Low-energy electroweak processes in the few nucleons in χEFT

- General considerations
- EM currents up to one loop
- A (sensitive) test case: radiative captures in A=3 and 4 systems
- Nuclear theory at 1%: μ-capture in d and ^{3}He
- Summary and outlook

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References:

Pastore *et al.*, PRC**80**, 034004 (2009); Girlanda *et al.*, PRL**105**, 232502 (2010);
Pastore *et al.*, PRC**84**, 024001 (2011); Marcucci *et al.*, arXiv:1109.5563

Nuclear χEFT approach

Weinberg, PLB**251**, 288 (1990); NPB**363**, 3 (1991); PLB**295**, 114 (1992)

- χEFT exploits the χ-symmetry exhibited by QCD to restrict the form of π interactions with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum Q, and $\mathcal{L}_{\rm eff}$ can be systematically expanded in powers of Q/Λ_χ ($\Lambda_\chi \simeq 1$ GeV)

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + ...$$

- χEFT allows for a perturbative treatment in terms of a Q–as opposed to a coupling constant–expansion
- The unknown coefficients in this expansion–the LEC's–are fixed by comparison with experimental data
- Nuclear χEFT provides a practical calculational scheme, capable (in principle) of systematic improvement

Work in nuclear χEFT: a partial listing

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A very incomplete list:

- NN and NNN potentials:
  - van Kolck *et al.* (1994–96)
  - Friar *et al.* (1996–04)
  - Kaiser, Weise *et al.* (1997–98)
  - Glöckle, Epelbaum, Meissner *et al.* (1998–2005)
  - Entem and Machleidt (2003, 2011)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al.* (1996–2009), hybrid studies in A=2–4
  - Meissner *et al.* (2001), Kölling *et al.* (2009–2011)
  - Phillips (2003), deuteron static properties and f.f.'s

Formalism

- Time-ordered perturbation theory (TOPT):

$$\langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid i \rangle$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

α_i = number of derivatives (momenta) and β_i = number of π's at each vertex

N_K = number of energy denominators with only nucleon kinetic energies (Q^2)

- Each of the $N - N_K - 1$ energy denominators expanded as

$$\frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[1 + \frac{E_i - E_I}{\omega_\pi} + \frac{(E_i - E_I)^2}{\omega_\pi^2} + \ldots \right]$$

- Power counting:

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots\,, \quad \text{and} \quad T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

From amplitudes to potentials

- Derive v such that

$$v + v\,G_0\,v + v\,G_0\,v\,G_0\,v + \ldots \qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

