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Estimating Producer Welfare Change when Input and Output Prices Change Simultaneously

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Economic analysis of a proposed policy change may require quantifying the gains or losses to an industry when input and product prices change simultaneously. Past studies have often measured such welfare changes by summing the effect of an input price change and the effect of a product price change, each taken in isolation of one another. For example, in published studies of the social benefits of rescinding tariff and quota protection of domestic textile and apparel industries, a comparative static analysis of the gains to trade liberalization has typically been performed for the two industries separately.[1] That is, the gains to trade liberalization were evaluated in the textile industry and in the apparel industry, but not for trade liberalization in both industries simultaneously. The authors most probably were not interested in estimating the effects of liberalization in just one of these industries, since such estimates would have questionable value due to political realities: It is unlikely that trade restraints on apparel would be removed without doing the same for textiles, since to do so would place domestic apparel producers at a competitive disadvantage vis a vis foreign producers.

Moreover, simply adding the gains calculated separately for trade liberalization in the two vertically related industries involves a potentially significant overestimation error.[2] This error arises in two ways: first, the gain to downstream (apparel) producers, i.e., the gain in "consumer surplus" in the upstream (textile) industry, would be based on the higher, pre-liberalization price in the downstream industry and so would be an overestimate; and, second, the producer loss in the downstream industry (apparel) would be based on the higher, pre-liberalization price in the upstream industry and so would be an underestimate.[3]

A more appropriate approach would be to analyze the gains to trade liberalization when trade restraints are (simultaneously) removed from both industries. Unfortunately, this necessitates estimating the net gain or loss to downstream producers as the prices of their input and

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1 See Cline (1987), Hufbauer et al. (1986).

2 Perhaps in recognition of this problem, some authors have left it to the reader to perform this addition. [Cline (1987).]

3 That is, the producer welfare change so estimated would be greater that the true change. Equivalently, any gain would be overestimated and any loss (in absolute value terms) would be underestimated. Similarly, this approach would underestimate the producer welfare change arising from the levying of tariffs in both upstream and downstream markets.

product fell simultaneously as a result of trade liberalization.[4] An estimation of this sort is complicated by the simultaneous shifting of both the industry's factor demand curve and product supply curve.

This paper develops analytically a graphic representation and a mathematical approximation for the change in producer welfare when both a factor price and a product price change simultaneously. The analysis assumes the producer operates in competitive factor and product markets, which permits the later generalization from the case of one firm to the case of multiple firms constituting an industry. Two additional simplifying assumptions are invoked: First, the ratio of the factor input to product output is a constant in the relevant portion of the expansion path of the firm, holding all factor prices constant. Second, the same ratio is relatively invariant to changes in the factor price.[5] While the latter assumption is not necessary, it not only simplifies the analysis and the graphical representation, it also introduces minimal estimation error in many real world applications.[6]

Define producer welfare as profit,

$$\Pi \quad = \quad pq - PQ - V(q) + F \ , \qquad (1)$$

where p and q are the price and quantity of the product, P and Q are the price and quantity of the factor, V is (other) variable costs, and F is fixed costs. If the ratio of Q to q is constant and equal to r, then

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[4] For example, see Anderson & Metzger (1990).

[5] The technology implicit in this analysis is essentially a modified Leontief, where the ratio of the factor input to product output is invariant to the price of that input, but where factor inputs (other than the one under study) can be assumed to exhibit the characteristic of diminishing marginal products necessary to obtain upward sloping marginal cost curves. This is admittedly a strong assumption, in that not all production technologies can be expected to be strictly consistent with this condition. However, it can be a generally accurate characterization of production whenever the firm is engaged in processing of some raw material (as opposed to manufacturing). A good example is that of petroleum refining where the ratio of crude oil input to refined product output is relatively constant over relevant ranges of refinery activity. The assumption may also be reasonable for apparel production, since the ratio of textile input to apparel product can be expected to be somewhat insensitive to the price of textile, i.e., there may be little potential for substitution between textile and other inputs.

