Radiation Singularities, Multiple Scattering and Diffusion in Multifractal Clouds

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Diffusion on One-Dimensional Multifractals

(P. Silas, S. Lovejoy, D. Schertzer)

Many geophysical and atmospheric fields exhibit multifractal characteristics over wide ranges of scale. These findings motivate a study of transport phenomena in multifractal media, particularly diffusion. As we study the diffusion properties of one-dimensional universal multifractal resistivity fields, a relation for the diffusion exponent d_w is derived and is found to depend only on K(-1), the value of the moment scaling function K(q) of the resistivity field for the $q = -1^{th}$ order statistical moment. This relation is subsequently verified through Monte Carlo simulations of diffusion on these systems.

The one-to-one correspondence between statistical moments and orders of singularity suggests that one order of singularity, namely γ_{-1}, is of special importance to diffusion on multifractals, as is confirmed by simulations performed using fields that have been thresholded. Although convergence is quite slow, in the limit of an infinitely large range of scales, a dynamical phase transition occurs about this particular singularity. The relation derived for the diffusion exponent breaks down for those multifractals where the $q = -1^{th}$ order moment diverges, which is typical of a multifractal phase transition. In these cases, d_w must be estimated by taking the sample size into account.

Scattering Statistics and Radiative Transfer in Lognormal Universal Multifractal Clouds

(G. Brösamlen, S. Lovejoy, B. Watson, D. Schertzer)

The study of radiative transfer in multifractal clouds is of great interest, an important application being to global climate models. In this work, we develop a formalism analogous to the multifractal singularity formalism for understanding photon scattering statistics in radiative transfer in lognormal universal multifractals, and test the results numerically. Although the results are only exactly valid in the thick cloud limit, the approximation is found to be quite accurate down to optical thickness of $\tau \approx 1 - 10$, so the results may be widely applicable. Furthermore, we show the possibility of "renormalizing" the multifractal by replacing it with a near equivalent homogeneous medium but with a "renormalized" optical thickness $\tau^{1/(1+C_1)}$ where C_1 is the codimension of the mean singularity of the cloud.

Finally, we argue that this approximation is likely to continue to be valid for multiple scattering and is also compatible with recent results for diffusion on lognormal multifractals.

Multifractal Analysis and Simulation of Interaction Between Clouds and Radiation Field

(C. Naud, D. Schertzer, S. Lovejoy)

In order to capture, at all scales and all intensities, the relationship between liquid water content (ρ) and radiative field (I), we study the relationship between their singularities γ_ρ and γ_I ($\rho_\lambda = \lambda^{\gamma_\rho}$; $= I_\lambda$; λ^{γ_I}) λ being the ratio of scale (λ=L/1 where L is the larger scale, 1 the scale of observation). We consider a multifractal cloud, defined by a universal multifractal model. For simplicity, we take a conservative model (H=0), with codimension of the mean C_1=0.75 and Levy index α=1.35 (as empirically observed) and apply to it the Radiative Transfer Equation in the perfect scattering case. We have analytical and numerical results for a plane cloud and a preliminary computation for the multifractal cloud.