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Micromagnetic Dynamics in Thin Films

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Web Pages

- Home Page:

  http://math.nist.gov/~MDonahue/

- OOMMF:

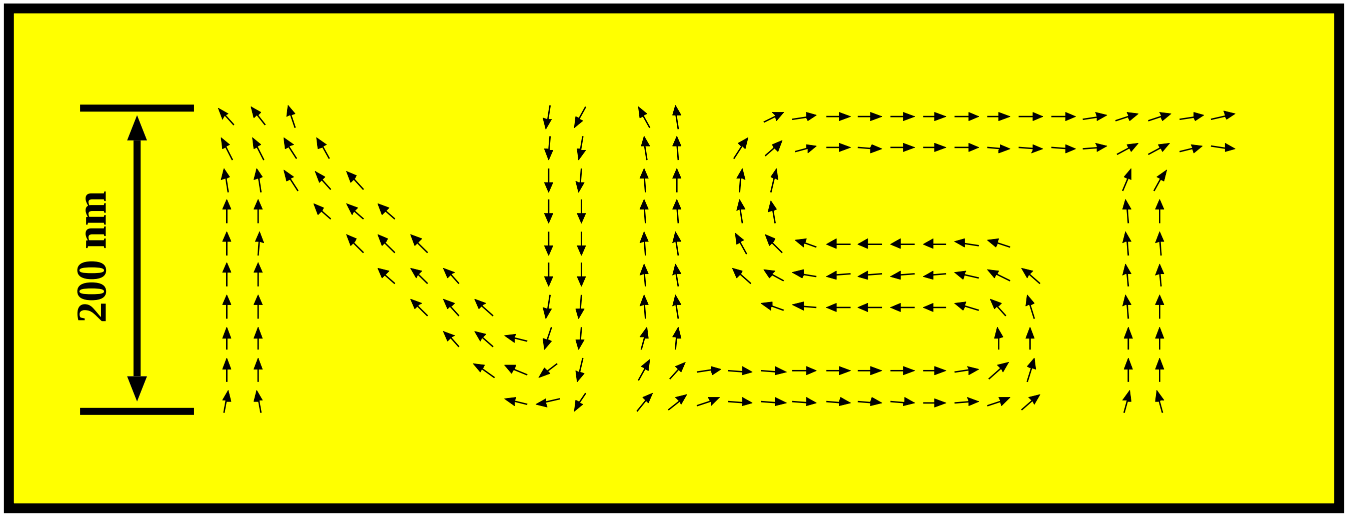
  http://math.nist.gov/oommf/

- μMAG:

  http://www.ctcms.nist.gov/~rdm/mumag.org.html

Outline

- Background
- μMAG Standard Problems
- High Speed Switching (Std. Prob. 4)
- Dynamics Near Quasi-Static Coercive Field
- Dynamics at Remanence



Micromagnetics is the study, modeling and simulation of magnetic materials and their behavior at the sub-micron scale.

Micromagnetic Equations

Landau-Lifshitz-Gilbert:

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\mathrm{eff}} - \frac{\gamma\,\alpha}{M_{\mathrm{s}}} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\mathrm{eff}})$$

$$\mathbf{H}_{\mathrm{eff}} = -\frac{1}{\mu_0} \frac{\partial E_{\mathrm{density}}}{\partial \mathbf{M}}$$

Energies:

$$E_{\mathrm{total}} = E_{\mathrm{exchange}} + E_{\mathrm{anisotropy}} + E_{\mathrm{Zeeman}} + E_{\mathrm{demag}}$$

Micromagnetic Energy Equations

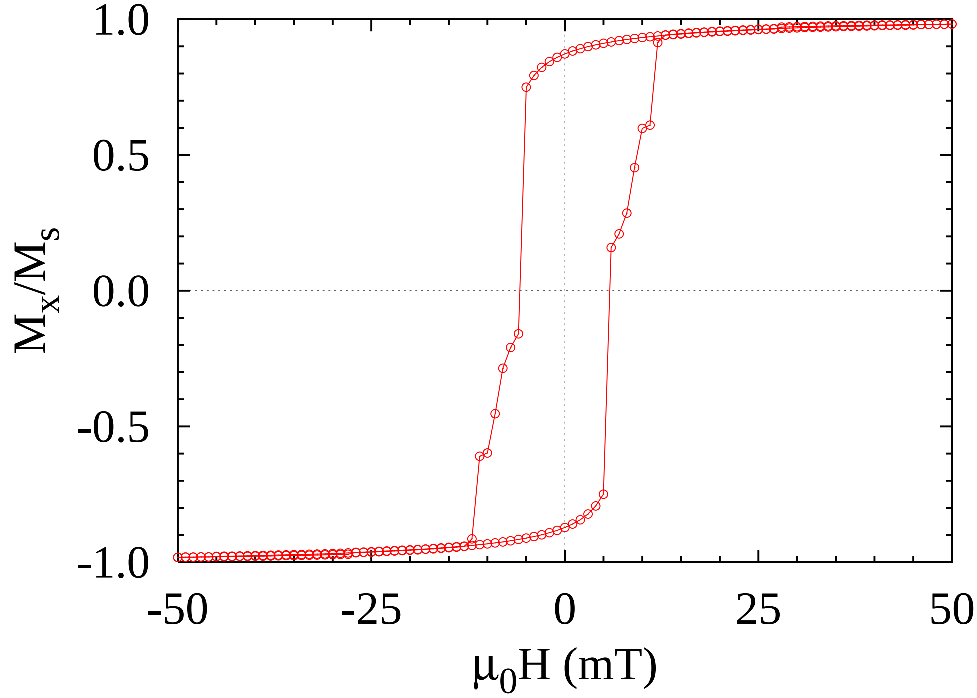
$$
\begin{aligned}
E_{\text{exchange}} &= \int_V \frac{A}{M_s^2} \left(|\nabla M_x|^2 + |\nabla M_y|^2 + |\nabla M_z|^2 \right) d^3 r \\
E_{\text{anisotropy}} &= -\int_V \frac{K_1}{M_s^2} (\mathbf{M} \cdot \mathbf{u})^2 \, d^3 r \\
E_{\text{Zeeman}} &= -\mu_0 \int_V \mathbf{M} \cdot \mathbf{H}_{\text{applied}} \, d^3 r \\
E_{\text{demag}} &= -\frac{\mu_0}{2} \int_V \mathbf{M} \cdot \mathbf{H}_{\text{demag}} \, d^3 r
\end{aligned}
$$

$$
\text{where} \quad \mathbf{H}_{\text{demag}}(r) = -\frac{1}{4\pi} \int_V \nabla \cdot \mathbf{M}(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \, d^3 r' + \frac{1}{4\pi} \int_S \hat{\mathbf{n}} \cdot \mathbf{M}(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \, d^2 r'
$$

