A=3–4 radiative captures in a hybrid χEFT approach

- EM currents in the conventional approach
- Potential and EM currents up to one loop in χEFT
- Boost corrections to the chiral potential
- ^{2}H$(n,\gamma)^3$H and ^{3}He$(n,\gamma)^4$He captures: a first set of calculations in the hybrid approach
- Summary and outlook

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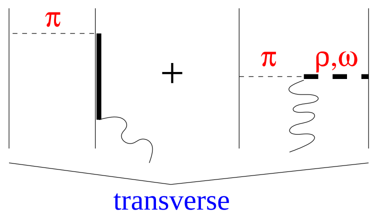
J. Goity   A. Kievsky   L. Marcucci   R. Wiringa

References: Pastore *et al.*, PRC**78**, 064002 (2008); PRC**80**, 034004 (2009); Girlanda *et al.*, PRC**81**, 034005 (2010)

Conventional Approach: EM Currents

Marcucci *et al.*, PRC**72**, 014001 (2005)

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(\mathrm{v}) + \ldots + \mathbf{j}^{(3)}(V^{2\pi})$$

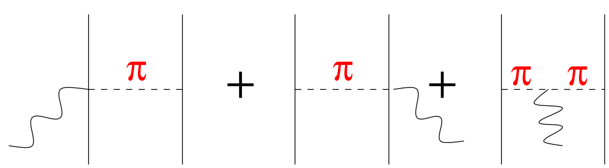


- Static part v_0 of v from π-like (PS) and ρ-like (V) exchanges
- Currents from corresponding PS and V exchanges, for example

$$\begin{aligned}
\mathbf{j}_{ij}(v_0; PS) &= \mathrm{i}\left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_z \Big[v_{PS}(k_j)\boldsymbol{\sigma}_i \left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right) \\
&+ \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} v_{PS}(k_i)\left(\boldsymbol{\sigma}_i \cdot \mathbf{k}_i\right)\left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right) \Big] + i \leftrightharpoons j
\end{aligned}$$

with $v_{PS}(k) = v^{\sigma\tau}(k) - 2\, v^{t\tau}(k)$ projected out from v_0 components

$$\mathbf{j}^{(2)}(\mathrm{v}) \xrightarrow[\text{long range}]{}$$



- Currents from v_p via minimal substitution in i) explicit and ii) implicit p-dependence, the latter from

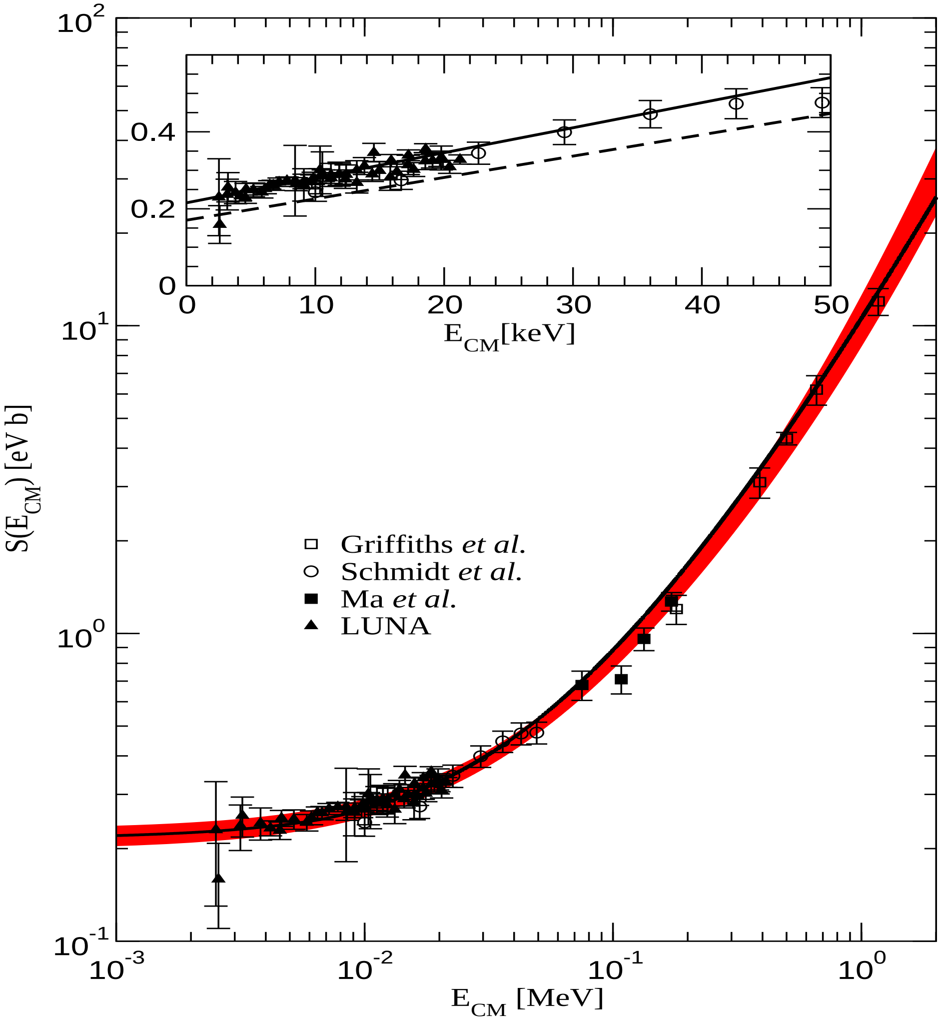
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 + (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, \mathrm{e}^{\mathrm{i}(\mathbf{r}_{ji}\cdot\mathbf{p}_i + \mathbf{r}_{ij}\cdot\mathbf{p}_j)}$$

- Currents are conserved, contain no free parameters, and are consistent with short-range behavior of v and $V^{2\pi}$, but are not unique

Variety of EM observables in A=2–7 nuclei well reproduced, including μ's and $M1$ widths, elastic and inelastic f.f.'s, inclusive response functions, ...

current predictions for ^{2}H$(n,\gamma)^3$H and ^{3}He$(n,\gamma)^4$He cross-sections shown later

$^2H(p,\gamma)^3He$ capture at low energies



Nuclear χEFT Approach

Weinberg, PLB**251**, 288 (1990); NPB**363**, 3 (1991); PLB**295**, 114 (1992)

- χEFT exploits the χ-symmetry exhibited by QCD to restrict the form of π interactions with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum Q, and $\mathcal{L}_{\text{eff}}$ can be systematically expanded in powers of Q/Λ_χ ($\Lambda_\chi \simeq 1$ GeV)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

- χEFT allows for a perturbative treatment in terms of a Q–as opposed to a coupling constant–expansion
- The unknown coefficients in this expansion–the LEC's–are fixed by comparison with experimental data
- Nuclear χEFT provides a practical calculational scheme, susceptible (in principle) of systematic improvement

Previous Work

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A *very* incomplete list:

- NN potentials:
  - van Kolck *et al.* (1994–96)
  - Kaiser, Weise *et al.* (1997–98)
  - Glöckle, Epelbaum, Meissner (1998–2005)
  - Entem and Machleidt (2002–03)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al.* (1996–2009), hybrid studies in A=2–4
  - Epelbaum, Meissner *et al.* (2001, 2009)
  - Phillips (2003), deuteron static properties and f.f.'s

Lots of work in pionless EFT too ...

