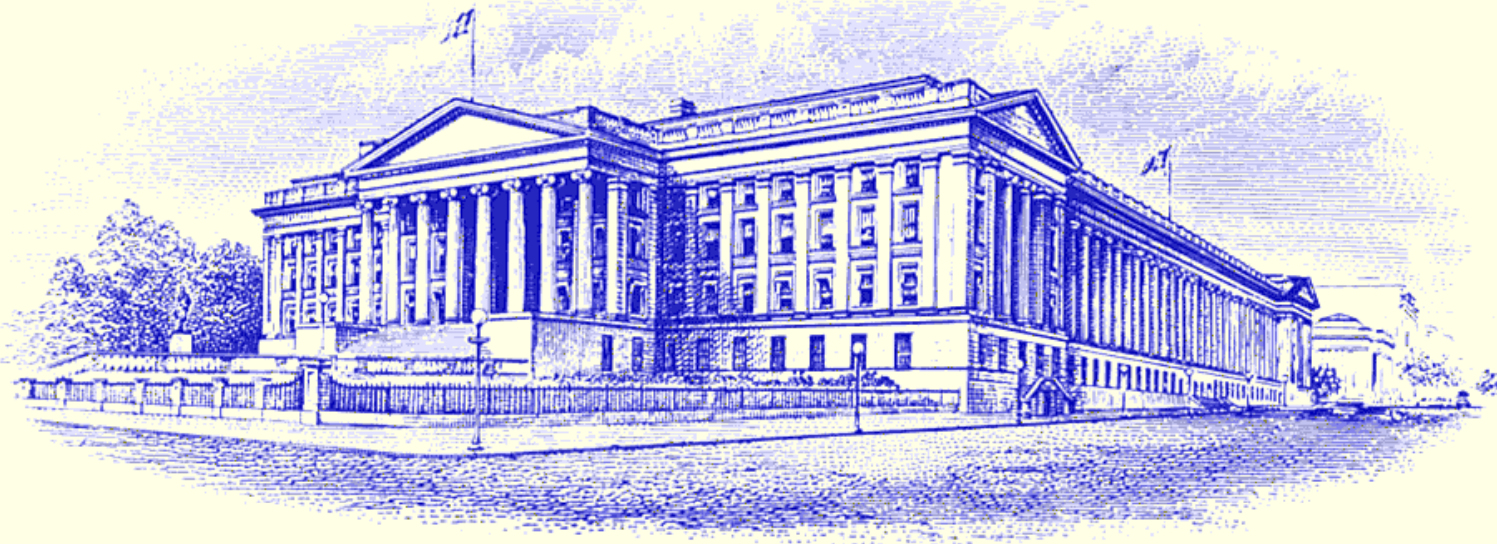


INTRODUCTION TO THE HQM YIELD CURVE



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Introduction

- The High Quality Market (HQM) Corporate Bond Yield Curve for the Pension Protection Act (PPA) uses a methodology developed at Treasury to construct corporate bond yield curves by using extended regressions on maturity ranges.
- This presentation describes the conceptual basis of the curve and the methodology for its construction. (For links to more detailed documentation about the curve, see the last slide.)
- The HQM methodology is general. It has been applied to Treasury inflation-indexed securities (TIPS) and has been used to construct other corporate bond yield curves; a sampling of those curves appears at the end of this presentation.

HQM Methodology Characteristics

The HQM methodology contains features and capabilities that do not appear in other yield curve approaches:

- It uses regression variables.
- It projects yield curves beyond the longest maturity date.
- It makes use of established bond market characteristics to help generate a stable curve.

HQM Methodology: Regression Variables

- The HQM methodology has the special capability of combining regression variables with the yield curve. The regression terms adjust for the particular attributes of individual bonds.
- The HQM yield curve represents the high quality corporate bond market, i.e., corporate bonds rated AAA, AA, or A. The HQM curve contains two regression terms. These terms are adjustment factors that blend AAA, AA, and A bonds into a single HQM yield curve that is the market-weighted average (MWA) quality of high quality bonds.
- Other yield curve approaches were typically developed for Treasury securities, and they do not take into account the distinctive characteristics of corporate bonds. Other approaches do not have regression variables and cannot blend bonds of different qualities.

HQM Methodology: Projections

- The HQM methodology projects yields beyond 30 years maturity (out to 100 years maturity to get discount rates for long-dated pension liabilities).
- The methodology ensures that the projections are consistent with yields before 30 years maturity and with long-term investment returns available in the market.
- Other yield curve approaches generally stop at 30 years maturity and contain no provision for projection.

HQM Methodology: Established Market Views

- The HQM methodology is derived from basic hypotheses about corporate bond markets. This informs how the parameters that underpin the analysis are set.
- Other yield curve approaches can be arbitrary inasmuch as they are not informed by basic market hypotheses. This makes it difficult to choose among them.

Developing the HQM Yield Curve: Summary

- The HQM yield curve is a spot yield curve.
- The HQM yield curve is calculated from an estimated discount function.
- The discount function is estimated using bond prices and other data and constraints consistent with relationships that hold in forward rates.
- Spot rates from the curve are used to value pension liabilities.

