A Game Theory Model of Celebrity Endorsements

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Abstract

Since Milgrom and Roberts (1986) game theorists studying advertising have generally assumed that aggregate advertising expenditures are perfectly observed by consumers. In the real world, however, consumers see only a small fraction of the commercials aired by a given firm and typically do not view the firm's total advertising expenditure. This gives the firm an incentive to make sure that each commercial has as much impact as possible. The impact of a commercial may be enhanced through extravagant production costs, or by purchasing a celebrity endorsement.

Using a signalling game this paper shows how a monopolist may attempt to balance the cost of production against the cost of air time to send a credible signal to consumers at the minimum possible cost. Several examples illustrate the extent to which extravagant production costs (or expensive celebrity endorsements) can substitute for additional spending on air time. Paradoxically, although it is the existence of signal loss (i.e. consumers viewing fewer commercials than were actually purchased) that makes a multifaceted advertising signal attractive, greater signal loss does not necessarily lead to greater production or endorsement expenditures. Rather as signal loss increases, the monopolist has a tendency to substitute expenditures on air time for expenditures on production or celebrity endorsements.

1. Introduction

The notion that advertising can be used to signal quality originates from Nelson (1970,1974). Nelson's argument is based on a distinction between search and experience goods. For experience goods, the actual quality can only be verified after purchase. Nelson's key insight is that even though the content of many advertisements conveys little or no direct information to consumers, the very existence of an expensive advertising campaign may convey information indirectly to consumers. As Milgrom and Roberts (1986) put it, the key message of many advertising campaigns seems to be: "We are spending an astronomical amount of money on this ad campaign."

Because a firm with a high-quality product can expect more repeat sales than a firm with a lower-quality product, such a firm can also afford to spend more money on advertising. The idea of signaling product quality through advertising is based upon the notion that after having viewed an expensive advertising campaign, the consumer is supposed to reason, "You must be a high-quality firm that expects a lot of repeat business, for otherwise, you could not possibly afford to spend all this money on advertising." As long as consumers draw this inference from advertising, the high-quality firms will, in fact, have an incentive to advertise, and consumers will, therefore, have an incentive to respond to advertising. In this way, a firm can credibly signal the actual quality of its product indirectly to consumers.

One limitation of previous models of advertising is that they often tacitly assumed that all dollars spent on advertising are equivalent. Such an assumption hinders any analysis of how firms divide up their advertising dollars. For example, why does one firm purchase an expensive celebrity endorsement from a famous rock star, while another firm spends almost all of its advertising dollars on air time? In Hertzendorf (1993) I argued that for advertising over various electronic media (i.e. television and radio) signal loss is an important factor which should not be ignored. Because most commercials will not actually be seen by consumers, consumers can only make imprecise estimates as to the firm's total expenditure on advertising. This inability of firms to convey directly to consumers their actual total expenditures on advertising hinders the firm's potential to credibly signal their high-quality. In fact, as shown in Hertzendorf (1993), the signal loss may provide an opportunity for a low-quality firm to trick consumers into thinking that it is high-quality firm by partially mimicking the strategy of such a firm.

Confronted with the problem of signal loss, firms have a strong incentive to increase the impact of each commercial actually viewed by consumers. This might be accomplished via extravagant production costs or by purchasing a celebrity endorsement. There are other approaches to increasing the impact of advertising. A firm might employ "saturation" advertising and purchase many ads during one show or during one evening. As part of a $45 million dollar campaign to introduce its new Mercury Mystique, Lincoln-Mercury bought 24 commercial spots during one evening (freezing out all automobile competitors that night) in the fall of 1994. This helps ensure that many consumers will see the commercial at one time and also increases the chances that a company's advertising will be noticed above the clutter of media message assaulting consumers each day.[1] Another approach to increasing the impact of commercials is to make them so controversial that, not only can they not be forgotten, but they also generate additional publicity for free. The Italian clothing manufacturer Benetton, for example, has developed controversial commercials that have included: an oil-soaked sea-bird, a dying AIDS patient, and most recently the bloody uniform of a Croatian soldier.[2]

Another effective method of overcoming signal loss is to advertise when the signal loss is low. This undoubtably explains why advertising during the Super Bowl is so attractive. In 1995 firms paid an average of 1 million dollars for each 30 second commercial aired during the Super Bowl. These commercials were viewed by about one-half of all Americans (130 million). Not only may advertisements during the Super Bowl be watched by a greater audience, it is also possible that they are more likely to be remembered.[3]

Of course, different approaches to overcoming signal loss may be combined. Advertising during the Super Bowl does not preclude extravagant production costs, rather it encourages it. Any firm that is spending a million dollars for 30 seconds of air time wants to be sure that its commercial will have an impact. Wilson Sporting Goods, for example, spent $1 million dollars to produce a 60 second commercial for Super Bowl XXIX (it cost $2 million to air it). The company has no plans to air the commercial on TV at any time in the future.[4] Nike, on the other hand, spent $3 million dollars to air one 90 second commercial whose production costs were reported to be in six figures.[5]

At some point it becomes difficult to distinguish between similar mechanisms. Celebrities, for example, might be used to reduce signal loss directly. A special one-hour episode of the television show "Friends" appeared immediately after Super Bowl XXX and drew 66.3 million viewers. On that same day a monthlong $30 million dollar promotion for Diet Coke was concluded. This commercial included the entire cast of "Friends," each of whom was paid an estimated $250,000 to $500,000.[6]

Although these examples highlight the tradeoff between various approaches to advertising, the same tradeoffs exist in all advertising campaigns. Each firm must decide to how to allocate advertising dollars to produce the maximum effect.

This paper uses a signalling game to study the tradeoff between expenditures on ad production and air time. Rather than focussing on the psychological or emotional impact of advertising, I treat high production costs and celebrity endorsements as components of a multifaceted advertising signal.[7]

In the game I develop, consumers attempt to infer a monopolist's total advertising expenditure from a stochastic advertising signal. The advertising signal is stochastic because consumers typically view fewer commercials than were purchased due to random signal loss. However, once at least one commercial is viewed by consumers, I assume that production costs are perfectly observed.

I view celebrity endorsements (as well as the production costs associated with elaborate commercials) as an attempt by firms to overcome signal loss. Without a celebrity endorsement, a consumer viewing one commercial might conclude with certainty, that the firm has spent tens of thousands of dollars to air the commercial that is being viewed.[8] However, if the same commercial includes Elton John, the consumer might, after viewing one commercial, conclude that the firm is spending millions of dollars on advertising, because that is how much Elton John costs. Hence, celebrity endorsements are a way of packing more information into each commercial, so that inferences about quality can be drawn from viewing fewer commercials.

The goal of this paper is show how a monopolist would attempt to minimize the cost of signaling product quality to consumers by exploiting the tradeoff between air time and high production costs (e.g. celebrity endorsements or special effects). The paper proceeds in 6 sections. Section 2 presents the basic model and the definition of a sequential equilibrium. In Section 3 I introduce a modest equilibrium refinement. The purpose of this refinement concept is to eliminate wildly implausible sequential equilibria. Section 4 provides a characterization of the refined equilibria, as well as examples. In Section 5 I show how the model can be easily extended to encompass price signaling. Section 6 concludes.

2. The Model

I assume that a monopolist manufactures either a high-quality or a low-quality good. Nature randomly selects between two possible quality values H and L according to some prior probability distribution $(1 \geq H \geq L \geq 0)$. I use ρ to denote the prior probability of Nature selecting a high-quality monopolist. Hence, following Harsanyi (1967), I am modeling a game of incomplete information as a game of imperfect information. A strategy for the monopolist is a function, $M(Q):[H,L] \rightarrow (A,S)$, that translates the actual quality of the firm into a nonnegative advertising pair. The first component of the advertising pair represents the number of advertisements purchased by the firm at price P_A. The second component of the advertising pair represents a dissipative advertising expenditure that is observed whenever at least one commercial is observed by consumers. We might think about this second advertising expenditure as the cost of a celebrity endorsement.

Since the "celebrity endorsement" expenditure is only observed when a commercial is seen, I draw an analogy to electronic communications and sometimes refer to the commercials as the "carrier signal" and the other expenditure as the "modulation." I assume that the carrier signal is stochastic due to the possibility of signal loss (not all the commercials will typically be seen by consumers). The modulation is essentially deterministic; once one commercial is seen by consumers, this expenditure has been perfectly observed. The carrier signal is chosen from the set of natural numbers N (i.e. N=0,1,2,3...), while the modulation is chosen from the positive reals R^+. Consumers are initially unaware of the type of monopolist they face, but form an expectation after observing one or more of the commercials and any associated celebrity endorsement. Hence, consumers only enter into the model indirectly through the function $EQ(\tilde{A},S):(N x R^+) \rightarrow [0,1]$ which translates their observations into an expectation of quality. The complete map which consumers use to translate an observation into an expectation of quality is sometimes referred to as the "beliefs of consumers." The exact nature of consumer beliefs cannot be specified until I introduce a specific stochastic process that will translate a monopolist's advertising expenditure into a probability density function for possible observations. The quality, which is bounded between 0 and 1, may be thought of as the probability that the good in question will perform satisfactorily for a given consumer. The number of advertisements observed by consumers will not, in general, be the same as the number of advertisements purchased by the monopolist. It will typically be less. This reflects the possibility that consumers will fail to view certain advertisements because of signal loss.

Throughout the paper, I use the convention of denoting a random variable in bold face and realizations of this random variable in regular type. Hence, $\boldsymbol{\tilde{A}}$ is the random variable observed by consumers and $\tilde{A}$ is a particular realization of this random variable. I use A_Q to denote the advertisements purchased by the type-Q monopolist (in equilibrium) and $g(\tilde{A}:A_Q)$ to denote the probability density function of $\tilde{\mathbf{A}}$. Hence $\tilde{A}$ is a random variable whose distribution depends on A_Q. Occasionally, I use a regular type "A" to indicate the argument of various functions.

