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ON DOMESTIC INDUSTRY:

A COMPARISON OF MARKET STRUCTURES

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THE EFFECT OF SUBSIDIZED IMPORTS ON DOMESTIC INDUSTRY: A COMPARISON OF MARKET STRUCTURES

by

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I. INTRODUCTION

Does the extent of injury suffered by a domestic industry from unfair imports depend on the type of competition that exists between domestic and foreign firms? Is injury more severe when domestic and foreign firms are perfect competitors or when they are oligopoly rivals? These questions have important implications for such issues as the administration of U.S. countervailing duty (CVD) law.

The Court for International Trade has recently upheld the use of economic analysis by the International Trade Commission (ITC) to estimate the injury suffered by domestic industry as a consequence of unfair imports.[2] However, the method used at the ITC to estimate injury in CVD investigations assumes domestic and

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[2] In its recent Trent Tube v. U.S. decision, the Court for International Trade (CIT) found that the economic model used by Chairman Brunsdale to estimate the injury suffered by domestic industry as a consequence of dumped imports "was not unreasonable" (p. 23). Although the case under review by the CIT involved dumping, the approach used by the Chairman to analyze CVD cases appears to be basically the same as that used to analyze dumping cases. In both, Chairman Brunsdale uses what has come to be called an "Elasticity Model." For a discussion of Chairman Brunsdale's analysis in CVD cases see Industrial Belts from Israel, Italy, Japan, Singapore, South Korea, Taiwan, the United Kingdom, and West Germany, Investigations 701-TA-293 (Final) and 731-TA-412/419 (Final), USITC Publication 2194, May 1989, (Views of Chairman Brunsdale, pp. 51-75), and New Steel Rails from Canada, Investigations 701-TA-297 (Final) and 731-TA-422 (Final), USITC Publication 2217, September 1989, (Dissenting Views of Chairman Brunsdale, pp. 83-124). For the recent CIT case, see United States Court for International Trade, Trent Tube Division, Crucible Materials Corporation, et al. v. United States, Slip Op. 90-58 (June 20, 1990).

See also two recent critical surveys of U.S. CVD law. Michael S. Knoll (1989), "An Economic Approach to the Determination of Injury under United States Antidumping and Countervailing Duty Law," New York University Journal of International Law and Politics, Vol. 22, No. 1, pp. 37-116; Alan O. Sykes (1989), "Countervailing Duty Law: An Economic Perspective," Columbia Law Review, Vol. 89, No. 2, pp. 199-263. Given that the law exists, Knoll argues that its administration requires economic analysis. However, Sykes finds that there are no plausible efficiency justifications for the existing law.

foreign firms are perfectly competitive.[3] For some investigations this method may be appropriate, particularly for cases involving agricultural products -- such as flowers and pork -- where there are a relatively large number of domestic and foreign suppliers. However, there are also investigations where there are a relatively small number of competitors[4] and oligopoly may be appropriate.[5] For these latter cases it is important to know how the injury estimates obtained assuming a competitive structure compare with the estimates that would have been obtained had the ITC used an oligopoly structure. This paper attempts to shed some light on this issue by comparing the injury caused by subsidized imports under five different market structures, perfect competition and four types of oligopolies.

One of our principal results is that, other things remaining the same, subsidized exports cause relatively more harm under perfect competition than under oligopoly. Harm is measured by the percent change in domestic industry revenue caused by a one percent increase in the subsidy granted to foreign firms. The factor that drives this result is the extent to which price of foreign product is affected by the subsidy, the "pass-through" issue. Under perfect competition (and with constant marginal costs) the full amount of the foreign subsidy is passed through to the price of the foreign product in the domestic market. However, under oligopoly there is a wedge between price and marginal cost (i.e., price exceeds marginal revenue) so that price of the foreign product in the domestic market does not fall by the full amount of the unit subsidy. As a consequence, the adverse effect of the subsidy on domestic industry is smaller under oligopoly. This result suggests that using the competitive market assumption to estimate injury yields upper bound estimates when the true market structure is oligopoly.

