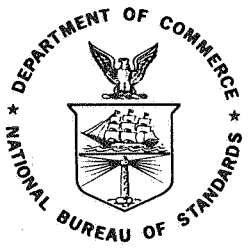


NBS MONOGRAPH 163

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Measurement Assurance for Gage Blocks

Measurement Assurance for Gage Blocks

Carroll Croarkin

Statistical Engineering Division
Center for Applied Mathematics
National Engineering Laboratory
National Bureau of Standards
Washington, DC 20234

John Beers and Clyde Tucker

Mechanical Processes Division
Center for Mechanical Engineering and
Process Technology
National Engineering Laboratory
National Bureau of Standards
Washington, DC 20234

With an Introduction by
J. M. Cameron
Retired
National Bureau of Standards

U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, Secretary
Jordan J. Baruch, Assistant Secretary for Science and Technology
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

Issued February 1979

Library of Congress Catalog Card Number 78—600162

National Bureau of Standards Monograph 163
Nat. Bur. Stand. (U.S.), Mono. 163, 78 pages (Feb. 1979)
CODEN: NBSMA6

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 1979

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402
Stock No. 003-003-02023-1 Price $2.50
(Add 25 percent additional for other than U.S. mailing)

TABLE OF CONTENTS

MEASUREMENT ASSURANCE FOR GAGE BLOCKS

by

M. Carroll Croarkin

and

John Beers and Clyde Tucker

With an Introduction by
J. M. Cameron

This monograph is intended for those who need to know on a continuing basis the uncertainty of their gage block calibration procedure. A general discussion of the philosophy of measurement assurance is given first. Then three levels of measurement assurance programs are outlined showing how control over the measurement process can be maintained and how the offset (or systematic error) from the unit of length maintained by the National Bureau of Standards can be made negligible.

Key Words: Calibration; gage blocks; length; measurement assurance; measurement process control; systematic error; uncertainty

1. What is a Measurement Assurance Program?

(An Introduction by J. M. Cameron)

1.1 Statement of the Problem

Measurement assurance is the name given to the procedures by which one ascertains that individual measurements are "good enough" for their intended purpose. Our focus has to be on individual measurements because a single measurement can be the basis for actions taken to maintain our health, safety or the quality of our environment. It is important that the errors of measurement be small enough so that the actions taken are only negligibly affected by these errors. We realize this necessity on a personal basis when we consider medical measurements, or our exposure to radioactivity. In any government regulatory action or measurement involved in legal actions it is also obvious that the shadow of doubt surrounding the measurements should be suitably small. But this is no less true for all other measurements in science and industry, and even though legal action may

not be involved, the validity of scientific inference, the effectiveness of process control, or the quality of production may depend on adequate measurements.

The measurement assurance programs described in this monograph relate specifically to measurement processes in which the length of a gage block is assigned relative to one or more reference standards. Calibration of customer's gage blocks by the National Bureau of Standards is an example of such a process as are calibrations done by other laboratories for the public or for production quality control within their own organization. The objective of these measurement assurance programs is to demonstrate on a continuing basis that the uncertainty of each measurement is suitably small relative to its end use.

The term "good enough" needs to be given operational meaning and this can only be done after one has a framework for describing the quality of measurements. To do this we will need to know

- the allowable limits of measurement error
- the reference base to which the measurements must be related
- the properties of the measurement process (extent of random variation, possible offset from the reference base)
- a means for assigning uncertainty to a measurement.

1.1.1 Allowable Limits of Measurement Error

How does one achieve the condition that the measurements are "good enough" for their intended use? It would seem obvious that one has to start with the need--i.e., deciding upon what is "good enough". There are a number of cases where the limits to be used arise quite naturally. For example, physiological restraints provide the definition for the allowable error in exposure to cobalt radiation in cancer treatment or in the amount of pollutant entering a lake. In nuclear materials control the allowable error is a function of the amount of material which would pose a hazard if diverted. In industrial production or commercial transactions, the error limit is determined by a balance between the cost of better measurement and the possible economic loss from poorer measurement.

In gage block calibration, the limits are often given by an announced "accuracy" claim or result from an hierarchal approach in which smaller accuracies are required of "higher level" laboratories. Ideally the limits would be based on actual need in production or quality control.

By whatever path such requirements are arrived at, let us begin with the assumption that the allowable error should not be outside the interval (-a, +b) relative to the quantity being measured. Our

problem is one of deciding whether the uncertainty of a single measurement is wholly contained in an interval of that size.

1.1.2 Reference Base to Which Measurements Must Be Related

It is instructive to contemplate what might happen if a measurement were to become an important element in a legal controversy. Two essential features should arise. First, that the contending parties would have to agree on what (actually realizable) measurement would be mutually acceptable. The logic of this seems unassailable--if one cannot state what measurement system would be accepted as "correct," then one would have no defensible way of developing specifications or regulations involving such measurements. Second, the uncertainty to be attached to the measurement would be established by a form of "cross-examination" by which one would determine the "shadow of doubt" relative to this acceptable value.

The consensus or generally accepted value can be given a particularly simple meaning in dealing with measurements of such quantities as length, mass, voltage, resistance, temperature, etc. One may require that uncertainties be expressed relative to the standards as maintained by a local laboratory or, when appropriate, to the national standards as maintained by NBS. In other cases, nationally accepted artifacts, standard reference materials or in some cases a particular measurement process may constitute a reference base. One basic quality of all these examples should not be overlooked--all are operationally realizable. The confusion engendered by introducing the term "true value" as the correct but unknowable value is thus avoided.

1.2 The Measurement Process

1.2.1 Properties of Measurement Processes

In discussing uncertainty, we must account for two characteristics of measurement processes. First, repeated measurements of the same quantity by the same measurement process will disagree and, second, the limiting means of measurements by two different processes will disagree. These facts lead to a perspective from which to view measurement, namely that the measurement be regarded as the "output" of a process analogous to an industrial production process. In defining the process, one must state the conditions under which a "repetition" of the measurement would be made, analogous to defining the conditions of manufacture in an industrial process.

The need for this specification of the process becomes clear if one envisions the "cross-examination" process. One would begin with such questions as

> Within what limits would an additional measurement by the same instrument agree?

Would the agreement be poorer if the time interval between repetitions were increased?

If two or more types (or manufacturers) of comparators were used, how much disagreement would be expected?

To these can be added questions related to the conduct of the measurement.

What about environmental conditions--temperature, moisture, etc.?

Is the result dependent on the procedure used?

Do different operators show persistent differences in values?

Are there instrumental biases or differences due to reference standards or calibrations?

The questions force one to define the measurement process--the process whose "output" we seek to characterize. Once the measurement method is agreed upon and set into operation, one then has the problem of sampling the output of the measuring process so as to be able to make statements about the health of the process relative to the needs. The needed redundancy can sometimes be achieved by remeasuring some of the items, or by measuring a reference artifact periodically. It is essential that the repetitions be done under the same diversity of conditions as the regular measurements, and that the items being measured be typical of the regular workload.

1.2.2 The Measurement Method

The current understanding of a scientific or industrial process or of a measurement process is embodied in a physical model which explains the interactions of various factors, corrections for environmental or other effects, and the probability models necessary to account for the fact that repetitions of the same event give rise to nonidentical answers.

One thus begins with the specification of a measurement method--the detailed description of apparatus, procedures and conditions by which one will measure some quantity. Once the apparatus is assembled and checked out, a measurement process exists whose output can be studied to see if it conforms to the requirement for which it was created.

1.2.3 Random Variation

One is accustomed to random variation as it occurs in industrial production in an attempt to produce identical items. In measurement,

each item to be measured is different and unless some redundancy is built in there will be no repetitions from which to disentangle the random error from differences between items.

A number of methods can be used to achieve the needed redundancy.

- Some measurements on test items could be repeated after a few minutes.
- Some measurements on test items could be repeated after one or more days.
- A reference object could be remeasured periodically.
- A check standard could be measured in parallel with the test item.

The crucial step in assessing the effects of random error is defining the set of repetitions over which the measurement is to apply. In the context of legal proceedings, one arrives at the degree of credibility of evidence by questions designed to find out how far the statement could be in error. In measurement, the uncertainty is arrived at by determining the amount of disagreement expected in the set of repetitions that would be appropriate for the intended use of the measurement. If repetitions are always done within a few minutes of each other, one would expect much better agreement than if they were run days or weeks apart. The short term variability is appropriate for process control purposes, but the uncertainty attached to the value for a test item must allow the day-to-day variation to be responsive to such questions as "Within what limits would one expect the value to lie if the item were re-submitted at a later date?"

1.2.4 The Concept of Measurement Repetition

Every measurement has a set of conditions in which it is presumed to be valid. At a very minimum, it is the set of repeated measurements with the same instrument-operator-procedure configuration. (This is the type of repetition expected in some process control operations.) If the measurement is to be interchangeable with one made at another location, the repetition would involve different instrument-operator-procedure-environment configurations. (This type of repetition occurs in producing items to satisfy a specification and in manufacturing generally.) In gage block calibration, the "repetition" involves at least the level of variability which would be encountered if the value were determined at intervals of one week or longer.

To evaluate a measurement process some redundancy needs to be built into the system to determine the process parameters. This redundancy should be representative of the set of repetitions to which the uncertainty statement is to apply. In the NBS gage block measurement program, a check standard is measured along with the unknowns submitted for calibration. One thus generates a sequence of measurements of the same object covering an extended time period.

From these results one can answer questions about the agreement expected in a recalibration and the operating characteristics of the measurement process. In this simple case the check standard is treated exactly the same way as the unknowns so that the properties of the process related to it are transferrable to the unknown.

1.2.5 Building Redundancy Into the System

The essential characteristic needed to establish the validity of measurement is predictability of the process, i.e., that the variability remains at the same level and that the process is not drifting or shifting abruptly from its established values. The evidence of predictability must come from redundant measurement of "control" or reference blocks of known value which have properties similar to those of the regular workload in order to verify this condition.

In measuring an "unknown" one gets a single value, but one still is faced with the need to make a statement that allows for the anticipated scatter of the results. If we had a sufficiently long record of measurements, we could predict the limits within which we were fairly certain that the next measurement would lie. Such a statement should be based on a collection of independent determinations, each one similar in character to the new observation, that is to say, so that each observation of the collection and also the new observation can be considered as random drawings from the same probability distribution. These conditions will be satisfied if the collection of points is from a sufficiently broad set of environmental and operating conditions to allow all the random effects to which the process is subject to have a chance to exert their influence on the variability. Suitable data collections can be obtained by incorporating an appropriate reference measurement into routine measurement procedures, provided such measurements are representative of the same variability to which the "unknown" is subject. The statistical procedures for expressing the results will depend on the structure of the data but they cannot overcome deficiencies in the representativeness of the values used.

Results from the control item provide the basis for determining the parameters of the measurement process and verifying that the properties are transferable to measurements on test items. One is saying, in effect, if we could have measured the "unknown" again and again, a sequence of values such as those for the control item would have been obtained. Whether our single value is above or below the mean we cannot say, but we are fairly certain it would not differ by more than the bounds to the scatter of the values on the control item.

The bound $\pm R$, to be used for the possible effect of random errors may be as simple as ± 3 standard deviations or may involve the combination of many components of variance. Once the set of repetitions over

which one's conclusions must apply is defined, the structure of the random error bound can be determined.

The question of how much redundancy is enough is difficult to answer. The observer could measure a "control" block after each 10 test blocks, he could measure every block against two standards or he could do an "experiment" (as NBS does) in which both a control is measured and a check on process variability is also made. Each of these approaches have computable operating characteristics relative to various forms which lack of process control could take--all give a high degree of protection against large changes in comparator scaling whereas a single "bad" value (e.g., due to a burr) on a test block would go unnoticed. If one knew which departures from ideal conditions were most likely to occur, he could design a procedure to protect against such occurrances.

1.2.6 Possible Offset of the Process

Once one has established that his measurement process is "in control" from the point of view of random variation, there remains the question of the possible offset of the process relative to other processes. It is not helpful to speak of the offset from a "true value" which exists only in the mathematical or physical model of the process. Considering measurement in the context of legal proceedings helps clear away some of the classical confusion about errors of measurement. In a legal or regulatory setting, one is forced to state what would be accepted as a correct answer such as one which agrees in a comparison (by a prescribed process) with national standards or with results from a designated laboratory or consensus of many laboratories.

The idea of defining uncertainty as the extent to which a measurement is in doubt relative to a standard or process defined as correct finds expression in the recent Nuclear Regulatory Commission statement [14]:

> 70.57(a) "Traceability" means the ability to relate *individual measurement results* to national standards or nationally accepted measurement systems ... (italics added)

In gage block measurement, the use of standards whose uncertainty is given relative to the length standards maintained by NBS eliminates the considerable effort required to document the uncertainty of measurements using interferometry alone. One still is confronted with the problem of setting bounds to the possible offset due to factors such as:

> Errors in the starting standards
>
> Departures from sought-after instrumentation (e.g., geometrical discrepancies)

Errors in procedures, environment, etc.

and other effects which are persistent. From properly designed experiments one can arrive at a limit to the possible extent of errors from these sources in answer to the question, "If the process were set up *ab initio*, how large a difference in the limiting means of the two processes would be reasonable?"

A bound to the possible offset from many of the important factors can be determined as part of a regular measurement process by running some of the controls under different conditions, operators, instruments and analyzing the results to see if significant differences appear.

From these measurements, one will have a set of bounds E_1, E_2, E_3, ... to the possible offset (systematic error) from the various factors. The question as to how to combine these into a single bound to the possible offset depends on knowledge of the joint effects of two or more factors and on the physical model assumed for the process. For example, if the bounds E_1 and E_2 arise from independent random error bounds, then it would be appropriate to combine them in quadrature, i.e., $\sqrt{E_1^2 + E_2^2}$. An error in the model (e.g., assumed linearity even when nonlinearity exists) would act as an additive error. The properties of proposed combination rules can be evaluated and a selection made of the most appropriate. The result will be an overall value, E, for the possible offset for the limiting mean of the process from that of the nationally accepted process.

1.2.7 Uncertainty

What can one say about the uncertainty of a measurement made by a process that may be offset from the nationally accepted process by some amount $\pm E$, and is subject to random errors bounded by $\pm R$? How should these values be combined? To begin with, one could raise the question, "If the random error could be made negligible, what uncertainty would one attach to a value from the process?" Clearly the answer is $\pm E$. The next question, "If, in addition, a random error of size R is possible, what do we now say about the uncertainty?" The answer seems obvious--E and R are added to give an uncertainty of $\pm[E + R]$.