The monopolist is only concerned about the expectations of consumers, because higher expectations of quality translate into greater profits for the monopolist. Given the beliefs of consumers $EQ(\tilde{\mathbf{A}},S)$, I represent the expected profits to the type-Q monopolist which purchases the advertising pair (A_Q,S_Q) as $E\pi(Q,A_Q,S_Q,EQ(\tilde{\mathbf{A}},S))-P_A A_Q-S_Q$. The advertising strategy (A_Q,S_Q) affects profits in two ways. First, holding fixed the beliefs of consumers, changing the advertising strategy will change the expectations that consumers are likely to have because the distribution of $\tilde{\mathbf{A}}$ will be altered. This in turn will change the monopolist's expected gross profits. Second, changing the advertising strategy will affect the advertising costs experienced by the monopolist. I assume that expected profits are increasing and continuous in expected quality so that, holding advertising expenditures fixed, the monopolist always wants consumer expectations of quality to be as high as possible. We are now ready to define the first equilibrium concept.[9]

Definition: A sequential equilibrium for this model is a strategy $M(Q)$ for the firm and system of beliefs for consumers $EQ(\tilde{A},S)$ such that:

(1) $M(H)=(A_H,S_H)$ maximizes $E\pi(H,A,S,EQ(\tilde{\mathbf{A}},S))-AP_A-S$ given $EQ(\tilde{\mathbf{A}},S)$;

(2) $M(L)=(A_L,S_L)$ maximizes $E\pi(L,A,S,EQ(\tilde{\mathbf{A}},S))-AP_A-S$ given $EQ(\tilde{\mathbf{A}},S)$;

(3) $EQ(\tilde{\mathbf{A}},S)$ is computed using Bayes' rule along the equilibrium path of play.

(4) Both A_Q and S_Q are nonnegative.

The exact nature of consumer expectations will depend upon the stochastic process by which the strategy of the monopolist is translated into a set of possible observations by consumers. In this paper I view the process of viewing commercials as analogous to the selection of balls from an urn without replacement. In particular we imagine that the monopolist purchases a given number of commercials and these commercials are placed in a large urn with all the other commercials purchased by all the other firms. B denotes the total number of commercials purchased by other firms. I refer to these commercials as the *irrelevant commercials.* I refer to the commercials or advertisements purchased by the monopolist as the *monopoly commercials.* V denotes the number of commercials randomly selected/viewed by consumers. As stated earlier A_H and A_L represent the number of commercials purchased by the high and low-quality type monopolist respectively. This setup is similar to Hertzendorf (1993).

We might imagine the urn to be shaped like a television set. The process of selecting balls from the urn without replacement would be equivalent to turning on the television at a randomly selected time, to a randomly selected channel. Since the same exact commercial cannot be observed twice (unless of course there is video tape involved), it seems appropriate to assume a stochastic process without replacement.

Clearly the consumers cannot view more commercials then were purchased by either type monopolist, nor can the number of monopoly commercials viewed exceed the total number of commercials viewed. Similarly, if the total number of commercials viewed, V, exceeds the number of irrelevant commercials, then the consumer must view at least $(V-B)$ of the monopoly commercials, unless of course, $A_Q < (V-B)$.[10] Let $A_{min} = \max\{0, \min\{V-B, A_Q\}\}$ and $A_{max} = \min\{V, A_Q\}$, then for $A_{min} \leq \tilde{A} \leq A_{max}$ we can define the probability of consumers viewing A monopoly ads using the hypergeometric distribution.

$$Prob(\tilde{A}=\tilde{A} : A_Q) = g(\tilde{A}:A_Q) = \frac{\binom{A_Q}{\tilde{A}}\binom{B}{V-\tilde{A}}}{\binom{B+A_Q}{V}} \quad for \quad A_{min} \leq \tilde{A} \leq A_{max}. \qquad (1)$$

For all other $\tilde{A}$, $g(\tilde{A}:A_Q)=0$. We are now ready for the first result.

Theorem 1: In any sequential equilibrium $A_L=0$ implies that $S_L=0$ while $A_L>0$ implies that $S_L=S_H$.

Proof: We employ a proof by contradiction. Suppose instead there was a sequential equilibrium with $A_L>0$ and $S_L \neq S_H$. If consumer beliefs are consistent with Bayes' rule then $EQ(\tilde{A}, S_L)=L$ for all $\tilde{A} \in \{A_{\min}, \ldots, A_{\max}\}$ while $EQ(\tilde{A}, S_L)=Q \geq L$ if $\tilde{A}=0$. However, given these beliefs the strategy $M(L)=(0,0)$ will induce a set of beliefs (in equilibrium) that are at least as favorable, possibly more favorable than the strategy $M(L)=(A_L, S_L)$. Because the strategy $M(L)=(0,0)$ is also less costly than the strategy $M(L)=(A_L, S_L)$, this later strategy could not have been optimal for type L and, therefore, could not have been part of a sequential equilibrium. The proof that $M(L)=(0, S_L)$ with $S_L>0$ cannot be part of an equilibrium is even easier. The alternative strategy $M(L)=(0,0)$ does not change any of the observations by consumers since the celebrity endorsement cannot be seen when $A_L=0$.. Hence, the alternative strategy reduces advertising expenditures of the low-type without affecting gross profits. This proves that $M(L)=(0, S_L)$ could not have been part of a sequential equilibrium. We might summarize the proceeding argument as follows: (1) if the low-quality monopolist isn't going to advertise, then it's certainly not going to waste money producing a commercial and (2) if advertising by the low-type monopolist doesn't trick consumers into believing its quality is higher than it really is, then advertising can't be worthwhile (hence, $S_L=S_H$). Q.E.D.

The importance of the previous result is that it tells us that if both types of monopolist advertise, they will also both use the same modulation (denoted by $\overline{S}$). This fact enables us to utilize Bayes' rule to define consumer expectations and the expected monopoly profits. Let the monopolist's strategy be given by $[M(H)=(A_H,\overline{S}),\ M(L)=(A_L,\overline{S})]$. Then we have

$$EQ(\tilde{A},\overline{S})=\frac{\rho Hg(\tilde{A}:A_H)+(1-\rho)Lg(\tilde{A}:A_L)}{\rho g(\tilde{A}:A_H)+(1-\rho)g(\tilde{A}:A_L)} \qquad for\ \ \tilde{A}<A_L,A_H \tag{2}$$

When $A_L<\tilde{A}<A_H$; $EQ(\tilde{A},\overline{S})=H$ and when $A_H<\tilde{A}<A_L$; $EQ(\tilde{A},\overline{S})=L$.

$$E\pi(Q,A_Q,\overline{S},EQ(\tilde{A},\overline{S}))=\sum_{\tilde{A}=A_{min}}^{A_{max}} g(\tilde{A}:A_Q)\left[\pi(Q,A_Q,\overline{S},EQ(\tilde{A},\overline{S}))-A_Q P_A-\overline{S}\right] \tag{3}$$

Hence to compute expected profits we just multiply the probability that $\tilde{A}$ commercials will be observed by the profits that will result from this realization of $\tilde{A}$, and sum up over all possible realizations. The expected quality incorporates Bayes' rule in the relevant region. Intuitively, the consumer is just comparing the relative likelihood that the realization of $\tilde{A}$ came from either of the two types. In doing so the consumer considers the strategy of each type and the prior probabilities ρ and $(1-\rho)$ that the firm is a high- or low-quality monopolist, respectively.

It is common practice to distinguish between two different kinds of equilibria. In a *pooling equilibrium* both the high and low-type monopolist pick identical strategies. In this situation observations of the strategy do not reveal anything new to consumers and so the expected quality of consumers must be $EQ(\tilde{A},S)=\rho H+(1-\rho)L$ (for all "in-equilibrium" observations.) In a *separating equilibrium* the high and low-type monopolist pick different strategies. In games were there is no signal loss, this would result in consumers knowing with certainty whether or not the quality is high or low. However, in the current game where there is signal loss this need not be the case. Observations by consumers may reveal imperfect information about quality. Only in the special case where an observation is consistent with the strategy of the high type $M(H)=(A_H,S_H)$ but inconsistent with the strategy of the low type $M(L)=(A_L,S_L)$ will the consumers know with certainty that they are facing a high-quality monopolist.

3. Pareto Optimal Equilibria

In general, there will not be a unique sequential equilibrium. This hinders one's ability to make testable predictions and also makes it difficult to do comparative statistics. However, many of the possible sequential equilibria are absurdly inefficient. The equilibria are inefficient in the sense that there are other sequential equilibria which reduce the cost of separating without affecting the "in-equilibrium" beliefs of consumers. These other equilibria are preferred by both the high type and the low-type, because they result in the same gross profits, but reduce the cost of advertising. To the extent that one has some faith in the ability of market forces to resolve the problem of incomplete information efficiently one would want to focus on this subset of the sequential equilibria. Let *SE* denote the set of all Sequential Equilibria.

Definition: Let $M(L)^*=(A_L^*,S_L^*)$ and $M(H)^*=(A_H^*,S_H^*)$, then $\{M(L)^*,M(H)^*,EQ^*(\tilde{\mathbf{A}},S)\}\in SE$ is **Pareto Optimal with respect to Sender Types** ("POST") if there does not exist another sequential equilibrium $\{M(L)',M(H)',EQ'(\tilde{\mathbf{A}},S)\}\in SE$ such that:

(A) $E\pi(H,A_H',S_H',EQ(\tilde{\mathbf{A}},S))-A_H'P_A-S_H'\geq E\pi(H,A_H^*,S_H^*,EQ(\tilde{\mathbf{A}},S))-A_H^*P_A-S_H^*$

(B) $E\pi(L,A_L',S_L',EQ(\tilde{\mathbf{A}},S))-A_L'P_A-S_L'\geq E\pi(L,A_L^*,S_L^*,EQ(\tilde{\mathbf{A}},S))-A_L^*P_A-S_L^*$

with one of the inequalities holding strictly.

This equilibrium refinement will enable me to rule out inefficient signaling by the high-quality monopolist. The notion of focusing on Pareto-dominant elements of the equilibrium set is not new. This approach was originally proposed by Spence in the 1970's. The reader may, however, wonder why I choose to use this approach rather than one of the more popular refinements that consider the plausibility of various out-of-equilibrium beliefs (i.e. Cho and Krep's (1987) *intuitive criterion* or Banks and Sobel's (1987) *universal divinity*.) In most signaling games the equilibrium is sustained by a collection of "out-of-equilibrium" beliefs. Consider the following example. The high-type monopolist earns a profit of 4 if it is believed to be a high type and a profit of 2 if it is believed to be a low type. The low type monopolist earns a profit of 2 if it is mistakenly believed to be the high type and a profit of 1 if it is believed to be the low type. The price of an advertisement is 1.1 and assume that there is no signal loss and no possibility for celebrity endorsements. The following is a sequential equilibrium $M(H)=(2)$, $M(L)=(0)$. The high type is known to be a high type in equilibrium and earns a profit of 4-2(1.1)=1.8. Consumer beliefs which are consistent with this equilibrium are $EQ(0)=EQ(1)=L$ and $EQ(2)=H$. However, because in equilibrium the consumer will never actually see one advertisement,[11] $EQ(1)=L$ is referred to as an "out-of-equilibrium" belief.

