

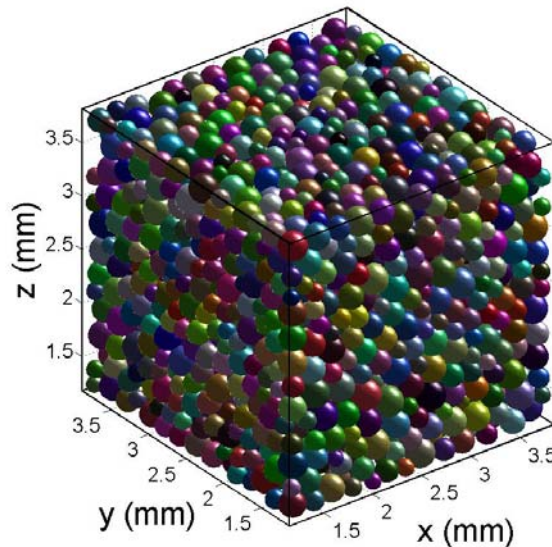
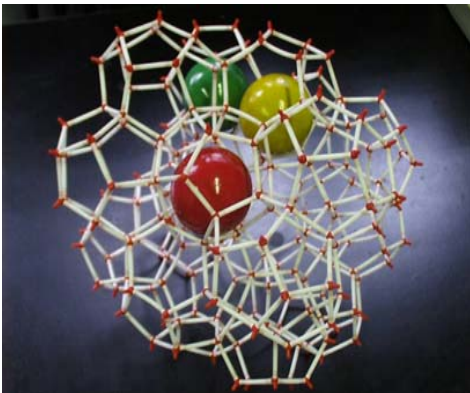


# Geomechanical Performance of Hydrate-bearing Sediments in Offshore Environments

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# Objectives

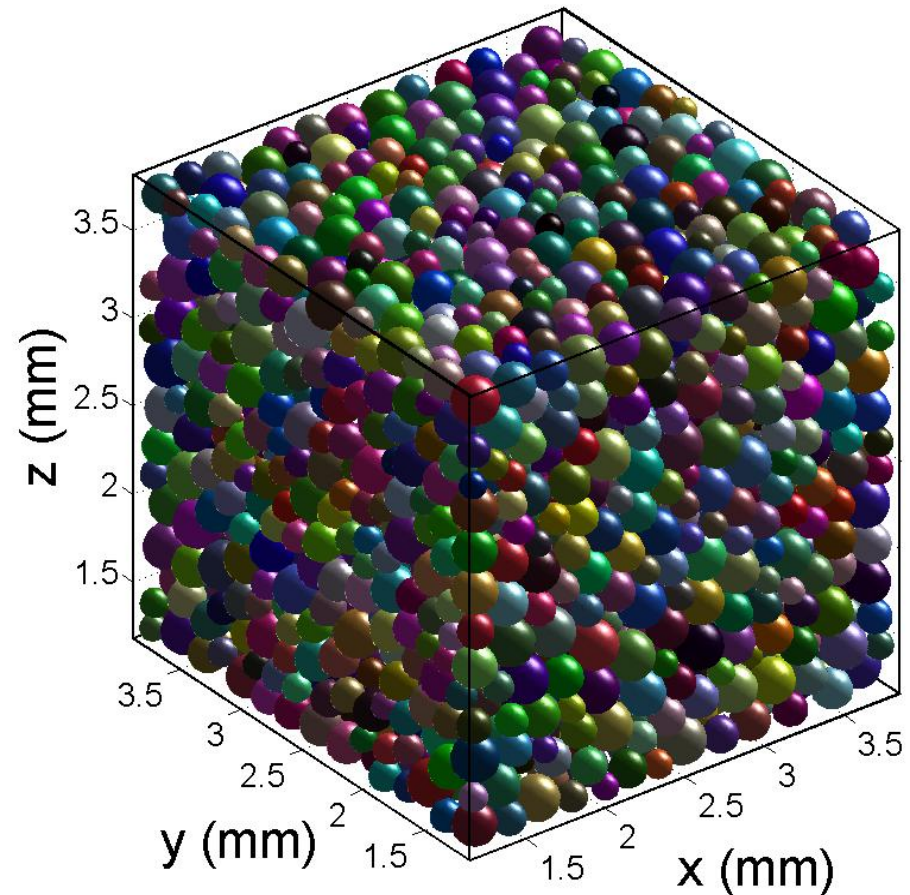
- **Motivation:** Quantify the impact of hydrate dissociation on the strength and elastic properties of hydrate-bearing sediments
- **The Task:** Estimate the changes of elastic moduli in order to populate the grid blocks for coupled TOUGH Fx/HYDRATE- FLAC3D simulations
  - **Inputs:** initial elastic moduli, variable pore pressure and hydrate saturation
  - **Tool:** Pore-scale quasi-static equilibrium model
  - **Method:** Verification of the model against available experiments and numerical simulations

