programs that simultaneously broaden education, decrease the teaching load of graduate assistants, and shorten time to Ph.D. NSF should encourage and, to the extent possible, fund programs at both the graduate and undergraduate levels that broaden exposure of students to mathematical problems in areas other than mathematics.

Provide funding for doctoral and postdoctoral students and younger researchers at levels comparable to those in the physical and biological sciences. Relatively low funding for research by graduate students in the mathematical sciences lowers morale and reduces the attractiveness of the discipline to bright young people. The scarcity of postdoctoral fellowships slows the professional development of potential academics. NSF should encourage postdoctoral students to immerse themselves in another discipline at the postdoctoral level.

Promote interactions between university-based mathematical scientists and users of mathematics in industry, government, and universities. NSF should increase support for programs that involve academic mathematical scientists in multidisciplinary and university/industry research. Particularly important is to support endeavors that distill mathematical challenges arising from new scientific and technological developments and to encourage research that addresses these challenges.

Strengthen research in abstract mathematics. A strong core of abstract mathematics is essential to the health of the mathematical sciences in the United States and in turn to all science. Because excellent abstract research is often motivated by problems encountered by users of the mathematical sciences, researchers need to maintain good communication with users.

Recognize its unique responsibility, as the principal federal funder of U.S. mathematical sciences, to sustain their position of leadership.

Milestones for NSF Activities

Over a period of three years, NSF should aim to:

Demonstrate substantial increases in funding for graduate students and postdoctoral fellows in mathematics. These increases would shorten the time to a graduate degree by requiring that students spend less time teaching, provide a broader and more flexible education, and make the field more attractive to U.S. students.

Show an increase in interdisciplinary activities involving mathematics. This increase would encourage the dissemination of mathematical concepts into communities of users, expose mathematicians to problems and opportunities outside conventional mathematics, and build partnerships between mathematical scientists and researchers in other disciplines.

Encourage activities aimed at broadening undergraduate and graduate curricula, with the objective of widening the range of curricular choices, raising the attractiveness of mathematical careers to students, and increasing the vocational flexibility of future mathematicians.

Show an increase in funding in the mathematical sciences to bring the number of mathematical scientists being funded to a level comparable to that in the physical and biological sciences. This is especially important for retaining young academic researchers.

Appendix 1

SENIOR ASSESSMENT PANEL MEMBERSHIP

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Convenor of Assessment Panel:

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Appendix 2

ASSESSMENT OF SUBFIELDS

In assessing the subfields of the mathematical sciences, we were greatly aided by the staff of the Division of Mathematical Sciences, which made an internal assessment based on information from leading researchers and on results from the peer review system. Individual members of the Panel assumed responsibility for assessing their own areas of expertise; they consulted with experts in the United States and abroad and used bibliographic and peer review data provided by staff of the Division. Although the resulting assessments are fundamentally subjective and qualitative, we have confidence in the data on which they are based and on the results.

We do, however, warn potential readers that the Appendix is more technical than the main body of the Report. It was compiled primarily for the use of the Panel in the short term and the Division of Mathematical Sciences in the medium term.

For assessment purposes the mathematical sciences were divided into nine subfields which mirror the program structure of the Division of Mathematical Sciences of NSF. Each subfield assessment is intentionally rather general; more specific evaluations within subfields are provided by the peer review system, which is regarded as very effective for this purpose.

Foundations

Foundations, or mathematical logic, is here subdivided into four areas: **set theory, model theory, recursion theory,** and **proof theory**. The United States has notable activities in this field, but Israel and Europe are, as a whole, dominant.

In **set theory** the United States shares leadership with Israel, with exciting results across the areas of determinacy, large cardinals, and combinatorics. However, the research community in set theory is aging, and younger mathematicians and graduate students are few and primarily foreign. The area of **model theory** is flourishing worldwide. The United States is a participant, but its activities are overshadowed by those in England, France, and Israel. There are notable interactions between this area and other fields of mathematics and with computer science. **Recursion (or computability) theory** is quiescent, with a substantial body of completed work. Barring a major breakthrough, or the further exploitation of connections with computational mathematics and computer science, the next decade is not expected to be very active. England plays a leading role, with the United States as a contributor, but the aging research population is not being replenished. In the United States, **proof theory** has, in large measure, moved to computer science. The United States is a minor contributor; research leadership is concentrated in France, Germany, Russia, and Israel. Major advances in computational complexity are expected to continue.