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[3] The CADIC model developed by Richard Boltuck at the ITC was initially designed to estimate the effects of dumped imports. However, CADIC can also be used to estimate the effects of subsidized imports, but in this application CADIC assumes domestic and foreign firms are perfectly competitive.

[4] For example, in the recent final ITC investigation of <u>New Steel Rails from Canada</u>, the ITC reported that (for 1988) there were only two domestic and two Canadian producers supplying the U.S. market. (pp. A-19 and A-68).

[5] Note that fewness of competitors is not sufficient for oligopoly and market power. Barriers to entry and exit must also be examined. Even though there are only two firms, if there are no barriers to entry and exit perfect competition is appropriate. In this paper we assume there are barriers, e.g., patent barriers.

Our second principal result is that we find that competitive industries are more sensitive to subsidies than oligopolies. In particular, a perfectly competitive industry is at least three times more sensitive to subsdidies than even a Bertrand oligopoly. Moreover, as the degree of rivalry in oligopoly decreases, domestic industries are less sensitive to subsidies.

II. ANALYTICAL FRAMEWORK AND NOTATION

We adopt a simple structure in which there are only two firms and two products. There is one domestic firm and one foreign firm. We use the conjectural variations approach to characterize competition between the two firms.[6] Each firm produces a differentiated product. The products are close but not perfect substitutes. The domestic firm sells only in domestic market. The foreign firm sells in the domestic market as well as in its own home market. The foreign firm receives constant per unit subsidy. The specific way we treat the subsidy is as an export subsidy. (Alternatively, we could have used a general production subsidy and would have obtained the same results.) The foreign country has barriers to prevent reimport of subsidized exports. Inverse demand functions exist and are

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[6] Several recent papers have adopted the conjectural variations approach to analyze international trade policy issues involving oligopoly structures. Two notable contributions are by Cheng and Dixit. Leonard Cheng (1988), "Assisting Domestic Industries under International Oligopoly: The Relevance of the Nature of Competition to Optimal Policies," American Economic Review, Vol. 78, No. 4, pp. 746-758. Avinash Dixit (1988), "Optimal Trade and Industrial Policies for the US Automobile Industry," in Empirical Methods for International Trade, (R. Feenstra, ed.), MIT Press, Cambridge, Mass., pp. 141-165.

For critical comments on the conjectures approach in oligopoly, see Carl Shapiro (1989), "Theories of Oligopoly Behavior," chap. 6 in Handbook of Industrial Organization, Vol. I, (R. Schmalensee and R.D. Willig eds.), Elsevier Science Publishers BV, Amsterdam, pp. 329-414. According to Shapiro (p. 356), oligopoly models using methods based on conjectural variations are generally inappropriate to study reactions and retaliations of rivals because these models are static and, instead, explicit dynamic models are required. Nonetheless, as indicated by the work of Cheng and Dixit, the conjectural variations approach provides a useful and convenient way of introducing different types of rivalry in oligopoly models, at least as a start to a possibly richer treatment that may emerge using explicitly dynamic models.

linear. Finally, marginal costs are assumed to be constant in relevant region.

In general, variables for the domestic firm are indicated by capital letters. Variables for the foreign firm are indicated by lower case letters.

P	=	price of domestic product in domestic market
X	=	quantity of domestic product sold in domestic market
R	=	total revenue of domestic firm
C(X)	=	total cost of domestic firm
C'	=	marginal cost of domestic firm
P*	=	total profits of domestic firm
$(dP/dX)_{cd}$	=	conjecture by domestic firm regarding effect of changes in its own output on the price it can charge
p	=	price of foreign product in domestic market
x	=	quantity of foreign product exported to domestic market
y	=	quantity of foreign product sold in foreign market
s	=	constant per unit subsidy received by foreign firm for each unit of exports
r	=	total revenue of foreign firm
$r_1(X,x)$	=	revenue of foreign firm from exports to domestic market
$r_2(y)$	=	revenue of foreign firm from sales to its own home market
c(x+y)	=	total cost of foreign firm
c'	=	marginal cost of foreign firm
p*	=	total profits of foreign firm
$(dp/dx)_{cf}$	=	conjecture by foreign firm regarding effect of changes in its exports to domestic market on the price it can charge in domestic market
E_{Rs}	=	elasticity of domestic industry revenue with respect to foreign subsidy
N_1	=	$(C'-A)[-b+(dp/dx)_{cf}] + k(c'-a-s)$
N_2	=	$[-B+(dP/dX)_{cd}](c'-a-s) + k(C'-A)$
D_1	=	$[-B+(dP/dX)_{cd}][-b+(dp/dx)_{cf}] - k^2$