# Approach

- Saturated sediment is modeled as a granular medium
- The skeleton can be either unconsolidated or cemented
- Some of the pores are filled with gas-hydrates
- Consequences of hydrate dissociation:
  - Increased pore pressure and reduced effective stress
  - Decrease of the skeleton strength by losing hydrate support
  - Possibly, thermal contraction/expansion

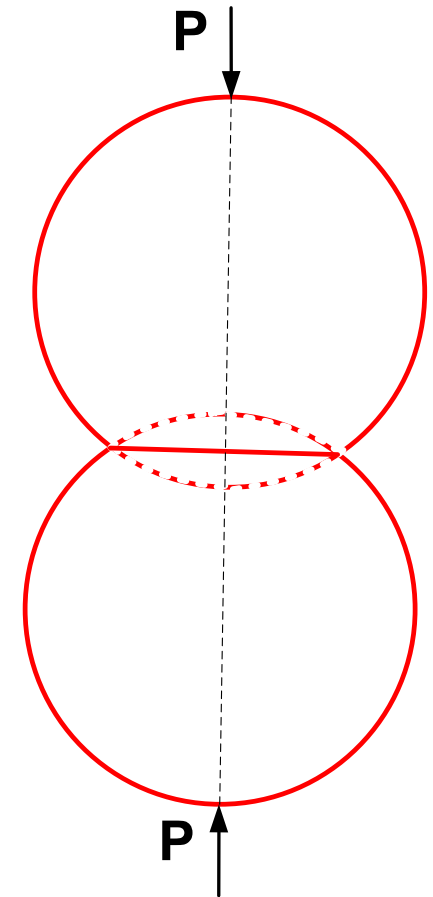
# Grain Pack Properties

- 3D random heterogeneous grain packing
  - Spherical grains, differ in radii and mechanical properties
  - Each grain is homogeneous, isotropic, and linearly elastic
- To be implemented
  - Adhesion or cement at grain-to-grain contacts
  - Different grain shapes



# Contact-Mechanics Model

- Hertz-Mindlin theory of elastic interaction of a grain pair
  - Small deformations localized around a small neighborhood of the contact area
  - Planar circular contact surface
  - Forces and moments at the contact



# Normal Contact: Hertzian Model

Contact force:

$$P_{ij} = \frac{4}{3} E_{ij}^* \sqrt{R_{ij}^*} h_{ij}^{3/2}$$

Elastic strain energy:

$$U_{ij} = \frac{8}{15} E_{ij}^* \sqrt{R_{ij}^*} h_{ij}^{5/2}$$

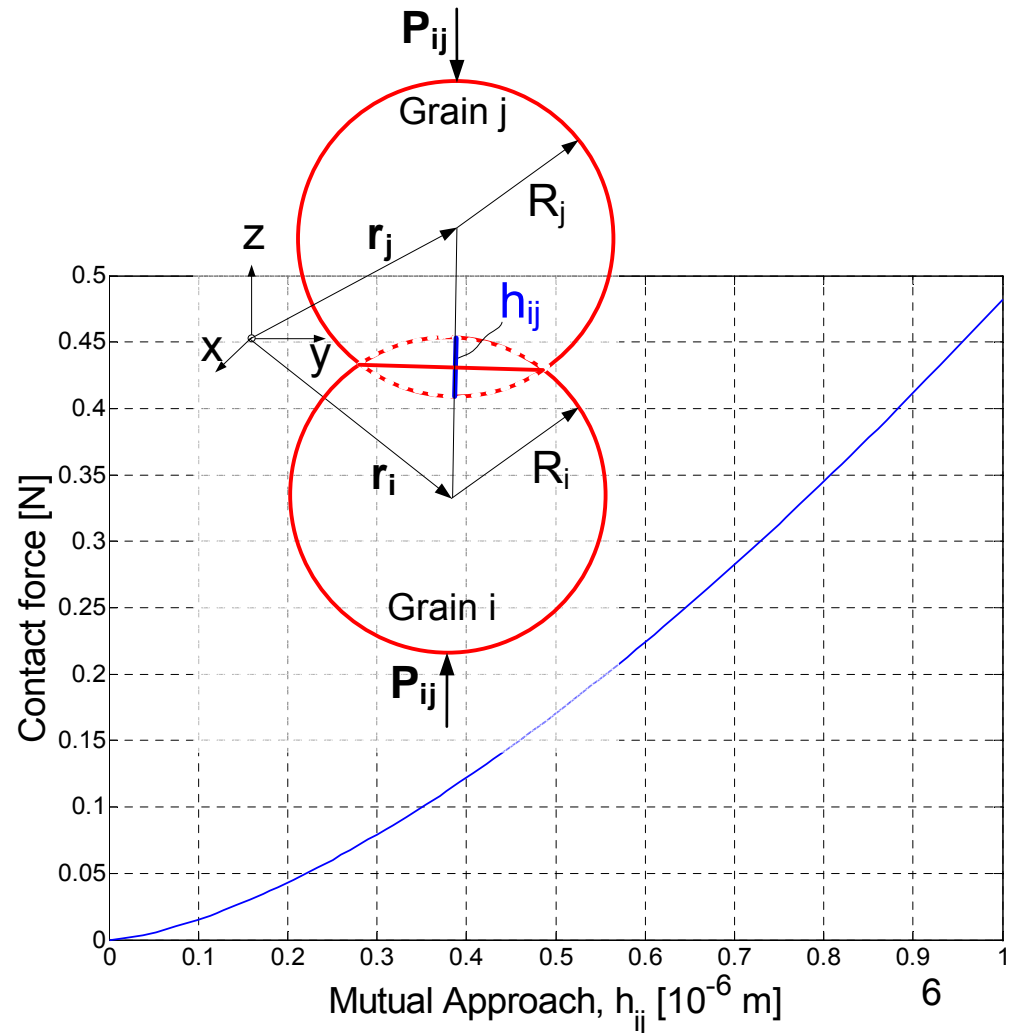
where

$$\frac{1}{E_{ij}^*} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j}$$

$$\frac{1}{R_{ij}^*} = \frac{1}{R_i} + \frac{1}{R_j}$$

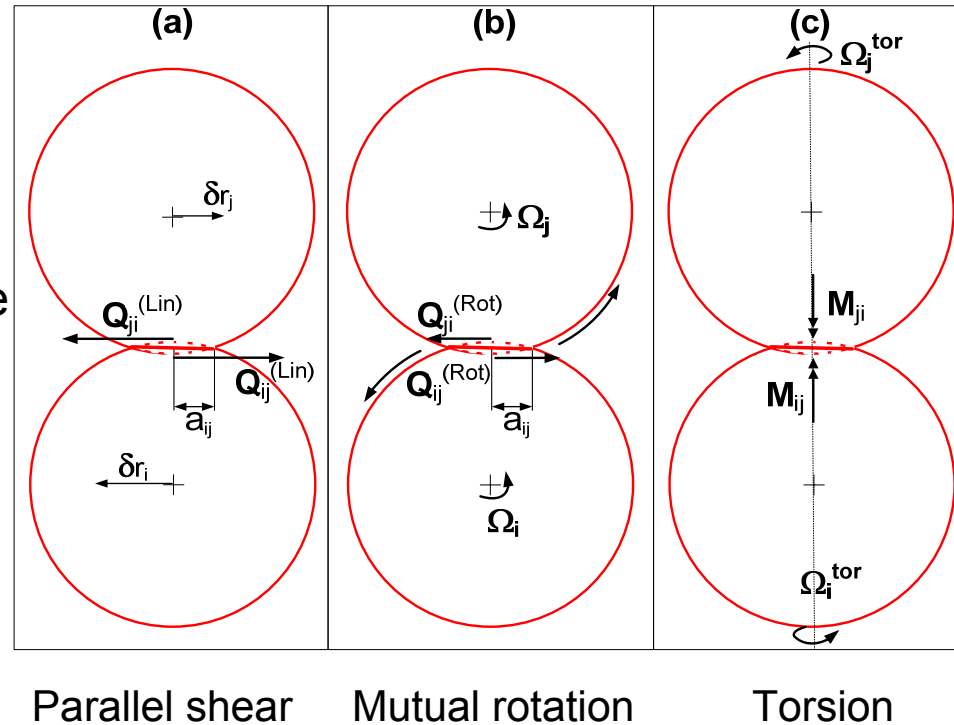
$$h_{ij} = R_i + R_j - \|\mathbf{r}_{ij}\|$$

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$$



# Frictional Contact: Mindlin's Theory

- Linear and rotational displacements introduce tangential tractions and moments
  - Tangential stiffness depends on normal pressure
  - Tangential tractions are path-dependent
  - Partial/complete slip can occur
  - Mindlin theory: normal tractions are not affected by the tangential components
- To eliminate some of these difficulties, we consider:
  - Pre-stressed pack
  - Small deformations
  - Static friction (no slip)



