Is It Always Optimal to “Sell the Firm” to a Risk-Neutral Agent?

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Abstract

The paper shows that the answer is no. Holmstrom (1979) and Shavell (1979) show that the sell the firm contract does not achieve the first best when the principal and the agent have different preferences over risk. This paper shows that the sell the firm contract does not achieve the first best when the principal and the agent have different preferences over time. In a dynamic decision making problem under uncertainty, if the agent's discount factor is less than the principal's, the agent will choose actions with relatively higher current payoffs and relatively lower continuation payoffs than the principal would prefer, even when the agent is sold the firm. When current and future payoffs are correlated, the principal can do better by offering the agent a contract that is even higher powered than the “sell the firm” contract. The paper shows that the principal can align the agent's incentives over time by offering the agent stock options. At their exercise date stock options are a liquid asset that pay the agent in the current period for the future value of his actions.

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1 Introduction

Obviously, this paper will claim that the answer is no. It is *not* always optimal for a risk-neutral principal to "sell the firm" to a risk-neutral agent. A classic result of principal-agent theory is that it *is* always optimal for a risk-neutral principal to offer a contract in which the agent pays the expected value of his actions and becomes the residual claimant on the outcome of his actions. In the vernacular of the literature this type of contract is as "high powered" as a contract can get (Kreps (1990)). Holmstrom (1979) and Shavell (1979) show that if the agent is more risk averse than the principal then the first best cannot be achieved and the sell the firm contract is not optimal. This paper provides an example of a situation in which the agent is risk-neutral but it is still not optimal to give the agent a sell the firm contract. This paper will present a simple dynamic environment in which it is not optimal for the risk-neutral principal to offer such a contract to the risk-neutral agent. Moreover, the paper shows that when current payoffs are correlated with future payoffs it is more efficient to offer a contract in which the power of the incentives is *greater* than in the "sell the firm" contract. Finally, the paper suggests that stock options could be used to align the agent's preferences over time with the time preferences of the principal. The value of stock options is that they are liquid at their exercise date. If the stock price reflects the future value of the firm, stock options can pay the agent the future value of current choices at the option's exercise date. This seems to be the way practitioners view the value of stock options (NCEO (2003)), the use of which is something of a puzzle (Hall and Murphy (2003)).

Consider the following illustrative example.[1] A senior executive of an oil company must regularly choose how much money to budget to oil exploration and drilling and how much money to budget to marketing related activities. In general assume that the executive prefers that more money be spent on marketing related activities.[2] Both expenditures can increase the firm's returns, however expenditure on exploration and drilling can also increase the

[1]Thank you to an anonymous reviewer for suggesting this example.

[2]Consider former RJR Nabisco executive F. Ross Johnson in *Barbarians at the Gate* (2003). Jensen (2001) has a detailed discussion of incentive problems inherent in budgeting.

firm's information about the state of the company's oil reserves. Obviously finding oil will increase the company's current profits and it will lead to new decisions in the future that increase future expected profits. However, it is also the case that *not* finding oil increases the firm's information and while it doesn't increase the firm's current profits, having better information about the company's oil reserves does lead to new decisions in the future that increase future expected profits. Given the executive's private information about the likelihood of finding oil (geological surveys and a like), the executive must choose between spending on oil exploration and drilling or spending on marketing. There is a moral hazard problem when the senior executive cares less about the future expected profits of the firm than the representative shareholder. The problem is that there may be cases where given the same private information the senior executive will choose to decrease expenditure on exploration and drilling, while the representative shareholder would have preferred that the executive increase this expenditure. A contract that "sells the firm" the executive is not optimal because the senior executive cares less about the future and thus the executive will still not make decisions that the shareholder considers optimal. Stock options may help to solve the problem if the stock market is able to recognize the value of the information discovered in exploration and drilling operations and incorporate that information into the stock price. The value that stock options may have over stock is that at their exercise date they may be more liquid than normal stock allowing the executive to reap the benefits of her decision to "invest" in the information discovered during in oil exploration and drilling.

The paper presents a principal-agent problem in which the contract is negotiated *ex ante* to the agent receiving his private information. The agent's decision making problem is similar to the monopoly pricing problem described by Rustichini and Wolinsky (1995). The paper takes an insight of Rustichini and Wolinsky (1995) and shows that the agent will choose the more costly action more often when his discount factor is higher. The reason is that given a set of beliefs about the current state of the world the more costly action leads to a higher future expected payoff. If the principal cares more about the future than the agent, the principal may want to give the agent incentives to choose the more costly action. In the example presented, the "good" outcome in the first period leads to higher returns in the second period. However the action which increases the likelihood of the good outcome has higher current costs. Therefore by paying the agent more today if the good outcome occurs, the agent will choose the correct action today. Increasing pay for the good outcome increases the power of the incentive contract beyond that of the "sell the firm" contract. Similarly, the paper shows that a contract that models the way stock options work can be used to provide the first best incentives to the agent. It is shown that stock options can solve the problem by paying the agent for the future value of his current choices in the current period.

The result occurs because current and future payoffs are positively correlated and so the fact that the agent doesn't care about future payoffs can be offset by increasing the importance of current payoffs. This is only a simple example, but it may be a good model of situations where costly current actions may lead to high current payoffs and high future payoffs. For example, investing in winning a drug development race to be first to market. If initial market success leads to long term success the Board of Directors may want the CEO earn higher returns for the initial successes. Another example may be winning a "standards" race in a technology market with network externalities. Or investing to become the internet auctioneer of choice, as eBay did (Cohen (2002)). It is interesting to note the importance of stock options in sectors of the economy where such races are important (NCEO (2003); Lazear (2003)).