  leads to T-matrix order by order in the power counting

- Assume

$$v = v^{(0)} + v^{(1)} + v^{(2)} + \ldots \qquad v^{(n)} \sim Q^n$$

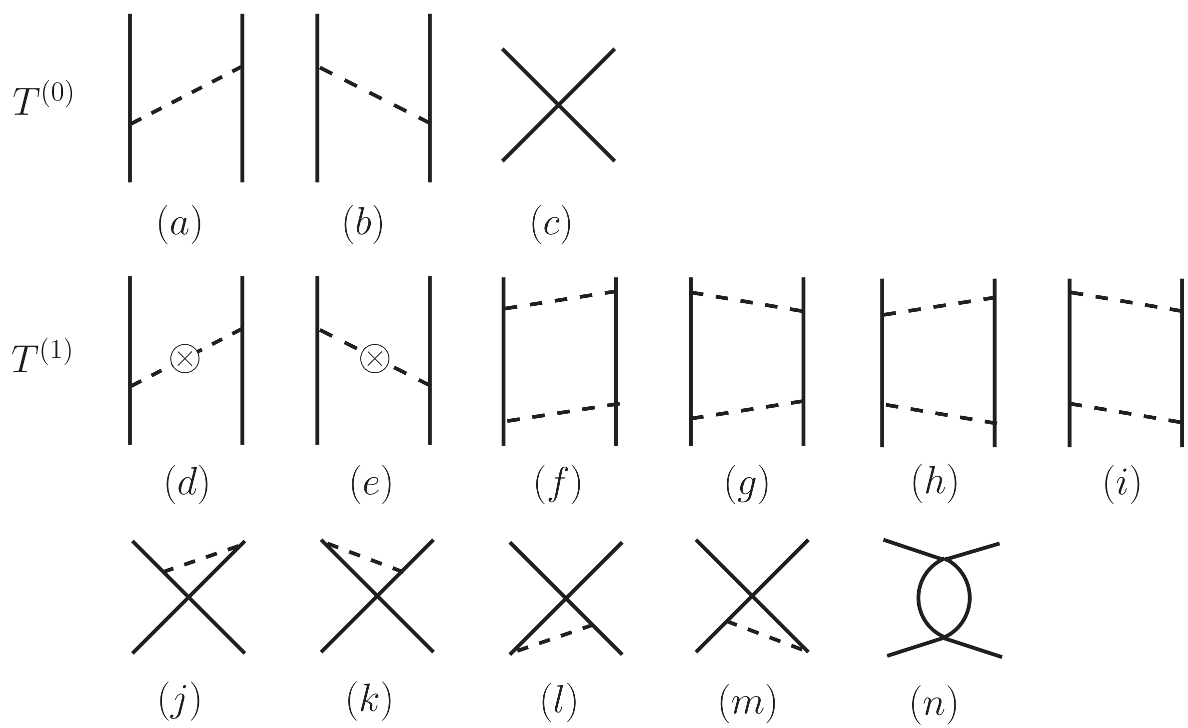
- Determine $v^{(n)}$ from

$$\begin{aligned}
v^{(0)} &= T^{(0)} \\
v^{(1)} &= T^{(1)} - \left[v^{(0)}\,G_0\,v^{(0)}\right] \\
v^{(2)} &= T^{(2)} - \left[v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\right] - \left[v^{(1)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v^{(1)}\right] \\
v^{(3)} &= T^{(3)} - \left[v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\right] - \left[v^{(1)}\,G_0\,v^{(0)}\,G_0\,v^{(0)} + \text{permutations}\right] \\
&\quad - \left[v^{(2)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v^{(2)}\right]
\end{aligned}$$

  where

$$v^{(m)}\,G_0\,v^{(n)} \sim Q^{m+n+1}$$

$T^{(n)}$ and $v^{(n)}$ up to order $n = 1$ (or Q^1)



- $v^{(0)} = T^{(0)}$ consists of (static) OPE and contact terms
- $v^{(1)} = T^{(1)} - \left[v^{(0)} \, G_0 \, v^{(0)} \right]$ vanishes

Including EM interactions

- In the presence of EM interactions (treated in first order)

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \ldots \qquad T_\gamma^{(n)} \sim e\,Q^n$$

- For $v_\gamma = A^0\,\rho - \mathbf{A}\cdot\mathbf{j}$ to match T_γ order by order

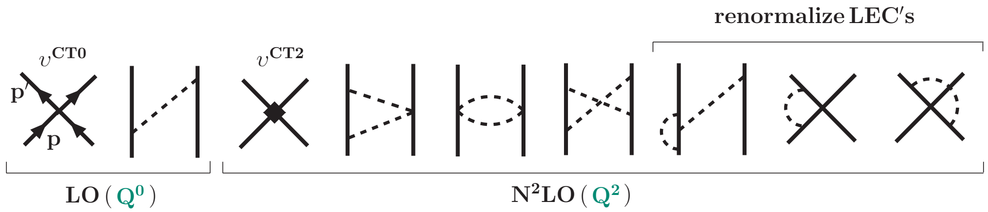
$$\begin{aligned} v_\gamma^{(-3)} &= T_\gamma^{(-3)} \\ v_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[v_\gamma^{(-3)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v_\gamma^{(-3)}\right] \end{aligned}$$

and up to $n = 1$ $(e\,Q)$

$$\begin{aligned} v_\gamma^{(1)} &= T_\gamma^{(1)} - \left[v_\gamma^{(-3)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)} + \text{permutations}\right] \\ &\quad - \left[v_\gamma^{(-2)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)} + \text{permutations}\right] \\ &\quad - \left[v_\gamma^{(-1)}\,G_0\,v^{(0)}\,G_0\,v^{(0)} + \text{permutations}\right] - \left[v_\gamma^{(0)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v_\gamma^{(0)}\right] \\ &\quad - \left[v_\gamma^{(-3)}\,G_0\,v^{(2)}\,G_0\,v^{(0)} + \text{permutations}\right] - \left[v_\gamma^{(-3)}\,G_0\,v^{(3)} + v^{(3)}\,G_0\,v_\gamma^{(-3)}\right] \end{aligned}$$

Recent developments based on these methods*

- NN potentials at order Q^2



- EM charge and current operators up to one loop ($e\,Q$)
- Form of charge operators at $e\,Q^0$ and $e\,Q^1$ depends on non-static corrections to OPE and TPE potentials
- While non unique, these non-static corrections to potentials and charge operators are unitarily equivalent†
- After (perturbative) renormalization, resulting operators still need to be regularized: $C_\Lambda(k) = {\rm e}^{-(k/\Lambda)^4}$

*Pastore *et al.* PRC**80**, 034004 (2009); PRC**84**, 024001 (2011)

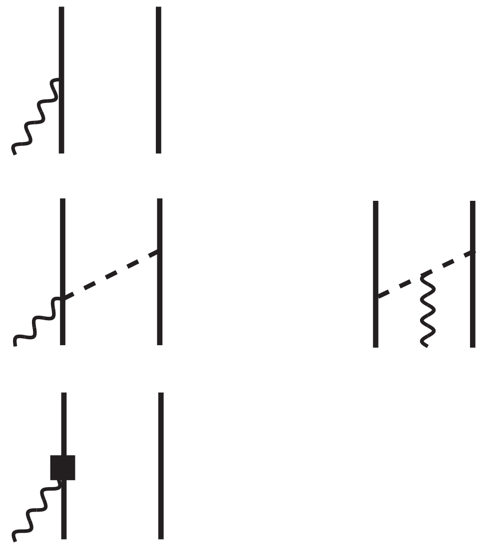
†Friar, Ann. Phys. **104**, 380 (1977) for similar considerations in the OPE sector

Two-body EM current operator in χEFT up to N^{2}LO ($e\,Q^0$)

