A state space model-based method of seasonal adjustment

A structural state space model-based method of seasonal adjustment presents certain advantages to seasonally adjust time series

Raj K. Jain

The Bureau of Labor Statistics publishes a very large number of economic time series such as the Consumer Price Index, the Producer Price Index, employment and unemployment statistics and many more. Most of these series are published as seasonally unadjusted series as well as seasonally adjusted series. More often, however, it is the seasonally adjusted data series that the business community and government agencies use in evaluating the economic situation. There are several reasons given for the use of seasonally adjusted series. It is suggested that the presence of seasonality[1] in time series obscures the stage of the business cycle that the economy is in. In addition, it obscures the effects of interventions,[2] such as a rapid cut in oil production, on a series. At the present time, BLS uses the Census X-11/X-12 ARIMA methods to seasonally adjust BLS indexes and series that have seasonality.[3] In the last 20 years or so, several ARIMA model-based methods have been proposed for seasonal adjustment.[4]

This article presents a structural model based method of seasonal adjustment called the state space model-based method.[5] This articles presents research conducted on this method and illustrates the advantages of the method. The research is part of the Bureau's ongoing efforts to explore relevent measurement issues of interest to the wider statistical community.

Raj K. Jain is a research economist in the Office of Prices and Living Conditions, Bureau of Labor Statistics. E-mail: jain_raj@bls.gov

A structural model

A time series is assumed to be the sum of four components. The first component is the time-trend, which reflects a long-term movement of a time series; the second component is the seasonal, which reflects a periodic movement in a series that repeats itself every year; the third component is the cyclical component, which tracks the course of the business cycle; and finally, the error component, which is the sum total of the effects of all those factors which are individually insignificant and are not included in the trend, the cyclical, or the seasonal components. If the time series is affected by interventions, which are the results of exogenous shocks to the series, then intervention components are included as separate components. This information is incorporated into an equation called the decomposition equation (illustrated as equation 1). To decompose the time series, each component of the time series is assumed to follow a model. The decomposition equation and the component models, together with the statistical properties assumed for the error terms is what constitutes a structural model of a time series. The following is an example of a structural model[6]:

$$y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t \tag{1}$$

$$\mu_t = 2\mu_{t-1} - \mu_{t-2} + \eta_t + \theta_1\eta_{t-1} + \theta_2\eta_{t-2} \tag{2}$$

$$\sum_{j=0}^{11}\gamma_{t-j} = \beta_{t-1} + \omega_t + \varphi_1\omega_{t-1} + \varphi_2\omega_{t-2} \tag{3}$$

$$\sum_{j=0}^{11}\beta_{t-j} = \varsigma_t \tag{4}$$

ψ_t (follows the Trigonmetric Cycle as in Harvey's equation 3.8)[7] (5)

In this model [8], y_t is the observed series, μ_t is the trend, γ_t is the seasonal, ψ_t is the cycle, and β_t is the slope of the seasonal, all at time *t.* The random errors, ε_t and ς_t in equations (1) and (4) and errors in (5), are assumed to be mutually uncorrelated, each having zero mean and constant variance. The random errors η_t, and ω_t in equations (2) and (3) respectively are mutually, but not serially uncorrelated. Each of these errors is assumed to follow a moving average process of order two, written as MA (2) process, and each has zero mean and constant variance. The $\theta_1, \theta_2, \varphi_1, \varphi_2$ are parameters of the MA (2) processes in these two equations to be estimated. Equation (1) is the decomposition equation, equation (2) is the component model for the trend, (3) and (4) are the equations representing the seasonal component model, and (5) represents the cyclical component model. The trend component model in (2) is a local polynomial of order two. The seasonal component model in (3) and (4) assumes that the seasonal component is not constant, but moving in the sense that the seasonal amplitude is not constant over the years. This adds greater flexibility to the estimation of seasonal component. In the structural model previously presented, the important parameters of interest to be estimated are the trend μ_t, the seasonal γ_t, and the cycle ψ_t. However, these are not constant parameters in the model above; these are assumed to be random parameters changing over time in the manner of their component models. This feature adds greater flexibility and realism to this kind of model for seasonal adjustment. The seasonally adjusted series is obtained by subtracting the seasonal component from the observed series.

Estimation of the model

Once we specify the structural model, the next step is to estimate the model. This is done by an iterative technique. To implement this technique, first, the model is put in the "state space" form.[9] In this form, the structural model resembles, but is not identical to, a linear model whose parameter vector is constrained by an auto-regressive process of order one written in short as AR (1) process. There are two parts to the estimation of the structural model in the state space form: (1) Estimation of the parameter vector called the "state vector" and its covariance matrix, given the initial values of the state vector, its covariance matrix, and the initial value of the matrix of error-variance parameters, called hyper-parameters. The estimation is done by the iterative technique called "Kalman Filtering and Smoothing"[10] and (2) Estimation of the matrix of hyper-parameters[11] is done by the Expectation Maximization[12] algorithm[13] and by a quasi-Newton[14] numerical optimization technique.

The Kalman filter is initialized with a zero state vector and a diagonal covariance matrix of the state vector, with the diagonal elements being very large. The very large initial variances of the elements of the state vector, indicates that the analyst has very little faith in the accuracy of these values. Also, the initial matrix of hyper-parameters is generally assumed to be diagonal, with very small but positive values. With initial values for the filter set, the Kalman filtering starts with the first observation and ends with the last observation of the sample. From the last observation, smoothing begins and goes backwards to the beginning of the sample period and one step more beyond that. These smoothed values of the state vector and its covariance matrix, one period before the sample period, are used as the new initial vales for starting the filter for the second iteration. The smoothed residuals and the filtered residuals are used to obtain the new estimate of the matrix of hyper-parameters for the next iteration. The filtered residuals of the model are also used to estimate the log-likelihood function of the model via the Prediction Error Decomposition.[15] This iterative process is continued until the decrease in the log-likelihood function is insignificant. At that point, the estimation of the hyper-parameters and the log-likelihood is switched to a quasi-Newton numerical optimization procedure.

