Non-uniform Thickness in Two-dimensional Micromagnetic Simulation

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2D Model Advantages

- Fewer cells
- Less memory required
- Shorter calculation time
- Simpler expressions – fewer bugs
- Easier to visualize and interpret
- Availability

2D Model Shortcomings

- No variation through thickness
- Limited to thin films
- Limited to single layers
- Limited to uniform thickness - Why?

2D Model with Variable Thickness

- For each cell k, store a relative thickness

$$0 \leq t_k \leq 1$$

- Absolute thickness:

$$0 \leq t_k t_{\max} \leq t_{\max}$$

- Energy terms: Zeeman, anisotropy, exchange, magnetostatic
- Adjust each field expression

$$\frac{d\mathbf{M}}{dt} = -\left|\gamma\right| \mathbf{M} \times \mathbf{H} - \frac{\lambda}{M_s} \mathbf{M} \times \mathbf{M} \times \mathbf{H}$$

- Adjust each energy expression
- Retain 2D advantages

Zeeman Energy

- Energy density and field unchanged

$$E_{\mathrm{k,Z}} = -\mu_0 \mathbf{M}_k \cdot \mathbf{H}_{\mathrm{k,Z}}$$

- Energy reduced by reduction in volume

$$\sum_k E_{\mathrm{k,Z}} \Delta^2 t_k t_{\mathrm{max}}$$

Anisotropy Energy (uniaxial)

- Energy density and field unchanged

$$H_{\mathrm{k,A}} = \frac{2K_1}{\mu_0 M_s^2}(\mathbf{M}\cdot\mathbf{u})\mathbf{u}$$

- Energy reduced by reduction in volume

$$-\sum_k \mu_0 \mathbf{M}_k \cdot \mathbf{H}_{\mathrm{k,A}} \Delta^2 t_k t_{\mathrm{max}}$$

Exchange Energy

- Original expression (eight-neighbor cosine)

$$E_{\mathrm{k,exch}} = \frac{A}{3\Delta^2}\mathbf{m}_{\mathbf{k}} \cdot \sum_{l \in N_k} (\mathbf{m}_k - \mathbf{m}_l)$$

- Each term: exchange energy between cells k and l
- Adjust energy density by relative thickness of both cells:

$$E_{\mathrm{k,exch}} = \frac{A}{3\Delta^2}\mathbf{m}_{\mathbf{k}} \cdot \sum_{l \in N_k} \frac{w(t_k, t_l)}{t_k} (\mathbf{m}_k - \mathbf{m}_l)$$

- Total exchange energy:

$$E_{\mathrm{exch}} = \frac{At_{\mathrm{max}}}{3} \sum_k \mathbf{m}_{\mathbf{k}} \cdot \sum_{l \in N_k} w(t_k, t_l)(\mathbf{m}_k - \mathbf{m}_l)$$

Exchange Energy Thickness Weighting

- Weights should have these properties

$$w(t_1, t_2) = w(t_2, t_1)$$

$$w(t, t) = t$$

$$\min(t_1, t_2) \leq w(t_1, t_2) \leq \max(t_1, t_2)$$

$$\lim_{t_2 \to 0} w(t_1, t_2) = 0$$

- Candidate weight functions:

$$w(t_1, t_2) = \min(t_1, t_2)$$

$$w(t_1, t_2) = \frac{2t_1 t_2}{t_1 + t_2}$$

Magnetostatic Energy

- Uniform thickness: Grid is periodic
- Demagnetization field is convolution of magnetization.
- Efficient FFT techniques available
- Variable thickness: efficient structure lost
- Preserve efficiency: represent reduced thickness as reduced moment.