A standard result of the principal-agent literature is that if the agent is risk-neutral the first best can be achieved by using a simple "sell the firm" contract (Kreps (1990)). Holmstrom (1979) and Shavell (1979) show that the sell the firm contract can be improved upon when the principal and the agent have different risk preferences. This paper similarly shows that the sell the firm contract can be improved upon when the principal and the agent have different time preferences. A number of papers have considered the principal agent problem in a dynamic setting, most notably Holmstrom and Milgrom (1987). The important characteristic of the dynamic model presented here is that the agent's time preferences affect his choices, and more specifically, they distort his actions away from those that the principal would prefer.

In this paper it is assumed that the agent has private information and that the contract is negotiated *ex ante* to this information being revealed.[3] This assumption follows, for example Holmstrom (1979), Adams (2002), Baker and Jorgensen (2003) and Raith (2004), all of which consider the case in a static setting. Adams (2002) and Raith (2004) argue that this assumption is what distinguishes principal-agent models where the agent makes decisions to models where the agent does not. A decision maker is an agent that observes private information and makes his choice contingent upon that information. To the author's knowledge this paper is first to consider the problem when the agent faces a dynamic decision making problem under uncertainty. That is, the paper analyzes a situation where the agent's private information changes from period to period and is affected by the action choices of the agent. The model is based on the model analyzed by Rustichini and Wolinsky (1995), and is but one example of the models analyzed in the literature on learning and dynamic decision making under uncertainty (Keller and Rady (1999); Mirman et al. (1993); Rothschild (1974)). This paper brings some of the insights of this literature to the principal-agent problem. The results suggest that the dynamic aspects of the agent's choices have important implications for contract theory if the principal and the agent have different preferences over time.

In recent years stock options have become quite popular, particularly in the high tech sector (NCEO (2003); Hall and Murphy (2003)). However, the use of such schemes represents something of a puzzle. Hall and Murphy (2003) discuss a number of explanations for why stock options are used, these can be categorized as follows; tax, accounting, borrowing, retention, and incentives. Stock options are taxed differently than normal stock grants, and, as many now realize, they have traditionally been accounted for differently in the company reports. The authors argue that even though the market can efficiently account for these reporting differences, managers seem to 'falsely perceive stock options to be inexpensive.' According to Lazear (2003) stock options do not seem to be a good way to finance a risky project as it would be cheaper for the firm to borrow from venture capitalists. However, stock options may be fine if employees have beliefs that are more optimistic than

[3]Note that all the analysis is in regards to the *ex ante* expected value of the contract.

the VCs (Hall and Murphy (2003)). Oyer (2003) argues that stock options may be a useful retention device because their value is correlated with the outside option of the employees, and so such a scheme would save on renegotiation costs. In regards to the incentive effects of options, the use of which is generally analyzed using standard static principal-agent model, the literature has found them to be a very costly method of providing incentives (Oyer and Schaefer (2003)). This paper presents a dynamic model and shows that stock options may be preferred to restricted stock grants because they can be immediately sold on the market. If the stock price provides a good signal of the future value of the employee's current actions, then stock options may be an efficient method of providing the appropriate incentives. According to NCEO (2003) the most important difference between stock options and ordinary stock is that the stock is sold immediately the option is exercised. Options allow the employee to "cash out" during a specific period of time, allowing employees to share in the "future growth of the firm" (NCEO (2003)).

The rest of the paper proceeds as follows. Section 2 presents a two-period model and shows that a "sell the firm" contract is not optimal. The section shows how stock options can be used to achieve the first best outcome. Section 3 presents a infinite-period model. Section 4 concludes.

2 Two-Period Model

This section presents a simple two-period model and gives an example of a situation in which it is not optimal to "sell the firm" to the agent. Proposition 1 shows that if the agent is less patient than the principal, it is not optimal for the principal to "sell the firm". Proposition 2 shows that the first best contract will be higher powered than "sell the firm" contract. The results follow in a straightforward manner from the realization that under some set of beliefs the value of choosing the costly action in the first period is that it leads to a higher expected payoff in the second period. The agent's valuation of the second period is lower than the principal's so there are a set of beliefs where the agent does not choose the costly action, even when the principal prefers that he did. The principal can solve the problem by paying the agent more if the "good" outcome occurs (less if the "bad" outcome). Note that the agent is risk-neutral so fixed amounts can be varied to achieve the optimal allocation between the principal and the agent over the two periods. Proposition 3 shows that a practical way to implement the first best contract is to offer the agent an option contract. This contract pays the agent a large positive amount if the good outcome occurs when the market "believes" that the agent chooses the costly task.

2.1 The Model

Consider a situation in which the principal and the agent are risk-neutral. The agent faces a two-period problem, $t \in \{1, 2\}$. The timing is as follows.

- Period 0.
  - The principal and the agent have a common *ex ante* distribution $f(w_1)$ on the agent's period 1 belief, w_1, of the state of the world $s_1 \in \{0, 1\}$, where $w_1 = Pr(s_1 = 1)$.
  - The principal makes a take-it-or-leave-it offer, $\{\pi_{0t}, \pi_{1t}\}_{t\in\{1,2\}}$. The agent's outside option is 0. The agent's discount factor is $\delta \in [0, 1)$ and the principal's discount factor is 1.
- Period 1.
  - The agent chooses an action $a_1 \in \{0, 1\}$, such that the agent bears the private cost $c > 0$ of choosing $a_t = 1$ and no cost of choosing $a_1 = 0$.
  - The outcome $y_1 \in \{0, 1\}$ is observed where

$$y_1 = \begin{cases} 1 & \text{w/ prob 1 if } a_1 = 1 \text{ and } s_1 = 1 \\ 1 & \text{w/ prob 0 if } a_1 = 1 \text{ and } s_1 = 0 \\ 1 & \text{w/ prob } p \text{ if } a_1 = 0 \end{cases} \tag{1}$$

    where $p \in (0, 1)$.
  - The agent is paid π_{11} if $y_1 = 1$ and π_{01} if $y_1 = 0$.

