Scale-Invariance, Non-Stationarity and Intermittency in the Structure of Cloudiness

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One of the main goals of the Atmospheric Radiation Measurement (ARM) Program is to better understand cloud-radiation interaction in order to improve our climate forecasting skills. We use data on the distribution of atmospheric liquid water from a variety of sources, including ARM's routine ground-based retrievals, and a number of carefully selected statistical properties to obtain simple and robust yet dynamically relevant characterizations of cloudiness. These characterizations are a prerequisite for developing more realistic stochastic cloud models which can be used to investigate numerically radiative transfer issues as in Cahalan et al. (1994a, 1994b), leading, in particular, to better GCM radiation schemes.

Consider a signal $\phi(x)$ where x represents time or a spatial coordinate. We are interested in the statistical properties of $\phi(x)$ that depend parametrically on scale r. In the following, we assume that $\phi(x)$ is statistically "scale-invariant," meaning that its statistics follow power-laws in r over some large range of scales. Such power-law behavior is expected from geophysical systems governed by strongly nonlinear dynamics over a large range of scales. Scale-invariant statistics are thus physically based and have proved invaluable in many branches of nonlinear science.

To discern these power-laws, we derive non-negative random quantities $\xi\,(r;x)$ from $\phi(x)$ which are dependent on scale r (examples to follow). In a *multi*scaling approach, we then seek $A(q)$ in

$$\left\langle \xi_\phi \right\rangle (r\,;x)^q \;\equiv\; \left\langle \xi_\phi(r)^q \right\rangle \;\propto\; r^{A(q)} \qquad (1)$$

where $\langle\cdot\rangle$ denotes an ensemble average and where the identity applies only if $\xi_\phi(\cdot)$ is stationary (i.e., statistically invariant under translation). If the proportionality constants in Equation (1) are only weakly dependent on the real parameter q, then the family of exponents $A(q)$ will account for most of the variability of $\phi(x)$, as captured by $\xi_\phi(\cdot)$, since r spans a wide range of values. In the following, we denote this range $[\eta,R]$.

In Davis et al. (1994b), we graphically explain step-by-step the two most popular "multifractal" statistics applicable to a non-stationary signal, namely: "structure functions" and "singular measures," both of which can be recast in a unified framework based on continuous wavelet transforms (Davis et al. 1994a). Here, we illustrate these analysis techniques using liquid water path (LWP) data retrieved from passive microwave radiometry at the Southern Great Plains ARM site.

Figure 1 shows a typical sample, one day long with a measurement approximately every 20 seconds. The ensemble-average is taken over 41 days free of suspicious behavior (e.g., missing data, unphysically large values, many negative values, rain and dew).

The ensemble-average energy spectrum (2)

$$E(k) \;\propto\; k^{-\beta}$$

for the LWP database is inset into Figure 1 and exhibits nearly a ß = 5/3 power-law in wavenumber $k \approx 1/r$. This means that, over the corresponding range of scales, the process is non-stationary *per se* (ß > 1) but has stationary increments (ß < 3); see Marshak et al. (1994) and Davis et al. (1994a) for theoretical arguments and Davis et al. (1994b), Marshak et al. (accepted), and Cahalan and Snider (1989) for empirical examples. This in turn guides our choices for $\xi_\phi(\cdot)$.

Structure Functions

We consider increments across scale *r*:

$$\xi_{\phi}(r;x) = |\Delta\phi(r;x)| = |\phi(x+r) - \phi(x)|. \qquad (3)$$

Figure 2a is a log-log plot of $\langle|\Delta\phi(r)|^q\rangle$ versus *r* ; the portion with linear behavior for all values of *q* determines the scaling range [η,*R*], spanning almost three decades. In this range of scales, we fit straight lines to the points for multiple *q* values, obtaining the exponents