It is notable that some of the acknowledged leaders in Foundations, both in the United States and abroad, are also active in other fields of mathematics or computer science. Several dynamic areas of model theory exemplify how a field can thrive both within its own core and in relation to other fields. Potential interactions exist for set theory and recursion theory. There is concern, however, that Foundations in general has not attracted enough young mathematicians in the United States during the past two decades. This has led to fragility in the field and concerns that the U.S. community cannot respond promptly to advances in other countries. In addition, there is uneasiness over the insularity of several key areas of Foundations, which results in the failure to explore opportunities arising both within mathematics and in science and technology, notably in computer and computational science.

Algebraic Geometry and Number Theory

The United States and Western Europe dominate research in these subfields, although Europe has had a much longer tradition in these areas. These fields have seen spectacular achievements over the last 50 years, as old, even ancient problems have been solved, culminating in the last decade with the solution of Fermat’s problem and the Mordell conjecture. In addition, these subfields have had significant impacts on and interactions with physics, cryptography, and other areas of mathematics.

Algebraic geometry is flourishing. Current problems of significance are likely to be solved in the next few years and to be replaced by others of equal significance. There are notable interactions with other areas of mathematics and theoretical physics. Leadership is shared by the United States, Japan, and Western Europe, with the United States having the most active researchers. In **computational algebraic geometry**, the United States lacks depth, and leadership is held by Europeans.

Number theory is dominated by the three “grand challenges”: the Riemann Hypothesis (RH), the Langlands Program, and the Block-Kato-Beilinson conjectures. There has been recent notable progress on the last two, but additional progress on the Langlands Program awaits a breakthrough. There is renewed activity in RH, but probably not at the depth needed. The United States, and Western Europe are the dominant centers, with Canada quite strong; at one time the Soviet Union was among the leaders.

Arithmetic geometry has experienced recent spectacular breakthroughs, notably the work on Fermat's Last Theorem. The United States has great strength, but Europe is probably stronger. **Analytic number theory** is relatively quiet at this time, needing a new major line of attack. The United States is the clear leader, with great depth but very few young researchers. **Computational number theory** is an area of very high activity, driven by increasing access to powerful computers and the link to cryptography. Europeans are major contributors to the open literature; there is also significant classified and proprietary work where the United States is regarded as the leader.

The entire subfield is considered to have notable opportunities for both internal development and for impact external to the area. In spite of this, the number of young Americans entering it has decreased significantly. A large proportion of current major U.S. contributors were educated abroad, which was not the case 20 years ago, and researchers move freely between the United States and Europe.

Algebra and Combinatorics

Algebra has undergone significant developments in the past decade. It is a very active subfield, with significant interaction with topology, geometry, and theoretical physics. The United States is regarded as the leader, with Western Europe a close second; both have benefited from the emigration of mathematicians from the former Soviet Union. There are also major centers of activity in Russia, Japan, Israel, and Australia.

In **algebraic representation theory,** research is enhanced by interactions with geometry, combinatorial methods, and theoretical physics. The United States is a leader, and Western Europe and Japan have significant strengths. U.S. leadership also holds in **finite and combinatorial group theory,** where the infusion of geometric ideas is leading to novel approaches and results. There are also strong centers of activity in Western Europe, Russia, and Israel. Rather exciting developments are underway in **noncommutative geometry and Lie theory,** with connections to algebraic geometry and to theoretical physics (quantum groups). The United States and Western Europe share research leadership. The United States is the clear leader in **ring theory**, where a breakthrough is needed for further major advances. **Computational algebra,** although still in its infancy, holds great promise. Europe has decidedly more depth and breath than the United States, and Australia is a strong participant.