An equilibrium refinement like the intuitive criterion would argue that the previous equilibrium is implausible because the out-of-equilibrium belief which sustains it (i.e. $EQ(1)=L$) is implausible. In particular, why would the low-quality firm ever purchase one advertisement? Even if after such a purchase the low type was mistakenly believed to be a high type it would only earn 0.9=2-(1.1). This is less than the low-type profits which are guaranteed by not advertising (i.e. 1). On the other hand, the high type would gladly defect to $A_H=1$ if this would cause consumers to believe it was the high type. Since only the high type would ever have an incentive to purchase one ad it is argued that consumer beliefs must be such that $EQ(1)=H$. Hence, consumers will interpret a disequilibrium message (i.e. 1 ad, instead of 2) as a signal that the monopolist is really a high quality. This, however, would then give the high-quality monopolist an incentive to defect and, thereby, overturn the previous equilibrium. In doing so, the high type will now earn 2.9 = (4 - 1.1) instead of 1.8 as in the previous equilibrium.

Hence, the more common approach to equilibrium refinements is to reduce the number of sequential equilibria by arguing that certain out-of-equilibrium beliefs (necessary to sustain some of the equilibria) are implausible. This approach will not work in my model because with signal loss present there is no longer a one-to-one correspondence between out-of-equilibrium strategies and out-of-equilibrium observations. Consider the following example where $V=1$ and $B>1$. In this situation consumers will view either one monopoly commercial or they will view no monopoly commercials. Regardless of what strategy either type selects, observing zero observations is always a possibility, while it also remains impossible for consumers to observe more than one monopoly commercial. If the type-Q monopolist switches to a strategy that involves a different number of commercials, the consumers will have no way of knowing that they are observing an out-of-equilibrium message that wasn't supposed to occur. Hence, there is no out-of-equilibrium number of commercials that can be observed. The use of a refinement that considers defections to out-of-equilibrium strategies can, therefore, only be used to limit the possible modulation (S_Q) that might exist in equilibrium. However, as the next example shows, such a refinement is of little use when trying to decide between two equilibria, especially when neither includes a celebrity endorsement.

Example 1: Consider a model with the following parameters: L=0, H=1, $\rho=1/2$, B=1, V=1 and $P_A=17/36$. The profit function for the high- and low-type monopolist are respectively:[12,13]

$E\Pi(H,A,S,EQ(\tilde{A},S)) = 4\,EQ(\tilde{A},S) - (A)P_A - S$

$E\Pi(L,A,S,EQ(\tilde{A},S)) = EQ(\tilde{A},S) - (A)P_A - S$

Since L=0, the low-type monopolist is selling junk which is sure to dissatisfy consumers. Because $\rho=(1-\rho)=1/2$, L=0, and H=1, the formula for expected quality (equation 2) is greatly simplified: $EQ(\tilde{A},S_Q) = g(\tilde{A}:A_H)/(g(\tilde{A}:A_H)+g(\tilde{A}:A_L))$. There are two separating equilibrium in this game with $S_Q=0$:[14]

Equilibrium 1: {M(H)=(1,0),M(L)=(0,0)} with EQ(0,0)=1/3, EQ(1,0)=1, and EQ(A,S) for all other (A,S) can be arbitrarily defined.

Equilibrium 2: {M(H)=(2,0), M(L)=(0,0)} with EQ(0,0)=1/4, EQ(1,0)=1, and EQ(A,S) for all other (A,S) can be arbitrarily defined.

The conditions for a sequential equilibrium can be easily verified. Consider equilibrium 1. Consumer expectations can be verified given the monopolist's strategy and the simplified formula above. The low type is earning a profit of 1/3 by not advertising. Purchasing one advertising will result in profits of $(1-P_A)=(1-17/36)$, 50% of the time (i.e. when the advertisement is viewed by consumers, recall that V=B=1) and profits of (1/3-17/36) the other 50% of the time. Therefore, the expected profit to the low type from mimicking the strategy of the high type is $E\Pi(L,1,0,EQ(\tilde{A},0))=7/36<1/3$. It can similarly be shown that any higher level of advertising will likewise result in expected profits of less than 1/3. Therefore, the low type is maximizing its expected profits by not advertising.

The high type earns an expected profit in equilibrium 1 of 24/9 - 17/36 (i.e. $4[(1/2)(1/3)+(1/2)(1)]-P_A$). If the high type decides not to advertise then no commercials will ever be seen and its profits are guaranteed to be 4(1/3)=12/9. If on the other hand, the high type purchases 2 ads instead of one, there is now a 2/3 chance that one of them will be viewed and so its expected profits are 28/9 - 34/36 ($4[(2/3)(1)+(1/3)(1/3)]-2P_A$). Simple calculation reveals that the high type will maximize its profits at M(H)=(1,0). In a similar manner the conditions for equilibrium 2 can also be verified.

We cannot discard either equilibrium because of implausible out-of-equilibrium beliefs. First, since $\tilde{A}\in[0,1]$, there are no out-of-equilibrium number of ads. There are only out-of-equilibrium celebrity endorsements. It is easy to see, however, that regardless of how we set $EQ(1,S)$, neither type would wish to ever defect from the equilibrium strategy. Defection would be most attractive when $EQ(1,S)=H=1$ for small S. Consider equilibrium 1. Given that the low type does not find it desirable to defect to (1,0), it certainly would not find it desirable to defect to (1,S), since both strategies induce the same expectations, and the second strategy is costlier. The same logic also applies to high type. Since the out-of-equilibrium celebrity endorsements may be set arbitrarily, both of these equilibria are "immune" to out-of-equilibrium belief refinements.

However, simple calculation reveals that equilibrium 1 is the only POST equilibrium. If the high type purchases one ad rather than two, then the expected quality (and also the profits of the low type) upon viewing zero ads will be 1/3 rather 1/4. The first equilibrium is, therefore, more profitable for the low type. Although by purchasing two ads instead of one the high type has a greater chance of being perceived as a high-quality firm (2/3 chance as opposed to a 1/2 chance) this benefit is more than offset by the additional advertising cost (and also by the reduction in profits when zero ads are observed by consumers). Figure 1 indicates the relative profitability of equilibrium 1 and equilibrium 2.

As games have become more complicated, the stories required to justify the implausibility of various out-of-equilibrium beliefs have also become more complicated. As discussed above, the introduction of noisy signaling is one such complication. Even in those situations where there actually exist out-of-equilibrium beliefs which are required to sustain the equilibrium, there will not in general be a one-to-one correspondence between out-of-equilibrium beliefs and out-of-equilibrium strategies. Although a POST equilibrium is no panacea, it does offer one simple framework within which to consider the plausibility of sequential equilibria.

The whole concept of a signaling game is that there are market mechanisms in place that can resolve incomplete information and, thereby, improve market performance. Advertising need not be viewed as an unproductive and manipulative attempt to increase profits. Rather advertising expenditures can be viewed as an important and efficient mechanism by which information is exchanged, even if it is the expenditure itself and not the content of the commercials that conveys the information. Although there may be numerous sequential equilibria there are obviously incentives for the monopolist to pick a reasonably efficient one. At a minimum, we should rule out those sequential equilibria which are not Pareto optimal with respect to the two types.

Although the consumers may also have an incentive to overturn inefficient equilibria, the benefits to each individual consumer are likely to be small. Unless consumers are somehow organized, it seems unlikely that they would provide the impetus for overturning an inefficient equilibrium (at least in this game).[15]

As in other recently proposed refinements, the POST equilibrium of a game will often be a pooling equilibrium rather than a separating equilibrium. In particular, I show in the next section that when the prior probability of facing a high-quality monopolist is sufficiently high, only a pooling equilibrium is possible. This is because as the prior probability increases, the consumer's (prior) expected quality also increases. This in turn reduces the incentives for both types of monopolist to incur the expense of advertising.

Other authors have also recently stressed the importance of pooling equilibria and critiqued the use of some refinements that restrict out-of-equilibrium beliefs. Mailath, Okuno-Fujiwara and Postlewaite (1993) propose a refinement concept called the *lexicographically maximum sequential equilibrium* ("LMSE"). By construction the LMSE is pareto optimal with respect to the sender types ("POST"). In the current game where there are only two types, the LMSE is simply the sequential equilibrium that is preferred by the high type. The idea of selecting the equilibrium preferred by the high type has also been proposed by Overgaard (1993) in his *Criterion A*. According to Overgaard, "At any point in the game the type L firm is deciding whether or not to mimic the H firm, hence, deciding whether to accept an equilibrium profile *proposed* by type H. When we take this viewpoint, the emphasis should be on the decision of type H..." Maskin and Laffont (1987), however, propose a different selection from the set of POST equilibria which they call *the best perfect Bayesian equilibrium.*[16] In particular, they argue in favor of selecting that equilibrium which maximizes the expected profits of the ex-ante monopolist (i.e. the profits of the monopolist before knowing its actual type).

In the next section, POST equilibria are characterized. However, as will be shown in later examples there may be multiple POST equilibria. I illustrate how the refinements discussed above can be utilized to derive a unique outcome.

4. Analysis

I now present several theorems which attempt to characterize the POST equilibria in my model.

Theorem 2: All Pooling POST Equilibria are of the form $M(H)=M(L)=(0,0)$ and $EQ(0,0)=\rho H+(1-\rho)L$.

Proof: Clearly if both firms are playing the same strategy then any observation consistent with that strategy reveals no information to consumers. Hence, the expected quality must be the same as the ex ante expected quality $\rho H+(1-\rho)L$. Consider first the case where $V\leq B$. In this situation, observing zero commercials is "in-equilibrium." I employ a proof by contradiction. Suppose that $M(H)=(\overline{A},\overline{S})$ and $M(L)=(\overline{A},\overline{S})$ are part of a pooling equilibrium. Then because (0,0) is an observation consistent with the equilibrium $EQ(0,0)=\rho H+(1-\rho)L$.[17] But then $M(Q)=(0,0)$ decreases the costs to type Q of signaling, while leaving the realized expectations of quality intact. This means that a defection by type-Q to the strategy $M(Q)=(0,0)$ will reduce the costs associated with signaling, while leaving the gross profits to the type-Q monopolist unaffected. Hence, a defection to $M(Q)=(0,0)$ increases the net profits of type-Q. This proves that conditions (1) and (2) of a sequential equilibrium are not satisfied. Hence $M(H)=(\bar{A},\bar{S})$ and $M(L)=(\bar{A},\bar{S})$ cannot be part of **any** pooling sequential equilibrium whenever $V \leq B$.

