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Evolution of Transverse Momentum Dependent distributions and experimental data

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Plot courtesy of Christian Weiss

Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions

Markus Diehl (2003) Matthias Burkardt (2003) and Transverse Momentum Dependent distributions

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al , 2011

Fundamental knowledge from 3D distributions

Cosmic Microwave Background

is the source of information on history of our universe, inflation, distribution of matter, dark matter etc

3 Dimensional partonic picture

gives us insights on the dynamics of the confined system of quarks and gluons.

It also gives information on fundamental properties of the nucleon

Spin is one of these properties

Hadron tomography

Conventional inclusive processes are sensitive to longitudinal momentum fraction of hadron momenta, they give no information on spatial or momentum 3D distribution of partons

Good knowledge of Parton Distribution Functions (PDFs) is acquired at HERA See Forte (2010)

However large-x behavior has still large uncertainties Data from Jlab 12 will be important

Transverse Momentum Dependent distributions

If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} << Q$$

it will be sensitive to quark transverse momentum k_{\perp}

 $\mathbf{l} + \mathbf{P}
ightarrow \mathbf{l}' + \mathbf{h} + \mathbf{X}$ TMD factorization

Ji, Ma, Yuan (2002) Collins(2011)

$$\Phi_{ij}(x,\mathbf{k}_{\perp}) = \int \frac{d\xi^{-}}{(2\pi)} \, \frac{d^{2}\xi_{\perp}}{(2\pi)^{2}} \, e^{ixP^{+}\xi^{-} - i\mathbf{k}_{\perp}\xi_{\perp}} \, \langle P, S_{P} | \bar{\psi}_{j}(0)\mathcal{U}(\mathbf{0},\xi)\psi_{i}(\xi) | P, S_{P} \rangle$$

Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_{\perp}) = \int \frac{d\xi^{-}}{(2\pi)} \frac{d^{2}\xi_{\perp}}{(2\pi)^{2}} e^{ixP^{+}\xi^{-} - i\mathbf{k}_{\perp}\xi_{\perp}} \langle P, S_{P} | \bar{\psi}_{j}(0) \mathcal{U}(\mathbf{0},\boldsymbol{\xi}) \psi_{i}(\boldsymbol{\xi}) | P, S_{P} \rangle |_{\boldsymbol{\xi}^{+} = 0}$$

SIDIS in Infinite Momentum Frame:

Transverse separation is due to presence of transverse parton momentum

Struck quark propagates in the gauge field of the remnant and forms gauge link

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Factorization theorems

- <u>Related:</u> Factorization Theorems:
 - Semi-Inclusive deep inelastic scattering.
 - Drell-Yan.√
 - e⁺/e⁻ annihilation.

$$-p + p - h_1 + h_2 + X$$

- TMD factorization $\Lambda^2_{\rm QCD} < P^2_{\rm h\perp} \ll Q^2$

Sensitive to parton transverse motion.

Ji, Ma, Yuan, Collins, Metz, Rogers, Mulders, etc

- <u>Related:</u> Factorization Theorems:
 - Semi-Inclusive deep inelastic scattering.
 - Drell-Yan.
 - e⁺/e⁻annihilation. ✓
 - $p + p \implies h_1 + h_2 + X$

- Collinear factorization $\Lambda^2_{\rm QCD} \ll P^2_{\rm h\perp}, Q^2$

Sensitive to multy parton correlations.

Qiu, Sterman, Efremov, Teryaev, Kanazava, Koike, etc

TMD and Collinear factorizations

Both factorizations are consistent in the overlap region

Collins, Mulders, Ji, Qiu, Yuan, Bacchetta, Metz, Kang, Boer, Koike, Vogelsang, Yuan etc

Relation of multyparton correlations and moments of TMDs

TMDs

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Mulders, Tangerman (1995), Boer, Mulders (1998)

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Sivers function

Let's consider unpolarised quarks inside transversely polarised nucleon

DISTRIBUTION

Sivers function

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times \hat{P}] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{k}_T^2)$$

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al , 2011; Duke workshop report: Anselmino et al Eur.Phys.J.A47:35,2011

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Access to 3D imaging

$$f(x, \mathbf{k}_T, S) = f_1(x, \mathbf{k}_T^2) - \frac{[\mathbf{k}_T \times P] \cdot S_T}{M} f_{1T}^{\perp}(x, \mathbf{k}_T^2)$$

Sivers function from experimental data HERMES and COMPASS

Anselmino et al 2005

Dipole deformation

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0, 1)$

Deformation in momentum space is: $k_x \cdot f(k_x^2 + k_y^2)$

This is so-called "dipole" deformation.

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

We calculate now average shift: $\langle k_x
angle$

$$\langle k_x \rangle = \int d^2 k_T \frac{\mathbf{k_T^2}}{2M} f_{1T}^{\perp}(x, \mathbf{k_T^2}) \equiv f_{1T}^{\perp(1)}(x)M$$

Average momentum shift is proportional to the **first moment** of Sivers function

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

The same statement in figures:

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$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$

The same statement in figures:

This is what we know from exerimental data already:

How do we measure Sivers function?

