ESTIMATING THE SAMPLING VARIANCE FOR ALTERNATIVE ESTIMATORS OF THE U. S. CONSUMER PRICE INDEX

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The official published United States Consumer Price Index (CPI) is a modified Laspeyres index. The Bureau of Labor Statistics has been investigating alternative forms of basic level and aggregate index estimators, and recently published a test series based on a geometric mean basic index (BLS, 1997.) BLS has also published index series based on 4 alternative aggregation formulae, two of which are termed superlative indices (Aizcorbe and Jackman, 1993.) This report presents estimates of the sampling variance for these alternative series.

In section one the estimators will be described. The variance estimates for the basic level geometric mean and the alternatively aggregated series will be given in section two and three, respectively. In section four comparisons between the stratified jackknife and stratified random groups variance estimators for the production Laspeyres series will be presented, and section five will discuss conclusions and future research directions.

1. Publication and Alternate Index Estimators

For a full discussion of the CPI the reader is referred to Chapter 19 of the *BLS Handbook of Methods*, (1992) and Leaver and Valliant (1995). However, we will describe certain features of the CPI pertinent to this study. The CPI is a modified Laspeyres index, which is a ratio of the costs of purchasing a set of items of fixed quality and quantity in two different time periods. The CPI is calculated monthly for the total US urban population for all consumer items, and it is also estimated at other levels defined by geographic area and item groups such as food, shelter, and apparel.

Prices for the CPI are collected in 88 primary sampling units (PSUs) in 85 geographic areas (the New York area consists of 3 PSUs and the Los Angeles area consists of 2 PSUs). Of these PSUs, 32 are self-representing. The remaining 56 were selected according to a stratified design in which one PSU was selected from each stratum for each of 12 non-self-representing index areas defined as medium-sized MSA's, small-sized MSA's and urban, non-MSA's in each of the four Census regions (Northeast, Midwest, South, and West.)

The CPI is estimated at the item stratum-index area level, although not all such indexes are published every month. It is constructed in two stages. In the first or basic level stage, the price index for an item-area is updated every 1 or 2 months via a function of sample price changes called a price relative. Let I_{ia}^{t} denote the index at time *t*, in item stratum *i*, area *a*, relative to time period *0*. Then

$$I_{ia}^{t} = RL_{ia}^{t,t-1} I_{ia}^{t-1}$$

where $RL_{ia}^{t,t-1}$ denotes the price relative between times *t* and *t-1*.

$$RL_{ia}^{t,t-1} = \frac{\sum_{j} w_{iaj}\left(\frac{P_{iaj,t}}{P_{iaj,c}}\right)}{\sum_{j} w_{iaj}\left(\frac{P_{iaj,t-1}}{P_{iaj,c}}\right)}, \text{ where}$$

P and w represent the price and sampling weight of sample item *j* , and *c* represents the outlet frame construction reference period for index area *a*.

The index for higher level item I and area A groupings is computed as a weighted sum of basic level indices:

$$I_{\mathrm{IA}}^{t} = \sum_{i \in \mathrm{I}} \sum_{a \in \mathrm{A}} r_{ia}^{b} I_{ia}^{t}, \text{ where}$$

r_{ia}^{b} = the item-area relative importance or relative consumer expenditure share, computed from the Consumer Expenditure Survey for reference period *b*.

The first alternative form of the index we examine is the basic level geometric mean index, published as an experimental index series in April 1997. At the basic level, this index is also a ratio of expenditure estimates which are updated each month with a geometrically averaged price relative. If IG_{ia}^{t} denotes the alternative index, called the *geo-index*, at time *t*, relative to time period *0*, then

$$IG_{ia}^{t} = RG_{ia}^{t,t-1} IG_{ia}^{t-1}$$

where $RG_{ia}^{t,t-1}$ denotes the one-period price relative which is the weighted geometric mean of sample unit price changes between times t and t-1.

$$RG_{ia}^{t,t-1} = \prod_{j \in I_i} \left(\frac{P_{iaj,t}}{P_{iaj,t-1}} \right)^{w'_{iaj}} = e^{\sum_{j \in I_{ia}} w'_{iaj} \ln\left(P_{iaj,t} / P_{iaj,t-1} \right)}$$

here P represents the price and w' represents the quote-level sampling weight of sample item j, normalized to the same sample rotation base for all quotes in an item-index area. Basic level geo-indices are aggregated in the same manner as the Laspeyres index described above.