The Spot Yield Curve

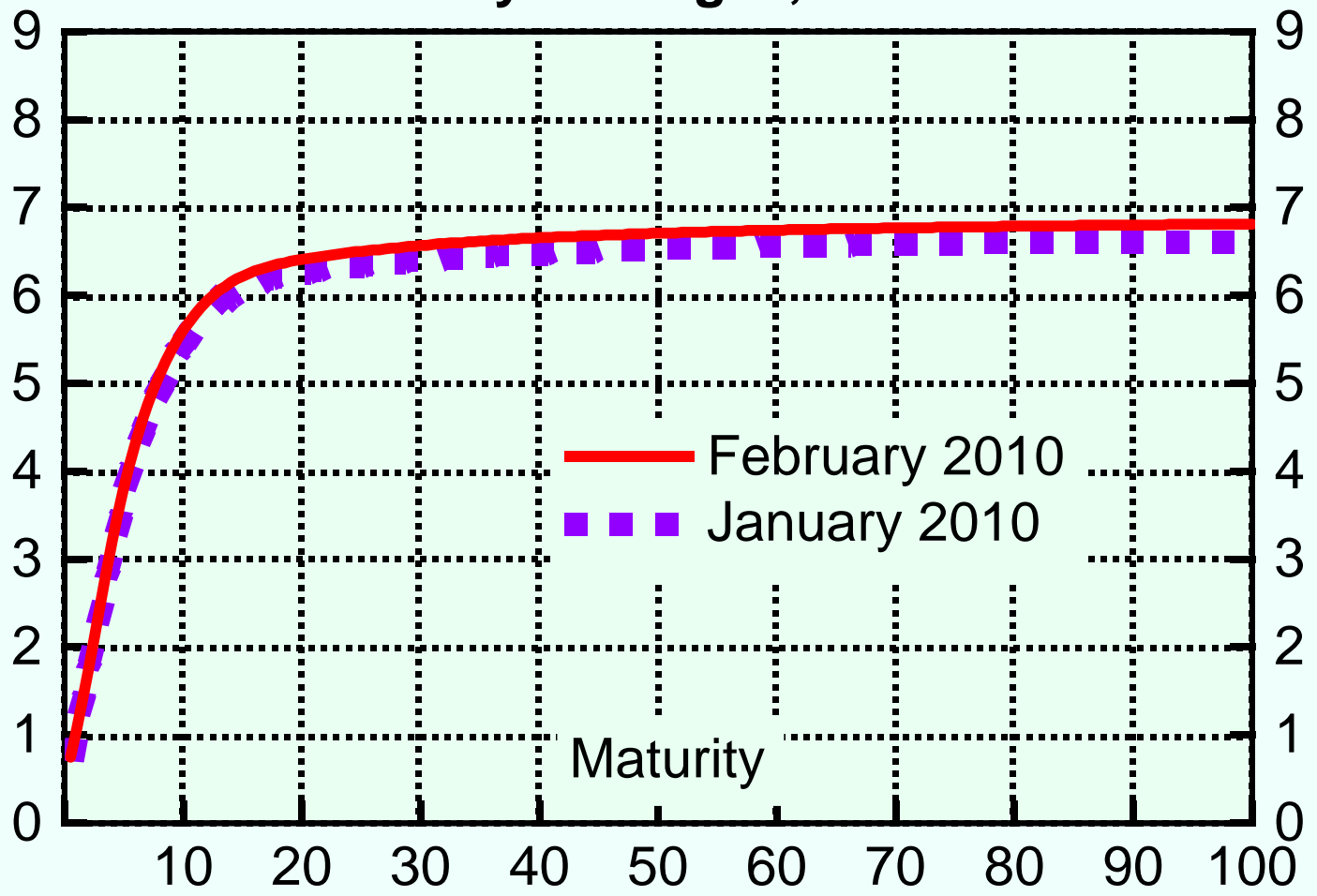
- The spot rate for any maturity is the yield on a bond that provides a single payment at that maturity. This is a zero coupon bond. Because each spot rate pertains to a single cash flow, it is the relevant interest rate concept for discounting a pension liability at the same maturity.
- The HQM Corporate Bond Yield Curve is a spot yield curve: it gives the spot rate for each maturity at the half-year maturities $\frac{1}{2}$ year up through 100 years, for a total of 200 spot rates.
- Zero coupon corporate bonds may not be generally available in the market, but spot rates can still be computed, because they can be inferred from market behavior.

The Spot Yield Curve, continued

- Each HQM yield curve pertains to a specific point in time. The curve is calculated late in the day for each business day, and the monthly HQM spot rates are averages of rates for all the business days of the month. Segment rates required by the PPA are derived from the monthly averages.
- The quality of the HQM curve is the market-weighted average (MWA) quality of the AAA, AA, and A bonds used to compute it.
- The next chart plots the monthly average spot rates out through 100 years maturity from the HQM yield curves for January and February of this year. The February curve is above January beyond the shortest maturities.

HQM SPOT YIELD CURVES

Monthly Averages, Percent



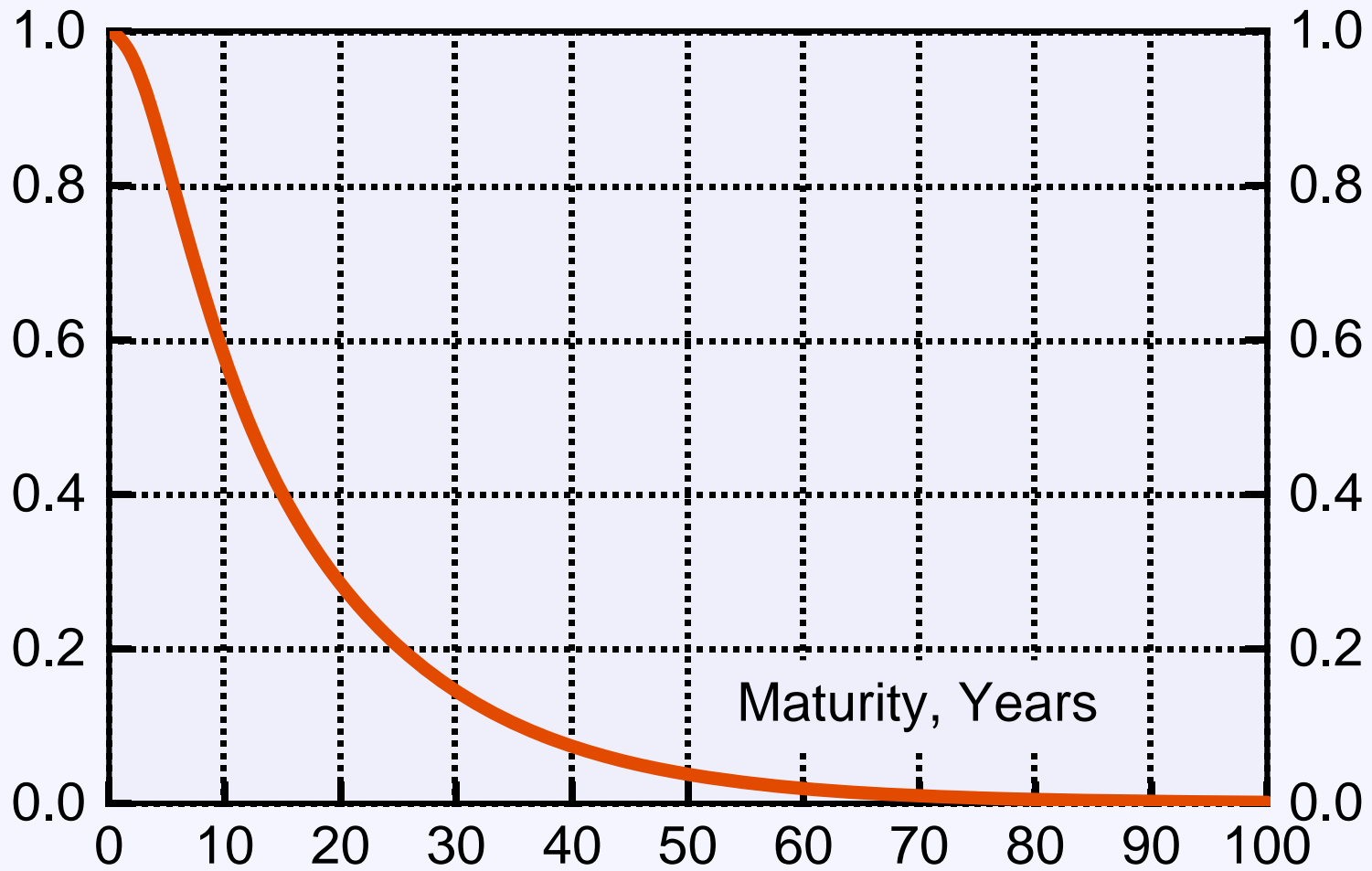
The Discount Function

- The HQM methodology uses the discount function, a well-known concept in finance.
- The discount function $\delta(\tau)$ gives for each maturity τ in years the present value of \$1 to be received at that maturity, that is, the amount that must be invested now to get \$1 τ years in the future.
- Markets do not provide an explicit discount function. But it can be extracted from market behavior through the yield curve, as in the case of spot rates. Estimating the discount function is a key goal of the HQM methodology.
- The discount function refers to the same point in time as the yield curve. It represents future payments whose credit quality is equal to the market-weighted average credit quality of high quality bonds.