Graph theory serves as a bridge between mathematics and areas of applications. The very strong researchers are in the United States, working not in mathematics departments but in computer science, electrical engineering, and industry. In Europe there is greater connectivity between mathematicians, computer scientists, and engineers; consequently, Europe is stronger in applications. The United States has pioneered the area of graph minors and remains the leader in this small subfield.

The United States has clear leadership and great depth in **algebraic combinatorics**, where significant challenges (MacDonald conjectures) and notable interactions with quantum cohomology are providing exciting results. In **probabilistic combinatorics** the United States has many research leaders but little depth. Recent breakthroughs have led to solutions of classical extremal problems. Western Europe and Israel are the other major centers of activity.

The subfields of algebra and combinatorics are central to the mathematical sciences and they attract good students and young researchers. In the last ten years, there has been a concerted effort to apply combinatorics in a wide set of areas such as crystallography, robotics, computational efficiency, DNA sequencing, and computer networks. In the United States, this has most frequently been done by theoretical computer scientists. Europe is quite strong in discrete mathematics. There is a paucity of interactions between mainline U.S. algebraists and other disciplines.

Topology and Geometric Analysis

Topology and Geometric Analysis have flourished in the last decade. This subfield is central to the mathematical sciences as an area of specific mathematical investigation, as a mode of thought that uses geometric and topological concepts in other branches of mathematics, and in the analysis of geometric patterns that arise in computing and the natural sciences. Perhaps the most exciting recent development is the manner in which geometry, topology, analysis, and theoretical physics have become intertwined and mutually reinforcing. The United States is regarded as the leader, with substantial strength in Western Europe and some strength in Japan.

Seiberg-Witten theory, originating in theoretical physics, has proven to be an effective tool in **symplectic topology/geometry,** stimulating much recent activity and leading to the solution of long-standing problems. There are active groups worldwide, most notably in the United States and Western Europe. The theory of algebraic invariants, especially the study of invariants for **low-dimensional manifolds** and **knots** in three-manifolds, has been exceptionally fast-moving, stimulated by its interaction with physics. The United States is especially dominant in this area, aided by recent emigration from the United Kingdom; Russia also has strength. Work continues on the **classification of three-dimensional manifolds**, driven by the Thurston Geometrization Conjecture, with the possibility of near-term success. Strength in this area is concentrated in the United States. **Homotopy theory** is playing an increasingly important role in algebraic geometry, but is generally mature; the United States is strong in this area. Progress has been made in providing powerful computations in **algebraic K-theory**, with major contributions from the United States, France, and Norway.

Riemannian geometry has experienced several major developments in the last decade; both the United States and Western Europe have significant depth in this area. **Regularity theory** for differential equations related to geometric objects has been an important area of research, with activity primarily in the United States. Recent work in the United States and Western Europe on **harmonic maps** has implications for super-rigidity and representation theory. **Noncommutative geometry**, involving a synthesis of geometry, analysis, algebra and topology in a quantization of mathematical entities, could lead to significant breakthroughs in the near future. Both the United States and Western Europe are leaders in this area. There has been steady, if not spectacular, progress in **geometric measure theory** and **minimal surfaces** over the last decade, with some applications to problems in materials research; this work has occurred mainly in the United States.

There is a strongly felt need, both in the United States and abroad, to actively stimulate interactions between topologists and geometers and members of the other sciences and technology in order to disseminate geometric ideas to prospective users and to stimulate new ideas in the subfield. A limited number of geometry and topology researchers are currently collaborating with specialists on DNA and polymers, control of mechanical systems, robotics, and image processing. A substantial number of young people are entering this subfield, although (as with other subfields) much activity in the United States is the product of the immigration of researchers trained abroad.

Analysis

Analysis is a subfield where theory and usage meet. The United States is regarded as a leader in this subfield, with very strong activities in Western Europe, Russia, and Japan. The recent past has seen rapid advances in broad areas of analysis, reflected by the award of six of the last eight Fields Medals in this subfield (three to U.S. residents). The international character of mathematics is well reflected in analysis, which features a very high level of international collaboration.

