Micromagnetic eddy currents in conducting cylinders

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Abstract

- Inclusion of eddy currents into micromagnetic programs permits proper analysis of dynamic effects in conducting magnetic media.
- We present a numerical implementation for eddy current calculations.
- We limit the geometry, but permit a thick wall.
- For a zero thickness wall, our results are directly comparable to an analytical model.
  - L Yanik, E. Della Torre, and M. J. Donahue, "A test bed for a finite difference time domain micromagnetic program with eddy currents," Physica B, 343/1-4, 2004, pp 216-221.
- Our calculations provide some results for testing more complex programs.

Outline

- Background
- Model
- Model Geometry
- Formulation of the Problem
- Model Results
- Conclusions

Background

- A solution of micromagnetic problems with eddy currents has been proposed by Della Torre and Eicke and implemented by Torres, et al.
- We have recently presented a model for testing limiting cases of a micromagnetic model, for a zero thickness wall, that included eddy currents.
- This model involves solving the coupled problems of eddy current and magnetization calculations.
- We intend to use this program to verify the accuracy of a more general programs.

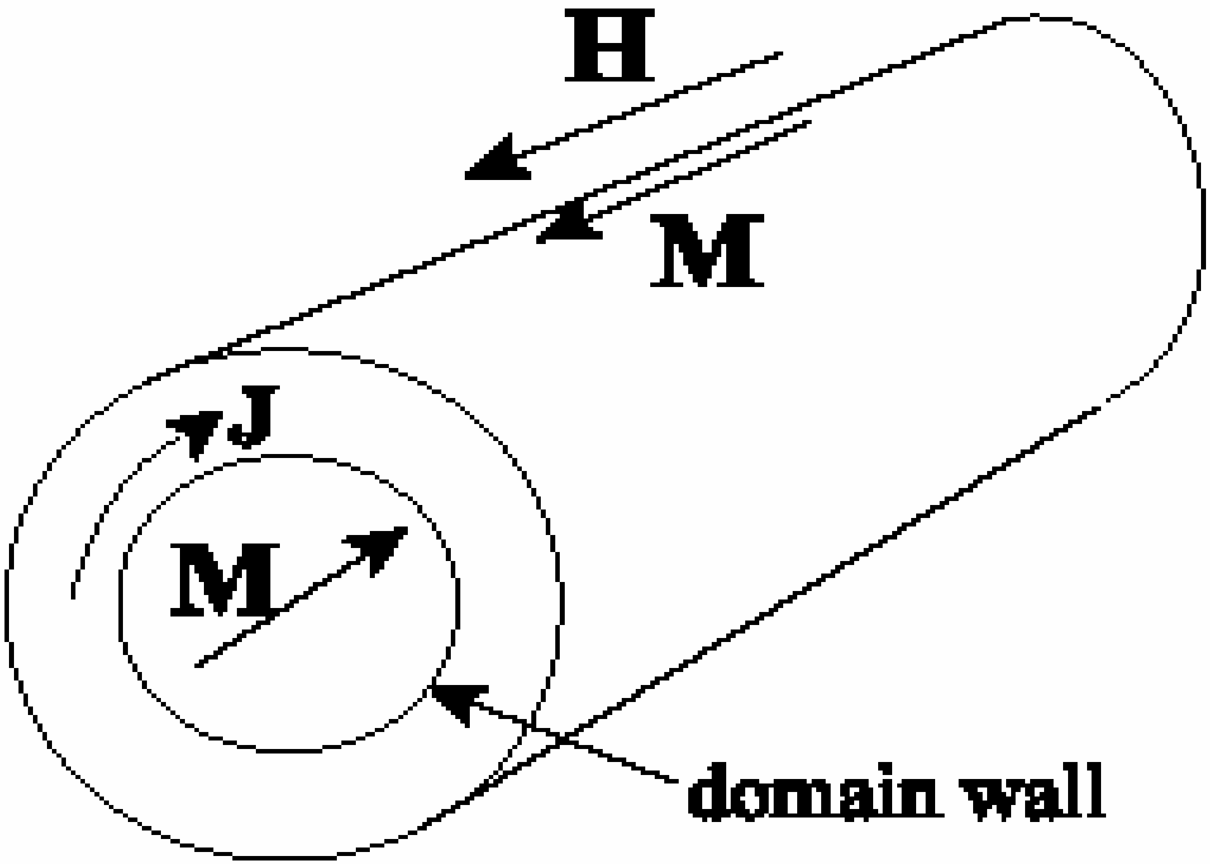
The Model

- We have developed a one dimensional model micromagnetic program to solve for the dynamic magnetization in conducting cylinders as a test bed for determining errors in these programs.
- Applying a magnetic field along the wire but opposite to the magnetization creates a Bloch wall that moves inwards and generates eddy currents that impedes its progress without creating demagnetizing fields.
- This model permits one to determine any effects of wall bending on its characteristics, since the wall's radius of curvature decreases as it approaches the center of the wire.