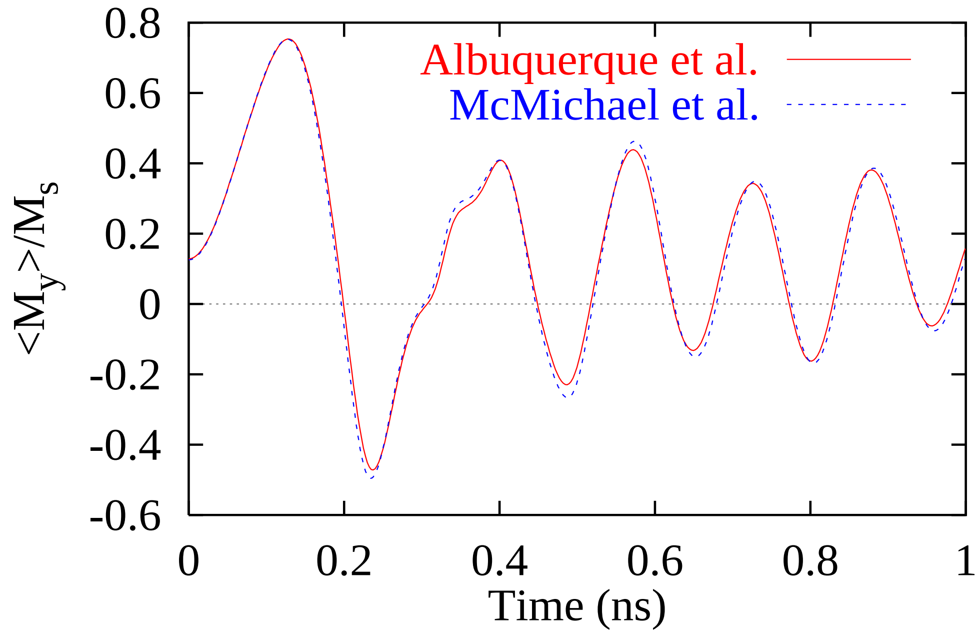
Comparative Material Constants

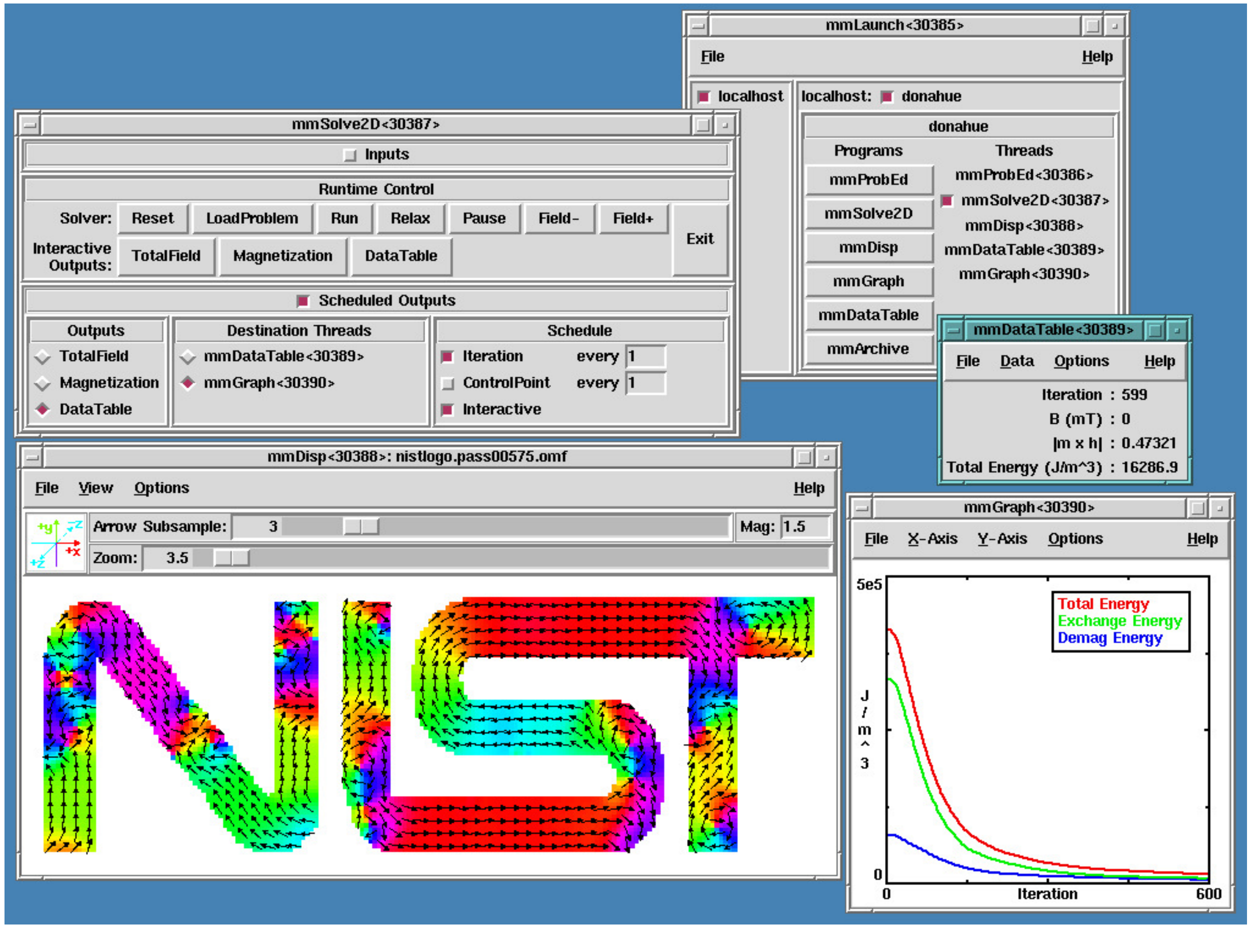
Material	M_s (kA/m)	K_1 (kJ/m^3)	A (pJ/m)	l_{ex} (nm)
Co	1400	520	30	4.9
Fe	1700	48	21	3.4
Ni	490	-5.7	9	7.7
Permalloy	860	0	13	5.3
$Nd_2Fe_{14}B$	1280	4500	13	3.5

Hysteresis Loops (quasi-static)



Time Series (dynamic)

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Magnetization
DataTable
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ControlPoint every 1
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|m x h| : 0.47321
Total Energy (J/m^3) : 16286.9
mmDisp<30388>: nistlogo.pass00575.omf
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Arrow Subsample: 3
Mag: 1.5
Zoom: 3.5
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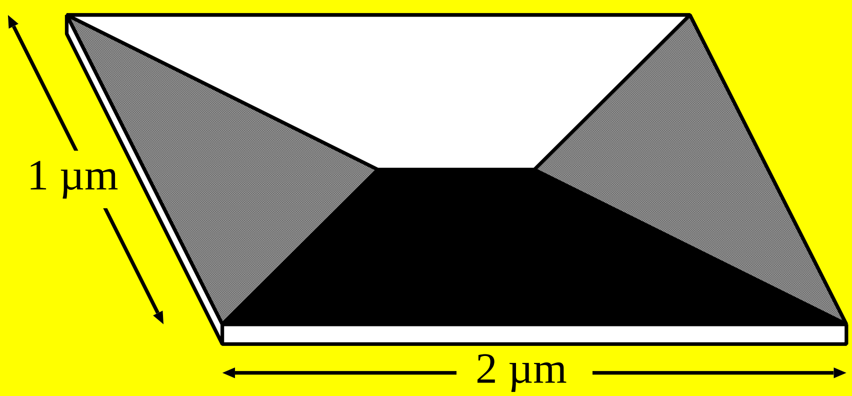
File X-Axis Y-Axis Options Help
5e5
0
J/m^3
Total Energy
Exchange Energy
Demag Energy
0
Iteration
600