The Discount Function, continued

- The values of the discount function depend upon many factors, including the purposes for which market participants borrow and lend, their expectations for the future, and their attitudes toward risk.
- The next chart illustrates the typical shape of recent discount functions. The discount function is unity at maturity zero, because the present value of \$1 received immediately is \$1, and normally decreases, because a payment received later in time is worth less today than one received earlier.
- The discount function says that $\delta(10)$ is the amount that must be invested now to get \$1 after 10 years. From the chart, this is about 58 cents. The value $\delta(20)$ falls to 28 cents, and $\delta(30)$ is 14 cents.

HQM DISCOUNT FUNCTION



The Spot Rate

- Once the discount function is known, the spot rate can be readily computed. Specifically, the discount function implies that an investment of $\delta(\tau)x$ gives a single payment of x at maturity τ , just like a zero coupon bond. The spot rate at maturity τ is the yield of such a bond.
- The formula for getting the spot yield curve $y(\tau)$ from $\delta(\tau)$ is as follows for maturities of at least $\frac{1}{2}$ year:

$$y(\tau) = 200 \times (1/\delta(\tau)^{\frac{1}{2\tau}} - 1)$$

- For example, at 10 years maturity, using the discount function from the last chart:

$$y(10) = 200 \times (1/0.5758^{\frac{1}{20}} - 1) = 5.60$$

The Price of a Bond

- To estimate the discount function, the price of a bond must be linked to the discount function.
- The price of a bond is the sum of three components: the present values of the bond's future cash flows discounted by the discount function, plus any adjustment factors for the bond's specific characteristics, plus a random error for characteristics of the bond that are not adjusted.
- In the case of the HQM curve, there are two adjustment factors, described later, that adjust each bond for the fact that the discount function pertains to market-weighted average (MWA) quality, while each individual bond is either AAA, AA, or A quality.

The Price of a Bond, continued

The formula for the price p of a bond is:

$$p = \sum_{i=1}^n \delta(\tau_i) c_i + \sum_{j=1}^m \zeta_j x_j + \varepsilon$$

where

p is the price of the bond (including accrued interest),

c_i for $i = 1, \dots, n$ are the n cash flows from the bond to be received at maturities τ_i ,

ζ_j for $j = 1, \dots, m$ are the m coefficients for the m regression variables x_j , giving m adjustment factors $\zeta_j x_j$, and

ε is the random error.

The Price of a Bond, continued

- With this price formula, the yield curve is estimated by choosing an appropriate set of bonds and calculating the discount function and adjustment factors which give the best fit of the formula to all the bonds. The spot yield curve is computed from the resulting discount function.
- The description of the discount function so far is not specific enough for estimation. To estimate, a precise form of the discount function must be chosen.
- The main source of differences among yield curve approaches is their different choices of discount function forms. The HQM methodology bases the discount function form on market maturity ranges.

The Forward Rate

- The concept of the forward rate is useful for picking a form for the discount function. The forward rate and the discount function are equivalent in that one can be derived from the other, and the forward rate is easier to work with.
- The forward rate is straightforward: for each maturity τ , the forward rate is the interest rate obtained by extending an investment maturing at τ by a small amount of time.
- The extended investment is in essence made up of two investments: the initial investment that yields a return through maturity τ , and the reinvestment of this return for the extension period. The forward rate is the future interest rate on the reinvestment.

The Forward Rate, continued

- An example of the forward rate is the calculation of future yields often done for Treasury securities. The February average 1-year yield in the Treasury market was 0.35 percent, and the 2-year yield was 0.86 percent. These yields imply that the 1-year forward yield 1 year in the future was about 1.38 percent. That is, an investor would be indifferent between buying a 2-year note and buying a 1-year note plus a second 1-year note at this yield a year from now.
- The forward rate for the HQM curve is analogous to this example, although it is defined as a short-term yield and applies to corporate bonds. In general, similar to the discount function, the market does not provide forward rates.

The Forward Rate, continued

- The forward rate curve $\xi(\tau)$ gives the forward rate for each maturity. Relative to the discount function, the forward rate is given by the following formula:

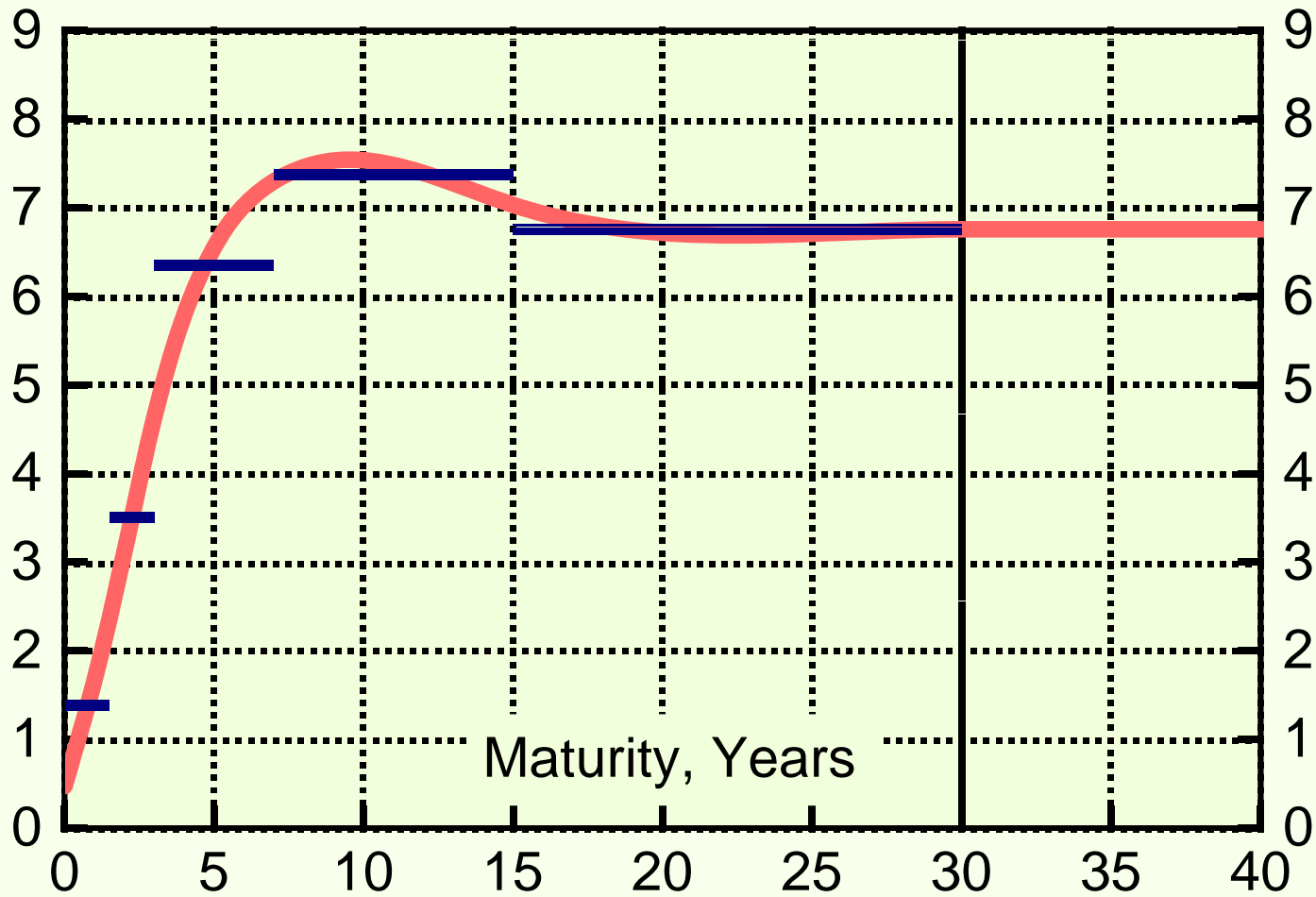
$$\xi(\tau) = -\frac{d\delta(\tau)}{d\tau} \frac{1}{\delta(\tau)}$$