The Model Geometry

- The model is an infinite solid circular cylinder of radius R
- We assume a perfect crystal of uniaxial with easy axis, z, coincides with the cylinder's axis.
- We assume magnetization is initially uniform in the positive z-direction.
- Applying a constant field in the negative *z*-direction eventually reverses the magnetization.
- To break the symmetry, we offset the surface magnetization by a small angle nucleating a Bloch wall that propagates towards the center.
- The moving wall induces eddy currents that impede the wall's progress.
- Due to symmetry as the magnetization changes it will remain cylindrically symmetric.

Cylinder's Geometry



Formulation of the Problem

- External magnetic field acts as a boundary condition on the magnetic field inside the material.
- Difference between the internal magnetic field from the surface magnetic field is due to the shielding effects of eddy currents.
- The magnetic field tries to penetrate the material and in doing so changes the magnetization which in turn generates the eddy currents that keep it from penetrating.
- At each time step, one has to simultaneously relax both the magnetization and the magnetic field.

Formulation of the Problem

- In Micromagnetic calculations, one normally assumes a continuous magnetization and approximates the exchange energy density as

$$w_{ex} = -\frac{A\,\mathbf{M}\cdot\nabla^2\mathbf{M}}{M_S^2}.$$

- This formula is indeterminate on the *z*-axis. We therefore go back to the definition of exchange energy between a pair of spins as