Public Code

μMAG Problem #1
Specification

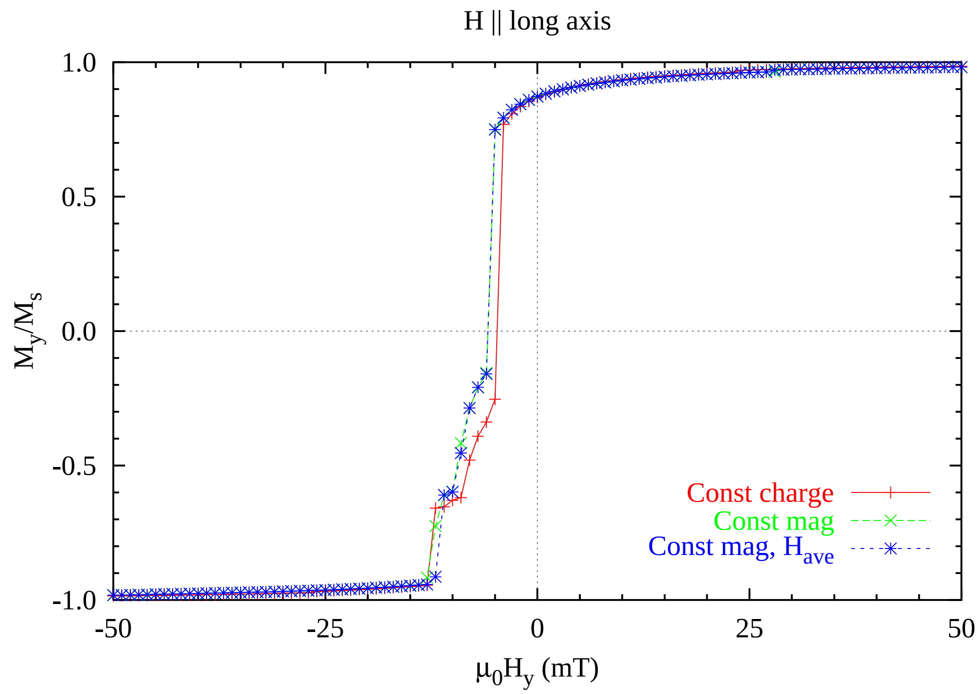
"Permalloy" rectangle

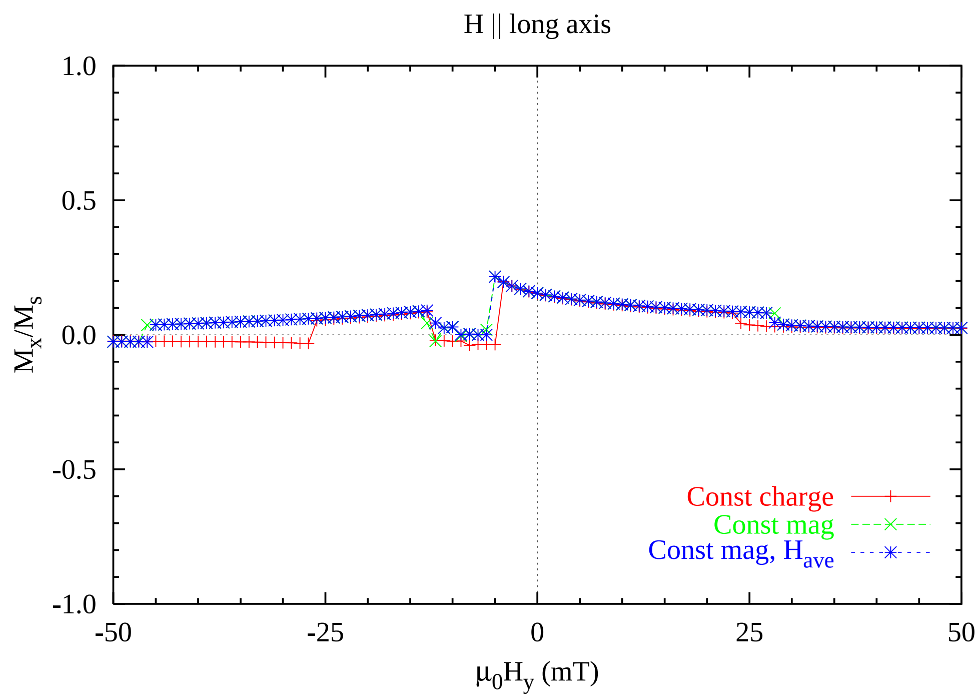


20 nm thick

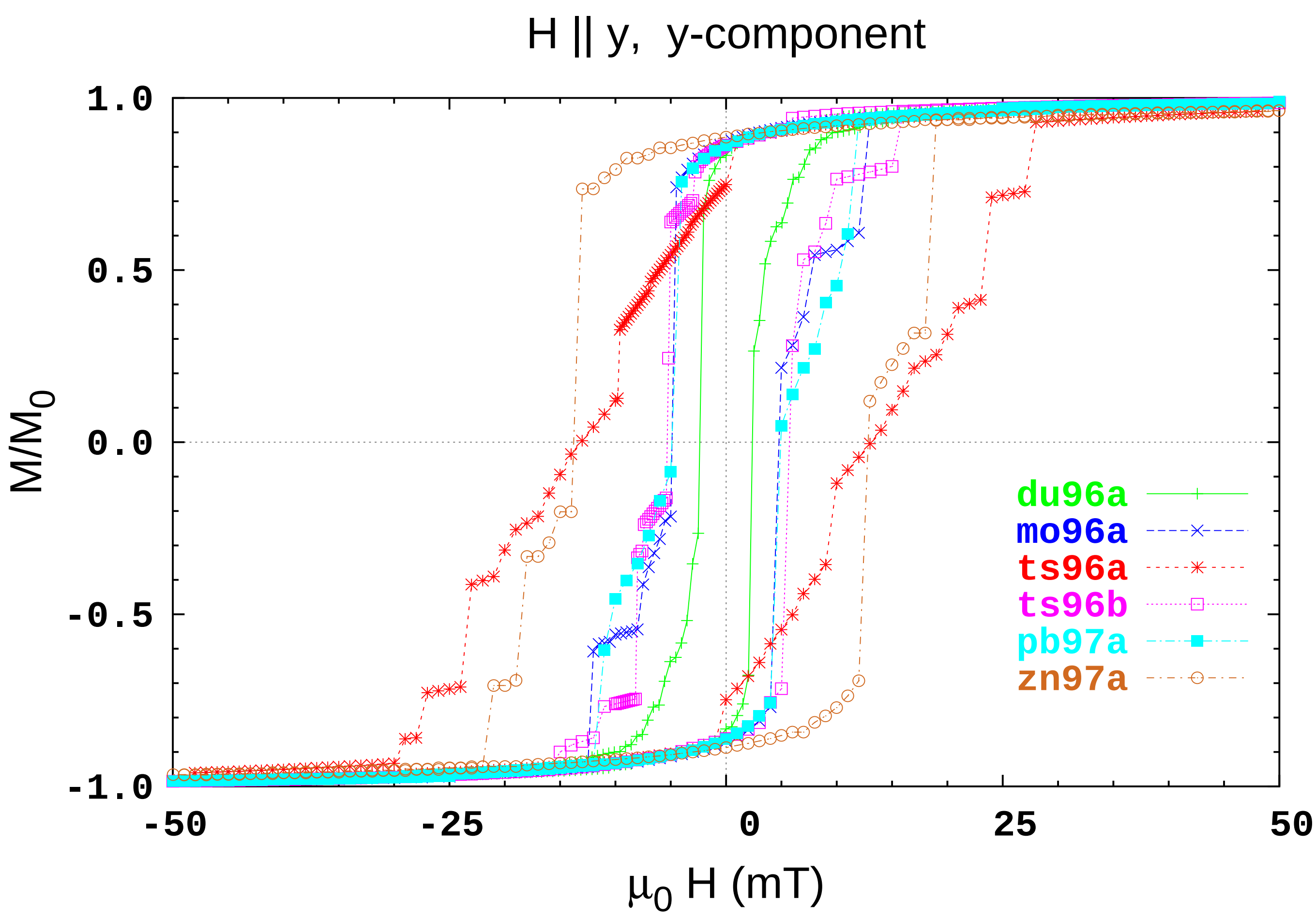
A = 1.3×10^{-11} J/m (1.3×10^{-6} erg/cm)
M = 8.0×10^{5} A/m (800 emu/cm^3)
K = 500 J/m^3 (5000 erg/cm^3)

μMAG Problem #1
OOMMF Results (20 nm cells)