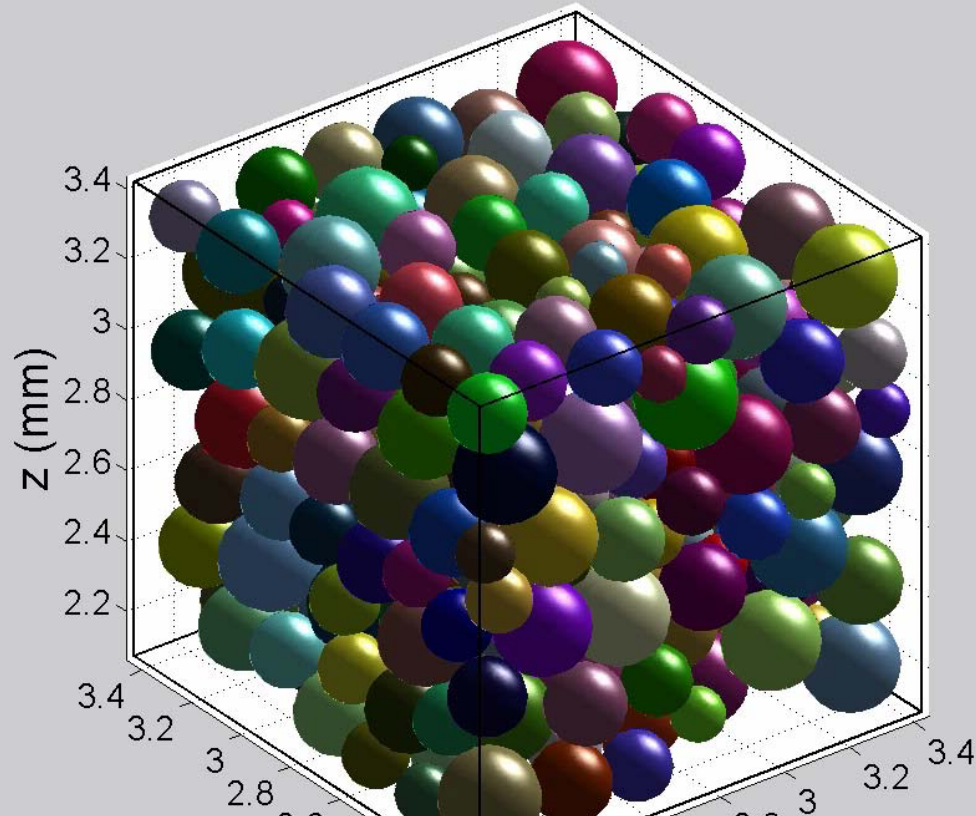
# Effective Properties *via* Simulations

- A grain pack is enclosed in a semi-rigid container
- Boundary conditions = wall displacements
- Macroscopic stress is generated by contact forces
- Quasi-static model: equilibrium configurations
- Equilibrium = minimum total elastic energy
- Conjugate Gradient minimization algorithm
  - Functional to minimize = total energy of the pack
  - Dynamic list of contacts
- Effective moduli using Hooke's law



# Example of Simulation

$\phi = 43.83\%$     $N = 4.16$



Parameters of the pack:

Radii of 0.07-0.13 mm

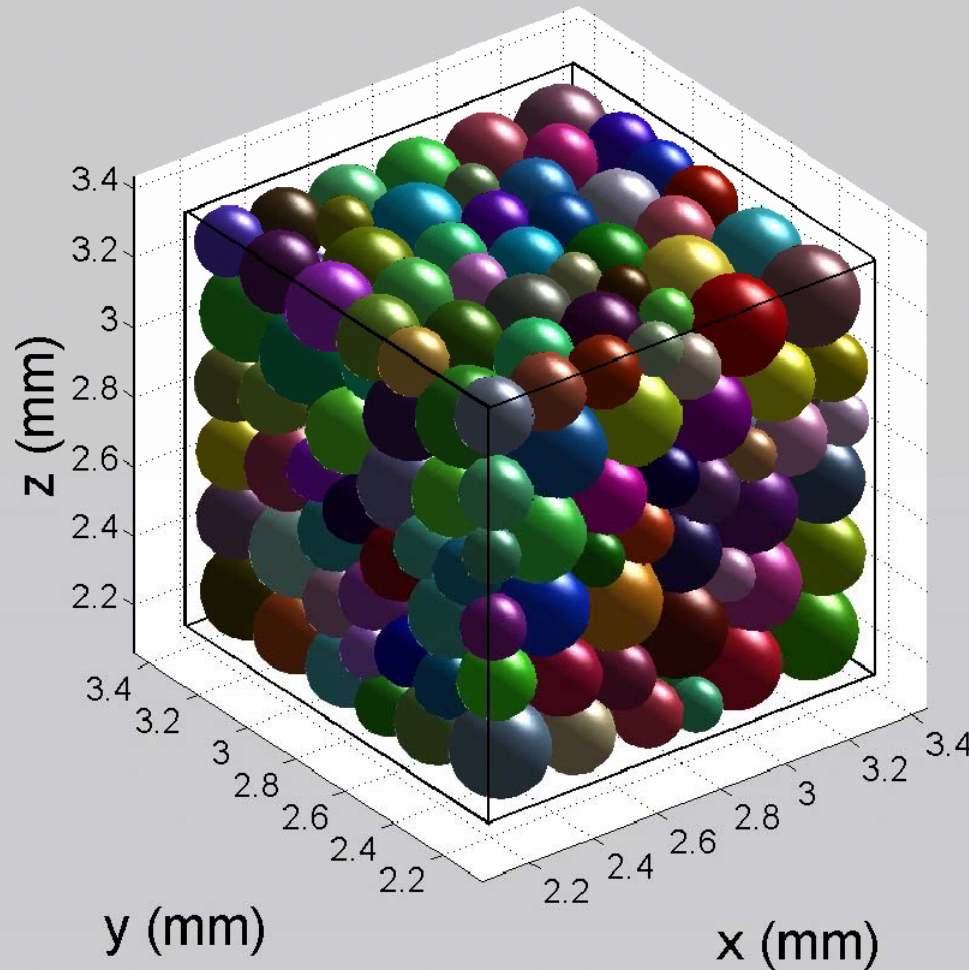
Moduli normally distributed

Mean values:  $E = 100$  GPa,  $\nu = 0.15$

Number of grains: 306 (small pack); 2,740 (large)