$$\mathbf{M}_k \to \mathbf{M}_k t_k$$

Magnetostatic Energy

- Moment reduction good “far-field” approximation.
- Self-demagnetization energy of a cell need correction

$$\mathbf{H}_k = -\mathbf{D}(\mathbf{M}_k t_k)$$

- Change in moment interpreted as change in demag factors

$$\mathbf{H}_k = -(t_k \mathbf{D})\mathbf{M}_k$$

$$\text{trace}(t_k \mathbf{D}) = t_k \neq 1$$

- Local correction, add

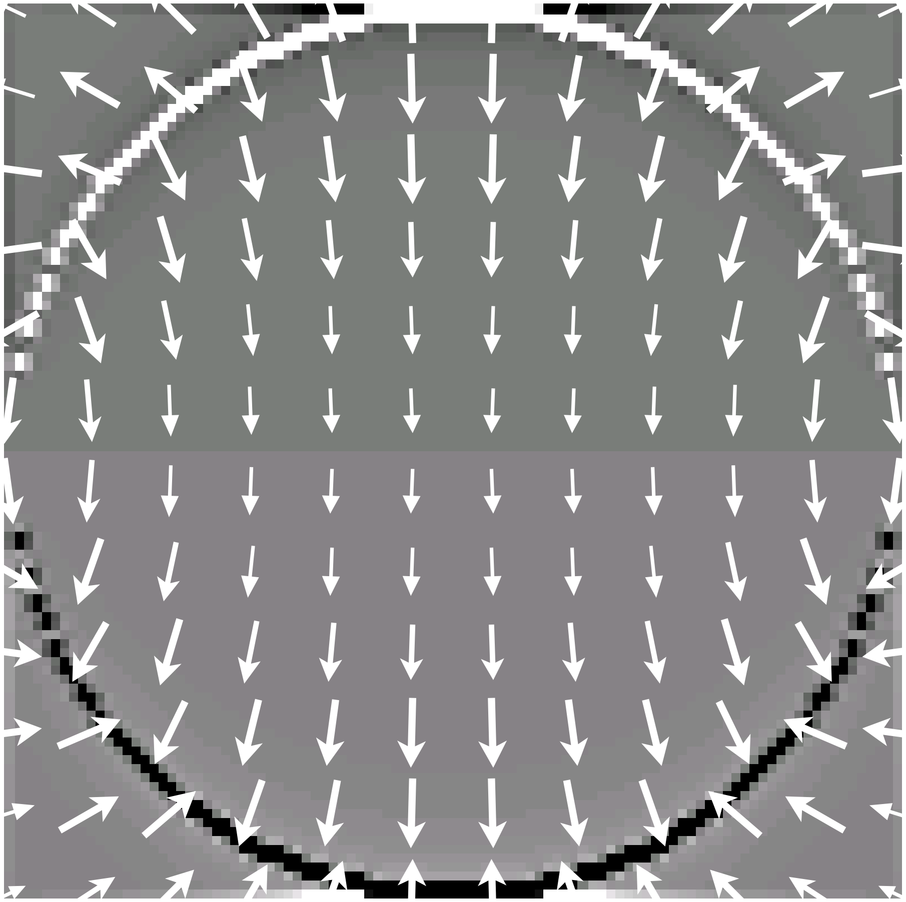
$$-(1 - t_k)\mathbf{M}_{\mathrm{k,z}}$$

as a correction to the out-of-plane demag field.