Preliminaries

- Degrees of freedom: pions (π) and nucleons (N)
- Time-ordered perturbation theory (TOPT):

$$
\begin{aligned}
-\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}} \cdot \mathbf{j} &= \langle N' N' \mid T \mid N N; \gamma \rangle \\
&= \langle N' N' \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid N N; \gamma \rangle
\end{aligned}
$$

- H_0 = free π and N Hamiltonians; H_1 = interacting π, N, and γ Hamiltonians implied by $\mathcal{L}_{\text{eff}}$
- Irreducible and recoil-corrected reducible contributions retained in T expansion

Power Counting

- In the chiral expansion the transition amplitude is expressed as

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots\,, \ \text{ and } \ T^{N^nLO} \sim (Q/\Lambda_\chi)^n \, T^{LO}$$

and power counting allows one to arrange contributions to T in powers of Q

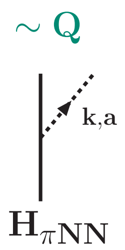
- A contribution with N interaction vertices and L loops scales as

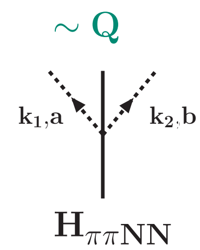
$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2}\right)}_{H_1\text{ scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

α_i = number of derivatives (momenta) and β_i = number of π's at each vertex

- This power counting also follows from considering Feynman diagrams, where loop integrations are in four dimensions

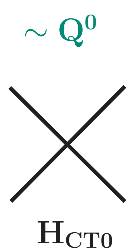
Strong Interaction Vertices up to Q^2

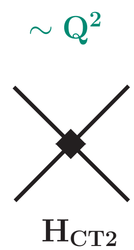




$$H_{\pi NN} = -i\frac{g_A}{F_\pi}\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\sqrt{2\,\omega_k}}\tau_a \qquad H_{\pi\pi NN} = -\frac{i}{F_\pi^2}\frac{\omega_{k_1}-\omega_{k_2}}{\sqrt{4\,\omega_{k_1}\omega_{k_2}}}\epsilon_{abc}\,\tau_c$$

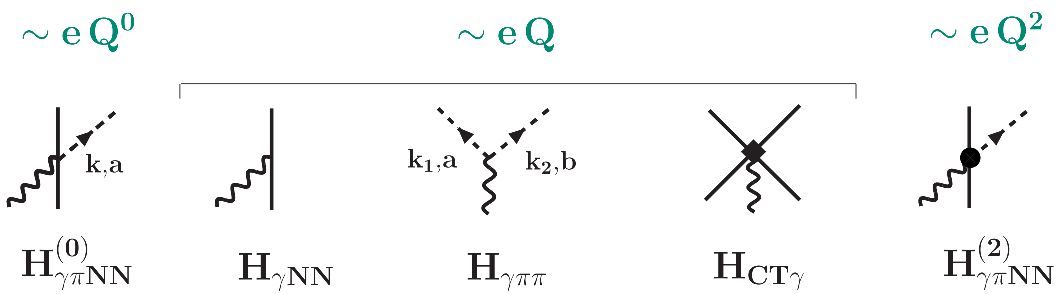
- $g_A = 1.29$ (via GT-relation) and $F_\pi = 184.8$ MeV





- $H_{\rm CT0}$: $4N$ contact terms, 2 LEC's
- $H_{\rm CT2}$: $4N$ contact terms with two gradients, 12 LEC's

Electromagnetic Interaction Vertices up to Q^2



- $H^{(0)}_{\gamma\pi NN}$, $H_{\gamma NN}$, and $H_{\gamma\pi\pi}$ known: depend on g_A, F_π, and proton and neutron μ's ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$)

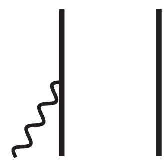
- $H_{\mathrm{CT}\gamma}$: terms from minimal substitution in H_{CT2} known, but 2 additional LEC's enter due non-minimal couplings

- $H^{(2)}_{\gamma\pi NN}$ from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al.* (1998): depends on 3 LEC's, two multiplying isovector structures ($\sim \gamma N\Delta$-excitation current) and one isoscalar structure ($\sim \gamma\rho\pi$ transition current)

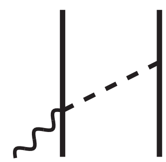
Two-Body Currents up to N^{2}LO

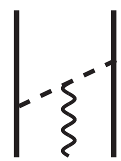
- Up to N^{2}LO

LO : $\mathbf{eQ^{-2}}$

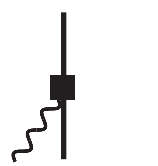


NLO : $\mathbf{eQ^{-1}}$

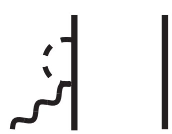


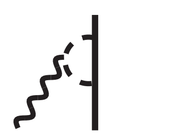


N^{2}LO : $\mathbf{eQ^{0}}$



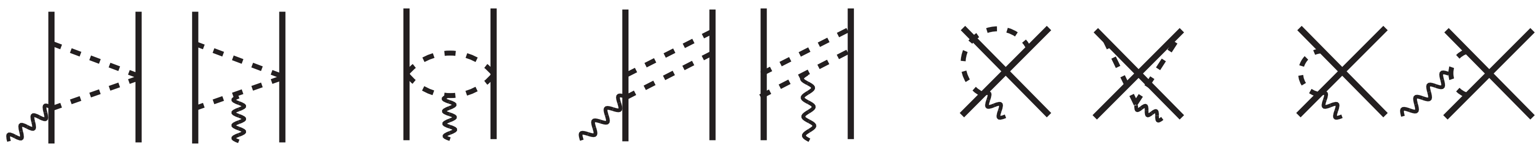
- One-loop corrections to one-body current absorbed into μ_N and $\langle r_N^2 \rangle$



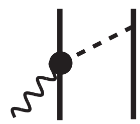


Two-Body Currents at N^{3}LO ($e\,Q^3$)

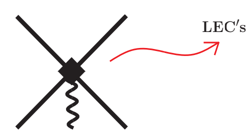
- One-loop corrections:



- Tree-level current with one $e\,Q^2$ vertex (3 LEC's):