- The forward rate is higher at a given maturity when investors who are trading at that maturity are less eager to lend based on their views of uncertainty and their expectations about the market, while borrowers are more eager to borrow based on their perceptions. The forward rate summarizes market views at each maturity in a single number.

The Forward Rate, continued

- The forward rate is especially useful because it can be compared across maturities, and so it can be used as a basis for developing a functional form for estimating the discount function.
- The next chart plots the forward rate curve up through 40 years maturity from the discount function in slide 13. (The averages of the forward rate in maturity ranges 0-1.5, 1.5-3, 3-7, 7-15, and 15-30 years are included, as discussed later.) The chart shows that the forward rate curve rose to a hump around 10 years maturity, then flattened out, and was constant beyond 30 years maturity.

HQM FORWARD RATE CURVE



Maturity Ranges

- Corporate bond trades tend to divide into maturity ranges. Prices of bonds in each range are related, because the traders who have chosen to trade in that range typically have shared preferences and expectations.
- As a consequence, the forward rates in each maturity range are related too, because they reflect the views of traders.
- The HQM methodology uses five maturity ranges, delineated by the maturity points 0, 1.5, 3, 7, 15, and 30 years maturity.

Maturity Ranges, continued

- The range 0 to 3 years reflects short-term trading, and there is enough activity that it should be further divided in two with a point at 1.5 years.
- The range 3 to 7 years contains bonds of somewhat longer term centering around 5 years maturity. The range 7 to 15 years contains the bulge of bonds frequently seen around 10 years maturity. The last range 15 to 30 years is for bonds with the longest maturities.
- For maturities beyond 30 years, the forward rate is projected from the five ranges, as described later.
- The five ranges are depicted in the previous chart on slide 22 as the maturities corresponding to the five forward rate averages.

Averages in the Ranges

- Given this choice of ranges, the forward rate in each range as a first approximation could be simply taken to be a constant, reflecting its average in the range.
- The previous chart on slide 22 shows what these averages would look like.
- However, the chart indicates that there is a significant amount of movement in the forward rate curve that is not captured by the averages and that needs to be included for the curve to be accurate.
- Moreover, the averages are not connected, and in order to get a well-behaved forward rate curve, the values of the curve in the ranges must join together smoothly.

Averages in the Ranges, continued

- A smooth forward rate curve can be obtained without departing too far from averages, and without introducing excessive volatility, by using a cubic polynomial to describe the curve in each range instead of a constant, and by smoothly joining the polynomials together at the end of each range.
- The forward curve specified in this manner is a cubic spline with knots at the six maturity points that delineate the ranges.
- The use of a cubic spline arises naturally as a parsimonious way to fit the components of the curve smoothly across the ranges. In particular, the spline is not imposed upon the forward curve without justification.

Averages in the Ranges, continued

- Moreover, the choice of fixed knots for the spline based on maturity ranges increases significantly the stability of the yield curve estimates over time.
- The next step is to estimate the spline and get the forward rate curve, the discount function, and finally the spot yield curve. However, before doing this, it is necessary to impose three constraints upon the forward curve, including one constraint at the short end and two at the long end.

Constraint in the Nearest Range

- Because there are no bonds with zero maturity, the forward rate curve has no anchor at the zero point in the first maturity range.
- For this reason, an approximate linear constraint is imposed on the movement of the forward rate at the earliest maturities. This is the constraint on the forward rate curve at the short end.
- This constraint sets the second derivative of the forward curve to zero at the beginning:

$$\frac{d^2\xi(0)}{d\tau^2} = 0$$

The Long-Term Forward Rate

- Forward rates can be estimated up through 30 years maturity, the maximum typical maturity of corporate bonds. However, for pension liability discounting it is necessary to derive spot rates up through 100 years maturity. Consequently, the forward rate curve has to be projected out through 100 years maturity.
- Because there are usually not enough data to estimate accurately the movements in the forward rate beyond 30 years maturity, the projected forward rate is set to a constant, which is an estimate of the average forward rate beyond 30 years maturity. This constant is called the long-term forward rate.

The Long-Term Forward Rate, continued

- Two constraints are needed for the last maturity range 15 to 30 years to derive the constant long-term forward rate and to ensure well-behaved projections beyond 30 years maturity.
- The first constraint makes the forward rate curve flatten out at maturity 30 years, and thereby connect smoothly with the constant long-term forward rate at 30 years.
- As a formula, this constraint sets the first derivative of the forward curve to zero at 30 years maturity:

$$\frac{d\xi(30)}{d\tau} = 0$$

The Long-Term Forward Rate, continued

- In addition, there is the critical question of how to determine the constant long-term forward rate.
- The HQM methodology postulates that forward rates are largely influenced by market aversion to risks that cannot be hedged and that generally grow over time.
- Based on these considerations, the HQM methodology assumes that the long-term forward rate is determined by the same factors that affect forward rates in the 15- to 30-year maturity range, since that range is likely sufficiently distant in time to reveal underlying long-term attitudes toward risk.

The Long-Term Forward Rate, continued

- Therefore, the constant long-term forward rate from 30 years maturity up through 100 years maturity is constrained to be the average forward rate in the 15- to 30-year maturity range. This is the third and final constraint on the forward rate curve.
- As a formula, this constraint is:

$$\frac{\int_{z=15}^{30} \xi(z) dz}{15} = \xi(30)$$

The Adjustment Factors

- The adjustment factors are estimated along with the discount function. This is a unique aspect of the HQM methodology. The HQM yield curve has two adjustment factors.
- The HQM curve needs the adjustment factors because the discount function pertains to market-weighted average AAA, AA, and A quality (MWA quality).
- Therefore, the price of each bond must be adjusted to reflect what it would be if the bond were of MWA quality. The adjustment makes the price consistent with the MWA quality of the discount function.

The Adjustment Factors, continued

- The adjustment factors enable the entire set of AAA, AA, and A bonds to be blended in the estimation, rather than estimating separate curves for each quality rating. The blending produces a more stable result, because the individual markets may be small and variable (especially the AAA market).
- Blending the bonds is also useful because yield curves for the three qualities tend to be related. Blending makes use of the common relationships among the individual curves to derive a more robust overall average yield curve.
- The first adjustment factor adjusts AAA and AA prices to be market-weighted average AAA-AA prices. The second factor adjusts average AAA-AA prices and A prices to be MWA prices for all three qualities.

