IWHSS 2012, 16 - 18 April 2012, Lisbon

Study of evolution of Transverse Momentum Dependent distributions

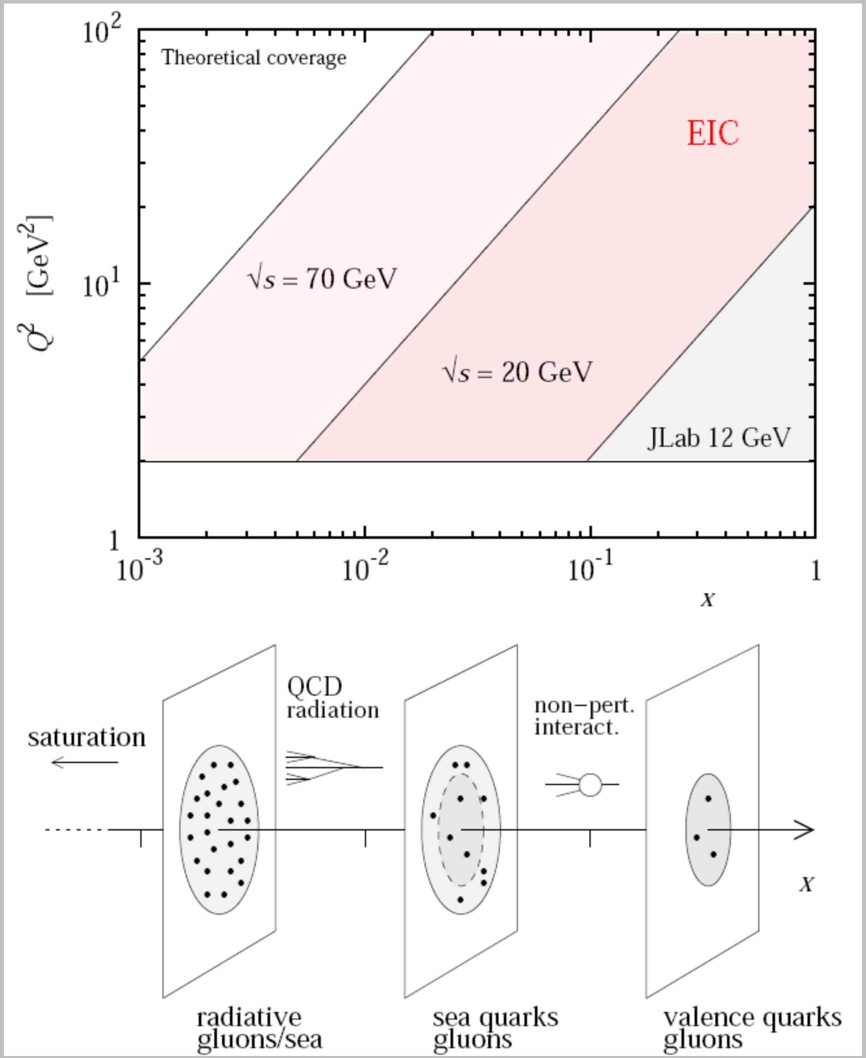
Alexei Prokudin
Jefferson Lab

In collaboration with
Ted Rogers and Mert Aybat
PRD83 (2011) 114042
PRD85 (2011) 034043
arXiv:1112.4423

Jefferson Lab
Thomas Jefferson National Accelerator Facility

See talks by
Werner, Alessandro, Barbara, Naomi and Cedric

Nucleon landscape



Plot courtesy of Christian Weiss

Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

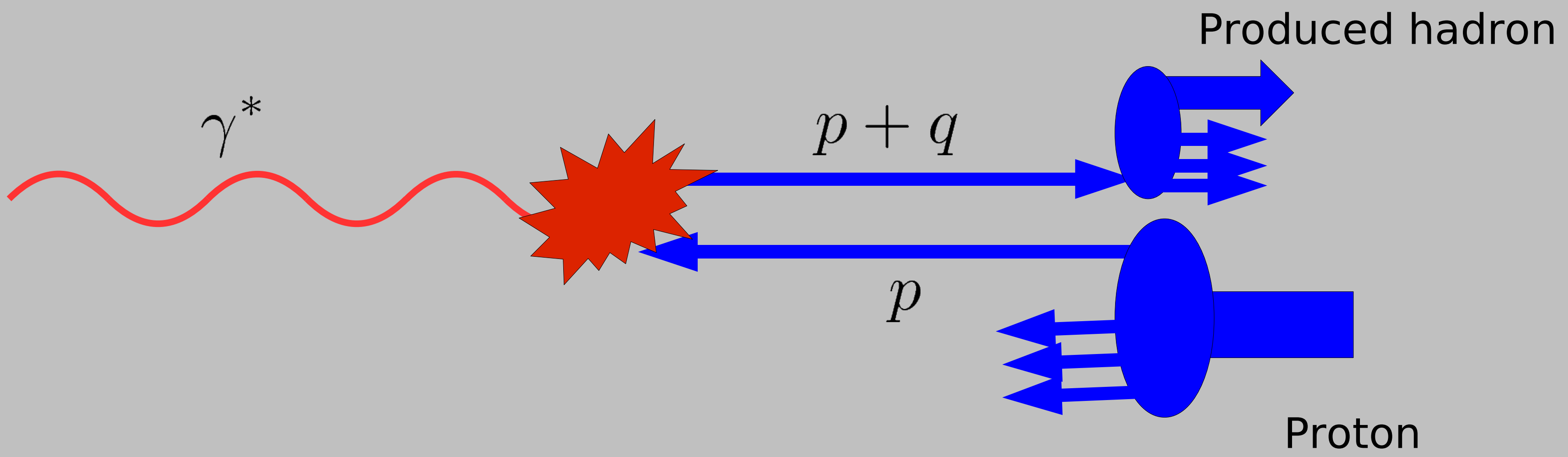
Markus Diehl (2003)
Matthias Burkardt (2003)

and Transverse Momentum Dependent distributions

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al , 2011

QCD and parton model

Let us calculate SIDIS cross section in parton model:

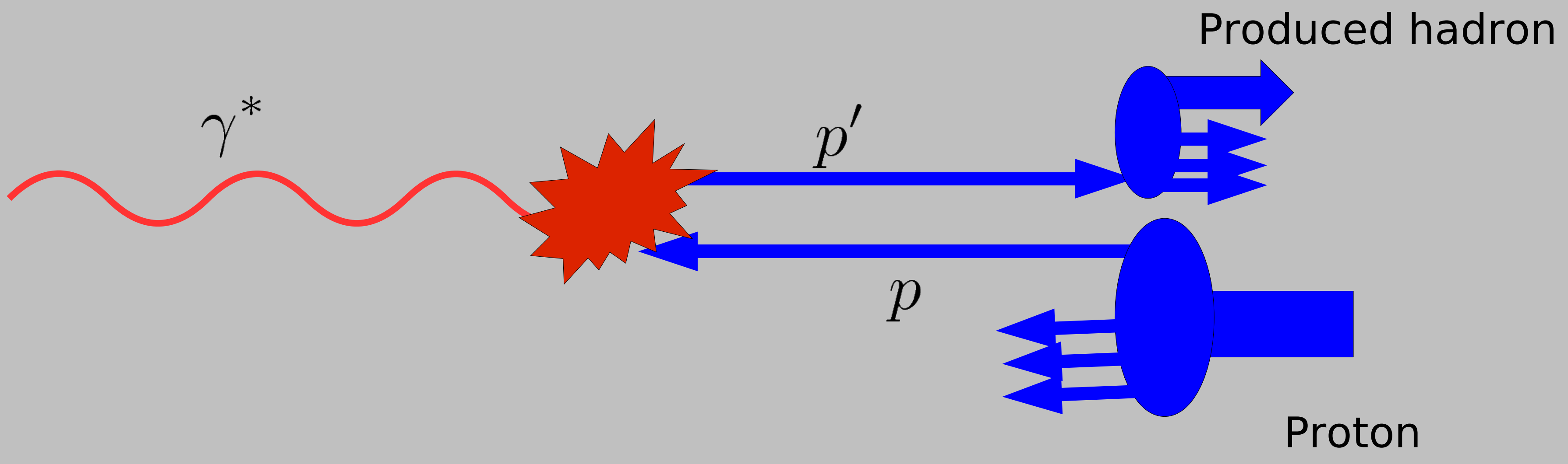


We work in Infinite Momentum Frame and all partons are collinear to the proton, thus