The area of **differential equations** is most important for its impact on other sciences and on technology. In ordinary differential equations, with related activities in **numerical analysis** and **dynamical systems**, the United States has a long tradition and very active groups which are challenged for leadership by groups in Western Europe. In **partial differential equations,** linear theory has reached maturity and nonlinear theory is developing very rapidly. The United States played a leading role in its early development, a role which is now shared with extremely strong groups in Western Europe, most notably in France. Unless more young researchers in the United States are attracted to this area, the United States will not be able to sustain its present position.

There is high promise for continuing major advances in **nonlinear partial differential equations**, **operator algebras**, **dynamical systems**, **representation theory** and solvable models of **mathematical physics,** and **harmonic analysis** (and applications). These advances are occurring throughout the world, with the United States playing a leading role. As in other areas, a high proportion of U.S. leading researchers, young faculty, and graduate students are recent immigrants. The subfield of analysis continues to have good interaction with applications arising in the physical sciences and engineering; many of its problems have been motivated by the study of phenomena from those fields.

There is a felt need for closer contact in the future, with the biological sciences; a paucity of contacts signifies lost opportunities of significance.

Probability

Probability arose from the study of gambling choices is relatively new as a rigorous discipline. Modern probability provides the foundation for statistical inference, and it is intimately associated with measure theory, a branch of analysis. Nowadays, the emphasis is on randomness and on indeterminate phenomena. Many new developments in probability are motivated by problems outside mathematics.

The United States is dominant in all aspects of probability, including theory, applications, and computational approaches. Bibliometric data indicate that approximately half the literature in probability theory is produced by U.S.-based researchers. Other centers of activity are France, the United Kingdom, Canada, and Japan. Activities in the U.S. community feature both breadth and depth, whereas activities abroad tend to be more narrowly specialized. Research in the former Soviet Union, once very strong, is now weak.

In general, probability theory is very vital today, both in the development of fundamental theory and in interactions with other branches of mathematics and the other sciences (areas of interaction within mathematics include stochastic partial differential equations, superprocesses, percolation, Yang-Mills equations, turbulence, statistical physics, and critical phenomena). A second strength is that U.S. probability has maintained close contact with a diverse set of areas of applications. Applied probability in the United States is profoundly influencing and drawing inspiration from problems in the biological sciences (genetics, DNA structure, competition processes), medicine (epidemiology), and the environmental sciences (hydrology, environmetrics). Contributions by applied probabilists underpin much applied work in operations research and management, stochastic networks in communications, and financial engineering. In all these areas, U.S. probabilists are at the forefront; the United Kingdom, Canada, and France are also active. There is also strong U.S. activity in computational probability, with a secondary strong center in the United Kingdom.

Probability permeates the sciences and technology, with notable activities in engineering, computer science, physics, management, and finance. Workers in these fields have close contacts with the academic probability community, resulting in substantial accomplishments in both theoretical and applied areas, many of which are stimulated by novel technological developments. Computation and simulation play an increasingly crucial role.

Applied Mathematics

Applied mathematics is the name given to the subfield of mathematics that is motivated by practical problems whose formulation and study is mathematical in method and spirit. Traditionally the term has been associated with applications of analysis to problems in the physical sciences. Nowadays all mathematics is being applied, so the term applied mathematics should be viewed as a different cross cut of the discipline.
The United States has a leadership position in some areas of applied mathematics, notably in the areas of **computer vision, financial engineering,** most aspects of **materials,** and some aspects of **mathematical biology**. The U.S. contingent of invited participants at the last Congress of Industrial and Applied Mathematics was the largest of any country.

Close interaction between applied mathematicians and practitioners in science and engineering is critical. In the United States, while those working in applied mathematicians often work alone, they are more involved in interdisciplinary research than mathematicians in other subfields, but much more needs to be done. The European community is making very large investments to this end. The United Kingdom is more adept in establishing close ties between industry and universities. And in France, engineering has closer relations with mathematics.