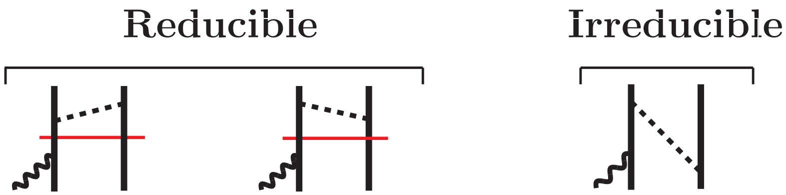
- Currents from contact interactions (12 LEC's from minimal and 2 LEC's from non-minimal couplings):



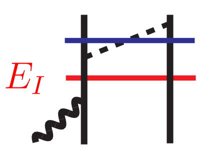
Recoil Corrections at N^{2}LO

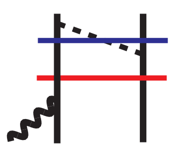
- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$

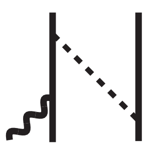


- Recoil corrections to the reducible contributions obtained by expanding in powers of $(E_i - E_I)/\omega_\pi$ the energy denominators





$$= v^\pi \left(1 + \frac{E_i - E_I}{2\,\omega_\pi}\right) \frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{LO}}$$

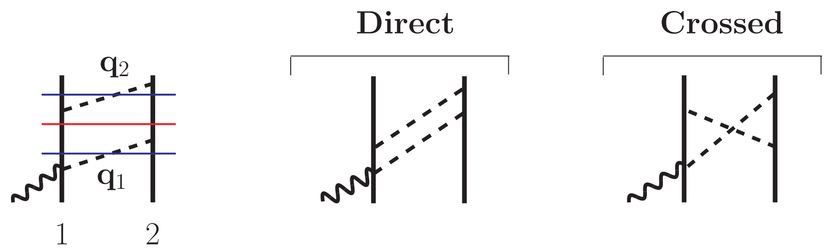


$$= -\frac{v^\pi}{2\,\omega_\pi}\,\mathbf{j}^{\mathrm{LO}}$$

- Recoil corrections to reducible diagrams cancel irreducible contribution

Recoil Corrections at N^{3}LO

$\mathbf{j}^{\mathrm{N^3LO}} \quad =$



- Reducible contributions

$$\mathbf{j}_{\rm red} \quad = \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\rm NLO}(\mathbf{q}_1)$$
$$-2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Irreducible contributions

$$\mathbf{j}_{\rm irr} \quad = \quad 2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$+ \quad 2\int \frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_1), V_{\pi NN}(2,\mathbf{q}_2)]_-\; V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Partial cancellations between recoil corrections to reducible diagrams and irreducible contributions

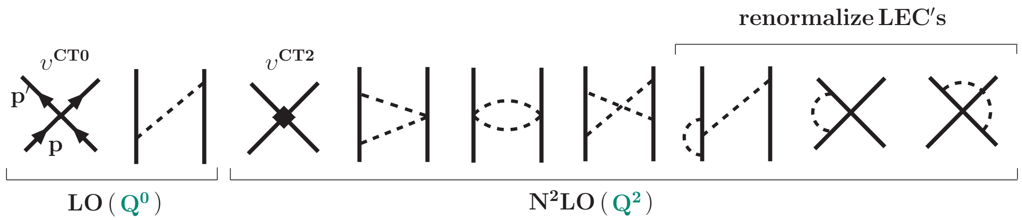
Magnetic Moment at N^{3}LO

$$
\begin{aligned}
\boldsymbol{\mu}^{\mathrm{N^3LO}}_{\mathrm{Sachs}} &= -\frac{i}{2}\, e\,(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z\, \mathbf{R} \times \nabla_k\, v_0^{2\pi}(k) + \frac{e}{4}\,\frac{\tau_{1,z} - \tau_{2,z}}{2}\,\mathbf{R} \\
&\times \Big[2\,(C_2 + C_4\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\,\mathbf{K} - i\, C_5\, \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \times \mathbf{k} \\
&+ \quad C_7\,(\boldsymbol{\sigma}_1\, \boldsymbol{\sigma}_2 \cdot \mathbf{K} + \boldsymbol{\sigma}_1 \cdot \mathbf{K}\; \boldsymbol{\sigma}_2) \Big] \\
\boldsymbol{\mu}^{\mathrm{N^3LO}}_{\mathrm{loop}} &= \quad \frac{e\, g_A^2}{8\,\pi^2 F_\pi^4}\, \tau_{2,z} \Big[F_0(k)\, \boldsymbol{\sigma}_1 - F_2(k)\, \frac{\mathbf{k}\, \boldsymbol{\sigma}_1 \cdot \mathbf{k}}{k^2} \Big] \\
&+ \quad \frac{e\, g_A^2}{2\,\pi^2 F_\pi^2}\, \tau_{2,z}\, (C_S\, \boldsymbol{\sigma}_2 - C_T\, \boldsymbol{\sigma}_1) + 1 \rightleftharpoons 2 \\
\boldsymbol{\mu}^{\mathrm{N^3LO}}_{\mathrm{tree}} &= e\,\frac{g_A}{F_\pi^2} \Big[\Big(d_1^V\, \tau_{2,z} + d^S\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \Big)\, \mathbf{k} \\
&- \quad d_2^V\, (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \boldsymbol{\sigma}_1 \times \mathbf{k} \Big] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}}{k^2 + m_\pi^2} + 1 \rightleftharpoons 2 \\
\boldsymbol{\mu}^{\mathrm{N^3LO}}_{\mathrm{CT}} &= -e\, c^S\, \boldsymbol{\sigma}_1 - e\, c^V (\tau_{1,z} - \tau_{2,z}) \boldsymbol{\sigma}_1 + 1 \rightleftharpoons 2
\end{aligned}
$$

Comparing to Park *et al.* (1996) and Kölling *et al.* (2009)

- Expressions for two-body currents (and potential) at one loop in agreement with those of Bonn group (derived via the UTM)
- Expression for $\boldsymbol{\mu}$ in Park *et al.* has different isospin structure: different treatment of box diagrams—only irreducible ones retained in Park *et al.*