LO : $\mathbf{eQ^{-2}}$

NLO : $\mathbf{eQ^{-1}}$

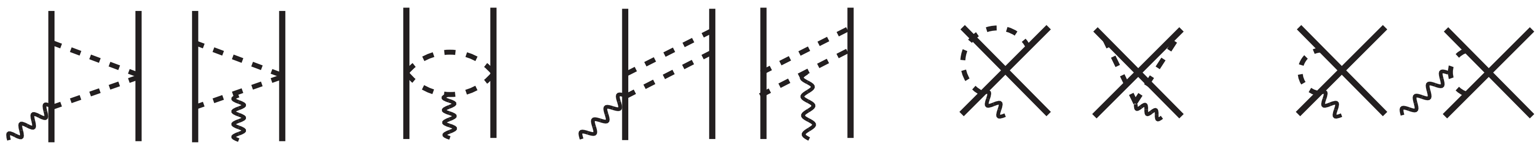
N^{2}LO : $\mathbf{eQ^{0}}$



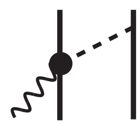
- These depend on the proton and neutron μ's ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$), g_A, and F_π
- One-loop corrections to one-body current are absorbed into μ_N and $\langle r_N^2 \rangle$

N^3LO $(e\,Q)$ corrections

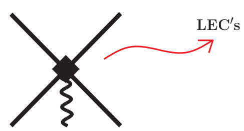
- One-loop corrections:



- Tree-level current with one $e\,Q^2$ vertex from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al.* (1998), involving 3 LEC's ($\sim \gamma N \Delta$ and $\gamma\rho\pi$ currents) :



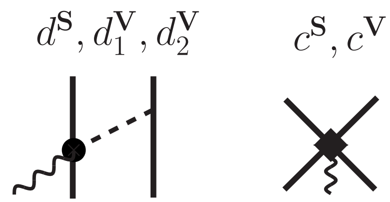
- Contact currents



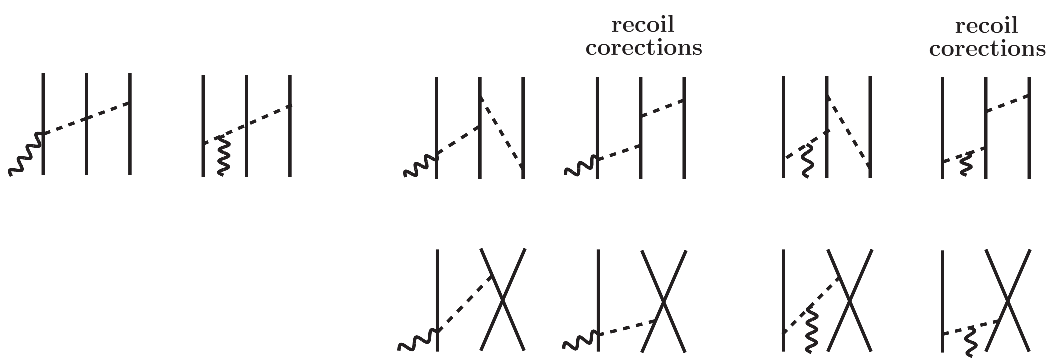
from i) minimal substitution in the interactions involving $\partial\, N$ (7 LEC's determined from strong-interaction sector) and ii) non-minimal couplings (2 LEC's)

EM observables at N^{3}LO

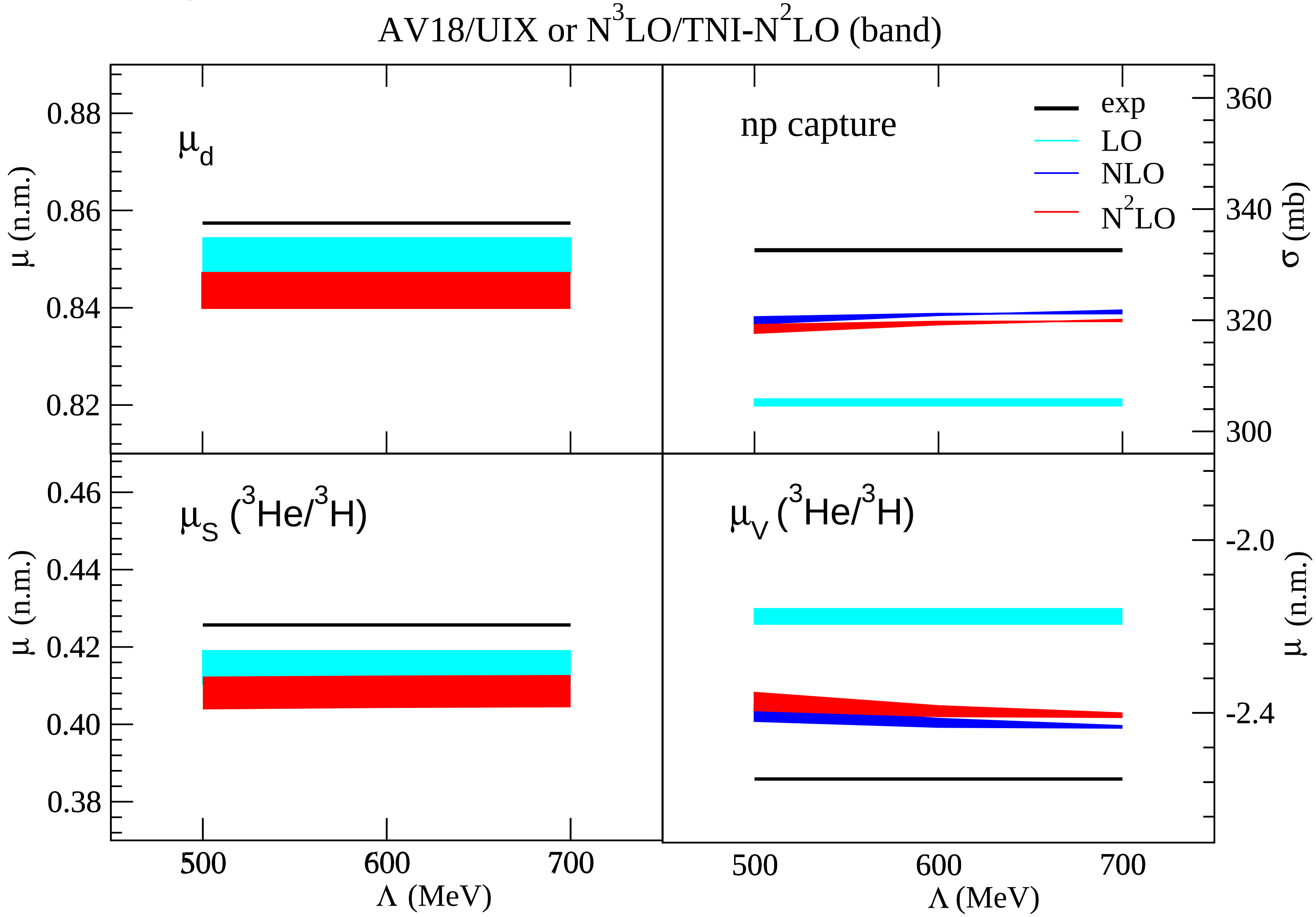
- Pion loop corrections and (minimal) contact terms known
- Five LEC's: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



