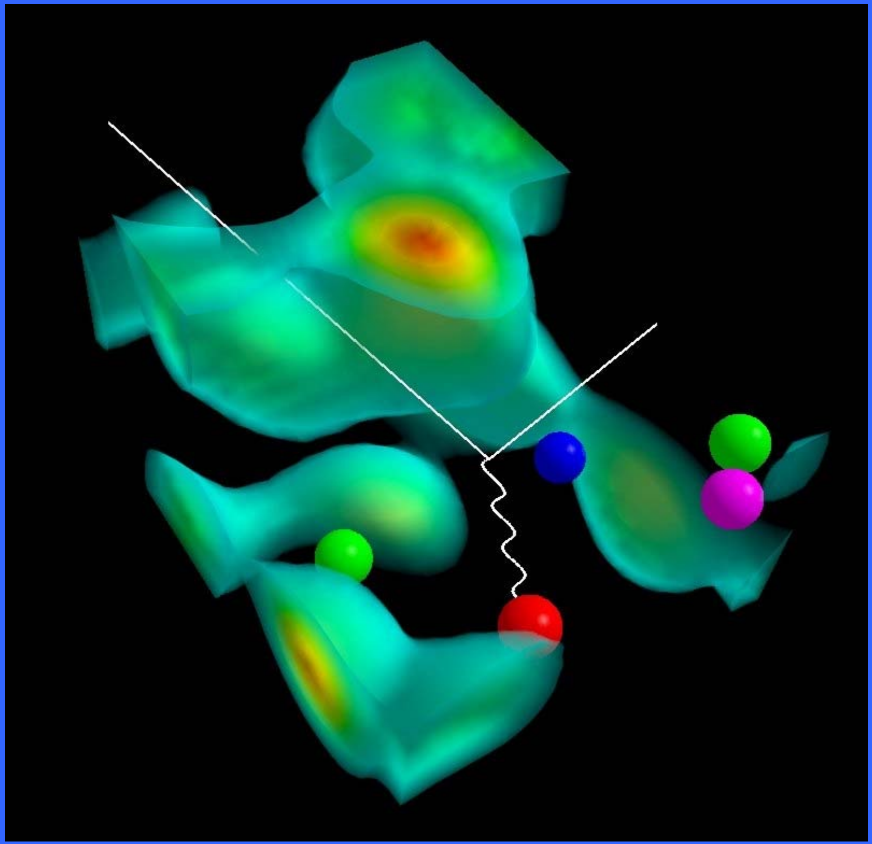
Recent Results for Hadron Structure from Lattice QCD



Anthony W. Thomas

Few Body 18, Santos Brazil : August 26th 2006

Outline

- Quantum Chromodynamics within the Standard Model

- Lattice QCD :
  there are problems ⇒ new opportunities!

  (and, by the way, some things CAN be calculated ACCURATELY)

- M_N , M_Δ , QQCD ⟺ QCD ⟺ pQQCD, M_ρ ; g_A, μ_N , $G_{E,M}^s$

Advances in Lattice QCD

Inclusion of Pion Cloud

Improvements in algorithms

χ PT allows accurate extrapolation

(needed because $t \sim m_{\pi}^{4} V^{1.25}$)

e.g. DWF $\Rightarrow$ Exact Chiral Symmetry

Precise computations at Physical Pion Mass

Advances in high-performance computing

from D. Richards

USQCD and the World

- Asqtad (Staggered) fermions:
  - Large scale generation on-going by MILC Collaboration.
  - Lattice spacing: a ~ 0.13fm, 0.09fm, 0.06fm
  - Suitable for valence Domain Wall (spin-physics) via partially quenched chiral perturbation theory
  - Not suitable for baryon spectrum program

- Clover (anisotropic):
  - Suitable for spectrum and simple form-factors
  - Anisotropy requires new calculation

- Chiral fermions (e.g., Domain-Wall/Overlap):
  - Algorithm investigations on-going at JLab
  - Large scale production by UKQCD and RBC
  - Too coarse lattice for JLab spectra

from R. Edwards

Anisotropic Clover: dynamical generation

Estimated cost of N_f=2+1 production (in TFlop-yrs) using z_π=4

$$\text{Cost(TFlop} - \text{yr)} = const \left(\frac{m_{PS}}{m_V}\right)^{-4} V(\text{fm})^{5/4} a(\text{fm})^{-7}$$

- Phase I – initial production + 10% analysis overhead
  - — Hybrid photo-couplings
  - — cost = 1.1 TF-yr + 10% analysis
- Phase II – all of 0.10fm and 0.125fm lattices
  - — Baryon spectra
  - — cost = 4.8 TF-yr + 50% analysis
- Phase III – a=0.08fm
  - — Light pion mass and continuum limit
  - — cost = 23 TF-yr + 50% analysis

Lattice Spacing	m_π (MeV)	2.4fm	3.2fm	4.0fm	Total (TFlop-yr)
$a = 0.08$ fm	181			7.9	7.9
	200		2.7	5.4	8.1
	254	0.4	1.1	2.3	3.8
	380	0.2	0.6	1.3	2.1
	485	0.05	0.1	0.3	0.5
				Total=	23 TF-yr
$a = 0.10$ fm	181			2.0	2.0
	220		0.4	1.0	1.4
	254		0.3	0.6	0.8
	300	0.05	0.1	0.3	0.5
	380	0.02	0.07	0.15	0.24
	485	0.01	0.03	0.07	0.12
		Sub-total=	1.0 TF-yr	Total=	5.1 TF-yr
$a = 0.125$ fm	200			0.3	0.3
	220			0.2	0.2
	254		0.04	0.1	0.15
	300		0.02	0.1	0.08
	380	0.005	0.01	0.06	0.04
	485	0.002	0.005	0.01	0.02
		Sub-total=	0.1 TF-yr	Total=	0.76 TF-yr

from R. Edwards

By the way….

Its NOT exp(- m_π L) that matters!

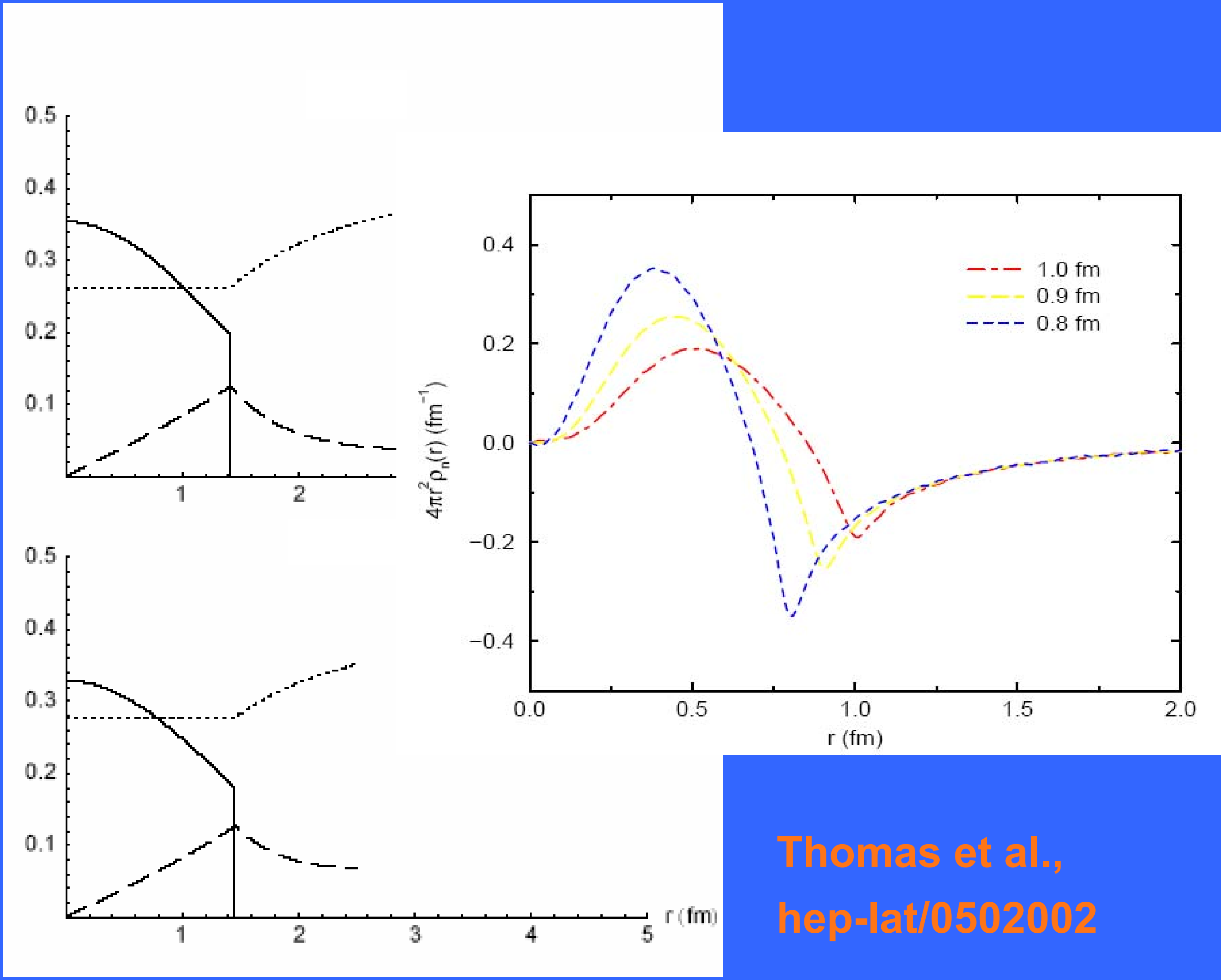
We must have

m_π (L/2 – R)>>1

with R ~ 0.8 fm

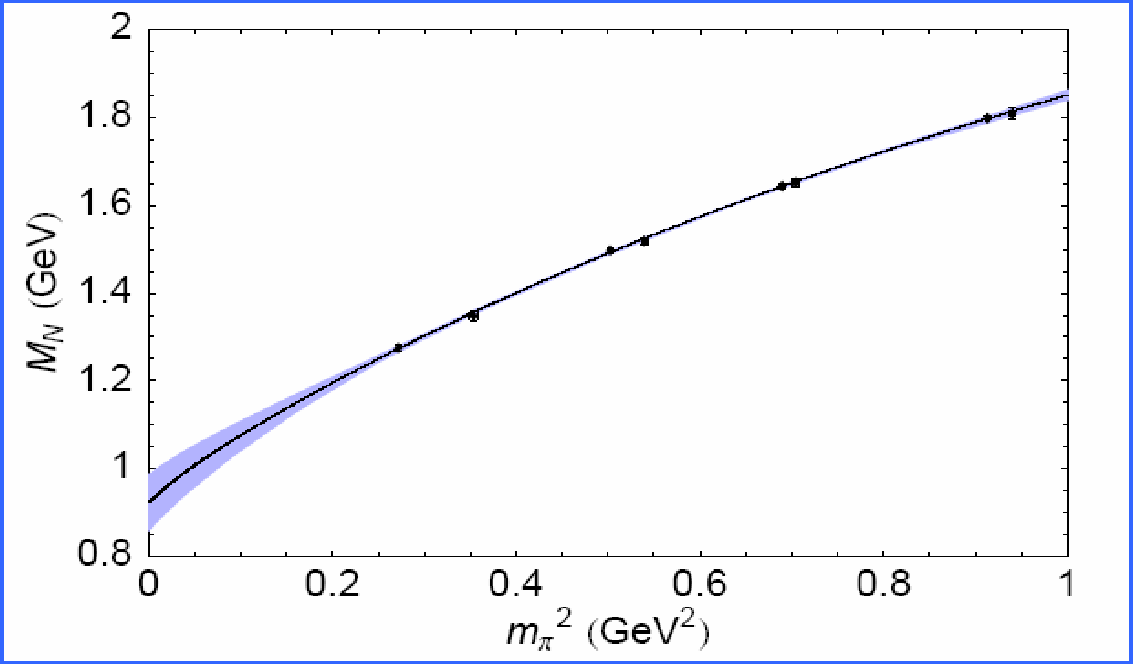
i.e. L> 2R+ 4/m_π

or L ~ 6 fm
when
m_π ~ 200 MeV



Thomas et al.,
hep-lat/0502002

χ'al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



FRR give same answer to <<1% systematic error!