But what if E were itself the result of only random errors? The answer depends on what one calls a repetition. By the way E is defined, it is the bound for the systematic offset of the process and although it may be arrived at from consideration of random errors, the factor involved keeps the same (unknown) value throughout. Our ignorance does not make it a random variable in our process.

The uncertainty of a measurement--the width of its "shadow of doubt" in a legal proceeding--must therefore be the sum of the random error and systematic error limits.

1.2.8 Measurement Process Control

The essential requirement for the validity of the uncertainty statement is that the process remain in a state of statistical control. Once an out-of-control condition occurs, one has lost predictability and the previous uncertainty statements are no longer valid.* The following sections of this monograph present techniques for monitoring the gage block measurement processes to assure that the process parameters have not changed. But one has to verify more than just those parameters related to random variations and possible offset from national values. One needs to build in tests of the adequacy of the physical model by a variety of tests on the process (e.g., by repeating measurements under different conditions to verify the adequacy of the corrections for such changes) as well as periodic redetermination of the bounds for systematic error. One thus tests that the assumed model is still acceptable and that the parameters assigned to that model have not changed.

When measurement requirements are stated in terms of system needs (number of correctly matching parts, number of correctly measured dosimeters, etc.), one can measure success of the measurement effort in terms of closeness to meeting those goals. Measurement efficiency is thus judged in terms of the output of the organization rather than by counting the number of significant digits. Also, one needs this measure of performance of the measurement effort to be able to identify those areas which need improvement. In gage block calibration, one ordinarily does not see the use to which the blocks are subjected and has only the requested uncertainty as a goal.

All measurements have some form of measurement assurance program associated with them although, as with quality control, we usually reserve the term for a formal program. In a formal program one treats the whole process--beginning with a study of the need, the development of a measuring process and a procedure for determining and monitoring its performance, and an evaluation of the effectiveness of the whole effort. A criterion of success is needed to determine whether more effort should be put into one's current measurement activity or whether perhaps some alternative would contribute more to the overall program. An appropriate criterion is not necessarily provided by the smallness of the uncertainty for a measurement.

*The practice of using the values of the statistical parameters from a single set of data in an uncertainty statement (e.g., confidence intervals based on the Student-t distribution) is not defendable unless there is a reasonable amount of evidence that the observed values can be regarded as random variables from the assumed stable probability distribution. The apparent exactness of these methods tends to obscure the fact that their validity is critically dependent on the need for randomness and independence of the measurements, qualities not easily demonstrated for a single isolated set of measurements.

2. Who Should Participate in an NBS-Sponsored Gage Block Measurement Assurance Program?

Measurement assurance allows participating laboratories who maintain a continuous and documented check on their calibration process to relate their proccess to the national unit of length maintained by NBS.

Laboratories who will benefit from this NBS service fall into two groups:

(1) Industrial laboratories who must prove the dimensional precision of their manufacturing process and its relationship to the length unit

(2) Laboratories who perform calibrations for customers and who are asked to prove that they provide an acceptable level of calibration accuracy.

It is possible that, where calibrations for customers are provided, two calibration levels could be offered; one employing the widely used "size check" (usually a single comparison which is adequate for many purposes) and the other employing the more involved procedures described in this monograph with documented uncertainty statements.

Many laboratories do not need direct intercomparison with NBS as part of their measurement assurance program either because their accuracy requirements are not stringent or because only "in-house" consistency is required. Instead, a modest program of intercomparisons involving their reference set and a working set will give evidence of control. See section 3.2 for a method of accomplishing this goal.

In making a decision about participating in the NBS program, the potential benefits should be weighed against the investment of time and effort necessary to make it effective.

3. Procedures for Gage Block Measurement Assurance

3.1 General

3.1.1 Choosing an Appropriate Program

Three different levels (options) of measurement assurance are described in the following sections. All three provide a format for the calibration of gage blocks and "controls" to assure the continued validity of uncertainty statements.

Briefly the three options are as follows: Option 1 (Section 3.2) describes the simplest technique in which a single measurement is made on each block, and an occasional control block is introduced into the calibration process. Option 2 (Section 3.3) describes a procedure in which duplicate measurements are made on each block by comparing each block to two standard blocks. Option 3 (Section 3.4) describes a program for calibrating two test blocks against two standard blocks by a drift eliminating design. The three options contain many common elements, and the sections are written to be self-complete.

The choice of a program for a particular laboratory depends on a number of factors including (1) the availability of gage block sets, equipment, and trained personnel; (2) the availability of time to spend on the program, and (3) the accuracy requirements for the calibrations. Options 1, 2, and 3 require progressively greater investments of time and effort, and generally yield progressively better process control. It may be advantageous to start with option 1 and upgrade to option 2 or 3 if necessitated by inadequate results. It is important in selecting the appropriate level to keep in mind that the program must become a regular part of the calibration procedure.

3.1.2 Methodology

The basic method is to incorporate the measurements of the "controls" into the regular working routine of the laboratory on a continuing basis so that the properties of the measurement process which are ascribable to this set of blocks can be attributed to the entire calibration procedure.

Included in each option are procedures for (1) establishing process parameters; (2) routine monitoring to assure process control; (3) updating process parameters; and (4) maintaining the tie of the process to national standards. A worksheet is provided for each of the foregoing procedures showing the necessary calculations and statistical tests.

Instead of a detailed explanation of the methods of process control, the worksheets are intended to guide the user through the various

procedures. Sample data and calculations are given in such a way that one can follow the example through the four steps outlined above that are the essence of a measurement assurance program.

3.1.3 Statistical Surveillance

Continuous monitoring of the process is necessary to insure that predictions based on the accepted values of the process parameters are valid and that the process remains in a state of control. Statistical surveillance is maintained on the accepted values for the controls and the associated random error components by the following means. After each calibration run, the observed value of the control block(s) is checked against its accepted value by comparing a test statistic, "t" to the critical value 3.0. This test corresponds to the .003 probability level for a normal distribution; i.e., assuming the observations come from a normal distribution with known variance. For observations from a normal distribution with unknown variance, the test statistic is distributed as Student's t. Similarly the random error components are checked against their accepted values using a test statistic based on the F distribution. Critical values of F which correspond to the .01 probability level are given in Table I (page 63). See reference [9] for a discussion of the applications of the t and F distributions in metrology.

If the criteria for both of these tests are satisfied, the process is regarded as being in control and the calibrated values for the unknown blocks and associated uncertainties are accepted as valid. Otherwise, some remedial action is indicated. Usually the calibration is repeated before more extensive steps are taken, but this is discussed in the appropriate sections.

3.1.4 Special Procedures for Interferometry and Long Gage Blocks (Over 4 Inches)

Procedures described in this monograph apply primarily to calibrations done with conventional electro-mechanical gage block comparators. Fringe counting interferometric gage block comparators are a special case because they can be used either as comparators in conjunction with standard blocks or they can be used to measure test blocks directly without reference to standard blocks. When one of these instruments is used as a comparator, all procedures in the body of this monograph apply. This is true even where only a few standard blocks are used to cover the full size range and the length differences between standard and test blocks are consequently large. When fringe counting interferometry is used to measure test blocks directly, procedures are somewhat different as described in Appendix B. The Appendix B procedures are also valid for static interferometry where test blocks are wrung to optical flats and measured in a Kosters type or a Fizeau type gage block interferometer.

The procedures outlined are intended for use primarily on short blocks (blocks up to 4 inches). The suggestions for measurement procedures, handling techniques and environmental controls which are covered in Section 4 deal with both short and long blocks. However, because long blocks are sometimes subject to rapid secular changes, the statistical analyses may have to be modified to allow for this condition. See reference [10] for a discussion of analysis of long block data.

3.2 One Set of Standards, Single Measurements on Unknowns, One Set of Control Blocks

3.2.1 The Measurement Process

In the simplest and perhaps most common procedure for gage block calibration the value for an unknown is assigned by measuring the difference in length between a standard block and the unknown using a comparator (which may be either a mechanical comparator or a fringe counting interferometric comparator). In some processes duplicate measurements are made as a check on gross errors. Unless the second set of measurements are separated far enough in time to be statistically independent, the differences should not be used in setting bounds for the process random error.

The random errors associated with the process are of two kinds; those arising from repetitions in the short term (a few minutes) and those involving long term differences (day-to-day, week-to-week, etc.) The random error appropriate for the calibration process is that associated with the repeated measurement of a control block over a sufficiently long time period to insure that all factors affecting variability have a chance to exert their influence.

Because of temperature effects, the amount of variation in the measurement process is usually length dependent. For this reason and for convenience of working group size, the usual set of blocks (80 or so blocks in the 0.050 to 4 in. range) should be treated as a number of subsets of up to 20 blocks each. At NBS the six groups listed below are used:

Group	Nominal Length (Inches)	Approximate No. of Blocks
I	0.050 to 0.09375	4
II	0.100 to 0.107	20
III	0.108 to 0.126	20
IV	0.127 to 0.146	20
V	0.147 to 0.500	15
VI	0.550 to 4.000	13

Redundancy is introduced into the system by repeatedly including the control block in the measurement procedure. The control blocks should

be treated exactly as test blocks measuring them in proper size sequence along with the test set. The control set should be made up of at least one block from each group with more blocks of the larger sizes. A suggested set of control blocks is:

Group	Nominal Size (Inches)	No. of Control Blocks K
I	0.05	1
II	0.1	1
III	0.125	1
IV	0.140	1
V	0.25, 0.50	2
VI	0.75, 1.0, 2.0, 4.0	4

One would expect the same variability for all control blocks in a group so that the standard deviations* computed for each of the control blocks in a group could be combined into one overall standard deviation for the group. If $s_1, \ldots s_k$ are the standard deviations for the k blocks in a group with degrees of freedom $\nu_1, \ldots \nu_k$ respectively, then

$$\text{s.d. (group)} = \sqrt{\frac{\nu_1 s_1^2 + \ldots + \nu_k s_k^2}{\nu_1 + \ldots + \nu_k}}$$

3.2.2 Establishing Process Parameters

To determine initial accepted values** for the controls and for the random error component, all the blocks in the control set should be measured by the usual process, say 6 times, with a few days between repetitions.

*The standard deviation of a single observation is given by

$$s = \sqrt{\frac{\Sigma\, r_i^2}{n-1}} \qquad \text{where}$$

the r_i is the difference between each observation and the average of n observations. The quantity n-1 is called the degrees of freedom associated with s.

**The term "value" for a gage block means the deviation from nominal size (measured length minus nominal length) at 20° C.

From that initial data, an average and a standard deviation should be computed for each control block. The accepted values for the control and for the random error component will be the average value for the block and the standard deviation for the group as indicated above. These will be the starting accepted values for the process. A portion of a typical worksheet showing starting values for two blocks in a control set is given in Figure 1. Examples given in the following sections are based on these two blocks and their parameters. All data in the examples is hypothetical and is intended only to explain the methodology.

3.2.3 Maintaining Process Control

After the process parameters have been established, control is maintained by checking the control value after each calibration run against the accepted control value.

A sample worksheet (see Figure 2) details how this is done and shows the appropriate statistical test for determining whether or not the process is in control.

3.2.4 Updating Process Parameters

As information collects on the regularly used control blocks, the accepted values for these blocks should be updated. If either of the process parameters has changed from its starting value as indicated by the t-test or F-test, a new process parameter must be computed based only on the current data. Otherwise the process will be considered to be continuous, and the data will be combined. A sample worksheet showing how this is done is given in Figure 3.

The frequency of updating will depend on the workload but should be done initially after 5 or 10 values and then at intervals dictated by convenience (e.g., every 6 months or a year).

3.2.5 Relationship to National Standards

It is important to know if a process produces values which are consistent with the national standards as maintained by NBS. A convenient method for testing the whole system is afforded by using two calibrated sets loaned to the laboratory by NBS. If each of these sets is measured as a test set in the calibration procedure, one will have the independent check needed to test for consistency with NBS. The data should be taken by the regular calibration process. Two complete calibrations should be done a day or two apart.

Laboratories with the capability of writing their own documented test report based on their measurements of the NBS sets are encouraged to do so. In this case NBS will provide the two calibrated sets of blocks, their current assigned values and associated uncertainties.

The table in Figure 4 summarizes the analysis of the data and shows whether or not there is a significant offset from the NBS process. If the offset is significant, the values for the laboratory reference standards and their associated uncertainties should be corrected as shown in the suggested "Report of Test" form in Figure 5. In effect this is a method for each laboratory to calibrate its own standards.

3.2.6 Uncertainty

The uncertainty, $\pm U$, associated with any test block calibrated by this process, assuming the process is in control, is given by

$$U = E + 3s_G$$

where E is the assigned uncertainty of the standard block, and s_G is the accepted group standard deviation for that block size (see Figure 4). This assumes that the standard blocks have documented uncertainties e.g., if they were calibrated at NBS . (For convenience one may use the maximum uncertainty found in the group so as to report a single uncertainty value for the whole group.)

3.2.7 Summary

In sections 3.2.3 and 3.2.5 tests are described which determine if the process is in control. The actions which can be taken when one of these tests indicates an out-of-control condition are described in section 4.6. In the absence of a specific format for out-of-control conditions, it should be noted that once predictability is lost, no statements should be made about the condition of the process. Until the process parameters have been re-established or some satisfactory corrective action has been taken, any uncertainty statements which are issued should be designated as provisional.

FIGURE 1--WORKSHEET 3.2.2

PROCESS PARAMETERS: ACCEPTED VALUES OF CONTROLS AND STANDARD DEVIATIONS

Values in Microinches at 20 °C

	Nominal Size (inches)	Average Value of Control	No. of Repetitions	Standard Deviation	Group* Standard Deviation
		L_c	n	s	s_G
Group II	0.10000	16.7	6	1.34	1.34
Group V	0.150	16.2	6	1.75	1.75

* $s_G = \sqrt{\frac{\sum_{i=1}^{k} s_i^2}{k}}$ k = no. of control blocks in the group (in this case k = 1).

The degrees of freedom associated with s_G are k(n-1).