[6] The results obtained when this assumption is not invoked are reported and discussed in footnote 7.

$$\Pi = pq - Prq - V(q) + F \; . \tag{2}$$

Any simultaneous change in prices p and P--for example as a result of tariffs or taxes--would affect producer welfare:

$$\begin{aligned}\Delta\Pi = \; & [p_0(\Delta q) + (\Delta p)q_1] \\ & - r[P_0(\Delta q) + (\Delta P)q_1] \\ & - [V(q_1)-V(q_0)] \; , \end{aligned} \tag{3}$$

where subscripts indicate the initial and subsequent time periods. Note that the first bracketed term is the change in revenues arising from the tariff, while the second and third terms capture the change in costs attributable to the factor and the other variable inputs, respectively. While the first two terms are readily estimable from observed prices and quantities, the last is more problematic.

Now, profit maximization implies

$$\partial\Pi/\partial q = p - rP - V'(q) = 0 \; , \tag{4}$$

where $V'(q)$ denotes the derivative of $V(q)$. If one defines an interval of integration $t=[0,1]$ where $q(t)$ assumes values $q(0)=q_0$ and $q(1)=q_1$, then the term $[V(q_1)-V(q_0)]$ can be expressed as

$$[V(q_1)-V(q_0)] = \int_0^1 V'(q)q' \, dt \; , \tag{5}$$

where q' is the rate at which q changes with respect to t over the integration path. Upon substituting for $V'(q)$ from equation (4),

$$[V(q_1)-V(q_0)] = \int_0^1 [p(t)-P(t)r]q' \, dt \; , \tag{6}$$

where $p(0)=p_0$, $p(1)=p_1$, $P(0)=P_0$, and $P(1)=P_1$. With no loss in generality, we assume the integration path between the initial and subsequent values is linear,[7] in which case q' is a constant equal to Δq. Then,

$$[V(q_1)-V(q_0)] = [(p_0+p_1) - (P_0+P_1)r](\Delta q)/2 \; .^{8} \tag{7}$$

Upon substituting equation (7) into equation (3), we obtain

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[7] See Burns (1973), pp. 342.

[8] Note that the area beneath a straight line connecting the two points $(0,p_0)$ and $(1,p_1)$ is $(p_0+p_1)/2$. Similarly for P_0 and P_1, the area is given by $(P_0+P_1)/2$.

$$\begin{aligned}\Delta\Pi &= [p_0(\Delta q) + (\Delta p)q_1] \\ &\quad - r[P_0(\Delta q) + (\Delta P)q_1] \\ &\quad - [(p_0+p_1) - (P_0+P_1)r](\Delta q)/2 \\ &= [(\Delta p)-(\Delta P)r](q_0+q_1)/2 \\ &= [(\Delta p)(q_0+q_1)/2] - [(\Delta P)(Q_0+Q_1)/2]\ .^{9} \qquad (8)\end{aligned}$$

Therefore, the change in producer welfare can be characterized as: (1) the change in the product price times the average quantity sold, pre- and post-price change; less (2) the change in the input price times the average quantity of the input purchased, pre- and post-price change. The first represents the gain or loss as a seller of the product, while the second is the gain or loss as a consumer of the input.

Figure A illustrates the case of a price increase arising because of tariffs levied on products of two vertically related industries, with the upstream market pictured in the upper panel and the downstream market pictured in the lower panel.[10] As noted, the change in welfare for the downstream producers has two components: Higher factor costs represent a loss, corresponding to the area in the upper panel bounded by the two price lines, the vertical axis and a line segment connecting the two points denoting factor demand pre- and post-tariff. Higher product price represents a gain, corresponding to the area in the lower panel bounded by the two price lines, the vertical axis, and a line segment connecting the points denoting supply pre- and post-tariff.