$$A(q) = \zeta(q) = qH(q) \geq 0,\ q \geq 0. \qquad (4)$$

Scaling stationary cases lead to scale independent increments hence $\zeta(q) \equiv 0$; for example, we notice in Figure 2a a transition from non-stationary to stationary behavior for scales $r > R$ which is known as the "integral" scale. The function $\zeta(q)$ plotted in Figure 3 is concave and smooth with $\zeta(0) = 0$. We interpret the exponent functions $\zeta(q)$ and $H(q)$ as equivalent ways of characterizing the *non-stationarity* of $\phi(x)$. Since $\zeta(1) = H(1) = H_1$ is > 0, we observe a tendency towards continuity (i.e., $|\Delta\phi(r;x)| \to 0$ as $r \to 0$). However, the LWP data do not appear to be differentiable since $|\Delta(r;x)|/r \to \infty$ as $r \to 0$. It is convenient to choose H_1, ranging from 0 to 1, as a simple measure of non-stationarity.

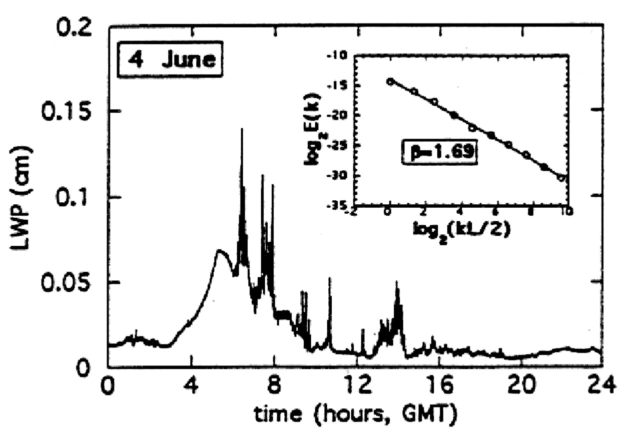

Figure 1. Column liquid water at the ARM Southern Great Plains site. A day-long sample retrieved from passive microwave radiometry, this and 40 other such files were used to obtain the statistics presented in Figures 2 and 3. The inset shows spectral energy $E(k)$ versus $kL/2$, where L=1 day, in log-log axes.

Singular Measures

We now compute

$$\varepsilon(\eta;x) = |\Delta\phi(\eta;x)| \qquad (5a)$$

where η denotes the lower end of the scaling regime for the structure functions or the energy spectrum. We then define