Next, assume that $V>B$. Again I employ a proof by contradiction. Suppose that $M(H)=(\bar{A},\bar{S})$ and $M(L)=(\bar{A},\bar{S})$ are part of a pooling equilibrium. Consider an alternative sequential equilibrium where $EQ(\tilde{A},S)=L$ for all $(\tilde{A},S)\neq(0,0)$. Clearly, $M(H)=M(L)=(0,0)$ and $EQ(0,0)=\rho H+(1-\rho)L$ form the only sequential equilibrium given these beliefs. Neither type has an incentive to defect because signaling is costly and can only worsen consumer expectations of quality. Hence, (1),(2) and (3) of a sequential equilibrium are satisfied. However, we will also have

$$E\Pi(H,0,0,EQ(0,0)) > E\Pi(H,\bar{A},\bar{S},\rho H+(1-\rho)L)-\bar{A}P_A-\bar{S} \text{ and}$$
$$E\Pi(L,0,0,EQ(0,0)) > E\Pi(L,\bar{A},\bar{S},\rho H+(1-\rho)L)-\bar{A}P_A-\bar{S}.$$

Hence, the pooling equilibrium with $M(Q)=(0,0)$ is Pareto preferred by both types to our conjectured equilibrium with $M(H)=(\bar{A},\bar{S})$ and $M(L)=(\bar{A},\bar{S})$. Roughly speaking, its mutually beneficial for both types to simultaneously abandon all advertising. This proves that all pooling POST equilibria are of the form $M(Q)=(0,0)$. Notice that in the first part of the proof ($V<B$) we exploited the definition of a sequential equilibrium, while in the second part of the proof ($V\geq B$) we exploited the additional requirements of a POST equilibrium. QED.

Theorem 3: For ρ sufficiently close to 1, all POST equilibria are pooling equilibria of the form $M(Q)=(0,0)$, $Q\in\{L,H\}$ and $EQ(0,0)=\rho H+(1-\rho)L$.[18]

Proof: Clearly $M(Q)=(0,0)$ and $EQ(0,0)=\rho H+(1-\rho)L$ is always a sequential equilibrium provided that out-of-equilibrium beliefs are defined such that $EQ(\tilde{A},S)=L$ for all $(\tilde{A},S)\neq(0,0)$. The expected profits to type-Q in the canonical pooling equilibrium are $E\Pi(Q,0,0,\rho H+(1-\rho)L)$. Consider any other sequential equilibrium where $M(H)=(A_H,S_H)$, $M(L)=(A_L,S_L)$ and consumer beliefs are $EQ(\tilde{\mathbf{A}},S)$. We will compare the profits of this arbitrary sequential equilibrium with the profits that both types will obtain in the canonical pooling equilibrium as ρ approaches 1. As $\rho\to 1$, $EQ(0,0)=\rho H+(1-\rho)L$ approaches H. By the continuity of expected profits in expected quality, as $\rho\to 1$, we also know that the expected profits to the type-H monopolist will approach $E\Pi(H,0,0,H)$. These are the best possible profits for type-H, since it has spent nothing on signaling, and yet consumers will perceive the product as being of the highest possible quality. Similarly, as $\rho\to 1$, the profits to the low type in the canonical pooling equilibrium will approach $E\Pi(L,0,0,H)$ and these are, likewise, the best possible profits for the low type. In other words, when we compare the expected profits in the canonical pooling equilibrium to the profits from some arbitrary sequential equilibrium we must have:

$$E\Pi(H,0,0,H) > E\Pi(H,A_H,S_H,EQ(\tilde{\mathbf{A}},S)) - P_A A_H - S_H \text{ and}$$

$$E\Pi(L,0,0,H) > E\Pi(L,A_L,S_L,EQ(\tilde{\mathbf{A}},S)) - P_A A_L - S_L$$

Hence, for ρ sufficiently close to 1, $M(H)=(A_H,S_H)$, $M(L)=(A_L,S_L)$ cannot be part of a POST equilibrium since the canonical pooling equilibrium with $M(Q)=(0,0)$ is more profitable for both types. Roughly speaking, when consumers think that a high quality monopolist is sufficiently likely then it doesn't pay for either type to incur the expense of signaling product quality. This is because the alternative is to sell goods under the consumers' prior expectations of quality, which are already quite high. QED.

Theorem 4: There are no POST equilibria of the form $M(H)=(A_H,\overline{S})$, $M(L)=(A_L,\overline{S})$ where $\overline{S}>0$.

Proof: Suppose instead that $M(H)=(A_H,\overline{S})$ and $M(L)=(A_L,\overline{S})$ are part of a sequential equilibrium. Without loss of generality we may assume that $EQ(\tilde{A},S)=L$ for all out-of-equilibrium observations. We will show that $M(H)=(A_H,0)$, $M(L)=(A_L,0)$ are part of a sequential equilibrium that is more profitable. By condition (1) of a sequential equilibrium we know that $E\Pi(H,A_H,\overline{S},EQ(\tilde{\mathbf{A}},S)) - P_A H_H - \overline{S} \geq E\Pi(H,A,S,EQ(\tilde{\mathbf{A}},S)) - P_A A - S$ for all

$(A,S)\geq(0,0)$, given $EQ(\tilde{\mathbf{A}},S)$. Next define $EQ'(\tilde{A},0)=EQ(\tilde{A},\overline{S})$ for all $A_{\min}\leq\tilde{A}\leq A_{\max}$ and $EQ'(\tilde{A},S)=L$ for all other $(\tilde{A},S)$. By construction we have $E\Pi(H,A,0,EQ'(\tilde{\mathbf{A}},S))=E\Pi(H,A,\overline{S},EQ(\tilde{\mathbf{A}},S))$ for all $A>0$. Substituting into condition (1) reveals $E\Pi(H,A_H,0,EQ'(\tilde{A},S))-P_AA_H-\overline{S}\geq E\Pi(H,A,0,EQ'(A,S))-P_AA-S$. But since $EQ(\tilde{A},S)=L$ for all out-of-equilibrium beliefs where $S\neq0$ we can also be sure that $E\Pi(H,A_H,0,EQ'(\tilde{A},S))-P_AA_H-\overline{S}\geq E\Pi(H,A,S,EQ'(A,S))-P_AA-S$ for all $(A,S)\geq(0,0)$. Adding $\overline{S}$ to the left side can only make it larger so we must also have $E\Pi(H,A_H,0,EQ'(\tilde{A},S))-P_AA_H\geq E\Pi(H,A,S,EQ'(A,S))-P_AA-S$ for all $(\tilde{A},S)$ holding fixed the beliefs of consumers. This implies that $M(H)=(A_H,0)$ satisfies condition (1) of a sequential equilibrium. Notice also that by construction $E\Pi(H,A_H,0,EQ'(\tilde{A},S))-P_AA_H\geq E\Pi(H,A,S,EQ(\tilde{A},S))-P_AA_H-\overline{S}$. We can do the same analysis for type-L and discover that $E\Pi(L,A_L,0,EQ'(\tilde{A},S))-P_AA_H\geq E\Pi(H,A,S,EQ'(A,S))-P_AA-S$ for all $(A,S)\geq(0,0)$ and that $E\Pi(L,A_L,0,EQ'(\tilde{A},S))-P_AA_L>E\Pi(L,A_L,\overline{S},EQ(\tilde{A},S))-P_AA_L-\overline{S}$. All of this implies that $\{M(H)=(A_H,0),M(L)=(A_L,0),EQ'(\tilde{A},S)\}$ is a sequential equilibrium that is preferred by both types to the sequential equilibrium $\{M(H)=(A_H,\overline{S}),M(L)=(A_L,\overline{S}),EQ(\tilde{A},S)\}$. Hence, the conjectured sequential equilibrium could not have been a POST equilibrium. QED.

Although technically challenging, the intuition behind this theorem is obvious. A POST equilibrium requires the absence of an alternative equilibrium where both types are better off. However, when both types adopt exactly the same modulation neither type achieves any advantage over the other. If both types can agree to suspend all celebrity endorsements each type gains $\bar{S}$, and consumer beliefs are unaffected. Combining theorem 1 and theorem 3 it is clear that all POST equilibria that involve celebrity endorsements involve no advertising signals by the low-type firm.

I now present two theorems that deal with separating equilibria that include celebrity endorsements (i.e. signal modulation). I will refer to these equilibria as "celebrity" equilibria. The previous theorems have shown that all celebrity equilibria are also separating equilibria. The next two theorems provide a further characterization of celebrity equilibria.

Theorem 5: A necessary and sufficient condition for there to exist a celebrity (separating) sequential equilibrium with $M(H)=(A_H,S_H)$, $M(L)=(0,0)$ is that there must exist (A_H,S_H) such that the following conditions (1), (2), and (3) hold.

(1) $Max_A \left(E\Pi(L,A,S_H,EQ(\tilde{\mathbf{A}},S))-P_A A-S_H\right) \le E\Pi(L,0,0,EQ(0,0))$

(2) $Max_A \left(E\Pi(H,A,S_H,EQ(\tilde{\mathbf{A}},S))-P_A A-S_H\right) = E\Pi(H,A_H,S_H,EQ(\tilde{\mathbf{A}},S))-P_A A_H-S_H$

(3) $E\Pi(H,A_H,S_H,EQ(\tilde{\mathbf{A}},S))-P_A A_H-S_H \ge E\Pi(H,0,0,EQ(0,0))$

Proof: Conditions (1)-(3) are sufficient for a sequential equilibrium provided that $EQ(\tilde{A},S)=L$ for all out-of-equilibrium observations. For example, given these out-of-equilibrium the low type would not be able to improve its profits by spending money on advertising unless it was prepared to select S_H, since any other modulation will be perceived as coming from the low type. Condition (1) just states that the strategy $M(L)=(0,0)$ is preferred by the low type to any strategy of the form $M(L)=(A,S_H)$ where $A>0$. If this condition is met, then the low type is maximizing profits, subject to the beliefs of consumers. For the same reason the high type would never have an incentive to defect from $M(H)=(A_H,S_H)$ to $M'(H)=(A,S)$ where $S\neq S_H$ since this would only result in consumers believing that the monopolist had a low-quality product. In other words, if the monopolist is considering a defection to another strategy with positive advertising it should only consider those defections where it maintains the in-equilibrium modulation level S_H. Condition (2) just says that the type-H has higher profits at $M(H)=(A_H,S_H)$ then at any strategy $M'(H)=(A,S_H)$ where $A\neq A_H$. The only other defection that the high type might consider is a defection to $M(H)=(0,0)$. Although this will reduce consumer expectations of quality it would also reduce advertising expenditures. Condition (3) just says that the high-quality monopolist prefers to separate. This proves sufficiency.