Magnetostatic Energy Adjustment Results

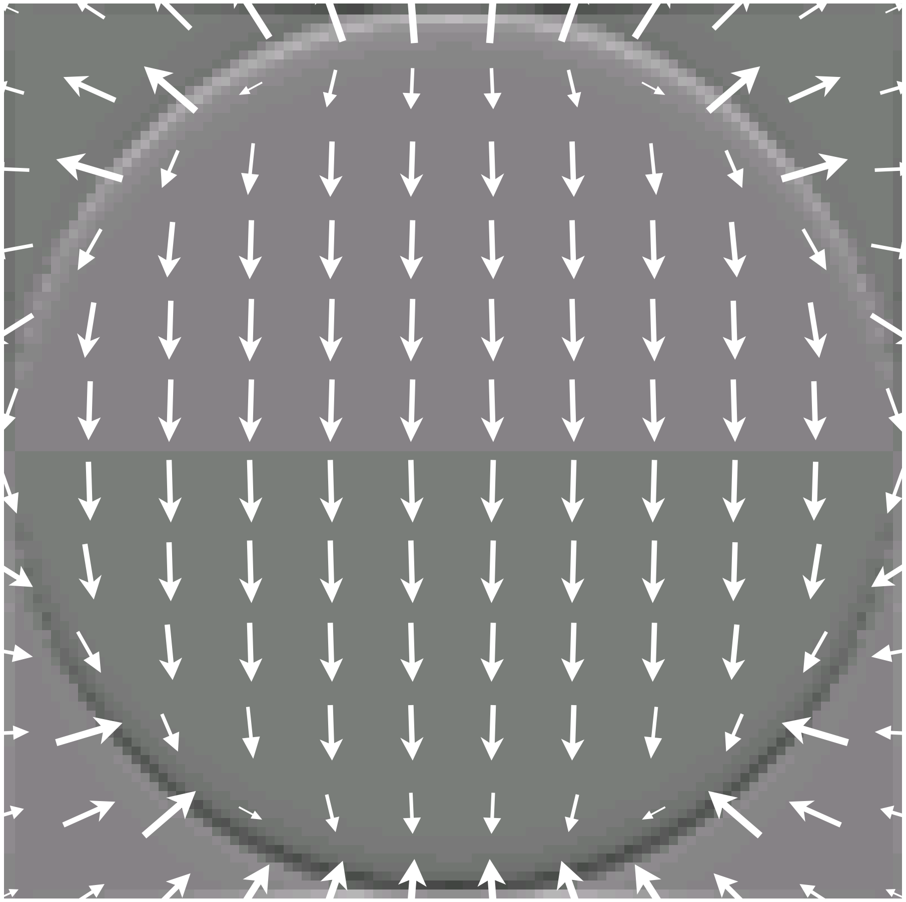
- Represent a 100 × 100 × 10 nm oblate ellipsoid with three models
  - 2D uniform thickness
  - 2D with variable thickness adjustments
  - 3D using 10 layers
- Cell size 1 nm for all.
- Check how well each produces uniform demag field.
- Check calculated demag factors
  - In-plane: 0.0696
  - Out-of-plane: 0.8608

2D Uniform Thickness



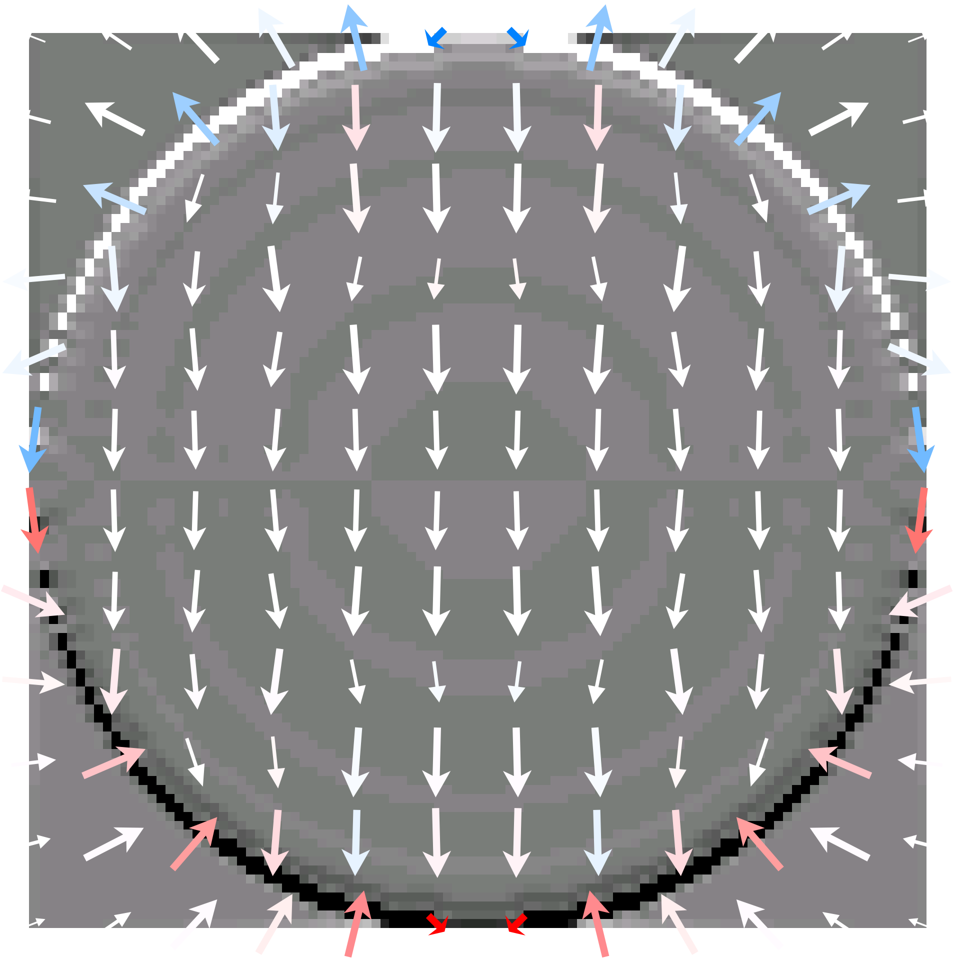
- In-plane: 0.1026
- Out-of-plane: 0.7947