$$\xi_{\phi}(r;x) = \varepsilon(r;x),\ r \geq \eta \qquad (5b)$$

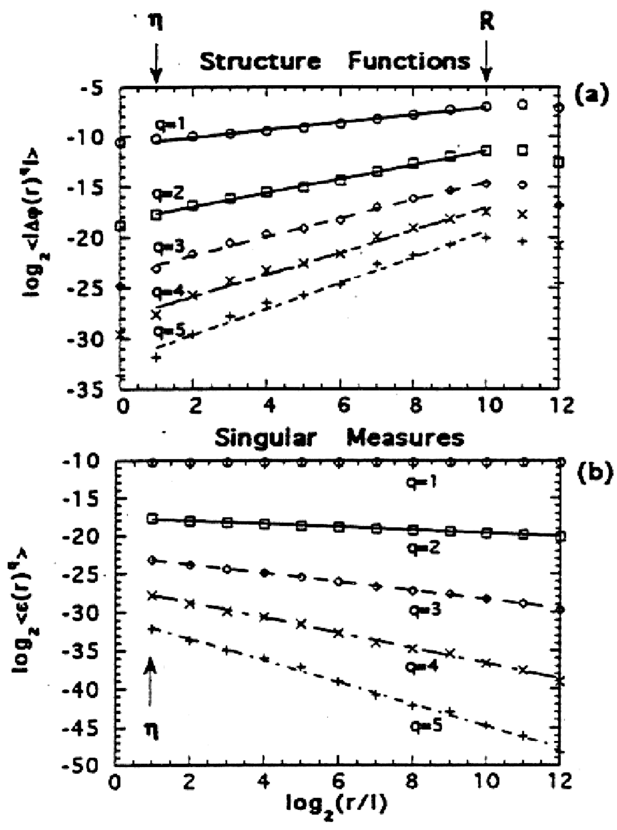

Figure 2. Scale-invariance of the statistical properties of LWP in the ARM database. (a) $\log_2 \langle|\Delta\phi(r)|^q\rangle$ versus $\log_2(r/l)$ where l = 20 s for $1 \leq q \leq 5$; the scaling regime is indicated between $\eta \approx 2l$ and $R \approx 2^{10}l$. It is noteworthy that the slope $\zeta(2) \approx 0.70$ accurately verifies the Wiener-Khintchine relation for scale-invariant non-stationary processes: $\beta = \zeta(2)+1$. (b) Same as in panel (*a*) but for $\log_2 \langle\varepsilon(r)^q\rangle$ versus $\log_2(r/l)$ for $\eta \leq r \leq L$, no need to stop at *R* in this case.

as the spatial averages of $|\Delta\,\phi(\eta;x)|$ over intervals $[x,x+r]$, up to $r = R$ at least. Figure 2b is a log-log plot of $\langle\varepsilon(r)^q\rangle$ versus r showing scaling for a range exceeding that of Figure 2a. Fitting straight lines, as before, gives the exponents

$$-A(q) = K(q) = (q-1)\,C(q),\ q \geq 0 \qquad (6)$$

which is convex and smooth with $K(0) = K(1) = 0$ (Figure 3). The non-increasing function $D(q) = 1\text{-}C(q)$ has been used to quantify the underlying *intermittency* of $\phi(x)$ in turbulence studies and the strangeness of the attractor in chaos theory. The exponent $C(1) = K'(1) = C_1$ (to be discussed in geometrical terms further on) ranges from 0 to 1 and provides a simple measure of intermittency.

In Davis et al. (1993 and 1994a), we discuss the possibility of $K(q) \leftrightarrow \xi(q)$ connections which have received considerable attention in the turbulence literature. As one aspect of such connections, we proposed the "mean multifractal plane" as a simple diagnostic device applicable to the intercomparison of geophysical datasets and their comparison with model calculations (Davis et al. 1993, 1994a, 1994b; also see footnote [a]). In this plane, we take H_1 as the horizontal axis, measuring non-stationarity; and C_1 as the vertical axis, measuring intermittency which is manifest in the small scale absolute gradient field $\varepsilon(\eta;x)$. Alternatively, we could consider the fractal dimension $D_g = 2 - H_1$ that describes the roughness of the graph of the data; and $D(1) = 1 - C_1$, known as the "information" dimension, which is the fractal dimension of the sparse set where we find the events that dominate the mean $\langle\varepsilon(\eta;x)\rangle$.

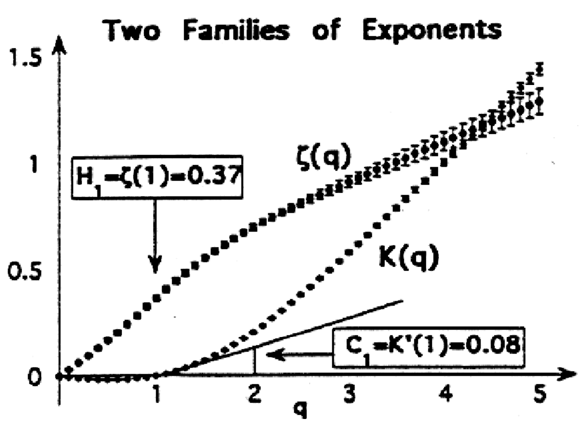

Figure 3. The $\zeta(q)$ and $K(q)$ Functions for LWP in the ARM database. The curvature of these two functions establishes the multifractality of the horizontal distribution of liquid water in the atmosphere. The mean exponents $H_1 = H(1)$ and $C_1 = K'(1)$ are highlighted and carried into Figure 4.