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Comparison with models

Light cone wf model Pasquini, Yuan (2011), Quark-diquark models Bacchetta et al (2010), Gamberg, Goldstein, Schlegel (2010) Yuan (2003), Avakian, Efremov, Schweitzer, Yuan (2010)

Good agreement.

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_{T1}}}{M}$$

The slice is at: x = 0.1

Low-x and high-x region is uncertain JLab 12 and EIC will contribute

No information on sea quarks

In future we will obtain much clearer picture

Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.

Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)

Brodsky,Hwang, Schmidt Belitsky,Ji,Yuan Collins Boer,Mulders,Pijlman, etc

One of the main goals is to verify this relation. It goes beyond "just" check of TMD factorization. Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou

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Kinematics

Kinematics $Q^2 \simeq sxy$

One needs a unique definition of TMDs

Foundations of perturbative QCD Collins 2011

$$W^{\mu\nu} = \sum_{f} |H_{f}(Q^{2}, \mu)|^{\mu\nu}$$

$$\times \int d^{2}\mathbf{k}_{1T} d^{2}\mathbf{k}_{2T} F_{f/P_{1}}(x_{1}, \mathbf{k}_{1T}; \mu, \zeta_{F}) F_{\bar{f}/P_{1}}(x_{2}, \mathbf{k}_{2T}; \mu, \zeta_{F})$$

$$\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_{T}) + Y(\mathbf{q}_{T}, Q)$$

 $F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$ **TMD distribution of partons in hadron** Report group (RG) renormalization

One needs a unique definition of TMDs

Foundations of perturbative QCD Collins 2011

Infinite rapidity of the gluon creates so called rapidity divergence

In collinear PDFs this divergence is cancelled between virtual and real gluon diagrams

It is not the case for TMDs Thus new regulator ζ_F is needed

 $F_{f/P_1}(x_1,\mathbf{k}_{1T};\mu,\zeta_F)$

Renorm group (RG) renormalization

Rapidity divergence regulator

Evolution of TMDs is done in coordinate space $\, {f b}_T \,$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{b}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Foundations of perturbative QCD Collins 2011

Why coordinate space?

Convolutions become simple products:

$$\begin{split} \tilde{W}^{\mu\nu} &= \sum_{f} |H_{f}(Q^{2},\mu)|^{\mu\nu} & \text{Idilbi, Ji, Ma, Yuan 2004} \\ &\times \int d^{2}\mathbf{b}_{T} e^{i\mathbf{b}_{T}\mathbf{q}_{T}} \tilde{F}_{f/P_{1}}(x_{1},\mathbf{b}_{T};\mu,\zeta_{F}) \tilde{F}_{\bar{f}/P_{1}}(x_{2},\mathbf{b}_{T};\mu,\zeta_{F}) \end{split}$$

In principle experimental study of functions in coordinate space Is possible

Boer, Gamberg, Musch, AP 2011

Collins, Soper 1982

Collins Soper Sterman 1985

Evolution of TMDs is done in coordinate space $\, {f b}_T \,$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{b}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Foundations of perturbative QCD Collins 2011

Complicated in case of Sivers function

Aybat, Collins, Qiu, Rogers 2012

$$F_{f/P^{\uparrow}}(x,\mathbf{k}_T,\mathbf{S}_T;\mu,\zeta_F) = F_{f/P}(x,\mathbf{k}_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,\mathbf{k}_T;\mu,\zeta_F) \frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

Unpolarised part:

$$\tilde{F}_{f/P}(x,b_T;\mu,\zeta_F) = (2\pi) \int_0^\infty dk_T k_T J_0(k_T b_T) F_{f/P}(x,k_T;\mu,\zeta_F)$$

Sivers function:

$$\tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F) = -(2\pi) \int_0^\infty dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_{\perp}, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_{\perp}, \mu)$$

 Collins-Soper kernel in coordinate space

Renormalization group equations

$$\frac{d\tilde{K}(b_{\perp},\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$
$$\frac{d\ln\tilde{F}(x,b_{\perp},\mu,\zeta)}{d\ln\mu} = -\gamma_F(g(\mu),\zeta)$$

TMD: Collins 2011 Rogers, Aybat 2011 Aybat, Collins, Qiu, Rogers 2011

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_{\perp}, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_{\perp}, \mu)$$

Collins-Soper kernel in coordinate space

At small \mathbf{b}_T perturbative treatment is possible

TMD: Collins 2011 Rogers, Aybat 2011 Aybat, Collins, Qiu, Rogers 2011

$$\tilde{K}(b_T,\mu) = -\frac{\alpha_s C_F}{\pi} \Big(\ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_E \Big) + \mathcal{O}(\alpha_s^2)$$

Large \mathbf{b}_T nonperturbative – matching via \mathbf{b}_* Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

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Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_{\perp}, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_{\perp}, \mu) \qquad \qquad \blacktriangleright \quad \text{Collins-Soper kernel in coordinate space}$$

Large \mathbf{b}_T nonperturbative – matching via \mathbf{b}_* Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

 $b_{max} = 0.5 \; (\text{GeV}^{-1})$

Brock, Landry, Nadolsky, Yuan 2003

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_{\perp}, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_{\perp}, \mu) \qquad \qquad \textbf{Collins-Soper kernel in coordinate space}$$

Large \mathbf{b}_T nonperturbative – matching via \mathbf{b}_* Collins Soper 1982

$$\tilde{K}(b_T, \mu) = \tilde{K}(b_*, \mu) - g_K(b_T)$$

Always perturbative

Non perturbative

$$g_{K}(b_{T}) = \frac{1}{2}g_{2}b_{T}^{2}$$

$$g_{2} \simeq 0.68 \ (GeV^{2})$$

This function is universal for different partons!