The First Adjustment Factor

- This factor assumes that for a bond of maturity τ , the difference between the price of the bond at AAA quality and the price at AA quality is $\zeta_1\tau$, that is, a constant amount ζ_1 multiplied by maturity τ .
- The reason for multiplying by maturity is that the higher rating is like insurance against risk, and the longer the maturity, the greater the amount of the insurance. The coefficient ζ_1 has been around 50 basis points recently.
- The first factor also uses the value ω_1 , which is the total par amount outstanding of AA bonds as a fraction of AAA plus AA par amounts outstanding. This value has been around 59 percent recently.

The First Adjustment Factor, continued

- The value ω_1 can be interpreted as the distance from AAA quality to market-weighted AAA-AA quality as a fraction of the distance from AAA to AA quality.
- Each AAA bond price is adjusted to market-weighted AAA-AA quality by subtracting $\zeta_1\omega_1\tau$ from the price. Subtracting the full amount $\zeta_1\tau$ would adjust the AAA price all the way down to AA. To adjust the AAA price down to AAA-AA quality, the full amount is multiplied by the fraction ω_1 .
- Similarly, each AA bond price is adjusted to market-weighted AAA-AA quality by adding $\zeta_1(1-\omega_1)\tau$ to the price.
- In the estimation, the regression variable x_1 is defined as $\omega_1\tau$ for AAA bonds, $(\omega_1-1)\tau$ for AA bonds, and zero for A bonds. The value ζ_1 is the regression coefficient to be estimated.

The Second Adjustment Factor

- The second adjustment factor transforms the prices of bonds of market-weighted AAA-AA quality and bonds of A quality into prices of MWA quality. It is entirely analogous to the first adjustment factor.
- For the second factor, the difference between the price of a bond at market-weighted AAA-AA quality and the price at A quality is $\zeta_2\tau$; again, the longer the maturity, the greater the amount of insurance against risk. The coefficient ζ_2 has been around 31 basis points recently.
- This factor also uses the value ω_2 , which is the total par amount outstanding of A bonds as a fraction of AAA, AA, and A par amounts outstanding. This value has been around 67 percent recently.

The Second Adjustment Factor, continued

- As before, the value ω_2 is the distance from AAA-AA quality to MWA quality as a fraction of the distance from AAA-AA to A quality.
- Therefore, a AAA-AA bond price is adjusted to MWA quality by subtracting $\zeta_2\omega_2\tau$ from the price. Similarly, an A bond price is adjusted to MWA quality by adding $\zeta_2(1-\omega_2)\tau$ to the price.
- In the estimation, the regression variable x_2 is defined as $\omega_2\tau$ for AAA and AA bonds, and $(\omega_2-1)\tau$ for A bonds. The value ζ_2 is the regression coefficient to be estimated.

Estimation

- The cubic spline with its three constraints and the two adjustment factors altogether contain seven parameters to be estimated.
- The estimation is done by using least squares to fit the price formula on slide 16 to the bond data.
- The next sections discuss the data and the weighting scheme that is applied to the data before estimation.

Data

- The HQM yield curve uses data from a set of high quality corporate bonds, rated AAA, AA, or A, that accurately represent the high quality corporate bond market.
- The bonds are selected from the universe of all high quality bonds based upon several types of characteristics.
- The HQM curve is meant to represent all bonds in the market. So a bond that is priced and available for trade is included even if its yield appears to be an outlier relative to other bonds. Such a bond is excluded only if there is some reason other than unusual yield for the exclusion.

Data, continued

- The basic type of selected bond is analogous to conventional Treasury coupon issues: a bond that pays a fixed semiannual nominal coupon denominated in U.S. dollars until maturity, when the principal is returned.
- Bonds that differ from the basic type are generally excluded. Bonds with floating coupons provide little if any information about future rates of return, since their own interest payments are changing over time. Convertible bonds do not have a clear price for cash flows because the price depends on equity values.
- Bonds must be issued by corporations. Asset-backed bonds are excluded, as well as bonds issued by U.S. sponsored agencies.

Data, continued

- The bond set covers maturities up through 30 years. Bid prices are used. Maturities below 1 year are filled in by Federal Reserve commercial paper rates.
- Each bond must meet a minimum size threshold in terms of par amount outstanding. The current minimum is \$250 million.
- Callable bonds are excluded at present, unless the call feature is make whole. Puttable bonds and bonds with sinking funds are also excluded.
- The number of corporate bonds that fulfill these specifications is sufficiently large at all maturities that there is no problem in computing the yield curve.

Weighting

- Before estimation, the HQM methodology applies weights to the bond data in two stages.
- The first stage weights the bonds by par amounts outstanding and weights the commercial paper rates equally with the bonds. This is done because bonds with larger par amounts outstanding are more important, and because the commercial paper rates must anchor the short end of the curve.
- In the second stage, for bonds with duration greater than unity, the bond data are divided by the square root of their (Macaulay) duration. The second stage corrects for the greater volatility of bonds with higher duration.

Results

- The following charts present sample results for the HQM yield curve for a single day, the last business day of 2009, December 31, 2009. As noted earlier, the published HQM curve for December is a monthly average.

Characteristics of the bonds included in the December 31 set:

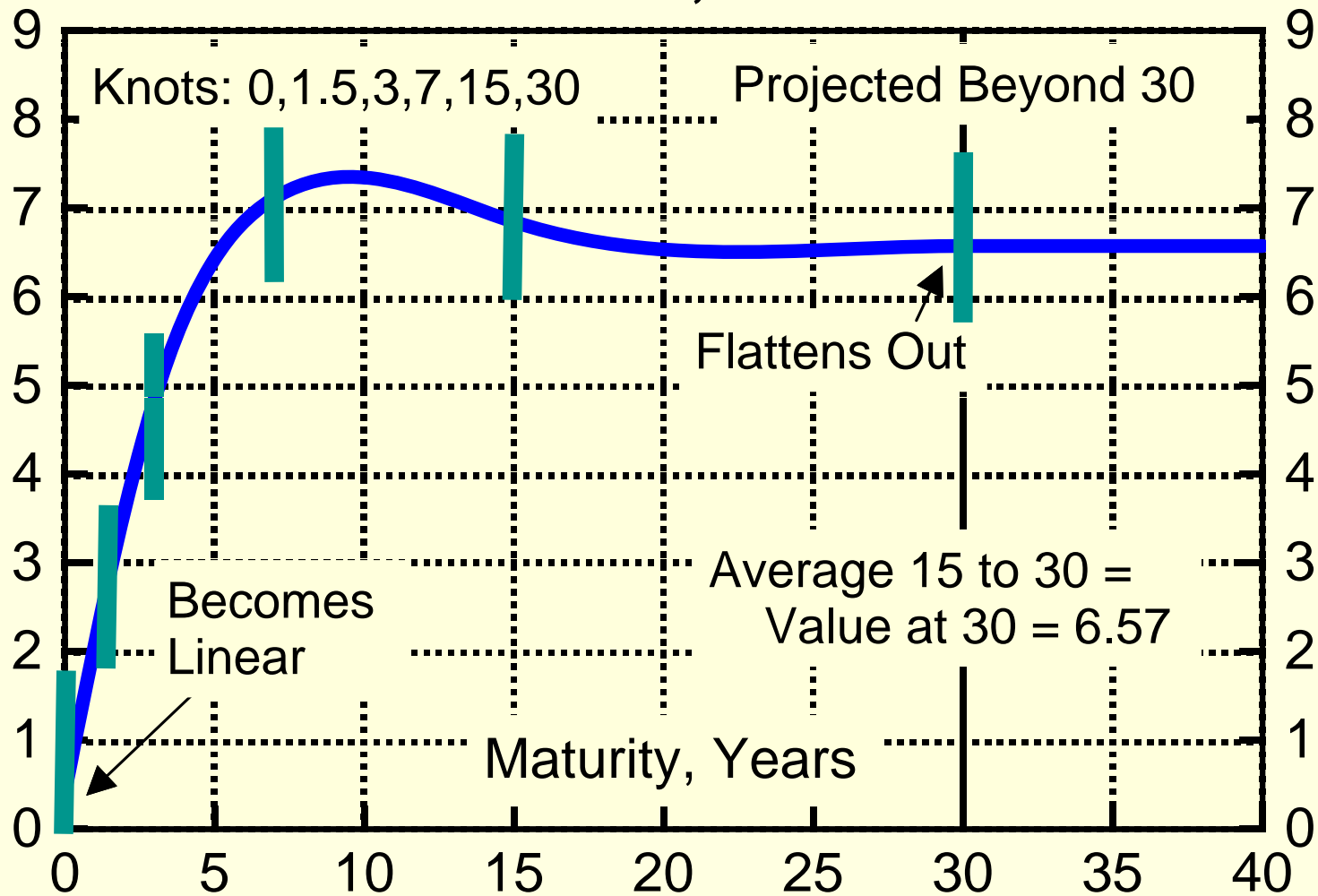
| Rating | Number of Bonds | Maturity Range (years) | Number of Bonds |
|------------------|-----------------|------------------------|-----------------|
| AAA | 101 | 0 – 1.5 | 147 |
| AA | 253 | 1.5 – 3 | 274 |
| A | 1,211 | 3 – 7 | 464 |
| Commercial paper | 7 | 7 – 15 | 327 |
| Total | 1,572 | 15 – 30 | 353 |

Results, continued

- The first chart shows the forward rate curve for December 31, 2009 plotted through 30 years maturity, and then projected out through 40 years maturity.
- The chart depicts the five cubics for the five maturity ranges, delineated by the vertical lines at the maturities 0, 1.5, 3, 7, 15, and 30 years. The cubics are knotted together to get a smooth curve. The near constraint dampens the curve at maturity zero.
- The forward curve is fixed at 30 years maturity and beyond at the long-term forward rate of 6.57 percent, which is given by the average of the forward rate in the 15–30 year maturity range. The second constraint flattens the forward curve at 30 years maturity.

HQM FORWARD RATE CURVE

12/31/2009, Percent

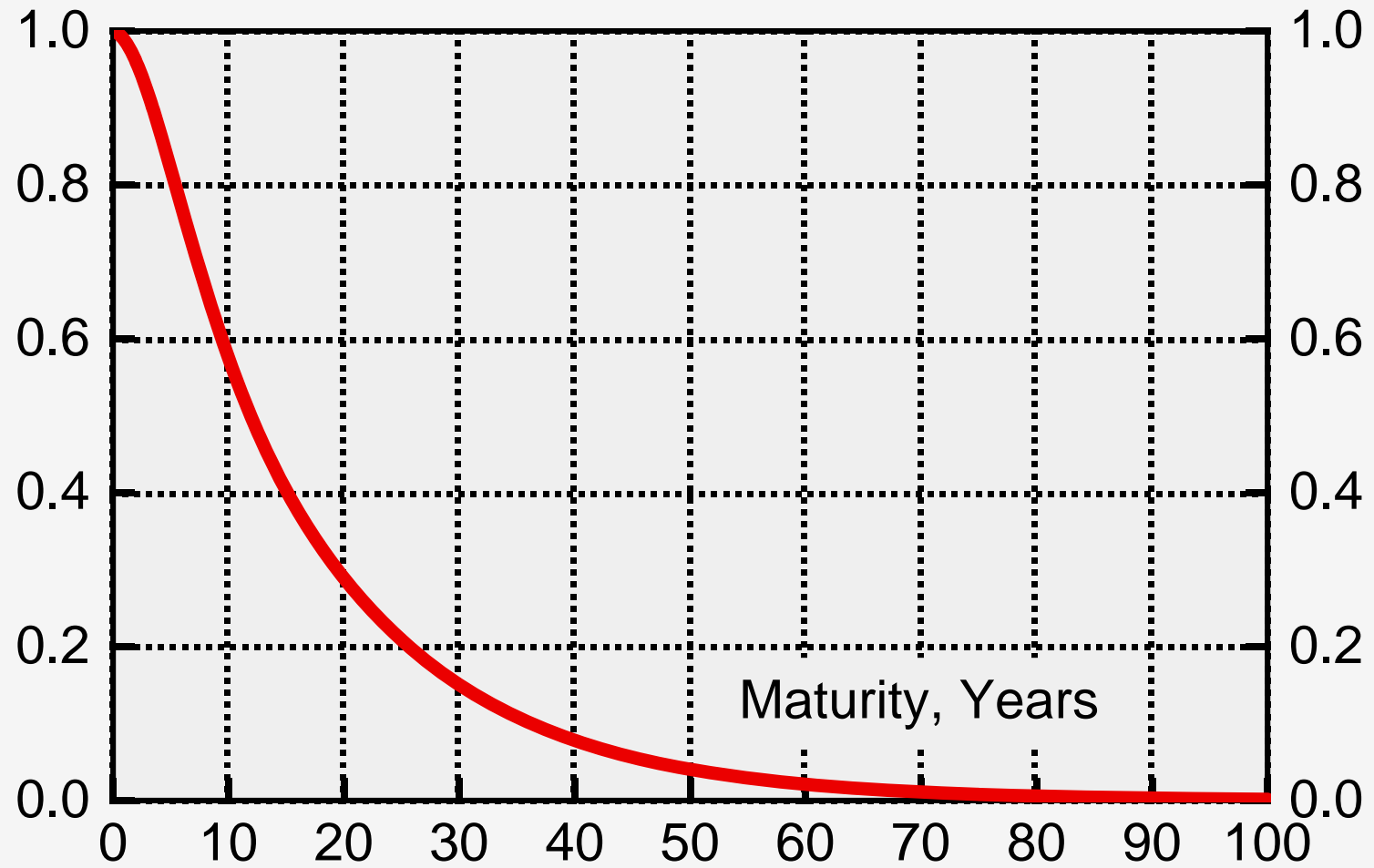


Results, continued

- The following chart plots the discount function for December 31, 2009.
- The chart shows that the discount function equals unity at maturity zero, because the present value of \$1 received immediately is \$1, and it declines, because the present value of a payment received later in time is smaller.

