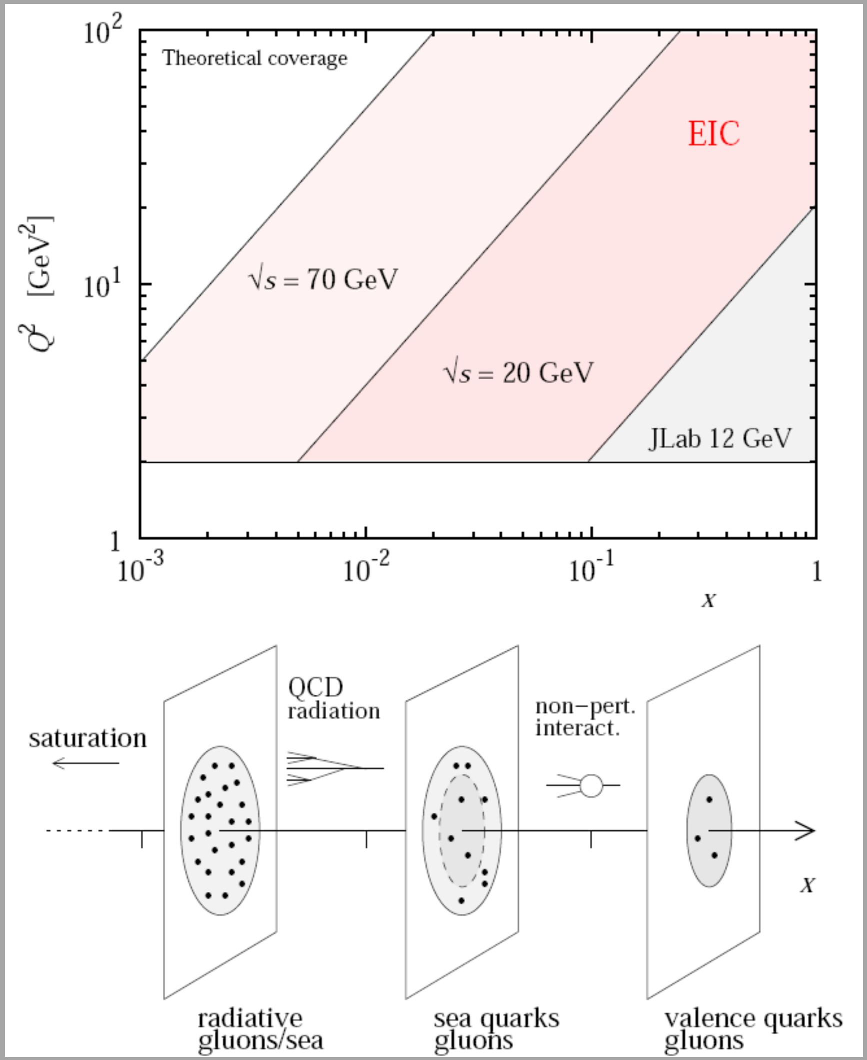
Temple University
Philadelphia

Transverse Momentum Dependent distributions and three-dimensional partonic structure of the proton

Alexei Prokudin
Jefferson Laboratory

Jefferson Lab

Nucleon landscape



Plot courtesy of Christian Weiss

Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

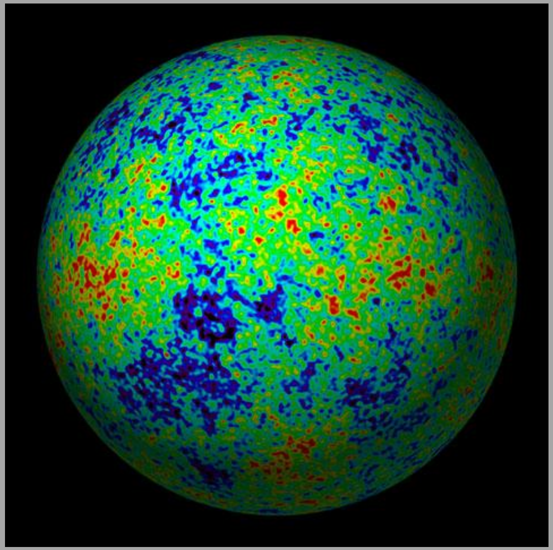
How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

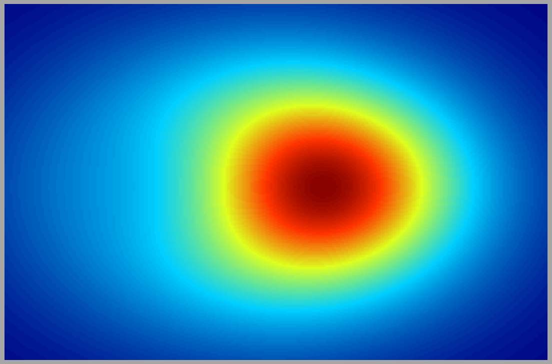
Markus Deihl (2003)
Matthias Burkardt (2003)

and Transverse Momentum Dependent distributions

Fundamental knowledge from 3D distributions



Cosmic Microwave Background is the source of information on history of our universe, inflation, distribution of matter, dark matter etc



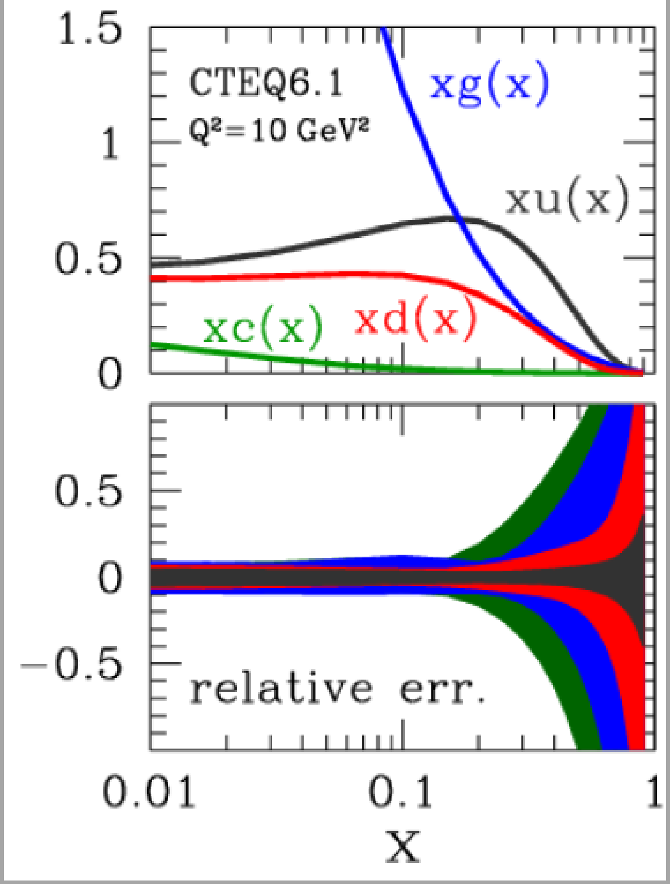
3 Dimensional partonic picture gives us insights on the dynamics of the confined system of quarks and gluons.

It also gives information on fundamental properties of the nucleon

Spin is one of these properties

Hadron tomography

Conventional inclusive processes are sensitive to longitudinal momentum fraction of hadron momenta, they give no information on spatial or momentum 3D distribution of partons



Good knowledge of
Parton
Distribution
Functions (PDFs)
is acquired at HERA

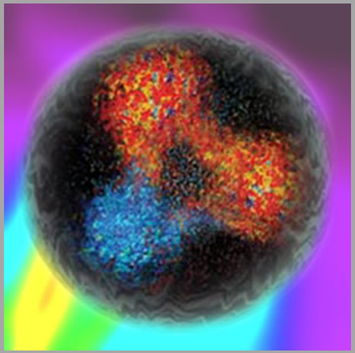
See Forte (2010)

However large-x behavior has still large uncertainties
Data from Jlab 12 will be important

Wigner distribution

Our goal is to understand 3 dimensional distributions of partons,
How they move, where they are located inside a nucleon

Meissner, Metz, Schlegel (2009) Lorce, Pasquini(2011) Ji, Xiong, Yuan (2012)



Wigner distribution (1933) is a possibility

$$W(\mathbf{p},\mathbf{r}) = \int d^3\eta\, e^{i\,\mathbf{p}\eta}\psi^*(\mathbf{r}+\eta/\mathbf{2})\psi(\mathbf{r}-\eta/\mathbf{2})$$

It gives both position and momenta

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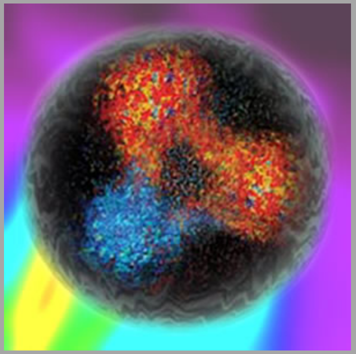

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Can it be measured?

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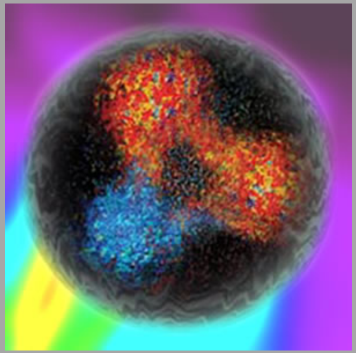

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$$W(\mathbf{p},\mathbf{r}) = \int d^3\eta \, e^{i\,\mathbf{p}\eta}\psi^*(\mathbf{r}+\eta/\mathbf{2})\psi(\mathbf{r}-\eta/\mathbf{2})$$

It gives both position and momenta

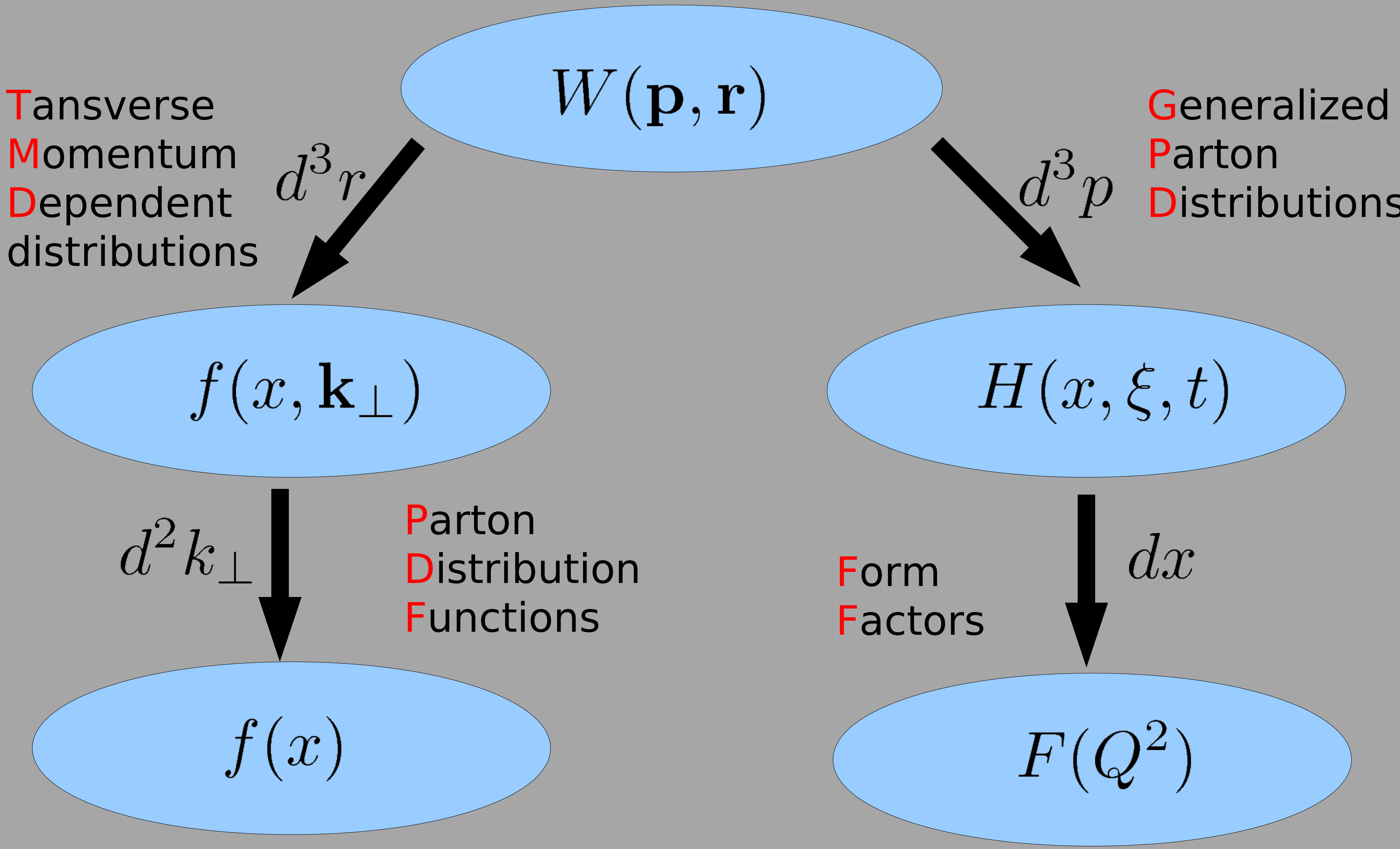
Can it be measured?

PROBABLY NOT!