Evaluation of the model

The next step in implementing the state space model-based method of seasonal adjustment is to evaluate the structural model and its components or their derivatives, especially the trend and the seasonally adjusted series. The structural model (described earlier) is evaluated for (1) its adequacy to explain the observed series; (2) its goodness of fit to the data series; and (3) the forecasting performance of the model with respect to the given series. The quality of seasonal adjustment is evaluated with respect to the smoothness of the trend and the presence of, and the identifiability of the stable and the moving seasonality in the observed series.

The adequacy of a structural model is tested, by using Ljung-Box statistics,[16] BDS statistics as developed by W. A. Brock, W. D. Dechert, and J. S. Scheinkman,[17] and MBDS statistics, a modification of BDS statistics by B. Mizrack.[18] The goodness of fit of a structural model is judged by the Akaike Information Criterion, (AIC)[19] and Adjusted Coefficients of Correlation (RBAR-SQUARE),[20] using regular sum of squares of residuals around their mean, differenced sum of squares around the mean of the differenced residuals, and the differenced sum of squares around the seasonal mean of the differenced residual series. For forecasting performance of the structural model, Root Mean Prediction Error Sum of Squares (RMPESS) is used. To evaluate the quality of seasonal adjustment, a test is conducted for the presence of stable or moving seasonality (or both), using *F*-tests constructed from the 2-way ANOVA on the trend-adjusted series. Another statistic developed by E. B. Dagum, called *m7*, which is a function of two *F* statistics constructed from 2-way ANOVA on the trend adjusted series, is used to test for identifiability of seasonality.[21] If the *m*7 value lies between zero and one, then the seasonality is identifiable; otherwise, it is not. The relative variance of the trend component is used to judge the smoothness of the trend. If the relative variance is zero or close to it, then the trend is judged as smooth.

The structural model presented earlier as an example is one of several models that can be used, depending on the choice of trend component model, choice of seasonal component model, assumptions on the error terms, presence or absence of interventions, and so forth. To determine which model best fits a time-series, Akaike Information Criterion estimated from each model are compared. The model with the minimum value of Akaike Information Criterion, assuming that other statistics are the same for all estimated models, is chosen as the best model for that series. In practice however, this assumption is not always satisfied. In that situation, one or two models which are acceptable, are further refined and estimated, and the choice for the best model, with the minimum criterion, is made from those models. These structural models have been estimated using 8 years of monthly, quarterly, and bimonthly BLS time series. A smaller sample size does not necessarily and significantly affect the quality of the estimated components. Moreover, these models are found to be robust with respect to new data for about 3 years; after that, it is safer to once again search for the best model. Of course, if a time series is subject to external shocks, the choice of model analysis for that series has to be done more frequently.

Advantages of structural model

This structural model-based approach to seasonal adjustment has several advantages. First, the structural model-based approach allows an analyst to use the existing statistical theory to test if a structural model represents the data generation process of a given time-series. Nonmodel-based methods lack formal statistical tests to evaluate the results of seasonal adjustment. Second, the structural model-based method estimates the variance of the seasonally adjusted series at the same time it estimates the seasonally adjusted series. This means that the estimation of the variance is also model based, and hence, subject to statistical scrutiny. In other methods, such as ARIMA model-based methods as well as nonmodel-based methods like X-11 and X-12 ARIMA methods, variance estimation is done separately from the seasonal adjustment and hence may be less reliable as a measure of the accuracy of the seasonally adjusted series. Third, many economic time-series such as the Consumer Price Indexes for gasoline, published by the Bureau of Labor Statistics, are affected by external interventions such as the limits placed on the production of crude oil by OPEC and hence, artificial upward increase in the prices of gasoline. In the structural model, a separate observable component is introduced to take account of the effect of an intervention. In other seasonal adjustment methods, the time-series is first subjected to *a priori* adjustment for those effects and then the intervention-adjusted series is seasonally adjusted. In the structural model-based method, all components are estimated simultaneously. A similar advantage lies with the method when a time series, such as retail sales published by the Census Bureau, is affected by the number of trading days[22] in a month or on the day the Easter[23] falls which varies from year to year. Fourth, many time-series are contaminated by sampling errors arising from the peculiarity of the sampling design in the collection of the sample data.[24] This problem is handled in the structural model-based method by introducing an unobserved component in the model.[25] That component is assumed to follow a moving average process of small order say two or three. There are no provisions to take care of this situation in other methods of seasonal adjustment. Fifth, trend and cycles can be decomposed in the structural model based method by introducing a separate component in the structural model for representing the effects of business cycles. This kind of flexibility, which is liked by many researchers, is not available in methods like X-11/X-12 ARIMA. Finally, the structural model-based method is a simple, versatile, and very elegant procedure. All the equations of a structural model are easy to understand. Economic time-series, which are affected by many different kinds of influences such as interventions, measurement error, or number of trading days, can be easily seasonally adjusted in one step. The estimation and evaluation designs of the state space model-based method also make it a very neat and elegant procedure.