HQM DISCOUNT FUNCTION

12/31/2009

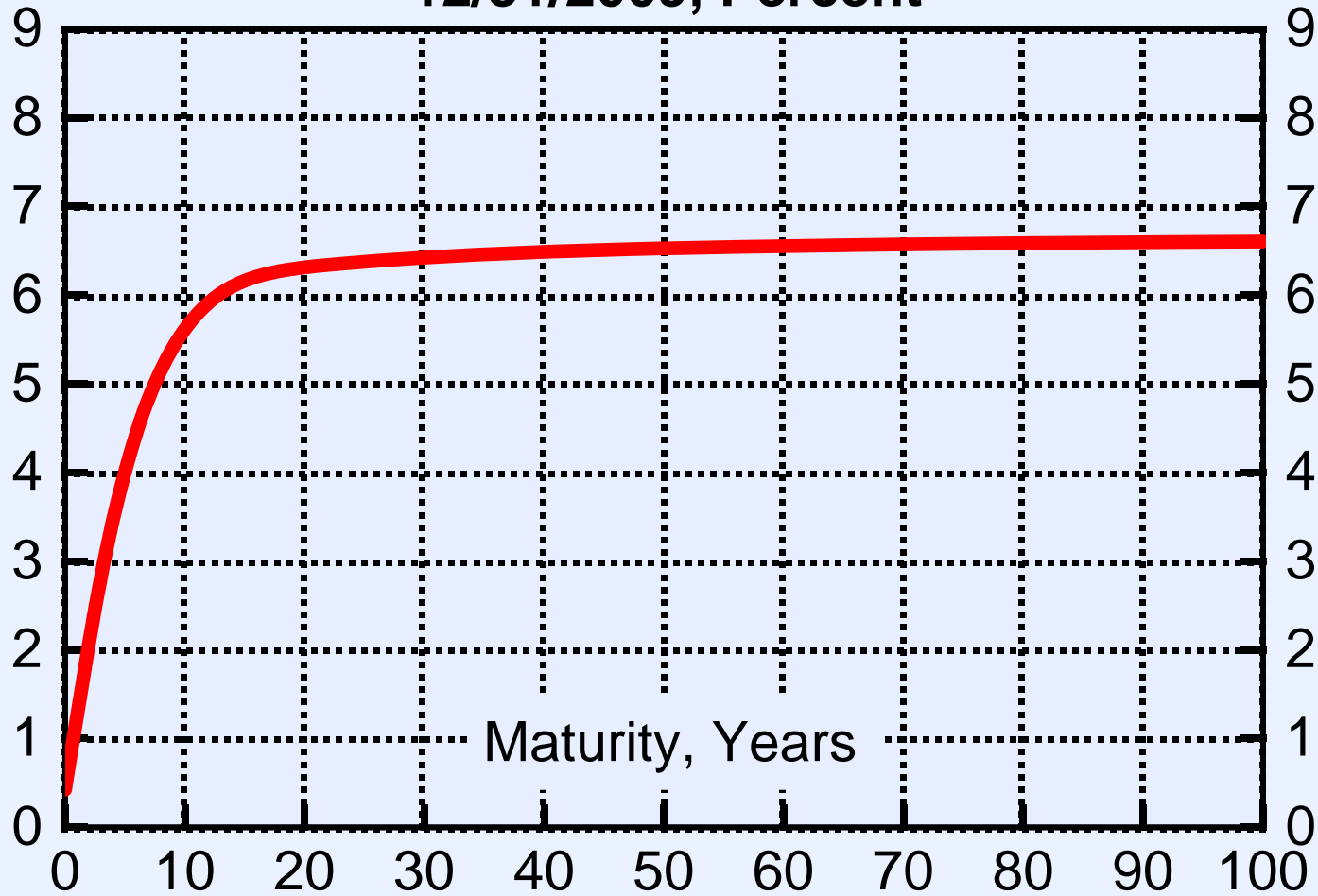


Results, continued

- The third chart shows the spot yield curve for December 31, 2009, derived from the discount function in the previous chart.
- The spot curve is projected out through 100 years. It slopes gently upward from about 15 years maturity onward. It reaches 6.42 percent at 30 years maturity, and 6.61 percent at 100 years maturity. It is near zero at the earliest maturities, reflecting the low short-term rates on this date.

HQM SPOT YIELD CURVE

12/31/2009, Percent



Results, continued

- The estimates of the two adjustment factor coefficients ζ_j for December 31, 2009 in basis points are as follows:

| <u>Variable</u> | <u>Coefficient</u> | <u>T-Ratio</u> |
|-----------------|--------------------|----------------|
| x_1 | 51 | 8.46 |
| x_2 | 32 | 10.91 |

- These coefficients say that on this date, the price of a AAA bond was 51 basis points higher per year of maturity than the price of a AA bond, and the price of a AAA-AA bond was 32 basis points higher per year of maturity than an A bond.
- So at 10 years maturity, the prices of AAA and AAA-AA bonds were 510 and 320 basis points, respectively, above the prices of AA and A bonds.

