Exploiting Effective Field Time Derivative Information to Improve Accuracy of a Norm-preserving Landau-Lifshitz Solver

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Background

- For fixed H, the Landau-Lifshitz equation

(1) $$\dot{m} = \frac{|\gamma|}{1+\alpha^2} H \times m + \frac{\alpha|\gamma|}{1+\alpha^2} m \times H \times m$$

has analytical solution.

- In spherical coordinates based on H and initial m,

(2) $$\phi(t) = |\gamma H| t$$

(3) $$\theta(t) = 2\tan^{-1}(\tan(\frac{\theta(0)}{2})\exp(-|\alpha\gamma H|t))$$

- While H remains fixed, exact trajectory $m(t)$ can be computed for any time step.

Semi-analytical Solution Technique

- Apply analytical solution only over time steps small enough that fixed H assumption remains an acceptable approximation.
- Computed trajectories satisfy $|m| = 1$.
- No renormalization scheme required.
- Naturally avoids errors in energy computations, dissipation rates, etc. that renormalization schemes can introduce.
- Semi-analytical step extends to predictor-corrector scheme.

Ben Van de Wiele, Femke Olyslager, and Luc Dupré, "Fast semianalytical time integration schemes for the Landau-Lifshitz equation", *IEEE Trans. Magn.*, vol. 43, no. 6, pp. 2917–2919, June 2007.

Semi-analytical predictor/corrector

Step 1a: Compute H_0 at m_0.

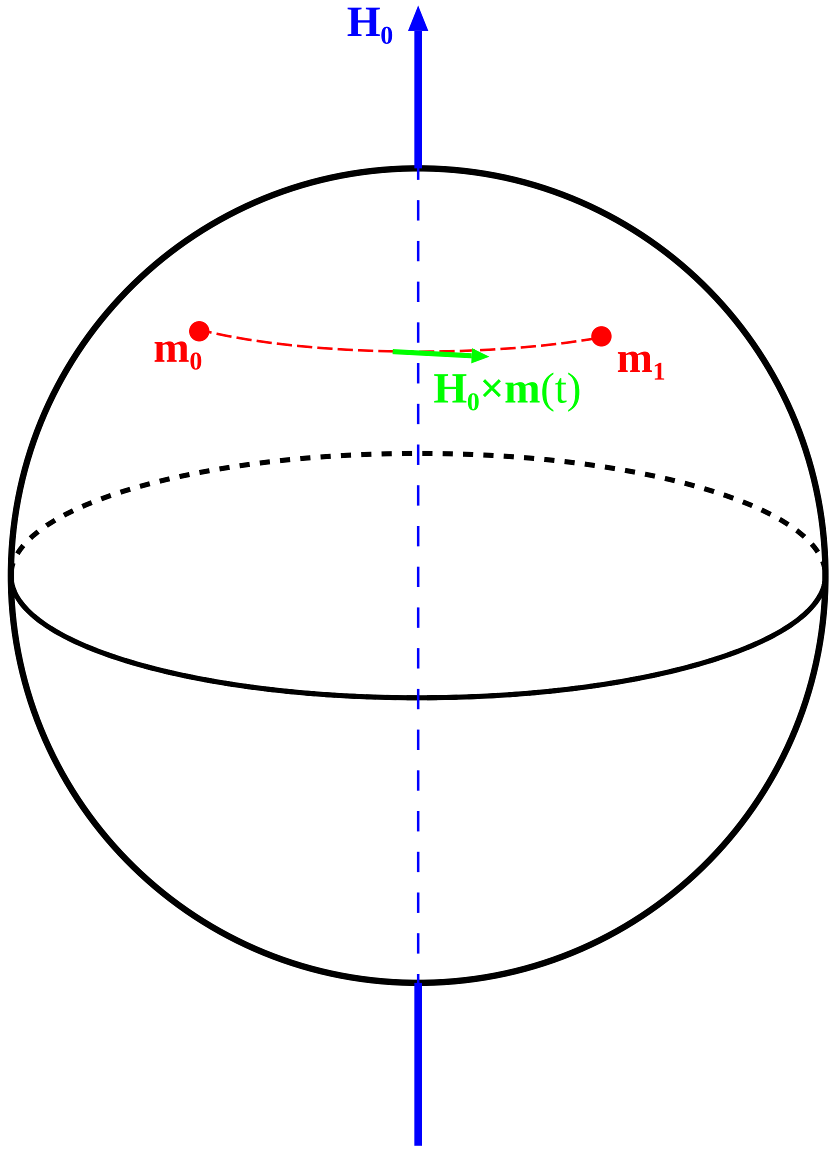
Step 1b: Assuming $H = H_0$ is fixed, compute m_1 using (2) and (3).

Step 1c: Compute H_1 at m_1.