FIGURE 2--WORKSHEET 3.2.3

OBSERVED VALUE OF CONTROL COMPARED TO ACCEPTED VALUE OF CONTROL

Values in Microinches at 20 °C

	Nominal Size (Inches)	Ident.	Comparator Readings Test	Standard	Diff. X-S	Value of Std.	Value of Block V+d
			X	S	d	V	L
Group II	0.10000	Test	19.0	17.0	2.5**	17.5	20.0
	0.10000	Control	17.0	16.0	0.5**	17.5	18.0
	0.10010	Test	13.5	10.5	3.0	11.0	14.0
	0.10020	Test	18.8	17.0	1.8	16.6	18.4
Group V	0.147	Test	20.0	18.5	1.5	18.3	19.8
	0.148	Test	19.5	17.8	1.7	18.2	19.9
	0.150	Test	19.5	18.0	0.9**	17.0	17.9
	0.150	Control	20.2	19.2	1.6**	17.0	18.6
	0.200	Test	20.0	18.3	1.7	19.5	21.2

*If $t \geq 3$, process is out of control for that group. Repeat entire group.

**Use average of S_{Test} and $S_{Control}$, i.e., $d = X - 1/2 (S_{Test} + S_{Control})$.

Values of V and E are assigned values (e.g., from NBS calibration). All other values are from participant's process.

FIGURE 2--continued

Accepted Value of Control	Accepted Group S.D.	t Test*	Uncertainty of Std.	Limit to Random Error	Uncertainty of Test Block
		$\lvert L-L_c \rvert / s_G$		$3s_G$	E + R
L_c	s_G	t	E	R	U
16.7	1.34	1.0	1.8	4.0	5.8
			1.8	4.0	5.8
			1.8	4.0	5.8
			2.3	5.2	7.5
			2.3	5.2	7.5
16.2	1.75	1.4 ~~1.7~~	2.3	5.2	7.5
			2.3	5.2	7.5

FIGURE 3--WORKSHEET 3.2.4

ACCEPTED VALUES OF PROCESS PARAMETERS COMPARED TO NEW VALUES FOR PROCESS PARAMETERS, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size	Starting Values			New Values		
		Control	Number	Group S.D.	Control	Number	Group S.D.
	(Inches)						
		L_c	n_1	s_G	L_c'	n_2	s_G'
Group II k=1	0.10000	16.7	6		15.2	12	
	Combined			1.34			2.12
	.						
	.						
Group V k=1	0.150	16.2	6		18.3	12	
	Combined			1.75			1.59

*If $t \geq 3$, the new value L_c' should replace L_c for the control; otherwise, use the combined value $\bar{L}$.

**If $F \geq F_{.01}$ for γ_1 and γ_2 degrees of freedom, the new value of s_G' should replace s_G as the group standard deviation; otherwise, use the combined standard deviation. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = k(n_2-1)$ and $\gamma_2 = k(n_1-1)$.

FIGURE 3--continued

t Test*	Combined Value Control	F Test**	Combined S.D.	Degrees of Freedom
$\frac{\lvert L_c - L'_c \rvert}{s_G\sqrt{1/n_1+1/n_2}}$	$\frac{n_1 L_c + n_2 L'_c}{n_1+n_2}$	s'^2_G/s^2_G	$\sqrt{\frac{(n_1-1)s^2_G+(n_2-1)s'^2_G}{n_1+n_2-2}}$	$k(n_1+n_2-2)$
t	$\bar{L}$	F	s_C	γ
2.2	15.7			
		2.5	1.91	16
2.4	17.6			
		0.8	1.64	16

FIGURE 4--WORKSHEET 3.2.5

VALUES OF NBS BLOCKS FROM LABORATORY PROCESS COMPARED TO VALUES ASSIGNED AT NBS, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size	Assigned Values		Calibrated Values		Differences		Accepted Group S.D.
	(Inches)	NBS_1	NBS_2	NBS_1	NBS_2	N_1-W_1	N_2-W_2	
				Average of 2 Values				
		W_1	W_2	N_1	N_2	d_1	d_2	s_G
Group II k=3	0.10000	51.5	50.3	53.08	51.82	1.58	1.52	
	0.1001	55.2	56.5	51.42	52.68	-3.78	-3.82	
	0.1002	52.8	51.4	50.20	48.90	-2.60	-2.50	
	Combined							1.91
Group V k=2	0.150	52.8	52.9	52.72	52.42	-0.08	-0.48	
	0.160	54.2	62.7	51.82	59.68	-2.38	-3.02	
	Combined							1.64

*If $t \geq 3$, process is out of control and a new value for the laboratory reference block should be determined if the standard deviation is in control.

**If $F \geq F_{.01}$ for k and γ degrees of freedom, process is out of control. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = k$ and $\gamma_2 = \gamma$.

FIGURE 4--continued

D.F.	t Test*	Difference	Observed Group S.D.	F Test**	Combined Group S.D.	D.F.
	$\lvert d_1+d_2\rvert/s_G$	d_1-d_2	$\sqrt{\frac{\sum_{i=1}^{k} D_i^2}{k}}$	s_N^2/s_G^2	$\sqrt{\frac{\gamma s_G^2+k s_N^2}{\gamma+k}}$	$\gamma+k$
γ	t	D	s_N	F	s_C	
	1.6	0.06				
	4.0*	0.04				
	2.7	-0.10				
16			0.07	0.0	1.75	19
	0.3	0.40				
	3.3*	0.64				
16			0.53	0.1	1.56	18

FIGURE 5

REPORT OF TEST
of
Length Calibrations From_____to_____ Inches
Using
NBS Gage Block Sets_____and_____

PARTICIPATING LABORATORY

_____

Duplicate measurements made by the participating laboratory on NBS Gage Block Sets_____ and _____ made by calibrating each NBS set against a standard set during a regular calibration procedure gave the following results at 20 °C (values are in microinches):

A. Standard Deviations

Group	Accepted SD s_G	DF λ_1	From Test s_N	DF λ_2	Test F	Combined SD s_C	DF λ_3
I	--	--	--	--	--	--	--
II	--	--	--	--	--	--	--
III	--	--	--	--	--	--	--
IV	--	--	--	--	--	--	--
V	--	--	--	--	--	--	--
VI	--	--	--	--	--	--	--

Conclusions from the F-test:

(a) If $F < F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is in control, and the standard deviation used in the uncertainty statement is s_C.

(b) If $F \geq F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is out of control, and the uncertainty statement is omitted.

NOTE: Standard deviations s_G, s_N and s_C are found in Figure 4.

FIGURE 5--continued

B. Offset of Participating Laboratory's Process from NBS

Nominal Size	From NBS Process: Assigned Values NBS_1	NBS_2	Uncertainties NBS_1	NBS_2	From This Test: Calibrated Values NBS_1	NBS_2	Offset	t Test
(Inches)							$1/2\{(N_1+N_2)-(W_1+W_2)\}$	
	W_1	W_2	R_1	R_2	N_1	N_2	Δ	t
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--

Nominal Size	Laboratory Standards: Assigned Value	Uncertainty	Corrected for Offset: Assigned Value	Uncertainty	Uncertainty Test Block
(Inches)			$V+\Delta$	$\frac{3s_c}{\sqrt{2}} + \frac{R_1+R_2}{2}$	
	V	E	V'	E'	U
--	--	--	--	--	--
--	--	--	--	--	--
--	--	--	--	--	--

Conclusions from t test:

(a) If $t < 3$, there is no offset from the NBS process, and the assigned value and the uncertainty of the standard is unchanged.

(b) If $t \geq 3$, there is an offset from the NBS process, and the assigned value of the standard should be corrected by Δ. The uncertainty of the standard is E'. The uncertainty for any test block is calculated from E or E' whichever is appropriate, i.e., $U = E' + 3s_c$.

NOTE: The computations of the F and t statistics are found in Figure 4.

3.3 Test Set Measured Against Two Standard Sets: Control on the Difference Between Standards

3.3.1 Measurement Process

To introduce redundancy into a calibration system, duplicate measurements are made on each unknown, X, by comparing X to two standard blocks S_1 and S_2 in the order X S_1 S_2 X. This will not only provide a check on gross errors, but the difference between the two standard blocks can be used to test process control. By continuous examination of this difference, the stability of the measurement process and of the standard blocks themselves can be kept under surveillance so that decisions about the condition of the process can be made after each calibration run.

The random errors associated with the process are of two kinds; those arising from repetitions in the short term, and those involving long term or day-to-day differences. The random error appropriate in this case is that associated with the repeated measurement of a control "block" over a sufficiently long time period to insure that all factors affecting total variability have a chance to exert their influence. (The control "block" is actually the difference between the two standard blocks.)

Because of temperature effects, the variation is usually length dependent. For this reason, and for convenience of working group size the usual set of blocks (80 or so blocks in the 0.05 to 4 inch range) should be treated as a number of subsets of up to 20 blocks each, and at NBS the following six groupings are used.

Group	Nominal Length (Inches)	Approximate No. of Blocks
I	0.050 to 0.09375	4
II	0.100 to 0.107	20
III	0.108 to 0.126	20
IV	0.127 to 0.146	20
V	0.147 to 0.500	15
VI	0.550 to 4.000	13

Variability should be approximately the same for all blocks in a group so that one can combine the standard deviations* computed for each of the blocks in a group into one overall value of the standard deviation for the group. If $s_1, \ldots s_k$ are the standard deviations for the k blocks in a group with degrees of freedom $\nu_1, \ldots \nu_k$ respectively,

$$\text{s.d. (group)} = \sqrt{\frac{\nu_1 s_1^2 + \ldots + \nu_k s_k^2}{\nu_1 + \ldots + \nu_k}}$$

3.3.2 Establishing Process Parameters

Before starting process control, it will be necessary to establish an initial accepted value** for the control, $S_1 - S_2$, and a value for the random error component. To do this the control set should be measured by the usual process, say 6 times, with a few days between repetitions. Then the average value for the control, $S_1 - S_2$, and the standard deviation associated with the difference should be computed for each pair of standard blocks.

The accepted starting values for the control and random error component will be the average and group standard deviation as indicated above. Figure 6 details a portion of a typical worksheet showing the starting values for two groups of blocks in the set. Examples given in the following sections are based on these two groups and their parameters. All data in the examples is hypothetical and is intended only to explain the methodology.

*The standard deviation of a single observation is given by

$$s = \sqrt{\frac{\Sigma\, r_i^2}{n - 1}}$$

where the r_i is the difference between each observation and the average of n observations. The quantity n-1 is called the degrees of freedom associated with s.

**The term "value" for a gage block means the deviation from nominal size (measured length minus nominal length) at 20 °C. The value for the control in this case is the difference in length between the two standards ($S_1 - S_2$) at 20 °C.

3.3.3 Maintaining Control

After the process parameters have been established, control is maintained by checking the observed value for each parameter after each calibration against its accepted value.

A portion of a typical worksheet is given in Figure 7 detailing the tests for determining whether or not the process is in a state of statistical control.

3.3.4 Updating Process Parameters

As information collects on the control blocks which are used regularly, the values for the process parameters should be updated as shown in Figure 8. If either of the process parameters has changed from its starting value as indicated by the t-test or F-test, a new process parameter must be computed based only on the current data. If the process parameters have not changed, the process will be considered to be continuous, and the data will be combined. A portion of a typical worksheet (Figure 8) details how this is done.

The frequency of updating will depend on the workload but should be done after 5 or 10 values initially and then at intervals dictated by convenience (e.g., every six months or a year).

3.3.5 Relationship to National Standards

The process should produce values which are consistent with the national system as maintained by NBS. In order to test the entire system, NBS is prepared to send each participating laboratory two calibrated sets when requested, presumably at intervals of one or two years. Each of these two sets should be calibrated twice by the laboratory using its regular calibration procedure thereby giving the independent check necessary for verifying consistency.

Laboratories with the capability of writing their own test reports uncertainties.
based on their measurements of the NBS sets are encouraged to do so. In this case NBS will provide the two calibrated sets of blocks, their current assigned values and associated The table in Figure 9 summarizes the analysis of the data and shows whether or not there is a signficant offset from the NBS process. If the offset is significant, the values for the laboratory reference standards and their associated uncertainties should be corrected as shown in the suggested "Report of Test" form in Figure 10. In effect this is a method for each laboratory to calibrate its own standards.

3.3.6 Uncertainty

The uncertainty, $\pm U$, associated with the average value of any test block which has been calibrated twice, assuming the measurement process is in control, is given by

$$U = \frac{U_1+U_2}{2} + \frac{3s_G}{2}$$

where U_1 and U_2 are the assigned uncertainties of the standard sets S_1 and S_2 respectively, and s_G is the accepted group standard deviation* for that block size (see Figure 9). This assumes that the standard blocks have documented uncertainties (e.g., if they were calibrated at NBS). For convenience one may use the maximum uncertainty in the group so as to report a single uncertainty for the entire group.

3.3.7 Summary

In the foregoing sections, tests are given to determine if the process average has shifted and if the variability of the process has changed. The actions which can be taken when one of these tests indicates an out-of-control condition are described in section 4.6. In the absence of a specific format for out-of-control conditions, it should be noted that once predictability is lost, no statements should be made about the condition of the process. In some cases the standard blocks may have actually changed and may need to be recalibrated. In others the process may have changed and new process parameters must be determined. Until the process parameters have been reestablished or some satisfactory corrective action has been taken, any uncertainty statements which are issued should be designated as provisional.

*Note that s_G is the standard deviation of the difference between the two standard blocks S_1 and S_2. The standard deviation of a single observation from the process is $s_G/\sqrt{2}$.