III. MEASURING INJURY TO DOMESTIC INDUSTRY

Injury to a domestic industry is measured by decline in total industry revenue. There are three reasons for selecting this measure as opposed to other measures, such as decline in producers' surplus. First, the U.S. CVD law requires the ITC to evaluate the effect of subsidized imports on several factors, including sales by the domestic industry.[7] Therefore, administration of existing CVD law requires an examination of domestic industry revenue. Second, under oligopoly with constant marginal costs, decline in industry revenue is positively related to the reduction in domestic industry profits. Third, even under perfect competition with constant marginal costs, decline in domestic industry revenue is positively related to adjustment costs borne by workers displaced by unfair imports. Thus, decline in industry revenue provides information about worker adjustment costs.

IV. EQUILIBRIUM IN THE DOMESTIC MARKET

The demand price equation for domestic product is

(1) $P = A - BX - kx$

where A, B, and k are positive constants.

The demand price equation for imported product is

(2) $p = a - kX - bx$

where a and b are positive constants.

The demand coefficients are subject to the following restrictions:

(3) $Bb > k^2$, $B > k$, and $b > k$.

If $Bb = k^2$, then domestic and foreign products are perfect substitutes (homogeneous). If $k = 0$, then domestic and foreign products are independent. The conditions $B > k$ and $b > k$ capture the notion that quantity changes of a particular product have a greater impact on that product's price compared to the price of a substitute product.

The total revenue of the domestic firm is

(4) $R = PX$.

The total revenue of the foreign firm is

(5) $r = r_1(X,x) + r_2(y) + sx = px + r_2(y) + sx$.

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[7] U.S.C. 19, sec. 1677, IV(7)(C)(iii). <u>United States Code, 1988 Edition</u>, U.S. Govt. Printing Office, 1989, p. 1108.

Total profits of the domestic firm are

(6) $P^* = R - C(X)$.

Total profits of the foreign firm are

(7) $p^* = r - c(x + y)$.

Profit maximization requires

(8) $dP^*/dX = dR/dX - dC/dX = A - BX - kx + X(dP/dX)_{cd} - C' = 0$,

and

(9) $dp^*/dx = dr/dx - dc/dx = a - kX - bx + x(dp/dx)_{cf} + s - c' = 0$.

Based on the profit maximizing conditions, the optimum values of X and x are[8]

(10) $X = [(C'-A)(-b+(dp/dx)_{cf})+k(c'-a-s)]/$

$[(-B+(dP/dX)_{cd})(-b+(dp/dx)_{cf}) \ -k^2] = N_1/D_1$,

(11) $x = [(-B+(dP/dX)_{cd})(c'-a-s)+k(C'-A)]/$

$[(-B+(dP/dX)_{cd})(-b+(dp/dx)_{cf}) \ -k^2] = N_2/D_1$.

V. CONJECTURAL VARIATIONS

The optimum quantities (and prices) depend on conjectural variations, i.e., on $(dP/dX)_{cd}$ and $(dp/dx)_{cf}$. These conjectures reflect the degree of rivalry between the domestic and foreign firms. At one extreme, perfect competition, rivalry is perceived to be so intense that individual firms believe they have no influence over price. With less intense forms of rivalry, firms believe they can influence prices by reducing sales. The less intense the rivalry, the stronger the perceived influence over price. Conjectures for the five types of competition are listed

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[8] Note that there are boundary conditions in equations (10) and (11) that impose restrictions on the coefficients and on the unit subsidy. Specifically, $A > C'$ and $(a+s) > c'$. These restrictions imply that threshold prices exceed marginal costs: the first buyer in the market is willing to pay a price that is higher than the firm's marginal cost. This ensures that both firms will find it profitable to supply the domestic market.