BLS has also computed index series based on alternative aggregations of yearly average basic level production Laspeyres indices, relative to different base years $\{b\}$:

$\{ I_{ia}^{t} / I_{ia}^{b} \}$, where $I_{ia}^{t} = \frac{1}{12} \sum_{u = year\ t,\ month\ 1}^{year\ t,\ month\ 12} I_{ia}^{u,8612}$, where the u's are in the same calendar year t. The formulae for the different ways of aggregating the basic Laspeyres indexes are given below:

Table 1: Alternative Index Aggregation Formulae

Laspeyres	$L_{t,b} = \sum_{i,a} r_{i,a}^{b} \left(\frac{I_{i,a}^{t}}{I_{i,a}^{b}} \right)$
Paasche	$P_{t,b} = = \left[\sum_{i,a} r_{i,a}^{t} \left(\frac{I_{i,a}^{b}}{I_{i,a}^{t}} \right) \right]^{-1}$
Fisher Ideal	$F_{t,b} = \left[L_{t,b} \times P_{t,b} \right]^{1/2}$
Tornqvist	$T_{t,b} = \prod_{i,a} \left(\frac{I_{i,a}^{t}}{I_{i,a}^{b}} \right)^{\left(\frac{1}{2} r_{i,a}^{b} + \frac{1}{2} r_{i,a}^{t} \right)}$ $= e^{\left[\sum_{i,a} \frac{1}{2} \left(r_{i,a}^{b} + r_{i,a}^{t} \right) \ln \left[\frac{I_{i,a}^{t}}{I_{i,a}^{b}} \right] \right]}$
Geometric Mean Aggregate	$G_{t,b} = \prod_{i,a} \left(\frac{I_{i,a}^{t}}{I_{i,a}^{b}} \right)^{r_{i,a}^{b}}$ $= e^{\left[\sum_{i,a} r_{i,a}^{b} \ln \left[\frac{I_{i,a}^{t}}{I_{i,a}^{b}} \right] \right]}$

where t indicates the current year, b indicates the base year, and $r_{i,a}^{t}$ is the item-area relative importance computed from the Consumer Expenditure Survey for year t.

Diewert (1995) discusses the properties of the different index forms. This paper will not discuss the relative merits of the alternative index forms but will only attempt to investigate their sampling variances.

Previous work in estimating the sampling variance of the CPI has largely been devoted to the Laspeyres estimator. Dippo and Wolter (1983) compared Taylor series approximations to jackknifing. In a series of papers, a hybrid random-groups-Taylor series approach was used to estimate the sampling variance of the CPI in Leaver (1990), Leaver et. al. (1991), and Leaver and Swanson (1992). Leaver and Valliant (1995) compare this hybrid estimator with a stratified random groups estimator using VPLX software. Baskin and Leaver (1996) explored variance estimation for the basic geometric means estimator for the housing component of the CPI. This paper builds on these previous studies and is the first to provide standard error estimates for the all items geo-index series and for alternatively aggregated series and their differences.

2. Test Geo-Indices on Production Data

The Bureau is currently producing a test geo-index, and recently published six years of back indices from January 1991 through February 1997.

The test geo-index is not being calculated in the same processing environment as the official Laspeyres index. For this reason, a test Laspeyres index is being computed simultaneously with the geo-index for purposes of comparison.

Estimates of variance for price change for both the test geo-index and the test Laspeyres index were produced using a stratified jackknife estimator implemented in VPLX. For a description of the VPLX software see Fay (1990). Details of this estimator are given below.

The stratified jackknife variance computations in this application are based on a segmentation of the CPI sample into 8 separate strata, one for each of 7 major item groups for self-representing index areas, and one for all items for non-self-representing index areas. The sample for each of the 7 major item group strata consists of 32 clusters, where each cluster is the sample from one of 32 self-representing index areas. The sample for the eighth stratum consists of 12 clusters, where each cluster is the sample from one of 12 non-self-representing index areas.