- $d_2^V/d_1^V = 1/4$ assuming Δ-resonance saturation
- Three-body currents at N^{3}LO vanish:



Fixing LEC's in EM Properties of A=2 and A=3 Nuclei



Fitted LEC values

- LEC's—in units of Λ—corresponding to $\Lambda = 500$–700 MeV for AV18/UIX (N3LO/N2LO)
- Isoscalar d^S (c^S) and isovector d_1^V (c^V) associated with higher-order $\gamma\pi N$ (contact) currents

Λ	$\Lambda^2 d^S \times 10^2$	$\Lambda^4 c^S$	$\Lambda^2 d_1^V$	$\Lambda^4 c^V$
500	–8.85 (–0.225)	–3.18 (–2.38)	5.18 (5.82)	–11.3 (–11.4)
600	–2.90 (9.20)	–7.10 (–5.30)	6.55 (6.85)	–12.9 (–23.3)
700	6.64 (20.4)	–13.2 (–9.83)	8.24 (8.27)	–1.70 (–46.2)

The nd and $n\,^3$He radiative captures

- Suppressed $M1$ processes:

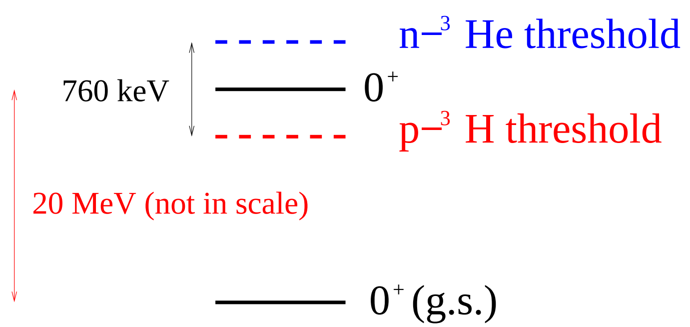
	$\sigma_{\rm exp}$(mb)
$^1{\rm H}(n,\gamma)^2{\rm H}$	334.2(5)
$^2{\rm H}(n,\gamma)^3{\rm H}$	0.508(15)
$^3{\rm He}(n,\gamma)^4{\rm He}$	0.055(3)

- The ^{3}H and ^{4}He bound states are approximate eigenstates of the one-body $M1$ operator, *e.g.* $\hat\mu({\rm IA})\,|^3{\rm H}\rangle \simeq \mu_p\,|^3{\rm H}\rangle$ and $\langle nd \mid \hat\mu({\rm IA})\,|^3{\rm H}\rangle \simeq 0$ by orthogonality

- A=3 and 4 radiative (and weak) captures very sensitive to i) small components in the w.f.'s and ii) many-body terms in the electro(weak) currents (80-90% of cross section!)