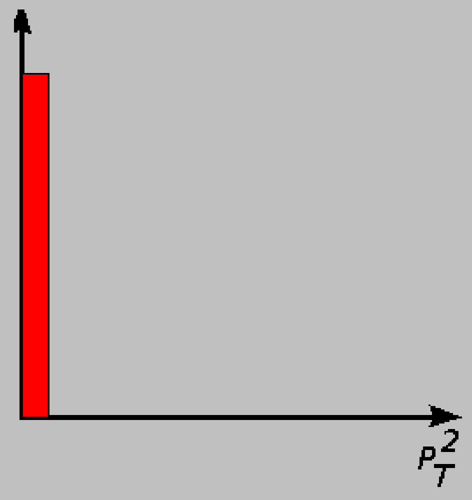
$$\frac{d\sigma}{dP_T^2} \sim \delta(P_T^2)$$

QCD and parton model

Let us calculate SIDIS cross section in parton model:

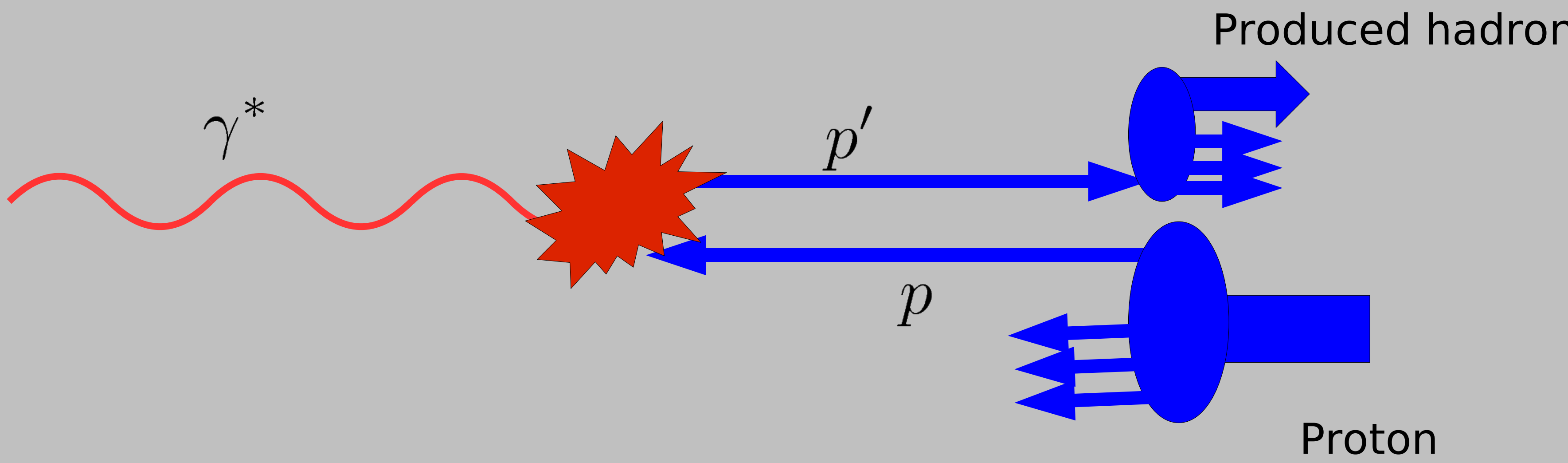


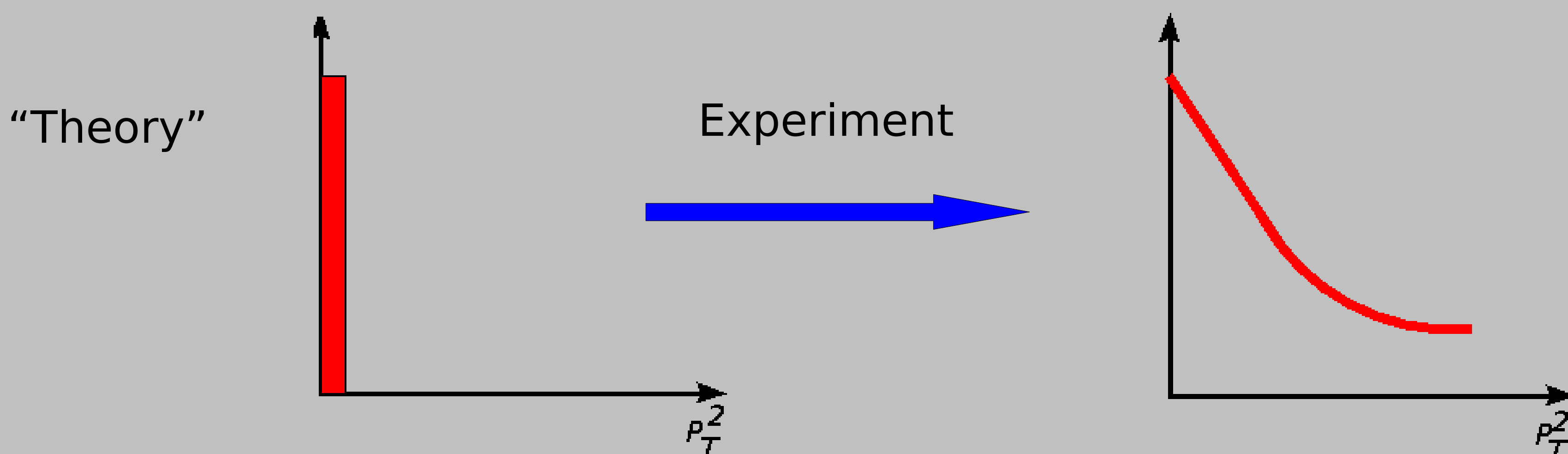
"Theory"



QCD and parton model

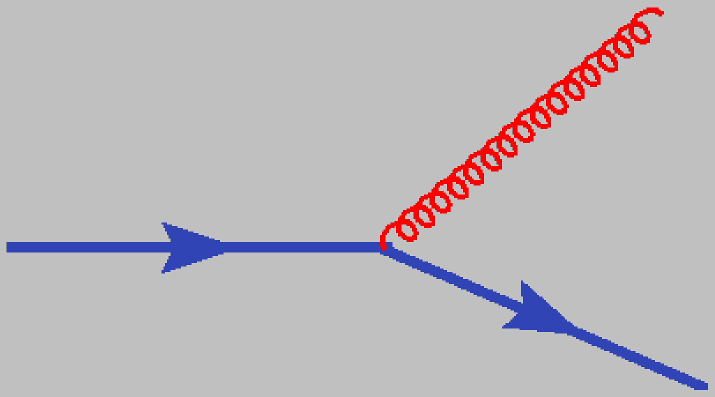
Let us calculate SIDIS cross section in parton model:





SIDIS and parton model

"QCD improved" parton model:



Radiation of gluons create transverse momenta

Terms like this appear

$$\left(\alpha_s\right)^n \left(\ln \frac{Q^2}{P_T^2}\right)^m$$

Result is singular as $P_T \longrightarrow 0$ and needs to be resummed

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Implementation of resummation
In QCD

Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS)is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{i q_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

Collinear distributions
are contained here

Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS) is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2b_T e^{iq_T\cdot b_T}\hat{W}(x_1,x_2,b_T)e^{-S(b_T,Q)} + Y(q_T,Q)$$

Collinear distributions
are contained here

Large logs (gluon radiation)
are resumed here

Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS) is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{iq_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

Collinear distributions are contained here

Large logs (gluon radiation) are resumed here

Corrections for large q_T

Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

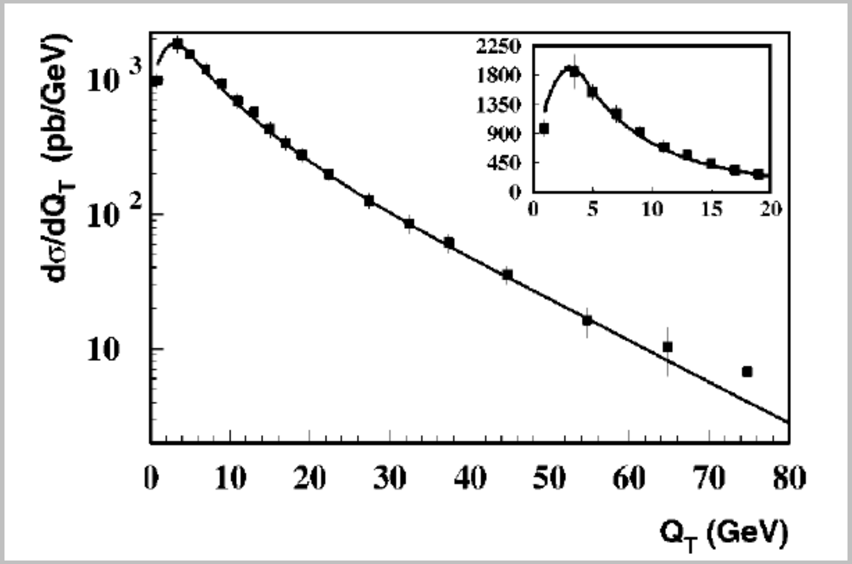
Resummation (CSS) is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{iq_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

A lot of phenomenology done. Energies from 20 GeV to 2 TeV.

Brock, Landry, Nadolsky, Yuan 2003
Qiu, Zhang 2001



Resummation

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS) is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{i q_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

A lot of phenomenology done. Energies from 20 GeV to 2 TeV.

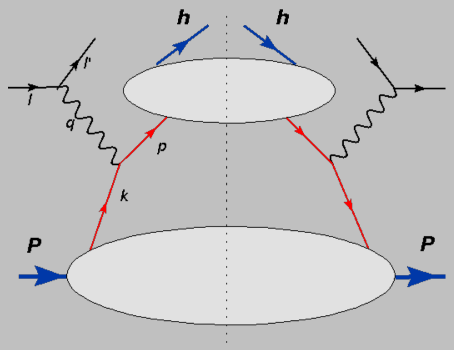
Brock, Landry, Nadolsky, Yuan 2003
Qiu, Zhang 2001

Drawbacks:

- Process dependent fits
- No direct connection to TMDs
- Designed for large energies

Transverse Momentum Dependent distributions

SIDIS



$$\mathbf{l + P \to l' + h + X}$$

Designed for low transverse momenta

If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} << Q$$

it will be sensitive to quark transverse momentum $k_\perp$

TMD factorization

Parton model: Field, Feynman (1977)
Polarised case: Kotzinian (1995)
Mulders, Tangerman (1995)

QCD: Ji, Ma, Yuan (2002)
Collins(2011)

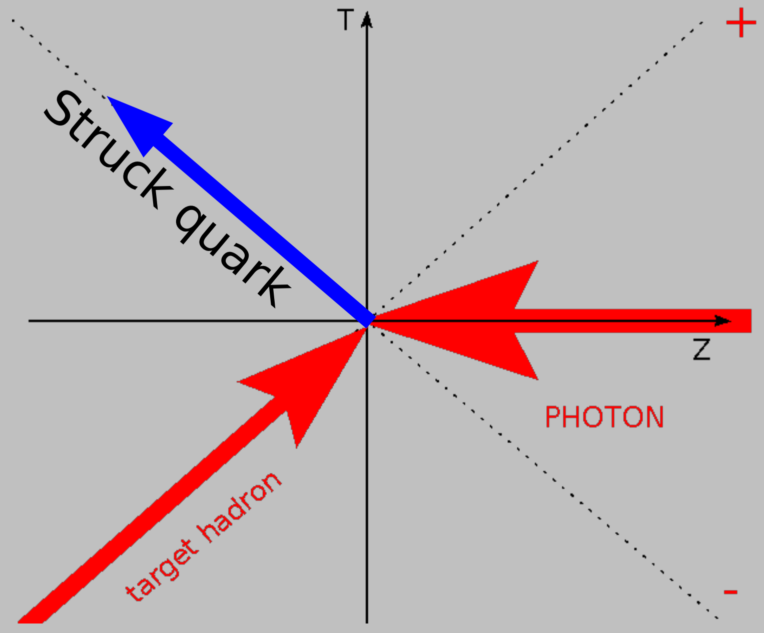
GAUGE INVARIANT

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp \xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle$$

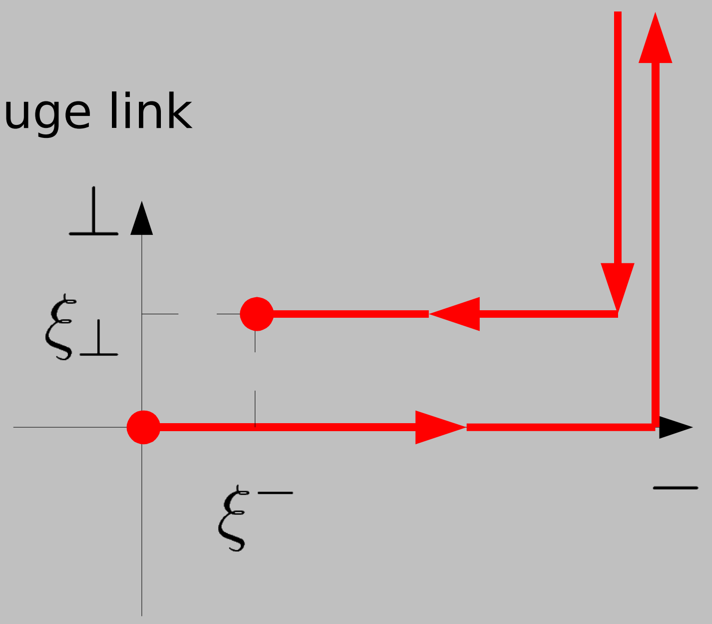
Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2}\, e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp} \langle P,S_P|\bar{\psi}_j(0)\mathcal{U}(\mathbf{0},\xi)\psi_i(\xi)|P,S_P\rangle|_{\xi^+=0}$$

SIDIS in Infinite Momentum Frame:



Gauge link



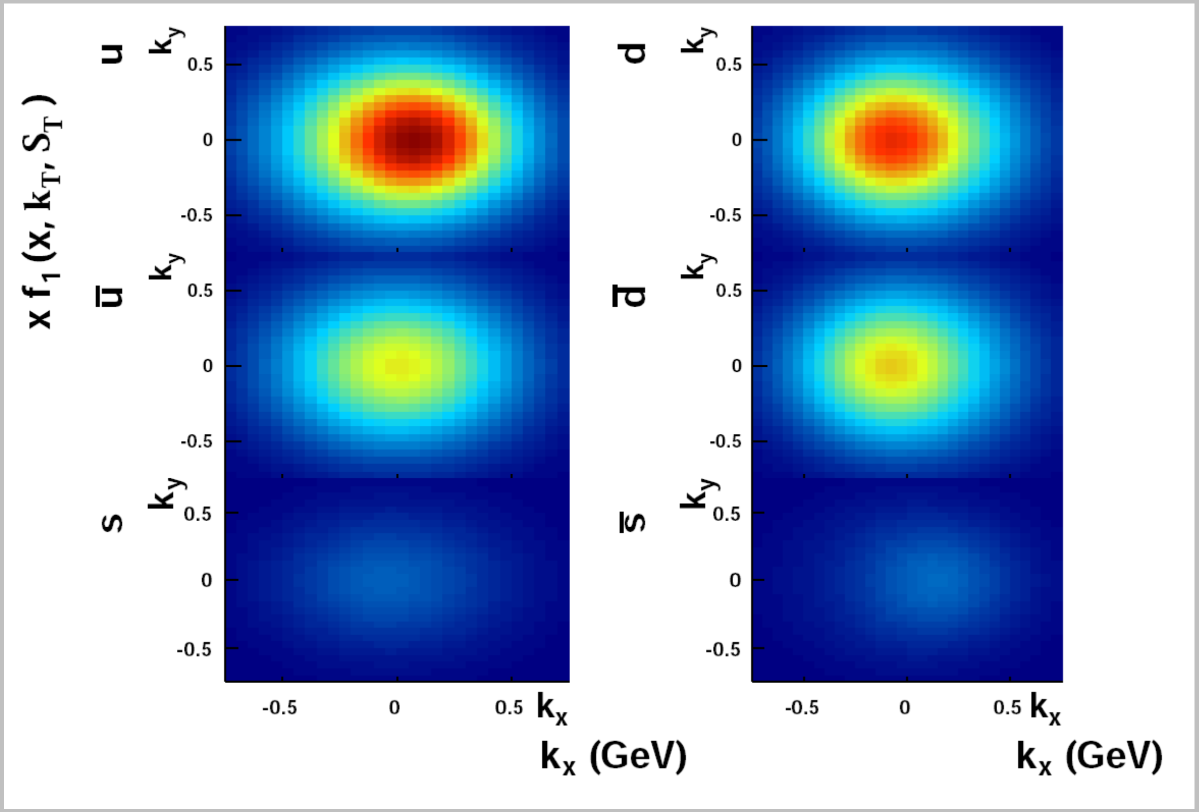
Transverse separation is due to presence of transverse parton momentum

Struck quark propagates in the gauge field of the remnant and forms gauge link

See talks by
Barbara, Naomi and Cedric

TMDs give us 3D distributions

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$



The slice is at:

$x = 0.1$

Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

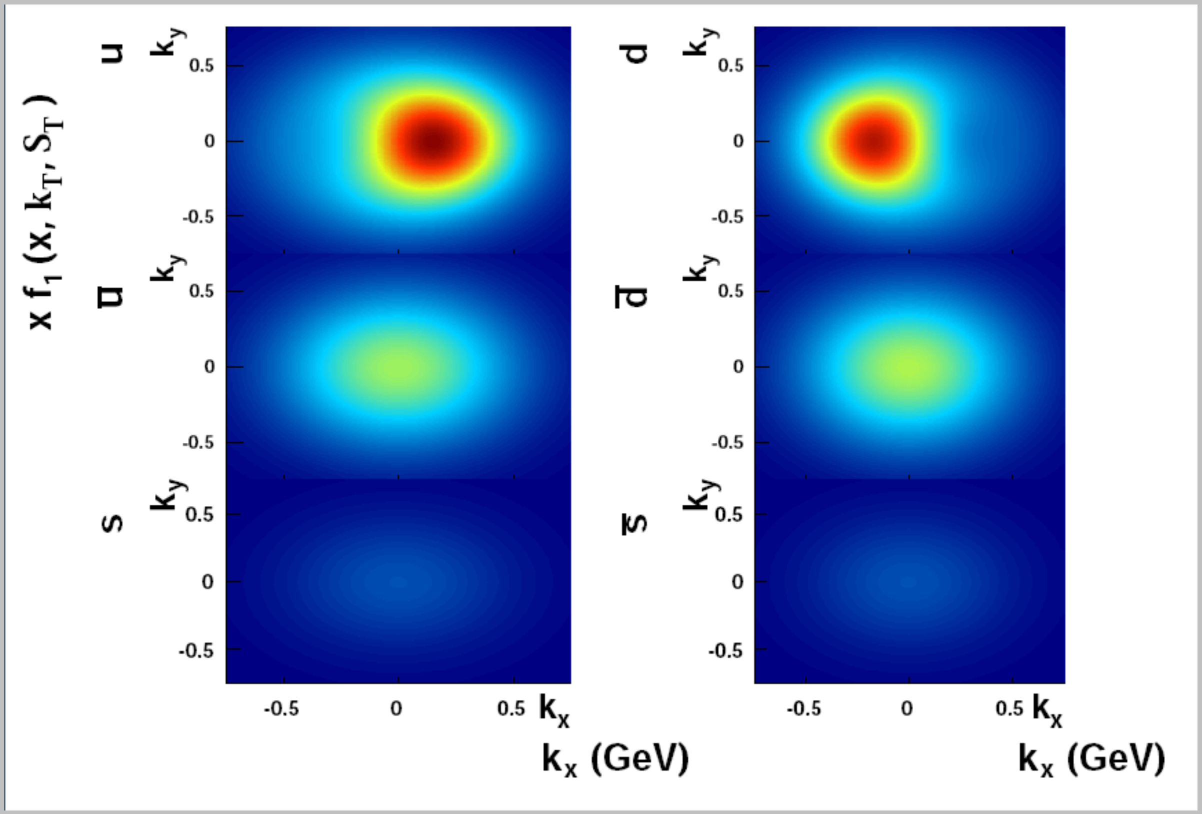
No information on sea quarks

Picture is still quite uncertain

See talks by
Barbara, Naomi and Cedric

TMDs give us 3D distributions

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$



The slice is at:

$x = 0.1$

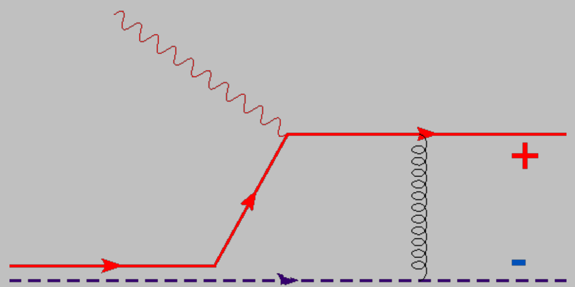
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

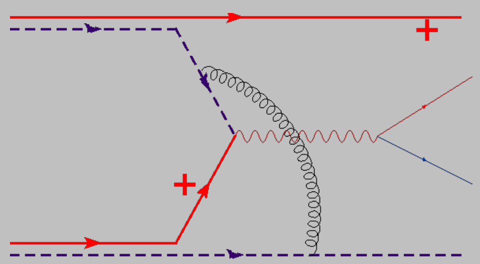
No information on sea quarks

In future we will obtain much clearer picture

Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)





Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman, etc

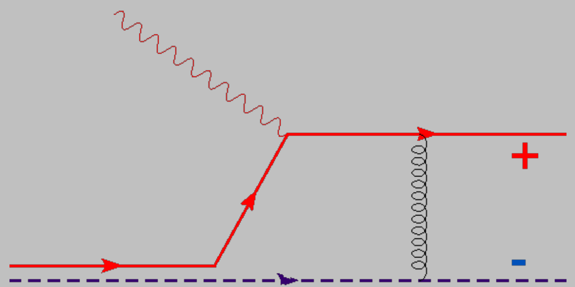
$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond “just” check of TMD factorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou

Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



man,

$$f_{1T}^{\perp SIDIS} = -$$

Drell-Yan is at much different resolution scale Q.
EIC will operate at higher Q.
What do we know about evolution of TMDs?