	Bare Coefficients				Renormalized Coefficients			
Regulator	a_0^Λ	a_2^Λ	a_4^Λ	Λ	c_0	c_2	c_4	m_N
Monopole	1.74	1.64	−0.49	0.5	0.923(65)	2.45(33)	20.5(15)	0.960(58)
Dipole	1.30	1.54	−0.49	0.8	0.922(65)	2.49(33)	18.9(15)	0.959(58)
Gaussian	1.17	1.48	−0.50	0.6	0.923(65)	2.48(33)	18.3(15)	0.960(58)
Sharp cutoff	1.06	1.47	−0.55	0.4	0.923(65)	2.61(33)	15.3(8)	0.961(58)
Dim. Reg. (BP)	0.79	4.15	+8.92	–	0.875(56)	3.14(25)	7.2(8)	0.923(51)

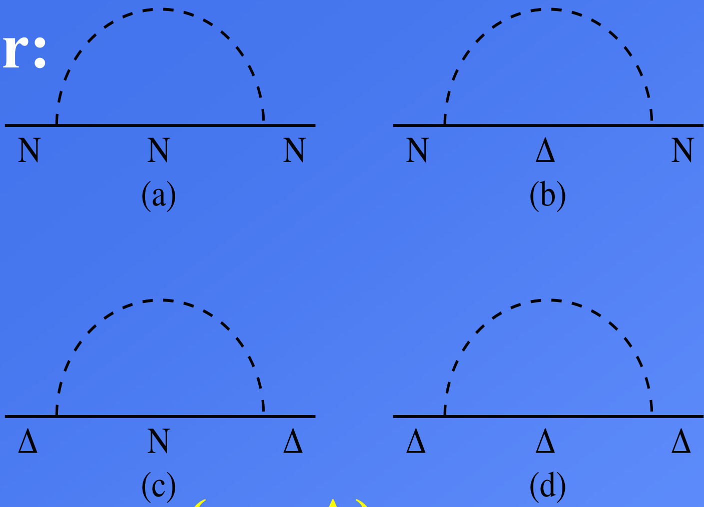
Leinweber et al., PRL 92 (2004) 242002

Extrapolation of Masses

At "large m_π" preserve observed linear (constituent-quark-<u>like</u>) behaviour: $M_H \sim m_\pi^2$

As $m_\pi \sim 0$: ensure LNA & NLNA behaviour:

(BUT must die as $(\Lambda / m_\pi)^2$ for $m_\pi > \Lambda$)

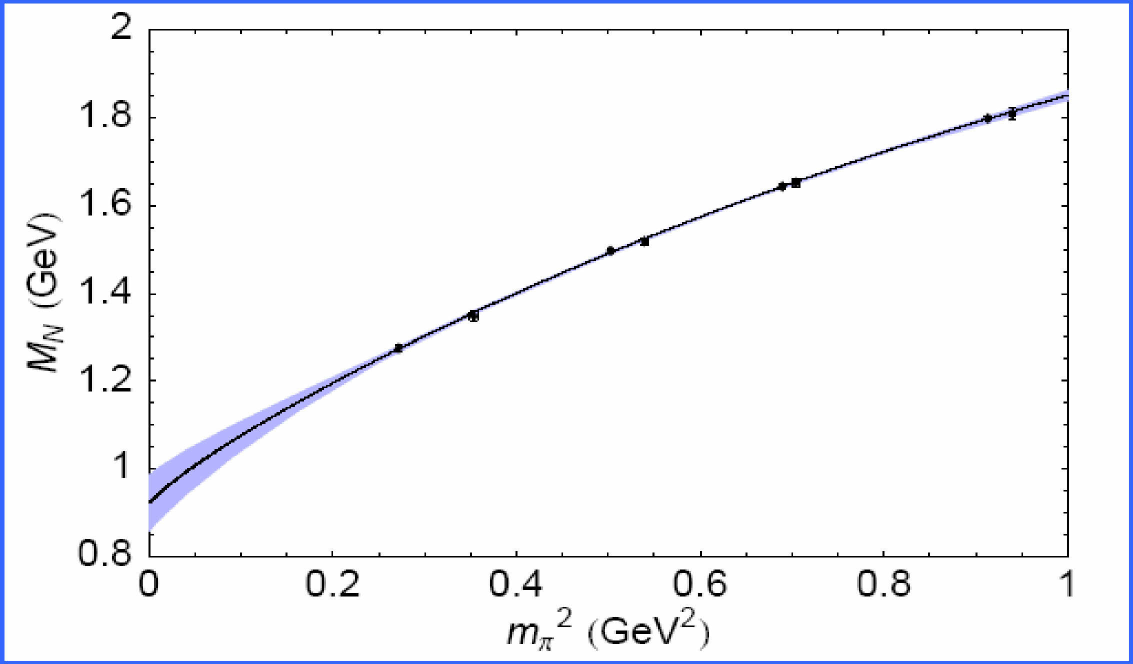


Hence use:

$$M_H = a_0 + a_2\, m_\pi^2 + a_4\, m_\pi^4 + \sigma_{LNA}(m_\pi, \Lambda) + \sigma_{NLNA}(m_\pi, \Lambda)$$

- **Evaluate self-energies with form factor , "finite range regulator", FRR, with $\Lambda \sim$ 1/Size of Hadron**

χ'al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



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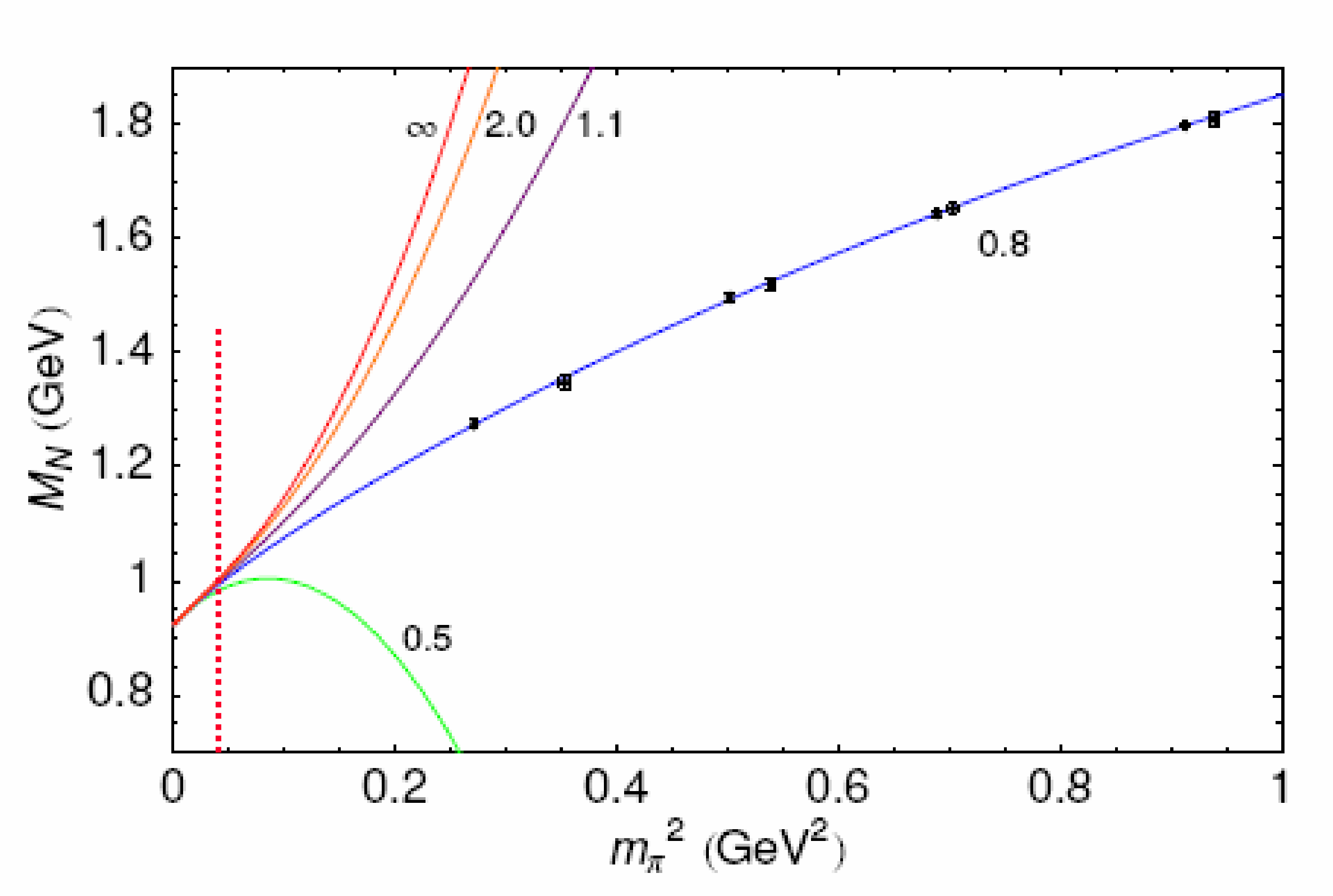
Leinweber et al., PRL 92 (2004) 242002

Power Counting Regime

Ensure coefficients c_0 , c_2 , c_4 all identical to 0.8 GeV fit



Leinweber, Thomas & Young, hep-lat/0501028

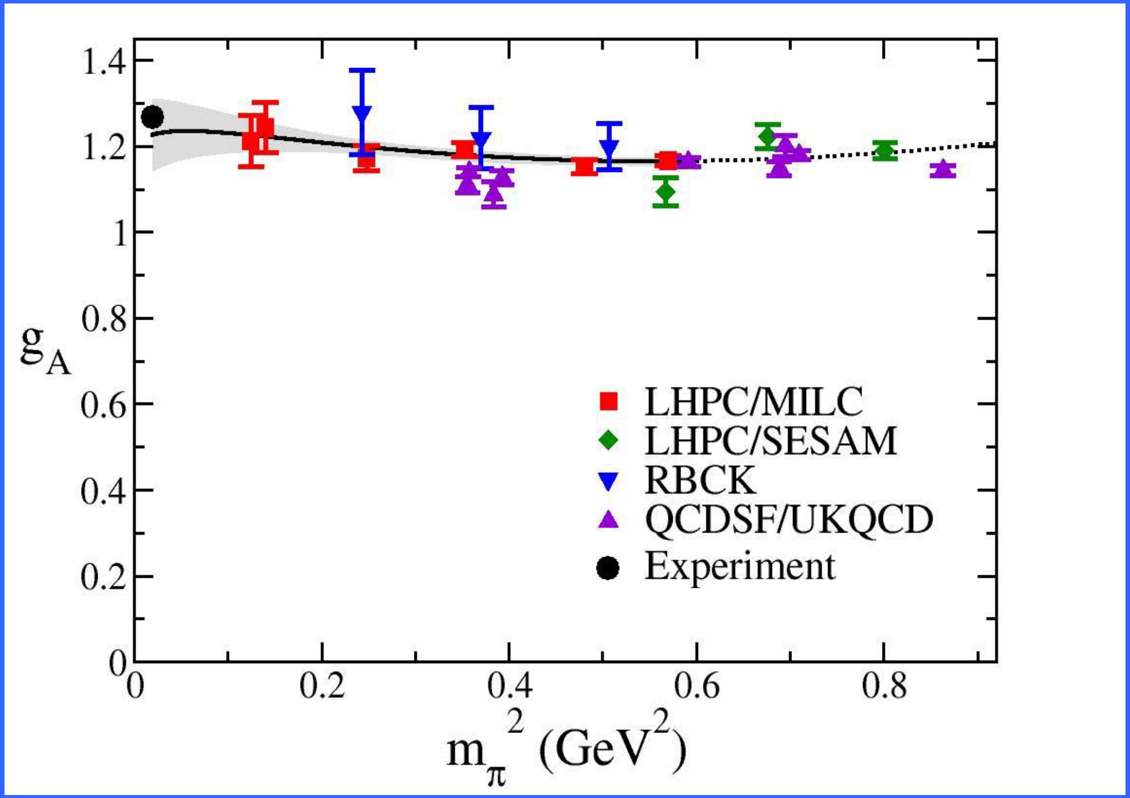
Convergence from LNA to NLNA is Rapid – Using Finite Range Regularization