Applications

In several studies, the author has applied the structural state space model-based method to several BLS series. This method was applied to the CPI for new cars, CPI for girls' apparel, CPI for gasoline, number of male agricultural workers 20 years and older, unemployment levels of civilians between 16 and 19, and the employment level in retail trade.[26] The state space model-based method with intervention analysis was applied to the CPI for gasoline, the CPI for women's dresses, CPI for women's suites, PPI for gasoline, and PPI for crude petroleum.[27] The state space model-based method with trading day and Easter adjustment was applied to two census series, the retail sales of men's and boys' clothing and wholesale sales of hardware, plumbing and heating equipment.[28] The state space model-based method with measurement errors was applied to the civilian unemployment rate and teenage unemployment rate in a previous study.[29] In this article, the model-based method is applied to the CPI of apples.[30] The CPI for apples is a monthly time-series, which is quite seasonal. The sample period chosen for application spans 8 years from January 1991 to December 1998. Several structural models were estimated using the apple data. The model presented earlier in equations (1) through (5) as an example of a structural model was found to be the best[31] amongst those models tested. This model was found to be adequate, had a good fit to the data, and had a good forecasting performance. It may be pointed out that the forecasting performance of a model is not critical for evaluation of that model for purposes of seasonal adjustment. As pointed out earlier, the adequacy of the model was checked by the Ljung-Box statistics Q*, BDS, and the MBDS tests. The Q* statistic, which is computed using 36 standardized residuals, has a Chi-Square distribution with 32 degrees of freedom. This statistic was found to be Q*(32)=34.43 and the corresponding p-value=0.35; hence it accepts the null hypothesis of uncorrelatedness of residuals. This implies that there is no systematic pattern left in the residuals because the model has captured all the systematic components in the series; hence the model is adequate. The BDS statistics were computed using all 96 smoothed residuals; the test value was BDS=1.39. This test also accepted the null hypothesis of independence and hence, uncorrelatedness of residuals. MBDS statistic, which is a modification of BDS statistic, also accepted the null hypothesis. The three adjusted coefficients of correlation[32] were found to be: RBARSQ=0.98, RBARSQ(DIFF)=0.89, and RBARSQ(SEAS)=0.56. These values indicate that the fit of the model is quite good; the closer these values are to one, the better the fit of the model. For this model, AIC=282.81. There is, however, no benchmark to compare this value with, except that this was one of the smallest values and hence, this model was judged to be a better model than other models under consideration.

Next, the estimated components of the structural model is analyzed, starting with the trend component for the structural model based method as presented in chart 1. The relative variance of the errors of the trend component model is estimated to be 0.43, which indicates that the trend ought to be very smooth. Chart 1 indicates that the trend is fairly smooth; it is smoother than the trend component obtained for the X-12 ARIMA method depicted in chart 2. In the structural model for the state space model-based method, the trend and cycle are estimated separately, whereas in the X-12 ARIMA method, the trend and cycle are estimated as one component because the latter method has no facility to estimate the two separately. However, even the combined trend plus cycle component of the structural model-based method was found to be smoother than the trend-cycle component of the X-12 ARIMA method. Empirically, the smoothness of trend has been found to be a good indicator of a good model.

Seasonal component is another important component of a seasonal time-series. The empirical results for the structural model-based method indicate that, based on *F*-tests from two-way ANOVA, the stable seasonality is significant at both the 5 percent and 1 percent level, but the moving seasonality is not significant at either the 5 percent or 1 percent level of significance. The amplitude of the structural model-based seasonal component in chart 3 varies from –17 to +19 at the beginning of the sample period; but then it keeps on diminishing throughout the sample period, and at the end, it varies from –8 to +8.

The seasonal component estimated by X-12 ARIMA method as shown in chart 4 gives somewhat similar results. As in the case of structural model base method, significant stable seasonality is present, but moving seasonality is not, in the case of X-12 ARIMA method. The amplitude of the seasonal component for the X-12 ARIMA method varies from –15 to +17 at the beginning of the sample period, but declines to the range between –8 and +9.

The statistic, *m*7,[33] which is found to be equal to 0.28 for the state space model-based method indicates that the seasonality is identifiable. The same is true for X-12 ARIMA method.

Finally, a comparison of the seasonally adjusted series obtained by the two methods is presented. The seasonally adjusted series for the state space model-based method is obtained by subtracting the seasonal component from the unadjusted series. Chart 5 displays the unadjusted sample series and the seasonally adjusted series obtained from applying the structural model-based method. The seasonally adjusted series has a pattern that is very similar to the trend, except that it has more kinks; but this is to be expected because, in addition to trend, it contains cyclical component and residual errors. The seasonally adjusted series for X-12 ARIMA depicted in chart 6 is also very similar to its trend, but with kinks. In comparison, the two seasonally adjusted series look very similar and more information is required to assess the superiority of one over the other. In applications to other BLS series, the author has shown significant differences in the seasonally adjusted series produced by the two methods.[34]

THIS STUDY PRESENTED a relatively new method of seasonal adjustment that incorporated several innovations. For example, in the specification of the structural models, the parameters of explanatory variables such as intervention variables or other exogenous or lag-dependent variables were not assumed to be constant as usual, but assumed to follow a random walk process. This added greater flexibility to the estimation of the effects of such variables. In the estimation of the structural models, the hyperparameters of the models were estimated by two methods: the Expectation Maximization (EM) algorithm and the quasi-Newton numerical optimization method. The Expectation Maximization algorithm takes the estimation towards optimization in a few iterations, but after that, its approach to optimization slows down to a snail's pace. At that point, a switch to a quasi-Newton method quickly leads to optimality. In the evaluation of the estimated structural model, two new test statistics, BDS and MBDS, were used. These tests are found to be very effective in testing the adequacy of the structural models. In several conference pa-

Chart 1. **Original sample series and the smooth trend component obtained by using state space model-based method, January 1991 through December 1998**