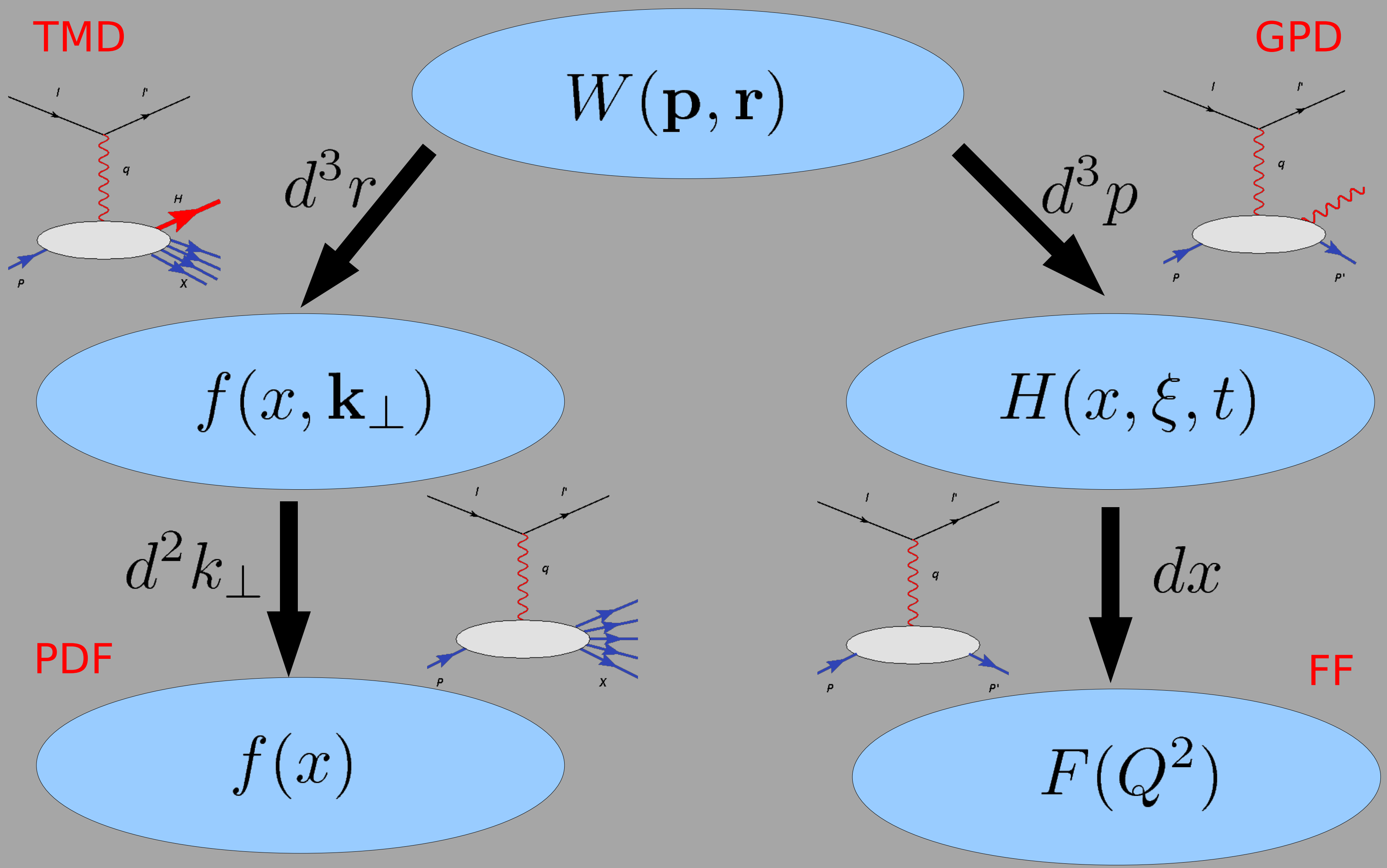
$$\Delta p \Delta r \geq \hbar/2$$

No simultaneous knowledge on position and momenta

Wigner distribution

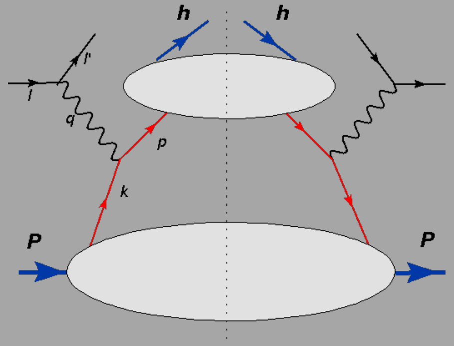


Wigner distribution



Transverse Momentum Dependent distributions

SIDIS



$$\mathbf{l} + \mathbf{P} \to \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} << Q$$

it will be sensitive to quark transverse momentum $k_\perp$

TMD factorization

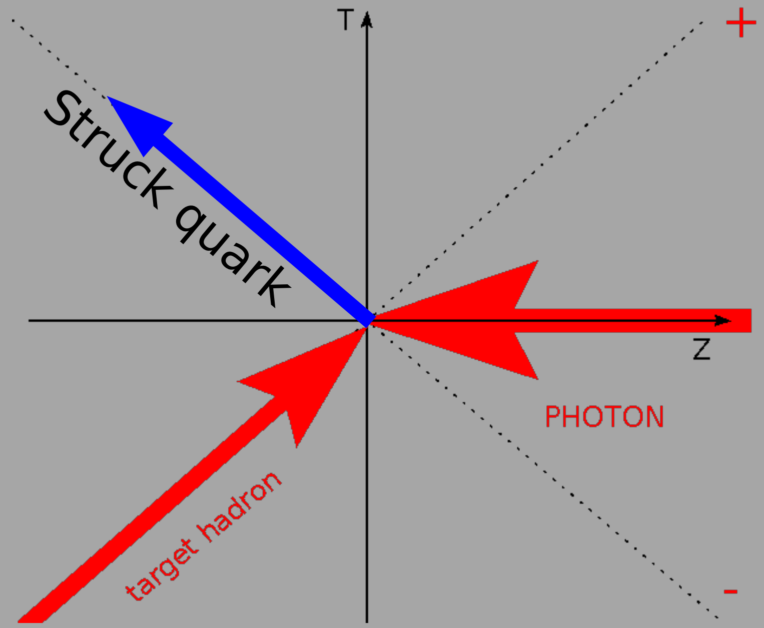
Ji, Ma, Yuan (2002)
Collins(2011)

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp \xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle$$

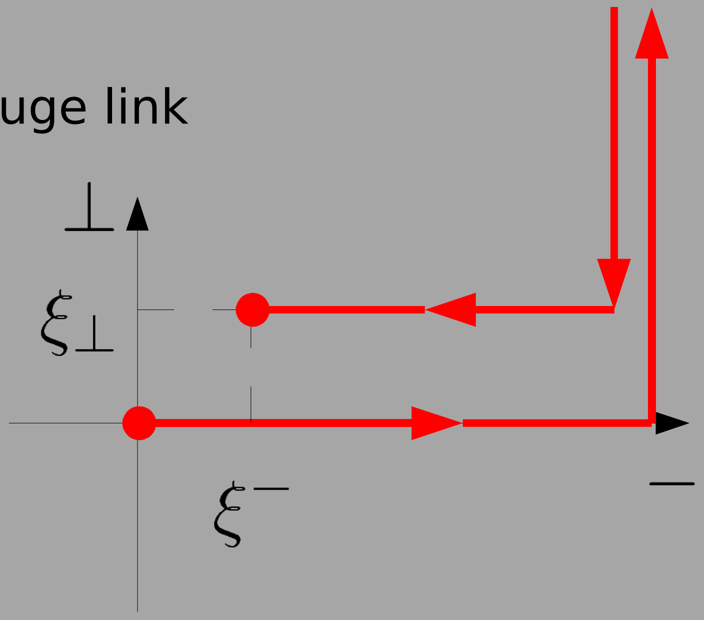
Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)}\,\frac{d^2\xi_\perp}{(2\pi)^2}\; e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp}\,\langle P,S_P|\bar{\psi}_j(0)\mathcal{U}(\mathbf{0},\xi)\psi_i(\xi)|P,S_P\rangle|_{\xi^+=0}$$

SIDIS in Infinite Momentum Frame:



Gauge link



Transverse separation is due to presence of transverse parton momentum

Struck quark propagates in the gauge field of the remnant and forms gauge link

Factorization theorems

- Related: Factorization Theorems:
  - Semi-Inclusive deep inelastic scattering. ✓
  - Drell-Yan. ✓
  - e^+/e^- annihilation. ✓
  - ~~p + p ➡ $h_1 + h_2 + X$~~ !!

• TMD factorization

$$\Lambda^2_{QCD} < P^2_{h\perp} \ll Q^2$$

Sensitive to parton transverse motion.

Ji, Ma, Yuan, Collins, Metz, Rogers, Mulders, etc

- Related: Factorization Theorems:
  - Semi-Inclusive deep inelastic scattering. ✓
  - Drell-Yan. ✓
  - e^+/e^- annihilation. ✓
  - p + p ➡ $h_1 + h_2 + X$ ✓

• Collinear factorization

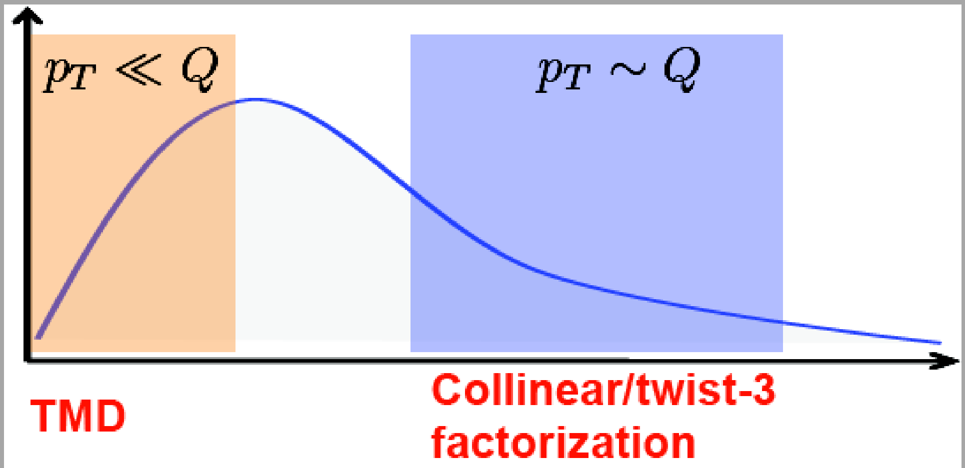
$$\Lambda^2_{QCD} \ll P^2_{h\perp}, Q^2$$

Sensitive to multy parton correlations.

Qui, Sterman, Efremov, Teryaev, Kanazava, Koike, etc

TMD and Collinear factorizations

Both factorizations are consistent in the overlap region

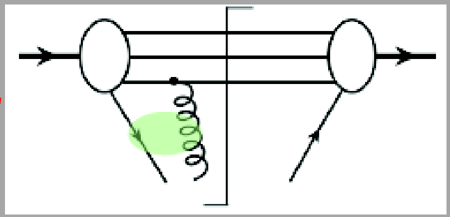


Collins, Mulders, Ji, Qui, Yuan, Bacchetta, Metz, Kang, Boer, Koike, Vogelsang, Yuan etc

Relation of multyparton correlations and moments of TMDs

$$\int d^2k_T \frac{k_T^2}{M} f_{1T}^{\perp}(x, k_T^2) + \mathrm{UVCT}(\mu^2) = T_F(x, x, \mu^2) \qquad f_{1T}^{\perp(1)} \equiv \int d^2k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2)$$

Sivers function



TMDs

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied

Mulders, Tangerman (1995), Boer, Mulders (1998)

Sivers function

Let's consider unpolarised quarks inside transversely polarised nucleon

DISTRIBUTION

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{k}_T^2)$$

Usual unpolarised distribution

This one is called SIVERS function
Correlation of transverse motion and transverse spin
Sivers (1990) Boer, Mulders (1998)

Sivers function

$$f(x,\mathbf{k}_T,S) = f_1(x,\mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x,\mathbf{k}_T^2)$$

This function gives access to 3D imaging

Spin-orbit correlation

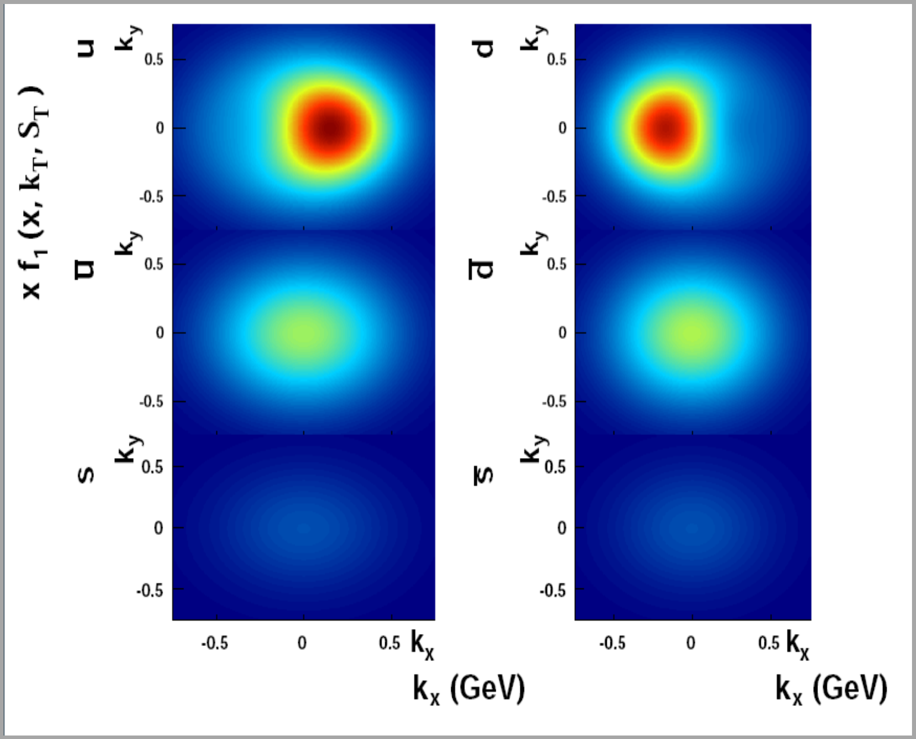
Physics of gauge links is represented

Requires Orbital Angular Momentum

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al , 2011;
Duke workshop report: Anselmino et al Eur.Phys.J.A47:35,2011

Access to 3D imaging

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{k}_T^2)$$



Dipole deformation

Sivers function from experimental data HERMES and COMPASS

Anselmino et al 2005

What do we learn from 3D distributions?

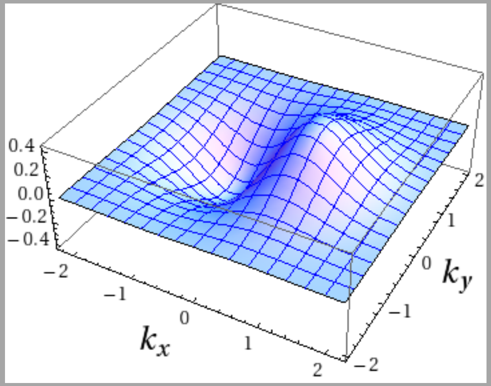
$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0,1)$

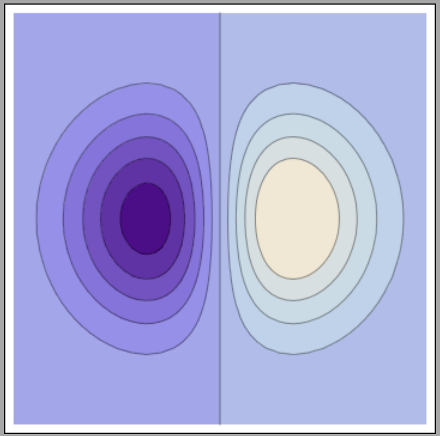
Deformation in momentum space is: $k_x \cdot f(k_x^2 + k_y^2)$

This is so-called “dipole” deformation.

3D picture:



Tomography:



What do we learn from 3D distributions?