Regulator	LNA	NLNA
Sharp	968	961
Monopole	964	960
Dipole	963	959
Gaussian	960	960
Dim Reg	784	884

M_N in MeV

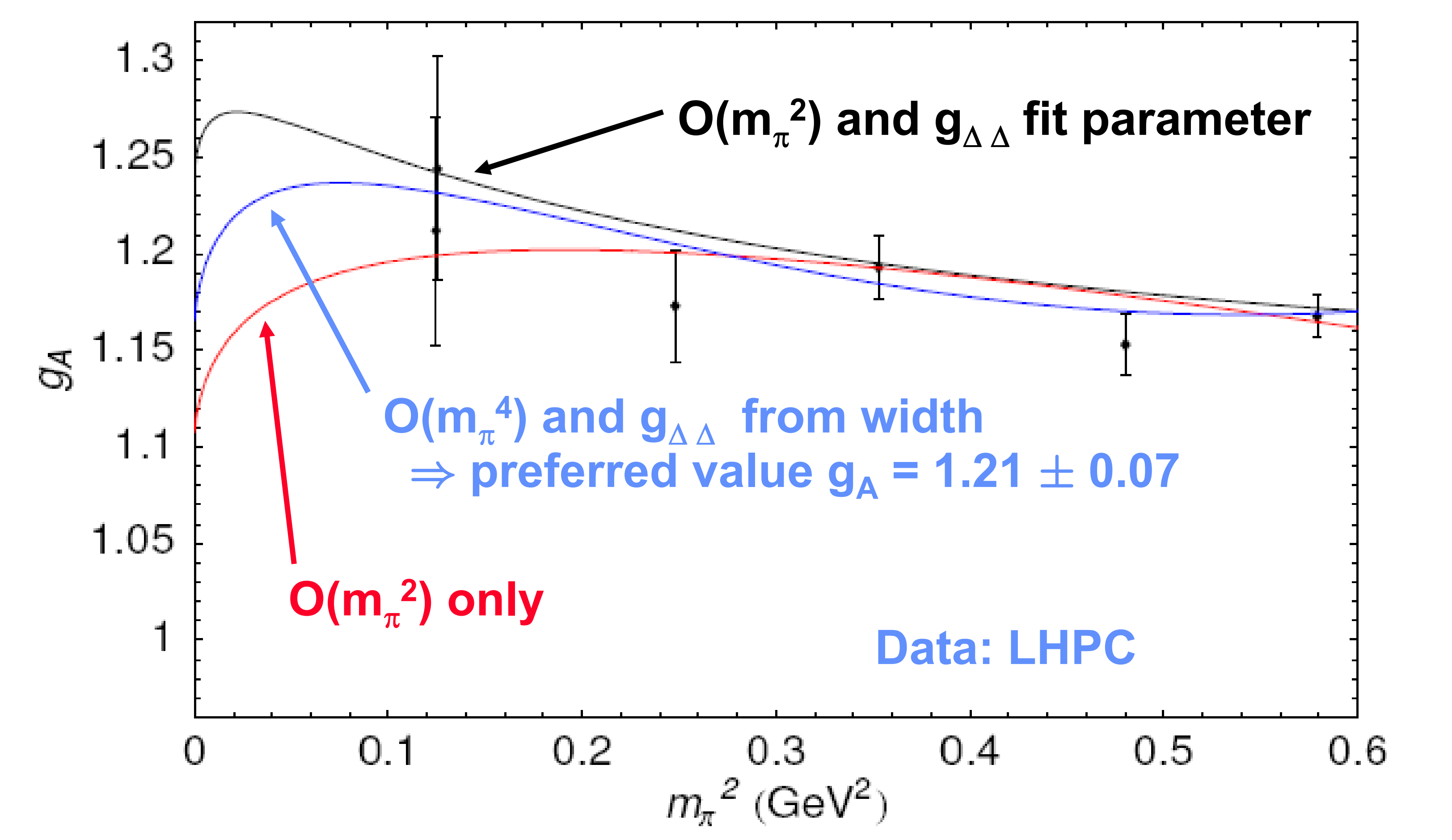
Axial Charge of the Nucleon

- **Hybrid Computation** of Hadron Structure using **MILC asqtad lattices** and **domain-wall-fermion valence quarks**
- Has enabled computations to be performed in full QCD at m_π approaching **350 MeV**



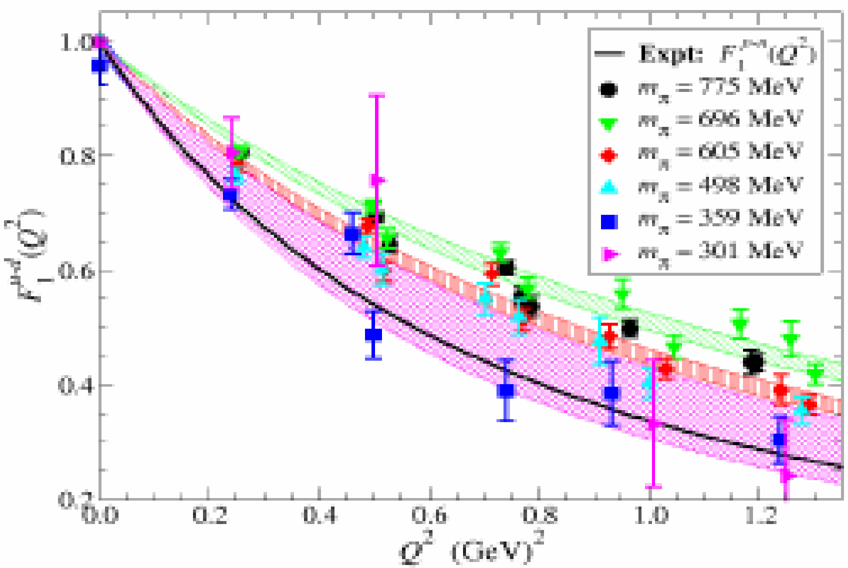
LHPC (Edwards *et al.*), PRL 96 (2006) 052001

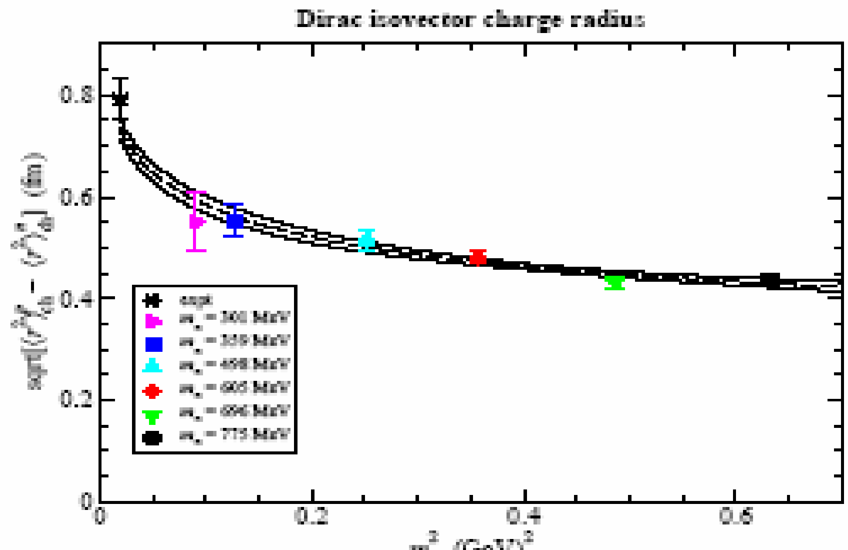
FRR Yields Essentially Identical Result



Young & Thomas, 2006

Proton EM Form Factors





- Lattice QCD computes the *isovector* form factor
- Hence obtain Dirac charge radius $\langle r^2\rangle^{u-d}_{ch}$ assuming dipole form
- Chiral extrap. Using LNA and LA terms and FRR
- As the pion mass approaches the physical value, the size approaches the correct value

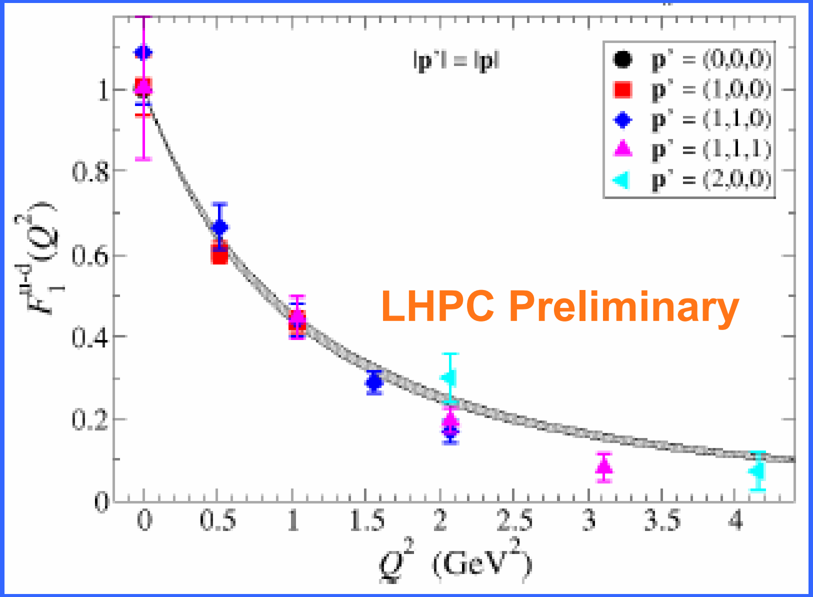
Data from LHPC Collaboration (Edwards et al.)

$$\langle r^2\rangle^{u-d}_{ch} = a_0 - 2\frac{(1+5g_A^2)}{(4\pi f_\pi)^2}\frac{1}{2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

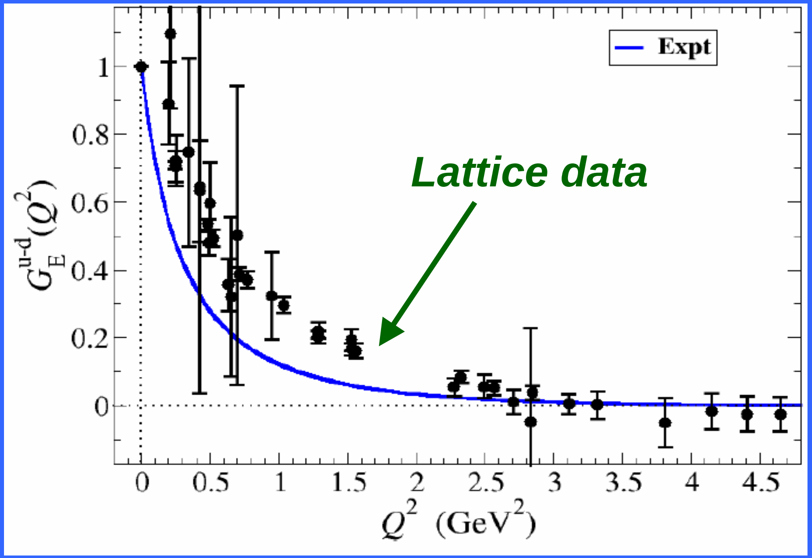
Leinweber, Thomas, Young, PRL86, 5011

from D. Richards

Isovector Form Factor at Higher Q^2



- Preliminary calculation with $m_\pi \simeq$ 600 MeV enabling us to reach $Q^2 \simeq 4$ GeV^2
- Fits of experimental data suggest G_E^{u-d} vanishes at $Q^2 \sim 4$ GeV^2
- Tantalizing suggestion of such behavior in lattice data.

