Electromagnetic Processes in Few-Nucleon Systems at Low Energies

- Nuclear EM currents in χEFT up to one loop
- Constraining the LEC's
- Predictions for radiative captures in A=3 and 4 systems
- Relativity constraints on chiral potentials
- Outlook

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References:

Pastore *et al.* PRC**80**, 034004 (2009); Girlanda *et al.* PRL**105**, 232502 (2010)

Work in Nuclear χEFT: a Partial Listing

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A very incomplete list:

- NN and NNN potentials:
  - van Kolck *et al.* (1994–96)
  - Kaiser, Weise *et al.* (1997–98)
  - Glöckle, Epelbaum, Meissner *et al.* (1998–2005)
  - Entem and Machleidt (2002–03)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al.* (1996–2009), hybrid studies in A=2–4
  - Meissner *et al.* (2001), Kölling *et al.* (2009–2010)
  - Phillips (2003), deuteron static properties and f.f.'s

Lots of work in pionless EFT too ...

Preliminaries

- Time-ordered perturbation theory (TOPT):

$$-\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}}\cdot\mathbf{j} = \langle N'N' \mid T \mid NN;\gamma\rangle$$

$$= \langle N'N' \mid H_1 \sum_{n=1}^{\infty}\left(\frac{1}{E_i - H_0 + i\,\eta} H_1\right)^{n-1} \mid NN;\gamma\rangle$$

- Power counting:

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots\,, \text{ and } T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

- Irreducible and recoil-corrected reducible contributions retained in T expansion
- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{H_1\text{ scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

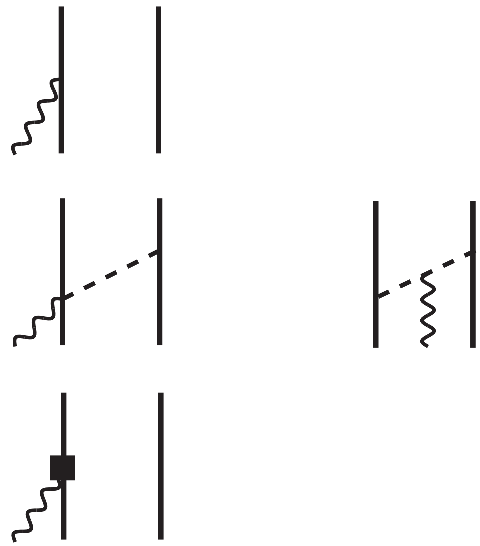
α_i = number of derivatives (momenta) and β_i = number of π's at each vertex

Two-Body EM Currents in χEFT up to N^{2}LO ($e\,Q^0$)

LO : $\mathbf{eQ^{-2}}$

NLO : $\mathbf{eQ^{-1}}$

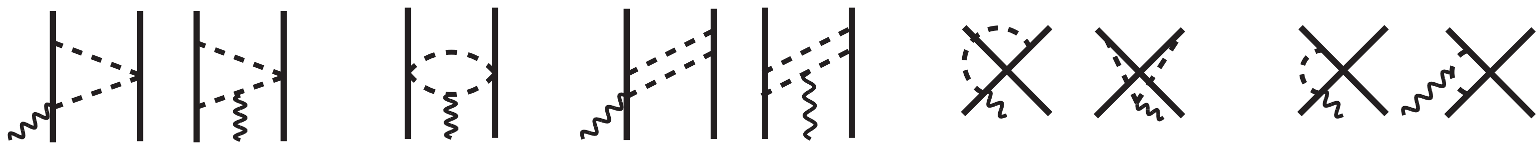
N^{2}LO : $\mathbf{eQ^{0}}$



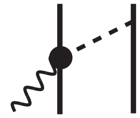
- These depend on the proton and neutron μ's ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$), g_A, and F_π
- One-loop corrections to one-body current are absorbed into μ_N and $\langle r_N^2 \rangle$

N^3LO $(e\,Q)$ Corrections

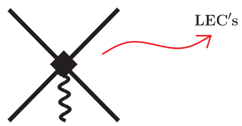
- One-loop corrections:



- Tree-level current with one $e\,Q^2$ vertex from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al.* (1998), involving 3 LEC's ($\sim \gamma N\Delta$ and $\gamma\rho\pi$ currents) :



- Contact currents

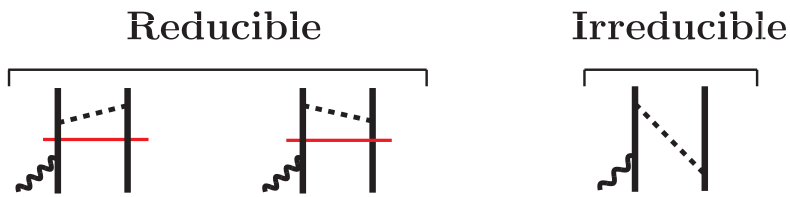


from i) minimal substitution in the interactions involving $\partial\, N$ (7 LEC's determined from strong-interaction sector) and ii) non-minimal couplings (2 LEC's)