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

We calculate now average shift: $\langle k_x \rangle$

$$\langle k_x \rangle = \int d^2 k_T \frac{\mathbf{k_T^2}}{2M} f_{1T}^{\perp}(x, \mathbf{k_T^2}) \equiv f_{1T}^{\perp(1)}(x) M$$

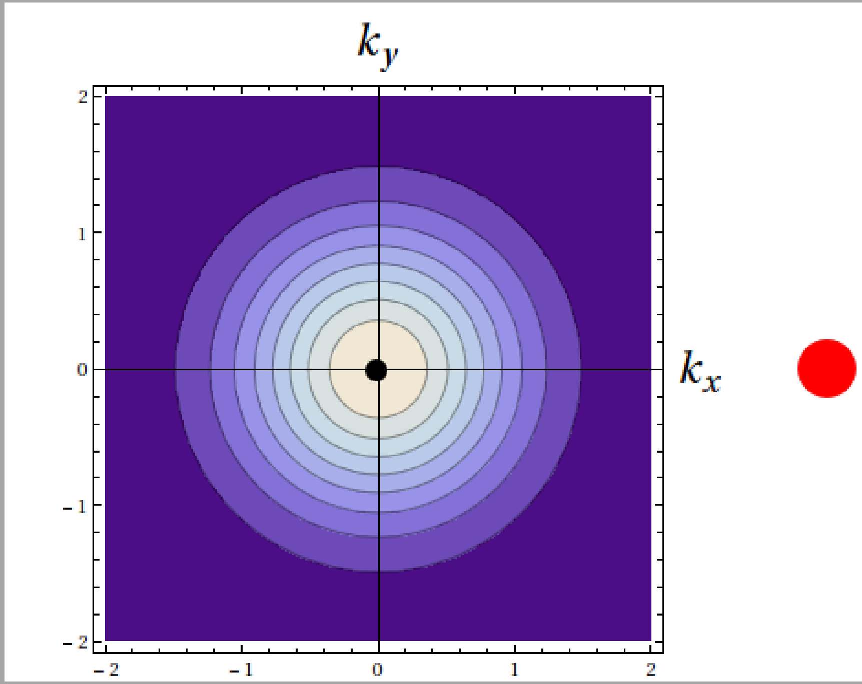
Average momentum shift is proportional to
the **first moment** of Sivers function

What do we learn from 3D distributions?

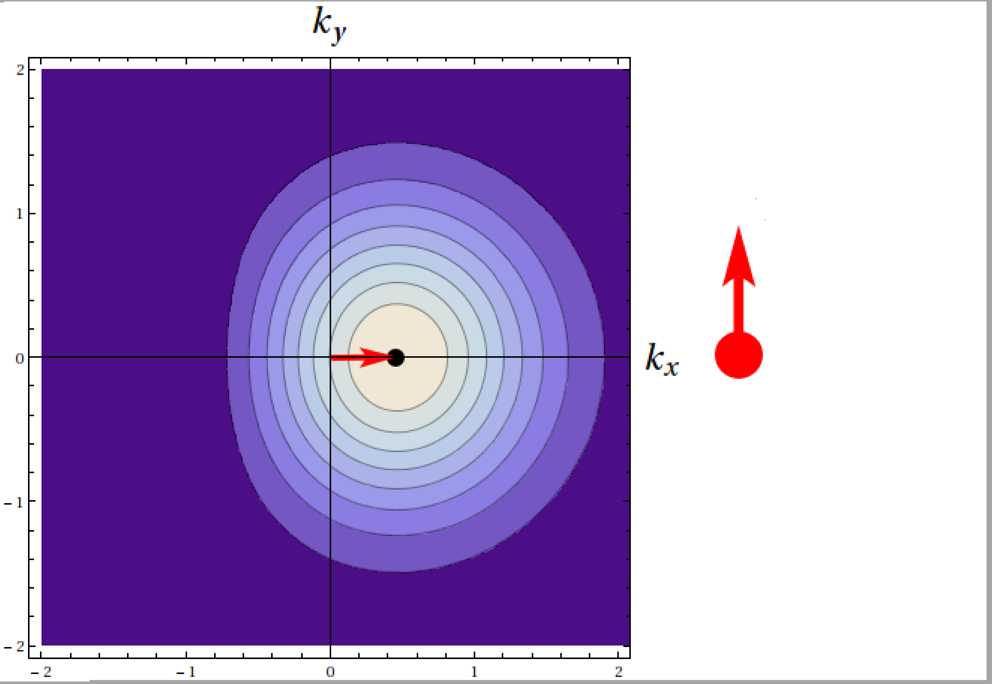
$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

The same statement in figures:

No polarisation:



Polarisation: $S_y \Rightarrow \langle k_x \rangle$

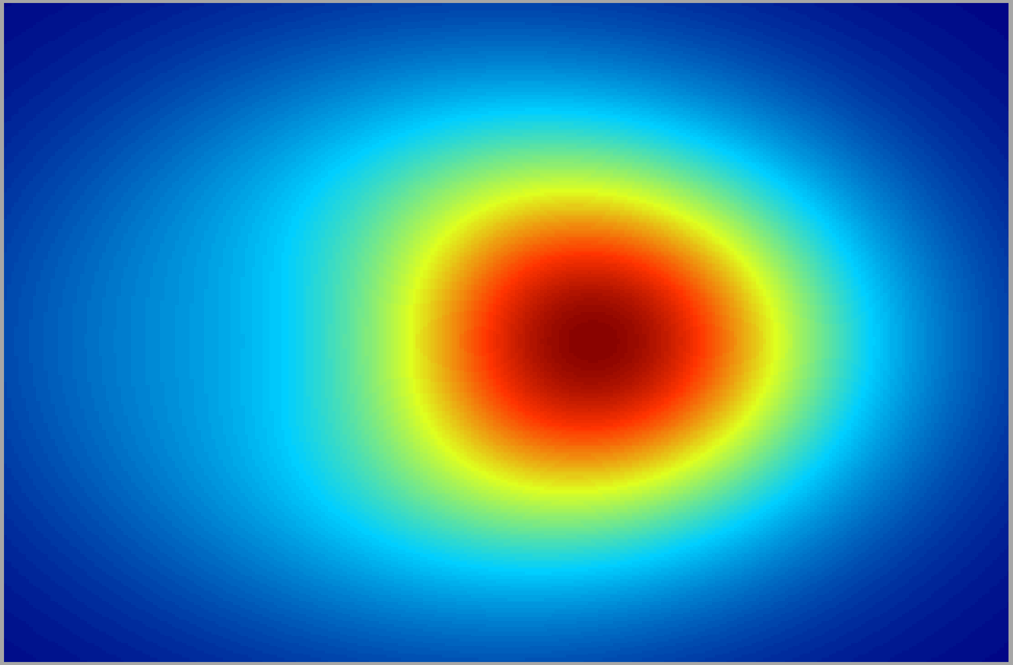


What do we learn from 3D distributions?

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$

The same statement in figures:

This is what we know from exerimental data already:



How do we measure Sivers function?

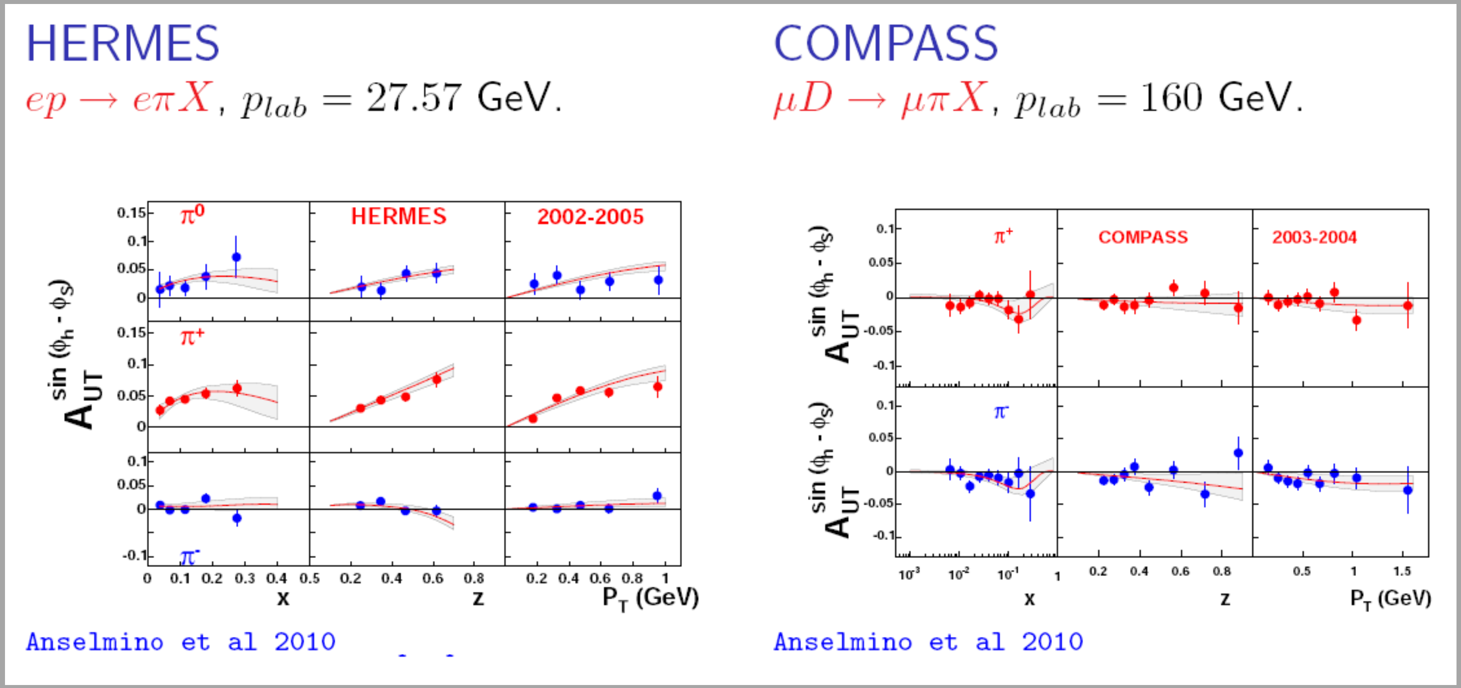
$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = \frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$$

$$\sigma^{\uparrow}-\sigma^{\downarrow} = -f_{1T}^{\perp}\otimes d\hat{\sigma}\otimes D_{h/q}\sin(\phi_h-\phi_S)$$

Unpolarised electron beam
Transversely polarised proton

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp}\otimes d\hat{\sigma}\otimes D_{h/q}}{\sum_q e_q^2 f_1\otimes d\hat{\sigma}\otimes D_{h/q}}$$

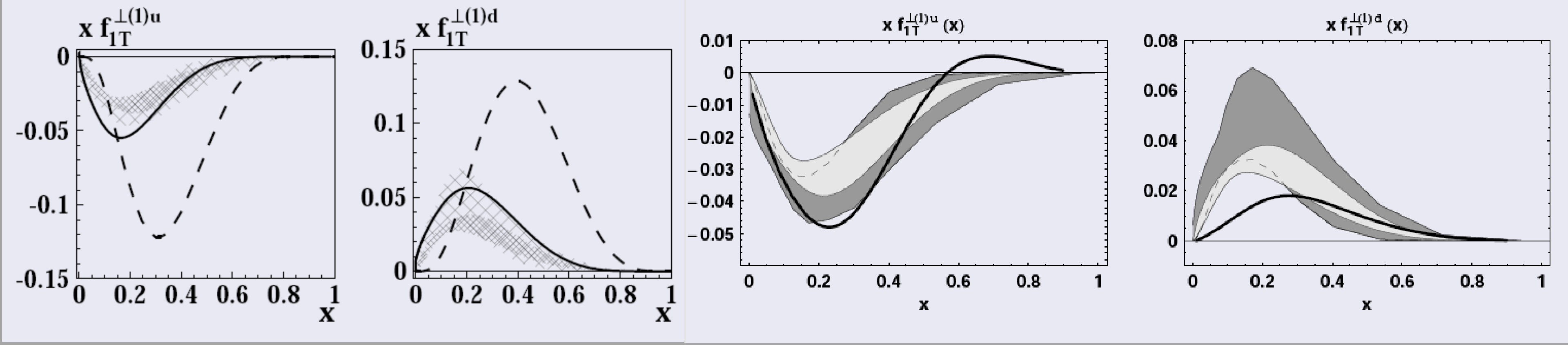
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel (2006)



Comparison with models

Light cone wf model Pasquini, Yuan (2011),
Quark-diquark models Bacchetta et al (2010),
Gamberg, Goldstein, Schlegel (2010)
Bag models Yuan (2003), Avakian, Efremov, Schweitzer, Yuan (2010)

Pasquini, Yuan (2011) Bacchetta et al (2010)

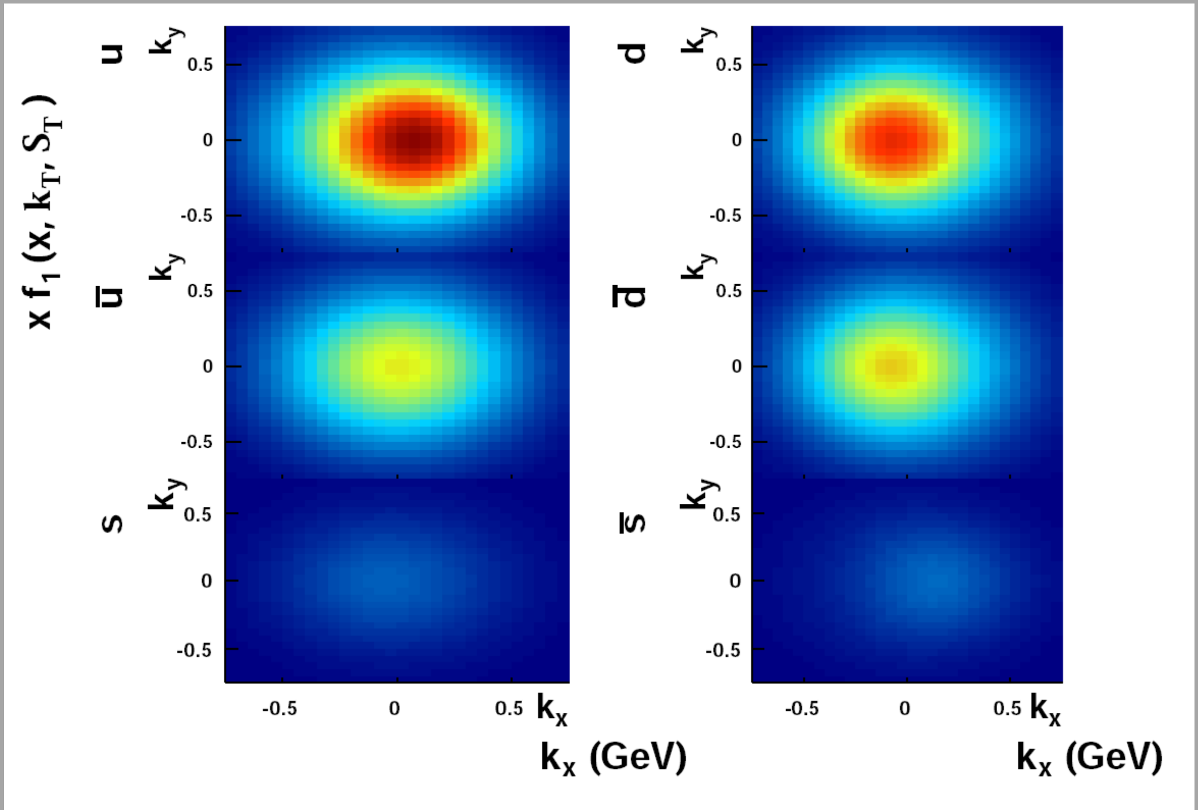


$f_{1T}^{\perp u} < 0$ $f_{1T}^{\perp d} > 0$

Good agreement.