Sample Yield Curves

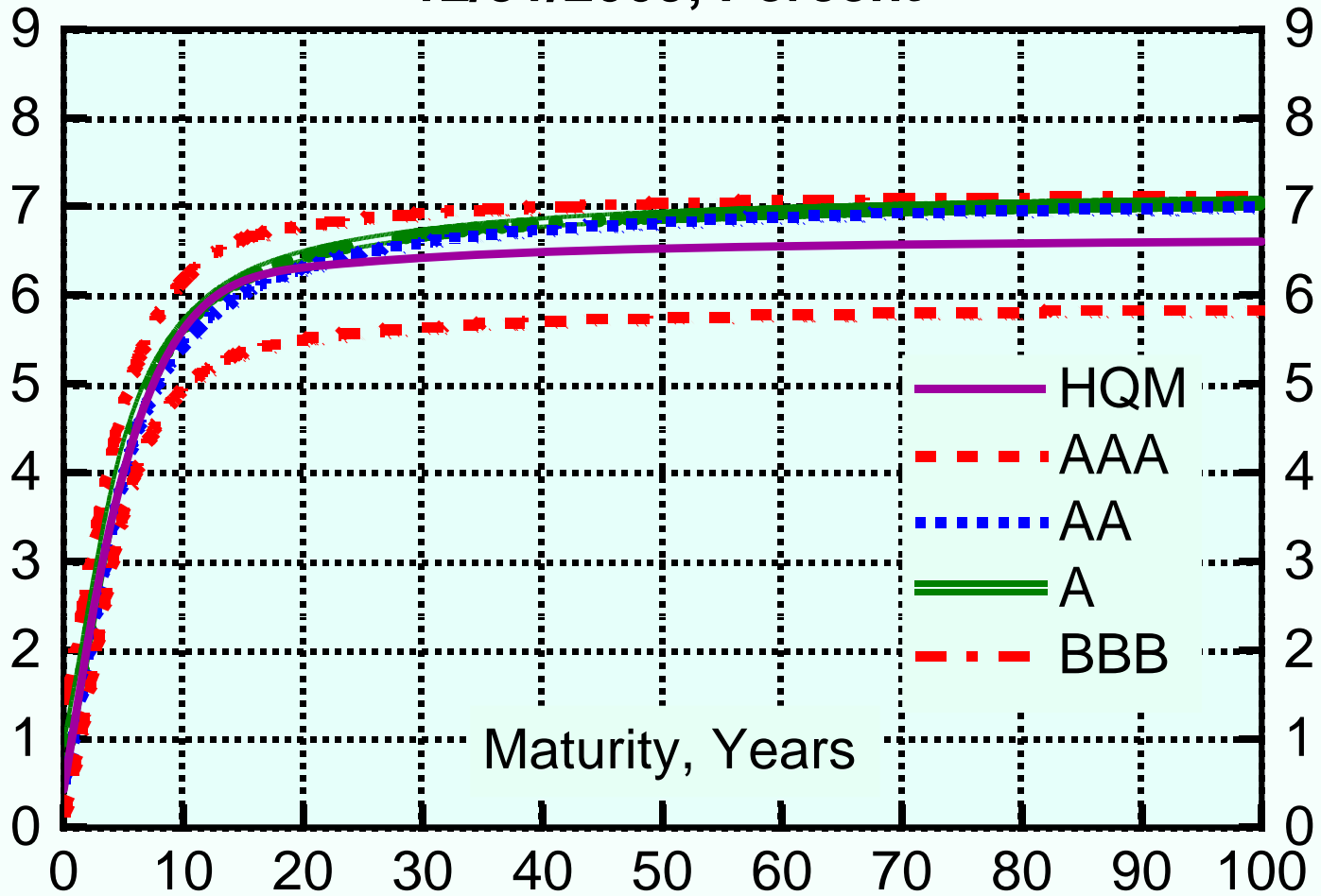
- The last two charts contain sample yield curves for December 31, 2009 at the four qualities AAA, AA, A, and BBB. Please note that these yield curves are prototypes, and they are not official Treasury yield curves.
- The sample curves were constructed using the HQM methodology modified to get these different qualities instead of the market-weighted average quality of the HQM curve.
- The curves illustrate the fact that the HQM methodology is a general-purpose yield curve methodology that can be applied to different kinds of yield curves.

Sample Yield Curves, continued

- The first chart displays the spot yield curves for the four qualities, with the HQM curve included for comparison. On December 31, 2009, the AA and A curves are rather close, with the AAA curve somewhat lower. The HQM curve is near the AA curve. The BBB curve is higher.
- The second chart displays the par yield curves for the four quality levels. These curves are shown for illustration, because par yield curves are standard in financial analysis.

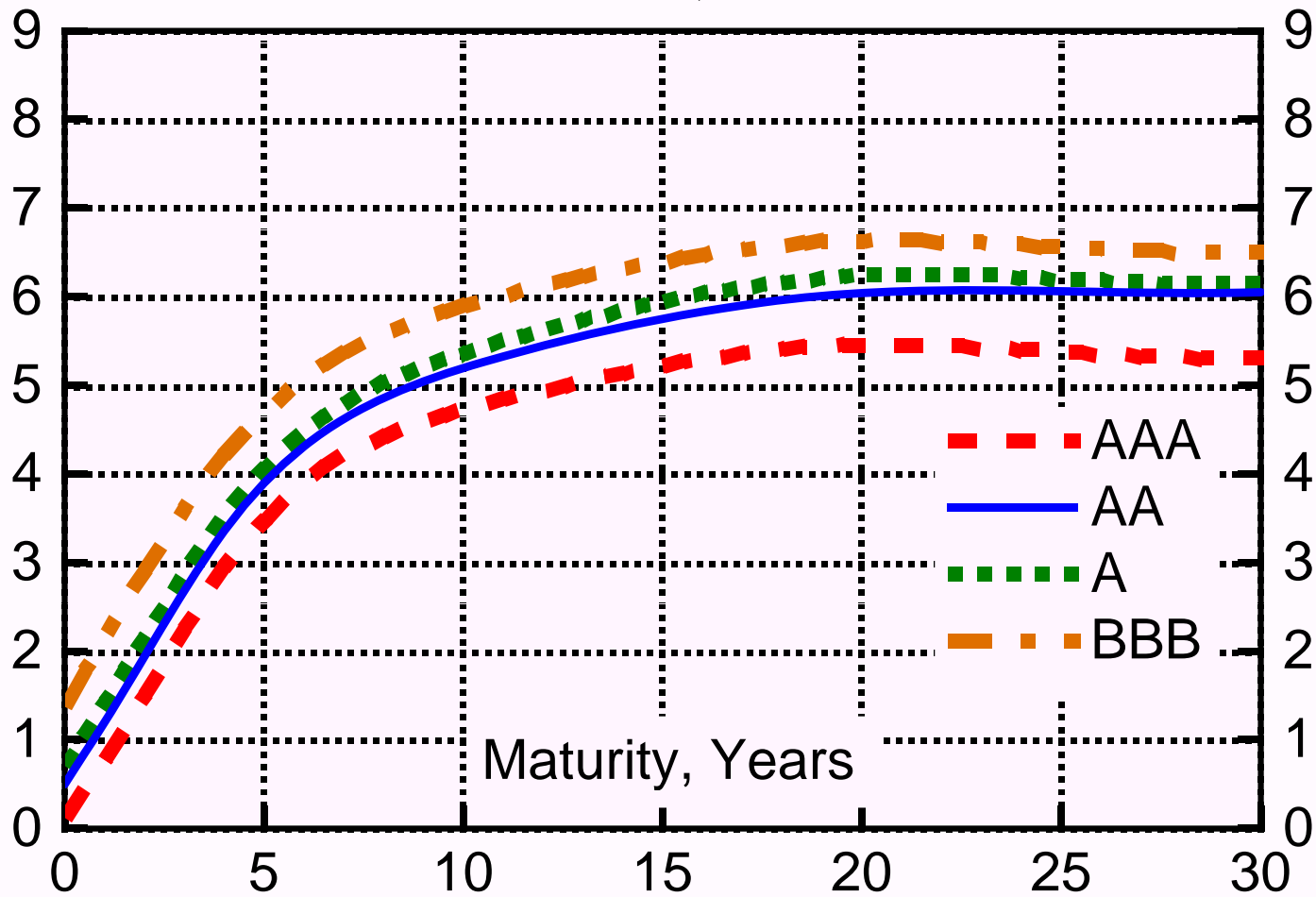
SPOT YIELD CURVES

12/31/2009, Percent



PAR YIELD CURVES

12/31/2009, Percent



For More Information

- The High Quality Market (HQM) Corporate Bond Yield Curve for the Pension Protection Act (PPA) is published by the IRS each month and is available on the IRS website.
- For more details on the mathematics behind the curve and for more documentation in general, visit the Office of Economic Policy website. Go to www.treas.gov, under “Offices” choose “Economic Policy,” then choose “The Corporate Bond Yield Curve.”
- The HQM methodology has been applied to Treasury inflation-indexed securities (TIPS). See papers 0601 and 0501 in the Economic Policy Research Paper Series on the Office of Economic Policy website