Technical Issues I: Recoil Corrections at N^{2}LO

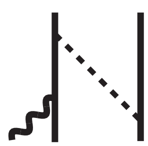
- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$



- Recoil corrections to the reducible contributions obtained by expanding in powers of $(E_i - E_I)/\omega_\pi$ the energy denominators

$$= v^\pi \left(1 + \frac{E_i - E_I}{2\,\omega_\pi}\right) \frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{LO}}$$

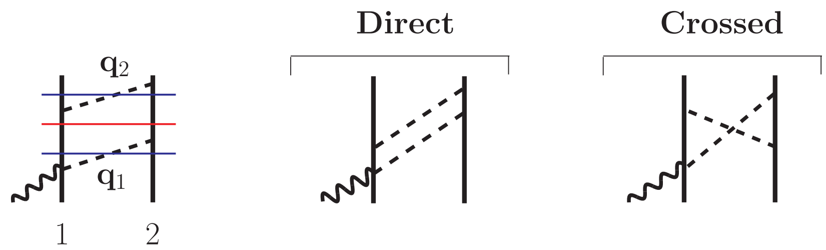


$$= -\frac{v^\pi}{2\,\omega_\pi}\,\mathbf{j}^{\mathrm{LO}}$$

- Recoil corrections to reducible diagrams cancel irreducible contribution

Technical Issues II: Recoil Corrections at N^3LO

$\mathbf{j}^{\mathrm{N^3LO}} =$



- Reducible contributions

$$\mathbf{j}_{\mathrm{red}} = \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1)$$
$$-2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

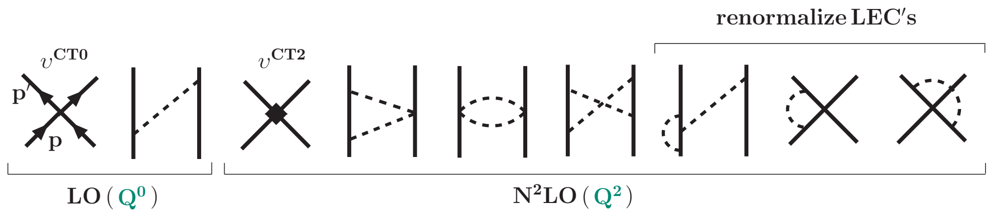
- Irreducible contributions

$$\mathbf{j}_{\mathrm{irr}} = 2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$+\ 2\int \frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_1), V_{\pi NN}(2,\mathbf{q}_2)]_-\ V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Partial cancellations between recoil corrections to reducible diagrams and irreducible contributions

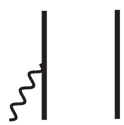
Nuclear χEFT (at Q^2 and $e\,Q$)

NN potential:

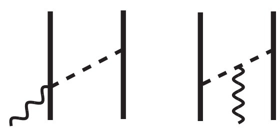


and accompanying set of conserved EM currents:

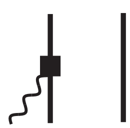
LO : eQ^{-2}



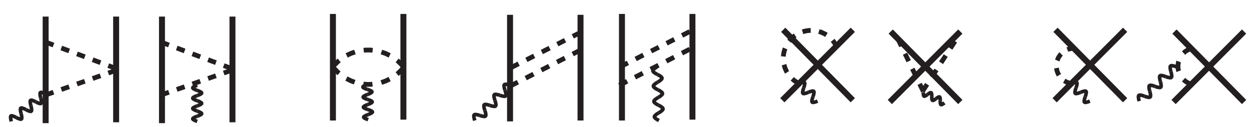
NLO : eQ^{-1}



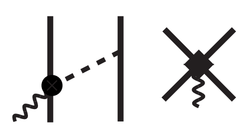
N^2LO : eQ^0



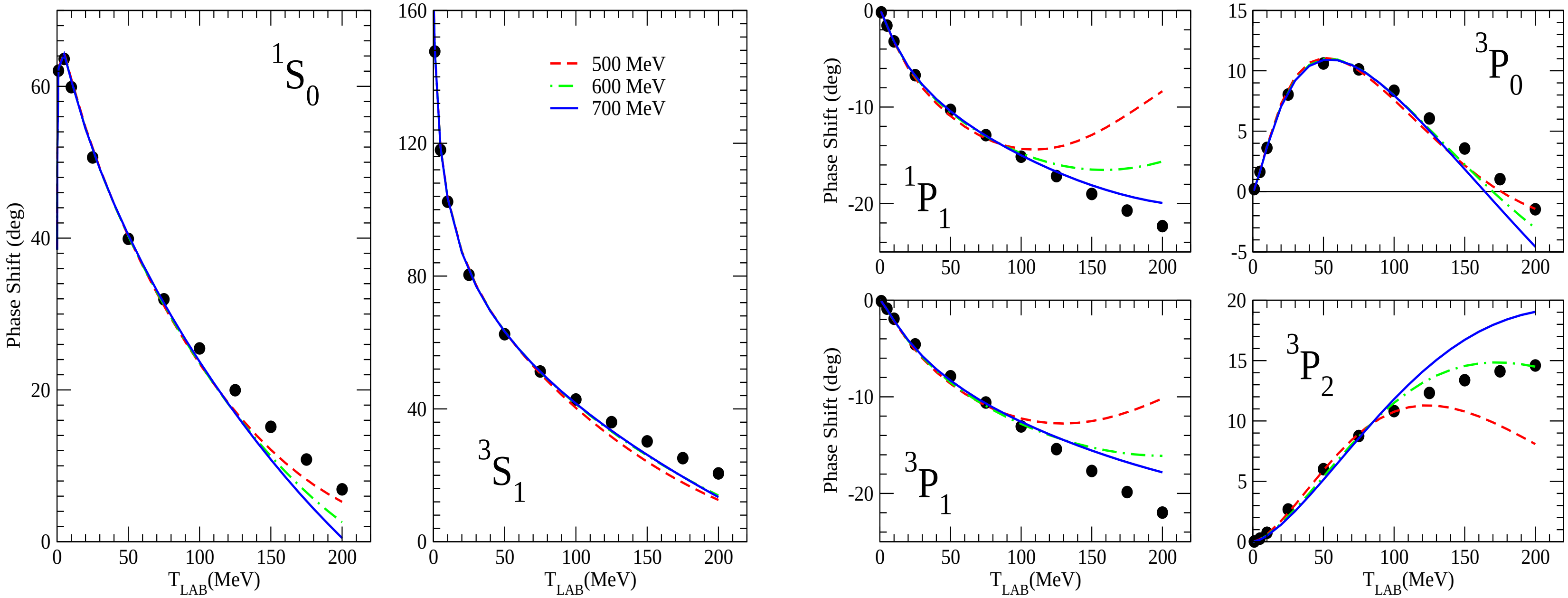
N^3LO : eQ



unknown LEC's

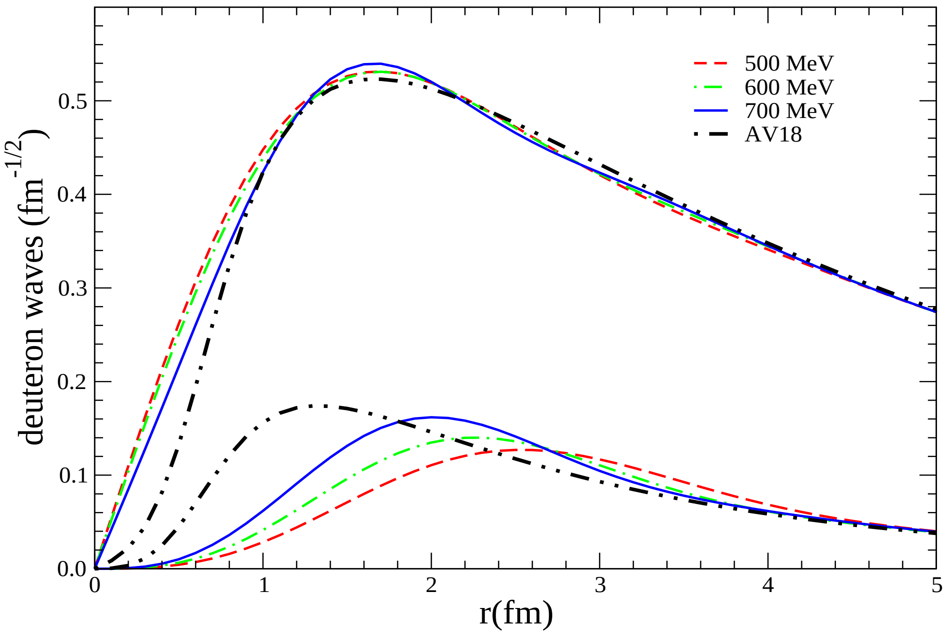


Fits to np Phases up to $T_{\mathrm{LAB}} = 100$ MeV



LS-equation regulator $\sim \exp(-Q^4/\Lambda^4)$ with Λ=500, 600, and 700 MeV (cutting off momenta $Q \gtrsim 3$–$4\ m_\pi$)