What do we learn from 3D distributions?

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$



The slice is at:

$x = 0.1$

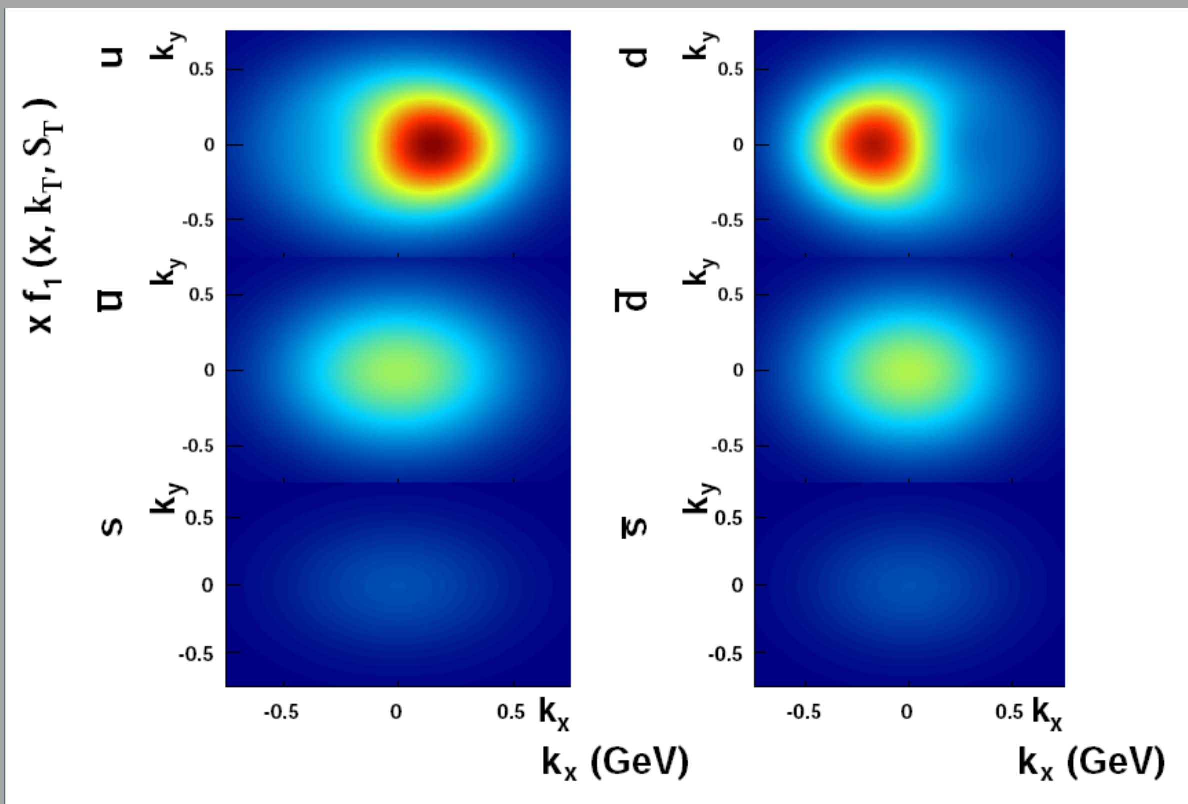
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

No information on sea quarks

Picture is still quite uncertain

What do we learn from 3D distributions?

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$



The slice is at:

$$x = 0.1$$

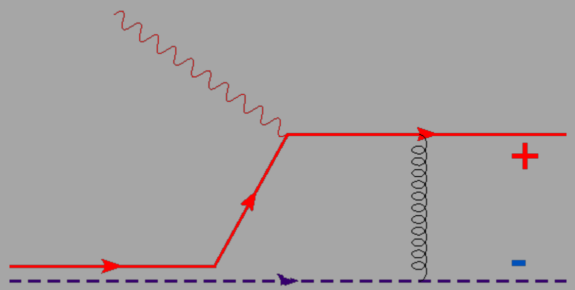
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

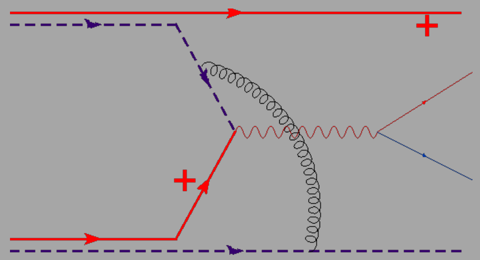
No information on sea quarks

In future we will obtain much clearer picture

Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)





Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman, etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond "just" check of TMD factorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou

TMD theoretical challenges

- Evolution and soft gluon resummation
- Global study at Next-to-Leading order
- Relation to Orbital Angular Momentum

Many more other questions

• What is the kt distributions of partons – gaussian, powerlike, sign changing?
• What is the difference of kt distributions of quarks and sea quarks?
• How to explore higher twist TMDs?
• How to explore distribution and fragmentation TMDs in a satisfactory way?
• etc

TMD evolution

Collins-Soper-Sterman factorization can be used

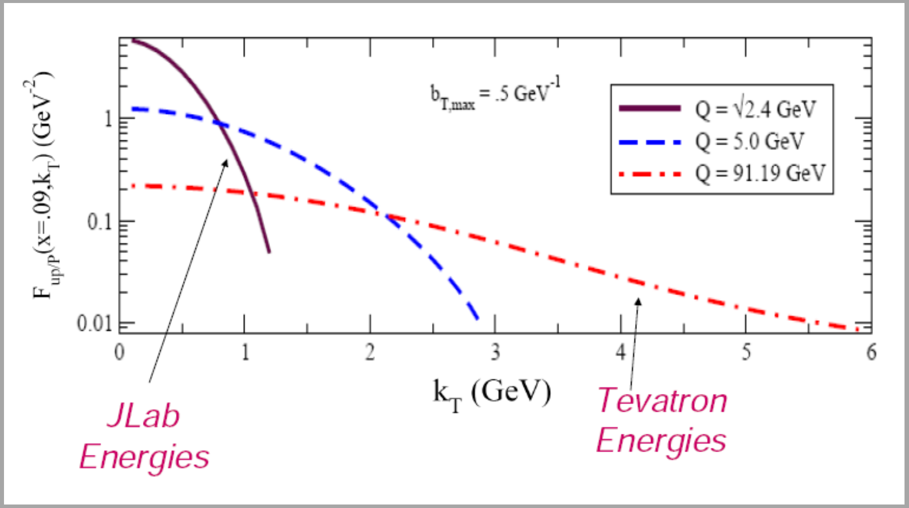
$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \zeta} = \tilde{K}(b_\perp, \mu)$$

CS kernel in coordinate space

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(\mu)$$

$$\frac{d\tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = \gamma_F(\mu, \zeta)$$

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011
Twist-3:
Kang, Xiao, Yuan 2011
Koike, Vogelsang 2011

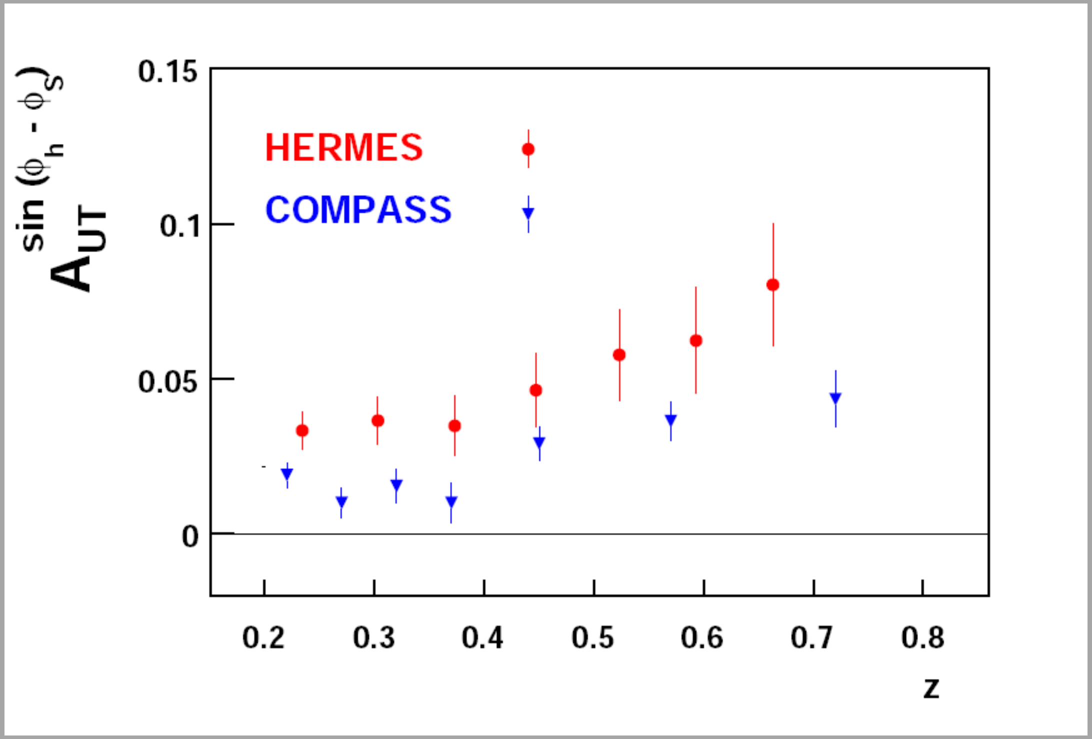


TMDs change with energy and resolution scale

Relevant to Electron Ion Collider

TMD evolution

Can we see signs of evolution in the experimental data?



Aybat, AP, Rogers 2011

COMPASS data is at

$$\langle Q^2 \rangle \simeq 3.6 \ (GeV^2)$$

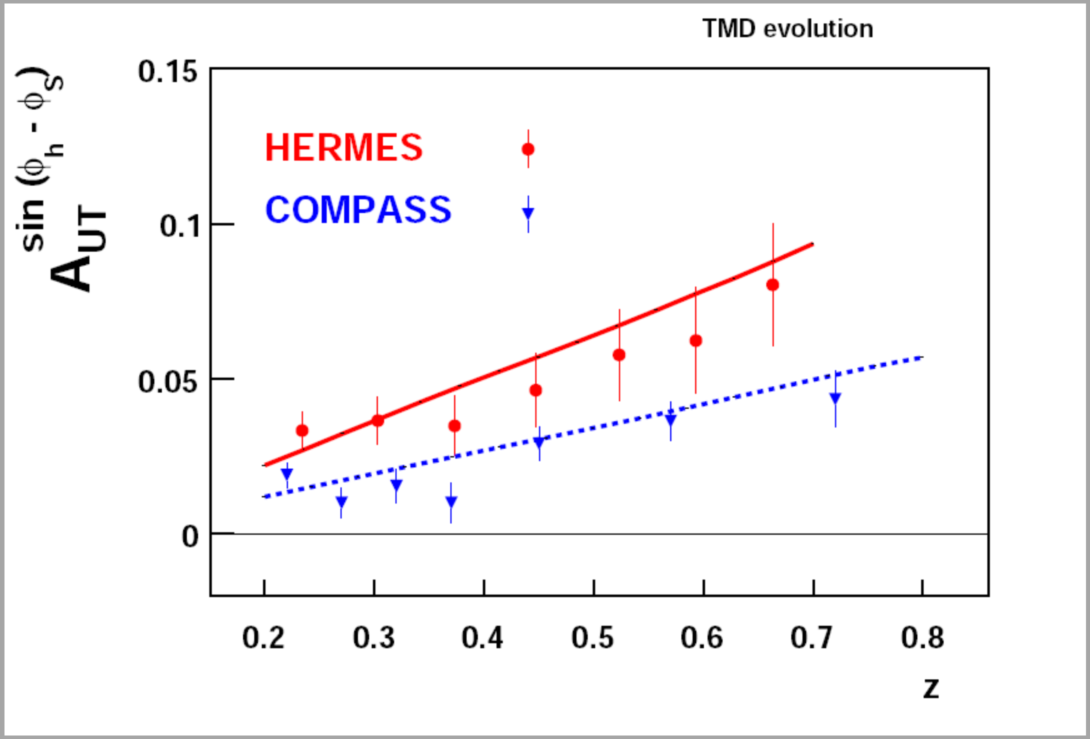
HERMES data is at

$$\langle Q^2 \rangle \simeq 2.4 \ (GeV^2)$$

TMD evolution

Can we explain the experimental data?

Full TMD evolution is needed!



Aybat, AP, Rogers 2011

COMPASS dashed line

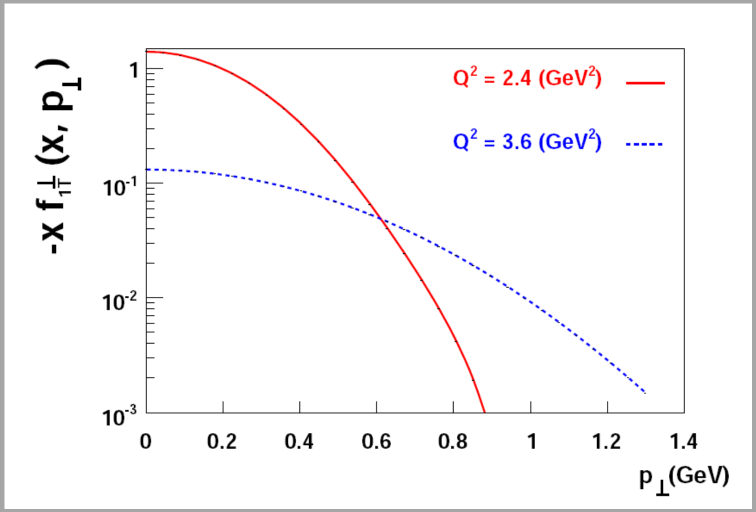
$$\langle Q^2 \rangle \simeq 3.6\ (GeV^2)$$