FIGURE 6--WORKSHEET 3.3.2

PROCESS PARAMETERS: ACCEPTED VALUE OF THE CONTROL AND GROUP STANDARD DEVIATION, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size (Inches)	Control Average Value of S_1-S_2	No. of Repetitions	Observed S.D. of Control	Group S.D. $\sqrt{\frac{\Sigma s_i^2}{k}}$	Degrees of Freedom k(n-1)
		L_c	n	s	s_G	γ
Group II	0.10000	-0.20	6	0.50		
k=4	0.10005	-1.25	6	0.37		
	0.10010	-0.32	6	1.12		
	0.10020	0.10	6	0.54		
	Combined				0.70	20
Group V	0.147	1.86	6	1.01		
k=5	0.148	-4.13	6	0.94		
	0.149	-1.38	6	0.92		
	0.150	1.00	6	1.46		
	0.200	0.43	6	0.54		
	Combined				1.02	25

FIGURE 7--WORKSHEET 3.3.3

OBSERVED VALUES OF CONTROL AND STANDARD DEVIATIONS
COMPARED TO ACCEPTED VALUES, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size	Comparator Readings						Control	Accepted Value of Control	
	(Inches)	Test	Std.	Std.	Test	$X-S_1$	$X-S_2$	d_2-d_1		$C-L_c$
		X	S_1	S_2	X	d_1	d_2	C	L_c	D
Group II k=4	0.10000	20.0	16.9	17.2	21.0	3.1	3.8	0.7	-0.2	0.9
	0.10005	23.1	22.0	22.9	23.8	1.1	0.9	-0.2	-1.2	1.0
	0.10010	19.0	16.7	16.0	20.1	2.3	4.1	1.8	-0.3	2.1
	0.10020	20.3	18.0	17.1	19.9	2.3	2.8	0.5	0.1	0.4
	Combined									
Group V k=5	0.147	21.3	20.7	16.0	21.2	0.6	5.2	4.6	1.9	2.7
	0.148	23.6	20.7	23.5	23.2	2.9	-0.3	-3.2	-4.1	0.9
	0.149	15.7	15.1	17.2	16.6	0.6	-0.6	-1.2	-1.4	0.2
	0.150	20.0	15.4	15.0	19.9	4.6	4.9	0.3	1.0	-0.7
	0.200	16.7	15.8	15.2	15.7	0.9	0.5	-0.4	0.4	-0.8
	Combined									

*If $t \geq 3$, the process is out of control. Remeasure blocks and test again. $|D|$ means the absolute value of D.

**If $F \geq F_{.01}$, for k and γ degrees of freedom, process is out of control. Remeasure all blocks in group and test again. The critical value, $F_{.01}$, can be found in Table I (page 63) where $\gamma_1 = k$ and $\gamma_2 = \gamma$. If $t < 3$ and $F < .01$, process is in control. Accept value for test block of $1/2(d_1+d_2+V_1+V_2)$ where V_1 and V_2 are assigned values for standards S_1 and S_2.

***U_1 and U_2 are the assigned uncertainties of standards S_1 and S_2.

FIGURE 7--continued

Observed S.D. of Group	Accepted S.D. of Group	D.F.	t Test*	F Test**	Uncertainty of Starting Std.	Limit to Random Error	Uncertainty of Test Block
$\sqrt{\frac{\Sigma D_i^2}{k}}$			$\lvert D \rvert / s_G$	s_I^2/s_G^2	$1/2(U_1+U_2)$***	$3s_G/2$	E + R
s_I	s_G	ν	t	F	E	R	U
			1.3		1.8	1.0	2.8
			1.4		1.8	1.0	2.8
			3.0*		1.8	1.0	2.8
			0.6		1.8	1.0	2.8
1.26	0.70	20		3.2			
			2.6		2.3	1.5	3.8
			0.9		2.3	1.5	3.8
			0.2		2.3	1.5	3.8
			0.7		2.3	1.5	3.8
			0.8		2.3	1.5	3.8
1.36	1.02	25		1.8			

FIGURE 8--WORKSHEET 3.3.4

NEW VALUES OF PROCESS PARAMETERS COMPARED TO ACCEPTED VALUES
k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size	Starting Values Control	Starting Values Group S.D.	Starting Values No.	New Values Control	New Values Group S.D.	New Values No.
	(Inches)						
		L_c	s_G	n_1	L_c'	s_G'	n_2
Group II k=4	0.10000	-0.2		6	-0.3		12
	0.10005	-1.2		6	-1.0		12
	0.10010	-0.3		6	-0.2		12
	0.10020	0.1		6	-0.9		12
	Combined		0.70			1.16	
Group V k=5	0.147	1.9		6	4.7		12
	0.148	-4.1		6	-3.5		12
	0.149	-1.4		6	-2.1		12
	0.150	1.0		6	0.4		12
	0.200	0.4		6	1.0		12
	Combined		1.02			1.34	

*If $t \geq 3$, the new value L_c' should replace L_c for the control; otherwise, use the combined value $\bar{L}_c$.

**If $F \geq F_{.01}$ for $k(n_2-1)$ and $k(n_1-1)$ degrees of freedom, the new value s_G' should replace s_G as the group standard deviation; otherwise, combine the new and starting value into a combined group standard deviation. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = k(n_2-1)$ and $\gamma_2 = k(n_1-1)$.

***If $F \geq F_{.01}$ for $k(n_1-1)$ and $k(n_2-1)$ degrees of freedom, the new value S_G' should replace s_G; otherwise, use the combined group standard deviation. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = k(n_1-1)$ and $\gamma_2 = k(n_2-1)$.

FIGURE 8--continued

t Test*	Combined Value Control	F Test**	F Test***	Combined Group S.D.	Degrees of Freedom
$\frac{\lvert L_c - L'_c \rvert}{s_G\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$	$\frac{n_1 L_c + n_2 L'_c}{n_1+n_2}$	$\frac{s'^2_G}{s_G^2}$	$\frac{s_G^2}{s'^2_G}$	$\sqrt{\frac{k(n_1-1)s_G^2+k(n_2-1)s'^2_G}{k(n_1+n_2-2)}}$	$k(n_1+n_2-2)$
t	$\bar{L}_c$	F	F	s_G	γ
0.3	-0.3				
0.6	1.1				
0.3	-0.2				
2.9	-0.6				
		2.7	0.4	1.04	64
5.5*	*				
1.2	-3.7				
1.4	-1.9				
1.2	0.6				
1.2	0.8				
		1.7	0.6	1.25	80

FIGURE 9--WORKSHEET 3.3.5

VALUES OF NBS BLOCKS FROM LABORATORY PROCESS COMPARED TO VALUES ASSIGNED BY NBS, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

Nominal Size	Values Assigned by NBS		Average of 4 Calibrated Values		Sum	Difference
(Inches)	NBS_1	NBS_2	NBS_1	NBS_2	$(N_1+N_2)-(V_1+V_2)$	$(N_1-N_2)-(V_1-V_2)$
	V_1	V_2	N_1	N_2	S	D
0.130	51.5	50.3	53.1	51.8	3.1	0.1
0.135	55.2	56.5	51.4	52.7	-7.6	0.0
0.145	52.8	51.4	50.2	48.9	-5.1	-0.1
0.200	52.8	52.9	52.7	52.4	-0.6	0.4
0.350	54.2	62.7	51.8	59.7	-5.4	0.6
Combined	(K=5)					

GROUP V

*If $t \geq 3$, process is out of control and a new value for the laboratory reference block should be determined if the standard deviation is in control. $|S|$ means the absolute value of S.

**If $F \geq F_{.01}$ for k and γ degrees of freedom, process is out of control. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = k$ and $\gamma_2 = \gamma$. If $F < F_{.01}$, combine the accepted and observed standard deviations as shown.

FIGURE 9--continued

Accepted Group S.D.	D.F.	Observed Group S.D.	t Test*	F Test**	Combined Group S.D.	Total D.F.
		$2\sqrt{\frac{\Sigma D_i^2}{k}}$	$\frac{2\lvert S\rvert}{s_G}$	$\frac{s_N^2}{s_G^2}$	$\sqrt{\frac{ks_N^2+\gamma s_G^2}{k+\gamma}}$	$k+\gamma$
s_G	γ	s_N	t	F	s_C	
			5.0*			
			12.2*			
			8.2*			
			1.0			
			8.6*			
1.25	80	0.66		0.3	1.23	85

FIGURE 10

REPORT OF TEST
of
Length Calibrations From _____ to _____ Inches
Using
NBS Gage Block Sets _____ and _____

PARTICIPATING LABORATORY

_____

Duplicate measurements made by the participating laboratory on NBS Gage Block Sets _____ and _____ made by calibrating each NBS set against two standard sets during a regular calibration procedure gave the following results at 20 °C (values are in microinches):

A. Standard Deviations

Group	Accepted SD s_G	DF λ_1	From Test s_N	DF λ_2	Test F	Combined SD s_C	DF λ_3
I	--	--	--	--	--	--	--
II	--	--	--	--	--	--	--
III	--	--	--	--	--	--	--
IV	--	--	--	--	--	--	--
V	--	--	--	--	--	--	--
VI	--	--	--	--	--	--	--

Conclusions from the F-test:

(a) If $F < F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is in control, and the standard deviation used in the uncertainty statement is s_C.

(b) If $F \geq F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is out of control and the uncertainty statement is omitted.

NOTE: Standard deviations s_G, s_N and s_C are found in Figure 9.

FIGURE 10--continued

B. Offset of Participating Laboratory's Process from NBS

Nominal Size	From NBS Process: Assigned Values NBS_1	NBS_2	Uncertainties NBS_1	NBS_2	From This Test: Calibrated Values NBS_1	NBS_2	Offset	t Test
(Inches)							$1/2\{(N_1+N_2)-(W_1+W_2)\}$	
	W_1	W_2	R_1	R_2	N_1	N_2	Δ	t
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--

Nominal Size	Laboratory Standards: Assigned Values S_1	S_2	Uncertainties S_1	S_2	Corrected for Offset: Assigned Values S_1	S_2	Uncertainties S_1	S_2	Uncertainty Test Block
(Inches)					$V_1 \mp \Delta$	$V_2 \mp \Delta$			
	V_1	V_2	U_1	U_2	V_1'	V_2'	U_1'	U_2'	U
--	--	--	--	--	--	--	--	--	--
--	---	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	---	--	--

Conclusions from t-test:

(a) If $t < 3$, there is no offset from the NBS process, and the assigned value and uncertainty of each standard are unchanged.

(b) If $t \geq 3$, there is an offset from the NBS process, and the assigned value of each standard should be corrected by Δ. The uncertainties associated with V_1' and V_2' are U_1' and U_2' where $U_1' = U_2' = \frac{1}{2}\{\frac{3s_c}{\sqrt{2}} + R_1 + R_2)\}$. The uncertainty for any test block is calculated from U_1 and U_2 or U_1' and U_2' whichever is appropriate, i.e., $U = \frac{U_1' + U_2'}{2} + \frac{3s_c}{2}$.

NOTE: The computations of the F and t statistics are found in Figure 9.

3.4 Two Test Sets and Two Standard Sets In A Drift Eliminating Design Involving Eight Observations, Difference Between Standards As Control

3.4.1 Measurement Process

Laboratories doing high precision work can benefit from a drift eliminating design using eight observations to intercompare two test blocks with two standard blocks. Redundancy is built into this design. In addition, any linear drift effect caused by comparator time dependence will be balanced out. The differences between the block pairs of the two standard sets S_1 and S_2 will serve as controls and will allow the stability of the measurement process and of the standard blocks themselves to be monitored.

The random errors associated with the process are of two kinds; namely, a short term component called "within" variability and a long term component called "total" variability.

The within variability, σ_W^2, is readily demonstrated in a repeated sequence of measurements made over a short time. This quantity is used for a day to day check on the process variability.

The total variability of the process, σ^2, reflects both short term and long term variability. It is found by examining the measurement process over a sufficiently long time period to insure that all factors affecting variability have a chance to exert their influence. It is used in a statistical t-test to identify any shift in the control value for the process.

Because of the effect of temperature on block length, variations are usually length dependent. For this reason, and for convenience of working group size, the overall set of blocks (80 or so blocks in the 0.05 to 4 inch range) should be treated as a number of groups of up to 20 blocks each. The following six groupings are recommended.

Group	Nominal Length (Inches)	Approximate No. of Blocks
I	0.050 to 0.09375	4
II	0.100 to 0.107	20
III	0.108 to 0.126	20
IV	0.127 to 0.146	20
V	0.147 to 0.500	15
VI	0.550 to 4.000	13

The group is selected so that variability of all blocks in a group is expected to be the same. Then the standard deviations computed for each block in a group can be combined into an overall group standard deviation. If $s_1, \ldots s_k$ are the standard deviations of the k blocks in a group with degrees of freedom $\nu_1, \ldots \nu_k$ respectively, then

$$\text{s.d. (group)} = \sqrt{\frac{\nu_1 s_1^2 + \ldots \nu_k s_k^2}{\nu_1 + \ldots + \nu_k}}$$

3.4.2 Establishing Process Parameters

Before starting the control process, it will be necessary to establish initial accepted values* for the controls and initial values for the random error components. To do this, at least six calibration runs should be made using the control blocks in the trend-eliminating design. The calibrations should be separated by several days. The measurement sequence for the trend-eliminating design along with the equations for finding the least squares estimates of the controls and the within standard deviation and the pooled within standard deviation are given in Appendix A. See [7] for a more complete analysis of trend-eliminating designs.

The average value of the control, the pooled within standard deviation, and the total standard deviation** are computed for each pair of standards. Then the random error components are combined into a group within standard deviation and a group total standard deviation as shown in 3.4.1. These values and the average value of the control are the starting process parameters.

Figure 6 details a portion of a typical worksheet showing the starting values for two groups of blocks in the set. Examples given in the following sections are based on these two groups and their parameters. All data in the examples is hypothetical and is intended only to explain the methodology.

*The term "value" for a gage block means the deviation from nominal size (measured length minus nominal length) at 20 °C. The value for the control in this case is the difference in length between the two standards ($S_1 - S_2$) at 20 °C.

**The total standard deviation for the control is given by

$$\hat{\sigma} = \sqrt{\frac{\Sigma\, r_i^2}{n - 1}}$$

where r_i is the difference between each value of the control and the average of n values. The quantity n-1 is called the degrees of freedom associated with $\hat{\sigma}$.

3.4.3 Maintaining Process Control

After the process parameters have been established, control is maintained by checking the observed value of the control $S_1 - S_2$ and the within standard deviation $\hat{\sigma}_W$ for each calibration run against their accepted values.

A typical section of a worksheet for making these comparisons showing the appropriate statistical tests is given in Figure 12.

3.4.4 Updating Process Parameters

Periodically as data accumulates on the control set, the output produced by the process is compared to the accepted process parameters. If one of the parameters has changed as indicated by a t-test or F-test, a new value for the parameters must be determined based only on the current data; otherwise, the process will be considered to be continuous, and the data will be combined. The worksheet in Figure 13 details this procedure and shows the appropriate statistical tests.

Initially the process should be updated after five or six calibrations and then as convenience dictates (every six months or yearly).

3.4.5 Relationship to National Standards

The process should produce values which are consistent with the national system as maintained by NBS. In order to test the entire system, NBS is prepared to send each participating laboratory two calibrated sets when requested, presumably at intervals of one or two years. These sets should each be calibrated twice by the laboratory using the trend-eliminating design procedure thereby giving the independent check necessary for verifying consistency.