Wave functions: recent progress

- 3 and 4 bound-state w.f.'s and 2+1 continuum routine by now
- Challenges with 3+1 continuum:
  1. Coupled-channel nature of scattering problem: n-^{3}He and p-^{3}H channels both open
  2. Peculiarities of ^{4}He spectrum (see below): hard to obtain numerically converged solutions



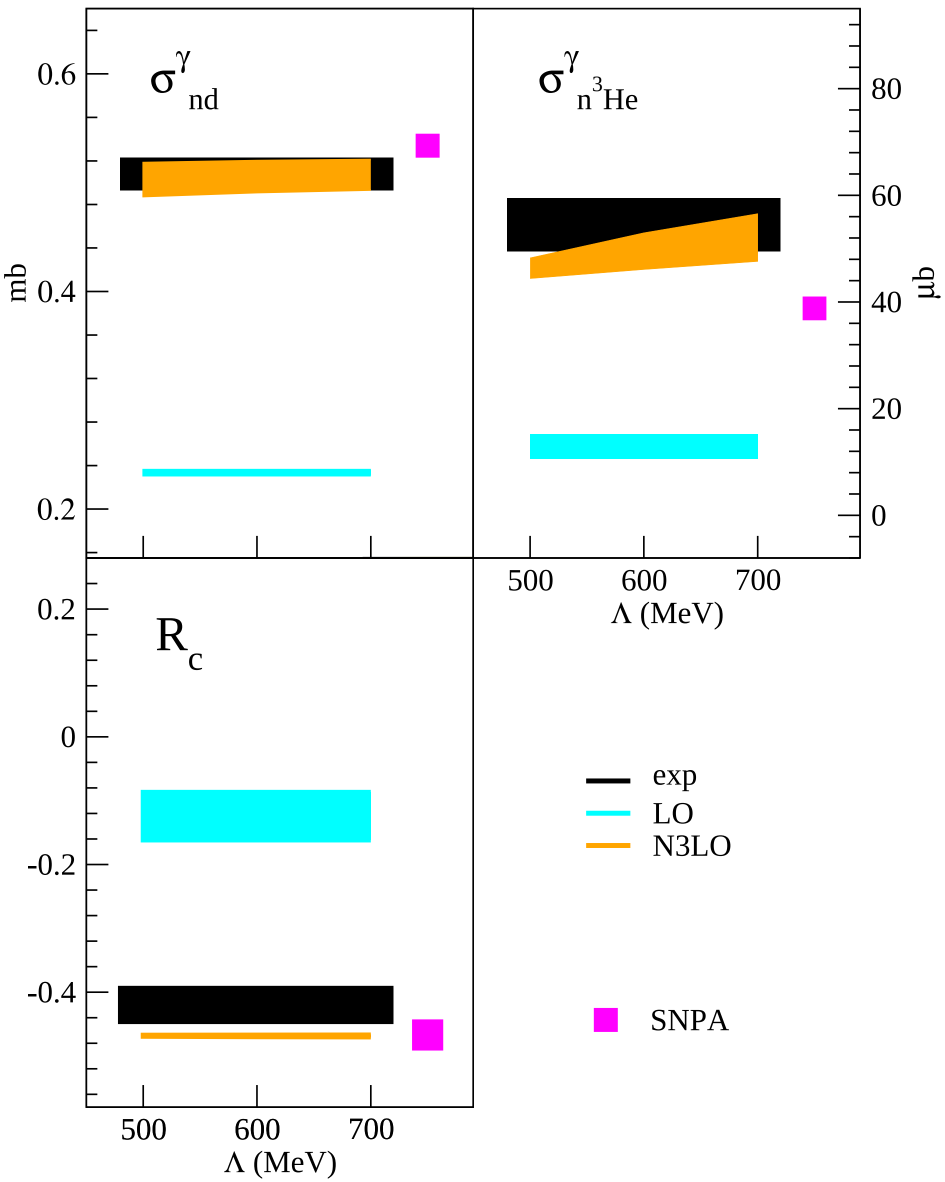
- Major effort by several groups*: both singlet and triplet n-^{3}He scattering lengths in good agreement with data

*Deltuva and Fonseca (2007); Lazauskas (2009); Viviani *et al.* (2010)

Triplet scattering length a_1 (fm)

Method	AV18	AV18/UIX
HH	$3.56 - i\,0.0077$	$3.39 - i\,0.0059$
RGM	$3.45 - i\,0.0066$	$3.31 - i\,0.0051$
FY	$3.43 - i\,0.0082$	$3.23 - i\,0.0054$
AGS	$3.51 - i\,0.0074$	
R-matrix	$3.29 \quad - i\,0.0012$	
EXP	$3.28(5) - i\,0.001(2)$	
EXP	$3.36(1)$	
EXP	$3.48(2)$	

Singlet scattering length a_0 (harder to calculate!) also in good agreement with experiment



σ^{γ}_{nd}
$\sigma^{\gamma}_{n^3He}$
R_c
mb
μb
0.6
0.4
0.2
80
60
40
20
0
0.2
0
-0.2
-0.4
500
600
700
Λ (MeV)
exp
LO
N3LO
SNPA

n-d radiative capture cross section* in μb: $\sigma_{\mathrm{nd}}^{\mathrm{EXP}} = 508(15)\ \mu\mathrm{b}$