Deuteron Properties



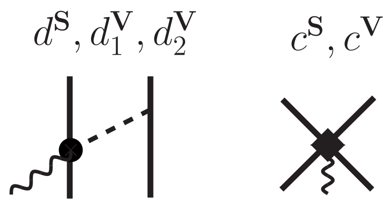
	Λ (MeV)			
	500	600	700	Expt
B_d (MeV)	2.2244	2.2246	2.2245	2.224575(9)
η_d	0.0267	0.0260	0.0264	0.0256(4)
r_d (fm)	1.943	1.947	1.951	1.9734(44)
μ_d (μ_N)	0.860	0.858	0.853	0.8574382329(92)
Q_d (fm^2)	0.275	0.272	0.279	0.2859(3)
P_D (%)	3.44	3.87	4.77	

Comparing to Park *et al.* (1996) and Kölling *et al.* (2009)

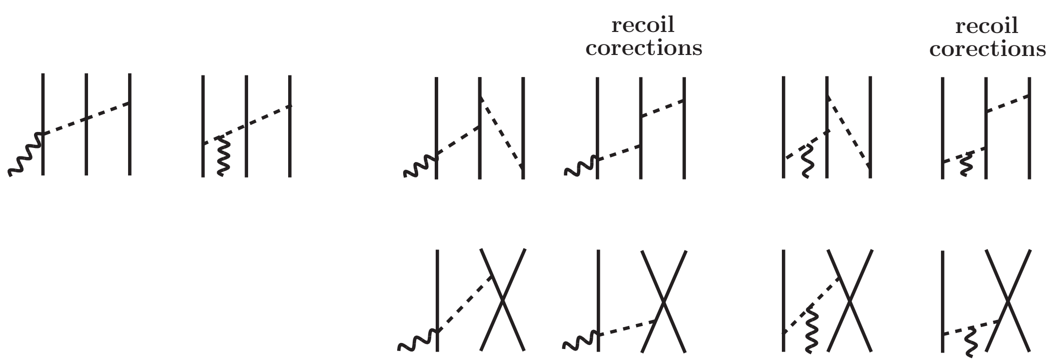
- Expressions for two-body currents (and potential, of course) at one loop in agreement with those of Bonn group (2009) derived via TOPT and the unitary transformation method
- Park *et al.* (1996) use covariant perturbation theory, but obtain different isospin structure for these loop currents: differences in treatment of box diagrams

EM Observables at N^{3}LO

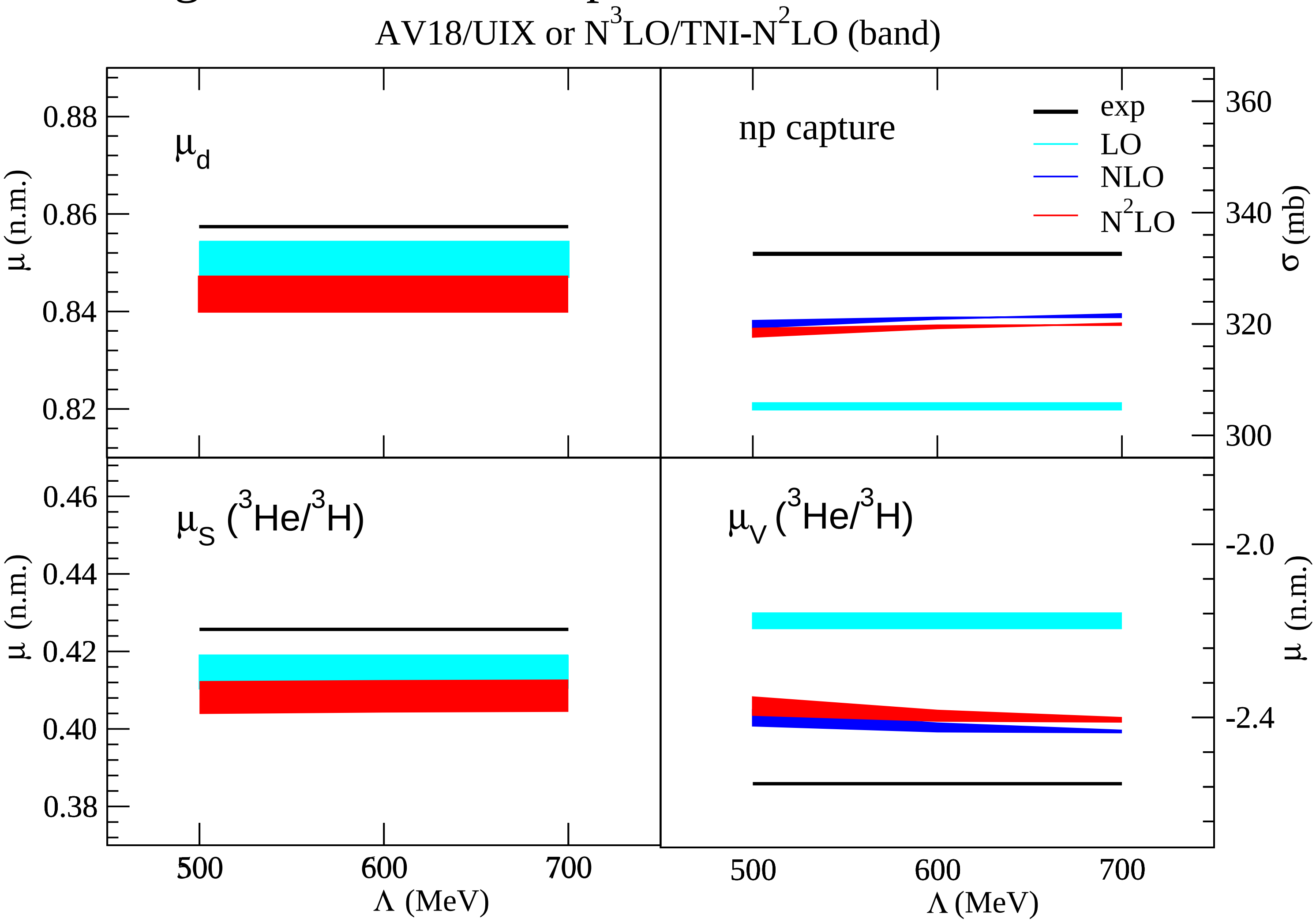
- Pion loop corrections and (minimal) contact terms known
- Five LEC's: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