Of course if condition (1) was not met, then the low type would start to mimic the strategy of the high type. If (2) was not met then the high type would switch its advertising level (but not its modulation) to some other level that increases profits. Finally, if (3) was not met, then the high type would switch to $M'(H)=(0,0)$. This proves necessity. QED.

Theorem 6: A necessary condition for a separating celebrity sequential equilibrium of the form $M(H)=(A_H,S_H)$, $M(L)=(0,0)$ to also be a POST equilibrium is that
$S_H=Max_A\left[E\Pi(L,A,0,EQ'(\tilde{A},0))-P_AA\right]-E\Pi(L,0,0,EQ(0,0))\equiv Z>0$ where
$EQ'(\tilde{A},0)=EQ(\tilde{A},S_H)$.[19]

Proof: Suppose that this theorem is false and that in our proposed POST equilibrium we had $S_H>Z$. Consider the following proposed alternative sequential equilibrium with $M(H)=(A_H,Z)$, $M(L)=(0,0)$ and $EQ''(\tilde{A},Z)=EQ(\tilde{A},S_H)$. Out-of-equilibrium observations are associated with the low type. Let A^* be the maximizer in the definition of Z. Then by construction we have $E\Pi(L,A^*,0,EQ'(\tilde{A},0))-P_AA^*-Z=E\Pi(L,0,0,EQ(0,0))$ and for all other A we must have $E\Pi(L,A,0,EQ'(\tilde{A},0))-P_AA-Z\leq E\Pi(L,0,0,EQ(0,0))$. Now by construction we know that
$E\Pi(L,A,Z,EQ''(\tilde{A},Z))=E\Pi(L,A,0,EQ'(\tilde{A},0))$ so we can also say that $E\Pi(L,A,Z,EQ''(\tilde{A},Z))-P_AA-Z\leq E\Pi(L,0,0,EQ(0,0))$ for any arbitrary A. This demonstrates that condition (1) of the previous theorem is satisfied.

Condition (2) of the previous theorem will clearly be satisfied with the modulation Z if it was satisfied at $S_H>Z$, since S_H is just being replaced by Z on both sides of condition (2). Finally condition (3) will be satisfied when S_H is replaced by $Z<S_H$ on the left side of condition (3). This

alternative equilibrium increases the net profits of the high-quality monopolist by $(S_H - Z)$. Hence, the original separating equilibrium with $S_H > Z$ could not have been a POST equilibrium. QED.

The previous two theorems have dealt with separating equilibria where there is a celebrity endorsement. By theorems 1 and 4 we know that these equilibria must involve no advertising by the low type. There may, however, also be a separating equilibrium without a celebrity endorsement. By specifying the relationship between the profits of low- and high-type monopolist, we can show that these alternative equilibria will have $A_H > A_L$.

Theorem 7: Suppose that $E\Pi(H,A,S,EQ) = cE\Pi(L,A,S,EQ)$[20] where $c>1$, then in any POST equilibrium with $A_L>0$ and $A_H>0$ we must also have $A_H>A_L$.

Proof: By theorems 1 and 4, we know that in any POST equilibrium with $A_L>0$ there will not be any celebrity endorsement. Because $(A_H,0)$ satisfies condition (1) of a sequential equilibrium we know:

$E\Pi(H,A_H,0,EQ(\tilde{\mathbf{A}},0)) - P_A A_H > E\Pi(H,A_L,0,EQ(\tilde{\mathbf{A}},0)) - P_A A_L$. Using the assumption about the relationship between the profit functions of the high and low types and rearranging terms we discover:

(A) $c\left[E\Pi(L,A_H,0,EQ(\tilde{\mathbf{A}},0)) - E\Pi(L,A_L,0,EQ(\tilde{\mathbf{A}},0)\right] > P_A A_H - P_A A_L$.

The fact that $(A_L,0)$ satisfies condition (2) of a sequential equilibrium implies that:

$E\Pi(L, A_L, 0, EQ(\tilde{\mathbf{A}}, 0)) - P_A A_L > E\Pi(L, A_H, 0, EQ(\tilde{\mathbf{A}}, 0) - P_A A_H$, or that $E\Pi(L, A_L, 0, EQ(\tilde{\mathbf{A}}, 0)) - E\Pi(L, A_H, 0, EQ(\tilde{\mathbf{A}}, 0)) > P_A A_L - P_A A_H$. Multiplying through by -1 yields

(B) $E\Pi(L, A_H, 0, EQ(\tilde{\mathbf{A}}, 0)) - E\Pi(L, A_L, 0, EQ(\tilde{\mathbf{A}}, 0)) < P_A A_H - P_A A_L$. Putting (A) and (B) together implies that

$\frac{P_A(A_H - A_L)}{c} < E\Pi(L, A_H, 0, EQ(\tilde{\mathbf{A}}, 0)) - E\Pi(L, A_L, 0, EQ(\tilde{\mathbf{A}}, 0)) < P_A(A_H - A_L)$. This last inequality tells us that $c>1$ implies that $A_H > A_L$. Hence, in any POST equilibrium where both types advertise, the high type will purchase more commercials than the low type. QED.

Example 2: Consider a model with the following parameters: L=0, H=1, ρ=1/2, B=1, V=2 and P_A=1/3. The profit function for the high- and low-type monopolist are respectively:

$E\Pi(H, A, S, EQ(\tilde{A}, S)) = 4\,EQ(\tilde{A}, S) - (A)P_A - S$

$E\Pi(L, A, S, EQ(\tilde{A}, S)) = EQ(\tilde{A}, S) - (A)P_A - S$

A key feature of this example is that V=2 and B=1. Hence, if the high type purchases one or more advertisements, at least one of these ads will be seen by consumers. Zero advertisements would no longer be an in-equilibrium observation. If the high type advertises, then the low type will have zero profits unless it also advertises. Given the previous analysis in this section, there are three different kinds of "potential" POST equilibria: a separating equilibrium (without any celebrity endorsement); a pooling equilibrium where neither type advertises; and a celebrity equilibrium. These three equilibria are listed below.

Equilibrium 1: {M(H)=(3,0), M(L)=(1,0)}, where EQ(1,0)=1/3, EQ(2,0)=1, and EQ(A,S)=0 for all other (A,S).

Equilibrium 2: {M(H)=(0,0), M(L)=(0,0)}, where EQ(0,0)=ρ=1/2 and EQ(A,S)=0 for all other (A,S).

Equilibrium 3: {M(H)=(1,2/3), M(L)=(0,0)}, where EQ(1,2/3)=H=1, EQ(0,0)=L=0, and EQ(A,S)=0 for all other (A,S).

In equilibrium 1, the low type is actually indifferent between advertising and not advertising since its profits are zero in either case. The expected profits to the high type in equilibrium 1 are: $(5/3)=4[(1/2)(1)+(1/2)(1/3)]-3P_A$.

If neither type advertises then the expected quality must be equal to the prior probability of facing a high quality firm (recall ρ=1/2). The high type's profits would always be 2, while the low type's profits would always be 1/2. A comparison of equilibrium 1 and equilibrium 2 is shown in figure 2.

Finally, in the celebrity equilibrium, the low type is deterred from advertising. Obviously EQ(0,0)=L=0, since the observation (0,0) is consistent with the strategy of the low type, but not consistent with the strategy of the high type. Therefore, the profits of the low type (in equilibrium) are zero. By theorem 6, S_H=2/3 is set just high enough to deter the low type from playing M(L)=(1, 2/3). Hence, when S_H=2/3, the expected profits to the low type from playing M(L)=(1,2/3) must also be zero.

The high type knows with certainty that exactly one of its advertisements will be seen by consumers along with the associated celebrity endorsement. This will be sufficient to convince consumers that it is the high type. Hence, the profits of the high type are $E\Pi(H,1,(2/3),EQ(1,2/3))=4-(1/3)-(2/3)=3$. A comparison between the pooling equilibrium and the celebrity equilibrium is shown in figure 3.

Using this example we can also see theorem 3 in action. In the celebrity equilibrium the high type gets 3, while the low type gets a payoff of 0. In the canonical pooling equilibrium the high type always receives a payoff of 4ρ, the low type receives a payoff of ρ.. A demarcation between the (celebrity) separating equilibrium and the pooling equilibrium is created at ρ=3/4. For ρ<3/4 both the canonical pooling equilibrium and the celebrity equilibrium are Pareto optimal with respect to sender types. This is because the low type prefers the pooling equilibrium, while the high type prefers the separating equilibrium. For ρ>3/4 both types prefer the pooling equilibria and this will be the unique Post equilibrium.

For ρ<3/4 the lexicographically maximum sequential equilibrium ("LMSE") is the celebrity equilibrium. For ρ>3/4 the LMSE is the canonical pooling equilibrium. Recall that the LMSE satisfies Overgaard's Criterion A and is simply the sequential equilibrium preferred by the high type.

5. Price Signaling

In this section I consider an extension of the previous model to include price signaling. I limit my analysis to the case where $V<B$. Hence, the strategy of the monopolist is now a function, $M(Q):[H,L]\to(P,A,S)$, that translates the actual quality of the firm into a nonnegative tuple. Gross profits and expectations of quality will likewise be a function of price, advertising and celebrity endorsements. Price is assumed to be completely deterministic and always observable. For the first result it is not necessary to specify the exact relationship between price and profits.

Theorem 8: Suppose that $V<B$. Let $M(H)=(P_H,A_H,S_H)$ and $M(L)=(P_L,A_L,S_L)$ be part of a sequential equilibrium with $A_H\neq 0$ or $A_L\neq 0$, then $P_H=P_L$.

Proof: I employ a proof by contradiction. Let $t\in\{L,H\}$ be the type with nonzero advertising, $A_t\neq 0$. Suppose that $P_t\neq P_s$. If consumer beliefs are consistent with Bayes' rule then $EQ(P_t,0,0)=t$, since the observation $(P_t,0,0)$ is consistent with the strategy of type t, but not with the strategy of type s. Given these beliefs, however, the strategy $M(t)=(P_t,0,0)$ will induce the same beliefs but increase type t's profits by

$P_tA_t + S_t$. This proves that $A_t > 0$ could not have been an optimal strategy for type t. Hence, it could not have been part of a sequential equilibrium. The assumption $V < B$ is required to ensure that viewing zero commercials is a possible in-equilibrium observation by consumers. Q.E.D.

By theorem 8 and theorem 1 we know that if advertising exists $P_H = P_L$ and that $S_L = S_H$ if $A_L > 0$. Hence, equations (1)-(3), which describe consumer expectations, can be extended in the obvious manner. I now discuss a more explicit model.