- Period 2.
  - The agent chooses a_2 given his updated belief (w_2) about the state of the world, s_2.

$$w_2 = \begin{cases} \alpha & \text{if } a_1 = 1 \text{ and } y_1 = 0 \\ 1-\alpha & \text{if } a_1 = 1 \text{ and } y_1 = 1 \\ (1-\alpha)w_1 + \alpha(1-w_1) & \text{if } a_1 = 0 \end{cases} \tag{2}$$

    where $\alpha \in (0, .5)$.
  - The agent is paid π_{12} if $y_2 = 1$ and π_{02} if $y_2 = 0$.

To give this simple two-period model some of the flavor of the infinite period model analyzed in the next section, it is assumed that the principal and the agent have a common *ex ante* belief regarding the *distribution* of the agent's first period belief (signal) of the state of the world ($f(w_1)$). The principal is assumed to choose an incentive contract that maximizers her *ex ante* expected profits. This assumption is implicit in the analysis that follows. Note that the agent's individual rationality constraint is also assumed to be *ex ante*. The agent's incentive compatibility constraint is defined as the optimal choices a_1 and a_2 given beliefs w_1 and w_2 and payoffs c, π_{01}, π_{11}, π_{02}, and π_{12}.

The important and interesting feature of this model is that the agent's belief (w_2) is a function of the agent's action in the previous period (a_1). By choosing $a_1 = 1$ the agent learns the state s_1 after observing the outcome y_1. If $a_1 = 0$, no information is revealed about the state. If the agent chooses the low cost action ($a_1 = 0$), the agent's belief at the beginning of period 2 will be $w_2 = (1-\alpha)w_1 + \alpha(1-w_1)$. While no additional information is learned, the agent's beliefs must adjust to account for the probability that the state of the world has changed, this probability is denoted, α.[4] If the agent chooses the high cost action then the agent "learns" the state by observing the outcome y_1. If the good outcome occurs ($y_1 = 1$) then it is known that the state in period 1 was $s_1 = 1$. Therefore, the agent's belief in period 2 is $w_2 = 1-\alpha$, where the probability that the state of the world is still good ($s_2 = 1$) is

[4] Below it is assumed that α is "small".

decreased by α to account for the probability that the state of the world changes between the two periods. If the bad outcome ($y_1 = 0$) is observed the agent's belief in period 2 is $w_2 = \alpha$. The value of learning the state in period 1 is that it may increase the expected value of the choice in period 2. It does so because the principal is better off if the high cost action is chosen *only* in the good state of the world. It is this "investment" in information that provides the link between today's choices and future payoffs. Thus, it is this investment that is subject to the vagaries of the agent's preferences.

In the first best case, the firm would be solving the following problem. In Period 2, the problem is

$$max_{a_2 \in \{0,1\}} \quad (1-a_2)p + a_2(w_2 - c) \tag{3}$$

If the agent chooses $a_2 = 0$, the expected return is p and there is no cost to the agent of choosing the action. If the agent chooses $a_2 = 1$, the expected return is w_2, the agent's belief about the state in period 2, minus the private cost of choosing the action (c). Let W_2^P characterize the agent's first best strategy, where P denotes "principal", such that if $w_2 \geq W_2^P$, then $a_2 = 1$ and if $w_2 < W_2^P$, $a_2 = 0$. From Equation (3) we have that $W_2^P = p + c$. The following three assumptions guarantee the interesting case, where the action choice in period 1 (a_1) affects the agent's information and his optimal action choice (a_2) in period 2.

Assumption 1 *The Interesting Case:*

1. *Let* $1 - c > p$
2. *Let* $w_1 < p + c$
3. *Let* α *be small.*

In this case, we see from Equation (2) that the second period's action is a function of the first period's action and the observed outcome y_1. Given Assumption (1), in the first best case the high cost action is chosen if the following inequality holds.

$$w_1 - c + w_1(1 - \alpha - c) + (1 - w_1)p \geq 2p \tag{4}$$

If the low cost action ($a_1 = 0$) is chosen in the first period nothing is learned about the state of the world and under Assumption 1, the optimal choice in period 2 is the low cost action ($a_2 = 0$) again. The expected payoff from the low action in period 1 is $2p$. However, if the high cost action is chosen there is some possibility that the good state occurs ($s_1 = 1$) and there will be a high payoff ($y_1 = 1$) and the beliefs will be such (given Assumption 1) that it will be optimal to choose the high cost action ($a_2 = 1$) in period 2. The expected payoff in this case is $1-\alpha-c$ where $1-\alpha$ is the belief that the state is still good ($s_2 = 1$). The other possibility is that the state is bad and the bad outcome occurs ($y_1 = 0$). Given Assumption 1, the best choice is the low cost action in period 2 ($a_2 = 0$). The expected payoff in this case is p. Given *ex ante* beliefs ($f(w_1)$) and Assumption 1, the expected first best profits are maximized when the low cost action is chosen for the agent's belief below a cutoff W_1^P and the high cost action is chosen for the agent's belief above the cutoff, where W_1^P is such that Equation (4) holds with equality.

2.2 Results

Let W_1 characterize the agent's optimal strategy given the contract, where $a_1^* = 1$ if $w_1 \geq W_1$ and $a_1^* = 0$ if $w_1 > W_1$. Proposition 1 is the main result of the paper. It states that if the agent's discount factor is less than 1, the cutoff belief W_1 is greater than the first best cutoff belief. This means that there is a set of beliefs $w_1 \in (W_1^P, W_1)$, such that the agent chooses the low cost action but *ex ante* the principal would prefer he choose the high cost action.

Proposition 1 *Given Assumption 1, if* $\pi_{t1} - \pi_{t0} = 1$, $\delta < 1$, *then* $W_1 > W_1^P$.