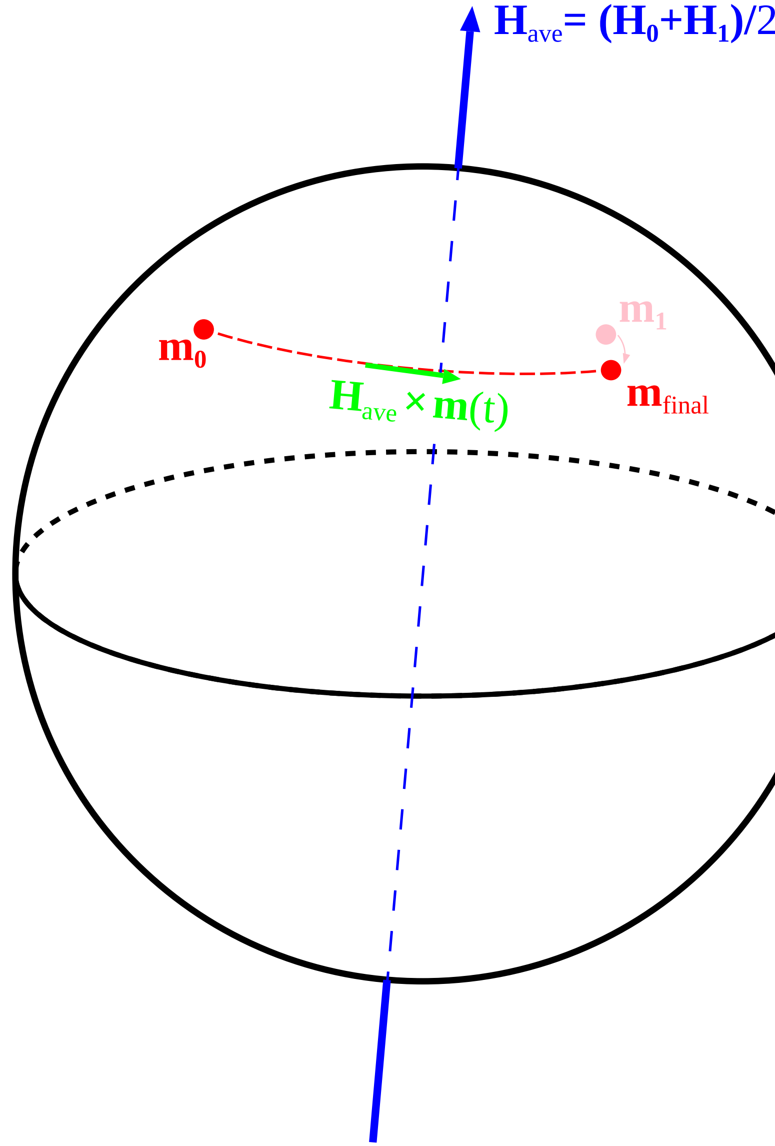
Step 2: Compute m_{final} using (2) and (3) and $H = (H_0 + H_1)/2$.

The resulting method is 2nd order in Δt.

Semi-analytic method (step 1)



Semi-analytic method (step 2)



Limitations

- H is a function of m; varies over simulation time scales.
- When exchange or demagnetization dominates, H is expected to vary at same rate as m.
- Semi-analytical technique only valid for small time steps

Predictor/corrector using dH/dt

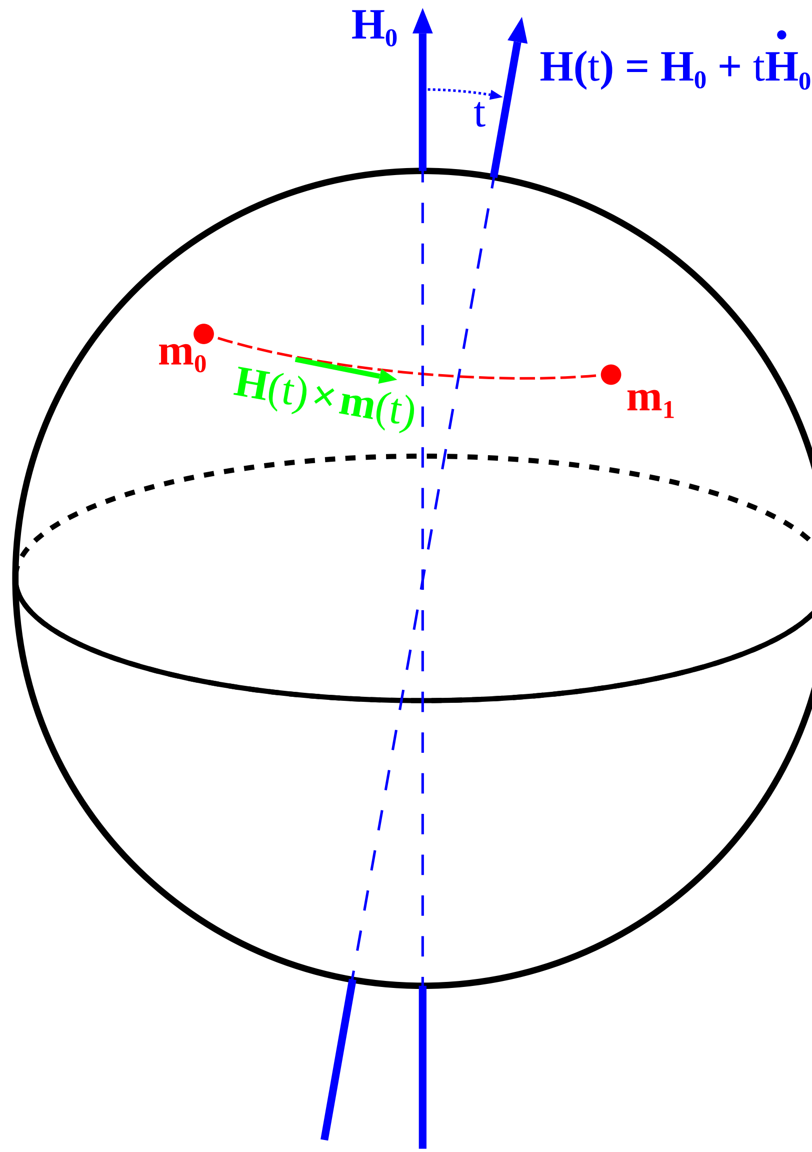
Idea: Use $\dot{H}_0$ to produce time-varying estimate $H(t)$.