$$W_{ex} = -J_{ex}\,\mathbf{s}_1\mathbf{s}_2.$$

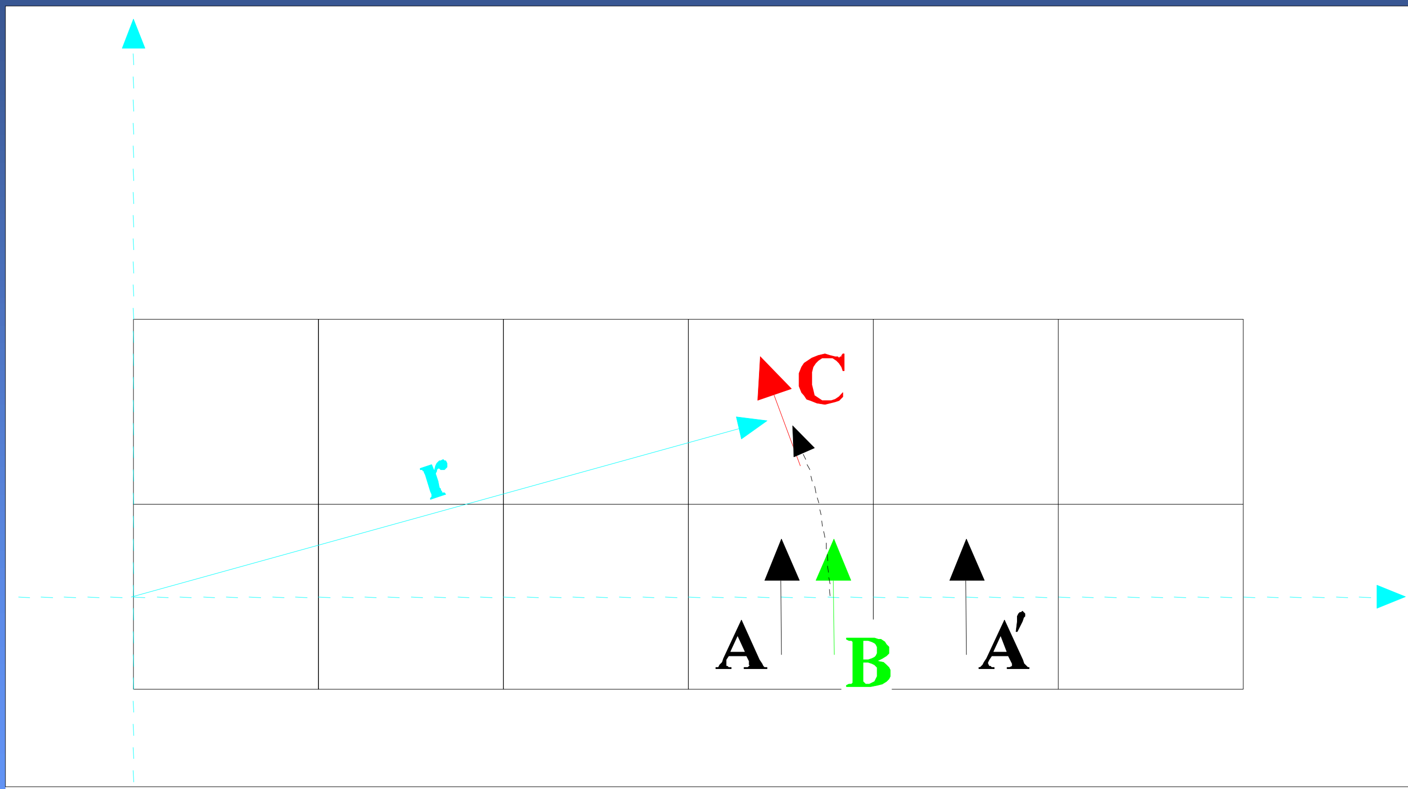
Formulation of the Problem

- If we assume that the magnetization varies linearly between a pair of calculation nodes *j* and *k*, then the exchange energy for the atoms in that row is given by

$$w_{ex,jk} = -J_{ex}\frac{h}{\delta}\cos\left[\frac{\delta}{h}(\alpha_j - \alpha_k)\right],$$

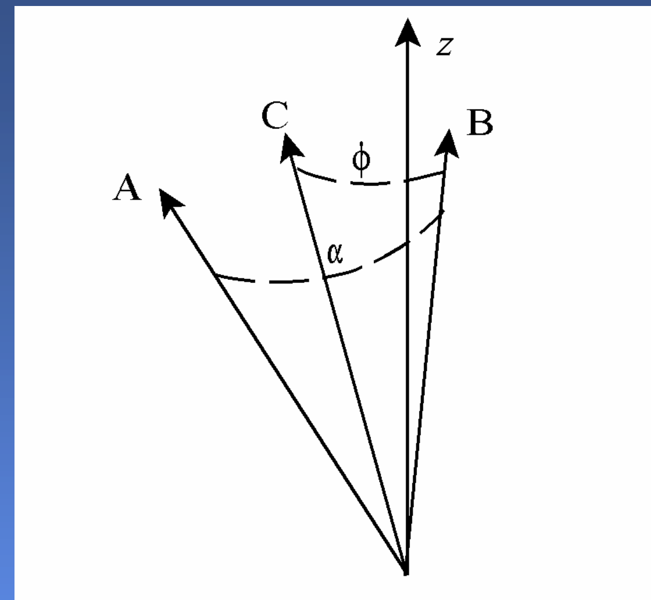
- Where
  - α_k is the angle that the magnetization M_k makes with respect to the *z*-axis,
  - Assume that $\varphi = 0$,
  - *h* is the distance between nodes,
  - δ is the distance between magnetic unit cells.
- To get the total exchange energy we have to sum this over all the computation points.

Computational grid



Relationship of spherical angles

- A is the spin on the computation row,
- B is the spin on the adjacent row,
- C is obtained by rotating the interpolated spin from the computation row by an angle φ
- Spin A makes an angle *A* with respect to the z-axis,
- Spin B makes an angle *B* with respect to the z-axis.
- The angle α between A and B is computed using the spherical angle formula



$$\cos\alpha = \cos A \cos B + \sin A \sin B \cos \phi .$$

- If φ = 0, valid for points along the *x*-axis.

Formulation of the Problem

- Exchange energy between two spins is ***J cos (A-B)***

$$w_{ex} = J(\cos A \cos B + \sin A \sin B \cos \phi).$$

- If there are *n* intervening atoms in between, then

$$w_{ex} = J\, n \cos\left[\frac{1}{n}\cos^{-1}(\cos A \cos B + \sin A \sin B \cos \phi)\right].$$

Formulation of the Problem

- Total anisotropy energy is given by

$$W_{anis} = 2\pi \int_0^R r K \sin^2\alpha \, dr.$$

- Zeeman energy is given by

$$W_{Zeeman} = -2\pi \int_0^R \mu_0 H_z(r) M_z(r)\, dr$$
$$= -2\pi \int_0^R \mu_0 H_z(r) M_s \cos[\alpha(r)]\, dr.$$