Proof. From Equation (4),

$$W_1^P = \frac{p+c}{2-\alpha-c-p} \tag{5}$$

For the agent, the equivalent inequality is

$$w_1 - c + \delta(w_1(1-\alpha-c) + (1-w_1)p) \geq p + \delta p \tag{6}$$

and so

$$W_1 = \frac{p+c}{1+\delta(1-\alpha-c-p)} \tag{7}$$

If $\delta = 1$ then $W_1 = W_1^P$. We have the result if $\frac{\partial W_1}{\partial \delta} < 0$. The derivative is negative if $1-c > p$ and α is small, which they are by Assumption 1. QED.

The proposition shows that in this example the first best cannot be achieved with the "sell the firm" contract if δ is less than 1. The intuition is the same as for the similar result in the infinite-period problem shown below. The sell the firm contract is represented by the difference between the payoffs ($\pi_{1t} - \pi_{0t} = 1$). The agent prefers to choose the low cost action "more often" than the principal would like because for some set of beliefs the action has a greater current period payoff and a smaller second period payoff. Given the "sell the firm" contract, the agent doesn't fully account for the extra "investment" value of choosing the high cost action. In expectation this extra amount is $1-\alpha-c-p$. As long as this is positive (which it is by assumption) and $\delta < 1$ the agent will choose $a_1 = 0$ more often than the principal would like. Proposition 2 states that the first best can be achieved with a contract that is higher powered than the "sell the firm" contract.

Proposition 2 *Given Assumption 1, if p is small, the first best can be achieved if $\pi_{11} - \pi_{01} > 1$.*

Proof. Let $\pi_{11} - \pi_{01} = D$ and $\pi_{12} - \pi_{02} = 1$, and so the agent's problem is

$$w_1 D - c + \delta(w_1(1-\alpha-c) + (1-w_1)p) \geq pD + \delta p \tag{8}$$

and so

$$W_1^D = \frac{pD+c}{D+\delta(1-\alpha-c-p)} \tag{9}$$

We have the result if $\frac{\partial W_1^D}{\partial D} < 0$. The derivative has the same sign as

$$p(D+\delta(1-\alpha-c-p)) - pD - c = p\delta(1-\alpha-c-p) - c \tag{10}$$

which is negative if p is small enough. QED.

The intuition is that the principal has the possibility of earning an extra $1-\alpha-c$ if the agent chooses the high cost action in the first period and the outcome is $y_1 = 1$. However the agent does not value this possibility as highly as the principal. The principal can replicate the first best incentives by increasing the relative value of choosing the high cost action in the first period, in this model this is done by putting a relatively higher payoff on the outcome $y_1 = 1$ (lower payoff on the outcome $y_1 = 0$). This works because the payoffs are positively correlated ($\alpha < .5$). This suggests that one solution to the incentive problem is to use the time series correlation in the payment instruments (the signals). Stock options may be a particular way to do this.[5]

2.3 An Option Contract

An option contract pays the agent the market's expected value of the firm in some future period less the strike price of the option, with a minimum value of zero. By paying the agent the expected value of the future value of the firm, the principal can solve the problem that the agent's time preferences distort their choices to actions that weight present returns greater than the principal would like. It is assumed that the stock bought in the option is immediately resold to the market when the option is executed.[6]

Consider a two-period problem similar to the problem described above. In period 2, the contract pays $\pi_{12} - \pi_{02} = 1$. In the first period the contract has two parts. First $\pi^p_{11} - \pi^p_{01} = 1$, this is the profit sharing part. The option part of the contract is, π^o.[7]

$$\pi^o = \begin{cases} r(1-\alpha-c) - rp & \text{if } w_1 > W^O_1 \text{ and } y_1 = 1. \\ rp - rp & \text{otherwise} \end{cases} \tag{11}$$

[5]Note that stock options may also work when there is no positive time series correlation if the stock market is able to observe other (more direct) signals of the agent's actions and information.

[6]This seems to be a reasonable characterization of what actually happens (Hall and Murphy (2003); NCEO (2003)). Note that what distinguishes stock options from normal stock is this characteristic that the stock is immediately sold when the option is exercised.

[7]The stock option gives the profit sharing contract an extra boost, and is thus a way of implementing the first best contract described above.

The option pays the agent the principal's "belief" about the value of the firm less a discount factor r and the strike price, which is assumed to be rp. The cutoff belief W_1^o is defined below (Equation (13)). The option is exercised after the action is chosen and the outcome is observed. The principal's belief about the value of the firm is consistent with the agent's equilibrium action if, given this contract, the agent chooses $a_1 = 1$ when $w_1 > W_1^O$. Note that the agent may choose $a_1 = 0$ and be paid $r(1-\alpha-c-p)$ if $y_1 = 1$. That is, the agent can cheat and choose the low cost action. If the good outcome ($y_1 = 1$) occurs the principal will still believe that the high cost action was chosen and that the good state ($s_1 = 1$) has been revealed. Therefore we have an equilibrium if the following inequality holds (the incentive compatibility constraint).

$$w_1-c+\delta(w_1((1-\alpha-c)+(1-w_1)p)+w_1r(1-\alpha-c-p)) \geq p+\delta(p+pr(1-\alpha-c-p)) \tag{12}$$

The cutoff value, W_1^O, is the w_1 such that this equation holds with equality. The following proposition shows that there exists an r such that $W_1^O = W_1^P$. That is, the first best can be achieved with an option contract of this form.

Proposition 3 *For small p, there $\exists r$ such that $W_1^O = W_1^P$.*

Proof. From Equation (12) we have

$$W^O = \frac{p + pr(1-\alpha-c-p) + c}{1+\delta(1-\alpha-c-p)+r(1-\alpha-c-p)} \tag{13}$$

From Proposition 1, if $r = 0$, $W_1^O > W_1^P$. We have the result if $\frac{\partial W_1^O}{\partial r}$ is negative. For small p this follows from Equation (13). QED.