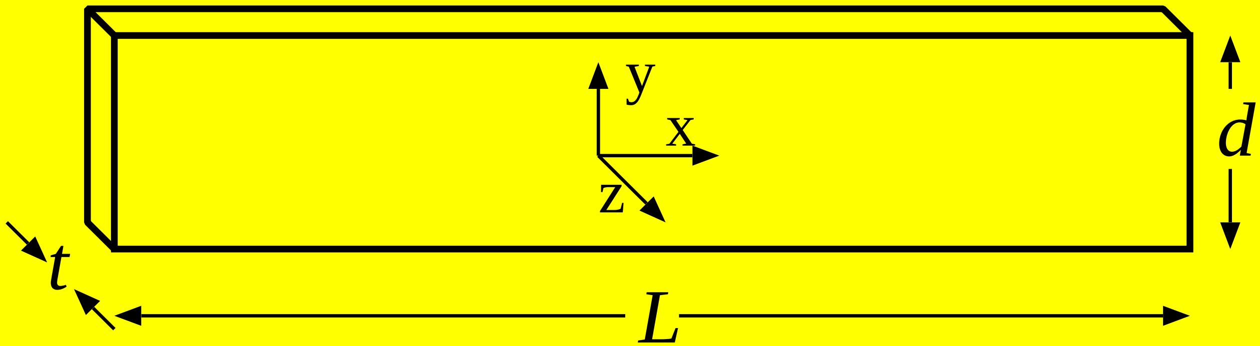




μMAG Problem #1 Results



μMAG Problem #2 Specification

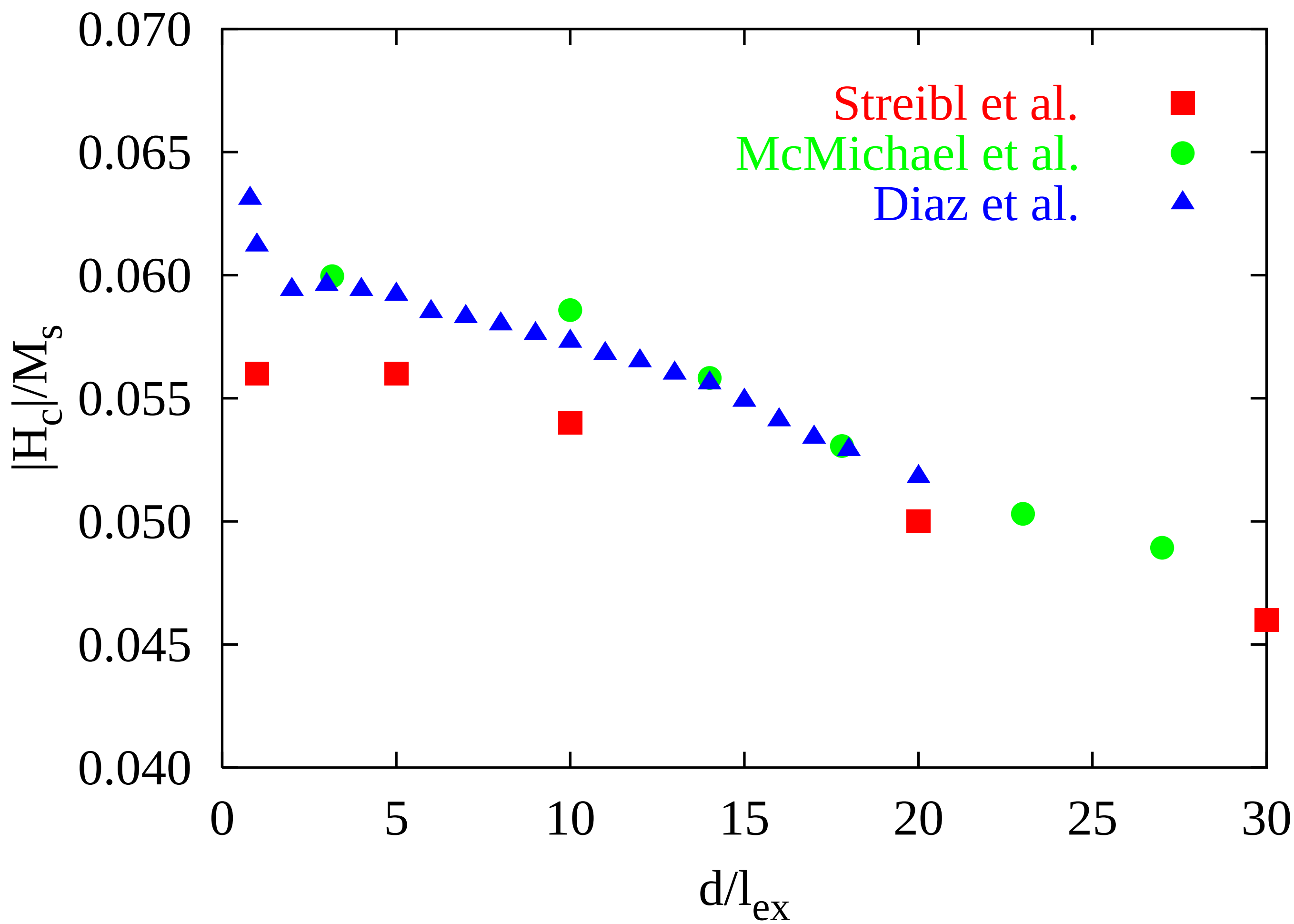


$L/d = 5.0, t/d = 0.1$

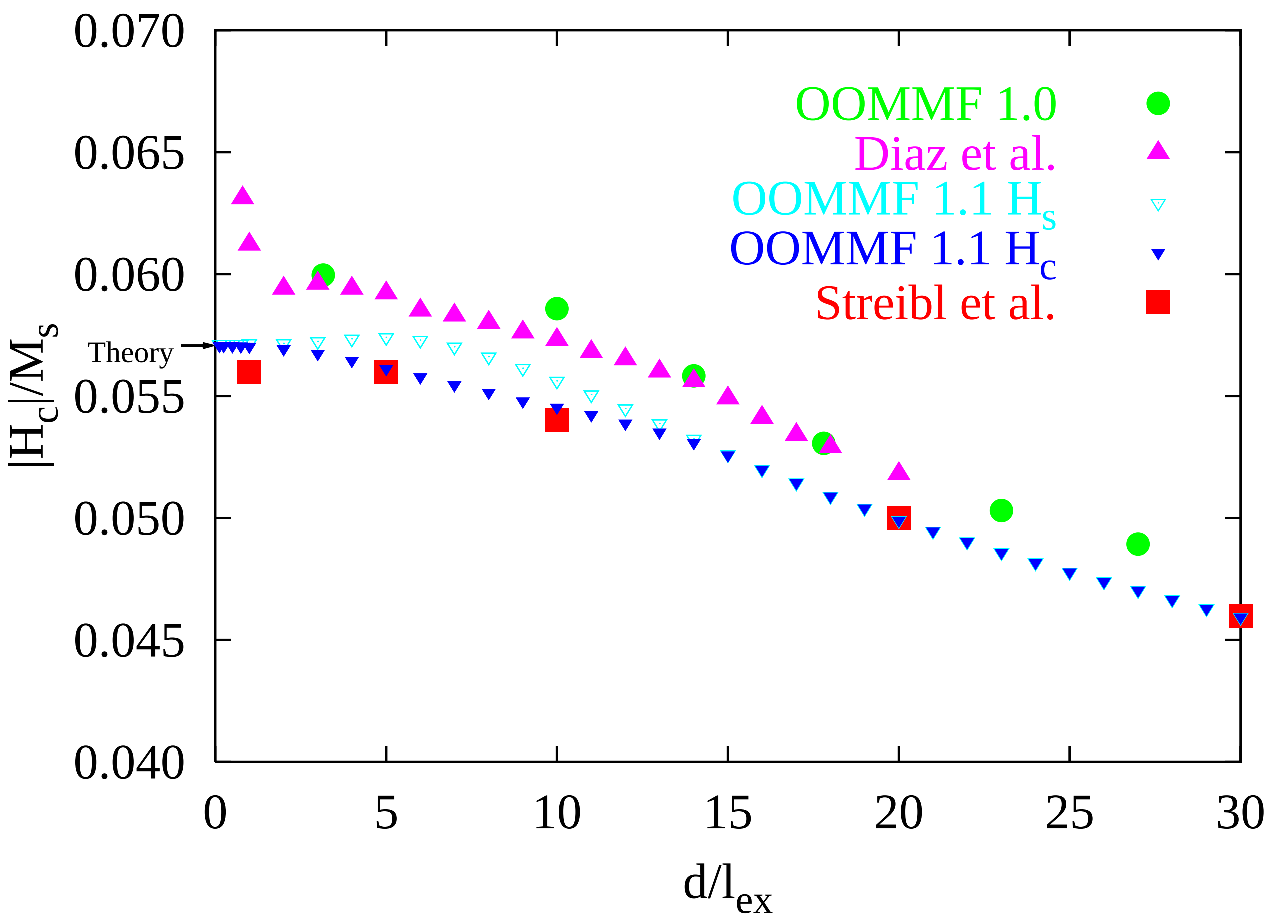
Exchange length $l_{ex} = (A/K_m)^{1/2}$