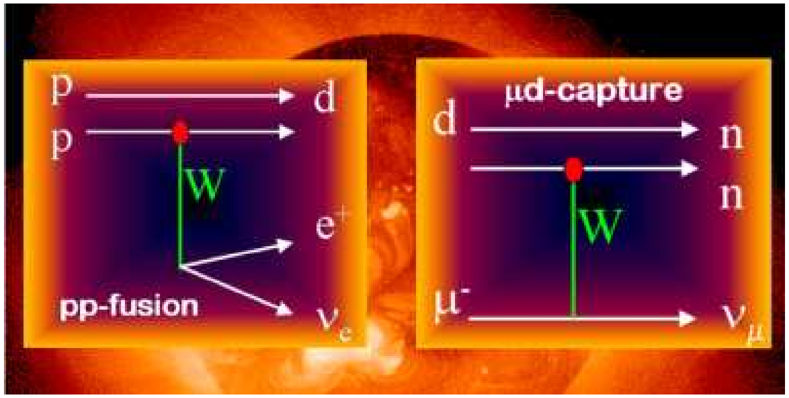
Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	231	343	322	272	487
600	231	369	348	306	491
700	231	385	362	343	493

n-^{3}He radiative capture cross section* in μb: $\sigma_{\mathrm{n\,^3He}}^{\mathrm{EXP}} = 55(4)\ \mu\mathrm{b}$

Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	15.2	5.95	0.91	1.36	48.3
600	15.2	10.2	2.87	0.04	53.0
700	15.2	11.5	3.56	0.38	56.6

*N3LO/N2LO potentials and HH wave functions

μ-Capture



From http://www.npl.illinois.edu/exp/musun/

Motivations:

- Test of first-principle (χEFT based) predictions for the μ-capture rates on d and ^{3}He
- Forthcoming measurement of the rate on d from MuSun Collaboration with projected error of 1%

Single-nucleon weak current

$$\langle n|\overline{d}\gamma_\mu(1-\gamma_5)u|p\rangle = \overline{u}_n\Big(F_1\,\gamma_\mu + \frac{i}{2m}F_2\,\sigma_{\mu\nu}q^\nu - G_A\,\gamma_\mu\gamma_5 - \frac{1}{m_\mu}\,G_{PS}\,\gamma_5 q_\mu\Big)u_p$$

- Additional scalar and pseudotensor f.f.'s, associated with second-class currents, possible (discussed later ...)
- $F_1(q^2)$ and $F_2(q^2)$ related to EM f.f.'s via CVC: well known
- $G_A(q^2) = g_A/(1+q^2/\Lambda_A^2)^2$: g_A known from neutron β-decay and $\Lambda_A \simeq 1$ GeV from π-electroproduction and $p(\nu_\mu,\mu^+)n$ data
- $G_{PS}(q^2)$ poorly known: PCAC and χPT predict

$$G_{PS}(q^2) = \frac{2m_\mu g_{\pi pn}F_\pi}{m_\pi^2 - q^2} - \frac{1}{3}g_A m_\mu m r_A^2$$

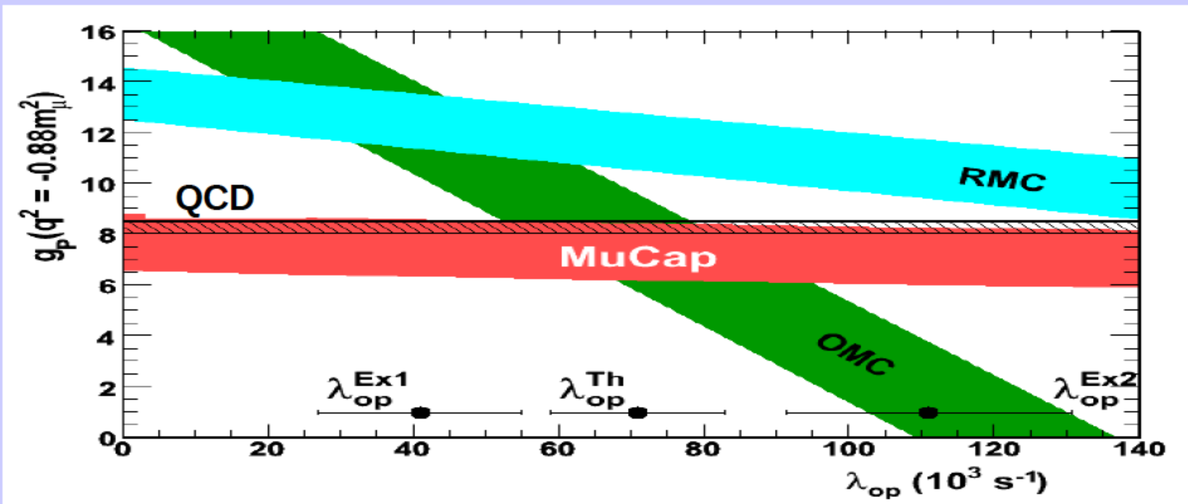
$G_{PS}(q_0^2) = 8.2 \pm 0.2$ at $q_0^2 = -0.88\,m_\mu^2$ relevant for $p(\mu^-,\nu_\mu)n$

Experimental situation I: $\mu^- + p$

MuCap Results

2005: Λ_S = 725.0 ± 13.7(stat) ±10.7(syst) s^{-1}

g_p (q^2 = -0.88m^2_μ) = 7.3±1.1



goal for 2006/2007 dataset is Λ_s to $\pm 5s^{-1}$

From Gorringe's talk at Elba XI (2010)

Experimental situation II: $\mu^- + d$

Two hyperfine states: 1/2 and 3/2 $\Rightarrow \Gamma^D$ and Γ^Q

From theory: $\Gamma^D \simeq 400$ s^{-1} and $\Gamma^Q \simeq 10$ s^{-1} $\Rightarrow$ only Γ^D

- Wang *et al.*, PR **139**, B1528 (1965): $\Gamma^D = 365(96)$ s^{-1}
- Bertini *et al.*, PRD **8**, 3774 (1973): $\Gamma^D = 445(60)$ s^{-1}
- Bardin *et al.*, NPA **453**, 591 (1986): $\Gamma^D = 470(29)$ s^{-1}
- Cargnelli *et al.*, Workshop on fundamental μ physics, Los Alamos, 1986, LA10714C: $\Gamma^D = 409(40)$ s^{-1}
- MuSun Collaboration: result to come!