# Example of Simulation

$\phi = 22.61\%$  Mean Coordination No. = 8.38



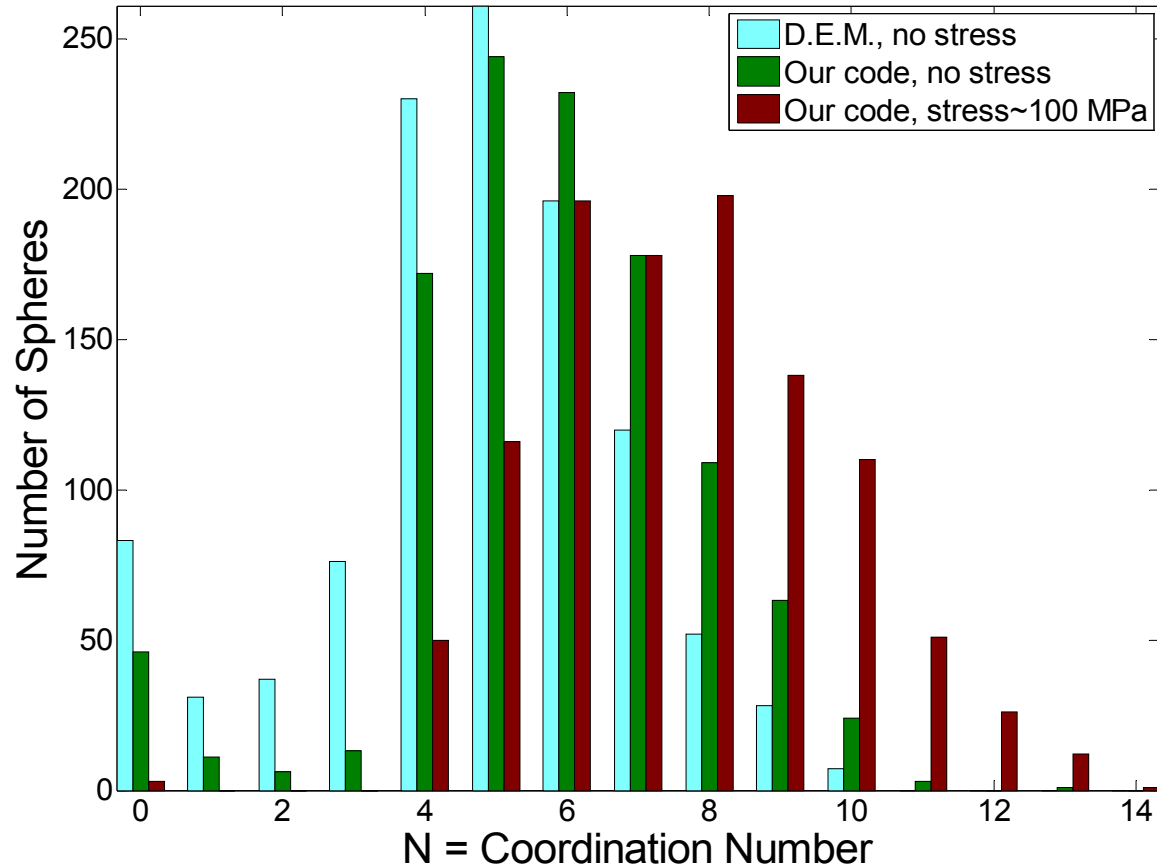
# Modeling Challenges

Difficulties are in the modeling of grain pairs and packs

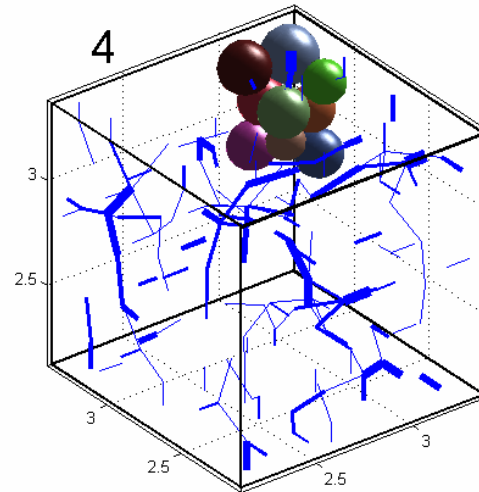
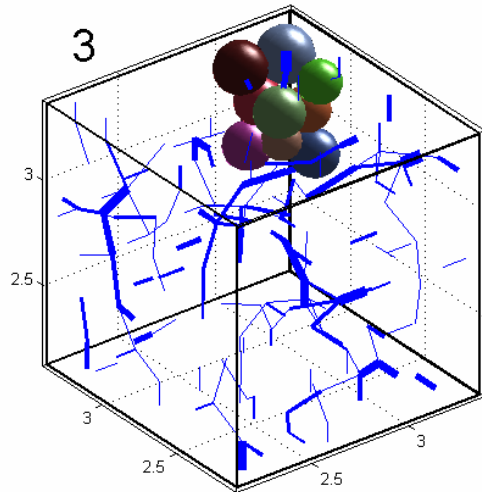
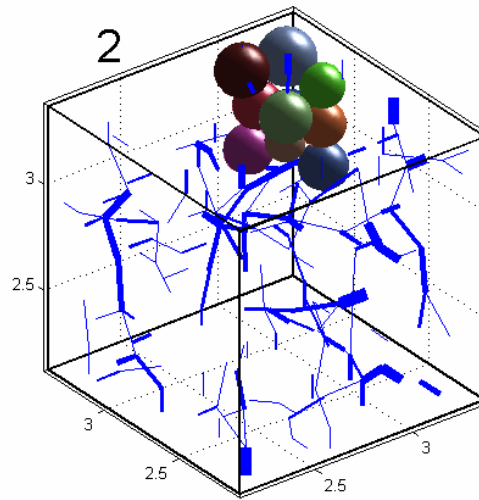
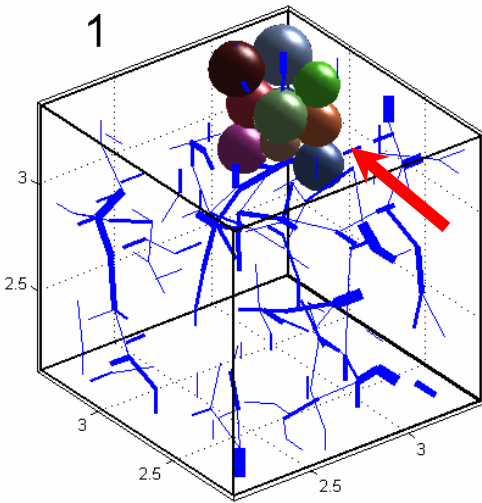
- Grain pairs
  - Nonlinearity of force-displacement relations
  - Stress depends on deformation history
  - Slip, partial or complete
- Grain packs
  - Complex contact geometry
  - Number of contacts, orientation and stiffness vary during the deformation
  - Deformation hysteresis even with fully elastic contacts

# Creating a Stable Pack: Rearrangement vs Deformation

- The minimum coordination number for a stable pack is 3 (with gravity)
- Pack produced by D.E.M. simulations is unstable, i.e. not in equilibrium
- Our algorithm eliminates most unstable structures, by mere rearrangements (no grain deformations)
- As stresses increase, the pack gets more stable



# Non-Linear Response



Local failure in a small pack (306 grains)

- Four consecutive configurations, similar macroscopic strains
- Force chains plotted are the top 10% contact forces
- Line width is scaled with the force magnitude
- Abrupt change from 2→3; brown grain is forced through a constriction