I assume that there is a continuum of consumers uniformly distributed with mass R, along the interval $[0,R]$. A consumer's address tells us how much he/she would value a product which performs satisfactorily. The quality of the product is operationalized as the probability that the good in question will perform satisfactorily in each period, for a randomly selected consumers. Throughout I make the simplifying assumption that consumers get no utility from an unsatisfactory product. I also assume that one purchase is sufficient (and necessary)[21] for each consumer to determine whether or not the good is satisfactory. The assumption that consumers become completely informed after the initial purchase enables a simplified analysis of repeat business. These assumption are similar to those made by Milgrom and Roberts (1986).

The end result of the above assumptions is a linear demand curve for a satisfactory product whose slope is -1 and whose price and quality intercepts are both R. The demand curve for a product whose expected quality is $Q<1$ can be derived from this by rotating the original demand curve counterclockwise around the point R on the quantity axis. The new demand curve will have slope $-Q$. As R increases, the demand curve shifts upward and to the right.

Consumers will purchase a maximum of one unit in each period, whenever the expected value of the product exceeds the market price. For simplicity, I assume that all consumers have identical prior beliefs. Because there is a continuum of agents, this assumption is equivalent to assuming that prior beliefs are independent of an individual's marginal valuations (i.e., their address).

I consider a very simple two-period game where the monopolist is not allowed to adjust its strategy in the second period. We can imagine that all the advertising takes place in period 1, when the new product is initially introduced. Consumers will then have an opportunity to again purchase the product in period 2 at the same price as in period 1. In fact, this would be the optimal response of a satisfied customer from the first period. Hence, if the true quality is Q, the fraction Q of the initial customers would now be repeat customers in period 2. Given the expected quality EQ, I define the profits of the firm that selects the price P and the advertising strategy (A,S) and whose actual quality is Q as

$$\pi(Q,P,A,S,EQ) = (1+Q)(R-(P/EQ))(P-C_Q) - P_A A - S. \qquad (4)$$

The $(1+Q)$ term reflects the additional profits from the second period. The next term $(R-(P/EQ))$ is a measure of the initial customers in period 1. The last two terms $-P_A A - S$ reflect the cost of advertising when A commercials are purchased at a price of P_A along with a celebrity endorsement S. Of course, as in the previous section, expected quality EQ, is actually a random variable whose distribution depends on the strategy of the monopolist. Knowing the distribution of expected quality enables one to calculated expected profits.

By theorem 8 we can see that the introduction of price signaling does not significantly complicate matters. Either all the signaling will be done by price or all the signaling will be done by advertising. Price signals and advertising signals will not be simultaneously employed by the monopolist. This is the same result as in Hertzendorf (1993).

This dichotomy between price and advertising signal creates a special role for the equilibrium refinement. Under price pooling or price separation it is the equilibrium refinement that determines the equilibrium price.[22] Generally speaking, if separating sequential equilibria exist, there will not be a unique POST equilibrium since the high and low types cannot agree on an optimal price. However, the unique LMSE will include the price preferred by the high type in any equilibrium.

Whether or not the monopolist will signal quality exclusively through price or advertising depends on the relationship between quality and marginal cost. When marginal cost is independent of quality (i.e. $C_H = C_L$) it is easy to show that price separation is impossible. This is because the profit functions of the high- and low-type monopolist will be maximized at the same price.[23] Roughly speaking, there aren't any pricing strategies that the low type finds too costly to mimic. In fact, when the difference between C_H and C_L is sufficiently small price signaling will not occur in equilibrium. When $|C_H - C_L|$ is sufficiently great the profit functions of the two types are adequately differentiated and price signaling becomes cheaper for the high type. (Please see Hertzendorf (1990) for a detailed discussion.)

These concepts are best illustrated with some examples. The program *Mathematica* was used to calculate equilibria in the following examples.[24]

<u>Example 3:</u> Consider the following model with V=2, B=2, H=1, L=0, P_A=1, ρ=1/4. Payoff functions to the low- and high-type monopolist are described above in equation (4) with $C_H = C_L = 0$ and R=4. There are three potential POST equilibria.

Equilibrium 1: {M(H)=(1/2, 0, 0}, M(L)=(1/2, 0, 0)} with EQ(1/4, 0, 0)= 1/4 and EQ(P,A,S)=0 for all other (P,A,S).

Equilibrium 2: {M(H)=(2, 1, 5/3), M(L)=(2,0,0)} with EQ(2, 1, 5/3)= 1 and EQ(P,A,S)< 1/4 for all other (P,A,S).

Equilibrium 3: {M(H)=(2, 2, 5/3), M(L)=(2,0,0)} with EQ(2, 1, 5/3)= 1 and EQ(P,A,S)< 1/4 for all other (P,A,S).

Equilibrium 1 is the canonical pooling equilibrium. Since there are no advertising or price signals of quality, consumer expectations of quality are fixed at their prior beliefs. Given that consumers are expecting quality to be 1/4, the Pareto optimal response of both types must be charge a price of 1/2. At this price the high type earns a profit of 2, the low type a profit of 1. This equilibrium is preferred by the low type.

Equilibria 2 and 3 are celebrity equilibria. The only difference between them is whether or not the high type purchases one ad or two ads. In both equilibria the low type is deterred from purchasing one commercial because of the celebrity endorsement (S_H=5/3). Because observing no commercials suggests to consumers that a low type monopolist is more likely, the corresponding expected quality (computed using Bayes' rule) must be less than 1/4 (the consumers' prior expectation of quality). Given P=2, no sales will be made if no commercials are seen. Since the low type purchases no commercials, its payoff is zero. When P=2 the expected quality must be at least 1/2 before any sales are made. Hence, we can think of $\overline{EQ}$=1/2 as the "reservation expectation." The POST equilibrium price is chosen so as to maximize the expected profits to the high type when expected quality is 1 (i.e. after viewing one or more ads). Although, in principle, the high type could select a price low enough so that sales will be made even when no commercials are seen, it will not find such a strategy optimal given the parameters above. Simple calculation reveals that equilibrium 3 is the LMSE and satisfies Overgaard's Criterion A.

If B increases from 2 to 3 so that there is more signal loss, the LMSE will be identical except that a celebrity endorsement of 1 (as opposed to 5/3) will now be sufficient to deter mimicry by the low type. This is because the extra noise reduces the value of mimicry. In particular, the extra noise implies that the likelihood that any low-type commercials will be seen is lower.

Example 4: Consider the following model with V=2, B=2, H=1, L=0, P_A=1, ρ=1/4. Payoff functions to the low- and high-type monopolist are described in equation (4) with C_H=C_L=1/2 and R=4. There is one POST equilibrium.

Equilibrium 1: {M(H)=(2, 1, 1), M(L)=(2,0,0)} with EQ(2, 1, 1)= 1 and EQ(P,A,S)< 1/4 for all other (P,A,S).

The canonical pooling equilibrium is no longer a POST equilibrium. Because C_H=C_L=1/2, the price must be set above 1/2 to create positive profits. However, given that the prior expected quality is only 1/4, this price is too high to result in sales. Expected profits to the high type are 2, expected profits to the low type are 0. As before, the price is chosen to maximize the profits of the high type, since the low type makes no sales. If B increases from 2 to 3, the LMSE will now include the strategy $M(H)=(2,2,1/2)$. Hence, the additional noise results in a smaller celebrity endorsement and a greater expenditure on air time.[25] Profits to the high type decline from 2 to 1.7.

6. Conclusions

This paper proposes a plausible theory of celebrity endorsements. I argue that because much advertising takes place over electronic media, signal loss could be a serious problem. In particular, signal loss interferes with the ability of consumers to determine the true advertising expenditures of firms. In an attempt to overcome this problem, firms may "modulate" their signal by including celebrity endorsements (or expensive special effects) in their commercials. This enables the firm to pack more information into each commercial and enables the consumer to receive a clearer signal. The end result is that market performance can be improved from both the standpoint of consumers and the firm.

For pedagogical reasons I have focussed on the special case where $C_H \neq C_L$ in order to rule out price separating equilibria. However, in equilibrium, price signals of quality would not exist simultaneously with stochastic advertising signals. Relaxing the assumption that marginal cost is independent of quality would simply introduce a fourth signaling option. The three different kinds of equilibria discussed in this paper (pooling, separating, and celebrity) would have to be compared to an equilibrium where the high type separates in price. A determination would then be made as to which equilibrium maximizes the profits of the high-quality monopolist. Although in the current article I treat quality as something that is exogenously determined, there is no reason to think that the importance of celebrity endorsements is confined to these models. Ippolito (1990) argues that in many markets where quality is endogenously determined, advertising has the capacity to "bond" performance. In this situation the message sent by advertising expenditures is not "This **is** a high-quality firm," but rather "This **will continue to be** a high-quality firm." Although in Ippolito's model the exact role of advertising has been altered, certainly the same consumer inference problems remain. The upshot of the matter is that celebrity endorsements can also provide a more efficient bonding mechanism by enabling consumers to get a clearer picture of the size of the bond that has actually been posted.

It is important to keep in mind that this is a *model* of advertising and not a *depiction* of it. A commercial which is quickly forgotten cannot influence consumer behavior. Special effects and celebrity endorsements are included in commercials to help make them memorable. If we assume that making unforgettable commercials is more expensive than making mundane commercials, the basic idea is still valid. Consumers may be responding to advertising simply because they can remember it, but they will only remember those commercials that were expensive to produce. Hence, the fact that no individual consumer is actually attempting to infer the monopolist's advertising expenditure is irrelevant. As Nelson put it, "Whatever their explicit reasons, the consumers' ultimate reason for responding to advertising is their self-interest in doing so . . . If it were not in consumer self-interest to respond to advertising, then the consumers' sloppy thinking about advertising would cost enough that they would reform their ways."

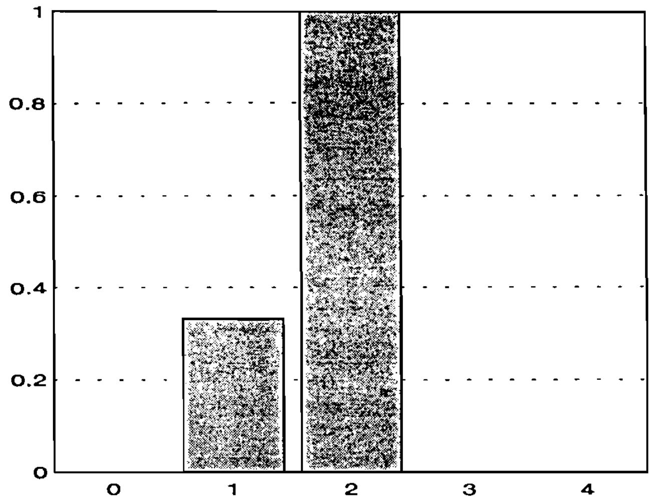
A Note On Figures 1-3

Figures 1-3 are bar graphs designed to facilitate a comparison among various equilibria. Graphs were drawn to similar scales to facilitate comparisons, even when some components on the advertising axis are irrelevant. For example, in figure 1, expectations for two, three and four advertisements are omitted since, as I point out in the paper, these expectations can be defined in any manner whatsoever. Also in figure 1, the negative profit regions for three and four advertisements were intentionally omitted.