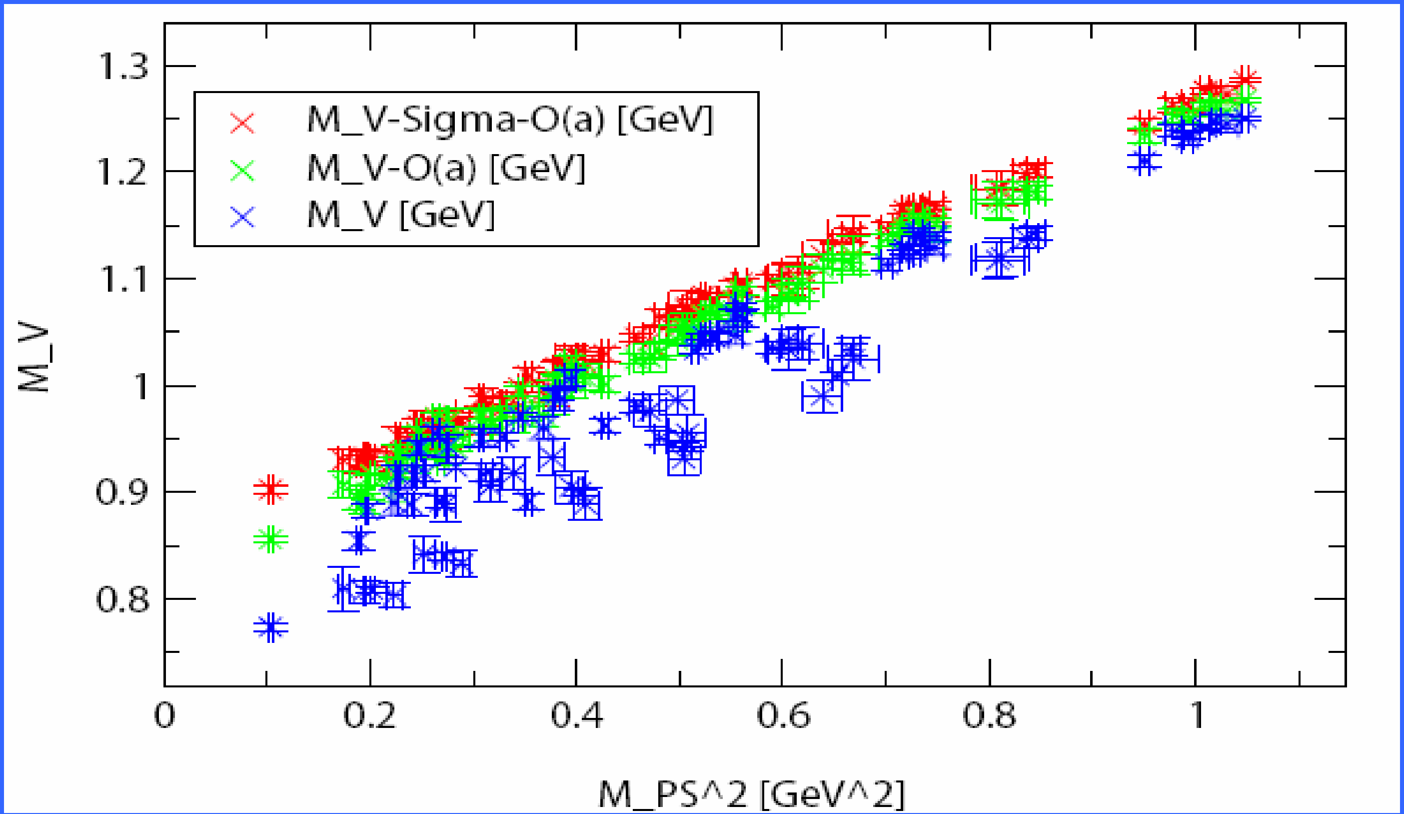


AIM: Form factor at $Q^2 > 10$ GeV^2, at pion masses down to 254 MeV in Asqtad/DWF Computation.

from D. Richards

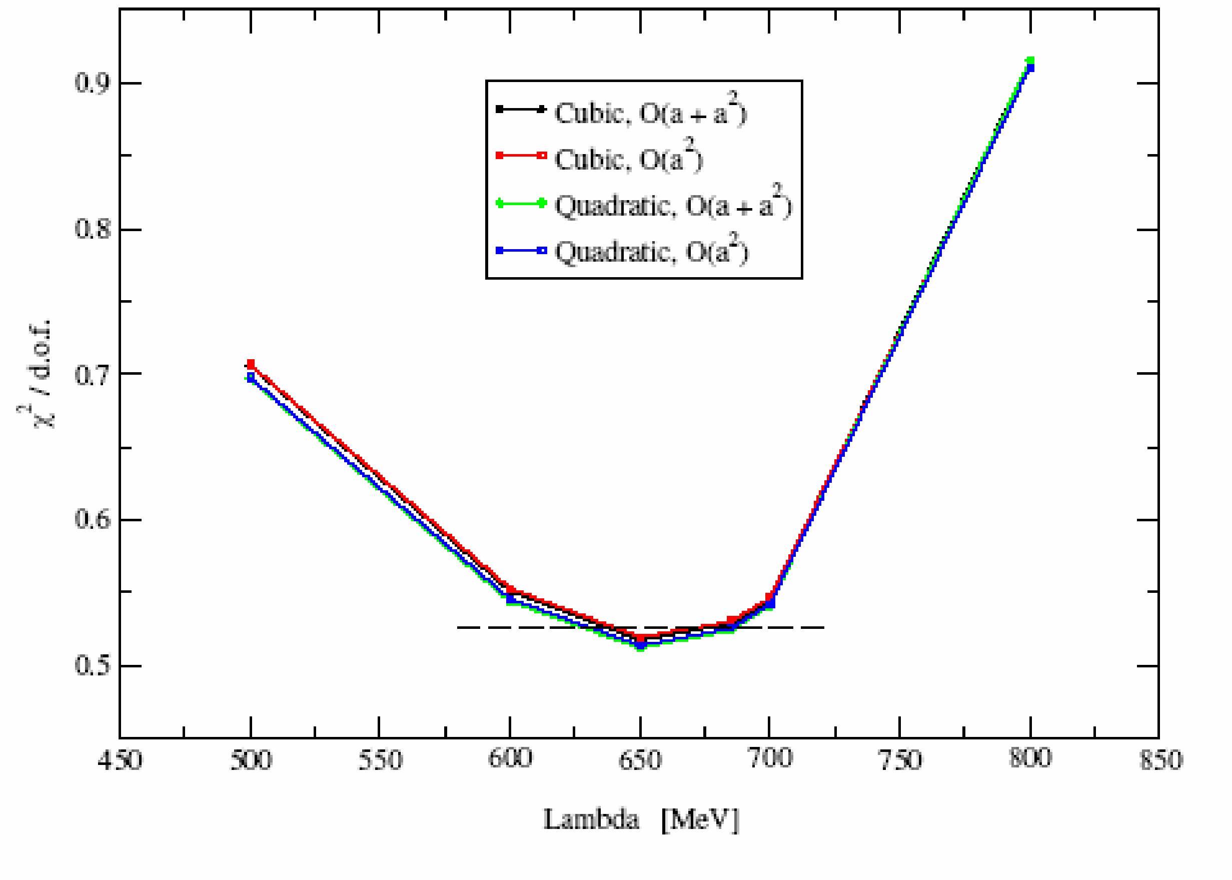
Analysis of pQQCD ρ data from CP PACS

i.e. $m_{val} \neq m_{sea}$



Fit with: $\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{deg})^2 + a_4 (M_{PS}^{deg})^4 + a_6 (M_{PS}^{deg})^6$

FRR Mass (in Σ_{TOT}) well determined by data

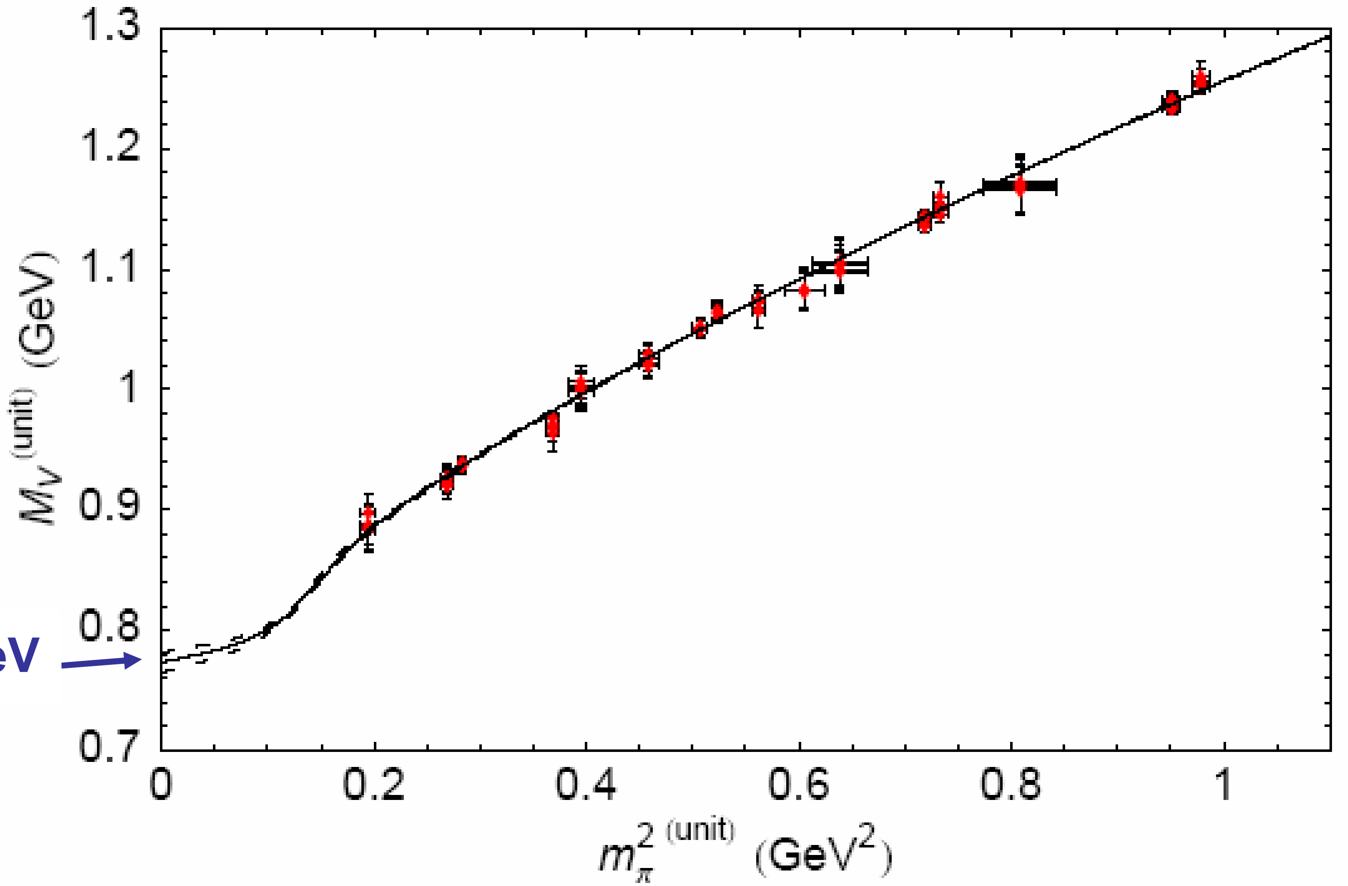


$$\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{deg})^2 + a_4 (M_{PS}^{deg})^4 + a_6 (M_{PS}^{deg})^6$$

Infinite Volume Unitary Results

$a \to 0$ and $m_{sea} = m_{val}$

All 80 data points drop onto single, well defined curve !