below, ranked in terms of degree of rivalry (from highest to lowest) or, equivalently, in terms of perceived influence over price (from lowest to highest).[9]

Perfect Competition:

(12) $$(dP/dX)_{cd} = 0$$
$$(dp/dx)_{cf} = 0;$$

Bertrand:

(13) $$(dP/dX)_{cd} = -B + k^2/b$$
$$(dp/dx)_{cf} = -b + k^2/B;$$

Consistent Conjectures:

(14) $$(dP/dX)_{cd} = -\text{sqroot}[Bb(Bb-k^2)]/b$$
$$(dp/dx)_{cf} = -\text{sqroot}[Bb(Bb-k^2)]/B;$$

Cournot:

(15) $$(dP/dX)_{cd} = -B$$
$$(dp/dx)_{cf} = -b;$$

Collusion:

(16) $$(dP/dX)_{cd} = -B - k(1-q)/q$$
$$(dp/dx)_{cf} = -b - kq/(1-q),$$

where q equals the ratio of the quantity of domestic firm sales to total units consumed domestically.

VI. EFFECT OF FOREIGN SUBSIDY ON DOMESTIC INDUSTRY

Introduction

The effect of an export subsidy on a domestic industry can be expressed in several ways, for example the marginal impact of a change in the subsidy on domestic industry revenue (i.e., dR/ds). However, to compare the effect of a subsidy across market structures we need to take account of the fact that initial equilibrium prices and quantities will differ across structures. We adjust for this by holding domestic industry

[9] The derivation of these conjectures is in the appendix.

revenue constant[10] and by constructing an elasticity measure, which by definition is independent of units. Specifically, we solve for the <u>percent</u> change in domestic industry revenue caused by a one <u>percent</u> increase in the subsidy rate. This is

(17) $E_{Rs} = (dR/ds)(s/R)$.

Domestic industry revenue depends on the optimum levels of X and x. Substituting equation (1) into equation (4) gives

(18) $R = AX-BX^2-kXx$.

To find the effect of s on R, differentiate equation (18) totally with respect to s, which gives[11]

(19) $dR/ds = -k[AD_1-BN_1-kN_2-N_1(dP/dX)_{cd}]/(D_1)^2 = -kC'/D_1$,

since $P - X(dP/dX)_{cd} = C'$.

Multiplying equation (19) by (s/R) gives E_{Rs},

(20) $E_{Rs} = -(sk/D_1)(C'/R)$.

Notice that E_{Rs} is inversely proportional to the subsidy rate and to the degree of substitution between domestic and foreign products. That is, as s increases, other things remaining the same, there is an increase in the percent decline of domestic industry revenue. Similarly, as k increases, the adverse effect on the domestic industry from a particular foreign subsidy also increases.

<u>Ranking of Market Structures</u>

To compare the E_{Rs}'s across market structures, note that they are inversely proportional to the D_1's. The D_1's are given below and listed according to size (from smallest to largest).

Perfect Competition:

(21) $D_1 = Bb - k^2$;

Bertrand:

[10] This assumes that the intercept terms in the demand equations (i.e., A and a) change so that initial domestic industry revenue is the same across market structures. This does not, however, affect the terms that enter conjectural variations.

[11] Note that X and x are replaced by equations (10) and (11).

$$(22) \qquad D_1 = [(4Bb - k^2)(Bb - k^2)]/Bb;$$

Consistent:

$$(23) \qquad D_1 = 2(Bb-k^2) + 2(\text{sqroot}(Bb(Bb-k^2)));$$

Cournot:

$$(24) \qquad D_1 = 4Bb - k^2;$$

Collusion:

$$(25) \qquad D_1 = 4Bb + 2Bkq/(1-q) + 2kb(1-q)/q.$$

The ranking of E_{Rs} across market structures is exactly the reverse of the order listed above for the D_1's. Specifically, foreign subsidies cause the greatest adverse effect on domestic industry revenue under perfect competition. Subsidies have progressively smaller effects under Bertrand, Consistent Conjectures, and Cournot. The relative effect of subsidies is smallest under collusion.