Laboratories with the capabilitiy of writing their own test reports based on their measurements of the NBS sets are encouraged to do so. In this case, NBS will provide the two calibrated sets of blocks, their current assigned values and associated uncertainties. The table in Figure 14 summarizes the analysis of the data and shows whether or not there is a significant offset from the NBS process. If the offset is significant, the values for the laboratory reference standards and their associated uncertainties should be corrected as shown in the suggested "Report of Test" form in Figure 15. In effect this is a method for each laboratory to calibrate its own standards.

3.4.6 Uncertainty

The uncertainty $\pm U$ associated with the value assigned to any test block which has been calibrated using the trend-eliminating design is given by

$$U = \frac{U_1 + U_2}{2} + \frac{3}{2}\sqrt{3\hat{\sigma}_G^2 - \frac{1}{6}s_G^2}$$

where U_1 and U_2 are the assigned uncertainties in the calibration of the standard sets S_1 and S_2 respectively, and S_G is the accepted total group standard deviation for that block size and s_G is the accepted within group standard deviation (see Figure 14). This assumes that the standard blocks have documented uncertainties (e.g., if they were calibrated at NBS.) For convenience one may use the maximum uncertainty in the group so as to report a single uncertainty for the entire group. An explanation of the computation of the uncertainty can be found in [7].

3.4.7 Summary

In the foregoing sections several statistical tests are described which determine if the process mean has shifted and if the variability of the process has changed. Actions which can be taken when one of these tests indicates an out-of-control condition are described in Section 4.6. In the absence of specific format instructions for out-of-control conditions, it should be understood that once predictability is lost, no statements should be made about the condition of the process. Any uncertainty statements which are issued should be designated as provisional until the process is again under control.

FIGURE 11--WORKSHEET 3.4.2

PROCESS PARAMETERS: ACCEPTED VALUES OF CONTROL, WITHIN AND TOTAL STANDARD DEVIATIONS, k BLOCKS IN A GROUP

Values in Microinches at 20 °C

	Nominal Size	Accepted Value of Control	No. of Reps.	Pooled Within S.D.	Group Within S.D.	Degrees of Freedom	Total S.D.	Group Total S.D.	Degrees of Freedom
	(Inches)	Average of $\hat{L}_c$		(See p 66)	$\sqrt{\frac{\Sigma\xi_i^2}{k}}$	4kn	$s(\hat{L}_c)$ (See p 42 footnote)	$\sqrt{\frac{\Sigma\hat{\sigma}_i^2}{k}}$	k(n-1)
		L_c	n	ξ	s_G	δ	$\hat{\sigma}$	$\hat{\sigma}_G$	γ
Group II k=4	0.10000	0.5	6	0.22			0.93		
	0.10005	-0.1	6	0.28			0.54		
	0.1001	-0.4	6	0.39			0.36		
	0.1002	-0.0	6	0.41			0.26		
	Combined		24		0.33	96		0.58	20
Group V k=5	0.147	0.10	6	0.65			0.34		
	0.148	0.20	6	0.23			0.62		
	0.149	0.04	6	0.48			0.64		
	0.150	0.21	6	0.51			0.34		
	0.200	0.20	6	0.30			0.48		
	Combined		30		0.46	120		0.50	25

FIGURE 12--WORKSHEET 3.4.3

OBSERVED VALUE FOR CONTROL AND WITHIN STANDARD DEVIATION COMPARED TO ACCEPTED VALUE
(SEE APPENDIX A FOR COMPUTATIONS)

Values in Microinches at 20 °C

	Nominal Size	Control	Accepted Value of Control	Observed Within S.D.	Accepted Values: Group Within S.D.	D.F.	Group Total S.D.	D.F.
	(Inches)		See Appendix A					
		$\hat{L}_c$	L_c	$\hat{\sigma}_W$	s_G	δ	$\hat{\sigma}_G$	γ
Group II	0.10000	2.5	0.5	0.48				
	0.10005	0.0	-0.1	0.16				
	0.1001	-0.5	-0.4	0.30				
	0.1002	-0.6	-0.0	0.50				
	Combined				0.33	96	0.58	20
Group V	0.147	0.8	0.1	0.59				
	0.148	-0.2	0.2	0.61				
	0.149	0.5	0.0	0.33				
	0.150	0.5	0.2	0.62				
	0.200	-0.5	0.2	0.43				
	Combined				0.46	120	0.50	25

*If $t \geq 3$, process is out of control for that block. Remeasure and test again.

**If $F \geq F_{.01}$ for 4 and δ degrees of freedom, process is out of control for that block. The critical value, $F_{.01}$, can be found in Table I where $\gamma_1 = 4$ and $\gamma_2 = \delta$.

***U_1 and U_2 are the assigned uncertainties of the two standard blocks.

FIGURE 12--continued

t Test*	F Test**	Uncertainty of Restraint	Limit to Random Error	Uncertainty of Test Block
$\lvert\hat{L}_c - L_c\rvert/\hat{\sigma}_G$	$\hat{\sigma}_W^2/s_G^2$	$1/2(U_1+U_2)$***	$\frac{3}{2}\sqrt{3\hat{\sigma}_G^2 - \frac{1}{6}s_G^2}$	E + R
t	F	E	R	U
3.4*	2.1	1.8	1.5	3.3
0.2	0.2	1.8	1.5	3.3
0.2	0.8	1.8	1.5	3.3
1.0	2.3	1.8	1.5	3.3
1.4	1.6	2.3	1.3	3.6
0.8	1.8	2.3	1.3	3.6
1.0	0.5	2.3	1.3	3.6
0.6	1.8	2.3	1.3	3.6
1.4	0.9	2.3	1.3	3.6

FIGURE 13--WORKSHEET 3.4.4

NEW VALUES FOR PROCESS PARAMETERS COMPARED TO ACCEPTED VALUES

Values in Microinches at 20 °C

	Nominal Size	Starting Values: Control	Starting Values: No.	Starting Values: Group Within S.D.	Starting Values: Group Total S.D.	New Values: Control	New Values: No.	New Values: Group Within S.D.	New Values: Group Total S.D.	t Test*	Average
	(Inches)									$\frac{\lvert L_c - L_c' \rvert}{\hat{\sigma}_G \sqrt{1/n_1 + 1/n_2}}$	$\frac{n_1 L_c + n_2 L_c'}{n_1 + n_2}$
		L_c	n_1	s_G	$\hat{\sigma}_G$	L_c'	n_2	s_G'	$\hat{\sigma}_G'$	t	$\bar{L}_c$
GROUP II	0.10000	0.5	6			1.0	12			1.7	0.8
	0.10005	-0.1	6			-0.8	12			2.4	-0.6
	0.1001	-0.4	6			0.1	12			1.7	-0.1
	0.1002	0.0	6			-0.2	12			0.7	-0.1
	Combined (k=4)			0.33	0.58			0.31	0.47		
GROUP V	0.147	0.1	6			-0.5	12			2.4	-0.3
	0.148	0.2	6			-1.3	12			6.0*	*
	0.149	0.0	6			0.5	12			2.0	0.3
	0.150	0.2	6			-0.2	12			1.6	-0.1
	0.200	0.2	6			-0.2	12			1.6	-0.1
	Combined (k=5)			0.46	0.50			0.23	0.65		

where $\delta_1 = 4kn_1$, $\delta_2 = 4kn_2$.

*If $t \geq 3$, the new value for the control L_c' is significantly different from L_c and should replace L_c; otherwise, use the average $\bar{L}_c$.

**If $F \geq F_{.01}$ for $4kn_2$ and $4kn_1$ degrees of freedom, replace s_G by s_G'; otherwise, use the combined value s_c.

***If $F \geq F_{.01}$ for $k(n_2-1)$ and $k(n_1-1)$ degrees of freedom, replace $\hat{\sigma}_G$ by $\hat{\sigma}_G'$; otherwise, use the combined value $\hat{\sigma}_c$.

√If $F \geq F_{.01}$ for $4kn_1$ and $4kn_2$ degrees of freedom, replace s_G by s_G'; otherwise use the combined value s_c.

√√If $F \geq F_{.01}$ for $k(n_1-1)$ and $k(n_2-1)$ degrees of freedom, replace $\hat{\sigma}_G$ by $\hat{\sigma}_G'$; otherwise, use the combined value $\hat{\sigma}_c$.

The critical values, $F_{.01}$, mentioned above can be found in Table I. The γ_1 in the table refers to the first number of degrees of freedom and γ_2 to the second.

FIGURE 13--continued

F Test**	F Test***	F Test ✓	F Test ✓✓	Combined Group Within S.D.	Degrees of Freedom	Combined Group Total S.D.	Degrees of Freedom
s'^2_G/s^2_G	$\hat{\sigma}'^2_G/\hat{\sigma}^2_G$	s^2_G/s'^2_G	$\hat{\sigma}^2_G/\hat{\sigma}'^2_G$	$\sqrt{\frac{\delta_1 s^2_G+\delta_2 s'^2_G}{(\delta_1+\delta_2)}}$	$4k(n_1+n_2)$	$\sqrt{\frac{(n_1-1)\hat{\sigma}^2_G+(n_2-1)\hat{\sigma}'^2_G}{n_1+n_2-2}}$	$k(n_1+n_2-2)$
F	F	F	F	s_c	δ	$\hat{\sigma}_c$	γ
0.9	0.7	1.1	1.5	0.32	288	0.51	64
0.2	1.7	4.0✓	0.6	✓	✓	0.61	80

FIGURE 14--WORKSHEET 3.4.5

VALUES OF NBS BLOCKS FROM LABORATORY PROCESS COMPARED TO VALUES ASSIGNED BY NBS

Values in Microinches at 20 °C

	Nominal Size	Values Assigned by NBS		Average of Two Calibrations		Sum
	(Inches)	NBS_1	NBS_2	NBS_1	NBS_2	$(N_1+N_2)-(W_1+W_2)$
		W_1	W_2	N_1	N_2	S
Group II	0.10000	19.00	19.56	19.00	18.32	-1.24
	0.10005	17.00	18.20	17.51	18.23	0.54
	0.1001	13.50	13.37	14.53	13.68	1.34
	0.1002	18.80	17.83	18.55	17.20	-0.88
	Combined					
Group V	0.147	21.30	18.77	21.20	18.01	-0.86
	0.148	23.60	27.71	23.29	25.69	-2.33
	0.149	15.70	16.22	16.62	15.45	0.15
	0.150	20.00	19.69	19.99	19.45	-0.25
	0.200	16.70	16.71	15.70	16.01	-1.70
	Combined					

*If $z \geq 3$, process is out of control and new values for the laboratory reference blocks should be determined if the standard deviations are in control.

**If $F \geq F_{.01}$ for 4 and γ degrees of freedom, process is out of control.

The critical value $F_{.01}$ can be found in Table I where $\gamma_1 = 4$ and $\gamma_2 = \gamma$.

FIGURE 14--continued

Observed Within S.D.	Accepted Within S.D.	D.F.	Accepted Total S.D.	Test*	F Test**
				$\frac{\lvert S \rvert}{\sqrt{\hat{\sigma}_G^2 - \frac{1}{12}s_G^2}}$	$\hat{\sigma}_W^2/s_G^2$
$\hat{\sigma}_W$	s_G	γ	$\hat{\sigma}_G$	z	F
0.35				2.5	1.2
0.59				1.1	3.4
0.79				2.7	6.1**
0.24				1.8	0.6
	0.32	288	0.51		
0.33				1.4	2.1
1.02				3.8*	19.7**
0.32				0.2	1.9
0.40				0.4	3.0
0.51				2.8	4.9**
	0.23	240	0.61		

FIGURE 15

REPORT OF TEST
of
Length Calibrations From_____to_____Inches
Using
NBS Gage Block Sets_____and_____

PARTICIPATING LABORATORY

_____

Duplicate measurements made by this laboratory on NBS Gage Block Sets_____and_____made by calibrating each NBS set against two standard sets during a regular calibration procedure gave the following results at 20 °C (values are in microinches):

A. Standard Deviations

Group	Accepted Standard Deviations				From This Test		F Test	Combined	
	Within	DF	Total	DF	Within	DF	$\hat{\sigma}_G^2/s_G^2$	Within	DF
	s_G	λ_1	$\hat{\sigma}_G$	λ_2	s_N	λ_3		s_C	λ_4
I	--	--	--	--	--	--	--	--	--
II	--	--	--	--	--	--	--	--	--
III	--	--	--	--	--	--	--	--	--
IV	--	--	--	--	---	--	--	--	--
V	--	--	--	--	--	--	--	--	--
VI	--	--	--	--	--	--	--	--	--

Conclusions from F-test:

(a) If $F < F_{.01}$ for λ_3 and λ_1 degrees of freedom, the process is in control, and the within standard deviation used in the uncertainty statement is S_C.

(b) If $F \geq F_{.01}$ for λ_3 and λ_1 degrees of freedom, the process is out of control, and the uncertainty statement is omitted.

NOTE: Standard deviations s_G, $\hat{\sigma}_G$, s_N and s_C are found in Figure 14.

FIGURE 15--continued

B. Offset of Participating Laboratory's Process from NBS

Nominal Size	From NBS Process: Assigned Values NBS_1	NBS_2	Uncertainties NBS_1	NBS_2	From This Test: Calibrated Values NBS_1	NBS_2	Offset	t Test
(Inches)							$1/2\{(N_1+N_2)-(W_1+W_2)\}$	
	W_1	W_2	R_1	R_2	N_1	N_2	Δ	t
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--

Nominal Size	Laboratory Standards: Assigned Values S_1	S_2	Uncertainties S_1	S_2	Corrected for Offset: Assigned Values S_1	S_2	Uncertainties S_1	S_2	Uncertainty Test Block
(Inches)					$V_1 \pm \Delta$	$V_2 \pm \Delta$			
	V_1	V_2	U_1	U_2	V_1'	V_2'	U_1'	U_2'	U
--	--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--	--

Conclusions from t-test:

(a) If $t < 3$, there is no offset from the NBS process, and the assigned values of the standards are unchanged. The uncertainties are U_1 and U_2 respectively.

(b) If $t \geq 3$, there is an offset from the NBS process, and the assigned value of each standard should be corrected by Δ. The uncertainties associated with V_1' and V_2' are U_1' and U_2' where:

$$U_1' = U_2' = \frac{3}{\sqrt{2}}\sqrt{\frac{\hat{\sigma}_G^2}{4}} + \frac{R_1+R_2}{2}.$$

The uncertainty for any test block is calculated from U_1 and U_2 or U_1' and U_2' whichever is appropriate, i.e.,

$$U = \frac{U_1'+U_2'}{2} + \frac{3}{2}\sqrt{3\hat{\sigma}_G^2 - \frac{1}{6}s_c^2}$$

NOTE: The computations of the F and t statistics are found in Figure 14.