Determining LEC's: NN Potential at N^{2}LO

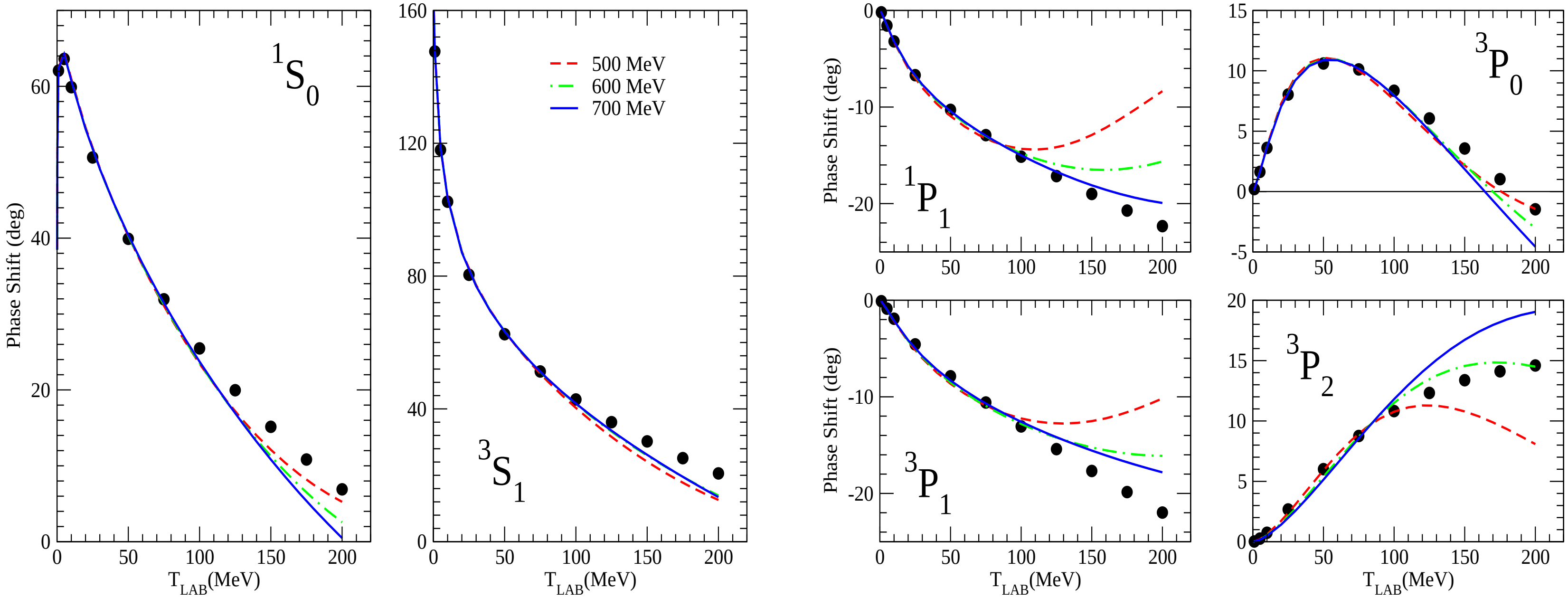


- Contact potential at N^{2}LO: $v^{\rm CT2}(\mathbf{k},\mathbf{K}) + v^{\rm CT2}_{\mathbf{P}}(\mathbf{k},\mathbf{K})$
  - Galilean-invariant term $v^{\rm CT2}$ depends on 7 LEC's (C_i's)
  - Pair-momentum dependent term $v^{\rm CT2}_{\mathbf{P}}$ depends on 5 LEC's:

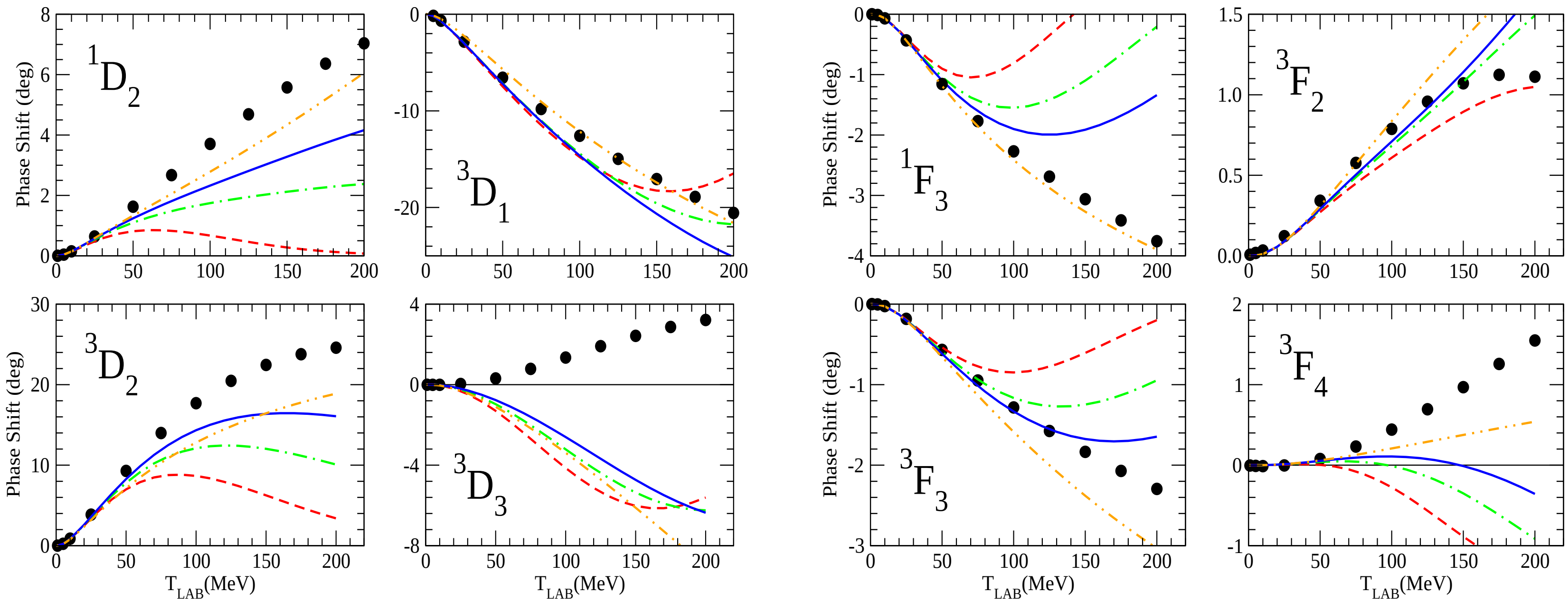
$$
\begin{aligned}
v^{\rm CT2}_{\mathbf{P}} &= i\, C_1^* \, \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k} + C_2^* \, (\boldsymbol{\sigma}_1 \cdot \mathbf{P} \; \boldsymbol{\sigma}_2 \cdot \mathbf{K} - \boldsymbol{\sigma}_1 \cdot \mathbf{K} \; \boldsymbol{\sigma}_2 \cdot \mathbf{P}) \\
&+ \quad (C_3^* + C_4^* \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\, P^2 + C_5^* \, \boldsymbol{\sigma}_1 \cdot \mathbf{P} \; \boldsymbol{\sigma}_2 \cdot \mathbf{P}
\end{aligned}
$$

- Interpretation of $v^{\rm CT2}_{\mathbf{P}}$: boost correction to LO (rest-frame) $v^{\rm CT0}$, then $C_1^* = (C_S - C_T)/(4m_N^2)$, $C_2^* = C_T/(2m_N^2)$, ...
- Retaining recoil corrections in both v and $\mathbf{j}$ ensures current conservation up to N^{3}LO