Formulation of the Problem

- By Faraday's law, the curl of electric field is given by

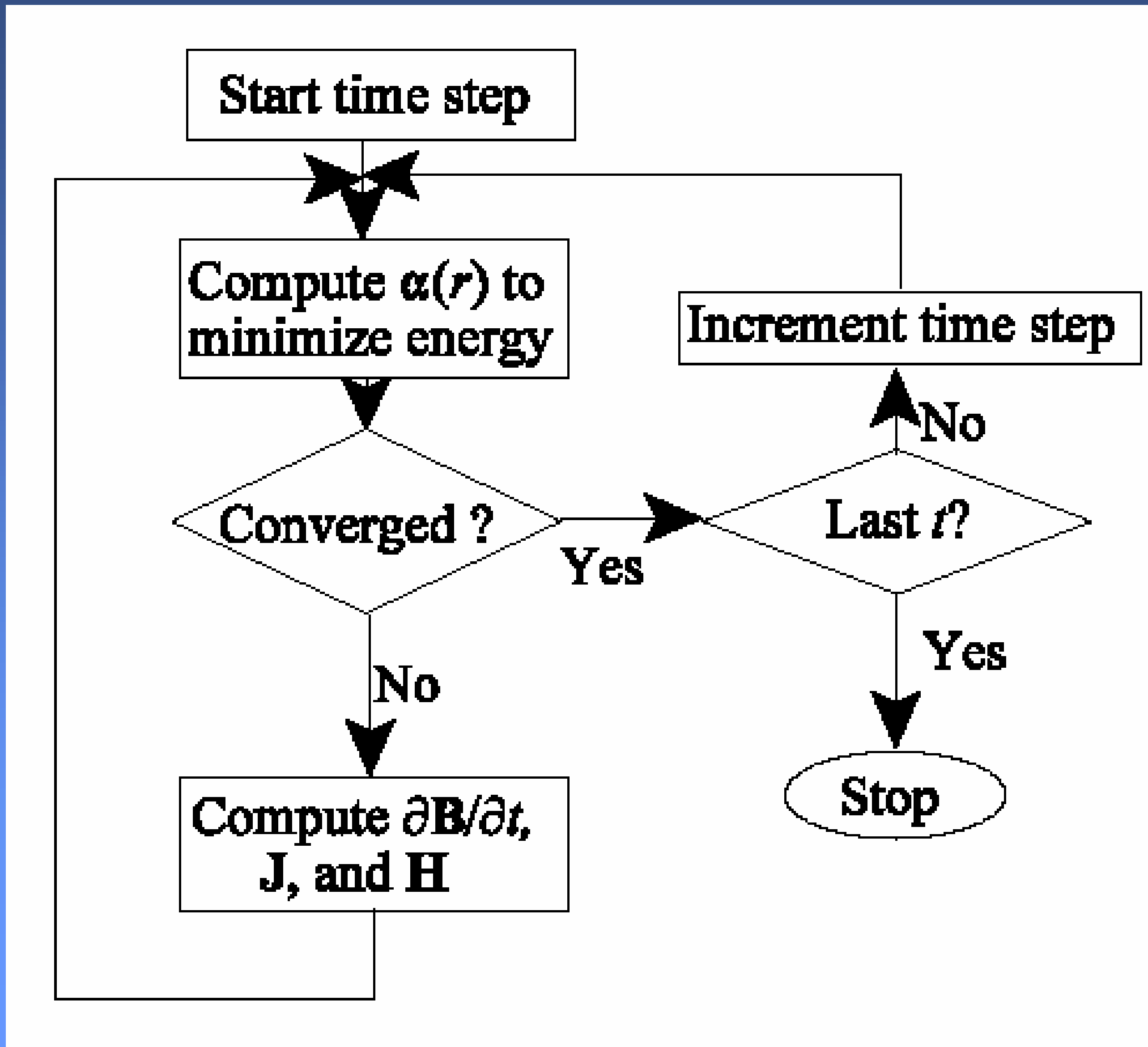
$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0\left[\frac{\partial \mathbf{H}}{\partial t} + \frac{\partial \mathbf{M}}{\partial t}\right].$$