Experimental situation III: $\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$

Total capture rate Γ_0:

- Folomkin *et al.*, PL **3**, 229 (1963): Γ_0=1410(140) s^{-1}
- Auerbach *et al.*, PR **138**, B127 (1967): Γ_0=1505(46) s^{-1}
- Clay *et al.*, PR **140**, B587 (1965): Γ_0=1465(67) s^{-1}
- Ackerbauer *et al.*, PLB **417**, 224 (1998): Γ_0=1496(4) s^{-1}

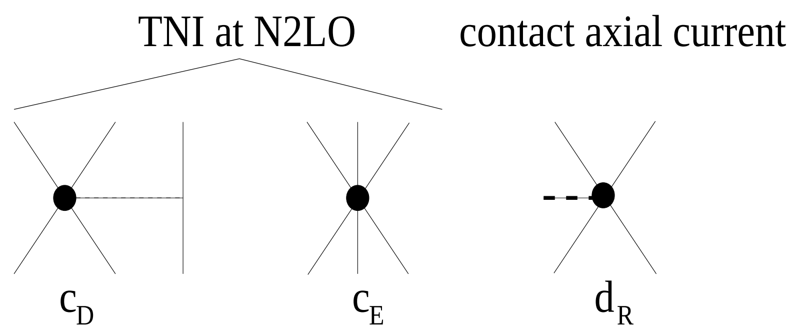
Angular correlation A_v:

- Souder *et al.*, NIMA **402**, 311 (1998): A_v=0.63 $\pm$ 0.09 (stat.)$^{+0.11}_{-0.14}$ (syst.)

Two-body weak currents

- Vector currents from isovector components of $\mathbf{j}_\gamma$ (CVC)
- Axial currents at N^{3}LO include pion-range terms as well as a single contact term (corresponding LEC denoted by d_R)
- One of the two LEC's in the TNI at N^{2}LO is related to d_R:

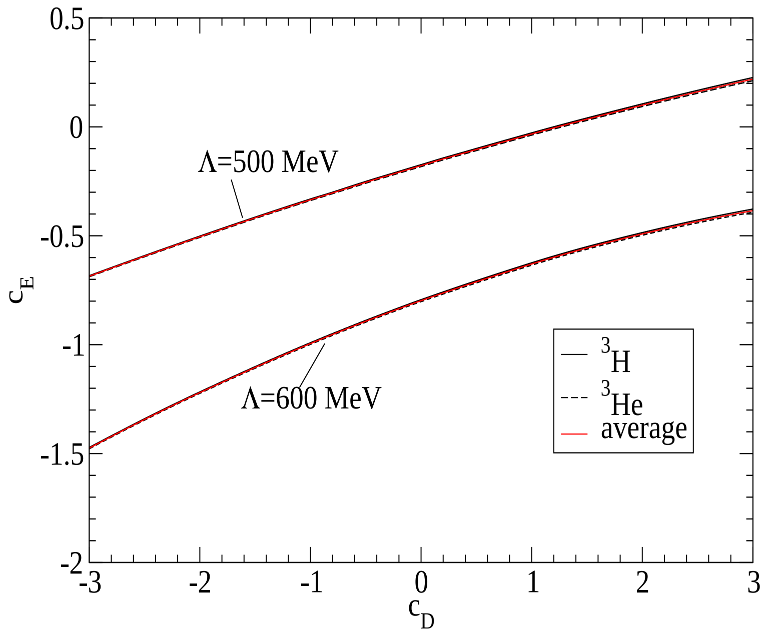
$$d_R = \frac{m_N}{\Lambda_\chi\, g_A} c_D + \frac{1}{3} m_N (c_3 + 2\, c_4) + \frac{1}{6} \ ,$$

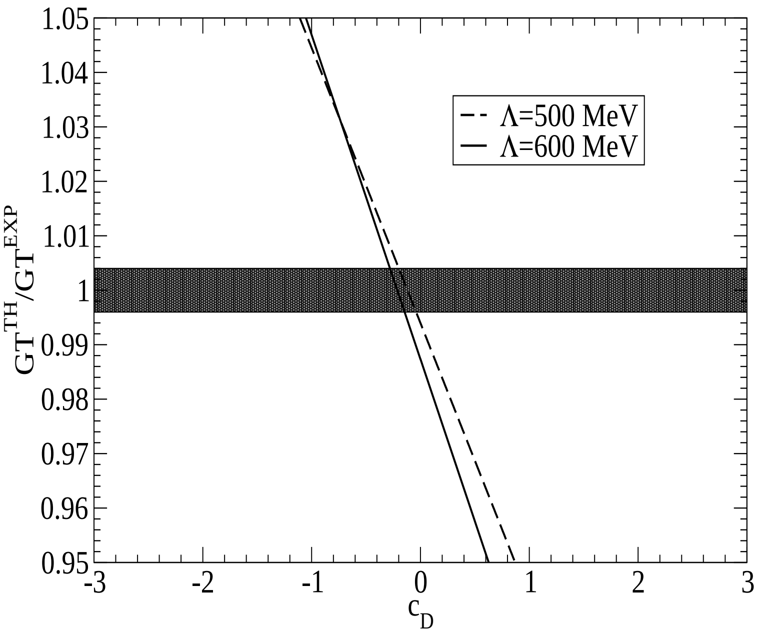


Fix d_R and c_E to reproduce the GT m.e. in ^{3}H β-decay and trinucleon BE*

*Gardestig and Phillips (2006), Gazit, Quaglioni, and Navratil (2009)