Step 1a: Compute H_0 and $\dot{H}_0$ at m_0.

Step 1b: Compute m_1 along sphere using linear estimate $H(t) = H_0 + t\dot{H}_0$.

Step 1c: Compute H_1 at m_1.

dH/dt method (step 1)



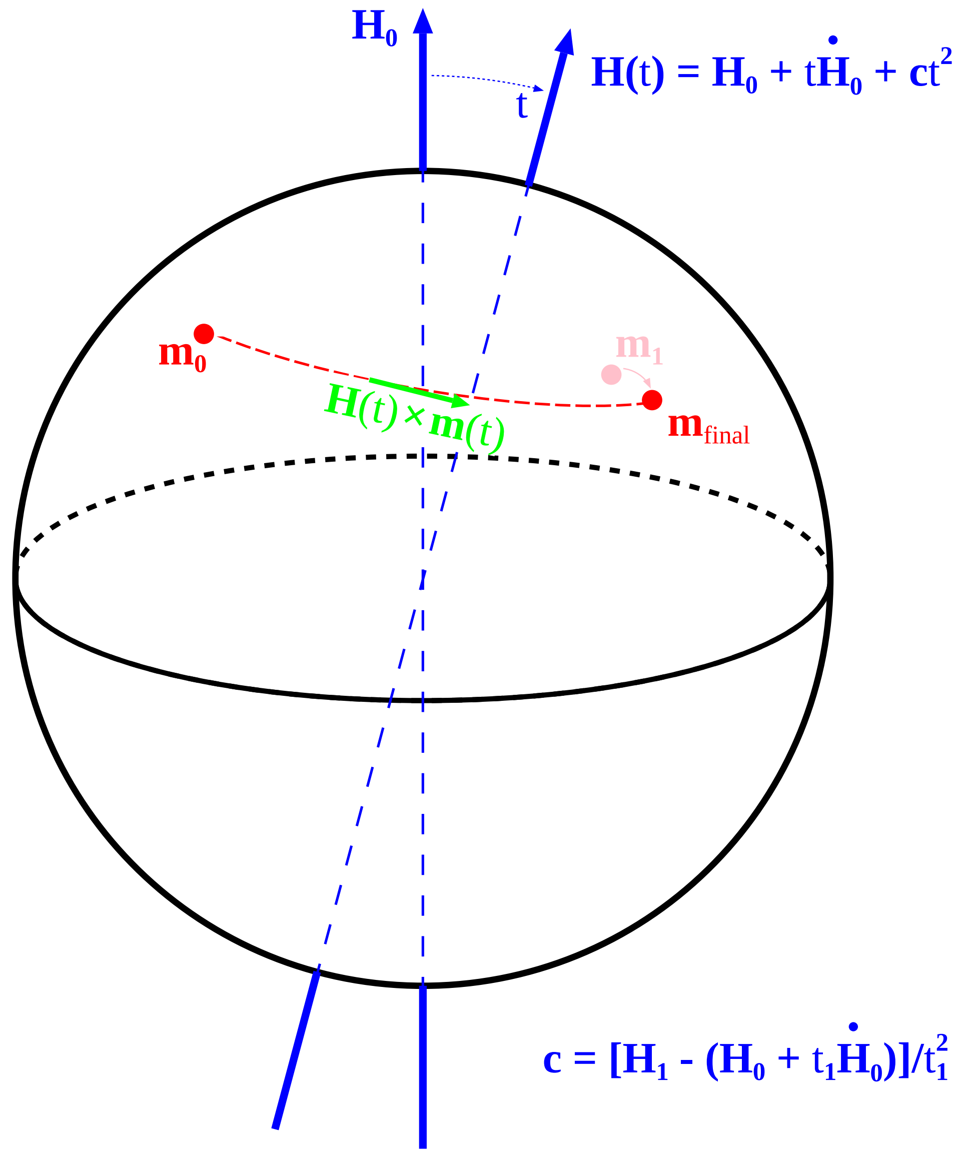
Predictor/corrector using dH/dt

Step 2a: Fit quadratic estimate
$H(t) = H_0 + t\dot{H}_0 + ct^2$ where
$c = \left[H_1 - \left(H_0 + t_1\dot{H}_0\right)\right] / t_1^2$.