- Electric field is then given by

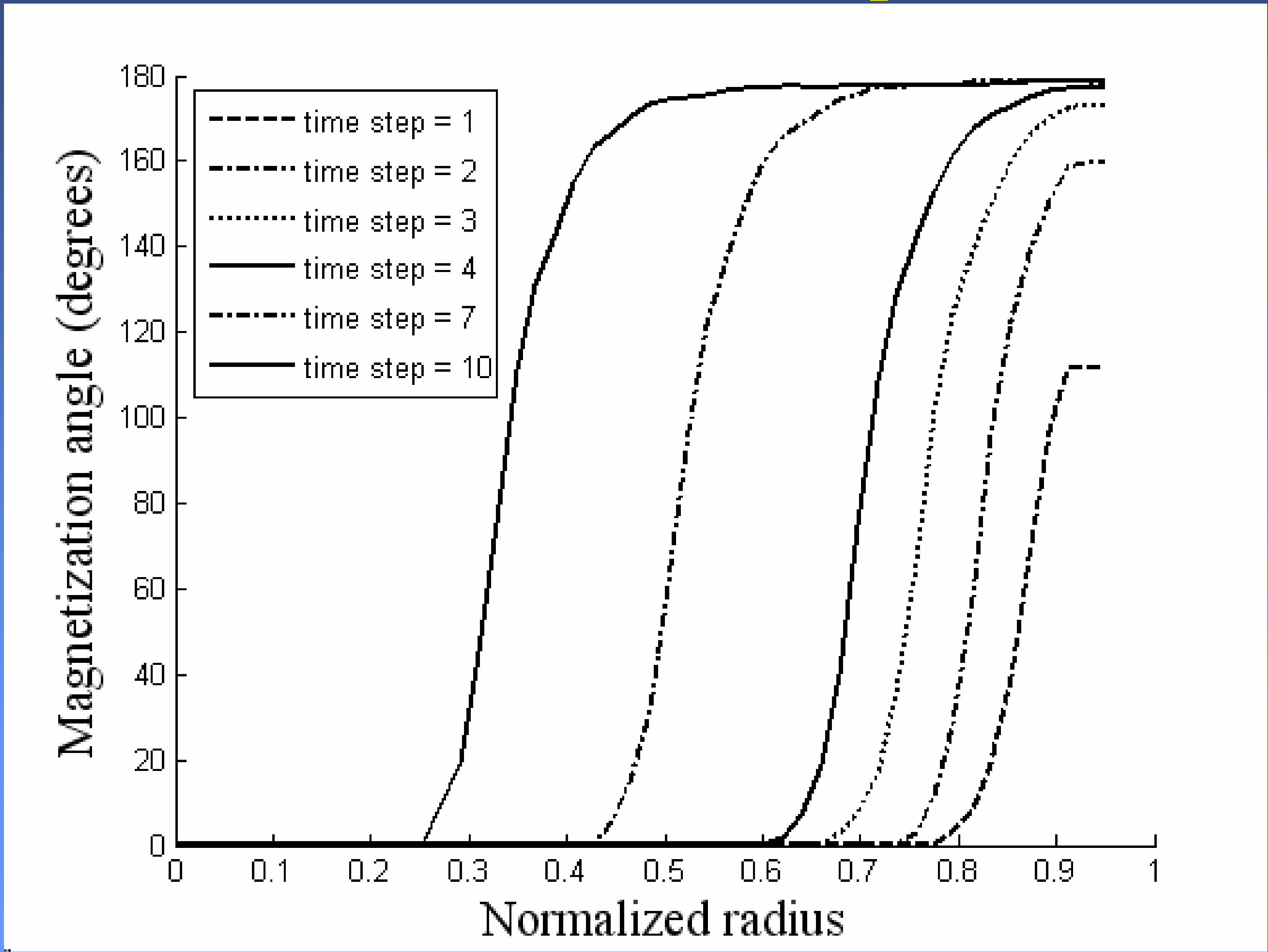
$$\mathbf{E}_{\mathbf{y}}(r) = -\frac{\mu_0}{r}\int_0^r \left[\frac{\partial H_z(\rho,t)}{\partial t} + \frac{\partial M_z(\rho,t)}{\partial t}\right]\rho\, d\rho\,.$$

- Electric field will induce eddy currents.
- Currents can be computed using Ohm's law $\mathbf{J} = \sigma\, \mathbf{E}$.