The above results also imply that there is a significant difference between perfect competition and oligopoly. The ratio of two E_{Rs}'s equals the reciprocal of the ratio of their D_1's. For example, the ratio of the E_{Rs} for perfect competition to the E_{Rs} for Bertrand is $4 - k^2/Bb$, which is greater than 3. Since Bertrand has the smallest E_{Rs} among the four oligopolies, this indicates perfect competition is at least three times more sensitive to foreign subsidies than oligopoly. Although it is not necessarily true that small firms are more competitive than large firms, this result might help explain why so many of the CVD cases that come before the ITC involve small firms.

Finally, the ranking of the E_{Rs}'s is explained by the effect of the subsidy on price of the subsidized import -- the pass-through" issue. The greater the effect of the foreign subsidy on foreign price (the pass-through), then the greater the adverse effect on domestic industry revenue. This is because the greater the decline in import price, the greater the switch of domestic consumers from domestic product to foreign product. The expression for the pass-through is obtained by totally differentiating equation (2), which gives

$$(26) \; dp/ds = [k^2 - b(B - (dP/dX)_{cd})]/D_1.$$

Substituting conjectures and D_1's gives the expressions for the pass-through (ranked from high to low (in absolute value)).[12]

Perfect Competition:

(27) $dp/ds = -1;$

Bertrand:

(28) $dp/ds = -2Bb/(4Bb-k^2);$

Consistent:

(29) $dp/ds = -(1/2);$

Cournot:

(30) $dp/ds = -(2Bb-k^2)/(4Bb-k^2);$

Collusion:

(31) $$dp/ds = -[2Bb - k^2 + bk(1-q)/q]/[4Bb+2Bkq/(1-q) + 2bk(1-q)/q].$$

Thus, under perfect competition, if the foreign firm receives a one dollar subsidy, it responds by lowering price charged U.S. importers by one dollar. In this case, competition forces the foreign firm to cut price to exactly match the subsidy. Under Bertrand competition and other forms of oligopoly competition, the pass-through is smaller because rivalry is less intense. For example, in the case of consistent conjectures, a one dollar subsidy lowers import price by fifty cents.

[12] Note that since the pass-through for collusion involves q (the share of the domestic firm), it is necessary for q to be greater than zero in order for the pass-through under collusion to be smaller than under Cournot.

VII. CONCLUSION

This paper examines how the adverse effect of subsidized imports on a domestic industry varies with market structure. Injury to the domestic industry is measured by decline in total revenue. Perfect competition is compared with four different oligopolies: Bertrand, consistent conjectures, Cournot, and collusion. Injury is most severe under perfect competition. Moreover, as the degree of rivalry between the domestic and foreign firms declines, relative injury also declines. Thus, using the competitive market assumption to estimate injury yields upper bound estimates if in fact oligopoly is the true market structure. Finally, we find that firms in perfectly competitive industries are much more sensitive to foreign subsidies than are oligopolies. Other things the same, the percent loss in revenue by competitive firms for a one percent increase in subsidies is at least three times higher under perfect competition than under oligopoly.

APPENDIX
DERIVATION OF CONJECTURES

This Appendix derives the conjectural variations for the five market structures examined in this paper.

Perfect Competition

Each firm conjectures that changes in its output will not affect its price. Thus

(A1) $(dP/dX)_{cd} = 0$

(A2) $(dp/dx)_{cf} = 0.$

Domestic and foreign firms view themselves as price takers.[13]

Bertrand

Each firm conjectures that its rival's price is given. For the domestic firm this means $(dp)_{cd} = -k(dX) - b(dx) = 0$, so $(dx/dX)_{cd} = -k/b$.

Thus if the domestic firm adjusts its output it anticipates the following effect on its price

(A3) $(dP/dX)_{cd} = d(A-BX-kx)/dX = -B-k(dx/dX)_{cd} = -B+k^2/b$

Similarly, conjectures by the foreign firm are found from $(dP)_{cf} = -B(dX) -k(dx) = 0$, which gives $(dX/dx)_{cf} = -k/B$.