One of the main goals is to verify ution.
It goes beyond “just” check of TMD ctorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou

Evolution of TMDs

One needs a unique definition of TMDs

Foundations of perturbative QCD
Collins 2011

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F) F_{\bar{f}/P_1}(x_2, \mathbf{k}_{2T}; \mu, \zeta_F)$$
$$\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) + Y(\mathbf{q}_T, Q)$$

$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$ **TMD distribution of partons in hadron**

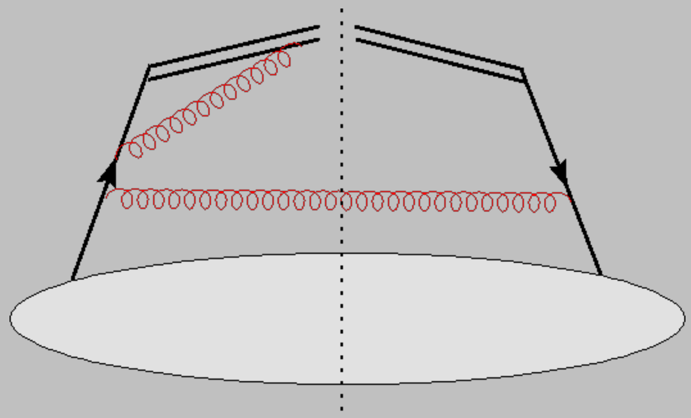
Rapidity divergence regulator

Renorm group (RG) renormalization

Evolution of TMDs

One needs a unique definition of TMDs

Foundations of perturbative QCD
Collins 2011



Infinite rapidity of the gluon creates so called rapidity divergence

In collinear PDFs this divergence is cancelled between virtual and real gluon diagrams

It is not the case for TMDs
Thus new regulator ζ_F is needed

$$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$$

Rapidity divergence regulator

Renorm group (RG) renormalization

Evolution of TMDs

Evolution of TMDs is done in coordinate space $\mathbf{b}_T$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Why coordinate space?

Convolutions become simple products:

Collins, Soper 1982
Collins, Soper, Sterman 1985
Idilbi, Ji, Ma, Yuan 2004
Boer, Gamberg, Musch, AP 2011

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu} \times \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \mathbf{q}_T} \tilde{F}_{f/P_1}(x_1, \mathbf{b}_T; \mu, \zeta_F) \tilde{F}_{\bar{f}/P_1}(x_2, \mathbf{b}_T; \mu, \zeta_F)$$

In principle experimental study of functions in coordinate space
Is possible

Boer, Gamberg, Musch, AP 2011

Evolution of TMDs

Evolution of TMDs is done in coordinate space $\mathbf{b}_T$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T\cdot\mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Complicated in case of Sivers function

Aybat, Collins, Qiu, Rogers 2012

$$F_{f/P^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T; \mu, \zeta_F) = F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, \mathbf{k}_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

Unpolarised part:

$$\tilde{F}_{f/P}(x, b_T; \mu, \zeta_F) = (2\pi) \int_0^\infty dk_T k_T J_0(k_T b_T) F_{f/P}(x, k_T; \mu, \zeta_F)$$

Sivers function:

$$\tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F) = -(2\pi) \int_0^\infty dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$

TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins-Soper kernel in coordinate space

Renormalization group equations

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu), \zeta)$$

TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins-Soper kernel in coordinate space

At small $\mathbf{b}_T$ perturbative treatment is possible

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{K}(b_T, \mu) = -\frac{\alpha_s C_F}{\pi}\Big(\ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_E\Big) + \mathcal{O}(\alpha_s^2)$$

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

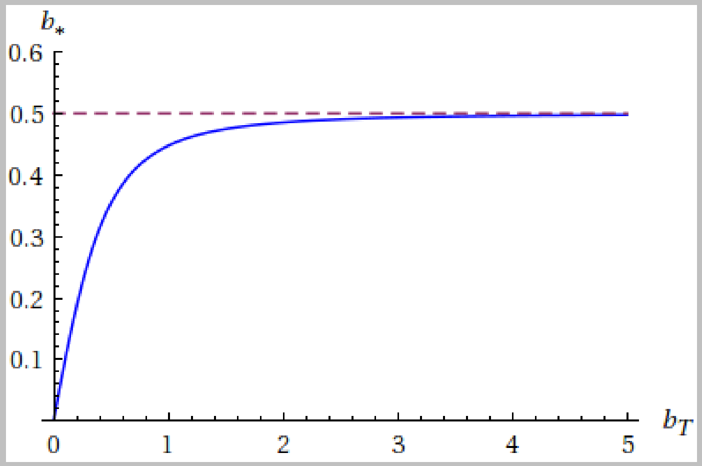
Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

$$b_{max} = 0.5\ (\mathrm{GeV}^{-1})$$

Brock, Landry, Nadolsky, Yuan 2003



TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

Always perturbative

$$\tilde{K}(b_T, \mu) = \tilde{K}(b_*, \mu) - g_K(b_T)$$

Non perturbative

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2$$

$$g_2 \simeq 0.68 \; (GeV^2)$$

This function is universal for different partons!

Brock, Landry, Nadolsky, Yuan 2003

TMD evolution

Relation to collinear treatment:

$$\tilde{F}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f}\left(\frac{x}{\hat{x}}, b_T, \mu, \zeta\right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD} b_T)$$

Collins Soper 1982

Valid at small $\mathbf{b}_T$, lowest order:

$$\tilde{C}_{j/f}(\frac{x}{\hat{x}}, b_T, \mu, \zeta) = \delta_{jf}\delta\left(\frac{x}{\hat{x}} - 1\right) + \mathcal{O}(\alpha_s)$$

Higher order for TMD PDFs

Aybat Rogers 2011

Higher order for Sivers function

Kang, Xiao, Yuan 2011

TMD evolution

Solution Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2)$$
$$\times \exp\Big[- g_K(b_T) \ln \frac{Q}{Q_0} \Big]$$

Non perturbative

$$\times \exp\Big[\ln \frac{Q}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \Big[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \Big]$$
$$+ \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{Q}{Q_0} \gamma_K(g(\mu')) \Big]$$