- $d_2^V / d_1^V = 1/4$ assuming Δ-resonance saturation
- Three-body currents at N^{3}LO vanish:



Fixing LEC's in EM Properties of A=2 and A=3 Nuclei



Fitted LEC Values

- LEC's—in units of Λ—corresponding to $\Lambda = 500$–700 MeV for AV18/UIX (N3LO/N2LO)
- Isoscalar d^S (c^S) and isovector d_1^V (c^V) associated with higher-order $\gamma\pi N$ (contact) currents

Λ	$\Lambda^2 d^S \times 10^2$	$\Lambda^4 c^S$	$\Lambda^2 d_1^V$	$\Lambda^4 c^V$
500	–8.85 (–0.225)	–3.18 (–2.38)	5.18 (5.82)	–11.3 (–11.4)
600	–2.90 (9.20)	–7.10 (–5.30)	6.55 (6.85)	–12.9 (–23.3)
700	6.64 (20.4)	–13.2 (–9.83)	8.24 (8.27)	–1.70 (–46.2)

The nd and $n\,^3$He Radiative Captures

- Suppressed $M1$ processes:

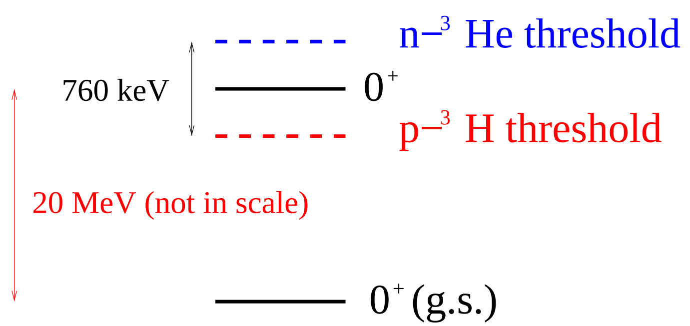
	$\sigma_{\rm exp}$(mb)
^{1}H$(n,\gamma)^2$H	334.2(5)
^{2}H$(n,\gamma)^3$H	0.508(15)
^{3}He$(n,\gamma)^4$He	0.055(3)

- The ^{3}H and ^{4}He bound states are approximate eigenstates of the one-body $M1$ operator, *e.g.* $\hat{\mu}(\text{IA})\,|^3\text{H}\rangle \simeq \mu_p\,|^3\text{H}\rangle$ and $\langle nd \mid \hat{\mu}(\text{IA})\,|^3\text{H}\rangle \simeq 0$ by orthogonality

- A=3 and 4 radiative (and weak) captures very sensitive to i) small components in the w.f.'s and ii) many-body terms in the electro(weak) currents (80-90% of cross section!)