The option contract pays the agent today as a function of the firm's future expected value. However, the option only pays out a positive amount if the good outcome occurs. It is not optimal to sell the firm to the agent, but the first best can be achieved by paying the agent more for the choice of $a_1 = 1$ in the first period. As shown above, the option contract is a practical way to do that. The option pays the agent today the discounted present extra value of choosing the costly action, $r(1-\alpha-c-p)$.

3 Infinite-Period Model

This section presents a more general model using results from Rustichini and Wolinsky (1995). Much of the intuition from the two-period model carries over to the infinite-period model. Although this model is more complicated than the model presented above, the results are more general than those for the particular case analyzed in Section 2. Proposition 4 states that as the agent's discount factor increases, the agent is more likely to choose the costly action ($a_t = 1$). The proposition also states that there is a positive relationship between the power of the incentives and the number of times the costly action is chosen in the optimal strategy. Corollary 1 shows that the first best can be achieved with a contract that is higher powered than a "sell the firm" contract.

3.1 The Model

Consider a situation in which the principal and the agent are risk-neutral. The agent faces a discrete but infinite period problem, $t \in \{1, 2, ...\}$. The timing is as follows.

- Period 0.
  - The principal and the agent have a common *ex ante* belief, w_0, of the state of the world $s_1 \in \{0, 1\}$, where $w_0 = Pr(s_1 = 1)$.
  - The principal makes a take-it-or-leave-it offer, $\{\pi_{0t}, \pi_{1t}\}_{t \in \{1,2,...\}}$. The agent's outside option is 0. For simplicity it is assumed that the *both* the agent and the principal negotiate the contract over the expected value of the stationary distribution generated by the actions of the agent.[8]
- Period 1.
  - The state of the world follows a simple Markov process.

$$\Pr(s_{t+1} = 1) = \alpha(1 - s_t) + (1 - \alpha)s_t \tag{14}$$

[8]Given that the principal can offer contracts that pay the agent more in earlier periods, this assumption does not seem to matter.

where $\alpha \in (0, .5)$.

- At the beginning of each period t, the agent has a belief $w_t \in [0, 1]$ about the state of the world $s_t \in \{0, 1\}$, such that $w_t = \Pr(s_t = 1)$.[9]
- The agent chooses an action $a_t \in \{0, 1\}$, such that the agent bears the private cost $c > 0$ of choosing $a_t = 1$ and no cost of choosing $a_t = 0$.
- The outcome $y_t \in \{0, 1\}$ is observed where

$$y_t = \begin{cases} 1 & \text{w/ prob 1 if } a_t = 1 \text{ and } s_t = 1 \\ 1 & \text{w/ prob 0 if } a_t = 1 \text{ and } s_t = 0 \\ 1 & \text{w/ prob } p \text{ if } a_t = 0 \end{cases} \tag{15}$$

  where $p \in (0, 1)$.
- The agent is paid π_{1t} if $y_t = 1$ and π_{0t} if $y_t = 0$.

• Period 2.

- The agent chooses a_2 given his updated belief (w_2) about the state of the world, s_2.

$$w_{t+1} = \begin{cases} \alpha & \text{if } a_t = 1 \text{ and } y_t = 0 \\ 1 - \alpha & \text{if } a_t = 1 \text{ and } y_t = 1 \\ (1 - \alpha) w_t + \alpha(1 - w_t) & \text{if } a_t = 0 \end{cases} \tag{16}$$

  where $w_t \in B$.[10]

• Period $t \in \{3, 4, ...\}$

- Follow in the same manner as 1 and 2.

[9] Unfortunately some clarity is lost because there are two different states, one is the state of the world s_t (in Rustichini and Wolinsky (1995) this is demand (d_t)), and the other is the state of the Markov decision process which is denoted σ.

[10] Note that B is an infinite and countable set given w_0. To see this let $a_1 = 1$ and $a_t = 0$ for all $t \in \{2, 3, ...\}$.

– At each period t the agent has a belief w_t and the agent chooses a_t^* such that

$$a_t^*(w_t) = \arg\max_{a_t \in \{0,1\}} (1 - a_t)((1-p)\pi_0 + p\pi_1)$$
$$+a_t((1-w_t)\pi_0 + w_t\pi_1 - c) + \delta \sum_{\{w_{t+1} \in B\}} q(w_{t+1}|w_t, a_t) v_{t+1}(w_{t+1}) \quad (17)$$

where $\delta \in (0,1)$ is the agent's discount factor, $q(w_{t+1}|w_t, a_t)$ is the transition probability, and $v(w_{t+1})$ is the expected value of future choices given w_{t+1}.

Period 0 represents the agent's individual rationality constraint. It is assumed that the agent chooses to accept or reject the contract based on the expected value of the stationary distribution (or the "long run expected value"). This assumption is made for simplicity and it also represents the idea that when employees negotiate an employment contract they consider a longer horizon than when they are making day-to-day decisions. The agent's incentive compatibility constraint is discussed below. Note further that in this case the *ex ante* beliefs and the principal and the agent are not relevant.

The agent's belief (w_{t+1}) is a function of the agent's action in the previous period (a_t). By choosing $a_t = 1$ the agent learns the state s_t after observing the outcome y_t. If $y_t = 1$ is observed then the agent knows that the state of the world is $s_t = 1$ and thus his belief about the state of the world in then next period is $w_{t+1} = 1 - \alpha$, where α is the probability that the state changes from period to period. Similarly if $y_t = 0$ is observed the agent's belief in period $t + 1$ is $w_{t+1} = \alpha$. If $a_t = 0$, no information is revealed about the state. Note however that the agent's belief changes to account for the probability that the state of the world changes. Therefore his belief is $w_{t+1} = (1 - \alpha)w_t + \alpha(1 - w_t)$. The value of knowing the state is that it increases the expected value of the choice in period $t+1$. While the expected value of choosing the low cost action ($a_t = 0$) doesn't change with the state of the world (it always equals p), the expected value choosing the high cost action is dependent on the state of the world. The high cost action pays more than p when the state of the world is $s_t = 1$ and less than p in the other state of the world.