HERMES solid line

$$\langle Q^2 \rangle \simeq 2.4\ (GeV^2)$$

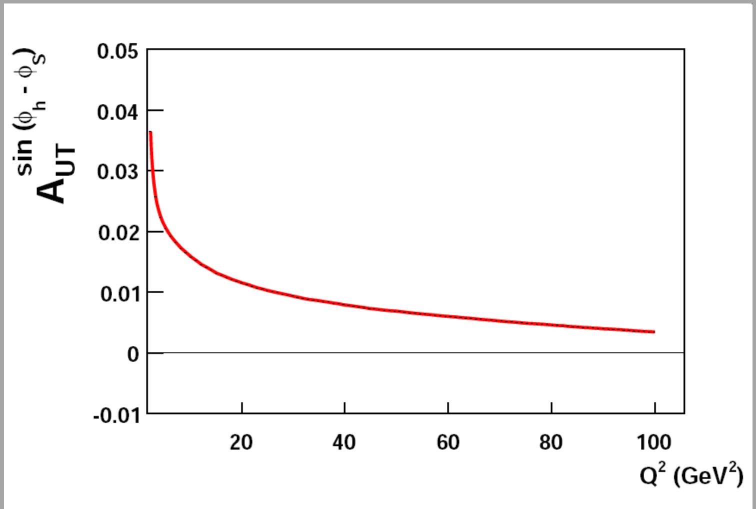
TMD evolution

This is the first implementation of TMD evolution for observables



Aybat, AP, Rogers 2011

Asymmetry changes with Q^2

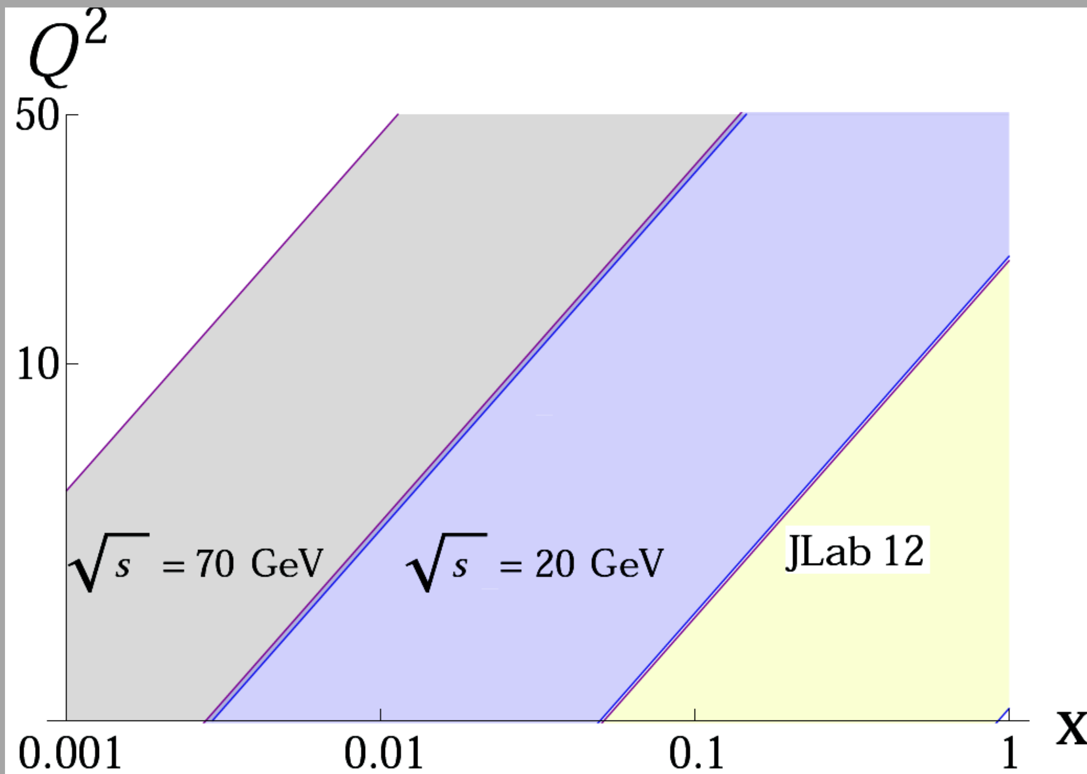


Functions change with energy

Phenomenological analysis with evolution is now possible

Kinematics

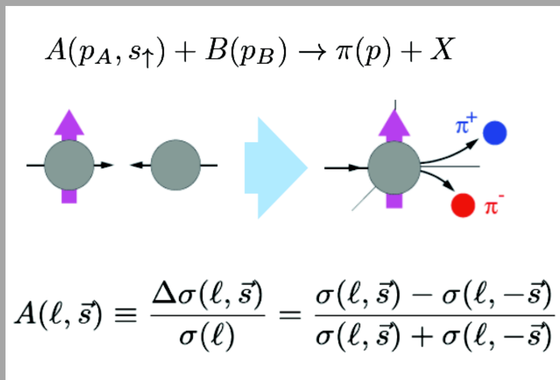
Kinematics $Q^2 \simeq sxy$

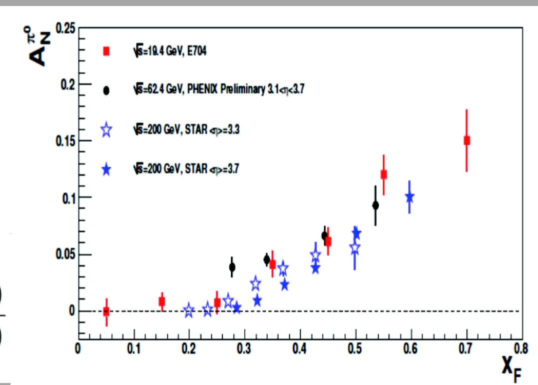


JLab 12 and future Electron Ion Collider are complimentary

Data analysis

Proton Proton Left -Right asymmetry





Only one scale P_T

Collinear analysis:

Kouvaris, Qiu, Vogelsang, Yuan (2006)

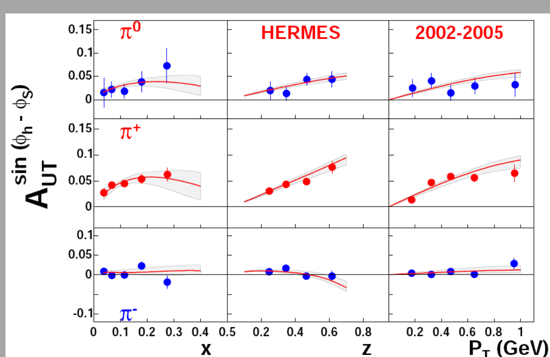
Kanazava, Koike (2010)

TMD analysis:

Anselmino et al (2006)

SIDIS

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \qquad d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S)}_{\text{Sivers effect}}$$



Two scales P_T, Q

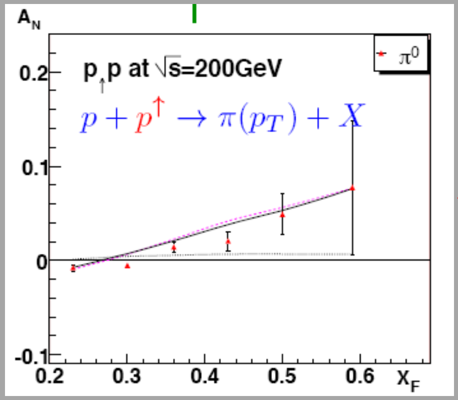
$$\Lambda^2_{QCD} < P^2_{h\perp} \ll Q^2$$

TMD analysis: Anselmino et al (2008); Collins et al (2007) ; Vogelsang, Yuan (2006)

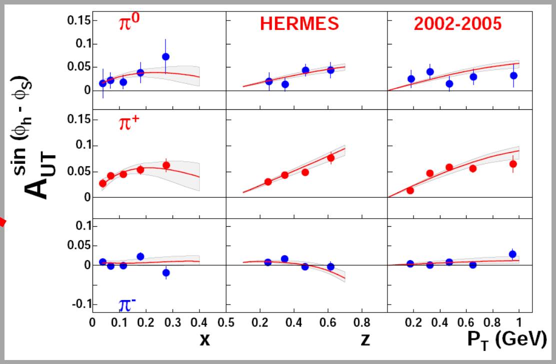
Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$



Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)



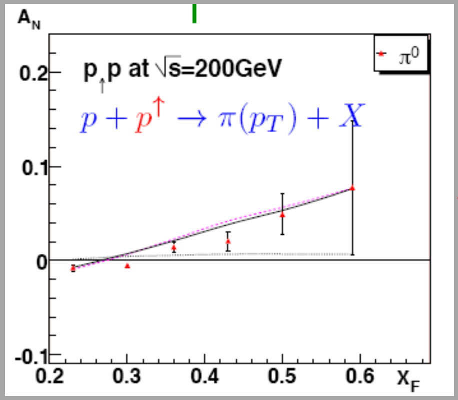
TMD analysis:

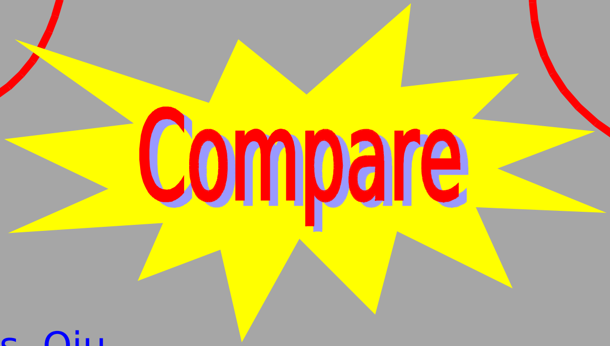
Anselmino et al (2008)

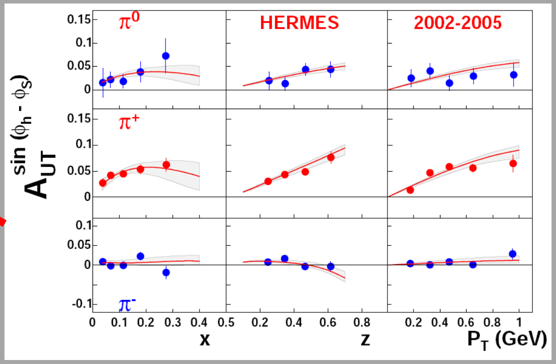
Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$







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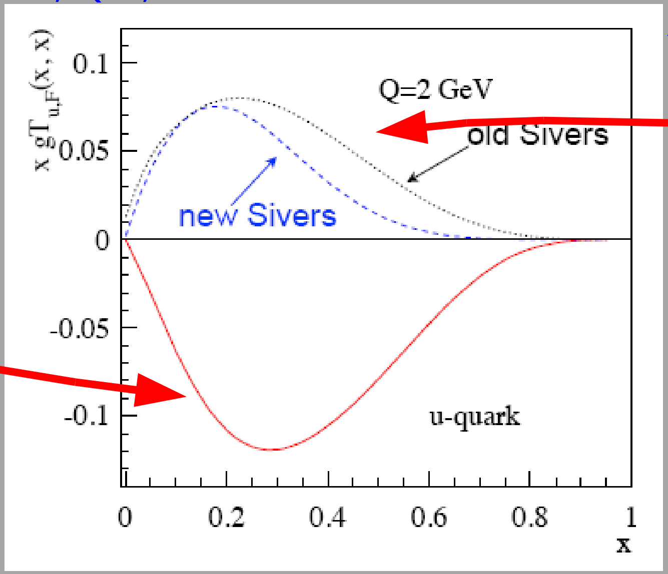

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Comparison of results

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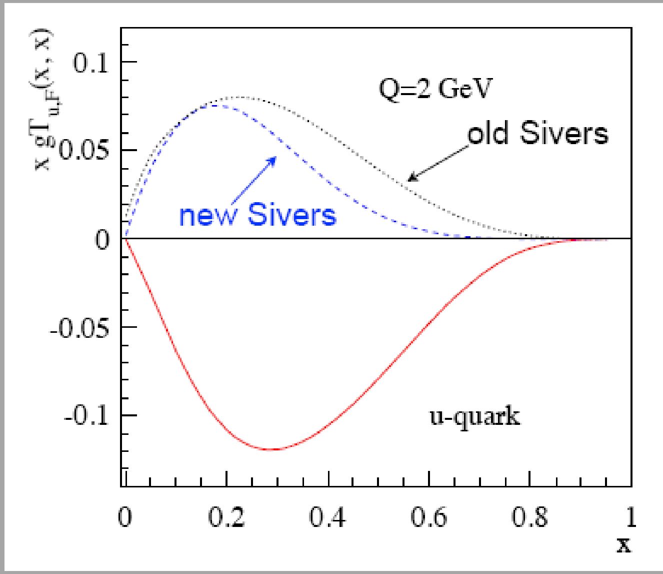

Kang, Qiu, Vogelsang, Yuan (2011)

- Magnitudes are similar
- Sign is opposite

Comparison of results

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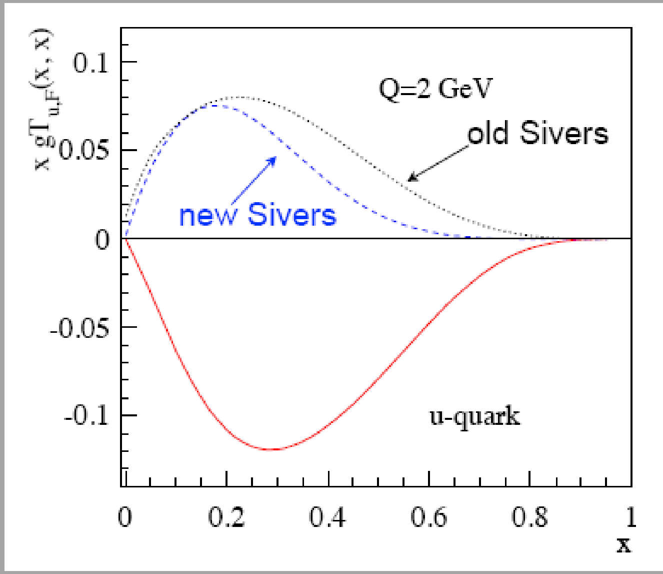


Kang, Qiu, Vogelsang, Yuan (2011)

- Magnitudes are similar
- Sign is opposite

It is a puzzle!

Possible explanations

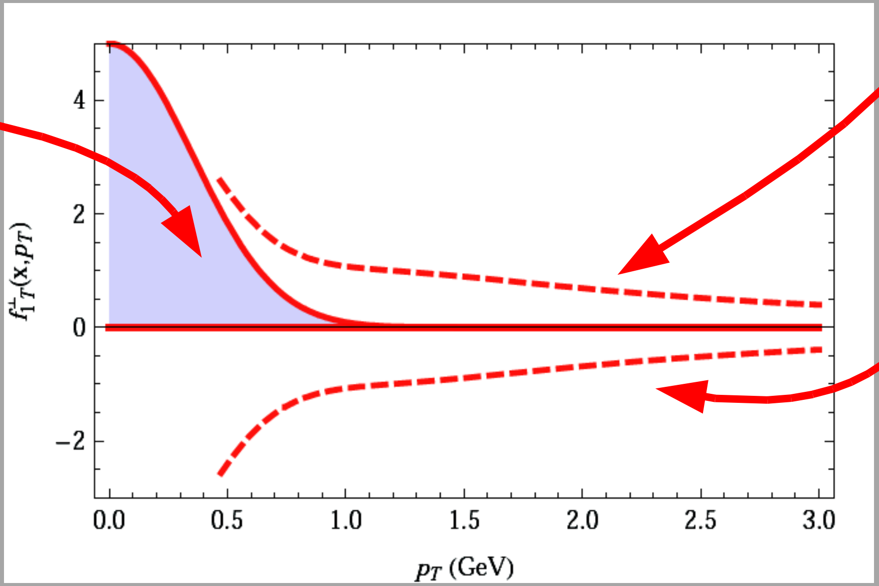
Sivers function can have nodes in $\mathbf{k_T}$.