2D Variable Thickness



- In-plane: 0.0635
- Out-of-plane: 0.8730

3D With 10 Layers

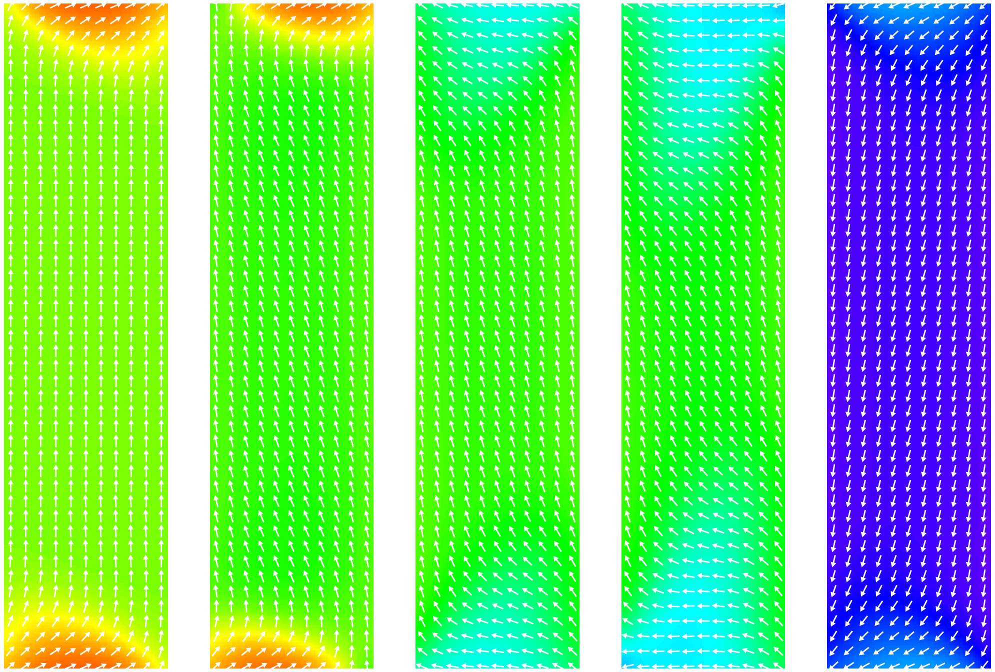


- In-plane: 0.0753
- Out-of-plane: 0.8559

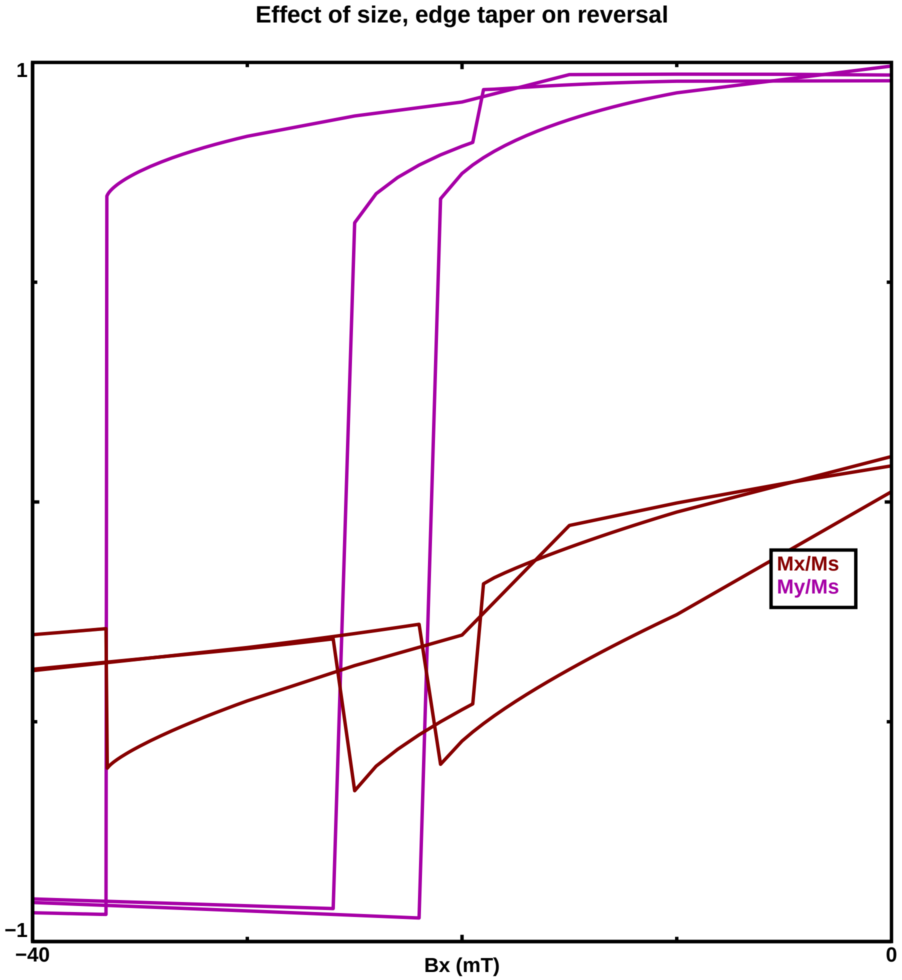
Shape Effects on Reversal

- Compare magnetic reversal of three permalloy samples
  - $500 \times 100 \times 10$ nm (Std. Prob. 2, $d/l \approx 19$)
  - $530 \times 130 \times 10$ nm
  - truncated pyramid, base: 530×130; top: 500×100 nm
- Reversal along [1 1 1] axis