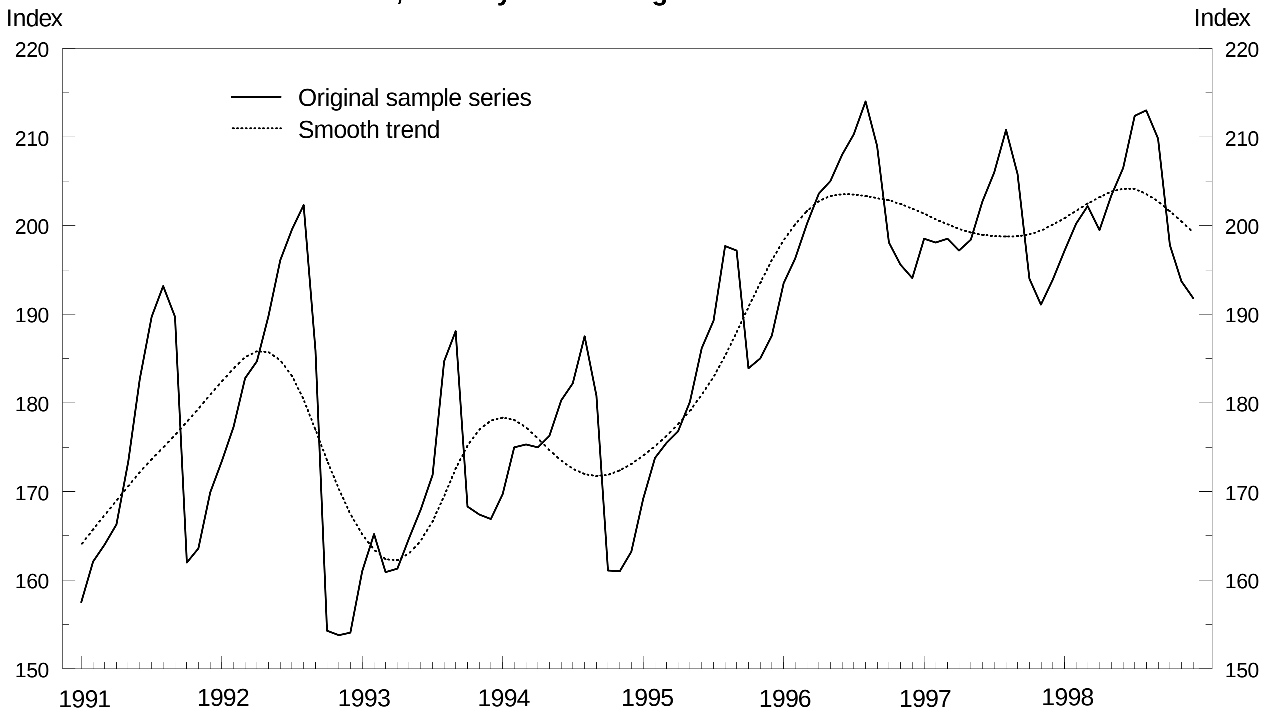


Chart 2. **Original sample series and the final trend component obtained by using X-12 ARIMA method, January 1991 through December 1998**

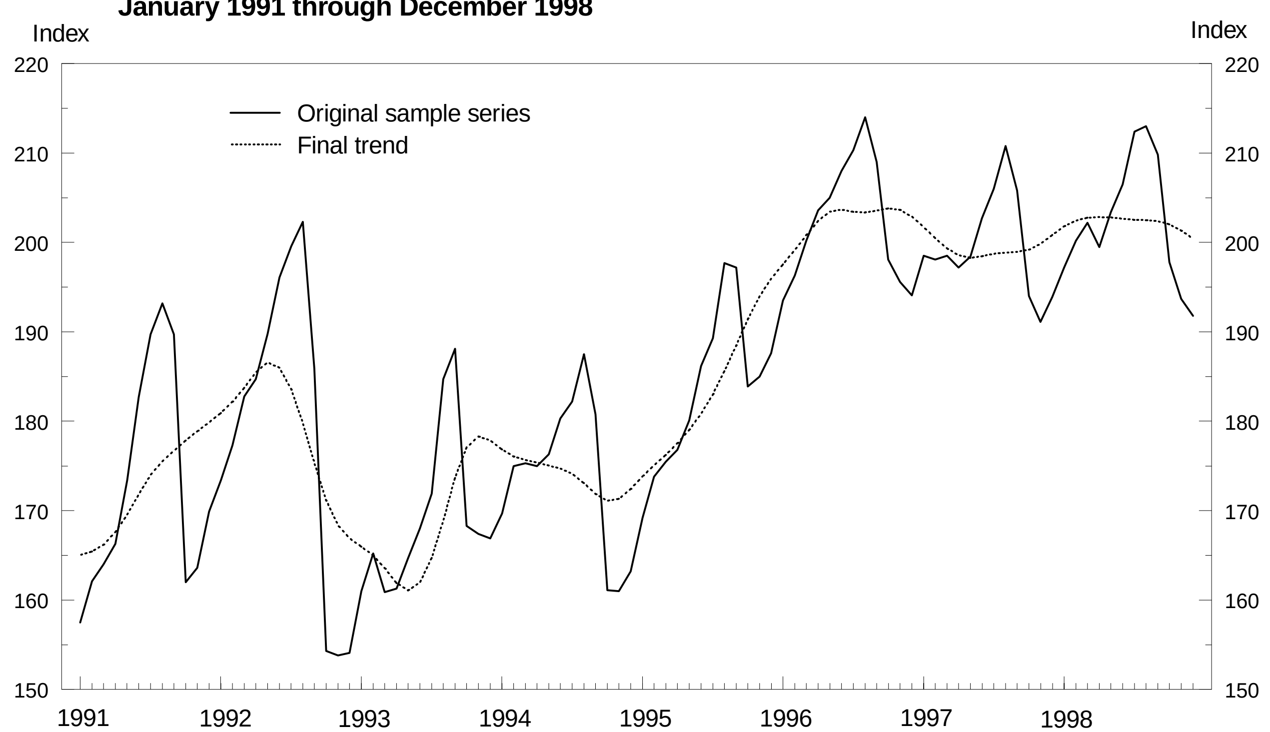


Chart 3. **Smooth seasonal component and its standard error obtained by using the state space model-based method, January 1991 through December 1998**