Kang, AP (2012)

SIDIS $\mathbf{T_F > 0}$

Perturbarive tail $\propto \frac{\mathbf{M^2}}{\mathbf{k_T^4}} \alpha_\mathbf{s}$

Bacchetta, Boer, Deihl, Mulders 2008



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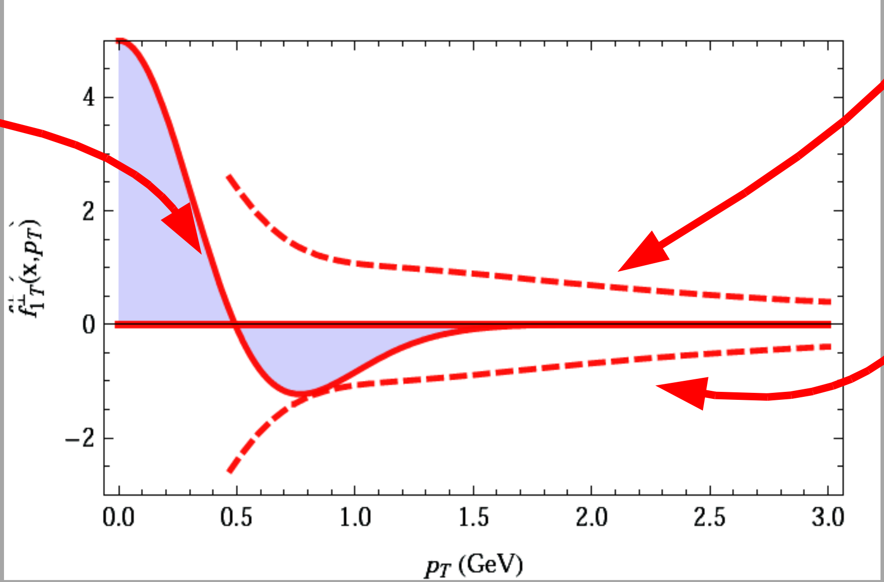
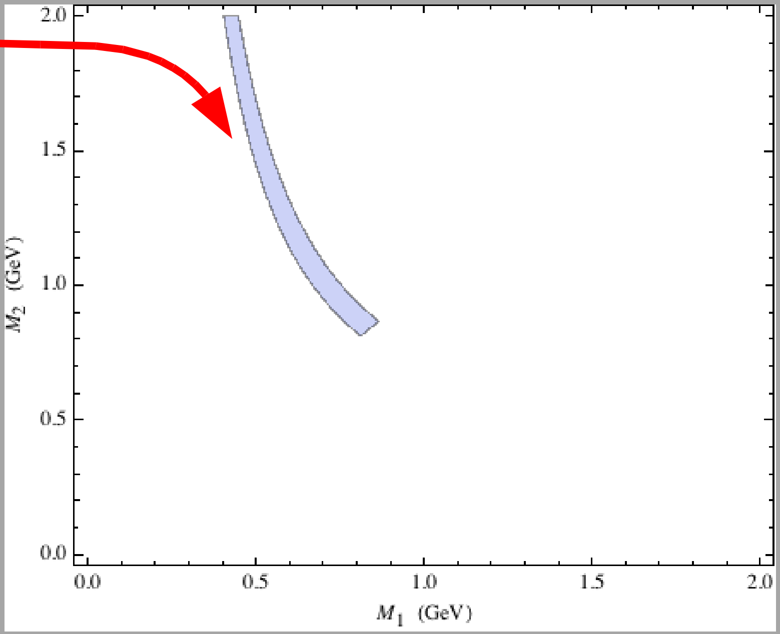


Bacchetta, Boer, Deihl, Mulders 2008

Possible explanations

Sivers function can have nodes in $\mathbf{k_T}$.
Kang, AP (2012)

Allowed region in parameter space



Appears to be not a natural solution!

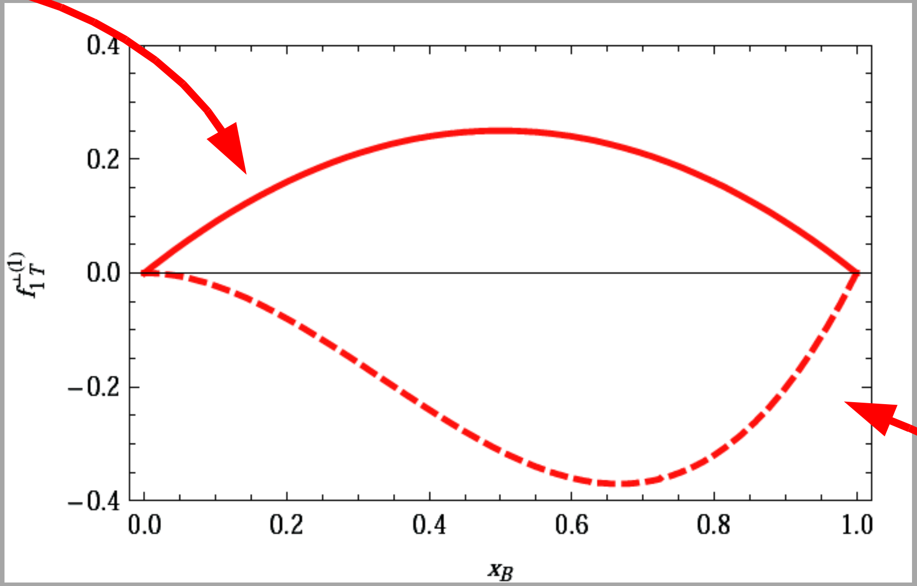
Possible explanations

Sivers function can have nodes in x.

Boer (2011)

Bacchetta et al, model calculation (2010), Kang, AP (2012)

SIDIS



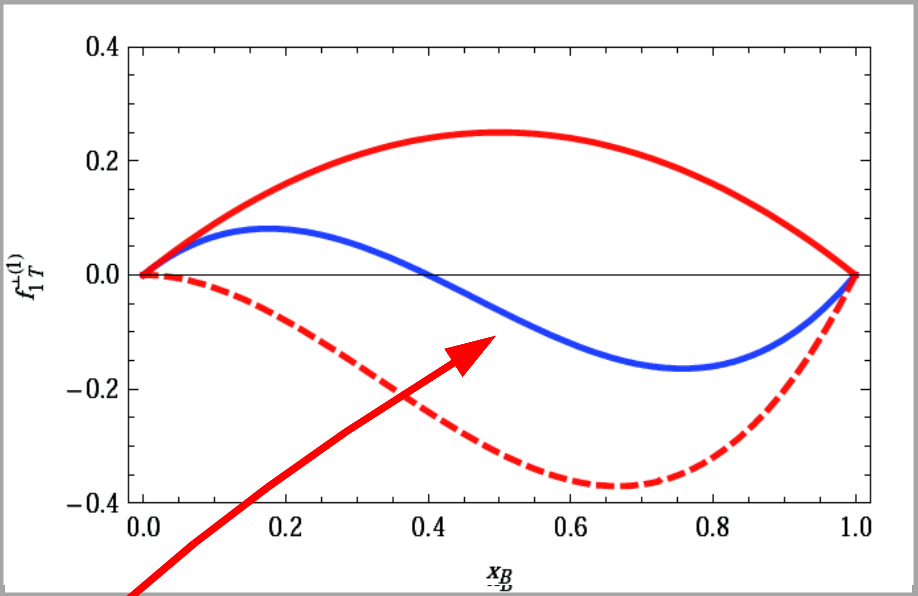
PP

Possible explanations

Sivers function can have nodes in x.

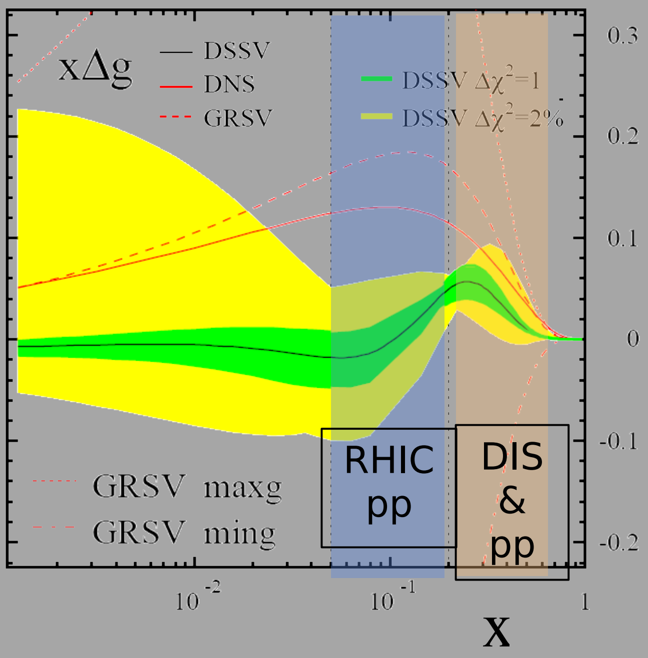
Boer (2011)

Bacchetta et al, model calculation (2010)



If PP and SIDIS probe different regions of x

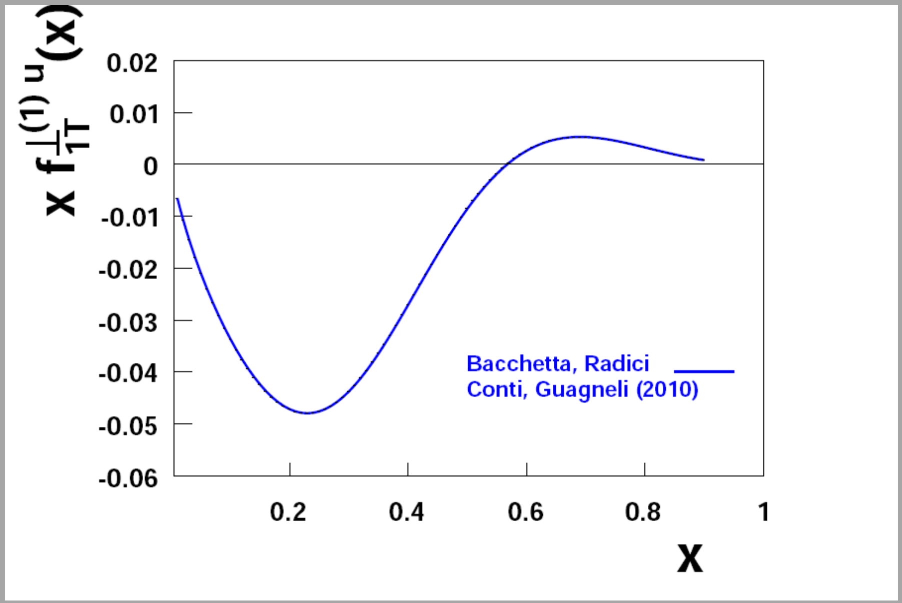
Are nodes so strange?



Node in $\Delta g(x)$ from DSSV global fit De Florian, Sassot, Stratmann, Vogelsang

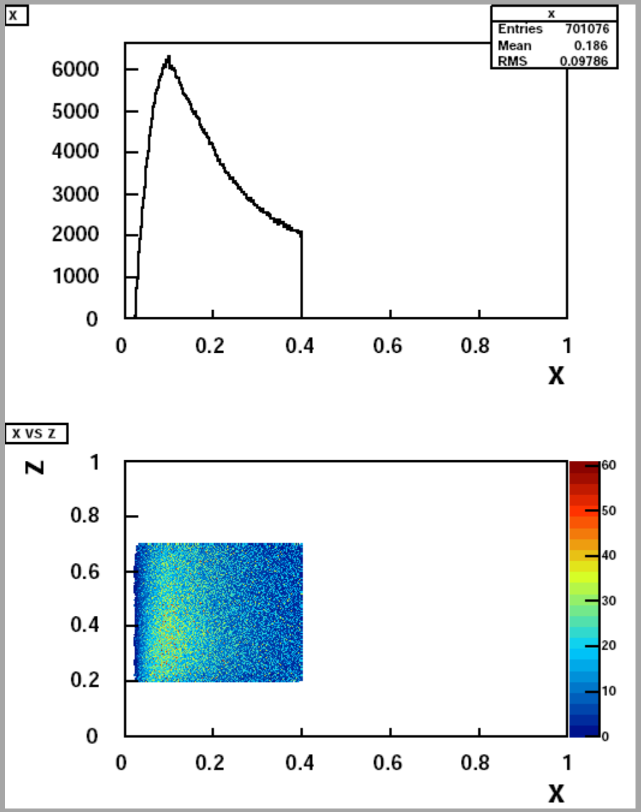
$$\Delta f \propto f(S) - f(-S)$$

Node in Sivers function
Bacchetta, Radici,
Conti, Guagneli (2010)