The U.S. research community has responded rapidly to opportunities in areas such as **fluid mechanics** and **materials science**, but the mathematics of these areas is still in its infancy. Researchers have responded much more slowly to problems arising from chemistry, the biological sciences, manufacturing, and design. Continuum mechanics, constructive gauge theory, and other aspects of theoretical/mathematical physics have long been active fields of research, more so in Europe than in the United States. **Optimization** is a very active field, with many applications, where the United States is very strong. In the United States it is often found in departments of industrial engineering and computer science. **Control theory** is another area where the United States has great strength, with activity in every field of engineering.

In the future, applied mathematics will be closer to computer modeling and simulation and farther from analytical theory. There is a need for more vigorous interactions with other fields of science and stronger contact with the industrial community.

Computational Mathematics

Computational Mathematics is the area of mathematics concerned with reliable and effective solutions to mathematical problems using the computer. **Numerical analysis, and approximation theory, which are closely linked to applied mathematics, as well as algorithms** and **data analysis,** are generally included, but the area also includes computational modeling and simulation of phenomena. Some would include symbolic manipulation and even the use of computers in an exhaustive delineation of cases in mathematical proofs. As has been noted, many subareas of the mathematical sciences now have a computational component. Because of the impact of computer architecture on effective computation (particularly various forms of parallelism) there is a strong link to computer science.

Factors that have affected the growth of Computational Mathematics include:

- Changes in computing capability and architecture;

- The emergence of standard packages from mathematical subroutines to structural analysis codes, making computation accessible to non-experts;

- The observation that underlying structure from particular disciplines may offer computational capability to broad classes of problems; and

- The desire of users of mathematics to more accurately simulate more detailed physical phenomena so as to replace costly testing.

Computational mathematics has become a mainstay in industry, finance, and public policy. The best example is the computational design of Boeing airplanes, which requires mathematicians well-trained in computational mathematics. Regretfully, good computational techniques, well studied by mathematicians, are seldom implemented in standard packages and, conversely, important ideas arising in applications are often not refined mathematically.

The literature in this area has grown enormously in the past decade, and its importance and impact continues to increase. However, the field as a whole remains fragmented and there is not enough synthesis and refinement of new techniques developed in industry and by other scientists.

The United States is the acknowledged leader in computational mathematics (especially in its commercial aspects), but not in all areas. The United States trails Western Europe in certain areas of numerical analysis and in symbolic computation, but the United States is the clear leader in providing commercial products.

The current strengths in computational mathematics draw on the widespread acceptance of computational modeling as the replacement for physical tests in a broad number of fields. Significant work in optimization has moved computer modeling close to the heart of analytic technique, thanks also to the easy availability of inexpensive, high-powered computers.

There are also weaknesses in computational mathematics, notably a general failure to synthesize new mathematics drawn from computational modeling using problem characteristics from various fields. As a result the field is more fragmented, and applications fields have not gained the mathematical expertise they need. This fragmentation has led some mathematicians to conclude that the area is in decline (e.g., the comment by a European review that "Nobody dominates, nobody is much interested anymore.”).

Statistics

The statistical sciences are very healthy across all subareas in the United States, which is the clear world leader. Statistics traditionally has been strong in the United Kingdom. It is now developing rapidly in continental Europe, so that the U.S. lead is shrinking. There are centers of significance in Australia and Japan.

Statistics has always been tied to applications, and the significance of results, even in theoretical statistics, is strongly dependent on the class of applications to which the results are relevant. In this aspect it strongly differs from all other subdisciplines of the mathematical sciences except computational mathematics.

The United States has both a high level of activity and a leadership role in **theoretical statistics.** Other centers are being developed in continental Europe and Australia. In **applied statistics** the United States is also the leader, with the United Kingdom in a very strong position, centers of excellence in Japan and Australia, and developing ones in Western Europe. In both of these areas, U.S. journals dominate the field. The United States is the clear leader in **computational statistics,** with the United Kingdom in a very strong position. Very rapid advances in this field have considerable significance to applications.