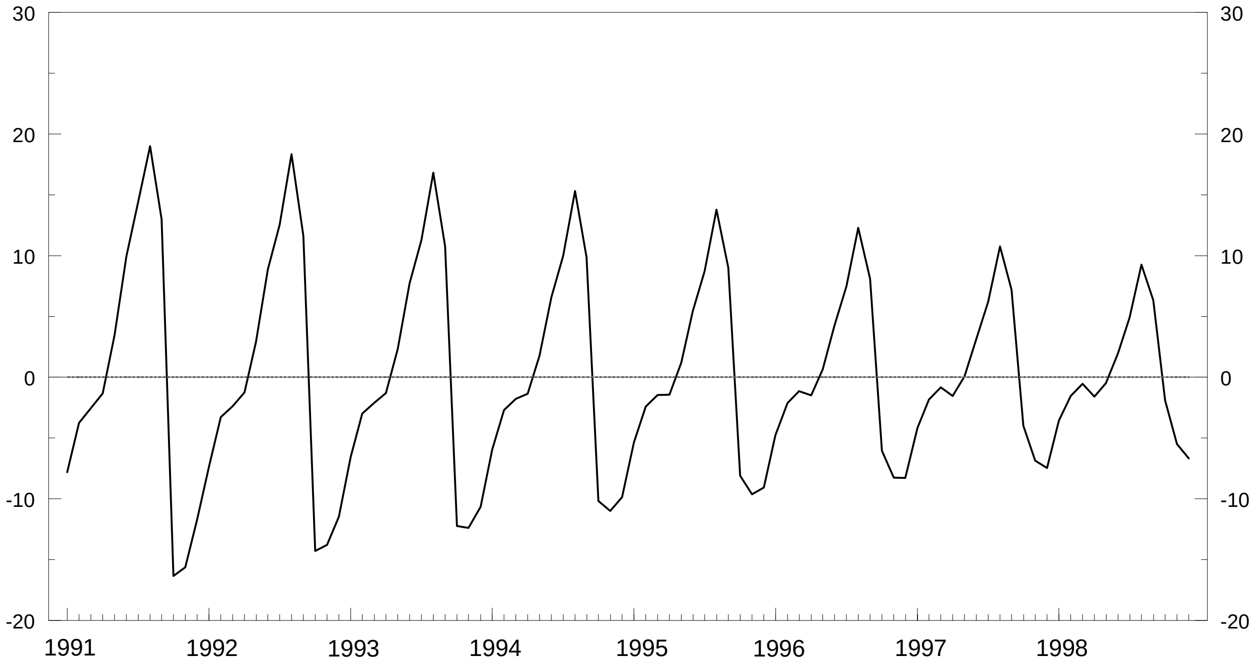


Chart 4. **Final seasonal component obtained by using X-12 ARIMA method, January 1991 through December 1998**