Then

(A4) $(dp/dx)_{cf} = d(a-kX-bx)/dx = -k(dX/dx)_{cf}-b = k^2/B-b.$

[13] When equations (A1) and (A2) are substituted into equations (10) and (11), the resulting equilibrium levels of X and x are such that price equals marginal cost.

Consistent conjectures[14]

Each firm conjectures correctly how its rival will react. Thus the conjecture held by the domestic firm about how the foreign firm will change its exports in reaction to an output change by the domestic firm is equal to the actual change by the foreign firm.

To find the domestic firm's consistent conjecture, solve for the actual response by the domestic firm to a change in imports, $(dX/dx)_{ad}$, in terms of the domestic firm's conjecture about how the foreign firm will change its exports in response to a change in the domestic firm's output, $(dx/dX)_{cd}$. The optimum X is

(A5) $$X = (C'-A+kx)/[-B+(dP/dX)_{cd}]$$

$$= (C'-A+kx)/[-2B-k(dx/dX)_{cd}].$$

Differentiate with respect to x (noting that $(dx/dX)_{cd}$ is treated as a parameter)

(A6) $$(dX/dx)_{ad} = (-k)/[2B+k(dx/dX)_{cd}].$$

Similarly, for the foreign firm, the optimum x is

(A7) $$x = (c'-s-a+kX)/[-2b-k(dX/dx)_{cf}]$$

Differentiate with respect to X

(A8) $$(dx/dX)_{af} = (-k)/[2b+k(dX/dx)_{cf}].$$

If conjectures are consistent

(A9) $$(dx/dX)_{cd} = (dx/dX)_{af} = dx/dX$$

and

(A10) $$(dX/dx)_{cf} = (dX/dx)_{ad} = dX/dx.$$

Rearranging equation (A6) and substituting equations (A8) (A9) and (A10) gives

(A11) $$(dX/dx)_{ad}[2B+k(dx/dX)_{cd}] + k$$

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[14] Consistent conjectures are discussed by Bresnahan and Kamien and Schwartz. Timothy Bresnahan (1981), "Duopoly Models with Consistent Conjectures," American Economic Review, Vol. 71, No. 5, pp. 934-945. Morton Kamien and Nancy Schwartz (1983), "Conjectural Variations," Canadian Journal of Economics, Vol. 16, No. 2, pp. 191-211.

$$= 2B(dX/dx) - [k^2(dX/dx)]/[2b+k(dX/dx)] + k$$

$$= 4Bb(dX/dx) + 2Bk(dX/dx)^2 - k^2(dX/dx) + 2bk + k^2(dX/dx)$$

$$= 2Bk(dX/dx)^2 + 4Bb(dX/dx) + 2bk = 0.$$

Using the quadratic formula, the solution for (dX/dx) is

(A12) $$dX/dx = [-4Bb + \text{sqroot}\{4Bb(4Bb - 4k^2)\}]/4Bk.$$

Similarly, rearranging equation (A8) and then substituting equations (A6) (A9) and (A10) gives

(A13) $$(dx/dX)_{af}[2b + k(dX/dx)_{cf}] + k$$

$$= 2b(dx/dX) - [k^2(dx/dX)]/[2B + k(dx/dX)] + k$$

$$= 4Bb(dx/dX) + 2kb(dx/dX)^2 - k^2(dx/dX) + 2Bk + k^2(dx/dX)$$

$$= 2kb(dx/dX)^2 + 4Bb(dx/dX) + 2Bk = 0.$$

The solution for (dx/dX) is

(A14) $$dx/dX = [-4Bb + \text{sqroot}\{4Bb(4Bb - 4k^2)\}]/4kb.$$

Thus

(A15) $$(dP/dX)_{cd} = d(A - BX - kx)/dX = -B -k(dx/dX)$$

$$= -B -k[-4Bb + \text{sqroot}(.)]/4kb = - \text{sqroot}(.)/4b$$

and

(A16) $$(dp/dx)_{cf} = d(a - kX - bx)/dx = -b - k(dX/dx)$$

$$= -b - k[-4Bb + \text{sqroot}(.)]/4Bk = - \text{sqroot}(.)/4B.$$

Finally, note that sqroot(.) in equations (A12) and (A14) has a real solution in view of the restrictions on the demand coefficients given in equation (3).[15]

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[15] Note also that only positive roots are used in equations (A12) and (A14). Negative roots cause outputs to be negative. To demonstrate this, it suffices to show that $D_1 < 0$ in equations (10) and (11).