The interaction between the academic community and users in industry and government is highly developed, and hence there is rapid dissemination of theoretical ideas and of challenging problems from applications, as well as a tradition of involvement in multidisciplinary work. Both in applications and in multidisciplinary projects, however, there exist serious problems in the misuse of statistical models and in the quality of education of scientists, engineers, social scientists, and other users of statistical methods. As observations generate more data, it will be essential to resolve this problem, perhaps by routinely including statisticians on research teams.

There are great opportunities for impact in data mining and in the analysis of very large data sets that information technology now demands. While data analysis is the essence of statistics, challenges in data mining demand new techniques that in all probability will need to come from mainstream mathematics. For example, concepts from quantum mechanics seem to provide promising tools.

There is ample professional opportunity for young people in statistics, both in academia, industry, and government. A very high proportion of graduate students are foreign-born and many remain in the United States upon graduation.

Appendix 3

POSSIBLE TRENDS IN THE COMING DECADES

By Michael Gromov

Here are a few brief remarks on possible trends in mathematics for the coming decades.

1. Classical mathematics is a quest for structural harmony. It began with the realization by ancient Greek geometers that our 3-dimensional continuum possessed a remarkable (rotational and translational) symmetry (groups O(3) and (R3) which permeates the essential properties of the physical world. (We stay intellectually blind to this symmetry no matter how often we encounter and use it in everyday life while generating or experiencing mechanical motion, e.g. walking. This is partly due to non-commutability of O(3) which is hard to grasp.) Then, deeper (non-commutative) symmetries were discovered: Lorentz and Poincare in relativity, gauge groups for elementary particles, Galois symmetry in the algebraic geometry and number theory, etc. And similar mathematics appears once again on a less fundamental level, e.g., in crystals and quasicrystals, in selfsimilarity for fractals, dynamical systems and statistical mechanics, in monodromies for differential equations, etc.

The search for symmetries and regularities in the structure of the world will stay at the core of the pure mathematics (and physics). Occasionally (and often unexpectedly) some symmetric patterns discovered by mathematicians will have practical as well as theoretical applications. We saw this happening many times in the past; for example, integral geometry lays at the base of the x-ray tomography (CAT scan), the arithmetic over prime numbers leads to generation of perfect codes and infinite dimensional representations of groups suggest a design of large economically efficient networks of a high connectivity.

2. As the body of mathematics grew, it became itself subject to a logical and mathematical analysis. This has led to the creation of mathematical logic and then of the theoretical computer science. The latter is now coming of age. It absorbs ideas from the classical mathematics and benefits from the technological progress in the computer hardware which leads to a practical implementation of theoretically devised algorithms. (Fast Fourier transform and fast multipole algorithm are striking examples of the impact of pure mathematics on numerical methods used every day by engineers.) And the logical computational ideas interact with other fields, such as the quantum computer project, DNA-based molecular design, pattern formation in biology, the dynamics of the brain, etc. One expects that in several decades computer science will develop ideas on even deeper mathematical levels which will be followed by radical progress in the industrial application of computers, e.g., a (long overdue) breakthrough in artificial intelligence and robotics.

3. There is a wide class of problems, typically coming from experimental science (biology, chemistry, geophysics, medical science, etc.) where one has to deal with huge amounts of loosely structured data. Traditional mathematics, probability theory, and mathematical statistics, work pretty well when the structure in question is essentially absent. (Paradoxically, the lack of structural organization and of correlations on the local level lead to high degree of overall symmetry. Thus the Gauss law emerges in the sums of random variables.) But often we have to encounter structured data where classical probability does not apply. For example, mineralogical formations or microscopic images of living tissues harbor (unknown) correlations which have to be taken into account. (What we ordinarily "see" is not the "true image" but the result of the scattering of some wave: light, x-ray, ultrasound, seismic wave, etc.) More theoretical examples appear in percolation theory, in selfavoiding random walk (modelling long molecular chains in solvents), etc. Such problems, stretching between clean symmetry and pure chaos, await the emergence of a new brand of mathematics. To make progress one needs radical theoretical ideas, as well as new ways of doing mathematics with computers and closer collaboration with scientists in order to match mathematical theories with available experimental data. (The wavelet analysis of signals and images, context dependent inverse scattering techniques, geometric scale analysis, and x-ray diffraction analysis of large molecules in crystallized form indicate certain possibilities.)