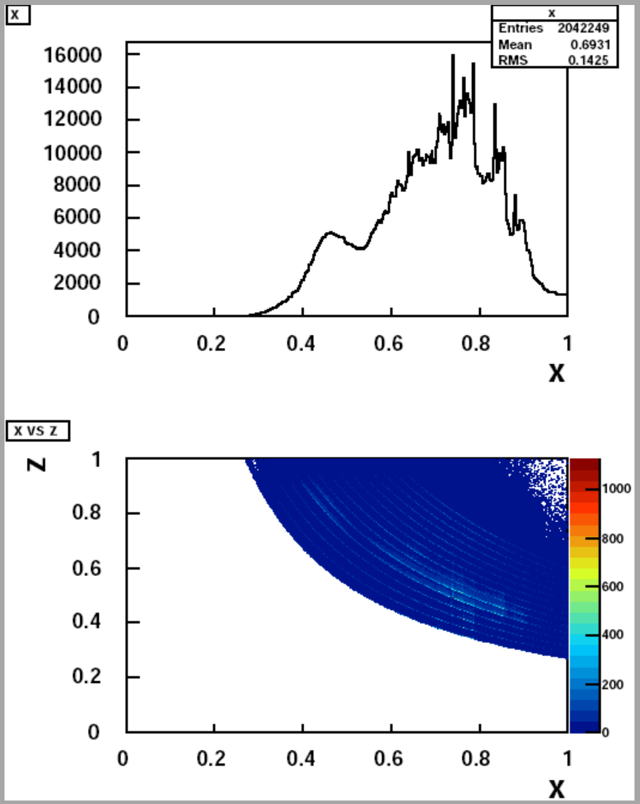
SIDIS vs PP kinematics

SIDIS HERMES



$$\mathbf{x < 0.4}$$

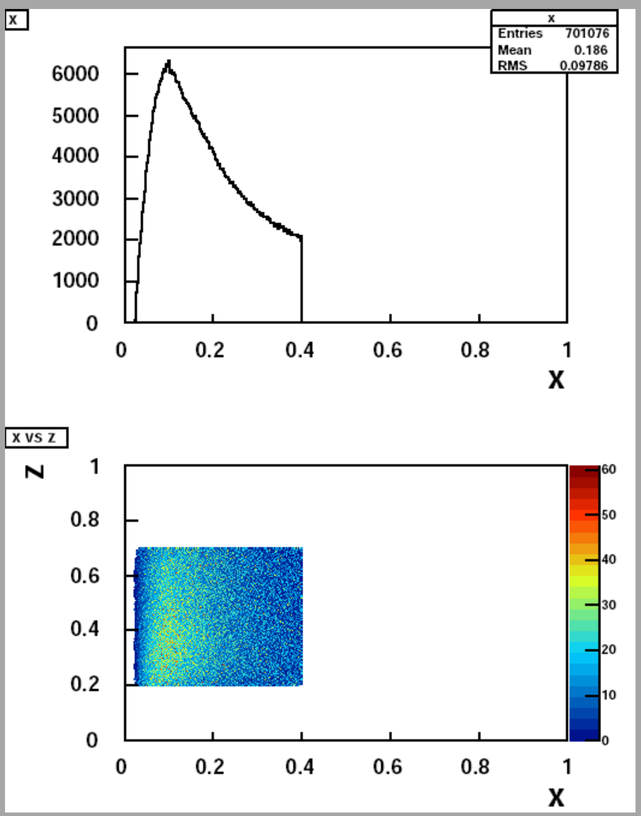
PP STAR



$$\mathbf{x > 0.4}$$

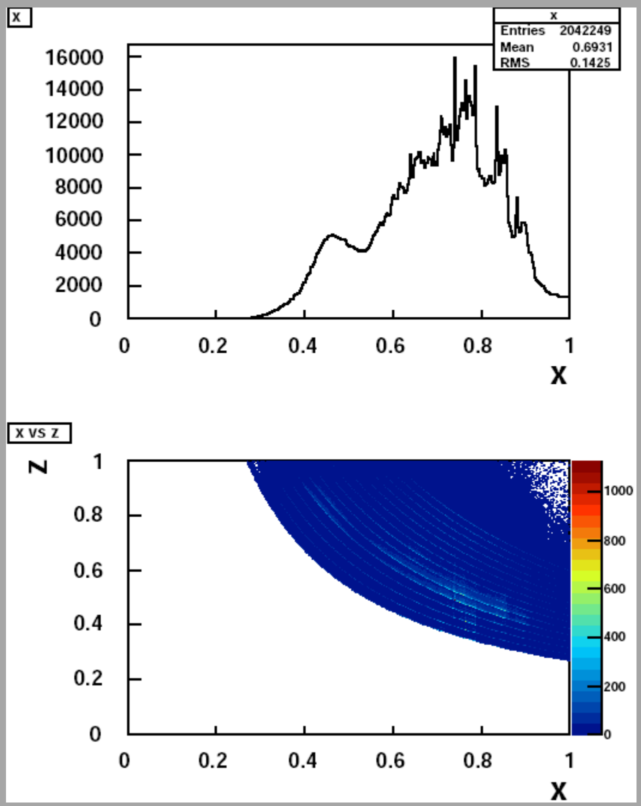
SIDIS vs PP kinematics

SIDIS HERMES



$$\mathbf{x} < \mathbf{0.4}$$

PP STAR



$$\mathbf{x} > \mathbf{0.4}$$

SIDIS and PP probe different regions in x !

Parametrization

$$f_{1T}^{\perp q} \propto x^{\alpha_q}(1-x)^{\beta_q}(1-\eta_q x)$$

as in De Florian, Sassot, Stratmann, Vogelsang (2009)

$1-\eta_q x$ has a node if $\eta_q > 0$

SIDIS: HERMES, COMPASS data $\pi^{\pm}$

TMD

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} \sim f_{1T}^{\perp} \otimes \sigma \otimes D_1$$

PP: STAR data π^0 **BRAHMS data** $\pi^{\pm}$

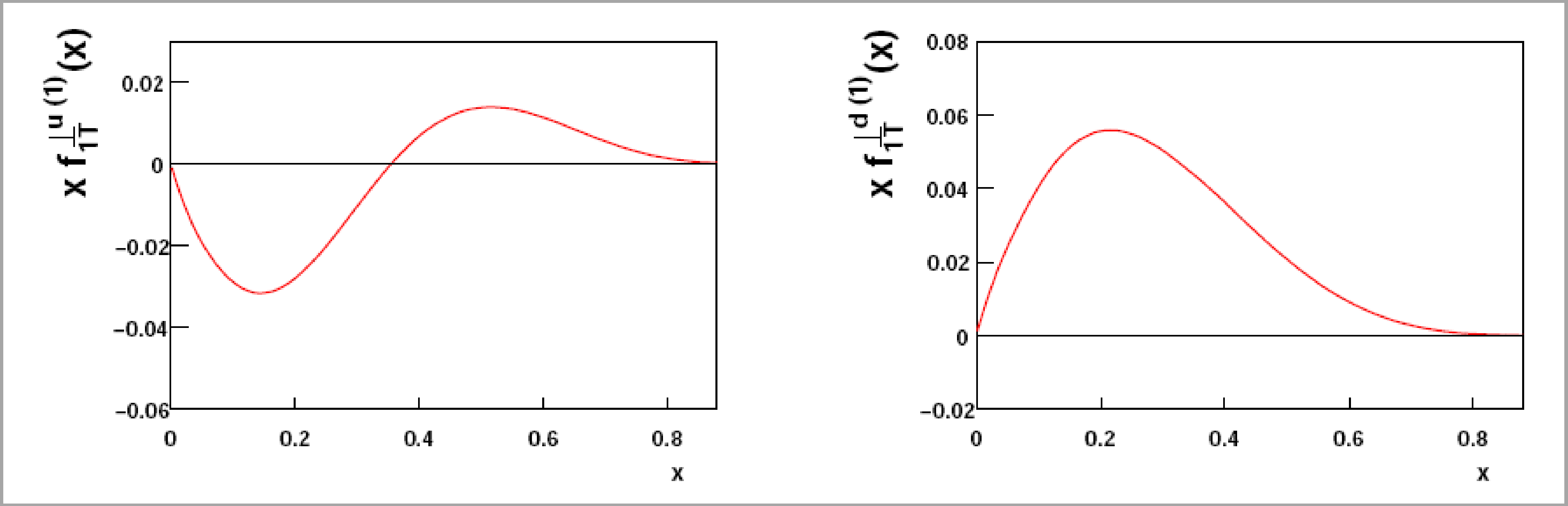
Twist-3

$$A_N \sim T_F \otimes \sigma \otimes D_1$$

- using PDF GRV98 and FF DSSV

Results: Sivers function

Kang, AP (2012)

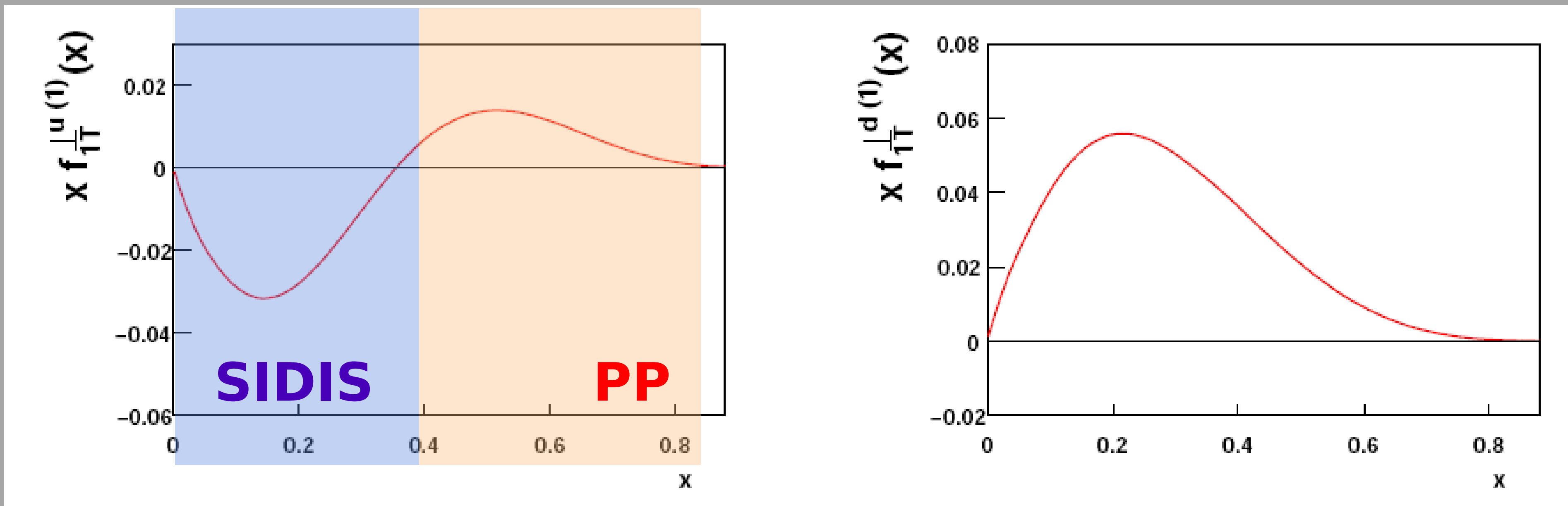


Sivers function can have a node!

$$\mathbf{x_{node} \sim 0.35}$$

Results: Sivers function

Kang, AP (2012)



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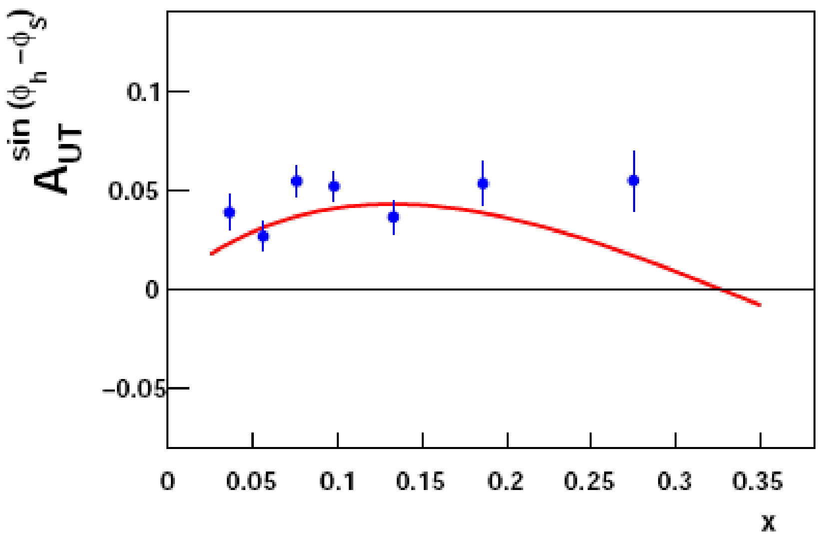
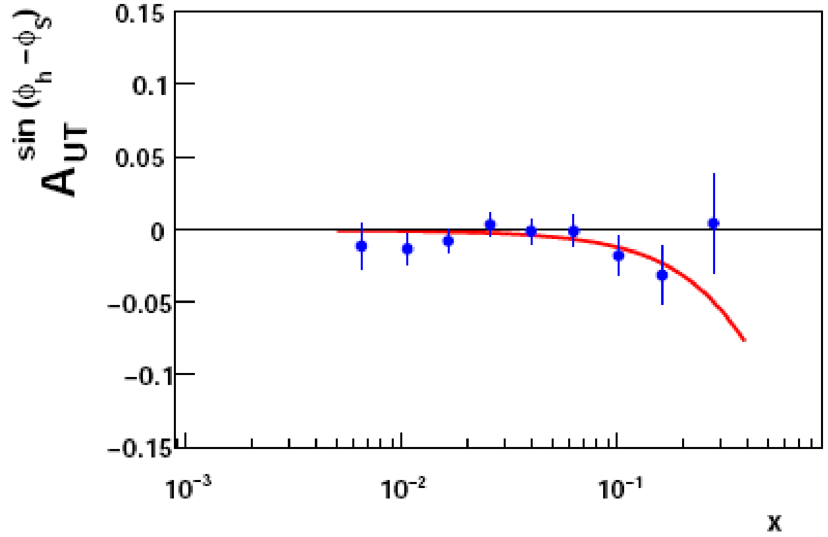
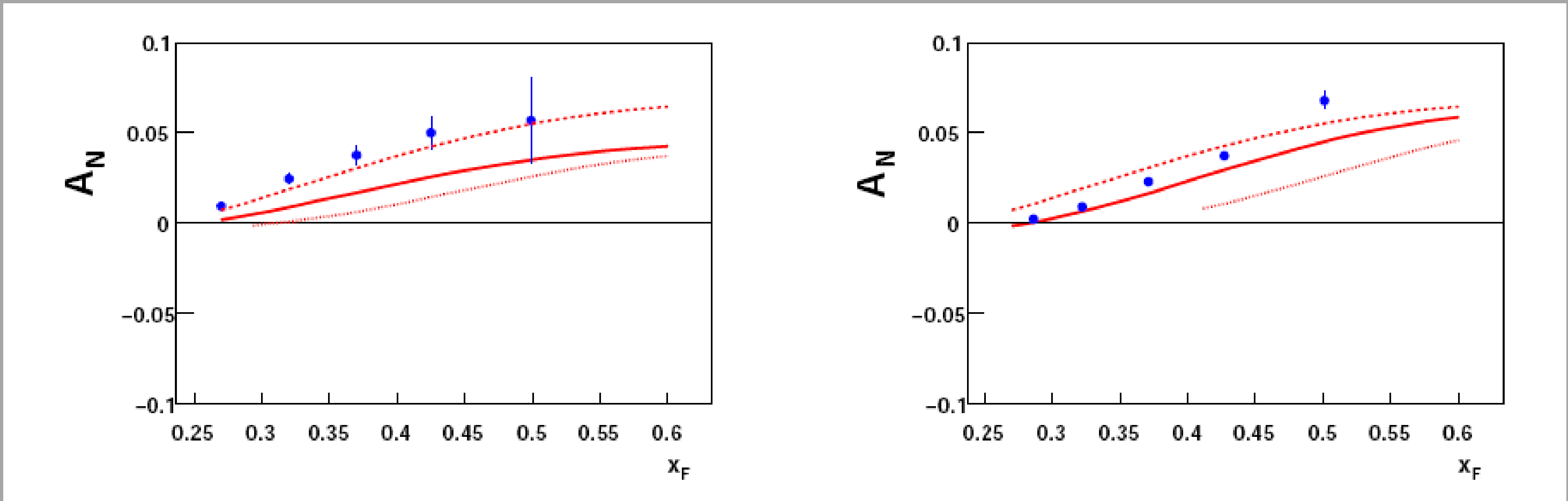