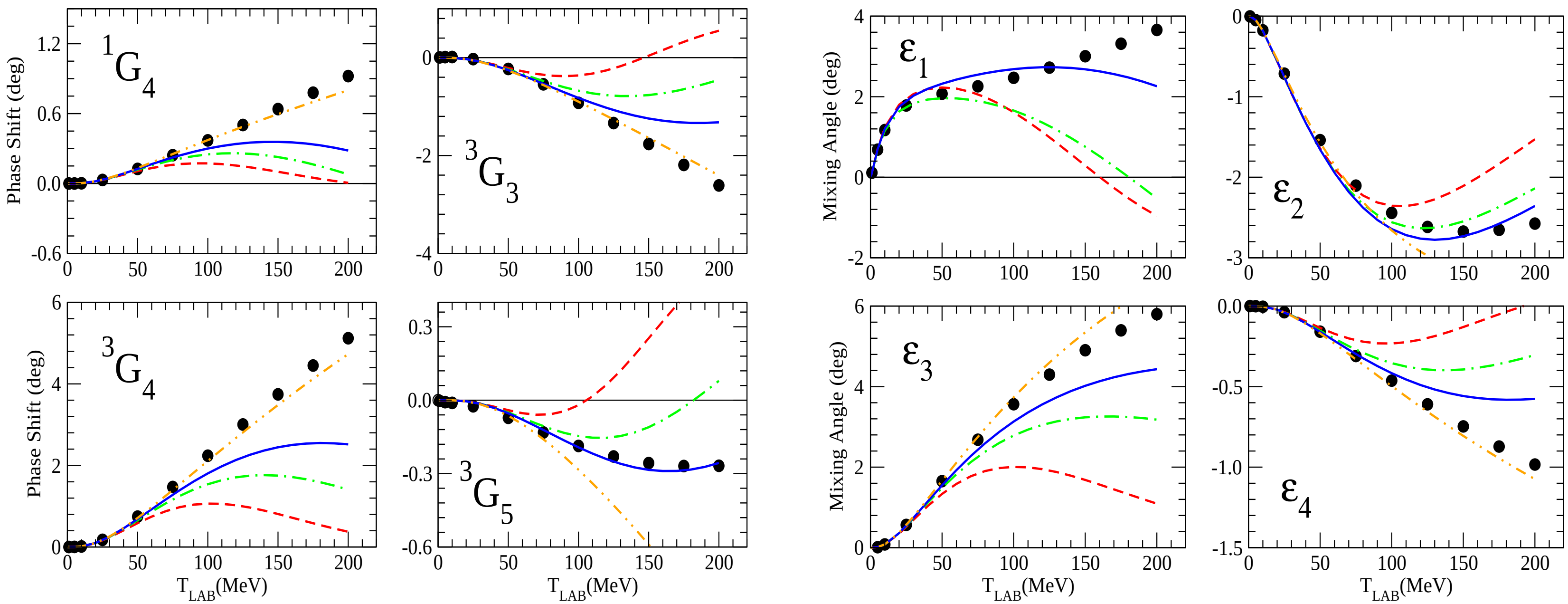
Fits to np Phases up to $T_{\mathrm{LAB}} = 100$ MeV



LS-equation regulator $\sim \exp(-2\,Q^4/\Lambda^4)$ with Λ=500, 600, and 700 MeV (cutting off momenta $Q \gtrsim 3$–$4\ m_\pi$)

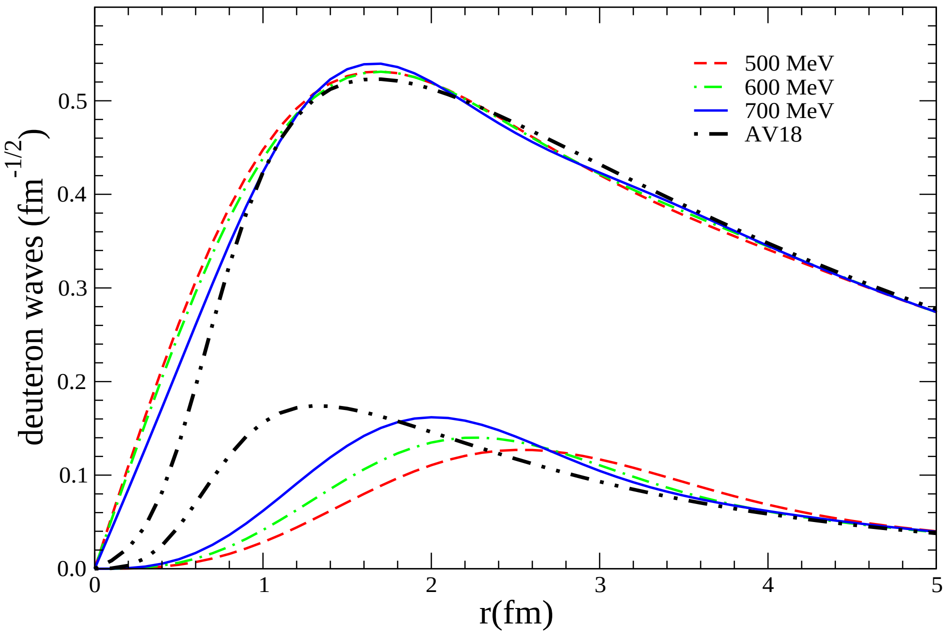


OPE+TPE chiral potential in first order PT, after Kaiser *et al.* (1997): orange dash-double-dot line



Phase Shift (deg)
1G4
3G3
Mixing Angle (deg)
ε1
ε2
3G4
3G5
ε3
ε4
TLAB(MeV)

Deuteron Properties



	Λ (MeV)			
	500	600	700	Expt
B_d (MeV)	2.2244	2.2246	2.2245	2.224575(9)
η_d	0.0267	0.0260	0.0264	0.0256(4)
r_d (fm)	1.943	1.947	1.951	1.9734(44)
μ_d (μ_N)	0.860	0.858	0.853	0.8574382329(92)
Q_d (fm^2)	0.275	0.272	0.279	0.2859(3)
P_D (%)	3.44	3.87	4.77	

Contact Lagrangian at Q^2

Ordóñez *et al.*, PRC**53**, 2086 (1996)