Both the theoretical and industrial impacts of this development will be enormous. For example, an efficient inverse scattering algorithm would revolutionize medical diagnostics, making ultrasonic devices at least as efficient as current x-ray analysis.

4. As the power of computers approaches the theoretical limit and as we turn to more realistic (and thus more complicated) problems, we face the "curse of dimension" which stands in the way of successful implementations of numerics in science and engineering. Here one needs a much higher level of mathematical sophistication in computer architecture as well as in computer programming, along with the ideas indicated above in 2. and 3. Successes here may provide theoretical means for performing computations with high power growing arrays of data.

5. We must do a better job of educating and communicating ideas. The volume, depth, and structural complexity of the present body of mathematics make it imperative to find new approaches for communicating mathematical discoveries from one domain to another and drastically improving the accessibility of mathematical ideas to non-mathematicians. As matters stand now, we, mathematicians, often have little idea of what is going on in science and engineering, while experimental scientists and engineers are in many cases unaware of opportunities offered by progress in pure mathematics. This dangerous imbalance must be restored by bringing more science to the education of mathematicians and by exposing future scientists and engineers to core mathematics. This will require new curricula and a great effort on the part of mathematicians to bring fundamental

mathematical techniques and ideas (especially those developed in the last decades) to a broader audience. We shall need for this the creation of a new breed of mathematical professionals able to mediate between pure mathematics and applied science. The cross-fertilization of ideas is crucial for the health of the science and mathematics.

6. We must strengthen financing of mathematical research. As we use more computer power and tighten collaboration with science and industry, we need more resources to support the dynamic state of mathematics. Even so, we shall need significantly less than other branches of science, so that the ratio of profit/investment remains highest for mathematics, especially if we make a significant effort to popularize and apply our ideas. So it is important for us to make society well aware of the full potential of mathematical research and of the crucial role of mathematics in near and long-term industrial development.

Appendix 4

Basic Research in Mathematics
Collaboration between Academia and Industry

Working Paper for the Discussion Meeting at
The Isaac Newton Institute on 4 November 1997

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BRIMS, Bristol, England

September 16, 1997

Mathematics is unique among the sciences in the depth and breadth of its contributions to humanity. It provides the infrastructure for all the exact sciences, some of the social sciences and all traditional engineering. This is supported by a corpus of pure mathematical knowledge of elegance, versatility and power[1]. The roots of the subject go back to antiquity but its great flowering took place in parallel with the first industrial revolution and the development of industrialised societies in the eighteenth and nineteenth centuries.

A second industrial revolution is now taking place, fuelled by development in computing, communications, finance and bioscience, as societies evolve from energy-based to information-based. Concomitant political and economic changes are altering the environment for basic research. Governments have changed the criteria for public funding of research, greater emphasis being placed on relevance, technology transfer and wealth creation. Lower defense spending, following the ending of the Cold War, has reduced another major source of research funding. Universities are struggling to prepare their students for a challenging job market while attempting to maintain their commitment to basic research[2]. What is emerging from this are Grand Challenges for mathematical science and new equilibria in the balance of basic research between governments, universities and industry.

Examples of these Grand Challenges include (1) the development of a mathematical infrastructure for computer engineering (2) the provision of universal access to the Internet and its services (3) the physics and engineering of electronic devices as feature

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[1] The abstracts of programmes at the Isaac Newton Institute, available in the Institute's information pack, give an idea of the scope of the subject. For its technological impact in the USA see James Glimm (editor), "*Mathematical Sciences, Technology and Economic Competitiveness*", Board on Mathematical Sciences, National Acadmey Press, 1991.