Step 2b: Compute m_{final} using quadratic $H(t)$ estimate.

The resulting method is 3rd order in Δt.

dH/dt method (step 2)



Computation of dH/dt

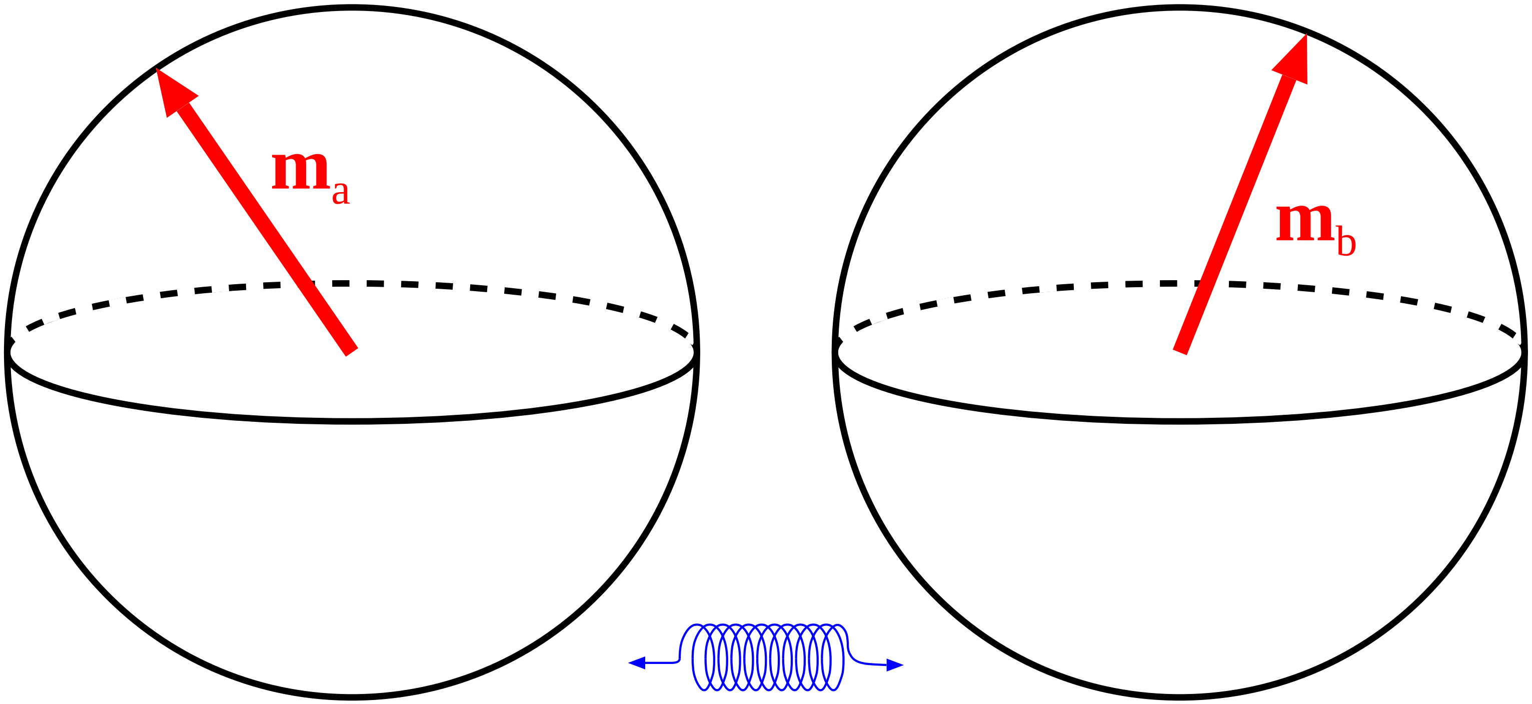
$$H_i^{\text{demag}} = -\sum_j N_{ij} m_j \qquad \dot{H}_i^{\text{demag}} = -\sum_j N_{ij} \dot{m}_j$$

$$H_i^{\text{exch}} = \frac{2A}{\mu_0 M_s \Delta^2} \sum_{j \in \mathcal{N}_i} m_j \qquad \dot{H}_i^{\text{exch}} = \frac{2A}{\mu_0 M_s \Delta^2} \sum_{j \in \mathcal{N}_i} \dot{m}_j$$

$$H_i^{\text{anis}} = \frac{2K_1}{\mu_0 M_s} (m_i \cdot u_i)\, u_i \qquad \dot{H}_i^{\text{anis}} = \frac{2K_1}{\mu_0 M_s} (\dot{m}_i \cdot u_i)\, u_i$$

$H^{\text{Zeeman}} =$? $\qquad \dot{H}^{\text{Zeeman}} =$ $\dot{?}$ (often 0)