O_1	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$
O_3	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$
O_4	$i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_6	$i\,(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{\nabla}} N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^k \overrightarrow{\nabla^j} N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^j \overrightarrow{\nabla^k} N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^j} \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^k} \sigma^j N)$
O_{13}	$(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma}\, \overrightarrow{\nabla^j} N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \sigma^j \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$

$$
\begin{aligned}
v^{\mathrm{CT2}}(\mathbf{k},\mathbf{K}) &= C_1\, k^2 + C_2\, K^2 + (C_3\, k^2 + C_4\, K^2)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + i\, C_5\, \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{K} \times \mathbf{k} \\
&+ C_6\, \boldsymbol{\sigma}_1 \cdot \mathbf{k}\ \boldsymbol{\sigma}_2 \cdot \mathbf{k} + C_7\, \boldsymbol{\sigma}_1 \cdot \mathbf{K}\ \boldsymbol{\sigma}_2 \cdot \mathbf{K} \\
v^{\mathrm{CT2}}_{\mathbf{P}}(\mathbf{k},\mathbf{K}) &= i\, C_1^*\, \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k} + C_2^*\, (\boldsymbol{\sigma}_1 \cdot \mathbf{P}\ \boldsymbol{\sigma}_2 \cdot \mathbf{K} - \boldsymbol{\sigma}_1 \cdot \mathbf{K}\ \boldsymbol{\sigma}_2 \cdot \mathbf{P}) \\
&+ (C_3^* + C_4^*\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\, P^2 + C_5^*\, \boldsymbol{\sigma}_1 \cdot \mathbf{P}\ \boldsymbol{\sigma}_2 \cdot \mathbf{P}
\end{aligned}
$$

Actually, 2 of the O_i's in original set are redundant …

Relativity Constraints

Girlanda *et al.*, PRC**81**, 034005 (2010)

- Reparametrization invariance: only 7 independent combinations of O_i's [Epelbaum *et al.*, PRC**65**, 044001 (2002)]
- What about the other 5 combinations?

Explore constraints that relativity imposes at order Q^2 in two ways:

- Write down the most general contact $\mathcal{L}$ up to Q^2 and carry out its NR reduction
- Enforce the CR's between the generators H and $\mathbf{K}$ directly in the NR theory within a consistent power counting scheme

Both lead to the same result:

$$C_1^* = \frac{C_S - C_T}{4m^2} \ , \ C_2^* = \frac{C_T}{2m^2} \ , \ C_3^* = -\frac{C_S}{4m^2} \ , \ C_4^* = -\frac{C_T}{4m^2} \ , \ C_5^* = 0$$

and $v_{\mathbf{P}}^{\mathrm{LO}}$ should be included in calculations of $A > 2$ properties

NR Reduction

Building blocks:

$$(\overline{\psi}\, i\overleftrightarrow{\partial}^{\alpha}\, i\overleftrightarrow{\partial}^{\beta} \cdots \Gamma_A\, \psi)\, \partial^\lambda\, \partial^\mu \cdots (\overline{\psi}\, i\overleftrightarrow{\partial}^{\sigma}\, i\overleftrightarrow{\partial}^{\tau} \cdots \Gamma_B\, \psi)/(2m)^{N_d}$$

∂ on whole bilinear is $\sim Q$; $\overleftrightarrow{\partial}$ inside bilinear is $\sim Q^0$ and, in principle, any number of $\overleftrightarrow{\partial}$ is allowed, however,

i) no two Lorentz indices can be contracted within a bilinear
ii) some of the most problematic terms of the type

$$(\overline{\psi}\, i\overleftrightarrow{\partial}^{\mu_1}\, i\overleftrightarrow{\partial}^{\mu_2} \cdots i\overleftrightarrow{\partial}^{\mu_n}\, \Gamma_A^\alpha\, \psi)\, (\overline{\psi}\, i\overleftrightarrow{\partial}_{\mu_1}\, i\overleftrightarrow{\partial}_{\mu_2} \cdots i\overleftrightarrow{\partial}_{\mu_n}\, \Gamma_{B\,\alpha}\, \psi)/(2m)^{2n}$$

do not introduce any new structures for $n > 1$, since

$$(\overline{u}_3\, \Gamma_A^\alpha\, u_1)\ (\overline{u}_4\, \Gamma_{B\,\alpha}\, u_2)\ [(p_1 + p_3) \cdot (p_2 + p_4)]^n/(2m)^{2n}$$

and to order Q^2 the $[\ldots]$ can be expanded as

$$1 + n\ \left[\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2 + \mathbf{p}_4^2 - (\mathbf{p}_1 + \mathbf{p}_3) \cdot (\mathbf{p}_2 + \mathbf{p}_4)\right]/(4m^2)$$

- 36 hermitian, $\mathcal{C}$- and $\mathcal{P}$-invariant terms
- NR reduction and use of EOM to remove time derivatives lead to 2 leading terms (Q^0), accompanied by specific $1/m^2$ corrections, and 7 subleading ones (Q^2)

$$\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{1}{2} C_S \left[O_S + \frac{1}{4m^2} (O_1 + O_3 + O_5 + O_6) \right] \\
& -\frac{1}{2} C_T \left[O_T - \frac{1}{4m^2} \left(O_5 + O_6 - O_7 + O_8 + 2\, O_{12} + O_{14} \right) \right] \\
& -\frac{1}{2} C_1 (O_1 + 2\, O_2) + \frac{1}{8} C_2 (2\, O_2 + O_3) - \frac{1}{2} C_3 (O_9 + 2\, O_{12}) \\
& -\frac{1}{8} C_4 (O_9 + O_{14}) + \frac{1}{4} C_5 (O_6 - O_5) - \frac{1}{2} C_6 (O_7 + 2\, O_{10}) \\
& -\frac{1}{16} C_7 (O_7 + O_8 + 2\, O_{13})
\end{aligned}$$

Poincaré Algebra Constraints

Girlanda *et al.*, PRC**81**, 034005 (2010)

- $H = H_0 + H_I$ and $\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I$

$$\left[K^i\,,\, K^j\right] = -i\,\epsilon^{ijk} J^k = \left[K_0^i\,,\, K_0^j\right] \qquad \left[\mathbf{K}\,,\, H\right] = i\,\mathbf{P} = \left[\mathbf{K}_0\,,\, H_0\right]$$

- Power counting:

$$\mathbf{K}_0 = \mathbf{K}_0^{(-1)} + \mathbf{K}_0^{(1)} + \dots \qquad H_0 = H_0^{(0)} + H_0^{(2)} + \dots$$

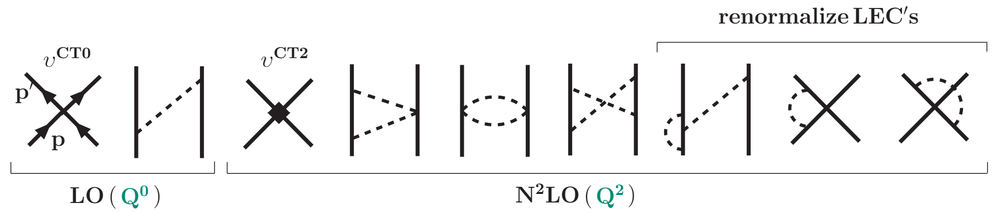
- Constraints arise on H_I and $\mathbf{K}_I$
- In terms of potentials: if $v = v^{\rm LO} + v^{\rm N^2LO}$ in CM ($\mathbf{P} = 0$), then boost correction is given by

$$\begin{aligned} \delta v^{\rm N^2LO}(\mathbf{P}) &= -\frac{P^2}{8\,m^2} v^{\rm LO} + \frac{i}{8\,m^2}\left[\mathbf{P}\cdot\mathbf{r}\;\mathbf{P}\cdot\mathbf{p}\,,\, v^{\rm LO}\right] \\ &\quad + \frac{i}{8\,m^2}\left[(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\times\mathbf{P}\cdot\mathbf{p}\,,\, v^{\rm LO}\right] \end{aligned}$$