It is straight forward to see that if $\delta = 0$ and $\pi_{1t} > \pi_{0t}$, that we have $a_t^* = 1$ if $w_t \geq W^M$ and $a_t^* = 0$ if $w_t < W^M$, where W^M is the agent's myopic cutoff belief

$$W^M = \frac{p(\pi_{1t} - \pi_{0t}) + c}{\pi_{1t} - \pi_{0t}} \tag{18}$$

We can also see that $a_t^* = 1$ is chosen "more often" when $\pi_{1t} - \pi_{0t}$ is larger, p is smaller and c is smaller. By increasing the difference $\pi_{1t} - \pi_{0t}$ the expected payoff from choosing $a_t = 1$ increases. If p increases the expected payoff from choosing $a_t = 0$ increases and if c increases the cost of choosing $a_t = 1$ increases relative to the expected value of choosing $a_t = 0$. That is, a myopic agent would react to incentives exactly as we would expect.

Lemma 1 *If $\pi_1 \geq \pi_0$, then the agent's optimal strategy exists and is characterized by a cutoff belief, $W \in [0, 1]$, such that if $w_t \leq W$, $a_t^* = 0$, and if $w_t > W$, $a_t^* = 1$.*

Lemma 1 shows that for all δ there exist a W such that $a_t^* = 1$ if $w_t \geq W$ and $a_t^* = 0$ otherwise. While Lemma 1 is similar to Rustichini and Wolinsky (1995) Claim 1, the proof is written out in full in Adams (2001) because B is infinite and countable and therefore it is not possible to appeal to results in Derman (1970). Rather, the proof is based on results presented in Puterman (1994). Rustichini and Wolinsky (1995) note that given Lemma 1, the agent's strategy can be summarized by the variable, N. The smaller N is, the greater the private cost born by the agent.

Definition 1 (Rustichini and Wolinsky (1995)) *N is the smallest non-negative integer if $w_t = 1 - \alpha$ and $a_t = ... = a_{t+N-1} = 0$, then $w_{N+t} < W$. $N = \infty$ if there is no such integer.*

By this definition, N is the number of times the agent chooses $a_t = 0$ after $y_t = 0$ when $a_t = 1$. That is, N is the number of times the low cost action is chosen after the agent receives a signal that the state of the world is bad. If N is large, $a_t^* = 1$ is chosen less often (W is high), while if N is small, $a_t^* = 1$ is chosen more often (W is low). Therefore at anytime t (after the initial couple of periods), the agent's incentive compatibility constraint is characterized by N.

As stated above it is assumed that the individual rationality constraint is defined over the expected value of the stationary distribution given N. Following Rustichini and Wolinsky (1995) it is straightforward to show that the total surplus of the firm is

$$Surplus = \frac{N\alpha p + 1 - \psi_N}{(N+1)\alpha + 1 - \psi_N} \tag{19}$$

where $\psi_N = \frac{1+(1-2\alpha)^{N+1}}{2}$. Assuming that the agent's outside option in terms of the expected value of the stationary distribution is 0, the first best contract would be one in which the principal maximizes Equation (19).

The following lemma states that as δ gets near to 1, the optimal solution to the agent's problem is equivalent to the N that optimizes the expected value of the stationary distribution (Equation (19)).

Lemma 2 *Let q_{it} be the marginal probability distribution at time, t, for Markov chain, $i \in I$, q_i is the stationary marginal probability distribution for Markov chain, i, and R_σ is the per period payoff in state σ. Then*

$$\arg\max_{i\in I} \lim_{\delta\to 1} \sum_{t=1}^{\infty} \delta^t \left(\sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma \right) = \arg\max_{i\in I} \sum_{\sigma\in\Sigma} q_i(\sigma) R_\sigma \tag{20}$$

Proof.

$$\begin{aligned}
&\arg\max_{i\in I} \lim_{\delta\to 1} \textstyle\sum_{t=1}^{\infty} \delta^t \sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma \\
&= \arg\max_{i\in I} \lim_{\delta\to 1} \textstyle\sum_{t=1}^{\infty} \frac{1}{T}\delta^t \sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma, \text{ when multiplied by a constant} \\
&= \arg\max_{i\in I} \lim_{\delta\to 1} \lim_{T\to\infty} \textstyle\sum_{t=1}^{T} \frac{1}{T}\delta^t \sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma \\
&= \arg\max_{i\in I} \lim_{T\to\infty} \textstyle\sum_{t=1}^{T} \frac{1}{T} \lim_{\delta\to 1} \delta^t \sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma \\
&\text{reversing the order of the limits} \\
&= \arg\max_{i\in I} \lim_{T\to\infty} \textstyle\sum_{t=1}^{T} \frac{1}{T} \sum_{\sigma\in\Sigma} q_{ti}(\sigma) R_\sigma, \text{ when } \delta \to 1 \\
&= \arg\max_{i\in I} \textstyle\sum_{\sigma\in\Sigma} (\lim_{T\to\infty} \sum_{t=1}^{T} \frac{1}{T} q_{ti}(\sigma)) R_\sigma
\end{aligned} \tag{21}$$

The Markov process has a stationary distribution (Rustichini and Wolinsky (1995)), and so

$$= \arg\max_{i\in I} \sum_{\sigma\in\Sigma} (\lim_{T\to\infty} \sum_{t=1}^{T} \frac{1}{T} \sum_{\sigma'\in\Sigma} q_{1i}(\sigma') Q^{t-1}(\sigma, \sigma')) R_\sigma \tag{22}$$

where $Q(\sigma, \sigma')$ is the probability of being state σ in period $t+1$ conditional upon being in σ' in period t

$$\begin{aligned} &= \arg\max_{i \in I} \textstyle\sum_{\sigma \in \Sigma} (\sum_{\sigma' \in \Sigma} q_{1i}(\sigma')(\lim_{T \to \infty} \sum_{t=1}^{T} \frac{1}{T} Q^{t-1}(\sigma, \sigma')))R_\sigma \\ &= \arg\max_{i \in I} \textstyle\sum_{\sigma \in \Sigma} (\sum_{\sigma' \in \Sigma} q_{1i}(\sigma') q_i(\sigma))R_\sigma \end{aligned} \tag{23}$$

(Hoel et al. (1972), p. 72)

$$= \arg\max_{i \in I} \sum_{\sigma \in \Sigma} q_i(\sigma) R_\sigma \tag{24}$$

QED.