With negative roots in equations (A12) and (A14)

(F1) $$(dP/dX)_{cd} = d(A - BX - kx)/dX$$

$$= -B - k(dx/dX)_{cd}$$

(continued...)

[15](...continued)

$= -B - k[-4Bb\text{-sqroot}(4Bb(4Bb-4k^2))]/4kb$

$= -B + 4kBb/4kb + (k)\text{sqroot}(.)/4kb$

$= -B + B + \text{sqroot}(.)/4b$

$= \text{sqroot}(.)/4b.$

(F2) $(dp/dx)_{cf} = d(a - kX - bx)/dx$

$= -b - k(dX/dx)_{cf}$

$= -b - k[-4Bb\text{-sqroot}(4Bb(4Bb-4k^2))]/4Bk$

$= -b + 4Bbk/4Bk + (k)\text{sqroot}(.)/4Bk$

$= -b + b + \text{sqroot}(.)/4B$

$= \text{sqroot}(.)/4B.$

(F3) $D_1 = [-B+(dP/dX)_{cd}][-b+(dp/dx)_{cf}] - k^2$

$= [-B+\text{sqroot}(.)/4b][-b+\text{sqroot}(.)/4B] - k^2$

$= [(-4Bb+\text{sqroot}(.))/4b][(-4Bb+\text{sqroot}(.))/4B] - k^2.$

Multiplying through by 16Bb

$= [-4Bb+\text{sqroot}(.)][-4Bb+\text{sqroot}(.)] - 16Bbk^2$

$= (-4Bb)^2+4Bb(4Bb-4k^2)-8Bb[\text{sqroot}(.)]-16Bbk^2.$

Dividing through by 4Bb

$= 4Bb+(4Bb-4k^2)-2(\text{sqroot}(.))-4k^2$

$= 8Bb-2[\text{sqroot}(.)]-8k^2.$

Dividing through by 8

$= (Bb-k^2)-\text{sqroot}[Bb(Bb-k^2)].$

This last expression is negative since

(F4) $(Bb-k^2)^2 < Bb(Bb-k^2)$

(continued...)

Cournot

Each firm conjectures that rival's output will not change if it changes its own output, so that $(dx/dX)_{cd} = (dX/dx)_{cf} = 0$. For the domestic firm this means

(A17) $(dP/dX)_{cd} = d(A-BX-kx)/dX = -B-k(dx/dX)_{cd} = -B.$

For the foreign firm

(A18) $(dp/dx)_{cf} = d(a-kX-bx)/dx = -b-k(dX/dx)_{cf} = -b.$

Collusion

Each firm conjectures that its output will be a constant proportion of total consumption. For the domestic firm this proportion is q where[16]

(A19) $q = X/(x+X).$

For the foreign firm the proportion is 1-q where

(A20) $1-q = x/(x+X).$

Then

(A21) $x = X(1-q)/q$ and $X = xq/(1-q)$

so that

(A22) $(dx/dX)_{cd} = (1-q)/q$

(A23) $(dX/dx)_{cf} = q/(1-q).$

In this case

(A24) $(dP/dX)_{cd} = d(A-BX-kx)/dX = -B-k(dx/dX)_{cd} = -B-k(1-q)/q$

and

(A25) $(dp/dx)_{cf} = d(a-kX-bx)/dx = -b-k(dX/dx)_{cf} = -b-kq/(1-q).$

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[15](...continued)

$(Bb-k^2) < Bb.$

Therefore, $D_1 < 0$ with negative roots and outputs are negative.

[16] We assume that units of measurement are selected so that P=p=1 in initial equilibrium. Therefore, we can add units of domestic and foreign products.