Perturbative

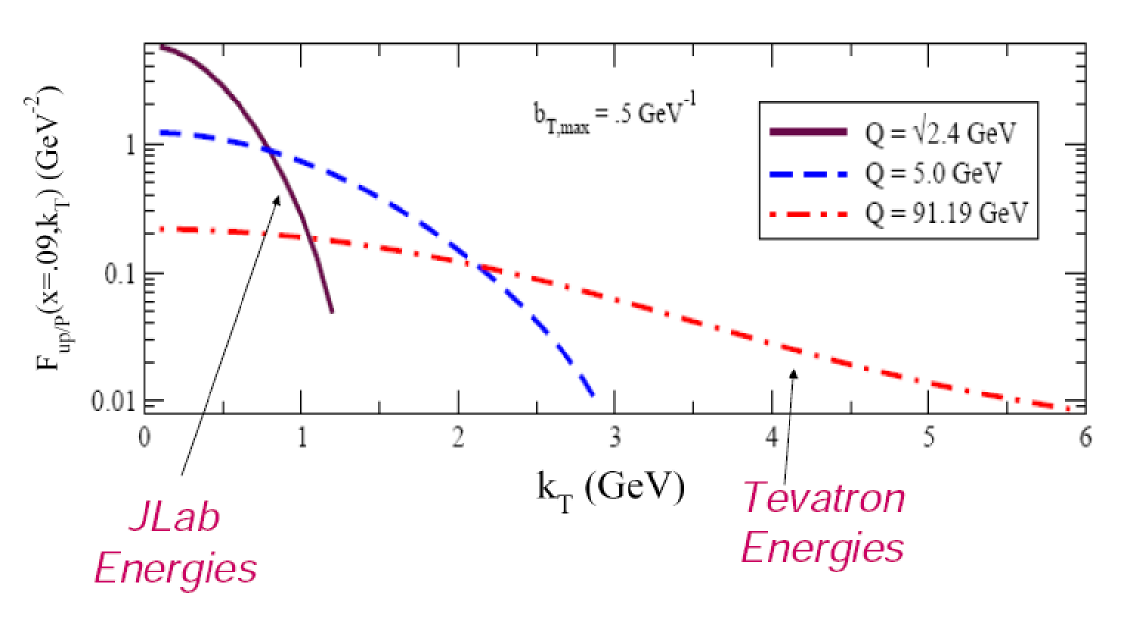
Typically for TMDs:

$$\tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) = F_{f/P}(x; Q_0) \exp\Big(- \frac{\langle k_T^2 \rangle}{4} b_T^2 \Big)$$

TMD evolution

Solution Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = F_{f/P}(x; Q_0) \exp\Big(- \underbrace{\Big[\frac{\langle k_T^2\rangle}{4} + \frac{g_2}{2} \ln\frac{Q}{Q_0}\Big]}_{\text{Non perturbative}} b_T^2 \Big)$$

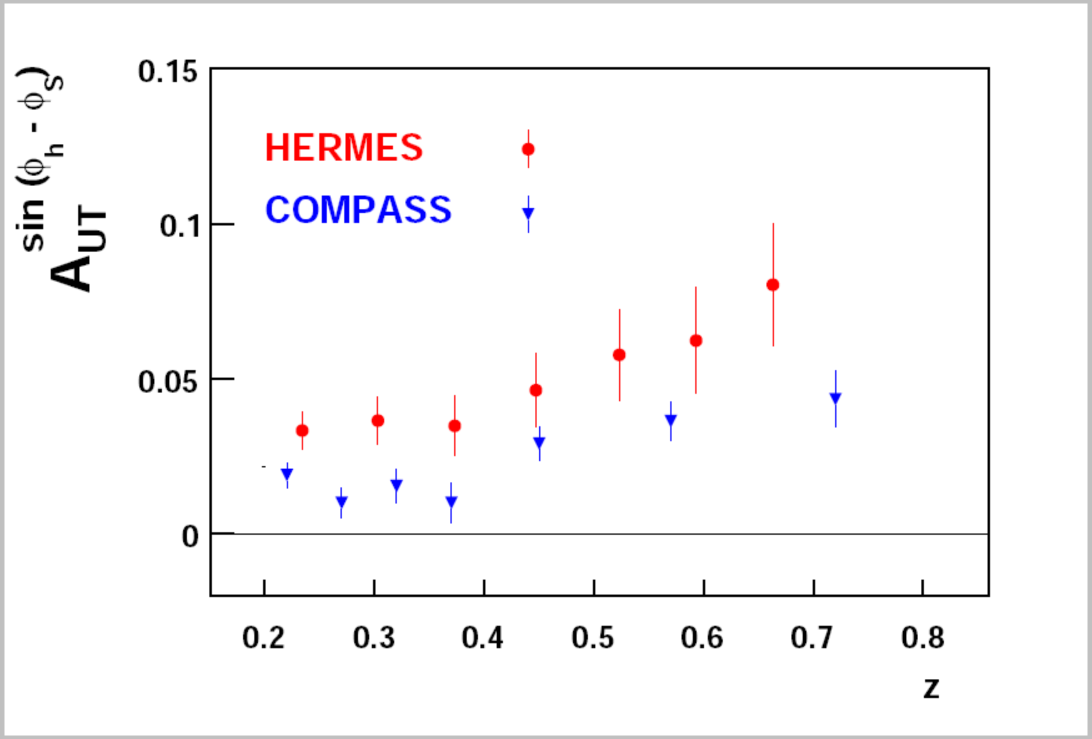


Gaussian behaviour is appropriate only in a limited range

TMDs change with energy and resolution scale

TMD evolution

Can we see signs of evolution in the experimental data?



Aybat, AP, Rogers 2011

COMPASS data is at

$$\langle Q^2 \rangle \simeq 3.6 \; (GeV^2)$$

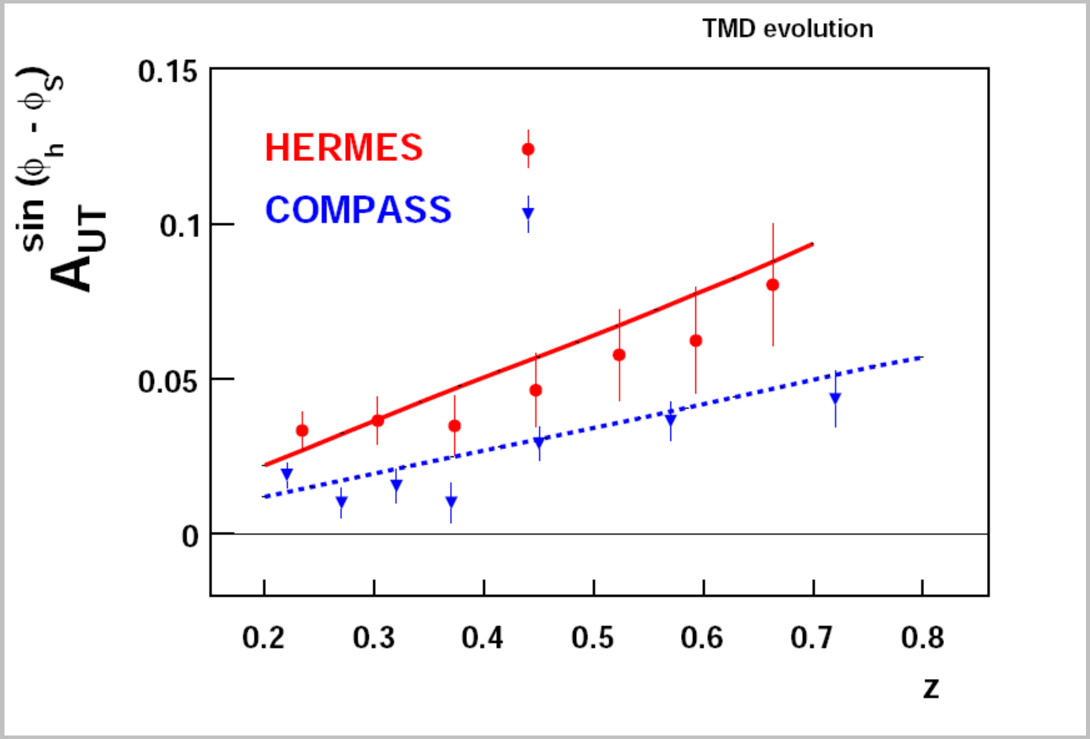
HERMES data is at

$$\langle Q^2 \rangle \simeq 2.4 \; (GeV^2)$$

TMD evolution

Can we explain the experimental data?