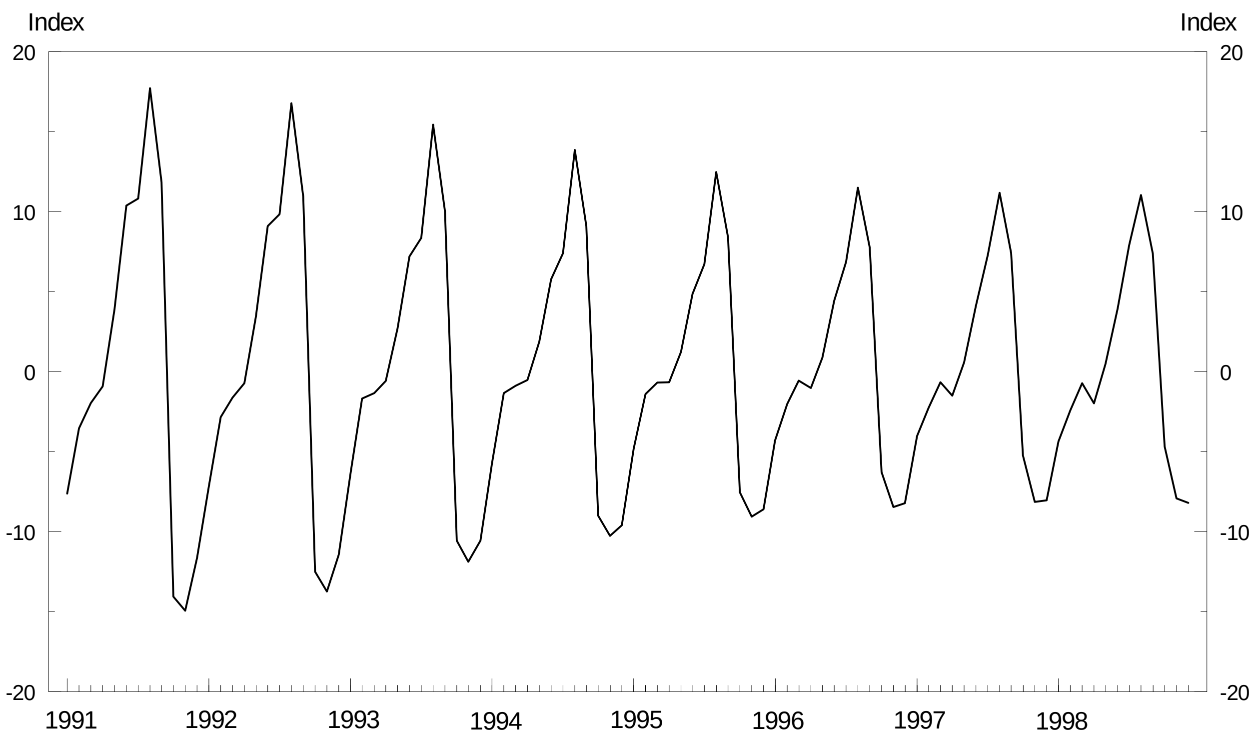


Chart 5. **Unadjusted sample series and the smooth seasonally adjusted series obtained by using the state space model-based method, January 1991 through December 1998**

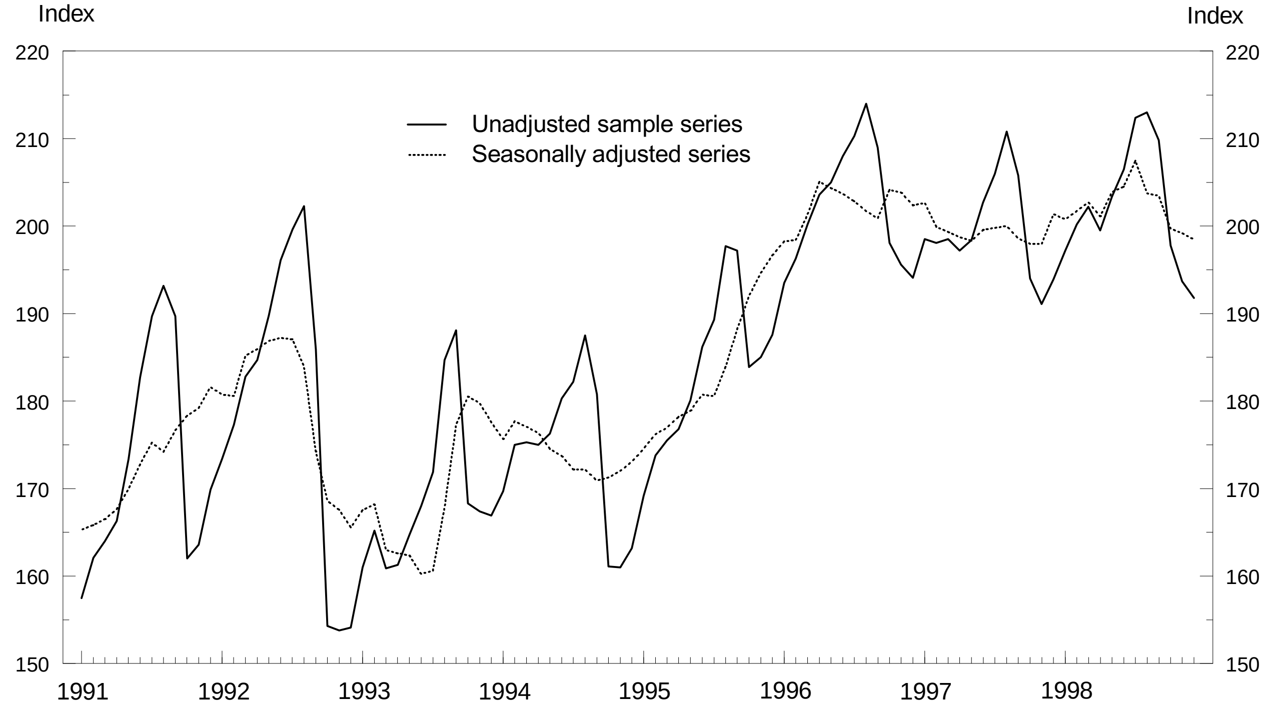
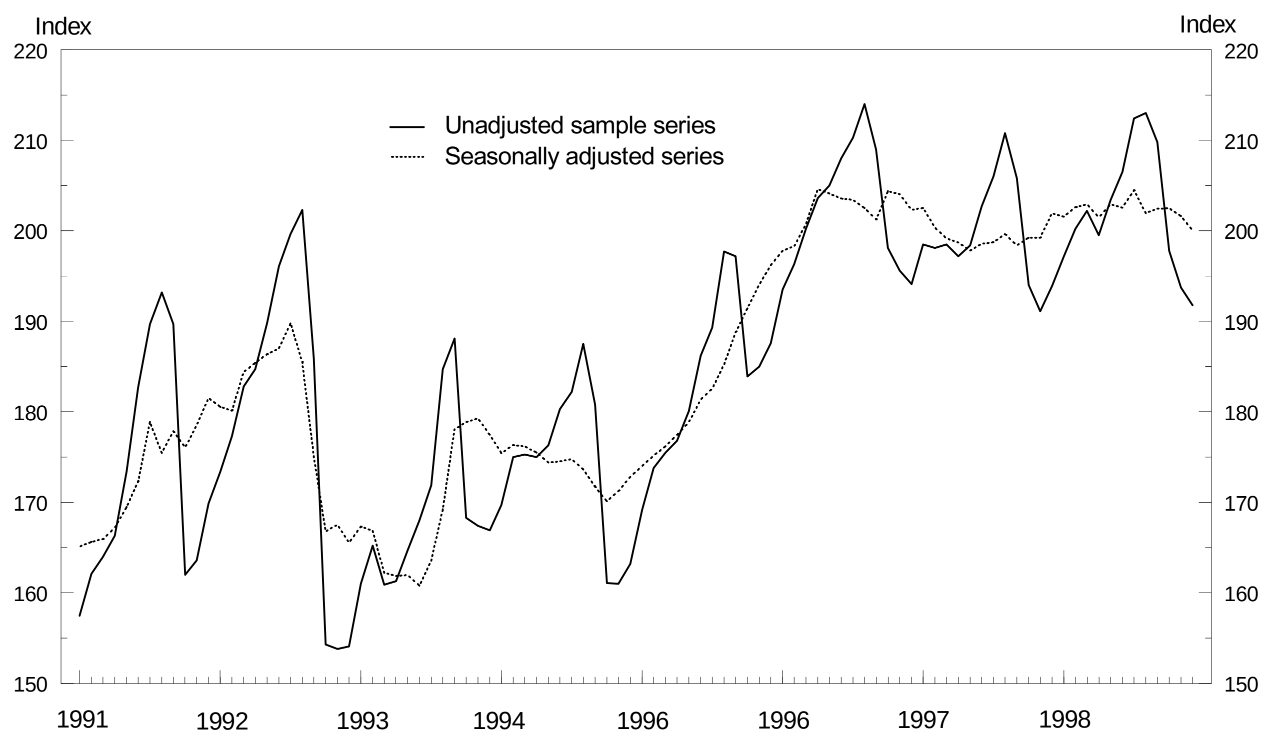