Mag. energy density $K_m = \mu_o M_s^2 / 2$

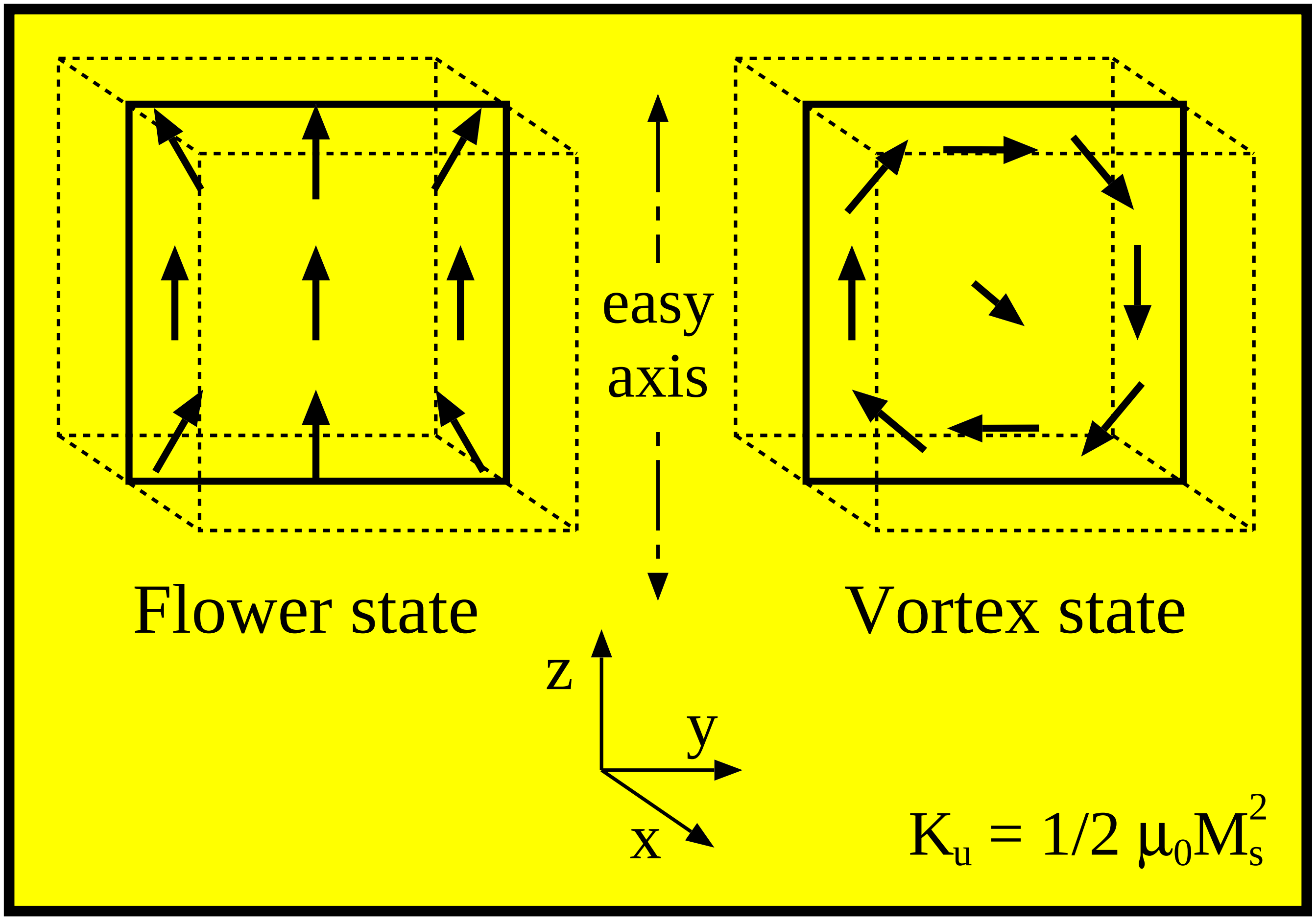
μMAG Problem #2, Coercivities



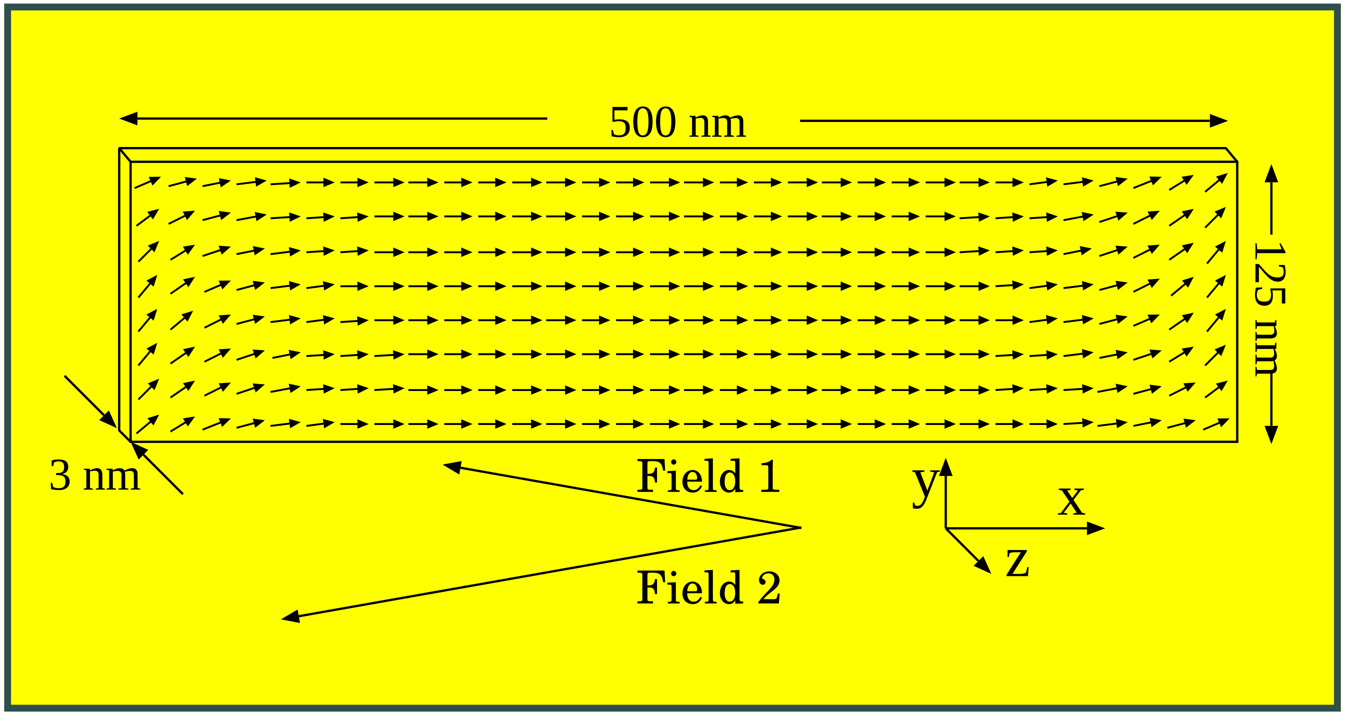
μMAG Problem #2, Coercivities



μMAG Problem #3 Specification



μMAG Problem #4 Specification

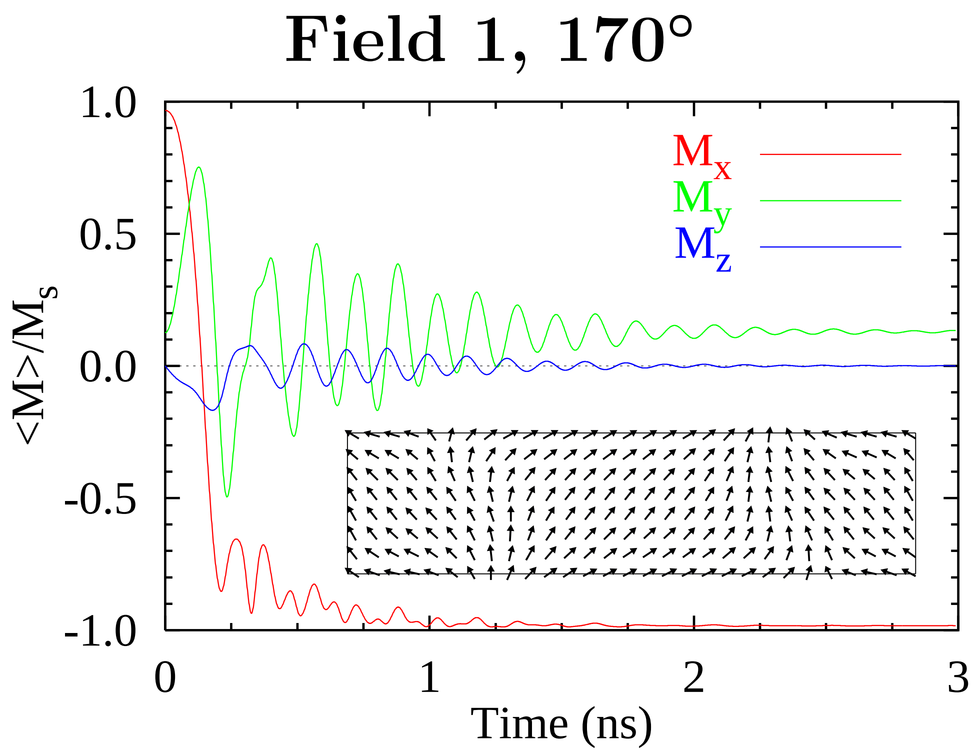


Permalloy parameters

Field 1: $\mu_0 H = 25$ mT at 170°

Field 2: $\mu_0 H = 36$ mT at 190°

(Fields are approximately 1.5 times the quasi-static coercivity.)


Field 1, 170°
1.0
0.5
0.0
-0.5
-1.0
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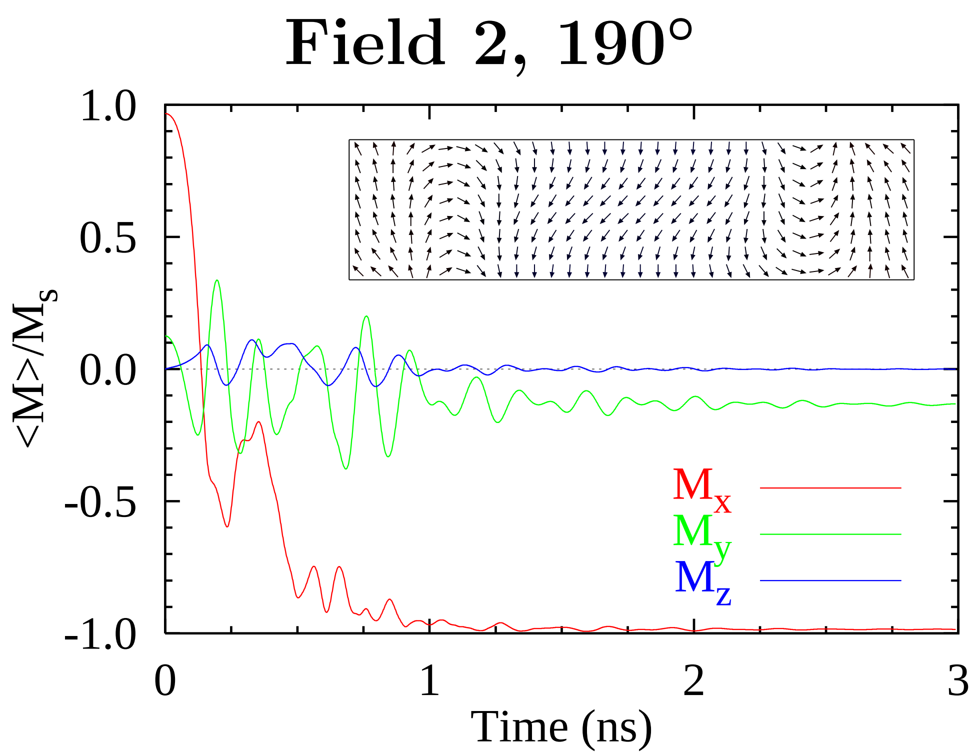
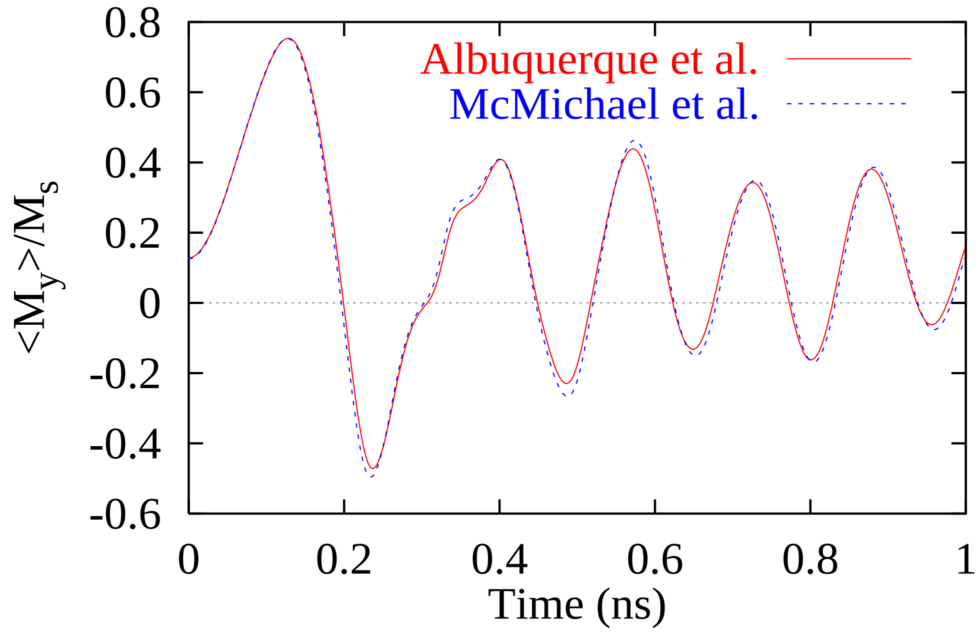
0
1
2
3
Time (ns)
M_x
M_y
M_z