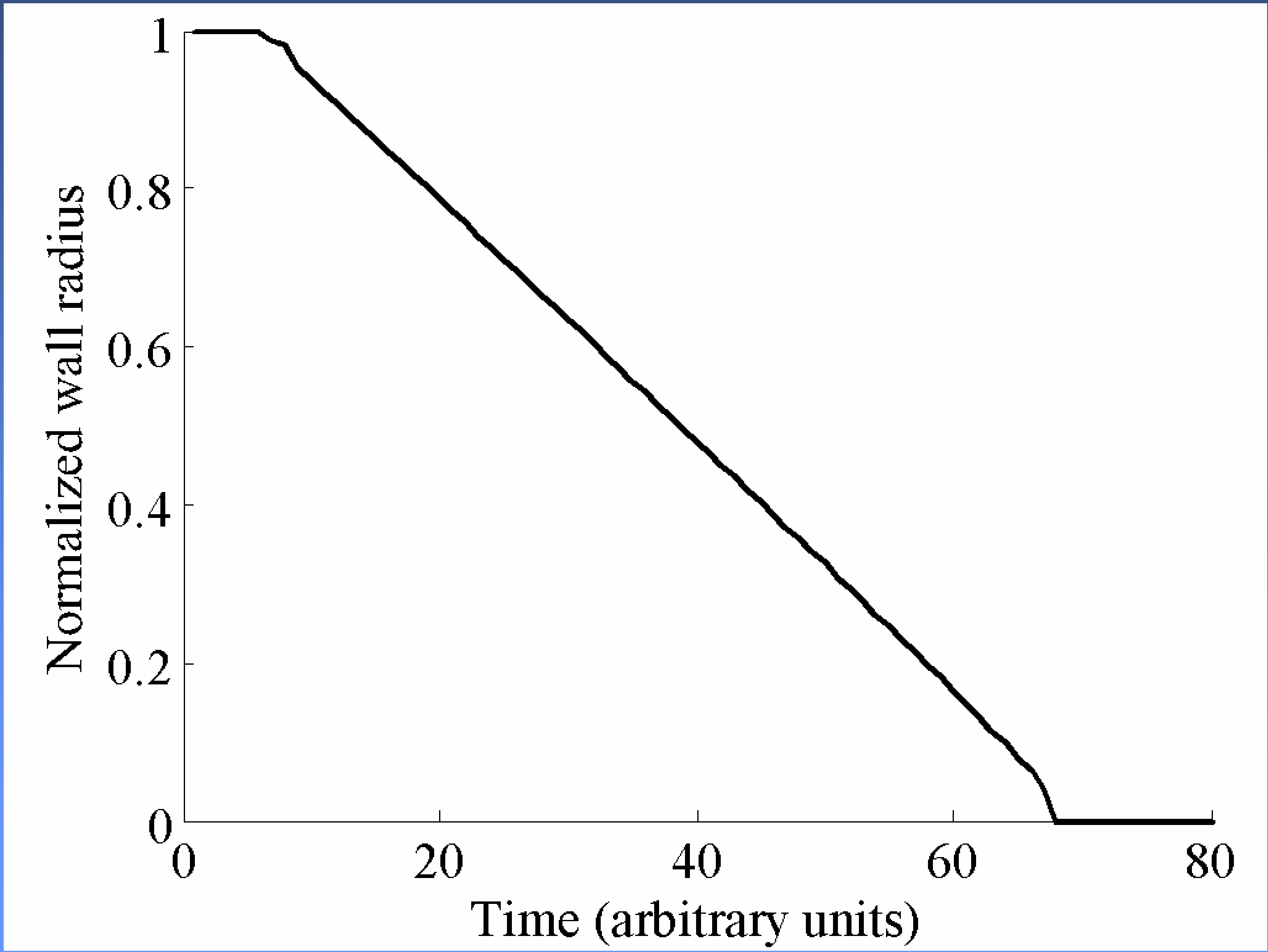
Flow chart of calculation



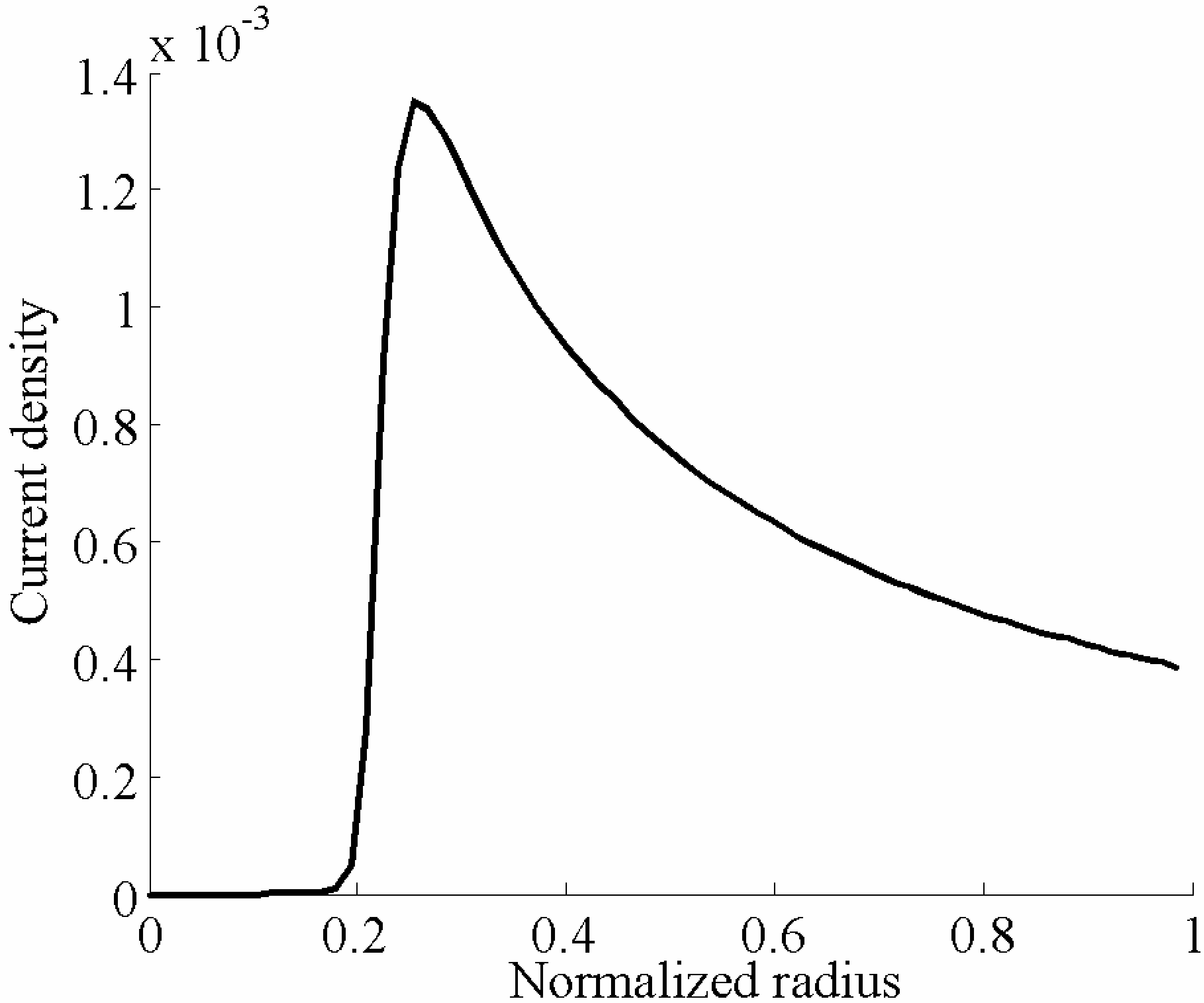
Magnetization angle as a function of radius for various time steps.