Full TMD evolution is needed!



Aybat, AP, Rogers 2011

COMPASS dashed line

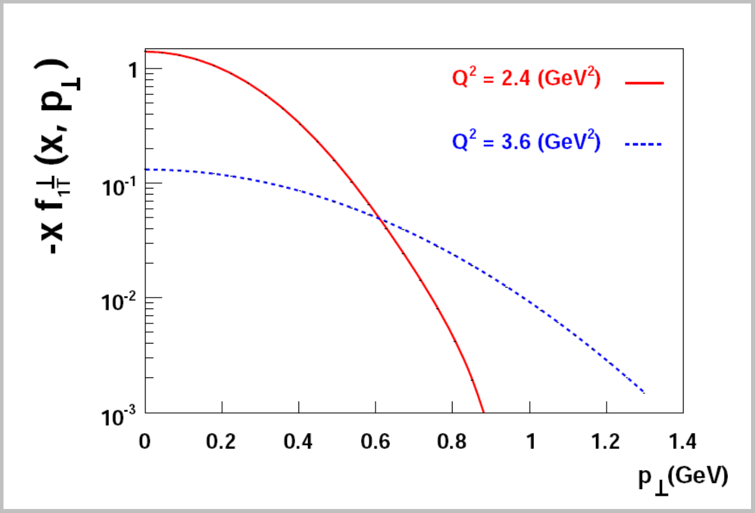
$$\langle Q^2 \rangle \simeq 3.6 \ (GeV^2)$$