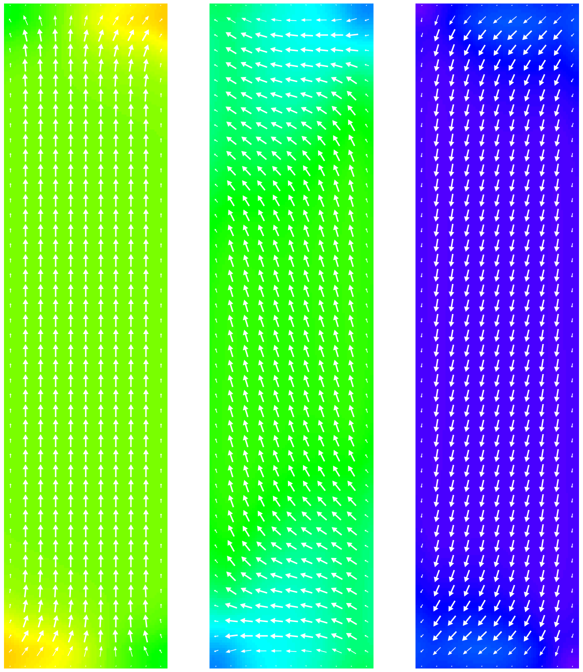
530 x 130 Reversal



Magnetization Reversal Curves



Truncated Pyramid Reversal



Comparison of 2D and 3D Results

- $530 \times 130 \times 10$ nm sample
  - 2D variable thickness model

$$H_s \approx 25.5\mathrm{mT}$$

  - 3D model – 5 layers

$$H_s \approx 24.5\mathrm{mT}$$

- Truncated pyramid sample
  - 2D variable thickness model

$$H_s \approx 21.5\mathrm{mT}$$

  - 3D model – 5 layers

$$H_s \approx 22.5\mathrm{mT}$$