Well known: Friar (1975); Carlson *et al.* (1993)

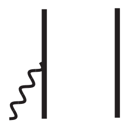
Nuclear χEFT (at Q^2 and $e\,Q^3$)

NN potential:

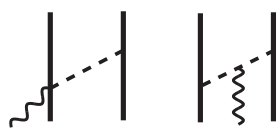


and consistent EM currents:

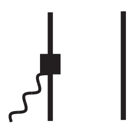
LO : eQ^{-2}



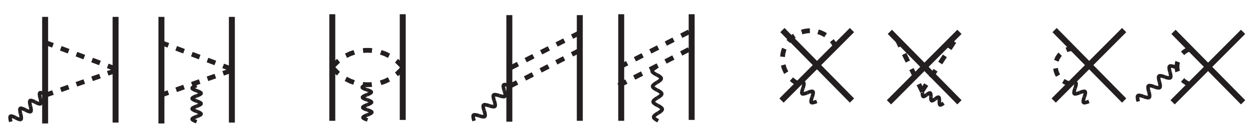
NLO : eQ^{-1}



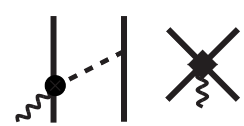
N^{2}LO : eQ^0



N^{3}LO : eQ

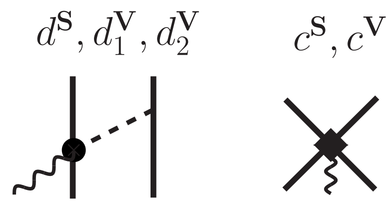


unknown LEC's

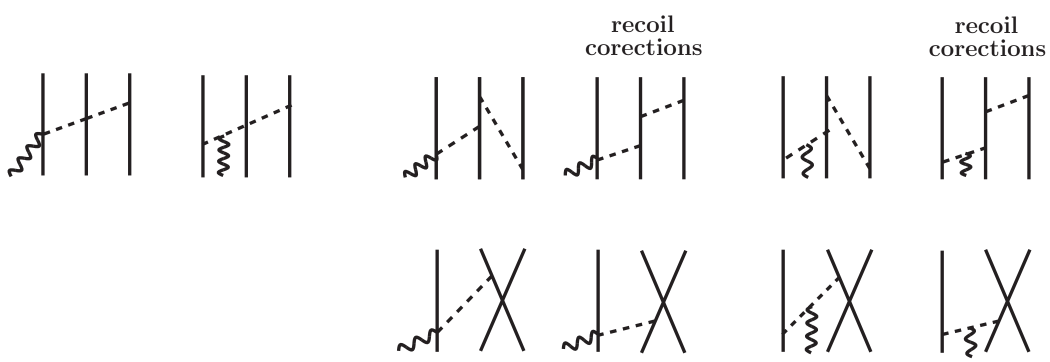


EM Observables at N^3LO

- Pion loop corrections known (g_A and F_π)
- Five LEC's: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon

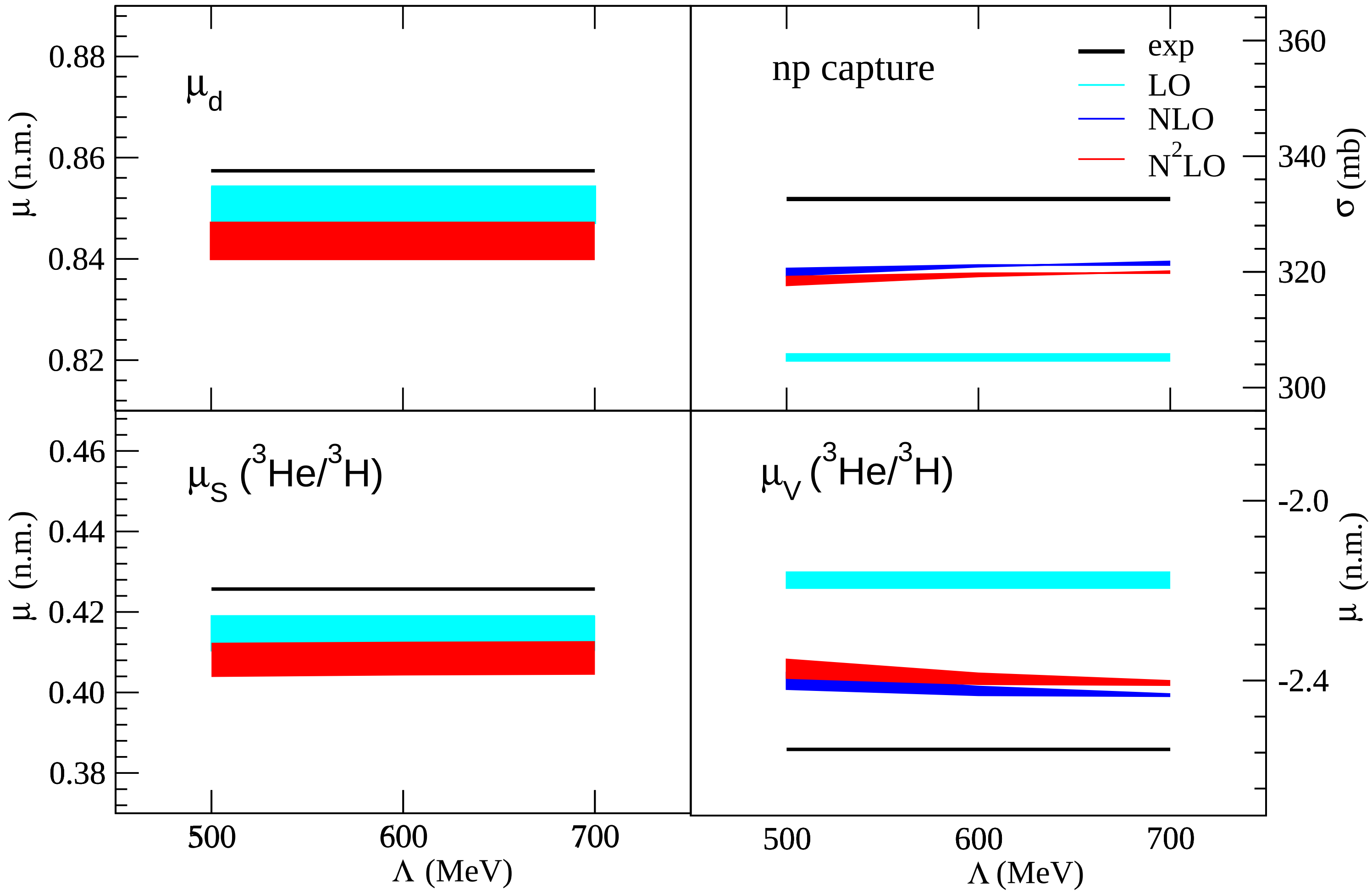


- $d_2^V/d_1^V = 1/4$ assuming Δ-resonance saturation
- Three-body currents at N^3LO vanish:



Fixing LEC's in EM Properties of A=2 and A=3 Nuclei

AV18/UIX or N^{3}LO/TNI-N^{2}LO (band)



The nd and $n\,^3$He Captures

- Suppressed $M1$ processes:

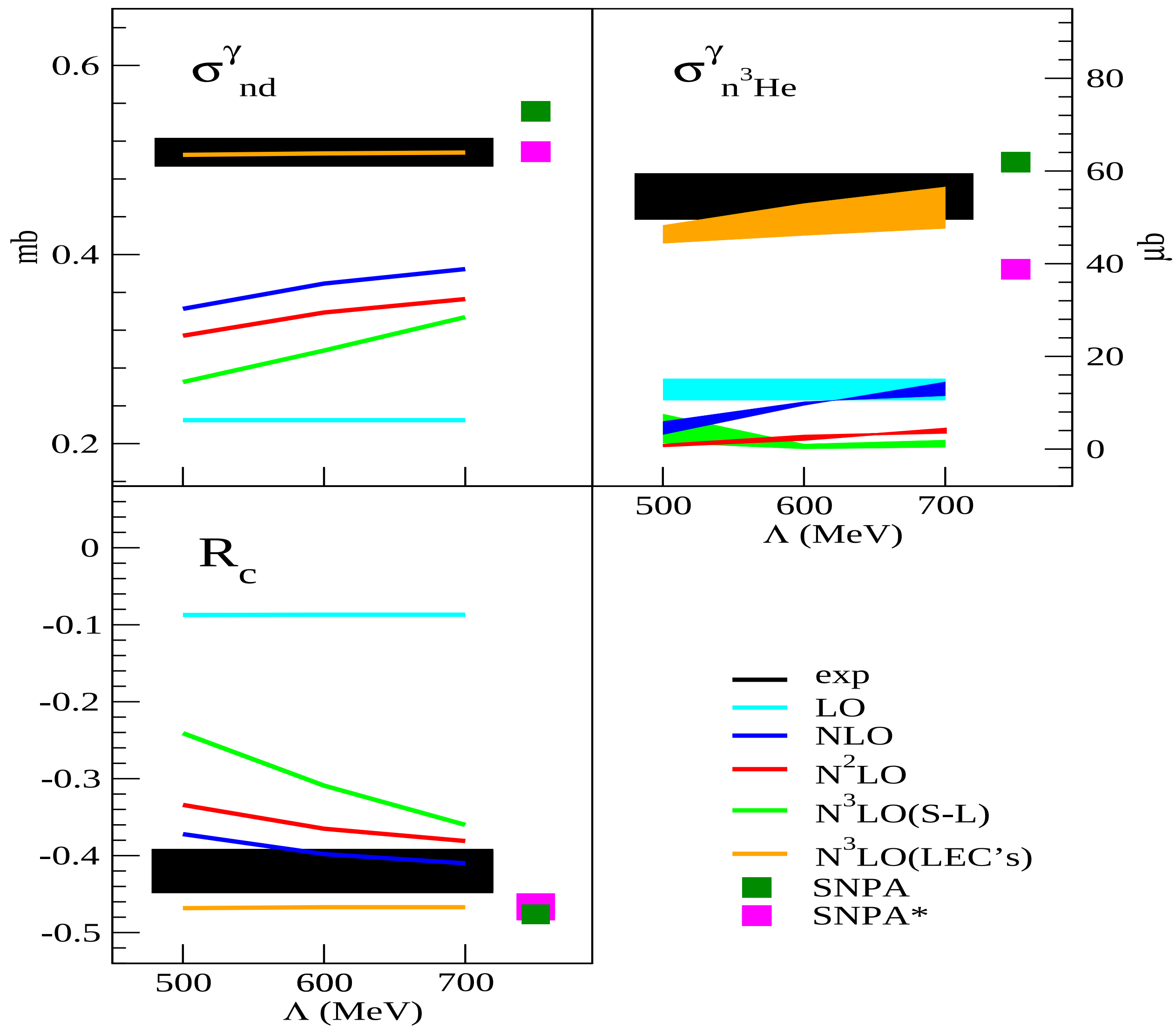
	$\sigma_{\rm exp}$(mb)
^{1}H$(n,\gamma)^2$H	334.2(5)
^{2}H$(n,\gamma)^3$H	0.508(15)
^{3}He$(n,\gamma)^3$He	0.055(3)

- The ^{3}H and ^{4}He bound states are approximate eigenstates of the one-body $M1$ operator, *e.g.* $\hat{\mu}({\rm IA})\,|^3{\rm H}\rangle \simeq \mu_p\,|^3{\rm H}\rangle$, since

$$|^3{\rm H}\rangle = \phi({\rm S})\mid (n\!\downarrow)_1, (n\!\uparrow)_2, (p\!\uparrow)_3\rangle + \ldots$$

Thus $\langle nd \mid \hat{\mu}({\rm IA})\,|^3{\rm H}\rangle \simeq 0$ by orthogonality

- A=3 and 4 radiative (and weak) captures very sensitive to i) small components in the w.f.'s and ii) many-body terms in the electro(weak) currents


σ^γ nd
σ^γ n³He
R c
mb
μb
0.6
0.4
0.2
80
60
40
20
0
-0.1
-0.2
-0.3
-0.4
-0.5
500
600
700
Λ (MeV)
exp
LO
NLO
N²LO
N³LO(S-L)
N³LO(LEC's)
SNPA
SNPA*

Summary and Outlook

- Hybrid predictions for nd ($n\,^3$He) capture in (reasonable) agreement with exp, and exhibit weak ($\simeq 10\%$) Λ-dependence
- Future work:
  1. Carry out consistent calculation—based on χEFT potential and currents—of A=2–4 EM observables (in progress)
  2. Incorporate boost corrections to chiral potential in calculations of bound and scattering state properties
  3. Extend hybrid studies to different combinations of 2N and 3N potentials and up to $A = 7$ systems (in progress)
  4. Include Δ-isobars in theory (might improve fits to phase shifts and reduce cutoff dependence)