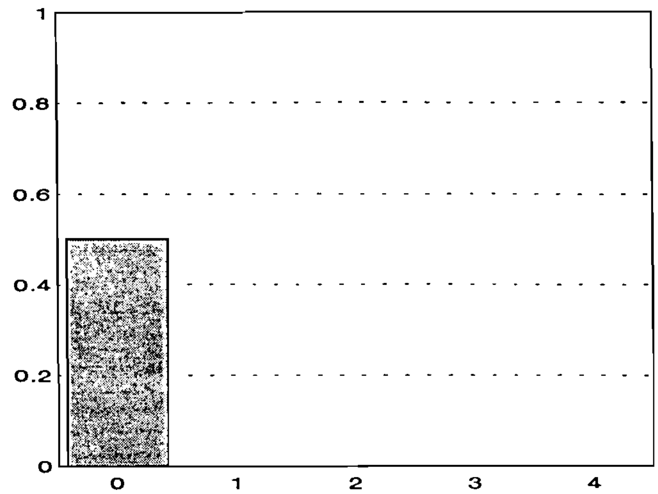
In figure 2, on the left-hand side (separating equilibrium), absence of a bar graph indicates a value of zero, rather than an intentional omission. On the right-hand side (pooling equilibrium), only the equilibrium strategies are plotted since other strategies are irrelevant.

In figure 3, on the right-hand side (celebrity equilibrium), the bar graphs are drawn under the assumption that S_Q=2/3, as is the case for this equilibrium. The lack of a bar indicates a value of zero, except for two, three and four advertisements on the graph of the low-type profits (they would all be negative).