[2] Congressman George E. Brown, "*Challenges Facing Mathematics in the Twenty-first Century*", Notices of the AMS, 44:576-579; 1997 A. Jackson, "*Downsizing at Rochester*", Notices of the AMS, 43:300-306, 1996; R. S. Rosenbloom and W. J. Spencer, "*The Transformation of Industrial Research*", Issues in Science and Technology, Spring 1996.

sizes shrink to atomic dimensions. These are not merely technological challenges: they require fundamental progress in several areas of mathematical science.
There is a distinguished tradition of industrial mathematics represented by the Society for Industrial and Applied Mathematics (SIAM) and its affiliated organisations[3]. While this will remain vital, the new industrial developments have begun to draw upon new areas of mathematics, including some hitherto regarded as pure (algebraic geometry, number theory, logic, etc), and have begun to throw up problems which lack adequate mathematical formulation in the first place[4].

Sustained collaborative research by engineers, mathematicians and others, both industrial and academic, seems essential to tackle Grand Challenge problems. There is a precedent for such research from the early days of telecommunications: the commercial and engineering challenges of providing universal access to the telephone stimulated fundamental developments in probability theory and other areas of mathematics[5]. The development of the World Wide Web present us with challenges of a similar scale and complexity, if not yet solutions of the same stature.

There are few modern environments in which such challenges can be successfully tackled. Much of the telecommunications work mentioned above was conducted at AT & T Bell Laboratories in a monopoly commercial environment which no longer exists[6]. Industry has only recently begun to experiment with new environments which attempt to merge academic and industrial philosophies and modes of operation[7].

The lack of organisational structures in matched by a lack of people. Many of the Grand Challenges of the second industrial revolution are not on academic research agendas or course curricula and there are consequently few people motivated to study them. Industry has been slow in articulating these challenges and in clarifying their intellectual depth and difficulty, a precondition for attracting the brightest academic talents. In recent years funding mechanisms have emerged which directly encourage collaboration in basic research[8]. Despite this development, the idea of collaborative research remains

[3] "*SIAM Report on Mathematics in Industry*", available from the Society for Industrial and Applied Mathematics; J. R. Ockendon, "*The Moving Interface between Mathematics and Industry*", Proceedings ICIAM-95.
[4] We know how to predict the stress in a girder but not the minimum redundancy needed to achieve a specified MTBF in a computer system, a problem first articulated by von Neumann but still awaiting an appropriate theoretical framework: J. von Neumann, "*Probabilistic logics and the synthesis of reliable organisms from unreliable components*", in C. E. Shannon and J. McCarthy (editors), Automata Studies, PUP, 1956.
[5] By, among others, Bode, Erlang, Heaviside, Kolmogorov, Nyquist, Rice, Shannon and Wiener: S. Millman (editor), "*A History of Engineering and Science in the Bell System: Communications Sciences* (1925-80), AT & T Bell Laboratories, 1984.
[6] S. Millman, op cit.
[7] Such as, Hewlett-Packard's BRIMS, Microsoft's Theory Group and Cambridge Research Laboratory and NEC's Princeton Research Institute.
[8] For example, NSF Mathematical Sciences Postdoctoral Industrial Fellowships, the UK's Realising our Potential Awards Scheme (ROPAs) and Royal Society Industry Fellowships.

uncomfortable to many on both sides of the academic-industrial divide. It is not always career-enhancing for an academic mathematician to spend time in industry. Postdoctoral students are particularly vulnerable to this pressure, which is unfortunate, since they are at a stage when the stimulation of new ideas could have the greatest impact. Mobility across the academic-industrial divide remains sluggish.

It is used to be taken for granted that mathematics was vital to society. This could be attributed, at least in part, to its immense contribution to the first industrial revolution. In recent times, in keeping with much else, the subject's role in society has come under searching scrutiny[9]. If it is to maintain its accustomed status in the future, can it afford not to be at the forefront of the second industrial revolution?

[9] Mathematics is not mentioned as a core area in "*Towards the Fifth Framework Programme: Scientific and Technological Objective*", European Commission Working Paper COM(97)47.

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