Chart 6. **Unadjusted sample series and the seasonally adjusted series obtained by using X-12 ARIMA method, January 1991 through December 1998**



pers mentioned earlier, the author has presented the empirical results of the application of this method with all the innovations mentioned, to various BLS and Census Bureau series. The author has written a complete computer program (in GAUSS) incorporating various aspects of seasonal adjustment such as "intervention and outlier analysis," "trading day and Easter adjustment," "survey sampling error adjustment," and all other innovations mentioned above. This study has presented a brief outline of this method of seasonal adjustment and its application to the CPI for apples. □

Notes

[1] Ted Jaditz, "Seasonality: economic data and model estimation," *Monthly Labor Review,* 1994, pp. 17–22.

Jaditz has discussed the factors that give rise to seasonality and the rationale of seasonally adjusting the economic time series.

[2] Interventions resulting from external events such as an OPEC decision to reduce total production of crude oil at a point in time that will almost immediately, or with a slight lag time, affect the retail prices and hence, the CPI of the gasoline at that time. Unless the effect of this intervention is separated from other components, the decomposition of the time series would produce components, which would include some effect of the intervention and hence be misleading. The approach to separating the effects of interventions at a certain point in time is called intervention analysis.

[3] *X-12 ARIMA Reference Manual* (Washington, DC, Time Series Staff, Bureau of the Census, 1999) and E. B. Dagum, *The X-11ARIMA/88 Seasonal Adjustment Method – Foundations And User's Manual* (Time Series Research and Analysis Division Statistics Canada, Ottawa, Canada, 1988).

[4] J. P. Burman, "Seasonal Adjustment by Signal Extraction," *Journal of the Royal Statistical Society*, 1980, series A, vol. 143, pp. 321–37 and S. C. Hillmer, and G. C. Tiao, "An ARIMA-Model-Based Approach to Seasonal Adjustment," *Journal of the American Statistical Association*, 1982, vol. 77, pp. 63–70.

[5] R. K., Jain, "A State Space Modeling Approach to the Seasonal Adjustment of the Consumer Price and other BLS Indexes: Some Empirical Results," BLS Working Paper, no. 229 (Bureau of Labor Statistics, 1992) and "Structural Model-Based Seasonal Adjustment of the Bureau of Labor Statistics Series," BLS Working Paper no. 236 (Bureau of Labor Statistics, 1992).

[6] Similar structural models, using Kalman Filtering and Smoothing for estimation, are currently being used by the Bureau of Labor Statistics to estimate State and Local Area Employment and Unemployment. For details, see R. Tiller, S. Brown, and A. Tupek, "Bureau of Labor Statistics' State and Local Area Estimates of Employment and Unemployment," ch. 5, in *Indirect Estimators in U.S. Federal Programs,* W.L. Schaible ed. (Springer, 1996). Also see R. K. Jain, "Comparative Performance of State Space Model Based and ARIMA Model Based Methods of Seasonal Adjustment," *1995 Proceedings of the Business and Economic Statistics Section-American Statistical Association*; A. C. Harvey, "A Unified View of Statistical Forecasting Procedures," *Journal of Forecasting*, 1984, vol. 3**,** pp. 245–75; A. C. Harvey, *Forecasting, Structural Time Series Models and the Kalman Filter* (Cambridge, MA, Cambridge University Press, 1990); and G. Kitagawa, and W. Gersch "A Smoothness Priors-State Space Modeling of Time Series with Trend and Seasonality," *Journal of American Statistical Association*, 1984, vol. 79, pp. 378–89.

[7] A. C. Harvey, "Trends and Cycles in Macroeconomic Time Series," *Journal of Business and Economic Statistics*, 1985, vol. 3, 216–27.

[8] In this structural model, no explanatory variable is used because we did not need one. In modeling some other time series, however, we can introduce observable economic variables as well as lag-dependent variables as independent variables to increase the explanatory power of the structural models.

[9] Harvey, *Forecasting, Structural Time Series Models,* 1990.

[10] See Harvey, *Forecasting, Structural Time Series Models,* 1990 and Kitagawa, and Gersch, "A Smoothness Priors-State Space Modeling of Time Series," *Journal of American Statistical Association*.

[11] Hyper-parameters are the variances of the errors of the component models.

[12] Expectation Maximization (EM) is an alternative nonlinear optimization algorithm, which was developed by A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," Journal of the *Royal Statistical Society,* 1977, Series B., vol. 39, pp. 1–38.