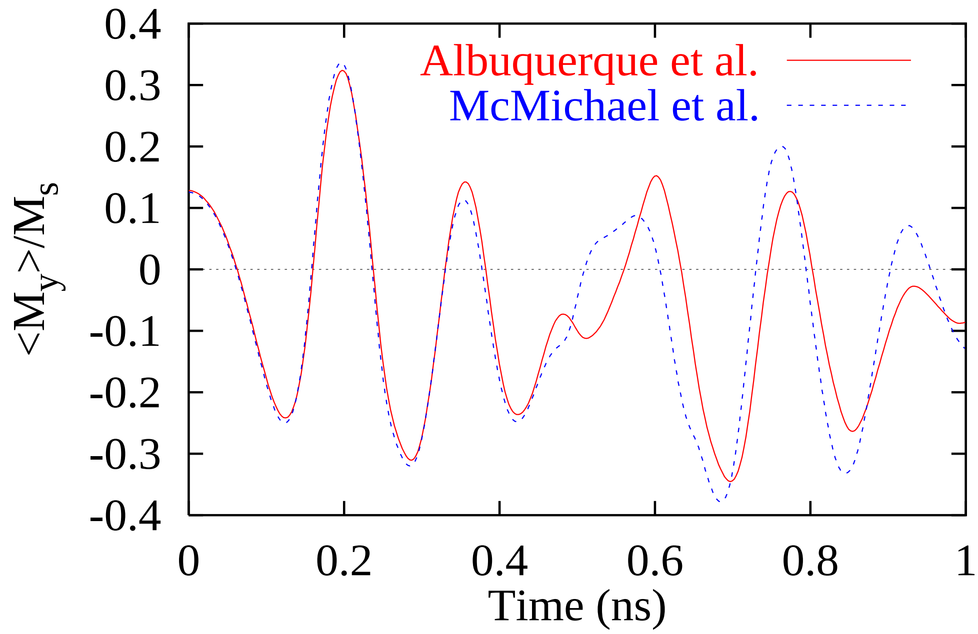

Field 2, 190°
1.0
0.5
0.0
-0.5
-1.0
<M>/M_s
0
1
2
3
Time (ns)
M_x
M_y
M_z

Prob. 4 Comparisons

Field 1 (170°)



Field 2 (190°)



Max angle data

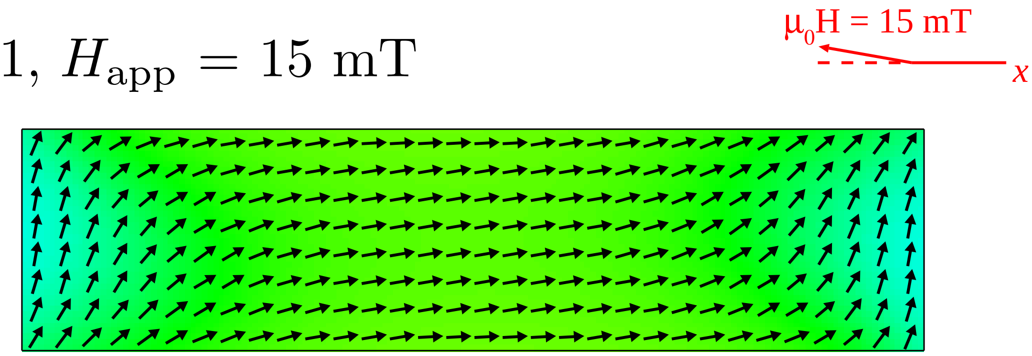
- Field 1, 2.5 nm cells, 8-dot exchange: 22.3°
- Field 2, 3.125 nm cells, 8-dot exchange: 118°
- Field 2, 2.5 nm cells, 8-dot exchange: 71.9°
- Field 2, 3.125 nm cells, 4-ang exchange: 48.1°

Exchange reference:

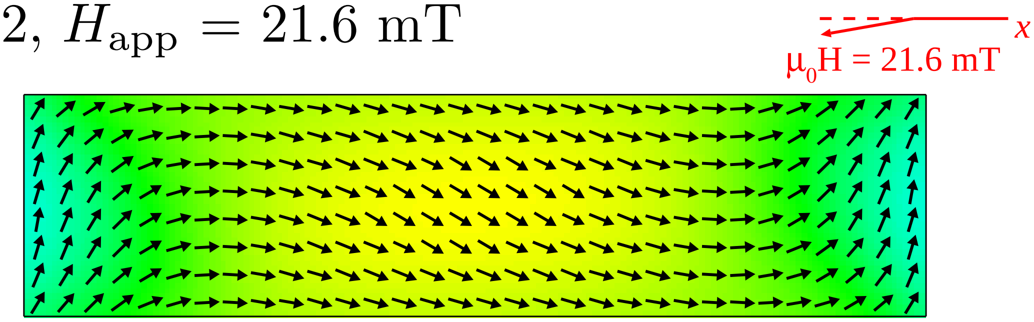
> **Exchange Energy Representations in Computational Micromagnetics,** M. J. Donahue and R. D. McMichael, *Physica B,* **233**, pp 272-278 (1997).

Quasi-Static Equilibria

Field 1, $H_{\text{app}} = 15$ mT



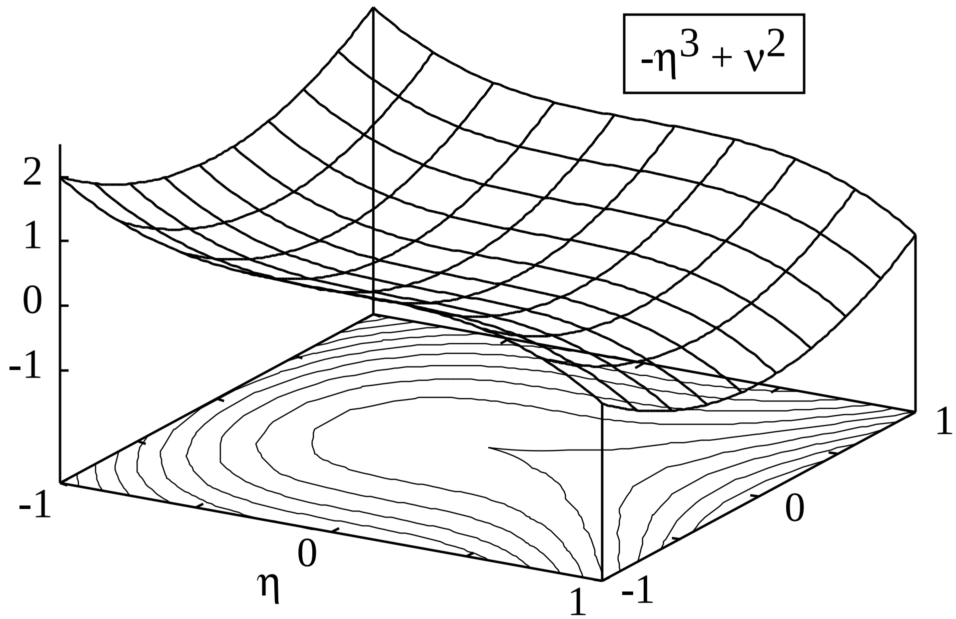
Field 2, $H_{\text{app}} = 21.6$ mT



Single Spin: Energy Surface

At critical field $\mathbf{H}_c$ assume

$$E_c = -U\eta^3 + K\nu^2$$



Let $\mathbf{h} = \mathbf{H} - \mathbf{H}_c$, then

$$\begin{aligned} E &= -U\eta^3 + K\nu^2 \\ &\quad -\mu_0 M\left[h_\eta \eta + h_\nu \nu + h_\zeta\left(1 - \frac{1}{2}\eta^2 - \frac{1}{2}\nu^2\right)\right] \end{aligned}$$

and

$$\eta_{\text{eq}} = \frac{\mu_0 M}{6U}\left[h_\zeta - \sqrt{h_\zeta^2 - \frac{12Uh_\eta}{\mu_0 M}}\right].$$

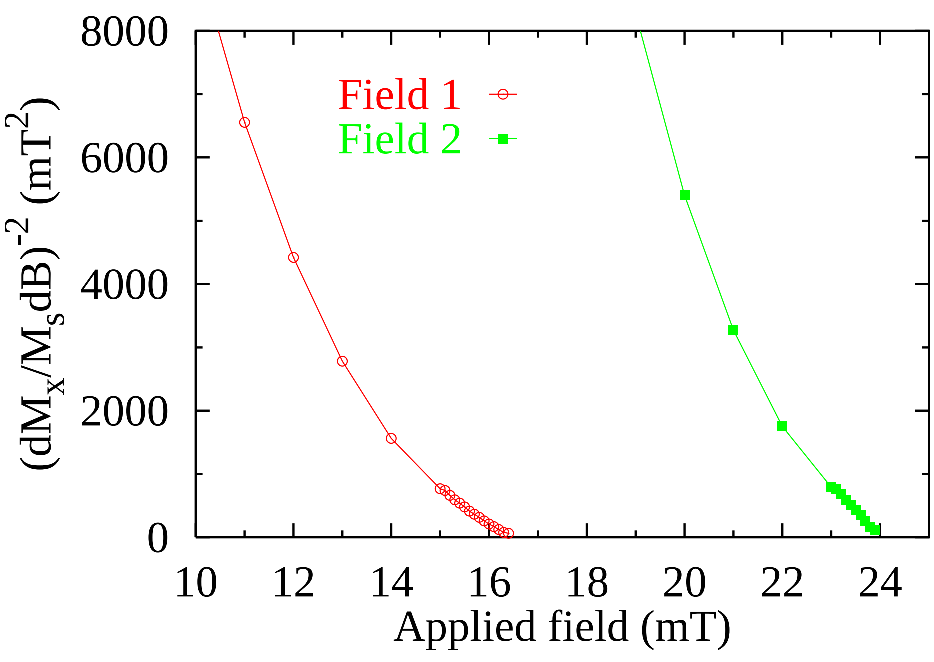
Critical Field Susceptibility

Divergent susceptibility $\chi_{\eta,\eta}$ is given by

$$\chi_{\eta,\eta} = M\frac{\partial \eta_{\rm eq}}{\partial h_\eta} = M\left[h_\zeta^2 - \frac{12U}{\mu_0 M}h_\eta\right]^{-1/2}.$$

If $h_\zeta^2 \ll 12Uh_\eta/\mu_0 M$, then

$$\chi_{\eta,\eta} \propto h_\eta^{-1/2} \qquad \text{as } |\mathbf{h}| \downarrow 0.$$



See also: A. Hubert and W. Rave, *Physica Status Solidi B*, **211**, pp 815–829 (1999).