777 ± 7 MeV

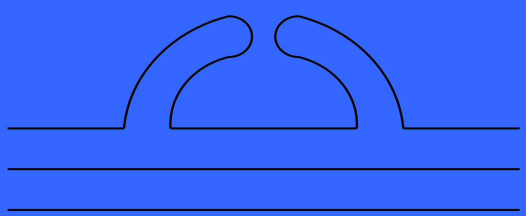
Allton, Young *et al.*, hep-lat/0504022

Baryon Masses in Quenched QCD

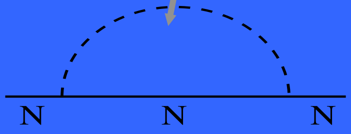
Chiral behaviour in QQCD quite different from full QCD

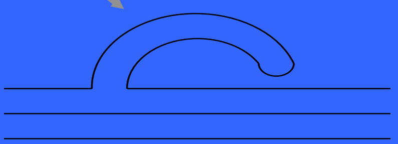
η' is an additional Goldstone Boson , so that:

$$m_N = m_0 + c_1 m_\pi + c_2 m_\pi^2 + c_3 m_\pi^3 + c_4 m_\pi^4 + m_\pi^4 \ln m_\pi + \ldots\ldots$$



Contribution from η' and π

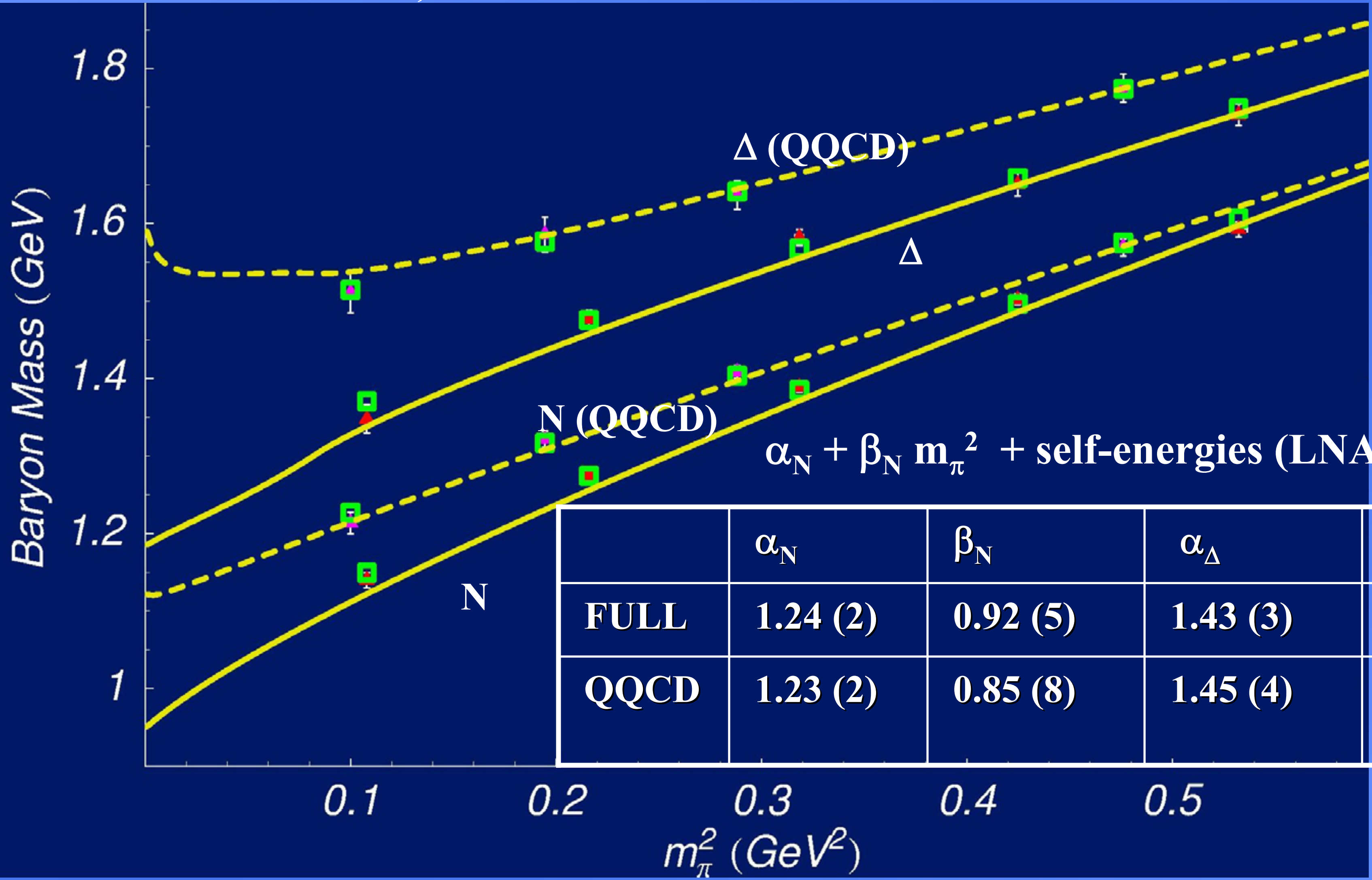




LNA term now $\sim m_q^{1/2}$

origin is η' double pole

- Lattice data (from MILC Collaboration) : red triangles
- Green boxes: fit evaluating σ's on same finite grid as lattice
- Lines are exact, continuum results



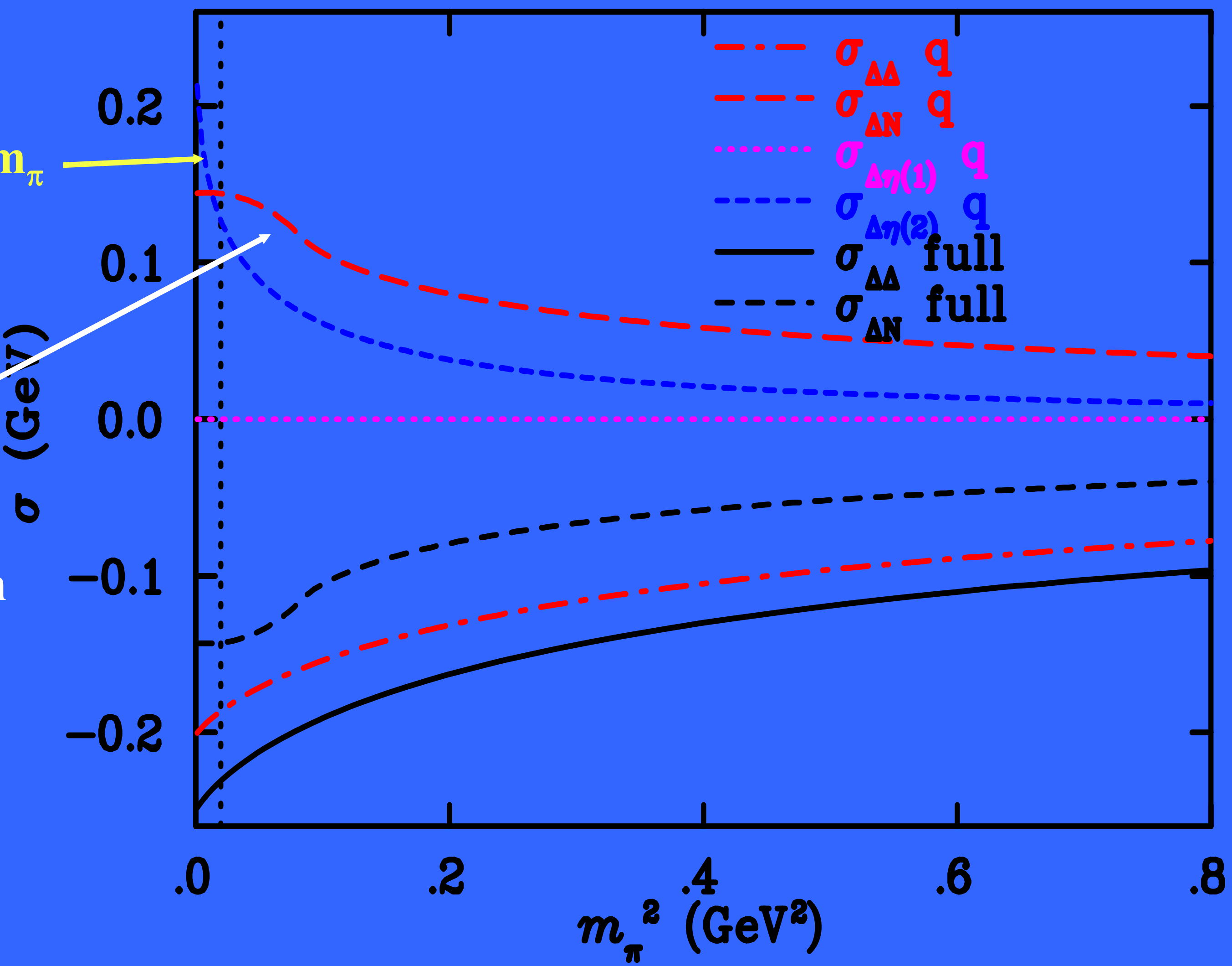
$\alpha_N + \beta_N m_\pi^2$ + self-energies (LNA+NLNA)

	α_N	β_N	α_Δ	β_Δ
FULL	1.24 (2)	0.92 (5)	1.43 (3)	0.75 (8)
QQCD	1.23 (2)	0.85 (8)	1.45 (4)	0.71 (11)

Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Δ in QQCD

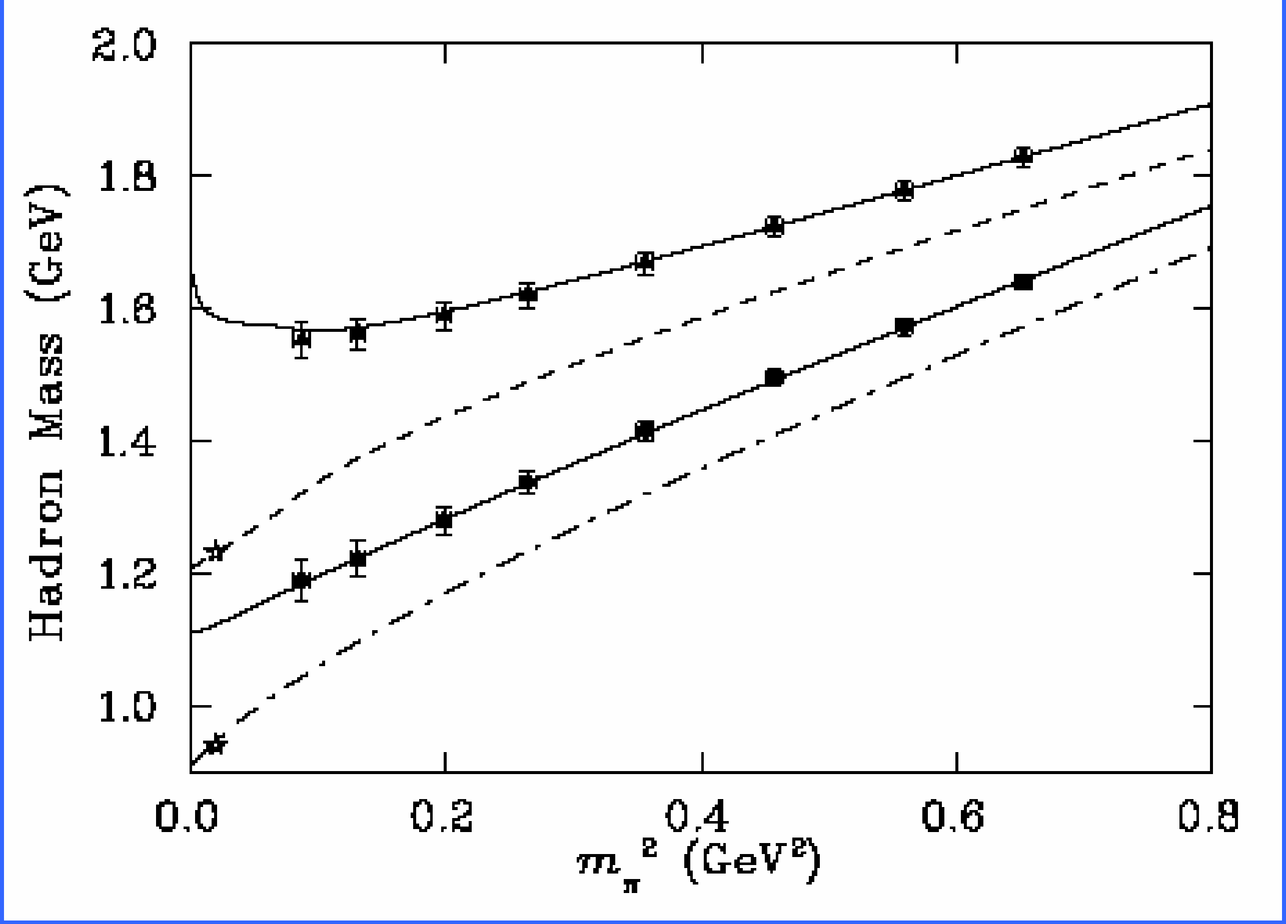
LNA term linear in m_π



Δ → N π contribution has opposite sign in QQCD (repulsive)

Overall σ_{QQCD} is repulsive !

Confirmation of Predicted Behavior of Δ



Zanotti et al., hep-lat/0407039

These results suggest following conjecture :

IF lattice scale is set using static quark potential (e.g. Sommer scale)

(insensitive to chiral physics)

Suppression of Goldstone loops for $m_\pi > \Lambda$ implies:

Analytic terms (e.g. $\alpha + \beta\, m_\pi^2 + \gamma\, m_\pi^4$)

representing "hadronic core" are the same in QQCD & QCD

Can then correct QQCD results by replacing LNA & NLNA behaviour in QQCD by corresponding terms in full QCD

Quenched QCD is then no longer an "uncontrolled approximation" !

Strangeness Widely Believed to Play a Major Role – Does It?

- As much as 100 to 300 MeV of proton mass:

$$M_N = \langle N(P)| -\frac{9\,\alpha_s}{4\,\pi}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d + m_s\bar{\psi}_s\psi_s|N(P)\rangle$$

$$\Delta M_N^{s-\mathrm{quarks}} = \frac{y m_s}{m_u + m_d}\sigma_N$$

y=0.2 ± 0.2

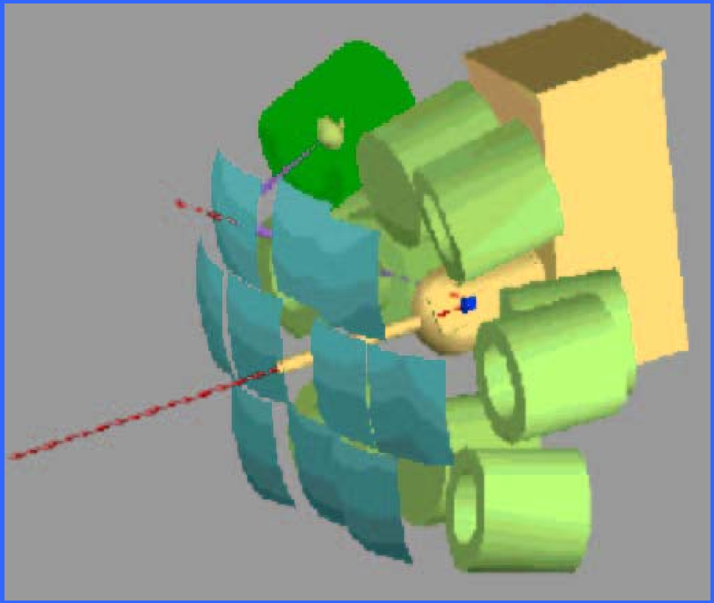
45 ± 8 MeV (or 70?)

Hence 110 ± 110 MeV (increasing to 180 for higher σ_N)

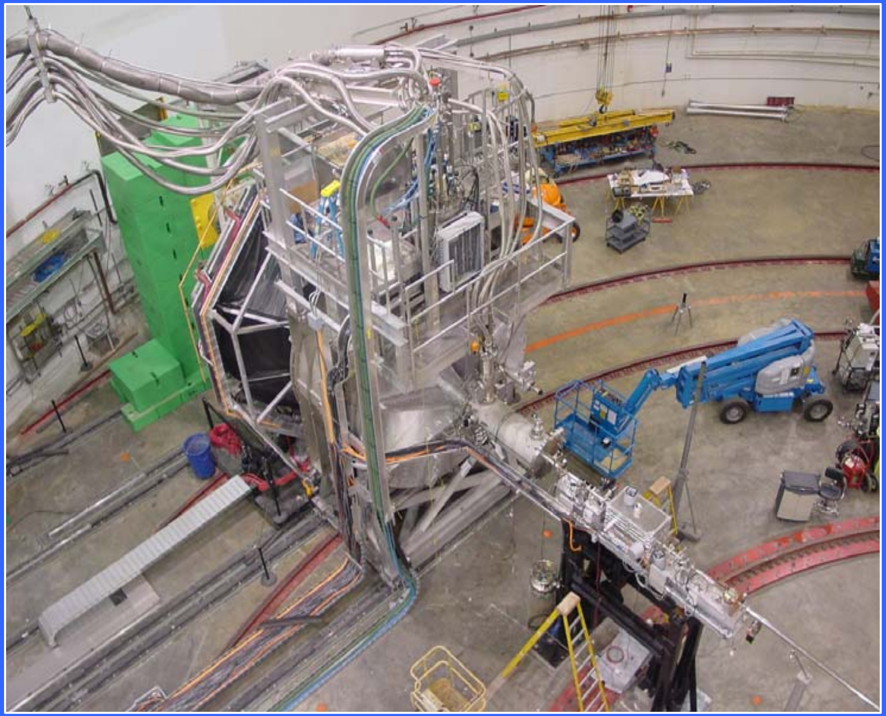
- Through proton spin crisis:
  As much as 10% of the spin of the proton

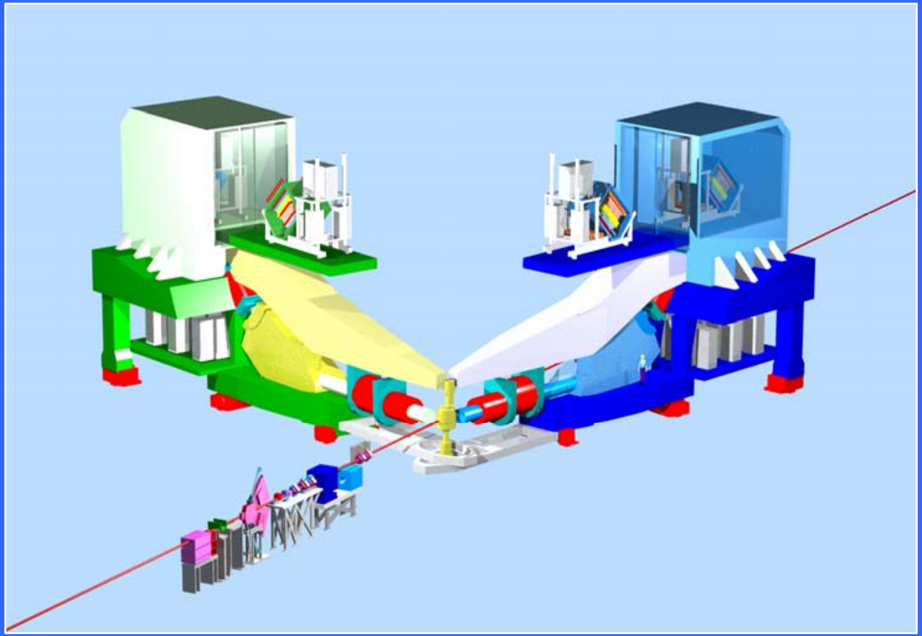
- HOW MUCH OF THE ELECTRIC and MAGNETIC FORM FACTORS ?

MIT-Bates & A4 at Mainz

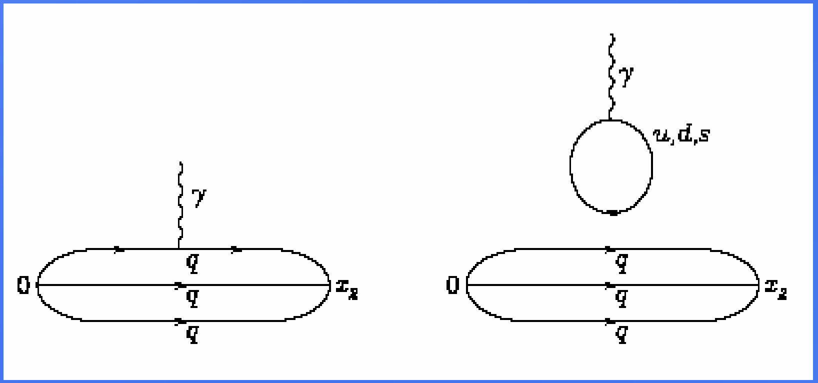


G0 and HAPPEx at Jlab





Magnetic Moments within QCD



CS

$p = 2/3\ u^p - 1/3\ d^p + O_N$

$n = -1/3\ u^p + 2/3\ d^p + O_N$

$2p + n = u^p + 3\ O_N$

(and $p + 2n = d^p + 3\ O_N$)