[13] R. H. Shumway, and D. S. Stoffer, "An Approach to Time Series Smoothing and Forecasting Using the EM Algorithm," *Journal of Time Series Analysis*, 1982, vol. 3, pp. 253–64.

[14] Quasi-Newton methods are numerical optimization methods in which, unlike that in Newton's method, the use of second derivatives in the approximation of the likelihood function, are altogether eliminated. These methods have excellent convergence properties even for ill-behaved functions.

[15] Prediction Error Decomposition is a fundamental result in time series. By using it, the joint density of observations can be written down in such a way that full maximum likelihood estimation of many complex series can be done easily. For details see Harvey, *Forecasting, Structural Time Series Models*, 1990, pp. 125–27.

[16] Harvey, *Forecasting, Structural Time Series Models,* 1990.

[17] W. A. Brock, W. D. Dechert, and J. Scheinkman, "A Test for Independence Based on the Correlation Dimension," *Econometric Review,* 1996, vol. 15, no.3, pp. 197–235.

[18] BDS and MBDS tests are specification tests applied to the residuals of linear or nonlinear models. The maintained hypothesis of these tests is that the true residuals are independent and independently distributed. These tests in the evaluation of structural models are used to test the adequacy of the models. See B. Mizrack, "A Simple Non parametric Test for Independence of Order (P)," Working Paper No. 1995-23 (New Jersey, Rutgers University, 1995).

[19] Akaike Information Criterion (AIC) is a model selection criterion. The model with the lowest AIC is presumed to be the best or optimal model from among the models analyzed. It is defined as: AIC = –2 (log likelihood of a model) + 2 (number of independent parameters estimated in the model). A model with large number of parameters is less likely to be chosen as the optimal model.

[20] Harvey, *Forecasting, Structural Time Series Models,* 1990.

[21] Dagum, *The X-11ARIMA/88 Seasonal Adjustment Method,* 1988.

[22] The number of trading days in a month varies from month to month. The total sales of a product in a month are therefore affected by this phenomenon. To correctly estimate various components of a time the effect of the number of trading days in a month has to be separated. The approach to doing that is called trading day adjustment.

[23] The Easter holiday falls at different dates each year any day between March 22 to April 22. Because sales of many consumer goods go up around Easter, the effect of this phenomenon on a time series has

also to be separated like that of number of trading days. The approach to doing that is called Easter day adjustment.

[24] J. A. Hausman, and M. W. Watson, "Errors in variables and seasonal adjustment procedures" *Journal of the American Statistical Association,* 1985, vol. 80, 541–52.

[25] An example of such a series is the teenage unemployment rate published by BLS. See R. K. Jain, "Measurement Errors and State Space Model Based Method of Seasonal Adjustment," *Proceedings of the Business and Economic Statistics Section-American Statistical Association,* 1998).

[26] See R. K. Jain, "A State Space Modeling Approach to the Seasonal Adjustment," 1992; "Structural Model-Based Seasonal Adjustment of the Bureau of Labor Statistics Series," BLS Working Paper, 1992; "Comparative Performance of State Space Model Based and ARIMA Model Based Methods of Seasonal Adjustment," 1995; and "Automatic Outlier Detection in Seasonal Adjustment Methods," *1997 Proceedings of the Business and Economic Statistics Section-American Statistical Association,* 1997.

[27] See R. K. Jain, "A State Space Model Based Approach to Intervention Analysis in the Seasonal Adjustment of the BLS Series: Some Empirical Results" BLS Working Paper no. 228, 1992 and "Structural Model-Based Seasonal Adjustment," BLS Working Paper, no. 229; and R. K. Jain, "The Seasonal Adjustment of the Consumer Price Indexes of Women's Apparel: An Application of State Space Model Based Approach to Intervention Analysis," *1993 Proceedings of the Business and Economic Statistics Section-American Statistical Association, 1993.*

[28] R. K. Jain, "Trading Day and Easter Adjustment in Seasonal Adjustment Methods," *1996 Proceedings of the Business and Economic Statistics Section-American Statistical Association* (Alexandria, VA, 1996).

[29] See R. K. Jain, "Measurement Errors and State Space Model Based Method of Seasonal Adjustment," *1998 Proceedings of the Business and Economic Statistics Section-American Statistical Association* (Alexandria, VA, 1998).

[30] Apple is an item-stratum (SEFK01) in the consumer price classification structure. It is part of "Fresh Fruits," which is a larger expenditure category. Although the CPI for apple is directly seasonally adjusted, this item stratum only indirectly enters the All-Item CPI via the aggregate category.

[31] The structural model in equations (1) through (5) is the best model in the sense that the Akaike Information Criterion (AIC) for this model was less than other structural models estimated for analysis. About six different structural models were used for comparison.

[32] For details see Harvey, *Forecasting, Structural Time Series Models,*1990, pp. 268–69.

[33] *m*7 is a statistic developed in the X-11ARIMA method. It is a function of the *F*-statistics for the stable seasonality and moving seasonality. If *m*7 lies between 0 and 1, then the two kinds of seasonality are identifiable. The experience of the author with this statistic is that almost every time *m*7 lies within the acceptable bounds for a model, that model turns out to be an acceptable model for that series. See Dagum, *The X-11ARIMA/88 Seasonal Adjustment Method,* 1988.

[34] R. K. Jain, "Structural Model-Based Seasonal Adjustment of the Bureau of Labor Statistics Series," Working Paper, 229; "The Seasonal Adjustment of the Consumer Price Indexes of Women's Apparel: *American Statistical Association*; and "Trading Day and Easter Adjustment in Seasonal Adjustment Methods," *American Statistical Association.*