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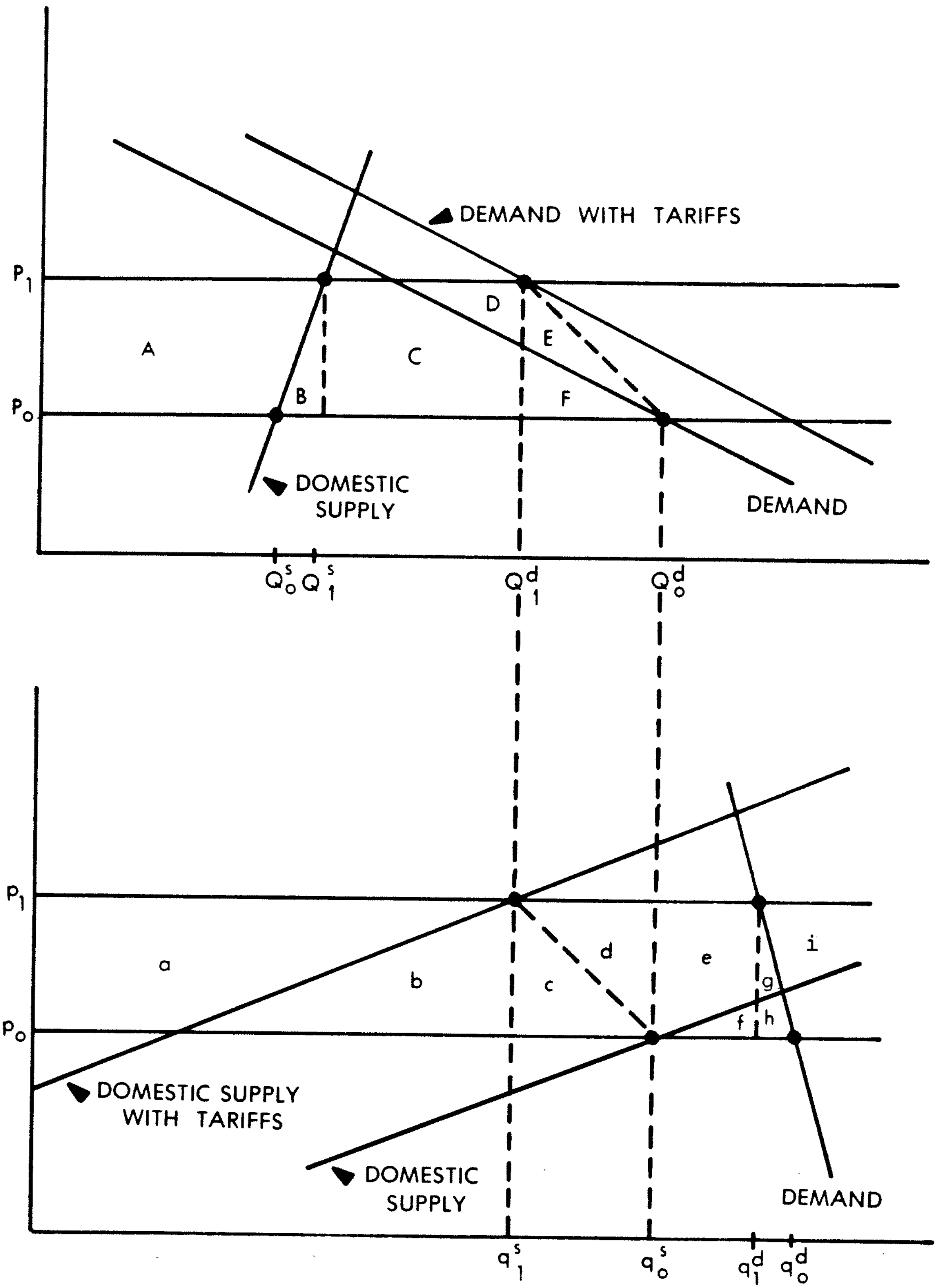
[9] Upon relaxing the assumption that the input:output ratio (r) is invariant to changes in the input price, an additional term, $-p_1q_1(\Delta r)$, is obtained. The term will be positive--i.e., this expression will underestimate the true change in producer welfare--whenever Δr is negative, or equivalently the price of the factor increases. Similarly, an overestimate would be obtained for decreases in the price of the factor. Since this term represents Δr% of revenues of the product, it has the potential to be large for those production processes in which there is significant factor substitutability. In the extreme case, in which the elasticity of substitution is infinite, the change in producer welfare would be comprised only of the gain or loss incurred as a seller of product, since the gain or loss associated with the use of the factor would be negligible. Accordingly, equation (8) would consist only of the first term.

[10] As noted previously, it has been implicitly assumed that the firm operates in competitive markets both upstream and downstream, so that it is possible to talk of an industry supply curve and an industry factor demand curve. Alternatively, the figure can be viewed as portraying an individual firm's supply and factor demand curves, where prices are determined exogenously, i.e. in world markets.