Ring Down Frequency

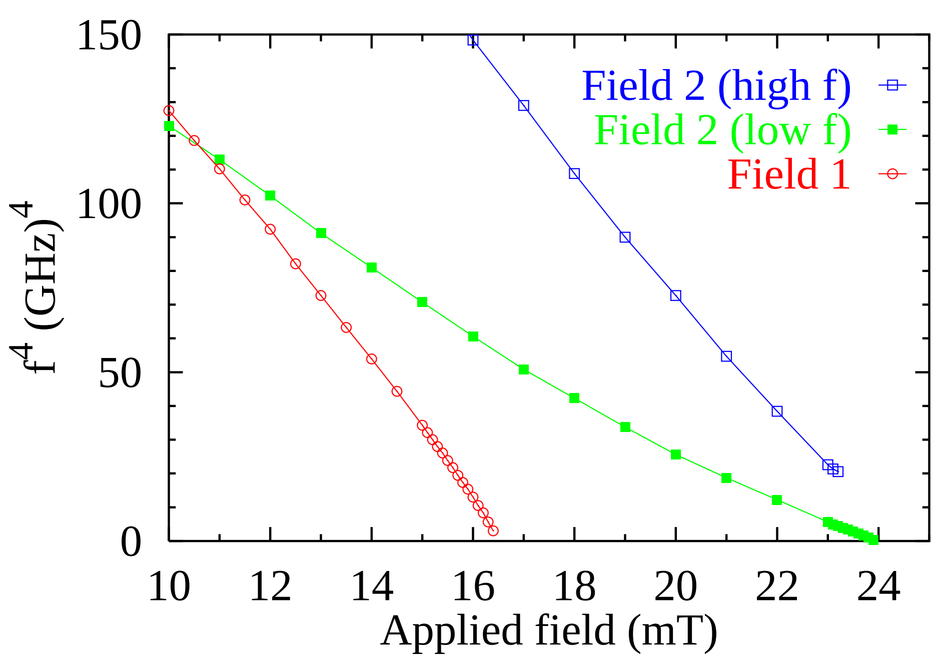
Precession frequency f is given by

$$f = \frac{\gamma}{2\pi\mu_0 M}\left[\frac{\partial^2 E}{\partial\eta^2}\frac{\partial^2 E}{\partial\nu^2} - \left(\frac{\partial^2 E}{\partial\eta\partial\nu}\right)^2\right]^{1/2}\Bigg|_{\eta_{\rm eq}\nu_{\rm eq}}$$

$$= \frac{\gamma}{2\pi}\left[h_\zeta + \frac{2K}{\mu_0 M}\right]^{1/2}\left[h_\zeta^2 - \frac{12U}{\mu_0 M}h_\eta\right]^{1/4}.$$

If $h_\zeta^2 \ll 12Uh_\eta/\mu_0 M$, then

$$f \propto h_\eta^{1/4} \qquad \text{as } |\mathbf{h}| \downarrow 0.$$



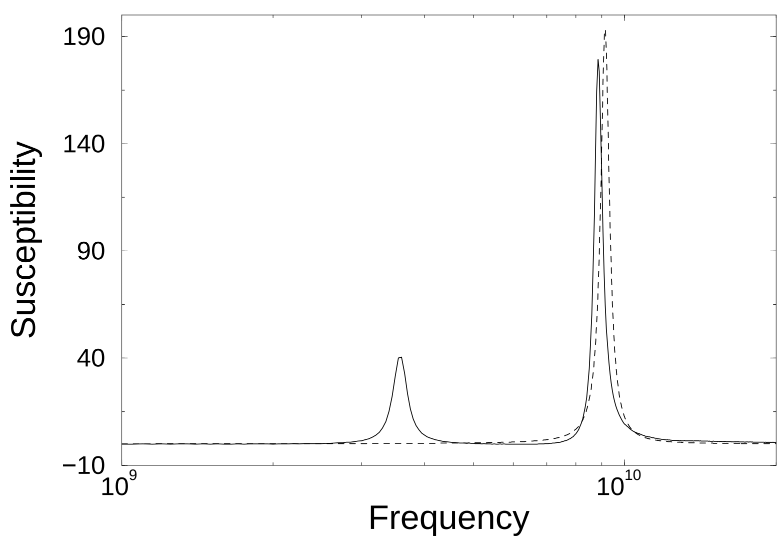
High Frequency Susceptibility

Define susceptibility χ by

$$< \mathbf{M}(t) > \cdot \mathbf{u} = \int_{-\infty}^{+\infty} \chi(t-t')[\mathbf{h}(t') \cdot \mathbf{u}]dt',$$

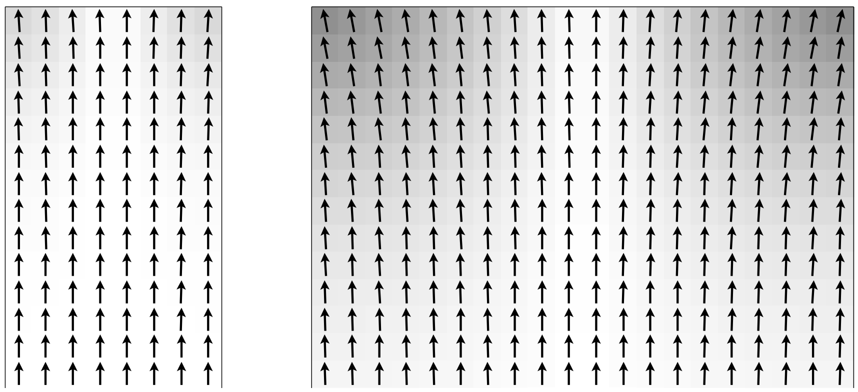
or in Fourier domain

$$M(t) = \chi(t) \star h(t) \leftrightarrow M(\omega) = \chi(\omega) \cdot h(\omega).$$



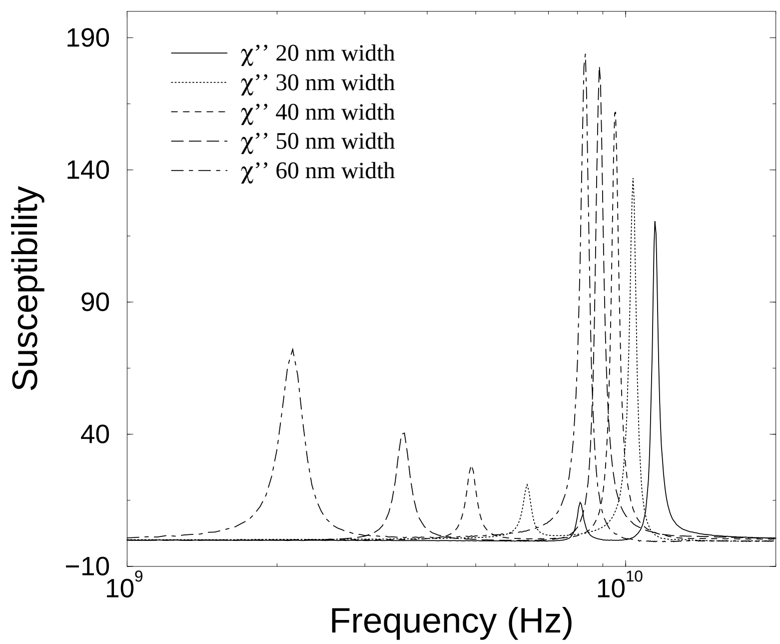
Permalloy stripe, 1 μm $\times$ 50 nm $\times$ 5 nm.
Solid line is micromagnetic simulation.