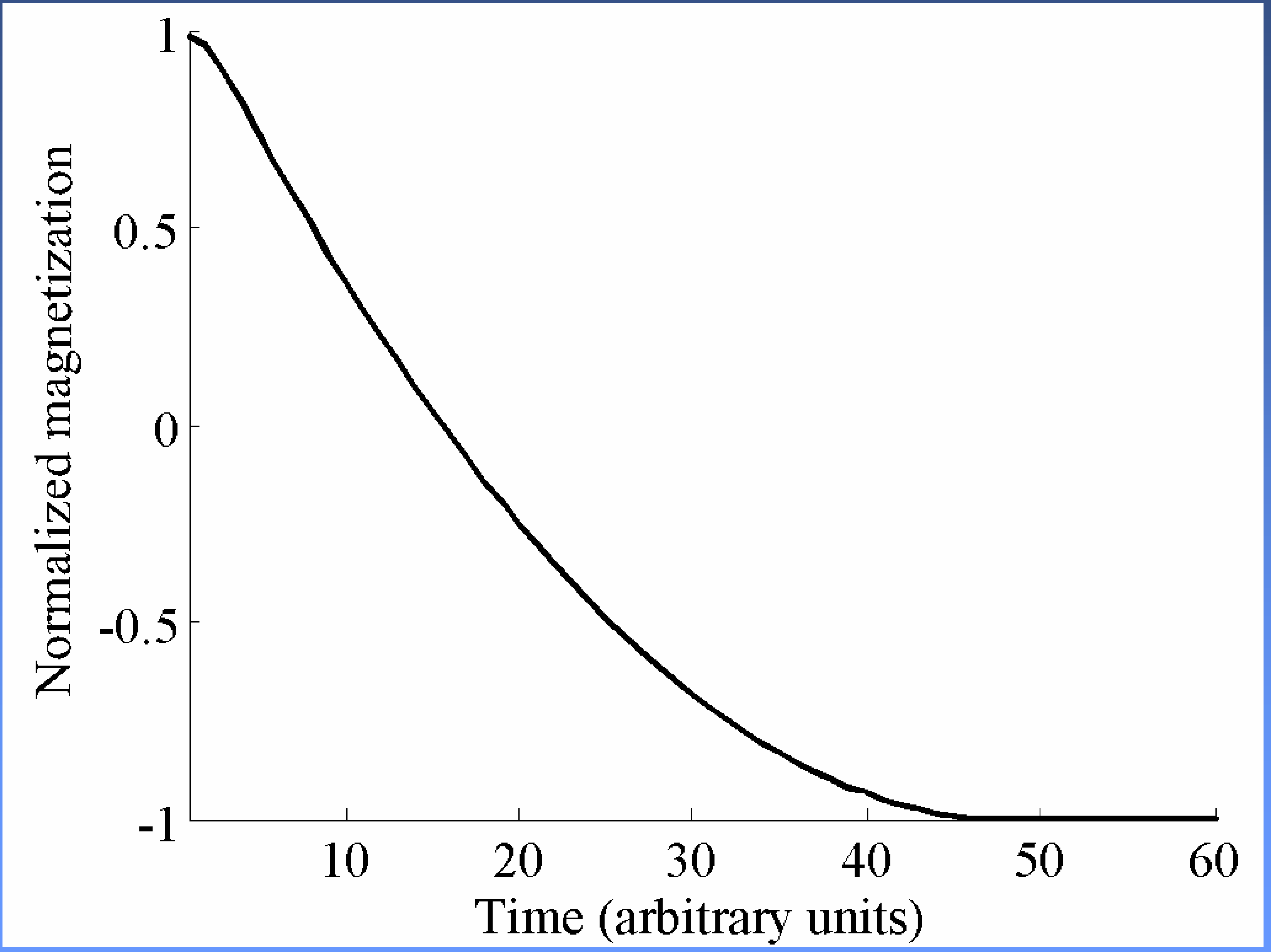
Normalized wall radius with time for a constant applied field.