Wave Functions: Recent Progress

- 3 and 4 bound-state w.f.'s and 2+1 continuum routine by now
- Challenges with 3+1 continuum:
  1. Coupled-channel nature of scattering problem: n-^{3}He and p-^{3}H channels both open
  2. Peculiarities of ^{4}He spectrum (see below): hard to obtain numerically converged solutions



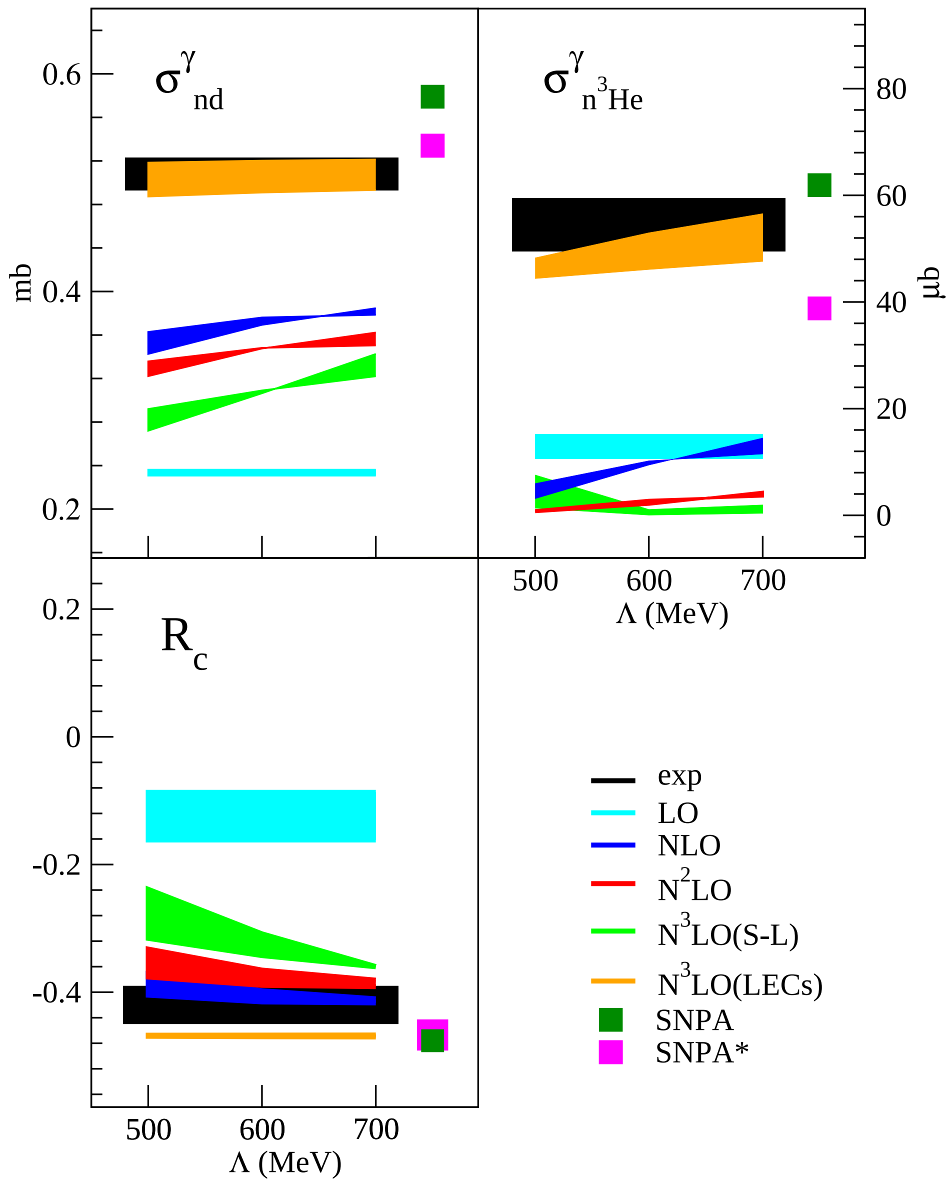
- Major effort by several groups*: both singlet and triplet n-^{3}He scattering lengths in good agreement with data

*Deltuva and Fonseca (2007); Lazauskas (2009); Viviani *et al.* (2010)

Triplet Scattering Length a_1 (fm)

Method	AV18	AV18/UIX
HH	$3.56 - i\,0.0077$	$3.39 - i\,0.0059$
RGM	$3.45 - i\,0.0066$	$3.31 - i\,0.0051$
FY	$3.43 - i\,0.0082$	$3.23 - i\,0.0054$
AGS	$3.51 - i\,0.0074$	
R-matrix	$3.29 \quad - i\,0.0012$	
EXP	$3.28(5) - i\,0.001(2)$	
EXP	$3.36(1)$	
EXP	$3.48(2)$	

Singlet scattering length a_0 (harder to calculate!) also in good agreement with experiment



σ^{γ}_{nd}
$\sigma^{\gamma}_{n^3He}$
R_c
mb
μb
0.6
0.4
0.2
80
60
40
20
0
0.2
0
-0.2
-0.4
500
600
700
Λ (MeV)
exp
LO
NLO
N^{2}LO
N^{3}LO(S-L)
N^{3}LO(LECs)
SNPA
SNPA*

n-d radiative capture cross section* in μb: $\sigma_{\rm nd}^{\rm EXP} = 508(15)\ \mu$b

Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	231	343	322	272	487
600	231	369	348	306	491
700	231	385	362	343	493

n-^{3}He radiative capture cross section* in μb: $\sigma_{\rm n\,^3He}^{\rm EXP} = 55(4)\ \mu$b

Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	15.2	5.95	0.91	1.36	48.3
600	15.2	10.2	2.87	0.04	53.0
700	15.2	11.5	3.56	0.38	56.6

*N3LO/N2LO potentials and HH wave functions

An aside: relativity constraints on v^{LO}

Contact Lagrangian at Q^2

Ordóñez *et al.*, PRC**53**, 2086 (1996)	
O_1	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$
O_3	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$
O_4	$i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_6	$i\,(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^k \overrightarrow{\nabla^j} N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^j \overrightarrow{\nabla^k} N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^j} \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^k} \sigma^j N)$
O_{13}	$(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \overrightarrow{\nabla^j} N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \sigma^j \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$

$$
\begin{aligned}
v^{\text{CT2}}(\mathbf{k},\mathbf{K}) &= C_1\,k^2 + C_2\,K^2 + (C_3\,k^2 + C_4\,K^2)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + i\,C_5\,\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{K}\times\mathbf{k} \\
&+ C_6\,\boldsymbol{\sigma}_1\cdot\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k} + C_7\,\boldsymbol{\sigma}_1\cdot\mathbf{K}\;\boldsymbol{\sigma}_2\cdot\mathbf{K} \\
v^{\text{CT2}}_{\mathbf{P}}(\mathbf{k},\mathbf{K}) &= i\,C_1^*\,\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\cdot\mathbf{P}\times\mathbf{k} + C_2^*\,(\boldsymbol{\sigma}_1\cdot\mathbf{P}\;\boldsymbol{\sigma}_2\cdot\mathbf{K} - \boldsymbol{\sigma}_1\cdot\mathbf{K}\;\boldsymbol{\sigma}_2\cdot\mathbf{P}) \\
&+ (C_3^* + C_4^*\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,P^2 + C_5^*\,\boldsymbol{\sigma}_1\cdot\mathbf{P}\;\boldsymbol{\sigma}_2\cdot\mathbf{P}
\end{aligned}
$$

Actually, 2 of the O_i's in original set are redundant ...

Relativity Constraints

Girlanda *et al.*, PRC**81**, 034005 (2010)

- Reparametrization invariance: only 7 independent combinations of O_i's [Epelbaum *et al.*, PRC**65**, 044001 (2002)]
- What about the other 5 combinations?

Explore constraints that relativity imposes at order Q^2 in two ways:

- Write down the most general contact $\mathcal{L}$ up to Q^2 and carry out its NR reduction
- Enforce the CR's between the generators H and $\mathbf{K}$ directly in the NR theory within a consistent power counting scheme

Both lead to the same result:

$$C_1^* = \frac{C_S - C_T}{4m^2}\ ,\ C_2^* = \frac{C_T}{2m^2}\ ,\ C_3^* = -\frac{C_S}{4m^2}\ ,\ C_4^* = -\frac{C_T}{4m^2}\ ,\ C_5^* = 0$$

and $v_{\mathbf{P}}^{\rm LO}$ should be included in calculations of $A > 2$ properties

NR Reduction I

Building blocks:

$$(\overline{\psi}\, i\overleftrightarrow{\partial}^{\alpha}\, i\overleftrightarrow{\partial}^{\beta} \cdots \Gamma_A\, \psi)\, \partial^{\lambda}\, \partial^{\mu} \cdots (\overline{\psi}\, i\overleftrightarrow{\partial}^{\sigma}\, i\overleftrightarrow{\partial}^{\tau} \cdots \Gamma_B\, \psi)/(2m)^{N_d}$$

∂ on whole bilinear is $\sim Q$; $\overleftrightarrow{\partial}$ inside bilinear is $\sim Q^0$ and, in principle, any number of $\overleftrightarrow{\partial}$ is allowed, however,

i) no two Lorentz indices can be contracted within a bilinear
ii) some of the most problematic terms of the type

$$(\overline{\psi}\, i\overleftrightarrow{\partial}^{\mu_1}\, i\overleftrightarrow{\partial}^{\mu_2} \cdots i\overleftrightarrow{\partial}^{\mu_n}\, \Gamma_A^{\alpha}\, \psi)\, (\overline{\psi}\, i\overleftrightarrow{\partial}_{\mu_1}\, i\overleftrightarrow{\partial}_{\mu_2} \cdots i\overleftrightarrow{\partial}_{\mu_n}\, \Gamma_{B\,\alpha}\, \psi)/(2m)^{2n}$$

do not introduce any new structures for $n > 1$, since

$$(\overline{u}_3\, \Gamma_A^{\alpha}\, u_1)\ (\overline{u}_4\, \Gamma_{B\,\alpha}\, u_2)\ [(p_1 + p_3) \cdot (p_2 + p_4)]^n/(2m)^{2n}$$

and to order Q^2 the $[\dots]$ can be expanded as

$$1 + n\ \left[\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2 + \mathbf{p}_4^2 - (\mathbf{p}_1 + \mathbf{p}_3) \cdot (\mathbf{p}_2 + \mathbf{p}_4)\right]/(4m^2)$$