$\Sigma^+ = 2/3\ u^\Sigma - 1/3\ s^\Sigma + O_\Sigma$

$\Sigma^- = -1/3\ u^\Sigma - 1/3\ s^\Sigma + O_\Sigma$

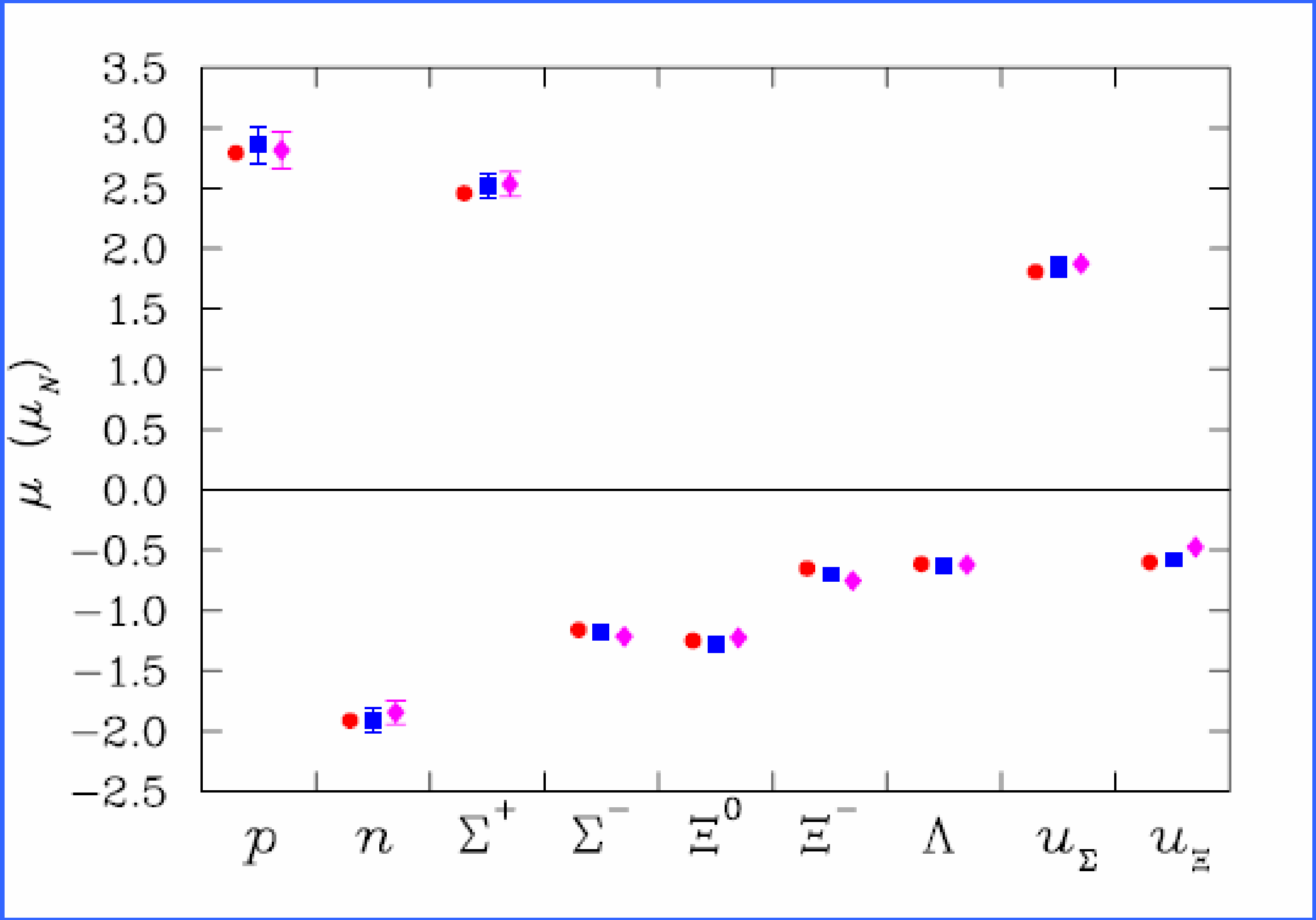
$\Sigma^+ - \Sigma^- = u^\Sigma$

HENCE: $O_N = 1/3\ [\ 2p + n - (\ u^p / u^\Sigma\ )\ (\Sigma^+ - \Sigma^-)\ ]$

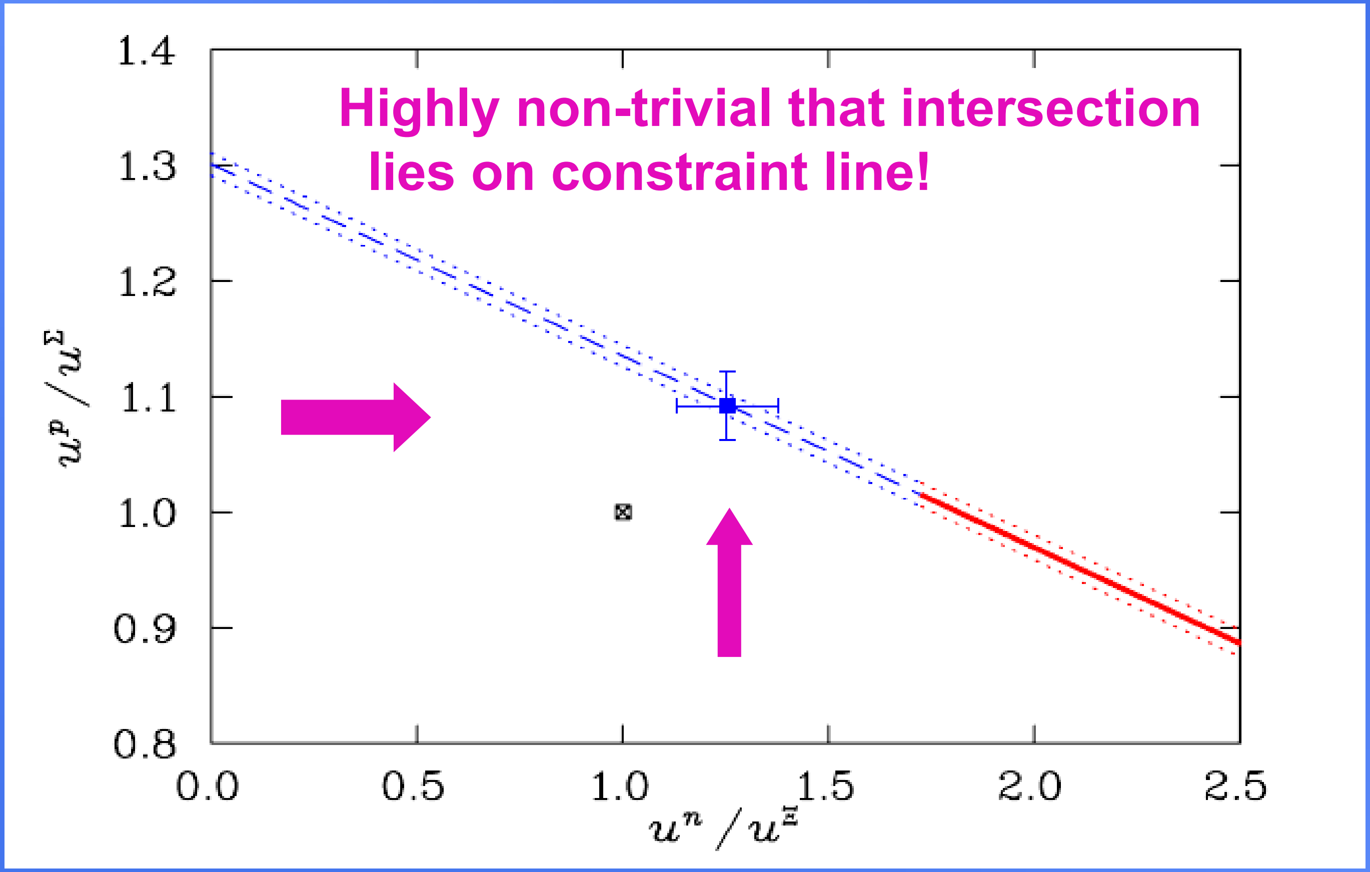
Just these ratios from Lattice QCD

OR $O_N = 1/3\ [\ n + 2p - (\ u^n / u^\Xi\ )\ (\Xi^0 - \Xi^-)\ ]$

Convergence LNA to NLNA Again Excellent (Effect of Decuplet)



Accurate Final Result for G_M^s



1.10±0.03

1.25±0.12

Yields : $G_M^s = -0.046 \pm 0.019\ \mu_N$

Leinweber et al., (PRL June '05) hep-lat/0406002

G_E^s by similar technique

In this case only know Σ^- radius (and p and n)
hence use absolute values of u and d radii:

$2p + n = u^p + 3\,O_N$ $\quad$ $p + 2n = d^p + 3\,O_N$

$\Rightarrow \langle r^2 \rangle_s = 0.000 \pm 0.006 \pm 0.007\ \mathrm{fm}^2$; $0.002 \pm 0.004 \pm 0.004\ \mathrm{fm}^2$

(c.f. using Σ^- : $-0.007 \pm 0.004 \pm 0.007 \pm 0.021\ \mathrm{fm}^2$)

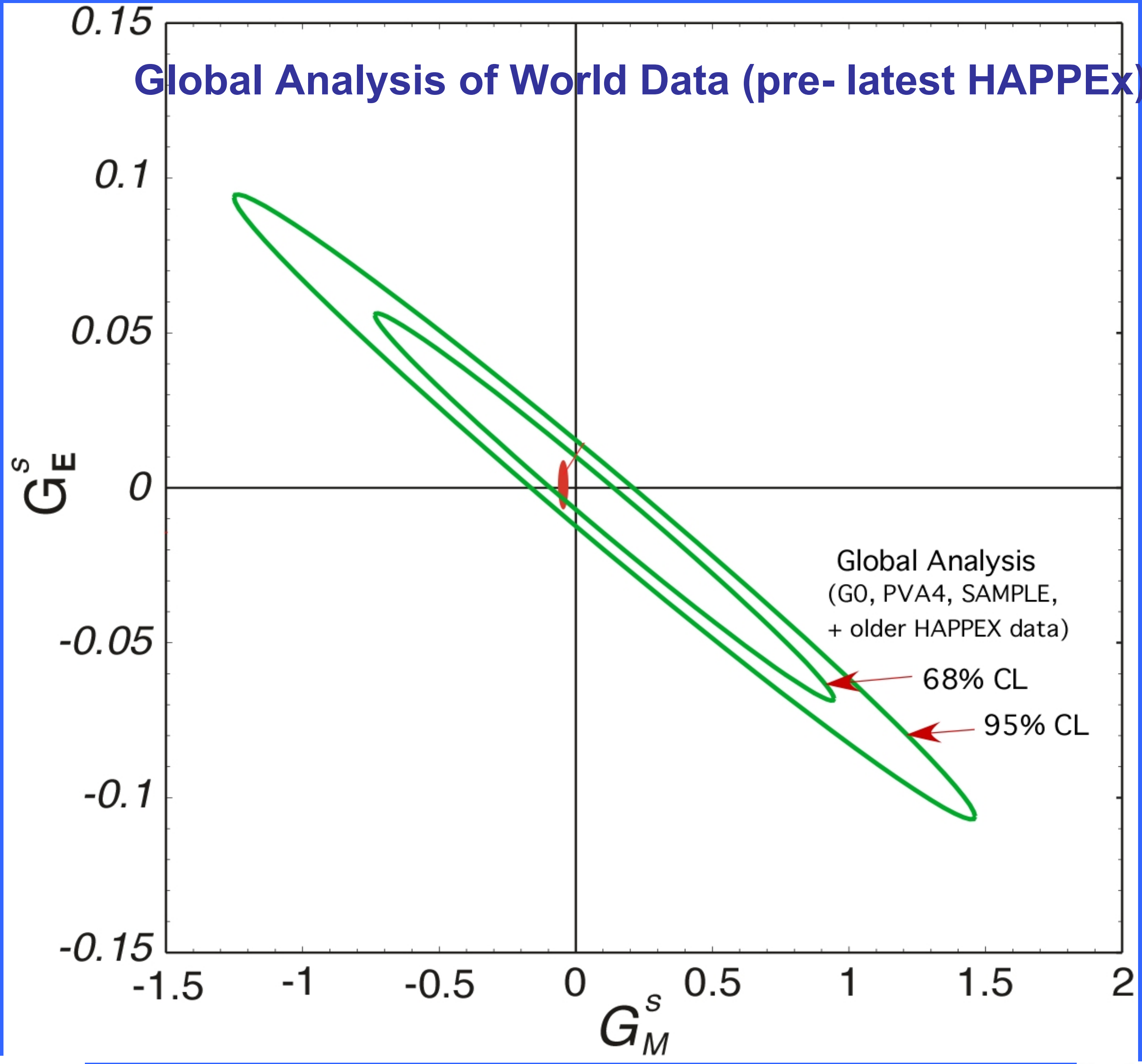
$$G_E^s(0.1\,\mathrm{GeV}^2) = +0.001 \pm 0.004 \pm 0.004$$

(up to order Q^4)