4 Operating a Measurement Assurance Program

4.1 Equipment and Environment

4.1.1 The Comparator

There are a number of suitable comparator designs and a typical one is shown schematically in Figure 16. An upper stylus and a lower stylus contact the gaging faces of a block supported on an anvil. Each stylus is attached to a differential transformer core. An integrated signal from these two transducers is displayed on a meter graduated in length units (usually microinches). In the simplest comparison, the difference in length between two gage blocks is obtained by inserting the blocks, one at a time, between the stylus tips and taking the difference between the two readings.

An important comparator feature is the point-to-point measurement along an axis through the gaging point of the block. Other suitable comparator designs use only one transducer but by various means insure the point-to-point measurement. Single-transducer comparators are susceptible to errors when measuring burred or non-flat blocks (see Figure 17) if they do not have provisions for point-to-point measurement.

For the most precise measurements, it is important that the comparator stylus contact the defined gage point of the block. A metal or plastic bar about 1/4 inch thick can be fastened to the anvil (with laboratory wax if no other means is provided) behind the stylus and positioned to stop the gage blocks so the stylus will contact the gage point. The bar can be L-shaped to position the gage block both laterally and transversely if desired and special configurations can be devised for comparing blocks of different shapes.

4.1.2 Gage Block Requirements

Nearly all gage blocks are either square or rectangular in cross section, but a few are circular in cross section. Any of these cross sections are easily handled in intercomparison procedures on short blocks (sizes up to 4 inches). Long blocks (over 4 inches) of the rectangular cross section are prone to tipping in vertical comparators.

Transferring the length unit to a gage block by intercomparison does not require rigorous constraints on the flatness and parallelism of its gaging faces because the transfer is made only along a single line through the block. Gage block use, however, may be more demanding on the geometry of the gaging faces. A separate paper covers the measurement of flatness and parallelism [1].

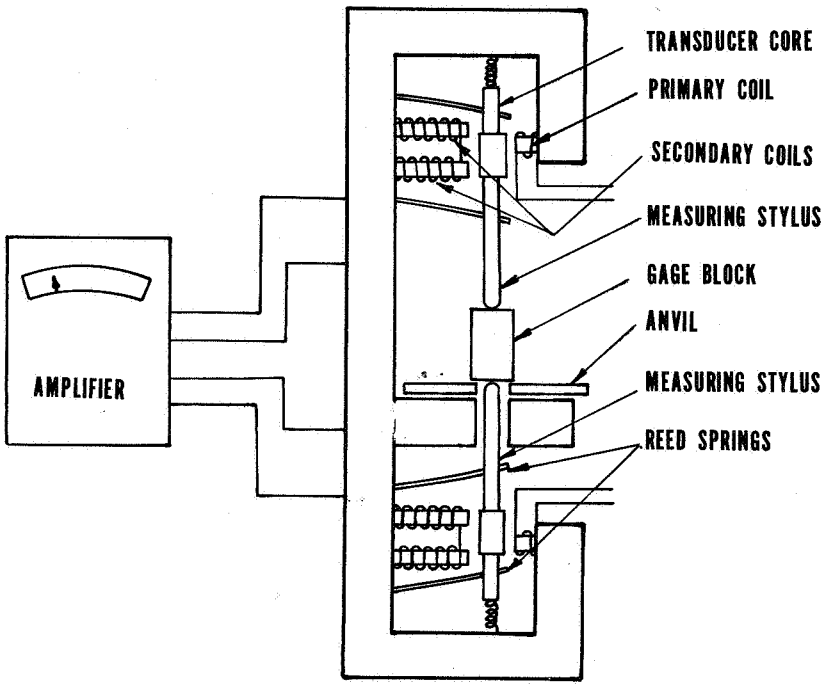


FIGURE 16: ELEMENTS OF A MECHANICAL COMPARATOR OF LENGTHS

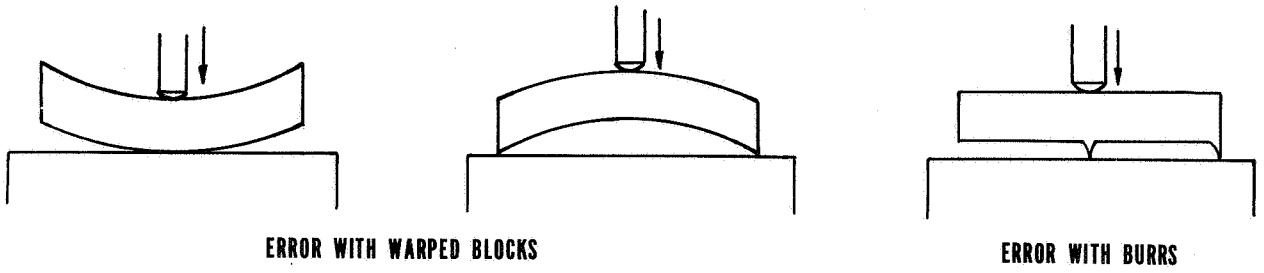


FIGURE 17: ERROR RESULTING FROM COMPARATOR DESIGN

Thermal expansion coefficients are generally taken from bulk values, and these may vary by as much as 10% from actual values for gage blocks. This problem can be circumvented in the intercomparison process by insuring that blocks being intercompared are close to 20°C. If non-standard temperatures are used, the coefficients must be known unless the attendant systematic errors are tolerable.

4.1.3 Environment

A temperature controlled laboratory is necessary for intercomparisons of the highest precision. The degree of temperature control needed depends on the length of the blocks being compared, differences in coefficients of thermal expansion among the blocks, and the limiting uncertainty required of the measurements. At NBS, short blocks are intercompared in a temperature controlled laboratory at 20°C + 0.25°C. Long blocks are intercompared in a laboratory at 20°C + 0.05°C.* Relative humidity should be held below 50% to prevent corrosion of blocks and instruments.

4.1.4 Temperature Effects and Their Control

A large uncertainty in the comparison process can be introduced by temperature effects. For example, a temperature difference of 0.5°C between two one-inch steel blocks will cause an error of nearly 6 micro-inches in the comparison. Two causes of temperature differences between blocks are sometimes overlooked:

(1) Room temperature gradients or nearby heat sources such as electronic equipment can cause significant temperature differences between blocks even when they are stored relatively close to each other before comparison.

(2) Blocks with different surface finishes on their non-gaging faces can absorb radiant heat at different rates and reach different equilibrium temperatures. The magnitude of these effects is proportional to gage block length.

A number of remedies are available to alleviate temperature gradients. For short blocks the remedies are quite simple. For example, store the blocks, both standards and unknowns, on a thermal equalization plate of smooth surface and good heat conductivity close to the comparator but away from heat sources. Also, use tweezers or tongs to handle the blocks and use a systematic, rythmic block handling technique in the comparison procedure to insure a nearly identical thermal environment for each block.

*Note: This does not mean that every laboratory needs this level of temperature control.

4.2 Measurement Techniques

The sequence of observations for each of the three program options has been described in sections 3.2.1, 3.3.1, and 3.4.1. This section will concentrate on techniques that have been found to be important in achieving good results.

4.2.1 Block Preparation

The master and test blocks must be thoroughly cleaned, examined, and deburred using procedures in reference [13]. The identification numbers are recorded for inclusion in the test report or records.

4.2.2 Comparator Preparation

The instrument anvil should be deburred and cleaned. The comparator transducer pressure, magnification and alignment should be checked. Periodic cleaning of the instrument anvil during the work day is recommended to help reduce the number of spurious readings that result from minute particles that can contaminate the anvil surface.

4.2.3 Block Storage

Short blocks are arranged on a thermal equalization plate next to the comparator. From here they can be moved to the comparator anvil by groups at the time of comparison. There is some thermal advantage, for sizes from about 0.3 inch to 4 inches, to leaving the blocks on the plate at all times except when the block is being inserted in the comparator for measurement. Alternatively, a group can be moved to the comparator anvil and allowed to equalize there for an appropriate period.

Long gage blocks are stored in a group on the comparator anvil after preparation. Additional long blocks that are to be measured during the day are prepared and placed on a thermal equalization plate next to the instrument. As comparisons of one group are completed a new size group can be moved from the plate to the anvil and allowed to equalize before comparison. All gage blocks are oriented on the comparator with the top surface uppermost.

4.2.4 Thermal Equalization Time

Equalization time varies with block size, treatment and allowable measurement uncertainty. Blocks prepared in advance and kept in the gaging area are placed on the equalization plate, or in the case of long blocks, moved from the equalization plate to the comparator anvil. They may then be intercompared using the following table as a first approximation.

Block Size (inches)	Equalization Time (minutes)
0.100 to 0.250	30
0.300 to 1.000	60
2.000 to 20.000	90

Experiments establishing optimum equalization times should be conducted in your own laboratory because of the many variables involved and differing measurement uncertainty requirements.

4.2.5 Temperature Measurement

Temperature measurements can be made with a calibrated mercury-in-glass thermometer. The thermometer is mounted on the block storage plate in the case of short blocks and on the instrument anvil for the long blocks. The more sophisticated temperature measuring devices such as thermocouples and thermistors are very useful for detecting gradients and inequalities.

4.2.6 Handling Techniques

The success of intercomparisons is largely dependent upon block handling techniques. Proper technique includes the insertion of all blocks between the styli in a like manner. The operator should develop a rhythm, after acquiring some experience with the process, that will ensure that each pair of blocks is handled for approximately the same length of time as all other pairs in the series.

A camel's hair brush or an air bulb is useful for sweeping or blowing dust particles from the blocks and the anvil just before insertion.

The short blocks are moved about by grasping them with rubber tipped 10-inch tweezers. When handling square style blocks, the tips of a pair of tweezers may be bent to accommodate this configuration.

The sequence of observations in option 3 was developed to compensate instrument and temperature drifts, but it still relies on equal handling of blocks for good results.

4.2.7 Temperature and Deformation Corrections

Deformation corrections for various stylus radii and pressures [3], can be applied to the observations when blocks of different materials are compared if the correction is of sufficient magnitude to be significant.

Temperature corrections are applied to all blocks above 0.350 inch in size when the blocks are of different materials. This size limit can be raised if larger uncertainties are acceptable.

4.3 Computation and Analysis of Data

NBS staff members will be available to help participants with computations and analysis of the results especially in the early stages of the program, but extensive services of this type must be done on an at-cost basis. Our general aim is to make participants as self sufficient as possible.

Laboratories wishing to receive a signed and documented NBS test report should submit their data and worksheets to Mr. Clyde Tucker, Room B104, Metrology Bldg., NBS, Washington, D.C. 20234. Responsibility for the calculations and resulting report will be assumed by NBS.

4.4 Control Charts

A useful tool for monitoring the progress of the measurement process is the control chart. This is a graphical presentation of the output from the calibration process on a continuing basis.

After each calibration the value of the control and the associated standard deviation(s) should be plotted against a time scale.

The chart of the control block values should have the control limits marked so that an out-of-control value is immediately visible (see Figure 18). It is also the quickest means of spotting a change in the size of the control block with time (see Figure 20) and should be carefully monitored especially for block sizes of one inch and longer.

The chart of the standard deviations provides visual evidence of the within day process variation (see Figure 19) and makes it possible to ascertain when a change in the process has occurred (see Figure 21).

4.5 Access to the National Standards

One or two sets of NBS reference standard gage blocks will be loaned to each participant at periodic intervals for measurement. The period will be governed by the performance and history of the participant's process. A time limitation may be imposed because of demand for these sets. Participants may chose to send their standard sets to NBS for periodic calibration instead of using the loaned sets.

4.6 Procedures for Correcting Out-of-Control Conditions

If the process is found to be out-of-control by an F-test, repeat the offending measurements to determine if the condition persists. If it persists, look for:

(1) Comparator malfunction
(2) Dust or other contamination on the gage blocks

FIGURE 18
OBSERVED DIFFERENCE BETWEEN 1–INCH GAGE BLOCKS USED AS CHECK STANDARD

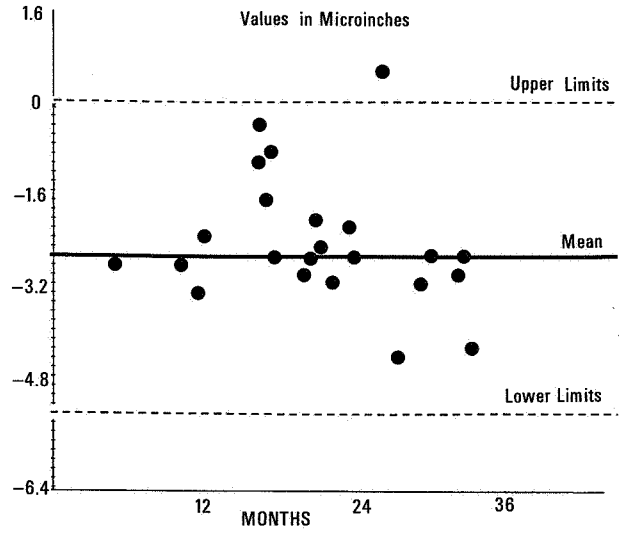


FIGURE 19
OBSERVED WITHIN STANDARD DEVIATIONS ON 1–INCH CHECK STANDARD

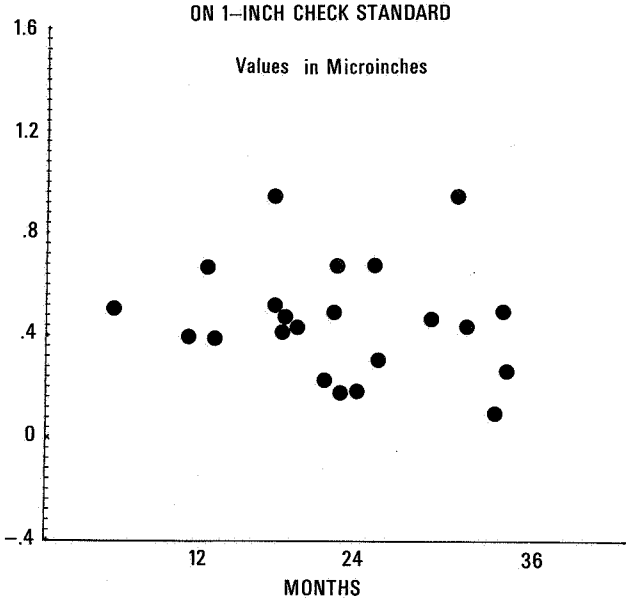


FIGURE 20
OBSERVED DIFFERENCES BETWEEN 6–INCH GAGE BLOCKS USED AS CHECK STANDARD

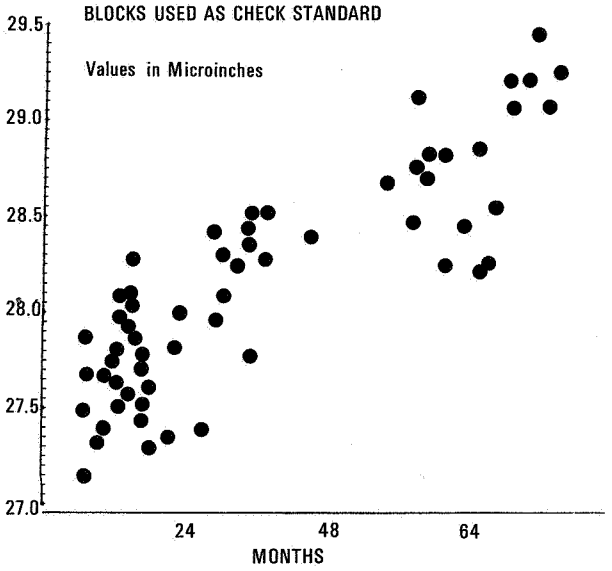
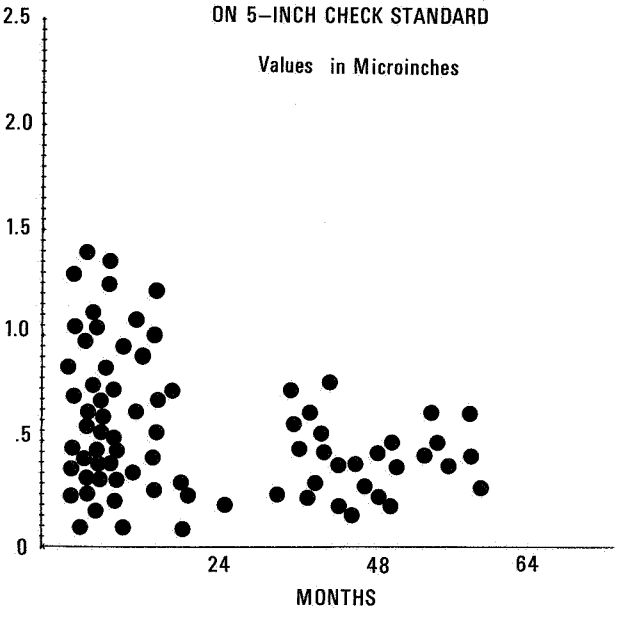


FIGURE 21
OBSERVED WITHIN STANDARD DEVIATIONS ON 5–INCH CHECK STANDARD



(3) Temperature problems, in blocks over 0.5 inch, such as too short an equalization time, heat source (including operator) too close to the comparator, and temperature difference between gage block equalization plate and the comparator.
(4) Lack of finesse in gage block handling during comparisons.

If the process is found to be out of control by a t-test:

(1) Look for dirt or burrs on the blocks (stoning may help).
(2) Look at block history for evidence of steady drift toward out-of-control condition. This would indicate a length change and a need to recompute the accepted difference and possibly a recalibration against NBS standards.
(3) If a large number of block sizes are out of control, check comparator calibration and function.
(4) For blocks larger than 0.5 inch, look for temperature differences between the blocks.

4.7 Upgrading the Process

A process may perform within the established control pattern but still not be adequate to the assigned goal. Some or all of the following changes may be made to improve the process:

(1) Upgrade the quality of the standard sets by purchasing new blocks.
(2) Obtain a better comparator if one is available.
(3) Improve temperature conditions by removing heat sources from vicinity of comparator, isolating operator from comparator by shielding, improving temperature control in lab, etc.
(4) Improve handling techniques during comparisons by equalizing time each block is handled, taking greater care about cleanliness, etc.
(5) Switch to option 3 (see Section 3.4) if not already using it.

ACKNOWLEDGEMENTS

The authors wish to thank Mr. J. M. Cameron for his help and guidance in preparing this document and Mrs. Kay Etzler and Mrs. Barbara Uglik for their typing skill and patience through many revisions of the manuscript.

TABLE I

F VALUES, UPPER 1% PROBABILITY LEVEL

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999·5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98·50	99·00	99·17	99·25	99·30	99·33	99·36	99·37	99·39	99·40	99·42	99·43	99·45	99·46	99·47	99·47	99·48	99·49	99·50
3	34·12	30·82	29·46	28·71	28·24	27·91	27·67	27·49	27·35	27·23	27·05	26·87	26·69	26·60	26·50	26·41	26·32	26·22	26·13
4	21·20	18·00	16·69	15·98	15·52	15·21	14·98	14·80	14·66	14·55	14·37	14·20	14·02	13·93	13·84	13·75	13·65	13·56	13·46
5	16·26	13·27	12·06	11·39	10·97	10·67	10·46	10·29	10·16	10·05	9·89	9·72	9·55	9·47	9·38	9·29	9·20	9·11	9·02
6	13·75	10·92	9·78	9·15	8·75	8·47	8·26	8·10	7·98	7·87	7·72	7·56	7·40	7·31	7·23	7·14	7·06	6·97	6·88
7	12·25	9·55	8·45	7·85	7·46	7·19	6·99	6·84	6·72	6·62	6·47	6·31	6·16	6·07	5·99	5·91	5·82	5·74	5·65
8	11·26	8·65	7·59	7·01	6·63	6·37	6·18	6·03	5·91	5·81	5·67	5·52	5·36	5·28	5·20	5·12	5·03	4·95	4·86
9	10·56	8·02	6·99	6·42	6·06	5·80	5·61	5·47	5·35	5·26	5·11	4·96	4·81	4·73	4·65	4·57	4·48	4·40	4·31
10	10·04	7·56	6·55	5·99	5·64	5·39	5·20	5·06	4·94	4·85	4·71	4·56	4·41	4·33	4·25	4·17	4·08	4·00	3·91
11	9·65	7·21	6·22	5·67	5·32	5·07	4·89	4·74	4·63	4·54	4·40	4·25	4·10	4·02	3·94	3·86	3·78	3·69	3·60
12	9·33	6·93	5·95	5·41	5·06	4·82	4·64	4·50	4·39	4·30	4·16	4·01	3·86	3·78	3·70	3·62	3·54	3·45	3·36
13	9·07	6·70	5·74	5·21	4·86	4·62	4·44	4·30	4·19	4·10	3·96	3·82	3·66	3·59	3·51	3·43	3·34	3·25	3·17
14	8·86	6·51	5·56	5·04	4·69	4·46	4·28	4·14	4·03	3·94	3·80	3·66	3·51	3·43	3·35	3·27	3·18	3·09	3·00
15	8·68	6·36	5·42	4·89	4·56	4·32	4·14	4·00	3·89	3·80	3·67	3·52	3·37	3·29	3·21	3·13	3·05	2·96	2·87
16	8·53	6·23	5·29	4·77	4·44	4·20	4·03	3·89	3·78	3·69	3·55	3·41	3·26	3·18	3·10	3·02	2·93	2·84	2·75
17	8·40	6·11	5·18	4·67	4·34	4·10	3·93	3·79	3·68	3·59	3·46	3·31	3·16	3·08	3·00	2·92	2·83	2·75	2·65
18	8·29	6·01	5·09	4·58	4·25	4·01	3·84	3·71	3·60	3·51	3·37	3·23	3·08	3·00	2·92	2·84	2·75	2·66	2·57
19	8·18	5·93	5·01	4·50	4·17	3·94	3·77	3·63	3·52	3·43	3·30	3·15	3·00	2·92	2·84	2·76	2·67	2·58	2·49
20	8·10	5·85	4·94	4·43	4·10	3·87	3·70	3·56	3·46	3·37	3·23	3·09	2·94	2·86	2·78	2·69	2·61	2·52	2·42
21	8·02	5·78	4·87	4·37	4·04	3·81	3·64	3·51	3·40	3·31	3·17	3·03	2·88	2·80	2·72	2·64	2·55	2·46	2·36
22	7·95	5·72	4·82	4·31	3·99	3·76	3·59	3·45	3·35	3·26	3·12	2·98	2·83	2·75	2·67	2·58	2·50	2·40	2·31
23	7·88	5·66	4·76	4·26	3·94	3·71	3·54	3·41	3·30	3·21	3·07	2·93	2·78	2·70	2·62	2·54	2·45	2·35	2·26
24	7·82	5·61	4·72	4·22	3·90	3·67	3·50	3·36	3·26	3·17	3·03	2·89	2·74	2·66	2·58	2·49	2·40	2·31	2·21
25	7·77	5·57	4·68	4·18	3·85	3·63	3·46	3·32	3·22	3·13	2·99	2·85	2·70	2·62	2·54	2·45	2·36	2·27	2·17
26	7·72	5·53	4·64	4·14	3·82	3·59	3·42	3·29	3·18	3·09	2·96	2·81	2·66	2·58	2·50	2·42	2·33	2·23	2·13
27	7·68	5·49	4·60	4·11	3·78	3·56	3·39	3·26	3·15	3·06	2·93	2·78	2·63	2·55	2·47	2·38	2·29	2·20	2·10
28	7·64	5·45	4·57	4·07	3·75	3·53	3·36	3·23	3·12	3·03	2·90	2·75	2·60	2·52	2·44	2·35	2·26	2·17	2·06
29	7·60	5·42	4·54	4·04	3·73	3·50	3·33	3·20	3·09	3·00	2·87	2·73	2·57	2·49	2·41	2·33	2·23	2·14	2·03
30	7·56	5·39	4·51	4·02	3·70	3·47	3·30	3·17	3·07	2·98	2·84	2·70	2·55	2·47	2·39	2·30	2·21	2·11	2·01
40	7·31	5·18	4·31	3·83	3·51	3·29	3·12	2·99	2·89	2·80	2·66	2·52	2·37	2·29	2·20	2·11	2·02	1·92	1·80
60	7·08	4·98	4·13	3·65	3·34	3·12	2·95	2·82	2·72	2·63	2·50	2·35	2·20	2·12	2·03	1·94	1·84	1·73	1·60
120	6·85	4·79	3·95	3·48	3·17	2·96	2·79	2·66	2·56	2·47	2·34	2·19	2·03	1·95	1·86	1·76	1·66	1·53	1·38
∞	6·63	4·61	3·78	3·32	3·02	2·80	2·64	2·51	2·41	2·32	2·18	2·04	1·88	1·79	1·70	1·59	1·47	1·32	1·00

with permission

Reprinted from Biometrika Tables for Statisticians, E. S. Pearson and H. O. Hartley, editors, Vol. 1, p. 161, The University Press, Cambridge (1956).

REFERENCES

[1] Beers, J. S. & Tucker, C. D., "Gage Block Flatness and Parallelism Measurement," NBSIR 72-239, 1973.

[2] Beers, J. S. & Tucker, C. D. "Intercomparison Procedures for Gage Blocks Using Electromechanical Comparators," NBSIR 76-979, 1976.

[3] Beers, J. S. & Taylor, J. E., "Contact Deformation in Gage Block Comparisons," NBS Technical Note 962, 1978.

[4] Beers, J. S., "A Gage Block Measurement Process Using Single Wavelength Interferometry," NBS Monograph 152, December 1975.

[5] Cameron, J. M., "Measurement Assurance," NBSIR 77-1240, April 1977.

[6] Cameron, J. M., "Measurement Assurance," Journal of Quality Technology, Vol. 8, No. 1, pp. 53-55, January 1976.

[7] Cameron, J. M. & Hailes, Geraldine E., "Designs for the Calibration of Small Groups of Standards in the Pressence of Drift," NBS Technical Note 844, August 1974.

[8] Cameron, J. M., Croarkin, M. C., Raybold, R. C., "Designs for the Calibration of Standards of Mass," NBS Technical Note 952, June 1977.

[9] Ku, Harry H., "Statistical Concepts in Metrology," Precision Measurement and Calibration, NBS SP 300, Vol. 1, pp. 296-330.

[10] Pontius, P. E., "Measurement Assurance Program--A Case Study: Length Measurements, Part 1. Long Gage Blocks (5 In. to 20 In.)," NBS Monograph 149, November 1975.

[11] Pontius, P.E., "Measurement Philosophy of the Pilot Program for Mass Calibration," Technical Note 288, May 1966.

[12] Pontius, P. E., "Notes on the Fundamentals of Measurement and Measurement as a Production Process," NBSIR 74-545, September 1974.

[13] Tucker, C. D., "Preparations for Gage Block Comparison Measurements," NBSIR 74-523, 1974.

[14] "Measurement Control Program for Special Nuclear Materials Control and Accounting (10 CFR 70.57)," Federal Register, Vol. 40, No. 155, pp. 33651-33653, August 1975.

APPENDIX A

TREND ELIMINATING DESIGN

Let the two test sets be designated by A and B and the two standard sets by S_1 and S_2. The design involves making the following observations in the order given:

$$z_1 = S_1 - S_2$$

$$z_2 = B - S_1$$

$$z_3 = A - B$$

$$z_4 = S_2 - A$$

$$z_5 = S_2 - B$$

$$z_6 = B - S_1$$

$$z_7 = S_1 - A$$

$$z_8 = A - S_2$$

The least squares estimates of the standards S_1 and S_2 are given by:

$$\hat{S}_1 = \frac{1}{24}\{5z_1 - 2z_2 - z_3 - 2z_4 - 3z_5 - 2z_6 + 3z_7 + 2z_8 + 12K\}$$

$$\hat{S}_2 = \frac{1}{24}\{-5z_1 + 2z_2 + z_3 + 2z_4 + 3z_5 + 2z_6 - 3z_7 - 2z_8 + 12K\}$$

The estimate of the difference $L_c = S_1 - S_2$ is

$$\hat{L}_c = \frac{1}{12}\{5z_1 - 2z_2 - z_3 - 2z_4 - 3z_5 - 2z_6 + 3z_7 + 2z_8\}$$

The estimates for the test blocks are:

$$\hat{A} = \frac{1}{24}\{-z_1 + 2z_2 + 5z_3 - 6z_4 - z_5 + 2z_6 - 7z_7 + 6z_8 + 12K\}$$

$$\hat{B} = \frac{1}{24}\{z_1 + 6z_2 - 5z_3 - 2z_4 - 7z_5 + 6z_6 - z_7 + 2z_8 + 12K\}$$

where K is the assigned value of S_1 plus the assigned value of S_2.