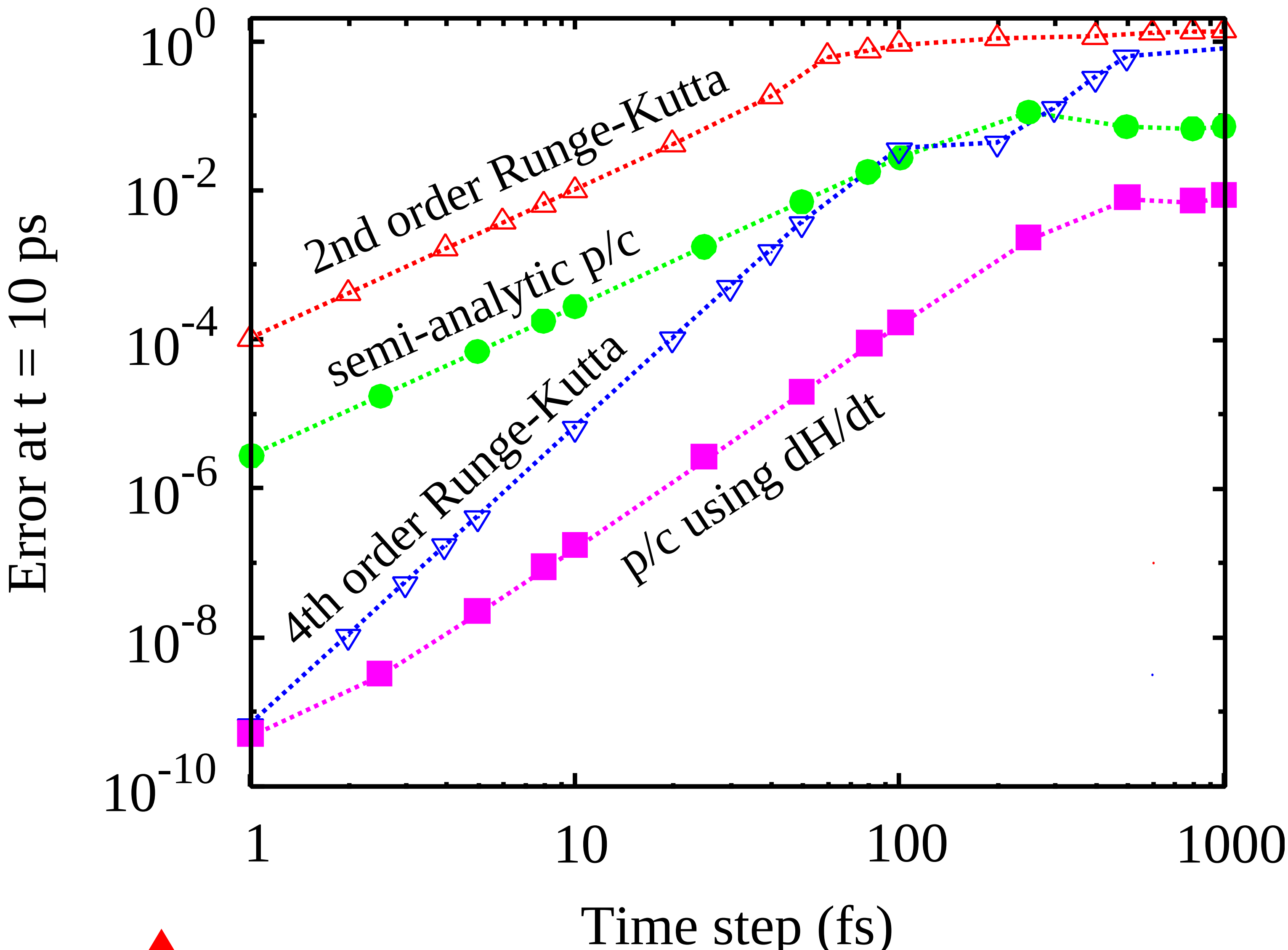
Notes: $\dot{m}$ is given by (1). Computational cost for dH/dt similar to cost for H.