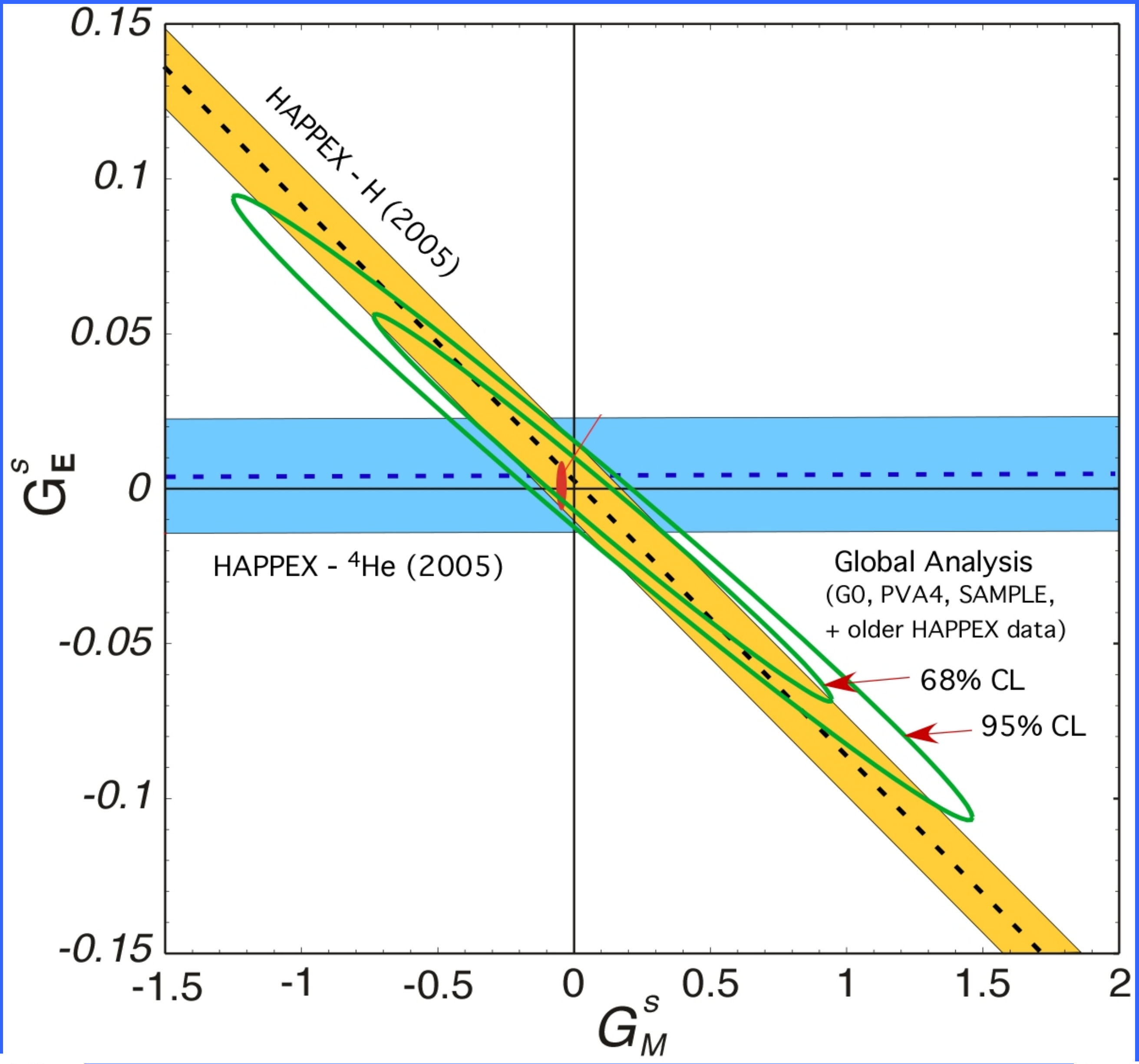
Note consistency and level of precision!

Leinweber, Young et al., hep-lat/0601025 (Jan 2006)

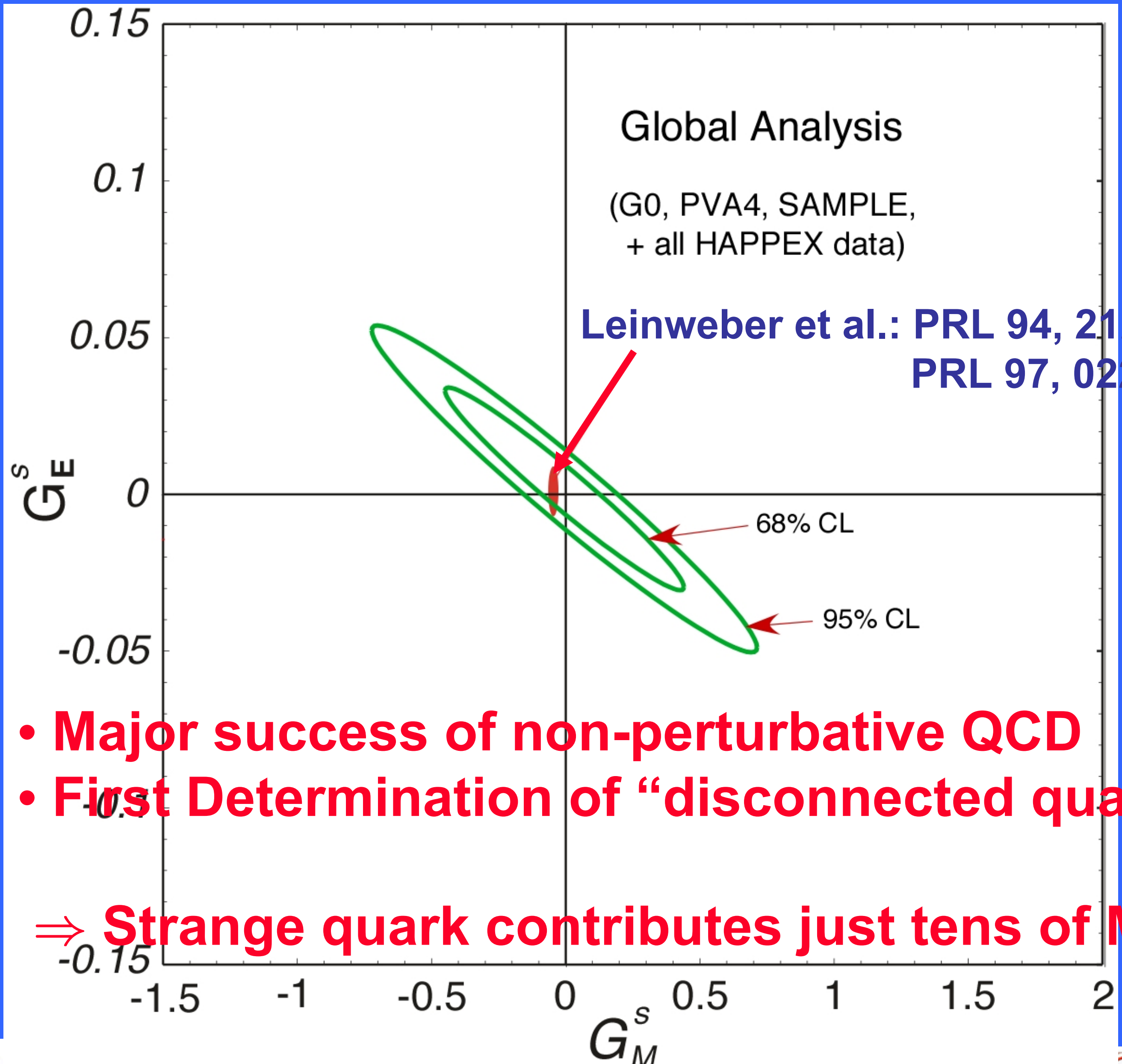
Young, Roche, Carlini, Thomas – nucl-ex/0604010 (PRL to appear)



Superimpose NEW HAPPEx Measurement (Dallas APS meeting, April 06)



Include new HAPPEx data : halves errors of previous world data !



Leinweber et al.: PRL 94, 212001 (2005)
PRL 97, 022001 (2006)

- Major success of non-perturbative QCD
- First Determination of "disconnected quark loop"

$\Rightarrow$ Strange quark contributes just tens of MeV to M_N

Conclusions

- Wonderful synergy between experimental advances at Jlab and progress using Lattice QCD to solve QCD

- Study of hadron properties as function of m_q using data from lattice QCD is extremely valuable…..
  (major qualitative advance in understanding)
  + TEST BEYOND STANDARD MODEL

- Inclusion of model independent constraints of χ PT to get to physical quark mass is essential
  FRR χPT resolves problem of convergence
- Insight enables: accurate, controlled extrapolation of all hadronic observables….
  (e.g. m_H , μ_H , $G_{E,M}^s$, $\langle r^2 \rangle_{ch}$, G_E,G_M, $\langle x^n \rangle$……..)

Conclusions….$_2$

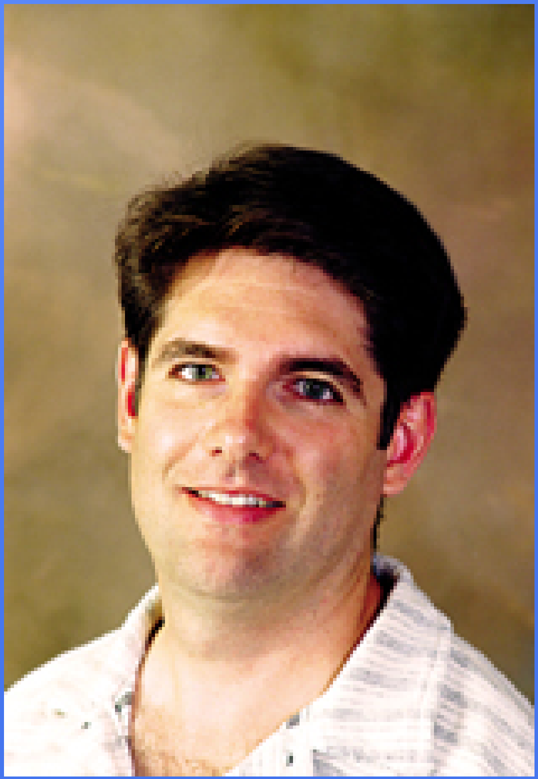
- In case where chiral coefficients are known, FRR enables accurate extrapolation to physical point

- Without chiral coefficients (e.g. spectroscopy of baryons and mesons) need data at very low pion mass (several points below $\sim$ 0.25 GeV)

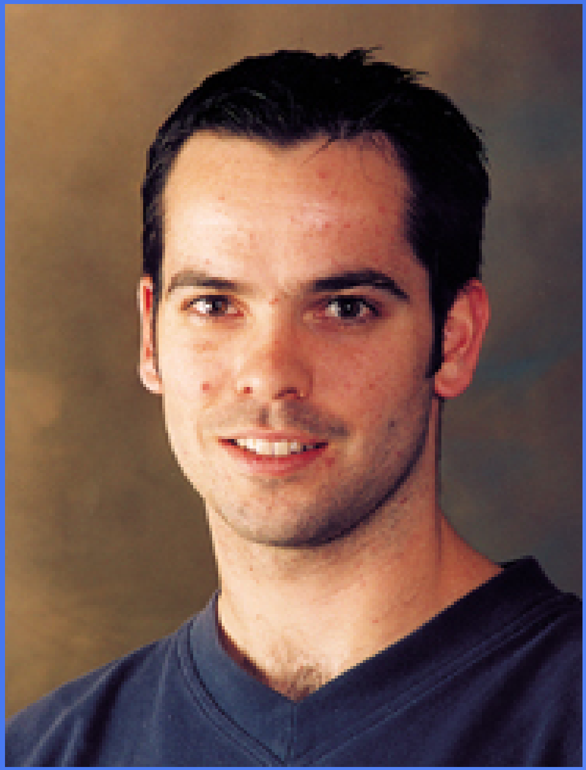
- It is a major challenge to obtain a reliable signal for "disconnected" loops directly in lattice QCD
  — this is a very important challenge

- For future there is a wonderful synergy with 12 GeV program at JLab and work on GPDs, form factors at high Q^2 , and higher moments of PDFs just beginning…..

Special Mentions……



Derek Leinweber



Ross Young