The variances of the parameters are given by:

$$V(\hat{A}) = V(\hat{B}) = \frac{13}{48}\sigma_w^2$$

$$V(\hat{L}_c) = \frac{5}{12}\sigma_w^2$$

where σ_w^2 is the within process variance whose least squares estimate is given by:

$$\hat{\sigma}_w^2 = \frac{\sum_{i=1}^{8} dev_i^2}{4}$$

$$dev_1 = \frac{1}{168}\{49z_1 - 7z_2 - 7z_3 + 21z_4 + 49z_5 + 49z_6 - 7z_7 + 21z_8\}$$

$$dev_2 = \frac{1}{168}\{-7z_1 + 87z_2 + 13z_3 - 5z_4 + 33z_5 - 41z_6 + 53z_7 + 35z_8\}$$

$$dev_3 = \frac{1}{168}\{-7z_1 + 13z_2 + 89z_3 + 25z_4 - 39z_5 + 37z_6 + 57z_7 - 7z_8\}$$

$$dev_4 = \frac{1}{168}\{21z_1 - 5z_2 + 25z_3 + 111z_4 - 27z_5 + 3z_6 - 23z_7 + 63z_8\}$$

$$dev_5 = \frac{1}{168}\{49z_1 + 33z_2 - 39z_3 - 27z_4 + 97z_5 + 25z_6 + 9z_7 + 21z_8\}$$

$$dev_6 = \frac{1}{168}\{49z_1 - 41z_2 + 37z_3 + 3z_4 + 25z_5 + 103z_6 + 13z_7 - 21z_8\}$$

$$dev_7 = \frac{1}{168}\{-7z_1 + 53z_2 + 57z_3 - 23z_4 + 9z_5 + 13z_6 + 73z_7 - 7z_8\}$$

$$dev_8 = \frac{1}{168}\{21z_1 + 35z_2 - 7z_3 + 63z_4 + 21z_5 - 21z_6 - 7z_7 + 63z_8\}$$

The pooled within standard deviation for n calibrations would be

$$\xi = \sqrt{\frac{\sum_{i=1}^{n} \hat{\sigma}_{w_i}^2}{n}}$$

The following data was taken on two test blocks A and B and two standard blocks S_1 and S_2 using the trend eliminating design over a six month period.

DATA ON FOUR BLOCKS OF NOMINAL SIZE 0.500 INCHES

Corrections in Microinches

Observations	Run	1		2		3	
S_1-S_2		55.9	51.7	54.0	51.1	53.9	50.2
B-S_1		56.3	55.9	52.1	54.2	52.0	53.5
A - B		56.0	56.5	51.2	52.2	55.7	51.0
S_2-A		51.9	55.2	51.1	52.2	50.8	56.0
S_2-B		52.0	56.8	51.2	52.8	51.0	51.4
B-S_1		57.0	56.0	52.7	54.3	51.7	54.9
S_1-A		56.1	56.0	54.7	52.0	54.7	54.9
A-S_2		56.2	52.2	52.1	51.5	56.3	51.8

	4		5		6	
	53.0	50.0	56.0	54.0	52.8	51.3
	54.0	53.2	55.0	56.0	55.3	53.0
	63.2	54.2	54.8	55.8	51.1	55.9
	50.5	63.0	52.2	54.2	52.2	51.8
	50.2	54.5	52.0	55.0	50.3	55.5
	54.8	53.8	55.0	55.5	56.0	53.1
	53.8	63.0	55.0	53.8	53.1	51.9
	63.0	50.2	53.8	52.2	52.0	50.3

The least squares estimates of the parameters are as follows:

Run	$\hat{L}_c$	$\hat{A}$	$\hat{B}$	$\hat{\sigma}_w$
1	4.00	54.11	54.91	.407
2	3.26	51.35	52.18	.283
3	3.60	55.21	51.21	.930
4	3.02	63.45	54.98	.537
5	2.82	52.54	53.32	.525
6	1.98	51.84	56.22	.729

If this group of measurements is the basis for the starting values in a measurement assurance program, then the value of the control $L_c = S_1 - S_2$ would be the average 3.11; the pooled within standard deviation would be $\xi = 0.606$ and the total standard deviation 0.696. $\hat{\sigma}$

Note: The restraint, K, is the assigned value of S_1 plus the assigned value of S_2. In the example, $K = 52.28$ μin.

APPENDIX B

B.1 Interferometric Gage Block Comparator, No Standards, One or More Measurements on Unknowns, One Set of Control Blocks

B.1.1 The Measurement Process

Test blocks can be measured directly in terms of light wavelengths. This can be done with fringe counting interferometric comparators, or static interferometers of the Kosters or Fizeau types. In most processes duplicate measurements are made. Unless the second set of measurements are separated far enough in time to be statistically independent, the differences should not be used in setting bounds for the process random error.

The random errors associated with the process are of two kinds: those arising from repetitions in the short term (a few minutes) and those involving long-term differences (day to day, week to week, etc.). The random error appropriate for regular calibration is that associated with the repeated measurement of a control block over a sufficiently long time period to insure that all factors affecting variability have a chance to use their influence. The amount of variation in the measurement process is usually length dependent because of temperature effects on the blocks and because of ambient air conditions affecting the wavelength. For this reason and for convenience of working group size, the usual set of blocks (80 or so blocks in the 0.050 to 4 in. range) should be treated as a number of subsets of up to 20 blocks each. At NBS the six groups listed below are used:

Group	Nominal Length (Inches)	Approximate No. of Blocks
I	0.050 to 0.09375	4
II	0.100 to 0.107	20
III	0.108 to 0.126	20
IV	0.127 to 0.146	20
V	0.147 to 0.500	15
VI	0.550 to 4.000	13

Redundancy is introduced into the system by repeatedly including a control block in the measurement procedure. The control set should be made up of at least one block from each group with more blocks of the larger sizes. A suggested set of control blocks is:

Group	Nominal Size (Inches)	No. of Control Blocks K
I	0.05	1
II	0.1	1
III	0.125	1
IV	0.140	1
V	0.25, 0.50	2
VI	0.75, 1.0, 2.0, 4.0	4

One would expect the same variability for all control blocks in a group so that the standard deviations* computed for each of the control blocks in a group could be combined into one overall standard deviation for the group. If $s_1, \ldots s_k$ are the standard deviations for the k blocks in a group with degrees of freedom $\nu_1, \ldots \nu_k$ respectively, then

$$\text{s.d. (group)} = \sqrt{\frac{\nu_1 s_1^2 + \ldots + \nu_k s_k^2}{\nu_1 + \ldots + \nu_k}}$$

B.1.2 Establishing Process Parameters

To determine initial accepted values** for the controls and for the random error component, all the blocks in the control set should be measured by the usual process, say 6 times, with a few days between repetitions.

From that initial data, an average and a standard deviation should be computed for each control block. The accepted values for the control and for the random error component will be the average value for the block and the standard deviation for the group as indicated above. These will be the starting accepted values for the process. A portion of a typical worksheet showing starting values for two blocks in a control set is given in Figure 1 (page 17). Examples given in the following sections are based on these two blocks and their parameters. All data in the examples is illustrative and is intended only to explain the methodology.

*The standard deviation of a single observation is given by

$$s = \sqrt{\frac{\Sigma\, r_i^2}{n-1}} \qquad \text{where}$$

the r_i is the difference between each observation and the average of n observations. The quantity n-1 is called the degrees of freedom associated with s.

**The term "value" for a gage block means the deviation from nominal size (measured length minus nominal length) at 20 °C.

B.1.3 Maintaining Process Control

After the process parameters have been established, control is maintained by checking the control value after each calibration run against the accepted control value. A sample worksheet (see Figure 1) details how this is done and shows the appropriate statistical tests for determining whether or not the process is in control.

B.1.4 Updating Process Parameters

As information collects on the regularly used control blocks, the accepted values for these blocks should be updated. A sample worksheet showing how this is done is given in Figure 3 (page 20).

The frequency of updating will depend on the workload but should be done after 5 or 10 values initially and then at intervals dictated by convenience (e.g., every 6 months or a year).

B.1.5 Relationship to National Standards

It is important to know if a process produces values which are consistent with the National standards as maintained by NBS. A convenient method for testing the whole system is afforded by using two calibrated sets loaned to the laboratory by NBS (or two NBS calibrated sets owned by the laboratory). These need not be complete sets but could be small sets consisting of a size distribution of blocks similar to the control set. If each of these sets is measured as a test set in the calibration procedure, one will have the independent check needed to test for consistency with NBS. The data should be taken by the regular calibration process. Two complete calibrations should be done a day or two apart.

Laboratories with the capability of writing their own documented test report based on their measurements of the NBS sets are encouraged to do so. In this case NBS will provide the two calibrated sets of blocks, and their current assigned values. The table in Figure 4 (page 22) summarizes the analysis of the data and shows whether or not there is a significant offset from the NBS process. If the offset is significant, the values for the laboratory reference standards and their associated uncertainties should be corrected as shown in the suggested "Report of Test" form in Figure B.2. In effect this gives each laboratory a method for determining the offset from the NBS process.

B.1.6 Uncertainty

The uncertainty, $\pm U$, associated with any test block calibrated by this process, assuming the process is in control, is given by

$$U = E + 3s_G$$

where E is the offset between the laboratory's process and the NBS process (see Figure B.2), and s_G is the accepted group standard deviation for that block size (see Figure 4). For convenience one may use the maximum uncertainty found in the group so as to report a single uncertainty value for the whole group.

B.1.7 Summary

In sections B.1.3 and B.1.5 tests are described which determine if the process is in control. In the absence of a specific format for out-of-control conditions, it should be noted that once predictability is lost, no statements should be made about the condition of the process. Until the process parameters have been re-established or some satisfactory corrective action has been taken, any uncertainty statements which are issued should be designated as provisional.

Nearly all of the statements made in section 4 (Operating a Measurement Assurance Program) apply to the interferometric technique as well. If the process is found to be out-of-control by an F-test or a t-test, the trouble sources listed in section 4.6 are valid, but a number of other sources need to be added if a length dependent systematic error is revealed when the NBS reference sets are measured. These error sources, unique to interferometers, are:

(1) Air temperature measurement
(2) Barometric pressure measurement
(3) Humidity measurement
(4) Vacuum wavelength of the light source
(5) Wavelength correction formula in which the above parameters are used
(6) Gage block temperature measurement
(7) Misalignment of the optical axis with the measurement axis of the interferometer
(8) Offset of the optical axis from the measurement axis of the interferometer (Abbe offset).

A full discussion of an interferometric measurement process and its error sources is in reference [4]

FIGURE B.1

OBSERVED VALUE OF CONTROL COMPARED TO ACCEPTED VALUE OF CONTROL

Values in Microinches at 20 °C

	Nominal Size	Ident.	Interferometer Readings	Value of Block	Accepted Value of Control	Accepted Group S.D.	t Test*	Limit to Random Error
	(Inches)						$\lvert L-L_c \rvert / s_G$	$3s_G$
			X	L	L_c	s_G	t	R
Group II	0.10000	Test	100020.0	20.0				4.0
	0.10000	Control	100019.0	19.0	16.7	1.34	1.0	
	0.10010	Test	100114.0	14.0				4.0
	0.10020	Test	100218.4	18.4				4.0
Group V	0.147	Test	147019.8	19.8				5.2
	0.148	Test	148019.9	19.9				5.2
	0.150	Test	150017.9	17.9				5.2
	0.150	Control	150018.6	18.6	16.2	1.75	1.4	
	0.200	Test	200021.2	21.2				5.2

*If $t \geq 3$, process is out of control for that group. Repeat entire group.

FIGURE B.2

REPORT OF TEST
of
Length Calibrations From_____to_____Inches
Using
NBS Gage Block Sets_____and_____

PARTICIPATING LABORATORY

_____

Duplicate measurements made by the participating laboratory on NBS Gage Block Sets_____ and _____made by calibrating each NBS set against a standard set during a regular calibration procedure gave the following results at 20 °C (values used are in microinches):

A. Standard Deviations

Group	Accepted SD s_G	DF λ_1	From Test s_N	DF λ_2	Test F	Combined SD s_C	DF λ_3
I	--	--	--	--	--	--	--
II	--	--	--	--	--	--	--
III	--	--	--	--	--	--	--
IV	--	--	--	--	--	--	--
V	--	--	--	--	--	--	--
VI	--	--	--	--	--	--	--

Conclusions from the F-test:

(a) If $F < F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is in control, and the standard deviation used in the uncertainty statement is s_C.

(b) If $F \geq F_{.01}$ for λ_1 and λ_2 degrees of freedom, the process is out of control, and the uncertainty statement is omitted.

NOTE: Standard deviations s_G, s_N and s_C are found in Figure 4.

FIGURE B.2--continued

B. Offset of Participating Laboratory's Process from NBS

Nominal Size	From NBS Process Assigned Values NBS_1	NBS_2	From This Test Calibrated Values NBS_1	NBS_2	Offset	t Test	Uncertainty Test Block
(Inches)					$1/2\{(N_1+N_2)-(W_1+W_2)\}$		
	W_1	W_2	N_1	N_2	E	t	U
--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--

Conclusions from t test:

(a) If $t < 3$, there is no offset from the NBS process, and the systematic error E is negligible.

(b) If $t \geq 3$, there is an offset from the NBS process, and the systematic error E should be added to the random error limit $3s_C$ to get the uncertainty for any test block.

NOTE: The computations of the F and t statistics are found in Figure 4.

Errata to accompany NBS Monograph 163

Measurement Assurance for Gage Blocks
by Carroll Croarkin, John Beers and Clyde Tucker

Page 19 - Column marked "t Test*" - Change value 1.7 to 1.4

Page 29 - Last paragraph - Line 4 should read "current assigned values and associated uncertainties."

Page 45 - Column marked "Group Within S.D." - Radical sign is missing. The formula should be

$$\sqrt{\frac{\Sigma \xi_i^2}{k}}$$

Page 63 - Credit at bottom of page should read, - "Reprinted with permission from Biometrika Tables for Statisticians, E.S. Pearson and H.O. Hartley, editors, Vol. 1, p. 161, The University Press, Cambridge (1956)."

Date prepared: April 4, 1979