DEMAND WITH TARIFFS
P1
P0
A
B
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D
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F
DOMESTIC SUPPLY
DEMAND
Q0s Q1s
Q1d
Q0d
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a
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e
f
g
h
i
DOMESTIC SUPPLY WITH TARIFFS
DOMESTIC SUPPLY
DEMAND
q1s
q0s
q1d q0d

The first area is the sum of areas A, B, C, D, E and F, while the second area is the sum of a, b, and c.

A different result would be obtained if one were to adopt the crude approach of simply summing the estimate of the gain/loss arising from an isolated price change in the upstream market, with the estimate of the gain/loss arising from an isolated price change in the downstream market. For example, consider the case of tariffs levied on textiles and apparels, as represented in Figure A, where the tariffs cause prices to simultaneously rise in the two vertically related markets. If the resulting downstream gain and upstream loss were estimated without taking into account the interrelationship of the two industries, the upstream loss would be calculated relative to the original demand curve (i.e., no tariff on apparel), and the downstream gain would be calculated relative to the original supply curve (i.e, no tariff on textiles). This estimate of the gain would be the sum of areas a, b, c, d, e, g, and i in the lower panel and the estimate of the loss would be A, B, C, and F in the upper panel. Upon summing these, a measure of the change in producer welfare (-a-2b-2c+A+B+C+F) would be obtained.[11] This is less than the true measure (-a-b-c+A+B+C+D+E+F) by the area (b+c+D+E) and so represents an underestimate. Alternatively, if the exercise were instead one of calculating the change in producer welfare when both prices decreased, i.e., tariffs were rescinded, an overestimate of the same amount would be obtained.

The error that would arise from this crude approach is potentially large. In fact, the error cannot be bounded so to assure that it would be small relative to the actual gain or loss to the producer(s). This can be seen by considering a simultaneous change in input and output prices that in fact leaves producer welfare unchanged. In this case, any gain or loss arrived at by the crude approach is entirely error. Consequently, the ratio of error to true gain/loss would be infinite.[12]

Finally, it might be argued that a crude approach requires less information in order to calculate the net change in producer welfare. Specifically, one would need the initial prices and quantities, the

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[11] Note that, due to similar triangles, the sum of the areas of triangles, b and c, must equal the sum of the areas of the triangles, d, e, g, and i.

[12] In order to reach this conclusion, it is necessary to show that the error, given by the area of (b+c+D+E), is nonzero. Now, from equations (9a) and (9b), (b+c) can be shown to be approximately equal to $(1/2)\epsilon_Q^D(\Delta P/P)(\Delta p)q$ (where ϵ_Q^D is defined in absolute value terms), while (D+E) is approximately equal to $(1/2)\epsilon_q^S(\Delta p/p)(\Delta P)Q$. For any (nonzero) Δp and (nonzero) ΔP that would leave the firm's profit unchanged, both terms would be nonzero and positive. Since the error of the estimate is nonzero, while the true value is defined to be zero, the ratio of error to the true value can not be bounded.

change in prices, and the elasticities of supply and factor demand. While the more accurate approach presented here would involve determining new quantities on shifted supply and factor demand schedules, the informational needs for this are in fact not any greater than for the crude approach. If one assumes that the factor is in more or less constant proportion with output, the only additional datum needed is that proportion (r). This can be computed from the initial quantities. Specifically, to arrive at the post-price change quantities, one would calculate the change in the two quantities as follows:

$$\Delta Q \quad = \quad - \epsilon_Q^D \, Q \, (\Delta P/P) \quad + \quad r \, \epsilon_q^S \, q \, (\Delta p/p) \qquad (9a)$$

$$\Delta q \quad = \quad \Delta Q/r \, , \qquad (9b)$$

where $\epsilon_Q^D > 0$ is the factor demand elasticity, $\epsilon_q^S > 0$ is the supply elasticity, and r is (as before) the ratio of factor input to factor output.[13]

[13] Note that the modified Leontief production function permits the elasticity of demand to be expressed in terms of the elasticity of supply. Specifically, it can be shown that $\epsilon_Q^D = rP\epsilon_q^S/p$. As a result, $\Delta Q = [rq\epsilon_q^S/p][\Delta p - r\Delta P]$.

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