Stripe Width Effects

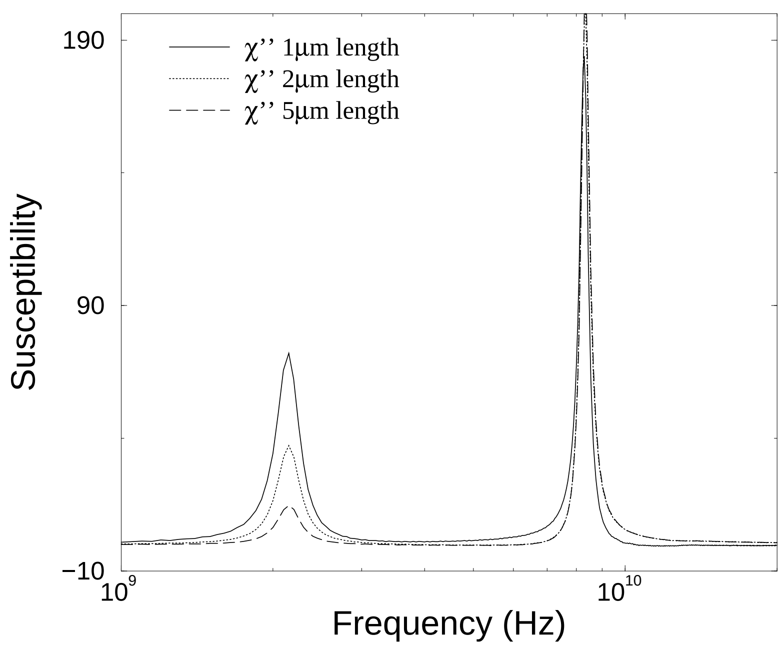


20 nm

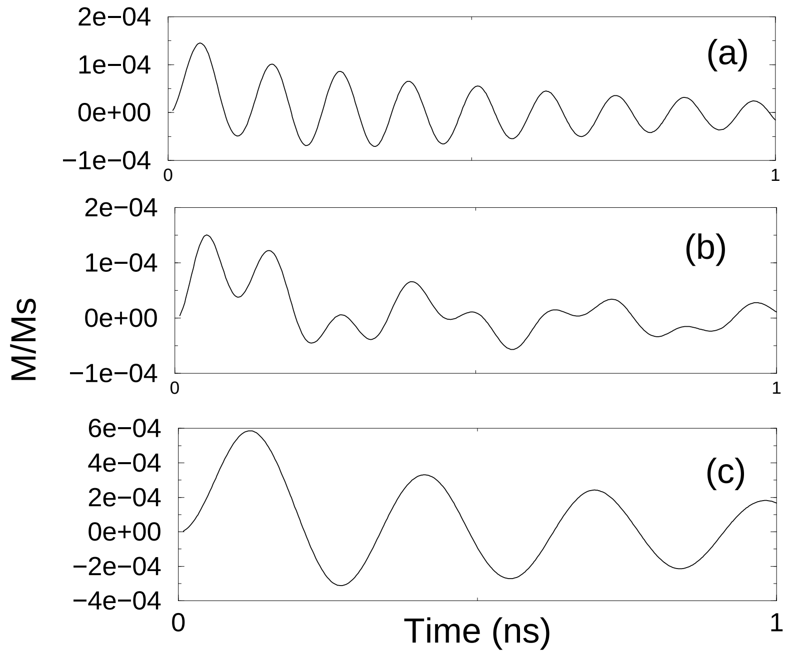
50 nm



Stripe Length Effects



Individual Spin Dynamics

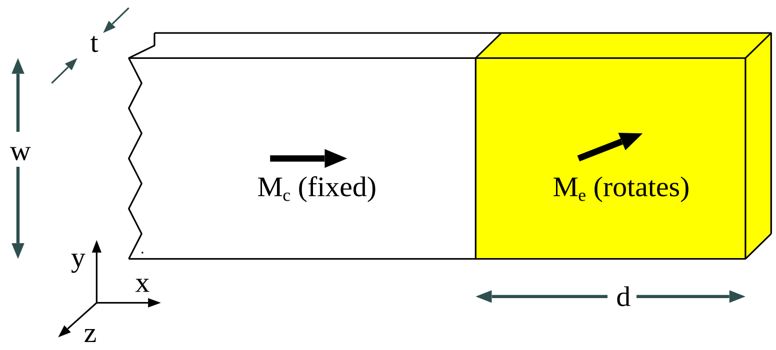


(a) Center spin

(b) Intermediate spin

(c) End spin

Simple Edge Spin Model



$$H_{\text{exch}} = \left(2A/(\mu_0 M_s d^2), 0, 0\right)$$
$$H_{\text{demag}} = (-N_x M_x/2, -N_y M_y, -N_z M_z)$$

Then

$$\text{Freq} = \frac{\gamma M_s}{2\pi}\sqrt{\left(\frac{2A}{\mu_0 M_s^2 d^2} - \frac{N_x}{2} + N_y\right)\left(\frac{2A}{\mu_0 M_s^2 d^2} - \frac{N_x}{2} + N_z\right)}$$

where

$$d \approx 4\, l_{\text{ex}} = 4\sqrt{2A/\mu_0 M_s^2}$$

Simple Model Results

Part width	Frequency (GHz)
20	9.6
30	7.8
40	6.5
50	5.4
60	4.5

Conclusions

- μMAG Problem 4 exhibits 2 reversal modes:

  $\boldsymbol{H}_{\mathbf{app,170^\circ}}$: End domains propagate towards center.

  $\boldsymbol{H}_{\mathbf{app,190^\circ}}$: Counter-rotating domains, 360° walls, complex dynamics.

- Near critical field H_c:
  - $\chi \propto (H_c - H)^{-1/2}$ for small range of H.
  - $f \propto (H_c - H)^{1/4}$ for wider range of H.

- Remanence dynamics:
  - Distinct central core and end domain behavior.
  - End domain frequency especially dependent on stripe width.

References

Micromagnetics of the Dynamic Susceptibility for Coupled Permalloy Stripes, O. Gérardin, J. Ben Youssef, H. Le Gall, N. Vukadinovic, P. M. Jacquart and M. J. Donahue, to appear in *Journal of Applied Physics.*

Behavior of muMAG Standard Problem No. 2 in the Small Particle Limit, M. J. Donahue, D. G. Porter, R. D. McMichael and J. Eicke, *Journal of Applied Physics,* **87**, pp 5520–5522 (2000).

Domain Wall Traps for Low-Field Switching of Sub-Micron Elements, R. D. McMichael, J. Eicke, M. J. Donahue and D. G. Porter, *Journal of Applied Physics,* **87**, pp 7058–7060 (2000).

High-Speed Dynamics, Damping, and Relaxation Times in Submicrometer Spin-Valve Devices, S. E. Russek, S. Kaka and M. J. Donahue, *Journal of Applied Physics,* **87**, pp 7070–7072 (2000).

OOMMF User's Guide, Version 1.0, M. J. Donahue and D. G. Porter, Interagency Report **NISTIR 6376,** National Institute of Standards and Technology, Gaithersburg, MD (Sept 1999).

Comparison of Magnetostatic Field Calculation Methods on 2-D Square Grids as Applied to a Micromagnetic Standard Problem, R. D. McMichael, M. J. Donahue, D. G. Porter and J. Eicke, *Journal of Applied Physics,* **85**, pp 5816–5818 (1999).

A Variational Approach to Exchange Energy Calculations in Micromagnetics, M. J. Donahue, *Journal of Applied Physics,* **83**, pp 6491–6493 (1998).

Head to head domain wall structures in thin magnetic strips, R. D. McMichael and M. J. Donahue, *IEEE Trans. Mag.*, **33**, pp 4167–4169 (1997).

Spin Reorientation Transitions and Domain Structure in Magnetic Multilayers, V. I. Nikitenko, L. M. Dedukh, V. S. Gornakov, Yu. P. Kabanov, L. H. Bennett, M. J. Donahue, L. J. Swartzendruber, A. J. Shapiro, H. J. Brown, *IEEE Trans. Mag.*, **33**, pp 3661–3663 (1997).

Exchange Energy Representations in Computational Micromagnetics, M. J. Donahue and R. D. McMichael, *Physica B*, **233**, pp 272–278 (1997).

Investigation of Domain Wall Formation and Motion in Magnetic Multilayers, L. H. Bennett, M. J. Donahue, A. J. Shapiro, H. J. Brown, V. S. Gornakov and V. I. Nikitenko, *Physica B*, **233**, pp 356–364 (1997).

Rates of Convex Approximation in Non-Hilbert Spaces, M. J. Donahue, L. Gurvits, C. Darken, and E. Sontag, *Constructive Approximation,* **13**, pp 187–220 (1997).

Experimental Study of Magnetization Reversal Processes in Nonsymmetric Spin Valve, V. S. Gornakov, V. I. Nikitenko, L. H. Bennett, H. J. Brown, M. J. Donahue, et al., *Journal of Applied Physics,* **81**, pp 5215–5217 (1997).

Magneto-Optical Indicator Film Study of the Magnetization of a Symmetric Spin Valve, V. I. Nikitenko, V. S. Gornakov, L. H. Bennett, P. J. Chen, et al., *IEEE Trans. Mag.,* **32**, pp 4639–4641 (1996).

Complementary Imaging of Granular Co-Ag Films with Magneto-Optical Indicator Film Technique and Magnetic Force Microscopy, M. J. Donahue, L. H. Bennett, V. I. Nikitenko et al., *Journal of Applied Physics,* **79**, pp 5315–5317 (1996).

Artifacts in Magnetic Resonance Imaging from Metals, L. H. Bennett, P. S. Wang and M. J. Donahue, *Journal of Applied Physics,* **79**, pp 4712–4714 (1996).

Additional References

A Generalization of the Demagnetizing Tensor for Nonuniform Magnetization, A. J. Newell, W. Williams, and D. J. Dunlop, *J. Geophysical Research*, **98**, pp 9551–9555 (1993).

Systematic Analysis of Micromagnetic Switching Processes, A. Hubert and W. Rave, *Physica Status Solidi B*, **211**, pp 815–829 (1999).

Web Pages

- Home Page:

  http://math.nist.gov/~MDonahue/

- OOMMF:

  http://math.nist.gov/oommf/

- μMAG:

  http://www.ctcms.nist.gov/~rdm/mumag.org.html