Returning to cloud structure issues, we have applied the whole spectral/multifractal analysis procedure to marine stratocumulus liquid water content (LWC) fields sampled during the First ISCCP[a] Regional Experiment (FIRE'87) (Davis et al.[a]) and the Atlantic Stratocumulus Transition EXperiment (ASTEX) (Davis et al. 1994b). In both cases, scale-invariance over three decades of spatial scale was uncovered. Figure 4 shows the (H_1,C_1) plane with points representing these two LWC analyses and the above ARM microwave LWP; for reference, the locus of turbulent velocity is indicated, as well as a plethora of models. We can draw the following conclusions:

- The proximity of the multifractal parameters for the two LWC points indicates that the dynamics that determine the internal structure of marine StCu depend little on the local climatology.
- Very similar parameters are found for daily LWP data retrieved at the Oklahoma ARM site and for LWC measured *in situ* over oceans; as expected, the integrated quantity, LWP, varies somewhat more smoothly: $H_1(\text{LWP}) > H_1(\text{LWC})$.
- The LWC, LWP and turbulence data points are incompatible with the simplest scale-invariant stochastic models. Additive processes lack intermittency, and multiplicative cascades are too stationary. We need a new class of model for cloud liquid water, along the lines traced out in Marshak et al. (1994) and Cahalan et al. (1994a).
- Dynamical cloud models may not have the same range of scales we can access through direct measurement, but they must nevertheless realistically reproduce the structures we observe in the data at the scales they do resolve. More precisely, they should yield similar values for $\langle|\Delta\phi(r)|^q\rangle$ and $\langle\varepsilon(r)^q\rangle$, and, we hope, even for the exponents such as (H_1,C_1).

(a) Davis, A., A. Marshak, W. Wiscombe, and R. Cahalan. The Scale-Invariant Structure of Marine Stratocumulus Deduced From Observed Liquid Water Distributions, 2 - Multifractal Properties and Model Validation. Submitted to *J. Atmos. Sci.*

(b) International Satellite Cloud Climatology Project.

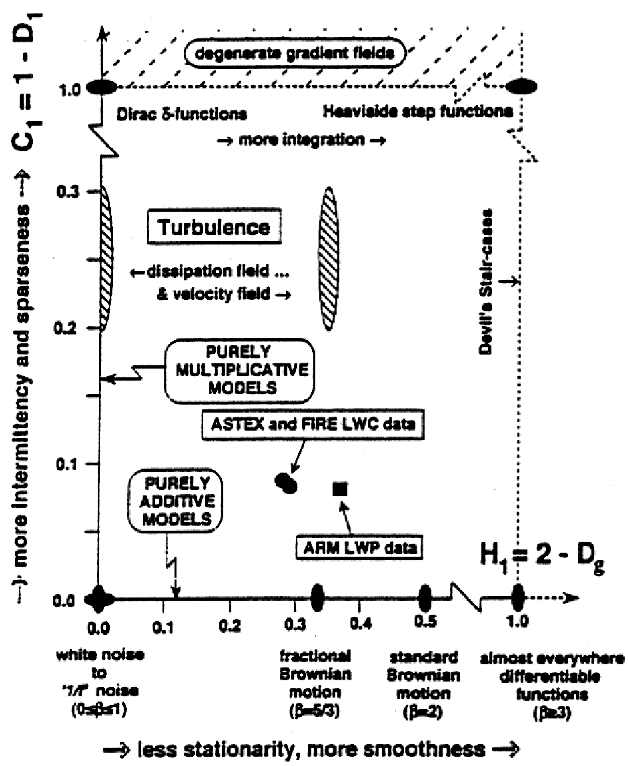

Figure 4. The "mean multifractal plane." Notice the natural bounds (0 and 1) for the two fundamental exponents; in particular, C_1 should be less than the dimensionality of the signal, in this case unity, otherwise the ε-field becomes "degenerate," i.e., vanishes in almost every realization! Geophysical signals tend to live inside the unit square, while the canonical ("multiplicative" and "additive") models live on the axes. It is of interest to recall that random "Devil's staircases" (integrals of random multiplicative cascades) live on the right boundary, and randomly positioned power-law singularities live on the upper boundary.

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