For the U.S. All Cities Index, *VPLX* constructs replicate indices $\hat{I}_{sm}^{u}$ ($u = t$ or t - k) for each of n_s clusters in each stratum s. This is done by deleting the index for cluster (m) in stratum (s), and aggregating the indices for the remaining clusters, while rescaling their weights (relative importances) to produce an estimate for the full stratum. These are then aggregated with full sample estimates for the remaining strata. The index corresponding to replicate (sm) is

$$\hat{I}^{u}_{(sm)} = \sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} r^{b}_{s'm'} \hat{I}^{u}_{s'm'} + \sum_{m' \in \{M_s - m\}} r^{b}_{sm'} \hat{I}^{u}_{sm'}$$

where $\{M_s\text{-}m\}$ denotes the set of clusters for stratum s omitting cluster m, and $r^{b}_{sm'}$ is the rescaled relative importance for the stratum-cluster sm' in replicate (sm). Replicate estimates of k-month price change were derived by taking ratios of replicate indices:

$$\hat{I}^{t,t-k}_{(sm)} = \hat{I}^{t}_{(sm)} \Big/ \hat{I}^{t-k}_{(sm)}$$

The stratified jackknife estimator of the variance of $\hat{I}^{t,t-k}$ is given by

$$v_{sjk}\left(\hat{I}^{t,t-k}\right) = \sum_{s=1}^{S} \frac{(n_s - 1)}{n_s} \sum_{m=1}^{n_s} \left[\hat{I}^{t,t-k}_{(sm)} - \hat{I}^{t,t-k}\right]^2$$

where the first sum is over all strata in the sample.

Price change variances were calculated for both one and twelve month periods, at the All Cities level for January 1991 through December 1996. Graphs of 12-month price change with two-standard error bands are presented in Figures 1-3. Index change estimates for the six-year period and their standard errors are given for All Items-All Cities in Table 3. The graphs show that the test indices are smoother and have smaller variances than the corresponding production index. The estimates of sampling variance for both test Laspeyres and test geo-indices are very similar.

Differences in price change estimates between the two test series and the production Laspeyres series and their standard errors were also estimated. A graph of the difference in cumulative price change between the test Laspeyres and test geo-index series with two-standard error bands is given in Figure 4. Differences in index change over the six-year period and their standard errors for All Items and 7 major groups are given in Table 4.

3. Higher Level Aggregates with Production Data

Estimates of variance for alternatively aggregated index series were also produced for 1987-1996 using a stratified jackknife estimator. Stratum and cluster definitions were the same as described for the basic level geo-index. The replicate index aggregate formulae for each alternative index are given below.

Price change variances were calculated for year to year and cumulative index change at the All Items, U.S. All Cities level. In general, the alternative indices reflect smaller price changes than the corresponding Laspeyres index. The estimates of sampling variance for all alternative indices are very similar.

Table 2: Replicates for Alternative Index Aggregation Formulae

Laspeyres	$\hat{L}^{u,b}_{(sm)} = \sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} \left(r^{b}_{s'm'} \hat{I}^{u,b}_{s'm'}\right) + \sum_{m' \in \{M_s - m\}} r^{b}_{sm'} \hat{I}^{u,b}_{sm'}$
Paasche	$\hat{P}^{u,b}_{(sm)} = \left[\left(\sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} \left(r^{u}_{s'm'} \big/ \hat{I}^{u,b}_{s'm'} \right)\right) + \left(\sum_{m' \in \{M_s - m\}} r^{u}_{sm'} \big/ \hat{I}^{u,b}_{sm'}\right)\right]^{-1}$
Fisher Ideal	$\hat{F}^{u,b}_{(sm)} = \left[\hat{L}^{u,b}_{(sm)} \times \hat{P}^{u,b}_{(sm)}\right]^{1/2}$
Tornqvist	$\hat{T}^{u,b}_{(sm)} = e^{\left[\sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} \left(\frac{r^{b}_{s'm'}}{2} \ln(\hat{I}^{u,b}_{s'm'})\right) + \sum_{m' \in \{M_s - m\}} \frac{r^{b}_{sm'}}{2} \ln(\hat{I}^{u,b}_{sm'})\right]} \times e^{\left[\sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} \left(\frac{r^{u}_{s'm'}}{2} \ln(\hat{I}^{u,b}_{s'm'})\right) + \sum_{m' \in \{M_s - m\}} \frac{r^{u}_{sm'}}{2} \ln(\hat{I}^{u,b}_{sm'})\right]}$
Geometric Mean Aggregate	$\hat{G}^{u,b}_{(sm)} = e^{\left[\sum_{s' \in \{S-s\}} \sum_{m' \in \{M_{s'}\}} \left(r^{b}_{s'm'} \ln(\hat{I}^{u,b}_{s'm'})\right) + \sum_{m' \in \{M_s - m\}} r^{b}_{sm'} \ln(\hat{I}^{u,b}_{sm'})\right]}$

Of particular interest were differences between alternative estimators in the “true index change,” computed as the average index change occurring over the year following a given index base year. Differences in “true one-year index change” estimates between the Laspeyres series and the four alternative index series, and their standard errors were also estimated using the same methodology and are given in Table 5.