NR Reduction II

- 36 (hermitian) $\mathcal{C}$- and $\mathcal{P}$-invariant terms
- NR reduction and use of EOM to remove time derivatives lead to 2 leading terms (Q^0), accompanied by specific $1/m^2$ corrections, and 7 subleading ones (Q^2)

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{2}C_S\left[O_S + \frac{1}{4m^2}(O_1 + O_3 + O_5 + O_6)\right] \\
& -\frac{1}{2}C_T\left[O_T - \frac{1}{4m^2}\left(O_5 + O_6 - O_7 + O_8 + 2\,O_{12} + O_{14}\right)\right] \\
& -\frac{1}{2}C_1(O_1 + 2\,O_2) + \frac{1}{8}C_2(2\,O_2 + O_3) - \frac{1}{2}C_3(O_9 + 2\,O_{12}) \\
& -\frac{1}{8}C_4(O_9 + O_{14}) + \frac{1}{4}C_5(O_6 - O_5) - \frac{1}{2}C_6(O_7 + 2\,O_{10}) \\
& -\frac{1}{16}C_7(O_7 + O_8 + 2\,O_{13})
\end{aligned}
$$

Poincaré Algebra Constraints

Girlanda *et al.*, PRC**81**, 034005 (2010)

- $H = H_0 + H_I$ and $\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I$

$$\left[K^i\,,\,K^j\right] = -i\,\epsilon^{ijk} J^k = \left[K_0^i\,,\,K_0^j\right] \qquad [\mathbf{K}\,,\,H] = i\,\mathbf{P} = [\mathbf{K}_0\,,\,H_0]$$

- Power counting—follows from $b_s(\mathbf{p})$ and $b_s^\dagger(\mathbf{p}) \sim Q^{-3/2}$:

$$\begin{aligned} \mathbf{K}_0 &= \mathbf{K}_0^{(-1)} + \mathbf{K}_0^{(1)} + \dots \qquad & H_0 = H_0^{(0)} + H_0^{(2)} + \dots \\ \mathbf{K}_I &= \mathbf{K}_I^{(2)} + \mathbf{K}_I^{(4)} + \dots \qquad & H_I = H_I^{(3)} + H_I^{(5)} + \dots \end{aligned}$$

- Constraints arise on H_I and $\mathbf{K}_I$; at order Q^2

$$\begin{aligned} H_I^{(5)} \sim\ & \frac{C_S}{8m^2}\int d\mathbf{x}\ (O_1 + O_3 + O_5 + O_6) \\ -\ & \frac{C_T}{8m^2}\int d\mathbf{x}\ (O_5 + O_6 - O_7 + O_8 + 2\,O_{12} + O_{14}) \end{aligned}$$

Boost Corrections to Potentials

- Well known result[1], v rest-frame potential:

$$\delta v(\mathbf{P}) = -\frac{P^2}{8\,m^2} v + \frac{i}{8\,m^2} \left[\mathbf{P} \cdot \mathbf{r}\; \mathbf{P} \cdot \mathbf{p}\,,\, v\right] + \frac{i}{8\,m^2} \left[(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times \mathbf{P} \cdot \mathbf{p}\,,\, v\right]$$

- Should be included in χEFT calculations–comparable to three-body force contributions

Expectation values in MeV[2]

	T	$v(NN)$	$V(NNN)$	$\delta v(NN)$
^{3}H	48.7	–55.9	–1.21	0.34
^{4}He	105.0	–127.4	–5.43	1.76

[1] Krajcik and Foldy (1974); Friar (1975); Carlson *et al.* (1993)

[2] Forest *et al.* (1995); for effects on A=3 continuum, see Witala *et al.* (2008)

Outlook

- An analysis of the parity-violating potential at N^2LO (Q) has just been completed: determined by h_π^1 plus 5 contact terms
- Include Δ-isobar degrees of freedom
- Study effects of boost corrections to chiral potentials in $A = 3$ and 4 bound- and scattering-state properties
- EM structure of light nuclei: $d(e, e')pn$ at threshold, charge and magnetic form factors, ...