Brock, Landry, Nadolsky, Yuan 2003

Relation to collinear treatment:

$$\tilde{F}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f}\left(\frac{x}{\hat{x}}, b_T, \mu, \zeta\right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD}b_T)$$
Colling Soper 1982

Valid at small $\, {f b}_T$, lowest order:

$$\tilde{C}_{j/f}(\frac{x}{\hat{x}}, b_T, \mu, \zeta) = \delta_{jf}\delta\left(\frac{x}{\hat{x}} - 1\right) + \mathcal{O}(\alpha_s)$$

Higher order for TMD PDFs

Aybat Rogers 2011

Higher order for Sivers function

Kang, Xiao, Yuan 2011

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$$\begin{array}{l} \text{Solution} \quad \begin{array}{l} \begin{array}{l} \text{Rogers, Aybat 2011} \\ \text{Aybat, Collins, Qiu, Rogers 2011} \end{array} \\ \tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) \\ \times \exp\left[-g_K(b_T) \ln \frac{Q}{Q_0} \right] \end{array} \right\} \quad \begin{array}{l} \text{Non perturbative} \\ \text{Non perturbative} \end{array} \\ \times \exp\left[\ln \frac{Q}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{Q_0}^Q \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right] \\ + \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{Q}{Q_0} \gamma_K(g(\mu')) \right] \quad \begin{array}{l} \text{Perturbative} \end{array}$$

Typically for TMDs:

$$\tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) = F_{f/P}(x; Q_0) \exp\left(-\frac{\langle k_T^2 \rangle}{4} b_T^2\right)$$

Solution Rogers, Aybat 2011 Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x,b_T;Q,\zeta_F) = F_{f/P}(x;Q_0) \exp\left(-\left[\frac{\langle k_T^2 \rangle}{4} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right]b_T^2\right)$$

Non perturbative

Gaussian behaviour is appropriate only in a limited range

TMDs change with energy and resolution scale

Can we see signs of evolution in the experimental data?

Aybat, AP, Rogers 2011

COMPASS data is at $\langle Q^2 \rangle \simeq 3.6 \; (GeV^2)$

HERMES data is at

 $\langle Q^2 \rangle \simeq 2.4 \; (GeV^2)$

Can we explain the experimental data? Full TMD evolution is needed!

Aybat, AP, Rogers 2011 COMPASS dashed line $\langle Q^2 \rangle \simeq 3.6 \; (GeV^2)$

HERMES solid line

 $\langle Q^2 \rangle \simeq 2.4 \; (GeV^2)$

This is the first implementation of TMD evolution for observables

Phenomenological analysis with evolution is now possible

Drell Yan

$$\mathbf{A_N} = \frac{\sum_{\mathbf{q}} \mathbf{f_{1T}^{\perp q}}(\mathbf{x_1}, \mathbf{p_T}) \otimes \mathbf{f_1^{\bar{q}}}(\mathbf{x_1}, \mathbf{p_T}) \sigma_{\mathbf{q}\bar{\mathbf{q}}}}{\sum_{\mathbf{q}} \mathbf{f_1^q}(\mathbf{x_1}, \mathbf{p_T}) \otimes \mathbf{f_1^{\bar{q}}}(\mathbf{x_1}, \mathbf{p_T}) \sigma_{\mathbf{q}\bar{\mathbf{q}}}}$$

Analysis at LO in hadronic cm frame Anselmino et al (2009)

$$\mathbf{x_1} = rac{\mathbf{x_F} + \sqrt{\mathbf{x_F^2} + 4\mathbf{M^2/s}}}{\mathbf{2}} pprox \mathbf{x_F}$$

In DY we probe Sivers function at $\mathbf{X}_{\mathbf{F}}$ Anselmino et al (2009)

Drell Yan

Asymmetry is suppesed with respect to LO analysis

CONCLUSIONS

- Three dimensional parton picture is achievable with GPD and TMD measurements
- TMD phenomenology is possible with evolution
- HERMES and COMPASS data are compatible with TMD evolution
- Future measurements at Electron Ion Collider and Drell-Yan experiments are important for both confirmation of sign change of function and TMD evolution effects.

QCD EVOLUTION 2012

http://www.jlab.org/conferences/qcd2012/ May 14 - 17, 2012 Jefferson Lab, Newport News, Virginia, USA

Organizing committee: Alexei Prokudin, Chair Anatoly Radyushkin Ian Balitsky Leonard Gamberg Harut Avakian

Deadline registration April 13th! Register soon! Alexei Prokudin