HERMES data **COMPASS data**

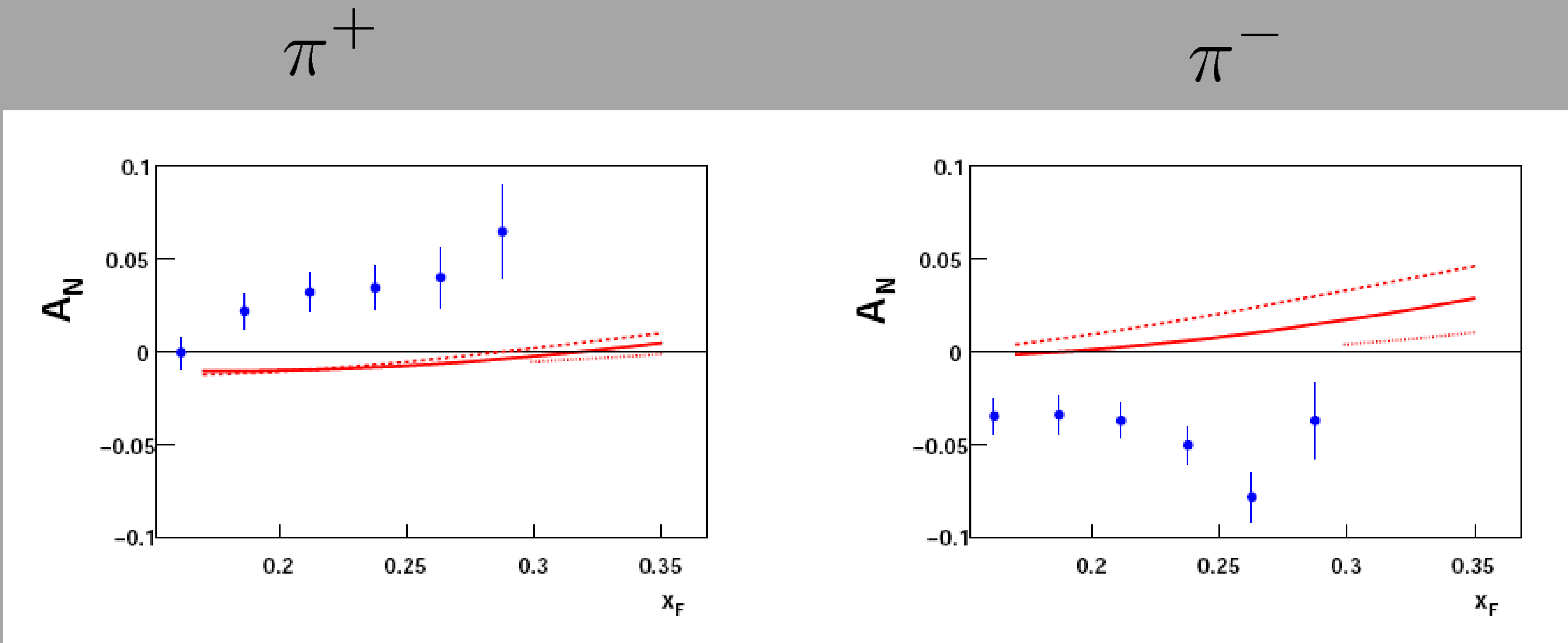
Results: PP



STAR data $\pi^0, \ \mathbf{y = 3.7}$**, reasonable description**

$$\mathbf{Q = P_T/2...2PT}$$

Results: PP



BRAHMS data $\theta = 4^o$ **,** **wrong sign**

SIGN PUZZLE IS STILL UNRESOLVED!

What is missing?

Twist-3 formalism:

$$\mathbf{A_N} \sim \mathbf{T_F} \otimes \sigma \otimes \mathbf{D_1} + \mathbf{h_1} \otimes \sigma \otimes \mathbf{H_F} + \ldots$$

We considered only Sivers effect, Soft Gluon Pole. Other parts should be added: sea-quarks, Soft Fermionic Pole contribution. Fragmentation part:Collins effect in particular.

For global analysis one should combine SIDIS, PP and e^+e^- data

TMD Collins effect in PP: Anselmino et al in preparation

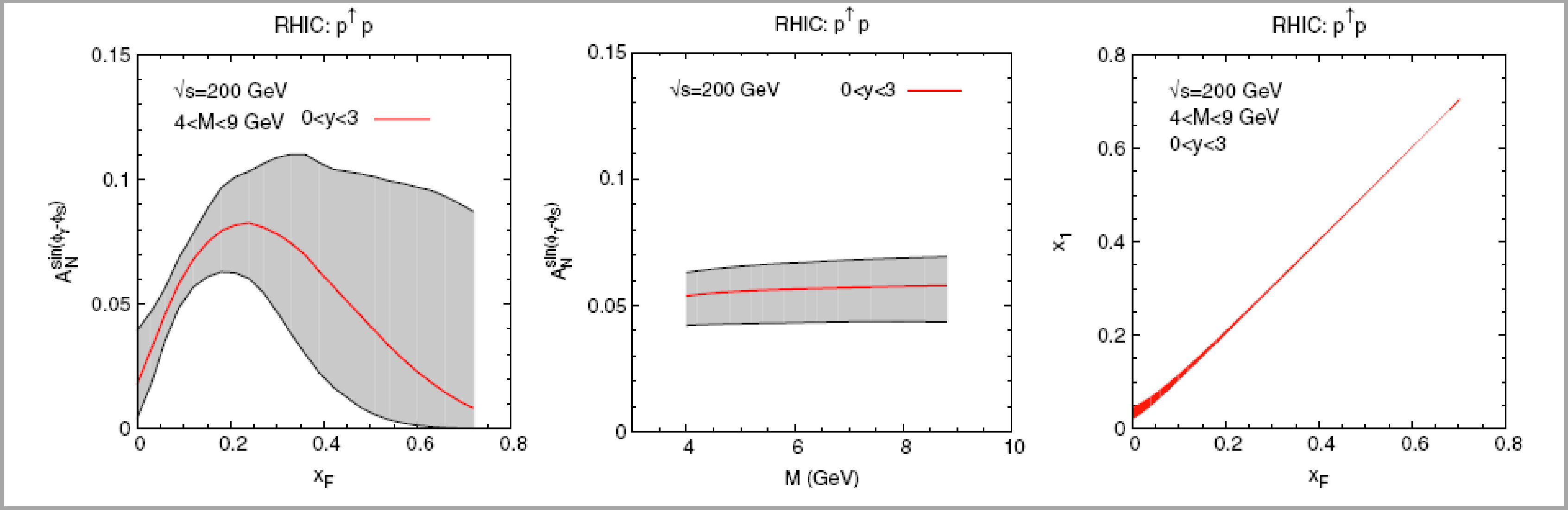
Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^q(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

Analysis at LO in hadronic cm frame
Anselmino et al (2009)

$$x_1 = \frac{x_F + \sqrt{x_F^2 + 4M^2/s}}{2} \approx x_F$$

In DY we probe Sivers function at x_F
Anselmino et al (2009)

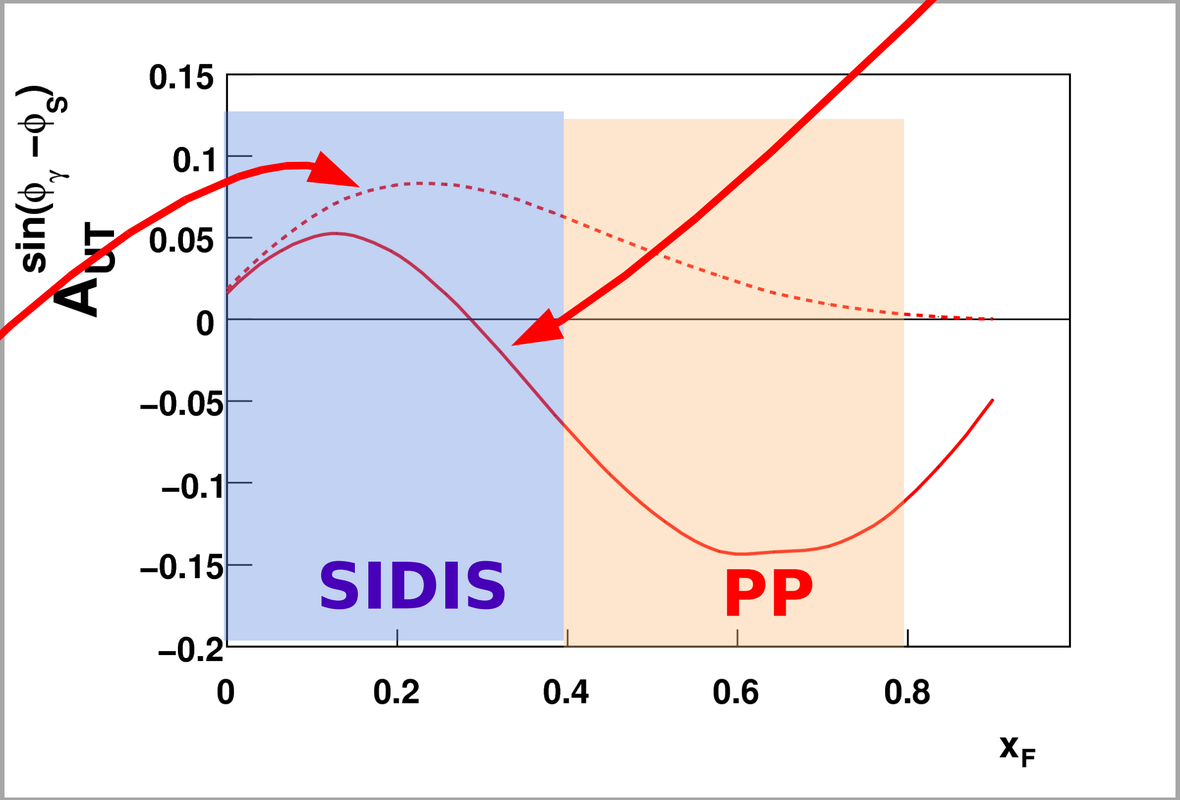


Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^{q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

Analysis at LO in hadronic cm frame

Kang, AP (2011)

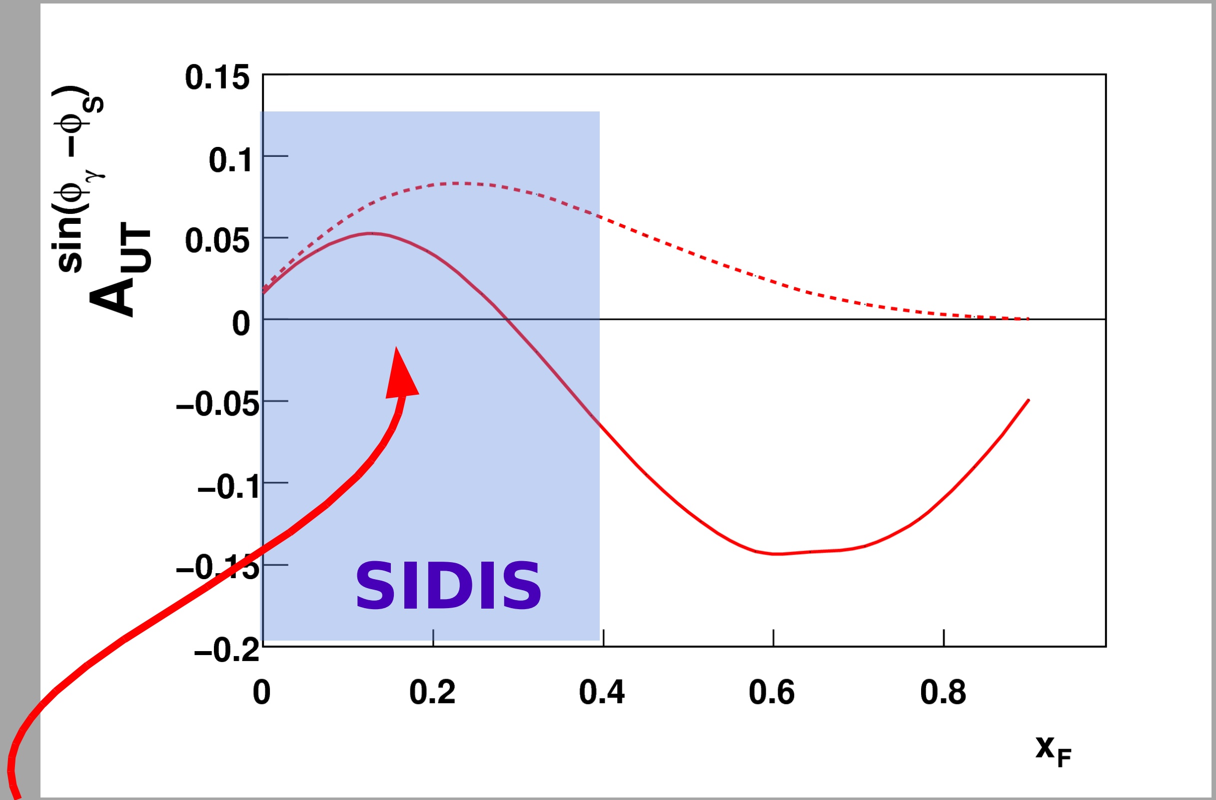


Anselmino et al (2009) no node

Drell Yan

$$\mathbf{A_N} = \frac{\sum_q \mathbf{f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{q\bar{q}}}{\sum_q \mathbf{f_1^{q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)}\sigma_{q\bar{q}}}$$

Analysis at LO in hadronic cm frame
Kang, AP (2011)



To measure in order to check

$$-\mathbf{f_{1T}^{\perp}}|_{\mathbf{DY}} = \mathbf{f_{1T}^{\perp}}|_{\mathbf{SIDIS}}$$

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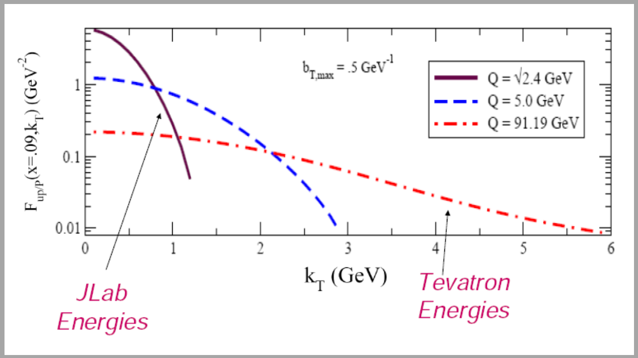
TMD&Twist-3 phenomenology

Global analysis of SIDIS, PP and e^+e^- data using TMD and twist-3 formalisms.
Kang, AP (2012), ...

TMD phenomenology:
NLO accuracy
Collins (2011), Aybat, Rogers (2011), ...



Twist-3 phenomenology:
NLO accuracy of hard functions
Vogelsang, Yuan (2009), ...

$$A_N \propto \Delta\sigma(Q, S_\perp) \propto T_f^{(3)}(x, x) \otimes \hat{H}_f \otimes \ldots$$

Beyond LO!

CONCLUSIONS

- Three dimensional parton picture is achievable with GPD and TMD measurements
- TMD phenomenology is possible with evolution
- Sivers function may have a node, however it **does not describe** BRAHMS data. Sign puzzle is still unresolved
- $\mathbf{x_F} \sim \mathbf{0...0.2}$ is “safe” for DY measurement
- Study of momentum dependence of TMDs, other observables such as inclusive pion production in SIDIS is needed in order to resolve sign puzzle