Table 3. Six-Year Price Change and Standard Errors for All Items,
U. S. All Cities CPI for Production Laspeyres, Test Laspeyres, and Test Geo-Index Estimators

Index Series	6-Year Percentage Price Change, 9012-9612	Standard Error
Production Laspeyres All Items	18.4841	.3822
Test Laspeyres All Items	17.8190	.4064
Test Geo-index All Items	15.5384	.3898

Table 4: % Differences between Estimator Series and Their Standard Errors for All Items and 7 Major Groups, U.S. All Cities Average CPI, December 1990-December 1996

Series Comparison	Test Laspeyres vs. Geo-index				Production Laspeyres vs. Geo-index			
Item	Difference, Six-Year Price Change	S. E.	6-Year Average % Difference, Dec-Dec Price Change	S. E.	Difference, Six-Year Price Change	S. E.	6-Year Average % Difference, Dec-Dec Price Change	S. E.
All Items	2.2806	.1127	.3252	.0159	2.9458	.1220	.4186	.0174
Food	3.1918	.0866	.4598	.0120	3.1898	.0920	.4595	.0127
Housing	.8516	.1822	.1222	.0260	2.4932	.2517	.3546	.0359
Apparel	7.8098	.3505	1.2919	.0548	7.7869	.3463	1.2883	.0542
Transportation	2.0359	.1064	.2995	.0158	2.0181	.1074	.2969	.0159
Medical Care	3.6469	.2361	.4511	.0289	3.6467	.2291	.4510	.0280
Entertainment	4.4196	.3600	.6258	.0523	4.3807	.3483	.6204	.0506
Other C&S	1.7078	.2153	.2151	.0272	1.6367	.2319	.2062	.0291

Table 5: % Differences between Paired Alternatively Aggregated Estimators of True One-Year Average Price Change and Their Standard Errors for All Items, U.S. All Cities , 1987-1995

Series Comparison	Laspeyres vs. Tornqvist		Laspeyres vs. Fisher Ideal		Geomean vs. Tornqvist		Paasche vs. Tornqvist	
Base Year, Index Year	% Difference, True One-Year Price Change	S.E.	% Difference, True One-Year Price Change	S.E.	% Difference, True One-Year Price Change	S.E.	% Difference, True One-Year Price Change	S.E.
8612, 1987	.1143	.0204	.1153	.0178	-.0489	.0203	-.1164	.0160
1987, 1988	.1309	.0185	.1302	.0183	-.0308	.0208	-.1297	.0183
1988, 1989	.0995	.0245	.1037	.0254	-.0547	.0213	-.1081	.0265
1989, 1990	.1485	.0337	.1443	.0327	-.0189	.0292	-.1403	.0318
1990, 1991	.1277	.0199	.1301	.0214	-.0259	.0176	-.1326	.0232
1991, 1992	.1319	.0175	.1332	.0184	-.0175	.0165	-.1346	.0195
1992, 1993	.0873	.0189	.0844	.0187	-.0488	.0183	-.0815	.0186
1993, 1994	.1025	.0161	.1045	.0159	-.0239	.0163	-.1066	.0161
1994, 1995	.1151	.0200	.1173	.0204	-.0188	.0197	-.1196	.0209
8-Year Avg (1987-95)	.1179	.0086	.1185	.0087	-.0299	.0062	-.1191	.0088

4. Comparison of Variance Estimators

Test geo-index and alternatively aggregated index variances were computed using a stratified jackknife estimator because within-index area replicate indices were not available for these series. The stratified jackknife estimates for the production Laspeyres series in Leaver and Valliant (1995) and for shelter geo-index series in Baskin and Leaver (1996) are based on availability of within-index area replication.

The stratified jackknife estimator assumes equal expected price change among the clusters within a stratum, so the resulting variances should on the average represent overestimates of the true variances for items for which this assumption does not hold. An example of this would be rent change, which varies substantially between index areas.

Graphs of the ratios of standard error estimates computed using a stratified random groups estimator (Leaver and Valliant, 1995) and stratified jackknife estimators for the production Laspeyres series for all items and apparel for January 1991 through December 1996 are given in Figures 5 and 6.