From Lemma 2 we can see that if δ is close to 1 and $\pi_{1t} - \pi_{0t} = 1$ the agent will choose the N that maximizes Equation (19). Therefore in this case the "sell the firm" contract achieves the first best. The next section discusses what happens when δ is not close to 1.

3.2 Results

This section shows that there exists a contract that is more efficient than the "sell the firm" contract. Proposition 4 presents an insight from Rustichini and Wolinsky (1995) that N decreases in δ. That is, as the discount rate falls the agent is "less likely" to choose the high cost action. Corollary 1 shows that the "sell the firm" contract is not optimal.

Following Rustichini and Wolinsky (1995), define Z as the expected value to the agent of his optimal strategy given that $y_t = 0$ and $a_t = 1$. Define Y as the expected value given $y_t = 1$ and $a_t = 1$. Further, let $\pi_{1t} = \pi_{0t} + D$. Note that the agent is paid at least π_{0t} in every period so its value is irrelevant for the maximization problem. Note further that for the case when $p = \frac{1}{D}$ and $c = 0$, the problem is as presented in Rustichini and Wolinsky (1995).

$$Z = \frac{\delta - \delta^{N+1}(pD)}{1-\delta} + \delta^{N+1}(\psi_N Z + (1-\psi_N)(D+Y) - c) \tag{25}$$

and

$$Y = \delta((1-\alpha)(D+Y) + \alpha Z - c) \tag{26}$$

Solving out for Y.

$$Y = \frac{\delta((1-\alpha)D + \alpha Z - c)}{1-\delta(1-\alpha)} \tag{27}$$

Substituting in, we have an equation that can be solve to give Z.

$$\begin{aligned} Z &= \tfrac{\delta-\delta^{N+1}(pD)}{1-\delta} \\ &+\delta^{N+1}\left(\psi_N Z + (1-\psi_N)\left(D + \tfrac{\delta((1-\alpha)D+\alpha Z-c)}{1-\delta(1-\alpha)}\right) - c\right) \end{aligned} \tag{28}$$

Define the implicit function, f.

$$\begin{aligned} f &= -Z + \tfrac{\delta-\delta^{N+1}(pD)}{1-\delta} \\ &+\delta^{N+1}\left(\psi_N Z + (1-\psi_N)\left(D + \tfrac{\delta((1-\alpha)D+\alpha Z-c)}{1-\delta(1-\alpha)}\right) - c\right) \end{aligned} \tag{29}$$

Using the implicit function theorem, the optimal solution for N is the N such that $\frac{\partial f}{\partial N} = 0$, and define f' as the implicit function of the solution.

$$\begin{aligned} f' &= -\tfrac{\log(\delta)\delta^{N+1}(pD)}{1-\delta} \\ &+\log(\delta)\delta^{N+1}\left(\psi_N Z + (1-\psi_N)\left(D + \tfrac{\delta((1-\alpha)D+\alpha Z-c)}{1-\delta(1-\alpha)}\right) - c\right) \\ &+.5\delta^{N+1}\log(1-2\alpha)(1-2\alpha)^{N+1}\left(Z - D - \tfrac{\delta((1-\alpha)D+\alpha Z-c)}{1-\delta(1-\alpha)}\right) \end{aligned} \tag{30}$$

The following proposition which is similar to Rustichini and Wolinsky (1995) Claim 3, characterizes the agent's strategy given parameters of the contract and the model.[11]

Proposition 4 *(i) For sufficiently small α and sufficiently large δ, N decreases in D. (ii) For sufficiently small α and sufficiently small p, N decreases in δ.*

Proof. (i) We have the result if $\frac{\partial f'}{\partial N}$ and $\frac{\partial f'}{\partial D}$ are the same sign. Let $\alpha = 0$, then f' is

$$f' = -\frac{\log(\delta)\delta^{N+1}(pD)}{1-\delta} + \log(\delta)\delta^{N+1}(Z-c) \tag{31}$$

Therefore

$$\frac{\partial f'}{\partial N} = -\frac{\log^2(\delta)\delta^{N+1}(pD)}{1-\delta} + \log^2(\delta)\delta^{N+1}(Z-c) \tag{32}$$

[11] The proposition is not the same is Rustichini and Wolinsky (1995) Claim 3 because of differences in the model specification, but the proof follows the same logic used by the authors.

and

$$\frac{\partial f'}{\partial D} = -\frac{\log(\delta)\delta^{N+1}(p)}{1-\delta} \tag{33}$$

We can see that for large enough δ both equations are negative and we have the result. (ii) In this case we have the result if $\frac{\partial f'}{\partial N}$ and $\frac{\partial f'}{\partial \delta}$ are the same sign.

$$\frac{\partial f'}{\partial \delta} = \left(-\frac{pD}{1-\delta} + Z - c\right)(N+2)\delta^N + \frac{\log(\delta)\delta^{N+1}pD}{(1-\delta)^2} \tag{34}$$

We see that if p is sufficiently small, then both Equation (32) and Equation (34) are positive. QED.

The proposition characterizes the agent's optimal strategy. It shows that as the power of the incentive increases, N decreases, meaning that the agent is going to choose the more costly action more often. Part (ii) shows that N decreases as the discount factor rises. This result has the same intuition as Proposition 1 in the two-period problem. When the agent cares more about the future, he will choose the costly action more often because for certain beliefs, it increases expected payoff in future periods.