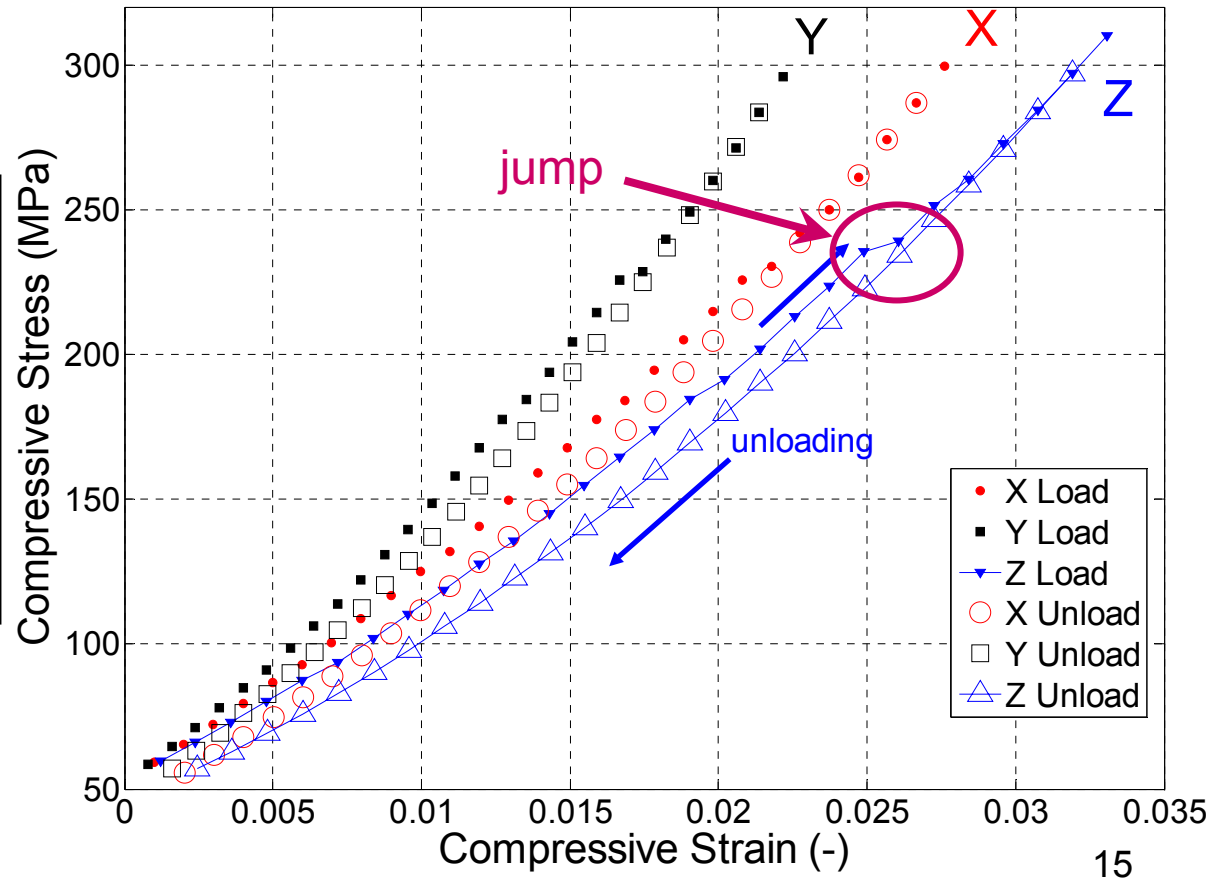
# Non-Linear Response

- At a particular combination of contact forces, some grains experience large, irreversible (inelastic) displacements:
  - Macroscopic stress is reduced by rearrangements of grain clusters
  - Local phenomenon, affecting only a small neighborhood of each rearranged cluster
  - More pronounced in smaller packs
  - Local failure, followed by stiffening of the pack
  - Similar to strain-hardening in metals