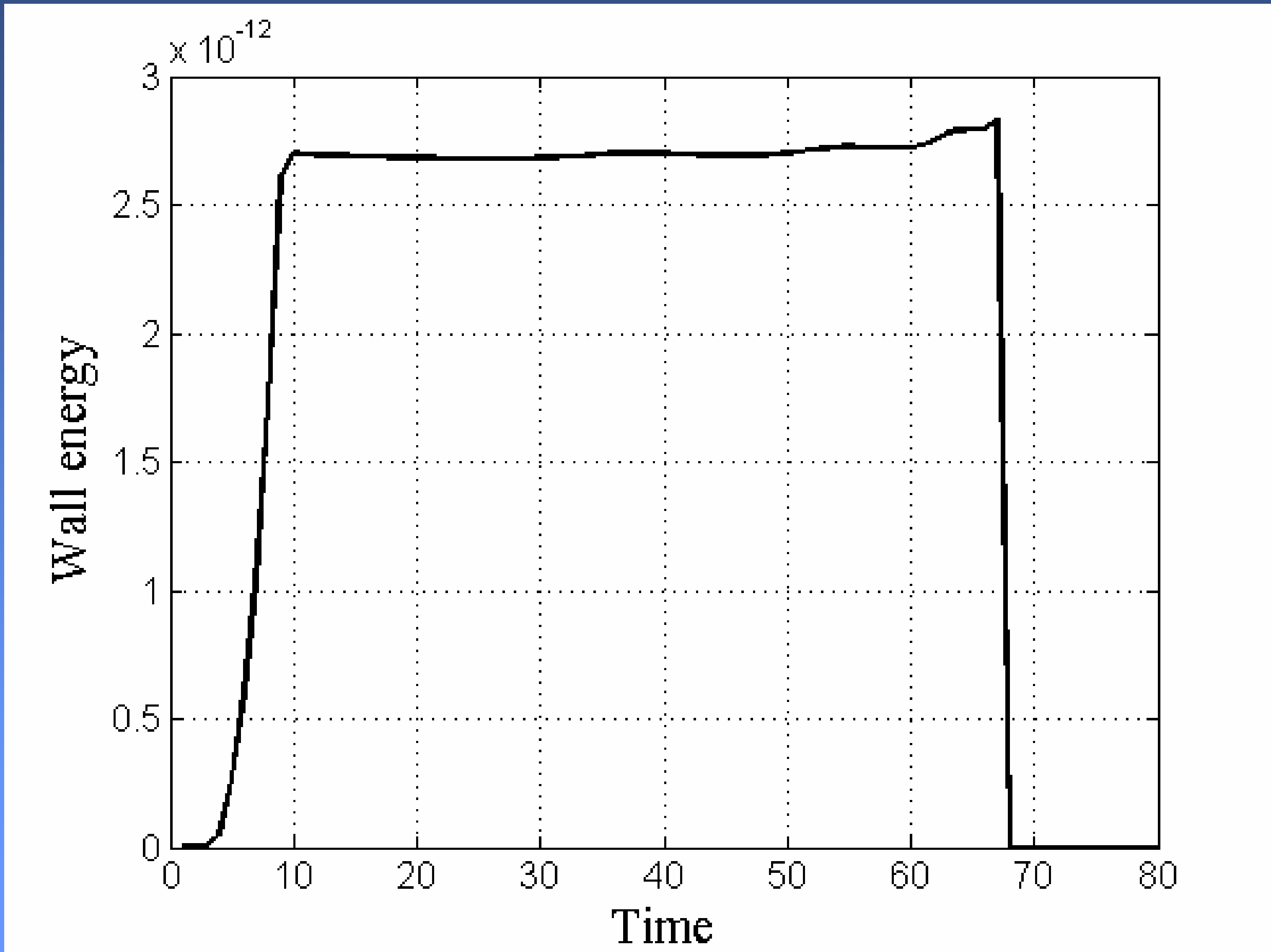
Eddy current density as a function of position part way through the magnetization reversal.



Normalized magnetization vs. time for a constant applied field.



Wall energy vs. time for a constant applied field.



Conclusions

- We present a numerical implementation of a one dimensional micromagnetic model for eddy current calculations.
- These calculations provide some computational results for testing more complex programs.
- We expected to see some effects of wall curvature on wall energy and wall thickness but could not find any, before the wall collapsed.
- The model was consistent with the model for a zero thickness wall presented previously.

References

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Computational grid used in the model