HERMES solid line

$$\langle Q^2 \rangle \simeq 2.4 \ (GeV^2)$$

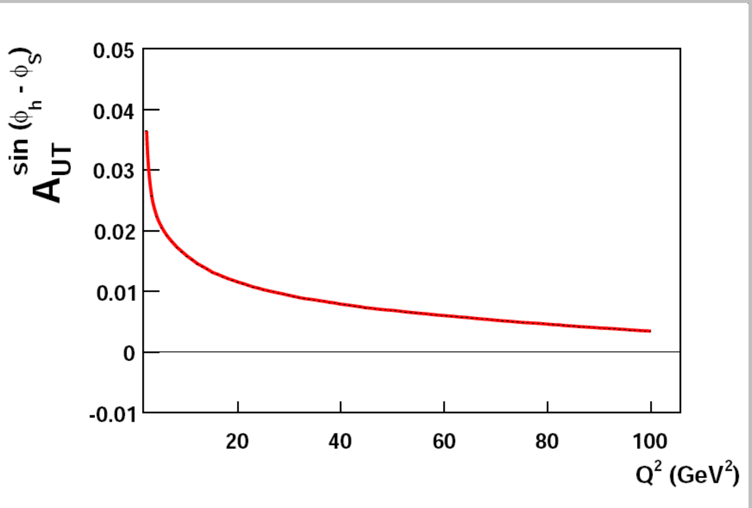
TMD evolution

This is the first implementation of TMD evolution for observables



Aybat, AP, Rogers 2011

Asymmetry changes with Q^2

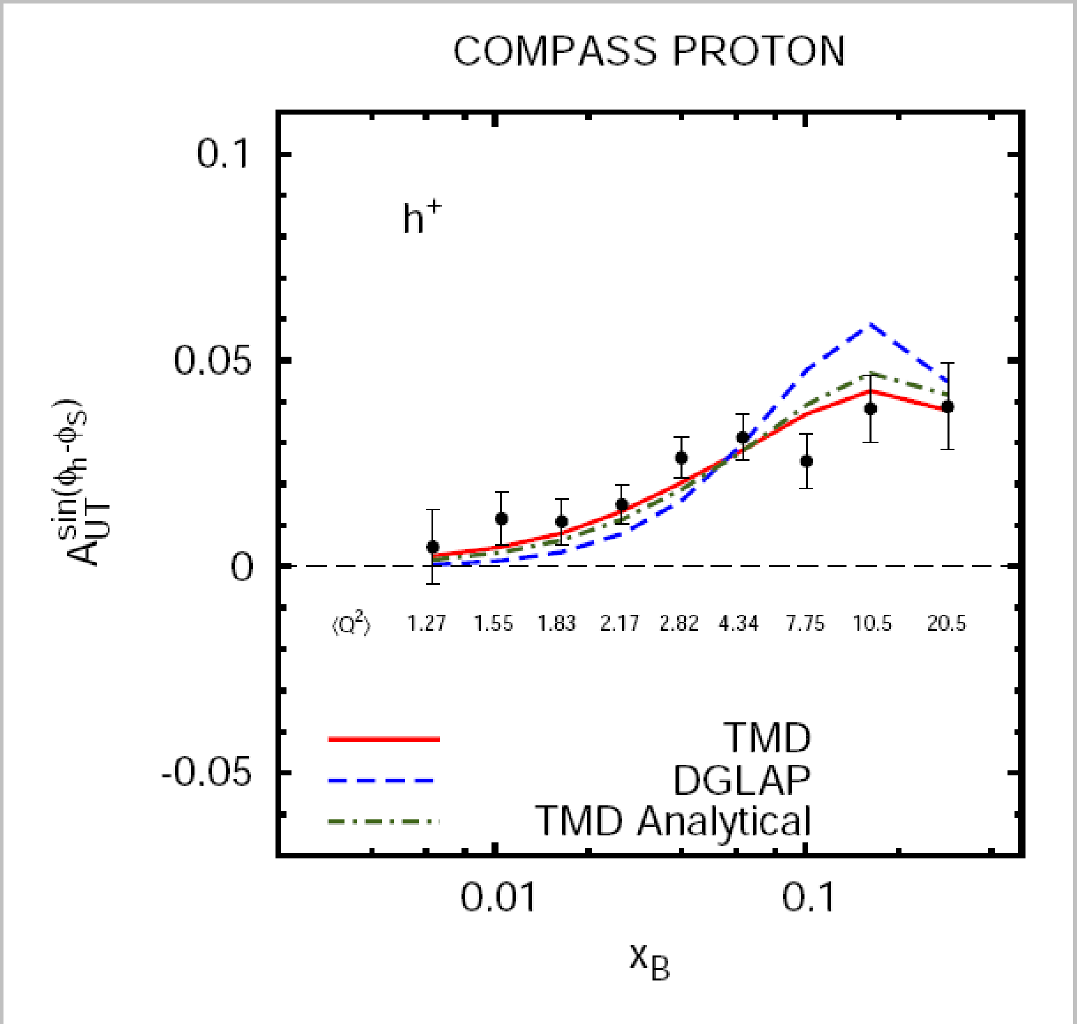


Functions change with energy

Phenomenological analysis with evolution is now possible

TMD evolution

The same conclusions in



Anselmino, Boglione, Melis 2012

Solid line – TMD evolution fit
Dashed line – DGLAP fit

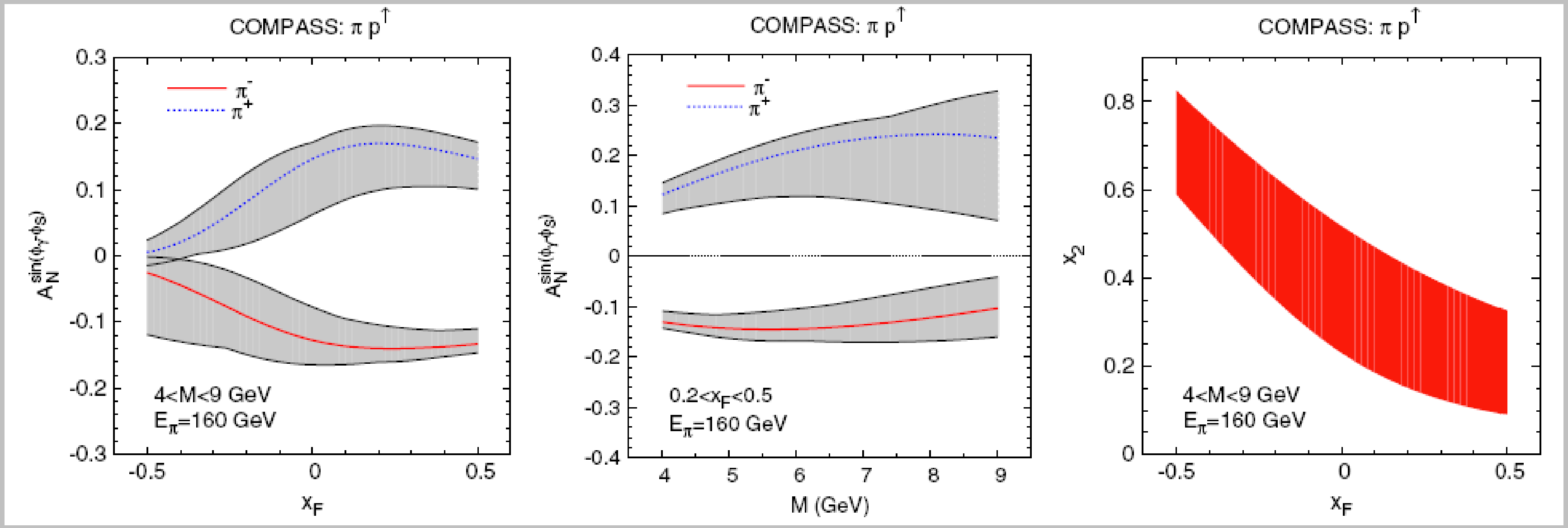
Drell Yan

$$\mathbf{A_N} = \frac{\sum_{\mathbf{q}} \mathbf{f_{1T}^{\perp q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)} \sigma_{\mathbf{q\bar{q}}}}{\sum_{\mathbf{q}} \mathbf{f_1^{q}(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)} \sigma_{\mathbf{q\bar{q}}}}$$

Analysis at LO in hadronic cm frame

Anselmino et al (2009)

$$\sqrt{s} = 17.4 \ \ (\mathrm{GeV})$$



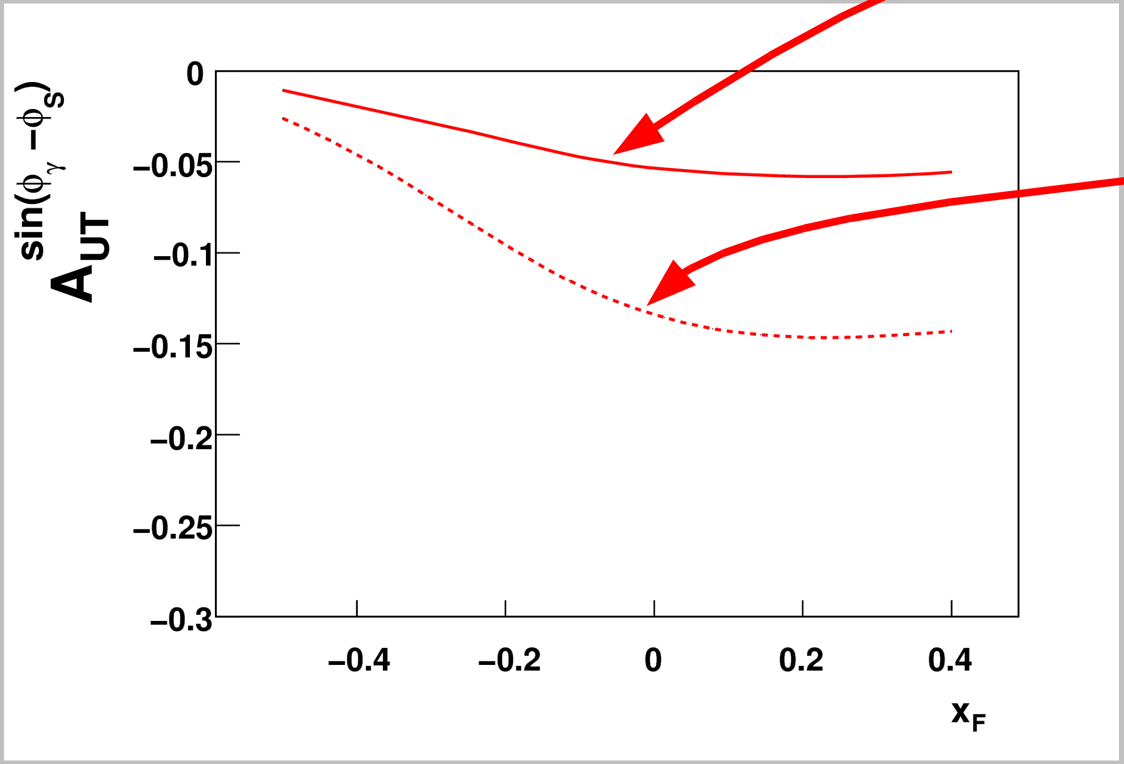
Drell Yan

$$\mathbf{A_N} = \frac{\sum_\mathbf{q} \mathbf{f}_{\mathbf{1T}}^{\perp\mathbf{q}}(\mathbf{x_2}, \mathbf{p_T}) \otimes \mathbf{f}_\mathbf{1}^{\bar{\mathbf{q}}}(\mathbf{x_1}, \mathbf{p_T})\sigma_{\mathbf{q}\bar{\mathbf{q}}}}{\sum_\mathbf{q} \mathbf{f}_\mathbf{1}^{\mathbf{q}}(\mathbf{x_2}, \mathbf{p_T}) \otimes \mathbf{f}_\mathbf{1}^{\bar{\mathbf{q}}}(\mathbf{x_1}, \mathbf{p_T})\sigma_{\mathbf{q}\bar{\mathbf{q}}}}$$

Analysis in hadronic cm frame

With TMD evolution

No TMD evolution



Asymmetry is suppesed with respect to LO analysis

What is needed?

In order to fix TMD evolution fits one would need to have

- Unpolarised cross-sections as a function of P_T for SIDIS, Drell-Yan and e^+e^-
- Unpolarised cross-sections at different energies and different values of Q^2
- Asymmetries at different energies and different values of Q^2

COMPASS will be source of a lot of information!

CONCLUSIONS

- Three dimensional parton picture is achievable with GPD and TMD measurements
- TMD phenomenology is possible with evolution
- HERMES and COMPASS data are compatible with TMD evolution
- Future measurements at Electron Ion Collider and Drell-Yan experiments at COMPASS are important for both confirmation of sign change of Sivers function and TMD evolution effects.