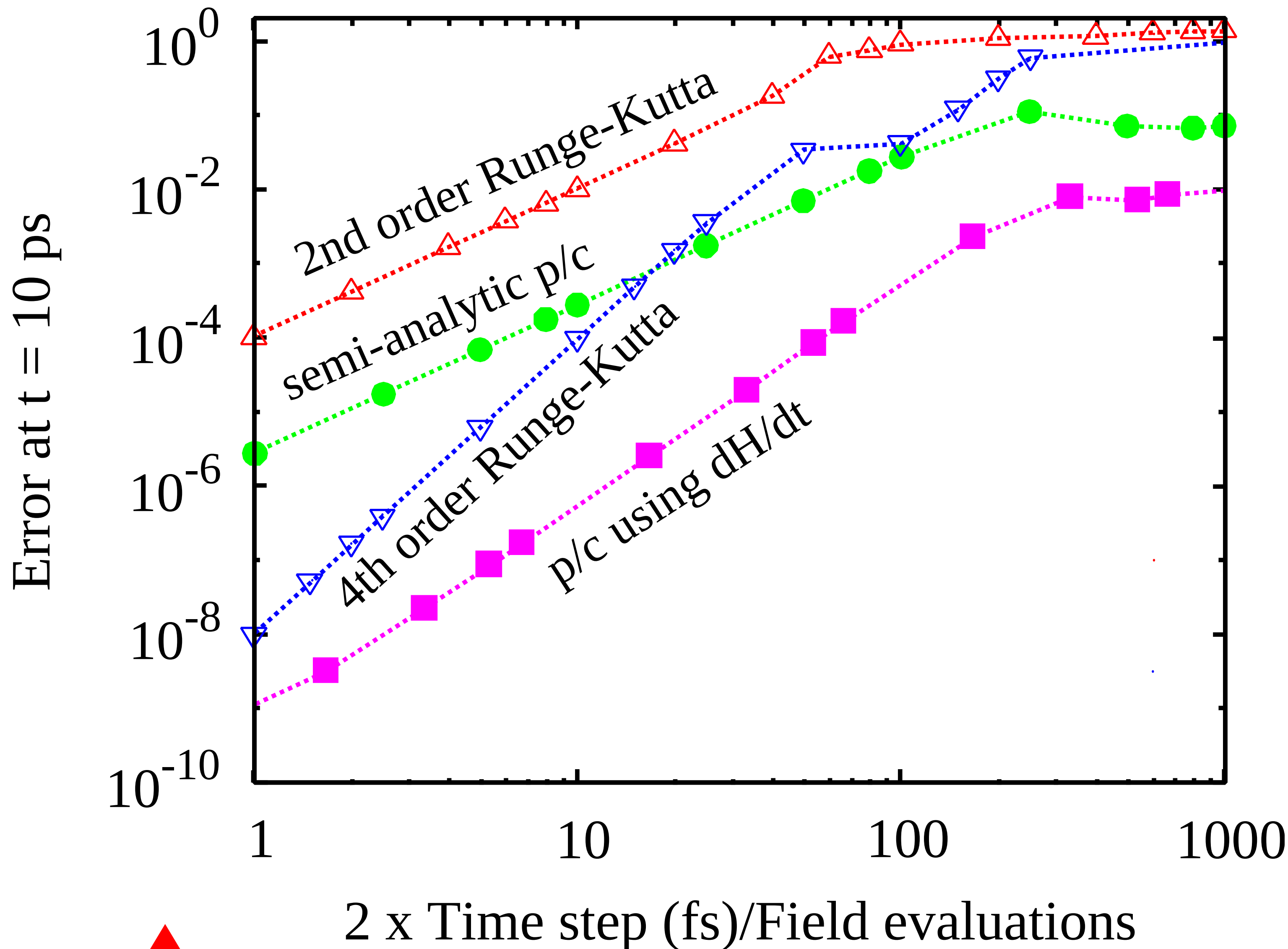
Coupled Two-spin System

(Includes exchange, demag, Zeeman, and cubic anisotropy energies)