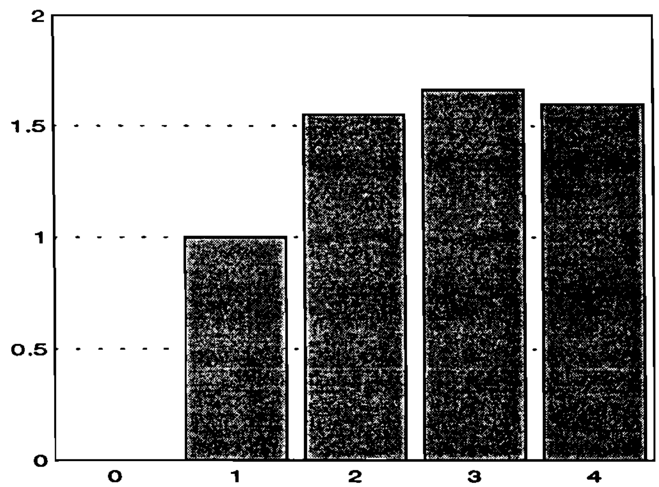
Separating Equilibrium



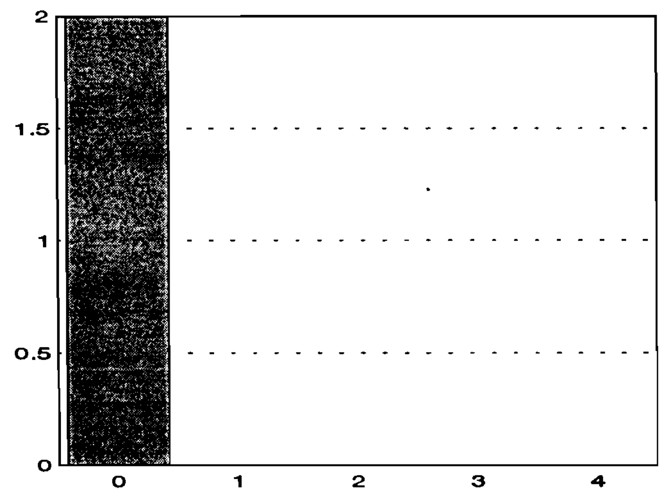
Pooling Equilibrium



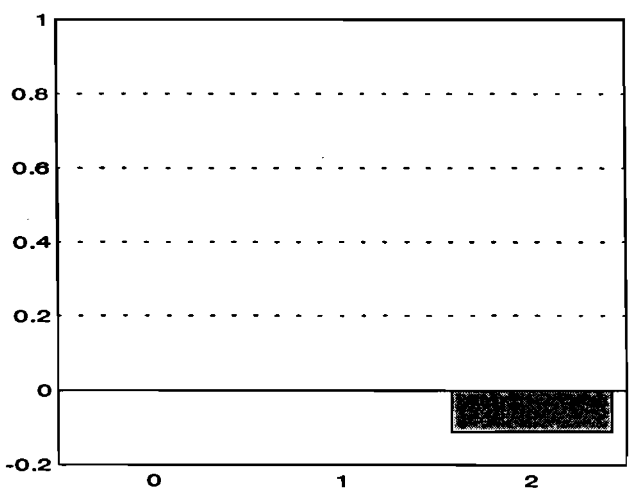
High-Type Profits



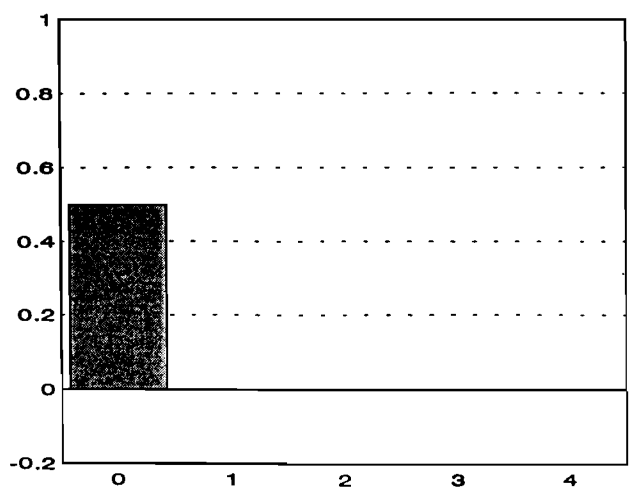
High-Type Profits



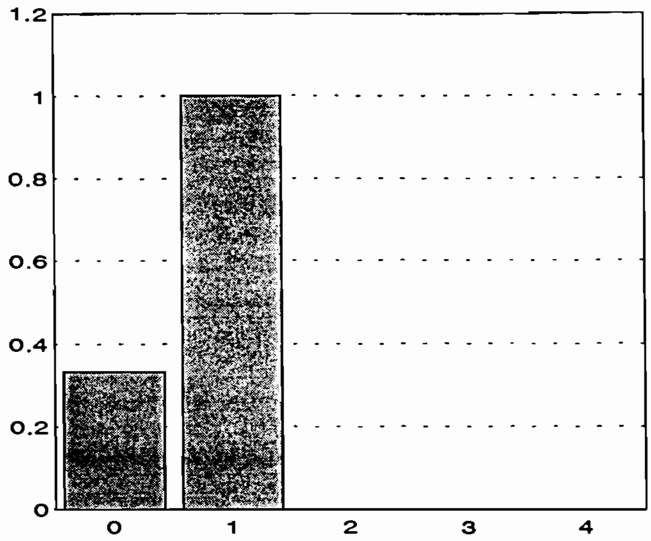
Low-Type Profits



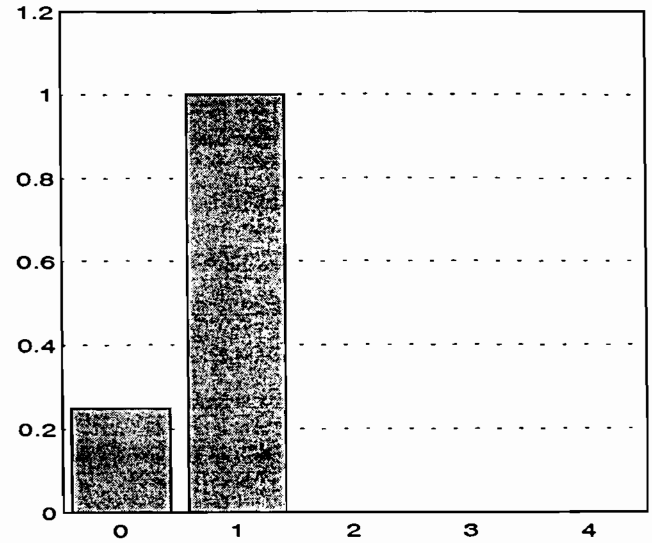
Low-Type Profits



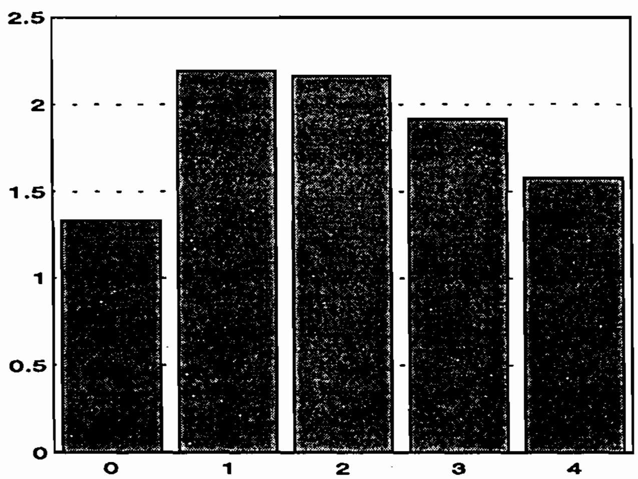
Equilibrium 1



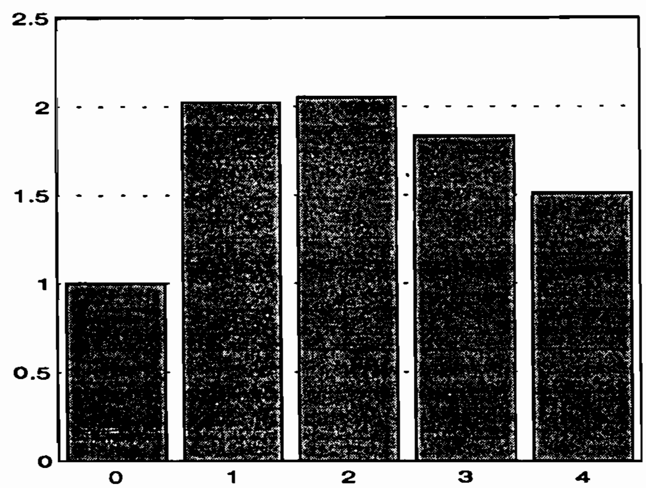
Equilibrium 2



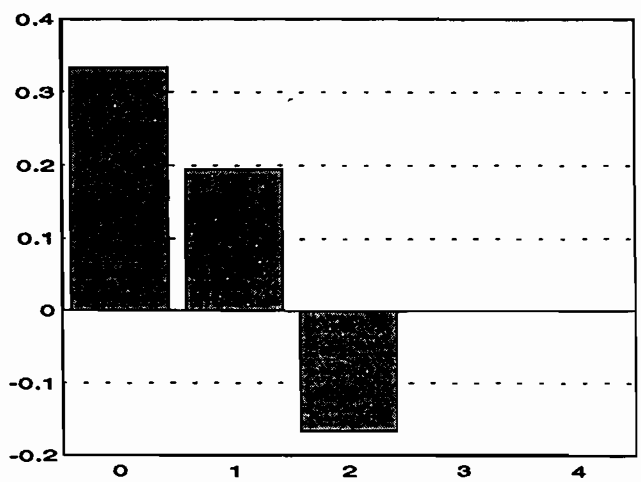
High-Type Profits



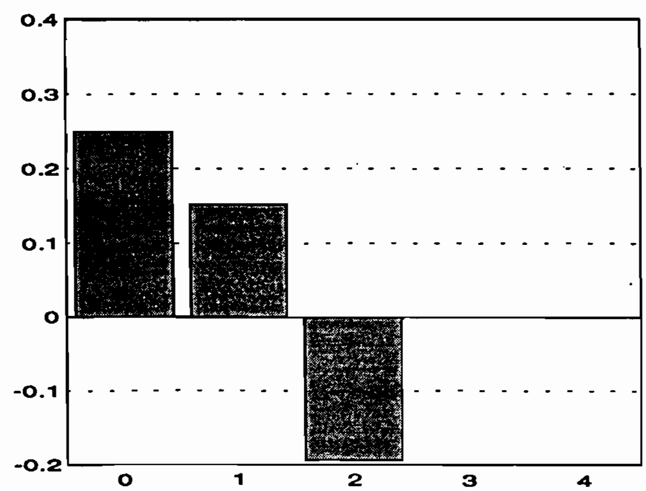
High-Type Profits

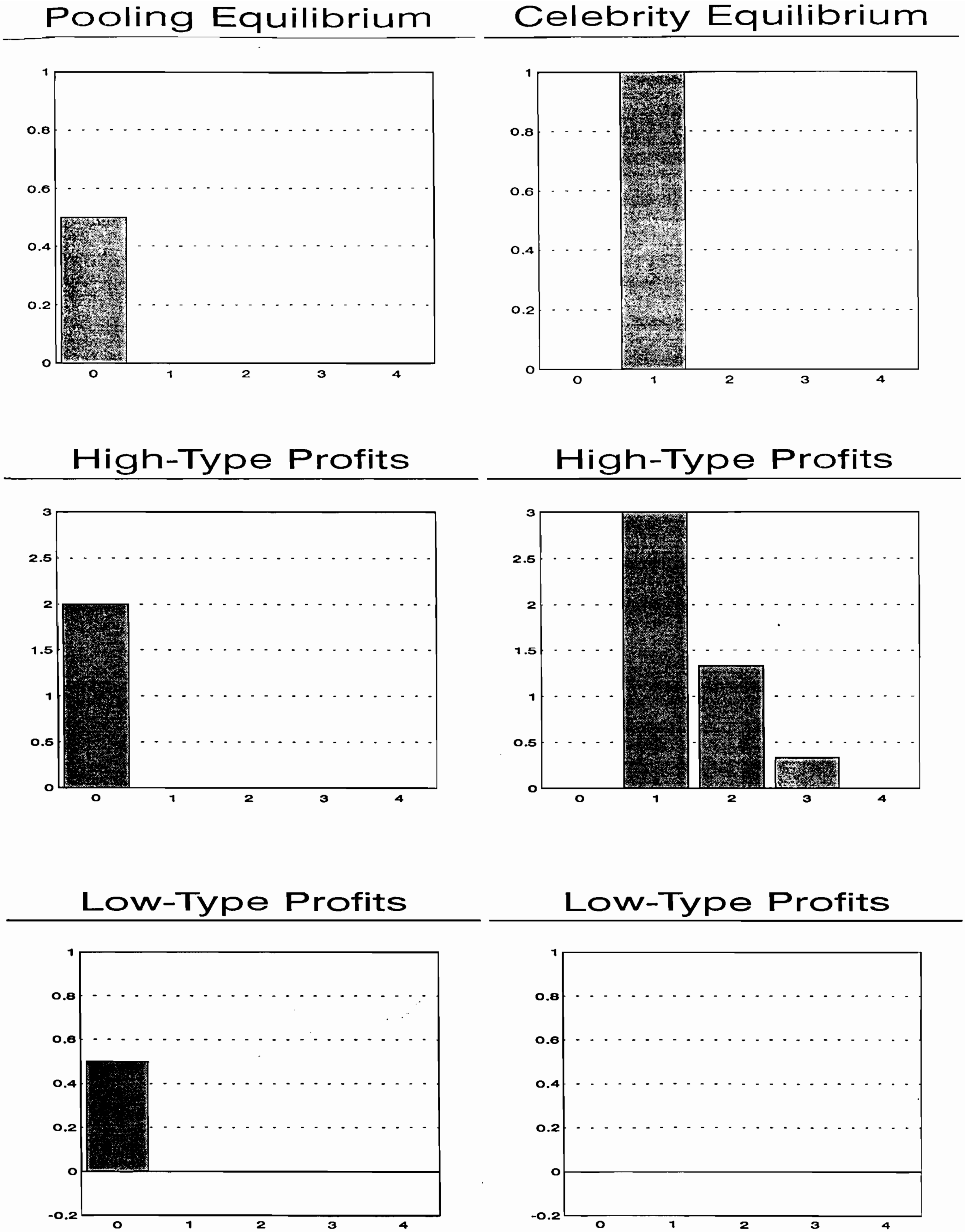


Low-Type Profits



Low-Type Profits




Pooling Equilibrium
Celebrity Equilibrium
High-Type Profits
High-Type Profits
Low-Type Profits
Low-Type Profits
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
2.5
3
-0.2
0 1 2 3 4

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1. "A Monopoly on Mystique, Lincoln-Mercury Uses Advertising Blitz to Market New Model," The Washington Post, September 1994.

2. "Benetton Touches a Nerve and Germans Protest," The New York Times, Friday 3, 1995.

3. See "Super Sunday: A Shared American Experience," The Washington Post, January 29, 1995.

4. See "Wilson, Once a Sports Goliath, Plays David," in The Wall Street Journal, January 1995.

5. See "Super Sunday: A Shared American Experience," in The Washington Post, January, 29, 1995.

6. See "Friends for Hire," in The New York Times, Friday, February 2, 1996.

7. The game I develop does not provide for any formal distinction between celebrity endorsements and high production costs. Rather I cite celebrity endorsements as an example of high production costs.

8. The cost of air time various considerably depending on the show. Air time during David Letterman goes for about $40-$50 thousand dollars for each commercial. Prime time shows sometimes charge hundreds of thousands of dollars per commercial. (See "Late-Night TV Becomes a Crowded Arena, The Wall Street Journal, Wednesday, August 31, 1994.)

9. Although the costs of advertising are explicitly modeled for obvious reasons, no attempt is made **at this time** to deal explicitly with the costs associated with production, nor with the determination of the optimal price. In specific examples, later in the paper, these other factors will be fleshed out in greater detail. All examples prior to section 5 are merely included for the purpose of illustrating important concepts.

10. In the model I consider, I do not rule out the possibility that $V > B + \min[A_L, A_H]$. V is simply the number of times that the consumer will stick his hand in the urn and attempt to remove a commercial. I allow the possibility that the consumer will come up empty-handed

if the urn is empty.)

11. The consumer will either observe zero ads or two ads depending on whether nature selects the low-quality or high-quality monopolist respectively.

12. These profit functions were chosen to illustrate the equilibrium concept with the minimum possible effort. Later examples utilize realistic payoff functions that are nonlinear in expectations.

13. In order to deter mimicry by the low-type, it is always necessary for the high-quality monopolist to be more profitable than the low-quality monopolist.

14. Although there are other sequential equilibria with celebrity endorsements, none of them can be **Pareto Optimal with respect to Sender Types** since they increase the costs of the high type without materially altering the induced set of consumer beliefs. Later in the paper, examples of POST-equilibria with celebrity endorsements will be presented.

15. I should point out here that consumers will not always wish to overturn the same equilibria that the monopolist views as inefficient. In the previous example, equilibrium 2 was better for consumers since they have better information about quality when no ads are viewed and also because they are more likely to discover the high-quality monopolist.

16. Although, strictly speaking, they have proposed a refinement of Bayesian Nash equilibria, there is no reason why their approach cannot also be used to refine sequential equilibria.

17. In other words we are using Bayes' rule along the equilibrium path of play.

18. From now on, I will refer to this pooling equilibrium as the canonical pooling equilibrium.

19. The reader should not be confused by the zero in $EQ'(\tilde{A},0)$. I have introduced a zero so that S_H can be defined explicitly rather than implicitly.

20. I am treating expected quality as a scaler rather than a random variable at this point. Although the linear relationship between the profit functions may seem a bit implausible, it is satisfied by examples illustrated later in the paper.

21. In other words, I am also assuming that consumers do not observe the quality level of the goods purchased by their neighbors.

22. I am speaking loosely here. More precisely, different equilibrium refinements determine different equilibria and these equilibria include different equilibrium prices.

23. This is because $\frac{\partial \pi}{\partial p}(H, P, 0, 0, EQ) = 2\frac{\partial \pi}{\partial p}(L, P, 0, 0, EQ)$.

24. Copies of the *Mathematica* programs used to calculate the following equilibria are available via email. Please send your request to: "MHertzendorf@ftc.gov" .

25. The fact that increasing noise results in a smaller celebrity endorsement may depend on the specific parameters in these examples. In particular, when the low type makes sales in the celebrity equilibrium this outcome might be reversed.