5. Conclusions

Regarding the test geo-index series and its test and production Laspeyres counterparts, Table 3 indicates very small differences in variance estimates among the three series. In terms of long term change, it is quite clear that the two test series are estimating different measures. The test geo-index estimator produced a significantly lower price change measure than the test Laspeyres index for every major group over the six year study. This difference is most remarkable in apparel, followed by entertainment, medical care, and food.

The difference in price change between the test Laspeyres and geo-index series has been regarded by some as a measure of the lower-level substitution bias inherent in the CPI. Previous BLS research has estimated this difference to be around 0.3 to 0.5 percentage points per annum since 1990, and approximately 0.25 since January 1996. Results here indicate that the estimate is .3252 percentage points per annum, over the six year period 1991-96, with a 95% confidence interval of (.294, .356).

It is also quite clear from the comparisons in Table 5 that the Fisher Ideal and Tornqvist series are estimating different measures from the Laspeyres aggregate. Differences in "true index change", their 9 year average, and cumulative price change are significant. In a similar manner the Paasche price change estimator consistently underestimates the Tornqvist series. The geomeans series also underestimates the Tornqvist series, though not significantly so with each yearly change. The 9-year average percentage difference between the two series, however, is significant.

The difference between the Laspeyres aggregation and the Tornqvist aggregation has been viewed as a measure of the upper-level substitution bias in the CPI. Most estimates of this difference cluster around 0.1 to 0.25. Results here indicate that the measure of .1175 percentage points per annum over the 9 year period 1987-95 with a 95% confidence interval of (0.101, 0.134).

Regarding variance estimators, the stratified jackknife fairly consistently overestimates the stratified random group variance for food, transportation and housing, though these graphs are not shown here due to space limitations. These findings are not entirely surprising, since these product groups are associated with larger local market effects. It underestimates the stratified random group estimator quite remarkably for apparel and this warrants further investigation.

The current research indicates the variance estimates for the geo-index and alternative indices behave similarly to the estimates for the Laspeyres index. There is little evidence that the differences in functional forms are producing different estimates of variance.

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Figure 1. Test Laspeyres Formula
U.S. All Cities, All Items

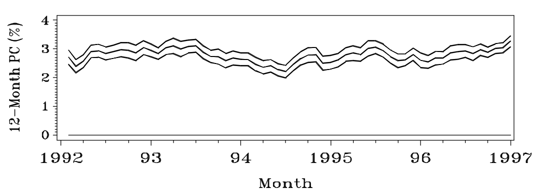

Figure 2: Test Geometric Means Formula
U.S. All Cities, All Items

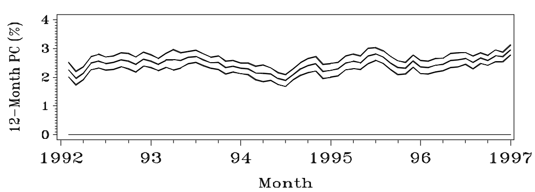

Figure 3. Production Laspeyres Formula
U.S. All Cities, All Items

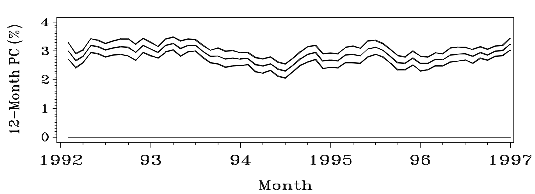

Figure 4. Test Laspeyres vs Test Geomeans
U.S. All Cities, All Items

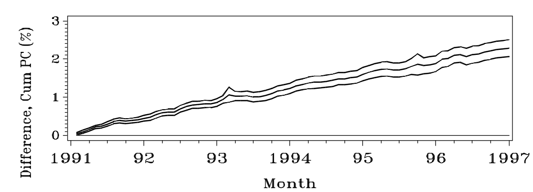

Figure 5: Standard Error Ratios
Stratified Random Groups to Stratified Jackknife
All Items

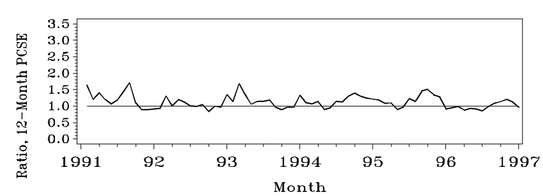

Figure 6: Standard Error Ratios
Stratified Random Groups to Stratified Jackknife
Apparel