Determining c_D and c_E



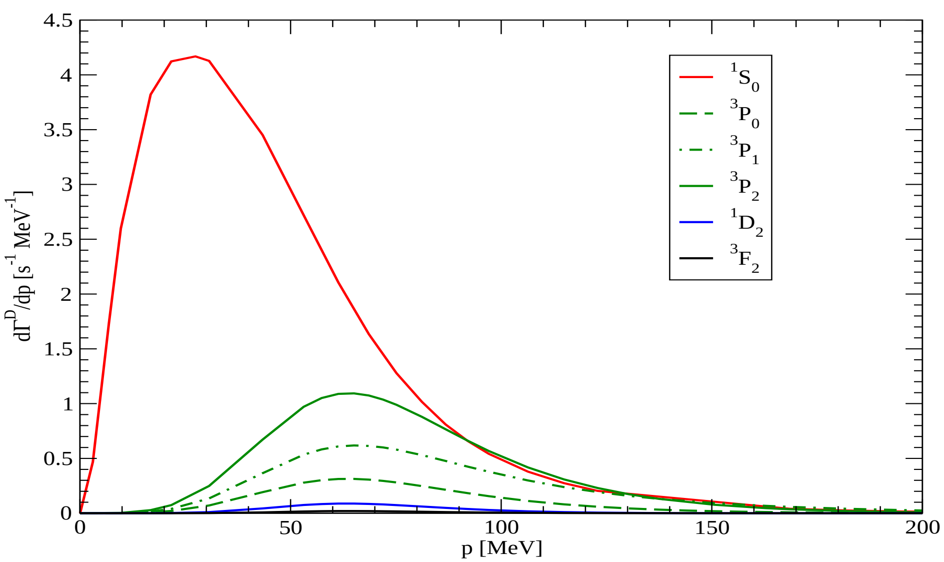


	$\Lambda = 500$ MeV	$\Lambda = 600$ MeV
$\{c_D; c_E\}$	$\{-0.20; -0.208\}$	$\{-0.32; -0.857\}$
$\{c_D; c_E\}$	$\{-0.04; -0.184\}$	$\{-0.19; -0.833\}$

$\mathrm{GT}^{\mathrm{EXP}} = 0.955 \pm 0.004$, error has conservatively been doubled

χEFT predictions for μ-capture on ^{2}H and ^{3}He

	$^1S_0$	$^3P_2$	$\Gamma(^2\mathrm{H})$	$\Gamma(^3\mathrm{He})$
IA($\Lambda = 500$ MeV)	238.8	72.4	381.7	1362
IA($\Lambda = 600$ MeV)	238.7	72.0	380.8	1360
FULL($\Lambda = 500$ MeV)	254.4±0.9	72.1	399.2±0.9	1488±9
FULL($\Lambda = 600$ MeV)	255.2±1.0	71.6	399.1±1.0	1499±9



Constraints on the induced pseudoscalar form factor

Theory predictions with conservative error estimates:

$$\Gamma(^2\mathrm{H}) = (399 \pm 3)\,\mathrm{sec}^{-1} \qquad\qquad \Gamma(^3\mathrm{He}) = (1494 \pm 21)\,\mathrm{sec}^{-1}$$

These errors are due primarily to:

- 0.5% experimental error on $\mathrm{GT}^{\mathrm{EXP}}$
- 0.4% uncertainties in electroweak radiative corrections—they increase the rates by $\simeq 3\%$ (Czarnecki *et al.*, 2007)
- cutoff dependence

Using $\Gamma^{\mathrm{EXP}}(^3\mathrm{He}) = (1496 \pm 4)$ sec^{-1}, one extracts

$$G_{PS}(q_0^2) = 8.2 \pm 0.7 \qquad q_0^2 = -0.954\, m_\mu^2$$

versus a χPT prediction of 7.99 ± 0.20 from Bernard *et al.* (1994) and Kaiser (2003)

Summary and outlook

- Nuclear χEFT in reasonable agreement with data for suppressed processes
- In some instances, such as μ-capture, it provides predictions with $\lesssim 1\%$ accuracy: extract information on nucleon properties
- Current efforts in χEFT aimed at:
  1. Completing an independent derivation of the parity-violating (PV) potential at N^{2}LO (Q), and an analysis of PV effects in A=2, 3, and 4 systems
  2. EM structure of light nuclei: $d(e, e')pn$ at threshold, charge and magnetic form factors, ...
  3. Including Δ d.o.f. explicitely in nuclear potentials and currents (to improve convergence)
  4. Loop corrections to axial current