The following corollary shows that the principal will get higher profits with $\pi_{1t} - \pi_{0t} > 1$ than she would get if $\pi_{1t} - \pi_{0t} = 1$.

Corollary 1 *If α and p are sufficiently small, and δ is sufficiently large, the first best contract is such that $\pi_{1t} - \pi_{0t} > 1$.*

Proof Let the contract be such that $\pi_{1t} - \pi_{0t} = 1$, and the agent optimal strategy of the agent be characterized by N_1. Given the agent's outside option is 0, the principal receives the expected value of the stationary distribution given N_1. Let N_2 be the N that optimizes the expected value of the stationary distribution. Given $\pi_{1t} - \pi_{0t} = 1$, by Lemma 2 N_2 is also the optimal choice for the agent as δ gets close to 1. By Part (ii) of Proposition 4, $N_2 < N_1$. By Part (i) of Proposition 4 for N_2 to be implemented the difference between $\pi_{1t} - \pi_{0t}$ must be increased. QED.

The proof of the corollary is based on the insight of Rustichini and Wolinsky (1995) that the N describing the optimal strategy of the agent is decreasing in the discount factor of the agent. The higher the discount factor, the greater the value to the agent of choosing the costly action $a_t = 1$. Although the agent bears higher current cost, the action increases the agent's information and thus the expected value of future choices. Given that the agent's discount factor is less than 1, under the "sell the firm" contract the agent does not reap the full benefits of choosing the costly action and "investing" in his information. The principal can achieve the first best by increasing the power of the incentive. In particular, by paying the agent more for the good outcome today the principal can align the agent's incentives to account for the higher future expected payoff. This result occurs because current and future payoffs are positively correlated, so the principal can replicate the first best be increasing the importance of current payoffs in the agent's decision making.

4 Conclusion

A standard result of the principal-agent literature is that it is optimal to sell the firm to a risk-neutral agent. Holmstrom (1979) and Shavell (1979) show that this isn't true when the agent is risk averse. This paper shows that it isn't true either when the agent and the principal have different preferences over time and the agent's choices are a function of the agent's discount factor. The agent doesn't care about the future as much as the principal would like, however because current and future payoffs are positively correlated the principal can replicate the first best by increasing the relationship between the agent's pay and current payoffs.

This paper presents an explanation for why a firm may use broad based stock options rather than ordinary stock to motivate its workers. The value of stock options is that they can exercised at a given date, paying the agent the expected future value of his current choices. In this way stock options can align the agent's and the principal's preferences over time, and enable the first best to be achieved. This explanation seems to line up with the way practitioners view the value of using broad based stock options (NCEO (2003)). In the model presented, the principal benefits in the future from costly actions. In particular the future payoffs are highly correlated with current payoffs. This may be a good representation of situations like R and

D races or "standards" races, and may account for the use of options in sectors where such races are important.

References

Adams, Christopher P., "Theory and Practice of Shopfloor Decision Making in Manufacturing." PhD dissertation, University of Wisconsin–Madison 2001.

_ , "Agent Discretion, Adverse Selection and the Risk-Incentive Trade-Off," December 2002. Federal Trade Commission Working Paper 255.

Baker, George and Bjorn Jorgensen, "Turbulence, Risk and Incentives," January 2003. Preliminary Draft, Harvard University.

Cohen, Adam, *The Perfect Store: Inside eBay*, first ed., Little, Brown and Company, 2002.

Derman, Cyrus, *Finite State Markovian Decision Processes*, Academic Press, 1970.

Hall, Brian J. and Kevin J. Murphy, "The Trouble with Stock Options," *Journal of Economic Perspectives*, 2003, *17* (3). Summer.

Hoel, Paul, Sidney Port, and Charles Stone, *Introduction to Stochastic Processes*, Waveland Press, 1972.

Holmstrom, Bengt, "Moral Hazard and Observability," *Bell Journal of Economics*, 1979, *10* (1), 74–91.

_ **and Paul Milgrom**, "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 1987, *55* (2), 303–328.

Jensen, Michael C., "Paying People to Lie: The truth about the budgeting process," September 2001. HBS Working Paper No. 01-072.

Keller, Godfrey and Sven Rady, "Optimal Experimentation in a Changing Environment," *The Review of Economic Studies*, July 1999, *66* (3), 475–507.

Kreps, David, *A Course in Microeconomic Theory*, Harvester Wheatsheaf, 1990.

Lazear, Edward P., "Output-Based Pay: Incentives, Retention or Sorting?," April 2003. IZA Discussion Paper No. 761.

Mirman, Leonard, Larry Samuelson, and Amparo Urbano, "Monopoly Experimentation," *International Economic Review*, 1993, *34* (3), 549–563.

NCEO, "www.nceo.org," web 2003. National Center for Employee Ownership.

Oyer, Paul, "Why Do Firms Use Incentives That Have No Incentive Effects?," *Journal of Finance*, 2003, *Forthcoming.*

_ **and Scott Schaefer**, "A Comparison of Options, Resticted Stock, and Cash for Employee Compensation," September 2003. Stanford.

Puterman, Martin, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*, John Wiley and Sons, 1994.

Raith, Michael, "Specific Knowledge and Performance Measurement," January 2004. University of Rochester.

Rothschild, Michael, "A Two-Armed Bandit Theory of Market Pricing," *Journal of Economic Theory*, 1974, *9*, 185–202.

Rustichini, Aldo and Asher Wolinsky, "Learning About Variable Demand in the Long Run," *Journal of Economic Dynamics and Control*, 1995, *19*, 1283–1292.

Shavell, S, "Risk Sharing and Incentives in the Principal and Agent Relationship," *Bell Journal of Economics*, 1979, *10* (1), 55–73.