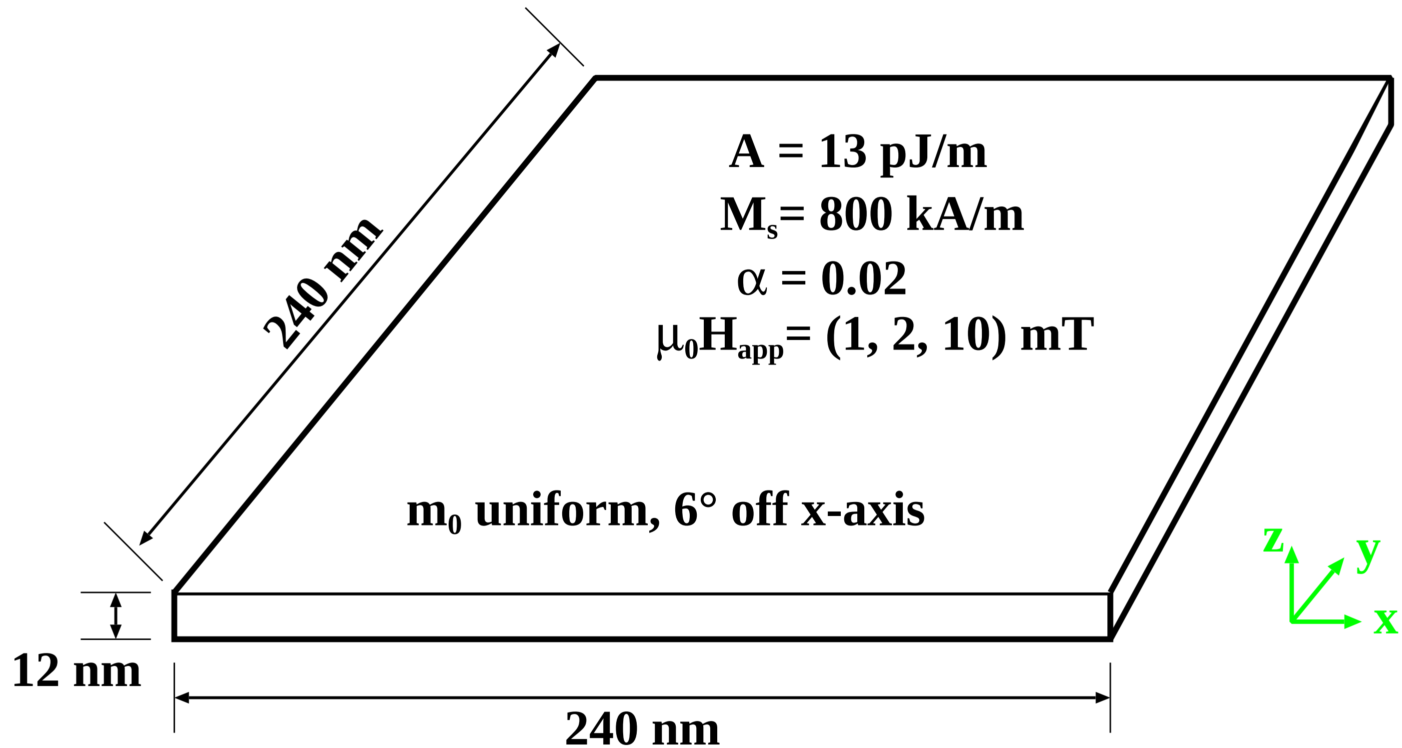
Two spin results



Two spin results, scaled

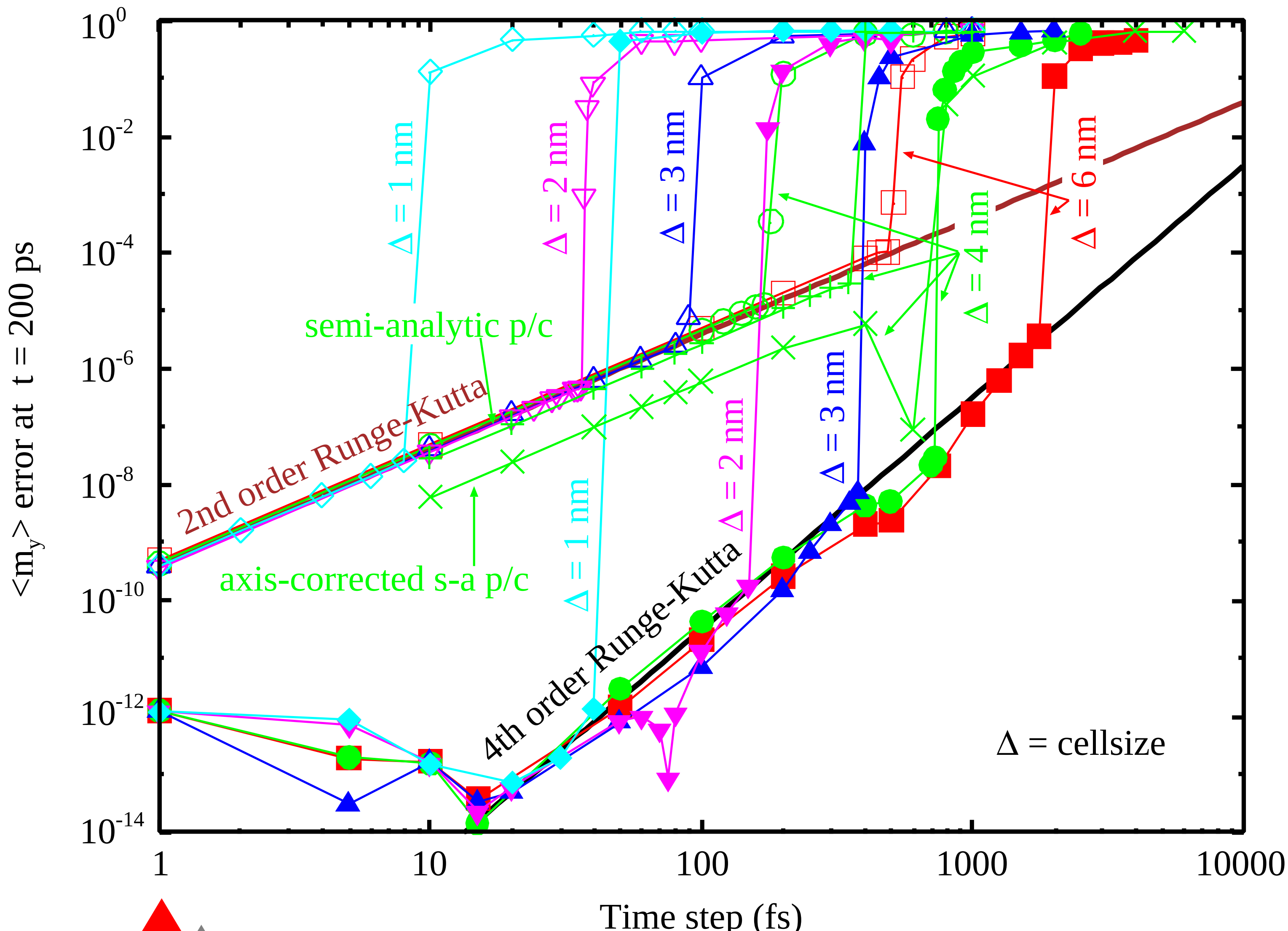


Multi-spin study



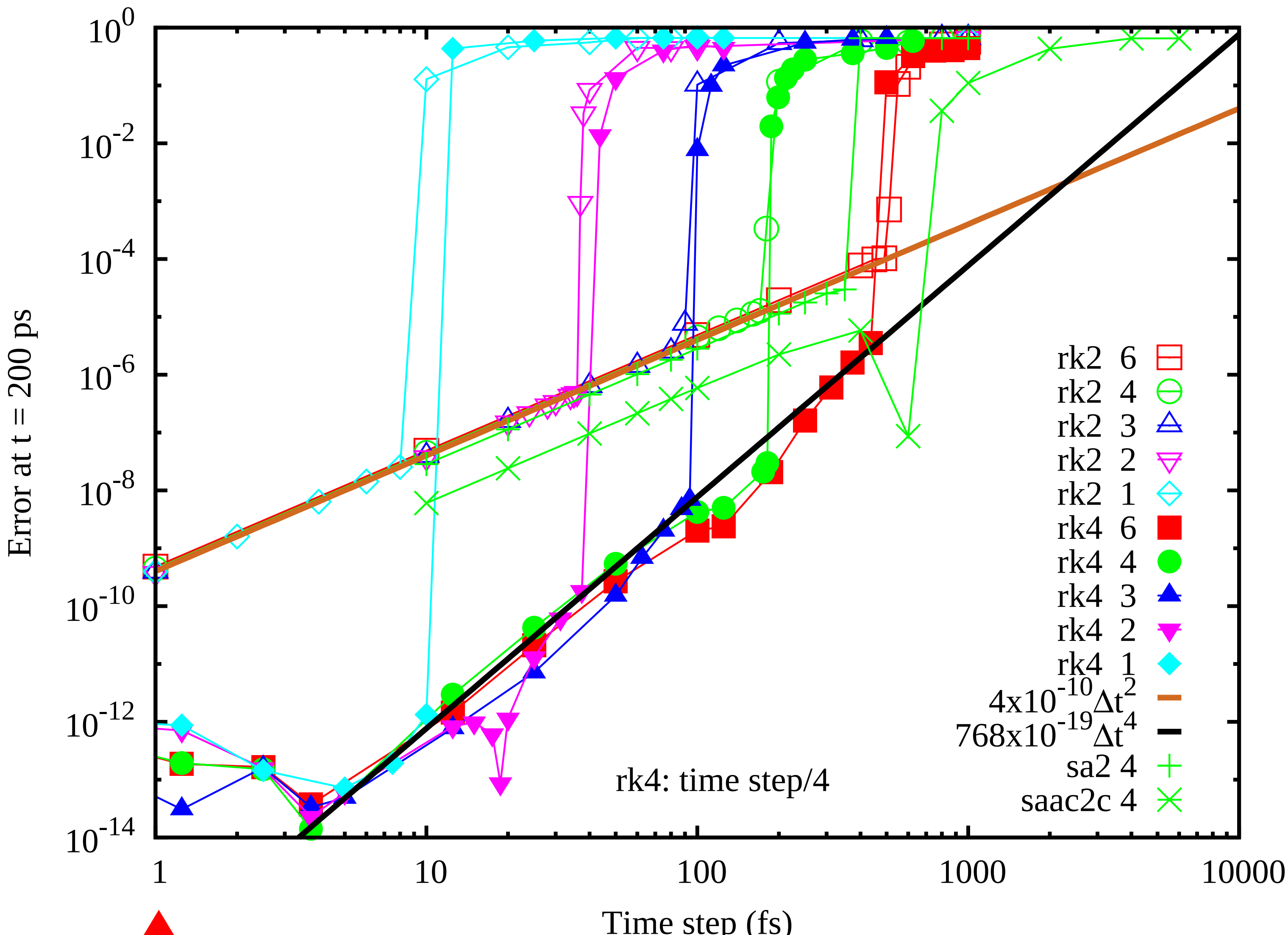
Cubic cells, edge dimension Δ (varies)

Multi-spin results



Multi-spin results

(RK4 timestep/4)



Summary: Two spin system

- LLG has been integrated on the sphere using H_0, $\dot{H}_0$, and H_1.
- Computing $\dot{H}_0$ costs similar to computing H_0.
- Proposed predictor/corrector method is 3rd order in Δt.
- On 2-spin test, proposed method yields smaller errors than 4th order Runge-Kutta for reasonable Δt.

Summary: Multi-spin test

- Smaller cells require smaller Δt to reach stable regime.
- In stable regime, integration error depends on Δt but not cellsize.
- Stability critical Δt for 4th order RK is 4 times larger than for 2nd order RK.
- The semi-analytic methods have good stability characteristics.