# Loading/Unloading Hysteresis

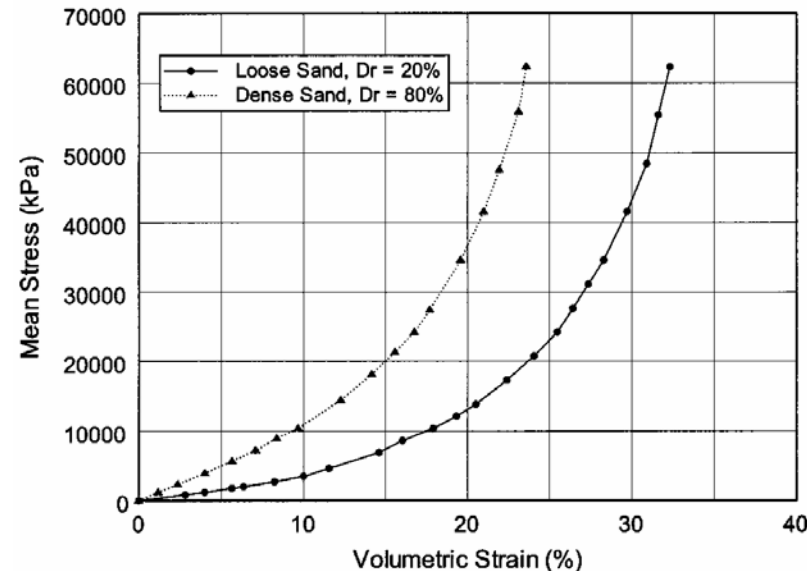
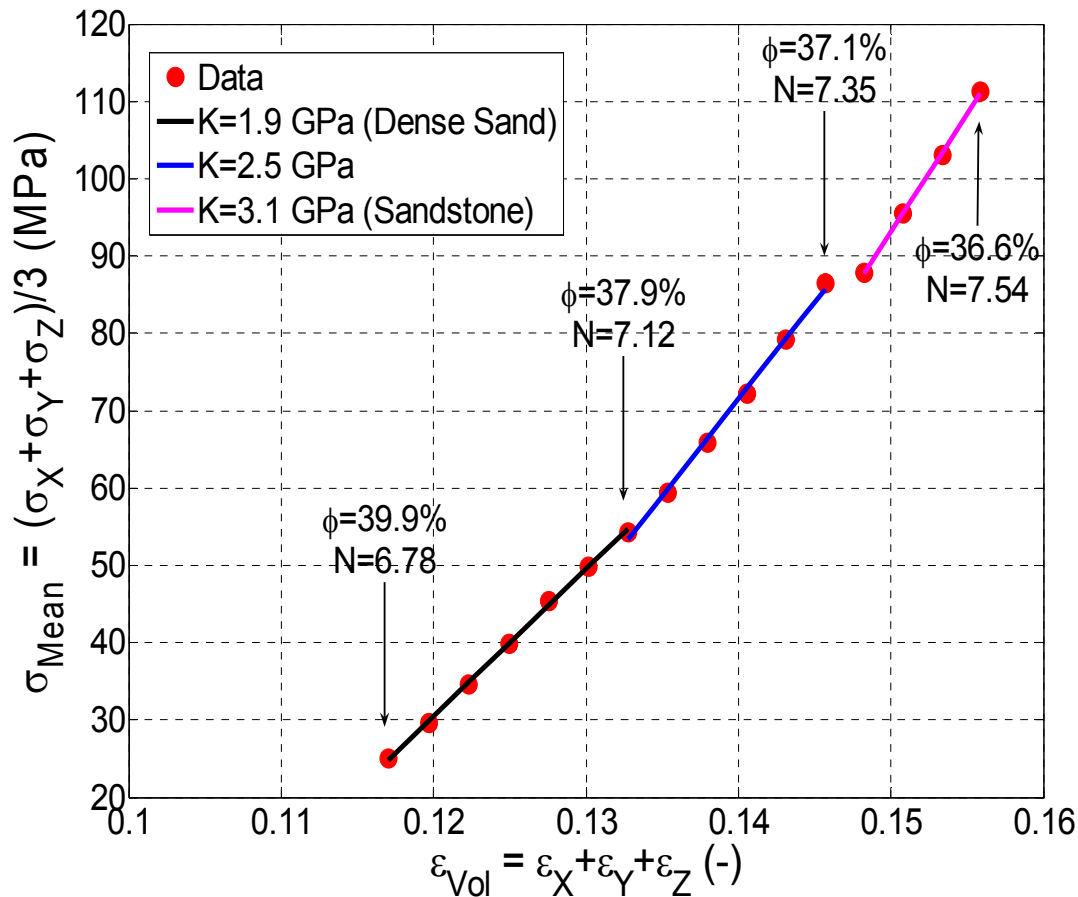
- Elastic response of a single contact
- Inelastic behavior of a grain assembly

Triaxial compression test :  
 $\phi = 35.8 \sim 30.8 \%$ ;  
 $N = 7.2 \sim 8.3$



# Stiffness vs. Compaction

- Bulk modulus  $K$  increases with pack density
- Density increases as porosity  $\phi$  decreases and coordination number  $N$  increases



Hydrostatic compression tests:

Left: Simulations;

Right: Experiment Vesic & Clough, 1968)



# Conclusions

- Granular media exhibit non-linear, path-dependent behavior, even for a frictionless contact model
- Because of grain rearrangement, macroscopic deformation is possible with little deformation of grains
- Hysteretic effects are more pronounced when grain ‘jumps’ occur
- Introduction of frictional contacts increases the path-dependency (hysteretic effects)

# Summary: Phase I

- A grain-scale model of rock
  - ✓ Hertz-Mindlin contact mechanics
  - ✓ Stable equilibrium grain packs
  - ✓ Simulation of loading-unloading hysteresis
  - ✓ Matching published laboratory measurements
  - ✓ Efficient numerical procedure

# Summary

- Grain-scale model provides a tool to estimate effective moduli
  - Irreversibility of deformations is captured
  - Stiffening ( $K \uparrow$ ) with compaction is evident
  - Values of  $K$  match physical experiments
  - Poisson's ratio high, due lack of friction, cement, and simplified grain shapes
  - Efficient algorithm based on conjugate gradient method

# Next...

- Use the already developed model to
  - Incorporate gas-hydrates and investigate
    - Solid skeleton support
    - Pore pressure and effective stress changes
  - Perform ensemble-averaging and investigate sample size effects
- Enhance the existing model by adding
  - Cementation/adhesion between grains
  - Failure criterion for cement
  - More complex grain shapes
  - Large strains

*Thank You!*

